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NETWORKS, BARRIERS, AND TRADE

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ABSTRACT

We study a non-parametric class of neoclassical trade models with global production networks and arbitrary distortions. We characterize their properties in terms of sufficient statistics useful for growth and welfare accounting as well as for counterfactuals. Using these sufficient statistics, we characterize societal losses from increases in tariffs and iceberg trade costs, and highlight the qualitative and quantitative importance of accounting for intermediates. Finally, we establish a formal duality between open and closed economies and use this to analytically quantify the gains from trade. Our results, which can be used to compute local and global counterfactuals, provide an analytical toolbox for studying large-scale trade models. Therefore, this paper helps bridge the gap between computation and theory.

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1 Introduction

Trade economists increasingly recognize the importance of using large-scale computational general equilibrium models for studying trade policy questions. One of the major downsides of relying on purely computational methods is their opacity: computational models can be black boxes, and it may be hard to know which forces in the model drive specific results. On the other hand, simple stylized models, while transparent and parsimonious, can lead to unreliable quantitative predictions when compared to the large-scale models.

This paper attempts to provide a theoretical map of territory usually explored by machines. It studies output and welfare in open economies with disaggregated and interconnected production structures and heterogeneous consumers. We address two types of questions: (i) how to measure and decompose, à la Solow (1957), the sources of output and welfare changes using ex-post sufficient statistics, and (ii) how to predict the responses of output, welfare, as well as disaggregated prices and quantities, to changes in trade costs or tariffs using ex-ante sufficient statistics. Our analysis is non-parametric and fairly general, which helps us to isolate the common forces and sufficient statistics necessary to answer these questions without committing to a specific parametric set up. We show how accounting for the details of the production structure can theoretically and quantitatively change answers to a broad range of questions in open-economy settings.

In analyzing the structure of open-economy general equilibrium models, we emphasize their similarities and differences to the closed-economy models used to study growth and fluctuations. To fix ideas, consider the following fundamental theorem of closed economies. For a perfectly-competitive economy with a representative household and inelastically supplied factors,

$$\frac{\mathrm{d}\log W}{\mathrm{d}\log A_i} = \frac{\mathrm{d}\log Y}{\mathrm{d}\log A_i} = \frac{sales_i}{GDP}, \tag{1}$$

where W is real income or welfare (measured by equivalent variation), Y is real output or GDP, and A_i is a Hicks-neutral shock to some producer i. Equation (1), also known as Hulten's Theorem, shows that the sales share of producer i is a sufficient statistic for the impact of a shock on aggregate welfare, aggregate income, and aggregate output to a first order. Specifically, Hulten's theorem implies that, to a first order, any disaggregated information beyond the sales share (the input-output network, the number of factors, the degrees of returns to scale, and the elasticities of substitution) is macroeconomically irrelevant.

In this paper, we examine the extent to which the logic of (1) can be transported into international economics. We provide the open-economy analogues of equation (1), and show that although versions of Hulten's theorem continue to hold in open-economies, the sales shares are no longer such universal sufficient statistics. Ultimately, there are two main barriers to naively applying Hulten's theorem in an open-economy: first, in an open-economy, output and welfare are no longer the same since welfare depends on terms-of-trade but output does not (see e.g. Burstein and Cravino, 2015); second, much of trade policy concerns the effects of tariffs, which knocks out the foundation of marginal cost pricing and Pareto efficiency that Hulten's Theorem is built on. Our generalizations make clear precisely the conditions under which a naive-application of (1) to an open-economy is valid. Even when not directly applicable, it proves helpful to think in terms of (1), and deviations from it.

Our framework allows for arbitrary distorting wedges (like tariffs or markups) in the initial equilibrium, and we derive comparative statics with respect to both wedges (like tariffs) and technologies (like iceberg costs of trade) in terms of model primitives. When the initial allocation is Pareto-efficient, because of the first welfare theorem, changes in wedges have no first-order effect on real GDP and world welfare. In this case, we can instead provide second-order approximations. We show that welfare losses to the world as a whole, and to the output of each country, from the imposition of tariffs or other distortions is approximately equal to a Domar-weighted sum of Harberger triangles. This result holds even in the absence of implausible compensating transfers, and we provide explicit formulas for what these Harberger triangles are equal to in terms of microeconomic primitives. We explain how to adjust these formulas to obtain welfare losses. We show that the existence of global value chains dramatically increases the costs of protectionism by inflating both the area of each triangle *and* the Domar weight used to aggregate the triangles. Simple (non-input-output) models, regardless of how they are calibrated, get either the area of the triangles or their weight wrong.

