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ABSTRACT

Since the 2001 recession, average core inflation has been below the Federal Reserve's 2% target. This deflationary bias is a predictable consequence of a low nominal interest rates environment in which the central bank follows a symmetric strategy to stabilize inflation. The deflationary bias increases if macroeconomic uncertainty rises or the natural real interest rate falls. An asymmetric rule according to which the central bank responds less aggressively to above-target inflation corrects the bias and allows inflation to converge to the central bank's target. We show that adopting this asymmetric rule improves welfare and reduces the risk of self-fulfilling deflationary spirals. This approach does not entail any history dependence in setting the policy rate or any commitment to overshoot inflation after periods in which the lower bound constraint was binding.

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1 Introduction

Since the 2001 recession, core inflation has been on average below the Federal Reserve's implicit 2% target. This phenomenon has become even more severe in the aftermath of the 2008 recession. In other words, the "conquest of US inflation" that started with the Volcker disinflation seems to have gone too far. Inflation, instead of stabilizing around the desired 2% inflation target, has kept falling down. This deflationary bias is a predictable consequence of a low nominal interest rate environment. We argue that a low inflation target should be combined with an asymmetric policy rule that allows persistent deviations of inflation above the central bank's target.

Figure 1 provides evidence for the stylized fact that we are interested in. The year-to-year PCE core inflation is reported with its ten-year moving average. In the early 1990s inflation was still well above 2%. By the end of the same decade, the Federal Reserve had completed the long process that had started with the Volcker disinflation. Around this time the Federal Reserve started discussing the possibility of moving to an explicit inflation targeting regime. While an explicit 2% target was only announced on 25 January 2012 by Federal Reserve Chairman Ben Bernanke, the existence of an implicit 2% target predates this historical shift.[1] However, as the graph illustrates, inflation has not stabilized around the desired target, instead it has kept on falling. As of today, the ten-year moving average is around 1.6%. Importantly, a similar picture emerges even when removing the 2001 and 2008 recessions. Furthermore, survey-based measures of long-term inflation expectations also declined in recent years. The University of Michigan's survey-based expectations on inflation five to ten years out has fallen by 80 basis points since the 2007. The survey of professional forecasters' ten-year-ahead expectations on CPI inflation has followed a similarly declining pattern since 2012.

The deflationary bias poses a serious challenge to the central bank. For instance, it may entail a considerable reputation loss if the private sector loses confidence in the Federal Reserve's ability to bring inflation back to target in an expansion. This outcome may be very costly as it could impair the central bank's capability to credibly commit to future actions, which is particularly critical to stimulate the economy when the current interest rate is at its zero lower bound (ZLB) constraint (Krugman 1998;

[1] There is some disagreement about what the Federal Reserve's effective inflation objective was before 2012 (Shapiro and Wilson 2019). However, there is a strong consensus that the objective has been 2 percent since 2010.

and Eggertsson and Woodford 2003). Furthermore, a prolonged period of low inflation might cast doubts about whether or not the Federal Reserve is in fact committed to a symmetric 2% inflation target, as opposed to a two-percent ceiling on the inflation rate.

The interaction of the following two factors explains the deflationary bias: (i) the remarkably low long-run nominal interest rates and (ii) the perfect symmetry of the current monetary policy framework, which treats positive and negative deviations of inflation from the central bank's target on equal footing. We formalize our argument using a standard non-linear New Keynesian model, which we use to show that in the absence of either one of these two factors the bias would have not emerged.

When the long-run nominal interest rate is calibrated to the low values that seem plausible today, the model predicts that average inflation will remain below target even during expansions. Forward-looking price setters anticipate that in case of a large negative shock the central bank will be unable to fully stabilize inflation due to the ZLB constraint on nominal rates. These beliefs bring about deflationary pressures and depress inflation dynamics even when the economy is away from the ZLB. All changes in the macroeconomic environment that make ZLB episodes more likely or more persistent also cause the deflationary bias to become more severe. Thus, a decline in the long-term real interest rate raises the probability of hitting the ZLB in the future and consequently makes the deflationary bias larger. Similarly, heightened macroeconomic uncertainty also causes or prolongs the ZLB and, hence, contributes to exacerbating the deflationary bias.

We argue that the symmetric approach to inflation stabilization, which is currently followed by the Federal Reserve, loses efficacy when the long-run nominal interest rate is low because it contributes to the formation of the deflationary bias. An example of the Federal Reserve's symmetric interpretation of its inflation objective is in the *Statement on Longer-Run Goals and Monetary Policy Strategy*, which maintains: "The Committee would be concerned if inflation were running persistently above or below this objective. Communicating this symmetric inflation goal clearly to the public helps keep longer-term inflation expectations firmly anchored [...]". We show that in the current low interest rate environment, it is advantageous for the Federal Reserve to be more concerned about inflation running below target than about inflation going above target.

The central bank can remove the deflationary bias and can raise social welfare by

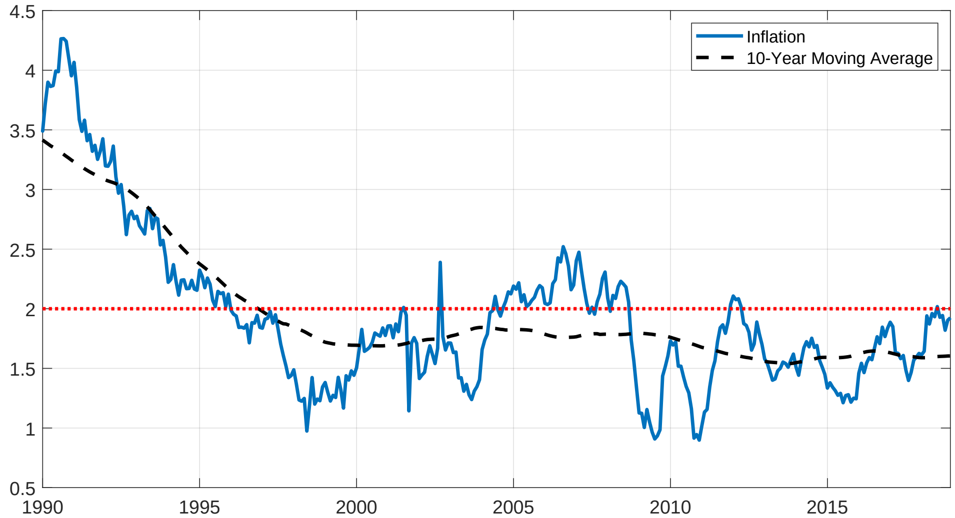

Figure 1: Year-to-year PCE core inflation and its ten-year moving average. Unit: Annualized percentage rates.

committing to adjust the policy rate less aggressively when inflation is above target than when inflation is below target. By removing the deflationary bias, this asymmetric strategy raises the long-term inflation expectations and hence makes self-fulfilling deflationary spirals less likely to happen. These deflationary spirals represent a pathological situation in which inflation may keep falling indeterminately. This asymmetric strategy removes the deflationary bias because it raises the risk of inflation on the upside and, in doing so, offsets the downside risk due to the ZLB. Thus, an apparent paradox emerges: In order to interpret its inflation target as symmetric, the central bank should follow an asymmetric strategy. The paradox is only apparent, because the asymmetric strategy corrects for the constraint represented by the ZLB.

Of course, in practice, it may not be so easy for the central bank to convince agents that it has adopted an asymmetric rule. When inflation is below target, announcing to be less aggressive in countering future upswings in inflation is time inconsistent.[2] In this context, the central bank can conduct an *opportunistic reflation* to demonstrate its commitment to the asymmetric strategy.[3] To conduct an opportunistic reflation,

[2] If the announcement is believed by the public, the deflationary bias disappears and once this happens, the central bank has an incentive to renege on its announcement and to respond aggressively to future upswings in inflation.

[3] This proposal is similar in spirit to what Governor Lael Brainard proposed in her speech on May 16, 2019: "One possibility we might refer to as "opportunistic reflation" would be to take advantage of a modest increase in actual inflation to demonstrate to the public our commitment to our inflation goal on a symmetric basis."

the central bank announces the adoption of the asymmetric rule in the aftermath of a shock that pushes inflation above target. Even though this action leads to a higher inflation rate in the short run, which entails a welfare loss, this rise in inflation offers the central bank the opportunity to show the public that the central bank is now committed to follow the asymmetric strategy, which raises welfare in the long-run by removing the deflationary bias.[4] We show that in our calibrated model opportunistic reflation improves welfare. Nevertheless, if the realized magnitude of the inflationary shock is big, this opportunistic move might give rise to a too high rate of inflation in the short run and hence might lower welfare compared to sticking to the symmetric rule. However, according to our model, the size of the shock has to be implausibly large for this to be a real concern.

If no opportunity to reflate the economy materializes, the central bank can cut the federal funds rate more aggressively when inflation is below target. If the central bank is understood to keep responding with the same strength as in the past when inflation will go above target, this action is shown to remove the deflationary bias. Interestingly, we find that cutting the federal funds rate more aggressively when inflation is below target turns out to reduce the risk of hitting the ZLB. This happens because the correction of the deflationary bias ends up raising the long-term nominal interest rate and hence the room for the central bank to respond to recessionary shocks.

We derive all our results in a prototypical New Keynesian model that features the zero lower bound constraint on nominal interest rates. We solve the model with global methods to take into account the non-linearity represented by the zero lower bound. We show that other non-linearities such as precautionary savings do not play an important role for the results that we discuss below. The model features demand shocks calibrated to match the level of macroeconomic volatility prevailing before the 2008 financial crisis. Given that this is a period of macroeconomic stability, our benchmark calibration for the volatility of the exogenous shocks can be considered a lower bound.

Below-target inflation and an overheating economy are two sides of the same coin, resolving an apparent puzzle recently observed in the U.S. data. To cope with the deflationary bias, standard monetary policy rules, such as the Taylor rule, induce the central bank to keep interest rates low, even when the economy is in an expansion.

[4]Under the asymmetric rule, the weaker systematic response to positive deviations of inflation from target raises agents' long-run uncertainty about inflation and hence, everything else being equal, lowers welfare in the long-run. However, in our model these losses are dominated by the gains from removing the deflationary bias.

This accommodative policy decreases the real interest rate and ends up overheating the economy, while inflation remains below target because of expectations of future low inflation.

