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ABSTRACT

This paper incorporates firm-level distortions into a Melitz model and characterizes welfare under misallocation. We derive an analogue to the well-known ACR result in an economy with distortions. We highlight a channel through which trade can reduce welfare by exacerbating misallocation. A key statistic to infer welfare is the gap between input and output shares. Using Chinese manufacturing data for quantitative analysis, we show that trade integration can lead to a 18% welfare loss coming from a reduction in allocative efficiency. The overall gains to trade is substantially smaller than implied by standard calculations.

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1 Introduction

The question of how much developing countries benefit from opening up to goods trade is a time-honoured subject, both of practical import and intellectual interest. Much has been understood about the nature and type of gains to trade, thanks to the remarkable progress made in the field of international trade in recent decades. Less clear, however, is why certain developing countries have benefited from trade more than others, and why certain countries have seemingly not benefitted much at all.¹ New trade theories suggest that developing countries have the most to gain from trade: if trade liberalization can induce reallocation of resources from less to more productive firms, aggregate productivity and welfare will rise in turn.

But developing countries are different in another respect: they are also subject to prevalent policy and institutional distortions. Examples include taxes and subsidies to certain firms, implicit guarantees and bailouts, preferential access to land and capital, and industrial policy and export promotion policies—common themes in developing countries. In the case of China, for instance, this explains why inefficient but politically-connected private firms and state-owned companies (SOE) have survived and even thrived. Implicit and explicit support for these firms combined with limited exit mechanisms for many SOEs have weakened firm selection effects, the upshot of which is a drag on aggregate productivity. Many believe that joining the WTO can potentially alleviate some of these problems by inviting more Foreign direct competition and helping the more productive firms expand.

How effective is this mechanism? Can trade necessarily improve allocations? Does trade necessarily lead to welfare gains for developing countries? These issues are far from obvious as alluded to by [Rodríguez-Clare \(2018\)](#), “ [a] complication that may matter for the computation of the gains from trade is the presence of domestic distortions.” Thus, the goal of the paper is to provide an answer to this question using the discipline of a general equilibrium model of trade that incorporates idiosyncratic firm-level distortions. We then use Chinese manufacturing data to quantify the effect of trade liberalization as well as to conduct empirical investigations relevant for our mechanisms.

¹For example, [Vaugh \(2010\)](#) shows, in large sample of countries, that poor countries do not systematically gain more from trade.

Our modelling framework incorporates firm-specific distortions into a two-country Melitz model. There are two dimensions of heterogeneity at the firm-level: productivity and distortions. These distortions are assumed to be exogenous output wedges or factor wedges. They drive differences in the marginal products across firms.

In this framework, we derive a general expression for welfare under trade that encompasses the case with and without distortions. We show that a special case in which there is only heterogeneity in productivity, which follows a Pareto distribution, yields the well-known result of ACR (after [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#)): a special case in which there is only heterogeneity in distortion with Pareto distribution yields an analogue to ACR under misallocation. These two are special cases in which firm selection is driven either solely by productivity or solely by distortions. This explicit expression for welfare yields insight on exactly why there could be a loss associated with trade. A crucial sufficient statistic is the gap between the aggregate input and output share. If the required inputs used for production is greater than the output share it yields, then the reduction in allocative efficiency arising from a reallocation of resources can bring about a welfare loss. In the special case, welfare is always lower in an open compared to a closed economy.

The welfare formula also nests a more general case without misallocation, corresponding to [Melitz and Redding \(2015\)](#). In the general case with distortions, productivity and distortions are competing in their impact on firm selection. The relative strength of the two depends on their joint distribution, and micro-level information still matters for welfare. The impact of allocative efficiency arising from a reallocation of resources can again be captured by the input-output gap, along with additional structural microeconomic parameters. To our knowledge, the paper is the first to theoretically characterize welfare gains to trade in a model with misallocation. The decomposition of welfare into a ‘pure technology effect’ and a ‘resource reallocation effect’ in this instance resonates with the decomposition in the recent work of [Baqae and Farhi \(2017\)](#)—an analysis that takes place in a closed-economy with distortions but without firm entry.

Contrary to the mechanism underpinning the [Melitz \(2003\)](#) model and its extensions, i.e. that trade can induce a reallocation of resources from low productivity to high productivity firms, the presence of distortions can bring about the opposite and exacerbate

misallocation. The reason is simple: distortions (for instance, tax and subsidies) act as a veil to a firm’s true productivity. A firm may be producing in the market not because it is inherently productive, but because it is sufficiently subsidized. A mass of highly-subsidized but not adequately productive firms will export and expand at the cost of other more productive firms. The high productivity/ high tax firms which were marginally able to survive in the domestic market would be driven out as the other firms gain market share and drive up costs. In other words, the selection effect which brings about efficiency gains in the Melitz-type model is no longer based solely on productivity; it is determined jointly by firm productivity and distortions. Trade may thus *lower* the average productivity of firms. Depending on the prevalence and nature of distortions, different countries may have different experiences with trade liberalization.

Second, we provide supportive empirical evidence on some of the key implications of our model. Although the study can apply to any group of countries, we focus on Chinese manufacturing data for the simple reason that this country is well known for its prevalent State interventions and policies; that a body of work has shown (see references below) that idiosyncratic distortions explain the majority of the dispersion in marginal products; and that trade liberalization has been an important recent phenomenon.

Employing Chinese manufacturing firm-level data, we first use a ‘measured wedge’ following the customary approach to examine some of its key features. These measured wedges reveal that there is substantial misallocation at the firm level. Their patterns are also consistent with common perception on what types of firms might enjoy special privileges, and hence have ‘lower taxes’. We then evaluate whether key implications of the model receive empirical support. Specifically, the model implies that selection will drive a positive relationship between productivity and taxes for the reason that firms facing higher taxes must be more productive to survive. This is true in the data—there is a large dispersion of the measured wedges across firms, as well as a robust positive relationship between wedges and productivity.² Second, the model indicates that given productivity, firms that

²Different from [Hsieh and Klenow \(2009\)](#), this correlation matters. Typically, this correlation is taken to be exogenous. In works that assume a perfect correlation between distortions and productivity (such as [Costa-Scottini \(2018\)](#) and [Ho \(2010\)](#)), there are always TFP and welfare gains to trade, in contrast to losses that can arise from our model. We demonstrate why this correlation cannot be taken to be exogenous when there is firm selection and fixed cost. Moreover, this assumption counters key empirical facts. We discuss this

face lower taxes will tend to export. Data indicates that controlling for productivity, exporters indeed have lower wedges.

Third, fully recognizing the limitations of the empirical methods to infer wedges (though it provides a robustness check and serves as supportive evidence), we employ a structural model and combine it with micro data to conduct a quantitative analysis of the impact of trade on welfare and aggregate productivity. We run counterfactual experiments for local changes in trade cost, as well as counterfactual experiments for domestic reforms. Our main conclusion is that welfare gains are much smaller when taking into account distortions; that there is a TFP loss of 3% as opposed to a TFP gain of 13.3% in the case without distortions, and that allocative inefficiency can induce a welfare loss of 18%.

It is important to point out that in the quantitative analyses we do not use directly the empirically-measured wedges, observed correlations, or distributions in the data to assess the impact of trade on welfare. The reason is that the *observed* statistics are not the *underlying* ones: existing firms have been subject to selection and thus their observed distributions are not the true ones. The same reasoning goes for the observed correlation between productivity and wedges. Both selection and fixed costs can drive a positive relationship between the two. For these reasons, the approach adopted in the quantitative exercises is to estimate the underlying joint distribution of wedges and productivity, costs of producing and exporting so as to match the observed patterns of firms' outputs, inputs, and exports. On this basis, we evaluate how the presence of distortions change the impact of trade on productivity and welfare, and how much trade has contributed to Chinese growth in a decomposition exercise. This contrasts with the reduced-form approach adopted in [Berthou, Chung, Manova, and Bragard \(2018\)](#), which uses empirically measured revenue productivity to assess the impact of trade reforms on aggregate productivity under misallocation.³ Our works are broadly complementary, as we focus on theoretical analyses and a structural approach to inferring welfare gains for China, whilst they focus on an empirical assessment of trade on aggregate productivities for 14 European countries.

at the end of Section 3.

³[Berthou et al. \(2018\)](#) theoretically and empirically assess the impact of trade reforms for 14 European countries and 20 industries over the period 1998-2011. They find that trade reforms have ambiguous effects on measured revenue productivity in the theory, while they are positive in the data.

An important point is the fact that ‘misallocation of resources’ goes beyond the observed misallocation among a set of operating firms. Because policy distortions also act as a barrier to entry (and exit), there is also misallocation among potential entrants and incumbents—firms that should have entered the market in an efficient economy that couldn’t, and firms that should have otherwise exited but have not. This reallocation along the entry/exit margin can also be significant. The endogenous mechanism of entry/exit and the attendant firm selection effect, along with trade, makes this different from the important works of Hsieh and Klenow (2009), Baily, Hulten, and Campbell (1992), Restuccia and Rogerson (2008), Bartelsman, Haltiwanger, and Scarpetta (2009), etc. Empirical works have also demonstrated the importance of entry and exit for China’s growth.⁴

The source of misallocation of resources is not the focus of our work, though we take the particular stance that firm-specific ‘distortions’ due to policy and institutional features is a major cause of misallocation. Distortions in China manifest themselves in the form of substantial privileges of state owned enterprises over private firms, of connected private firms, or of firms belonging to particular locations. Specific policies that can drive these wedges include implicit subsidies such as soft budget constraints, favorable costs of capital, preferential tax treatments and implicit guarantees. Firms with political connections having access to special deals and receiving substantial benefits are also widely documented (see Guo, Jiang, Kim, and Xu (2013) and Bai, Hsieh, and Song (2019)). Wu (2018) conducts an empirical analysis and finds that policy distortions can be explained by investment promoting programs that favor such firms.

There is also substantial evidence coming from a number of papers that idiosyncratic firm-distortions due to policy and institutional features account for a large part of the observed dispersion in marginal products across firms in China. In principle, misallocation can arise from a variety of factors; but different approaches to disentangle them have come to similar conclusions that policy distortions are elemental.⁵ Furthermore, we con-

⁴Brandt, Van Biesebroeck, and Zhang (2012) find that net entry accounts for roughly half of Chinese manufacturing productivity growth. The creation and selection of new firms in China’s non-state sector has been particularly important.

⁵These could be technological frictions, such as adjustment costs, information frictions, financial frictions, or markups. Wu (2018) finds that policies account for the majority of the observed misallocation of capital, as opposed to financial frictions. Using a different approach and modeling framework, David and Venkateswaran (2017) find also that firm-specific distortions, rather than technological or information fric-

duct empirical analyses to show how these wedges systematically relate to certain firm characteristics in a way that echoes prior findings. To ensure that measured dispersion is not due to measurement error, we use three alternative approaches utilizing panel data to demonstrate the minor role it plays.⁶

Many of these distortions are presumably idiosyncratic and unrelated to trade. Examples are in the legion: Chinese firms such as the car manufacturing company Chery, have enjoyed easy access to land and capital from the Wuhu government. Foxconn, the world's largest electronics contractor manufacturer, has enjoyed substantial tax breaks from many provinces including industrial land at significantly discounted prices. Tesla has recently received free land and subsidies from the local government of Shanghai. A recent study by [Chen and Kung \(2018\)](#) demonstrate the firms that are connected with political elites were able to obtain land at 80 to 90 percent discount over the period 2004-2016.

For the reasons above, the baseline model in the paper is to focus on domestic policy distortions. Still, one can ask whether some of the large dispersion of marginal products reflects endogenous distortions—those that can potentially change with trade liberalization. As a robustness check we examine a model of endogenous distortions with variable markup, and ask whether trade can mitigate these distortions and the misallocation of resources. Section 5.1 takes up a variable markup model. We show that these models 1) yield some obvious counterfactual predictions on the relationship between exporters and wedges; 2) that markup alone also explains little of the dispersion in wedges. To match the observed correlation and dispersion one would still need to include exogenous distortions. Moreover, the attendant pro-competitive effects in a model with endogenous markup may be 'elusive' as pointed out by [Arkolakis, Costinot, Donaldson, and Rodríguez-Clare \(2018\)](#).⁷

In this framework, positive firm selection is the central driving force for gains to trade. As such, it abstracts from other types of gains to trade, such as trade-induced technologi-

tions, account for the majority of the observed dispersions in marginal products. [Bai, Lu, and Tian \(2018\)](#) disciplines financial frictions with firms' financing patterns, sales distribution and change of capital. They find that financial frictions cannot explain the observed relation between firms' measured distortions and size.

⁶However, [Bils, Klenow, and Ruane \(2017\)](#) cannot rule out multiplicative measurement errors.

⁷This paper makes the point that when more productive firms expand at the expense of less productive ones, thanks to trade, the aggregate markup tends to rise. Thus, overall, trade models with endogenous markups do not necessarily generate higher gains from trade.

cal diffusion (Alvarez, Buera, and Lucas Jr (2013) and Buera and Oberfield (2016)), adoption (Perla, Tonetti, and Waugh (2015) and Sampson (2015)) and innovation (Atkeson and Burstein (2010)). While these mechanisms in principle work to increase the gains to trade, with its quantitative significance a subject to debate,⁸ it does not detract from the fact that the distortionary impact on allocation efficiency still induces large welfare losses, which is what we are interested in. Of course, distortions can also interact with some of these additional channels. For instance, in a model with firm innovation, one would need to consider the fact that distortions not only affect production decisions, but potentially also innovation decisions. These considerations go beyond the scope of this paper but deserve further consideration. We also do not consider how trade can reduce domestic distortions, for example if concurrent domestic reforms are requisite for joining the WTO or if quotas are removed (see Khandelwal, Schott, and Wei (2013)). As a robustness check, however, we allow for firms to face a different distribution of distortions when they start to export and examine welfare and efficiency gains therein.

A key message of this paper is that in order for developing countries to reap the full gains of trade, simultaneous or antecedent domestic reforms aimed at reducing policy distortions may be crucial. Based on our structural model, trade contributed only 8% to growth over the period of 1998 to 2005 in China. This echoes with the findings of Tombe and Zhu (2019) that find similar numbers despite employing a very different framework with migration flows. The policy implication drawn from this framework is consistent with works indicating that policies aimed to neutralize domestic distortions may be complementary to trade liberalization (Chang, Kaltani, and Loayza (2009) and Harrison and Rodríguez-Clare (2010)). It counters other claims that trade liberalization should take precedence owing to positive firm selection (Asturias, Hur, Kehoe, and Ruhl (2016)).⁹

⁸Perla, Tonetti, and Waugh (2015) and Atkeson and Burstein (2010), for instance, find that trade gains are not too different from ACR gains. In Perla, Tonetti, and Waugh (2015), there is trade-induced within-firm productivity improvements. However, their aggregate growth effects come with costs—losses in variety and reallocation of resources away from goods production. Thus, the aggregate effect on welfare is similar to ACR gains. Atkeson and Burstein (2010) show that general equilibrium effects limits the first-order effects on aggregate productivity even when there is firm-level innovation.

⁹They show that the best sequence of reforms is to first decrease trade costs, then to improve contract enforcement, and, finally, to decrease the cost of firm creation. The reason is that an increase in competition leads to an expansion of productive firms and crowding out of less efficient ones. By liberalizing international trade first so as to impose firm selection early, inefficient firms are prevented from entering later when contract enforcement and firm entry costs are reformed. In contrast, we show that the selection mechanism

In sum, this paper shows that experiences of trade liberalization in developing countries should not be considered to be independent of micro-level distortions to which they are subject. Our paper demonstrates that the presence of policy distortions have a first-order quantitative effect on the gains to trade. The organization of the paper is as follows: Section 2 derives a theoretical framework of trade gains under misallocation. Section 3 investigates patterns of measured wedges among Chinese manufacturing firms. Section 4 provides a quantitative assessment on the impact of trade liberalization under misallocation, with various extensions of the benchmark framework. Section 5 discusses a model with endogenous distortions. Section 6 concludes.