Our comparative static results generalize the local *hat-algebra* of Jones (1965) beyond frictionless $2 \times 2 \times 2$ no input-output economies. These local results can also be numerically integrated to arrive at exact global comparative statics. This provides an alternative to *exact* hat-algebra (e.g. Dekle et al., 2008) common in the literature. Whereas exact hat-algebra requires solving a large nonlinear system of equations once, this differential approach requires solving a smaller linear system repeatedly. Computationally, for large and highly nonlinear models, this differential equation approach is significantly faster.[1]

Finally, we analytically characterize the gains from trade by considering how welfare changes as a country moves towards autarky. To do so, we show that under some conditions, there exists a useful isomorphism between open and closed economies. In particular, for any open-economy with nested-CES import demand there exists a companion (dual) closed economy, and the welfare effects of trade shocks in the open-economy are equal to

[1] We also provide flexible Matlab code for performing these loglinearizations and numerically integrating the results.

the output effects of productivity shocks in the closed economy. Hence, we can use results from the closed-economy literature, principally Hulten (1978) and Baqaee and Farhi (2017a), to characterize the effects of trade shocks on welfare. Our formulas provide a generalization of some of the influential insights of Arkolakis et al. (2012) to environments with disaggregated, non-loglinear (non-Cobb-Douglas) input-output connections. Compared to the loglinear (Cobb-Douglas) production networks common in the literature (e.g. Costinot and Rodriguez-Clare, 2014; Caliendo and Parro, 2015), we find that accounting for nonlinear production networks significantly raises the gains from trade. Accounting for nonlinear input-output networks is as, or more important, as accounting for intermediates in the first place. For example, for the US, the gains from trade increase from 4.5% to 9% once we account for intermediates with a loglinear network, but they increase further to 13% once we account for realistic complementarities in production. The numbers are even more dramatic for more open economies, for example, the gains from trade for Mexico go from 11% in the model without intermediates, to 16% in the model with a loglinear network, to 44.5% in the model with a non-loglinear network.

In Section 7, we present a series of worked-out analytical examples. These examples show how our general results can be applied to study a range of different applied questions, like Dutch disease, the incidence of tariffs on different factors, how global value chains can amplify the losses from protectionism, and how the presence of universal intermediate inputs, like foreign energy, amplify the welfare loss of moving towards autarky.

The outline of the paper is as follows. In Section 2, we set up the model and define the objects of interest. In Section 3, we derive some first-order growth-accounting results useful for measurement and decompositions. In Section 4, we derive first-order comparative statics in terms of microeconomic primitives, useful for prediction. In Section 5, we apply the results in Section 4 to approximate societal losses from tariffs and other wedges to the second order. In Section 6, we establish a (global) dual relationship between closed and open economies and use it to study the gains from trade. Section 7 contains analytical examples. Section 8 contains quantitative examples, calibrated using nested-CES functional forms, to show the sorts of questions our results can be used to answer and to check the accuracy of our local approximations. Section 9 concludes.

Related Literature. This paper is related to three literatures: the literature on the gains from trade, the literature on production networks, and the literature on growth accounting. We discuss each literature in turn starting with the one on the gains (or losses) from trade. Our results generalize some of the results in Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2014) to environments with non-linear input-output connections. Our framework generalizes the input-output models emphasized in Caliendo and Parro (2015), Caliendo et al. (2017), Morrow and Trefler (2017), Fally and Sayre (2018), and Bernard et al. (2019). Our results about the effects of trade in distorted economies also relates to Berthou et al. (2018) and Bai et al. (2018). Our results also relate to complementary work with non-parametric or semi-parametric models of trade like Adao et al. (2017), Lind and Ramondo (2018), and Allen et al. (2014). Whereas they study reduced-form general equilibrium demand systems, we show how to construct these general equilibrium objects from microeconomic primitives. The cost is that our approach requires more data, but the benefit is that our analysis does not rely on the invertibility or stability of factor demand systems or gravity equations, assumptions that can be easily violated in models with intermediates or wedges. Our characterization of how factor shares and prices respond to shocks is related to a large literature, for example, Trefler and Zhu (2010), Davis and Weinstein (2008), Feenstra and Sasahara (2017), Dix-Carneiro (2014), Galle et al. (2017), among others. Finally, our computational approach, which, instead of solving a nonlinear system of equations, numerically integrates derivatives, is similar to the way computational general equilibrium (CGE) models are solved (for a survey, see Dixon et al., 2013).

The literature on production networks has primarily been concerned with the propagation of shocks in closed economies, typically assuming a representative agent. For instance, Long and Plosser (1983), Acemoglu et al. (2012), Atalay (2017), Carvalho et al. (2016), Baqaee and Farhi (2017a,b), and Baqaee (2018), among others. A recent focus of the literature, particularly in the context of open economies, has been to model the formation of links, for example Chaney (2014), Lim (2017), Tintelnot et al. (2018), and Kikkawa et al. (2018). Our approach, which builds on the results in Baqaee and Farhi (2017a,b), is different: rather than modelling the formation of links as a binary decision, we use a Walrasian environment where the presence and strength of links is determined by cost minimization subject to some production technology.

Finally, our growth accounting results are related to closed-economy results like Solow (1957), Hulten (1978), as well as to the literature extending growth-accounting to open economies, including Kehoe and Ruhl (2008) and Burstein and Cravino (2015). Perhaps closest to us are Diewert and Morrison (1985) and Kohli (2004) who introduce output indices which account for terms-of-trade changes. Our real income and welfare-accounting measures share their goal, though our decomposition into pure productivity changes and reallocation effects is different. In explicitly accounting for the existence of intermediate inputs, our approach also speaks to how one can circumvent the double-counting problem and spill-overs arising from differences in gross and value-added trade, issues studied by Johnson and Noguera (2012) and Koopman et al. (2014). Relative to these other papers, our approach has the added bonus of easily being able to handle inefficiencies and wedges.