Adam and Billi (2007) and Nakov (2008) were the first to show that the deflationary bias and the corresponding output bias arise in New Keynesian models in which the nominal interest rate is occasionally constrained by the zero lower bound. In this paper, we emphasize that the symmetry of standard monetary policy rules (e.g., the Taylor rule) plays an important role for these biases to arise and show that adopting an asymmetric rule can remove these biases. Amano et al. (2018) and Hills et al. (2016) estimate structural models to quantify the deflationary bias and the output bias in the U.S. economy. Even though the estimated magnitude of these gaps differs across these studies, both studies find that the size of these bias is non-negligible. For instance, Hills et al. (2016) show that the deflationary bias can be as large as 40 basis for the U.S. economy.

Kiley and Roberts (2017), Mertens and Williams (2019), and, Bernanke et al. (2019) evaluate a large varieties of monetary policy rules (including dynamic rules such as price-level-targeting rules, average-inflation-rate rules, and shadow-rate rules) and conclude that dynamic rules, which make up for forgone accommodation after the ZLB episode, can eliminate the deflationary biases and deliver better macroeconomic outcomes than static rules (such as the Taylor rule). In contrast, the asymmetric strategy we propose does not rely on history dependence to solve the deflationary bias. Therefore, the central bank is not committed to engineer deflation following a period of above-target inflation. Similarly, the asymmetric strategy does not contemplate inflation overshooting; that is, a contingency in which the central bank sets the policy rate so as to engineer positive deviations of inflation from its target.

Mertens and Williams (2019) study a rule in which the central bank enforces an upper bound on the FFR. Compared to our proposal, this strategy has the drawback to exacerbate the problem of indeterminacy in dynamic general equilibrium models. Nakata and Schmidt (2016) show that lowering the intercept of the interest-rate rule would eliminate the deflationary bias in a model calibrated to match the key features of the U.S. economy. Nakata and Schmidt (2019a) show that the deflationary bias can be resolved by appointing a conservative central banker á la Rogoff. Appointing a central banker whose inflation target is slightly higher than the social optimum will also resolve the bias as shown by Seneca (2019). Nakata and Schmidt (2019b) show that modifying the objective function of a discretionary central bank to include an interest-rate smoothing mitigates the deflationary bias. None of these four contributions shows the importance of asymmetric strategies to remove the deflationary bias.

Our paper is related to the debate about the necessity of increasing the inflation target to reduce the probability of encountering the ZLB (Coibion et al. 2012). The main differences are in the focus of the paper and in the policy prescription. First, we focus on the deflationary bias and its sources. We also argue that the probability of encountering the zero lower bound is in part endogenous because of the deflationary bias. While Coibion et al. (2012), following Coibion and Gorodichenko (2011), log-linearize the model around a positive trend inflation, we solve the model with global methods. This is what allows us to derive the deflationary bias.[5] Finally, our policy prescription does not involve a change in the target or price targeting or inflation overshooting and thereby does not call for a radical reform of the current monetary policy framework of the Federal Reserve.

Our work is also related to the literature that aims at identifying the behavior of the natural rate of interest. Because of the difficulties faced by the Federal Reserve in raising inflation, estimates of the natural interest rate have become central for the policy debate (Laubach and Williams 2003, and, Del Negro et al. 2017). Our results suggest that in a low interest rate environment, estimates of the natural interest rate obtained using linear models can be biased.

The paper is organized as follows. In Section 2, we present a prototypical New Keynesian model to study the deflationary bias. The calibration of the model to the U.S. data and its solution is discussed in Section 3. In Section 4, we show that given the low long-run real interest rate, inflation fails to converge to the Federal Reserve's 2-percent inflation target in the long run. We also explain that average inflation is lower than the central bank's target in Section 5. In Section 6, we study a simple policy proposal that aligns average inflation with the desired target and show that this proposed strategy may be introduced in the aftermath of an inflationary shock (opportunistic reflation) or by simply cutting the rate strategically when inflation is below target (strategic interest rate cut). In Section 7 we conclude.

[5] The deflationary bias would also arise in a model that is conditionally linear, as long as the zero lower bound constraint is retained. But in a fully linearized model, the bias would not arise.

2 The Model

In this section, we introduce a prototypical New Keynesian model in the tradition of Clarida, Galí, and Gertler (2000), Woodford (2003), and Galí (2008) augmented with a zero lower bound constraint for the nominal interest rate set by the monetary authority.

The economy consists of households, final goods producers, a continuum of monopolistic intermediate goods firms, a monetary authority, and a fiscal authority. Households buy and consume the final goods from producers, trade one-period government bonds, and supply labor to firms. The final goods producers buy intermediate goods and aggregate them into a homogenous final good using a CES aggregation technology. The intermediate goods firms set the price of their differentiated good subject to price adjustment costs a la Rotemberg. They demand labor to produce the amount of differentiated goods to be sold to households in a monopolistic competitive market. Labor is the only factor of production. The fiscal authority balance its budget in every period. The monetary authority sets the interest rate for the government bonds.

The model is solved with global methods in its non-linear specification.

The Representative Household In every period, the representative household chooses consumption C_t, labor H_t, and government bonds B_t so as to maximize the expected discounted stream of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^d \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{1}$$

subject to the flow budget constraint

$$P_t C_t + B_t = P_t W_t H_t + R_{t-1} B_{t-1} + T_t + P_t Div_t \tag{2}$$

where P_t is the price level, W_t is the real wage, R_t is the gross interest rate, T_t are lump-sum taxes and Div_t are real profits from the intermediate good firms. B_t denotes the one-period government bonds in zero net supply. The preference shock ζ_t^d follows an AR(1) process in logs $\ln(\zeta_t^d) = \rho_\zeta \ln(\zeta_{t-1}^d) + \sigma^{\zeta^d} \epsilon_t^{\zeta^d}$.

Solving the representative household's problem yields the Euler equation

$$1 = \beta R_t E_t \frac{\zeta_{t+1}^d}{\zeta_t^d} \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{1}{\Pi_{t+1}}, \tag{3}$$

where $\Pi_t = P_t/P_{t-1}$ is gross inflation, and the labor supply

$$W_t = \chi N_t^{\eta} c_t^{\sigma}, \tag{4}$$

Final Goods Producers Final goods producers transform intermediate goods into the homogeneous good, which is obtained by aggregating intermediate goods using the following technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{5}$$

where $Y_t(j)$ is the consumption of the good of the variety produced by firm j.

The price index for the aggregate homogeneous good is:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}, \tag{6}$$

and the demand for the differentiated good $j \in (0,1)$ is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t. \tag{7}$$

Intermediate Goods Firms The firm j produces output with labor as the only input

$$Y_t(j) = A \ H_t(j)^{\alpha} \tag{8}$$

where A_t denotes the total factor productivity, which follows an exogenous process. The firm j sets the price $P_t(j)$ of its differentiated goods j so as to maximize its profits:

$$Div_t(j) = P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - \alpha \ mc_t \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t - \frac{\varphi}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right) Y_t, \tag{9}$$

subject to the downward sloping demand curve for intermediate goods. The parameter $\varphi > 0$ measures the cost of price adjustment in units of the final good.

The first order condition is

$$(\epsilon-1)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t}=\epsilon\ \alpha\ MC_t\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t}-\varphi\left(\frac{P_t(j)}{\Pi P_{t-1}(j)}-1\right)\frac{Y_t}{\Pi P_{t-1}(j)}+$$
$$\varphi E_t\Lambda_{t,t+1}\left(\frac{P_{t+1}(j)}{\Pi P_t(j)}-1\right)\frac{P_{t+1}(j)}{\Pi P_t(j)}\frac{Y_{t+1}}{P_t(j)} \tag{10}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is

$$\Lambda_{t,t+1}=\beta E_t\left[\left(\frac{\zeta^d_{t+1}}{\zeta^d_t}\right)\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\right] \tag{11}$$

In equilibrium all firms choose the same price. Thus, the New Keynesian Phillips curve is

$$\varphi\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_t}{\Pi}=(1-\epsilon)+\epsilon\ \alpha\ MC_t+\varphi E_t\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_{t+1}}{\Pi}\frac{Y_{t+1}}{Y_t} \tag{12}$$

Monetary Authority The monetary authority sets the interest rate R_t responding to inflation and output from their corresponding targets. The monetary authority faces a zero lower bound constraint. The policy rule reads as follows

$$R_t=\max\left[1,R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}\right]. \tag{13}$$

where Π and Y denote the inflation target which pins down the inflation rate in the deterministic steady state and the natural output level, which is the level output that would arise if prices were flexible. The fiscal authority sets taxes to balance the budget in every period

$$T_t=B_t-R_{t-1}B_{t-1}. \tag{14}$$

Resource Constraint The resource constraint is

$$C_t=Y_t\left[1-\frac{\varphi}{2}\left(\frac{\Pi_t}{\Pi}-1\right)^2\right] \tag{15}$$

Parameters		Value
β	Steady state discount rate	0.9975
α	Production Function	1
σ	Relative risk aversion	1
η	Inverse Frisch elasticity	1
ϵ	Price elasticity of demand	7.6670
χ	Disutility labor	0.8696
φ	Rotemberg pricing	79.41
ϕ_{Π}	MP inflation response	2.0000
ϕ_Y	MP output response	0.2500
$\bar{\Pi}$	Inflation target	1.0050
ρ_{ζ^d}	Persistence preference shock	0.60
$100\sigma_{\zeta^d}$	Std. dev. preference shock	1.175

Table 1: Benchmark calibration: Parameter Values

3 Model Solution and Calibration of Parameters

We solve the model with time iterations and linear interpolation as in Richter et al. (2014). Expectations are evaluated with Gauss-Hermite Quadrature. The state variable is the preference shock ζ_t^d and the policy functions are labor N_t and inflation Π_t.

We set the discount factor β to 0.9975 that corresponds to an annualized real interest rate of one percent, which is in line with the Federal Open Market Committee (FOMC) Summary of Economic Projections (SEP). The standard deviation of preference shocks σ^{ζ^d} is chosen in line with the standard deviation of the U.S. real GDP growth rate over a period ranging from the first quarter of 1985Q1 through the fourth quarter of 2007. This period has been characterized by record low macroeconomic volatility and therefore the calibrated value of the standard deviation of preference shocks should be regarded as low by historical standards. For instance, the standard deviation of the U.S. real GDP growth rate was twice as big in the 1970s. We will show how trend inflation and the long-term real interest rate vary under different assumptions about the Post-Great Recession macroeconomic volatility.

The persistence of preference shocks ρ_ζ is set to 0.60. Higher values for this parameter prevents us from solving the model. The same problem occurs as well if one raises the variance of the shock too much. Both parameters lift the unconditional volatility of preference shocks and hence the number of future periods agents expect monetary policy to be passive because of the ZLB constraint. The remaining parameters are standard and are listed in Table 1.

