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CRYPTOCURRENCIES, CURRENCY COMPETITION, AND THE IMPOSSIBLE
TRINITY

Pierpaolo Benigno
Linda M. Schilling
Harald Uhlig

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Cryptocurrencies, Currency Competition, and the Impossible Trinity
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ABSTRACT

We analyze a two-country economy with complete markets, featuring two national currencies as well as a global (crypto)currency. If the global currency is used in both countries, the national nominal interest rates must be equal and the exchange rate between the national currencies is a risk-adjusted martingale. We call this result Crypto-Enforced Monetary Policy Synchronization (CEMPS). Deviating from interest equality risks approaching the zero lower bound or the abandonment of the national currency. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise. Thus, the classic Impossible Trinity becomes even less reconcilable.

Pierpaolo Benigno
Dipartimento di Economia e Finanza
Luiss Guido Carli
Viale Romania 32
00197 Rome
ITALY
pbenigno@luiss.it

Harald Uhlig
Department of Economics
University of Chicago
1126 E 59th Street
Chicago, IL 60637 and
NBER
huhlig@uchicago.edu

Linda M. Schilling
Ecole Polytechnique CREST
5 Avenue le Chatelier
Palaiseau, Fran 91120
France
lin.schilling@gmail.com

1 Motivation

Globally usable cryptocurrencies are on the rise. 10 years after the introduction of Bitcoin, Facebook is seeking to launch Libra designed to appeal to its more than 2 billion world-wide members. Other companies are not far behind. While other means of payments have been in worldwide use before, the ease of use and the scope of these new cryptocurrencies are about to create global currencies of an altogether different quality. How will they alter the financial landscape? How will this affect exchange rates and monetary policies of traditional currencies? We shall argue: considerably so.

Global currencies are not a new phenomenon. The Spanish Dollar in the 17th and 18th century, gold during the gold standard period, the Pound Sterling prior to 1944 and the U.S. Dollar since then may provide prior examples, and often served as an internationally accepted unit of account. The new cryptocurrencies, however, seek to become a means of payment, thus directly competing with national currencies for transaction purposes. We argue that this feature together with the consequences for national monetary policies is an entirely new phenomenon: in any case, this phenomenon and its consequences certainly deserves proper analysis. This is our aim.

We analyze a two-country economy featuring a home, a foreign and a global (crypto)currency. We adopt a general framework and assume that these currencies provide liquidity services. We show in section 7 that this framework encompasses a number of standard approaches in the monetary economics literature. For the benchmark case that markets are complete, that liquidity services are rendered immediately and that the global currency is used in both countries, we show that nominal interest rates must be equal and that the exchange rate between the home and the foreign currency is a risk-adjusted martingale, see proposition 4.1. We call this phenomenon a *crypto-enforced monetary policy synchronization (CEMPS)*. The home central bank, say, may seek to regain independence from this forced synchronization by moving interest rates down or up relative to the foreign interest rate. In the first case, it risks being trapped in too low-interest-rate policies, approaching the zero lower

bound. In the second case, it risks that its own national currency is abandoned as a medium of exchange. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise, see section 6. In particular, the central bank may be forced to the zero lower bound, when the global currency consortium seeks to keep its currency in use per selecting appropriately low and competitive fees.

Our results can be understood as a strengthened version of the Mundell-Fleming Trilemma (Fleming, 1962; Mundell, 1963) or *Impossible Trinity*. According to this cornerstone result in international economics, it is impossible to ensure a fixed exchange rate, free capital flows and an independent monetary policy, all at the same time. In our framework, we allow the exchange rate to be flexible and assume free capital flows: nevertheless, monetary policy becomes perfectly synchronized. More broadly, our results reach the same conclusions as Rey (2015) that the trilemma is transformed into a 'dilemma' or an 'irreconcilable' duo. While the global financial cycle is the culprit in her analysis, it is the worldwide diffusion of a global currency in ours.

1.1 Literature

Our analysis adds to the literature on the implications of currency competition for exchange rates and monetary policy. We provide a general framework based on asset-pricing considerations, which can nest classical monetary models used in the open-economy literature, like those presented in Obstfeld and Rogoff (1996), adding a global currency and assuming complete markets.

Our paper is related but complementary to the literature on the internationalization of currency and vehicle currencies from the perspective of currency substitution. Our result can be read as a sharp contrast to Obstfeld and Rogoff (2002), who argue that international monetary policy coordination is of minor importance compared to national considerations: in our framework, the introduction of a global currency leaves the central bank with little or only unattractive choices. Krugman (1979), Goldberg and Tille (2008) and Rey (2001) study a three country, three currency foreign exchange model

where transaction costs may give rise to a 'vehicle currency', i.e. the usage of the third country's currency to avoid direct foreign exchange between two countries. Here, instead, a third, global currency acts as substitute for either country's currency, thus allowing for currency competition on the local level. Therefore, currencies can be abandoned here despite symmetry in liquidity services. Further, the main focus of our analysis is on the implications for monetary policy of competing currencies. Like us, [Casas et al. \(2016\)](#) examine the impact of a global or dominant currency. In contrast to us, they focus on exchange rate pass-through under stickiness of prices, while we focus on the usage of a currency as a means of payment. We thus view our research as complementary to theirs. [Benigno \(2019\)](#) focuses on a one-country model and shows that under competition to cryptocurrencies, the central bank can face some bounds on interest rate and inflation if government currency has to retain some role as medium of exchange. We differ from his analysis by analyzing the consequence of cryptocurrency competition for the international monetary system by building on a general stochastic framework.

There is a large body of the literature which focuses on monetary policy under currency competition while abstracting from competition between interest-bearing assets (bonds) and currency. A classic contribution is [Kareken and Wallace \(1981\)](#) and its stochastic version [Manuelli and Peck \(1990\)](#). [Garraatt and Wallace \(2018\)](#) provide an extension to cryptocurrencies. [Schilling and Uhlig \(2018\)](#) focus on implications of competition between a cryptocurrency and traditional fiat money for the price evolution of the cryptocurrency and for monetary policy. [Schilling and Uhlig \(2019\)](#) analyze implications of goods-specific transaction costs for currency substitution. Here, instead, the nominal interest rates are decisive for currency substitution due to competition between bonds and currencies. [Fernández-Villaverde and Sanches \(2016\)](#) analyze currency competition and monetary policy in a Lagos-Wright model. Our framework is considerably more general than all these contributions, allowing interest-bearing bonds and encompassing a number of monetary models. Like us, [Brunnermeier and Niepelt \(2019\)](#) pursues the implications of the equivalence between private and public money, though our emphasis is on the

international context and has a different focus. Our analysis in section 6 shares common themes and conclusions with [Marimon et al. \(2003\)](#), who likewise emphasize that cheap inside monies place tight upper bounds on inflation rates there or nominal interest rates here.

2 A simple framework

This section uses a simple framework with a minimalist and non-stochastic structure to provide intuition and to preview the main results.

There are two countries, home and foreign, and three currencies: currency h and f of their respective countries and a global (crypto) currency g . While currency h can be used for transactions only in country h and the currency f only in country f , the global currency can be used in both countries. Money, either in a physical or digital form, provides non-pecuniary benefits, which we call liquidity services and yield liquidity premia. Let us assume that the two currencies are perfect substitutes in providing liquidity services and that these services are delivered at the same time in which money is held.¹

Let S_t be the exchange rate between currency h and f in date t , i.e. the amount of currency h needed to buy one unit of currency f . Let Q_t denote the amount of currency h needed to buy one unit of the global (crypto) currency. Likewise, let Q_t^* the amount of currency f needed to buy one unit of the global (crypto) currency. Therefore,

$$Q_t = S_t Q_t^* \tag{1}$$

At a generic time t , a resident in country h can acquire $M_{h,t}$ units in currency h and $M_{g,t}$ units in the global currency at the exchange rate Q_t implying an

¹In a non-stochastic economy, it does not matter whether the liquidity services for holding money in t are provided at time t or at time $t+1$. As it will be shown in the next sections and in [Appendix B.1](#), results in the stochastic economy are different and specific to the timing of liquidity services. The assumption of perfect substitutability between the currencies is stark and chosen to provide clear-cut results. The generalization to imperfect substitutability is discussed in [Appendix C](#).

overall expenditure or total money holding

$$M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}, \quad (2)$$

expressed in units of the domestic currency. Note that we are assuming perfect substitutability here and in the main text: the extension to imperfect substitutability is taken up in Appendix C. Since liquidity services are delivered immediately in t , the investor in country h receives non-pecuniary benefits from the overall money expenditure $M_{h,t}^{\text{tot}}$ deflated by the price of some generic consumption good (either tradeable or non-tradeable) for which money is exchanged for. At time $t + 1$, the two monies deliver an overall pay-off $M_{h,t} + Q_{t+1} M_{g,t}$, in units of the domestic currency. Since liquidity services provided by each currency are substitutes, the amount of services received is independent of the portfolio choice. Only if the returns on money are equal, then agents are willing to hold both currencies in their portfolio. This is equivalent to saying that the exchange rate Q should be constant, $Q_{t+1} = Q_t$. Otherwise, one currency would dominate the other as a means of payment. This result is nothing more than a restatement of Kareken and Wallace (1981), additionally allowing the monies to provide liquidity services. The analysis can equivalently be applied to country f to obtain that the exchange rate Q^* should also be constant.

