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**ABSTRACT**

This paper formalizes a classic idea that in second-best environments trade can induce welfare losses. In a framework that incorporates distortion wedges into a Melitz model, we analyze a channel in which trade can reduce allocative efficiency arising from the reallocation of resources. A key aggregate statistics that captures this negative selection is the gap between input and output shares. We derive sufficient conditions for reallocation loss due to trade under important distributions. Using Chinese manufacturing data, we show that this reallocation term is significantly negative, largely offsetting conventional gains to trade.

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# 1 Introduction

The question of how much developing countries benefit from opening up to goods trade is a time-honoured subject, both of practical import and intellectual interest. Much has been understood about the nature and type of gains to trade, thanks to the remarkable progress made in the field of international trade in recent decades. Less clear, however, is why certain developing countries have benefited from trade more than others, and why certain countries have seemingly benefited less—or not much at all.[1] New trade theories suggest that developing countries have the most to gain from trade: if trade liberalization can induce reallocation of resources from less to more productive firms, aggregate productivity and welfare will rise in turn.

But developing countries are different in another respect: they are also subject to prevalent policy and institutional distortions. Examples include taxes and subsidies to certain firms, implicit guarantees and bailouts, preferential access to land and capital, and industrial policy and export promotion policies—common themes in developing countries. In the case of China, this can explain why many less productive firms—such as certain state companies—have survived and even thrived. Many believe that joining the WTO can potentially alleviate some of these problems by inviting direct competition from abroad.

But how effective is the role of trade and can it necessarily improve allocations and lead to welfare gains? These issues are far from obvious as alluded to by Rodríguez-Clare (2018), " [a] complication that may matter for the computation of the gains from trade is the presence of domestic distortions." This argument that trade may exert a different impact in a second-best environment has been an old age question posed by Bhagwati and Ramaswami (1963). Even in classic textbook analysis, there are discussions on the "domestic market failure argument against trade", that " [when] the theory of second best [is applied] to trade policy..., imperfections in the internal function of an economy may justify interfering in its external economic relations" (Krugman, Obstfeld, and Melitz (2015)). We formalize these ideas in the context of the new trade model variety. A main contribution in this paper is to derive a general theoretical welfare formula to analyze

[1] For example, Waugh (2010) shows, in large sample of countries, that poor countries do not systematically gain more from trade.

additional channels of trade in a second-best environment. A second contribution is to take advantage of firm-level data to gauge how much these effects matter.

Our modelling framework incorporates firm-specific distortions into a two-country Melitz model. There are two dimensions of heterogeneity at the firm-level: productivity and distortions. The distortions in the benchmark model are assumed to be exogenous output wedges or factor wedges, which can arise from various kinds of policy and institutional distortions in developing countries. These distortions drive differences in the marginal products across firms.

Incorporating distortions changes the nature of firm selection. Contrary to the mechanism underpinning the Melitz (2003) model and its extensions—that trade can induce a reallocation of resources from low productivity to high productivity firms—the presence of distortions can bring about the opposite and exacerbate misallocation. The reason is simple: distortions (for instance, tax and subsidies) act as a veil to a firm's true productivity. A firm may be producing in the market not because it is inherently productive, but because it is sufficiently subsidized. A mass of highly-subsidized but not adequately productive firms will export and expand at the cost of other more productive firms. The high productivity/ high tax firms which were marginally able to survive in the domestic market would be driven out as the other firms gain market share and drive up costs. In other words, the selection effect which brings about gains in the Melitz-type model is no longer based solely on productivity; it is determined jointly by firm productivity and distortions. Trade may thus *lower* the average productivity of firms.

To formalize this argument, we derive a general welfare formula in Section 2 that relates to canonical trade models, such as Arkolakis, Costinot, and Rodríguez-Clare (2012) and Melitz and Redding (2015). We show that a key statistic in capturing this negative reallocation channel in the aggregate is the gap between aggregate input share in producing domestic goods and aggregate expenditure share on domestic goods. If the required inputs used for producing export is greater than the output share it yields, then the reduction in allocative efficiency arising from a reallocation of resources, occasioned by trade, can bring about a welfare loss.

To our knowledge, the paper is the first to theoretically characterize welfare loss to trade, and derive sufficient conditions for welfare. The decomposition of welfare into a 'pure technology effect' and a 'resource reallocation effect' in this instance resonates with the decomposition in the recent work of Baqaee and Farhi (2020). Whereas they focus on network effects, we focus on how distortions determine the welfare impact of trade.

The second contribution is to operationalize our results in the context of China. We choose China because it is known to be an economy with a variety of distortions, and also because it experienced an important trade liberalization event in the early 2000's. In our quantitative analysis, we expand upon the basic framework to one that incorporates additional wedges, including on fixed and variable costs to exporting. We use micro data from Chinese manufacturing, and examine how much departure there is between our model and the standard trade models without pre-existing domestic distortions. We find that welfare gains are much smaller when taking into account distortions. For China, allocative inefficiency led to a welfare loss of 12.9%, more than offsetting the conventional 'ACR' gains.

It is important to point out that in the quantitative analyses we do not use directly empirically-measured wedges, observed correlations, or distributions in the data. The reason is that the *observed* statistics are not the *underlying* ones: existing firms have been subject to selection and thus their observed distributions are not the true ones. The same reasoning goes for the observed correlation between productivity and wedges. As we show in Section 3.2, the presence of fixed costs and/or firm selection can drive a positive relationship between the two. For these reasons, the approach adopted in the quantitative exercises is to estimate the underlying joint distribution of wedges and productivity, costs of producing and exporting so as to match the observed patterns of firms' outputs, inputs, and exports.

This contrasts with the approach adopted in Berthou, Chung, Manova, and Sandoz (2018), Costa-Scottini (2018) and Ho (2010) which are other papers that incorporate firm-level distortions to trade models.[2] All three papers measure firm wedges or productivity directly from the data using common measures of marginal revenue product. Our work with Berthou et al. (2018) is broadly complementary. Theoretically, they derive a model

[2]Theoretically, Costa-Scottini (2018) and Ho (2010) assume a log linear relationship, and hence perfect correlation of (log) productivity and (log) wedges. This assumption of perfect correlation is both limiting in its scope of analysis, and also inconsonant with patterns in the data.

with distortions to marginal costs on the input, and establish gains from trade that is either positive or negative with misallocation. They illustrate these ambiguous results, and empirically assess the impact of trade on measured productivity for 14 European countries. By contrast, our theoretical work derives sufficient conditions for welfare loss due to reallocation, and quantitatively assess welfare gains using Chinese data.

Our model matches well moments in the data, including a number of observed differences between exporters and non-exporters, and among exporters alone. We also show that output distortions are the main drivers behind the welfare loss. As a robustness check, we also examine a model of endogenous distortions in Section 3.5.[3] We show that these distortions alone fail to match key aspects of the data: 1) they yield some obvious counterfactual predictions on the relationship between exporters and wedges; 2) endogenous distortions alone also generates too high correlation between measured productivity and wedges and explains little of the dispersion in wedges. To match the observed correlation and dispersion one would still need to include exogenous distortions.

What makes our paper different from the important works of Hsieh and Klenow (2009) (henceforward HK), Baily, Hulten, and Campbell (1992), Restuccia and Rogerson (2008), Bartelsman, Haltiwanger, and Scarpetta (2009) is first of all, the open economy nature of our model, and secondly, the endogenous mechanism of entry/exit and the attendant firm selection effect. Yang (2021) pointed out the importance of endogenous entry and selection in distorted HK closed economy, while we focus on the trade effects with firm level distortions. Empirical works have also demonstrated the importance of entry and exit for China's growth.[4]

On the one hand, China is well suited for the study for multiple reasons: for its prevalent State interventions and policies;[5] and that a body of work has shown that idiosyncratic

[3] In Appendix J, we show that a model with variable markups shares similar features with the endogenous distortion model. Moreover, the attendant pro-competitive effects in a model with endogenous markup may be 'elusive' as pointed out by Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2018).

[4] Brandt, Van Biesebroeck, and Zhang (2012) find that net entry accounts for roughly half of Chinese manufacturing productivity growth. The creation and selection of new firms in China's non-state sector has been particularly important.

[5] Specific policies that can drive these wedges include implicit subsidies such as soft budget constraints, favorable costs of capital, preferential tax treatments and implicit guarantees. Firms with political connections having access to special deals and receiving substantial benefits are also widely documented (see Guo, Jiang, Kim, and Xu (2013) and Bai, Hsieh, and Song (2019)). Wu (2018) conducts an empirical analysis and finds

distortions explain the majority of the dispersion in marginal products.[6] On the other hand, in this framework, positive firm selection is the central driving force for gains to trade. As such, it abstracts from other types of gains to trade, such as trade-induced technological diffusion (Alvarez, Buera, and Lucas Jr (2013) and Buera and Oberfield (2016)), adoption (Perla, Tonetti, and Waugh (2015) and Sampson (2015)) and innovation (Atkeson and Burstein (2010)). While these mechanisms in principle work to increase the gains to trade, with its quantitative significance a subject to debate,[7] it does not detract from the fact that the distortionary impact on allocation efficiency still induces large welfare losses, which is what we are interested in. Of course, distortions can also interact with some of these additional channels. For instance, in a model with firm innovation, one would need to consider the fact that distortions not only affect production decisions, but potentially also innovation decisions. Policy distortions can be introduced to serve other purposes, a consideration which is important but beyond the scope of this paper. We also do not consider how trade can reduce domestic distortions, for example if concurrent domestic reforms are requisite for joining the WTO or if quotas are removed (see Khandelwal, Schott, and Wei (2013)). However, in our quantitatively analysis, we do allow for firms to face a different distribution of distortions when they export and examine welfare gains therein.

Taken together, our quantitative analysis is meant to highlight the first-order effects of a particular channel—allocative inefficiency, and also to compare it with benchmark results in the workhorse models of international trade. An implication of this paper is that in order for developing countries to reap the full gains of trade, simultaneous or antecedent

that policy distortions can be explained by investment promoting programs that favor such firms. A recent study by Chen and Kung (2018) demonstrate the firms that are connected with political elites were able to obtain land at 80 to 90 percent discount over the period 2004-2016.

[6] Wu (2018) finds that policies account for the majority of the observed misallocation of capital, as opposed to financial frictions. Using a different approach and modeling framework, David and Venkateswaran (2017) find also that firm-specific distortions, rather than technological or information frictions, account for the majority of the observed dispersions in marginal products. Bai, Lu, and Tian (2018) disciplines financial frictions with firms' financing patterns, sales distribution and change of capital. They find that financial frictions cannot explain the observed relation between firms' measured distortions and size.

[7] Perla, Tonetti, and Waugh (2015) and Atkeson and Burstein (2010), for instance, find that trade gains are not too different from ACR gains. In Perla, Tonetti, and Waugh (2015), there is trade-induced within-firm productivity improvements. However, their aggregate growth effects come with costs—losses in variety and reallocation of resources away from goods production. Thus, the aggregate effect on welfare is similar to ACR gains. Atkeson and Burstein (2010) show that general equilibrium effects limits the first-order effects on aggregate productivity even when there is firm-level innovation.

domestic reforms aimed at reducing policy distortions may be crucial.[8]

# 2 Theoretical Framework

## 2.1 Baseline Model

The world consists of two large open economies, Home and Foreign, with heterogenous firms. The two economies can differ in the size of labor and distribution of firms. Labor is immobile across countries and inelastic in supply.

**Consumers.** A representative consumer in the Home country chooses the amount of final goods $C$ in order to maximize utility $u(C)$, subject to the budget constraint

$$PC = wL + \Pi + T,$$

where $P$ is the price of final goods, $L$ is labor, $w$ is wage rate, $\Pi$ is dividend income, and $T$ is the amount of lump-sum transfers received from the government.

**Final Goods Producers.** Final goods producers are perfectly competitive. A CES production function implies that aggregate output $Q$ and price index $P$ take the form

$$Q = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \qquad P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

where $\sigma$ is the elasticity of substitution across intermediate goods, $\Omega$ is the endogenous set of goods, $p(\omega)$ is the price of good $\omega$ in the market. The individual demand for the good is thus given by

$$q(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma} Q. \tag{1}$$

Henceforward, $\omega$ is suppressed for convenience.

**Intermediate Goods Producers.** There is a competitive fringe of potential entrants (in both countries) that can enter by paying a sunk entry cost of $f_e$ units of labor. Potential entrants

[8] The policy implication drawn from this framework is consistent with works indicating that policies aimed to neutralize domestic distortions may be complementary to trade liberalization (Chang, Kaltani, and Loayza (2009) and Harrison and Rodríguez-Clare (2010)).

face uncertainty about their productivity in the industry. They also face a stochastic revenue wedge $\tau$, which can be seen as a tax (>1) or subsidy (<1) on every revenue earned.[9] Once the sunk entry cost is paid, a firm draws its productivity $\varphi$ and $\tau$ independently from a joint distribution, $g(\varphi, \tau)$ over $\varphi \in (0, \infty), \tau \in (0, \infty)$.[10] Firms are monopolistically competitive. Those that sell domestically solve

$$\max_{p,q} \ \frac{pq}{\tau} - \frac{w}{\varphi} q - wf. \tag{2}$$

Production of $q$ units entails fixed cost of production $f$ and constant variable costs such that total labor required is $\ell = f + q/\varphi$.[11] If firms decide to export, they face a fixed exporting cost of $f_x$ units of labor and iceberg variable costs of trade $\tau_x > 1$ such that the exporting firm's problem is

$$\max_{p_x,q_x} \ \frac{p_x q_x}{\tau} - \frac{w}{\varphi} \tau_x q_x - w f_x,$$

where foreign demand is $q_x = \left(p_x/P_f\right)^{-\sigma} Q_f$, with $P_f$ and $Q_f$ denoting the aggregate price index and demand abroad. Firms with the same productivity and distortion behave identically, and thus we can index firms by their $(\varphi, \tau)$ combination. Let the optimal production and profit for domestic market be $q(\varphi, \tau)$ and $\pi(\varphi, \tau)$ and for the foreign market be $q_x(\varphi, \tau)$ and $\pi_x(\varphi, \tau)$.

Given the fixed cost of production, there is a zero-profit cutoff productivity below which firms would choose not to produce, or service the foreign market.[12] The cutoff productivi-

[9] It is equivalent to an input wedge on all the input a firm uses.

[10] The model equilibrium is equivalent to a stationary equilibrium of a model allowing for constant exogenous probability of death $\delta$ and entry cost $f_e/\delta$.

[11] We can easily extend the production to include capital, i.e. $k^\alpha \ell^{1-\alpha}$. The unit cost for producing $q$ or fixed cost is $\alpha^{-\alpha}(1-\alpha)^{\alpha-1} w^{1-\alpha} r_k^\alpha$ where $r_k$ is the rental cost of capital. In our simple model, we introduce one heterogeneous distortions at the firm level, and our $\tau$ is an output distortion, it includes all input distortions that increase the marginal products of capital and labor by the same proportion as an output distortion. In our quantitative exercises, we include both capital and labor, and also extend the model to consider heterogenous distortions in foreign markets that could be different from in the domestic market.

[12] Equilibrium price is the standard result $p = [\sigma/(\sigma-1)]\,(w\tau/\varphi)$, and thus domestic producing firm profits are $\pi(\varphi, \tau) = \sigma^{-\sigma}(\sigma-1)^{\sigma-1} P^\sigma Q w^{1-\sigma} \varphi^{\sigma-1} \tau^{-\sigma} - wf$. If firms export, the optimal export price is $p_x = [\sigma/(\sigma-1)]\,(w\tau_x\tau/\varphi)$, and exporting profits are $\pi_x(\varphi, \tau) = \sigma^{-\sigma}(\sigma-1)^{\sigma-1} P_f^\sigma Q_f (w\tau_x)^{1-\sigma} \varphi^{\sigma-1} \tau^{-\sigma} - wf_x$.

ties for servicing the domestic and foreign markets are

$$\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{wf}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}} w\tau^{\frac{\sigma}{\sigma-1}}, \qquad \varphi_x^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{wf_x\tau_x^{\sigma-1}}{P_f^\sigma Q_f}\right]^{\frac{1}{\sigma-1}} w\tau^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

These cutoffs are different for firms facing different levels of distortions. Low productivity firms that would have been otherwise excluded from the market can now enter the market and survive if sufficiently subsidized.

The government's budget is balanced so that the lump-sum transfers is given by

$$T = \int_{\omega\in\Omega_H}\left(1-\frac{1}{\tau}\right) p(\omega)q(\omega)d\omega,$$

where the endogenous set of goods $\Omega_H$ includes Home firms goods selling to both domestic and foreign markets.

**Equilibrium Conditions.** The equilibrium features a constant mass of entrants $M_e$ and producers $M$, along with an ex-post distributions of productivity and distortion among operational firms $\mu(\varphi,\tau) = g(\varphi,\tau)/\int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\, d\varphi d\tau$ if $\varphi \geq \varphi^*(\tau)$; and $\mu(\varphi,\tau) = 0$ otherwise. The probability of successful entry is $\omega_e = \int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\, d\varphi d\tau$, and of exporting conditional on entry is $\omega_x = \int\int_{\varphi_x^*(\tau)}^{\infty} \mu(\varphi,\tau)\, d\varphi d\tau$. In equilibrium, the measure of producing firms equals the product of measure of entrants and the probability of entering: $\omega_e M_e = M$.

Foreign economy has a distribution $g_f(\varphi,\tau)$ on productivity and distortion. Its measure of entrants and producers are given by $M_{ef}$ and $M_f$, the cutoff productivities are $\varphi_f^*(\tau)$ and $\varphi_{xf}^*(\tau)$, and its ex-post distributions of operational firms is $\mu_f(\varphi,\tau)$.

In equilibrium, the Home price index $P$ satisfies:

$$P = \frac{\sigma}{\sigma-1}\left[M\int\int_{\varphi^*(\tau)}^{\infty}\left(\frac{w\tau}{\varphi}\right)^{1-\sigma}\mu(\varphi,\tau)\, d\varphi d\tau + M_f\int\int_{\varphi_{xf}^*(\tau)}^{\infty}\left(\frac{w_f\tau_x\tau}{\varphi}\right)^{1-\sigma}\mu_f(\varphi,\tau)\, d\varphi d\tau\right]^{\frac{1}{1-\sigma}}. \tag{4}$$

Another key equations is free entry condition:

$$\int \int_{\varphi^*(\tau)} \pi(\varphi, \tau) g(\varphi, \tau) d\varphi d\tau + \int \int_{\varphi_x^*(\tau)} \pi_x(\varphi, \tau) g(\varphi, \tau) d\varphi d\tau = w f_e, \tag{5}$$

which, combined with labor market clearing implies an equation for the measure of producing firms:

$$M = \frac{L}{\sigma \left( \frac{f_e}{\omega_e} + f + \omega_x f_x \right)}. \tag{6}$$

The equilibrium conditions of price index $P_f$, free entry, and labor market clearing in Foreign take similar forms as those in Home. In addition, the assumption of balanced trade yields

$$P_f^{\sigma} Q_f M \int \int_{\varphi_x^*(\tau)}^{\infty} \left( \frac{w \tau_x \tau}{\varphi} \right)^{1-\sigma} \mu(\varphi, \tau) \, d\varphi d\tau = P^{\sigma} Q M_f \int \int_{\varphi_{xf}^*(\tau)}^{\infty} \left( \frac{w_f \tau_x \tau}{\varphi} \right)^{1-\sigma} \mu_f(\varphi, \tau) \, d\varphi d\tau, \tag{7}$$

Normalizing the Home country wage rate to 1, there are eleven equations, the two zero cutoff productivities for domestic production and exporting (3), the definition of price indices (4), the free entry conditions (5), the labor market clearing condition (6) and all of their Foreign counterparts, along with a goods market clearing/balanced trade equation (7). These equations yield the equilibrium consisting of eleven unknowns $\{\varphi^*(\tau)$, $\varphi_x^*(\tau)$, $\varphi_f^*(\tau)$, $\varphi_{xf}^*(\tau)$, $P$, $P_f$, $Q$, $Q_f$, $M$, $M_f$, $w_f\}$. A detailed derivation of the model is provided in Appendix A.

**Proposition 1.** *The allocations, entrants, and cutoff functions* $\{\varphi^*(\tau)$, $\varphi_x^*(\tau)$, $\varphi_f^*(\tau)$, $\varphi_{xf}^*(\tau)$, $Q$, $Q_f$, $M$, $M_f\}$ *are independent of mean wedge* $\bar{\tau}$*. Prices* $\{P, P_f, w_f\}$ *change proportionally with* $\bar{\tau}, \bar{\tau}^f$*, i.e.* $P(\bar{\tau}_1)/P(\bar{\tau}_2) = \bar{\tau}_1/\bar{\tau}_2$*, and similarly for* $P_f$ *and* $w_f$*.*

The proposition shows that increasing the mean of the wedges doesn't affect real variables. Hence misallocation of resources is not because of the average wedge across firms but heterogenous wedges.

## 2.2 Theoretical Comparative Static

We proceed to analyze welfare with distortions. Welfare, denoted as $W$, is evaluated with the final consumption per capita $C/L$, which equals $Q/L$ in equilibrium. Simple algebra has it that $Q/L = PQ/PL = (PQ/L)(1/P)$, where $PQ/L$ is the revenue-based total factor productivity of the economy, i.e. $PQ/L = \overline{TFPR}$. Using the price index (4) and the balanced trade condition (7), we get an expression for welfare,

$$W = \frac{\sigma-1}{\sigma} M_e^{\frac{1}{\sigma-1}} \left[ \int\int_{\varphi^*(\tau)} \left( \varphi \frac{\overline{TFPR}}{MRPL_\tau} \right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \int\int_{\varphi_x^*(\tau)} \left( \frac{\varphi}{\tau_x} \frac{\overline{TFPR}}{MRPL_\tau} \right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau \right]^{\frac{1}{\sigma-1}}, \tag{8}$$

where $MRPL_\tau$ is the firm-specific marginal revenue product of labor, $MRPL_\tau = w\tau$. This expression shows that welfare is related to a weighted firm productivity using relative distortions as weights. In an efficient case without distortions, all firms have the same marginal revenue product, $MRPL_\tau = \overline{TFPR} = w$. The source of welfare loss here can arise from a misallocation of resources, captured by dispersions in $\overline{TFPR}/MRPL_\tau$, and a misallocation caused by selection and entry mechanisms captured by $M_e$, $\varphi^*, \varphi_x^*$ being different from their respective efficient levels.

A change in productivity or trade cost affects the economy through selection, entry and misallocation. To see this, we first list our notations. Let $p_i q_i$ and $\ell_i$ be the total sales and variable labor of firm with productivity $\varphi_i$; let $\lambda$ be the share of the expenditure on domestic goods (as in Arkolakis, Costinot, and Rodríguez-Clare (2012)):

$$\lambda = \frac{\int\int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau}{\int\int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int\int_{\varphi_x^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau}. \tag{9}$$

We also define $S$ to be the share of variable labor used in producing domestic goods,

$$S = \frac{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int\int_{\varphi_x^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}. \tag{10}$$

It is easy to see from the above two definitions that without distortions, S is the same as $\lambda$. With distortion, a firm's variable labor is not proportional to its sales, and so $S$ and $\lambda$ are not the same.

As in Arkolakis, Costinot, and Rodríguez-Clare (2012) (henceforward ACR) and Melitz and Redding (2015) (henceforth MR) , a concept capturing the extensive margins is

$$\gamma_\lambda(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi,\tau) d\varphi d\tau\right]}{d\ln\hat{\varphi}}, \quad \gamma_s(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau\right]}{d\ln\hat{\varphi}}, \tag{11}$$

where $\gamma_\lambda(\hat{\varphi})$ denotes the elasticity of the cumulative sales of firms above any cutoff $\hat{\varphi}$ within a market, with respect to the cutoff. In our setup with distortions, we also need $\gamma_s(\hat{\varphi})$ which is the elasticity of the cumulative variable labor of firms above any cutoff $\hat{\varphi}$ within a market, with respect to the cutoff.[13]

To build intuition, we first analyze the impact of a firm-specific technology shock in a closed economy and a trade cost shock in an open economy.

**Proposition 2.** *In the presence of distortions,*

1. *In a closed economy, the change in welfare associated with an exogenous productivity shock to firms with productivity $\varphi_i$ is*

$$dlnW = -d\ln P + (\sigma-1)\left[\frac{p_i q_i}{PQ} - \frac{\ell_i}{L}\right] d\ln\varphi_i + \left[\gamma_s(\hat{\varphi}^*) - \gamma_\lambda(\hat{\varphi}^*)\right] d\ln\hat{\varphi}^*.$$

2. *In an open economy, the change in welfare associated with an exogenous iceberg cost shock is*

$$d\ln W = -d\ln P + \left[-d\ln\lambda + d\ln S\right] + \left[\gamma_s(\hat{\varphi}^*) - \gamma_\lambda(\hat{\varphi}^*)\right] d\ln\hat{\varphi}^*.$$

PROOF: Appendix B.1.

Technology shocks can have two effects on welfare. The first is through a change in the aggregate price index $P$, where a positive productivity shock or a negative trade shock

[13]Using these definitions, we get the same formula for the partial trade elasticity as in ACR,

$$-\frac{\partial\ln(1-\lambda)/\lambda}{\partial\ln\tau_x} = \sigma - 1 + \gamma_{\lambda f}(\hat{\varphi}^*_{xf}) + (\gamma_{\lambda f}(\hat{\varphi}^*_{xf}) - \gamma_\lambda(\hat{\varphi}^*))\frac{\partial\ln\hat{\varphi}^*}{\partial\ln\tau_x}, \tag{12}$$

where $\hat{\varphi}^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{wf}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}} w$ and $\hat{\varphi}^*_{xf} = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{w_f f_x \tau_x^{\sigma-1}}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}} w_f$. Here $\sigma-1$ captures the intensive-margin elasticity, and $\gamma_\lambda$ and $\gamma_{\lambda f}$ evaluated at the cutoffs capture extensive margins. The definition of $\gamma_\lambda(\hat{\varphi})$ functions are the same as in ACR and MR. But with distortions, trade share and their elasticities are not sufficient to reflect welfare change, labor share $S$ and $\gamma_s(\hat{\varphi})$ also matter.

lowers $P$ and leads to a welfare gain. The second effect is coming from a change in the resources going into each firm. Holding fixed the labor used by each firm, the productivity shock increases the producer's sales. But the shock also changes relative prices, and in turn demand, which causes a reallocation of labor among firms. If firm $i$ is relatively subsidized, then $1/\tau_i$ is larger than the average level of distortion, and its labor share is larger than its output share. This producer is too large relative to the efficient allocation, and thus, reallocating labor towards this firm worsens allocative efficiency. This has a similar flavor to findings in Baqaee and Farhi (2020) (henceforward BF), which shows that the effect of a micro shock to productivity or wedge on output can be decomposed into a "pure technology effect" and a "resource allocation effect". The former is the change in output holding fixed the share of resources going to each user; the latter is the change in output resulting from the reallocation of shares of resources across users.[14]

The same reasoning applies to trade cost shocks in an open economy. If trade induces a reshuffling of resources, and the relatively subsidized firms expand thanks to this 'technology' shock, allocative efficiency can deteriorate. Thus, apart from a first-order welfare gain working through lowering the price index, a reduction in trade costs also can incur a first-order loss through the reallocation of resources. This open-economy case is more complex compared to the closed-economy setting because trade has a differential impact on firms. Some firms that remain domestic producers are not directly affected by the trade cost shock, while others may be selected into exporting, and some may be ousted from producing altogether. Despite these heterogeneous effects, a neat result arises: the gap between aggregate input share and aggregate sales share is informative of the allocative efficiency. If the change in aggregate domestic labor share $S$ is greater than the change in domestic expenditure share $\lambda$, then the trade cost shock could be welfare-reducing.