The left plot of Figure 2 shows the percentage of periods spent at the ZLB when the

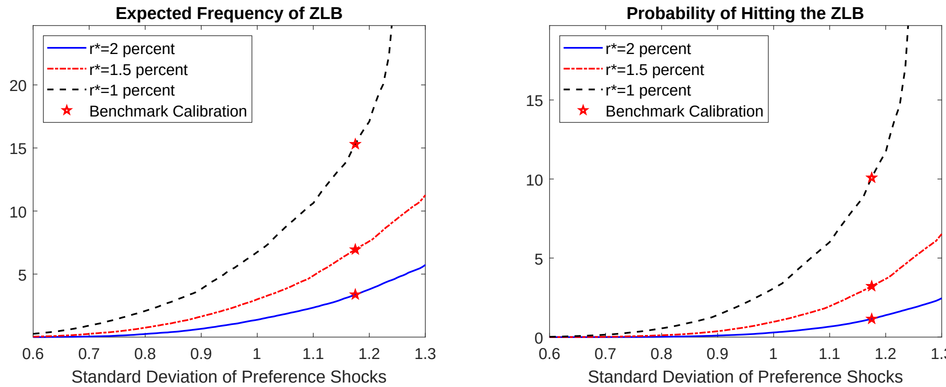

Figure 2: The risk of the zero lower bound. Left graph: Expected frequency of the zero lower bound as the variance of preference shocks varies and for different values of the long-run real rate. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (300,000). Right graph: Probability of hitting the zero lower bound next period conditional on being at the stochastic steady state in the current period for different values of the variance of preference shocks and of the steady-state real rate. The probability is expressed in percentage points.

model is simulated for a long period of time (300,000 periods). In technical jargon, this is the ergodic probability of being constrained by the ZLB. As shown in the figure, this probability is affected by how volatile the shocks are (x-axis). The different lines are associated with different assumptions about the long-run annualized real rate of interest $r^* = \beta^{-1}$. Our benchmark calibration for this parameter is one percent, which is in line with the FOMC SEP. The red stars on the lines denote the standard deviation of the shock that allows the model to match the observed volatility of real GDP growth rate during the Great Moderation and should be thereby considered as an historically-low value.

A lower long-term real interest rate raises the expected frequency of the ZLB as it shrinks the central bank's room of maneuver to counter the deflationary effects of recessionary shocks. We are closer to the bound on average so the central bank is expected to hit the lower bound more often. Note that the expected frequency of the ZLB grows at an increasing speed as the long-term real interest rate r^* falls. Furthermore, the speed is higher, the larger is the volatility of the shock. Quite clearly, the more volatile shocks are, the higher the expected frequency of the ZLB. Symmetrically, as the steady-state real rate of interest $r^* = \beta^{-1}$ declines, a given increase in volatility implies exponential increases in the frequency of the zero lower bound.

The graph on the right shows how likely it is for monetary policy to become constrained by the ZLB in the next period conditional on being currently at the (stochastic) steady state. As for the expected frequency of the ZLB, we study how this probability varies as we change the standard deviation of the preference shocks and the steady-state real rate of interest r^*. The larger the volatility of the shock, the more likely it is that the ZLB will be binding in the next period. It should be noted that the probability rises exponentially with the volatility of the shock. Lowering the long-term real rate of interest leads to similar results.

The worrying finding highlighted by both graphs is that in a low real-interest rate environment (low r^*, black dashed lines) the two functions are very steep. This means that even a small increase in the volatility of the shocks can lead to substantial increases in the probability of encountering the zero lower bound. Recall that our benchmark calibration for the volatility of the preference shock is arguably very low for the U.S., given that it was chosen to match the level of volatility during the Great Moderation. The results above imply that even a small increase in the macroeconomic volatility may lead agents to believe that the ZLB constraint is a pervasive problem for monetary policy. As we shall show in the next section, these beliefs cause serious macroeconomic biases and distortions and can potentially lead to deflationary spirals. These scenarios may arise for levels of macroeconomic volatility that are only slightly larger than those observed during the Great Moderation period and are way below the high levels observed in the 1970s. Symmetrically, forces that contribute to lower the natural interest rate can lead to similar outcomes for a given level of volatility.

4 ZLB Risk and Macroeconomic Biases

Hitting the inflation target is harder for the central bank when the probability of encountering the ZLB is non-negligible. Even in tranquil times and away from the ZLB, the mere risk that monetary policy might become constrained in the future hinders the convergence of inflation to the central bank's inflation target (Adam and Billi 2007 and Nakov 2008). This is because forward-looking price setters anticipate that in case of a large negative shock the central bank will be unable to fully stabilize inflation due to the ZLB constraint. These beliefs cause inflation expectations to become disanchored from the central bank's target and depress inflation dynamics.

The existence of this inflationary bias constitutes an important anomaly that should concern policymakers. Failure to acknowledge this anomaly leads the central bank to conduct an overstimulative monetary policy that ends up overheating the economy and creating more macroeconomic biases. These macroeconomic biases are broadly consistent with the recent performance of the U.S. economy.[6] Moreover, the size of these biases increases exponentially as the volatility of the macroeconomic environment rises and the natural rate of interest declines. In the subsequent sections, we will show that the symmetric approach by the central bank to inflation stabilization is responsible for these biases.

To show that inflation fails to converge to the central bank's target in absence of inflationary shocks, it is useful to define the stochastic steady-state equilibrium of the model.[7] We define the inflationary bias as the difference between the rate of inflation at the stochastic steady-state equilibrium and the central bank's inflation target, which coincides with the rate of inflation at the deterministic steady state. The deflationary bias arises when inflation at the stochastic steady state is lower than the central bank's target. A large deflationary bias implies serious hurdles for the central bank to hit its inflation objective.

Deterministic and stochastic steady state Both the deterministic and stochastic steady state define an economy that has not been hit by shocks for a sufficiently long number of periods, so that their variables have stabilized around their steady state values and do not vary anymore (unless a shock suddenly hits). However, in the deterministic steady state, agents fail to appreciate the macroeconomic risk due to future realizations of the shocks. Instead, in the stochastic steady state, agents appreciate the macroeconomic risks due to future realizations of the shocks and adjust their behavior accordingly. While in a linear model these two concepts of steady-state equilibria lead to the same macroeconomic outcome, in non-linear models whether agents act in response to future macroeconomic risks matters.

Unlike the stochastic steady state equilibrium, the deterministic steady-state equilibrium of our model can be characterized analytically. The real interest rate in the deterministic steady state, r^*, coincides with β^{-1} and captures the long-run level of the real interest rate in absence of risk. Importantly, $r^* = \beta^{-1}$ also coincides with the deterministic steady state of the natural interest rate. The deterministic steady state of inflation is pinned down by the inflation target of the central bank, Π, and can be effectively dealt with as a parameter. Since the price adjustment cost function takes into

[6]See Hills et al. (2016) for an empirical investigation of the magnitude of the deflationary bias and the associated output bias.

[7]Some scholars use the terms "risky steady state" to refer to what we call stochastic steady state. See, for instance, Coeurdacier et al. (2011).

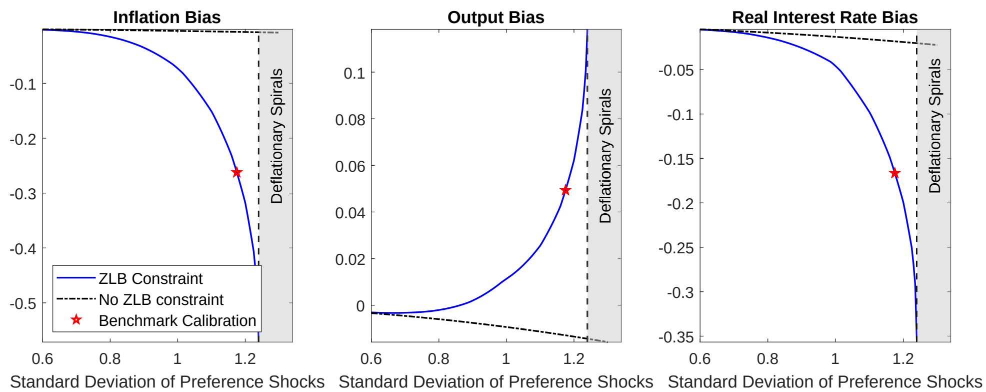

Figure 3: Macroeconomic distortions due the zero lower bound as the volatility of the preference shocks varies. Left graph: The inflationary bias due to model's non-linearities. The red star denotes the calibrated value of the standard deviation of this shock. The difference between the blue solid line and the black dot-dashed line captures the deflationary effects of a risk of a recession that pushes the nominal interest rate to its lower bound. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. The gray area marks the region of the values for the standard deviation of the preference we cannot solve the model for. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

account the deterministic steady state inflation rate, the chosen value of the inflation target does not affect any macroeconomic outcomes either at the deterministic steady state or away from the deterministic steady state. Thus, the deterministic steady state for output Y is purely determined by the level of TFP.

Unlike the stochastic steady state, the deterministic steady state is not affected by macroeconomic uncertainty, which influences the optimal behaviors of rational agents in non-linear models. Such volatility drives a wedge between the outcomes of these two steady-state equilibria and hence fuels the inflationary bias. In this section, we will show that among the many sources of non-linearity in the model (e.g., the non-linearities that give rise to precautionary savings), the zero lower bound constraint is the main culprit behind the formation of the deflationary bias and all the associated macroeconomic biases.

The Deflationary Bias The left graph of Figure 3 shows the difference between the inflation rate at the stochastic steady state and inflation at the deterministic steady state with (blue solid line) and without the zero lower bound constraint (black dash-dotted line). Comparing the blue solid line with the black dash-dot line allows us to isolate the effects of the ZLB constraint. From the figure, it is easy to conclude that when removing the ZLB constraint, the gap between the deterministic and stochastic steady state is quite low. Instead, the risk of hitting the zero lower bound can lead to large discrepancies between the desired and realized level of inflation.

The red star denotes the inflationary bias that arises at the calibrated value of the standard deviation of the preference shock, which is set to match the volatility of the U.S. real GDP growth rate during the Great Moderation. Hence it can be regarded as a value lying at the low-end of the spectrum of plausible values for the U.S. economy. This finding suggests that in absence of shocks, inflation undershoots the central bank's inflation target by 27 basis points because of the risk of hitting the ZLB in the future. This happens even when the macroeconomic volatility is as low as that observed during the Great Moderation. As the macroeconomic volatility increases, the bias widens up exponentially. A one-percentage-point increase in the standard deviation of shocks causes a 15-basis-points reduction in the model's long-run inflation rate. This is because the curve is very steep on the right of the red star, which denotes the benchmark value for the standard deviation of preference shocks. We consider these results a reason of concern for policymakers.

