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ABSTRACT

Inversions—in which the popular vote winner loses the election—have occurred in 4 US Presidential elections. We show that rather than being statistical flukes, inversions have been ex ante likely since the 1800s. In elections yielding a popular vote margin within one percentage point (which has happened in one-eighth of Presidential elections), 40% will be inversions in expectation. Inversion probabilities are asymmetric, in various periods favoring Whigs, Democrats, or Republicans. Feasible policy changes—including awarding each state's Electoral College ballots proportionally between parties rather than awarding all to the state winner—could substantially reduce inversion probabilities, though not in close elections.

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1 Introduction

Over the last two hundred years, the US Presidential candidate with the most votes has lost the election about 8% of the time. These *electoral inversions* are a consequence of the two-tiered system of voting in the Electoral College. Presidential inversions have generated a long history of controversy. Disagreement over the desirability of electoral reform among theorists, politicians, and the public often hinges on whether inversions are likely and on whether the Electoral College generates some advantage for one party or another relative to the popular vote outcome. Practical proposals to repeal the Electoral College system—including over a hundred constitutional amendments proposed by members of Congress over the past two centuries—are often predicated on the belief that inversions are probable enough to warrant major constitutional change (Edwards, 2011; Peirce and Longley, 1968). The National Popular Vote Interstate Compact—which as of 2019 has been signed into law by 16 states, covering 196 of the 270 electoral votes needed for activation—is motivated in part by the increasing closeness of recent elections and the historical frequency of inversions when elections are close.[1]

Yet, despite this, an important set of empirical facts remains understudied. How ex ante probable, in fact, are electoral inversions in US Presidential races? How do inversion probabilities vary with the closeness of the national popular vote—could one occur at much wider popular vote margins than in the four historical instances of inversion (1876, 1888, 2000, and 2016)? And is it merely coincidence that all inversions have involved a popular vote majority for the Democrat and an Electoral College win for the Republican? The answers to such questions are not obvious because the sample of Presidential elections is small (only 25 observations per century), making it difficult to distinguish statistical flukes from events that were ex ante probable.

In this paper, we address these questions. To do so, we estimate probability distributions over Presidential election outcomes using election data extending back to 1836. By 1836, citizens rather than state legislatures voted in Presidential elections in all but one state. We begin by defining a statistical model (i.e., a data generating process) for state vote totals in Presidential races. We then estimate the model using state-level voting data. Sampling from the estimated model yields probable outcomes for each state's vote tally. Aggregating across the states yields a probable national election outcome in terms of citizen votes and Electoral College ballots. Sampling many thousands of these

[1] See nationalpopularvote.com, "Introduction: What It Is - Why It's Needed."

probable national elections produces a joint probability distribution over the national popular vote and the national Electoral College outcome. Our statistical model is flexible enough to nest the standard approaches to election modeling from the positive political science literature (Gelman and King, 1994 and Katz, Gelman and King, 2004), recent advances in election forecasting (Silver, 2016), as well as other non-parametric approaches.

Because different estimation approaches and different structural restrictions on the data generating process correspond to different substantive assumptions about the complex and interacting behaviors of voters, campaigns, and other social and economic forces, we generate results under many alternative models. Rather than preferring a particular set of assumptions, we show that the envelope of results described by *any plausible model* generates an informative lower bound on inversion probabilities in close elections. This is true even though the models differ substantially in their implied probability distributions over the national popular vote and differ substantially in the covariance structure that links state shocks within an election. The latter determines, for example, whether Florida and Ohio tend to move together in an election year. Key results hold even when, in place of *estimating* model parameters, we iterate over a large grid of exogenously-set variances and covariances representing uncertainty in state-level voting outcomes. We explain in Section 5 how the structure of the Electoral College's aggregation mechanism, which translates citizen votes into a Presidential winner, can account for this invariance to model and parameter uncertainty.

We show that in elections decided by a percentage point or less (equal to 1.3 million votes by 2016 turnout), the probability of inversion is about 40%. For races decided by two percentage points or less, the probability of inversion is about 30%. Significant likelihood of inversion persists at larger vote margins. To our knowledge, no other study has produced an empirical estimate of these parameters. These findings are robust not only to model and parameter uncertainty, but also to excluding from our sampling frame the election-year observations in which an inversion actually occurred: In a split sample, our models predict that the 1876, 1888, 2000, and 2016 inversions were likely events. For example, our models—armed only with the information that the two-party popular vote outcome in the 2000 election was 49.7%R / 50.3% D, and estimated from a sample that excludes 2000—predict that an inversion was *more likely than not* for a generic Republican and Democrat candidate pair.

The high probability of inversion at narrow vote margins is an across-history property of the Electoral College system. It is not a modern phenomenon. Nor is it dependent on extraordinary political circumstances or the idiosyncrasies of a particular candidate. Even as the Union has grown over the past two centuries from 24 states in 1836 to 51 with DC today; even as larger shares of the population (non-whites, the poor, women) have been granted and exercised the right to vote for President; and even as the dominant political parties have been born, changed, and died (see Figure 1), our work shows that the high probability of an inversion has remained a constant feature of US Presidential politics.

We also show that inversion probabilities are not symmetric across political parties. The asymmetry arises because states are heterogeneous in Electoral College representation, and this heterogeneity is correlated with political alignment. In the past 30 years, this has favored Republicans: conditional on an inversion occurring, the ex ante probability that it will be won by a Republican ranges from 69% to 93% across models (in contrast to the ex post realization of 100%). This is in part driven by the fact that Republican candidates have been more likely to have had a narrow popular vote loss than Democrats over this time period. But *conditional* on a narrow popular vote loss for Democrats, modern Democratic candidates have had about a 35% chance winning the Presidency via inversion. In the Reconstruction period, although inversions were likely for both parties and although no Democrat has ever in fact won an inversion, the ex ante probabilities of inversions favored Democrats.

After establishing our main results, we decompose how various features of the Electoral College's aggregation rules contribute to inversion probabilities. Candidate sources of mismatch between the popular vote and Electoral College outcome include the winner-takes-all awarding of elector ballots by most states, the inclusion of two senator-derived ballots that are not in proportion to population, the rounding errors inherent in dividing the US population across just a few hundred indivisible electors (today, there are 538), and the substantial demographic differences between residents-at-last-Census (of any age and citizenship status) and voters-on-election-day. By changing the implicit aggregation function to remove one or more of these features—for example, subtracting two elector ballots per state—we show how each contributes mechanically to inversion probabilities. This exercise provides new insights into exactly *how* the Electoral College system adds random—though not mean zero—noise to the popular vote outcome.

The Electoral College is a distinguishing feature of the U.S. political system, and so has been widely studied across many fields (e.g., May, 1948; Peirce and Longley, 1968; Ball and Leuthold, 1991; Garand and Parent, 1991; Katz, Gelman and King, 2004; see Miller, 2012 for a complete review).

Nonetheless, the facts we document here are new. Much prior attention in the economics, law, and positive political science literatures has been focused on demographic inequalities and other facts about the Electoral College—such as the effective voting power by geography or race (Banzhaf III, 1968; Sterling, 1978; Blair, 1979), the strategic deployment of campaign resources across states (Bartels, 1985), or the probability of a single voter being individually pivotal (Gelman, Silver and Edlin, 2012; Gelman and Kremp, 2016). However, very few papers have quantified any aspect of the probability of an inversion. An important feature of our analysis is the modeling of election uncertainty. This distinguishes our work from the many studies in political science that characterize Presidential elections deterministically, such as via uniform partisan swing analysis, and therefore cannot assess the *probability* of an inversion (e.g., Garand and Parent, 1991; Grofman, Koetzle and Brunell, 1997). Although a recent theoretical literature has specifically examined the probability of Electoral College inversions (Lepelley et al., 2014; Kikuchi, 2017; Bakthavachalam and Fuentes, 2017; de Mouzon et al., 2018; Kaniovski and Zaigraev, 2018), the toy mathematical models underlying these studies do not take as inputs actual election-related data.[2]

A much smaller prior literature and some recent data journalism have made empirically sophisticated probabilistic assessments of particular Presidential races—most recently as forecasts around future elections (e.g., Silver, 2016; Cohn, 2019) but sometimes to understand probabilities underlying past events (Katz, Gelman and King, 2004) or counterfactuals (Morris, 2019). Nonetheless, there exists in the literature no data-driven estimate of the probability of an inversion as an across-history property of the Electoral College. Instead, predictions of ex ante inversion probabilities exist only for some 20th and 21st century elections, often made by forecasters in the weeks preceding the election. Our claims are broader, not tied to a particular pair of candidates or snapshot in time.

Our study provides important new facts for state and national policymakers weighing proposals that could lead to a national popular vote. We also advance a long literature on the Electoral College that spans many fields, including law, mathematics, and various social science disciplines. In economics, our study connects to recent work applying econometric techniques to issues at the intersection of economic demography and US politics (Vogl, 2014; Allcott and Gentzkow, 2017; Boxell, Gentzkow and Shapiro, 2017), and particularly to studies with an historic focus (Gentzkow et al., 2015; Kuziemko and Washington, 2018). In documenting partisan asymmetry in inversion probabilities,

[2] Or in some cases, even that there are 50 heterogeneous US states.

our work is also related to studies that have estimated partisan bias in Congressional representation due to Congressional district gerrymandering (Gelman and King, 1994; Fryer and Holden, 2011). Our study is the first to estimate inversion probabilities and partisan asymmetry across the history of the US Electoral College.

2 Background and Data

2.1 The Electoral College and Party Systems

Figure 1 describes the periods in US history that we study. Political scientists have identified several stable Party Systems, characterized by competition between a fixed pair of parties with stable political properties. We estimate results within spans of years defined by stable party systems. This avoids grouping together, for example, election outcomes for the Democratic party before and after the early 1960s partisan realignment of the North and South. We begin our study in 1836, after the Twelfth Amendment changed the rules of the Electoral College (hereafter EC) and after various state-level reforms rendered the Presidential election somewhat familiar to our system today. Most importantly, we begin only after all states (other than South Carolina) had transitioned to allowing their citizens to vote in Presidential elections (rather than having state Legislatures decide on awarding EC ballots). We do not study the Civil War era when Confederate secession changed the set of states. Nor do we include in our main sample the first half of the 20th century, which generated decades of consecutive landslide victories—first for Republicans, then for Democrats (though see Appendix E.5 for supplementary analyses on this period). Landslides are less informative of the probability distribution of vote share outcomes around the 50% threshold of interest. Given these restrictions, we study the Antebellum (1836-1852), Reconstruction (1872-1888), and Modern (1964/1988-2016) periods.[3]

There have been four inversions during our sample periods: in 1876, 1888, 2000, and 2016. There are also compelling arguments that Kennedy's 1960 victory (outside of our main sample) was in fact an inversion as well (see Gaines, 2001 and Appendix B). In mechanical terms, inversions can occur because of several features inherent in the two-tier system of the Electoral College. EC representation is linked to congressional apportionment. The number of each state's EC ballots is equal to the number of its US Representatives plus two, for the two Senators of each state. Following the Twenty-

[3]In Appendix A.2 we provide further historical context as it relates to sample definition.

third Amendment, Washington DC also receives as many electors as the least populous state. States individually determine how to award their EC ballots. Currently, all states except Maine and Nebraska award ballots as a statewide winner-takes-all contest (also known as the unit rule).

2.2 Data

The key inputs to our analysis are the historical election returns by state for each Presidential election year. Data on state-level vote tallies for each candidate and the size of the state's EC delegation in each election come from the Leip (2018) compilation of state election returns. Where possible, we check these data against Federal Election Commission records. In the few state $\times$ election year instances of disagreement (which are generally small), we rely on state government election records, where available. Further details on election data are documented in Appendix C. For data on state populations, we use IPUMS extracts from decennial Censuses (Ruggles et al., 2018) and the American Community Survey.

Following the literature, we normalize vote shares as a fraction of the total won by the two major parties. The major parties were the Democrats and Whigs from 1836 to 1852 and Democrats and Republicans for the later periods we examine. The 50% share of the two party vote is the relevant threshold for our analysis. For example, in 2000 the Republican candidate (Bush) garnered 48.847% of Florida citizen votes, or 50.005% of votes cast for either of the two major parties. By crossing the 50%-two-party threshold, Bush took all Florida EC ballots. This two-party normalization simplifies the graphical presentation, but does not substantively impact our analysis, as third party candidates won no EC ballots over our study periods.[4] A related but distinct issue is that a third party candidate could be pivotal in determining which of the two major party candidates wins the two-party majority. We examine sensitivity to various ways of handling third party votes below.

3 Methods

We construct probability distributions over national election outcomes. We proceed in two stages: first estimating the statistical model (i.e., the data generating process) for Presidential elections, and then sampling from estimated model to build distributions of likely outcomes. We do this many times, for

[4]We ignore faithless electors, which could further impact mismatch between the popular and Electoral College outcomes. See Appendix C.

many models.

3.1 Data Generating Process / Statistical Model

We flexibly model the data generating process for a state-by-election-year (st) outcome as consisting of a state expectation, $\overline{\alpha_s}$, and a mean-zero shock (ϵ_{st}), which may be correlated across states:

$$V_{st} = \overline{\alpha_s} + \epsilon_{st} \tag{1}$$
$$\epsilon_{st} = \gamma_t + \phi_{st} + \mathbf{X}_s \boldsymbol{\delta}_t$$

The outcome variable of interest is V_{st}, the two-party vote share for the indexed party (normalized to be Whigs before the Civil War and Republicans afterward) in the state-year, or the log-odds transformation of this vote share.

The compound shock ϵ_{st} includes an election year shock γ_t that is common to all states and independent across years. It also includes a state-specific shock ϕ_{st} that varies independently across states within each election year. The last component of ϵ_{st} is a vector $\boldsymbol{\delta}_t$ that accommodates correlation in the shocks experienced by different states in the same election year on the basis of common state characteristics—for example because some issue or candidate appeals to Western states (in which $\mathbf{X}_s$ is a vector of region indicators) or states with large non-white populations (in which $\mathbf{X}_s$ is the fraction of each state's population that is non-white). We defer parameterizing the distributions of γ_t, ϕ_{st}, and $\boldsymbol{\delta}_t$ until we discuss estimation below.

Equation (1) serves as both a model to be estimated, and—post-estimation—the process from which we sample Monte Carlo draws to generate distributions of probable elections. In the context of estimation, t corresponds to a particular election, like Hayes v Tilden 1876. In the context of simulation, t is a probable election that could have occurred during the period from which the parameters were estimated. In other words, t is a single simulation run, which contains S state realizations. (S=51 in the Modern sampling frame, which includes DC.) We also transform V_{st} to a number of EC ballots cast by each state's electors for each candidate, according to the mechanical rules of the Electoral College system. Aggregating V_{st} across states for a fixed t yields a national popular vote (NPV). Aggregating EC ballots across states for a fixed t yields an EC winner. For each model we estimate, we generate 100,000 election simulations to yield smooth joint distributions of NPV and EC outcomes.

The statistical model in (1) is a generalization of the consensus approach to modeling uncertainty in US election outcomes in political science. It nests the "unified method of evaluating electoral systems" in Gelman and King, 1994 and its more recent applications (e.g., Katz, Gelman and King, 2004). The unified method, as it is typically applied to legislative elections such as US House seats, estimates the variances of legislative district shocks (σ_γ^2) and a common shock (σ_ϕ^2). Often, the cross-district correlation terms (analogous in our application to the cross-state correlation terms) are restricted to be zero. With alternative assumptions on the structure of ϵ_{st}, our statistical model can accommodate any typical approach in the positive political science literature.[5] It also nests contemporary election forecasting models, applying these to *backwards*-looking uncertainty. And, finally, Equation (1) also nests the various ad-hoc approaches to estimation that we introduce below.

This flexibility is central to our approach. Specification uncertainty is an important challenge in this context: The sample of elections is too small to be confident of any one model for the distribution of potential Presidential election outcomes. Therefore, we estimate results under alternative sets of assumptions on the structure of ϵ_{st}, including on the permitted or assumed correlation structure that links outcomes across states in an election year, and on whether the distribution is parametrically estimated following the literature, or built up from bootstrap draws that avoid parametric assumptions.

Below, we report results produced by hundreds of parametric and non-parametric models. For tractability, we sometimes focus attention on 25 named models that span much of the relevant space of model uncertainty. Rather than preferring any single model or estimation approach, we show that the envelope of results described by *any plausible model* generates an informative lower bound on inversion probabilities. Demonstrating robustness to model uncertainty is a key contribution of the paper.

3.2 Parametric Estimation

We estimate several parametric versions of Equation (1) over three study periods. In the $x1$ set of models—A1, R1, M1, for application to the Antebellum, Reconstruction and Modern periods, respectively—shocks are assumed to be distributed as independent normals, with $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$. $\mathbf{X}\delta$ is restricted to zero. Thus, each state draws an idiosyncratic shock from the same distribution, and all states receive a common national shock from a separate distribution in each election t. This error structure, in which common national shocks are the only source of correlated

[5] See Appendix B for a complete discussion of how our statistical model nests other models in the literature and how it relates to deterministic methods like uniform partisan swing analysis.

shocks across states, aligns with the stylized fact in the elections literature that common, national shocks are an important component of the across-election-year variance. The $x1$ models are similar to the model in the Katz, Gelman and King (2004) analysis of the Electoral College, though applied to different study periods and to answer a somewhat different set of questions. We estimate the parameter vector $\theta = \{\overline{\alpha}_{s=1}, \ldots \overline{\alpha}_{s=51}, \sigma_\gamma, \sigma_\phi\}$ via maximum likelihood.