Our first result in the paper follows directly from the above analysis: when a global currency is used in both local markets, the exchange rate, S , between currency h and f has to be constant too although h and f do not compete directly since h and f are not simultaneously accepted in the same local market. The monies h and f , however, compete indirectly through the global currency g which has worldwide acceptance, by this creating a link between the two local currencies. This indirect competition then enforces equal returns on h and f . To see this result, use the constancy of Q and Q^* into (1).

Our second result states that simultaneous trade in a global and local currencies requires synchronization of monetary policies across countries, i.e the nominal interest rates are equalized across countries. To see this result, allow

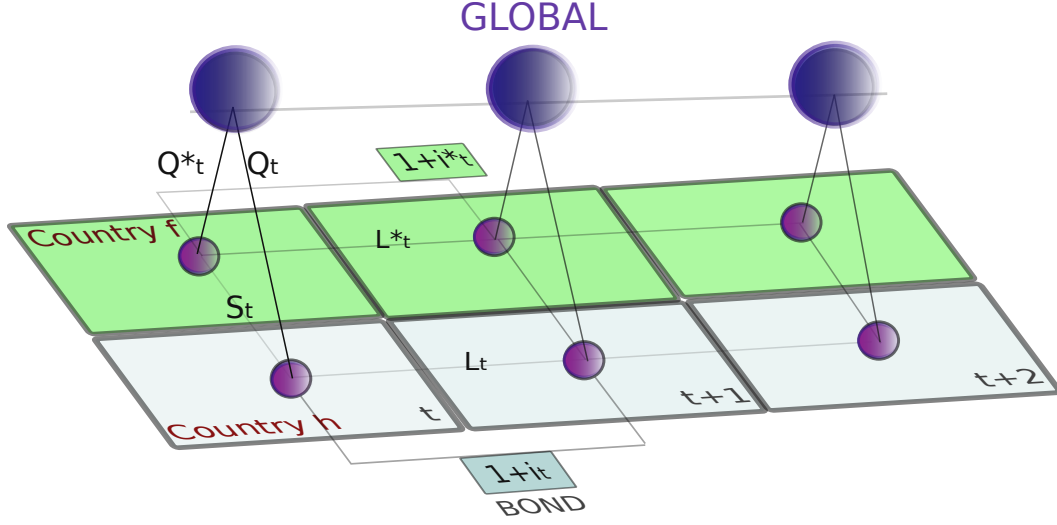


Figure 1: International trade and money flow in time

investors in each country to trade also in two nominal bonds denominated in currency h and f , respectively. In a non-stochastic economy, with frictionless capital markets, uncovered interest rate parity holds

$$\frac{1 + i_t}{1 + i_t^*} = \frac{S_{t+1}}{S_t} \quad (3)$$

in which i_t and i_t^* denote respectively the nominal interest rate in country h and f from period t to $t + 1$ on one-period bonds denominated in the respective currencies. Since the exchange rate S is constant, interest rates should be equal. Figure 1 summarizes the key relationship between interest rates and exchange rates.

As the next section will show, the result of equal nominal interest rates extends unchanged to a stochastic economy in the case liquidity services of money are delivered at the same time money is held in the portfolio. The result of constant exchange rates generalizes to a stochastic economy with the qualification that the exchange rate between currency h and f follows instead a martingale in the risk-neutral measure. In the stochastic setting, we will further show the equalization of the liquidity premia of money across

countries.²

3 A general framework

We present our main results through a general framework relying only on asset-pricing considerations, in a stochastic multi-period economy. Our structure is broad enough to encompass a large variety of models.³ Both agents can trade either bond and can hold the global currency. The agent in country h can in addition hold currency h but not currency f . Vice versa, the agent in country f can hold currency f but cannot trade currency h .

A key assumption for obtaining our result is that markets are complete, arbitrage-free and frictionless. As a consequence, a stochastic discount factor exists and is unique. Let \mathcal{M}_{t+1} denote the nominal stochastic discount factor in units of currency h for the agent in country h , and let likewise \mathcal{M}_{t+1}^* denote the nominal stochastic discount factor in units of currency f for the agent in country f . An implication of complete markets is that the nominal discount factors in units of the two local currencies are connected through their exchange rate since they are equalized once expressed in the same unit of account.⁴

Assumption 3.1 (Complete Markets:).

$$\mathcal{M}_{t+1} = \mathcal{M}_{t+1}^* \frac{S_t}{S_{t+1}}. \quad (4)$$

² Liquidity premia are in general monotone in the opportunity cost of holding money, i.e. the interest rate. Equal interest rates directly imply equal liquidity premia.

³The framework applies to one or multi-good exchange or production economies. Thus, we do not pin down these features specifically. Agents may live for two periods in an OLG model or be infinitively lived. In Section 7 we map our general framework into specific examples drawn from classical monetary models in which we specify preferences, constraints and maximization problem. These classical monetary models have been examined in a large body of literature, including existence of equilibria and their properties: for these reasons, we can sidestep these issues here.

Time is discrete and there is uncertainty. The economy does feature the same two countries as introduced in previous section, and at least the same three currencies and two nominal bonds.

⁴For the generality of this result see Obstfeld (2007).

Consider a (non-monetary) asset offering a (possibly random) nominal return R_{t+1} in currency h . Since returns have a price of one (Cochrane, 2009), standard asset pricing considerations imply, that the stochastic discount factor, \mathcal{M}_{t+1} , and return, R_{t+1} , are such that

$$1 = E_t[\mathcal{M}_{t+1}R_{t+1}]. \quad (5)$$

Thus, since a nominal one-period bond in country h pays the return $R_{t+1} = 1 + i_t$,

$$\frac{1}{1 + i_t} = E_t[\mathcal{M}_{t+1}] \quad (6)$$

and likewise for the bond in country f

$$\frac{1}{1 + i_t^*} = E_t[\mathcal{M}_{t+1}^*]. \quad (7)$$

While nonmonetary assets are used for the intertemporal transfer of resources, money offers some liquidity services above and beyond the intertemporal transfer. We shall therefore assume that currency h as well as the global currency pays a non-monetary liquidity service L_t to agents in country h per unit of currency, in addition to the intertemporal payoff. Likewise, we assume that currency f pays a liquidity premium L_t^* to agents in country f per unit of currency. For clarity and simplicity, we here assume that currency h and g in country h as well as currency f and g in country g are perfect substitutes, postponing the generalization and discussion of imperfect substitutability to Appendix C.

In a full model specification such as given in Section 7, these liquidity services are endogenously determined by the optimal consumption choices of households. In all of these models, money is held across periods from t to $t + 1$, and the particular model structure determines, whether the services are rendered in period t (“immediately”) or in $t + 1$ (“with delay”). For the benchmark case here, we assume the former, but return to the later in the Appendix B.1.

Assumption 3.2 (Liquidity immediacy). *The purchase of currency h and g*

in country h at t yields an immediate liquidity premium L_t receivable in t . Analogously, the time t purchase of currency f and g in country f at t yields an immediate liquidity premium L_t^* receivable in t .

The date- t (post-liquidity) price of a unit of currency h , expressed in units of the same currency equals unity, by definition. Standard asset pricing considerations then deliver

$$1 \geq L_t + E_t[\mathcal{M}_{t+1}]. \quad (8)$$

Whenever (8) holds with equality, agents in country h are willing to accept currency h at its unitary price, since they are exactly compensated by the liquidity premium plus the discounted future value of the payoff, which are both terms on the right hand side of equation (8). In case of a strict inequality, the current price of currency h is too high compared to expectations on future price development such that agents are not willing to hold the currency. Note that we do not allow short sale.

Likewise, for a unit of the global (or crypto) currency, trading at a price of Q_t in terms of units of currency h , we obtain

$$Q_t \geq L_t Q_t + E_t[\mathcal{M}_{t+1} Q_{t+1}], \quad (9)$$

where this equation holds with equality, if the global currency is used in country h , i.e. if agents are compensated for the price Q_t of a global currency exactly by the liquidity premium plus the discounted future value of the payoff, on the right hand side of equation (9). The price cannot be lower than the right hand side, since otherwise agents in country h would seek to acquire the currency and bid up its value. The price can be higher, however, if the global currency is not used in country h . We implicitly rule out short sales or, more precisely, rule out that short-sold currencies render negative liquidity premia. Combining (6) and (8), we obtain

$$\frac{i_t}{1 + i_t} \geq L_t, \quad (10)$$

which holds with equality when currency h is used and therefore describes

a monotone relationship between the nominal interest rate and the liquidity services. For the foreign country, we likewise obtain

$$1 \geq L_t^* + E_t[\mathcal{M}_{t+1}^*] \quad (11)$$

$$Q_t^* \geq L_t^* Q_t^* + E_t[\mathcal{M}_{t+1}^* Q_{t+1}^*], \quad (12)$$

$$\frac{i_t^*}{1 + i_t^*} \geq L_t^*. \quad (13)$$

In the analysis that follows we stick to the following assumptions.