For the benchmark calibration of the standard deviation of the shock, the line of the inflationary bias is very steep. Consequently, it would take just a two-percentage-point increase in the standard deviation of preference shocks to make self-fulfilling deflationary spirals, which will be analyzed in greater detail below, possible. Given that our benchmark calibration reflects a record-low macroeconomic volatility, this result is a reason for concern. As we shall show, this finding is driven by the low long-term real interest rate r^*, which is at an historical low according to the *Survey of Economic Projections* (SEP). A higher long-term real rate of interest r^* makes the function of the inflationary bias less steep and therefore would increase the threshold of the volatility that triggers the deflationary spirals.

It should also be noted that the steepness of the function of the long-term inflationary bias has to be chiefly imputed to the presence of the ZLB constraint. Indeed, the slope of the black dashed dotted line, which capture the counterfactual case where the ZLB constraint is not enforced and nominal rates are allowed to become negative, is tiny and constant for different values of the standard deviation of the shocks.

What if the central bank realizes that inflation is in general below the desired target and decides to lower its inflation target to make it coincide with average inflation? The long-run deflationary bias induced by the ZLB constraint would become even larger because lowering the target would make the probability of encountering the zero lower bound even larger. We discuss below what the central bank can do to bring inflation in line with the desired target.

The Real Interest Rate Bias The right graph of Figure 3 shows the bias on the real interest rate due to the zero lower bound. This negative bias emerges because the central bank is conducting an active monetary policy with the objective of closing the negative inflation gap, which is shown in the graph on the left. As inflation is persistently below target, so is the real interest rate.

This result is quite important. It shows that for a given level of the deterministic steady state of the natural interest rate r^*, the risk of encountering the zero lower bound can lead to a substantial decline in the stochastic steady state for the real interest rate. Thus, if an econometrician were to use a moving average or the trend component of the real interest rate to back out r^*, she would reach the conclusion that r^* is in fact lower than what really is. Estimates of the natural interest rate have become central for the policy debate (Laubach and Williams 2003 and Del Negro et al. 2017). Our results suggest that when the zero lower bound risk is relevant, estimates of the natural rate of interest obtained using linear models can be biased. We consider this an interesting direction for further research.

To sum up, the inflationary bias brought about the presence of the ZLB can generate first-order distortion for a central bank that tries to stabilize inflation around the target. Furthermore, we noticed that the combination of low real rate of interests, r^*, and moderate macroeconomic risk can trigger the long-run bias in inflation and output or even worse deflationary spirals.

The Output Bias The center graph shows the effects of the risk of hitting the ZLB on long-run output. As before, the long-term output bias due to the zero lower bound is given by the vertical difference between the blue solid line and solid dashed-dot line, which gives us the bias when the non-negativity constraint on the nominal interest rate is not imposed. The reason why the output bias is positive is because the central bank has a two percent inflation target but inflation fluctuates around its stochastic steady state that is lower than the central bank's target (see the left graph of Figure 3). As a result the central bank keeps the interest rate lower than its deterministic steady-state level to close the negative inflation gap. This can be seen in the right graph of Figure 3. This monetary stimulus drives a positive wedge between the level of output at the stochastic steady state and that at the deterministic steady state.

It should be noted that if we relax the ZLB constraint, the other non-linearities

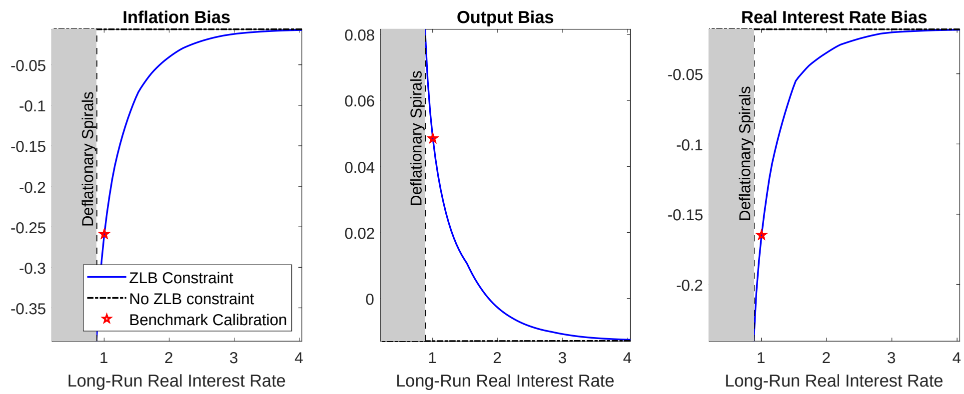

Figure 4: Macroeconomic distortions due the zero lower bound as the long-real real rate of interest varies. Left graph: The inflationary bias due to model's non-linearities. The red star denotes the calibrated value of the standard deviation of this shock. The difference between the blue solid line and the black dot-dashed line captures the deflationary effects of a risk of a recession that pushes the nominal interest rate to its lower bound. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. The gray area marks the region of the values for the standard deviation of the preference we cannot solve the model for. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

in the model would imply a level of output *lower* than the deterministic steady state value. The difference between the two would be increasing in the volatility of the shock. In absence of the ZLB constraint, the long-run inflationary bias becomes tiny (see graph on the left) and therefore the central bank will respond to this by lowering the interest rate only by a little. Moreover, in this case precautionary motives, which prompt households to save more to shelter themselves against future risks, become the driver of the negative long-term output bias. The positive bias introduced by the lower bound constraint dominates these effects for our benchmark calibration of the standard deviation of preference shocks (red star). As we shall see, if the long-term real interest rate r^* is higher, the ZLB bias on long-term output would be dominated by the effects of the other non-linearities in the model. This is because a higher real rate of interest expands the central bank's room of maneuver in case of negative preference shocks. Consequently, deflationary spirals are less likely.

Deflationary Spirals The gray areas in Figure 3 denote the region of values of the standard deviation of preference shocks for which the zero lower bound constraint sparks self-fulfilling deflationary spirals. The mechanism goes as follows. As inflation expectations fall, current inflation tends to fall because of the Phillips curve. If monetary policy is not constrained by the zero lower bound, the central bank can effectively counteract these deflationary pressures so as to make them not supported in equilibrium. When monetary policy is constrained by the ZLB, the central bank has a limited capability of lowering the nominal rate if needed to stabilize prices. This limitation sets the stage for self-fulfilling deflationary spirals. When preference shocks become excessively volatile, the probability of hitting the ZLB increases and so does the probability that these deflationary spiral are supported in equilibrium.

Implications of a Low Natural Real Interest Rate The results we have discussed so far rely on the assumption that the long-run natural rate of interest is equal to one percent. This seems to be close to the current consensus of policymakers. In this section, we show that the inflationary bias is triggered by the combination of a low interest rates environment and the presence of the zero lower bound that can constrain the central bank's ability of countering the deflationary effects of shocks. In fact, higher values of the real rate of interest would mitigate or even completely eliminate the bias on inflation and the real interest rate because it would be less likely that monetary policy will become constrained by the ZLB.

Figure 4 precisely illustrates these results. A long-term real rate of interest r^* equal to 3 percent or higher effectively zeroes the bias on inflation and that on the real interest rate due to the risk of hitting the ZLB in the future. The intuition is straightforward: when the long-term real interest rate is higher the central bank has more room to counteract the deflationary effects of a contractionary shock and hence is less likely to become constrained by the zero lower bound (see Figure 2). Nevertheless, a slightly lower real interest rate r^* than that of our benchmark calibration can lead to deflationary spirals (the gray area). In such an unfavorable state of the world, the central bank loses control over inflation expectations because the expected probability of hitting the ZLB is so large that deflationary spirals can arise.

It is worth emphasizing that the size of the bias due to non-linearities in the model other than the ZLB does not vary with the long-term real interest rate, suggesting that the long-term macroeconomic biases linked to a low-interest-rate environment is entirely due to one specific source of non-linearity in the New Keynesian model: the zero lower bound. Furthermore, the other non-linearities imply a small negative bias for both inflation, and output, as well as the real interest rate: -0.5 bps, -1.2 bps, -1.7 bps, respectively.

Figure 5 shows the effects of changing both the standard deviation of shocks and the long-term real rate of interest r^*. The important takeaway from this graph is that

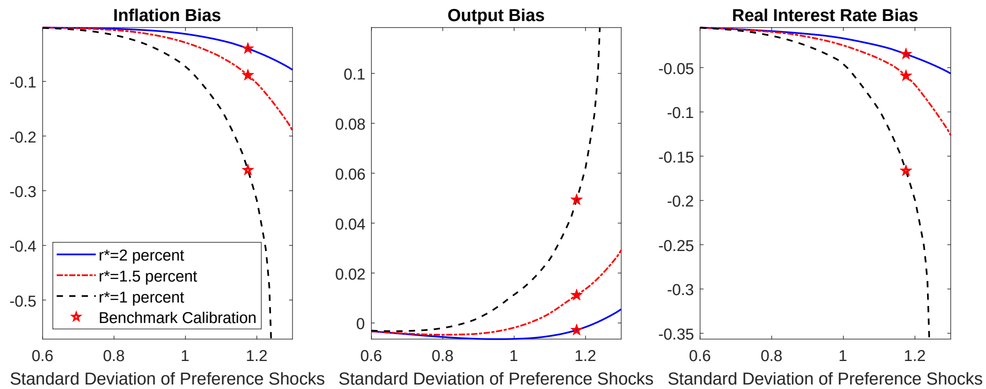

Figure 5: Macroeconomic distortions due the zero lower bound as the standard deviation of preference shocks varies (x-axis) and for alternative values of the steady-state real rate of interest. Left graph: The inflationary bias due to the zero lower bound constraint. The red star denotes the calibrated value of the standard deviation of this shock. Center graph: the same as the left graph but the bias is computed with respect to output (level). Right graph: the same as the left graph but the bias is computed with respect to the real interest rate. Units: Inflation and real interest bias is measured in percentage points of annualized rates while the output bias and the standard deviation of the preference shocks are in percent.

as the long-term real interest rate r^* increases sufficiently, the long-term inflation and output biases disappear. Moreover, a higher real rate of interest r^* would make the function of the inflationary bias less steep and therefore would increase the threshold of the volatility of shocks that triggers the deflationary spirals.