In other models, we allow subnational correlations in state outcomes, though there are important constraints on our ability to estimate a variance-covariance matrix for state vote shares. For example, the modern study period includes observations for 51 states over the 8 elections that fall between 1988 and 2016. The unconstrained covariance matrix would be 51×51 triangular. Therefore, when effectively constraining this matrix by choosing the $\mathbf{X}$ vector—i.e., making substantive assumptions about which state characteristics could link the shocks between states—we follow the elections literature and recent advances in election forecasting.

In models M2, R2, and A2, we follow FiveThirtyEight's methodology (Silver, 2016) in using fatter-tailed distributions and an alternative process for correlated shocks. In particular, we use t distributions with one degree of freedom fewer than the number of election years in the sample period. And, in addition to independent state and national shocks described by σ_γ and σ_ϕ, we specify an $\mathbf{X}$ vector that includes region indicators, fraction non-white in the state, and fraction with a college degree in the state. Other parametric models ($x5$, $x7$, $x8$, $x9$, $x10$) vary the set of characteristics $\mathbf{X}$ permitted to link the state shocks, as indicated below. Parameters $\theta = \{\overline{\alpha}_{s=1}, \ldots \overline{\alpha}_{s=51}, \sigma_\gamma, \sigma_\phi, \sigma_{\delta_{\text{Region}}}, \sigma_{\delta_{\text{Ed}}}, \sigma_{\delta_{\text{Race}}}\}$ are estimated via maximum likelihood.

It is important to understand that the unknowns of interest here are parameters describing the *uncertainty* in election outcomes—i.e., the shock process described by $\epsilon_{st} = \gamma_t + \phi_{st} + \mathbf{X}_s\delta_t$—rather than parameters describing the *expectations* of state election outcomes in past elections. The best unbiased predictor of, for example, the expected Republican vote share in Ohio over elections in the last thirty years is arguably the observed mean of the Republican vote share in Ohio over that period. The challenge lies in statistically describing uncertainty around how these historical elections could have unfolded differently. Our focus on estimating spread is in contrast to studies investigating, for example, how changing demographics could shift states' future partisan alignment.[6] Nonetheless,

[6]Such studies might estimate the marginal effects coefficient vector $\boldsymbol{\beta}$ in a model like $V_{st} = \mu_s + \mathbf{X}_{st}\boldsymbol{\beta} + \xi_{st}$ and then project the model onto a different $\mathbf{X}$ vector. In our models, δ represents a mean zero random shock that multiplies $\mathbf{X}$. This is in contrast to a constant parameter $\boldsymbol{\beta}$ that multiplies $\mathbf{X}$.

for completeness, we estimate versions of Equation (1) that additionally allow for time-varying state demographics to shift state vote share *expectations*, in place of capturing state expectations with fixed effects $\overline{\alpha_s}$. Appendix E.3 shows this has no impact on our results of interest.

3.3 Hyperparameter grid

A challenge for any study of the EC is that with only a few elections per party system, it is impossible to be confident that estimates precisely reflect the true variance-covariance matrix of random shocks across states. We therefore investigate the sensitivity of our results to *assuming* model parameters that cover a large grid of national shock variances (σ_γ), state shock variances (σ_ϕ), and Census-region shock variances (σ_r). This exercise allows us to assess the importance of parameter uncertainty. Although arbitrarily specifying model parameters would in most settings, and for most questions, generate uninformative bounds, we show that for the probability of inversions in close Presidential elections, these bounds are informative (far from zero). We discuss why in Section 5.

3.4 Non-Parametric, Bootstrap-Based Monte Carlo

Beyond assessing parameter uncertainty, we further address model uncertainty by investigating sensitivity to the parametric assumptions on the error process. To do so, we perform a bootstrap Monte Carlo in several forms. The bootstrap procedures conform to the DGP described in Equation (1), but rather than making parametric assumptions on the shocks and estimating these parameters, we draw ϵ_{st} directly from the discrete distributions of historical events.

To generate a single counterfactual election (t), an actual election year outcome is drawn for each state from among the election years in the sampling frame. In the baseline, these draws of election years are independent across states and are made with equal probability among the election years included in the sample frame. Thus, a simulated election during the Antebellum era might include the Whig vote share in Alabama in 1836, in Arkansas in 1852, in Connecticut in 1840, and so on. Combining a draw from each state yields a counterfactual election. Generating many such elections yields a probability distribution over election outcomes.

Independent sampling (models M3, R3, A3) constitutes a naive baseline. It tends to generate under-dispersed vote share distributions because it implicitly neglects a stylized fact of US elections: there is a large common (i.e., national) component to vote share deviations from long run state means.

Therefore, we perform variants on the bootstrap procedure that preserve within-year, across-state correlation in outcomes to various degrees. In one set of simulations, we include a tunable parameter that, within a simulated election t, places excess probability weight on drawing state outcomes from the same realized election year. Within each simulation t, we first randomly (with uniform probability) draw a focal year on which to apply the excess probability mass.[7] In models M4, R4, and A4, we set this excess mass parameter to 0.50, so that for each state, there is a 50% chance that the draw comes from the randomly selected focal year for that simulation. The remaining 50% probability is divided uniformly across the other years in the sample frame to generate one simulated election. A new focal year is drawn for the next election iteration t. In models M5, R5, and A5, we use wild bootstrap draws (Cameron, Gelbach and Miller, 2008) from a common pool of discrete shocks experienced by all states over the sample period. Other bootstrap variants are reported below. Among these are cases in which we allow for swing state bootstrap draws to be correlated to various degrees.

4 Results

Using the parameter estimates $\widehat{\theta}$, or taking bootstrap draws in the case of non-parametric simulations, we sample from the data generating process in (1) to find the joint distributions of national popular votes and Electoral College outcomes. The summary statistics of interest from these distributions are the conditional inversion function, $\mathrm{Inv}(\mathrm{NPV})$, which expresses the probability of an electoral inversion at each level of the index party's share of the national popular vote, and $\mathrm{Win}(\mathrm{NPV})$, which expresses the probability that the index party wins the Presidency as a function of the NPV.

4.1 Inversion Rates

We begin in Figure 2 by displaying Monte Carlo election simulations corresponding to the $x1$ models for the Modern (M1), Reconstruction (R1), and Antebellum (A1) periods. This set of models provides a baseline. The left panels show the probability distribution over the national popular vote share won by the index party. (Table A1 reports parameter estimates for each model presented in this section, though we primarily communicate spreads by drawing the implied distributions.)

If the EC and the national popular vote outcome always agreed, then $\mathrm{Win}(\mathrm{NPV})$ would follow a step function that increased from 0 to 1 as the national popular vote share crossed 0.50. For each

[7] See Appendix D for complete details.

of the historical periods, Figure 2 shows that Win(NPV) evolves smoothly across the 0.50 vote share threshold. The slope $\frac{\partial \text{Win}}{\partial \text{NPV}}$ is roughly constant over a wide range—one or two percentage points depending on the historical period. Thus the electoral system is similarly "responsive" to votes cast over these ranges in the sense of marginal impacts on the probability of victory. The broad pattern is not surprising given the history or known mechanics of the Electoral College, though these probability functions have not been previously reported.

In the modern period, Win(NPV) crosses 0.50 Republican vote share at about 65% probability, implying that Republicans should be expected to win 65% of Presidential contests in which they narrowly lose the popular vote. For Republican/Whig vote shares less than 0.50, any Republican/Whig victory is an inversion, so $\text{Inv}(\text{NPV}) = \text{Win}(\text{NPV})$ for NPV$<$0.50. For Republican/Whig vote shares greater than 0.50, $\text{Inv}(\text{NPV}) = 1 - \text{Win}(\text{NPV})$.

Panels B, D, and E of Figure 2 restrict the axes to focus-in on close elections and plot Inv(NPV). In Panel B, $\text{Inv}(50.01) \approx 0.35$ indicates that the generic Democrat wins 35% of simulated elections in which they narrowly lose the national popular vote. Inversion probabilities peak very close to the NPV=0.50 threshold, but mismatch persists even at wide popular vote loss margins for Whigs, Democrats, and Republicans. For example in Panel B, a 3.0 point margin favoring the Democrat (i.e., 48.5% Republican vote share, or a gap of about 4 million votes by 2016 turnout) is associated with a 16% inversion probability. To calculate the overall probability of an inversion, we find the integral of Inv(NPV) over the displayed probability distributions.

In order to summarize and compare results across models, Figure 3 reports key summary statistics from several parametric and non-parametric models, each with different assumptions and constraints on the DGP. Each row in the figure corresponds to a distinct model, with the first row summarizing M1 (Panel A from Figure 2). 100,000 simulated elections are drawn for each. For each model, we report various centiles of the popular vote distribution and the probability that a race is decided by within one or two percentage points of the national popular vote. The middle graphical panel displays the main findings: conditional and unconditional inversion probabilities across models and sample periods.

Figure 3 shows that in the three periods that we study, electoral inversions have been ex ante probable. Inversions are expected in more than 30% of elections within a two percentage point NPV margin. In elections within a one percentage point margin—about 1.3 million votes, based on 2016 turnout—the probability of an inversion is about 40%, though higher in some models. In historical fact, six Presidential elections (of the 48 since 1828) have yielded a popular vote margin within 1 percentage point. Two of these have been inversions (three if one counts Kennedy/Nixon 1960). If elections continue to remain close—as they have been in recent years—then inversions will occur with substantially higher frequency than their overall historical mean.

It is important to understand that despite model agreement on Inv(NPV), there is little model agreement on the probability of a close election. Different assumptions on the correlation structure in the error process or different sample constructions (see M12) cause dramatic differences in the predicted NPV distribution. This implies that, to the extent our results have applicability to a future election, they are most informative combined with an external projection of the closeness of the race.

Nonetheless, using the unconditional probability of an inversion (that is, irrespective of the popular vote margin) we compute that a voter who votes in 15 Presidential elections over a 60-year voting lifetime has a high ex ante probability of voting in at least one inverted election. If elections continue to be as close as they have been in the last 30 years, it is a near certainty that Americans who vote for the first time in 2020 will eventually experience an inversion over a 60-year voting lifetime.

4.2 Asymmetry

The probabilities of inversion are asymmetric across parties. In the past 30 to 60 years, this has favored Republicans: across the 12 modern-era models, conditional on an inversion occurring, the probability that it is won by a Republican ranges from 69% to 93% (Table A2). One can also ask, conditional on a Presidential victory occurring, what is the probability that the victory was generated by an inversion rather than by a popular vote majority? Here there is less model agreement on the precise parameter, though all models show a modern Republican advantage: the probability that any single Presidential win arises from a popular vote loss ranges from 28% to 71% across models for Republicans, compared to 3% to 14% across models for Democrats (Table A3).

The partisan asymmetry arises because states are heterogeneous in both EC representation and in the strength of partisan alignment. For example, in the modern period, Democrats have tended to win large states by large margins and lose them by small margins (see Figure 5). As long as geography predicts partisan affiliation (which it always has in US history), asymmetry is a general property of the Electoral College system. In the Reconstruction period, although inversions were likely for both parties and although no Democrat has ever in fact won an inversion, the *ex ante* probabilities of inversions favored Democrats (Table A3). Whether Whigs or Democrats were probabilistically advantaged in the Antebellum period is sensitive to model choice.

4.3 Robustness to Model and Parameter Uncertainty

The high probability of an inversion in a close election is a robust result. The conditional occurrence rate is similar across time periods in which the United States was comprised of different states and different political parties. It is similar in parametric (M1, M2, M5, R1, R2, R5, A1, A2, A5) and non-parametric (M3, M4, M6, R3, R4, R6, A3, A4, A6) models, models that omit data from the elections in which an inversion actually occurred (M10, M11, R10), and models that extend the modern sample backward to include the 1960s (M12)—the widest possible timeframe in which "Democrat" and "Republican" are at least arguably stable identities in the Modern period.

To fully understand the impact of the inferential challenge in estimating and simulating election outcomes with relatively few data points, we next report results that iterate over a grid of exogenously-specified variances and correlations in the data generating process. This hyperparameter approach, which assumes (rather than estimates) parameters in a model that includes state shocks, regional shocks, and national shocks is described in full detail in Appendix E.2 and displayed across Figures A2 and A3. The resulting models (about 300) span distributions from an underdispersed across-simulation minimum standard deviation of 0.53 popular vote percentage points to an overdispersed maximum standard deviation of 7.16 percentage points. Across these simulations, the probability of an inversion in a close election is entirely robust, with its Modern-era minimum never falling below 30% for elections decided by within two percentage points of the NPV.

Panel A of Figure 4 provides an overall summary of robustness to model and parameter uncertainty by overlaying the conditional win rate functions, Win(NPV), for about one hundred models of the modern period. Gray lines in the figure correspond to a subset of the hyperparameter models from Appendix E. Purple lines in the figure correspond to various parametric results. Among the purple lines, the overlay expands the set of models from Figure 3 by including parametric models that either assign all third-party votes to Democrats prior to estimation or assign all third-party votes to Republicans prior to estimation. See Figure A1 for an in-depth treatment of these counterfactuals.

The brown lines in Figure 4 expand on the bootstrap results from Figure 3 to vary the implied correlation structure of the bootstrap procedure. We do this by tuning the probability that state draws come from the same election year in 5% steps from 15% up to 50%. We also repeat this procedure for swing states only, sampling non-swing states independently. We also repeat this procedure for "safe" states only, sampling non-safe states independently. Safe states here means those that are in the top quintile of vote share margin (Democrat or Republican leaning) averaged over the sample period.[8] These types of exercises, which correspond to 44 bootsrap-based results in Figure 4, widen the probability distribution of NPV outcomes relative to the baseline bootstrap model (M3), because they build in the empirical within-election, across-state correlations.

Despite similarity in Win(NPV) in Panel A, the models in Figure 4 are substantially different in terms of the simulated elections they produce. Panel B shows that the probability densities over the national popular vote differ. The cross-state correlations also differ. For example, in model M1, the Massachusetts–Connecticut correlation in vote outcomes across simulated elections and the Massachusetts–Oklahoma correlation are both about 0.31. In M2, which adds correlated shocks based on observable similarity, these correlations are 0.59 and 0.24, respectively. By model design, all of these correlations are zero in M3, which assumes that election year shocks are independent across states. To provide a visual summary of these types of differences, Panel C displays the correlations between the simulated Michigan outcome and the outcome for each of 11 other swing states.[9] The same set of models from Panel A are included. Panel D shows these correlations between Texas and the 11 other largest non-swing states. Models are substantially different by these metrics.

In sum, Figures 4, A2, and A3 indicate that even if it is not possible to fully identify the data generating process for Presidential elections from the small set of observed election years, our main results are robust to any plausible alternative parameter set. Importantly, models with shocks linked by election year, region, racial composition, and educational characteristics produce similar inversion probabilities to models that assume that state shocks are completely independent. This suggests that smaller changes are unlikely to affect the conclusions here. We illustrate this in Appendix E.4 where we alter the parameterizations of the race and education variables in a model like M2.

[8] See Appendix D.5 for additional details.

[9] We follow *Politico* and *FiveThirtyEight* in defining swing states. See Appendix D.5.

5 Alternative Aggregation Rules

What can account for the stability of the inversion probabilities across models and parameter estimates? Although models differ in their assumptions on the DGP behind popular vote outcomes, the results from all models are fed through the same mechanical aggregation rules that translate state-level popular vote outcomes into a national EC winner. In particular, the preferences of voters in small states are upweighted by the two EC ballots corresponding to Senators (Figure 5, Panel A), and the winner-takes-all method of awarding states' EC ballots means that any votes beyond what is needed for one-vote plurality can contribute to a wedge between the NPV and the EC outcome.[10] Panel B of Figure 5 shows that in the modern period, Democrats have tended to win large states by large margins and lose them by small margins.

In Figure 6 we examine the impacts of mechanically altering the EC's aggregation formula. Holding fixed the votes cast, we change the rules of the EC system to either (i) eliminate the two elector ballots that each state receives for its Senators, (ii) allocate each state's EC ballots proportional to each state's popular vote outcome (up to the nearest whole ballot), or (iii) do both simultaneously. Under (i), DC and Wyoming are each apportioned one EC ballot instead of three. Under (ii) a party that wins 49.99% of the vote in a state with 25 EC elector ballots would win 12 ballots instead of zero. This exercise is not intended as a policy evaluation that could account for the endogenous responses of voters to a changing electoral system. Instead, it is meant to shed light on, for example, whether the popular press is correct in asserting that modern Republicans have a statistical advantage due to the current Republican alignment of lower-population (higher EC representation) states (e.g., The New York Times, 2016; The Economist, 2018).