2 Theoretical Framework

2.1 Baseline Model

The world consists of two large open economies, Home and Foreign, with heterogeneous firms. The two economies can differ in the size of labor and distribution of firms. Labor is immobile across countries and inelastic in supply.

Consumers. A representative consumer in the Home country chooses the amount of final goods C in order to maximize utility $u(C)$, subject to the budget constraint

$$PC = wL + \Pi + T,$$

where L is labor, w is wage rate, Π is dividend income, and T is the amount of lump-sum transfers received from the government.

Final Goods Producers. Final goods producers are perfectly competitive, and combine intermediate goods using a CES production function

$$Q = \left[\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

is substantially weakened in the presence of distortions.

where σ is the elasticity of substitution across intermediate goods, and Ω is the endogenous set of goods. The corresponding final goods price index is thus

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},$$

where $p(\omega)$ is the price of good ω in the market. The individual demand for this good is thus given by

$$q(\omega) = \frac{p(\omega)^{-\sigma}}{P^{-\sigma}} Q.$$

Intermediate Goods Producers. There is a competitive fringe of potential entrants (in both countries) that can enter by paying a sunk entry cost of f_e units of labor. Potential entrants face uncertainty about their productivity in the industry. They also face a stochastic revenue wedge τ , which can be seen as a tax (>1) or subsidy (<1) on every pq earned.¹⁰ Once the sunk entry cost is paid, a firm draws its productivity φ and τ from a fixed joint distribution, $g(\varphi, \tau)$ over $\varphi \in (0, \infty), \tau \in (0, \infty)$.¹¹

Firms are monopolistically competitive. Production of each intermediate good entails fixed production cost of f units of labor and a constant variable cost that depends on firm productivity. The total labor required to produce $q(\varphi)$ units of a variety is therefore:¹²

$$\ell = f + \frac{q}{\varphi}.$$

Productivity φ is idiosyncratic and independent across firms. The existence of a fixed production cost means that only a subset of firms produces—those that draw a sufficiently low productivity cannot generate enough variable profits to cover the fixed production cost. If firms decide to export, they face a fixed exporting cost of f_x units of labor and iceberg

¹⁰It is equivalent to an input wedge on all the input a firm uses.

¹¹The model equilibrium is equivalent to a stationary equilibrium of a model allowing for constant exogenous probability of death δ and entry cost f_e/δ .

¹²We can easily extend the production to include capital, i.e. $k^\alpha \ell^{1-\alpha}$. The unit cost for producing q or fixed cost is $\alpha^{-\alpha} (1-\alpha)^{\alpha-1} w^{1-\alpha} r_k^\alpha$ where r_k is the rental cost of capital. In our model, we introduce one heterogeneous distortions at the firm level, and our τ is an output distortion, but it includes all input distortions that increase the marginal products of capital and labor by the same proportion as an output distortion. In the data, there are distortions that affect both capital and labor and distortions that change the marginal product of one of the factors relative to the other. In our quantitative exercises, we include both capital and labor, and the distortions on both factors.

variable costs of trade τ_x , which is greater than 1. Firms with the same productivity and distortion behave identically, and thus we can index firms by their (φ, τ) combination.

An intermediate goods firm thus solves the following problem

$$\max_{p,q} \frac{pq}{\tau} - \frac{w}{\varphi}q - wf$$

subject to the demand function $q = \frac{p^{-\sigma}}{P^{1-\sigma}}Q$, henceforward suppressing ω for convenience. From here it is clear that a revenue tax is equivalent to a tax on all input costs incurred by the firm.

Firms are infinitesimally small, and thus take the aggregate price index as given. Equating the after-tax marginal revenue with marginal costs yields the standard result that equilibrium prices are a mark-up over marginal costs:

$$p = \frac{\sigma}{\sigma-1} \frac{w\tau}{\varphi}. \quad (1)$$

Optimal profits are then

$$\pi = \sigma^{-\sigma}(\sigma-1)^{\sigma-1}P^\sigma Q\tau^{-\sigma}w^{1-\sigma}\varphi^{\sigma-1} - wf. \quad (2)$$

It immediately follows that given the fixed cost of production, there is a zero-profit cutoff productivity below which firms would choose not to produce, and exit the market. Thus, a firm would choose to produce only if $\varphi \geq \varphi^*(\tau)$. This cutoff productivity level satisfies

$$\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left[\frac{wf}{P^\sigma Q} \right]^{\frac{1}{\sigma-1}} w\tau^{\frac{\sigma}{\sigma-1}}. \quad (3)$$

The cutoff productivity is now a function of the firm-specific distortion, and differs across firms facing different levels of distortions. Firms with a higher tax τ will have a higher cutoff for productivity. This means that low productivity firms that would have been otherwise excluded from the market can now enter the market and survive if sufficiently subsidized.

The government's budget is balanced so that

$$T = \int_{\omega \in \Sigma} \left(1 - \frac{1}{\tau}\right) p(\omega) q(\omega) d\omega,$$

where Σ is the endogenous set of home products. With trade, firms now have the option of exporting abroad. If a Home firm exports to the Foreign economy, it solves the following problem

$$\max \frac{p_x q_f}{\tau} - \frac{w}{\varphi} \tau_x q_f - w f_x$$

subject to the Foreign demand function $q = \frac{p_x^{-\sigma}}{P_f^{-\sigma}} Q_f$, where P_f and Q_f denote the aggregate price index and demand in Foreign. Given the same constant elasticity of demand in the domestic and export markets, equilibrium prices in the export market are a constant multiple of those in the domestic market:

$$p_x(\varphi, \tau) = \frac{\sigma}{\sigma - 1} \frac{w \tau_x \tau}{\varphi},$$

The optimal profit from servicing the Foreign market,

$$\pi_x = \sigma^{-\sigma} (\sigma - 1)^{\sigma-1} P_f^\sigma Q_f \tau^{-\sigma} (w \tau_x)^{1-\sigma} \varphi^{\sigma-1} - w f_x, \quad (4)$$

yields an optimal cutoff for exporting:

$$\varphi_x^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left[\frac{w f_x \tau_x^{\sigma-1}}{P_f^\sigma Q_f} \right]^{\frac{1}{\sigma-1}} w \tau^{\frac{\sigma}{\sigma-1}}. \quad (5)$$

Consumer love of variety, a fixed production cost and additional fixed cost of exporting, mean that firms would never export without also selling in the domestic market. There are, hence, two cutoff productivities relevant for the domestic economy: one for entering the domestic market as given by (3) and one for entering the Foreign market, as given by (5). To the extent that taxes τ are constant across firms, the ratio $\varphi_x^*(\tau)/\varphi^*(\tau)$ is a constant and is greater than 1 so long as $\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P_f^\sigma Q_f}{P_f^\sigma Q_f} > 1$. Analogously, firms in the Foreign country, which draw their productivity from a distribution $g_f(\varphi, \tau)$, are subject to two cutoff productivities, one for servicing their domestic market, and one for exporting to the

Home economy

$$\varphi_f^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left[\frac{w_f f}{P_f^\sigma Q_f} \right]^{\frac{1}{\sigma-1}} w_f \tau^{\frac{\sigma}{\sigma-1}}, \quad (6)$$

$$\varphi_{xf}^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left[\frac{w_f f_x \tau_x^{\sigma-1}}{P^\sigma Q} \right]^{\frac{1}{\sigma-1}} w_f \tau^{\frac{\sigma}{\sigma-1}}. \quad (7)$$

where w_f denotes the Foreign wages, and the fixed cost of producing and exporting are assumed to be identical in the two economies.

The equilibrium features a constant mass of firms entering M_e and producing M , along with a ex-post distributions of productivity and distortion among operational firms $\mu(\varphi, \tau)$. The ex-post distribution $\mu(\varphi, \tau)$ is a truncation of the ex-ante productivity-distortion distribution, $g(\varphi, \tau)$, at the zero-profit cutoff productivity given by Eq.3:

$$\mu(\varphi, \tau) = \frac{g(\varphi, \tau)}{\int \int_{\varphi^*(\tau)}^{\infty} g(\varphi, \tau) d\varphi d\tau} \quad (8)$$

if $\varphi \geq \varphi^*(\tau)$; and $\mu(\varphi, \tau) = 0$ otherwise. The denominator is the probability of successful entry, denoted as

$$\omega_e = \int \int_{\varphi^*(\tau)}^{\infty} g(\varphi, \tau) d\varphi d\tau. \quad (9)$$

In equilibrium, the measure of producing firms equals the product of measure of entrants and the probability of entering, i.e.

$$\omega_e M_e = M.$$

We define the probability of exporting conditional on entry as

$$\omega_x = \int \int_{\varphi_x^*(\tau)}^{\infty} \mu(\varphi, \tau) d\varphi d\tau = \frac{\int \int_{\varphi_x^*(\tau)}^{\infty} g(\varphi, \tau) d\varphi d\tau}{\int \int_{\varphi^*(\tau)}^{\infty} g(\varphi, \tau) d\varphi d\tau}.$$

In an equilibrium with positive entry, the free entry condition requires that

$$\int \int_{\varphi^*(\tau)} \pi(\varphi, \tau) g(\varphi, \tau) d\varphi d\tau + \int \int_{\varphi_x^*(\tau)} \pi_x(\varphi, \tau) g(\varphi, \tau) d\varphi d\tau = w_f e. \quad (10)$$

The first term is the expected profits from domestic sales conditional on entry, multiplied by

the probability of entry. The second term is the expect profits from export sales conditional on exporting, multiplied by the probability of exporting. The free entry condition requires that their sum be equal to the entry costs (in terms of labor).

The free entry condition (10), combined with optimal profit functions (2) and (4) gives an expression for the price index P :

$$P = \frac{\sigma}{\sigma - 1} \left[M \int \int_{\varphi^*(\tau)}^{\infty} \left(\frac{w\tau}{\varphi} \right)^{1-\sigma} \mu(\varphi, \tau) d\varphi d\tau + M_f \int \int_{\varphi_{xf}^*(\tau)}^{\infty} \left(\frac{w_f \tau_x \tau}{\varphi} \right)^{1-\sigma} \mu_f(\varphi, \tau) d\varphi d\tau \right]^{\frac{1}{1-\sigma}}, \quad (11)$$

where M and M_f denote the measure of operating firms in Home and Foreign. The Foreign price index P_f takes a similar form.

Goods market clearing. The assumption of a balanced trade results in

$$P_f^\sigma Q_f M \int \int_{\varphi_x^*(\tau)}^{\infty} \left(\frac{w\tau_x \tau}{\varphi} \right)^{1-\sigma} \mu(\varphi, \tau) d\varphi d\tau = P^\sigma Q M_f \int \int_{\varphi_{xf}^*(\tau)}^{\infty} \left(\frac{w_f \tau_x \tau}{\varphi} \right)^{1-\sigma} \mu_f(\varphi, \tau) d\varphi d\tau. \quad (12)$$

Labor market clearing. In an equilibrium in which the mass of firms M are constant in both economies, the labor market condition yields

$$M = \frac{L}{\sigma \left(\frac{f_e}{\omega_e} + f + \omega_x f_x \right)}. \quad (13)$$

Normalizing the Home country wage rate to 1, there are eleven equations, the zero cutoff productivities for domestic production and exporting (3), (5), and its Foreign counterparts, the free entry conditions (10) along with its Foreign counterpart, the definition of the Home and Foreign price indices (11), and a goods market clearing/balanced trade equation (12), along with the measure of firms (13) and its Foreign counterpart. These equations yield the equilibrium consisting of eleven unknowns $\{\varphi^*(\tau), \varphi_x^*(\tau), \varphi_f^*(\tau), \varphi_{fx}^*(\tau), P, P_f, Q, Q_f, w_f, M, M_f\}$. A detailed derivation of the model is provided in Appendix A.

Proposition 1. *The allocations, entrants, and cutoff functions $\{Q, Q_f, M, M_f, \varphi^*(\tau), \varphi_f^*(\tau), \varphi_x^*(\tau), \varphi_{xf}^*(\tau)\}$ are independent of mean wedge $\bar{\tau}$. Prices $\{P, P_f, w_f\}$ change proportionally with $\bar{\tau}$, $\bar{\tau}^f$, i.e. $P(\bar{\tau}_1)/P(\bar{\tau}_2) = \bar{\tau}_1/\bar{\tau}_2$, and similarly for P_f and w_f .*

The proof of the proposition is straightforward.

2.2 Theoretical Comparative Static

We proceed to analyze welfare and efficiency with distortions. The welfare, or consumption, is given by¹³

$$W = \frac{\sigma - 1}{\sigma} \left[M_e \int \int_{\varphi^*(\tau)} \left(\varphi \frac{\overline{MRPL}}{MRPL_\tau} \right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau + M_e \frac{P_f^\sigma Q_f}{P^\sigma Q} \int \int_{\varphi_x^*(\tau)} \left(\frac{\varphi}{\tau_x} \frac{\overline{MRPL}}{MRPL_\tau} \right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau \right]^{\frac{1}{\sigma-1}} \quad (14)$$

where $MRPL_\tau$ is the firm specific marginal revenue product of labor $MRPL_\tau = w\tau$, and \overline{MRPL} is the economy wide marginal revenue product of labor, $\overline{MRPL} = PQ/wL$. Hence the welfare relates some weighted productivity of firms with relative distortions as the weight. In an efficient case without distortions, all firms have the same marginal revenue product, $MRPL_\tau = \overline{MRPL}$ for any τ . Hence, Equation (14) shows that the source of welfare loss in the presence of firm-level distortions can arise from a misallocation of resources, captured by dispersions in $\overline{MRPL}/MRPL_\tau$, and a misallocation caused by selection and entry mechanisms as captured by M_e , φ^* , φ_x^* being different from their respective efficient levels. Change in trade cost affects the economy through the selection and entry and the misallocations.

Next, we derive a general expression for changes in welfare associated with changes in trade costs. [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) demonstrates that in the absence of distortions, welfare changes across a wide class of models can be inferred using two variables: (i) changes in the share of expenditure on domestic goods; and (ii) the elasticity of bilateral imports with respect to variable trade costs (the trade elasticity). Different trade models can have different micro-level predictions, sources of welfare gains, and different structural interpretations of the trade elasticity. But conditional on observed trade flows and an estimated trade elasticity, the welfare predictions are the same. The generality of this formulation, however, relies on a certain set of macro-level restrictions. [Melitz and Redding \(2015\)](#) (henceforth MR) show that under more general distribution functions for productivity, the trade elasticity is no longer invariant to trade costs and across markets,

¹³Home's welfare depends on foreign exports, which are rewritten by using the balanced trade condition.

and therefore no longer a sufficient statistic for welfare. Micro-level information is still important for welfare.