Assumption 3.3 (Nonnegative liquidity premia). *The liquidity premia are non-negative, i.e. $L_t \geq 0$ and $L_t^* \geq 0$.*

This assumption together with equations (10) and (13) implies that $i_t \geq 0$ and $i_t^* \geq 0$, i.e. imply a zero lower bound for nominal interest rates. Moreover, we assume that at least one currency is used in each country while bonds cannot serve as medium of exchange.

Assumption 3.4 (Currency usage). *In country h , at least one of (8) and (9) holds with equality. In country f , at least one of (11) and (12) holds with equality.*

Additionally, it is reasonable to impose that at least one of (9) or (12) holds with equality, but we are not making use of that restriction. We make the assumption that the global currency has a positive value in the time period t under consideration.

Assumption 3.5 (Global currency is valued).

$$Q_t > 0 \text{ and } Q_t^* > 0 \quad (14)$$

Given the triangular relationship among exchange rates, $Q = SQ^*$, it follows that $Q > 0$ if and only if $Q^* > 0$. Thus, the currency being valued in one country necessarily spills over to the other country.

4 Main Results

Some additional terminology shall prove useful. For a generic random variable X_{t+1} , define risk adjusted expectation $\tilde{E}_t[X_{t+1}]$ in country h as

$$\tilde{E}_t[X_{t+1}] \equiv \frac{E_t[\mathcal{M}_{t+1}X_{t+1}]}{E_t[\mathcal{M}_{t+1}]}, \quad (15)$$

and the risk adjusted expectation $\tilde{E}_t^*[X_{t+1}]$ in country f as

$$\tilde{E}_t^*[X_{t+1}] \equiv \frac{E_t[\mathcal{M}_{t+1}^*X_{t+1}]}{E_t[\mathcal{M}_{t+1}^*]}. \quad (16)$$

We now obtain our main result.

Proposition 4.1 (Stochastic Economy under Liquidity Immediacy)

In a stochastic economy, under the assumption of liquidity immediacy, complete markets, and the global currency being valued. If all currencies are used in both countries, i.e. if equations (9), (12) and (8), (11) hold with equality, then

- i) the nominal interest rates are equalized $i_t = i_t^*$;*
- ii) the liquidity premia are equal $L_t = L_t^*$;*
- iii) the nominal exchange rate S_t between currency h and f follows a martingale, using risk adjusted expectations of country h ;*
- iv) the nominal exchange rate $S_t^* = 1/S_t$ between currency f and h follows a martingale, using risk adjusted expectations of country f ;*

Proof. [Proposition 4.1] The competition between currency h and the global currency, i.e. (8) and (9) with equality, the complete-market assumption (4) and finally the competition between currency f and global currency, i.e. (11) and (12) with equality, deliver

$$E_t[\mathcal{M}_{t+1}] = E_t\left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t}\right] = E_t\left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*}\right] = E_t[\mathcal{M}_{t+1}^*]$$

Equations (8) and (11) now imply $L_t = L_t^*$ and thus $i_t = i_t^*$, per equations (10) and (13). Exploit the equality (17) and use the complete market assumption (4) to obtain

$$E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1} \frac{S_{t+1}}{S_t} \right] \quad (17)$$

implying the country- h risk adjusted martingale property for the exchange rate,

$$S_t = \tilde{E}[S_{t+1}].$$

Proceeding instead per replacing \mathcal{M}_{t+1} on the left hand side of (17) with $\mathcal{M}_{t+1}^* S_t / S_{t+1}$ implies the country- f risk adjusted martingale property for $S_t^* = 1/S_t$,

$$S_t^* = \tilde{E}^*[S_{t+1}^*]. \quad \square$$

Proposition 4.1 says that, with complete markets, global usage of the global currency, and simultaneous usage of local currency, monetary policies must be perfectly synchronized: their nominal interest rates must be equal. This result, which we call *Crypto-Enforced Monetary Policy Synchronization (CEMPS)*, constitutes a constraint on the impossible trinity. Under free capital flows and without a global currency, uncovered interest parity and the classic Impossible Trinity result provides the home central bank with a choice: it can either give up on a pegged exchange rate or the independence of monetary policy. Our result shows that adding a global currency implies a further restriction, when it becomes a perfect substitute of the local currencies. Now, the monetary policy of the central banks can no longer be independent. Additionally, the exchange rate is now a risk-adjusted martingale and not necessarily a peg. The classical Impossible Trinity thus becomes even less reconcilable.

It is instructive to examine the special case of a non-stochastic economy as a benchmark. The following corollary immediately follows from Proposition 4.1. It is a version of the celebrated result in Kareken and Wallace (1981).

Corollary 4.1 (Deterministic Economy under Liquidity Immediacy)

In a deterministic economy, under the assumption of liquidity immediacy, complete markets, the global currency being valued and global currency usage, the

nominal exchange rate S_t between currency h and f is constant, $S_t \equiv \bar{S}$.

Proof. Immediate. □

With currency substitution, the countries' nominal interest rates are equalized independently of whether the economy is stochastic or deterministic. However, the result of constant exchange rate in the deterministic case is replaced by the martingale behavior of the stochastic economy. Still in this case it is possible to say something on the probability of deviations from a constant exchange rate by exploiting the Markov inequality, i.e. that any nonnegative random variable X satisfies

$$P(X \geq a) \leq \frac{E[X]}{a}$$

for any $a > 0$. As is well known, this bound is sharp only, if X has point masses at either zero or at a . The argument is applied to the change of the exchange rate $X = S_{t+1}/S_t$ and the result may be useful for bounding the probabilities of extreme events in a distribution-free manner.

Proposition 4.2 (Deviations)

For any $K > -1$, the percentage deviation from constancy of the nominal exchange rate satisfies

$$P_t \left(\frac{S_{t-1}}{S_t} - 1 > K \right) \leq \frac{1 + (1 + i_t) \sigma_{t|M} \sigma_{t|S/S}}{K + 1} \quad (18)$$

Proof. [Proposition 4.2] Proof in Appendix A. □

Since K is allowed to be negative, the likelihood that $\frac{S_{t-1}}{S_t}$ drops below one can be estimated as well.

4.1 Regaining monetary policy independence

Revisiting the result from Proposition 4.1, does this mean that the central banks in the two countries have no choice but to accept this fate of coordinated monetary policy? Or can, country h deviate from the monetary policy in

country f , and if so, in which direction? Proposition 4.1 can also be read the other way around. If $i_t \neq i_t^*$, then either the global currency is not used in at least one country or one of the national currencies is not in use or both. The central bank in country h may then contemplate pursuing a policy that makes sure that the global currency is not used in country h , i.e. that (9) remains an inequality.

Proposition 4.3 (Escaping global currency adoption)

In a stochastic economy, under the assumption of liquidity immediacy, complete markets, and the global currency being valued. Assume that both local currencies are used in their corresponding countries, i.e. equations (8) and (11) hold with equality. Independently of whether the global currency is used or not in country f , if $i_t < i_t^$, then*

- i) the global currency is not adopted in country h ;*
- ii) the liquidity premia satisfy $L_t < L_t^*$;*
- iii) the nominal exchange rate S_t between currency h and f follows a supermartingale, using risk adjusted expectations in country h ;*
- iv) the nominal exchange rate $S_t^* = 1/S_t$ between currency f and h follows a submartingale, using risk adjusted expectations in country f ;*

Proof. [Proposition 4.3] Proof in Appendix A. □

To understand the economics behind this result, it is important to acknowledge not only the competition between the currency h respectively f and the global currency but also the countrywise competition between the bond and currency and the role of the free foreign exchange market. The proof has 3 parts. First, since the nominal interest rate in country f is higher than the nominal interest rate in country h , currency liquidity services in country f are higher than in country h . Second, the competition between the national currencies and the global currency yields upper bounds on the risk-adjusted return of the global currency. The bound is sharper, if the nominal interest rate is higher, i.e. in country f , and it binds, if the global currency is adopted.

Third, by free foreign exchange markets and the no arbitrage condition, the risk-adjusted return on the global currency has to be equal in countries h and f . As a consequence, the country with the weaker constraint on that return does not adopt the global currency.

The proposition shows, that there is an escape hatch indeed, but only to one side. Starting from an equilibrium in which the global currency is used in both countries, by lowering the risk-free interest rate in currency h below that in currency f , the central bank in country h lowers the opportunity costs of holding the domestic currency and thus makes it more attractive than the global currency as a means of payment, crowding out global currency in country h .

This escape hatch is not particularly attractive, however. Nominal interest rates can only be lowered to zero. Furthermore, a rat race between the two central banks may then eventually force both to stick at the zero lower bound forever or at quite low interest rates.⁵ Some may applaud this as the ultimate and global implementation of the Friedman rule, while others may fear deflationary spirals and macroeconomic damages. Either way, these surely would also count as dramatic consequences of the presence of a global currency.