Figure 6 plots inversion probabilities under alternative aggregation rules. We implement the alternative rule sets on top of underlying vote shares generated by two parametric models (M1, M2) and two non-parametric models (M3, M4). We focus here on the Modern party system. Results for all three periods are reported in Figures A4, A5, and A6.

The alternative aggregation rules variously shrink or shift the range over which inversions are likely. Consistent with popular perception of a Republican EC advantage due to small states' alignment, removing two EC ballots per state for the Senators shifts the Inv(NPV) function right

[10] Further, all models share the same underlying data informing state expectations of party margins of victory. For example, Washington DC and Utah are won by large margins by Democrats and Republicans, respectively, in the modern period. That fact enters all models.

across all panels/models in the figure. This implies a reduction in the probability of an inversion that awards the Presidency to the Republican. However, the shift is moderate, and merely changes the partisan balance without markedly reducing the overall inversion probability (the integral under the displayed function). For example, in M1, the probability of an inversion within a one percentage NPV margin changes negligibly from 43% to 42% with the removal of the "plus two," even as the composition of those inversions moves closer to partisan balance.

A policy of awarding ballots proportionally across the candidates—alone or in combination with removing the two Senator-linked ballots per state—generates a more substantial overall reduction in inversions. However, the two proposals in combination have a relatively small impact on inversion probabilities in very close elections. In fact, across all panels/models, the two reforms in combination actually *increase* the probability that inversion occurs following a narrow popular vote loss by a Democrat.

Why would some inversions persist under these alternative aggregation rules? One reason is that electors are few in number and indivisible: it is not possible to win 0.47 of an EC ballot under these rules or current rules. In Figures A4 through A6, we show that inflating the number of electors by a factor of 100 (post-apportionment) to reduce these rounding "errors" further reduces inversions. Though this still fails to cause convergence between the EC outcome and national popular vote for any historical period.

The final remaining source of divergence is that electors are apportioned according to population-last-Census, which includes all residents of all ages, measured up to a decade prior to election day. In contrast, election-day turnout includes only some adults of the current population. Besides differences in turnout of eligible voters, states differ in age pyramids, in net migration between Census rounds, and in the proportion of non-citizens, disenfranchised felons, and other non-voting adults. Because of this, the turnout to population-last-Census ratio varies from about 26% (Nevada) to 53% (Maine) today (Figure 5, Panel D). During the Antebellum and Reconstruction periods—which, of course, include years in which slaves counted towards EC apportionment at a rate of three-fifths—the gaps between the number of voters in a state and the state's Census-relevant population were greater. In 1852, the ratio spanned 5% (Alabama) to 63% (California). Accordingly, we find less convergence between the popular vote and EC outcome under the alternative aggregation mechanisms for these periods.

6 Conclusion

A robust finding of every model considered is that inversions are likely in close elections—where "close" includes elections with popular vote margins in the millions. Close elections are a game-theoretic equilibrium for two-party competition Downs (1957), which may be why US Presidential popular vote margins have often been small in stable party systems.

Ultimately, the EC system adds random (though not mean zero) noise to the popular vote outcome. Feasible policy changes shrink the variance of—but do not eliminate—this noise, reducing the range over which inversions are likely, though in some cases actually increasing the probability of inversions in close elections. We conclude that electoral inversions are enduringly fundamental to the Electoral College system.

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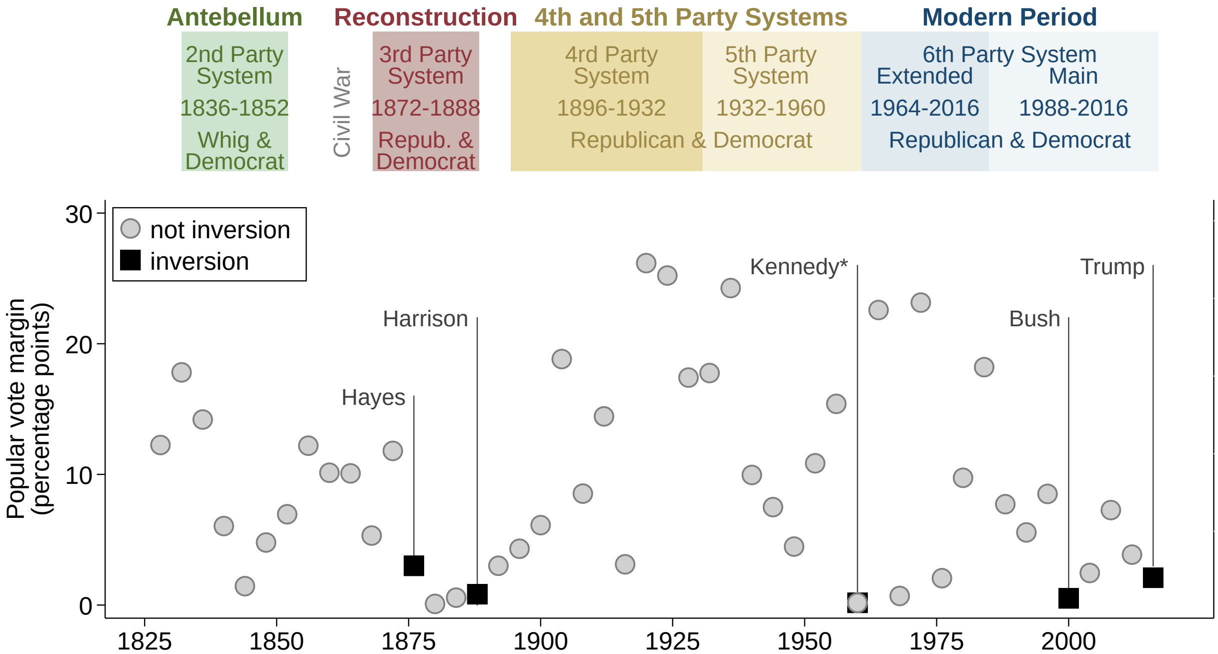
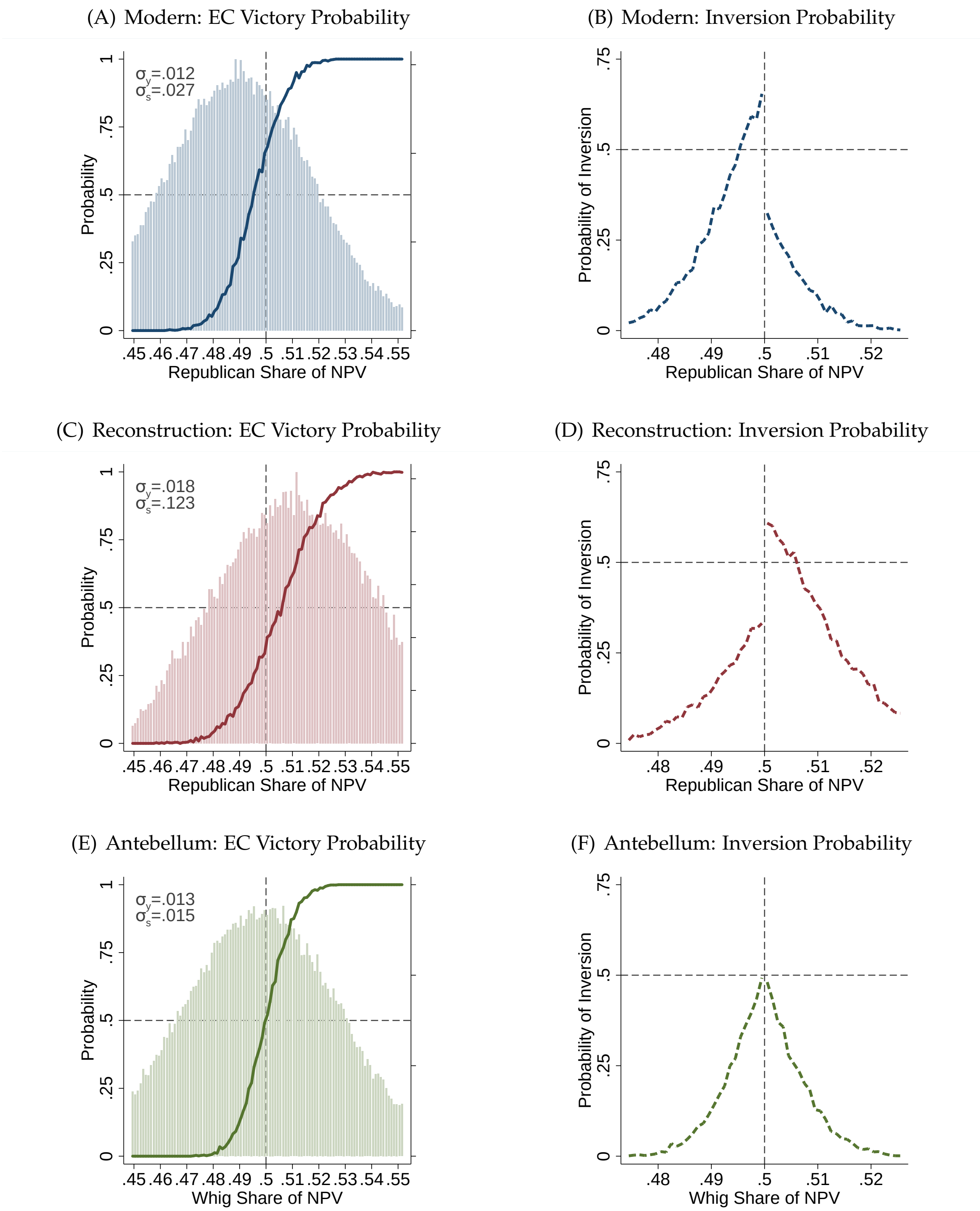
Figure 1: Background: Parties, Victory Margins, and Inversions in US Presidential Elections



Note: Timeline shows the periods of stable "party systems." We use this external characterization to determine our sample periods. The plot in the bottom panel displays the national popular vote margin for each US Presidential election between 1828 and 2016. The margin is measured as the absolute value of: the Whig or Republican vote share minus the Democrat vote share, each measured as a fraction of the two party vote. There are 4 widely acknowledged inversions: 1876, 1888, 2000, and 2016. All were won by Republicans.

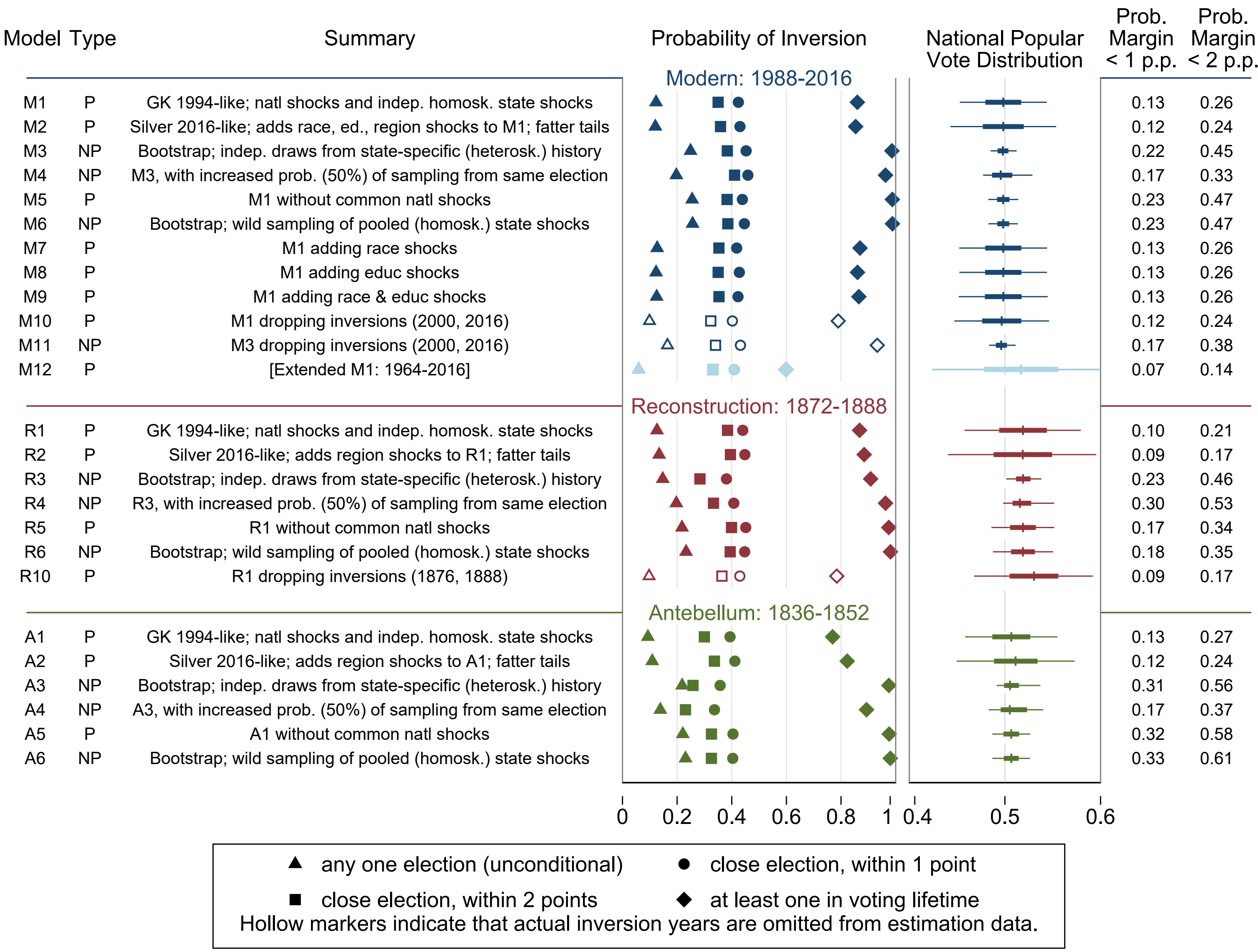
*In the 1960 election, Kennedy arguably lost the popular vote to Nixon despite winning the Electoral College; see Gaines (2001) and Appendix B.

Figure 2: Simulated Election Distributions Based on Parametric Estimates: Three Periods



Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of the baseline model (M1, R1, A1). Rows correspond to different historical periods, as indicated. Each panel consists of 100,000 simulated election draws. The Whig and Republican national popular vote shares run along the horizontal axes. The solid lines in the left panels (A, C, E) trace the conditional probability of a Whig/Republican electoral win at each level of the Whig/Republican vote share. In the left panels, win rates greater than zero for Whig/Republican vote shares $<$ 50% indicate inversions in favor of the Whig/Republican candidate. Win rates less than one for Whig/Republican vote shares $>$ 50% indicate inversions in favor of the Democrat candidate. The right panels (B, D, F) plot the inversion probabilities at each level of the vote share.

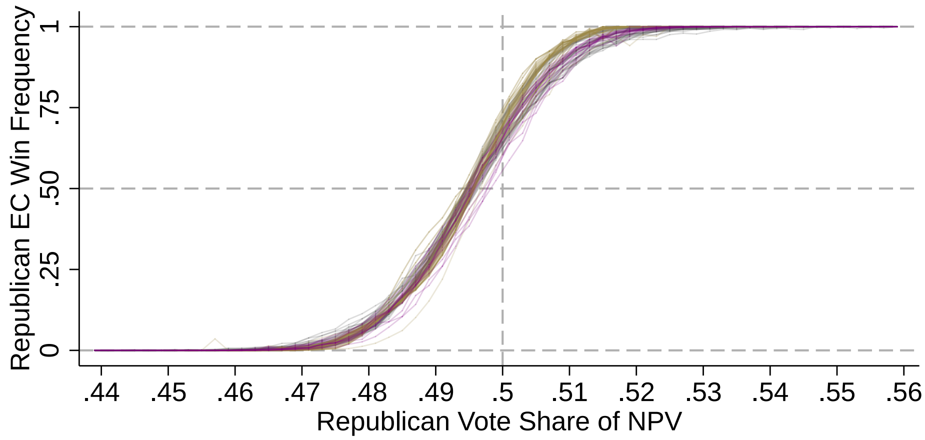
Figure 3: Main Result: Inversion Probabilities Across Models, Methodologies, and Time Periods



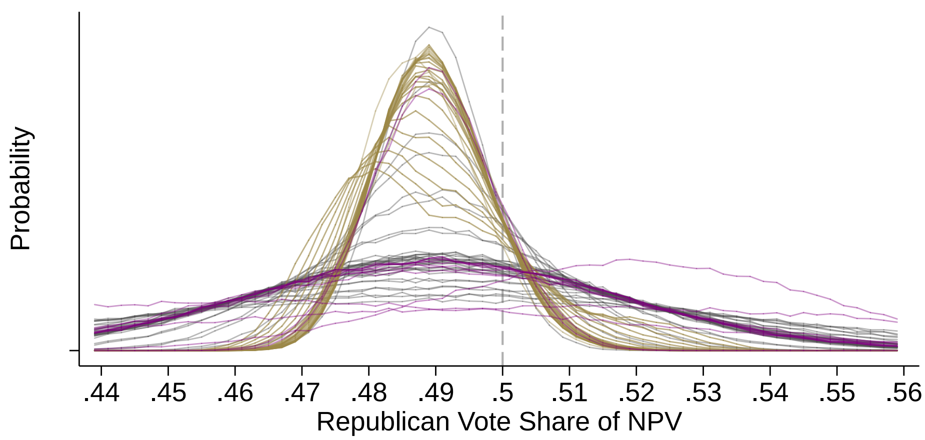
Note: Figure reports summary statistics from 25 election models. Repeated numerals in model names indicate that the same specification is used across time periods—as in M1, R1, A1. The popular vote distribution presents 5th, 25th, 50th, 75th, and 95th centiles of the simulated elections. P/NP denotes parametric/non-parametric. Parametric models are estimated by maximum likelihood; non-parametric models use bootstrap resampling of past election outcomes. P(mar. <1pp) and P(mar. <2pp) report the probability that the popular vote margin is within 1 and 2 percentage points, respectively.