Our approach is general, and relies on duality, along the lines of Dixit and Norman (1980). We differ from the classic analysis, however, in that, in extending Hulten's theorem to open economies, we state our comparative static results in terms of observable sufficient statistics: expenditure shares, changes in expenditure shares, the input-output table, and elasticities of substitution. Our approach relies heavily on the notion of the allocation matrix, which helps give a physical interpretation to the theorems, and is convenient for analyzing inefficient economies. In inefficient economies, the abstract approach that relies on macro-level envelope conditions, like Dixit and Norman (1980) and Chipman (2008), runs into problems. However, our results and their interpretation in terms of the allocation matrix readily extend to inefficient economies.

2 Framework

In this section, we set up the model and define the equilibrium and statistics of interest.

2.1 Model Environment

There is a set of countries C, a set of producers N producing different goods, and a set of factors F. Each producer and each factor is assigned to be within the borders of one of the countries in C. The sets of producers and factors inside country c are N_c and F_c. The set F_c of factors physically located in country c may be owned by any household, and not necessarily the households in country c. To streamline the exposition, we assume that there is a representative agent in each country.[2]

Distortions. Since tax-like wedges can implement any feasible allocation of resources in our model, including inefficient allocations, we use wedges to represent distortions in the model. These tax wedges may be explicit, like tariffs, or they may be implicit, like markups or financial frictions. For ease of notation, to represent a wedge on i's purchases of inputs from k, we introduce a fictitious middleman k' that buys from k and sells to i at a "markup" $\mu_{k'}$. The revenues collected by these markups/wedges are rebated back to the households in a way we specify below.[3]

[2] See Appendix M for a discussion of how to extend the results to models with heterogeneous households within countries.

[3] These fictitious middlemen are convenient for writing compact formulas, but adding them to the model explicitly is computationally inefficient. In the computational appendix, Appendix K, we discuss these issues in more detail.

Factors. Income is earned by primary factors and revenues generated by the wedges. A primary factor is simply a non-produced (endowment) good, and earns Ricardian rents.[4] To model revenues earned by wedges, for each country $c \in C$, we introduce a "fictitious" factor that collects the markup/wedge revenue accruing to residents of country c. We denote the set of true primary factors by F and the set of true and fictitious factors by F^*. The $C \times (N + F)$ matrix Φ is the ownership matrix, where Φ_{ci} is the share of i's value-added (sales minus costs) that goes to households in country c.

Households. The representative household in country c has homothetic preferences[5]

$$W_c = \mathcal{W}_c(\{c_{ci}\}_{i \in N}),$$

and faces a budget constraint given by

$$\sum_{i \in N} p_i c_{ci} = \sum_{f \in F} \Phi_{cf} w_f L_f + \sum_{i \in N} \Phi_{ci} \left(1 - 1/\mu_i\right) p_i y_i + T_c,$$

where c_{ci} is the quantity of the good i consumed by household c, w_f and L_f is the wage and quantity of factor f, p_i is the price and y_i is the quantity of good i, and T_c is an exogenous lump-sum transfer. The right-hand side is country c's income: the first summand is income earned by primary factors, the second summand is income earned from wedges ("fictitious" factors), and the final summand is net transfers.

Producers. Good $i \in N$ belongs to some country $c \in C$ and is produced using a constant-returns-to-scale production function[6,7]

$$y_i = A_i F_i \left(\{x_{ik}\}_{k \in N}, \{l_{if}\}_{f \in F_c} \right),$$

[4]That is, we assume factors are inelastically supplied. In Appendix L, we discuss how to endogenize factor supply by using a Roy model and discuss the connection of our results with those in Galle et al. (2017).

[5]In mapping our model to data, we interpret domestic "households" as any agent which consumes resources without producing resources to be used by other agents. Specifically, this means that we include domestic investment and government expenditures in our definition of "households".

[6]This is more general than it might appear. First, production has constant returns to scale without loss of generality, because non-constant-returns can be captured via fixed factors. Second, the assumption that each producer produces only one output good is also without loss of generality. A multi-output production function is a single output production function where all but one of the outputs enter as negative inputs. Finally, productivity shifters are Hicks-neutral without loss of generality. To represent input-augmenting technical change for i's use of input k, introduce a fictitious producer buying from k and selling to i, and hit this fictitious producer with a Hicks-neutral shock.

[7]We rule out fixed costs in our analysis. Our results accommodate an extensive margin of product entry-exit, but only if it operates according to a choke-price, rather than a fixed cost. For an analysis of general equilibrium models with fixed costs see Baqaee and Farhi (2020).

where y_i is the total quantity of good i produced, x_{ik} is intermediate inputs from k, l_{if} is factor inputs from f, and A_i is an exogenous Hicks-neutral productivity shifter. Producer i chooses inputs to minimize costs and sets prices equal to marginal cost times a wedge $p_i = \mu_i \times mc_i$.