In the analysis below, we first consider a fall in trade costs in an open economy equilibrium. The following proposition provides a general representation of welfare:

Proposition 2. *The change in welfare associated with an iceberg cost shock is*

$$\begin{aligned}
 d \ln W = \frac{1}{\gamma_s + \sigma - 1} & \left[-d \ln \lambda \quad (\text{ACR}) \right. \\
 & + d \ln M_e \quad (\text{MR}) \\
 & + \frac{\sigma}{\sigma - 1} (\gamma_s - \gamma_d) d \ln M_e \\
 & \left. - \left(\sigma - 1 + \frac{\sigma \gamma_s}{\sigma - 1} \right) d \ln \lambda + \left(\sigma - 1 + \frac{\sigma \gamma_d}{\sigma - 1} \right) d \ln S \right] \quad (\text{Reallocation})
 \end{aligned} \tag{15}$$

where λ is the share of domestic output; S is the share of variable labor used in producing domestic goods; γ_d is the elasticity of the cumulative domestic market share for firms above the domestic cutoff, with respect to the cutoff; and γ_s is the same elasticity for S .¹⁴

1. Without domestic distortions, $S = \lambda$ and $\gamma_s = \gamma_d$. If productivity follows a Pareto distribution with parameter θ , $\gamma_d = \theta - \sigma + 1$ and $d \ln M_e = 0$. Hence, $d \ln W = \frac{1}{\theta} [-d \ln \lambda]$ as in ACR.

2. Under a general distribution function and without domestic distortions, $S = \lambda$, $\gamma_s = \gamma_d$, but they are not constant, and $d \ln M_e \neq 0$. Hence, $d \ln W = \frac{1}{\gamma_d(\varphi^*) + \sigma - 1} [-d \ln \lambda + d \ln M_e]$. Micro structure matters for $\gamma_d(\varphi^*)$ and welfare as in MR.

3. With homogenous productivity and Pareto-distributed domestic distortion $1/\tau$ with parameter θ , $\gamma_d = \frac{\sigma-1}{\sigma}(\theta - \sigma + 1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta - \sigma)$. Hence, $d \ln W = \frac{\sigma}{\sigma-1} [d \ln S - d \ln \lambda]$.

¹⁴The elasticities are also known as the hazard functions. The elasticity related to outputs or the hazard function of the distribution of log firm size within the domestic market is given by

$$\gamma_d = \frac{\int \left(\frac{\varphi^*(\tau)}{\tau} \right)^{\sigma-1} g(\varphi^*, \tau) \varphi^* d\tau}{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau} \right)^{\sigma-1} g(\varphi, \tau) d\varphi d\tau}.$$

The elasticity related to inputs or the hazard function for the distribution of log after-tax firm size, which is also proportional to the distribution of firm variable input, within the domestic market is given by

$$\gamma_s = \frac{\int \left(\frac{\varphi^*(\tau)}{\tau} \right)^{\sigma-1} / \tau g(\varphi^*, \tau) \varphi^* d\tau}{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau} \right)^{\sigma-1} / \tau g(\varphi, \tau) d\varphi d\tau}.$$

4. In general, for a local change in trade cost, information on the change of domestic (sales and variable labor) shares and the measure of entrants, the joint distribution of firms sales and variable inputs, and the marginal firms (hence we know γ_d and γ_s) are sufficient for computing the associated welfare change.

PROOF: Appendix B.1.

The above proposition encapsulates welfare results for four different cases. The special case in which there is only heterogeneity in productivity, Pareto-distributed, recovers the ACR formula (point 1). The case without distortion and under a more general productivity distribution gives rise to MR (point 2). With misallocation, the special case in which there is only heterogeneity in distortions which is Pareto distributed gives an analogue formula to ACR: the difference in the change in the domestic labor and output share provides a sufficient statistics for welfare (point 3). In the general welfare representation, the first term is referred to as ACR, the third and fourth terms are brought about by distortions, and is referred to as an *entry* and *reallocation* effect.¹⁵

Note that the proposition applies to both symmetric and asymmetric countries and takes into consideration the impact of the Foreign distribution of firms on the Home country. It also shows the effect of domestic distortions on a Foreign country. In the case that the Foreign economy is devoid of distortions, the third and fourth terms in Foreign's welfare formula go to zero. Thus, Home's domestic distortions affect Foreign only through Foreign's λ , M_e , the cutoffs, and hence γ .

In the two special cases, firm selection is either driven solely by productivity, or solely by distortions. The former implies that there is always a welfare improvement when the economy opens up to trade, whereas the latter implies that there is an unambiguous loss (see Lemma below). In the more general case, both productivity and distortions jointly determine firm selection. The resource reallocation is both one amongst existing firms (last term), and along the entry dimension (third term). Without distortions, a firm's share of input is equal to its share of output, so that in aggregate, $S = \lambda$. In the presence of

¹⁵Note that we can write the welfare change using different general equilibrium variables, e.g. $d \ln W = \frac{1}{\gamma_d + \sigma - 1} [-d \ln \lambda + d \ln M_e] + (\frac{\gamma_d / (\sigma - 1)}{\gamma_d + \sigma - 1} + 1) d \ln PQ$. Trade changes the allocation and misallocation of the resources, which in turn affects the aggregate expenditure (and thus selection), and hence welfare in the economy. Replacing $d \ln PQ$ by equilibrium conditions, one arrives at Proposition 2, which provides more transparent indications of how exporting/selection reallocates resources and affects misallocation.

distortions, the two are no longer equal. The gap between input and output shares is informative about changes in allocative efficiency. If the change in required inputs exceeds the change in output it produces, i.e. $d \ln S < d \ln \lambda$, resources reallocation has induced an efficiency loss.¹⁶ This reallocation comes from the flows that take place among a given set of producers/exporters on the intensive margin, captured by $(\sigma - 1)(d \ln S - d \ln \lambda)$; it also comes from a reallocation on the extensive margin, induced by the fact that cutoffs have changed. This effect is summarized by the terms with the hazard rates γ_s and γ_d .

Lemma 1. *Under homogenous productivity and Pareto-distributed domestic distortion $1/\tau$ with parameter θ ,*

1. *Moving from a closed economy to an open economy entails a welfare loss.*
2. *In the open-economy equilibrium, the reallocation term is always negative.*

PROOF: Appendix B.2.

This Lemma presents two important features under the special case. First, compared to the close economy, an open economy with any level of finite iceberg trade cost always has a lower welfare as long as there is selection to export. Second, among the equilibria of the open economy, for a marginal reduction of iceberg cost, as long as there are larger fraction of exporters, the reallocation term is always negative, implying a worsening of misallocation after a reduction in trade costs.

The intuition for why the open economy has a lower welfare than in the closed economy is made transparent by this special case: under homogenous productivity, the efficient allocation for the closed economy dictates that firms have identical market shares— either all firms export or none of them export when the economy opens up to trade. Hence, the ex-post efficient allocation should also be equal shares for all firms. However, with distortions, the relatively subsidized firms produce more than in the efficient case, with the dispersion of sales (employment) reflecting the distortions. When opening up to trade, it is the relatively subsidized firms that export, in turn making the firm's distribution even more skewed—misallocation is exacerbated. The share of labor required in producing domestic goods ends up being less than the domestic output share. When firm selection is purely

¹⁶Note that $d \ln S \leq 0$ and $d \ln \lambda \leq 0$ in response to a trade cost reduction.

driven by distortions, allocative efficiency deteriorates when moving from autarky to an open economy.

Point 2 in the above Lemma focuses on a local change in trade costs in the open economy equilibrium, where $d \ln W = \frac{\sigma}{\sigma-1} [d \ln S - d \ln \lambda]$. The difference in the change in the share of expenditure on domestic goods and the share of variable labor used in producing domestic goods constitutes a sufficient statistics for the welfare change. This change of welfare reduces to two terms: the standard ACR term, where $-d \ln \lambda > 0$, and the reallocation term, which is negative.¹⁷ Overall, the change in welfare in the open economy equilibrium displays a U-shape pattern: for high levels of trade cost, there is a welfare loss; and for low levels of trade cost, there is a welfare gain associate with a marginal change in trade cost. The reason is that firms selection driven by distortions are less significant when trade costs are small (at zero trade cost all firms export), and the welfare gains dominate the losses associated with reallocation.

These implications carry a similar flavor to findings in [Baqaee and Farhi \(2017\)](#), to which we draw an analogy in order to further fix ideas in our open economy. Their work studies the macroeconomic impact of microeconomic shocks to productivity and wedges, and show that the effect on output can be decomposed into a “pure technology effect” and a “resource allocation effect”. The former is the change in output holding fixed the share of resources going to each user; the latter is the change in output resulting from the reallocation of shares of resources across users. To invoke a closer comparison to theirs, one can do a similar analysis in our closed-economy setting—that is, decomposing the aggregate effect of a firm-specific productivity shock to a pure technology effect and a resource reallocation effect occasioned by distortions.¹⁸ This decomposition demonstrates why an increase in a firm’s productivity may not raise aggregate productivity: if a firm is relatively subsidized, then firm i ’s labor share is larger than its sales share, and its expansion means that the

¹⁷In the welfare expression, the second and third terms cancel out, and the fourth term $-(\sigma - 1 + \frac{\sigma \gamma_s}{\sigma - 1}) d \ln \lambda + (\sigma - 1 + \frac{\sigma \gamma_d}{\sigma - 1}) d \ln S = (1 - \theta) d \ln \lambda + \theta d \ln S < 0$.

¹⁸The aggregate impact of an exogenous firm-specific productivity shock is given by $\frac{d \ln Q}{d \ln \varphi_i} = \frac{q_i^{\sigma-1}}{\int q_j^{\sigma-1} d j} + (\sigma - 1) \left[\frac{q_i^{\sigma-1}}{\int q_j^{\sigma-1} d j} - \frac{q_i^{\sigma-1} \frac{1}{\tau_i}}{\int q_j^{\sigma-1} \frac{1}{\tau_j} d j} \right] = \frac{p_i q_i}{P Q} + (\sigma - 1) \left[\frac{p_i q_i}{P Q} - \frac{\ell_i}{L} \right]$, which shows that it depends on firm i ’s sales share and the gap between its sales share and labor share. A firm is relatively subsidized if $1/\tau_i$ is larger than the average level of distortion. In a closed-economy, with no free entry and exogenous markup, the gap is the change of labor income share.

resource allocation component is negative.

The same reasoning applies for the trade cost shock. Associated with a lowering of trade costs is a positive "pure technology" effect (the first two terms of welfare in Proposition 2), as well as a distortion-induced entry and reallocation effect (the last two terms), where γ_d and γ_s and the change in S summarize the reallocation of resources towards or away from more distorted firms. When a relatively subsidized firm expands due to trade, allocative efficiency deteriorates, and it is welfare-reducing.

Having established sufficiency conditions in the special case, we can also now examine the general case. The necessary condition for the resource allocation term to be negative is: either $\gamma_s \leq \gamma_d$ or $d \ln S \leq d \ln \lambda$. Intuitively, misallocation happens when the input elasticity is smaller than the output elasticity, i.e. more resources are used to produce the same unit of output. The following Lemma presents a sufficient condition for $\gamma_s \leq \gamma_d$ for a more general distribution for productivity and distortions.

Lemma 2. *Suppose (τ, ϕ) are jointly log-normally distributed with standard deviations of σ_τ and σ_ϕ and correlation ρ . When $\sigma_\tau \geq \frac{\sigma-1}{\sigma} \rho \sigma_\phi$,*

1. *the cumulative labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order.*
2. *the hazard functions $\gamma_s \leq \gamma_d$ and shares $S \leq \lambda$ at any cutoff, hence moving from a closed economy to an open economy, the reallocation term is always negative.*

PROOF: Appendix B.3.

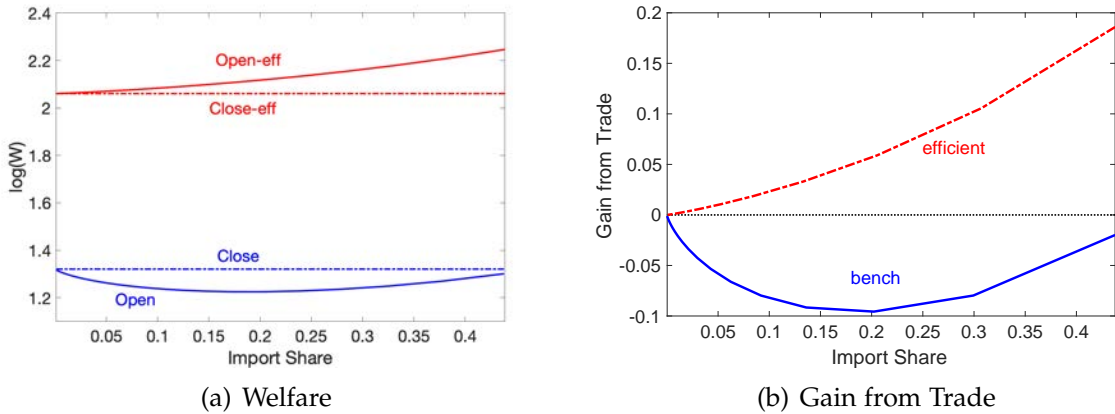
Under the condition $\sigma_\tau \geq \frac{\sigma-1}{\sigma} \rho \sigma_\phi$, the cumulative labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order. We can prove the input elasticity is always smaller than the output elasticity at any cutoff, and the share of variable labor used in producing domestic goods is always smaller than the share of expenditure on domestic goods, which implies from close to open $d \ln S$ is more negative than $d \ln \lambda$. The reallocation term is always negative.

2.3 Numerical Example

To unpack the theoretical results and to provide more intuition for the mechanisms that underpin these results, we next turn to a numerical example of the benchmark model with symmetric countries. The joint distribution between productivity and distortions is taken to be joint log-normal with standard deviations of $\sigma_\tau = \sigma_\varphi = 0.5$ and correlation of φ and τ of $\rho = 0.8$. The elasticity of substitution $\sigma = 2$, and the fixed costs are $f = 0.03$, $f_x = 0.035$, $f_e = 0.01$.

First, the example illustrates that welfare (Eq. 14) can fall when the economy opens up to trade. Figure 1.1 (a) plots the level of welfare against import shares under the alternative scenarios: the efficient case without distortions, the case with distortions, and when the economy is closed or open. Three observations immediately follow: 1) that there is a welfare loss in the case with distortions compared to the case without; 2) opening up to trade leads to welfare gains in the efficient case; however, 3) opening up engenders a welfare *loss* in the presence of distortions. Taking the differences between the open and close economy in either case, with or without distortion, we plot the welfare change after trade in Figure 1.1 (b). It is clear that there is welfare loss with distortions in our benchmark.

Figure 1.1: Welfare and the Change from Trade

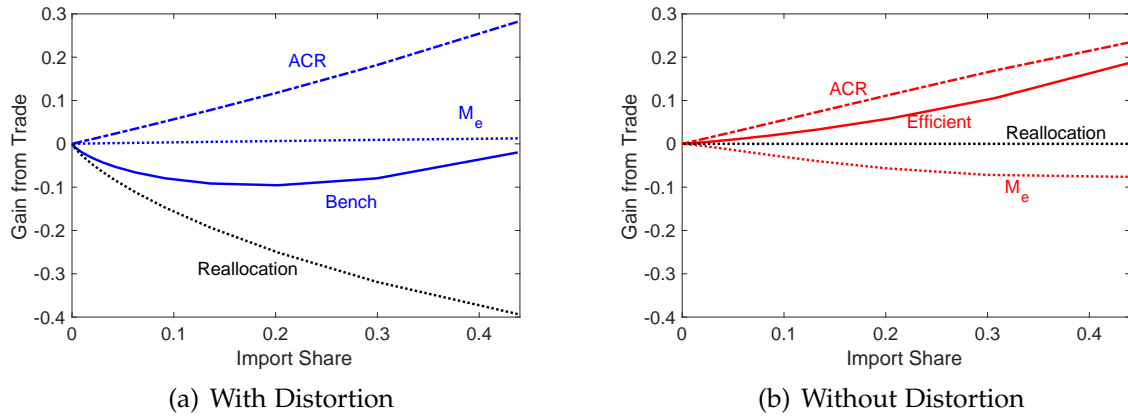


Second, we show the welfare decompositions according to our welfare formula (15) in Proposition 2. Figure 1.2 (a) displays the three components in our benchmark model—the ACR term, entry M_e , and reallocation. Through the impact of technology, welfare increases with lower trade cost, i.e the ACR term increases with the import share. Trade, however,

exacerbates the misallocation and reduces allocative efficiency. Hence, the reallocation term becomes more and more negative as the import share increases. Overall, the resource misallocation effect dominates, and the benchmark shows welfare losses with trade. In the current example, the entry effect plays very small role. Figure 1.2 (b) shows the decomposition without distortions. In this case, there is no misallocation and the resource allocation term remains zero. The entry effect becomes more negative with trade, which mitigates the increase in ACR. Overall, there is a welfare gain after trade.