Figure 4: Robustness: Win(NPV) Function is Invariant to Model & Parameter Uncertainty, Despite That Other Statistics Are Not

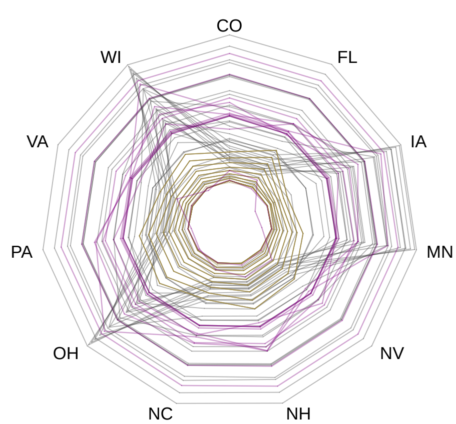
(A) Probability of Republican EC Win at Each NPV, Across 92 Models



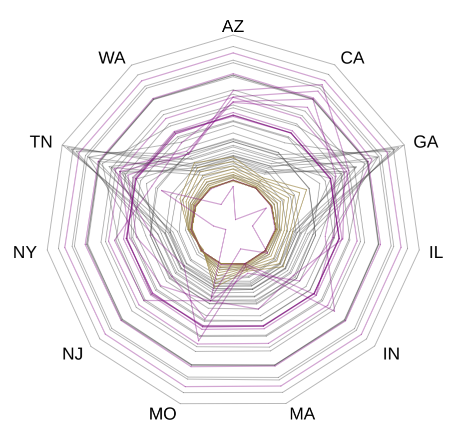
(B) National Popular Vote Distributions (Same Models as in A)



(C) Within-Election Correlation: MI with Other Swing States (Same Models as in A)

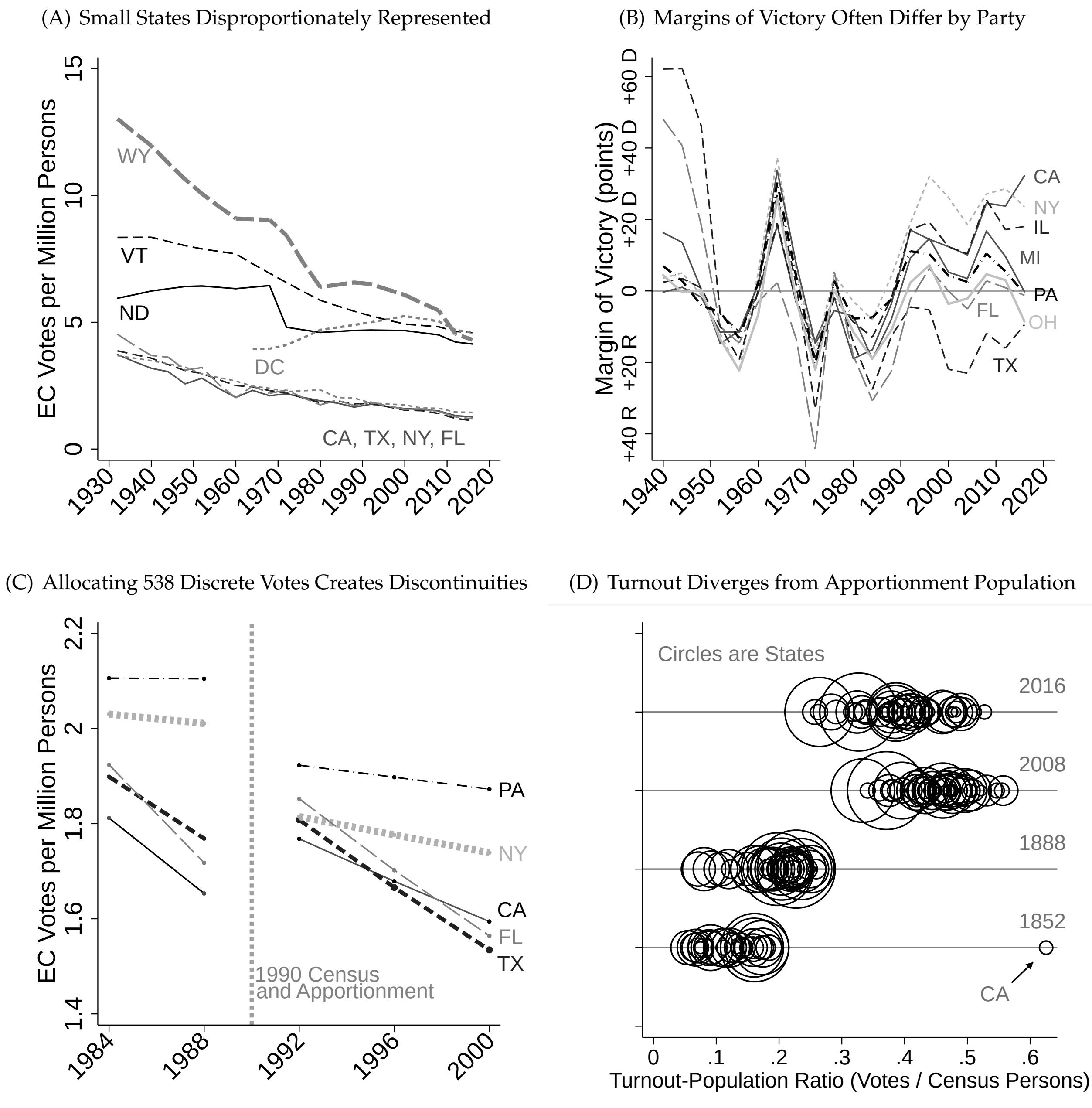


(D) Within-Election Correlation: TX with Other Large States (Same Models as in A)



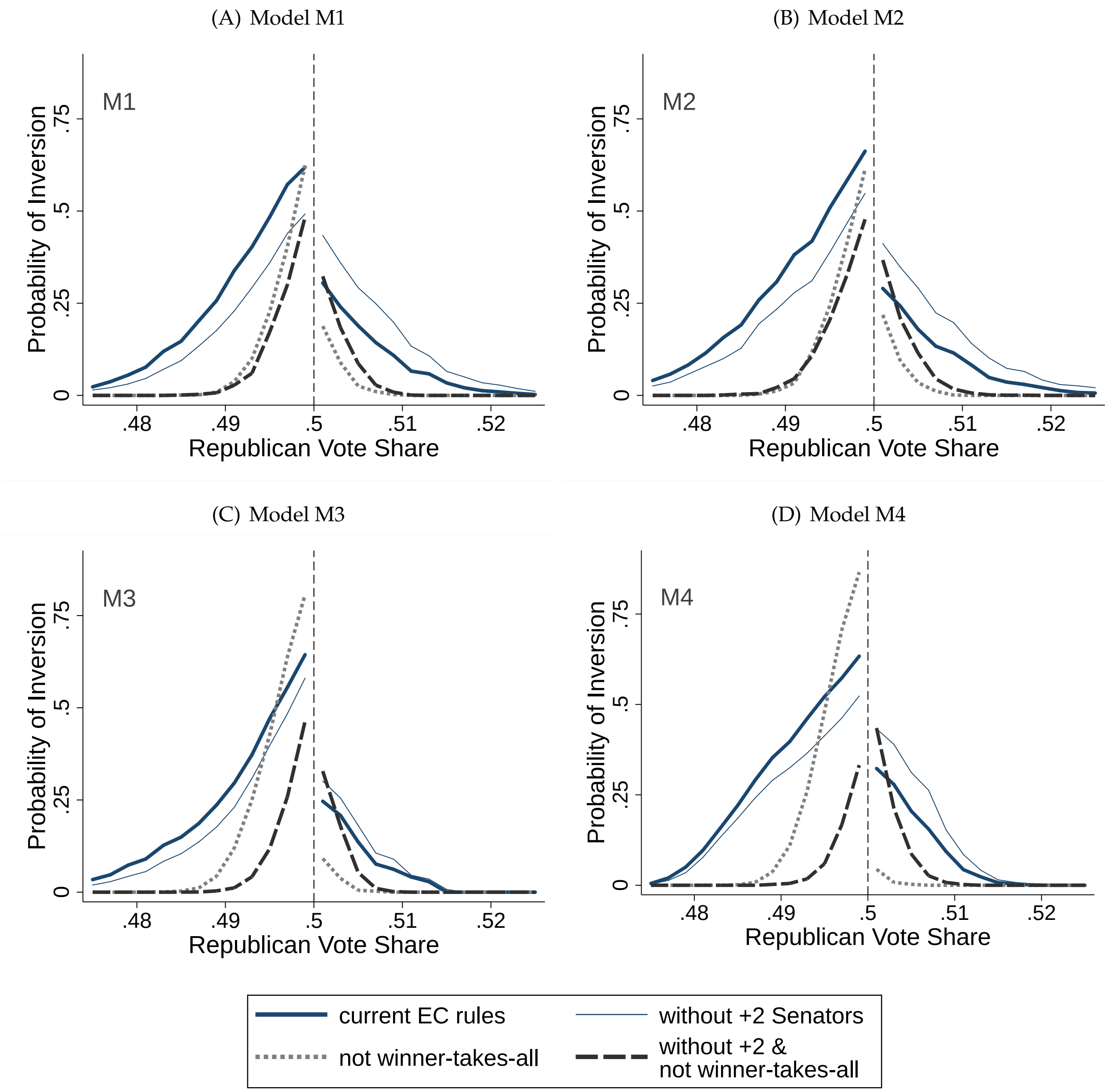
Note: Figure shows statistics and distributions under various model restrictions and estimation methods. All models are for the Modern sampling frame. Purple, brown and grey indicate results from parametric models, bootstrapped distributions, and the hyperparameter exercise, respectively. Radar plots in Panels C and D compare the within-year, across-state correlations between Michigan and 11 other swing states and between Texas and 11 other large, non-swing states.

Figure 5: Possible Sources of Inversion in the Electoral College



Note: Figure shows four features of the EC that could cause the victor in a Presidential contest to differ from the winner of the national popular vote. Panel A plots, for the four largest and four smallest states by today's population, the state's allotment of EC ballots divided by its population. Panel B plots the average vote margins over time by Democrat and Republican candidates for the largest states. Panel C focuses on reapportionment among several large states following the 1990 Census. Panel D highlights such geographic differences between turnout and apportionment-relevant population, with each marker corresponding to a state. The various horizontal lines in Panel D compare different election years as indicated.

Figure 6: Decomposing Impacts of the EC's Aggregation Rules



Note: "Without +2 Senators" allocates each state a number of electors equal to its number of US Representatives, without the two electors for its US Senators. Washington DC gets one elector ballot in this case. "Not winner-takes-all" divides the whole number of electors per state between parties. Model numbers correspond to Figure 2.

ONLINE APPENDIX for "Inversions in US Presidential Elections: 1836-2016"

by Geruso, Spears, and Talesara

A Background

A.1 Provisions of the Electoral College

The general provisions for the Electoral College system are established in Article One, Section 2 of the Constitution, though the particular method for determining the number of electors and allocating these across states has varied over time. EC electors are linked to Congressional apportionment, and so their number and geographic distribution has been affected by the various Apportionment Acts of Congress that have set the rules for allocating congressional seats within and across states over the past two centuries. In particular, the number of EC electors allocated to each state is equal to the number of voting members of the US House of Representatives plus two (for the two Senators of each state). Today there are 538 electors in total: 435 votes corresponding to US Representatives, 100 corresponding to US Senators, and three EC ballots for Washington DC. The present cap at 435 US Representatives and the method for apportionment of congressional seats across states was established by the Reapportionment Act of 1929.

A.2 The Party Systems in the 19th and 20th Centuries

Our earliest sampling frame consists of Antebellum elections from 1836 to 1852. This range includes all years in which Democrats and Whigs were the predominant political parties in national politics. Political scientists typically classify the range 1828 to 1854 as the Second Party System, and consider 1852 to be the last Presidential election year prior to the Civil War in which the parties were stable. We cannot use 1828 and 1832 in our two-major-party procedure, as these years contain Presidential candidates earning Electoral College votes from parties other than the Whigs and Democrats (National Republican, Nullifier, Anti-Masonic).

Our second sampling frame consists of the Reconstruction Era, 1876-1892. Political scientists typically classify the range 1854 to 1892 as the Third Party System. We drop the Civil War years and elections before 1876 as these were characterized by a multiplicity of competing parties that earned EC ballots as well as Republican landslide victories. There was also a changing roster of states gradually rejoining the union in this postwar period. We end our Reconstruction sample period at 1892 because the next election in 1896 represented a major political realignment. The realignment is typically recognized as the end of the Third Party System and the beginning of the Fourth Party System.

Other time periods are less useful in providing identifying variation in electoral outcomes. For example, the period 1896 to 1932, the so-called "Fourth Party System" mostly yielded Presidential landslide victories. These create degenerate distributions for the purposes of understanding inversion probabilities. We nonetheless present results for this period, see Appendix Section E.5 and Figure A9.

B Further Discussion of the Related Literature

Because of the considerable importance of the EC to US politics — as well as the importance of multi-tiered elections to democratic systems worldwide — the EC has received extended attention in the literature. However, because no prior study has investigated the same question we ask here, which is about the fundamental statistical nature of EC inversions, no prior paper has used the same materials and methods. Here we detail how our approach is distinguished from prior literature that: (i) studies empirical facts about the EC other than about inversions in the EC (B.1), or that (ii) studies properties of inversions other than their conditional and unconditional probabilities across historical periods (B.4).

In one striking example of the richness of the EC literature, political scientists and historians have even debated which elections should count as an inversion — a debate that is possible because of the complexity, and therefore ambiguity, of the implementation of the EC in practice across states, parties, and centuries (Kallina, 1985; Rakove, 2004; Estes, 2011). Gaines (2001), for example, argues that the 1960 election should be counted as an inversion because over 175,000 popular votes in Alabama (a number in excess of Kennedy's national popular vote margin of victory) were for Democratic electors who were opposed to Kennedy.

A more recent literature considers potential advantages and disadvantages of a national popular vote compact (DeWitt and Schwartz, 2016; Koza, 2016; de Mouzon et al., 2019). Because these studies often either take a normative or legal focus or do not use empirical data, and because they consider aspects of Presidential elections other than the probability of inversion (such as the probability or difficulty of a recount, or incentives for strategic voting), we do not consider them further here.

B.1 Empirical facts about the EC, but not about inversions

One of the oldest empirical literatures about the EC documents empirical facts about the distribution of electoral votes across states. In particular, much of this literature describes the allocation of average electoral influence (in the sense of EC ballots per popular vote or EC ballots per person) across states or across population groups. For example, Blair (1979) computes that, by such metrics, whites have more average voting power than blacks. Warf (2009) maps differences in average voting power across states.

Another category of descriptive analysis computes facts about average "voting power" in a way that is distinct from the mere probability of being pivotal, which is the focus of the next section. Banzhaf III (1968), for example, makes computations that compare the size of each state with its number of electoral votes, in order to compute a state-specific index of voting power. A follow-up literature has considered properties of Banzhaf's index and proposed alternatives (Owen, 1975; Dubey and Shapley, 1979).

B.2 Pivotal voters and the EC

A long literature in political science and economics considers the relative costs and benefits of voting, in particular focusing on the probability of being pivotal in deciding the election (Riker and Ordeshook, 1968; Gelman, Katz and Tuerlinckx, 2002). Several papers have applied these ideas to the Electoral College context, including Gelman, King and Boscardin (1998); Gelman, Silver and Edlin (2012); Miller (2013). Our paper is not concerned with the probability that a voter, or a voter in a particular state, or a voter in an election of particular closeness, will be pivotal.