Iceberg Trade Costs. We capture changes in iceberg trade costs as Hicks-neutral productivity changes to specialized importers or exporters whose production functions represent the trading technology. The decision of where trading technologies should be located is ambiguous since they generate no income. It is possible to place them in the exporting country or in the importing country, and this would make no difference in terms of the welfare of agents or the allocation of resources.[8]

Equilibrium. Given productivities A_i, wedges μ_i, and a vector of transfers satisfying $\sum_{c \in C} T_c = 0$, a general equilibrium is a set of prices p_i, intermediate input choices x_{ij}, factor input choices l_{if}, outputs y_i, and consumption choices c_{ci}, such that: (i) each producer chooses inputs to minimize costs taking prices as given; (ii) the price of each good is equal to the wedge on that good times its marginal cost; (iii) each household maximizes utility subject to its budget constraint taking prices as given; and, (iv) the markets for all goods and factors clear so that $y_i = \sum_{c \in C} c_{ci} + \sum_{j \in N} x_{ji}$ for all $i \in N$ and $L_f = \sum_{j \in N} l_{jf}$ for all $f \in F$.

2.2 Definitions and Notation

In this subsection, we define the statistics of interest and introduce useful notation.

Nominal Output and Expenditure. Nominal output or Gross Domestic Product (GDP) for country c is the total final value of the goods produced in the country. It coincides with the total income earned by the factors located in the country:

$$GDP_c = \sum_{i \in N} p_i q_{ci} = \sum_{f \in F_c} w_f L_f + \sum_{i \in N_c} (1 - 1/\mu_i) \, p_i y_i,$$

where $q_{ci} = y_i 1_{\{i \in N_c\}} - \sum_{j \in N_c} x_{ji}$ is the "final" or net quantity of good $i \in N$ produced by country c. Note that q_{ci} is negative for imported intermediate goods.

Nominal Gross National Expenditure (GNE) for country c, also known as domestic absorption, is the total final expenditures of the residents of the country. In our model, it

[8] We do not need to take a precise stand at this stage, but we note that this will matter for our conclusions regarding country-level real GDP changes (as pointed out by Burstein and Cravino, 2015).

coincides with nominal Gross National Income (GNI) which is the total income earned by the factors owned by a country's residents adjusted for international transfers:

$$GNE_c = \sum_{i\in N} p_i c_{ci} = \sum_{f\in F} \Phi_{cf} w_f L_f + \sum_{i\in N} \Phi_{ci} \left(1 - 1/\mu_i\right) p_i y_i + T_c.$$

To denote variables for the world, we drop the country-level subscripts. Nominal GDP and nominal GNE are *not* the same at the country level, but they are the same at the world level:

$$GDP = GNE = \sum_{f\in F} w_f L_f + \sum_{f\in N} (1 - 1/\mu_i) p_i y_i = \sum_{i\in N} p_i q_i = \sum_{i\in N} p_i c_i,$$

where, for the world, final consumption coincides with net output $c_i = q_i$ because $c_i = \sum_{c\in C} c_{ci} = \sum_{c\in C} q_{ci} = q_i$, net transfers are zero $T = 0$ because $T = \sum_{c\in C} T_c$. Let world GDP be the numeraire, so that $GDP = GNE = 1$. All prices and transfers are expressed in units of this numeraire.

Real Output and Expenditure. To convert nominal variables into real variables, as in the data, we use Divisia indices throughout. The change in real GDP of country c and the corresponding GDP deflator are defined to be

$$\mathrm{d}\log Y_c = \sum_{i\in N} \Omega_{Y_c,i} \,\mathrm{d}\log q_{ci}, \quad \mathrm{d}\log P_{Y_c} = \sum_{i\in N} \Omega_{Y_c,i} \,\mathrm{d}\log p_i,$$

where $\Omega_{Y_c,i} = p_i q_{ci}/GDP_c$ is good i's share in final output of country c.[9]

The change in real GNE of country c and the corresponding deflator are

$$\mathrm{d}\log W_c = \sum_{i\in N} \Omega_{W_c,i} \,\mathrm{d}\log c_{ci}, \quad \mathrm{d}\log P_{W_c} = \sum_{i\in N} \Omega_{W_c,i} \,\mathrm{d}\log p_i,$$

where $\Omega_{W_c,i} = p_i c_{ci}/GNE_c$ is good i's share in country c's consumption basket. By Shephard's lemma, changes in real GNE are equal to changes in welfare for every country .

As with the nominal variables, real GDP and real GNE are *not* the same at the country level. However, these differences vanish at the world level so that, for the world, $\mathrm{d}\log Y = \mathrm{d}\log W$ and $\mathrm{d}\log P_Y = \mathrm{d}\log P_W$.[10] Conveniently, changes in country real GDP and real

[9] Our definition of real GDP coincides with the double-deflation approach to measuring real GDP, where the change in real GDP is defined to be the sum of changes in real value-added for domestic producers. We also slightly abuse notation since, at the initial equilibrium, $q_{ci} = 0$ for new goods and $q_{ci} < 0$ for imported intermediates. In these cases, we define $\mathrm{d}\log q_{ci} = \mathrm{d}\, q_{ci}/q_{ci}$.