The numerical example also shows using import share to infer welfare changes when distortions are present gives rise to markedly different results. Figure 1.2 (a) shows that ACR invariably predicts gains to trade, rather than losses. Figure 1.2 (b) shows that in the absence of distortions, ACR is a good approximation for welfare in the efficient case. But using ACR under distortions leads to a large departure: our benchmark results in this example predicts welfare losses rather than gains. Using aggregate observables to infer welfare gains as in ACR can thus be very misleading in the presence of distortions.

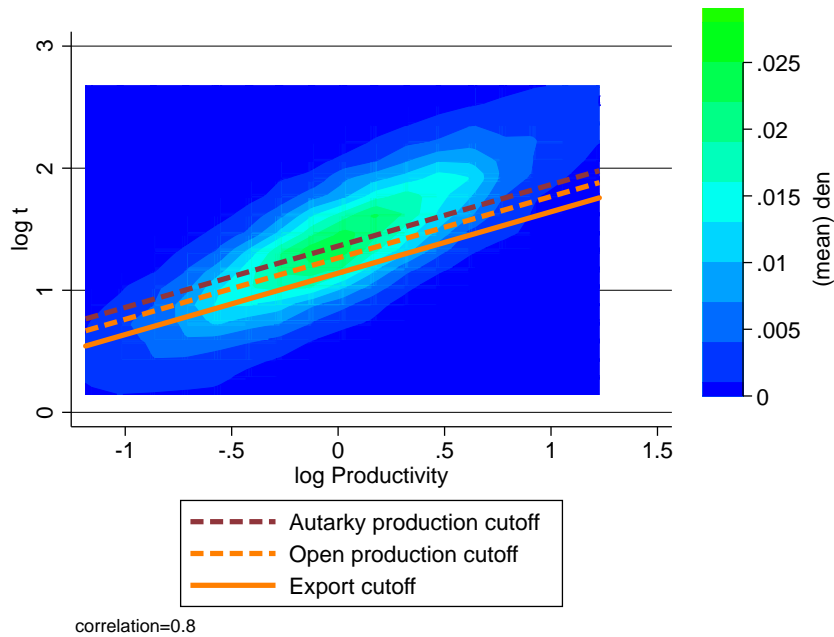
Figure 1.2: Welfare Decomposition



Third, we examine the precise mechanisms that explain how trade can reduce aggregate efficiency. In the same numerical example, Figure 1.3 illustrates how distortions affect firm selection. The density of firms is shown by a heat map of firms that lie along a positively sloped distortion-productivity line. It is clear that the productivity cutoff for production and exports is no longer determined solely by productivity, but also by domestic distortion. Only firms below the cutoff line can operate. In this figure, a large mass of highly-productive firms are excluded from servicing the market altogether. As the econ-

omy opens up, the cutoff line is shifted further downward. Even if firms have the same level of productivity, some with higher taxes may be displaced while those with lower ones will survive. This downward shift of the cutoffs allows for some low productivity and high subsidy firms to survive and gain market share.

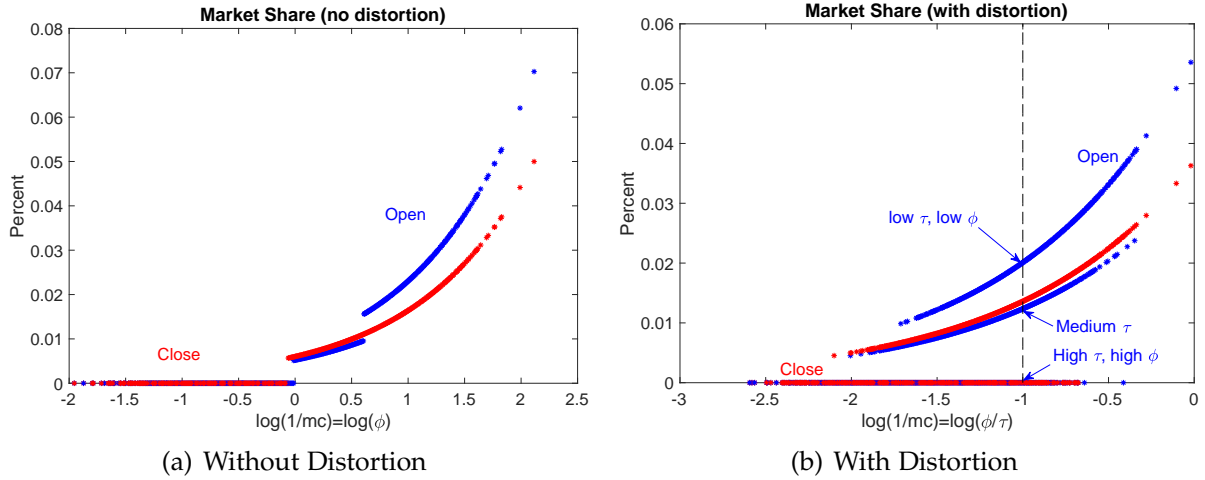
Figure 1.3: Selection Effects



Another way to show the impact on selection is to examine firms' market share. The two panels in Figure 1.4 plot the market share of firms, both in the closed and open economy. The left panel is the case without distortions. Firms with the same productivity level have the same marginal cost; their market share, above a cutoff productivity, rises with their productivity. Comparing the blue and red lines show that above the export cutoff, more productive firms have higher market shares in the open economy than in the closed economy, demonstrating that these firms expand under trade liberalization. This happens at the cost of displacing other less productive firms' market share, or driving them out of the market entirely. Here, the example clearly demonstrates that resources move from less productive to more productive firms as an economy opens up to trade.

The right panel shows the firm's market share in the case with distortions. Firms may share the same marginal cost and face the same potential revenues. However, their after-tax profits may differ, and thus their market share can also differ. Consider the point at which

Figure 1.4: Selection Effects



the (log) effective productivity level (ϕ/τ) is at -1 . At this point, a firm with high, medium and low level of productivity face the same marginal costs. However, the high productivity firm is also subject to high taxes and thus low after-tax profit, and does not make the cut for production. The medium-tax-medium-productivity firm has positive market share but loses out to the low-tax-low-productivity firm when the economy opens up. Resources are reallocated from the more productive to the less productive firms. Also, there is no longer a neat line up of market shares according to productivity: there is a wide range of productivities for which production is excluded.¹⁹

Distribution of Distortions. The distribution of distortions is an important determinant to the gains to trade. There are two key parameters: ρ , the correlation of τ and ϕ , and σ_τ , the dispersion of τ . The correlation of distortion and productivity is important insofar as a higher correlation means that more productive firms are more likely to be excluded from the market. But reductions in welfare is possible even when the correlation is negative. The reason is that for any given productivity, it is always the more subsidized firms can export and highly taxed firms exit, leading to a possible welfare loss. Figure 1.5 (a) illustrates this. It compares the gain from trade for $\rho = 0.8$, under our benchmark numerical example, and for $\rho = -0.8$, where productivity and distortion are highly negatively correlated and other parameters are the same as in the benchmark example. Under $\rho = -0.8$, the welfare gain

¹⁹This is also true if the distortions are input wedge on all the labor a firm uses. Firms face higher input wedge would have a lower profit in a market.

(loss) from trade is always larger (smaller) than that in the case of $\rho = 0.8$. But when the import share is below 20%, there are still losses from trade.

Figure 1.5: Gains/Loss from Trade

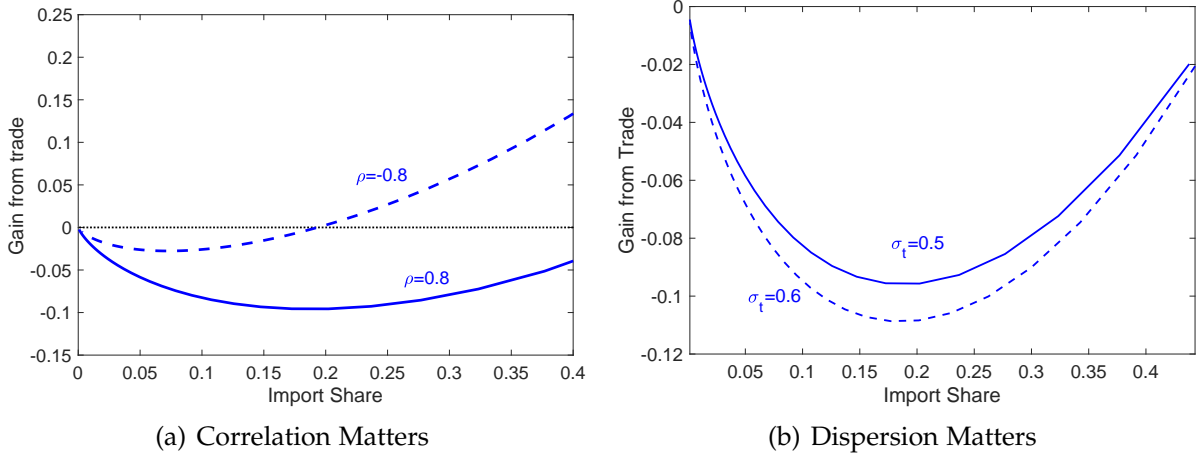


Figure 1.5 (b) compares the gain from trade under different σ_τ and other parameters are the same as in the benchmark example. The welfare gain (loss) from trade is always larger (smaller) when σ_τ is smaller.

In summary, the size of welfare loss after opening up depends on the correlation of φ and τ and the dispersion of τ . The firm level data helps us identify these parameters. Specifically, in the quantitative section, we will use the firm-level output and use its dispersion and its correlation with firm inputs to estimate ρ and σ_τ .

3 Empirical Results

We proceed to investigate whether key empirical implications of our model is supported by the data. Inferring firm-level distortion directly from the data has its challenges. Still, key properties and relationships of these measured wedges and productivity support core implications of our model. Though our focus is to estimate a structural model to infer welfare and productivity gains to trade, without directly using these empirical measures, the exercise that follows is of independent value, as well as providing a robustness check.

Data. Our data for Chinese firms are from an annual survey of manufacturing enterprises collected by the Chinese National Bureau of Statistics. The dataset includes non-state firms with sales over 5 million RMB (about 600,000 US dollars) and all of the state firms for the 1998-2007 period. We have information from the balance sheet, profit and loss statements, and cash flow statements, which incorporate more than 100 financial variables. The raw data consist of over 125,858 firms in 1998 and 306,298 firms by 2007.

Customary Measured Distortions. To back out factor and output distortions we adopt a Cobb-Douglas production function for a firm i in industry j , $y_{ji} = \varphi_{ji} k_{ji}^\alpha \ell_{ji}^{1-\alpha}$, where y_{ji} , ℓ_{ji} , k_{ji} represent the output, labor, and capital stock of firm i in sector j . The marginal revenue product of labor and capital is $\partial(p_{ji}y_{ji})/\partial(\ell_{ji})$ and $\partial(p_{ji}y_{ji})/\partial(k_{ji})$, and with firm profit maximization, yields

$$MRPL_{ji} \equiv \frac{\sigma-1}{\sigma}(1-\alpha_j) \frac{p_{ji}y_{ji}}{\ell_{ji}} = \tau_{ji}^\ell w_j$$

$$MRPK_{ji} \equiv \frac{\sigma-1}{\sigma} \alpha_j \frac{p_{ji}y_{ji}}{k_{ji}} = \tau_{ji}^k r_j,$$

where w_j and r_j denote industry-level wages and interest rates. These marginal products are proportional to the average products, assuming common markups and capital elasticities within the industry, and no fixed cost as in [Hsieh and Klenow \(2009\)](#) (HK). Firms equalize the after-tax marginal revenue products of factors. In the absence of distortions, revenue per person should be equalized across firms. In the presence of distortions, a firm that faces higher taxes will end up with a higher marginal revenue product and less capital/labor than an otherwise identical firm facing a subsidy. Equilibrium allocations yield

$$p_{ji}y_{ji} \propto \left[\frac{\varphi_{ji}}{(\tau_{ji}^k)^{\alpha_j} (\tau_{ji}^\ell)^{1-\alpha_j}} \right]^{\sigma-1},$$

from which firm-level productivity can be inferred as

$$\varphi_{ji} = \left(P_j^{\sigma-1} X_j \right)^{\frac{1}{1-\sigma}} \frac{(p_{ji}y_{ji})^{\frac{\sigma}{\sigma-1}}}{k_{ji}^{\alpha_j} \ell_{ji}^{1-\alpha_j}}, \quad (16)$$

which is also referred as $TFPQ$ as HK.

It is the relative marginal revenue and relative productivities—deviations from the industry mean—that matters. Thus, the measured relative marginal revenue product, or the relative average product ($ARPK_{ji}$), is calculated as $\log(p_{ji}y_{ij}/k_{ij}) - \log(\overline{ARPK_j})$ where $\overline{ARPK_j}$ is the industry mean of the average product. The same holds for the measured marginal revenue of labor. The elasticity of output with respect to capital in each industry is taken to be 1 minus the labor share in the corresponding industry in the U.S, following HK. The reason that labor shares are not computed from Chinese data is that the prevalence of distortions would affect these elasticities, and industry-level elasticities and distortions cannot be separately identified. The U.S. is taken to be the benchmark as the relatively undistorted economy. These labor share comes from the U.S. NBER productivity database, which is based on the Census and the Annual Survey of Manufactures (ASM). We take the benchmark elasticity of substitution parameter σ to be 3, but experiment with other values within the conventional range. Different from HK, we take a firm's employment to measure ℓ_{ji} rather than the firm's wage bill. This addresses the problem that Chinese wage data implies too low of a labor share as measured by input-output tables and the national accounts. We define the capital stock as the book value of fixed capital net of depreciation.

We find large dispersions in measured distortions in China, similar to the levels in HK for the year 1998 and 2007. Measured distortions have come down over time, between 1998 and 2007, as evident in Table 1. There is also greater dispersion in the average product of capital than there is in the average product of labor.

We next turn to investigating further what factors are systematically related to measured distortions. Table 2 reports the regression results of the relative average product of capital of a firm on a set of variables. The coefficient on firm-productivity is large and significant; 1 percent increase in relative productivity ($TFPQ$) is associated with a 0.7 percent increase in relative distortion. Moreover, more than half of the variation in distortions is explained by productivity alone. The positive relationship is consistent with the predictions of our model, as it predicts that firms that face higher tax have to be more productive to survive. The same is true for the results on exporters: given productivity, firms must have

lower taxes on average in order to export. What these wedges are and where they might come from is suggested by its systematic relationship with firm characteristics: state-owned enterprises and Foreign-owned firms are subject to lower taxes on average, given productivity. If we perform these regressions using the average product of labor instead, in Table 3, results are similar.