More precisely, we can view the reallocation effect as arising from an intensive margin: holding selection fixed, i.e., $d \ln \hat{\varphi}^* = 0$, the shifting of resources among an existing set of firms is captured by $(d \ln S - d \ln \lambda)$. From an extensive margin, where selection and cutoffs $\hat{\varphi}^*$ change, the gap of $S$ and $\lambda$ and $\gamma_\lambda$ and $\gamma_S$ summarize the reallocation of resources towards or away from more distorted firms (more on this below).

[14] The difference between BF and our closed-economy model is the endogenous firm selection and entry captured by the last term.

We now go one step further in understanding these reallocation effects, and how the welfare expression in Proposition 2 relates to trade gains arising from canonical trade models. We derive a general expression for changes in welfare associated with changes in trade costs. Arkolakis, Costinot, and Rodríguez-Clare (2012) demonstrate that in the absence of distortions, welfare changes across a wide class of models can be inferred using two variables: (i) changes in the share of expenditure on domestic goods; and (ii) the elasticity of bilateral imports with respect to variable trade costs (the trade elasticity). Different trade models can have different micro-level predictions, sources of welfare gains, and different structural interpretations of the trade elasticity. But conditional on observed trade flows and an estimated trade elasticity, the welfare predictions are the same. The generality of this formulation, however, relies on a certain set of macro-level restrictions. Melitz and Redding (2015) show that under more general distribution functions for productivity, the trade elasticity is no longer invariant to trade costs and across markets, and therefore no longer a sufficient statistic for welfare. Micro-level information is still important for welfare.

In the analysis below, we first consider a fall in trade costs in an open economy equilibrium. Substituting the trade balance condition (7) into the price index equation (4), and the labor market condition (6) into the free entry condition (5), while combining the differentiation of the two conditions yield a general representation of welfare:

**Proposition 3.** *(**General Welfare Expression**) The change in welfare associated with an iceberg cost shock is*

$$
\begin{aligned}
d\ln W = \frac{1}{\gamma_s+\sigma-1}\Big\{ & \overbrace{-d\ln\lambda}^{ACR} + \overbrace{d\ln M_e}^{MR} && (13)\\
& + \frac{\sigma}{\sigma-1}(\gamma_s-\gamma_\lambda)\,d\ln M_e && (\textit{Entry Distortion})\\
& - \left(\sigma-1+\frac{\sigma\gamma_s}{\sigma-1}\right)d\ln\lambda + \left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right)d\ln S\Big] && (\textit{Reallocation})
\end{aligned}
$$

PROOF: Appendix B.2.

The above proposition encapsulates welfare results for three different cases:

1. Without domestic distortions, $S=\lambda$ and $\gamma_s=\gamma_\lambda$. If productivity follows a Pareto

distribution with parameter $\theta$, $\gamma_\lambda = \theta - \sigma + 1$ and $d\ln M_e = 0$. Hence, as in ACR

$$d\ln W = \frac{1}{\theta}\left[-d\ln\lambda\right].$$

2. Under a general distribution function and without domestic distortions, $S = \lambda$, $\gamma_s = \gamma_\lambda$ (non constants), and $d\ln M_e \neq 0$. Hence,

$$d\ln W = \frac{1}{\gamma_\lambda(\hat{\varphi}^*) + \sigma - 1}\left[-d\ln\lambda + d\ln M_e\right].$$

Here, the micro structure matters for $\gamma_\lambda$ and hence welfare, as in MR.

3. With homogenous productivity and Pareto-distributed domestic distortion $1/\tau$ with parameter $\theta$, $\gamma_\lambda = \frac{\sigma-1}{\sigma}(\theta - \sigma + 1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta - \sigma)$. Hence,

$$d\ln W = \frac{\sigma}{\sigma - 1}\left[-d\ln\lambda + d\ln S\right].$$

In this general welfare representation, we label the first term as *ACR*,[15] the second term *MR*, the third term *entry distortion*, and the fourth term *reallocation*. The first two terms relate to $d\ln\lambda$ and $d\ln M_e$, so we label them as ACR and MR. The last two terms are absent in ACR or MR. Note that all of the equilibrium terms are affected by distortions. We label the last term as reallocation as it singles out the key gap—between sales and labor share. Information on the change of domestic shares (sales and variable labor), the measure of entrants, the joint distribution of firms sales and variable inputs, and the cutoff firms (from which we know $\gamma_\lambda$ and $\gamma_s$) are sufficient for computing the associated welfare change for a local change in trade cost.

It's useful to examine the special cases embedded therein. If there is only heterogeneity in productivity, Pareto-distributed (case 1), the ACR formula is recovered. The case without distortion and under a more general productivity distribution gives rise to MR. The second special case is that under misallocation, where there is only heterogeneity in distortions which is Pareto distributed, an analogue formula to ACR can be obtained: the difference in the change in the domestic labor and sales share provides a sufficient statistics for welfare.

[15] To be precise, the coefficient is not a trade elasticity and no longer a structure constant.

In the two special cases (1) and (3), firm selection is either driven solely by productivity, or solely by distortions. The former implies that there is always a welfare improvement when the economy opens up to trade, whereas the latter implies that there is an unambiguous loss (see corollary below). In the more general case, productivity and distortions jointly determine firm selection. The resource reallocation is both one amongst existing firms (last term), and along the entry dimension (third term). Without distortions, a firm's share of input is equal to its share of sales, so that in aggregate, $S = \lambda$. In the presence of distortions, the two are no longer equal. The gap between input and sales shares is informative about changes in allocative efficiency. If the change in required inputs exceeds the change in revenue it produces, i.e. $d \ln S < d \ln \lambda$, it means with further opening up, the input share used to produce for exports exceeds the export revenue share. Resources reallocation has induced an efficiency loss.[16]

**Corollary 1.** *(**Welfare Loss**) Under homogenous productivity and Pareto-distributed domestic distortion $1/\tau$ with parameter $\theta$,*

$$d \ln W = \frac{\sigma}{\sigma - 1} \left[ d \ln S - d \ln \lambda \right].$$

1. *Moving from a closed economy to an open economy always entails a welfare loss, as $\lambda > S$.*
2. *In the open-economy equilibrium, the reallocation term is $\frac{\sigma}{(\sigma-1)\theta}[(1-\theta) d \ln \lambda + \theta d \ln S]$ and is always negative.*[17]

PROOF: Appendix B.3. This corollary presents two important features under the special case (3). First, compared to the closed economy, an open economy with any level of finite iceberg trade cost always has a lower welfare— so long as there is selection into exporting. Second, in an open economy equilibrium, a marginal reduction of iceberg cost always brings about a negative reallocation effect, so long as it results in a higher fraction of exporters in equilibrium, thus worsening misallocation with the reduction in trade costs.

[16] We also study the welfare expression in the Ricardian model and Armington model with distortions in Bai, Jin, and Lu (2020). The gap between domestic input share and sales share in total expenditure is still informative of resource reallocation. A proposition similar to Corollary 1 holds for these two models with distortions.

[17] In the welfare expression, the second and third terms cancel out, and the welfare change is $\frac{\sigma}{(\sigma-1)\theta}[-d \ln \lambda - \left(\sigma - 1 + \frac{\sigma \gamma_s}{\sigma - 1}\right) d \ln \lambda + \left(\sigma - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1}\right) d \ln S] = \frac{\sigma}{(\sigma-1)\theta}[-d \ln \lambda + (1-\theta) d \ln \lambda + \theta d \ln S]$, where the sign of $(1-\theta) d \ln \lambda + \theta d \ln S$ is always negative.

The intuition for why the open economy has a lower welfare than the closed economy is made transparent by this special case: under homogenous productivity, efficient allocation is that either all firms export or none of them export, and hence firms should have identical market shares. However, with distortions, the relatively subsidized firms produce more than others, with the dispersion of sales (employment) reflecting the distortions. Trade exacerbates misallocation as the relatively subsidized firms export and expand, which makes the firm distribution even more skewed. Furthermore, the share of labor required in producing domestic goods ends up being less than the domestic output share. When firm selection is purely driven by distortions, allocative efficiency deteriorates when moving from autarky to an open economy.

Point 2 in the above Corollary focuses on a local change in trade costs in the open economy equilibrium. This change of welfare reduces to two terms: the standard ACR term, where $-d\ln\lambda > 0$, and the reallocation term, which is always negative. Overall, the change in welfare in the open economy equilibrium displays a U-shape pattern. Hence, for a marginal change in trade cost, the welfare change can be negative or positive: for high levels of trade cost, there is a welfare loss; and for low levels of trade cost, there is a welfare gain. The reason is that firm selection driven by distortions is less significant when trade costs are small (at zero trade cost all firms export), and the welfare gains dominate the losses associated with reallocation.

Having established sufficiency conditions in the special case, we can now examine a more general case. The necessary condition for the reallocation term to be negative is: either $\gamma_s \leq \gamma_\lambda$ or $d\ln S \leq d\ln\lambda$. Intuitively, misallocation happens when the input elasticity is smaller than the revenue elasticity or when more resources are used to produce the same unit of revenue. The following corollary presents a sufficient condition for $\gamma_s \leq \gamma_\lambda$ for a more general distribution for productivity and distortions.

**Corollary 2.** *Suppose $(\tau, \varphi)$ are jointly log-normal with standard deviations of $\sigma_\tau$ and $\sigma_\varphi$ and correlation $\rho$. When $\sigma_\tau \geq \frac{\sigma-1}{\sigma}\rho\sigma_\varphi$, then;*

1. *The cumulative variable labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order, with the cumulative labor (I) and*

*sales (O) share in the domestic market defined as* [18]

$$I(\hat{\varphi}) = \frac{\int \int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int \int_0^{\inf} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}, \quad O(\hat{\varphi}) = \frac{\int \int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{1-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int \int_0^{\inf} \varphi^{\sigma-1}\tau^{1-\sigma} g(\varphi,\tau) d\varphi d\tau}. \tag{14}$$

2. *The hazard functions $\gamma_s \leq \gamma_\lambda$ and shares $S \leq \lambda$ at any cutoff, hence moving from a closed economy to an open economy, the reallocation term is always negative.*

PROOF: Appendix B.4.

Under the condition $\sigma_\tau \geq \frac{\sigma-1}{\sigma}\rho\sigma_\varphi$, the cumulative labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order. Theoretically, we can prove that moving from a closed to open economy, the labor share used to produce exports is always greater than the export share of total sales, or in other words, the share of variable labor used in producing domestic goods is always smaller than the share of expenditure on domestic goods, i.e., $S \leq \lambda$. This implies that going from a closed to open economy $d\ln S$ is more negative than $d\ln\lambda$.

Intuitively, recall that cutoffs for production or exporting are related to firm profits, which now depend on $(\varphi, \tau)$, and the cumulative labor share and sales share distribution are functions of different values for $\hat{\varphi}$, as in (Eq.14). Since likelihood dominance implies first-order stochastic dominance, under the above condition the cumulative labor share distribution has more mass among higher profit firms than the cumulative output share distribution. Thus, when the economy opens to trade, higher profit firms start to export, the share of labor used to produce exports would exceed the export share.

The condition $\sigma_\tau \geq \frac{\sigma-1}{\sigma}\rho\sigma_\varphi$ holds for sure when the correlation is negative $\rho < 0$. In this case, more productive firms are more likely to be subsidized. It is even so for exporters, who end up with larger labor shares than sales share. Hence the reallocation term is always negative, and loss from trade is possible even when the correlation is negative. See Appendix C for numerical results with different $\rho$.

Proposition 3 applies to both symmetric and asymmetric countries and takes into con-

[18] The hazard functions of these distributions, $-\frac{d\ln(1-I(\hat{\varphi}))}{d\ln\hat{\varphi}}$ and $-\frac{d\ln(1-O(\hat{\varphi}))}{d\ln\hat{\varphi}}$, are exactly the elasticities $\gamma_s$ and $\gamma_\lambda$ defined before as (11).

sideration the impact of the Foreign distribution of firms on the Home country. It also shows the effect of domestic distortions on a Foreign country. In the case where the Foreign economy does not have distortions, the third and fourth terms in Foreign's welfare formula go to zero. The Foreign welfare is provided in the following proposition:

**Proposition 4.** *(**Foreign Welfare**) In the case that the Foreign economy is devoid of distortions,*

$$d \ln W_f = \frac{1}{\gamma_f + \sigma - 1}[-d \ln \lambda_f + d \ln M_{ef}].$$

Thus, Home's domestic distortions affect Foreign only through Foreign's $\lambda_f$, $M_{ef}$, the cutoffs $\varphi_f^*$, and hence $\gamma_f$. Appendix H.1 discusses how Home distortions affect Foreign in details using the estimated model in Section 3.

Proposition 3 is useful for making transparent the key mechanism that underlies how trade and misallocation affect welfare. It also easily relates to the existing literature, such as the ACR and MR formulations. Thus, it is the main decomposition we emphasize. However, the welfare decomposition can take on various guises. For example, using the equilibrium conditions, we can decompose welfare changes into domestic sales share, entry, and aggregate $\overline{TFPR}$ in the form[19]

$$d \ln W = \frac{1}{\gamma_\lambda + \sigma - 1}[-d \ln \lambda + d \ln M_e] + \left( \frac{\gamma_\lambda/(\sigma - 1)}{\gamma_\lambda + \sigma - 1} + 1 \right) d \ln \overline{TFPR}, \tag{15}$$

where the aggregate wedge is defined as the revenue product of labor, $\overline{TFPR} = PQ/L$. With a constant labor, the change in the aggregate wedge is the same as the change of aggregate expenditure $PQ$, i.e. $d \ln \overline{TFPR} = d \ln PQ$. Trade changes the aggregate expenditure (and thus selection) and hence welfare in the economy. The change in the aggregate wedge $d \ln \overline{TFPR}$ maps onto the reallocation effect in Proposition 3. The gap between $d \ln S$ and $d \ln \lambda$, and the extensive margins help us to understand how trade affect the resource reallocation and aggregate wedge. A fall in aggregate wedge implies welfare losses—as resources are further reallocated towards more subsidized firms.

It worths noting that potential welfare losses from trade for a country with distortions are not because this country subsidizes exports and the foreign country benefits from it.

[19] See Appendix B.5.

To highlight this point and abstract from the terms of trade effect, we study a numerical example in Appendix C with symmetric countries, both of which have domestic distortions. In this example, both countries suffer from trade. The example demonstrates that using ACR term under distortions leads to a large departure: welfare *losses* become welfare gains in this case. Thus, using aggregate observables to infer welfare gains as in ACR can be very misleading in the presence of distortions, unlike in the efficient case where ACR is a good approximation.

# 3 Quantitative Analysis

This section presents estimates of the quantitative effects of trade liberalization when including domestic distortions, estimating the model to data corresponding to China and the U.S.. We extend the benchmark model to incorporate additional heterogeneity in distortions, allowing firms facing different distortions in the foreign market. We proceed to discuss an important issue surrounding measurement—why distortions and productivity *cannot* be measured directly from the data—a customary approach adopted in past and present works. Instead, we take the moments related to distortion and productivity in the data to estimate our model, and then evaluate welfare following trade liberalization. Our aim is not to provide a full-fledged quantitative account of China's trade liberalization experience. For that, one would need a much richer model with a complex set of mechanisms. Instead, the main purpose is to use China as an example to demonstrate the large quantitative and qualitative differences that may arise under a model with distortions, compared to the standard model without distortions. A substantial negative reallocation effect can offset much of the gains to trade commonly understood.

## 3.1 Extended model

The benchmark model in Section 2 is expanded upon to include two additional wedges, an export wedge $\tau_{ex}$ on foreign sales, and a wedge on the fixed cost of exporting $\tau_{fx}$. These two wedges allow firms to face different distortions in the foreign market. A firm draws a quadruple $(\varphi, \tau, \tau_{ex}, \tau_{fx})$ from a cumulative distribution $G(\varphi, \tau, \tau_{ex}, \tau_{fx})$. The optimization problem for domestic production is the same as in (2). The exporting problem becomes

$$\max \frac{p_x q_x}{\tau_{ex}} - \frac{w}{\varphi}\tau_x q_x - w\tau_{fx} f_x,$$

where the last term reflects additional wedge on fixed exporting cost. Firms pay $w\tau_{fx}f_x$, but workers only receive $wf_x$. The firm exports if and only if its productivity is higher than the exporting cutoff $\varphi_x^*(\tau_{ex}, \tau_{fx})$ given by

$$\varphi_x^*(\tau_{ex}, \tau_{fx}) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1}\left[\frac{f_x \tau_x^{\sigma-1}}{P_f^{\sigma} Q_f}\right]^{\frac{1}{\sigma-1}} w^{\frac{\sigma}{\sigma-1}} \tau_{fx}^{\frac{1}{\sigma-1}} \tau_{ex}^{\frac{\sigma}{\sigma-1}}.$$

Either a low wedge on sales or a low wedge on fixed cost of exporting raises the export participation of the firm. A detailed derivation of the extended model is provided in Appendix D.

**Proposition 5.** *The change in welfare associated with an iceberg cost shock is*

$$\begin{aligned} d\ln W = \frac{1}{\gamma_s + \sigma - 1}\Big\{ & \overbrace{-d\ln\lambda}^{ACR} + \overbrace{d\ln M_e}^{MR} && (16)\\ & + \frac{\sigma}{\sigma-1}(\gamma_s - \gamma_\lambda) d\ln M_e && (\textit{Entry distortion})\\ & - \left(\sigma - 1 + \frac{\sigma\gamma_s}{\sigma-1}\right) d\log\lambda + \left(\sigma - 1 + \frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln S && (\textit{Reallocation})\\ & + \left(\sigma - 1 + \frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln\left(1 + \frac{M_e f_x \int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty} (\tau_{fx} - 1)\, dG}{L}\right)\Big\}. && (\textit{Fixed Cost}) \end{aligned}$$

Proof: see Appendix E.

As it turns out, the welfare decomposition takes on a similar form as in the benchmark model provided in Proposition 3. It also holds for asymmetric countries and for general distributions of $G(\varphi, \tau, \tau_{ex}, \tau_{fx})$. The additional term reflects the fixed cost wedges (*Fixed Cost*). As can be seen, when there are no extra heterogenous wedges on fixed cost, i.e. $\tau_{fx} = 1$ for all firms, the last term becomes zero and the main Proposition 3 holds exactly as before, even with different levels of distortions in domestic markets $\tau$, and in foreign markets $\tau_{ex}$. We quantitatively assess the importance of distortions to output versus distortions to exporting fixed costs below.

## 3.2 Data and Measurement

The data for Chinese firms comes from an annual survey of manufacturing enterprises collected by the Chinese National Bureau of Statistics. The dataset includes non-state firms with sales over 5 million RMB (about 600,000 US dollars) and all of the state firms for the 1998-2007 period. Information is derived from the balance sheet, profit and loss statements, and cash flow statements, which incorporate more than 100 financial variables. The raw data consist of over 125,858 firms in 1998 and 306,298 firms by 2007.

Our strategy is to use the observed distributions of inputs, outputs, export participation, and export intensity from Chinese firm-level data to estimate the underlying joint distribution of distortions and productivity in conjunction with other parameters in the model. We do not recover the joint distribution of productivity $\varphi$ and domestic distortion $\tau$ directly from the data for two reasons. First, neither a firm's productivity nor its distortions can be measured directly. Second, firm selection affects the observed joint distribution of wedge and productivity. The observed joint distribution in the data is the ex-post one after selection, rather than the underlying one. The importance of these two issues merits a full elaboration below. The model estimation is then detailed in Section 3.3.

The customary way to recover a firm's productivity or distortion is to use its value added per input. This measurement, however, could be contaminated with the presence of distortions or fixed cost of producing. Consider a non-exporting firm. Using the first order condition of a firm's optimization problem (2), we can write the valued added per input as

$$\frac{pq}{\ell} \propto \tau \left[ 1 - \frac{f}{\ell(\varphi, \tau)} \right], \tag{17}$$

where $f$ is the fixed cost of producing. This corresponds to what is referred to as 'TFPR'. If there are no wedges, $\tau = 1$, the value added per input increases with input $\ell$, which in turn increases with a firm's physical productivity, as in Melitz. If there are no fixed costs, $f = 0$, the value added per input actually measures the firm's wedges, as in HK. With both wedges and fixed costs, the value added per input not only depends on the productivity of the firm but also on the true wedge $\tau$. For this reason, the valued added per input cannot be used to directly recover the firm's productivity or its wedges.

The second reason for which one cannot recover the joint distribution directly from the data is firm selection. The observed dispersion and correlation of some measured wedge and productivity pertains to operating firms only. Hence, it embodies an endogenous selection mechanism. For instance, even if the underlying correlation were negative, the selection mechanism can induce the observed correlation to *become positive*, for the simple reason that high-taxed firms must be more productive in order to stay in the market. The selection mechanism will strengthen any underlying correlation between the two variables. For the same reason, the observed dispersions of the two variables are also the ones after selection has taken place. In order to compute the impact of distortions on welfare and productivity gains, one would need to know the underlying correlation and dispersion, and therefore one would need micro data and a structural model to uncover it. This is exactly the approach we adopt, detailed in the following subsection.[20]

### 3.3 Parameterization and moments

We assume that the joint distribution $G$ in the home country follows a multivariate log normal distribution with zero mean $\mu$ and a variance-covariance matrix $\Sigma$, which is characterized by four standard deviations $(\sigma_\varphi, \sigma_\tau, \sigma_{ex}, \sigma_{fx})$ and six correlations $(\rho_{\varphi,\tau}, \rho_{\varphi,\tau_{ex}}, \rho_{\varphi,\tau_{fx}}, \rho_{\tau,\tau_{ex}}, \rho_{\tau,\tau_{fx}}, \rho_{\tau_{ex},\tau_{fx}})$. We set the elasticity of substitution between varieties $\sigma$ to be 3 as in HK. This value is consistent with the estimates from plant-level US manufacturing data in Bernard, Eaton, Jensen, and Kortum (2003). The Home labor $L$ and the entry cost $f_e$ are normalized to 1. We choose foreign labor $L_f$ to be 0.2 to match the relative labor force of the US to China. Given that Foreign affects Home only though aggregate variables, we can assume that Foreign is without distortions, while taking the fixed costs $f_e$, $f$, and $f_x$, iceberg cost $\tau_x$, and the dispersion of productivity $\sigma_\varphi$ to be the same as those in Home. Then we estimate the mean of foreign productivity $\mu_{f\varphi}$ to match the relative GDP of US to China.

[20] Note that our approach is different from the existing literature, for example Costa-Scottini (2018) and Ho (2010), which assume a perfect correlation of (log) productivity and (log) wedges and take the joint distribution of measured productivity and average revenue product of labor directly from the data.

Table 1: Parametrization and Moments

Panel A: Parameters

| *Endogenously chosen* | Value |
|---|---|
| Fixed cost of producing $f$ | 0.07 |
| Fixed cost of export $f_x$ | 0.14 |
| Iceberg trade cost $\tau_x$ | 2.61 |
| Mean foreign prod $\mu_{f\varphi}$ | 3.32 |
| Std. productivity $\sigma_\varphi$ | 1.31 |
| Std. distortion on home sales $\sigma_\tau$ | 1.05 |
| Std. distortion on export sales, exporters $\sigma_{\tau_{ex}}$ | 0.95 |
| Corr(prod., domestic distortion) $\rho_{\varphi,\tau}$ | 0.89 |
| Corr(prod., foreign sale distortion) $\rho_{\varphi,\tau_{ex}}$ | 0.65 |
| Corr($\tau$, $\tau_{ex}$) $\rho_{\tau,\tau_{ex}}$ | 0.68 |
| Std. distortion on export fixed cost $\sigma_{\tau_{fx}}$ | 0.65 |
| Corr($\varphi$, $\tau_{fx}$) $\rho_{\varphi,\tau_{fx}}$ | 0.30 |
| Corr($\tau$, $\tau_{fx}$) $\rho_{\tau,\tau_{fx}}$ | −0.10 |
| Corr($\tau_{ex}$, $\tau_{fx}$) $\rho_{\tau_{ex},\tau_{fx}}$ | 0.00 |

Panel B: Moments

| *Targeted moments* | Data | Model |
|---|---|---|
| Fraction of firms producing | 0.85 | 0.86 |
| Fraction of firms exporting | 0.30 | 0.29 |
| Import share | 0.23 | 0.23 |
| Relative GDP of U.S. to China | 1.79 | 1.79 |
| Std. TFPQ | 1.32 | 1.29 |
| Std. TFPR | 0.93 | 0.91 |
| Std. TFPR, exporters | 0.89 | 0.84 |
| Corr (TFPR, TFPQ) | 0.91 | 0.91 |
| Corr (TFPR, TFPQ), exporters | 0.90 | 0.90 |
| Std. export intensity | 0.35 | 0.30 |
| Corr (ex. participation, TFPQ) | 0.06 | 0.07 |
| Corr (ex. participation, TFPR) | −0.03 | −0.05 |
| Corr (ex. intensity, TFPQ) | 0.01 | 0.01 |
| Corr (ex. intensity, TFPR) | −0.04 | −0.05 |
| | | |
| *Non-targeted moments* | | |
| TFPQ gap (ex−nonex) | 0.17 | 0.20 |
| TFPR gap (ex−nonex) | −0.06 | −0.11 |
| Export intensity | 0.47 | 0.47 |
| Std. value added | 1.20 | 1.18 |
| Corr (value added, TFPQ) | 0.76 | 0.78 |
| Corr (value added, TFPR) | 0.41 | 0.46 |
| Corr (value added, ex-int) | 0.08 | 0.09 |
| Corr (value added, ex-part) | 0.17 | 0.23 |
| *Among Exporters* | | |
| Std. value added | 1.20 | 1.31 |
| Std. TFPQ | 1.25 | 1.31 |
| Corr (ex. intensity, TFPQ) | −0.13 | −0.15 |
| Corr (ex. intensity, TFPR) | −0.06 | −0.02 |
| *Among Non-Exporters* | | |
| Std. value added. | 1.16 | 1.08 |
| Std. TFPQ. | 1.34 | 1.28 |
| Std. TFPR | 0.96 | 0.93 |
| Corr (TFPR, TFPQ) | 0.93 | 0.93 |

Note: Data moments are for 2005 Chinese National Bureau of Statistics. Value added, TFPR, and TFPQ are logged. Corr denotes correlation, Std for standard deviation, ex for export, ex-int for export intensity, ex-part for export participation. TFPR gap is the difference between the average TFPR of exporters and that of non-exporters. Similarly for TFPQ gap.