[10] Real GDP and real GNE for the world are defined by aggregating across all countries, so $\mathrm{d}\log Y = \sum_{i\in N}(p_i q_i/GDP)\,\mathrm{d}\log q_i$, $\mathrm{d}\log P_Y = \sum_{i\in N}(p_i q_i/GDP)\,\mathrm{d}\log p_i$, $\mathrm{d}\log W = \sum_{i\in N}(p_i c_i/GNE)\,\mathrm{d}\log c_i$, and $\mathrm{d}\log P_W = \sum_{i\in N}(p_i c_i/GNE)\,\mathrm{d}\log p_i$,.

GNE aggregate up to their world counterparts.[11]

Finally, infinitesimal changes in real GDP and real GNE can be integrated or *chained* into discrete changes by updating the corresponding shares along the integration path. We denote the corresponding discrete changes by $\Delta \log Y$, $\Delta \log Y_c$, $\Delta \log W$, and $\Delta \log W_c$. In the case of GDP, this is how these objects are typically measured in the data, and in the case of GNE, this coincides with the nonlinear change in the welfare of each agent c.

Input-Output Matrices. The Heterogenous-Agent Input-Output (HAIO) matrix is the $(C+N+F) \times (C+N+F)$ matrix Ω whose ijth element is equal to i's expenditures on inputs from j as a share of its total revenues/income

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i} \mathbf{1}_{\{i \in N\}} + \frac{p_j c_{ij}}{GNE_i} \mathbf{1}_{\{i \in C\}}.$$

The HAIO matrix Ω includes the factors of production and the households, where factors consume no resources (zero rows), while households produce no resources (zero columns). The Leontief inverse matrix is

$$\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \dots .$$

Whereas the input-output matrix Ω records the *direct* link from one agent or producer to another, the Leontief inverse matrix Ψ records instead the *direct and indirect* exposures through the production network.

Denote the diagonal matrix of wedges by μ (where non-taxed quantities have wedge $\mu_i = 1$) and define the *cost-based* HAIO matrix and Leontief inverse to be

$$\tilde{\Omega} = \mu \Omega, \qquad \tilde{\Psi} = (I - \tilde{\Omega})^{-1}.$$

It will sometimes be convenient to treat goods and factors together and index them by $k \in N + F$ where the plus symbol denotes the union of sets. To this effect, we slightly extend our definitions. We interchangeably write y_k and p_k for the quantity L_k and wage w_k of factor $k \in F$.

Exposures. Each $i \in C + N + F$ is *exposed* to each $j \in C + N + F$ through revenues Ψ_{ij} and through costs $\tilde{\Psi}_{ij}$. Intuitively, Ψ_{ij} measures how expenditures on i affect the sales of j (due to backward linkages), whereas $\tilde{\Psi}_{ij}$ measures how the price of j affects the marginal cost of

[11] Namely, $d \log Y = \sum_{c \in C} (GDP_c / GDP)\, d \log Y_c$ and $d \log W = \sum_{c \in C} (GNE_c / GNE)\, d \log W_c$.

i (due to forward linkages). In the absence of wedges, $\mu_i = 1$ for every i, these two objects coincide.

When i is a household, we use special notation to denote *backward* and *forward* exposure. In particular, let

$$\lambda_k^{W_c} = \Psi_{c,k} = \sum_{i \in N} \Omega_{c,i} \Psi_{ik}, \quad \tilde{\lambda}_k^{W_c} = \tilde{\Psi}_{c,k} = \sum_{i \in N} \tilde{\Omega}_{c,i} \tilde{\Psi}_{ik}.$$

In words, c's exposure to k is the expenditure share weighted average of the exposure of c's suppliers to k. By analogy, the forward and backward exposure of country c's GDP (as opposed to welfare) is defined as

$$\lambda_k^{Y_c} = \sum_{i \in N} \Omega_{Y_c,i} \Psi_{ik}, \quad \tilde{\lambda}_k^{Y_c} = \sum_{i \in N} \Omega_{Y_c,i} \tilde{\Psi}_{ik}, \tag{2}$$

where recall that $\Omega_{Y_c,i} = p_i q_{ci} / GDP_c$ is the share of a good i in GDP. As usual, the world-level backward and forward exposure to k are denoted by suppressing the country subscript: that is, λ_k^Y and $\tilde{\lambda}_k^Y$ respectively.

We sometimes denote exposure to factors with capital Λ or $\tilde{\Lambda}$ to distinguish them from non-factor producers λ or $\tilde{\lambda}$. In other words, when $f \in F^*$, we write $\Lambda_f^{Y_c} = \lambda_f^{Y_c}, \Lambda_f^{W_c} = \lambda_f^{W_c}, \tilde{\Lambda}_f^{W_c} = \tilde{\lambda}_f^{W_c}, \tilde{\Lambda}_f^{W_c} = \tilde{\lambda}_f^{W_c}$ to emphasize that f is a factor.