Table 1: Dispersion of Distortions

	1998	2001	2004	2007
std(ARPK)	1.348	1.306	1.241	1.185
std(ARPL)	1.184	1.039	0.940	0.923

Table 2: ARPK Regressions

VARIABLES	(1) ln(ARPK)	(2) ln(ARPK)	(3) ln(ARPK)	(4) ln(ARPK)	(5) ln(ARPK)	(6) ln(ARPK)
ln(TFPQ)	0.652*** (147.7)	0.697*** (153.0)	0.706*** (154.8)	0.705*** (153.9)	0.707*** (160.3)	0.711*** (168.1)
age				-0.00178*** (-8.772)	-0.00191*** (-9.477)	-0.00174*** (-9.386)
1.soe					-0.116*** (-3.388)	-0.109*** (-3.313)
1.foreignown					-0.460*** (-19.74)	-0.379*** (-20.60)
exporters						-0.233*** (-13.82)
Constant	-3.617*** (-134.6)	-3.280*** (-60.38)	-3.204*** (-54.16)	-3.173*** (-53.37)	-3.049*** (-44.45)	-3.042*** (-44.88)
Observations	1,616,507	1,616,507	1,506,572	1,505,657	1,505,657	1,505,657
R-squared	0.566	0.628	0.640	0.640	0.655	0.659
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	No	Yes	Yes	Yes	Yes	Yes
Location FE	No	No	Yes	Yes	Yes	Yes

Robust t-statistics clustered at the four-digit industry level in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Relationship between productivity and distortion. There is no apriori reason to believe that more productive firms are associated with higher wedges, as is shown in the regression

Table 3: ARPL Regressions

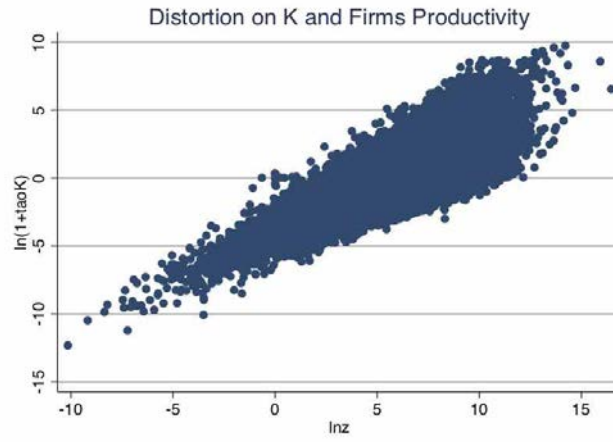
VARIABLES	(1) ln(<i>ARPL</i>)	(2) ln(<i>ARPL</i>)	(3) ln(<i>ARPL</i>)	(4) ln(<i>ARPL</i>)	(5) ln(<i>ARPL</i>)	(6) ln(<i>ARPL</i>)
ln(<i>TFPQ</i>)	0.530*** (110.7)	0.570*** (228.5)	0.569*** (222.5)	0.568*** (224.2)	0.565*** (228.4)	0.567*** (229.4)
age				-0.00161*** (-9.072)	-0.00140*** (-8.783)	-0.00128*** (-8.440)
1.soe					-0.0840*** (-7.136)	-0.0787*** (-7.057)
1.foreignown					0.0615*** (3.925)	0.123*** (8.317)
exporters						-0.175*** (-27.08)
Constant	-3.593*** (-123.2)	-3.274*** (-109.1)	-3.229*** (-103.2)	-3.201*** (-100.5)	-3.172*** (-95.80)	-3.167*** (-97.30)
Observations	1,616,507	1,616,507	1,506,572	1,505,657	1,505,657	1,505,657
R-squared	0.619	0.691	0.699	0.700	0.701	0.705
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	No	Yes	Yes	Yes	Yes	Yes
Location FE	No	No	Yes	Yes	Yes	Yes

Robust t-statistics clustered at the four-digit industry level in parentheses

*** p<0.01, ** p<0.05, * p<0.1

results. To show this graphically, we next plot the relationship between $TFPQ$ and $ARPK$ in Figure 1.6; a similar relationship holds for $TFPQ$ and $ARPL$. Though this relationship was not important in the special case of HK assuming a joint log normal distribution between the two variables—it does matter for more general cases. Moreover, it is vital for our quantitatively analysis.

Figure 1.6: Correlation Between Measured Distortion and Measured Productivity



Note: $\ln(1 + \tau K)$ denotes the logged measured distortion, $ARPK$, and $\ln z$ is the logged measured productivity, $TFPQ$.

The *observed* correlation between $TFPQ$ and $ARPK$ (or $ARPL$) cannot be treated to be the same as the *underlying* correlation between φ and τ for three reasons. First, the observed relationship may be an outcome of a selection process as our model demonstrates. Even if the underlying correlation is negative, the selection mechanism can induce the observed correlation to become positive, for the simple reason that high-taxed firms must be more productive in order to stay in the market. The selection mechanism will strengthen any underlying correlation between the two variables. In order to compute the impact of distortions on welfare and productivity gains, one would need to know the underlying correlation ρ , and therefore one would need micro data and a structural model to uncover it. What is also interesting about the relationship in the data is the line that seems to cut firms from above—reminiscent of the cutoff line endogenously generated in the model.

Costa-Scottini (2018) and Ho (2010) assume a perfect correlation of (log) productivity and (log) wedges in their analysis. We have shown why the assumption of an exogenous

correlation may be misplaced. Moreover, this assumption counters findings in the data in that 1) measured (log) productivity and (log) wedges are far from perfectly correlated, and 2) exporters have lower wedges. As long as these two variables are not perfectly correlated, selection will affect their measured correlation and one would need to estimate it from the model.

Second, the observed dispersion is not the underlying dispersion because it is among the operating firms. Lastly, with fixed cost we can only measure average revenue product. This measured revenue productivity does not map onto actual productivity or wedge as it is affected by productivity, fixed costs and distortions altogether.

For these reasons, we do not use directly the empirically-measured wedges, observed correlations, or distributions in the data to assess the impact of trade on welfare. Instead, we estimate the underlying joint distribution of distortions and productivity, costs of producing and exporting so as to match the observed patterns of firms' outputs, inputs, and exports. Then we evaluate how the presence of distortions change the impact of trade on productivity and welfare, and how much trade has contributed to Chinese growth in a decomposition exercise. We turn to this analysis in the subsequent section.

4 Quantitative Results

This section presents estimates of the quantitative effects of trade liberalization when taking into account the existence of domestic distortions. The two countries, Home and Foreign, are calibrated to data corresponding to China and the U.S.. The quantitative exercise is important for gauging welfare gains to trade because measured distortions following the conventional approach, though with independent value, has its limitations. Table 4 reports the calibrated and chosen parameters. The Home labor L is normalized to 1 and Foreign labor L_f to 0.2 to match the relative labor force of US to China. Productivity levels are set to match the relative GDP of US to China. Given that Foreign affects Home only through aggregate variables, we can assume that Foreign is absent of distortions, while taking $f_e, f, f_x, \tau_x, \sigma_\varphi$ to be the same as those in Home. We set the elasticity of substitution between varieties σ to be 3, the one taken in HK. This value is consistent with the estimates

from plant-level US manufacturing data in [Bernard, Eaton, Jensen, and Kortum \(2003\)](#).

Table 4: Model Parameters

<i>Parameter</i>	<i>Value</i>	<i>Identification</i>
Elasticity of substitution σ	3	
Home labor L	1	Normalization
Foreign Labor L_f	0.2	Relative labor size of US to China
<i>Internal Estimation</i>		
Entry cost f_e	0.2	Fraction of firms producing (one year survive rate in the data)
Fixed cost of producing f	0.015	Mean-lowest 5% $\ln(k^\alpha \ell^{1-\alpha})$
Fixed cost of export f_x	0.12	Fraction of firm exporting
Iceberg trade cost τ_x	1.5	Export intensity
Std. productivity σ_ϕ	1.2	Std of existing firms $\ln VA$
Std. distortion σ_τ	0.9	Std of existing firms $\ln(k^\alpha \ell^{1-\alpha})$
Corr(distortion, productivity) ρ	0.86	Corr($\ln VA, \ln(VA/k^\alpha \ell^{1-\alpha})$)
Mean Foreign prod $\mu_{f\phi}$	5.5	Relative GDP of U.S. to China
Note: VA denotes value added, k capital, ℓ employment, <i>Std</i> standard deviation, <i>Corr</i> correlations.		

Table 5: Data and Model Moments

Target Moments	Data(2005)	Model
Fraction of firms producing	0.85	0.85
Mean – lowest 5% for $\ln(k^\alpha \ell^{1-\alpha})$	1.82	1.53
Fraction of firm exporting	0.30	0.28
Export intensity	0.41	0.42
Std of existing firms $\ln(VA)$	1.20	1.26
Std of existing firms $\ln(VA/k^\alpha \ell^{1-\alpha})$	0.93	0.84
Corr($\ln VA, \ln(VA/k^\alpha \ell^{1-\alpha})$)	0.41	0.35
Relative real GDP of US to China	1.79	1.77

Note: VA denotes value added, k capital, ℓ employment, *Std* standard deviation, *Corr* correlations.

The remaining 8 parameters are estimated jointly, to match the model moments with their data counterparts. Table 4 and 5 reports the estimated parameters and the moments in the data and model. The moments we choose are the ones that are most relevant and sensitive to variations in model parameters. Clearly, every parameter matters for the general equilibrium and affects other moments. However, there is by and large a clear correspondance between certain parameters and moments. The parameter most relevant for matching the fraction of surviving firms is the entry cost f_e , as $\omega_e E[\pi(\phi, \tau)] = w f_e$. Lower

entry costs induces more entrants to pay the costs, and the result is a lower fraction of survivors. Next, to identify the fixed cost f , one needs only to turn to the smallest firms, which have their profit just about cover fixed cost. That is, the after-tax profit $\pi = wf$ and $w\ell_{min} = (\sigma - 1)wf$, and the mean of firms' labor $w\ell_{mean} = (\sigma - 1)w(f_e/\omega_e + f)$. Hence, the difference between mean and lowest 5% of input helps identify f .

We calibrate f_x and τ_x to match the export participation and intensity in Chinese manufacturing. The resulting parameter $\tau_x = 1.5$ is inline with the estimate of 1.7 in [Anderson and Van Wincoop \(2004\)](#), and the 1.83 in [Melitz and Redding \(2015\)](#). The dispersions in productivity and distortions, and their correlation are important for matching the observed joint distribution between value-added and inputs in the data. Table 5 shows that the discrepancy between our model and data moments is reasonably small, though we underestimate the dispersion in distortions and slightly overestimate the dispersion in size. An important variable is the correlation between value added and distortions, $Corr(\ln VA, \ln(VA/k^\alpha \ell^{1-\alpha}))$. This variable is more positive the higher is $\rho \frac{\sigma_\varphi}{\sigma_\tau}$, where ρ is the underlying correlation between distortions and productivity. A higher underlying correlation and a lower dispersion in distortions raise the observed correlation between value added and inputs.

4.1 Implied Gains from Trade and Loss in TFP

Table 6 reports the gains from trade and efficiency losses for both Home and Foreign. The upper panel compares welfare and TFP in the open economy to those in the closed economy. In the benchmark estimation, the gains from trade for Home is 4.4%. Without distortions, the gains from trade is more than doubled (9.8%). Foreign's gain from trade is about 8.2% when Home has domestic distortions. Eliminating Home distortions allows Foreign to benefit more—a 19% of welfare gain.

Note that Proposition 2 holds for asymmetric countries, so that we can decompose the welfare change according to its main equation, (Eq.15). In our benchmark, the import share is 30.8%, which implies that the change in domestic output share is $-d \ln \lambda = 0.368$, and the ACR term in the equation is 17.9%. But there is a large and negative reallocation term showing up in China, amounting to -18.2% . Taking only the ACR component would

Table 6: Welfare and TFP

	Open relative to close			Decomposition	
	Welfare	TFP	Import Share	ACR	Reallocation
<i>Home (%)</i>					
Benchmark	4.4	-2.9	30.8	17.9	-18.2
No-distortion	9.8	13.3	20.8	11.4	0
<i>Foreign(%)</i>					
Benchmark	8.2	12.9	17.9	9.0	0
No-distortion	18.9	13.3	35	19.1	0
TFP loss: Distortion relative to no-distortion					
	Overall loss	Misallocation	Entry-selection		
Benchmark	140.4	119.2	21.2		
Home Closed-Economy	124.2	118.7	5.4		

overestimate the gains to trade by 407%, according to our model. Moreover, if one were to ignore the presence of distortions and followed the usual approach to ACR using a trade elasticity– for instance of 4, estimated in [Simonovska and Waugh \(2014\)](#), then the resulting welfare gains would be 9.2% – more than doubling the gains that our model predicts. As [Melitz and Redding \(2015\)](#) show, trade elasticities vary with trade costs, so whether a partial, a full or an average theoretical trade elasticity is used will matter for the size of the ACR gain. But regardless, the ACR term is positive no matter which elasticity is utilized. What we show here is that our channel leads to a sizeable welfare loss.

We assume here that Foreign does not face any distortions, and thus the reallocation term for Foreign is zero according to our Proposition 2. Since Foreign has a smaller import share than Home, one would arrive at the conclusion that more gains would accrue to Home than to Foreign, according to ACR. But our benchmark model reveals the opposite: Foreign has gains to trade that is almost double that of Home, whereas using conventional ACR, Foreign’s gain is only half of that of Home. Without distortions, the gains to trade would double for both economies, suggesting that countries would gain more from trade by undertaking domestic reforms.

We also compare TFP before and after trade liberalization. To compute TFP, we construct the real GDP using producer price index following [Burstein and Cravino \(2015\)](#). Our benchmark results show that opening up leads to a 3% loss. In contrast, without distortions, TFP increases by 13.3%. Hence, contrary to the standard predictions, trade liberalization

can exacerbate rather than improve resource allocation, causing a decline rather than a rise in TFP. As Foreign has no distortions, the TFP levels are basically the same between the two models.

The lower panel of Table 6 reports Home's TFP losses due to distortions, both for a closed and open-economy case scenario. TFP loss is defined as the difference between efficient TFP when there are no distortions and that when there are distortions. Different from Hsieh and Klenow (2009), our efficient TFP considers endogenous entry. We therefore can decompose the deviation of TFP from its efficient level into a misallocation effect among a fixed set of operating firms, and a misallocation effect generated by entry and selection into producing and export:

$$\log TFP_{eff} - \log TFP = \underbrace{\log TFP_{FX} - \log TFP}_{\text{misallocation loss}} + \underbrace{\log TFP_{eff} - \log TFP_{FX}}_{\text{entry and selection loss}},$$

where TFP_{eff} pertains to the case without distortions, TFP_{FX} corresponds to the level in the case with distortions but where M , φ^* , and φ_x^* are fixed, in a closed economy.²⁰

Not surprisingly, there are large TFP losses for Home under domestic distortions. In the closed-economy case, eliminating these distortions would increase China's TFP by 124%. The TFP losses are larger, 140%, in the open economy benchmark model where China has an import share of more than 30%. Looking at its decomposition, the majority of the losses appears to come from misallocation among existing firms. One reason is that the share of surviving firms is high according to available data, but the survival rate could be overestimated as the dataset includes firms only of a certain scale. If smaller firms were observed, then it is possible that a lower survival rate would make the losses coming from the entry and selection margin larger.