It is also interesting to notice that an increase in the long-term real rate of interest of one percentage point more than halves the inflationary bias due to the ZLB constraint. This gain increases with the size of the macroeconomic volatility, which is captured by the volatility of the preference shocks (x-axis).

5 The Unconditional Bias

The previous section has shown that even when the economy is at the stochastic steady and thus away from the zero lower bound, a deflationary bias arises because of the risk of encountering the zero lower bound in the future. This, in turn, triggers a bias in the real interest, as the central bank tries to lift inflation closer to the target and drives a wedge between actual output and optimal output Y. In this section, we go one step further and show that the inflationary bias is even larger when focusing on average inflation as opposed to the stochastic steady state.

To show this result, we define the unconditional deflationary bias as the difference between the model's unconditional mean of inflation and inflation at the deterministic steady-state equilibrium, which coincides with the central bank's inflation target Π.

This alternative concept of deflationary bias can be observed more directly in the data and is indeed more closely related to the bias shown in Figure 1 than the one used in the previous section.

Both definitions of inflation bias are useful. For instance, the definition based on the stochastic steady state allowed us to assess the extent to which the ZLB risk hinders the central bank's ability to close the inflation and output gaps away from the ZLB. Furthermore, the fact that inflation is subdued at the stochastic steady state allows us to clarify the following important result: When our model predicts that inflation is on average below the central bank's target, this result is not mechanically driven by the periods actually spent at the ZLB. Rather, this result reflects the distortions imposed by the existence of the ZLB on inflation dynamics, which were discussed in the previous section.

Recall, that the stochastic steady state corresponds to a situation in which the economy has not been hit by any shock for a long time, but agents are aware of risk. In a linear model, the stochastic steady state values would coincide with the unconditional means of the corresponding variables, but this is not the case in a non-linear environment. Thus, we simulate the model for several periods and then compute the mean of the variables of interest. In this setting, the risk of encountering the zero lower bound is in fact materialized.

Figure 6 reports the average bias as the volatility of the preference shock varies. The bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 1,000,000 periods. We drop the first 100,000 observations to minimize the effects of initial conditions. The biases are reported on the same scale used in Figure 3. The deflationary bias is now even larger. The zero lower bound is not a mere possibility, but an event that is in fact realized. Thus, average inflation is even further away from the desired inflation target because the economy experiences the very low inflation associated with the zero lower bound period.

This pattern for the behavior of inflation seems consistent with what is reported in Figure 1. In the late 1990s, the conquest of US inflation was completed. The central bank was successful in convincing agents about the 2% inflation target. In terms of the model, this event can be captured as convergence toward the stochastic steady state associated with a 2% inflation target. Such a low target, combined with a low real natural interest rate environment leads to a negative inflationary bias, even if the zero lower bound is not binding, and inflation drifts below the desired 2% target. In

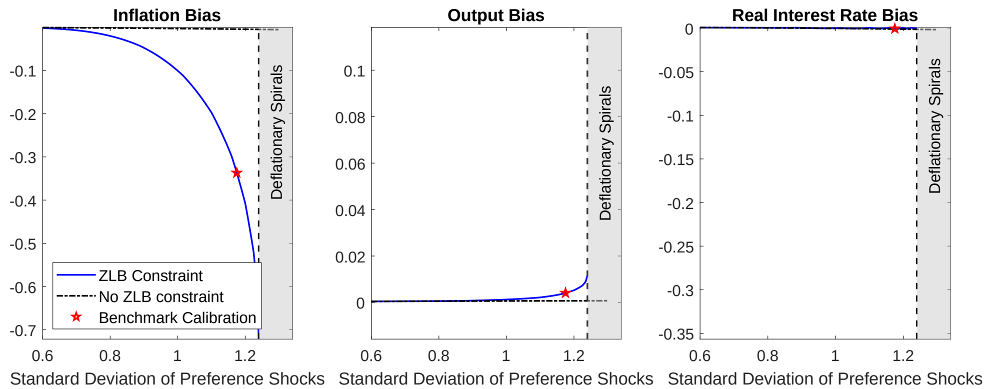

Figure 6: Average bias as the volatility of the preference shock varies. The bias is computed by taking the mean of inflation, output, and the real interest based on a simulation lasting 1,000,000 periods. We drop the first 100,000 observations to minimize the effects of initial conditions. The biases are reported on the same scale used in Figure 3.

fact, during those years the Federal Reserve was genuinely concerned about the risk of deflation (Krugman 2003). With the 2008 recession, the ZLB risk materialized. The model predicts in this case a further reduction in inflation, as in the data. Finally, as the economy recovers, the model predicts that inflation would not move to a 2% target, but it stabilizes around a lower value corresponding to the stochastic steady state.

When it comes to the behavior of output and the real interest rate, the bias is largely gone. When looking at the average bias for the real interest rate, there is a countereffect that pushes the bias to be positive. This countereffect is brought about by the presence of the ZLB itself that truncates the left tail of the distribution of the nominal interest rate. Thus, the negative bias that arises away from the zero lower bound is compensated by the fact that at the zero lower bound the central bank cannot further lower the interest rate, making the effective real interest rate too high. Importantly, the two phenomena are just the two sides of the same coin: The negative bias away from the zero lower bound is generated by the deflationary pressure that arises exactly because at the zero lower bound the central bank is not able to lower the interest rate to mitigate the fall in inflation.

6 The Asymmetric Rule

We have shown that the deflationary bias induced by the zero lower bound is reduced when the natural interest rate r^* increases or the macroeconomic volatility is lower. We now turn our attention to what the central bank can do to address the deflationary bias.

6.1 The Policy Proposal

In the literature and in policy circles, there has been an ample discussion about the possibility of increasing the inflation target as a way to avoid the perils of the zero lower bound (Coibion et al. 2012). An increase in the target would reduce the possibility of hitting the zero lower bound and the associated bias, as shown by Nakata and Schmidt (2016). However, policymakers have been quite reluctant to follow this path. Arguments that have been proposed against this solution are the loss of reputation and the fact that higher inflation seems to be associated with more volatile inflation. Another line of research has proposed price or nominal GDP targeting and average-inflation targeting (Mertens and Williams 2019). However, such policies are perceived as risky because they may require the central bank to engineer a deflation over certain periods of time in order to bring the price level down.

In this paper, we are advocating a different approach that does not require the central bank to explicitly aim at hitting a time-varying inflation target. The central bank can commit to react less aggressively to positive deviations of inflation from target than to negative deviations. We will show that embracing this asymmetric strategy can effectively remove the macroeconomic biases.

The policy rule that we propose implies a smaller response to inflation when inflation is above target. Such an asymmetric policy rule takes into account the risk of encountering the zero lower bound when inflation is below target by tolerating longer periods of inflation above target. Specifically, we consider the following modified policy rule:

$$R_t = \max\left[1, \mathbf{1}_{\Pi_t<\Pi}\left(\frac{\Pi_t}{\Pi}\right)^{\underline{\theta_\Pi}}(1-\mathbf{1}_{\Pi_t<\Pi})\left(\frac{\Pi_t}{\Pi}\right)^{\overline{\theta_\Pi}}\left(\frac{Y_t}{Y}\right)^{\theta_Y}R\right] \tag{16}$$

where $\underline{\theta_\Pi}$ denotes the response of inflation when inflation is below target, $\overline{\theta_\Pi}$ stands for the response to inflation when inflation is above target, and $\mathbf{1}_{\Pi_t<\Pi}$ is an indicator function that is equal to one when inflation is below target ($\Pi_t < \Pi$). In what follows, we set $\underline{\theta_\Pi} = 2$ as in the benchmark calibration of Section 3 and study how the average and stochastic steady state biases vary in response to changes in $\overline{\theta_\Pi}$.

The asymmetric rule in equation (16) can be interpreted as a strategy according to which the central bank promises to be slower in raising the rates when inflation goes above target. This rule reduces the risk of the zero lower bound and its effects. It is therefore particularly effective in a low-interest rate environment, like the current one, in which the biases on key macroeconomic variables can be sizable.

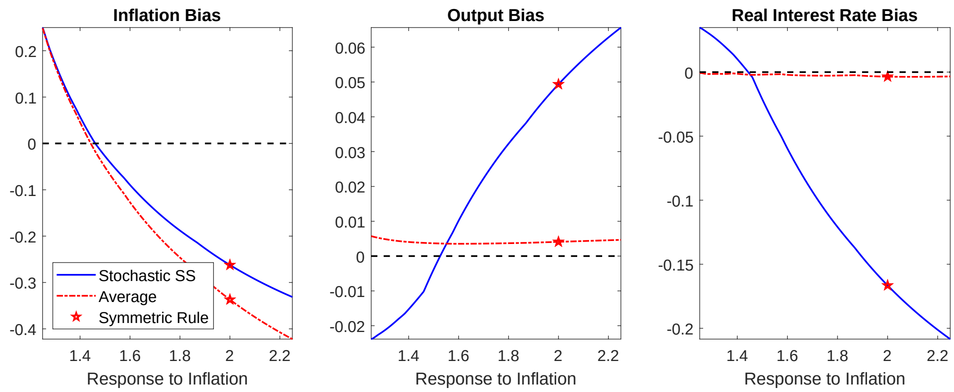

Figure 7: Macroeconomic biases due to ZLB as the central bank varies its response to positive deviations of inflation from target. The inflation bias (left plot), the output bias (center plot), and the real interest rate bias (the right plot) are computed by taking the difference between these variables at the stochastic steady state and their value at the deterministic steady state (blue solid line). These biases are also computed as the difference between the average value of these three variables and their value at the deterministic steady state (red dashed-dotted line). The response when inflation is below target is always equal to 2 as in the benchmark calibration. The red star marks the symmetric case in which the central bank responds with equal strength to inflation or deflation. Units: The inflation and the real interest rate biases are expressed in annualized percentage points and the output gap in percentage points.

Figure 7 shows how the macroeconomic distortions due to the zero lower bound varies as the central bank promises to be less aggressive when inflation is above its target. As before, we examine the behavior of the bias away from the zero lower bound (stochastic steady state, blue solid line) and its mean (black dashed line). The red stars denote the distortion under a symmetric rule with a response to inflation equal to two, as in the benchmark calibration.

We observe that being less aggressive when inflation is above target helps mitigate all the three biases. Specifically, for a response $\overline{\theta_{\Pi}}$ around 1.5, the ZLB-driven macroeconomic distortions become negligible. In a nutshell, to remove the macroeconomic distortions due to the ZLB constraint, policymakers need to be willing to tolerate inflation above the target for longer periods of time. By raising inflation expectations and removing deflationary bias, the asymmetric rule also makes deflationary spirals less likely to happen. This important result manifests itself with smaller gray areas in Figures 3 and 4. This is an important point to which we will return in Section 6.3.