What can force central banks to lower interest rates rather than raise them is the risk of entering in unknown territories in which their currency is abandoned as mean of exchange in favor of the global currency. These worries can limit in a significant way the room of manoeuvring of the central bank in stabilizing economy. The next Proposition depicts a case in which the global currency is used in country f and spreads to country h when its central bank raises rates above the foreign ones.

Proposition 4.4 (Losing medium-of-exchange property)

In a stochastic economy, assume liquidity immediacy, complete markets, and the global currency being used in country f , i.e. equation (12) holding with

⁵In a one-country model Benigno (2019) has shown that if the central bank keeps the inflation target below the growth rate of private currency, then it can maintain the monopoly power as medium of exchange. However, cryptocurrencies' issuance is in general engineered with quite low, or zero, growth rates so that inflation targets set by central banks should be close to zero or below.

equality. If the central bank in country h sets $i_t > i_t^*$, then currency h is abandoned in country h and the global currency takes over (currency substitution).

Currency h would also be abandoned in country h if the central bank sets $i_t = i_t^*$ and only currency g is used in country f .

Proof. [Proposition 4.4] Proof in Appendix A. □

4.2 Global currency pricing

We now collect results on the stochastic process driving the global currency. Conclusions depend on which currency is adopted for liquidity services and in which country.

Proposition 4.5 (Global currency's stochastic process based on usage)

In a stochastic economy, under the assumption of liquidity immediacy, complete markets, and the global currency being valued.

- i) If both the global currency and currency h (f) are used in country h (f), equations (9) ((12)) and (8) ((11)) hold with equality, then the global currency's exchange rate in units of currency h (f) follows a martingale in the country- h (f) risk-adjusted measure*

$$\tilde{E}_t [Q_{t+1}] = Q_t, \quad (\tilde{E}_t^* [Q_{t+1}^*] = Q_t^*). \quad (19)$$

- ii) If in country h (f), the only currency used is currency h (f), then the global currency's exchange rate in units of currency h (f) follows a supermartingale in the country- h (f) risk-adjusted measure*

$$\tilde{E}_t [Q_{t+1}] < Q_t \quad (\tilde{E}_t^* [Q_{t+1}^*] < Q_t^*). \quad (20)$$

- iii) If in country h (f), the only currency used is the global currency, then the global currency's exchange rate in units of currency h (f) follows a*

submartingale in the country- h (f) risk-adjusted measure

$$\tilde{E}_t [Q_{t+1}] > Q_t \quad (\tilde{E}_t^* [Q_{t+1}^*] > Q_t^*). \quad (21)$$

Proof. Proof in Appendix A. □

Note, part (i) of Proposition 4.1 is similar to the fundamental pricing equation in Schilling and Uhlig (2018).

5 Benchmark: no global currency

It is helpful to compare the result of Proposition 4.1 to those that would obtain without a global currency. In the latter case, the competition between domestic bond and money implies the relationship between interest rate and liquidity premia as shown in equations (10) and (13), for country h and f , respectively. The competition, instead, between the two nominal one-period bonds denominated in currency h and f together with completeness of markets yields uncovered interest parity:

$$E_t \left[\mathcal{M}_{t+1} \frac{S_{t+1}}{S_t} \right] = \frac{1 + i_t}{1 + i_t^*}, \quad (22)$$

which can be also written using country- h risk-adjusted expectation as

$$\frac{\tilde{E}_t [S_{t+1}]}{S_t} = \frac{1 + i_t}{1 + i_t^*}. \quad (23)$$

Lacking the competition induced by the global currency, there is nothing that ex-ante restricts liquidity premia across countries and synchronizes interest rates. Monetary policymakers are free to choose their policies and the exchange-rate regime. The interest rate can be set to react to macroeconomic variables, and the exchange rate is let to float. Alternatively, one of the two countries can even decide to fix or manage the exchange rate but in this case it has to relinquish its independence in setting monetary policy, as the Impossible Trinity would say. Competition from a global currency makes this trinity even

harder to reconcile. As discussed in the previous section, one should expect more synchronization of policies or a pressure to set rates low in order to keep medium-of- exchange properties for government currencies.

6 Special case: asset-backed global currencies

This section is motivated by the recent announcement that Facebook is going to launch a new global currency, Libra. The main characteristic of Libra that we are going to investigate is its backing through a basket of risk-free securities denominated in government currencies. In our framework, suppose that the consortium issuing the global currency backs it by safe bonds denominated in currency h . Moreover, assume that the consortium is ready to buy and sell any amount of the global currency at a fixed price Q_t . When issuing the amount Δ_t of the global currency at some date t , the consortium invests the proceeds $\Delta_t Q_t$ in the safe bonds of country h . In period $t + 1$, the consortium receives the interest payments on the bonds. The consortium keeps a portion of the date $t + 1$ portfolio value as a per-period asset management fee, assumed to be $\phi_t \Delta_t Q_t$ for some $\phi_t \geq 0$ set in t . One may wish to think of these fees as profits paid to the shareholders of the consortium.

The consortium then sets the new price Q_{t+1} , again trading any amount of global currency at that price and investing their client's funds in home safe bonds. The bond returns after management fee which accrues between t and $t + 1$ to the global currency investors can be redeemed at the global currency's price Q_{t+1} or are reinvested.

In order to credibly promise the repurchase of the global currency for Q_{t+1} at $t + 1$ and assuming no profits beyond the asset management fee, assets and liabilities have to grow at the same rate,

$$Q_{t+1} = (1 + i_t - \phi_t) Q_t \tag{24}$$

Note that for $i_t \geq \phi_t$ the price of the global currency then increases over time $Q_{t+1} \geq Q_t$.

Proposition 6.1 (Asset backed global currency)

Assume the global currency is valued.

- (i) If $\phi_t < i_t$, then currency h is crowded out and only the global currency is used in country h . Moreover, $L_t = \frac{\phi_t}{1+i_t}$.*
- (ii) If $\phi_t = i_t$, both the currency h and the global currency coexist in country h .*
- (iii) If $\phi_t > i_t$, then only currency h is used in country h .*

Proof. Proof in Appendix A. □

From the results in proposition 6.1, we can retrieve more striking implications if we assume the fee to take the form of a fixed portion of the interest payments, $\phi_t = \kappa i_t$ for some parameter $0 \leq \kappa \leq 1$. Then

1. If $\kappa < 1$, then $i_t \leq \phi_t$ only holds for $i_t = 0$. Moreover, $i_t = 0$ implies $\phi_t = 0$ and the global currency is used together with the local currency in country h .
2. If $\kappa = 1$ (or $\phi_t = i_t$), then the price Q_t for the global currency is fixed (Stablecoin) and both currencies are used.

A useful reading of the above results from a central-banking perspective is the following. For the local currency h to maintain usage, the nominal interest rate has to undercut or match the management fee ϕ . The proposition therefore suggests that an interesting Bertrand-type game could unfold. The home central bank may seek to undercut the fee charged by the consortium, in order to drive the global currency out of usage at home. But without usage, the global currency consortium could not earn any revenue from the fees: it would be better off and might in turn respond by lowering its fees in response⁶. In the limit, this dynamic could result in both parties ratcheting down the "price" for its currencies to their marginal costs of issuance. If these marginal costs are zero or near zero, an assumption often made in the literature, one obtains a zero interest rate policy and a zero fee in the limit. Put differently, the

⁶The consortium may not care, if country h is small. It presumably would care, though, if this is a large and economically important country or a large and important currency union.

currency competition between the currency h and the global currency leads to the establishment of the celebrated Friedman rule to keep interest rates at zero, thereby setting the private costs of holding the currencies equal to the social cost of producing it. There is a large literature establishing conditions under which the Friedman rule is optimal, see [Woodford \(1990\)](#). More generally, if one currency has higher marginal costs of production than the other, then the resulting zero profit condition for this higher-cost currency will dictate the resulting limit.

These results are also reminiscent of view in [Hayek \(1978\)](#), that unfettered competition can align private incentives with social objectives. To extract rents from liquidity services, currency issuers have to supply a better money than others, by keeping its value high and therefore inflation low. But then competition kicks in to eradicate rents to zero and eliminate liquidity premia, so that the better money serves also the social benefits. Benigno (2019) presents a model of currency competition obtaining the same result under free entry. Our insights are related to the analysis in [Marimon et al. \(2003\)](#), who likewise emphasize that cheap inside monies place tight upper bounds on inflation rates there or nominal interest rates here.

In a nutshell, Libra may push central banks back to the zero lower bound. In essence, an asset backed global currency employs bonds to finance liquidity services, thus combining both the advantages of the liquidity services of money with the interest payments of bonds. Using the home currency can now not be more costly than the asset management fee charged by the consortium.

7 Examples

In the previous sections, we have presented our results using a general framework with a generic notation for the stochastic discount factors and the liquidity services. We now provide several examples of models which put more structure on preferences and constraints. We consider three different models: 1) a money-in-utility function model; 2) a cash-in-advance-constraint model in which the “credit” market opens before the “cash” market; 3) a cash-in-

advance-constraint model in which the “cash” market opens before the “credit” market. The first two models can be casted in the framework of Section 3 in which liquidity services are received at the same time money is held in agents’ portfolio. Model 3) deals with the case of delayed liquidity services, which is discussed in its more general form in Appendix refsec:delay.