²⁰ Here are the definitions of TFP_{eff} and TFP_{FX}

$$TFP_{eff} = \frac{\sigma - 1}{\sigma} \left[M^{eff} \int_{\varphi_{eff}^*}^{\infty} \varphi^{\sigma-1} \mu^{eff}(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, TFP_{FX} = \frac{\sigma - 1}{\sigma} \left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}.$$

4.2 Decomposing China's Growth from 1998-2005

The rapid growth in China over the last four decades has been one of the most remarkable phenomena the world has witnessed in recent history. In between 1998 and 2005, its real GDP increased by 57%. Accompanying this development was a combination of domestic reforms and opening up programs—policies that fostered trade and FDI inflows. As a result, both trade and technological progress increased over time, while measured domestic distortions concurrently fell. A natural question is how much of the growth is attributed to trade over this period. Other competing factors include technological improvement, factor accumulation, and domestic reforms—that is, the allocative gains associated with a reduction in distortions. In what follows, we perform a quantitative analysis to answer this question. Specifically, we recalibrate the model parameters for the year 1998 and compare the implied GDP and TFP levels to those in the benchmark year, 2005. Overall, our results attribute the majority of China's GDP and TFP growth to technological improvement, capital accumulation, and a mitigation of distortions. Trade alone contributes to only about 8% of GDP growth.

Table 7 reports the moments for both 1998 and 2005. The starting year is taken to be 1998, as it is the first year in which firm-level data is available, and three years before China joined the WTO. Compared to the year 2005, trade intensity was significantly lower in 1998, both in terms of the fraction of firms that export, and also the export intensity of these firms. Distortions were large in the earlier years, as seen by the fact that the dispersion of measured distortion was about 20% higher in 1998 compared to 2005. This implies a higher trade cost τ_x and dispersion of distortion σ_τ in 1998—at about 43% and 20% higher than the level in 2005. The mean TFP in 2005 is about 45% higher than that in 1998, reflecting technological improvements and factor accumulation over time.

These estimates are then used to run counterfactual experiments, in order to decompose China's growth in between 1998 and 2005. The factors considered include technological progress, input accumulation, and the reduction of trade costs and domestic distortions. In each experiment, the parameters for the year 1998 remain fixed, while each of the following parameters—mean TFP μ_φ , trade cost τ_x , or dispersion of distortion σ_τ —are allowed to vary to its 2005 level. Table 8 shows that the increase of technology and inputs alone

Table 7: Data, 1998 and 2005

Target Moments	Data (1998)	Data (2005)
Fraction of firms producing ω_e	0.77	0.85
Mean – lowest 5% for $\ln(K^\alpha L^{1-\alpha})$	2.04	1.82
Fraction of firm exporting	0.25	0.30
Export intensity	0.30	0.41
std of existing firms $\ln(VA)$	1.33	1.20
std of existing firms $\ln(VA/K^\alpha L^{1-\alpha})$	1.12	0.93
Corr($\ln VA$, $\ln(VA/K^\alpha L^{1-\alpha})$)	0.47	0.41
Relative real GDP of US to China	2.50	1.79
Change of China's real GDP		57%

lead to a 44% increase in GDP and a 46% increase in TFP. Reduction in trade costs would independently boost GDP by 8% and TFP by only 3%. In contrast, lowering the dispersion of distortions increases GDP by 66% and TFP by 69%.²¹

A notable point of comparison is with [Tombe and Zhu \(2019\)](#), which, despite adopting an altogether different approach, finds also small gains to trade. In their model that features migration across regions and sectors in China, international trade contributes to only 7% of productivity growth in between 2000 and 2005. In other words, international trade has led to very little allocative benefits of labor across regions and sectors—as compared to direct reforms that lower migration costs or reductions in internal trade costs. Their model does not feature distortions at the firm level that can render trade's allocative benefits even smaller. This leads us to find an even smaller effect of trade on productivity in China over roughly the same period.

Of course, a caveat is that trade may also help reduce domestic distortions. If, say, the WTO requires certain kind of domestic reforms as a pre-condition for entry, then some of the technological improvement and reductions in the level of distortions could be partially induced by opening up policies. We do not consider this here—but examine this issue in the following subsection. Also, this quantitative exercise of course also ignores other potential channels of gains to trade, such as pro-competition effect of trade, or potentially transfers of technology ([Ramondo and Rodríguez-Clare \(2013\)](#))—though these effects may

²¹Note that the contributions to GDP or TFP increase don't add up to 100% because the productivity distribution and fixed costs have also changed from 1998 to 2005. Furthermore, there are interacting effects on mean TFP, trade cost, and distortion dispersions.

still be quantitatively small. The point we make here is that in our benchmark framework, the contribution of trade pales in comparison to the contribution of domestic policies and technological progress in accounting for China’s growth experience. Part of this reason may be attributed to the lingering presence of distortions.

Table 8: Decomposition of China’s Growth between 1998-2005

	Change of Real GDP	Change of TFP
Benchmark	57%	56%
Counterfactual Change from 1998-2005:		
Technology and inputs alone (Increase mean φ)	44%	46%
Trade alone (Decrease τ_x)	8%	3%
Distortion alone (Decrease σ_τ)	66%	69%

4.3 Selection through Export: an Out of Sample Test and Extension

To examine whether Chinese firm characteristics change when they become exporters, we examine the relationship between measured distortions and firm export status in both the cross-section and the time series dimension. According to the model, given productivity—exporters face a lower distortion due to selection. This implication, as we have shown previously, is broadly consistent with the data. We then use the time series data to check whether ‘measured’ distortions change when firms enter the export market. Model implications are qualitatively consistent with the data. Finally, we consider model extensions with export rebate and allow for different distortions when exporting.

The first two columns of Table 9 reports the data and the model regressions of measured distortion on measured productivity $TFPQ$ and a dummy of exporters. Both the model and the data exhibit a pattern whereby exporters face a lower marginal product. Note that the differences between exporters and non-exporters were not targeted. There is a stronger selection effect in the model than in the data—exporters’ marginal product is about 64% lower than non-exporters in the model, compared to 26% in the data.

We further consider whether the measured marginal product or distortion vary with entering or exiting the Foreign market. This will help us to understand whether differences

between exporters and non-exporters stem from selection or additional/different distortions when exporting. Throughout the sample period, we sort exporters into three types: ‘always exporters’ are those who are exporting throughout the sample years 1998 to 2007, ‘starters’ are those who started to export after 1998, and ‘stoppers’, who stop exporting sometime in the interim years. Entry effect measures the percentage difference of measured distortions for starters, between the post- and pre-exporting entry periods. Exit effect measures percentage difference of measured distortion for stoppers, between the post- and pre-exporting exit periods.

As shown in Column 3 of Table 9, in the data, ‘always exporters’ have a lower marginal product. Firms’ measured marginal products decrease when they start exporting and increase when they exit. Note that although the firm-level distortions are exogenous in the model, the measured distortions do change after firms start exporting— due to the fixed costs of exporting.

In our benchmark model, if trade costs decreases from their 1998 level to their 2005 level, some firms would start to export, and their measured distortion would decrease—as shown in the entry-effect of Column 4. This is true in the data, as ‘always exporters’ and ‘starters’ have lower measured marginal product. Our benchmark model shows a relatively large selection effect since the model does not have as large heterogeneity as in the data, and a relative small entry effect. In reality, the change in a firm’s marginal product after exporting could be driven by multiple factors. For example, exporters face different distortions from non-exporters, or there are endogenous distortions that change with trade liberalization. We proceed to explore these possibilities.

In Column 5, we introduce a 10% tax rebate after exporting. This generates an even larger selection effect and the measured wedge further decrease upon exporting. In this case, export subsidies from tax rebate benefit the Foreign country, and Home welfare gains from trade decreases to 1.6% and TFP loss increases to 4.5%.

Column 6 considers an extension that firms face different distribution of distortion when exporting. We use the standard deviation of export intensity to discipline the dispersion of the additional wedges when exporting. To be more precise, we estimate the mean and standard deviation of the extra export distortions to match the standard deviation of export

intensity and the regression coefficient on always exporters. When the trade cost is reduced as in the benchmark, the coefficient on starter and the entry effect are similar to the data. Not surprisingly, the loss from trade is much larger in this extension than in the benchmark. The reason is because there are additional direct distortions on the selection of firms to Foreign market when open up to trade.

Table 9: Measured Marginal Products, data vs model

VARIABLES	(1) Data	(2) Benchmark	(3) Data	(4) Benchmark	(5) Export rebate	(6) Different τ in export
entry effect			-0.104*** (-12.69)	-0.050	-0.103	-0.09
exit effect			0.0315*** (4.574)			
starter			-0.101*** (-21.74)	-0.429	-0.400	-0.08
stopper			-0.0891*** (-20.98)			
always exporters			-0.301*** (-23.47)	-0.768	-0.791	-0.324
log(TFPQ)	0.636*** (250.5)	0.652	0.638*** (254.9)	0.653	0.654	0.613
exporters	-0.264*** (-24.14)	-0.637				
Constant	-3.258*** (-106.2)	0.401	-3.255*** (-107.0)	0.414	0.425	0.714
Observations	1,587,629		1,584,242			
R-squared	0.823		0.826			
Time FE	Yes		Yes			
Industry FE	Yes		Yes			

Note: Robust t-statistics in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The dependent variable is measured average product, which is $\log(TFPR)$ in the data and $\log(VA/(\ell + f))$ in the model.

5 Discussion

In this section, we explore a model with endogenous distortion arising from endogenous markup, which has been extensively studied in the standard trade literature. We show that the endogenous markup model runs counter with the data in that exporters in the model face a higher markup and distortion. We then address the issue that misallocation could be driven by mismeasurement of inputs or outputs. We show that even taking out the standard measurement errors, there are still large distortions remaining among Chinese firms.

5.1 Endogenous distortions

Here we build a model with endogenous markup as in [Edmond, Midrigan, and Xu \(2018\)](#). The consumer's problem is the same as before.

Final goods producer Final goods producers are competitive and produce with intermediate goods with a Kimball aggregator

$$\int_{\omega \in \Omega} \gamma \left(\frac{q}{Q} \right) d\omega = 1,$$

where $\gamma(\cdot)$ follows [Klenow and Willis \(2016\)](#) specification as

$$\gamma \left(\frac{q}{Q} \right) = 1 + (\sigma - 1) \exp \left(\frac{1}{\varepsilon} \right) \varepsilon^{\frac{\sigma}{\varepsilon} - 1} \left[\Gamma \left(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon} \right) - \Gamma \left(\frac{\sigma}{\varepsilon}, \frac{(q/Q)^{\frac{\varepsilon}{\sigma}}}{\varepsilon} \right) \right], \quad (17)$$

$\sigma > 1, \varepsilon \geq 0$ and $\Gamma(s, x)$ denotes the upper incomplete Gamma function $\Gamma(s, x) = \int_x^\infty t^{s-1} e^{-t} dt$. The demand function for each intermediate good producer is therefore given by

$$p(\omega) = \gamma' \left(\frac{q(\omega)}{Q} \right) PD, \quad (18)$$

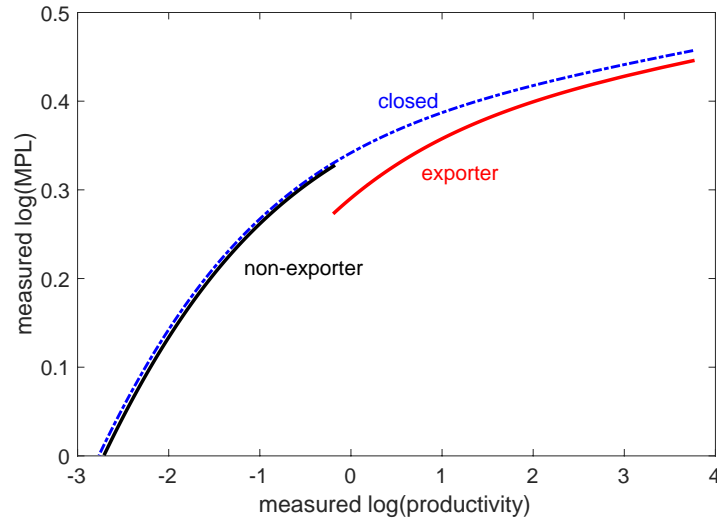
where D is a demand index, $D = \left[\int_{\omega \in \Omega} \gamma' \left(\frac{q(\omega)}{Q} \right) \frac{q(\omega)}{Q} d\omega \right]^{-1}$.

Intermediate good producer The problem of an intermediate good producer is similar as before except it faces a demand function as in equation (18). The firm will choose the price as a markup over the marginal cost,

$$p = \frac{\sigma}{\sigma - (q/Q)^{\frac{\varepsilon}{\sigma}}} \frac{w\tau}{\varphi}.$$

Note that the markup is endogenous and depends on the size of the firm, the higher the quantity a firm sells, the higher the markup it charges. The firm's optimal production and profit increase with φ and decrease with τ . Hence there exists a cutoff $\varphi^*(\tau)$, firms produce when $\varphi \geq \varphi^*(\tau)$.

Figure 1.7: Measured MPL and Productivity in an Endogenous Markup Model



To compare with the benchmark model, we choose ε as 0.08 to match the aggregate marginal product of labor of 1.45 as in [Edmond, Midrigan, and Xu \(2018\)](#), while keeping other parameters the same as in the benchmark. Figure 1.7 plots the relationship between the measured $\log(MPL)$ and the measured \log productivity in this model. First, higher productivity firms produce more and end up with a higher endogenous markup. The measured MPL is therefore higher. Hence, we observe an upward sloping line for the closed economy. Second, this upward sloping patterns also show up in the open economy. Moreover, exporters are more productive and face a higher wedge. Non-exporters face a more competitive market after opening up and charge a lower markup, the measured wedge is smaller. Around the exporting cutoff, exporters face a lower MPL due to the

fixed cost of exporting. Overall, exporters face higher measured *MPL*.

In summary, if the observed wedges are purely driven by markups and they endogenously change with trade, we should see that: 1) exporters on average have higher markups, hence higher—rather than lower—wedges; and given productivity, they should have the same wedge; 2) measured log (*MPL*) and log (*VA*) will be almost perfectly correlated. These implications are at odds with the regression results shown in section 4.3, where exporters face lower wedges. Thus, even in this extended model, similar exogenous distortions are needed to match the observed dispersion and correlation. This is consistent with Song and Wu (2015) and David and Venkateswaran (2017) that the heterogeneity in markup explains very limited MPK dispersion in China. Moreover, Arkolakis et al. (2018) show that the gains from trade in a model with endogenous markup is similar to ACR. In the current model, the excess welfare gain from trade when removing domestic distortions is 3.2%, while it is 5.4% (9.8%-4.4%) in the benchmark model.

5.2 Detecting measurement error

Differences in measured average products need not imply differences in true marginal products. The presence of fixed costs in producing and exporting in our model, for instance, means that the average revenue products differ from the marginal revenue products. For this reason, a model estimation is used to back out the true dispersion in marginal products. Nevertheless, other types of mismeasurements in output and input may also generate a dispersion in the average revenue products, and thereby affect the measured TFPR—as shown in Bils, Klenow, and Ruane (2017) and Song and Wu (2015). In this section, we address the issues surrounding measurement error following the practices adopted in the literature, and provide more robustness checks for the empirical results.

The main approach involves using panel data to improve the estimates on the true marginal product dispersion, rather than simply employing cross-sectional data. With this method, we find that the measurement errors are small in China, accounting for only 18% of the variation in the average product.²² This 18% includes the mismeasurement of pro-

²²Bils, Klenow, and Ruane (2017) finds measurement errors can explain about half of variation of average products in Indian, and about 80% of that in the U.S, but little for China.

duction inputs in the presence of fixed cost, which is accounted for in our benchmark.