It is worth emphasizing that this policy does not simply remove the bias by taking the average of high and low inflation. Instead, it effectively reduces the probability of hitting the ZLB. This is obvious when the bias is measured as the distance between the inflation target (deterministic steady state) and the stochastic steady state of inflation. As explained above, in this case, the economy is always away from the zero lower bound. The reduction in the bias is therefore a result of a lower risk of hitting the zero lower

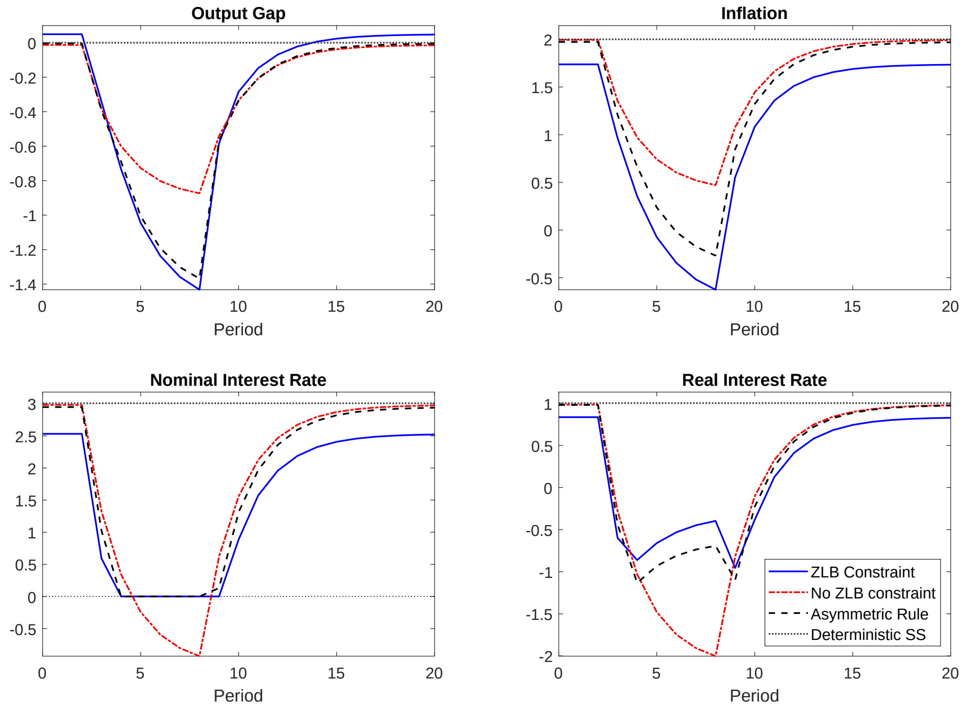

Figure 8: Simulations of the output gap, inflation, nominal and real interest rate during an artificial recession. The economy is at its stochastic steady state in period 0, 1, and 2. From period 3 until period 8, the economy is hit by a one-standard-deviation negative preference shock in every period. Starting from period 9 no more shocks occur and the economy evolves back to its stochastic steady state. These simulations are shown under three cases: (1) The benchmark model with the ZLB constraint (blue solid line); (2) the benchmark model in which the ZLB constraint is not imposed, allowing negative nominal interest rates model (red dashed line); (3) the model with the asymmetric policy rule that allows the central bank to remove the deflationary bias by responding less aggressively to positive deviations of inflation from target (red dashed-dotted line). Units: Inflation and interest rates are measured in percentage points of annualized rates while the output gap is expressed in percent.

bound in the future.

To further elaborate on this last point, we simulate the economy under a sequence of negative shocks large enough to bring the economy to the zero lower bound. Three cases are considered: (1) The benchmark model with the zero lower bound (solid blue line); (2) the same model without imposing the zero lower bound constraint (dashed line); (3) the model with the asymmetric policy rule that allows the central bank to remove the macroeconomic biases (dotted line). Figure 8 reports the path for the endogenous variables in the three cases. We assume that the economies are initially at their corresponding stochastic steady states and the size of the each shock is one standard deviation. In period 3, a sequence of negative demand shocks hits the economy. Starting from period 9 no more shocks occur and the preference shock slowly goes back to its own steady state.

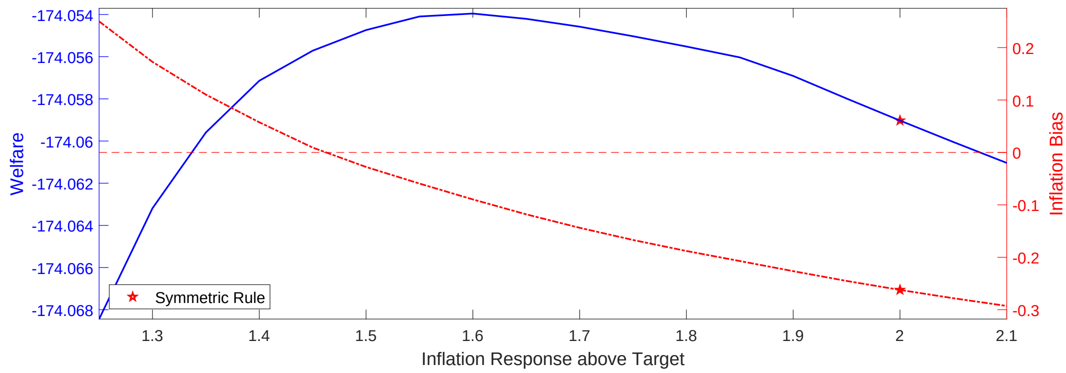

Figure 9: Welfare and inflation bias as the response to positive deviations of inflation from target varies in magnitude. Welfare is evalutated at the stochastic steady state and reported on the left axis. The inflation bias is defined as the difference between the annualized percentage rate of inflation at the stochastic steady state and the annualized percentage rate of inflation at the deterministic steady state and is reported on the right axis.

Before the sequence of negative shocks occurs, the economies are at their stochastic steady state and the three lines are horizontal. The distance of these lines from the deterministic steady state, which is denoted by the black dotted horizontal line, exactly captures the macroeconomic biases, which are also shown in Figure 7. Furthermore, we can see that inflation under the asymmetric rule is quite similar to the level of inflation that would prevail in the counterfactual economy that does not impose the zero lower bound constraint. In line with the results presented above, when the asymmetric rule is implemented, the deflationary pressure coming from the possibility of entering the zero lower bound is compensated by the inflationary pressure stemming from the fact that the central bank is willing to accept more prolonged periods of inflation above the target. As the sequence of negative shocks starts hitting the economy, the beneficial effects of the asymmetric rule relative to the symmetric rule are shown in Figure 8. Inflation falls by less and recovers more quickly. Consistently with such path, the time spent at the zero lower bound is reduced.

Importantly, Figure 8 illustrates that the asymmetric strategy removes the deflationary bias and, at the same time, does not lead inflation to overshoot the central bank's target after the ZLB period as the adoption of makeup strategies (e.g., the Krugmann-Eggertsson-Woodford "lower-for-longer" policies) would entail. In this respect, our proposal calls for a much less radical reform of the current U.S. monetary policy framework.

6.2 Welfare Analysis

We evaluate the appeal of the asymmetric rule by measuring its impact on households' welfare W_0, which reads as follows:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^d \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{17}$$

Figure 9 shows welfare W_t (left axis) and the inflationary bias (right axis). As the central bank deviates from the symmetric rule (the red star) by lowering the response to above-target inflation, welfare increases. When this response is around 1.6, the welfare peaks and then it declines as the response to positive inflation deviations from target is further decreased. It should be noticed that to close the inflationary bias, the central bank has to respond more weakly to inflation than optimal. The asymmetric rule that removes the deflationary bias completely, is suboptimal in that it allows too large and persistent positive deviations of inflation from the central bank's target. The optimal asymmetric rule solves the following trade-off. On the one hand, by tolerating some persistent positive deviations of inflation from its target the central bank manages to mitigate the deflationary bias. On the other hand, by responding strongly to positive deviations of inflation from its target the central bank retains the necessary commitment to rule out episodes of high inflation.

Announcing that the central bank will respond less aggressively to inflation when inflation will be above target is time inconsistent if this announcement is made when inflation is below target. Therefore, the central bank needs an opportunity to show the public its commitment to the new asymmetric rule. The arrival of an inflationary shock that pushes inflation above target is such an opportunity. We call this scenario opportunistic reflation. We now investigate the implications for welfare and the macroeconomic outcomes of a central bank pursuing an opportunistic reflation.

Opportunistic Reflation Let us assume that the economy is initially at the stochastic steady state associated with the symmetric rule when it gets hit by a positive preference shock that boosts consumption and aggregate demand. The central bank receives now the opportunity to show to the private sector that it is willing to commit to the optimal asymmetric rule by responding less aggressively to the inflation consequences of that shock. It is assumed that by observing the muted response to inflation, the private sector immediately believes that the central bank will follow the

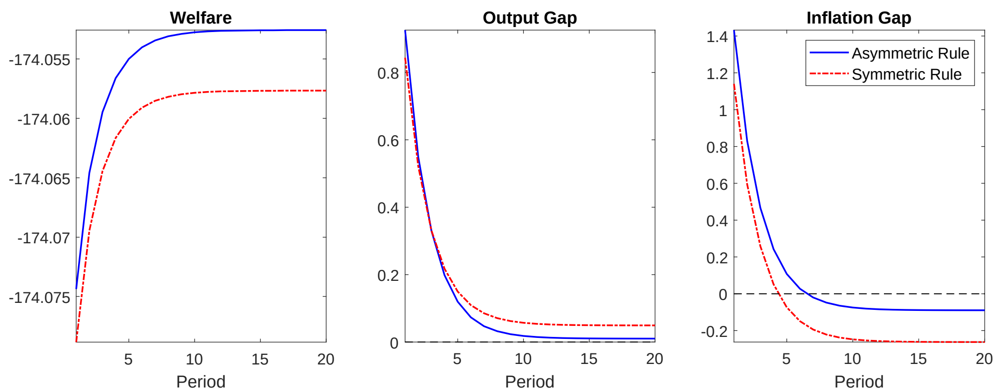

Figure 10: The dynamics of welfare, the output gap, and the inflation gap after a two-standard-deviation positive preference shock hits the economy in period 1. Two cases are reported: the case in which the central bank adopts the optimal asymmetric rule and conducts an opportunistic reflation of the economy and the case in which the central bank does not take this opportunity and sticks to the symmetric rule. In both cases, the economy is initialized at its stochastic steady state. Units: Inflation gap is measured in percentage points of annualized rates while the output bias is expressed in percentage points.

asymmetric rule forever.