7.1 Money-in-the-utility-function model

The model follows the Sidrauski-Brock framework extended to allow for multiple currencies. Consumers in Home country have preferences of the form

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ U(C_t) + V \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) \right\} \quad (25)$$

where $M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}$ as in equation (2), where β is the rate of time preferences with $0 < \beta < 1$, C is a consumption good and P its price in units of currency h . We can also assume more generally that C represents a bundle of goods. Consumers get utility from consumption, through a concave function $U(\cdot)$ strictly increasing in C and from real money balances by holding currency h , M_h , and the global currency M_g . The utility $V(\cdot)$ is weakly increasing in real money balances but may exhibit a satiation point at a finite level of real money balances; Q_t is the price of the global currency in units of currency h .

Consistently with the general framework of Section 3, consumers can invest in four securities: *i*) a risk-free bond denominated in currency h , B_h , paying an interest rate i ; *ii*) a risk-free bond denominated in currency f , B_f , paying an interest rate i^* ; *iii*) money in units of currency h , M_f , and *iv*) the global money, M_g . Consumers can also trade in a set of state-contingent securities able to span all states of nature. We omit them from the presentation of the budget constraint of the consumer. The nominal exchange rate between currency h and f is denoted by S , as in the main text; let T denote lump-sum transfers received from the government in units of currency h while T_g are transfers from the issuer of global money in units of global currency. Finally, Y is the home endowment of good C . Preferences in country f are specular,

with appropriate starred variables. Consumers are subject to the following budget constraints

$$B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} = W_t + P_t(Y_t - C_t) + T_t + Q_t T_{g,t},$$

in which

$$W_t \equiv M_{h,t-1} + Q_t M_{g,t-1} + (1 + i_{t-1})B_{h,t-1} + (1 + i_{t-1}^*)S_t B_{f,t-1}.$$

In the preferences (25) domestic and global money are perfect substitutes. While we shall allow short sales of bonds, as in the main text, we impose a short-sale constraint on the global currency and currency h , i.e. $M_g \geq 0$ and $M_h \geq 0$. The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\begin{aligned} \frac{U_C(C_t)}{P_t} \frac{1}{1 + i_t} &= E_t \left\{ \beta \frac{U_C(C_{t+1})}{P_{t+1}} \right\} \\ \frac{U_C(C_t)}{P_t} \frac{1}{1 + i_t^*} &= E_t \left\{ \beta \frac{U_C(C_{t+1})}{P_{t+1}} \frac{S_{t+1}}{S_t} \right\} \\ \frac{U_C(C_t)}{P_t} &\geq \frac{1}{P_t} V_m \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) + E_t \left\{ \beta \frac{U_C(C_{t+1})}{P_{t+1}} \right\} \\ \frac{Q_t U_C(C_t)}{P_t} &\geq \frac{Q_t}{P_t} V_m \left(\frac{M_{h,t}^{\text{tot}}}{P_t} \right) + E_t \left\{ \beta \frac{Q_{t+1} U_C(C_{t+1})}{P_{t+1}} \right\}, \end{aligned}$$

with the last two equations holding with equality for an interior solution in which $M_h \geq 0$ and $M_G \geq 0$, respectively. As in main text, at least one should hold with equality. In the above conditions, $U_C(\cdot)$ and $V_m(\cdot)$ are the derivatives of the respective functions with respect to their argument.

These equations can be casted in the notation of Section 3 by noting that the stochastic discount factors are

$$\mathcal{M}_{t+1} = \beta \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{P_t}{P_{t+1}} \quad \mathcal{M}_{t+1}^* = \beta \frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*}$$

and the liquidity premia are

$$L_t = \frac{V_m\left(\frac{M_{h,t}^{\text{tot}}}{P_t}\right)}{U_C(C_t)} \quad L_t^* = \frac{V_m\left(\frac{M_{f,t}^{\text{tot},*}}{P_t^*}\right)}{U_C(C_t^*)},$$

where $M_{f,t}^{\text{tot},*} = M_{f,t}^* + Q_t^* M_{g,t}^*$, analogously to (2). Note that complete markets imply that

$$\frac{U_C(C_t)}{P_t} = k \frac{U_C(C_t^*)}{S_t P_t^*}$$

for some positive parameter k which can be set equal to one. In the case purchasing power parity holds, $P_t = S_t P_t^*$, marginal utilities of consumption are proportional across countries. When all currencies are used, Proposition 4.1 applies and therefore $L_t = L_t^*$. Another implication is that the marginal utilities of real money balances $V_m(\cdot)$ are equalized across countries.

7.2 Cash-in-advance model, type I

Consider a cash-in-advance model with the timing of Lucas and Stokey (1987), in which the “credit” market opens before the “cash” market. Consumers living in country h have the following preferences

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(C_{T,t}, C_{N,t}) \quad (26)$$

in which C_T and C_N are, respectively, a traded and non-traded good and β , with $0 < \beta < 1$ is the intertemporal discount factor; $U(\cdot, \cdot)$ is a concave function, strictly increasing in both arguments. Preferences in country f are similar with variables denoted by a star.

Each period is divided in two sub-periods. In the first sub-period financial markets are open and consumer’s budget constraint is given

$$B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} = W_t + T_t + Q_t T_{g,t} \quad (27)$$

in which W_t is the nominal wealth, which remains after taking into account the purchases of goods in the previous period

$$W_t = (1 + i_{t-1})B_{h,t-1} + (1 + i_{t-1}^*)S_t B_{f,t-1} + M_{h,t-1} + Q_t M_{g,t-1} + (28) \\ + P_{T,t-1}(Y_{T,t-1} - C_{T,t-1}) + P_{N,t-1}(Y_{N,t-1} - C_{N,t-1}).$$

Y_T and Y_N are the endowments of the traded and non-traded goods, and P_T and P_N the respective prices. In the second subperiod of period t , the “cash” market opens and non-traded goods can be purchased following this constraint

$$M_{h,t}^{\text{tot}} \geq P_{N,t} C_{N,t}. \quad (29)$$

where $M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}$ as in equation (2). Budget constraints can be written specularly for the consumers living in country f .

The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\frac{\lambda_t}{1 + i_t} = E_t \{ \beta \lambda_{t+1} \} \quad (30)$$

$$\frac{\lambda_t}{1 + i_t^*} = E_t \left\{ \beta \lambda_{t+1} \frac{S_{t+1}}{S_t} \right\} \quad (31)$$

$$\lambda_t \geq \mu_t + \beta E_t \{ \lambda_{t+1} \} \quad (32)$$

$$\lambda_t Q_t \geq \mu_t Q_t + \beta E_t \{ \lambda_{t+1} Q_{t+1} \} \quad (33)$$

with the last two equations holding with equality for an interior solution $M_{h,t} > 0$ and $M_{g,t} > 0$, respectively; λ_t and μ_t are the multipliers associated with the constraints (27) and (29), respectively. Moreover the first-order conditions with respect to C_N and C_T implies that

$$\frac{U_{C_N}(C_{T,t}, C_{N,t})}{P_{N,t}} = \mu_t + \beta E_t \{ \lambda_{t+1} \}, \quad (34)$$

$$\frac{U_{C_T}(C_{T,t}, C_{N,t})}{P_{T,t}} = \beta E_t \{ \lambda_{t+1} \}, \quad (35)$$

where $U_{C_T}(\cdot, \cdot)$ and $U_{C_N}(\cdot, \cdot)$ are the derivatives of the function $U(\cdot, \cdot)$ with respect to the first and second arguments, respectively. We can now map this model in the notation of the general framework of Section 3, by noting that the stochastic discount factors are given by

$$\mathcal{M}_{t+1} = \frac{\beta \lambda_{t+1}}{\lambda_t} \quad \mathcal{M}_{t+1}^* = \frac{\beta \lambda_{t+1}^*}{\lambda_t^*}$$

while the liquidity premia can be written instead as

$$L_t = \frac{\mu_t}{\lambda_t} \quad L_t^* = \frac{\mu_t^*}{\lambda_t^*}.$$

Using the first-order conditions (32), (34) and (35), we can also write the nominal stochastic discount factors

$$\begin{aligned} \mathcal{M}_{t+1} &= \beta \frac{U_{C_N}(C_{T,t+1}, C_{N,t+1})}{U_{C_N}(C_{T,t}, C_{N,t})} \frac{P_{N,t}}{P_{N,t+1}} \\ \mathcal{M}_{t+1}^* &= \beta \frac{U_{C_N}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_N}(C_{T,t}^*, C_{N,t}^*)} \frac{P_{N,t}^*}{P_{N,t+1}^*} \end{aligned}$$

and the liquidity premia as

$$\begin{aligned} L_t &= \frac{U_{C_N}(C_{T,t}, C_{N,t}) - \frac{P_{N,t}}{P_{T,t}} U_{C_T}(C_{T,t}, C_{N,t})}{U_{C_N}(C_{T,t}, C_{N,t})} \\ L_t^* &= \frac{U_{C_N}(C_{T,t}^*, C_{N,t}^*) - \frac{P_{N,t}^*}{P_{T,t}^*} U_{C_T}(C_{T,t}^*, C_{N,t}^*)}{U_{C_N}(C_{T,t}^*, C_{N,t}^*)}. \end{aligned}$$