We exploit three alternative methods to detect measurement error: average annual observations within firms, first differences over years within firms, and covariance between first differences and average products. All three approaches point to the same conclusion: that 1) there is a large dispersion in marginal products in China; 2) measurement error only accounts for a small fraction of the dispersion in the measured marginal products (i.e. average products).

First, if measurement error were idiosyncratic across firms and over time, one can take the time average of annual observations within firms to wash out these errors, drastically reducing the dispersion of average products. The upper panel of Table 10 reports the statistics when we take the average within firms. The average standard deviation is 1.19 for the average marginal product of capital and 0.96 for the average marginal product of labor. The standard deviations of value added and the average product of inputs are 1.19 and 0.94, where the correlation between the two variables is 0.4. These results mimic our benchmark moments. In particular, the dispersions of average products of inputs are still high. This implies that measurement errors of the iid type cannot explain the observed dispersions in the average products.

Second, as pointed out by Bils, Klenow, and Ruane (2017), the dispersion of first differences reflect the true distortion if marginal products are constant over time. Calculating the first differences of value added ΔVA , capital ΔK , and labor ΔL , and then taking the ratio $\Delta VA/\Delta K$ and $\Delta VA/\Delta L$ gives us an alternative measure of marginal products. The 1% tails of both ratios are trimmed, and the results are displayed in the middle panel of Table 10 for the year of 2001, 2004, and 2007. The dispersions are even higher than those in Table 1 for the standard measured marginal product of inputs using this measure.

Moreover, the alternative measured marginal products are highly correlated with our average products. Figure 1.8 plots the $\ln(\Delta VA/\Delta I)$ against the benchmark average product of input $\ln(VA/I)$ where I is the composite of inputs, $I = K^\alpha L^{1-\alpha}$, where each dot corresponds to one of 100 percentiles of $\ln(VA/I)$. The regression coefficient at the firm level is 0.72, see Table A-1. Note that without measurement errors, the two measures are perfectly correlated. For the case with only measurement error, the two measures have no

Table 10: Detecting Measurement Errors

Average annual observation within firm				
$\text{std}(\ln(\text{APK}))$	$\text{std}(\ln(\text{APL}))$	$\text{std}(\ln \text{VA})$	$\text{std}(\ln(\text{VA}/I))$	$\text{corr}(\ln \text{VA}, \ln(\text{VA}/I))$
1.19	0.96	1.19	0.94	0.4
First level differences				
	2001	2004	2007	
$\text{std}(\ln(\Delta \text{VA}/\Delta K))$	1.82	1.78	1.76	
$\text{std}(\ln(\Delta \text{VA}/\Delta L))$	1.68	1.60	1.61	
Regression				
	Ψ	$\Psi(1 - \lambda)$		
	0.53***	-0.0997***		
	(34.58)	(-20.65)		

Note: This table reports three ways to detect measurement errors.

The upper panel reports the average annual levels within firms.

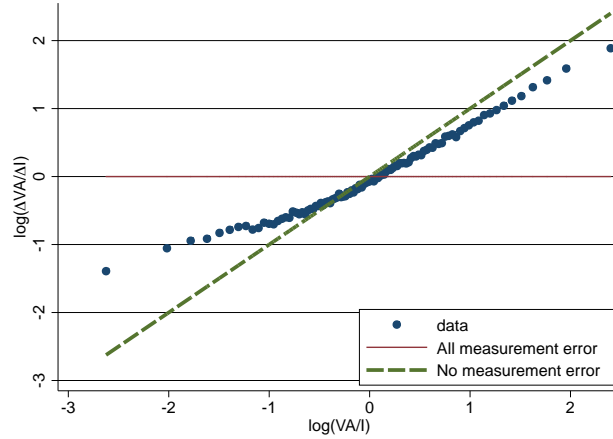
The middle panel reports the ratio of first differences as another measure of marginal product, where ΔVA denotes the first difference of value added.

The lower panel reports regression coefficient as in equation (19).

Robust t-statistics in parentheses.

correlation. Hence, the high correlation between the alternative measure and the average products suggest small measurement errors and a large distortion-induced misallocation.

Figure 1.8: Measured Marginal Product using First Differences vs TFPR



Lastly, we follow [Bils, Klenow, and Ruane \(2017\)](#) and run the following regression to further quantify the extent to which measured average products reflect true marginal products:

$$\widehat{\Delta \text{VA}}_i = \Phi \cdot \log(\text{TFPR}_i) + \Psi \cdot \Delta \hat{I}_i - \Psi(1 - \lambda) \cdot \log(\text{TFPR}_i) \cdot \Delta \hat{I}_i + D_s + \zeta_i \quad (19)$$

where $\Delta \widehat{VA}_i$ and $\Delta \hat{I}_i$ are the growth rate of measured value added and inputs respectively, and $\log(TFPR_i)$ is the measured average products. The underlying assumption here is that the measurement errors are additive. The variable of interest in the regression is λ , the variance of distortions relative to that of $TFPR$: $\lambda = \frac{\sigma_{\ln \tau}^2}{\sigma_{\ln(TFPR)}^2}$. The regression coefficient for Ψ is 0.53 and for the interaction of $\log(TFPR_i)$ and $\Delta \hat{I}_i$ is -0.0997. Both are significant, and the robust t-statistics are reported in Table 10. The implied λ is therefore 0.81. Hence, 81% of variation in $TFPR$ or average products is accounted for by distortion τ and 19% is due to measurement errors. The results are robust if we weight the observations with their share of aggregate value added or if we control for higher orders of $\ln(TFPR)$ to allow for stationary shocks to firms productivity and distortions. See the Appendix D for details.²³

In summary, the three alternative ways of sifting out measurement errors using panel data all point to the result that the dispersion in the average product of inputs are mainly driven by distortions rather than measurement error typically conceived.

6 Conclusion

This paper evaluates the impact of trade liberalization when the economy is subject to firm-level distortions. Given its prevalence and importance in developing countries, it is reasonable to ask how trade might affect welfare when these distortions are taken into account. This paper shows theoretically and quantitatively that opening an economy may in fact reduce allocative efficiency and exacerbate the misallocation of resources, by helping firms that are more subsidized (rather than those who are more productive) to expand. The findings in these papers, nevertheless, in no way disclaims the potential wide variety of sources and the magnitude of gains to trade beyond what is conventionally modelled and taken up in the current framework. But it does highlight that these losses could be sizeable

²³Bils, Klenow, and Ruane (2017) also considers the following extension to allow for stationary shocks to firms productivity and distortions:

$$\begin{aligned} \Delta \widehat{VA}_i = & \Phi \cdot \log(TFPR_i) + \Psi \cdot \Delta \hat{I}_i - \Psi(1 - \lambda) \cdot \log(TFPR_i) \\ & + \Gamma \cdot [\log(TFPR_i)]^2 + \Lambda(1 - \lambda) \cdot [\log(TFPR_i)]^2 \Delta \hat{I}_i \\ & + \Upsilon \cdot [\log(TFPR_i)]^3 + \Lambda(1 - \lambda) \cdot [\log(TFPR_i)]^3 \Delta \hat{I}_i. \end{aligned}$$

and comparable to major sources of welfare gains. We use Chinese manufacturing data in a period of the economy's rapid integration to demonstrate quantitatively that standard calculations for welfare may grossly overestimate the gains.

The paper serves as a first attempt to understand the interactions between trade and idiosyncratic firm level distortions on a theoretical level. The assumption of exogenous distortions is both reasonable empirically but also helpful in laying bare the key theoretical mechanisms on their interaction with trade. Of course, the next step would be to examine more thoroughly how trade might partially change the distribution and nature of these distortions. Extensions of the work are inherently numerous and promisingly fruitful. One can examine how distortions interact with other channels of gains to trade, such as innovation. One can also examine a dynamic model and the sequence of trade and domestic reforms. Our work joins the growing body of work and interest on why developing countries' experience with trade liberalization might have been so curiously diverse and uneven. Our work hopefully lends itself as one explanation to such a question.

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ONLINE APPENDIX TO “MISALLOCATION UNDER TRADE LIBERALIZATION”

BY YAN BAI, KEYU JIN, AND DAN LU

A Model Derivation

Closed Economy Equilibrium. In a closed economy, the free entry condition requires that the present value of producing equals the entry cost. The probability of entry ω_e is given by

$$\omega_e = \int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt, \quad (\text{A.1})$$

and the distribution of operating firms $\mu(\varphi, t)$

$$\mu(\varphi, t) = \frac{g(\varphi, t)}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt} = \frac{g(\varphi, t)}{\omega_e}$$

if $\varphi \geq \varphi^*(t)$; and 0 otherwise. Let the per-period expected profit conditional on producing be

$$E[\pi(\varphi, t)] = \int \int_{\varphi^*(t)}^{\infty} \pi(\varphi, t) \mu(\varphi, t) d\varphi dt.$$

The free entry condition is given by

$$\omega_e E[\pi(\varphi, t)] = wf_e. \quad (\text{A.2})$$

The optimal profit function (2) combined with the above expression in the free entry condition yields an equation for P ,

$$\frac{PQ}{\sigma} \left(P \frac{\sigma-1}{\sigma} \right)^{\sigma-1} w^{1-\sigma} \int \int_{\varphi^*(t)}^{\infty} \left[\varphi^{\sigma-1} t^{-\sigma} \right] g(\varphi, t) d\varphi dt - wf \int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt = wf_e. \quad (\text{A.3})$$

Let M_e be the measure of new entrant. The labor market clearing condition requires

$$L = ME \left[\frac{q}{\varphi} + f \right] + M_e f_e,$$

where the average labor demanded by firms $E \left[\frac{q}{\varphi} + f \right]$ is given by

$$E \left[\frac{q}{\varphi} + f \right] = \int \int_{\varphi^*(t)}^{\infty} \left[\frac{q}{\varphi} + f \right] \mu(\varphi, t) d\varphi dt.$$

In a stationary equilibrium, the number of entrants equals number of exits, such that $\omega_e M_e = M$ and

$$M = \frac{L}{\sigma \left(\frac{f_e}{\omega_e} + f \right)}. \quad (\text{A.4})$$

Open Economy Equilibrium. Optimal prices and cutoff functions are straightforward analogues of the closed economy case. The free entry condition for Home and Foreign implies

$$\begin{aligned} & \frac{PQ}{\sigma} \left(P \frac{\sigma-1}{\sigma} \right)^{\sigma-1} w^{1-\sigma} \int \int_{\varphi^*(t)}^{\infty} \left[\varphi^{\sigma-1} t^{-\sigma} \right] g(\varphi, t) d\varphi dt - w f \int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt \\ & + \left[\frac{P_f Q_f}{\sigma} \left(P_f \frac{\sigma-1}{\sigma} \right)^{\sigma-1} (\tau_x w)^{1-\sigma} \int \int_{\varphi_x^*(t)}^{\infty} \left[\varphi^{\sigma-1} t^{-\sigma} \right] g(\varphi, t) d\varphi dt - w f_x \int \int_{\varphi_x^*(t)}^{\infty} g(\varphi, t) d\varphi dt \right] \\ & = w f_e. \quad (\text{A.5}) \end{aligned}$$

$$\begin{aligned} & \frac{P_f Q_f}{\sigma} \left(P_f \frac{\sigma-1}{\sigma} \right)^{\sigma-1} w_f^{1-\sigma} \int \int_{\varphi_f^*(t)}^{\infty} \left[\varphi^{\sigma-1} t^{-\sigma} \right] g_f(\varphi, t) d\varphi dt - w_f f \int \int_{\varphi_f^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt \\ & + \left[\frac{PQ}{\sigma} \left(P \frac{\sigma-1}{\sigma} \right)^{\sigma-1} (\tau_x w_f)^{1-\sigma} \int \int_{\varphi_{xf}^*(t)}^{\infty} \varphi^{\sigma-1} t^{-\sigma} g_f(\varphi, t) d\varphi dt - w_f f_x \int \int_{\varphi_{xf}^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt \right] \\ & = w_f f_e. \quad (\text{A.6}) \end{aligned}$$

It follows that the aggregate prices of Home and Foreign are

$$\begin{aligned} P^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left[M w^{1-\sigma} \frac{\int \int_{\varphi^*(t)}^{\infty} \left(\frac{\varphi}{t} \right)^{\sigma-1} g(\varphi, t) d\varphi dt}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt} + M_f (\tau_x w_f)^{1-\sigma} \frac{\int \int_{\varphi_{xf}^*(t)}^{\infty} \left(\frac{\varphi}{t} \right)^{\sigma-1} g_f(\varphi, t) d\varphi dt}{\int \int_{\varphi_f^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt} \right] \\ P_f^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left[M_f w_f^{1-\sigma} \frac{\int \int_{\varphi_f^*(t)}^{\infty} \left(\frac{\varphi}{t} \right)^{\sigma-1} g_f(\varphi, t) d\varphi dt}{\int \int_{\varphi_f^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt} + M w^{1-\sigma} \tau_x^{1-\sigma} \frac{\int \int_{\varphi_{xf}^*(t)}^{\infty} \left(\frac{\varphi}{t} \right)^{\sigma-1} g(\varphi, t) d\varphi dt}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt} \right]. \end{aligned}$$

Next we derive the measure of operating firms at Home and Foreign M and M_f . Let M_e be the measure of new entrant, ω_e be the entry probability given by (A.1), and ω_x be the export probability conditional on entry

$$\omega_x = \int \int_{\varphi_x^*(t)}^{\infty} \mu(\varphi, t) d\varphi dt = \frac{\int \int_{\varphi_x^*(t)}^{\infty} g(\varphi, t) d\varphi dt}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt}.$$

The expected per-period profit includes the profit from both domestic production and international production,

$$\begin{aligned} & \int \int_{\varphi^*(t)} \pi(\varphi, t) \mu(\varphi, t) d\varphi dt + \int \int_{\varphi_x^*(t)} \pi_x(\varphi, t) \mu(\varphi, t) d\varphi dt \\ & \equiv E\pi + \omega_x E\pi_x \end{aligned}$$

where $\mu_x(\varphi, t) = \mu(\varphi, t)/\omega_x$. Using the average profits, free entry, stationary equilibrium condition, we have

$$M = \frac{L}{\sigma \left(\frac{f_e}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt} + f + \frac{\int \int_{\varphi_x^*(t)}^{\infty} g(\varphi, t) d\varphi dt}{\int \int_{\varphi^*(t)}^{\infty} g(\varphi, t) d\varphi dt} f_x \right)} \quad (\text{A.7})$$

and

$$M_f = \frac{L_f}{\sigma \left(\frac{f_e}{\int \int_{\varphi_f^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt} + f + \frac{\int \int_{\varphi_{xf}^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt}{\int \int_{\varphi_f^*(t)}^{\infty} g_f(\varphi, t) d\varphi dt} f_x \right)}. \quad (\text{A.8})$$

B Proofs

B.1 Proof for Proposition 2

To derive Proposition 2, we use the following equilibrium equations,

(a) Free entry condition:

$$\begin{aligned} & \frac{1}{\sigma} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} w^{1-\sigma} \left[P^\sigma Q \int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau + P_f^\sigma Q_f \tau_x^{1-\sigma} \int \int_{\varphi_x^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau \right] \\ & = w f_e + w f H + w f_x H_x \end{aligned}$$

(b) The labor market clearing condition:

$$M_e = \frac{L}{\sigma (f_e + f + H_x f_x)}.$$

(c) Price index:

$$P^{1-\sigma} = con_p \times \left[M_e \int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{1-\sigma} g(\varphi, \tau) d\varphi d\tau + \frac{P_f^\sigma Q_f}{P^\sigma Q} M_e \tau_x^{1-\sigma} \int \int_{\varphi_x^*(\tau)} \varphi^{\sigma-1} \tau^{1-\sigma} g(\varphi, \tau) d\varphi d\tau \right]$$

where con_p is a constant that depends on the model parameters.