The rest 14 parameters, including $\{f, f_x, \tau_x, \mu_{f\varphi}\}$, the four standard deviations, and the six correlations, are estimated jointly to match the model moments with their data counterparts. The key moments used to estimate the productivity and distortions are the joint distribution of firms' value-added and inputs. More precisely, they are used to construct firms' measured revenue-based total factor productivity (TFPR) and quantity-based total factor productivity (TFPQ) in our model[21] and to match them with corresponding moments in the data. We use total inputs instead of variable inputs when constructing TFPR and TFPQ both in the data and in the model. Thus, TFPQ and TFPR as discussed above, do not correspond to $\varphi$ or $\tau$. However, it is roughly the case for operating firms if $f$ or $f_x$ are relatively small, as shown in (Eq. 17).

The composite inputs with capital and labor taken are $k_{ji}^{\alpha_j}\ell_{ji}^{1-\alpha_j}$ for firm $i$ in industry $j$ with industry labor share $\alpha_j$.[22] Following HK, labor shares are not computed from Chinese data due to the prevalence of distortions. These industry labor shares come from the U.S. NBER productivity database, which is based on the Census and the Annual Survey of Manufactures (ASM). Different from HK, we take a firm's total employment to measure $\ell_{ji}$ rather than the firm's wage bill. We define the capital stock as the book value of fixed capital net of depreciation. TFPR, the value added over total composite inputs, for firm $i$ in industry $j$, and TFPQ—related to physical productivity—are measured by

$$TFPR_{ji} = \frac{p_{ji}q_{ji}}{k_{ji}^{\alpha_j}\ell_{ji}^{1-\alpha_j}}, \qquad TFPQ_{ji} \propto \frac{(p_{ji}q_{ji})^{\frac{\sigma}{\sigma-1}}}{k_{ji}^{\alpha_j}\ell_{ji}^{1-\alpha_j}}. \tag{18}$$

Both TFPR and TFPQ are measured with their deviations from the industry mean. We find large dispersions in TFPR in China, similar to the levels in HK for the years from 1998 to 2007. Measured TFPR dispersions have come down over time, between 1998 and 2007, as evident in Table A-2.

Table 1 reports the estimated parameters and the moments in the data and model. The moments we choose are the ones that are most relevant to firm productivity and distor-

[21]In our model, TFPR is the value-added over total inputs which include both inputs for production and fixed costs, i.e. $TFPR = pq/\ell$. TFPQ is output per input, i.e. $TFPQ = q/\ell$, which also equals $(P^\sigma Q)^{\frac{1}{1-\sigma}}(pq)^{\frac{\sigma}{\sigma-1}}/\ell$ using the demand function (Eq. 1).

[22]We don't observe variable and fixed costs separately. Follow Bernard, Redding, and Schott (2007), we assume fixed costs take the same composite of capital and labor as variable cost.

tion, and firm selection in the open economy. These include the moments of the joint distributions of TFPR and TFPQ across both non-exporters and exporters, the extensive and intensive margin of producing and exporting, and their correlations with the firms' TFPR and TFPQ. Clearly, every parameter matters for the general equilibrium and affects all the moments. However, there is by and large a clear correspondence between certain parameters and moments.

The parameter most relevant for matching the fraction of surviving firms is the fixed cost $f$. A lower fixed cost leads to a higher fraction of survivors. The first-year firm survival rate is used to match the share of producing firms. Firm-level data of the sample periods reveals that roughly an average of 85% of entrants survive into the second year. The estimated value of $f$ is low about 0.07. The export costs $f_x$ and $\tau_x$ determine the export participation and import share in Chinese manufacturing. Export participation is measured as the fraction of firms exporting among the sample firms. Export intensity of each firm is the ratio of the export sales over the sales of the firm. Both are in nominal terms. Lastly, we calculate the import share as total exports over total sales across all the firms, given the balanced trade assumption. The sensitivity analysis of the case without balanced trade is explored in Appendix H.2. The resulting parameter $\tau_x = 2.6$ is higher than the estimate of 1.7 in Anderson and Van Wincoop (2004), and the 1.83 in Melitz and Redding (2015), reflecting China's large trade costs in 2005. The estimated mean foreign productivity $\mu_{f\varphi}$ is 3.32, which produces a relative US-China GDP of about 1.79.

We estimate the underlying distributions of productivity and distortions so that the model generates firm selection as observed in the data–including 1) firm export participation and intensity; 2) their correlations with firm TFPR and TFPQ, and 3) firms' joint distribution of TFPR and TFPQ for exporters versus non-exporters.

The dispersions in productivity and distortions, and their correlations are important for matching the observed joint distribution between TFPR and TFPQ in the data. As we show in (Eq.17), TFPR increases with both productivity and output wedges. In the model, a firm's TFPQ is given by $q/\ell = \varphi[1 - f/\ell(\varphi, \tau)]$, which implies TFPQ increases with productivity but decreases with output distortions. Hence the standard deviations, $\sigma_\varphi$ for productivity, $\sigma_\tau$ for domestic sale distortion, and $\sigma_{\tau_{ex}}$ for foreign sale distortion, shape the standard deviations of TFPQ and TFPR of non-exporters and exporters. The estimation calls for a smaller dispersion of exporting wedge $\sigma_{\tau_{ex}}$ (0.95) than that of domestic wedge $\sigma_{\tau}$ (1.05), to match the lower dispersion of TFPR among exporters than that among non-exporters. The correlations of productivity and distortions are linked to the correlations of TFPQ and TFPR among exporters and non-exporters. Both $\rho_{\varphi,\tau}$ and $\rho_{\varphi,\tau_{ex}}$ are positive—0.89 and 0.65 respectively.

Under the estimated value of fixed cost $f$ and $f_x$, $\tau_x$, and foreign productivity, underlying distributions should generate firm selection observed in the data: export participation and intensity, and their correlation with firm TFPR and TFPQ. In the model, the export intensity of a firm is given by $\frac{p_x q_x}{pq+p_x q_x} = \frac{1}{1+(P^{\sigma}Q/(P_f^{\sigma}Q_f))(\tau_x\tau_{ex}/\tau)^{\sigma-1}}$, which depends on the iceberg cost $\tau_x$ and the relative distortion of selling to foreign and domestic market, $\tau_{ex}/\tau$. The average export intensity is affected by the iceberg cost. The standard deviation of export intensity is affected by $\rho_{\tau,\tau_{ex}}$, the correlation between $\tau$ and $\tau_{ex}$ and selection. When they are perfectly correlated, the export intensity is constant across firms. A low standard deviation value of 0.35 in the data implies that the two are highly correlated—about 0.68. Evidently, the correlations of export intensity with TFPR and TFPQ are also informative about the underlying distributions on productivity and distortions.

Lastly, heterogenous wedges on fixed exporting cost also affect selection. The standard deviation of the export fixed cost, $\tau_{fx}$, affects selection, and hence also the standard deviation of TFPQ for exporters. It also affects the relation between export participation and intensity with TFPR and TFPQ. The correlation between fixed wedges and productivity and output wedges further affect selection. Our estimation calls for a positive $\rho_{\varphi,\tau_{fx}}$ as 0.3, and a negative $\rho_{\tau,\tau_{fx}}$ as $-0.1$. The two exporting wedges, $\tau_{ex}$ and $\tau_{fx}$, are almost not correlated.

**Model fit.** Panel B of Table 1 reports the targeted moments in the model and the data. Our model matches well all of the empirical targets. First, our model produces the observed fraction of firms producing (0.85) and exporting (0.3), and the import share (0.23). Second, our model successfully replicates the distributions of TFPR and TFPQ, among all firms and across exporters. The overall standard deviation of TFPQ is 1.32 in the data compared to 1.29 in the model. The standard deviation of TFPR is 0.93 for all of the firms and 0.89 for exporters in the data, compared to 0.91 and 0.84 in the model. Our model matches the correlation of TFPR and TFPQ for exporters and the correlation across all firms, around 0.9, despite the fact that the underlying correlation $\rho_{\varphi,\tau_{ex}}$ is 0.65, which is much lower than 0.89 for $\rho_{\varphi,\tau}$. The larger gap between TFPR-TFPQ correlation and $\rho_{\varphi,\tau_{ex}}$ among exporters reflects the stronger selection effect into the exporting market.

The distortions significantly impact both the extensive and intensive margins of trade. We proceed to examine trade correlations, i.e., how export participation and intensity vary with TFPR and TFPQ. The export participation is weakly positively correlated with TFPQ, 0.06 in the data and 0.07 in the model. It is weakly negatively correlated with TFPR, about $-0.03$ in the data and $-0.05$ in the model. With small fixed costs, $\varphi$ influences more TFPQ, and $\tau$ or $\tau_{ex}$ influences more TFPR. The signs of these trade correlations show that the more productive and low wedge firms are more likely to become an exporter. Distortion on the export fixed cost $\sigma_{fx}$ and its correlations with productivity as well as with other distortions help generate these correlations.

Table 1 also lists other moments related to value added and the differences between exporters and non-exporters. These moments are also close to the data as they contain similar information as our targeted moments. For example, when constructing TFPR and TFPQ, we use both value added and inputs, as shown in (Eq.18). In particular, the log of value added is proportional to the log-difference between TFPR and TFPQ. Once we match the joint distribution of TFPR and TFPQ, we also generate the observed standard deviation of value added. Similarly, the fact that we target the correlations of TFPR and TFPQ for all the firms and for exporters means that the model's implication for the correlation for non-exporters is not far from the data. Export intensity (0.47) is the same as in the data, because we match the aggregate import share (export share) and the difference between exporter and non-exporters.

Overall the model matches tightly the standard deviation of value added among all the firms, and among exporters and non-exporters. It generates the observed correlations of value added with TFPR, TFPQ, export intensity, and export participation.[23] On average, exporters have 6% lower TFPR and 17% higher TFPQ than non-exporters in the data. Our

[23] Our model matches the observed overlap of exporters' and non-exporters' size distributions. Hsieh, Li, Ossa, and Yang (2020) also emphasized the importance of matching this overlap.

model generates similar magnitudes. Among exporters, the export intensity is negatively correlated with both TFPR and TFPQ. Our model successfully reproduces these negative correlations in the data, $-0.06$ for correlation between export intensity and TFPR, and $-0.13$ for export intensity and TFPQ.

In summary, our estimations uncover the underlying distributions of productivity and distortions. There are large dispersed firm level distortions and they are higher correlated with firms productivity, which can generate large inefficiency. Distortions in exporting market are relatively less dispersed and less correlated with productivity, but still—exporters are the relatively subsidized ones after selection.

### 3.4 Implied Gains from Trade and Role of Distortions

In this section, we examine the gains from trade in our benchmark. We contrast them with the case when there are no distortions. To further understand the source of gain from trade, we decompose welfare according to our theory in the extended model, given in Proposition 5. We also study the role of each distortion and the role of key moments in shaping the gains from trade.

**Welfare and decomposition** Table 2 reports the welfares of Home and Foreign and their gains from trade when Home has or has no distortions. Each country's welfare is express as relative to its welfare under a closed economy without distortions. In the benchmark with distortions, Home welfare is substantially lower than in the no-distortion case. Both closed and open economy feature a significant negative welfare, around $-170\%$. An open economy has an even lower number, implying a loss from trade about of 1.18%. Without distortions, Home welfare would be greatly improved and the gains from trade is positive–around 3.31%.

The gains from trade for Home would be greatly overestimated if we ignore the presence of distortions and follow the usual approach of ACR, where gains is the ratio of the change in logged domestic output share $-d\ln\lambda$ to a trade elasticity. Note that in the decomposition in Proposition 5, the first component *ACR term* carries $\gamma_s$. If we use a partial trade elasticity in the benchmark, the ACR gain from trade would be 12.07%. As MR points out, whether a

Table 2: Welfare Implications

| | Home | | | Foreign | |
|---|---|---|---|---|---|
| | benchmark | no distortions | | benchmark | no distortions |
| *Welfare (relative to closed, no distortions)* | | | | | |
| Closed economy | −169.56 | 0.00 | | 0.00 | 0.00 |
| Open economy | −170.74 | 3.31 | | 8.94 | 9.40 |
| *Gain from trade* | | | | | |
| Model | −1.18 | 3.31 | | 8.94 | 9.40 |
| *Home welfare decomposition* | ACR term | Reallocation | MR | Entry distortion | Fixed cost |
| *with distortions* | 12.53 | −12.94 | −1.43 | 0.18 | 0.06 |

Note: All numbers are in percent. In the benchmark model, Home has distortions $(\tau, \tau_{ex}, \tau_{fx})$, Foreign faces no distortions. The welfares of each country are relative to its own welfare under closed-economy and no distortions. Welfare decomposition is conducted according to (Eq.16).

partial or a full trade elasticity is used will matter for the size of the ACR gain. But the point here is that in the case with distortions, a 'reallocation' channel leads to a sizeable welfare loss that counters the welfare gains. Taking only the ACR component would significantly overestimate the gains to trade according to our model. Without distortions, the ACR term would give a welfare gain of 3.73%, close to the model value of 3.31%.

The Table shows a full decomposition of the change in Home welfare according to equation (16) in Proposition 5.[24] The loss from trade comes from the large and negative reallocation term showing up in China, amounting to −12.94%. Even though the ACR term is positive, 12.53%, the negative reallocation term dominates. The other two terms, welfare associated with entry and fixed cost, are small–about 0.18% and 0.06% respectively. Without distortions, the reallocation and fixed cost term would be zero. Hence, the loss in the Home country results from negative welfare associated with misallocation from distortions. Trade exacerbates the misallocation through endogenous selection of less 'taxed' firms.[25]

Foreign country is absent distortions and its gain from trade is about 9% with or without Home distortions, though its gains is slightly lower when Home features distortions. The

[24]The welfare formula in Proposition 5 holds accurately for small changes in trade cost. For our decomposition of counterfactual comparing to autarky, there is a small difference between $d \ln W$ (the left-hand side) and our decomposition, about 0.004. We can always use an integration of small changes from open to autarky to do a precise decomposition.

[25]Note that all of the terms are equilibrium endogenous variables and are affected by distortions. The reallocation term also includes the extensive margin and the effects of fixed cost wedges, in later this subsection and the following subsection, we will do counterfactual to show how each wedges affect welfare.

ACR formula approximates well the gains from trade for Foreign, also around 9%. Home distortion affects foreign welfare through the import price and the relative wage. In particular, the price Foreign faces is lower were Home firms to be taxed less; on the other hand, some high marginal cost Home firms will be selected into exporting, making the Foreign's import prices higher. In terms of the relative wage, when there are distortions, the relative higher demand for foreign products induces a higher wage. Without Home distortions, its efficiency improves, and the Foreign wage would be lower, but with a lower import price as well. The overall effect on Foreign depends on the relative magnitude of these forces. See Appendix H.1 for an illustration of the two forces in our benchmark model.

**Welfare and distortions.** To understand the sources of welfare loss, we consider three comparative statics. In the second column of Table 3, we shut down the distortions on fixed exporting cost $\tau_{fx}$ and keep all the other distortions and parameters the same as in the benchmark. In this case, the welfare loss after trade becomes smaller, from 1.18% to 0.67%. However, the country still suffers a loss from trade and the reallocation term is still highly negative, $-12.57\%$. Hence, the distortions on the fixed cost of exporting affects does not affect the overall welfare and reallocation by much. To examine further the impact of $\tau_{fx}$ on the firms' distribution of TFPR, TFPQ, and export, we report the key moments in the second column of Table 3. This distortion affects mostly the correlation of export participation with TFPQ, which increases from 0.07 to 0.18. The overall export intensity with TFPQ also rises from 0.01 to 0.12. The distortion $\tau_{fx}$, however, changes little other moments, especially the dispersion of TFPR and TFPQ and their correlations–which is critical for overall welfare.

In the third column of Table 3, we shut down the output wedges $\tau$ and $\tau_{ex}$ but keep $\tau_{fx}$, while keeping all other parameters the same as in the benchmark. Without output wedges, the model matches poorly the dispersion of TFPR for both non-exporters (going from 0.93 to 0.11) and exporters (from 0.84 to 0.05) . With low levels of distortions, the overall welfare gain from trade becomes positive, 3.27%, close to the efficient case gains of 3.31%. The ACR term 3.71% is also close to the true welfare gain of 3.27%. The reallocation term changes from $-12.94\%$ to zero. Without output wedges, the export intensity does not vary across firms, and the standard deviation of export intensity is zero. Export participation is driven

Table 3: Welfare, Distortions, and Moments

| | Benchmark | Bench Parameters | | | Reetimation (no $\tau_{fx}$) | |
|---|---|---|---|---|---|---|
| | | No $\tau_{fx}$ | No output wedges | No wedges | $\tau \neq \tau_{ex}$ | $\tau = \tau_{ex}$ |
| *Home welfare gains (%)* | | | | | | |
| Overall | −1.18 | −0.67 | 3.27 | 3.31 | −0.73 | 1.38 |
| | | | | | | |
| ACR term | 12.53 | 12.59 | 3.71 | 3.73 | 12.53 | 12.54 |
| Reallocation | −12.94 | −12.57 | 0.00 | 0.00 | −12.67 | −7.39 |
| MR | −1.43 | −1.26 | −0.44 | −0.42 | −1.11 | −4.53 |
| Entry distortion | 0.18 | 0.16 | 0 | 0 | 0.13 | 0.41 |
| Fixed cost | 0.06 | 0.00 | 0.02 | 0 | 0 | 0 |
| *Key Moments* | | | | | | |
| Std. TFPQ | 1.29 | 1.27 | 0.83 | 0.83 | 1.32 | 1.31 |
| Std. TFPR | 0.91 | 0.90 | 0.11 | 0.11 | 0.93 | 0.91 |
| Corr (TFPR, TFPQ) | 0.91 | 0.91 | 0.87 | 0.87 | 0.92 | 0.91 |
| *Among Exporters* | | | | | | non-targeted |
| Std. TFPQ. | 1.31 | 1.23 | 0.64 | 0.52 | 1.27 | 1.31 |
| Std. TFPR. | 0.84 | 0.85 | 0.05 | 0.02 | 0.87 | 0.90 |
| Corr (TFPR, TFPQ) | 0.90 | 0.92 | 0.81 | 0.89 | 0.92 | 0.97 |
| Std. export intensity | 0.30 | 0.28 | 0 | 0 | 0.28 | 0 |
| *Among Non-Exporters* | | | | | | |
| Std. TFPQ. | 1.28 | 1.26 | 0.56 | 0.53 | 1.30 | 1.31 |
| Std. TFPR | 0.93 | 0.92 | 0.11 | 0.11 | 0.93 | 0.89 |
| Corr. (TFPR, TFPQ) | 0.93 | 0.93 | 0.96 | 0.97 | 0.94 | 0.98 |
| *Trade Correlations* | | | | | non-targeted | |
| Corr (export int., TFPQ) | 0.01 | 0.12 | 0.72 | 0.77 | 0.12 | 0.06 |
| Corr (export int., TFPR) | −0.05 | −0.01 | 0.44 | 0.47 | 0.01 | −0.30 |
| Corr (export part., TFPQ) | 0.07 | 0.18 | 0.72 | 0.77 | 0.21 | 0.06 |
| Corr (export part., TFPR) | −0.05 | −0.01 | 0.44 | 0.47 | 0.02 | −0.30 |

Note: Welfare decomposition is conducted according to (Eq.16). TFPR and TFPQ are logged. Corr denotes correlation, Std for standard deviation, 'export int' for export intensity, 'export part' for export participation. The case 'No $\tau_{fx}$' shuts down $\tau_{fx}$, $\tau_{fx} = 1$. The case 'No output wedges' shut down both $\tau$ and $\tau_{ex}$, $\tau = \tau_{ex} = 1$. The case 'No wedges' shuts down all distortions $(\tau, \tau_{ex}, \tau_{fx})$. The other parameters in these three case are the same as the benchmark. For 'Reestimation $\tau \neq \tau_{ex}$', we estimate the model with no $\tau_{fx}$ but allowing for differential $\tau_{ex}$ and $\tau$. In this case, we do not target the four trade correlations. For 'Reestimation $\tau = \tau_{ex}$', we estimate the model with no $\tau_{fx}$ and $\tau = \tau_{ex}$. In this case, we do not target within-group distributions of TFPR and TFPQ and the four trade correlations. Both the cases under 'Reestimation' also match the fraction of firms producing and exporting, import share, and relative GDP, which are not listed in the table. ACR, Reallocation, MR, Entry distortion, and Fixed cost in the welfare decomposition are constructed according to (Eq.20).

by productivity and $\tau_{fx}$. The export participations and intensities are largely positively correlated with TFPR and TFPQ, which are inconsistent with the data.

The fourth column of Table 3 shows the results without any distortions. The only heterogeneity of firms comes from productivity. The gain from trade is the highest in this case. Without distortions, the reallocation term becomes zero. Even without distortions, there is still some dispersion in TFPR. The reason, as stated in Section 3.2, is that with fixed cost TFPR is also affected by productivity and is not distortion in the model. However, productivity dispersion can only generate about one-tenth of TFPR dispersion in the benchmark, given the low fixed cost.

In summary, our benchmark model shows that opening up to trade can generate a welfare loss that derives from a worsened misallocation of resources due to trade. Between the two types of distortions, output wedges is most important for driving these results. Distortions on the fixed cost of exporting help generate the co-movement in exports, TFPR, and TFPQ, but contributes little to reallocation and the overall welfare.

**Welfare and moments.** To understand the role of the chosen moments on the welfare implications, we conduct two alternative estimations in the last two columns of Table 3. In these estimations, we shut down some moments related to TFPR and TFPQ and their attendant distortions while reestimating all the other parameters.

In the first estimation, we target the same set of moments as in the benchmark except for the trade correlations, i.e. the co-movements of export intensity and participation with TFPR and TFPQ. Given fewer moments than the benchmark, we shut down $\tau_{fx}$ but allow for differential output wedges on domestic and foreign sales, $\tau \neq \tau_{ex}$ (fifth column of Table 3). The model successfully produces the moments of average extensive margins of producing and trade, the standard deviations of TFPR, TFPQ, and their correlations among exporters and non-exporters. The export participation is too correlated with TFPQ, raising it from 0.07 in the benchmark to 0.21. Its correlation with TFPR also increases from $-0.05$ to 0.02. The correlations of export intensity with TFPQ and TFPR follow a similar pattern. The overall welfare after trade is higher, $-0.73\%$ compared to $-1.18\%$ in the benchmark. However, the welfare is still lower than the case without distortions since the reallocation term is still large and negative, −12.67%, close to the benchmark value −12.94%. Note that in this case the welfare under no distortions is different from the benchmark due to the reestimation of the parameters.

The last column of Table 3 further shuts down heterogeneity between the output distortions on domestic and foreign sales, i.e. $\tau = \tau_{ex}$. In this case, we give up generating the group-specific distributions of TFPR and TFPQ, and consider only the overall dispersions of TFPR, TFPQ, and their correlations, which the estimation successfully produces. Even though the correlation of TFPR and TFPQ across all firms matches the data, the model over-estimates these correlations for both exporters and non-exporters. It also misses the trade correlations with TFPR and TFPQ. The ACR term still greatly overestimates the gain from trade. With fewer distortions, the welfare gain from trade is higher, 1.38% since the reallocation is less negative, about −7.4%.

These two analyses demonstrate the importance of matching the overall and the group-specific distributions of TFPR and TFRQ in accounting for the welfare gain from trade liberalization.

## 3.5 Endogenous Wedge

The benchmark model assumes exogenous distortions correlated with firm-level productivity. A possible scenario is that distortions are size-dependent, for instance, on firm revenue. David and Venkateswaran (2019) discusses how size-dependent policies can lead to some isomorphism with policies that are correlated with underlying productivity. In what follows, we assume that the distortions on domestic and foreign sales, $\tau$ and $\tau_{ex}$, positively depend on a firm's sales with an elasticity of $\beta$:

$$\ln \tau = \beta \ln(pq) + \ln \varepsilon, \qquad \ln \tau_{ex} = \beta \ln(p_x q_x) + \ln \varepsilon_{ex}, \tag{19}$$

where $\varepsilon$ and $\varepsilon_{ex}$ are idiosyncratic distortions on domestic and export revenue and potentially correlate with the firm's productivity or the distortion on the fixed exporting cost. In this case, a firm's distortions endogenously change as firms expand or shrink. In equilibrium with the optimal production, we can show that

$$\ln \tau = \mu_c + \frac{\beta(\sigma - 1)}{1 - \beta + \sigma\beta} \ln \varphi + \frac{1}{1 - \beta + \sigma\beta} \ln \varepsilon + \frac{\beta}{1 - \beta + \sigma\beta} \ln(P^{\sigma} Q),$$

where $\mu_c$ is a constant. The case with $\beta = 0$ gets us back to the benchmark model. With size dependent policies, the distortion $\tau$ is endogenously correlated with productivity even if $\varepsilon$ and $\varphi$ are uncorrelated.[26] See Appendix F for model details and G for derivation of welfare formulas.

**Proposition 6.** *The change in welfare associated with an iceberg cost shock is*

$$\begin{aligned} d \ln W = \frac{1}{\gamma_s + \tilde{\sigma} - 1} \Big\{ & - d \ln \lambda + d \ln M_e \\ & + \left( \frac{\sigma}{\sigma - 1} (\gamma_s - \gamma_\lambda) - \tilde{\sigma}\beta \right) d \ln M_e \\ & - \left( \tilde{\sigma}(1 - \beta) - 1 + \frac{\sigma \gamma_s}{\sigma - 1} \right) d \ln \lambda + \left( \tilde{\sigma} - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) d \ln S \\ & + \left( \tilde{\sigma} - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) d \ln \left( 1 + \frac{M_e f_x \int_{\varphi_x^*(\varepsilon_{ex}, \tau_{fx})}^{\infty} \left( \tau_{fx} - 1 \right) dG}{L} \right) \Big\}, \end{aligned} \tag{20}$$

*where* $\tilde{\sigma} = \sigma / (1 - \beta + \sigma\beta)$.