In Figure 10, we show the impulse response function of welfare and the macroeconomic gaps (inflation and output) to a two standard deviation positive preference shock under the symmetric rule and under the optimal asymmetric rule. The output gap is measured in deviations from the flexible price economy whereas the inflation gap is expressed in deviations from the central bank's two-percent target. The optimal asymmetric rule raises the output and inflation gaps in the short run relative to the symmetric rule whereas it mitigates the macroeconomic gaps in the longer run (10 quarters out). Welfare is reported in the left graph of Figure 10, which shows that the optimal asymmetric rule unambiguously raises welfare both in the short run and in the longer run.

Why is welfare higher in every period when the central bank adopts the asymmetric rule even though this rule causes output and inflation gaps to widen more at the beginning? In every period, welfare does not depend only on the current inflation and output gaps but it is also affected by the expected discounted stream of welfare gains that will be accrued over time. The short-term responses of social welfare to a two-standard-deviation positive preference shock implies that the long-term welfare gains associated with the mitigation of the macroeconomic biases outweigh the short-term welfare losses.[8]

[8]Under the asymmetric rule, the weaker systematic response to inflation raises agents' long-run uncertainty about inflation and hence, everything else being equal, lowers welfare in the long-run. However, in our model these losses are dominated by the gains from removing the deflationary bias.

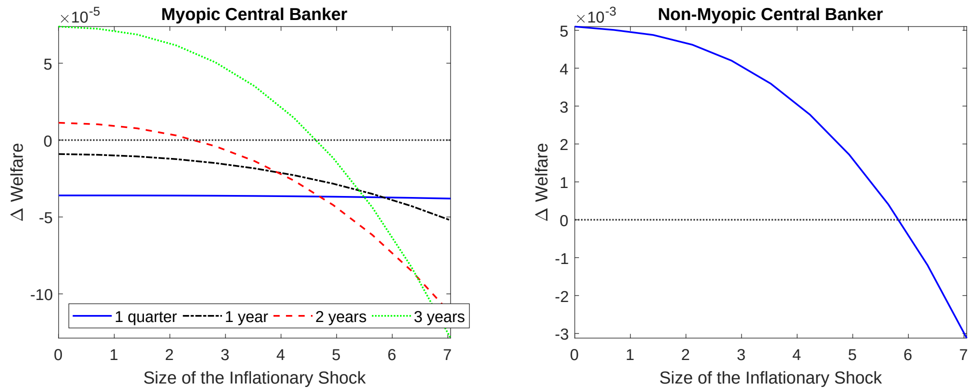

Figure 11: Welfare gains/losses from carrying out an opportunistic reflation as the size of the inflationary shock varies under different assumptions about how forward looking the central banker is. The left plot shows the myopic central banker's case and the different lines refer to different degrees of myopia; that is, the horizon k the central banker cares about when computing welfare gains/losses. The right plot shows the case of the benevolent central banker who maximizes the households' utility and thereby cares about the welfare gains at all horizons. Welfare gains/losses are computed as the difference between the welfare associated with adopting the optimal asymmetric rule and the welfare associated with sticking to the benchmark symmetric rule in the period when the inflationary shock hits the economy.

The opportunistic reflation involves a trade-off between short-term and long-term macroeconomic stabilization. Hence, a myopic central bank may refrain from seizing this opportunity as welfare costs are front-loaded.[9] To further investigate this issue, we tweak the welfare function (17) to study the behaviors of a myopic central banker who only cares about the welfare gains accrued up to a finite time horizon k. The welfare of the myopic central banker is denoted by $\widetilde{W}_0^k$, which is defined as follows:

$$\widetilde{W}_0^k = E_0 \sum_{t=0}^{k} \beta^t \zeta_t^d \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right] \tag{18}$$

The left plot of Figure 11 shows the myopic central bank's welfare gains from carrying out an opportunistic reflation following a positive preference shock as the size of the shock varies. The gains are computed by comparing the welfare under the optimal asymmetric rule for the non-myopic/benevolent central banker and that under the symmetric rule at the time the inflationary shock hits the economy. The different lines are associated with four degrees of the central banker's myopia, which is captured by the relevant horizons $k = 1$, 4, 8, and 12 quarters. The shorter the horizon k is, the more myopic the central banker is. The gains are shown as a function of the size of the shock. The myopic central banker's gains decline as the size of the preference shocks increases and, hence, the short-run response of inflation to the shock is more

[9]In what follows, a myopic central bank can also be interpreted as a conservative central bank that cares too much about the short-term inflation consequences of its actions.

pronounced. The speed of this decline increases as the myopia of the central banker becomes less severe.

If the relevant horizon is less or equal than one year ($k < 4$), gains are negative for all positive shock sizes. Such high levels of myopia dissuade the central bank from seizing the opportunity of reflating the economy as the policymaker is more allured by the short-run welfare gains, which stem from mitigating the immediate inflationary consequences of the shock. If the myopic central bank has a horizon of two years, it will opportunistically reflate the economy if the standard deviation of preference shocks is lower than two. Lower degree of myopia (higher k) leads the central bank to carry out the opportunistic reflation even when the magnitude of the shock is very large and the likely short-run inflationary consequences of the shock are considerable.

Going back to the case of the non-myopic/benevolent central banker ($k \longrightarrow \infty$), the right plot of Figure 11 shows the welfare gains from opportunistic reflation in this case as the size of the positive preference shock varies. The optimal asymmetric rule dominates the symmetric rule if the size of the shock is less than 5 times the calibrated standard deviations of the shocks (i.e., $100\sigma_{\zeta^d} = 1.175$). We consider this value as fairly high, which suggests that opportunistic reflation increases the economy's welfare by removing the deflationary bias.

Strategic Interest Rate Cuts We showed that if the central bank seizes the opportunity of reflating the economy by adopting an asymmetric rule after an inflationary shock arises, social welfare generally increases. If no opportunity to reflating the economy arises, the central bank can still remove the inflationary bias and improves welfare by cutting more aggressively the interest rate if inflation is below target while clarifying that the response to inflation above target is unchanged.

This alternative asymmetric rule also eliminates the macroeconomic biases. The upper panels of Figure 12 report the behavior of the macroeconomic biases defined with respect to the stochastic steady state (blue solid lines) and the observable averages (red dashed lines) as the response to below-target inflation, $\underline{\theta_\Pi}$, varies. The response to positive deviations of inflation from the target is the same as in the symmetric rule ($\overline{\theta_\Pi} = 2$). The red star denotes the distortions under a symmetric rule ($\underline{\theta_\Pi} = \overline{\theta_\Pi} = 2$) as in the baseline calibration. The response to inflation below target that zeroes the biases is approximately three.

The effects of adopting this asymmetric rule on the probability of hitting the ZLB

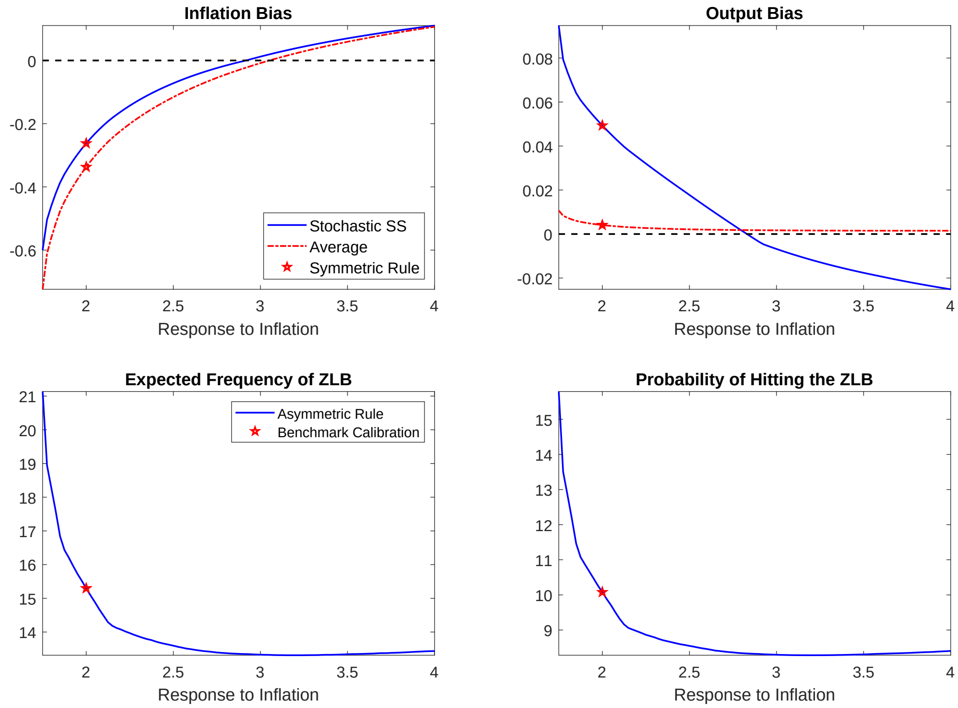

Figure 12: Macroeconomic biases due to risk of hitting ZLB under the asymmetric rule. The biases are computed relatively to the stochastic steady state (blue solid line) or the average inflation (red dashed-dotted line) and are shown in the upper panels. The output gap is expressed in percentage points and inflation gap is expressed in percentage points of annualized rates. The lower panels show the risk of hitting the ZLB in the next period (left) and the expected frequency of the ZLB (right) as the response to inflation below target varies. The frequency is in percentage points and it is computed as the ratio between the number of periods spent at the zero lower bound and the total sample size (300,000). The probability of hitting the zero lower bound in the next period is conditional on being at the stochastic steady state in the current period and is expressed in percentage points.

and the frequency of ZLB episodes is ambiguous ex ante. On the one hand, lowering more vigorously the nominal interest rate to fight against deflationary pressures could increase the probability of hitting the zero lower bound. On the other hand, committing to respond more aggressively to negative deviations of inflation from target eliminates the deflationary bias and thereby raises the long-term nominal interest rate. Higher nominal rates cause the likelihood of hitting the ZLB to fall. As shown in the lower panels of Figure 12, the asymmetric rule that allows the central bank to remove the macroeconomic bias ($\theta_\Pi = 3$) lowers the probability of hitting the ZLB and the expected frequency of ZLB episodes.