The results of Proposition 4.1 applies in the case all currencies are used. Additional results can be derived in this particular example. Note first that complete market implies that $\lambda_t = \kappa \lambda_t^*$ for some positive constant κ and at all t , which in the context of the above model can be also written as

$$\frac{U_{C_N}(C_{T,t}, Y_{N,t})}{P_{N,t}} = k \frac{U_{C_N}(C_{T,t}^*, Y_{N,t}^*)}{S_t P_{N,t}^*}. \quad (36)$$

Under appropriate conditions on the initial distribution of wealth, the constant

k can be set equal to 1.⁷ In (36), we have substituted equilibrium in the non-traded goods market, $C_{N,t} = Y_{N,t}$ and $C_{N,t}^* = Y_{N,t}^*$. Moreover, combining the first-order conditions (30), (32), (34) and (35) it is possible to obtain that

$$\frac{U_{C_N}(C_{T,t}, Y_{N,t})}{U_{C_T}(C_{T,t}, Y_{N,t})} = (1 + i_t) \frac{P_{N,t}}{P_{T,t}} \quad \frac{U_{C_N}(C_{T,t}^*, Y_{N,t}^*)}{U_{C_T}(C_{T,t}^*, Y_{N,t}^*)} = (1 + i_t^*) \frac{P_{N,t}^*}{P_{T,t}^*},$$

Using $i_t = i_t^*$ and (36) with $k = 1$ in the above conditions, we obtain that

$$\frac{U_{C_T}(C_{T,t}, Y_{N,t})}{U_{C_T}(C_{T,t}^*, Y_{N,t}^*)} = \frac{P_{T,t}}{S_t P_{T,t}^*}. \quad (37)$$

Assume that the law-of-one price holds for traded goods, then $P_{T,t} = S_t P_{T,t}^*$, and consider the special case in which $Y_{N,t} = Y_{N,t}^*$, then (37) implies perfect cross-country risk-sharing of the consumption of traded goods, $C_{T,t} = C_{T,t}^*$. Using this result in (36), we also obtain that the law-of-one price holds for non-traded goods $P_{N,t} = S_t P_{N,t}^*$, for which it is key the equalization of the nominal interest rates.

7.3 Cash-in-advance model, type II

Consider a cash-in-advance model with a different timing, in which the “cash” market now opens before the “credit” market. Preferences of consumers living in country h are similar to (26). Each period is divided in two sub-periods. In the first sub-period the non-traded good can be purchased subject to the following constraint

$$M_{h,t-1} + Q_t M_{g,t-1} \geq P_{N,t} C_{N,t} \quad (38)$$

in which variables follow previous definitions. After the “cash” market closes, in the second sub-period of period t the “credit” market opens and consumers

⁷The result that $\lambda_t = \kappa \lambda_t^*$ implies (36) depends on the fact that money allows to insure any movement in the price of non-traded good in the cash constraint (29).

are subject to the following constraint

$$\begin{aligned} B_{h,t} + S_t B_{f,t} + M_{h,t} + Q_t M_{g,t} + P_{T,t} C_{T,t} + P_{N,t} C_{N,t} = \\ + P_{T,t} Y_{T,t} + P_{N,t} Y_{N,t} + T_t + Q_t T_{g,t} + W_t \end{aligned} \quad (39)$$

where

$$W_t \equiv (1 + i_{t-1}) B_{h,t-1} + (1 + i_{t-1}^*) S_t B_{f,t-1} + M_{h,t-1} + Q_t M_{g,t-1}.$$

Budget constraints can be specularly written for the consumers in country f .

The first-order conditions with respect to B_h , B_f , M_h , M_g are

$$\begin{aligned} \frac{\lambda_t}{1 + i_t} &= E_t \{ \beta \lambda_{t+1} \} \\ \frac{\lambda_t}{1 + i_t^*} &= E_t \left\{ \beta \lambda_{t+1} \frac{S_{t+1}}{S_t} \right\} \\ \lambda_t &\geq \beta E_t \{ \mu_{t+1} + \lambda_{t+1} \} \\ \lambda_t Q_t &\geq \beta E_t \{ (\mu_{t+1} + \lambda_{t+1}) Q_{t+1} \} \end{aligned}$$

with the last two equations holding with equality for an interior solution $M_{h,t} > 0$ and $M_{g,t} > 0$, respectively. In the above conditions, λ_t and μ_t are the multipliers associated with the constraints (39) and (38), respectively. Moreover the first-order conditions with respect to C_N and C_T implies that

$$\frac{U_{C_N}(C_{T,t}, C_{N,t})}{P_{N,t}} = \mu_t + \lambda_t, \quad (40)$$

$$\frac{U_{C_T}(C_{T,t}, C_{N,t})}{P_{T,t}} = \lambda_t. \quad (41)$$

Note that in this model liquidity premia are received with one-period delay. Therefore this example can be mapped in the notation of the general framework presented in Appendix B.1, by noting that the stochastic discount

factors are given by

$$\mathcal{M}_{t+1} = \frac{\beta \lambda_{t+1}}{\lambda_t} \quad \mathcal{M}_{t+1}^* = \frac{\beta \lambda_{t+1}^*}{\lambda_t^*}$$

while liquidity premia are

$$L_{t+1} = \frac{\mu_{t+1}}{\lambda_{t+1}} \quad L_{t+1}^* = \frac{\mu_{t+1}^*}{\lambda_{t+1}^*}.$$

Using the first-order conditions (40) and (41), we can further write the stochastic discount factors and the liquidity premia as

$$\begin{aligned} \mathcal{M}_{t+1} &= \beta \frac{U_{C_T}(C_{T,t+1}, C_{N,t+1})}{U_{C_T}(C_{T,t}, C_{N,t})} \frac{P_{T,t}}{P_{T,t+1}} \\ \mathcal{M}_{t+1}^* &= \beta \frac{U_{C_T}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_T}(C_{T,t}^*, C_{N,t}^*)} \frac{P_{T,t}^*}{P_{T,t+1}^*} \end{aligned}$$

and

$$\begin{aligned} 1 + L_{t+1} &= \frac{U_{C_N}(C_{T,t+1}, C_{N,t+1})}{U_{C_T}(C_{T,t+1}, C_{N,t+1})} \frac{P_{T,t+1}}{P_{N,t+1}} \\ 1 + L_{t+1}^* &= \frac{U_{C_N}(C_{T,t+1}^*, C_{N,t+1}^*)}{U_{C_T}(C_{T,t+1}^*, C_{N,t+1}^*)} \frac{P_{T,t+1}^*}{P_{N,t+1}^*}. \end{aligned}$$

The results of Proposition B.1 and Corollary B.1 apply to this model.

8 Conclusion

Starting from a general framework, we have analyzed a two-country economy featuring a home, a foreign and a global (crypto)currency. For the benchmark case that markets are complete, that the global currency is used in both countries and that currency liquidity services are immediate, we have shown that nominal interest rates must be equal and that the exchange rate between the home and the foreign currency is a risk-adjusted martingale. We call this

phenomenon *Crypto-Enforced Monetary Policy Synchronization (CEMPS)*. It adds a further restriction to the classic Impossible Trinity. We have discussed the dangers for monetary policies, seeking to escape this restriction. We have characterized the implications for the exchange rate dynamics and the pricing dynamics of the global currency. If the global currency is backed by interest-bearing assets, additional and tight restrictions on monetary policy arise. We demonstrate, how our general framework encompasses a number of classic monetary models in the literature such as money-in-the-utility function and cash-in-advance. In the appendix, we have extended our results to the case of delayed liquidity services, where additional correlation terms arise, and when currencies are not perfect substitutes.

We conclude that the introduction of a globally used currency may therefore substantially change the landscape of international monetary policy. We leave to future research to further investigate how the dominant role of traditional currencies as safe assets in international markets will be challenged by the presence of global crypto currencies.

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APPENDIX

A Proofs of Propositions

In this Appendix, we collect the proofs of the Propositions.