Appendix G exemplifies this formula for different distributions. For some distributions, $\beta$ does not show up explicitly, and if one can gauge the change of output share $\lambda$ and input share $S$ from the data, the endogenous wedge model forecasts the same gain from trade as the exogenous wedge model.

Table 4 reports quantitative impact of endogenous wedges with $\beta = 0.2$ and contrast them with our benchmark model under exogenous wedges. To highlight the role of endogenous wedge, we consider three cases. The first case shuts down all exogenous distortions $(\varepsilon, \varepsilon_{ex}, \tau_{fx})$ and keeps only the endogenous ones. We reestimate all the parameters using the relevant moments. The second case includes both endogenous and exogenous wedges and adopts benchmark parameters. The third case reestimates this model with

[26] In equilibrium, a firm chooses its price as $p = \frac{\sigma}{(\sigma-1)(1-\beta)} \tau(pq, \varepsilon) \frac{w}{\varphi}$, as if it faces a variable markup $\tau$ increasing with its revenue $pq$. Hence, the endogenous wedge model relates to the literature studying the welfare implications of variable markup, for example Feenstra and Weinstein (2017), Dhingra and Morrow (2019), and Edmond, Midrigan, and Xu (2018).

Table 4: Endogenous Wedge

| | Endogenous wedge $\beta = 0.2$ | | | Benchmark |
|---|---|---|---|---|
| | Only endo. wedge (recali) | Bench para. | Endo. & exog. wedge (recali) | |
| *Parameters for distribution* | | | | |
| Std. productivity $\sigma_\varphi$ | 1.44 | 1.31 | 1.33 | 1.31 |
| Std. distortion on home sales $\sigma_\tau$ | | 1.05 | 1.15 | 1.05 |
| Corr(prod., domestic distortion) $\rho_{\varphi,\tau}$ | | 0.89 | 0.75 | 0.89 |
| Std. distortion on export sales $\sigma_{\tau_{ex}}$ | | 0.95 | 0.99 | 0.95 |
| Corr(prod., foreign sale distortion) $\rho_{\varphi,\tau_{ex}}$ | | 0.65 | 0.25 | 0.65 |
| Corr($\tau$, $\tau_{ex}$) $\rho_{\tau,\tau_{ex}}$ | | 0.68 | 0.40 | 0.68 |
| Std. distortion on export fixed cost $\sigma_{\tau_{fx}}$ | | 0.65 | 0.55 | 0.65 |
| Corr(prod., exporting fixed cost) $\rho_{\varphi,\tau_{fx}}$ | | 0.30 | 0.30 | 0.30 |
| Corr($\tau$, $\tau_{fx}$) $\rho_{\tau,\tau_{fx}}$ | | −0.10 | −0.20 | −0.10 |
| Corr($\tau_{ex}$, $\tau_{fx}$) $\rho_{\tau_{ex},\tau_{fx}}$ | | 0.00 | −0.25 | 0.00 |
| *Key momentss* | | | | |
| Std. TFPQ | 1.29 | 1.23 | 1.26 | 1.29 |
| Std. TFPR | 0.43 | 0.93 | 0.91 | 0.91 |
| Corr (TFPR, TFPQ) | 0.98 | 0.95 | 0.89 | 0.91 |
| Export intensity | 0.24 | 0.40 | 0.42 | 0.47 |
| Std. export intensity | 0.00 | 0.23 | 0.30 | 0.30 |
| *Among Exporters* | | | | |
| Std. TFPQ. | 0.77 | 1.21 | 1.25 | 1.31 |
| Std. TFPR. | 0.27 | 0.85 | 0.80 | 0.84 |
| Corr (TFPR, TFPQ) | 0.998 | 0.95 | 0.91 | 0.90 |
| Corr (export intensity, TFPQ) | *n.a.* | 0.20 | −0.18 | −0.15 |
| Corr (export intensity, TFPR) | *n.a.* | 0.27 | −0.08 | −0.02 |
| *Among Non-Exporters* | | | | |
| Std. TFPQ. | 0.82 | 1.24 | 1.31 | 1.28 |
| Std. TFPR | 0.34 | 0.96 | 0.89 | 0.93 |
| Corr. (TFPR, TFPQ) | 0.997 | 0.96 | 0.92 | 0.93 |
| *Trade correlations* | | | | |
| Corr (export intensity, TFPQ) | 0.78 | 0.02 | 0.01 | 0.01 |
| Corr (export intensity, TFPR) | 0.66 | −0.03 | −0.09 | −0.05 |
| Corr (export participation, TFPQ) | 0.78 | −0.10 | 0.10 | 0.07 |
| Corr (export participation, TFPR) | 0.66 | −0.21 | −0.09 | −0.05 |
| *Home welfare gains from trade* | | | | |
| Overall | 6.52 | −0.08 | −1.66 | −1.18 |
| ACR | 22.10 | 23.92 | 22.11 | 12.53 |
| Reallocation | −12.49 | −21.85 | −22.96 | −12.94 |
| No-distortion | 8.46 | 3.31 | 1.89 | 3.31 |

Note: TFPR and TFPQ are logged. Corr denotes correlation and Std for standard deviation. In all estimations, fraction of firms producing and exporting, import share, and relative GDP are perfectly matched and not listed in the table. ACR and reallocation are constructed according to (Eq.20). 'Only endo. wedge (recali)' estimates the model with only endogenous wedge. 'Bench para' uses benchmark parameters and $\beta = 0.2$. 'Endo. & exog. wedge (recali)' estimates the model with both endogenous and exogenous wedges.

both exogenous and endogenous wedges.

The first column of Table 4 reports the results with only endogenous wedges. We reestimate all the parameters $(f, f_x, \tau_x, \mu_{\varphi f})$ and the standard deviation of productivity $\sigma_\varphi$. The fraction of firms producing and exporting, import share, and relative GDP are perfectly matched and omitted from the table. With only heterogeneity on productivity, we target only the overall standard deviation of TFPQ but not TFPR moments or cross-group TFPQ distributions.

The results with only endogenous wedges have the counterfactual feature of 1) generating almost perfectly correlated TFPR and TFPQ among exporters and among non-exporters, 0.998 and 0.997 respectively; 2) the selection into exporting market is based solely on productivity, and thus exporters have higher TFPQ and TFPR than non-exporters, as reflected by their correlation with export participation — 0.78 and 0.66, respectively. However, in the data, they are 0.06 and $-0.03$. Exporters, in particular, have a higher TFPR in the model, contrary to the data.[27] Lastly, 3) the endogenous wedge alone generates less than half of the observed TFPR dispersions. Therefore, even the endogenous wedge distorts the production incentive of highly productive firms and leads to a negative reallocation term; this effect is not large enough. The resultant magnitude of reallocation is about half the size of ACR, $-12.49\%$ versus 22.10%. In contrast, these two terms have similar magnitudes in the benchmark. Hence, with less dispersed TFPR, the endogenous wedge model generates higher overall gain from trade than the benchmark, but it remains lower than without distortions.

Note that these counterfactual features are generic and present also for higher levels of $\beta$ as long as there are no exogenous distortions. Again, the perfect correlation between $\ln \tau$ ($\ln \tau_{ex}$) and $\ln \varphi$ leads to almost perfect within-group correlations between TFPR and TFPQ and causes the counterfactual selection into the exporting market.

We now add exogenous distortions $(\varepsilon, \varepsilon_{ex}, \tau_{fx})$ to this model with $\beta = 0.2$ and other parameters as the benchmark. See the second column of Table 4. Adding exogenous distortions helps the model in resolving the aforementioned counterfactual features. The

[27] Apendix J studies a model with endogenous markup as in Edmond, Midrigan, and Xu (2018). The model has a similar counterfactual implication: exporters end up with higher TFPR since they are more productive and charge a higher endogenous markup.

dispersions of TFPR are nearly identical to those in the benchmark. Exogenous distortions also break the selection solely on the basis of productivity $\varphi$. Both export participation and export intensity are less correlated with TFPR and TFPQ. Under more dispersed TFPR, the reallocation term becomes significantly more negative than when only endogenous wedges are present. As a result, the overall gain from trade decreases from 6.52% to $-0.08$%.

This model with endogenous and exogenous wedges under benchmark parameters still has some moments that differ from the data. TFPR and TFPQ are still more correlated than the benchmark and the data, for both exporters and non-exporters. The correlation of export participation with TFPR and TFPQ are both too negative relative to the benchmark.

We therefore reestimate this model with endogenous and exogenous wedges in a similar fashion as in the benchmark. See the third column of Table 4. Given the high correlations of TFPR and TFPQ due to the endogenous wedge, the estimation calls for a larger dispersion of the underlying exogenous wedges $(\varepsilon, \varepsilon_{ex})$ to reduce the correlations. The resultant standard deviations of $\varepsilon$ and $\varepsilon_{ex}$ are 1.15 and 0.99 respectively, both higher than in the benchmark. The estimation also calls for lower correlations of productivity with $\varepsilon$ or $\varepsilon_{ex}$ than the benchmark, bringing them down from 0.89 and 0.68 to 0.75 and 0.25. With all the moments close to the benchmark, the magnitude of reallocation term is now similar to that of ACR. The consequent gains from trade is therefore negative, about $-1.66$%, though with the reestimation the result under no distortion is also smaller. Overall, a negative reallocation effect offset the gains and leave a small loss from trade.

### 3.6 Decomposing China's Growth from 1998-2005

The rapid growth in China over the last four decades has been one of the most remarkable phenomena the world has witnessed in recent history. In between 1998 and 2005, its real GDP increased by 57%. Accompanying this development was a combination of domestic reforms and opening up programs—policies that fostered trade and FDI inflows. As a result, both trade and technological progress increased over time, while domestic distortions concurrently fell. A natural question is how much of the growth is attributed to trade over this period. Other competing factors include technological improvement, factor accumulation, and domestic reforms—that is, the allocative gains associated with a reduction in distortions. In what follows, we perform a quantitative analysis to answer this question. Specifically, we reestimate the model parameters for the year 1998 and compare the implied GDP those in the benchmark year, 2005. Overall, our results attribute the majority of China's GDP growth to technological improvement, capital accumulation, and a mitigation of distortions. With only reduction in iceberg trade cost, GDP could only increase by 5% instead of 57%.

Table 5 reports the moments for both 1998 and 2005. The starting year is taken to be 1998, as it is the first year in which firm-level data is available, and three years before China joined the WTO. Compared to the year 2005, trade intensity was lower in 1998, both in terms of the fraction of firms that export, and also the export intensity of these firms. Distortions were large in the earlier years, as partly seen by the fact that the overall dispersion of TFPR is about 20% higher in 1998 compared to 2005. The trade correlations with TFPR or TFPQ are more positive in 1998 than 2005.

Estimations show a higher trade cost $\tau_x$ and dispersion of distortion $\sigma_\tau$ and $\sigma_{\tau_{ex}}$ in 1998—at about 53%, 24%, and 18% higher than the level in 2005. The standard deviation of $\tau_{fx}$ is smaller in 1998, and according to our analysis in previous section, this change affects little the welfare. The estimated correlations of productivity with distortions in 1998 are almost the same as those in 2005 given the similar correlation of TFPR and TFPQ in these two years. The mean productivity in 2005 is about 44% higher than that in 1998, reflecting technological improvements and factor accumulation over time.

These estimates are then used to run counterfactual experiments, in order to decompose China's growth in between 1998 and 2005. The factors considered include technological progress (and capital accumulation), and the reduction of trade costs and domestic distortions. In each experiment, the parameters for the year 1998 remain fixed, while each set of the following parameters– mean productivity $\mu_\varphi$, trade cost $\tau_x$, or distribution of productivity and distortions—are allowed to vary to its 2005 level. Table 6 shows that the increase of technology and inputs alone lead to a 43% increase in GDP. Reduction in trade costs would independently boost GDP by 5%. In contrast, lowering the dispersion of distortions increases GDP by 17%.[28]

[28]Note that the contributions to GDP increase don't add up to 100% because fixed costs have also changed

Table 5: Data, 1998 and 2005

| Target Moments | Data (1998) | Data (2005) |
|---|---|---|
| Fraction of firms producing | 0.85 | 0.85 |
| Fraction of firms exporting | 0.25 | 0.30 |
| Import share | 0.16 | 0.23 |
| Relative GDP of U.S. to China | 2.60 | 1.79 |
| Std. TFPQ | 1.55 | 1.32 |
| Std. TFPR | 1.12 | 0.93 |
| Std. TFPR, exporters | 1.01 | 0.89 |
| Corr (TFPR, TFPQ) | 0.93 | 0.91 |
| Corr (TFPR, TFPQ), exporters | 0.92 | 0.90 |
| Std. export intensity | 0.38 | 0.35 |
| Corr (export participation, TFPQ) | 0.08 | 0.06 |
| Corr (export participation, TFPR) | −0.01 | −0.03 |
| Corr (export intensity, TFPQ) | 0.04 | 0.01 |
| Corr (export intensity, TFPR) | 0.00 | −0.04 |

Note: Data is from Chinese National Bureau of Statistics. TFPR and TFPQ are logged. Corr denotes correlation and Std for standard deviation.

A notable point of comparison is with Tombe and Zhu (2019), which, despite adopting an altogether different approach, finds also small gains to trade. In their model that features migration across regions and sectors in China, international trade contributes to only 7% of productivity growth in between 2000 and 2005. In other words, international trade has led to very little allocative benefits of labor across regions and sectors—as compared to direct reforms that lower migration costs or reductions in internal trade costs. Their model does not feature distortions at the firm level that can render trade's allocative benefits even smaller. This leads us to find an even smaller effect of trade in China over roughly the same period.

Of course, a caveat is that trade may also help reduce domestic distortions. If, say, the WTO requires certain kind of domestic reforms as a pre-condition for entry, then some of the technological improvement and reductions in the level of distortions could be partially induced by opening up policies. We do not consider this here. Also, this quantitative exercise of course also ignores other potential channels of gains to trade, such as pro-competition effect of trade, or potentially transfers of technology (Ramondo and Rodríguez-

from 1998 to 2005. Furthermore, there are interacting effects on mean productivity, trade cost, and distortion dispersions.

Clare (2013))–though these effects may still be quantitatively small. The point we make here is that in our benchmark framework, the contribution of trade pales in comparison to the contribution of domestic policies and technological progress in accounting for China's growth experience.

Table 6: Decomposition of China's Growth between 1998-2005

| | Change of Real GDP (%) |
|---|---|
| Benchmark | 57 |
| Counterfactual Change from 1998-2005: | |
| Technology and inputs alone (Increase mean $\varphi$) | 43 |
| Trade alone (Decrease $\tau_x$) | 5 |
| Distortion alone (Same distortion as 2005) | 17 |

# 4 Conclusion

This paper evaluates the impact of trade liberalization when the economy is subject to firm-level distortions. Given its prevalence and importance in developing countries, it is reasonable to ask how trade might affect welfare when these distortions are taken into account. This paper shows theoretically and quantitatively that opening an economy may in fact reduce allocative efficiency and exacerbate the misallocation of resources, by helping firms that are more subsidized (rather than those who are more productive) to expand. The findings in this paper does not disclaim the potential wide variety of sources and the magnitude of gains to trade beyond what is taken up in the current framework. But it does highlight that these losses could be sizeable and comparable to major sources of welfare gains. We use Chinese manufacturing data in a period of the economy's rapid integration to demonstrate quantitatively that standard calculations for welfare may grossly overestimate the gains.

The paper serves as a first attempt to understand the interactions between trade and idiosyncratic firm level distortions on a theoretical level. Extensions of the work can examine how distortions interact with other channels of gains to trade, such as innovation. One can also examine a dynamic model and the sequence of trade and domestic reforms. Our work joins the growing body of work and interest on why developing countries' experience with trade liberalization might have been so curiously diverse and uneven. Our work hopefully lends itself as one explanation to such a question.

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# Online Appendix to "Misallocation Under Trade Liberalization"

by Yan Bai, Keyu Jin, and Dan Lu

This appendix is organized as follows.



# A Model Derivation

**Closed Economy Equilibrium.** In a closed economy, taking as given the aggregates prices $(P, w)$ and demand $Q$, the problem of a firm with $(\varphi, \tau)$ implies the optimal price

$$p(\varphi, \tau) = \frac{\sigma}{\sigma - 1} \frac{w\tau}{\varphi} \tag{A.1}$$

and optimal profit $\pi(\varphi, \tau) = [\sigma^{-\sigma}(\sigma-1)^{\sigma-1}P^{\sigma}Qw^{1-\sigma}]\varphi^{\sigma-1}\tau^{-\sigma} - wf$. The cutoff of production is given by $\varphi^*(\tau) = con_v \times P^{-1}(PQ)^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{\sigma-1}}$ with the normalization of $w = 1$ and the constant $con_v = \sigma^{\frac{\sigma}{\sigma-1}}(\sigma-1)^{-1}f^{\frac{1}{\sigma-1}}$.

Let $\mu(\varphi,\tau)$ be the distribution of operating firms $\mu(\varphi,\tau) = \frac{g(\varphi,\tau)}{\int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)d\varphi d\tau} = \frac{g(\varphi,\tau)}{\omega_e}$ if $\varphi \geq \varphi^*(\tau)$; and 0 otherwise. Define $M_e$ and $M$ as a measure of entrants and operative firms, respectively.

An equilibrium is characterized by an aggregate price index, a free entry condition, and a labor market clearing condition. The aggregate price index is the weighted average of the prices (A.1) of the operating firms:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \int\int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau)d\varphi d\tau. \tag{A.2}$$

The free entry condition requires that the present value of producing equals the entry cost, i.e.,

$$\omega_e E[\pi(\varphi,\tau)] = w f_e, \tag{A.3}$$

where $\omega_e$ is the probability of entry, $\omega_e = \int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\, d\varphi d\tau$, and the expected profit is given by $E[\pi(\varphi,\tau)] = \int\int_{\varphi^*(\tau)} \pi(\varphi,\tau)\mu(\varphi,\tau)d\varphi d\tau$.

The labor market clearing condition requires

$$L = ME\left[\frac{q}{\varphi} + f\right] + M_e f_e, \tag{A.4}$$

where the average labor demanded by firms is $E\left[\frac{q}{\varphi}+f\right] = \int\int_{\varphi^*(\tau)}^{\infty}\left[\frac{q}{\varphi}+f\right]\mu(\varphi,\tau)\, d\varphi d\tau$. In equilibrium, the number of producers equals the number of entrants multiplying the probability of producing, such that

$$\omega_e M_e = M. \tag{A.5}$$

Noting that $\omega_e E(q/\phi) = (\sigma-1)(\omega_e f + f_e)$, which can be obtained through optimal profit function and the free entry condition, we arrive at

$$M_e = \frac{L}{\sigma\left(f_e + \omega_e f\right)}. \tag{A.6}$$

**Open Economy Equilibrium.** Optimal prices and cutoff functions are straightforward analogues of the closed economy case. An equilibrium of the open economy consists of seven aggregate conditions: two free entry conditions for Home and Foreign, two aggregate price index for Home and Foreign, two labor market conditions for Home and Foreign, and one balanced-trade condition.

Home's free entry condition is given by

$$\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\int\int_{\varphi^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right] g(\varphi,\tau)\,d\varphi d\tau - wf\int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\,d\varphi d\tau$$
$$+\left[\frac{P_f Q_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w)^{1-\sigma}\int\int_{\varphi_x^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right] g(\varphi,\tau)\,d\varphi d\tau - wf_x\int\int_{\varphi_x^*(\tau)}^{\infty} g(\varphi,\tau)\,d\varphi d\tau\right]$$
$$= wf_e. \quad \text{(A.7)}$$

Rewriting this equation

$$w^{1-\sigma}\left[P^{\sigma}Q\int\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau + P_f^{\sigma}Q_f\tau_x^{1-\sigma}\int\int_{\varphi_x^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau\right]$$
$$= \sigma^{\sigma}(\sigma-1)^{1-\sigma}\left(wf_e + \omega_e wf + \omega_x\omega_e wf_x\right)$$

where $\omega_e = \int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\,d\varphi d\tau$ and $\omega_x = \int\int_{\varphi_x^*(\tau)}^{\infty}\mu(\varphi,\tau)\,d\varphi d\tau = \frac{\int\int_{\varphi_x^*(\tau)}^{\infty} g(\varphi,\tau)d\varphi d\tau}{\int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)d\varphi d\tau}$ are the entry probability and the export probability conditional on entry, respectively. Similarly, we can write Foreign's free entry condition

$$\frac{P_f Q_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w_f^{1-\sigma}\int\int_{\varphi_f^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right] g_f(\varphi,\tau)\,d\varphi d\tau - w_f f\int\int_{\varphi_f^*(\tau)}^{\infty} g_f(\varphi,\tau)\,d\varphi d\tau$$
$$+\left[\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w_f)^{1-\sigma}\int\int_{\varphi_{xf}^*(\tau)}^{\infty}\varphi^{\sigma-1}\tau^{-\sigma}g_f(\varphi,\tau)\,d\varphi d\tau - w_f f_x\int\int_{\varphi_{xf}^*(\tau)}^{\infty} g_f(\varphi,\tau)\,d\varphi d\tau\right]$$
$$= w_f f_e. \quad \text{(A.8)}$$

Home and foreign aggregate prices are

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Big[M \int \int_{\varphi^*(\tau)}^{\infty} (\frac{w\tau}{\varphi})^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau + M_f \int \int_{\varphi^*_{xf}(\tau)}^{\infty} (\frac{w_f \tau \tau_x}{\varphi})^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau\Big], \quad \text{(A.9)}$$

$$P_f^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Big[M_f \int \int_{\varphi^*_f(\tau)}^{\infty} (\frac{w_f\tau}{\varphi})^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau + M \int \int_{\varphi^*_{x}(\tau)}^{\infty} (\frac{w \tau \tau_x}{\varphi})^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau\Big]. \quad \text{(A.10)}$$

Using the free entry and labor market clearing, we have the home and foreign analogue:

$$M_e = \frac{L}{\sigma\left(f_e + \omega_e f + \omega_x \omega_e f_x\right)}. \quad \text{(A.11)}$$

Lastly, the balanced trade condition requires

$$P_f^{\sigma} Q_f M \int \int_{\varphi^*_x(\tau)}^{\infty} \left(\frac{w\tau_x\tau}{\varphi}\right)^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau = P^{\sigma} Q M_f \int \int_{\varphi^*_{xf}(\tau)}^{\infty} \left(\frac{w_f\tau_x\tau}{\varphi}\right)^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau. \quad \text{(A.12)}$$

# B Proofs

## B.1 Proof for Proposition 2

*Proof.* (1) In a closed economy, using the free entry condition and the labor market clearing condition (normalizing $w = 1$)

$$\omega_e E[\pi(\varphi, \tau)] = f_e \tag{A.13}$$

$$\int \int_{\varphi^*(\tau)}^{\infty} \pi(\varphi, \tau) g(\varphi, \tau) d\varphi d\tau = f_e \tag{A.14}$$

$$\int \int_{\varphi^*(\tau)}^{\infty} \sigma^{-\sigma}(\sigma-1)^{\sigma-1} P^{\sigma} Q (\frac{\varphi}{\tau})^{\sigma-1} \frac{1}{\tau} g(\varphi, \tau) d\varphi d\tau = f\omega_e + f_e = \frac{L}{\sigma M_e} \tag{A.15}$$

$$\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} M_e \int \int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} \frac{1}{\tau} P^{\sigma-1} PQ g(\varphi, \tau) d\varphi d\tau = L \tag{A.16}$$

Log differentiating,

$$d \ln W = d \ln Q = -d \ln P - d \ln M_e - (\sigma-1) d \ln P - d \ln \int \int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} \frac{1}{\tau} g(\varphi, \tau) d\varphi d\tau. \tag{A.17}$$

Note that a firm's sales $p(\varphi, \tau) q(\varphi, \tau)$ is proportional to $(\frac{\varphi}{\tau})^{\sigma-1}$, and a firm's labor usage $\ell(\varphi, \tau)$ is proportional to $\varphi^{\sigma-1}\tau^{-\sigma}$. Hence $p_i q_i = M_e \int (\frac{\varphi_i}{\tau})^{\sigma-1} (P^{\sigma} Q) g(\varphi_i, \tau) d\tau$ and $\ell_i = M_e \int (\varphi_i^{\sigma-1} \tau^{-\sigma})(P^{\sigma} Q) g(\varphi_i, \tau) d\tau$ are, respectively, the sum of firm sales and labor with the same productivity $\varphi_i$. Also, $\int \int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi, \tau) d\varphi d\tau$ and $\int \int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau$ are proportional to the cumulative sales and labor share of firms above the cutoff within the market, respectively. And the elasticities of these variables with respect to the cutoff are

$$\gamma_\lambda = -\frac{d \ln \left[\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG(\varphi, \tau)\right]}{d \ln \varphi^*}, \quad \gamma_s = -\frac{d \ln \left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG(\varphi, \tau)\right]}{d \ln \varphi^*}. \tag{A.18}$$

Log-differentiating the price index $P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \int \int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi, \tau) d\varphi d\tau$:

$$(1-\sigma) d \ln P = d \ln M_e + (\sigma-1) \frac{p_i q_i}{PQ} d \ln \varphi_i - \gamma_\lambda(\hat{\varphi}^*) d \ln \hat{\varphi}^*,$$

noting that $\varphi^*(\tau) = \hat{\varphi}^* \tau^{\frac{\sigma}{\sigma-1}}$.

$$\hat{\varphi}^* = con_v \times P^{-1} (PQ)^{\frac{1}{1-\sigma}} \tag{A.19}$$

Substituting the above equation into (A.17):

$$d \ln W = -d \ln P + (\sigma - 1) \left[ \frac{p_i q_i}{PQ} - \frac{\ell_i}{L} \right] d \ln \varphi_i + (\gamma_s(\hat{\varphi}^*) - \gamma_\lambda(\hat{\varphi}^*)) d \ln \hat{\varphi}^*.$$

(2) To derive the effect of trade cost shock in the economy, let let $\lambda$ be the share of the expenditure on domestic goods as in ACR,

$$\lambda = \frac{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau}{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int \int_{\varphi_x^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau}. \tag{A.20}$$

We also define $S$ to be the share of variable labor used in producing domestic goods,

$$S = \frac{\int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau}{\int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int \int_{\varphi_x^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau}. \tag{A.21}$$

Note that without distortions, $\lambda = S$.