B.3 Inversion analyses using "uniform partisan swing" method

Uniform partisan swing analysis—originating in Butler (1951) and Gudgin and Taylor (1979)—has become a standard tool for understanding the relationship between electorate votes and election outcomes, such as congressional seats. The method takes an observed election outcome and, in the classic application, "swings" all legislative districts by the same common vote share. By varying the vote share in a deterministic way in small increments, the method can trace when seats flip, and so can trace the relationship between swings in the common, across-district component of votes and the aggregated election outcomes. Primarily applied to estimating seats-votes curves in legislative elections such as for the US Congress (e.g., in Gelman and King, 1990), the method has been ported to analyzing EC. In particular, several studies map the relationship between electorate votes and EC ballots (Garand and Parent, 1991; Miller, 2012). The important differentiator of our study is the incorporation of uncertainty. In uniform swing analysis, there is no probability distribution over the aggregate vote share. In addition, there is no uncertainty in the way that contests across states (or legislative districts) resolve differently. They are assumed to commove perfectly. Therefore, these studies—which do not estimate probability distributions—do not address the goals of our paper, which are the computation of a set of important conditional and unconditional probabilities.

B.4 Inversions: Theoretical computations and election-specific predictions

Our paper uses data from many elections in the 19th through 21st centuries to estimate the unconditional and conditional probability of an inversion, abstracting away from the features of any particular pair of parties or candidates. The wide set of methods that we employ has not previously been applied to this question, and no prior set of estimates of these probabilities exists in the literature.

One of the richest existing literatures about the EC, from the game theory and formal political science literatures, theoretically computes the probability of inversions in mathematical models that abstract away from any data about the actual EC (Kikuchi, 2017; de Mouzon et al., 2018). Many of these papers, like ours, are focused on the stochastic properties of electoral systems, but unlike ours they are not grounded in voting data—for example, how partisan alignment and voting patterns in New York differ from those in Texas.

Another set of papers considers the probability of an inversion in one or more particular elections. Here, we have been able to build upon the methods of prior studies focused on single-election predictions or postdictions. For example, our M1 model is structurally analogous to the model that Katz, Gelman and King (2004) apply to specific years,[11] and our M2 model is similar to the model that Silver (2016) used to predict the distribution of potential outcomes prior to the 2016 Presidential election. Some papers in this election-specific literature consider counterfactual policies, as in our Figure 3, but without a probabilistic approach. Cervas and Grofman (2019), for example, apply a set of counterfactuals to determine whether they would have yielded an inversion in several actual historical inversions, assuming that vote totals were the same as what historically occurred.

Among the literature that considers the statistical properties of Presidential elections in particular time periods, two of the papers closest to ours in methodology are Merrill (1978) and Ball and Leuthold (1991), which are in dialogue with one another. Neither paper computes or discusses the probability of a close election, which plays a central role in our analysis. The interpretation of these papers is somewhat limited by the details and specificity of their modeling choices. Their sample selection differs from ours and from one another: where Merrill, in the mathematics literature, pools elections from 1900 to 1976 (which ignores the mid-20th-century partisan realignment, and therefore ignores the fact that a vote for a Democrat in the time of Wilson had different economic and racial correlates than a vote for a Democrat in the time of Carter), Ball and Leuthold (1991) (like Katz, Gelman and King, 2004) compute statistics for each of a series of years from 1920 to 1984, but also pool problematically across distinct periods of partisan realignment (e.g., their 1984 estimates pool data from 1944 to 1984). Methodologically, each paper makes analytic computations, assuming a single parametric form which specifies that each state shares the same distribution: a symmetric normal distribution in the case of

[11] Thomas et al. (2013) use essentially the same model as Katz, Gelman and King (2004) to estimate partisan bias in the EC in 14 specific elections, but do not estimate the probability of inversions. Partisan bias is indeed important to quantify, but is distinct from the probability of an inversion: for example, a two-tiered system that added high-variance, mean-zero noise to election outcomes would generate zero *ex ante* partisan bias, but would yield a high probability of inversion.

Merrill (1978) and a parameterized beta distribution in the case of Ball and Leuthold. Neither paper explores robustness to these assumptions—Ball and Leuthold suggest that a non-parametric approach, such as we use in M3 and M4, would be "difficult to conceptualize." Despite these limitations, these papers are important for their early anticipation that state-indexed models could be used to describe statistical properties of Presidential elections.

C Data and Estimation

C.1 Data Sources

For data on state populations, we use IPUMS extracts from decennial Censuses (Ruggles et al., 2018). For intercensal election years, we follow the standard practice of exponentially interpolating state populations. Where needed, we convert whole populations to apportionment-relevant populations using the appropriate period-specific apportionment algorithm—most notably, applying the three-fifths rule to black populations prior to 1868.

The key inputs to our analysis are the historical election returns by state for each Presidential election year. For each Presidential election, we assemble data on vote tallies for each candidate in each state, as well as data on EC elector ballots cast for each candidate by the EC delegation from each state. Data on state-level election returns and on EC ballots cast come from the Leip (2018) compilation of state returns. We check these data against Federal Election Commission records. In the few state $\times$ election year instances of disagreement, we rely on state government election records, where available. Further details on election data cleaning are documented below.

For most simulations, we retain information on only the two major parties—Democrats and Whigs from 1836 to 1852 and Democrats and Republicans for the later periods we examine. This normalization, which is standard in the literature (see, e.g., Gelman and King, 1994) does not substantively impact our analysis of inversion probabilities, as third party candidates won no EC ballots over our study periods.[12,13] Of course, a third party candidate could be pivotal in determining which of the two major party candidate wins a state $\times$ year. The building blocks of our estimation and Monte Carlo exercise are actual state $\times$ year election outcomes. We primarily take these outcomes as basic data and make no assumptions on how a state return might have differed if not for a third party candidate. Thus, most of our statistics describe a typical election outcome over our sample period, rather than elections in which we remove the influence of third parties. However, in Figure A1 we assess sensitivity to two extreme and opposite assumptions on the impact of third parties. First we reestimate our baseline model reassigning all third-party votes in each state $\times$ year to the Democratic candidate. Then we reestimate our baseline model reassigning all third-party votes in each state $\times$ year to the Republican candidate.

C.2 Data Cleaning and Restrictions

Here we catalogue our handling of various special cases and anomalies that arise in the election data:

- We ignore the few historical instances of faithless electors, who cast EC ballots for a candidate other than the candidate to whom they were pledged. In the cases of faithless electors, we award

[12] In particular, one could relabel the horizontal axes in our figures below to center on the state $\times$ year specific threshold, with ticks on the axis indicating distance from that state-election-specific threshold. In the Florida 2000 example, Bush and Gore won 48.847% and 48.838% of votes respectively, with 2.315% going to other candidates. The state $\times$ year specific threshold for a Republican victory in this case would be 0.488425 ($= (1 - 0.02315)/2$).

[13] The last third party candidate to win a single pledged Electoral College vote was Wallace in 1968, which predates our modern sampling frame. The Antebellum and Reconstruction Era elections produced no third party EC ballots, other than via faithless electors.

Electoral College ballots as they would have been awarded based on state popular vote results.

- We exclude third parties from our analysis. No third-party candidate won EC ballots in any of the election years we use for sampling (except through faithless electors), so excluding third parties has no effect on electoral outcomes. However, when we scale popular vote outcomes by turnout, we include third party voters in our measure of total turnout. Our results are robust to excluding third party voters from total turnout.
- In each election year, we drop states where EC ballots were allocated by state legislatures rather than by the state popular vote. This includes South Carolina in 1836 to 1852 and Colorado in 1876.
- In 1836, the Whig party ran multiple candidates across the country. All states which held a citizen vote for President (as opposed to awarding EC ballots via the state legislature) had one Whig candidate on their ballot, and no states had more than one. We treat all Whig candidates as one candidate in the 1836 election.
- We start the reconstruction era in 1872 because in 1868, not all of the Confederate states had rejoined the Union.
- In 1872, Horace Greeley, the Democratic candidate, died after the general election but before electors had formally cast votes. Officially, no EC ballots were cast for Greeley. We use popular vote data from the general election, and award EC ballots as if Greeley had not died.
- In 1872, the electors of Arkansas and Louisiana were not certified by Congress. We use the popular vote outcomes in these states in 1872 to award their EC ballots to Ulysses S. Grant.
- We end the reconstruction era in 1888 because there was a major third party in 1892 (Populists). Additionally, not all states had both major party candidates on their ballots.
- In the modern era, Maine (since 1972) and Nebraska (since 1992) have split their EC ballots between the state popular vote winner and congressional district popular vote winners. In practice, both states have only split their EC ballots once each. We ignore this rule and allocate Maine and Nebraska's electoral votes by a winner-takes-all rule. Our results are robust to using their method of splitting votes.

In a few instances, the Leip tallies of individual citizen votes do not align with the Federal Election Commission tallies. We investigate these cases and use states' election commissions to resolve the disputed numbers. The differences are often small, within a mere handful of votes. The cases for which we update Leip's tallies on the basis of state election commission records are:

- 2016 California
- 2016 Minnesota
- 2016 New York
- 2016 Ohio
- 2012 North Dakota
- 2012 Ohio
- 2004 Rhode Island

- 1996 South Carolina
- 1992 Nebraska
- 1988 Louisiana
- 1988 Massachusetts
- 1988 Nebraska

C.3 Sampling Frames

Figure 1 of the main text indicates the periods in US history that we study. Political scientists have identified several stable Party Systems, characterized by competition between a fixed pair of parties with stable political properties.[14] We take these groupings as a starting point for our sample definitions. We further restrict attention to spans of years that include electoral victories for both parties because consecutive landslide victories of a single party do not generate useful variation for our purposes of studying inversion probabilities in close elections. Given these criteria, we study the Second, Third, and Sixth Party Systems, corresponding to the Antebellum, Reconstruction, and Modern eras, as indicated in the figure.

Our earliest study period consists of elections between 1836 and 1852. This range includes all years in which Democrats and Whigs were the predominate political parties in national politics, and spans through the last Presidential election year prior to the Civil War in which the parties were stable. In the Reconstruction Era, we study years 1872-1888. Like today, the parties during this period were Republicans and Democrats, though the political alignment of states was rather different, with Democrats dominant in the Southeastern US, and Republicans dominant in the North, West and Mid-West.[15]

Finally, we treat 1988 to 2016 as our baseline modern period, although model M12 in Figure 2 demonstrates that our estimates of the probability of an inversion conditional on a close election are robust to extending the modern sampling frame further back to the 1960s.

We do not additionally focus on elections between 1900 and 1960 because over this period there was little usable variation for our purposes. With the exception of Woodrow Wilson's terms, Republicans won landslide victories from 1900 to 1928. This was followed by consecutive Democratic landslide victories (four of them by Franklin Roosevelt) beginning in 1932, and then Republican landslide victories again in the 1950s. Sampling or estimating from periods of consecutive landslide victories of one party generates landslide counterfactuals, leading to degenerate distributions with little to no probability density around the 50% national popular vote share, which is our threshold of interest. Nonetheless, in Appendix E.5, we show results for the 1900-1960 period for completeness.

As an initial indication of the plausibility of our results, Figure 1 highlights the four inversions that have occurred. The figure shows that during periods of close elections, inversions have been relatively likely. Two of the five elections in the Reconstruction era were inversions, as well as two of the four closest elections in the Modern period. For both periods in which inversions have occurred, we present results that exclude inversion years from our sampling frames.

[14] See Appendix A.2 for further discussion of how our sampling frames align with conventional treatments of the historical US party systems in political science.

[15] Although the Third Party System includes the 1892 election, we exclude it from our analysis as the two major parties were not on the ballot in every state.

D Additional Details on Methods

D.1 Parametric Analysis

Table A1 reports the maximum likelihood estimates of the parameters in Equation [1] of the main text. Model numbers (such as M1) correspond to Figure 2 in the main text. Estimates in the table are grouped by period. Within each period, the first model (M1) corresponds to the baseline, preferred estimate, following the Gelman and King (1994) "unified method of evaluating electoral systems." It includes a national shock and independent state shocks, with state shocks drawn from a common distribution. The parameters of particular interest are the variances of the national and state shocks, σ_γ^2 and σ_ϕ^2. Either 31, 38, or 51 expected state vote share parameters, $\overline{\alpha}_s$, are also estimated, depending on the data period.

Other columns compute estimates for alternative samples or model restrictions. Column 2 (M2) estimates additional state covariance terms on the basis of geographic region, race, and education. States within a region receive a common, independent shock. The race term multiplies the fraction of each state that is nonwhite by a random, common coefficient drawn from a mean-zero t distribution. The education term multiples the fraction of each state's adult population that is college-educated by a random, common coefficient drawn from a mean-zero t distribution. Thus, these two shocks allow nationally correlated dispersion by education and race. Data on these demographics come from published, state-level summary statistics of the American Community Survey. The M2 model closely follows the Silver (2016) approach to modeling uncertainty in election forecasting.[16] The next column (M5) estimates the model assuming no national shocks, counter to the stylized facts from the elections literature about the importance of a common, national component to the uncertainty. Columns 7 through 9 add race and education covariance terms singly and together. M10 drops from the sampling frame the two historical instances of inversions in 2000 and 2016. The last model in the modern period (M12) extends the sample to 1964, which walks the data period backward to the partisan realignment of the North and South in the early 1960s (Kuziemko and Washington, 2018). Additional columns repeat these estimates for the Antebellum and Reconstruction eras. Model R10 drops the inversion instances (1876 and 1888) from the Reconstruction period; there were no inversions in the Antebellum period. The requisite data for estimating the demographic covariance terms (models M7, M8, M9) exist only for the Modern period.

D.2 Bootstrap Monte Carlo: Scaling Turnout

One practical consideration that arises when sampling state election outcomes from different years is that the raw vote counts of later years tend to be larger, reflecting population growth. This creates a problem when summing citizen votes across states to yield a national popular vote. We address this by scaling the each party's vote tally in a state by that state's turnout in some common reference year before summing across states. For example, we use 2016 turnout for the modern period. In practice, using alternative reference years for the turnout weights make almost no difference to the simulation results.

Another complication is that EC apportionment across the states varies across election years. In our earliest historical period, the Union itself was changing: There were were 25 states in 1836, with 6 new states joining by 1852. Therefore, when performing Monte Carlo simulations, for each iteration t,

[16] FiveThirtyEight's probability distributions over elections account for three potential types of error and uncertainty, relative to the best mean predicted vote share in each state—a common national error, a set of demographic and regional errors, and, independent state-specific errors. For demographic and regional errors: "The following characteristics are considered in the simulations: religion (Catholic, mainline Protestant, evangelical, Mormon, other, none); race (white, black, Hispanic, Asian, other); region (Northeast, South, Midwest, West); party (Democrat, Republican, independent); and education (college graduate or not)." (Silver, 2016)

we first draw an election year from the period (independent, uniform probability, with replacement). We then sample from states present in that year and set the count of EC electors allocated to each state according to the allocation in that year and set the turnout equal to the turnout in that year.

D.3 Bootstrap Monte Carlo: Generating Correlation Between State Outcomes

A downside of the independent sampling in our baseline bootstrap is that the lack of a common election-year component to the variation leads to under-dispersion relative to the actual span of election outcomes. To better capture the fact that national sentiment (or the characteristics of a particular pair of candidates) tends to moves states together in a given election year, we generate a variant in which we sample state outcomes with probability weights that attach extra probability mass to being drawn from the same election year. In particular, for each simulation we first draw a focal year, y^*, uniformly, independently, with replacement. Let M denote the probability that the outcome from the y^* election is sampled for each state in a given simulated national election t. Increasing M increases the within-year, across-state correlation in voting patterns without imposing parametric assumptions on the distribution of the shocks.

This addition to the bootstrap procedure brings the dispersion closer to the actual dispersion of observed elections. For Figure 3, we set the probability of drawing from the same focal year at 50%. In Figure 4, we vary M in 0.05 steps from 0.15 to 0.50. In our baseline sample, which contains 8 elections from 1988 to 2016, an equal-probability draw would be 0.125 weight on each year.

D.4 Bootstrap Monte Carlo: Wild Pooled Error Sampling

Models M6, R6, A6 create a larger pool of empirical error terms for bootstrap drawing, consisting of all state deviations from their period means over all elections in the period. Each state is first assigned its empirical sample-period mean two party vote share. Then for each state, there is an independent wild bootstrap draw from this common pool, so that the ϕ_{st} term is drawn identically across states. The "wild" here is in the sense of Cameron, Gelbach and Miller (2008). It refers to multiplying each draw by a random 1 or -1, effectively doubling the sampling frame and imposing symmetry on the empirical distribution.

D.5 Bootstrap Monte Carlo: Swing State and Safe State Correlations

In Section 4.3 we discuss how in Figure 4 we include models that vary the implied correlation structure of the bootstrap procedure, tuning the probability that state draws come from the same election in 5% steps from 15% up to 50%. We do this overall, and also for swing states separately and "safe" states separately. We define swing states following recent convention: Colorado, Florida, Iowa, Michigan, Minnesota, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Virginia, and Wisconsin.[17] If, for example, the 1992 outcome is drawn with excess probability mass for Colorado, then the 1992 outcomes are also drawn with the same excess mass for the other 11 swing states. In this approach, counterfactual elections necessarily come closer the true equilibrium processes by which campaigns are making joint decisions on allocating investments across swing states as they anticipate factors like voter responsiveness to advertising and candidate visits. The primary source of variation in this set of simulations is the margin by which reliably red or reliably blue states are won (according to state-specific historical variability).