Sales and Income. Exposures of GDP to a good or factor k at the country and world levels have a direct connection to the sales of k:

$$\lambda_k^{Y_c} = 1_{\{k \in N_c + F_c\}} \frac{p_k y_k}{GDP_c}, \quad \lambda_k = \frac{p_k y_k}{GDP}. \tag{3}$$

Hence, the exposure of world GDP λ_k^Y to k is just the sales share (or *Domar weight*) of k in world output $\lambda_k = p_k y_k / GDP$. Similarly, the exposure of country c's GDP to k is the *local Domar weight* of k in country c, that is $\lambda_k^{Y_c} = 1_{\{k \in N_c + F_c\}} (GDP/GDP_c) \lambda_k$.

We also define *factor income shares*: the share of a factor f in the income of country c and of the world are denoted

$$\Lambda_f^c = \frac{\Phi_{cf} w_f L_f}{GNE_c}, \quad \Lambda_f = \frac{w_f L_f}{GNE}.$$

Since world GNE is equal to world GDP, it follows from (2) and (3) that $\Lambda_f = \Lambda_f^Y = \sum_{i \in N} \Omega_{Y,i} \Psi_{if}$.

3 Comparative Statics: Ex-Post Sufficient Statistics

In this section, we characterize the response of real GDP and welfare to shocks. We state our results in terms of changes in endogenous, but observable, sufficient statistics. In the next section, we solve for changes in these endogenous variables in terms of microeconomic primitives.

Allocation Matrix. To better understand the intuition for the results, we introduce the allocation matrix, which helps give a physical interpretation to the theorems. Following Baqaee and Farhi (2017b), define the $(C+N+F) \times (C+N+F)$ *allocation matrix* $\mathcal{X}$ as follows: $\mathcal{X}_{ij} = x_{ij}/y_j$ is the share of good j used by i, where i and j index households, factors, and producers. Every feasible allocation is defined by a feasible allocation matrix $\mathcal{X}$, a vector of productivities A, and a vector of factor supplies L. In particular, the equilibrium allocation gives rise to an allocation matrix $\mathcal{X}(A, L, \mu, T)$ which, together with A, and L, completely describes the equilibrium.[12]

We decompose changes in any quantity X into changes due to the technological environment, for a given allocation matrix, and changes in the allocation matrix, for given technology. In vector notation:

$$\mathrm{d}\log X = \underbrace{\frac{\partial \log X}{\partial \log A}\,\mathrm{d}\log A + \frac{\partial \log X}{\partial \log L}\,\mathrm{d}\log L}_{\Delta \text{ technology}} + \underbrace{\frac{\partial \log X}{\partial \mathcal{X}}\,\mathrm{d}\,\mathcal{X}}_{\Delta \text{ reallocation}}.$$

Real GDP. The response of real GDP to shocks, stated in terms of country c variables, is given by the following.

Theorem 1 (Real GDP)**.** *The change in real GDP of country c in response to productivity shocks, factor supply shocks, transfer shocks, and shocks to wedges is:*

$$\mathrm{d}\log Y_c = \underbrace{\sum_{i\in N_c} \tilde{\lambda}_i^{Y_c}\,\mathrm{d}\log A_i + \sum_{f\in F_c} \tilde{\Lambda}_f^{Y_c}\,\mathrm{d}\log L_f + \sum_{i\in N-N_c} \left(\tilde{\Lambda}_i^{Y_c} - \Lambda_i^{Y_c}\right)\mathrm{d}\log(q_{ci})}_{\Delta \text{ technology}}$$

$$\underbrace{- \sum_{i\in N_c} \tilde{\lambda}_i^{Y_c}\,\mathrm{d}\log \mu_i - \sum_{f\in F_c}^{F} \tilde{\Lambda}_f^{Y_c}\,\mathrm{d}\log \Lambda_f^{Y_c} + \sum_{i\in N-N_c} \left(\Lambda_i^{Y_c} - \tilde{\Lambda}_i^{Y_c}\right)\mathrm{d}\log \Lambda_i^{Y_c}}_{\Delta \text{ reallocation}}, \quad (4)$$

[12] Since there may be multiplicity of equilibria, technically, $\mathcal{X}(A, L, \mu, T)$ is a correspondence. In this case, we restrict attention to perturbations of isolated equilibria. As shown by Debreu (1970), we can generically expect equilibria to be locally isolated.

where, for imported intermediates $i \in N - N_c$, the term $\Lambda_i^{Y_c} = \sum_{i \in N_c} \Omega_{Y_c,i} \Psi_{ik} = -p_i q_{ci}/GDP_c$ is expenditure on imported intermediate i as a share of GDP and $\tilde{\Lambda}_i^{Y_c} = \sum_{i \in N_c} \Omega_{Y_c,i} \tilde{\Psi}_{ik}$. The change in world real GDP $\mathrm{d}\log Y$ *can be obtained by simply suppressing the country index c. That is,*

$$\mathrm{d}\log Y = \underbrace{\sum_{i \in N} \tilde{\lambda}_i^Y \mathrm{d}\log A_i + \sum_{f \in F} \tilde{\Lambda}_f^Y \mathrm{d}\log L_f}_{\Delta\ technology} - \underbrace{\sum_{i \in N} \tilde{\lambda}_i^Y \mathrm{d}\log \mu_i - \sum_{f \in F}^{F} \tilde{\Lambda}_f^Y \mathrm{d}\log \Lambda_f^Y}_{\Delta\ reallocation}.$$