First, we make use of the following equations: the price index (A.9), and the balance trade condition (A.12), we get

$$P^{1-\sigma} = con_p M_e w^{1-\sigma} \left[ \int \int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi, \tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int \int_{\varphi_x^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi, \tau) d\varphi d\tau \right]. \tag{A.22}$$

Combine with the definition of $\lambda$,

$$P^{1-\sigma} = con_p M_e w^{1-\sigma} \frac{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau}{\lambda}.$$

Take log and differentiation of the above equation:

$$(1 - \sigma) d \ln P = d \ln M_e + d \ln \left[ \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{1-\sigma} dG(\varphi, \tau) \right] - d \ln \lambda \tag{A.23}$$

Second, use the free entry condition (A.7), the labor market condition, hence the number of firms (A.11) to get

$$w^{1-\sigma}\left[P^{\sigma}Q\int\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau+P_f^{\sigma}Q_f\tau_x^{1-\sigma}\int\int_{\varphi_x^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau\right]$$
$$=\sigma^{\sigma}(\sigma-1)^{1-\sigma}\frac{wL}{\sigma M_e}$$

Combine with the definition of $S$,

$$w^{1-\sigma}P^{\sigma}Q\frac{\int\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau}{S}=\sigma^{\sigma}(\sigma-1)^{1-\sigma}\frac{wL}{\sigma M_e}$$

Take log and differentiation of the above equation:

$$d\ln P^{\sigma}Q+d\ln\left[\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}dG(\varphi,\tau)\right]-d\ln S=-d\ln M_e \tag{A.24}$$

In sum, we have two equations, and using the definition of $\gamma$ (A.18):

$$(1-\sigma)d\ln P=d\ln M_e-d\ln\lambda-\gamma_{\lambda}(\hat{\varphi}^*)d\ln\hat{\varphi}^* \tag{A.25}$$
$$d\ln(PQ)=(1-\sigma)d\ln P-d\ln M_e+d\ln S+\gamma_s(\hat{\varphi}^*)d\ln\hat{\varphi}^*. \tag{A.26}$$

and

$$d\ln Q=-d\ln P+(-d\ln\lambda+d\ln S)+(\gamma_s(\hat{\varphi}^*)-\gamma_{\lambda}(\hat{\varphi}^*))d\ln\hat{\varphi}^*, \tag{A.27}$$

where from the cutoff equation (A.19), we have

$$d\ln\hat{\varphi}^*=-d\ln P-\frac{1}{\sigma-1}d\ln(PQ). \tag{A.28}$$

□

## B.2 Proof for Proposition 3

*Proof.* Solving equations (A.25)-(A.28) gives Proposition 3:

$$
\begin{aligned}
d\ln W = \frac{1}{\gamma_s+\sigma-1}\Big[ & -d\ln\lambda \qquad (ACR) \\
& + d\ln M_e \qquad (MR) \\
& + \frac{\sigma}{\sigma-1}(\gamma_s-\gamma_\lambda) d\ln M_e \qquad (Entry) \\
& - \left(\sigma-1+\frac{\sigma\gamma_s}{\sigma-1}\right) d\ln\lambda + \left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln S \Big] \qquad (Reallocation)
\end{aligned}
\tag{A.29}
$$

□

## B.3 Proof for Corollary 1

*Proof.* Under the special case, $\gamma_\lambda = \frac{\sigma-1}{\sigma}(\theta-\sigma+1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta-\sigma)$, and the change in welfare becomes $d\ln W = \frac{\sigma}{\sigma-1}\left[d\ln S - d\ln\lambda\right]$.

1. Welfare change from a closed to an open economy:

   Because domestic shares are

$$
\lambda = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q}\tau_x^{1-\sigma}\left(\frac{\tau_x^{\sigma-1} f_x}{f}\frac{P^\sigma Q}{P_f^\sigma Q_f}\right)^{\frac{\sigma-1-\theta}{\sigma}} + 1\right]^{-1}
$$

$$
S = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q}\tau_x^{1-\sigma}\left(\frac{\tau_x^{\sigma-1} f_x}{f}\frac{P^\sigma Q}{P_f^\sigma Q_f}\right)^{\frac{\sigma-\theta}{\sigma}} + 1\right]^{-1},
$$

we know that $\lambda > S$ as long as there is selection to export, i.e., $\frac{\tau_x^{\sigma-1} f_x}{f}\frac{P^\sigma Q}{P_f^\sigma Q_f} > 1$. In an open economy, the input share used to produce for exports exceeds the export share under the special case where reallocation is driven purely by distortions. Thus, $d\ln S$ is more negative than $d\ln\lambda$ when moving from a closed to open economy. Hence, the open economy has an unambiguously lower welfare.

2. The reallocation term is always negative:

   In the welfare expression of Prop 2, the second and third terms cancel out, and the welfare change becomes

$$\begin{aligned}&\frac{\sigma}{(\sigma-1)\theta}[-d\ln\lambda-\left(\sigma-1+\frac{\sigma\gamma_s}{\sigma-1}\right)d\ln\lambda+\left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right)d\ln S]\\=&\frac{\sigma}{(\sigma-1)\theta}[-d\ln\lambda+(1-\theta)d\ln\lambda+\theta d\ln S],\end{aligned}$$

where the reallocation term is proportional to $(1-\theta)d\ln\lambda+\theta d\ln S$.

$$\begin{aligned}d\ln\lambda&=(1-\lambda)\frac{\theta+1}{\sigma}d\ln\frac{\tau_x^{\sigma-1}P^\sigma Q}{P_f^\sigma Q_f}\\d\ln S&=(1-S)\frac{\theta}{\sigma}d\ln\frac{\tau_x^{\sigma-1}P^\sigma Q}{P_f^\sigma Q_f}\end{aligned}$$

Substitute for $d\ln\lambda$ and $d\ln S$, the reallocation term is

$$(1-\theta)d\ln\lambda+\theta d\ln S=\frac{\theta^2(1-S)-(\theta^2-1)(1-\lambda)}{\sigma}d\ln\frac{\tau_x^{\sigma-1}P^\sigma Q}{P_f^\sigma Q_f}.$$

Substitute for $\lambda$ and $S$, it can be shown that $\theta^2(1-S)-(\theta^2-1)(1-\lambda)>0$, hence as long as the trade cost reduction induces larger fraction of exporters, the reallocation term is always negative. Q.E.D. □

## B.4 Proof for Corollary 2

*Proof.* Recall the producing cutoff is given by $\varphi^*(\tau)=\hat{\varphi}^*\tau^{\frac{\sigma}{\sigma-1}}$ where $\hat{\varphi}^*=\frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{wf}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}}w$. Recall $I(\hat{\varphi})$ and $O(\hat{\varphi})$ where $I$ is the cumulative input/labor share in the domestic market, and $O$ is the cumulative sales share in the domestic market.

$$I(\hat{\varphi})=\frac{\int\int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau}{\int\int_0^{\inf}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau}$$

$$O(\hat{\varphi})=\frac{\int\int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}}\varphi^{\sigma-1}\tau^{1-\sigma}g(\varphi,\tau)d\varphi d\tau}{\int\int_0^{\inf}\varphi^{\sigma-1}\tau^{1-\sigma}g(\varphi,\tau)d\varphi d\tau}.$$

Let $i(\hat{\varphi}) = I'(\hat{\varphi})$ and $o(\hat{\varphi}) = O'(\hat{\varphi})$. The hazard functions $\gamma_s$ and $\gamma_\lambda$ are

$$\gamma_s = -\frac{d\ln(1 - I(\hat{\varphi}))}{d\ln\hat{\varphi}} = \frac{i(\hat{\varphi})}{1 - I(\hat{\varphi})},$$

$$\gamma_\lambda = -\frac{d\ln(1 - O(\hat{\varphi}))}{d\ln\hat{\varphi}} = \frac{o(\hat{\varphi})}{1 - O(\hat{\varphi})},$$

When $\frac{i(\hat{\varphi})}{o(\hat{\varphi})}$ increases with $\hat{\varphi}$, i.e. $I$ is likelihood ratio dominates $O$, then

$$\frac{1 - I(\hat{\varphi})}{i(\hat{\varphi})} = \int_{\hat{\varphi}} \frac{i(\hat{\varphi}')}{i(\hat{\varphi})} d\hat{\varphi}' \geq \int_{\hat{\varphi}} \frac{o(\hat{\varphi}')}{o(\hat{\varphi})} d\hat{\varphi}' = \frac{1 - O(\hat{\varphi})}{o(\hat{\varphi})},$$

that is, $\gamma_s \leq \gamma_\lambda$.

Let $x = \log\varphi, y = \log\tau$, then $x = \hat{\varphi} + \frac{\sigma}{\sigma-1}y$. Under joint-normal distribution of $(x, y)$, define

$$V(\hat{\varphi}) \equiv \frac{i(\hat{\varphi})}{o(\hat{\varphi})} = \frac{\int \exp(\sigma x(\hat{\varphi}, y) - \sigma y) g(x(\hat{\varphi}, y), y) dy}{\int \exp(\sigma x(\hat{\varphi}, y) + (1-\sigma) y) g(x(\hat{\varphi}, y), y) dy}$$

where

$$g(x, y) = \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_\varphi^2} + \frac{y^2}{\sigma_\tau^2} - \frac{2\rho xy}{\sigma_\varphi\sigma_\tau}\right)\right].$$

When $\sigma_\tau \geq \frac{\sigma-1}{\sigma}\rho\sigma_\varphi$, $V'(\hat{\varphi}) \geq 0$. Then the cumulative labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order, and the hazard functions satisfy $\gamma_s \leq \gamma_d$.

Furthermore,

$$\frac{d\ln\frac{1-I(\hat{\varphi})}{1-O(\hat{\varphi})}}{d\ln\hat{\varphi}} = \frac{d\ln(1 - I(\hat{\varphi}))}{d\ln\hat{\varphi}} - \frac{d\ln(1 - O(\hat{\varphi}))}{d\ln\hat{\varphi}} = -\gamma_s + \gamma_d \geq 0$$

then, it follows

$$\frac{1 - I(\hat{\varphi}_x^*)}{1 - I(\hat{\varphi}^*)} \geq \frac{1 - O(\hat{\varphi}_x^*)}{1 - O(\hat{\varphi}^*)}$$

and $S \leq \lambda$. Moving from a closed economy to an open economy, the reallocation term is always negative. Q.E.D. □

### B.5 Proof for Equation (15)

*Proof.* From (A.25) and (A.28)

$$(1-\sigma)d\ln P = d\ln M_e - d\ln\lambda - \gamma_\lambda(\hat{\varphi}^*)d\ln\hat{\varphi}^*,$$

$$d\ln\hat{\varphi}^* = -d\ln P - \frac{1}{\sigma-1}d\ln(PQ),$$

we get

$$d\ln P = -\frac{1}{\gamma_\lambda+\sigma-1}[-d\ln\lambda + d\ln M_e] - \left(\frac{\gamma_\lambda/(\sigma-1)}{\gamma_\lambda+\sigma-1}\right)d\ln PQ,$$

hence,

$$d\ln W = d\ln Q = d\ln PQ - d\ln P = \frac{1}{\gamma_\lambda+\sigma-1}[-d\ln\lambda + d\ln M_e] + \left(\frac{\gamma_\lambda/(\sigma-1)}{\gamma_\lambda+\sigma-1}+1\right)d\ln PQ.$$

□

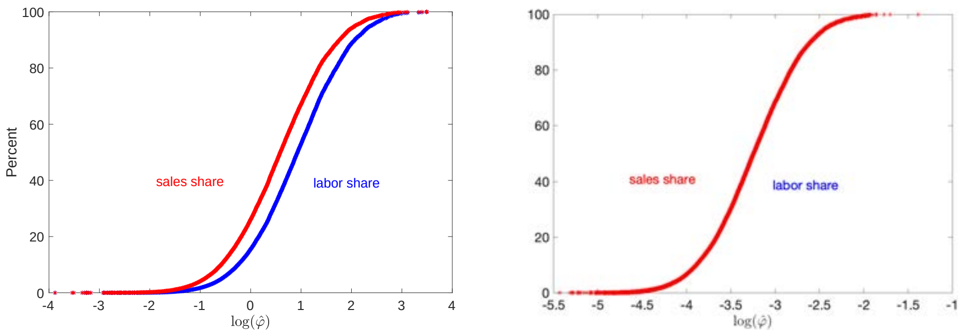
# C Numerical Example

To unpack the theoretical results and to provide more intuition for the mechanisms that underpin these results, we next turn to a numerical example of the benchmark model with symmetric countries, i.e., both face domestic distortions. The assumption of symmetry abstracts from terms of trade effect and highlights the role of misallocation in generating loss from trade. Specifically, If Home suffers a loss from trade is not because Home is subsidizing firms' exports and Foreign gains due to a terms of trade effect. This symmetric example emphasizes that loss from trade comes from the deterioration of resource allocations.

The joint distribution between productivity and distortions is taken to be joint log-normal with standard deviations of $\sigma_\tau = \sigma_\varphi = 0.5$ and correlation of $\varphi$ and $\tau$ of $\rho = 0.8$. The elasticity of substitution $\sigma$ equals 3, the entry cost and fixed costs of domestic producing are 1, and the fixed cost for exporting $f_x$ is 1.5.

Corollary 2 applies here as the distribution of $(\varphi, \tau)$ and the parameters satisfies its conditions. We plot the cumulative variable input and sales share under any $\log(\hat{\varphi})$ in panel (a) of Figure A-1. According to Corollary 2, the cumulative variable input share

Figure A-1: Accumulated Labor Share vs Sales Share in a Market



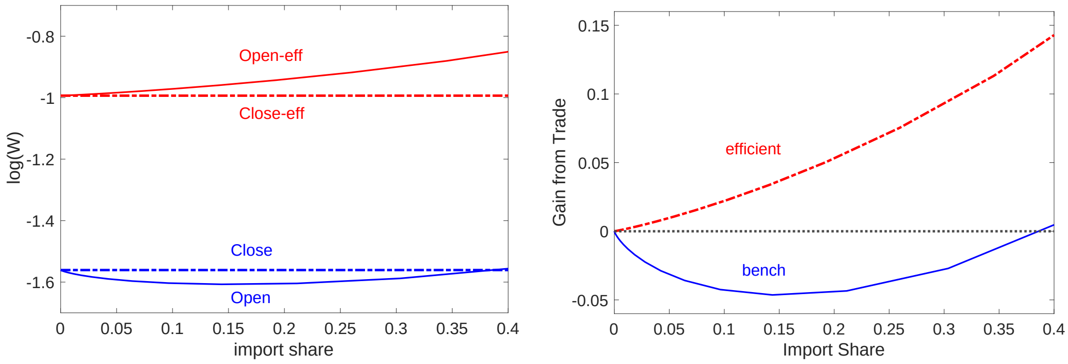

(a) With Distortion (b) Without Distortion

distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order, which implies first-order stochastic dominance. In contrast, without distortions with $\tau = 1$, these two distributions are identical, as shown in panel (b) of Figure A-1. When the economy opens to trade, firms that export are those with high profit and also use a large share of labor to produce. Overall, the share of labor used to produce exports would exceed the export share, worsening the misallocation of resources.

The example helps illustrate a few points. First, welfare (Eq. 8) can fall when the economy opens up to trade. Figure A-2 (a) plots the level of welfare against import shares under the alternative scenarios: the efficient case without distortions, the case with distortions, and when the economy is closed or open. Three observations immediately follow: 1) that there is a welfare loss in the case with distortions compared to the case without; 2) opening up to trade leads to welfare gains in the efficient case; however, 3) opening up engenders a welfare *loss* in the presence of distortions. Taking the differences between the open and close economy in either case, with or without distortion, we plot the welfare change after trade in Figure A-2 (b). It is clear that there is welfare loss with distortions.

Second, the numerical example also demonstrates that using import shares to infer welfare changes can gives rise to markedly different results when there are distortions, as in Figure A-3(a), which decomposes welfare into ACR and a reallocation term, compared against the benchmark. Using ACR under distortions leads to a large departure: welfare *losses* become welfare gains in this case. Thus, using aggregate observables to infer welfare gains as in ACR can thus be very misleading in the presence of distortions, unlike in the

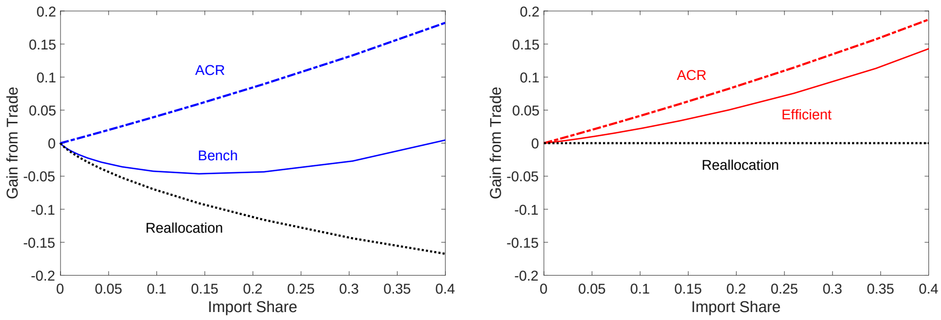
Figure A-2: Welfare and the Change from Trade




(a) Welfare (b) Gain from Trade

efficient case where ACR is a good approximation (Figure A-3(b)).

Figure A-3: Welfare Decomposition

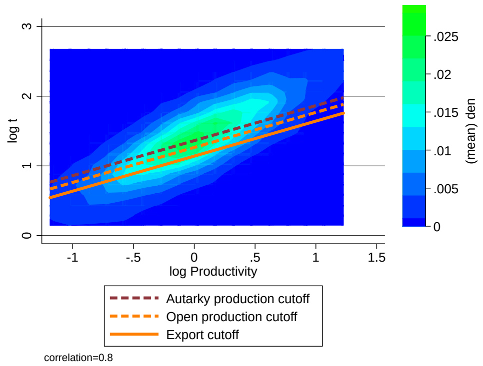



(a) With distortion (b) Without distortion

Next, we examine how distortions affect the selection mechanism, in the same numerical example (Figure A-4). The density of firms is shown by a heat map of firms that lie along a positively sloped distortion-productivity line. The productivity cutoff for production and exports is no longer determined solely by productivity, but also by domestic distortion. Only firms below the cutoff line can operate. In this figure, a large mass of highly-productive firms are excluded from servicing the market altogether. As the economy opens up, the cutoff line is shifted further downward. Even if firms have the same level of productivity, some with higher taxes may be displaced while those with lower ones will survive. This downward shift of the cutoffs allows for some low productivity and high subsidy firms to survive and gain market share.

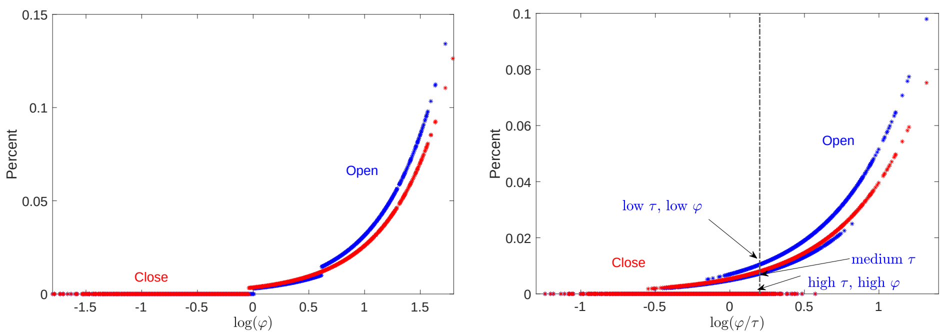
Another way to show the impact on selection is to examine firms' market share. The two panels in Figure A-5 plot the market share of firms, both in the closed and open economy.

Figure A-4: Cutoffs




The left panel is the case without distortions. Without distortion, the marginal cost is the inverse of the productivity $\varphi$. Firms with the same productivity level have the same marginal cost; their market share, above a cutoff productivity, rises with their productivity. Comparing the blue and red lines show that above the export cutoff, more productive firms have higher market shares in the open economy than in the closed economy, demonstrating that these firms expand under trade liberalization. This happens at the cost of displacing other less productive firms' market share, or driving them out of the market entirely. Here, the example clearly demonstrates that resources move from less productive to more productive firms as an economy opens up to trade.

Figure A-5: Selection Effects




(a) Without distortion (b) With distortion

The right panel shows the firm's market share in the case with distortions. Firms may share the same marginal cost $\tau/\varphi$ and face the same potential revenues. However, their after-tax profits may differ, and thus their market share can also differ. Consider the point at which $\log(\varphi/\tau)$ is at 0.2. At this point, a firm with high, medium and low level of productivity faces the same marginal costs. However, the high productivity firm is also subject to high taxes and thus low after-tax profit, and does not make the cut for production. The medium-tax-medium-productivity firm has positive market share but loses out to the low-tax-low-productivity firm when the economy opens up. Resources are reallocated from the more productive to the less productive firms. Also, there is no longer a neat line up of market shares according to productivity: there is a wide range of productivities for which production is excluded.[29] Aggregate welfare effect depends on how trade alters the aggregate domestic labor share and sales share.

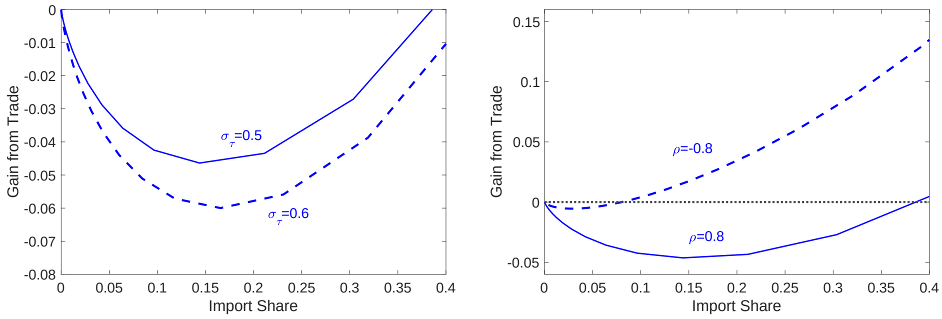
**Distribution of Distortions.** The distribution of distortions is an important determinant to the gains to trade. There are two key parameters: $\rho$, the correlation of $\tau$ and $\varphi$, and $\sigma_\tau$, the dispersion of $\tau$. Figure A-6 (a) compares the gains from trade under different $\sigma_\tau$, while the other parameters remain the same as in the benchmark example. The welfare gain (loss) from trade is always larger (smaller) when $\sigma_\tau$ is smaller.

The correlation of distortion and productivity is important insofar as a higher correlation means that more productive firms are more likely to be excluded from the market. But reductions in welfare is possible even when the correlation is negative. The reason is that for any given productivity, it is always the more subsidized firms that can export, and the highly taxed ones that exit— leading to a possible worsen of misallocation. In fact, as shown in Corollary 2, when the correlation is negative, more productive firms are highly subsidized. Exporters are those more productive and highly subsidized ones, hence their labor share are larger than sales share, and the reallocation term is always negative. Overall effects combine the positive "technology" effect and the negative reallocation effect. Figure A-6(a) illustrates this. It compares the gains from trade for $\rho = 0.8$, under our benchmark numerical example, and for $\rho = -0.8$, where productivity and distortion are highly negatively correlated. Under $\rho = -0.8$, the welfare gain (loss) from trade is always larger (smaller) than that in the case of $\rho = 0.8$. But when the import share is below 20%, there are still losses from trade even under a negative correlation.

In sum, the size of welfare loss after opening up depends on the correlation of $\varphi$ and $\tau$

[29]This is also true if the distortions are input wedge on all the labor a firm uses. Firms face higher input wedge would have a lower profit in a market.

Figure A-6: Gains/Loss from Trade




(a) Dispersion Matters (b) Correlation Matters

and the dispersion of $\tau$. The firm level data helps us identify these parameters. Specifically, in the quantitative section, we will use the firm-level output and use its dispersion and its correlation with firm inputs to estimate the underlying distribution of productivity and distortions.

# D Extended model with Heterogenous Exporting Wedges

In the open economy, an entrant firm draws from a quadruple of productivity $\varphi$, wedge of domestic sales $\tau$, wedge of foreign sales $\tau_{ex}$, and wedge of fixed cost in foreign sales $\tau_{fx}$, i.e. $(\varphi, \tau, \tau_{ex}, \tau_{fx})$, from a distribution with pdf $g(\varphi, \tau, \tau_{ex}, \tau_{fx})$ and cdf $G(\varphi, \tau, \tau_{ex}, \tau_{fx})$. Foreign firms draw the quadruple from a pdf $g_f$ and cdf $G_f$. The foreign country has total labor $L_f$ and endogenous prices of $P_f$ and $w_f$. Export is subject to an iceberg exporting cost $\tau_x$ and $f_x$, which are the same for all the firms.