[17] *Politico* published a swing state list leading up to the 2016 election that included: Colorado, Florida, Iowa, Michigan, Nevada, New Hampshire, North Carolina, Ohio, Pennsylvania, Virginia and Wisconsin. FiveThirtyEight adds Minnesota to this list to generate a list of "traditional swing states."

Conversely, when we sample "safe" states from the same election with excess probability, we primarily vary the state victor and the margin by which swing states and other potentially contestable states are won. Safe states, in the context of this analysis include the top quintile of states (12) in terms of the average Democrat or Republic margin of the victory over the sample period.

E Supplementary Results and Robustness

E.1 Asymmetry in Inversion Probabilities

Tables A2 reports, conditional on an inversion occurring, which party was likely to have won the EC (and to have lost the popular vote). Table A3 reports the probability that an inversion accounts for the expected wins of each party. All models agree that for the modern period, inversions favor Republicans. Across all 12 Modern Era models, the probability that an inversion is won by a Republican ranges from 69% to 93%. For the Reconstruction period, Democrats were favored in inversions according to the standard, parametric models—though models based on bootstrap draws disagree. In the Antebellum period, there is no consensus across models as to whether Whigs or Democrats were favored.

E.2 Robustness to Gridded Parameter Values

With only a few elections per party system, it is impossible to be confident that estimates of the true parameter values underlying the data generating process are precise. To examine the extent to which our main results could be sensitive to errors in these estimates, we calculate our outcomes of interest under a set of exogenously-specified variances and correlations. In these simulations, we take only the state historical means of vote shares as data. Uncertainty around these means is assumed to follow $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$ as in our baseline models (M1, R1, A1). But here, we cycle over a grid of values for σ^2_γ and σ^2_ϕ, rather than relying on estimates.

Figure A2, which is described in the main text, presents results from this procedure. The procedure generates 96 unique, assumed parameter combinations in each period. In addition to iterating over national and state variances, each combination is used while including or omitting a shared shock by geographic region. The implied election outcome distributions range from an underdispersed minimum standard deviation of 0.53 popular vote percentage points to an overdispersed maximum standard deviation of 7.16 percentage points.

In Figure A3, we present supplementary detail for a subset of the assumed parameter combinations in Figure A2. These simulations include state and national shocks. The variance of the national shock increases along the horizontal axis in each panel. The variance of the state shocks are traced in several contour lines in each panel, as indicated. In the panels on the left, we report the probability of close elections within a 2 percentage point margin. In the panels on the right, we report inversion probabilities, conditional on close elections within the same margins.

The slopes of contours in Panels A, C and E indicate that the probability of a close election outcome is sensitive in each period to the gridded parameter values. In particular it is sensitive to the variance of the common, national shock. However, in all cases the inversion probabilities (Panels B, D, and F) remain high. In the modern period, the probability of an inversion—conditional on a margin less than 2 percent—never drops below 35%, regardless of the parameters exogenously set. The graph thus traces the same lower envelope on inversion probabilities for the Modern period as Figure 3.

We can summarize Figures 3, 4, A2, and A3 as indicating that our finding of high inversion probabilities in close elections is robust to: (i) parametric approaches that vary the assumptions on the DGP across those adopted by the political science literature and election forecasting practitioners, (ii) non-parametric bootstrap approaches that include both independent and highly-correlated sampling of state outcomes, (iii) approaches that omit from estimation or bootstrap sampling the actual historical instances of electoral inversions, and (iv) searching over a wide grid of potential parameters, including parameters that are likely to be outside of the true parameter space.

E.3 Results Allowing for State Characteristics to Shift Expected Vote Shares

As we describe in Section 3, we model the data generating process for a state-by-election-year (*st*) outcome as consisting of a state expectation, $\overline{\alpha_s}$, and a mean-zero shock (ϵ_{st}), which may be correlated across states:

$$\begin{aligned} V_{st} &= \overline{\alpha_s} + \epsilon_{st} \\ \epsilon_{st} &= \gamma_t + \phi_{st} + \mathbf{X}_s \delta_t \end{aligned} \tag{2}$$

We view the important unknown parameters here to be the mean-zero shocks, not the state-specific $\overline{\alpha_s}$ terms that describe the expected vote share in each state. This reflects our interest in understanding how past elections might have unfolded differently, rather than how we expect a future election to unfold, if circumstances (such as demographics) change.

Nonetheless, for completeness, we here examine robustness to allowing for state characteristics to shift state vote share expectations. We estimate:

$$\begin{aligned} V_{st} &= \mu_s + \mathbf{X}_{st} \boldsymbol{\beta} + \epsilon_{st} \\ \epsilon_{st} &= \gamma_t + \phi_{st} \end{aligned} \tag{3}$$

This specification adds a vector of coefficients $\boldsymbol{\beta}$ that multiplies $\mathbf{X}$. The characteristics included in $\mathbf{X}$ in (3) are education (as %college graduates) and race (as %non-white). These vary at the state $\times$ election year level. Figure A7 presents the results overlaid with model M1.

E.4 Within-Year, Across State Shocks Linked by State Characteristics

In Section, 4.3, we note that because we show that models with shocks linked by election year, region, racial composition, and educational characteristics produce similar inversion probabilities to models that assume that state shocks are completely independent, it is unlikely that smaller tweaks will affect our main findings. Here we demonstrate this.

For the model plotted in Figure A8, we allow for race-linked shocks to multiply an $\mathbf{X}$ vector that includes region indicators, % non-hispanic white, % non-hispanic black, % hispanic, % college degree, and % high school completion in the state. This contrasts with M2, where $\mathbf{X}$ includes only % non-white and % college degree. In the figure we overlay a plot of this more flexible model with M2. The two are statistically indistinguishable in terms of the conditional probability of an inversion they imply (right panel).

E.5 Results for the Fourth and Fifth Party Systems (1896-1960)

Our main analysis samples do not include elections in the first half of the twentieth century, which was characterized by landslide victories for both Democrats and Republicans. For completeness, we estimate inversion probabilities for this time period here. We divide the timeframe according to a standard typology of party systems. We analyze separately elections in the Fourth Party System (1896–1932) and the Fifth Party System (1936–1956). For the Fifth Party System, we do not include 1960, because doing so would add the complication that it would be the *only* election in this span during which Alaska or Hawaii were states. For the Fourth Party system, we begin in 1916 in order to generate a stable set of states over the sample period, and we drop 1912 and 1924 because in each of these election years, a third party won EC ballots.

Figure A9 presents results for the $x1$ and $x2$ class of models over the Fourth and Fifth Party System periods. These models apply the same structural assumptions and estimation procedures used for M1 and M2 in the Modern period (see Figure 3). The characteristic $\text{Win}(\text{NPV})$ curves are similar to other periods. Further, the ex ante probabilities of an inversion in a close election are high in these models. An important difference between these results and results from the Antebellum, Reconstruction, and Modern periods is that the probability of a close election was much lower, making the unconditional probability of an inversion lower.

Table A1: ML Parameter Estimates for Variance Terms

	Modern Period: 1988-2016								Reconstruction: 1872-1888				Antebellum: 1836-1852		
	State and National Shocks M1 (1)	State, National, & Other Shocks M2 (2)	State Shocks Only M5 (3)	State, National, & Other Shocks M7 (4)	State, National, & Other Shocks M8 (5)	State, National, & Other Shocks M9 (6)	Omitting Inversions 2000 & 2016 M10 (7)	Extended M1: 1964-2016 M12 (8)	State and National Shocks R1 (9)	State, National, & Other Shocks R2 (10)	State Shocks Only R5 (11)	Omitting Inversions 1876 & 1888 R10 (12)	State and National Shocks A1 (13)	State, National, & Other Shocks A2 (14)	State Shocks Only A5 (15)
σ_y	0.012	0.012		0.012	0.012	0.012	0.014	0.053	0.018	0.003		0.018	0.013	0.011	
σ_s	0.027	0.016	0.039	0.023	0.021	0.019	0.022	0.051	0.123	0.075	0.140	0.131	0.015	0.018	0.032
Other shocks															
region		0.005								0.051				0.002	
race		0.183		0.154		0.125									
education		0.568			1.090	0.908									
# of α_s estimates (states)	51	51	51	51	51	51	51	51	38	38	38	38	31	31	31

Note: Table reports maximum likelihood estimates of the parameters in Equation [1] of the main text under various model specifications, as indicated in the column headers. Model estimates in the table are grouped by period. Either 25 (because we only use states that were present throughout the Antebellum period for estimating variances), 37, or 51 expected state vote share parameters, $\overline{\alpha}_s$, are also estimated by joint maximum likelihood but not reported here. Depending on the data period, some states were not present for all election years within the sample frame. These, including Colorado in the Reconstruction frame, are simply assigned their mean for $\overline{\alpha}_s$.

Table A2: Asymmetry: Who Wins from an Inversion?

	Model	Probability of Republican/Whig Win, Conditional on Inversion Occurring					
		Conditional on Margin of Victory					Any
		<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin
Republicans and Democrats	M1	0.68	0.70	0.73	0.75	0.76	0.77
	M2	0.70	0.71	0.74	0.76	0.77	0.78
	M3	0.79	0.84	0.90	0.92	0.92	0.93
	M4	0.68	0.71	0.75	0.80	0.82	0.82
	M5	0.66	0.72	0.80	0.84	0.85	0.85
	M6	0.69	0.74	0.82	0.85	0.86	0.87
	M7	0.68	0.71	0.74	0.76	0.77	0.78
	M8	0.71	0.72	0.76	0.78	0.79	0.79
	M9	0.71	0.73	0.76	0.78	0.79	0.80
	M10	0.62	0.63	0.66	0.68	0.68	0.69
	M11	0.75	0.81	0.88	0.90	0.90	0.90
	M12	0.61	0.60	0.61	0.61	0.62	0.62
Republicans and Democrats	R1	0.36	0.34	0.31	0.30	0.28	0.27
	R2	0.39	0.39	0.38	0.36	0.36	0.33
	R3	0.77	0.75	0.73	0.71	0.70	0.70
	R4	0.87	0.88	0.90	0.90	0.90	0.90
	R5	0.36	0.33	0.30	0.28	0.26	0.24
	R6	0.45	0.43	0.40	0.37	0.36	0.34
	R10	0.50	0.48	0.46	0.45	0.44	0.43
Whigs and Democrats	A1	0.51	0.51	0.51	0.51	0.51	0.51
	A2	0.32	0.31	0.29	0.27	0.26	0.25
	A3	0.48	0.51	0.54	0.51	0.44	0.37
	A4	0.41	0.41	0.42	0.39	0.36	0.31
	A5	0.51	0.51	0.52	0.52	0.51	0.51
	A6	0.49	0.49	0.49	0.48	0.48	0.48

Note: Table reports statistics describing the probability that inversions were won by the index party (Republican/Whig). Columns condition on various two-party popular vote share margins. Statistics for Democrats are one minus the indicated value in the table. Model estimates in the table are grouped by period: (M)odern, (R)econstruction, (A)ntebellum.

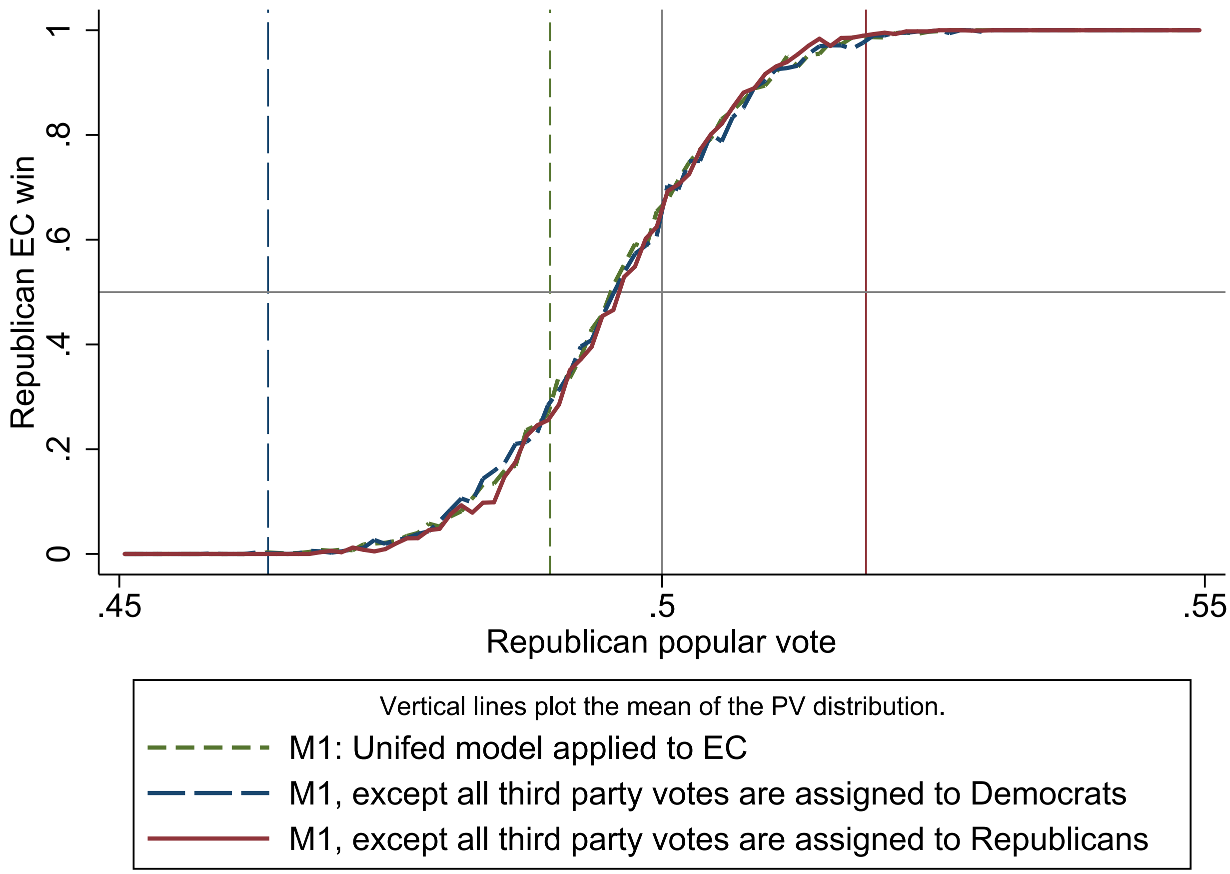
Table A3: Asymmetry: Probability that an Observed EC Win Was Caused by an Inversion

		Probability that Inversion Occurrs, Conditional on a Republican/Whig Win					
		Conditional on Margin of Victory					Any
	Model	<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin
Republicans and Democrats	M1	0.47	0.45	0.41	0.36	0.32	0.22
	M2	0.48	0.45	0.41	0.37	0.34	0.21
	M3	0.54	0.59	0.66	0.70	0.71	0.72
	M4	0.53	0.53	0.51	0.52	0.51	0.43
	M5	0.52	0.56	0.62	0.65	0.67	0.68
	M6	0.55	0.58	0.64	0.67	0.69	0.69
	M7	0.47	0.45	0.41	0.37	0.33	0.23
	M8	0.48	0.46	0.41	0.37	0.33	0.23
	M9	0.48	0.45	0.41	0.37	0.33	0.23
	M10	0.45	0.43	0.37	0.32	0.28	0.18
	M11	0.54	0.59	0.64	0.67	0.68	0.68
	M12	0.47	0.42	0.35	0.28	0.23	0.06
Republicans and Democrats	R1	0.45	0.40	0.32	0.25	0.20	0.06
	R2	0.46	0.43	0.36	0.30	0.26	0.08
	R3	0.43	0.36	0.25	0.19	0.15	0.12
	R4	0.46	0.47	0.44	0.41	0.38	0.28
	R5	0.44	0.39	0.30	0.23	0.18	0.09
	R6	0.45	0.40	0.31	0.25	0.21	0.12
	R10	0.45	0.41	0.33	0.26	0.21	0.06
Whigs and Democrats	A1	0.45	0.39	0.30	0.24	0.19	0.10
	A2	0.42	0.38	0.27	0.20	0.15	0.06
	A3	0.48	0.46	0.42	0.41	0.38	0.25
	A4	0.46	0.43	0.33	0.24	0.19	0.10
	A5	0.46	0.42	0.35	0.31	0.28	0.26
	A6	0.46	0.42	0.35	0.30	0.28	0.26