A.1 Proof of Proposition 4.2

Proof. [Proposition 4.2] Let $K > -1$ arbitrary, e.g. $K = 0$. From (17), and again using the definition of the covariance,

$$0 = cov_t \left(\mathcal{M}_{t+1}, \frac{S_{t+1}}{S_t} \right) + E_t[\mathcal{M}_{t+1}] E_t \left[\frac{S_{t+1}}{S_t} - 1 \right]. \quad (\text{A.1})$$

Solving for $E[S_{t+1}/S_t]$,

$$E \left[\frac{S_{t+1}}{S_t} \right] = 1 + \frac{cov_t(\mathcal{M}_{t+1}, \frac{S_{t+1}}{S_t})}{E_t[\mathcal{M}_{t+1}]} = 1 + (1 + i_t) cov_t \left(\mathcal{M}_{t+1}, \frac{S_{t+1}}{S_t} \right). \quad (\text{A.2})$$

In what follows, define $\sigma_{t|M}$ the conditional standard deviation of the nominal stochastic discount factor, $\sigma_{t|S}$ the conditional standard deviation of the nominal exchange rate between home and foreign, and $\rho_{(M_{t+1}, S_{t+1})}$ the correlation coefficient between the two. Then, the likelihood that the percentage change

of the exchange rate exceeds K is bounded from above by

$$\begin{aligned}
P_t \left(\frac{S_{t+1}}{S_t} - 1 > K \right) &\leq \frac{E_t \left[\frac{S_{t+1}}{S_t} \right]}{K + 1} \\
&= \frac{1 - (1 + i_t) \text{cov}_t(\mathcal{M}_{t+1}, \frac{S_{t+1}}{S_t})}{K + 1} \\
&= \frac{1 - (1 + i_t) \rho_t(\mathcal{M}_{t+1}, \frac{S_{t+1}}{S_t}) \sigma_{t|\mathcal{M}} \sigma_{t|S}}{K + 1} \\
&\leq \frac{1 + (1 + i_t) \sigma_{t|\mathcal{M}} \sigma_{t|S}}{K + 1} \\
&\rightarrow 0 \quad \text{as } K \rightarrow \infty
\end{aligned}$$

where the first inequality holds by the conditional Markov inequality, the following equality holds by (A.2), the consecutive equality holds by definition of the correlation coefficient and the inequality thereafter holds since always $\rho \in [-1, 1]$ and since standard deviations are positive. \square

A.2 Proof of Proposition 4.3

Proof. [Proposition 4.3] It holds

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = E_t \left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] \quad (\text{A.3})$$

$$\leq E_t[\mathcal{M}_{t+1}^*] = \frac{1}{1 + i_t^*} \quad (\text{A.4})$$

$$< \frac{1}{1 + i_t} = E_t[\mathcal{M}_{t+1}] = 1 - L_t \quad (\text{A.5})$$

The first step follows by market completeness, the second step holds since the global currency may or may not be in use in country f , by this yielding a weakly lower return than currency f in country f . The third step uses equation (7). The fourth step, the inequality sign, is implied by the assumption $i_t < i_t^*$, the fifth step uses equation (6) and the final step follows from the assumption that currency h is used in country h , i.e. equation (8) with the equality sign.

Thus,

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] < 1 - L_t \quad (\text{A.6})$$

and the global currency is not used in country h . Further, we directly see $L_t < L_t^*$ from our derivation. Thus, by market completeness

$$E_t \left[\mathcal{M}_{t+1} \frac{S_{t+1}}{S_t} \right] = E_t[\mathcal{M}_{t+1}^*] < E_t[\mathcal{M}_{t+1}] \quad (\text{A.7})$$

where the last step follows from the derivation above. Therefore, S_t follows a supermartingale in country- h risk-adjusted measure. Vice versa,

$$E_t[\mathcal{M}_{t+1}^*] < E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1}^* \frac{S_t}{S_{t+1}} \right] \quad (\text{A.8})$$

Thus, with $S^* = 1/S$, $E_t[\mathcal{M}_{t+1}^*] < E_t \left[\mathcal{M}_{t+1}^* \frac{S_{t+1}}{S_t^*} \right]$ and also the exchange rate from the perspective of country f follows a submartingale. \square

A.3 Proof of Proposition 4.4

Proof. [Proposition 4.4] We have

$$E_t[\mathcal{M}_{t+1}] = \frac{1}{1 + i_t} < \frac{1}{1 + i_t^*} = E_t[\mathcal{M}_{t+1}^*] \quad (\text{A.9})$$

$$\leq 1 - L_t^* = E_t \left[\mathcal{M}_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] \quad (\text{A.10})$$

$$= E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] \leq 1 - L_t \quad (\text{A.11})$$

Here the first step uses equation (6), the second step uses the policy set in the two countries, $i_t \geq i_t^*$, the third step equation (7). The fourth step and inequality follows because currency f may or may not be used in country f . The fifth step uses that the global currency is used in country f , the sixth step uses completeness of markets and the last step uses that the global currency may or may not be adopted in country h . Altogether, $E_t[\mathcal{M}_{t+1}] < 1 - L_t$ for

$i > i^*$. Alternatively, $E_t[\mathcal{M}_{t+1}] < 1 - L_t$ for $i = i^*$ if currency f is not used in country f , $E_t[\mathcal{M}_{t+1}^*] < 1 - L_t^*$. \square

A.4 Proof of Proposition 4.5

Proof. [Proposition 4.5] (1) Assume in country h that the global currency and currency h are used. Then

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = 1 - L_t = E_t[\mathcal{M}_{t+1}] \quad (\text{A.12})$$

Multiplication by $Q_t/E_t[\mathcal{M}_{t+1}]$ yields the result.

(2) Assume in country h that currency h is used and the global currency is not used then we have

$$E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] < 1 - L_t = E_t[\mathcal{M}_{t+1}] \quad (\text{A.13})$$

As before, multiplication by $Q_t/E_t[\mathcal{M}_{t+1}]$ yields the result. The parallel result for country f follows similar steps, as well as case (3). \square

A.5 Proof of Proposition 6.1

Proof. [Proposition 6.1] (i) Assume $\phi_t < i_t$. Then

$$1 - L_t \geq E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] > E_t[\mathcal{M}_{t+1}]. \quad (\text{A.14})$$

The first inequality holds by (9), the second step holds by (24), the third step follows from $i_t > \phi_t$. Since $1 - L_t > E_t[\mathcal{M}_{t+1}]$, the local currency h is not used. Given the assumption that at least one currency is used in country h , (9) has to hold with equality, $1 - L_t = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}]$, and the global currency is used in h . By no arbitrage, a comparison between the return on the global currency and the bond through (6) yields

$$\frac{1 - L_t}{1 + i_t - \phi_t} = \frac{1}{1 + i_t} \quad (\text{A.15})$$

and thus $L_t = \frac{\phi_t}{1+i_t}$.

(ii) Assume $\phi_t = i_t$, then

$$1 - L_t \geq E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] = E_t[\mathcal{M}_{t+1}] \quad (\text{A.16})$$

and since at least one currency has to be in use, we have

$$1 - L_t = E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] = E_t[\mathcal{M}_{t+1}] \quad (\text{A.17})$$

implying that both currencies are used.

(iii) Assume $\phi_t > i_t$, then

$$1 - L_t \geq E_t[\mathcal{M}_{t+1}] > (1 + i_t - \phi_t) E_t[\mathcal{M}_{t+1}] = E_t \left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t} \right] \quad (\text{A.18})$$

Thus, the global currency is not used. But since once currency has to be used, it has to be currency h , $1 - L_t = E_t[\mathcal{M}_{t+1}]$. \square

B Robustness analysis

In this section, we present some robustness analysis of our main results. First, we investigate the case in which liquidity services are delayed one period with respect to when money is held in the agents' portfolio. Second, we sketch out the implications of imperfect substitutability between monies.

B.1 Delayed liquidity services

An important assumption of our framework is liquidity immediacy, i.e. that the liquidity services provided by currency occur at the same date t that money is added to the portfolio of agents. However, some models, like the third example in Section 7, instead postulate liquidity premia to be received a period after portfolio choices are made, i.e. with delay in $t + 1$:

Assumption B.1 (Liquidity delay). *The purchase of the global currency and*

currency h in country h at t yields delayed liquidity premia L_{t+1} receivable in $t + 1$. Analogously, the time t purchase of global currency and currency f in country f at t yields delayed liquidity premia L_{t+1}^* receivable in $t + 1$.

In this case, equations (8), (9) and (10) need to be replaced with

$$1 \geq E_t[\mathcal{M}_{t+1}(1 + L_{t+1})], \quad (\text{B.19})$$

$$Q_t \geq E_t[\mathcal{M}_{t+1}(1 + L_{t+1})Q_{t+1}], \quad (\text{B.20})$$

$$\frac{i_t}{1 + i_t} \geq E_t[\mathcal{M}_{t+1}L_{t+1}]. \quad (\text{B.21})$$

The liquidity premia are appropriately discounted by the stochastic discount factor. Since we will focus on equilibria in which all currencies are used, we set (B.19), (B.20), (B.21) with an equality sign.