A domestic exporting firm solves the following problem

$$\max_{p_x, q_x} \frac{1}{\tau_{ex}} p_x q_x - \frac{w}{\varphi} \tau_x q_x - \tau_{fx} w f_x$$

subject to the foreign demand function $q_x = \frac{p_x^{-\sigma}}{P_f^{-\sigma}} Q_f$. The optimal exporting price is

$$p_x = \frac{\sigma}{\sigma - 1} \frac{w \tau_x \tau_{ex}}{\varphi},$$

and the optimal sales is

$$p_x q_x = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} w^{1-\sigma}\tau_x^{1-\sigma}(P_f^{\sigma}Q_f)\left(\frac{\varphi}{\tau_{ex}}\right)^{\sigma-1}.$$

The optimal exporting profit is

$$\pi_x = \sigma^{-\sigma}(\sigma-1)^{\sigma-1}P_f^{\sigma}Q_f(w\tau_x)^{1-\sigma}\varphi^{\sigma-1}\tau_{ex}^{-\sigma} - \tau_{fx}wf_x.$$

**Cutoffs** The two cutoff productivities in the home country entering the domestic market, $\varphi^*(\tau)$, and foreign markets, $\varphi_x^*(\tau_{ex}, \tau_{fx})$, are:

$$\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{wf}{P^{\sigma}Q}\right]^{\frac{1}{\sigma-1}} w\tau^{\frac{\sigma}{\sigma-1}}, \qquad \varphi_x^*(\tau_{ex}, \tau_{fx}) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{\tau_{fx}wf_x\tau_x^{\sigma-1}}{P_f^{\sigma}Q_f}\right]^{\frac{1}{\sigma-1}} w\tau_{ex}^{\frac{\sigma}{\sigma-1}}. \quad \text{(A.30)}$$

Similarly, the two cutoffs for the foreign country are

$$\varphi_f^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{w_f f}{P_f^{\sigma}Q_f}\right]^{\frac{1}{\sigma-1}} w_f\tau^{\frac{\sigma}{\sigma-1}}, \qquad \varphi_{xf}^*(\tau_{ex}, \tau_{fx}) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{\tau_{fx}w_f f_x\tau_x^{\sigma-1}}{P^{\sigma}Q}\right]^{\frac{1}{\sigma-1}} w_f\tau_{ex}^{\frac{\sigma}{\sigma-1}}. \quad \text{(A.31)}$$

**Free Entry Conditions**

$$\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG - wf\int_{\varphi^*(\tau)}^{\infty} dG$$
$$+\Big[\frac{P_fQ_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG - wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG\Big] = wf_e, \quad \text{(A.32)}$$

and similarly for the foreign country:

$$\frac{P_f Q_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w_f^{1-\sigma}\int_{\varphi_f^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG_f - w_f f\int_{\varphi_f^*(\tau)} dG_f$$
$$+\left[\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w_f)^{1-\sigma}\int_{\varphi_{xf}^*(\tau_{ex},\tau_f)}\varphi^{\sigma-1}\tau_{ex}^{-\sigma}dG_f - w_f f_x\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG_f\right] = w_f f_e \tag{A.33}$$

**Measure $M$ and $M_f$** Define the fraction of firms operating for the domestic market and the fraction exporting, conditional on producing to be:

$$\omega_e = \int_{\varphi^*(\tau)} dG(\varphi,\tau,\tau_{ex},\tau_{fx}), \qquad \omega_x = \frac{\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} dG\left(\varphi,\tau,\tau_{ex},\tau_{fx}\right)}{\int_{\varphi^*(\tau)} dG\left(\varphi,\tau,\tau_{ex},\tau_{fx}\right)},$$

$$\omega_{ef} = \int_{\varphi_f^*(\tau)} dG_f(\varphi,\tau,\tau_{ex},\tau_{fx}), \qquad \omega_{xf} = \frac{\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})} dG_f\left(\varphi,\tau,\tau_{ex},\tau_{fx}\right)}{\int_{\varphi_f^*(\tau)} dG_f\left(\varphi,\tau,\tau_{ex},\tau_{fx}\right)}.$$

Home's free entry condition implies

$$\int_{\varphi^*(\tau)}\left(\frac{1}{\sigma-1}\frac{q}{\varphi}-f\right)dG + \int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{1}{\sigma-1}\tau_x\frac{q_x}{\varphi}-\tau_{fx}f_x\right)dG = f_e,$$

where we replaced the optimal profits $\pi$ with $\frac{1}{\sigma-1}\frac{wq}{\varphi} - wf$ and $\pi_x$ with $\frac{1}{\sigma-1}\frac{\tau_x w q_x}{\varphi} - w\tau_{fx}f_x$. Home's labor market clearing condition requires

$$L = M_e\left[\int_{\varphi^*(\tau)}\left(\frac{q}{\varphi}+f\right)dG + \int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\tau_x\frac{q_x}{\varphi}+f_x\right)dG + f_e\right].$$

Using the free-entry condition and the labor market clearing condition, we have

$$M_e = \frac{L}{\sigma\left[f_e + \omega_e f + \omega_x\omega_e f_x + \frac{\sigma-1}{\sigma}f_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}(\tau_{fx}-1)dG\right]}, \tag{A.34}$$

and similarly for foreign:

$$M_{ef} = \frac{L_f}{\sigma \left[ f_e + \omega_{ef} f + \omega_{xf}\omega_{ef} f_x + \frac{\sigma-1}{\sigma} f_x \int_{\varphi^*_{xf}(\tau_{ex},\tau_{fx})} (\tau_{fx} - 1) dG_f \right]}. \tag{A.35}$$

We can then get $M = \omega_e M_e$ and $M_f = \omega_{ef} M_{ef}$.

**Aggregate price level** We can write the the aggregate prices of home and foreign as:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[ M w^{1-\sigma} \frac{\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG}{\int_{\varphi^*(\tau)} dG} + M_f (\tau_x w_f)^{1-\sigma} \frac{\int_{\varphi^*_{xf}(\tau_{ex},\tau_{fx})}^{\infty} (\frac{\varphi}{\tau_{ex}})^{\sigma-1} dG_f}{\int_{\varphi^*_f(\tau)}^{\infty} dG_f} \right] \tag{A.36}$$

$$P_f^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[ M_f w_f^{1-\sigma} \frac{\int_{\varphi^*_f(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG_f}{\int_{\varphi^*_f(\tau)} dG_f} + M (\tau_x w)^{1-\sigma} \frac{\int_{\varphi^*_x(\tau_{ex},\tau_{fx})}^{\infty} (\frac{\varphi}{\tau_{ex}})^{\sigma-1} dG}{\int_{\varphi^*(\tau)}^{\infty} dG} \right]. \tag{A.37}$$

**Summary of equilibrium conditions** The equilibrium consists of $(P, P_f, M, M_f, Q, Q_f, w_f)$ with $w = 1$ as normalization. The equations consist of two free entry conditions (A.32) and (A.33), two labor clearing conditions (A.34) and (A.35), two price indices (A.36) and (A.37), and the balanced trade condition

$$P_f^{\sigma} Q_f M_e \int_{\varphi^*_x(\tau_{ex},\tau_{fx})} \left(\frac{w \tau_x \tau_{ex}}{\varphi}\right)^{1-\sigma} dG \;=\; P^{\sigma} Q M_{ef} \int_{\varphi^*_{xf}(\tau_{ex},\tau_{fx})} \left(\frac{w_f \tau_x \tau_{ex}}{\varphi}\right)^{1-\sigma} dG_f. \tag{A.38}$$

Finally, the cutoff functions are given by (A.30) and (A.31).

# E Proof of General Welfare Formula in the Extended Model

Proposition 5: in the extended model, the change in welfare associated with an iceberg cost shock is

$$
\begin{aligned}
d\ln W = \frac{1}{\gamma_s+\sigma-1}\Big\{ & -d\ln\lambda + d\ln M_e \\
& + \frac{\sigma}{\sigma-1}(\gamma_s-\gamma_\lambda) d\ln M_e \\
& - \left(\sigma-1+\frac{\sigma\gamma_s}{\sigma-1}\right) d\log\lambda + \left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln S \\
& + \left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln\left(1+\frac{M_e f_x \int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}(\tau_{fx}-1)\,dG}{L}\right)\Big\}.
\end{aligned}
\tag{A.39}
$$

*Proof.* 1. Define input $S$ and output $\lambda$ shares

$$
\lambda = \frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG}{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right] + \frac{P_f^\sigma Q_f}{P^\sigma Q}\tau_x^{1-\sigma}\left[\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \varphi^{\sigma-1}\tau_{ex}^{1-\sigma} dG\right]}
$$

$$
S = \frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG}{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] + \frac{P_f^\sigma Q_f}{P^\sigma Q}\tau_x^{1-\sigma}\left[\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \varphi^{\sigma-1}\tau_{ex}^{-\sigma} dG\right]}
$$

2. Define $\gamma_\lambda(\hat{\varphi})$ and $\gamma_s(\hat{\varphi})$

   $\gamma_\lambda(\hat{\varphi})$—the elasticity of the cumulative sales within the domestic market for firms above a cutoff, and $\gamma_s(\hat{\varphi})$—the elasticity of the cumulative domestic (variable) labor for firms above any cutoff $\hat{\varphi}$, both with respect to the cutoff.

$$
\gamma_\lambda(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} (\frac{\varphi}{\tau})^{\sigma-1} dG\right]}{d\ln\hat{\varphi}}, \quad \gamma_s(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{-\sigma} dG\right]}{d\ln\hat{\varphi}}. \tag{A.40}
$$

   Note $\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} (\frac{\varphi}{\tau})^{\sigma-1} dG$ is proportional to the cumulative market share (in any given market) of firms above any cutoff $\hat{\varphi}$. Therefore $\gamma_\lambda(\hat{\varphi})$ represents the hazard function for the distribution of log firm sales within a market. Similarly, $\gamma_s(\hat{\varphi})$ represents the hazard function for the distribution of log firm variable labor within a market. $\gamma_\lambda(\hat{\varphi}^*)$ and $\gamma_s(\hat{\varphi}^*)$ are these elasticity evaluated at the domestic production cutoff.

3. Free entry condition

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}w^{1-\sigma}\left[\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG+\frac{P_f^{\sigma}Q_f}{P^{\sigma}Q}(\tau_x)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG\right]$$
$$=wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG+w\omega_e f+wf_e$$

We can rewrite the equilibrium condition (A.34) of $M_e$

$$M_e=\frac{L}{\sigma f_e+\omega_e\sigma f+\left[(\sigma-1)\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\tau_{fx}dG+\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}dG\right]f_x},$$

as the following one

$$\omega_e wf+wf_e=\frac{wL}{\sigma M_e}-\left[\frac{\sigma-1}{\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\tau_{fx}dG+\frac{1}{\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}dG\right]wf_x. \tag{A.41}$$

Replacing $\omega_e wf+wf_e$ in the free-entry condition using (A.41), we have

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}w^{1-\sigma}\left[\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG+\frac{P_f^{\sigma}Q_f}{P^{\sigma}Q}(\tau_x)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG\right]$$
$$=\frac{1}{\sigma}wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG+\frac{wL}{\sigma M_e}$$

Using the definition of $S$ and normalizing $w=1$, we reach the following equation:

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{\left[\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}dG\right]}{S}=\frac{L}{\sigma M_e}\left[1+\frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG\right]$$

4. Price index:

$$P^{1-\sigma}=\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left[M_e w^{1-\sigma}\int_{\varphi^*(\tau)}(\frac{\varphi}{\tau})^{\sigma-1}dG+M_{ef}(\tau_x w_f)^{1-\sigma}\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}^{\infty}(\frac{\varphi}{\tau_{ex}})^{\sigma-1}dG_f\right]$$

Replacing the second-term with the following balance trade condition

$$P_f^{\sigma}Q_f M_e\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{w\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma}dG=P^{\sigma}QM_{ef}\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}\left(\frac{w_f\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma}dG_f$$

we have

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG + (\tau_x)^{1-\sigma} \frac{P_f^\sigma Q_f}{P^\sigma Q} \int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \left(\frac{\varphi}{\tau_{ex}}\right)^{\sigma-1} dG\right].$$

Using the definition of $\lambda$, the above equation becomes

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG}{\lambda}\right].$$

5. Summary of two equations: from free-entry and pricing index, we have

$$\frac{P^\sigma Q}{\sigma} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right]}{S} = \frac{L}{\sigma M_e}\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx}-1\right) dG\right]$$

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG(\varphi,\tau)}{\lambda}\right]$$

Taking log and differentiation of the above two equations:

$$\begin{aligned} d\ln P^\sigma Q + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] - d\ln S \\ = -d\ln M_e + d\ln\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx}-1\right) dG\right] \end{aligned}$$

$$(1-\sigma) d\ln P = d\ln M_e + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right] - d\ln\lambda$$

The term $d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right]$ from the first equation above is

$$\begin{aligned} d\log\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] &= -\frac{\int \varphi^*(\tau)^\sigma \tau^{-\sigma} g(\varphi^*(\tau),\tau) d\log\varphi^*(\tau) d\tau}{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG(\varphi,\tau)} \\ &= -\frac{\int \varphi^*(\tau)^\sigma \tau^{-\sigma} g(\varphi^*(\tau),\tau) d\tau}{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG(\varphi,\tau)} \frac{1}{1-\sigma}\left(\sigma d\ln P + d\ln Q\right) \\ &= \gamma_s \frac{1}{\sigma-1}\left(\sigma d\ln P + d\ln Q\right) \end{aligned}$$

where the last equality uses the cutoff condition: $\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[wf\right]^{\frac{1}{\sigma-1}} w(P^\sigma Q)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{\sigma-1}}$.

Similarly, in the second equation, $d\ln\left[\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{1-\sigma}dG\right]$ is such that

$$d\ln\left[\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{1-\sigma}dG\right]=\gamma_\lambda\frac{1}{\sigma-1}\left(\sigma d\ln P+d\ln Q\right).$$

6. Plugging $\gamma_s$ and $\gamma_\lambda$ back into the two equations we have

$$\begin{aligned}\sigma d\ln P+d\ln Q+\gamma_s\frac{1}{\sigma-1}\left(\sigma d\ln P+d\ln Q\right)-d\ln S&\\ =-d\ln M_e+d\ln&\left[1+\frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG\right]\end{aligned}\tag{A.42}$$

$$(1-\sigma)d\ln P=d\ln M_e+\gamma_\lambda\frac{1}{\sigma-1}\left(\sigma d\ln P+d\ln Q\right)-d\ln\lambda\tag{A.43}$$

7. Finally, solve the above two equations, we have $d\ln W=d\ln Q$ and

$$\begin{aligned}d\ln W=&\frac{1}{\gamma_s+\sigma-1}\Big\{-d\ln\lambda+d\ln M_e\\&+\frac{\sigma}{\sigma-1}(\gamma_s-\gamma_\lambda)d\ln M_e\\&-\left(\sigma-1+\frac{\sigma\gamma_s}{\sigma-1}\right)d\log\lambda+\left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right)d\ln S\\&+\left(\sigma-1+\frac{\sigma\gamma_\lambda}{\sigma-1}\right)d\ln\left(1+\frac{M_e f_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG}{L}\right)\Big\}.\end{aligned}$$

□

# F Endogenous Wedges

A new entrant firm draws a quadruple of productivity $\varphi$, wedge of domestic sales $\varepsilon$, wedge of foreign sales $\varepsilon_{ex}$, and wedge of fixed cost in foreign sales $\tau_{fx}$, i.e. $(\varphi,\varepsilon,\varepsilon_{ex},\tau_{fx})$, from a distribution with pdf $g(\varphi,\varepsilon,\varepsilon_{ex},\tau_{fx})$ and cdf $G(\varphi,\varepsilon,\varepsilon_{ex},\tau_{fx})$. Foreign firms draw the quadruple from a pdf $g_f$ and cdf $G_f$. Assume that

$$\ln\tau=\beta\ln pq+\ln\varepsilon,\qquad\ln\tau_{ex}=\beta\ln p_x q_x+\ln\varepsilon_{ex},$$

i.e, firms wedges also depend on their size.

An exporting firm at Home solves the following problem

$$\max_{p_x,q_x} \frac{1}{\tau_{ex}} p_x q_x - \frac{w}{\varphi}\tau_x q_x - \tau_{fx} w f_x$$

subject to the foreign demand function $q_x = \frac{p_x^{-\sigma}}{P_f^{-\sigma}} Q_f$.

Let $\tilde{\sigma} \equiv \frac{\sigma}{1-\beta+\sigma\beta}$. The optimal exporting price and quantity is

$$p_x = \left[\frac{\tilde{\sigma}}{(\tilde{\sigma}-1)} \frac{\varepsilon_{ex} w \tau_x}{\varphi} (P_f^{\sigma} Q_f)^{\beta}\right]^{\frac{\tilde{\sigma}}{\sigma}}$$
$$q_x = \left[\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right]^{\tilde{\sigma}} \left(P_f^{\sigma} Q_f\right)^{\frac{\tilde{\sigma}-1}{\sigma-1}} \left[\frac{\varepsilon_{ex} w \tau_x}{\varphi}\right]^{-\tilde{\sigma}},$$

and the optimal profit of exporting is given by

$$\pi_x = \frac{1}{\tilde{\sigma}-1} \left[\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right]^{\tilde{\sigma}} (P_f^{\sigma} Q_f)^{\frac{\tilde{\sigma}-1}{\sigma-1}} (\frac{\varphi}{w\tau_x})^{(\tilde{\sigma}-1)} \varepsilon_{ex}^{-\tilde{\sigma}} - \tau_{fx} w f_x.$$

**Cutoffs** There are two cutoff productivities in home country, $\varphi^*(\tau)$ for entering the domestic market and $\varphi_x^*(\tau_{ex}, \tau_{fx})$ for entering the foreign market:

$$\varphi^*(\varepsilon) = \frac{\tilde{\sigma}^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}}}{\tilde{\sigma}-1} [wf]^{\frac{1}{\tilde{\sigma}-1}} [P^{\sigma} Q]^{-\frac{1}{\sigma-1}} w \varepsilon_{ex}^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}}$$

$$\varphi_x^*(\varepsilon_{ex}, \tau_{fx}) = \frac{\tilde{\sigma}^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}}}{\tilde{\sigma}-1} \left[w f_x \tau_{fx}\right]^{\frac{1}{\tilde{\sigma}-1}} \left[P_f^{\sigma} Q_f\right]^{-\frac{1}{\sigma-1}} (w\tau_x)\, \varepsilon_{ex}^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}}.$$

**Free entry conditions** The free entry condition for home implies

$$\frac{1}{\tilde{\sigma}-1} \left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)^{\tilde{\sigma}} (P^{\sigma} Q)^{\frac{\tilde{\sigma}-1}{\sigma-1}} w^{1-\tilde{\sigma}} \int_{\varphi^*(\tau)} \left[\varphi^{\tilde{\sigma}-1} \varepsilon^{-\tilde{\sigma}}\right] dG - wf \int_{\varphi^*(\tau)} dG$$
$$+ \left[\frac{1}{\tilde{\sigma}-1} \left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)^{\tilde{\sigma}} (P_f^{\sigma} Q_f)^{\frac{\tilde{\sigma}-1}{\sigma-1}} (\tau_x w)^{1-\tilde{\sigma}} \int_{\varphi_x^*(\varepsilon_{ex}, \tau_{fx})} \left[\varphi^{\tilde{\sigma}-1} \varepsilon_{ex}^{-\tilde{\sigma}}\right] dG - w f_x \int_{\varphi_x^*(\tau_{ex}, \tau_{fx})}^{\infty} \tau_{fx} dG\right] = w f_e. \tag{A.44}$$

A similar equation holds for the foreign economy.

$$\frac{1}{\tilde{\sigma_f}-1}\left(\frac{\tilde{\sigma_f}-1}{\tilde{\sigma_f}}\right)^{\tilde{\sigma_f}}(P_f^{\sigma}Q_f)^{\frac{\tilde{\sigma_f}-1}{\tilde{\sigma}-1}}w_f^{1-\tilde{\sigma_f}}\int_{\varphi_f^*(\tau)}\left[\varphi^{\tilde{\sigma_f}-1}\varepsilon^{-\tilde{\sigma_f}}\right]dG_f - w_f f\int_{\varphi_f^*(\tau)}dG_f$$
$$+\Big[\frac{1}{\tilde{\sigma_f}-1}\left(\frac{\tilde{\sigma_f}-1}{\tilde{\sigma_f}}\right)^{\tilde{\sigma_f}}(P^{\sigma}Q)^{\frac{\tilde{\sigma_f}-1}{\tilde{\sigma}-1}}\left(\tau_x w_f\right)^{1-\tilde{\sigma_f}}\int_{\varphi_{xf}^*(\varepsilon_{ex},\tau_{fx})}\left[\varphi^{\tilde{\sigma_f}-1}\varepsilon_{ex}^{-\tilde{\sigma_f}}\right]dG_f$$
$$- w_f f_x\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG_f\Big] = w_f f_e. \quad \text{(A.45)}$$

**Measure** $M$ **and** $M_f$ Conditional on entry, the expected per-period profit includes the profit from both domestic production and exporting, where the average profits for domestic and foreign sales are given by

$$E\pi = E\left[\frac{1}{\tilde{\sigma}-1}\frac{wq}{\varphi} - wf\right], \qquad E_x\pi_x = E_x\left[\frac{1}{\tilde{\sigma}-1}\frac{\tau_x w q_x}{\varphi} - w\tau_{fx}f_x\right].$$

Free entry implies

$$E\left[\frac{1}{\tilde{\sigma}-1}\frac{wq}{\varphi} - wf\right] + \omega_x E_x\left[\frac{1}{\tilde{\sigma}-1}\frac{\tau_x w q_x}{\varphi} - w\tau_{fx}f_x\right] = \frac{wf_e}{\omega_e}.$$

The labor market clearing condition implies

$$L = M\left(E\frac{q}{\varphi}+f\right) + M\omega_x\left(E_x\frac{\tau_x q_x}{\varphi}+f_x\right) + M_e f_e.$$

Hence, we can write $M_e$ as

$$M_e = \frac{L}{\tilde{\sigma}f_e + \tilde{\sigma}\omega_e f + \omega_x\omega_e f_x\left[(\tilde{\sigma}-1)E_x\tau_{fx}+1\right]}. \quad \text{(A.46)}$$

A similar equation holds for the Foreign economy.

$$M_{ef} = \frac{L_f}{\tilde{\sigma_f}f_e + \tilde{\sigma_f}\omega_{ef} + \omega_{xf}\omega_{ef}f_x\left[(\tilde{\sigma_f}-1)E_{xf}\tau_{fx}+1\right]}. \quad \text{(A.47)}$$

**Aggregate price level**

$$P^{1-\sigma} = \Big[ \left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} (P^{\sigma}Q)^{\beta\frac{1-\tilde{\sigma}}{1-\beta}} M w^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*(\varepsilon)} \left(\frac{\varphi}{\varepsilon}\right)^{\frac{\tilde{\sigma}-1}{1-\beta}} dG}{\int_{\varphi^*(\varepsilon)} dG} + \left(\frac{\tilde{\sigma}_f}{\tilde{\sigma}_f-1}\right)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} (P^{\sigma}Q)^{\beta_f\frac{1-\tilde{\sigma}_f}{1-\beta_f}} M_f (\tau_x w_f)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} \frac{\int_{\varphi^*_{xf}(\varepsilon_{ex},\tau_{fx})}^{\infty} \left(\frac{\varphi}{\varepsilon_{ex}}\right)^{\frac{\tilde{\sigma}_f-1}{1-\beta_f}} dG_f}{\int_{\varphi^*_f(\varepsilon)}^{\infty} dG_f} \Big] \tag{A.48}$$

$$P_f^{1-\sigma} = \Big[ \left(\frac{\tilde{\sigma}_f}{\tilde{\sigma}_f-1}\right)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} \left(P_f^{\sigma}Q_f\right)^{\beta_f\frac{1-\tilde{\sigma}_f}{1-\beta_f}} M_f w_f^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} \frac{\int_{\varphi^*_f(\varepsilon)} \left(\frac{\varphi}{\varepsilon}\right)^{\frac{\tilde{\sigma}_f-1}{1-\beta_f}} dG_f}{\int_{\varphi^*_f(\varepsilon)} dG_f} + \left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} \left(P_f^{\sigma}Q_f\right)^{\beta\frac{1-\tilde{\sigma}}{1-\beta}} M (\tau_x w)^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*_x(\varepsilon_{ex},\tau_{fx})}^{\infty} \left(\frac{\varphi}{\varepsilon_{ex}}\right)^{\frac{\tilde{\sigma}-1}{1-\beta}} dG}{\int_{\varphi^*(\varepsilon)}^{\infty} dG} \Big] \tag{A.49}$$

**Summary of equilibrium conditions** The equilibrium consists of $(P, P_f, M, M_f, Q, Q_f, w_f)$ with $w = 1$ as normalization. In addition to the free entry conditions (A.44) and (A.45), the pricing equations (A.48) and (A.49), and measure of firms (A.46) and (A.47), there is one balanced trade condition

$$\left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} \left(P_f^{\sigma}Q_f\right)^{\frac{\tilde{\sigma}}{\sigma}} M \tau_x^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*_x(\varepsilon_{ex},\tau_{fx})} \left(\frac{w\tau_x\varepsilon_{ex}}{\varphi}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} dG}{\int_{\varphi^*(\varepsilon)} dG}$$

$$= \left(\frac{\tilde{\sigma}_f}{\tilde{\sigma}_f-1}\right)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} (P^{\sigma}Q)^{\frac{\tilde{\sigma}_f}{\sigma}} w_f^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} \tau_x^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} M_f \frac{\int_{\varphi^*_{xf}(\varepsilon_{ex},\tau_{fx})} \left(\frac{w_f\tau_x\varepsilon_{ex}}{\varphi}\right)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} dG_f}{\int_{\varphi^*_f(\varepsilon)} dG_f},$$

along with the associated cutoffs given above.

# G Proof of Welfare with Endogenous Wedges

**Proposition 6.** The change in welfare associated with an iceberg cost shock is

$$
\begin{aligned}
d\ln W = \frac{1}{\gamma_s + \tilde{\sigma} - 1}\Big\{ & - d\ln\lambda + d\ln M_e \\
& + \left(\frac{\sigma}{\sigma-1}(\gamma_s - \gamma_\lambda) - \tilde{\sigma}\beta\right) d\ln M_e \\
& - \left(\tilde{\sigma}(1-\beta) - 1 + \frac{\sigma\gamma_s}{\sigma-1}\right) d\ln\lambda + \left(\tilde{\sigma} - 1 + \frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln S \\
& + \left(\tilde{\sigma} - 1 + \frac{\sigma\gamma_\lambda}{\sigma-1}\right) d\ln\left(1 + \frac{M_e f_x \int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx} - 1\right) dG}{L}\right)\Big\}
\end{aligned}
\tag{A.50}
$$

where $\tilde{\sigma} = \sigma/(1-\beta+\sigma\beta)$.

1. With homogenous productivity, and a distortion that positively depends on a firm's sales with an elasticity of $\beta$ and a Pareto-distributed distortion $1/\varepsilon$ with parameter $\theta$:

$$
dlnW = \frac{\sigma}{\sigma-1}\left[d\ln S - d\ln\lambda\right],
$$

   which is the same expression as the case without endogenous wedges.

2. With Pareto-distributed productivity with parameter $\theta$, and a distortion that positively depends on a firm's sales with an elasticity of $\beta$, and no exogenous distortions, the welfare becomes

$$
d\ln W = \frac{1}{\theta}\Big[ - d\ln\lambda + \left(\frac{\sigma}{\sigma-1}\theta - 1\right)\left(-d\ln\lambda + d\ln S\right)\Big].
$$

Under a general distribution, $\beta$ shows up explicitly in the formula (Eq. 20). On the one hand, it changes the elasticity by changing $\sigma$ to an effective elasticity, $\tilde{\sigma} = \sigma/(1-\beta+\sigma\beta) \leq \sigma$ if $\beta \geq 0$. On the other hand, the endogenous wedge also affects the elasticity of $\gamma_\lambda$ and $\gamma_s$.

*Proof.* We now prove the general welfare formula under the endogenous wedge.