		Probability that Inversion Occurrs, Conditional on a Democrat Win					
		Conditional on Margin of Victory					Any
	Model	<0.5pp	<1pp	<2pp	<3pp	<4pp	Margin
Republicans and Democrats	M1	0.43	0.37	0.25	0.18	0.13	0.05
	M2	0.45	0.38	0.27	0.19	0.14	0.05
	M3	0.32	0.21	0.08	0.05	0.03	0.03
	M4	0.46	0.40	0.27	0.18	0.11	0.06
	M5	0.38	0.28	0.15	0.09	0.07	0.06
	M6	0.37	0.27	0.14	0.09	0.06	0.05
	M7	0.43	0.36	0.25	0.18	0.13	0.05
	M8	0.42	0.36	0.24	0.17	0.12	0.04
	M9	0.41	0.36	0.24	0.17	0.12	0.04
	M10	0.43	0.36	0.26	0.18	0.14	0.05
	M11	0.33	0.20	0.08	0.04	0.03	0.02
	M12	0.45	0.40	0.31	0.24	0.19	0.05
Republicans and Democrats	R1	0.47	0.46	0.42	0.39	0.35	0.21
	R2	0.50	0.46	0.42	0.39	0.36	0.20
	R3	0.47	0.46	0.42	0.41	0.40	0.40
	R4	0.49	0.42	0.31	0.24	0.22	0.21
	R5	0.49	0.49	0.47	0.45	0.44	0.40
	R6	0.50	0.49	0.48	0.47	0.45	0.43
	R10	0.48	0.45	0.40	0.36	0.32	0.19
Whigs and Democrats	A1	0.45	0.39	0.30	0.23	0.18	0.09
	A2	0.45	0.43	0.37	0.32	0.28	0.16
	A3	0.39	0.29	0.18	0.15	0.17	0.20
	A4	0.42	0.34	0.23	0.18	0.17	0.17
	A5	0.44	0.39	0.30	0.25	0.22	0.19
	A6	0.44	0.39	0.31	0.26	0.23	0.21

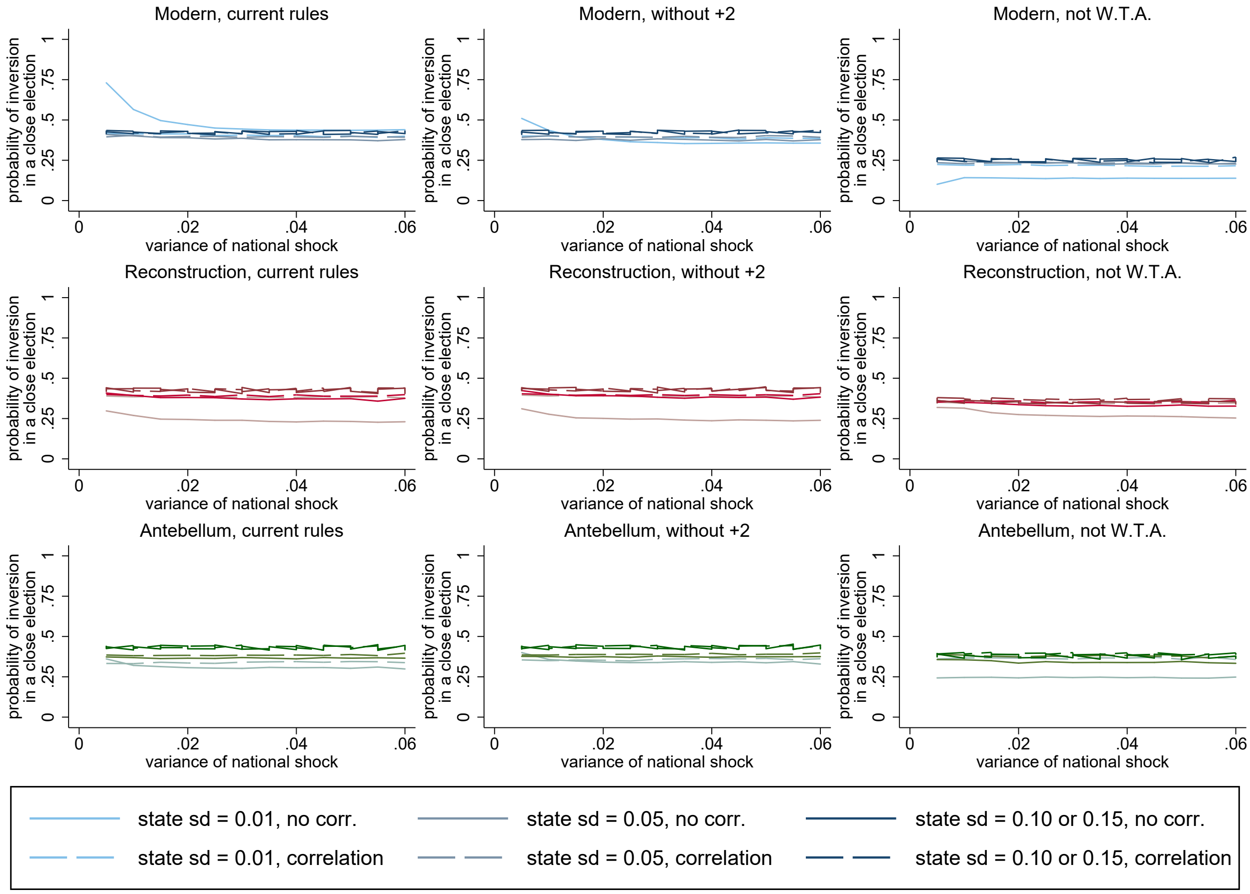
Note: Table reports statistics describing the probability of inversions, conditional on the Electoral College win going to the indicated party. Columns additionally condition on various two-party popular vote share margins. Model estimates in the table are grouped by period: (M)odern, (R)econstruction, (A)ntebellum.

Figure A1: Results Are Robust to the Full Range of Possible Consequences of Third-Party Votes



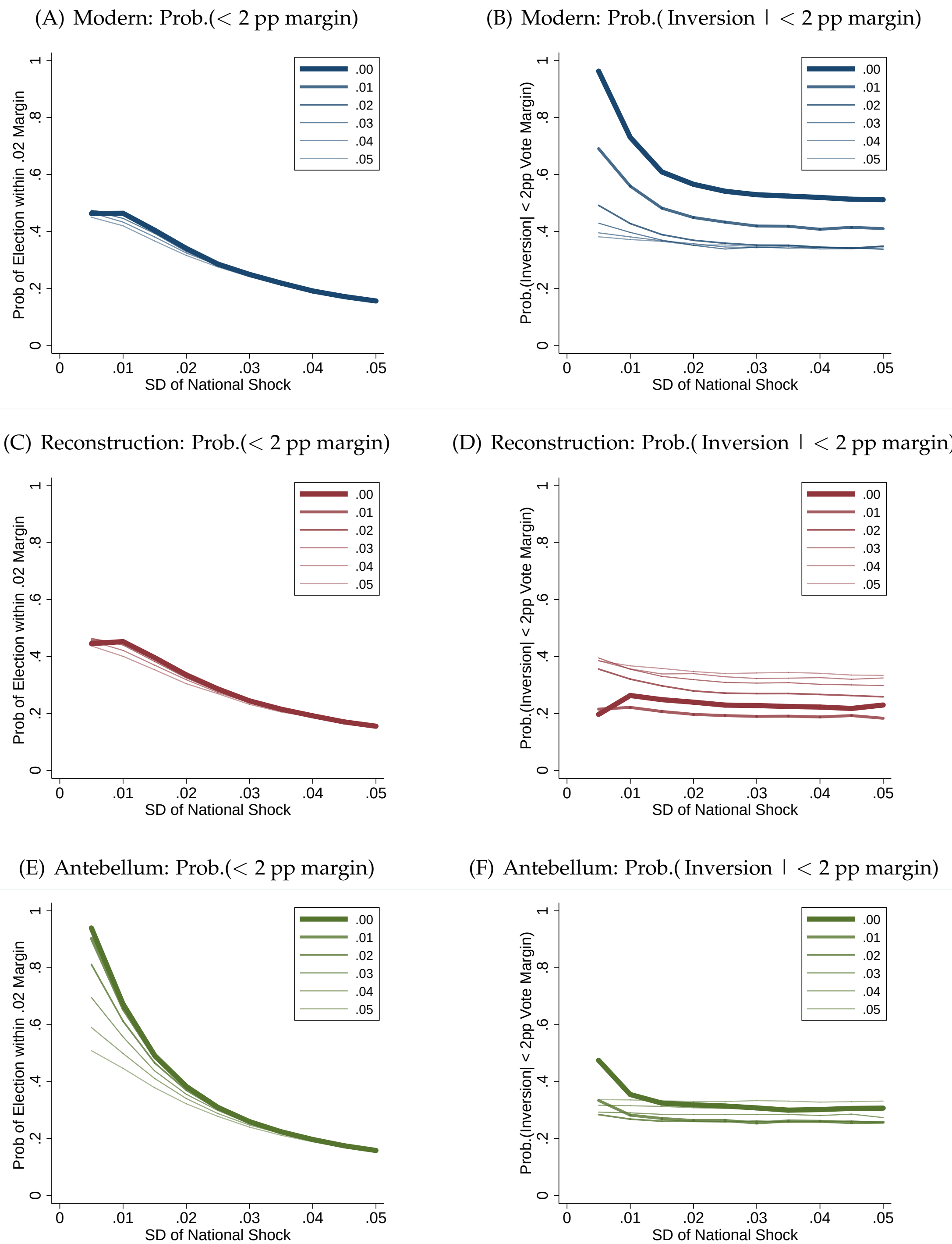
Note: This figure demonstrates robustness of our M1 estimates to extreme treatments of third-party votes. In particular, we present simulation results that use parameter estimates from estimating M1 after assigning all third-party votes to the Democratic candidate in that year, and after assigning all third-party votes to the Republican candidate in that year. As the vertical lines show, this counterfactual assignment makes a large difference to the central tendency of the distribution of popular votes. However, it does not change the object of interest: the conditional probability of winning the EC, as a function of the popular vote outcome.

Figure A2: Robustness: Iterating Over a Grid of Exogenously-Set Parameters



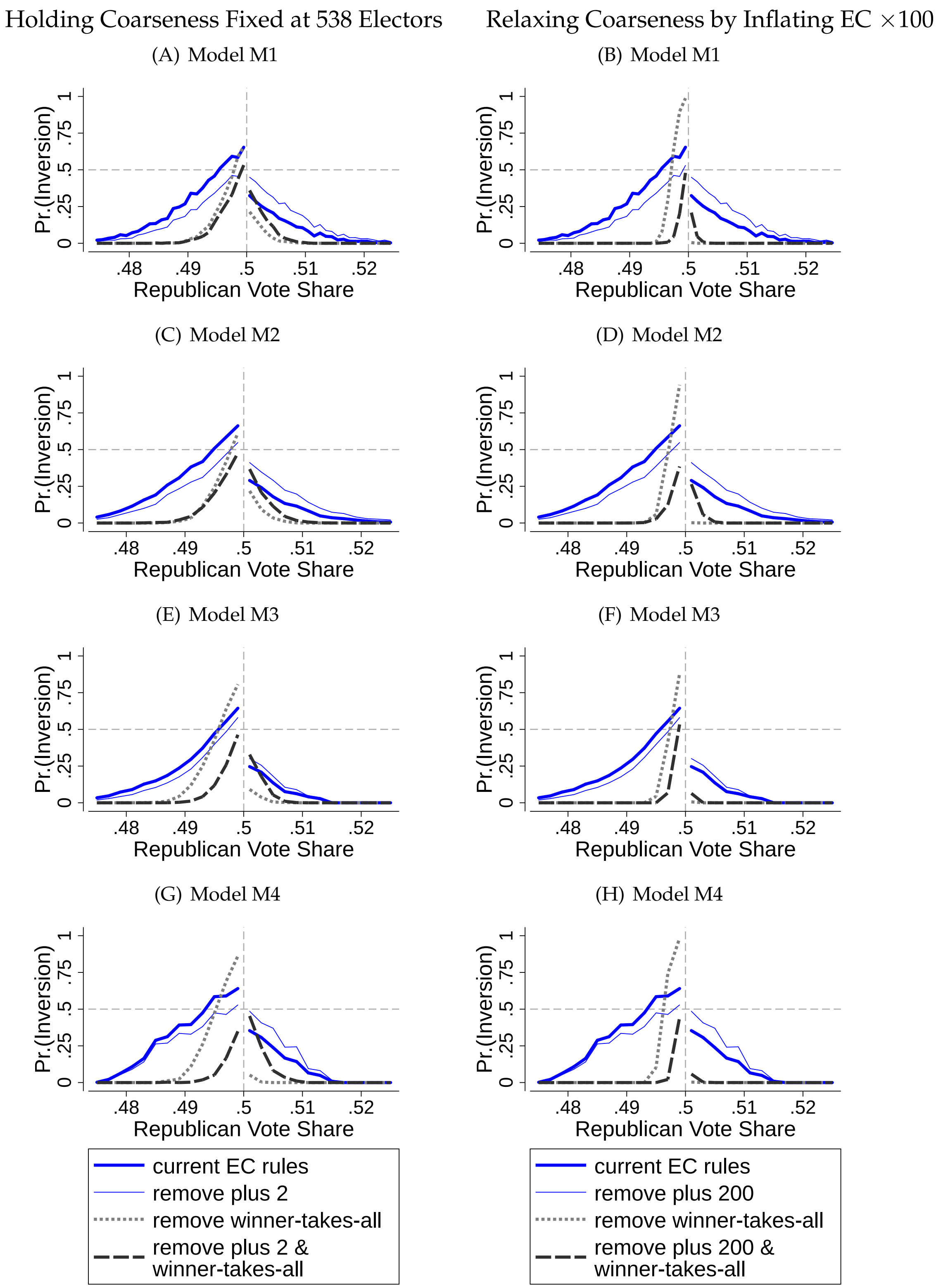
Note: Each panel plots the probability of an inversion conditional on a 1.55 percentage point popular vote margin or less (which corresponds to 2 million popular votes at 2016 turnout) for a set of additive logit models, each with: observed state means; a random national shock; random state-specific shocks at a standard deviation of 1, 5, 10, or 15 percentage points; and (for some models) random regionally-correlated shocks at a standard deviation of 5 percentage points (see Methods). Note that these 288 models use only state-specific means from past election data, and exogenously specify variances as hyperparameters. "Without +2" allocates each state a number of electors equal to its number of Representatives, without two electors for Senators. "Not W.T.A." divides the whole number of electors per state between parties.

Figure A3: Robustness: Iterating Over a Grid of Exogenously-Set Parameters (further detail)



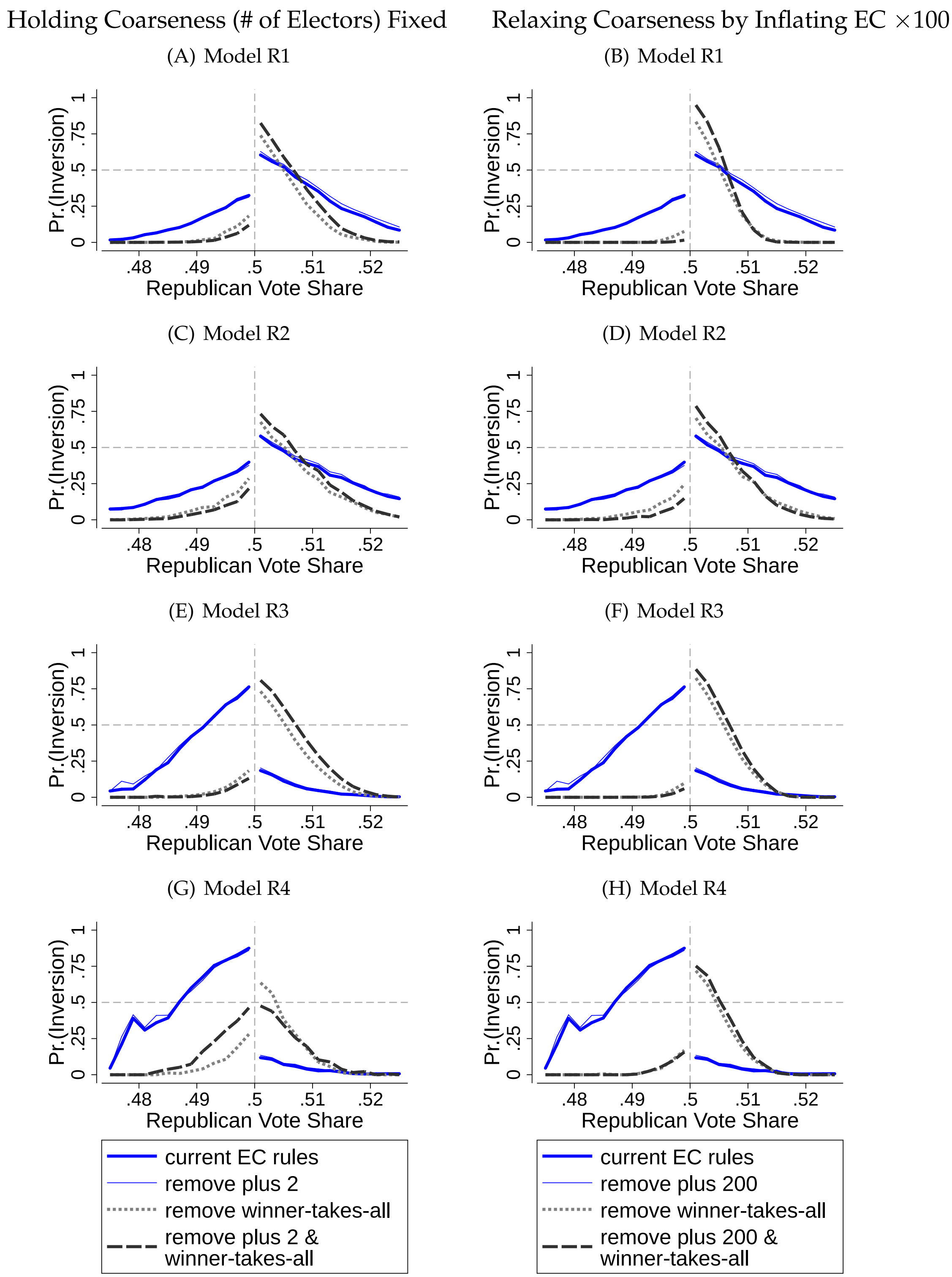
Note: Figure calculates inversion probabilities under a set of exogenously-specified variances and correlations. The error terms from the data generating process in Equation [1] of the main text is assumed to follow $\gamma_t \sim N(0, \sigma_\gamma)$ and $\phi_{st} \sim N(0, \sigma_\phi)$ as in the baseline models (M1, R1, A1). We cycle over a grid of values for σ_γ^2 and σ_ϕ^2, rather than relying on estimates. The variance of the national shock increases along the horizontal axis in each panel. The variance of the state shocks are traced in several contour lines in each panel, as indicated. In the panels on the left, we report the probability of close elections. In the panels on the right, we report inversion probabilities, conditional on close elections within the same margins.