In country f , one must likewise replace (11), (12) and (13) with

$$1 \geq E_t[\mathcal{M}_{t+1}^*(1 + L_{t+1}^*)], \quad (\text{B.22})$$

$$Q_t^* \geq E_t[\mathcal{M}_{t+1}^*(1 + L_{t+1}^*)Q_{t+1}^*], \quad (\text{B.23})$$

$$\frac{i_t^*}{1 + i_t^*} \geq E_t[\mathcal{M}_{t+1}^*L_{t+1}^*]. \quad (\text{B.24})$$

And again in what follows, we are going to assume that the above equations hold with an equality sign. Define the conditional covariance under the home country risk-adjusted measure as

$$\widetilde{cov}_t(X, Y) \equiv \widetilde{E}_t[XY] - \widetilde{E}_t[X] \widetilde{E}_t[Y] \quad (\text{B.25})$$

For a random variable X , define the risk-adjusted expectation in country f as the equivalent to $\widetilde{E}_t[\cdot]$ via

$$\widetilde{E}_t^*[X] \equiv \frac{E_t[\mathcal{M}_{t+1}^*X]}{E_t[\mathcal{M}_{t+1}^*]} \quad (\text{B.26})$$

Let

$$\Delta_t \equiv i_t - i_t^*$$

be the differences between the nominal interest rates. Maintaining all other assumptions, we now turn to derive the implications for the exchange rate. The next results apply in general independently of whether liquidity premia are or are not delayed, and they just need as input the interest rate differential, like in (B.29).

Proposition B.1 (Delayed Liquidity Services and Exchange Rates)

In a stochastic economy, assuming liquidity delay, complete markets, all currencies being used, the expected liquidity services differences and exchange rates then satisfy

$$\Delta_t = \tilde{E}_t[L_{t+1}] - \tilde{E}_t^*[L_{t+1}^*] \quad (\text{B.27})$$

and

$$\frac{\tilde{E}_t[S_{t+1}]}{S_t} = 1 + \frac{\Delta_t}{1 + i_t^*} \quad (\text{B.28})$$

This corollary is a consequence strictly of the given interest differential: the presence of the global currency is not necessary to establish these consequences. Note how the results here are adjusted relative to the expressions in our benchmark result. The (expected) liquidity services now differ by the interest rate differential. If that is zero as in the main result, so is the (expected) liquidity service difference. The exchange rate is no longer a risk-adjusted martingale: instead, there is an adjustment term that depends on the interest rate differential. If that interest rate differential is zero as in the main result, we are back to the risk-adjusted martingale.

Proof. [Proposition B.1] Note that (B.19) and (B.21) can be written as

$$i_t = \tilde{E}_t[L_{t+1}]$$

Likewise, (B.22) and (B.24) can be written as

$$i_t^* = \tilde{E}_t^*[L_{t+1}^*]$$

The combination yields (B.27). Finally, consider the uncovered-interest-parity relationship (23) to obtain (B.28). \square

Corollary B.1 (Stochastic Economy under Delayed Liquidity Premia)

In a stochastic economy, assuming liquidity delay, complete markets, and all currencies being used, the nominal interest rate differential satisfies

$$i_t^* - i_t = \frac{\widetilde{cov}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t[Q_{t+1}]} + \frac{\widetilde{cov}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]} \quad (\text{B.29})$$

Note that the benchmark result of interest rate equality in case of liquidity immediacy is a direct consequence of (B.29), since the conditional covariance terms must be zero, if L_{t+1} and L_{t+1}^* are known in t . In the general case, nonzero covariance terms arise and equation (B.29) informs us, in which direction one needs to adjust the interest differential.

Proof. [Corollary B.1.] Since all currencies are used, (B.21) and (B.24) hold with equality. With (6) and (7), rewrite (B.21) and (B.24) using the risk-adjusted measures as

$$i_t = \tilde{E}_t[L_{t+1}] \quad (\text{B.30})$$

and

$$i_t^* = \tilde{E}_t^*[L_{t+1}] = \frac{\tilde{E}_t[L_{t+1}^* S_{t+1}]}{\tilde{E}_t[S_{t+1}]} \quad (\text{B.31})$$

where in the latter we have also used the assumption of complete markets. Combining the two equations above, we can write the interest-rate differential as

$$i_t^* - i_t = \tilde{E}_t[L_{t+1}^*] - \tilde{E}_t[L_{t+1}] + \frac{\widetilde{cov}_t(L_{t+1}^*, S_{t+1})}{\tilde{E}_t[S_{t+1}]}, \quad (\text{B.32})$$

Note that this equation holds, regardless of whether there is a global currency or not. The presence of the global currency, however, delivers a restriction on the difference between the expected liquidity services. Use (B.23) together with the assumption of complete markets and the equivalence $Q_t = S_t Q_t^*$ to obtain

$$Q_t = E_t[\mathcal{M}_{t+1}(1 + L_{t+1}^*)Q_{t+1}] \quad (\text{B.33})$$

This can be written under the risk-adjusted measure as

$$(1 + i_t)Q_t = \tilde{E}_t [(1 + L_{t+1}^*)Q_{t+1}]. \quad (\text{B.34})$$

Also write (B.20) using the risk-adjusted measure

$$(1 + i_t)Q_t = \tilde{E}_t [(1 + L_{t+1})Q_{t+1}], \quad (\text{B.35})$$

and compare it with the equation above to obtain that

$$0 = \tilde{E}_t [(L_{t+1}^* - L_{t+1})Q_{t+1}] \quad (\text{B.36})$$

and thus

$$\tilde{E}_t [L_{t+1}^*] - \tilde{E}_t [L_{t+1}] = \frac{\widetilde{\text{cov}}_t(L_{t+1} - L_{t+1}^*, Q_{t+1})}{\tilde{E}_t [Q_{t+1}]}, \quad (\text{B.37})$$

Plug (B.37) into (B.32) to deliver (B.29). \square

Note that equation (B.37) determines the expected difference in the liquidity premia, by which we can retrieve the result of the benchmark case of equal liquidity premia when L_{t+1} and L_{t+1}^* are known at time t .

C Imperfect substitutability of currencies

Our analysis easily generalizes with suitable modification to the situation, where the currencies are not perfect substitutes. As in section 2, let $M_{h,t}^{\text{tot}}$ denote the total money holding in country h at time t , expressed in units of the domestic currency. In section 2 and implicitly in the general framework of section 3, we have assumed that $M_{h,t}^{\text{tot}}$ is the sum of the nominal value of the home currency as well as the global currency used at home,

$$M_{h,t}^{\text{tot}} = M_{h,t} + Q_t M_{g,t}, \quad (\text{C.38})$$

see equation 2. More generally, assume that

$$M_{h,t}^{\text{tot}} = f(M_{h,t}, Q_t M_{g,t}) \quad (\text{C.39})$$

for some constant returns to scale function $f(\cdot, \cdot)$. This captures the idea that the national currency may be relatively more useful for certain transactions, while the global currency is more useful for others. A fully spelled out version of this idea is in Schilling and Uhlig (2019). Due to constant returns to scale, (C.39) can alternatively be written in terms of real units as

$$\frac{M_{h,t}^{\text{tot}}}{P_t} = f\left(\frac{M_{h,t}}{P_t}, \frac{Q_t M_{g,t}}{P_t}\right) \quad (\text{C.40})$$

Equation (C.38) arises for the linear specification

$$f(M_{h,t}, Q_t M_{g,t}) = M_{h,t} + Q_t M_{g,t}$$

Total home money holdings provide the total liquidity services $L_t M_{ht}^{\text{tot}}$. Via (C.39), a marginal unit of home currency therefore provides liquidity services $L_t f_{1,t}$, while a marginal unit of global currency provides liquidity services $L_t f_{2,t} Q_t$, where $f_{1,t}$ and $f_{2,t}$ are the partial derivatives of the function f with respect to their first and second argument, evaluated at $(M_{h,t}, Q_t M_{g,t})$. Equations (8) and (9) now become

$$1 \geq L_t f_{1,t} + E_t[\mathcal{M}_{t+1}] \quad (\text{C.41})$$

and

$$1 \geq L_t f_{2,t} + E_t\left[\mathcal{M}_{t+1} \frac{Q_{t+1}}{Q_t}\right] \quad (\text{C.42})$$

These equations and the usual properties of constant-returns-to-scale functions make clear, that the marginal liquidity services $L_t f_{1,t}$ and $L_t f_{2,t}$ provided by

either currency now depend on the ratio⁸ of their nominal values

$$\rho_t = \frac{Q_t M_{g,t}}{M_{h,t}} \quad (\text{C.43})$$

Propositions 4.1, 4.3 and 4.4 require appropriate modification. If f is not linear, then $i_t < i_t^*$ generally results in a tilt towards home currency and decrease in ρ_t rather than a complete elimination of the global currency, though the latter is still a possibility, if $f_2(\dots, 0) < \infty$, i.e. if the two currencies are substitutes, and the interest rate difference $i_t^* - i_t$ is sufficiently large. Likewise, $i_t > i_t^*$ generally results in a tilt towards the global currency and increase in ρ_t , rather than a complete elimination of the home currency. Once again, the latter can happen in the economically plausible case of substitutes and $f_1 < \infty$ as well as a sufficiently large interest rate differential $i_t - i_t^*$. These considerations add nuance to the main analysis, without changing its core message.

⁸For completeness and as usual, define the function $g(\rho) = f(1, \rho)$. Note that $M_{h,t}^{\text{tot}} = g(\rho_t)M_{h,t}$. Calculate that $f_{1,t} = g(\rho_t) - g'(\rho_t)\rho_t$ and $f_{2,t} = g'(\rho_t)$.