To understand equation (4), first consider the case where there are no wedges in the initial equilibrium. Then forward and backward exposures are the same $\tilde{\Lambda}_i^{Y_c} = \Lambda_i^{Y_c}$. Furthermore, since revenues generated by wedges exactly offset the reduction in primary factor income shares $\sum_{i \in N_c} \tilde{\lambda}_i^{Y_c} \mathrm{d}\log \mu_i = -\sum_{f \in F_c}^{F} \Lambda_f^{Y_c} \mathrm{d}\log \Lambda_f^{Y_c} = -\sum_{f \in F_c}^{F} \tilde{\Lambda}_f^{Y_c} \mathrm{d}\log \Lambda_f^{Y_c}$, Theorem 1 simplifies to

$$\mathrm{d}\log Y_c = \sum_{i \in N_c} \lambda_i^{Y_c} \mathrm{d}\log A_i + \sum_{f \in F_c} \Lambda_f^{Y_c} \mathrm{d}\log L_f.$$

That is, when there are no initial (domestic) wedges, country c's real GDP is equal to a Domar-weighted sum of *domestic* productivity shocks and *domestic* factor endowments. In this case, changes in the allocation matrix have no effect on real GDP.[13] Intuitively, when there are no domestic wedges, there is an envelope theorem for real GDP (the competitive equilibrium maximizes the joint profits of all domestic firms for given prices). Hence, without wedges, reallocations cannot affect real GDP to a first-order. Furthermore, in the absence of wedges, foreign shocks, like shocks to iceberg costs outside c's borders, have no effect on real GDP.

Now, suppose there are pre-existing wedges. There are two major changes. First, on the first line of equation (4), there are "mechanical" technology effects (holding fixed the distribution of resources). As in the efficient benchmark, shocks to domestic productivity $\mathrm{d}\log A_i$ and domestic factor-endowments $\mathrm{d}\log L_f$ move real GDP. However, when there are pre-existing wedges, changes in the quantity of imported intermediate inputs $\mathrm{d}\log q_{ci}$ also change real GDP. This happens because expenditure on imported intermediates $\Lambda_i^{Y_c}$ is not equal to the shadow value of imported intermediates $\tilde{\Lambda}_i^{Y_c}$. Imported intermediates are netted out of GDP using expenditures $\Lambda_i^{Y_c}$ and not their shadow-values, so, when there are initial wedges, changes in the quantity of imported intermediates changes real GDP.[14]

[13] Theorem 1 generalizes Hulten (1978), Burstein and Cravino (2015), and Baqaee and Farhi (2017b).

[14] For example, if there is a tariff on an imported intermediate $i \in N - N_c$, then expenditure on the import is less than its shadow-value $\Lambda_i^{Y_c} < \tilde{\Lambda}_i^{Y_c}$. In this case, ceteris paribus, an increase in intermediate input usage $d\log q_{ci} = dq_{ci}/q_{ci} > 0$ will boost real GDP and TFP. Therefore, even in an economy where the allocation of resources across producers is efficient, trade shocks can alter aggregate TFP by changing the quantity of imported intermediates. For an example, see Gopinath and Neiman (2014).

The second line of (4) reflects changes in the allocation of resources. When there are preexisting wedges, reallocation can have first-order effects on real GDP even holding fixed microeconomic productivities, factor endowments, and the quantity of imports. These are genuine changes in efficiency, and they occur because resources are not being used efficiently at the initial equilibrium. The intuition for the second line of (4) is similar to that described in Baqaee and Farhi (2017b) and Baqaee and Farhi (2019) for closed economies.

Welfare. We now turn our attention to changes in welfare (real GNE).[15]

Theorem 2 (Welfare)**.** *The change in welfare of country c in response to productivity shocks, factor supply shocks, and transfer shocks can be written as:*

$$\mathrm{d}\log W_c = \underbrace{\sum_{f\in F}\tilde{\Lambda}_f^{W_c}\,\mathrm{d}\log L_f + \sum_{i\in N}\tilde{\lambda}_i^{W_c}\,\mathrm{d}\log A_i}_{\Delta\ technology}$$
$$\underbrace{-\sum_{i\in N}\tilde{\lambda}_i^{W_c}\,\mathrm{d}\log\mu_i + \sum_{f\in F^*}\left(\Lambda_f^c - \tilde{\Lambda}_f^{W_c}\right)\mathrm{d}\log\Lambda_f + (GNE/GNE_c)\,\mathrm{d}\,T_c}_{\Delta\ reallocation},$$

where $\mathrm{d}\,T_c$ is the change in net transfers. The change $\mathrm{d}\log W$ of world real GNE is obtained by suppressing the country index c. That is,

$$\mathrm{d}\log W = \underbrace{\sum_{f\in F}\tilde{\Lambda}_f^{W}\,\mathrm{d}\log L_f + \sum_{i\in N}\tilde{\lambda}_i^{W}\,\mathrm{d}\log A_i}_{\Delta\ technology} \underbrace{-\sum_{i\in N}\tilde{\lambda}_i^{W}\,\mathrm{d}\log\mu_i - \sum_{f\in F^*}\tilde{\Lambda}_f^{W}\,\mathrm{d}\log\Lambda_f}_{\Delta\ reallocation}.$$

As with real GDP, changes in welfare can be broken into technological effects (holding fixed the distribution of resources) and reallocation effects (holding fixed technology). However, unlike real GDP, reallocation effects are first-order even when there are no wedges. This is because unlike real GDP, even in the absence of wedges, there is no envelope theorem for the welfare of a given country. We discuss the intuition for the technology and reallocation effects in turn.