(1) Let the labor share be

$$S = \frac{(P^{\sigma}Q)^{\frac{\tilde{\sigma}-1}{\sigma-1}} \int_{\varphi^*(\varepsilon)} \varphi^{\tilde{\sigma}-1}\varepsilon^{-\tilde{\sigma}} dG}{\left[(P^{\sigma}Q)^{\frac{\tilde{\sigma}-1}{\sigma-1}} \int_{\varphi^*(\varepsilon)} \varphi^{\tilde{\sigma}-1}\varepsilon^{-\tilde{\sigma}} dG + (P_f^{\sigma}Q_f)^{\frac{\tilde{\sigma}-1}{\sigma-1}} \tau_x^{1-\tilde{\sigma}} \int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})} \varphi^{\tilde{\sigma}-1}\varepsilon_{ex}^{-\tilde{\sigma}} dG\right]} \tag{A.51}$$

The labor market clearing condition is

$$\begin{aligned} L &= M\left(E\frac{q}{\varphi} + f\right) + M\omega_x\left(E_x\frac{\tau_x q_x}{\varphi} + f_x\right) + M_e f_e \\ &= M\tilde{\sigma}\left[\frac{f_e}{\omega_e} + f + \omega_x f_x \frac{(\tilde{\sigma}-1)E_x\tau_{fx} + 1}{\tilde{\sigma}}\right] \end{aligned}$$

Hence,

$$M = \frac{L}{\tilde{\sigma}\frac{f_e}{\omega_e} + \tilde{\sigma} f + \omega_x f_x\left[(\tilde{\sigma}-1)E_x\tau_{fx} + 1\right]}.$$

Combined with the free entry condition A.44, and using the definition of S,

$$\begin{aligned} &\frac{1}{\tilde{\sigma}-1}\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)^{\tilde{\sigma}} (P^{\sigma}Q)^{\frac{\tilde{\sigma}-1}{\sigma-1}} w^{1-\tilde{\sigma}} \int_{\varphi^*(\varepsilon)} \left[\varphi^{\tilde{\sigma}-1}\varepsilon^{-\tilde{\sigma}}\right] dG \\ &+ \frac{1}{\tilde{\sigma}-1}\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)^{\tilde{\sigma}} (P_f^{\sigma}Q_f)^{\frac{\tilde{\sigma}-1}{\sigma-1}} (\tau_x w)^{1-\tilde{\sigma}} \int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})} \left[\varphi^{\tilde{\sigma}-1}\varepsilon_{ex}^{-\tilde{\sigma}}\right] dG \\ &= wf_e + wf\int_{\varphi^*(\varepsilon)} dG + wf_x \int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})}^{\infty} \tau_{fx} dG \end{aligned}$$

$$\frac{1}{\tilde{\sigma}-1}\left(\frac{\tilde{\sigma}-1}{\tilde{\sigma}}\right)^{\tilde{\sigma}} \frac{(P^{\sigma}Q)^{\frac{\tilde{\sigma}-1}{\sigma-1}} \int_{\varphi^*(\varepsilon)} \varphi^{\tilde{\sigma}-1}\varepsilon^{-\tilde{\sigma}} dG}{S} = \frac{L}{\tilde{\sigma}M_e}\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})}^{\infty} (\tau_{fx}-1)\, dG\right]$$

and log differentiating, we have

$$\frac{\tilde{\sigma}-1}{\sigma-1} d\ln(P^{\sigma}Q) - \gamma_S d\ln\varphi^* - d\ln S = -d\ln M_e + d\ln\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\varepsilon_{ex},\tau_{fx})}^{\infty} (\tau_{fx}-1)\, dG\right]$$

(2) Define the total spending $E$ as

$$E = \left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} (P^{\sigma}Q)^{\beta\frac{1-\tilde{\sigma}}{1-\beta}+1} Mw^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*(\varepsilon)} \left(\frac{\varphi}{\varepsilon}\right)^{\frac{\tilde{\sigma}-1}{1-\beta}} dG}{\int_{\varphi^*(\varepsilon)} dG}$$
$$+ \left(\frac{\tilde{\sigma}_f}{\tilde{\sigma}_f-1}\right)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} (P^{\sigma}Q)^{\beta_f\frac{1-\tilde{\sigma}_f}{1-\beta_f}+1} M_f(\tau_x w_f)^{\frac{1-\tilde{\sigma}_f}{1-\beta_f}} \frac{\int_{\varphi^*_{xf}(\varepsilon_{ex},\tau_{fx})}^{\infty} \left(\frac{\varphi}{\varepsilon_{ex}}\right)^{\frac{\tilde{\sigma}_f-1}{1-\beta_f}} dG_f}{\int_{\varphi^*_f(\varepsilon)}^{\infty} dG_f}$$

hence the sales share is

$$\lambda = \frac{\left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} (P^{\sigma}Q)^{\beta\frac{1-\tilde{\sigma}}{1-\beta}+1} Mw^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*(\varepsilon)} \left(\frac{\varphi}{\varepsilon}\right)^{\frac{\tilde{\sigma}-1}{1-\beta}} dG}{\int_{\varphi^*(\varepsilon)} dG}}{E} \tag{A.52}$$

Substituting into the price index (A.48),

$$P^{1-\sigma} = \frac{\left(\frac{\tilde{\sigma}}{\tilde{\sigma}-1}\right)^{\frac{1-\tilde{\sigma}}{1-\beta}} (P^{\sigma}Q)^{\beta\frac{1-\tilde{\sigma}}{1-\beta}} Mw^{\frac{1-\tilde{\sigma}}{1-\beta}} \frac{\int_{\varphi^*(\varepsilon)} \left(\frac{\varphi}{\varepsilon}\right)^{\frac{\tilde{\sigma}-1}{1-\beta}} dG\left(\varphi,\varepsilon,\varepsilon_{ex},\tau_{fx}\right)}{\int_{\varphi^*(\varepsilon)} dG\left(\varphi,\varepsilon,\varepsilon_{ex},\tau_{fx}\right)}}{\lambda}$$

and log differentiating, we have

$$(1-\sigma)d\ln P = \frac{\beta(1-\tilde{\sigma})}{1-\beta} d\ln(P^{\sigma}Q) + d\ln M_e - \gamma_{\lambda} d\ln\varphi^* - d\ln\lambda$$

where the cutoff if $d\ln\varphi^* = -\frac{1}{\sigma-1}d\ln(P^{\sigma}Q)$ as before.

(3) Summary of three equations

$$\frac{\tilde{\sigma}-1}{\sigma-1} d\ln(P^{\sigma}Q) - \gamma_S d\ln\varphi^* - d\ln S = -d\ln M_e + d\ln\left[1 + \frac{M_e f_x}{L}\int_{\varphi^*_x(\varepsilon_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx}-1\right) dG\right]$$

$$\frac{\beta(\tilde{\sigma}-1)}{1-\beta} d\ln(P^{\sigma}Q) + (1-\sigma)d\ln P = d\ln M_e - \gamma_{\lambda} d\ln\varphi^* - d\ln\lambda$$

$$d\ln\varphi^* = -\frac{1}{\sigma-1} d\ln(P^{\sigma}Q)$$

Combining the above three equations, the change in welfare associated with an iceberg cost shock is

$$d \ln W = \frac{1}{\gamma_s + \tilde{\sigma} - 1} \Big\{ - d \ln \lambda + d \ln M_e$$
$$+ \left( \frac{\sigma}{\sigma - 1} (\gamma_s - \gamma_\lambda) - \tilde{\sigma}\beta \right) d \ln M_e$$
$$- \left( \tilde{\sigma}(1-\beta) - 1 + \frac{\sigma \gamma_s}{\sigma - 1} \right) d \ln \lambda + \left( \tilde{\sigma} - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) d \ln S$$
$$+ \left( \tilde{\sigma} - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) d \ln \left( 1 + \frac{M_e f_x \int_{\varphi_x^*(\varepsilon_{ex}, \tau_{fx})}^{\infty} \left( \tau_{fx} - 1 \right) dG}{L} \right) \Big\} \tag{A.53}$$

where $\tilde{\sigma} = \sigma / (1 - \beta + \sigma\beta)$.

The expression can also be written as

$$d \ln W = \frac{1}{\gamma_s + \tilde{\sigma} - 1} \Big[ \big( \frac{\tilde{\sigma} - 1}{\sigma - 1} + \frac{\sigma}{\sigma - 1} (\gamma_S - \gamma_\lambda) \big) (-d \ln \lambda + d \ln M_e)$$
$$\left( \tilde{\sigma} - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) \left( -d \ln \lambda + d \ln S + d \ln \left( 1 + \frac{M_e f_x \int_{\varphi_x^*(\varepsilon_{ex}, \tau_{fx})}^{\infty} \left( \tau_{fx} - 1 \right) dG}{L} \right) \right) \Big]$$

If $\beta = 0$, $\tilde{\sigma} = \sigma$, and the benchmark result is restored. Otherwise, $\tilde{\sigma}$ includes $\beta$.

**Special Case I.** Recall that $\ln \tau = \beta \ln pq + \ln \varepsilon$. With homogenous productivity and Pareto-distributed domestic distortion $1/\varepsilon$ with parameter $\theta$, the original result is again restored:

$$d \ln W = \frac{\sigma}{\sigma - 1} \left[ d \ln S - d \ln \lambda \right].$$

Proof: Assume $x = 1/\varepsilon$ follows a Pareto distribution, then

$$\frac{(1 - \tilde{\sigma})}{1 - \sigma} d \ln \left( P^\sigma Q \right) = d \ln S - d \ln M_e - d \ln \int_{x*} x^{\tilde{\sigma}} x^{-\theta - 1} dx$$

$$\frac{\beta(\tilde{\sigma} - 1)}{1 - \beta} d \ln \left( P^\sigma Q \right) + (1 - \sigma) d \ln P = -d \ln \lambda + d \ln M_e + d \ln \int_{x*} x^{\frac{\tilde{\sigma} - 1}{1 - \beta}} x^{-\theta - 1} dx$$

Plugging in the cutoff:

$$\frac{(1 - \beta)\theta}{\sigma} d \ln \left( P^\sigma Q \right) = d \ln S - d \ln M_e$$

$$(\frac{\sigma-1}{\sigma}-\frac{(1-\beta)\theta}{\sigma})d\ln(P^{\sigma}Q)+(1-\sigma)d\ln P=-d\ln\lambda+d\ln M_e.$$

Thus, we have

$$d\ln P=-\frac{1}{\theta(1-\beta)}\Big[(-d\ln\lambda+d\ln M_e)+\left(\frac{\theta(1-\beta)}{\sigma-1}-1\right)(-d\ln\lambda+d\ln S)\Big]$$

$$d\ln Q=\frac{\sigma}{\sigma-1}\left[d\ln S-d\ln\lambda\right]$$

□

**Special Case II.** The case with $\varphi$ and $\beta$, but no other type of $\varepsilon$, and a Pareto distributed productivity with parameter $\theta$, welfare becomes

$$d\ln W=\frac{1}{\theta}\Big[-d\ln\lambda+\left(\frac{\sigma}{\sigma-1}\theta-1\right)(-d\ln\lambda+d\ln S)\Big]$$

Proof: From the definition of $S$ in A.51, the integral in the numerator become $\int_{\varphi*}\varphi^{\tilde{\sigma}-1}\varphi^{-\theta-1}d\varphi$, and for $\lambda$ in A.52, it becomes $\int_{\varphi*}\varphi^{\frac{\tilde{\sigma}-1}{1-\beta}}\varphi^{-\theta-1}d\varphi$. Thus:

$$\gamma_S=\theta-(\tilde{\sigma}-1),\quad \gamma_\lambda=\theta-\tilde{\sigma}+\frac{\tilde{\sigma}}{\sigma}.$$

Plug into the three equations:

$$\frac{\tilde{\sigma}-1}{\sigma-1}d\ln(P^{\sigma}Q)-\gamma_S d\ln\varphi^*-d\ln S=-d\ln M_e$$

$$\frac{\beta(\tilde{\sigma}-1)}{1-\beta}d\ln(P^{\sigma}Q)+(1-\sigma)d\ln P=-d\ln\lambda-\gamma_\lambda d\ln\varphi^*+d\ln M_e$$

$$d\ln\varphi^*=-\frac{1}{\sigma-1}d\ln(P^{\sigma}Q)$$

We have:

$$
\begin{aligned}
d\ln P &= -\frac{1}{\theta}\Big[(-d\ln\lambda + d\ln M_e) + \left(\frac{\theta}{\sigma-1} - 1\right)(-d\ln\lambda + d\ln S)\Big] \\
d\ln PQ &= (-d\ln\lambda + d\ln S) \\
d\ln Q &= \frac{1}{\theta}\Big[(-d\ln\lambda + d\ln M_e) + \left(\frac{\sigma}{\sigma-1}\theta - 1\right)(-d\ln\lambda + d\ln S)\Big]
\end{aligned}
$$

# H Discussions

## H.1 Impact of Home Distortions on Foreign Welfare

In the benchmark, foreign gain from trade is about 9% with or without Home distortions, though its gains is slightly lower when Home features distortions. Without distortions at Foreign, Proposition 4 shows that Foreign welfare still satisfies MR decomposition. But Home distortions have impact on Foreign's domestic sales share, entry, and cutoffs. To understand the impact, let's revisit Foreign welfare. From consumers' budget constraint and firms' free-entry condition, we can write Foreign welfare as

$$
W_f = C_f = \frac{w_f L_f}{P_f},
$$

where $w_f$ and $P_f$ are Foreign wage and consumer price, respectively. We can further write down Foreign aggregate price index as

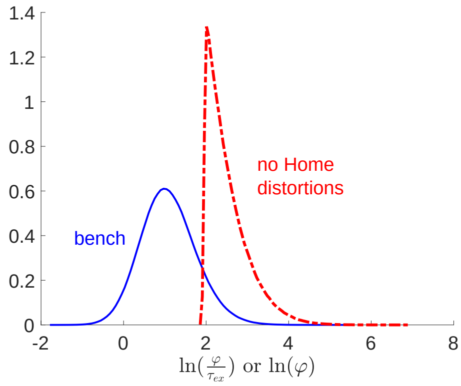
$$
P_f = \left[M_{ef}\int_{\varphi_f^*}\left(\frac{\sigma}{\sigma-1}\frac{w_f}{\varphi}\right)^{1-\sigma} dG_f + M_e\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{\sigma}{\sigma-1}\frac{w\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma} dG\right]^{\frac{1}{1-\sigma}}.
$$

Plugging $P_f$ back to the welfare equation and reorganizing it, we have

$$
W_f = \frac{\sigma-1}{\sigma}L_f\left[M_{ef}\int_{\varphi_f^*}\varphi^{\sigma-1}dG_f + M_e\tau_x^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{w_f}{w}\frac{\varphi}{\tau_{ex}}\right)^{\sigma-1} dG\right]^{\frac{1}{\sigma-1}}. \qquad \text{(A.54)}
$$

Hence, Home distortion affects foreign welfare through the import prices, the relative wage $w_f/w$, Foreign producing cutoff $\varphi_f^*$, and Home exporting cutoff $\varphi_x^*$. The import prices are proportional to firms' marginal cost of producing $\tau_{ex}/\varphi$, or they are inversely related to firms' effective productivity $\varphi/\tau_{ex}$. The higher the average effective productivities, the lower the import prices, the higher the Foreign welfare. Also, the higher the relative wage, the higher the Foreign welfare.

Figure A-7: Distribution of Foreign Imported Goods




The prices Foreign faces are lower were Home firms to be taxed less (low $\tau_{ex}$); on the other hand, some low-$\varphi$ hence high-marginal-cost Home firms will be selected into exporting, making the Foreign's import prices higher. Figure A-7 depicts the distribution of the effective productivities ($\varphi/\tau_{ex}$) of Foreign country's imported goods from Home country. The blue-solid line is for the benchmark, and the red-dashed line is for no Home distortions. The differences of the two lines reflect the different underlying distributions of $\varphi/\tau$ and $\varphi$, as well as the different cutoffs of Home exporting $\varphi_x^*$ with and without distortions. The benchmark distribution is to the left of that when Home faces no distortions. These low effect productivity (or high marginal costs) tend to reduce Foreign welfare.

Meanwhile, Home distortions also have general equilibrium effect on relative wages. When there are Home distortions, the relative higher demand for foreign products induces a higher Foreign wage. Without Home distortions, its efficiency improves, and the Foreign wage would be lower. Under our estimation, the relative Foreign wage under Home distortions is about twice higher than that under no distortions.

In summary, Home distortions have two opposing effects on Foreign welfare. On the one hand, distortions push up the import prices (through low effective productivity or high marginal cost) of Foreign and lower Foreign welfare. On the the hand, distortions raise Foreign wage and welfare. These two effects cancel out in our estimation and lead to a similar welfare gain for Foreign country with or without Home distortions. One factor affects the race of the two effects is the dispersion of $\tau_{ex}$. More dispersed $\tau_{ex}$ pushes up more the import prices of Foreign and leads to a lower Foreign welfare under Home distortions.

### H.2 Imbalanced Trade

To see the quantitative impact of trade imbalances between China and U.S., we follow Dekle, Eaton, and Kortum (2007) and impose the observed imbalances in our equilibrium condition. Due to wealth transfer from trade imbalance, we would expect a decrease in Home import share and welfare and an increase in foreign wage. Quantitatively, under our benchmark parameters and trade surplus at Home (China), foreign wage increases by 1.7% and Home welfare in the open economy decreases by 4.7% relative to our benchmark. This decline in welfare mainly comes from the wealth transfers from Home's trade surplus, as in Dekle, Eaton, and Kortum (2007) . Adding trade surplus to Home country slightly affects our model moments. We also reestimate the model parameters, and the quantitative results are similar as the case without reestimation. (i.e., Home has lower welfare than our benchmark.)

Note that our model is a static one. Under a dynamic model, a country that runs trade surplus in the current period should run trade deficits in the future. Thus, the net present value of trade imbalance should be close to zero. Let $\beta$ be countries' discount factor and $r$ the world interest rate. Under a complete market model and $\beta(1+r) = 1$, the country's overall welfare gain or loss from trade would be roughly the same as our benchmark result.

### H.3 Iceberg Cost vs Tariff

The benchmark model considers idiosyncratic distortions that are taxes/subsidies and trade costs that are pure resource losses. Here we discuss some alternatives.

### H.3.1 Domestic distortion takes the form of iceberg cost

We'd like to point out that the model with an iceberg-cost type of distortion does not produce any wedges. In this case, distortion works like a productivity shock. Hence, the welfare decomposition is equivalent to ACR or MR. There is alway gains from trade. Most importantly, the welfare decomposition has no reallocation term. In other words, using aggregates as in the literature can capture well the gains from trade. Let us elaborate on these points below.

To clearly make the point, we now present a closed economy, where distortions are modelled in the same way as the iceberg trade cost. Specifically, to produce $q$ units, the firm has to use $\ell_v = \tau q/\varphi$ units of variable labor plus the fixed cost, where $\tau$ is the distortion and $\varphi$ is the productivity. A intermediate-good firm $(\varphi, \tau)$ solves the following problem

$$\max_{p,q} \;\; pq - \frac{w\tau}{\varphi} q - wf,$$

subject to the demand function $q = \frac{p^{-\sigma}}{P^{-\sigma}} Q$. We can characterize the optimal price $p$, variable labor $\ell_v$, output $q$, and revenue $pq$ as

$$p = \frac{\sigma}{\sigma - 1} w \left(\frac{\varphi}{\tau}\right)^{-1}, \tag{A.55}$$

$$\ell_v = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma - 1}, \tag{A.56}$$

$$q = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma}, \tag{A.57}$$

$$pq = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} (P^\sigma Q) w^{1-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma - 1}, \tag{A.58}$$

It is easy to see that all the endogenous variables here $(p, \ell_v, q, pq)$ only depend on the ratio of $\varphi$ to $\tau$, or the effective productivity $\tilde{\varphi} = \varphi/\tau$. Note that in our benchmark model with 'tax' style of distortion, the optimal $p$, $q$, and $pq$ take the same formula as above. However, optimal variable labor is given by,

$$\ell_v^{bench} = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \varphi^{\sigma - 1} \tau^{-\sigma}. \tag{A.59}$$

The distortion in our benchmark model is equivalent to a labor wedge. For one unit of labor, households receive $w$ unit of payment, but firms pay for $w\tau$. With this one unit of labor, firms produce $\varphi$ unit of goods. The marginal product of variable labor $pq/\ell_v^{bench} = \frac{\sigma}{\sigma-1}w\tau$ is firm-specific and is not equalized across firms. In contrast, the iceberg cost behaves like a productivity. For one unit of labor, households receive $w$ and firms pay for $w$, there is no wedge between them. All firms have the same marginal product of variable labor, $pq/\ell_v = \frac{\sigma}{\sigma-1}w$. However, with one unit of labor, firms can only produce $\varphi/\tau$ unit of goods, which costs extra resources. However, there is no efficiency loss from misallocation (wedge) as in HK.

Hence an open-economy model under iceberg-type of distortion is equivalent to a Melitz model with productivity distribution on $\tilde{\varphi} = \varphi/\tau$. If $\tilde{\varphi}$ follows a Pareto distribution, we reach the ACR result, where the import share and trade elasticity can forecast the gain from trade. We do not need the underlying distribution of physical productivity $\varphi$ and true distortion $\tau$ for measuring the gain from trade. If $\tilde{\varphi}$ follows a general distribution, the MR results hold. Still, there is no reallocation term as in our theory.

In summary, the iceberg type of distortion shows up like a technology shock. It lowers welfare because firms have to use more labor to produce the same unit of output. There are deadweight losses. However, the iceberg cost does not generate misallocations across firms. Hence the welfare decomposition does not consist a reallocation term to reflect such misallocation. In contrast, our benchmark aims to examine the implication of HK type of distortion on gain from trade. This type of distortion generates misallocation showing up as wedges across firms.

### H.3.2 Tariff instead of iceberg trade cost

Tariff works like the distortion in our benchmark and generates a wedge between sales and input share, which shows up in the welfare decomposition. In this part, we first show how tariff affects our equilibrium conditions. We then present the welfare decomposition under tariff. Lastly, we compare quantitatively the results under tariff and under iceberg trade cost.

Let $\tau_m$ denote tariff. First, tariffs enter the price index the same way as an iceberg trade cost:

$$P^{1-\sigma} = con_p \times \left[ M \frac{\int \int_{\varphi^*(\tau)}^{\infty} (\frac{\varphi}{w\tau})^{\sigma-1} g\left(\varphi, \tau\right) d\varphi d\tau}{\int \int_{\varphi^*(\tau)}^{\infty} g\left(\varphi, \tau\right) d\varphi d\tau} + M_f \frac{\int \int_{\varphi^*_{xf}(\tau)}^{\infty} (\frac{\varphi}{w_f \tau_m \tau})^{\sigma-1} g_f\left(\varphi, \tau\right) d\varphi d\tau}{\int \int_{\varphi^*_f(\tau)}^{\infty} g_f\left(\varphi, \tau\right) d\varphi d\tau} \right].$$

Second, the free entry condition is different. Now tariff enters the formula in a similar way as output distortions. In particular, it is $\tau_{mf}^{-\sigma}$ that enters, and it is $\tau_x^{1-\sigma}$ in the iceberg cost case.

$$\begin{aligned} & w^{1-\sigma} \left[ P^\sigma Q \int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau + P_f^\sigma Q_f \tau_{mf}^{-\sigma} \int \int_{\varphi_x^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau \right] \\ & = \sigma^\sigma (\sigma - 1)^{1-\sigma} \left( w f_e + w f \omega_e + w f_x \omega_x \right). \end{aligned}$$

If we assume the two countries charge the same $\tau_m$, the trade balance condition is the same as before. With different tariffs across countries, total expenditure in Home could be different from its total revenues. In this case, the balanced trade condition becomes

$$\begin{aligned} & P_f^\sigma Q_f M \int \int_{\varphi_x^*(\tau)}^{\infty} \left( \frac{w\tau}{\varphi} \right)^{1-\sigma} \tau_{mf}^{-\sigma} \mu\left(\varphi, \tau\right) d\varphi d\tau \\ & \quad = P^\sigma Q M_f \int \int_{\varphi_{xf}^*(\tau)}^{\infty} \left( \frac{w_f \tau_m \tau}{\varphi} \right)^{1-\sigma} \tau_m^{-\sigma} \mu_f\left(\varphi, \tau\right) d\varphi d\tau, \end{aligned}$$

where $\tau_{mf}$ is Foreign tariff on imported Home goods.

**Welfare decomposition** To show that tariff is different from iceberg trade cost, we drive a formula without any other distortions but tariff. This model is the same as Melitz except that we replace the iceberg trade cost with tariff. In this case, the welfare formula becomes

$$\begin{aligned} d \ln W = \frac{1}{\gamma_\lambda + \sigma - 1} \Big[ & - d \ln \lambda + d \ln M_e \\ & + \left( \sigma - 1 + \frac{\sigma \gamma_\lambda}{\sigma - 1} \right) \left( - d \ln \lambda + d \ln S \right) \Big] \end{aligned}$$

where $\lambda$ is domestic sales share, $S$ domestic input share, $\gamma_\lambda$ is the elasticity to cumulated sales share with respect to cutoff and is evaluated at domestic cutoff.

On the one hand, tariff creates a gap between domestic sales and input share, which does not show up in the case with iceberg cost as in ACR or MR. With tariff, even if firms' productivities follow a Pareto distribution, we still cannot only use the change of domestic sales share (or import share) and trade elasticities to infer the welfare change after trade.

On the other hand, tariff is a tax and incentivized Home firms to shift labor toward domestic production. This leads to a positive reallocation term since the change of domestic input share tend to be larger than that of sales share, i.e. $d \ln S \geq d \ln \lambda$. In contrast, in our benchmark model, by selection, firms with large export 'subsidies' $\tau_{ex}$ will employ more labor and export more. This leads to larger increase in labor share than sales share for exporting firms, or the change of domestic labor share is smaller sales share, $d \ln S \leq d \ln \lambda$. Hence, tariff works opposite to the export distortions in our benchmark model.

We can also prove that the welfare expression is the same as in our benchmark when there are various distortions $(\tau, \tau_{ex}, \tau_{fx})$.

**Quantitative results** We now compare quantitatively the welfare impact of tariff and iceberg trade cost. To illustrate their differences, we first follow Baqaee and Farhi (2021) and compare two counterfactuals on our benchmark results, a 10% universal increase in tariff and a 10% universal increase in iceberg trade cost. These changes are for both the home and foreign country. We also consider a model with both tariff and the iceberg trade cost, and we take the tariffs from the data and reestimate the iceberg trade cost together with other parameters.

Table A-1 reports the gain from trade and welfare decomposition for the home country in these experiments together with our benchmark. Without distortions, both the increase in iceberg trade cost and the increase in tariff lower the gain from trade. The reduction is larger for the iceberg cost, reflecting its deadweight loss. With distortions, higher iceberg cost lowers the trade share, which in turn lowers the magnitude of ACR and reallocation term. Overall, the welfare gain from trade is lower than the benchmark, decreasing to $-1.66\%$ from the benchmark $-1.18\%$.