Figure A4: Electoral Inversions Under Counterfactual Policies: Modern Period



Note: Figure illustrates inversions under counterfactual EC allocation rules. Counterfactuals that remove the plus 2 senators assign each state EC ballots equal to their US Representatives. Counterfactuals that remove the winner-takes-all condition allocates state EC ballots according to each candidate's vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed at 538. The right panel inflates the congressional delegation size to 53,800 to examine the impact of relaxing the rounding error ("coarseness") constraint.

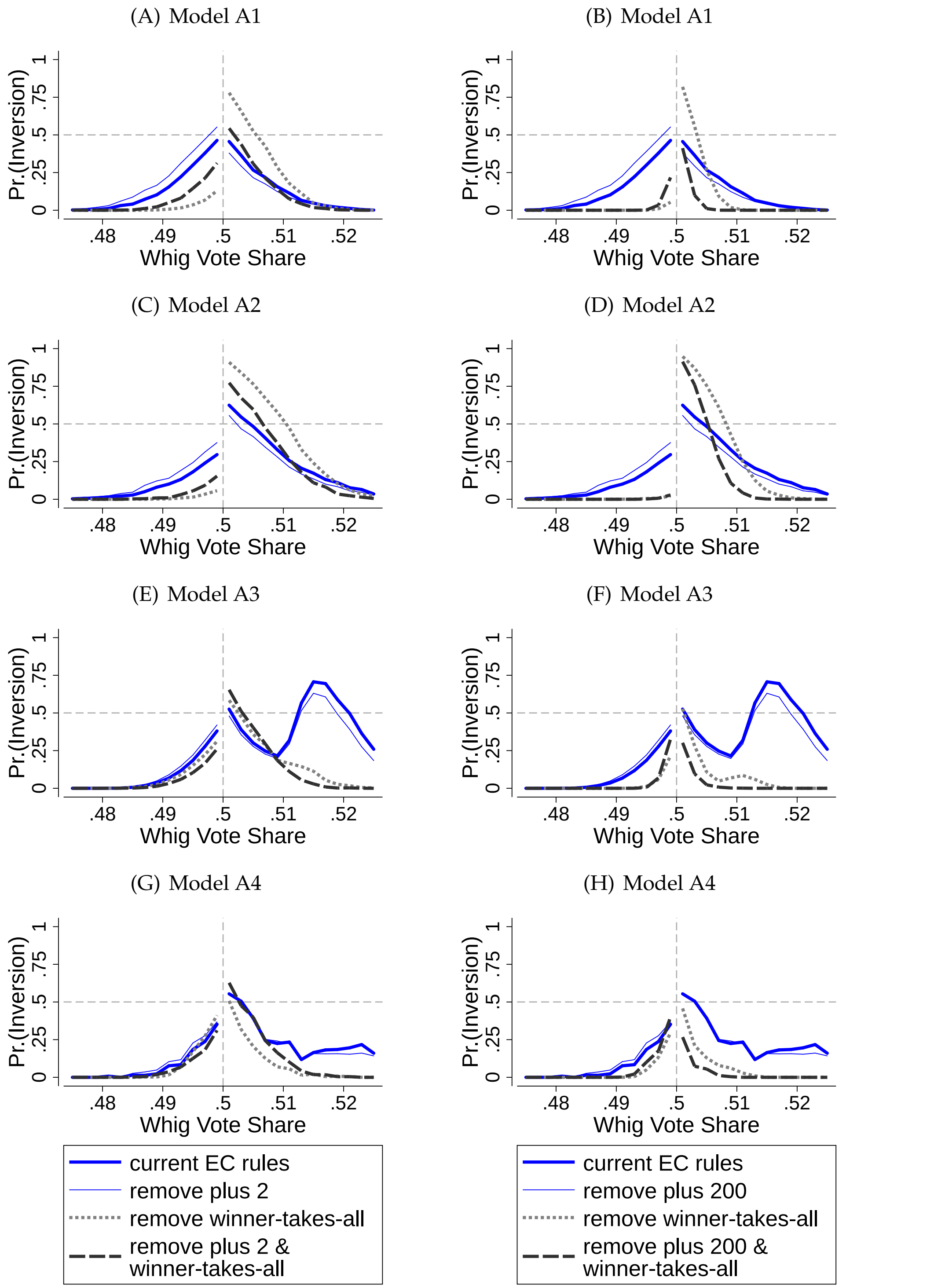
Figure A5: Electoral Inversions Under Counterfactual Policies: Reconstruction Period



Note: Figure illustrates inversions under counterfactual EC allocation rules. Counterfactuals that remove the plus 2 senators assign each state EC ballots equal to their US Representatives. Counterfactuals that remove the winner-takes-all condition allocates state EC ballots according to each candidate's vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed . The right panel inflates the congressional delegation size by 100 times to examine the impact of relaxing the rounding error ("coarseness") constraint.

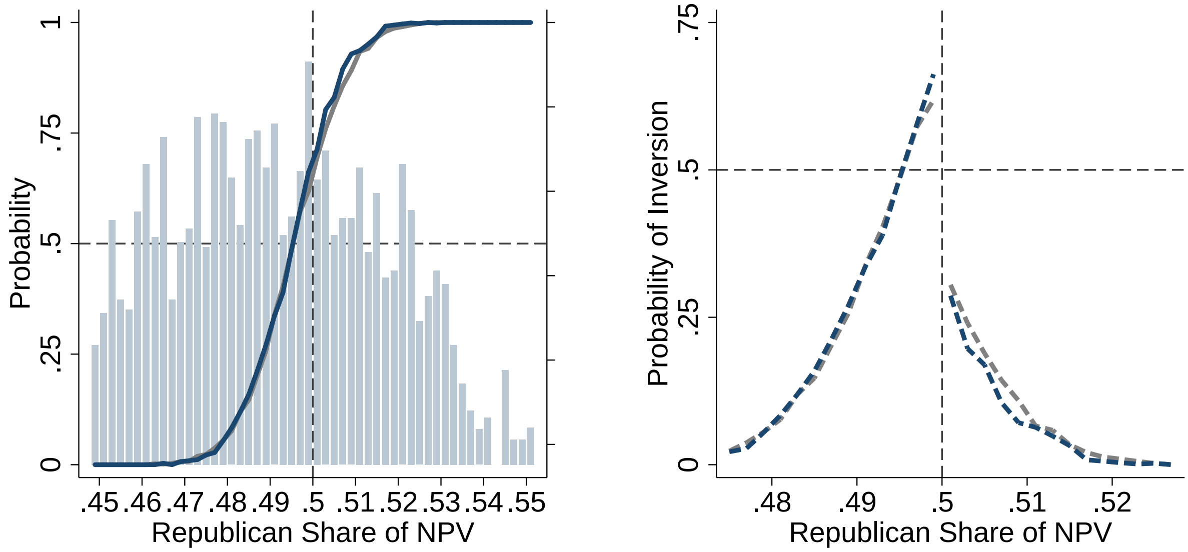
Figure A6: Electoral Inversions Under Counterfactual Policies: Antebellum Period

Holding Coarseness (# of Electors) Fixed | Relaxing Coarseness by Inflating EC $\times 100$



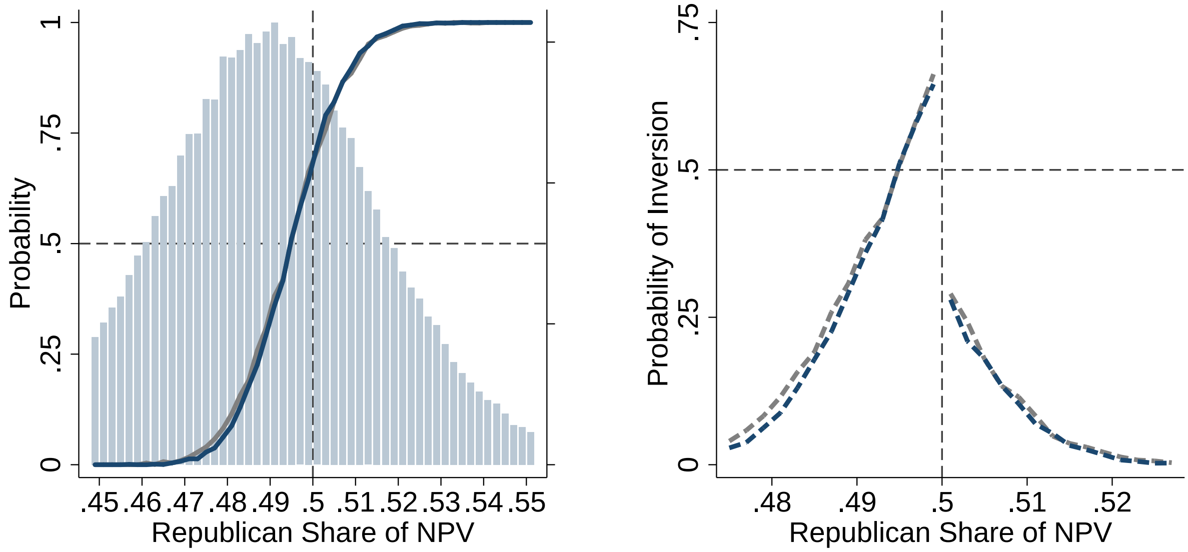
Note: Figure illustrates inversions under counterfactual EC allocation rules. Counterfactuals that remove the plus 2 senators assign each state EC ballots equal to their US Representatives. Counterfactuals that remove the winner-takes-all condition allocates state EC ballots according to each candidate's vote share in the state, up to a rounding error. The left panels holds the congressional apportionment fixed . The right panel inflates the congressional delegation size by 100 times to examine the impact of relaxing the rounding error ("coarseness") constraint.

Figure A7: Results Allowing for State Characteristics to Shift Expected Vote Shares



Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of model M1 compared to a new model (M13) that allows for state characteristics (region, education, race) to shift state vote share expectations. This is in contrast to models like M2 that allow correlations based on these characteristics in the across-state, within-election *shocks*. Each model simulation consists of 100,000 simulated election draws. The M1 model is plotted in gray for reference behind the M13 model in blue. The Republican share of the national popular vote runs along the horizontal axis. The solid blue line is the conditional probability of a Republican electoral win at each level of the national popular vote share. See Figure 2 and Appendix E.3 for additional notes.

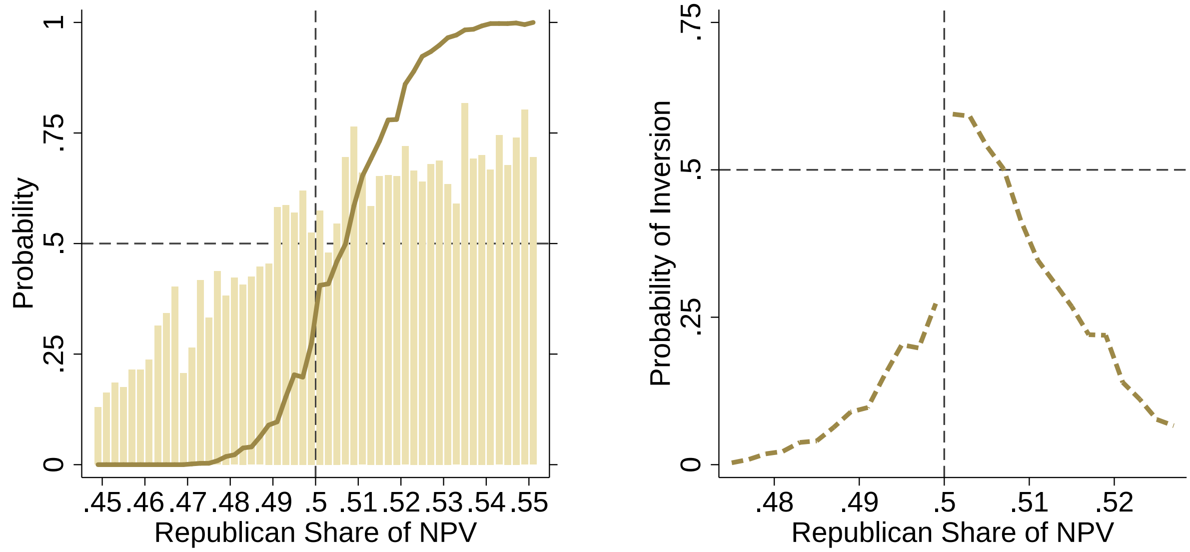
Figure A8: Alternative Parameterizations of Race- and Education-Linked Shocks



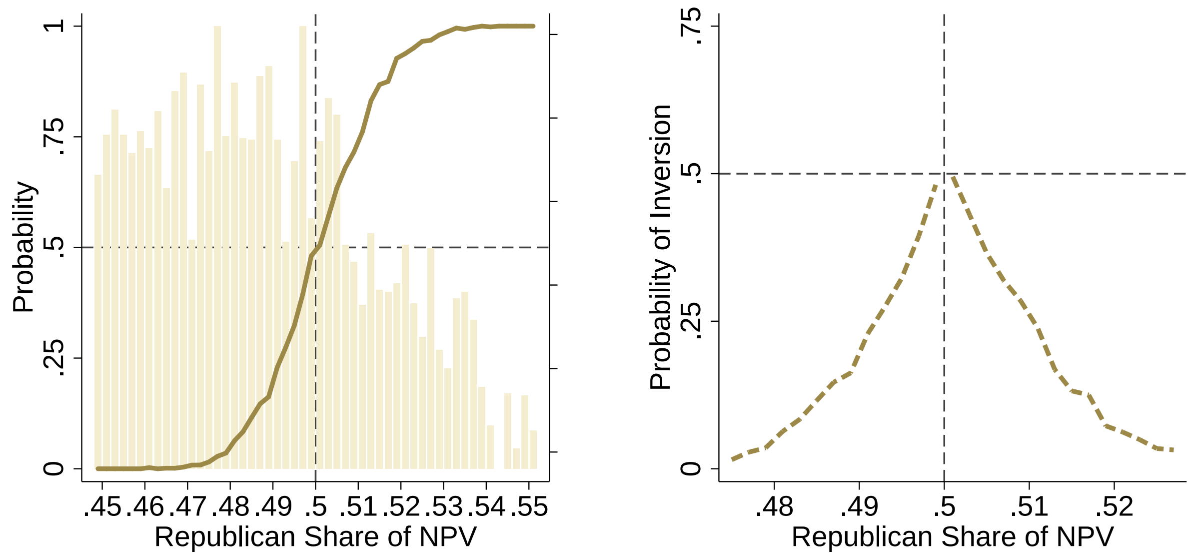
Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of model M2 compared to an alternative model that changes how state characteristics are parameterized in the shock term. We allow for race-linked shocks to multiply an **X** vector that includes region indicators, % non-hispanic white, % non-hispanic black, % hispanic, % college degree, and % high school completion in the state. This contrasts with M2, where **X** includes only % non-white and % college degree. Each model simulation consists of 100,000 simulated election draws. The M2 model is plotted in gray for reference behind the alternative model in blue. The Republican share of the national popular vote runs along the horizontal axis. The solid blue line is the conditional probability of a Republican electoral win at each level of the national popular vote share. See Figure 2 and Appendix E.4 for additional notes.

Figure A9: Results for the Fourth and Fifth Party Systems: 1896–1956

(A) Fourth Party System



(B) Fifth Party System



Note: Figure shows inversion probabilities and probability distributions over national popular vote outcomes implied by the parametric estimates of the $x1$ family of models estimated for sample periods in the first half of the twentieth century—the sand-colored regions of the party system history described in Figure 1. Rows correspond to different historical periods, as indicated. Each model simulation consists of 100,000 simulated election draws. The Republican share of the national popular vote runs along the horizontal axis. The solid line is the conditional probability of a Republican electoral win at each level of the national popular vote share. See Figure 2 for additional notes and Appendix E.5.