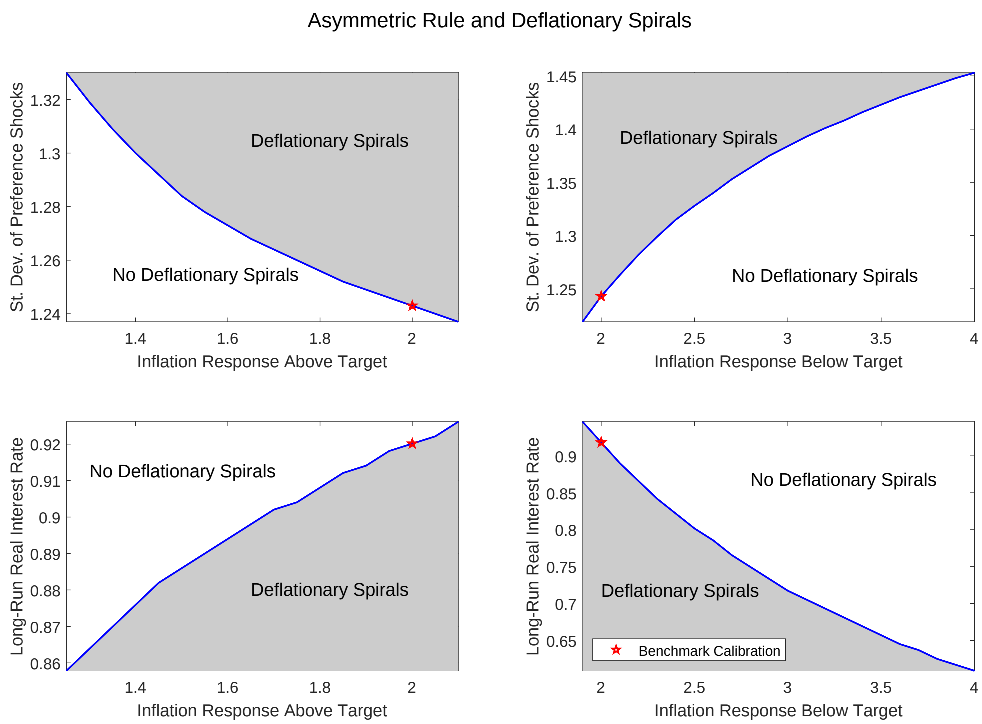

Figure 13: Asymmetric Rule and Deflationary Spirals. Upper left plot: the values of the standard deviation of preference shocks above which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.0. Upper right plot: the value of the standard deviation of preference shocks above which deflationary spirals arise as the below-target response to inflation varies and the above-target response is set to be equal to 2.0. Lower left plot: the values of the real long-term interest rate below which deflationary spirals arise as the above-target response to inflation varies and the below-target response is set to be equal to 2.0. Lower right plot: the values of the real long-term interest rate below which deflationary spirals arise as the below-target response to inflation varies and the above-target response is set to be equal to 2.0. The red stars mark the the thresholds for the standard deviation of the preference shock and for the real interest rate under the benchmark calibration (symmetric rule).

6.3 Asymmetric Rules and Deflationary Spirals

As already discussed in Section 4, adopting an asymmetric rule does not remove only the deflationary bias but it also lowers the risk for the economy to experience self-fulfilling deflationary spirals. While welfare is not directly affected by this risk since in our model parameters are fixed, falling into a self-fulfilling deflationary spirals may be very costly for the economy. The gray areas in Figure 13 denote the values of the standard deviation of preference shocks (upper panels) and the values of the long-term real interest rate (lower panels) that trigger the deflationary spirals for any given above-target response to inflation (left panels) and for any given below-target response to inflation (right panels). The bigger the asymmetry in the parameters of the rule, the bigger (smaller) the macroeconomic uncertainty (the real rate of interest) has to be to trigger self-fulfilling deflationary spirals. This is because asymmetric rules make the risk of encountering the ZLB lower and hence diminish the likelihood and the expected duration of passive monetary policies in the future.

Mertens and Williams (2019) study a rule according to which the Federal Reserve enforces an upper bound on the federal funds rate to resolve the deflationary bias. This rule, while correcting the bias, would imply an increase in the probability of self-fulfilling inflationary spirals because effectively monetary policy becomes passive when inflation goes above a certain level. Therefore, such a rule reduces the risk of deflationary spirals at the cost of increasing the risk of triggering inflationary spirals. Instead, our asymmetric rule always implies active responses to inflation deviations from the target and hence does not expose the economy to the risk of indeterminately large increases in inflation.

7 Conclusions

An environment in which monetary policy faces the risk of encountering the zero lower bound, inflation tends to remain persistently below target, even if monetary policy is not constrained. This is because agents anticipate the possibility of low inflation in the future. We showed an asymmetric policy strategy that allows more prolonged spell of inflation above target eliminates the macroeconomic biases due to the ZLB. A strategy according to which the central bank reacts less aggressively to positive deviations of inflation from target than to negative deviations can effectively remove the macroeconomic biases, improve social welfare, and reduce the risk for the economy to fall into highly costly self-fulfilling deflationary spirals.

We argue that convincing agents that the central bank will abandon the old symmetric strategy to embrace the asymmetric one is non-trivial when inflation is below target. Once the central bank has removed the bias by announcing the asymmetric strategy, it has an incentive to renege on its announcement and to stick to the symmetric rule, which lowers the volatility of inflation. A way to address this time inconsistency is to conduct an opportunistic reflation; that is, to wait for an inflationary shock that will give the central bank the opportunity to show the public that its response to higher than target inflation is muted.

We show that carrying out an opportunistic reflation is welfare improving in a standard New Keynesian model. Nevertheless, the welfare gains are back-loaded and hence the policymaker needs to be sufficiently forward looking to be willing to conduct an opportunistic reflation.

If no opportunity to reinflate the economy occurs and inflation keeps staying below target, the central bank can cut the rate more aggressively. This action shows to the public that the central bank has credibly adopted an asymmetric rule according to which the policy rate will be adjusted more aggressively when inflation is below target while the central bank keeps its anti-inflation attitude unchanged when inflation is above target. This second asymmetric strategy is shown to be able to remove the macroeconomic biases as well and lowers the probability of hitting the ZLB.

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A Non-linear Solution Method

The model is solved with global methods to handle the non-linear nature of the zero lower bound. The agents take the presence of the zero lower bound into account and form their expectations accordingly. Therefore, the possibility of hitting the zero lower bound in the future affects potentially the equilibrium outcome in times of unconstrained monetary policy. We use time iteration (Coleman 1990 and Judd 1998) with piecewise linear interpolation of policy functions as in Richter et al. (2014).[10] Expectations are calculated using numerical integration based on Gauss-Hermite quadrature.

In general, the whole equilibrium system can be solved if the value of the state variable and policy variables are known. The state variable is ζ_t^d, while the policy variables are Π_t and labor H_t. However, a unknown policy function g associates the state variable with the policy variables:

$$\Pi_t = g^1(\zeta_t^d) \tag{19}$$

$$H_t = g^2(\zeta_t^d) \tag{20}$$

where $g = (g^1, g^2)$ and $g^i : R^1 \to R^1$. To solve the model, we approximate the unknown policy functions with piecewise linear functions that can be written as:

$$\Pi_t = \tilde{g}^1(\zeta_t^d) \tag{21}$$

$$H_t = \tilde{g}^2(\zeta_t^d) \tag{22}$$

where we restrict $\tilde{g}$ to the class of piecewise linear policy functions. The time iteration algorithm to solve for the policy functions is summarized below:

1. Define a discretized state grid $[\underline{\zeta_t^d}, \overline{\zeta_t^d}]$ and integration nodes $\epsilon^{\zeta^d} = [\underline{\epsilon_t^{\zeta^d}}, \overline{\epsilon_t^{\zeta^d}}]$.

2. Guess the policy functions $\tilde{g}(\zeta_t^d)$.

[10] This approach can handle the non-linearities associated with zero lower bound. Richter et al. (2014) demonstrate that linear interpolation outperforms Chebyshev interpolation, which is a popular alternative, for models with zero lower bound. The kink in the policy functions is more accurately located which gives a more precise solution.

3. Solve for all time t variables for a given state vector ζ_t^d. The policy variables are:

$$\Pi_t = \tilde{g}^1(\zeta_t^d) \tag{23}$$
$$H_t = \tilde{g}^2(\zeta_t^d) \tag{24}$$

so that the remaining variables are given as:

$$Y_t = AH_t^{1-\alpha} \tag{25}$$
$$C_t = Y_t(1 - \varphi\left(\frac{\Pi_t}{\Pi} - 1\right)^2/2) - g \tag{26}$$
$$R_t = \max\left[1, R\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}\right] \tag{27}$$
$$W_t = \chi N_t^\eta c_t^\sigma \tag{28}$$
$$MC_t = \frac{W_t}{(1-\alpha)AH_t(j)^{-\alpha}} \tag{29}$$

Calculate the state variable for period $t+1$ at each integration node i:

$$\zeta_{t+1}^{i,d} = \exp\left(\rho_\zeta \log(\zeta_t^d) + \epsilon_{t+1}^{i,\zeta^d}\right) \tag{30}$$

where the superscript i indicates the variable at integration node i. For each integration node $\zeta_{t+1}^{i,d}$, calculate the policy variables and solve for output and consumption:

$$\Pi_{t+1}^i = \tilde{g}^1(\zeta_t^{i,d}) \tag{31}$$
$$H_{t+1}^i = \tilde{g}^2(\zeta_t^{i,d}) \tag{32}$$
$$Y_{t+1}^i = A{H_{t+1}^i}^{1-\alpha} \tag{33}$$
$$C_{t+1}^i = Y_{t+1}^i(1 - \varphi\left(\frac{\Pi_{t+1}^i}{\Pi} - 1\right)^2/2) - g \tag{34}$$

Calculate the errors for the Euler Equation and the New Keynesian Phillips curve

$$err_1 = 1 - \beta R_t E_t\Big[\frac{\zeta_{t+1}^d}{\zeta_t^d}\Big(\frac{C_t}{C_{t+1}}\Big)^\sigma\frac{1}{\Pi_{t+1}}\Big],$$
$$err_2 = \varphi\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - (1-\epsilon) - \epsilon MC_t(1-\alpha) - E_t\varphi\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}}{\Pi}\right)\frac{Y_{t+1}}{Y_t}.$$

where the expectations are numerically integrated across the integration nodes. The nodes and weights are based on Gaussian-Hermite quadrature.

4. Use a numerical root finder to minimize the errors for the equations. The policy function of period t is adjusted, while the policy functions for the next period $t+1$ are fixed.

5. Update the policy functions until the errors at each point of the discretized state are sufficiently small.