An increase in tariff from the benchmark also reduces the incentive to trade and lowers the ACR term in a similar magnitude as in the case of increasing trade cost. However, the reallocation term becomes less negative, $-8.62\%$ versus $-11.16\%$ in the trade cost case.

Hence the gain from trade becomes larger, 0.75%. Tariff increases Home welfare because it corrects some of the distortions at Home. By selection, exporters tend to have low $\tau_{ex}$ and use too larger share of labor relative to the output share that exporters produce. Hence a positive tax like tariff helps cancel out the exporters' benefit from $\tau_{ex}$ and reduces the gap between input and sales share.

The last column of Table A-1 reestimates the model with tariffs from the data. The average Chinese tariff is 9.55% and the US tariff is 3.33%, both for the manufacturing sector in 2005. We estimate the iceberg trade cost together with other parameters in our model, similar in our benchmark. In particular, the reestimation guarantees an import share of 22.5% as in the data. And the ACR term is similar to the benchmark model, both around 12.5%. As we discussed above, tariff tends to correct the exacerbated misallocation from trade, the reallocation becomes smaller, about 3% higher than the benchmark. In total, the gain from trade is 1.82%.

Overall, all these results have the outcome of large negative reallocation. Using the aggregates only with ACR term will greatly overestimate the gain from trade. The overestimation are about 10 times the true gain from trade in the model.

Table A-1: Welfare Implications of Trade Cost and Tariff

| | Bench | Increase in trade cost (10% universal) | Increase in tariff (10% universal) | 2005 Tariff (reestimation) |
|---|---|---|---|---|
| Home country | | | | |
| Gain from trade | −1.18 | −1.66 | 0.75 | 1.82 |
| *Welfare decomp.* | | | | |
| ACR term | 12.53 | 10.34 | 10.22 | 12.49 |
| Reallocation | −12.94 | −11.16 | −8.62 | −10.14 |
| No Home distortions | | | | |
| Gain from trade | 3.31 | 2.72 | 3.29 | 4.02 |

Note: All numbers are in percent.

# I TFPR and TFPQ in the Data and Measurement Error

We find large dispersions in measured TFPR in China, similar to the levels in HK for the year 1998 and 2007. TFPR can be written into two terms: revenue product of labor $ARPL_{ji} = p_{ji}q_{ji}/\ell_{ji}$ and revenue product of capital $ARPK_{ji} = p_{ji}q_{ji}/k_{ji}$, i.e. for any firm $i$ in industry $j$,

$$\log(TFPR_{ji}) = \alpha_j \log(ARPL_{ji}) + (1 - \alpha_j) \log(ARPK_{ji}).$$

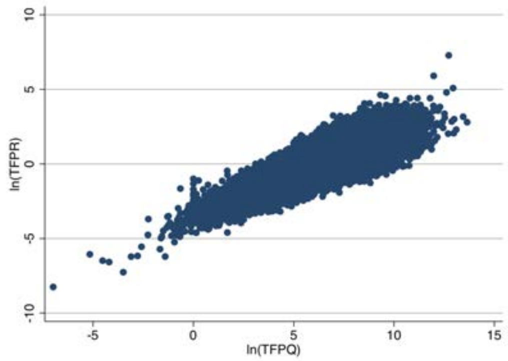
where $\alpha_j$ is the industry specific labor share. Both measured ARPL and ARPK have come down over time, between 1998 and 2007, as evident in Table A-2. There is also greater dispersion in the average product of capital than there is in the average product of labor.

Table A-2: Dispersion of ARPK and ARPL

| | 1998 | 2001 | 2004 | 2007 |
|---|---|---|---|---|
| std(ARPK) | 1.348 | 1.306 | 1.241 | 1.185 |
| std(ARPL) | 1.184 | 1.039 | 0.940 | 0.923 |

We next turn to investigating further what factors are systematically related to measured TFPR. First, TFPR is highly correlated with TFPQ, as shown graphically in Figure A-8. Second, we conduct the regression analyses of measured TFPR on TFPQ and a set of variables like age, ownership, exporter dummy with or without industry and location fixed effect. See Table A-3.

Figure A-8: Measured TFPR and TFPQ




In all these regressions, the coefficient on firm TFPQ is large and significant; 1 percent increase in TFPQ is associated with about 60 percent increase in TFPR. Moreover, more

Table A-3: TFPR Regressions

| VARIABLES | (1) ln($TFPR$) | (2) ln($TFPR$) | (3) ln($TFPR$) | (4) ln($TFPR$) | (5) ln($TFPR$) | (6) ln($TFPR$) | (7) Extended model ln($TFPR$) |
|---|---|---|---|---|---|---|---|
| ln($TFPQ$) | 0.574*** | 0.630*** | 0.635*** | 0.635*** | 0.635*** | 0.639*** | 0.648 |
| | (243.5) | (235.9) | (243.2) | (241.6) | (248.4) | (261.6) | |
| Age | | | | -0.00165*** | -0.00163*** | -0.00148*** | |
| | | | | (-9.736) | (-10.10) | (-10.05) | |
| SOE | | | | | -0.100*** | -0.0930*** | |
| | | | | | (-4.577) | (-4.481) | |
| Foreign owned | | | | | -0.230*** | -0.156*** | |
| | | | | | (-25.96) | (-24.60) | |
| Exporters | | | | | | -0.213*** | -0.241 |
| | | | | | | (-24.96) | |
| Constant | -3.502*** | -3.296*** | -3.236*** | -3.209*** | -3.131*** | -3.129*** | |
| | (-243.5) | (-106.2) | (-89.23) | (-87.12) | (-75.08) | (-77.04) | |
| Observations | 1,587,629 | 1,587,629 | 1,479,528 | 1,478,648 | 1,478,648 | 1,478,648 | |
| R-squared | 0.739 | 0.812 | 0.822 | 0.823 | 0.831 | 0.837 | |
| Time FE | Yes | Yes | Yes | Yes | Yes | Yes | |
| Industry FE | No | Yes | Yes | Yes | Yes | Yes | |
| Location FE | NO | NO | YES | YES | YES | YES | |

Robust t-statistics in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

than half of the variation in TFPR is explained by TFPQ alone. The positive relationship is consistent with the predictions of our model as showing in the model regression (Column 7). The same is true for the results on exporters: given TFPQ, firms must have lower taxes on average in order to export, and have a lower TFPR. TFPR differences are also systematic related to firm characteristics: state-owned enterprises and Foreign-owned firms are subject to lower TFPR on average, given TFPQ.

**Measurement error** With the presence of fixed costs in producing and exporting in our model, the measured TFPR does not perfectly relate to the true wedges. In the data, there are other types of mismeasurements in output and input, which may also generate a dispersion in the average revenue products, and thereby affect the measured TFPR— as shown in Bils, Klenow, and Ruane (2017) and Song and Wu (2015). Here we use Bils, Klenow, and Ruane (2017)'s method to detect measurement errors. We find that even taking out the standard measurement errors, there are still large distortions remaining among Chinese firms.

The main approach involves using panel data to estimate the true marginal product dispersion among operating firms, rather than simply employing cross-sectional data. With this method, we find that the measurement errors are small in China, accounting for only 18% of the variation in the average product.[30] This 18% includes the mismeasurement of production inputs in the presence of fixed cost, which is accounted for in our benchmark.

We exploit three alternative methods to detect measurement error: average annual observations within firms, first differences over years within firms, and covariance between first differences and average products. All three approaches point to the same conclusion: that 1) there is a large dispersion in marginal products in China; 2) measurement error only accounts for a small fraction of the dispersion in the measured marginal products (i.e. average products).

First, if measurement error were idiosyncratic across firms and over time, one can take the time average of annual observations within firms to wash out these errors, drastically reducing the dispersion of average products. The upper panel of Table A-4 reports the

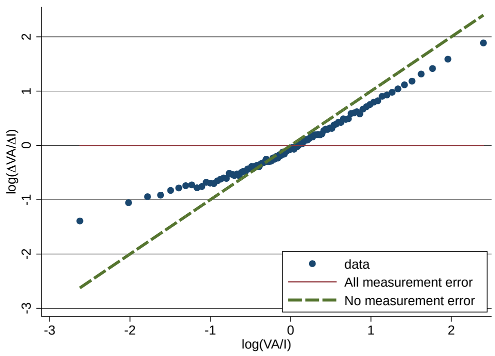
[30] Bils, Klenow, and Ruane (2017) finds measurement errors can explain about half of variation of average products in Indian, and about 80% of that in the U.S, but little for China.

Table A-4: Detecting Measurement Errors

| Average annual observation within firm | | | | |
|---|---|---|---|---|
| std(ln(ARPK)) | std(ln(ARPL)) | $std(lnVA)$ | $std(ln(VA/I))$ | $corr(lnVA, ln(VA/I))$ |
| 1.19 | 0.96 | 1.19 | 0.94 | 0.4 |
| | | First level differences | | |
| | | 2001 | 2004 | 2007 |
| | $std(ln(\triangle VA/\triangle K))$ | 1.82 | 1.78 | 1.76 |
| | $std(ln(\triangle VA/\triangle L))$ | 1.68 | 1.60 | 1.61 |
| | | Regression | | |
| | | $\Psi$ | $\Psi(1-\lambda)$ | |
| | | $0.53^{***}$ | $-0.0997^{***}$ | |
| | | (34.58) | $(-20.65)$ | |

Note: This table reports three ways to detect measurement errors. The upper panel reports the average annual levels within firms. The middle panel reports the ratio of first differences as another measure of marginal product, where $\triangle VA$ denotes the first difference of value added. The lower panel reports regression coefficient as in equation (A.60). Robust t-statistics in parentheses.

Figure A-9: Measured Marginal Product using First Differences vs TFPR



statistics when we take the average within firms. The average standard deviation is 1.19 for the average product of capital and 0.96 for the average product of labor. The standard deviations of value added and the average product of inputs are 1.19 and 0.94, where the correlation between the two variables is 0.4. These results mimic the moments in year 2005. In particular, the dispersions of average products of inputs are still high. This implies that measurement errors of the iid type cannot explain the observed dispersions in the average products.

Table A-5: Measured Marginal Products using First Differences vs TFPR

| | (1) | (2) | (3) |
|---|---|---|---|
| VARIABLES | $\log(\frac{\triangle VA}{\triangle I})$ | $\log(\frac{\triangle VA}{\triangle I})$ | $\log(\frac{\triangle VA}{\triangle I})$ |
| $\log(TFPR)$ | 0.718*** | 0.715*** | 0.718*** |
| | (135.3) | (158.6) | (135.3) |
| Constant | 1.410*** | 0.331*** | 1.410*** |
| | (78.31) | (17.49) | (78.31) |
| Observations | 624,659 | 624,699 | 624,659 |
| R-squared | 0.173 | 0.269 | 0.173 |
| Time FE | Yes | Yes | Yes |
| Industry FE | Yes | Yes | Yes |

Robust t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Specification (2) weights all the observations with the absolute value of composite input growth.

Specification (3) weights all the observations with the share of aggregate value added.

Second, as pointed out by Bils, Klenow, and Ruane (2017), the dispersion of first differences reflect the true distortion if marginal products are constant over time. Calculating the first differences of value added $\triangle VA$, capital $\triangle K$, and labor $\triangle L$, and then taking the ratio $\triangle VA/\triangle K$ and $\triangle VA/\triangle L$ gives us an alternative measure of marginal products. The 1% tails of both ratios are trimmed, and the results are displayed in the middle panel of Table A-4 for the year of 2001, 2004, and 2007. The dispersions are even higher than those in Table A-2 for the measured average product of inputs.

Moreover, the alternative measured marginal products are highly correlated with average products. Figure A-9 plots the $\ln(\triangle VA/\triangle I)$ against the benchmark average product of input $\ln(VA/I)$ where $I$ is the composite of inputs, $I = K^{\alpha}L^{1-\alpha}$, where each dot corresponds to one of 100 percentiles of $\ln(VA/I)$. The regression coefficient at the firm level is

0.72, see Table A-5. Note that without measurement errors, the two measures are perfectly correlated. For the case with only measurement error, the two measures have no correlation. Hence, the high correlation between the alternative measure and the average products suggest small measurement errors and a large distortion-induced misallocation.

Table A-6: Estimate Measurement Error

| VARIABLES | (1) $\triangle\widehat{VA}$ | (2) $\triangle\widehat{VA}$ | (3) $\triangle\widehat{VA}$ |
|---|---|---|---|
| $\log(TFPR)$ | 0.0376*** | 0.0144*** | 0.0616*** |
| | (22.62) | (9.170) | (16.07) |
| $[\log(TFPR)]^2$ | | | -0.0128*** |
| | | | (-6.110) |
| $[\log(TFPR)]^3$ | | | 0.00152*** |
| | | | (4.008) |
| $\triangle\widehat{input}$ | 0.530*** | 0.523*** | 0.524*** |
| | (34.58) | (33.03) | (31.13) |
| $\log(TFPR) \times \triangle\widehat{input}$ | -0.0997*** | -0.0954*** | -0.0893*** |
| | (-20.65) | (-19.16) | (-6.420) |
| $[\log(TFPR)]^2 \times \triangle\widehat{input}$ | | | -0.00611 |
| | | | (-0.919) |
| $[\log(TFPR)]^3 \times \triangle\widehat{input}$ | | | 0.00108 |
| | | | (1.040) |
| Constant | -0.0207*** | 0.0551*** | -0.0241*** |
| | (-3.125) | (8.231) | (-3.592) |
| Observations | 1,106,982 | 1,106,914 | 1,106,982 |
| R-squared | 0.044 | 0.042 | 0.044 |
| Time FE | Yes | Yes | Yes |
| Industry FE | Yes | Yes | Yes |

Robust t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Specification (2) weights all the observations with the share of aggregate value added.

Lastly, we follow Bils, Klenow, and Ruane (2017) and run the following regression to further quantify the extent to which measured average products reflect marginal products:

$$\triangle\widehat{VA}_i = \Phi \cdot \log(TFPR_i) + \Psi \cdot \triangle\hat{I}_i - \Psi(1-\lambda) \cdot \log(TFPR_i) \cdot \triangle\hat{I}_i + D_s + \xi_i \tag{A.60}$$

where $\triangle\widehat{VA}_i$ and $\triangle\hat{I}_i$ are the growth rate of measured value added and inputs respectively, and $\log(TFPR_i)$ is the measured average products. The underlying assumption here is that the measurement errors are additive. The variable of interest in the regression is $\lambda$, the variance of distortions relative to that of $TFPR$: $\lambda = \frac{\sigma^2_{\ln\tau}}{\sigma^2_{\ln(TFPR)}}$. The regression coefficient for

$\Psi$ is 0.53 and for the interaction of $\log(TFPR_i)$ and $\triangle \hat{I}_i$ is -0.0997. Both are significant, and the robust t-statistics are reported in Table A-4. The implied $\lambda$ is therefore 0.81. Hence, 81% of variation in $TFPR$ or average products is accounted for by distortions and 19% is due to measurement errors.

The results are robust if we weight the observations with their share of aggregate value added or if we control for higher orders of $\ln(TFPR)$ to allow for stationary shocks to firms productivity and distortions.[31] See Table A-6.

In summary, the three alternative ways of sifting out measurement errors using panel data all point to the result that the dispersion in the average product of inputs are mainly driven by distortions rather than measurement error typically conceived.

# J Endogenous markup

In this section, we explore a model with endogenous distortion arising from endogenous markup, which has been extensively studied in the standard trade literature. We show that the endogenous markup model runs counter with the data in that exporters in the model face a higher markup and distortion.

Here we build a model with endogenous markup as in Edmond, Midrigan, and Xu (2018). The consumer's problem is the same as before.

**Final goods producer** Final goods producers are competitive and produce with intermediate goods with a Kimball aggregator

$$\int_{\omega \in \Omega} \gamma \left( \frac{q}{Q} \right) d\omega = 1,$$

[31] Bils, Klenow, and Ruane (2017) also consider the following extension to allow for stationary shocks to firms productivity and distortions:

$$\begin{aligned}\triangle \widehat{VA}_i = \Phi \cdot \log(TFPR_i) + \Psi \cdot \triangle \hat{I}_i - \Psi(1-\lambda) \cdot \log(TFPR_i) + \Gamma \cdot [\log(TFPR_i)]^2 \\ + \Lambda(1-\lambda) \cdot [\log(TFPR_i)]^2 \triangle \hat{I}_i + \Upsilon \cdot [\log(TFPR_i)]^3 + \Lambda(1-\lambda) \cdot [\log(TFPR_i)]^3 \triangle \hat{I}_i.\end{aligned}$$

where $\gamma(\cdot)$ follows Klenow and Willis (2016) specification as

$$\gamma\left(\frac{q}{Q}\right) = 1 + (\sigma - 1)\exp\left(\frac{1}{\varepsilon}\right)\varepsilon^{\frac{\sigma}{\varepsilon}-1}\left[\Gamma\left(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon}\right) - \Gamma\left(\frac{\sigma}{\varepsilon}, \frac{(q/Q)^{\frac{\varepsilon}{\sigma}}}{\varepsilon}\right)\right], \tag{A.61}$$

$\sigma > 1$, $\varepsilon \geq 0$ and $\Gamma(s, x)$ denotes the upper incomplete Gamma function $\Gamma(s, x) = \int_x^\infty t^{s-1}e^{-t}dt$. The demand function for each intermediate good producer is therefore given by

$$p(\omega) = \gamma'\left(\frac{q(\omega)}{Q}\right)PD, \tag{A.62}$$

where $D$ is a demand index, $D = \left[\int_{\omega\in\Omega}\gamma'\left(\frac{q(\omega)}{Q}\right)\frac{q(\omega)}{Q}d\omega\right]^{-1}$.

**Intermediate good producer** The problem of an intermediate good producer is similar as before except it faces a demand function as in equation (A.62). The firm will choose the price as a markup over the marginal cost,

$$p = \frac{\sigma}{\sigma - (q/Q)^{\frac{\varepsilon}{\sigma}}}\frac{w\tau}{\varphi}.$$

Note that the markup is endogenous and depends on the size of the firm, the higher the quantity a firm sells, the higher the markup it charges. The firm's optimal production and profit increase with $\varphi$ and decrease with $\tau$. Firms face the same fixed cost and exporting costs as in the Benchmark model, hence there exists a cutoff $\varphi^*(\tau)$, firms produce when $\varphi \geq \varphi^*(\tau)$.

**Equilibrium under Endogenous Markup** A closed-economy equilibrium consists of aggregate $(P, Q, M)$ that satisfy:

$$M = \frac{\omega_e L}{Q\int_0^{\sigma^{\sigma/\varepsilon}}\int^{\hat{\tau}(\hat{q})}\left[\frac{\sigma}{\sigma-\hat{q}^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}}{\varphi}\right]g(\varphi(\tau,\hat{q}),\tau)\frac{d\varphi(\tau,\hat{q})}{dq}d\tau d\hat{q}}$$

$$Q\int_0^{\sigma^{\sigma/\varepsilon}}\int^{\hat{\tau}(\hat{q})}\left[\frac{\hat{q}^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}}{\varphi}\right]g(\varphi(\tau,\hat{q}),\tau)\frac{d\varphi(\tau,\hat{q})}{dq}d\tau d\hat{q} = \omega_e f + f_e$$

$$\frac{M}{\omega_e}\int_0^{\sigma^{\sigma/\varepsilon}}\int^{\hat{\tau}(\hat{q})}\gamma\left(\hat{q}\right)g(\varphi(\tau,\hat{q}),\tau)\frac{d\varphi(\tau,\hat{q})}{dq}d\tau d\hat{q}=1,$$

where

$$\omega_e=\int_0^{\sigma^{\sigma/\varepsilon}}\int^{\hat{\tau}(\hat{q})}g(\varphi(\tau,\hat{q}),\tau)d\tau d\hat{q}$$

and

$$\gamma'\left(\frac{q}{Q}\right)=\frac{\sigma-1}{\sigma}\exp\left(\frac{1-(q/Q)^{\frac{\varepsilon}{\sigma}}}{\varepsilon}\right).$$

The open equilibrium consists of unknowns $(P,Q,M,P_f,Q_f,M_f,w_f)$ that satisfy:

$$\frac{\sigma}{\sigma-\hat{q}_x^{\frac{\varepsilon}{\sigma}}}\frac{w\tau_x\tau}{\varphi}=\gamma'(\hat{q}_x)P_fD_f$$

$$\pi_x=\left[\frac{\hat{q}_x^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}_x^{\frac{\varepsilon}{\sigma}}}\frac{\tau_x\hat{q}_x}{\varphi}Q_f-f_x\right]w,$$

where we get the zero profit cutoff. The free entry condition becomes:

$$\int\int_{\varphi^*(\tau)}\left[\frac{\hat{q}^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}}{\varphi}Q-f\right]g(\varphi,\tau)d\tau d\varphi$$
$$+\int\int_{\varphi_x^*(\tau)}\left[\frac{\hat{q}_x^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}_x^{\frac{\varepsilon}{\sigma}}}\frac{\tau_x\hat{q}_x}{\varphi}Q_f-f_x\right]g(\varphi_x,\tau)d\tau d\varphi=f_e \tag{A.63}$$

The labor market clearing condition is:

$$M=\frac{\omega_eL}{\int\int_{\varphi^*(\tau)}\left(\frac{\sigma}{\sigma-\hat{q}^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}}{\varphi}Q\right)g(\varphi,\tau)d\tau d\varphi+\int\int_{\varphi_x^*(\tau)}\left(\frac{\sigma}{\sigma-\hat{q}_x^{\frac{\varepsilon}{\sigma}}}\frac{\tau_x\hat{q}_x}{\varphi}Q_f\right)g(\varphi,\tau)d\tau d\varphi} \tag{A.64}$$

$$\left[\frac{M}{\omega_e}\int\int_{\varphi^*(\tau)}\gamma\left(\hat{q}\right)g(\varphi,\tau)d\tau d\hat{q}+\frac{M_f}{\omega_{ef}}\int\int_{\varphi_{xf}^*(\tau)}\gamma\left(\hat{q}_{xf}\right)g_f(\varphi,\tau)d\tau d\varphi\right]=1 \tag{A.65}$$

For Foreign,

$$\int\int_{\varphi_f^*(\tau)}\left[\frac{\hat{q}_f^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}_f^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}_f}{\varphi}Q_f-f\right]g_f(\varphi,\tau)d\tau d\varphi$$
$$+\int\int_{\varphi_{xf}^*(\tau)}\left[\frac{\hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}}{\sigma-\hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}}\frac{\tau_x\hat{q}_{xf}}{\varphi}Q-f_x\right]g_f(\varphi,\tau)d\tau d\varphi=f_{ef} \tag{A.66}$$

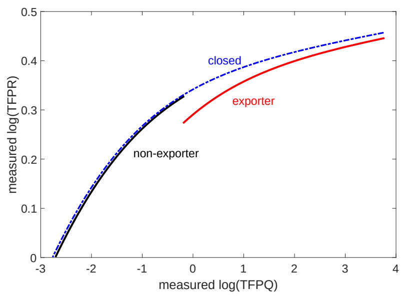
$$M_f=\frac{\omega_e L_f}{\int\int_{\varphi_f(\tau)}\left(\frac{\sigma}{\sigma-\hat{q}_f^{\frac{\varepsilon}{\sigma}}}\frac{\hat{q}}{\varphi}Q_f\right)g_f(\varphi,\tau)d\tau d\varphi+\int\int_{\varphi_{xf}^*(\hat{q})}\left(\frac{\sigma}{\sigma-\hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}}\frac{\tau_x\hat{q}_{xf}}{\varphi}Q\right)g_f(\varphi,\tau)d\tau d\hat{q}} \tag{A.67}$$

$$\left[\frac{M_f}{\omega_{ef}}\int\int_{\varphi_f^*(\tau)}\gamma\left(\hat{q}_f\right)g_f(\varphi,\tau)d\tau d\hat{q}+\frac{M}{\omega_e}\int\int_{\varphi_x^*(\tau)}\gamma\left(\hat{q}_x\right)g(\varphi,\tau)d\tau d\varphi\right]=1 \tag{A.68}$$

Finally, the goods market clearing condition is:

$$\frac{M}{\omega_e}\int\int_{\varphi_x^*(\tau)}\left[\frac{\sigma}{\sigma-\hat{q}^{\frac{\varepsilon}{\sigma}}}\frac{w\hat{q}}{\varphi}Q_f\right]g(\varphi,\tau)d\tau d\varphi=\frac{M_f}{\omega_{ef}}\int\int_{\varphi_{xf}^*(\tau)}\left[\frac{\sigma}{\sigma-\hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}}\frac{w_f\hat{q}_{xf}}{\varphi}Q\right]g_f(\varphi,\tau)d\tau d\varphi \tag{A.69}$$

Figure A-10: Measured TFPR and TFPQ in an Endogenous Markup Model




Notes: TFPQ is measured with $q/(\ell_v+f)$ and TFPR is $pq/(\ell_v+f)$ where $\ell_v$ is the variable input.

To compare with the benchmark model, we choose $\varepsilon$ as 0.08 to match the aggregate marginal product of labor of 1.45 as in Edmond, Midrigan, and Xu (2018), while keeping other parameters the same as in the benchmark. Figure A-10 plots the relationship between the measured $\log(TFPR)$ (which again is ARPL, $pq/(\ell_v + f)$ in the model and $f$ includes exporting fixed cost if firm exports) and the measured $\log(TFPQ)$ (which is $q/(\ell_v + f)$) in the model). First, higher productivity firms produce more and end up with a higher endogenous markup. The measured TFPR is therefore higher. Hence, we observe an upward sloping line for the closed economy. Second, this upward sloping patterns also show up in the open economy. Moreover, exporters are more productive and face a higher wedge. Non-exporters face a more competitive market after opening up and charge a lower markup, the TFPR is smaller. Around the exporting cutoff, exporters face a lower TFPR due to the fixed cost of exporting. Overall, exporters face higher TFPR.

In summary, if the observed wedges are purely driven by markups and they endogenously change with trade, we should see that: 1) exporters on average have higher markups, hence higher—rather than lower—TFPR; and given TFPQ, they should have the same TFPR; 2) measured log (TFPR) and log (VA) will be almost perfectly correlated. These implications are at odds with the regression results, where exporters face lower TFPR. Thus, even in this endogenous markup model, similar exogenous distortions are needed to match the observed dispersion and correlation. This is consistent with Song and Wu (2015) and David and Venkateswaran (2017) that the heterogeneity in markup explains very limited MPK dispersion in China. Moreover, Arkolakis et al. (2018) show that the gains from trade in a model with endogenous markup is similar to ACR.