(d) Cutoff of producing:

$$\varphi^* = con_v \times P^{-1} (PQ)^{\frac{1}{1-\sigma}} \tau^{\frac{\sigma}{\sigma-1}}$$

where con_v is a constant that depends on the model parameters.

(e) Definition of domestic share λ :

$$\frac{1-\lambda}{\lambda} = \frac{\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \left[\int \int_{\varphi_x^*} \varphi^{\sigma-1} \tau^{1-\sigma} g(\varphi, \tau) d\varphi d\tau \right]}{\left[\int \int_{\varphi^*} \varphi^{\sigma-1} \tau^{1-\sigma} g(\varphi, \tau) d\varphi d\tau \right]}$$

(f) Definition of the share of variable labor used in producing domestic goods S :

$$\frac{1-S}{S} = \frac{\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \int \int_{\varphi_x^*(t)} \varphi^{\sigma-1} t^{-\sigma} g(\varphi, t) d\varphi dt}{\int \int_{\varphi^*(t)} \varphi^{\sigma-1} t^{-\sigma} g(\varphi, t) d\varphi dt}$$

Differentiating the above system of equations and using the relationship $d \ln W = d \ln(PQ) - d \ln P$, we get Proposition 2 with the following definitions:

$$\gamma_d = \frac{\int (\varphi^*(\tau))^{\sigma-1} \tau^{1-\sigma} g(\varphi^*(\tau), \tau) \varphi^*(\tau) d\tau}{\int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{1-\sigma} g(\varphi, \tau) d\varphi d\tau}$$

$$\gamma_s = \frac{\int (\varphi^*(\tau))^{\sigma-1} \tau^{-\sigma} g(\varphi^*(\tau), \tau) \varphi^*(\tau) d\tau}{\int \int_{\varphi^*(\tau)} \varphi^{\sigma-1} \tau^{-\sigma} g(\varphi, \tau) d\varphi d\tau}.$$

With homogenous productivity and if domestic distortion $1/\tau$ follows a Pareto distribution with parameter θ , $\gamma_d = \frac{\sigma-1}{\sigma}(\theta - \sigma + 1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta - \sigma)$, hence $d \ln W = \frac{\sigma}{\sigma-1} [d \ln S - d \ln \lambda]$. Changes in the share of expenditure on domestic goods and the share of variable labor used in producing domestic goods are sufficient statistics for welfare change.

B.2 Proof for Lemma 1

Under the special case, $\gamma_d = \frac{\sigma-1}{\sigma}(\theta - \sigma + 1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta - \sigma)$, and the change of welfare becomes $d \ln W = \frac{\sigma}{\sigma-1} [d \ln S - d \ln \lambda]$.

Welfare change from close to open:

$$\lambda = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \left(\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P^\sigma Q}{P_f^\sigma Q_f} \right)^{\frac{\sigma-\gamma-1}{\sigma}} + 1 \right]^{-1}$$

$$S = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \left(\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P^\sigma Q}{P_f^\sigma Q_f} \right)^{\frac{\sigma-\gamma}{\sigma}} + 1 \right]^{-1}$$

as long as there is selection to export, $\lambda > S$, and $d \ln S$ is more negative than $d \ln \lambda$ from closed to open. The open economy has an unambiguously lower welfare than in the closed economy. Because with reallocation purely driven by distortions, in the open economy, the input share used to produce for exports exceeds the export share.

The reallocation term is always negative: Substitute in γ_d and γ_s , the last term, the

reallocation becomes $(1 - \theta)d \ln \lambda + \theta d \ln S$. Furthermore,

$$\begin{aligned}
d \ln \lambda &= -(1 - \lambda) \left(1 - \sigma - \frac{\sigma - 1}{\sigma}(\theta - \sigma + 1) \right) d \ln \tau_x + \frac{(1 - \lambda)(1 - \sigma - \gamma_x)}{\sigma - 1} d \ln(P_f^\sigma Q_f / P^\sigma Q) \\
&= (1 - \lambda) \frac{\theta + 1}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f} \\
d \ln S &= -(1 - S) \left(1 - \sigma - \frac{\sigma - 1}{\sigma}(\theta - \sigma) \right) d \ln \tau_x + \frac{(1 - S)(1 - \sigma - \gamma_{sx})}{\sigma - 1} d \ln(P_f^\sigma Q_f / P^\sigma Q) \\
&= (1 - S) \frac{\theta}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f}
\end{aligned}$$

Substitute in $d \ln \lambda$ and $d \ln S$, the reallocation term

$$(1 - \theta)d \ln \lambda + \theta d \ln S = \frac{\theta^2(1 - S) - (\theta^2 - 1)(1 - \lambda)}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f}$$

$\theta^2(1 - S) - (\theta^2 - 1)(1 - \lambda) > 0$, hence as long as the trade cost reduction induces larger fraction of exporters, the reallocation term is always negative. Q.E.D.

B.3 Proof for Lemma 2

Recall the producing cutoff is given by $\phi^*(c, \tau) = c\tau^{\frac{\sigma}{\sigma-1}}$ where $c = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left[\frac{wf}{P^\sigma Q} \right]^{\frac{1}{\sigma-1}} w$. Define two functions $F(c)$ and $H(c)$ where F relates to the cumulative labor share, and H relates to the cumulative output share,

$$F(c) = \int \int_{\phi^*(c, \tau)} \phi^\sigma \tau^{-\sigma} g(\phi, \tau) d\phi d\tau$$

$$H(c) = \int \int_{\phi^*(c, \tau)} \phi^\sigma \tau^{1-\sigma} g(\phi, \tau) d\phi d\tau.$$

Let $f(c) = F'(c)$ and $h(c) = H'(c)$. The hazard functions of γ_d and γ_s are

$$\gamma_d = \frac{h(c)}{1 - H(c)}, \quad \gamma_s = \frac{f(c)}{1 - F(c)}.$$

When f/h increases with c , i.e. f is likelihood ratio dominates h , then $\gamma_d \geq \gamma_s$. Let $x = \log \varphi, y = \log \tau$, then $x = c + \frac{\sigma}{\sigma-1}y$. Under joint-normal distribution of (x, y) ,

$$V(c) \equiv \frac{f(c)}{h(c)} = \frac{\int \exp(\sigma x^* - \sigma y) g(\varphi^*(c, \tau), y) dy}{\int \exp(\sigma x^* + (1 - \sigma)y) g(\varphi^*(c, \tau), y) dy}$$

where

$$g(x, y) = \exp \left[-\frac{1}{2(1 - \rho^2)} \left(\frac{x^2}{\sigma_\varphi^2} + \frac{y^2}{\sigma_\tau^2} - \frac{2\rho xy}{\sigma_\varphi \sigma_\tau} \right) \right].$$

When $\rho \frac{\sigma_\varphi}{\sigma_\tau} \leq \frac{\sigma}{\sigma-1}$, $V'(c) \geq 0$. Q.E.D.

C Equilibrium under Endogenous Markup

A closed-economy equilibrium consists of aggregate (P, Q, M) that satisfy:

$$\begin{aligned} M &= \frac{\omega_e L}{Q \int_0^{\sigma^{\sigma/\varepsilon}} \int^{\hat{\tau}(\hat{q})} \left[\frac{\sigma}{\sigma - \hat{q}^{\frac{\varepsilon}{\sigma}} \varphi} \right] g(\varphi(\tau, \hat{q}), \tau) \frac{d\varphi(\tau, \hat{q})}{d\tau} d\tau d\hat{q}} \\ Q \int_0^{\sigma^{\sigma/\varepsilon}} \int^{\hat{\tau}(\hat{q})} \left[\frac{\hat{q}^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}^{\frac{\varepsilon}{\sigma}} \varphi} \right] g(\varphi(\tau, \hat{q}), \tau) \frac{d\varphi(\tau, \hat{q})}{d\tau} d\tau d\hat{q} &= \omega_e f + f_e \\ \frac{M}{\omega_e} \int_0^{\sigma^{\sigma/\varepsilon}} \int^{\hat{\tau}(\hat{q})} \gamma(\hat{q}) g(\varphi(\tau, \hat{q}), \tau) \frac{d\varphi(\tau, \hat{q})}{d\tau} d\tau d\hat{q} &= 1, \end{aligned}$$

where

$$\omega_e = \int_0^{\sigma^{\sigma/\varepsilon}} \int^{\hat{\tau}(\hat{q})} g(\varphi(\tau, \hat{q}), \tau) d\tau d\hat{q}$$

and

$$\gamma' \left(\frac{q}{Q} \right) = \frac{\sigma - 1}{\sigma} \exp \left(\frac{1 - (q/Q)^{\frac{\varepsilon}{\sigma}}}{\varepsilon} \right).$$

The open equilibrium consists of unknowns $(P, Q, M, P_f, Q_f, M_f, w_f)$ that satisfy:

$$\frac{\sigma}{\sigma - \hat{q}_x^{\frac{\varepsilon}{\sigma}}} \frac{w \tau_x \tau}{\varphi} = \gamma'(\hat{q}_x) P_f D_f$$

$$\pi_x = \left[\frac{\hat{q}_x^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}_x^{\frac{\varepsilon}{\sigma}}} \frac{\tau_x \hat{q}_x}{\varphi} Q_f - f_x \right] w,$$

where we get the zero profit cutoff. The free entry condition becomes:

$$\begin{aligned} & \int \int_{\varphi^*(\tau)} \left[\frac{\hat{q}^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}^{\frac{\varepsilon}{\sigma}}} \frac{\hat{q}}{\varphi} Q - f \right] g(\varphi, \tau) d\tau d\varphi \\ & + \int \int_{\varphi_x^*(\tau)} \left[\frac{\hat{q}_x^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}_x^{\frac{\varepsilon}{\sigma}}} \frac{\tau_x \hat{q}_x}{\varphi} Q_f - f_x \right] g(\varphi_x, \tau) d\tau d\varphi = f_e \end{aligned} \quad (\text{A.9})$$

The labor market clearing condition is:

$$M = \frac{\omega_e L}{\int \int_{\varphi^*(\tau)} \left(\frac{\sigma}{\sigma - \hat{q}^{\frac{\varepsilon}{\sigma}}} \frac{\hat{q}}{\varphi} Q \right) g(\varphi, \tau) d\tau d\varphi + \int \int_{\varphi_x^*(\tau)} \left(\frac{\sigma}{\sigma - \hat{q}_x^{\frac{\varepsilon}{\sigma}}} \frac{\tau_x \hat{q}_x}{\varphi} Q_f \right) g(\varphi, \tau) d\tau d\varphi} \quad (\text{A.10})$$

$$\left[\frac{M}{\omega_e} \int \int_{\varphi^*(\tau)} \gamma(\hat{q}) g(\varphi, \tau) d\tau d\hat{q} + \frac{M_f}{\omega_{ef}} \int \int_{\varphi_{xf}^*(\tau)} \gamma(\hat{q}_{xf}) g_f(\varphi, \tau) d\tau d\varphi \right] = 1 \quad (\text{A.11})$$

For Foreign,

$$\begin{aligned} & \int \int_{\varphi_f^*(\tau)} \left[\frac{\hat{q}_f^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}_f^{\frac{\varepsilon}{\sigma}}} \frac{\hat{q}_f}{\varphi} Q_f - f \right] g_f(\varphi, \tau) d\tau d\varphi \\ & + \int \int_{\varphi_{xf}^*(\tau)} \left[\frac{\hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}}{\sigma - \hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}} \frac{\tau_x \hat{q}_{xf}}{\varphi} Q - f_x \right] g_f(\varphi, \tau) d\tau d\varphi = f_{ef} \end{aligned} \quad (\text{A.12})$$

$$M_f = \frac{\omega_e L_f}{\int \int_{\varphi_f(\tau)} \left(\frac{\sigma}{\sigma - \hat{q}_f^{\frac{\varepsilon}{\sigma}}} \frac{\hat{q}_f}{\varphi} Q_f \right) g_f(\varphi, \tau) d\tau d\varphi + \int \int_{\varphi_{xf}^*(\hat{q})} \left(\frac{\sigma}{\sigma - \hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}} \frac{\tau_x \hat{q}_{xf}}{\varphi} Q \right) g_f(\varphi, \tau) d\tau d\hat{q}} \quad (\text{A.13})$$

$$\left[\frac{M_f}{\omega_{ef}} \int \int_{\varphi_f^*(\tau)} \gamma(\hat{q}_f) g_f(\varphi, \tau) d\tau d\hat{q} + \frac{M}{\omega_e} \int \int_{\varphi_x^*(\tau)} \gamma(\hat{q}_x) g(\varphi, \tau) d\tau d\varphi \right] = 1 \quad (\text{A.14})$$

Finally, the goods market clearing condition is:

$$\frac{M}{\omega_e} \int \int_{\varphi_x^*(\tau)} \left[\frac{\sigma}{\sigma - \hat{q}_{\sigma}^{\frac{\varepsilon}{\sigma}}} \frac{w \hat{q}}{\varphi} Q_f \right] g(\varphi, \tau) d\tau d\varphi = \frac{M_f}{\omega_{ef}} \int \int_{\varphi_{xf}^*(\tau)} \left[\frac{\sigma}{\sigma - \hat{q}_{xf}^{\frac{\varepsilon}{\sigma}}} \frac{w_f \hat{q}_{xf}}{\varphi} Q \right] g_f(\varphi, \tau) d\tau d\varphi \quad (\text{A.15})$$

D Regressions for measurement errors

Table A-1: Measured Marginal Products using First Differences vs TFPR

VARIABLES	(1) $\log(\frac{\Delta VA}{\Delta I})$	(2) $\log(\frac{\Delta VA}{\Delta I})$	(3) $\log(\frac{\Delta VA}{\Delta I})$
$\log(TFPR)$	0.718*** (135.3)	0.715*** (158.6)	0.718*** (135.3)
Constant	1.410*** (78.31)	0.331*** (17.49)	1.410*** (78.31)
Observations	624,659	624,699	624,659
R-squared	0.173	0.269	0.173
Time FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes

Robust t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Specification (2) weights all the observations with the absolute value of composite input growth.

Specification (3) weights all the observations with the share of aggregate value added.

Table A-2: Estimate Measurement Error

VARIABLES	(1) $\Delta \widehat{VA}$	(2) $\Delta \widehat{VA}$	(3) $\Delta \widehat{VA}$
$\log(TFPR)$	0.0376*** (22.62)	0.0144*** (9.170)	0.0616*** (16.07)
$[\log(TFPR)]^2$			-0.0128*** (-6.110)
$[\log(TFPR)]^3$			0.00152*** (4.008)
$\Delta \widehat{input}$	0.530*** (34.58)	0.523*** (33.03)	0.524*** (31.13)
$\log(TFPR) \times \Delta \widehat{input}$	-0.0997*** (-20.65)	-0.0954*** (-19.16)	-0.0893*** (-6.420)
$[\log(TFPR)]^2 \times \Delta \widehat{input}$			-0.00611 (-0.919)
$[\log(TFPR)]^3 \times \Delta \widehat{input}$			0.00108 (1.040)
Constant	-0.0207*** (-3.125)	0.0551*** (8.231)	-0.0241*** (-3.592)
Observations	1,106,982	1,106,914	1,106,982
R-squared	0.044	0.042	0.044
Time FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes

Robust t-statistics in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Specification (2) weights all the observations with the share of aggregate value added.