The direct technology effect of a shock depends on each household's exposures to the technology shock. Since households consume foreign goods, either directly or indirectly through supply chains, this means that technology shocks outside of a country's borders affect the household in that country holding fixed the allocation matrix.

[15]Throughout the paper, if the wedges associated with a fictitious factor f are 1, then we have $\Lambda_f = 0$ and $\mathrm{d}\log\Lambda_f/\,\mathrm{d}\log\mu_i$ is not defined. In this case, elasticities can be replaced with semi-elasticities in a straightforward way, but we omit the details for brevity.

The second line in Theorem 2 captures reallocation effects. The first term is the direct effect of wedges on consumer prices. To see the intuition for the second term on the second line, recall that $\tilde{\Lambda}_f^{W_c} = \tilde{\Psi}_{cf}$ is the cost-based forward exposure of household c to factor f. This captures the total reliance of household c on f, taking into account direct and indirect exposures through supply chains. Intuitively, the reallocation effects consider, for each factor f, how the income earned by the factor changes $\mathrm{d}\log\Lambda_f$, and whether household c is a net seller $\Lambda_f^c - \tilde{\Lambda}_f^{W_c} > 0$ or a net buyer $\Lambda_f^c - \tilde{\Lambda}_f^{W_c} < 0$ of factor f. The final term on the second line is the change in net transfers.

Once we aggregate to the level of the world, if there are no pre-existing wedges, the reallocation effects are zero. In other words, for efficient models, reallocation effects are zero-sum distributive changes. On the other hand, when there are pre-existing wedges, reallocation effects are no longer zero-sum, since they can make everyone better or worse off by changing the efficiency of resource allocation. Appendix H contains a detailed and formal discussion of the reallocation effects. Appendix H emphasizes that these reallocation effects are not the same as changes in the terms-of-trade.

Simple Example. To see the difference between Theorems 1 and 2, consider a productivity shock $\mathrm{d}\log A_i$ to a foreign producer $i \notin N_c$. Suppose there are no wedges and all production and utility functions are Cobb-Douglas. Since there are no wedges, Theorem 1 implies that domestic real GDP does not respond to the foreign productivity shock $\mathrm{d}\log Y_c = 0$.

Now, consider the change in welfare in Theorem 2. The Cobb-Douglas assumption implies that factor income shares do not respond to productivity shocks $\mathrm{d}\log\Lambda_f = 0$. Hence, there are no reallocation effects for welfare either. Nevertheless, domestic welfare does respond to the foreign productivity shock $\mathrm{d}\log W_c = \lambda_i^{W_c}\,\mathrm{d}\log A_i$. Intuitively, even though there are no reallocation effects, an increase in foreign productivity increases the overall amount of goods the world economy can produce and this increases the welfare of country c to the extent that the consumption basket of country c relies on i (directly and indirectly through global supply chains).

Uses of Theorems 1 and 2. Since Theorems 1 and 2 depend on endogenous movements in factor income shares, they cannot be used directly to make predictions. However, despite this fact, they are useful for three reasons: (i) they provide intuition about why and how Hulten's theorem fails to describe welfare and real GDP in open economies, (ii) they can be used to measure and decompose changes into different sources *conditional* on observing the changes in factor shares (extending growth-accounting to open and distorted economies), and (iii) they can be combined with the results in Section 4 to perform counterfactuals.

Outline of the Rest of the Paper. In Section 4, we provide a full characterization of how disaggregated sales shares, prices, and quantities change in terms of microeconomic primitives (ex-ante sufficient statistics) to a first-order. In Section 5, we use these first-order results to approximate the losses to society from the imposition of tariffs and other distortions to a second-order. In Section 6, we use these results to study the effect on welfare of large external shocks, for instance, the cost of moving the economy to autarky. We end the paper with analytical and quantitative examples in Sections 7 and 8.

4 Comparative Statics: Ex-Ante Sufficient Statistics

Section 3 shows that the response of welfare and real GDP to shocks depend on changes in ex-post sufficient statistics (like changes in factor shares). In this section we characterize these ex-post sufficient statistics in terms of microeconomic primitives: the HAIO matrix and elasticities of substitution in production and in consumption (ex-ante sufficient statistics). The results of this section can then be combined with Theorems 1 and 2 to answer counterfactual questions about welfare and real GDP. We focus on two types of shocks: productivity shocks, which nest shocks to factor supply and iceberg costs, and wedge shocks, which nest shocks to tariffs and markups.

4.1 Set Up

To clarify exposition, we specialize production and consumption functions to be nested-CES aggregators, with an arbitrary number of nests and elasticities. This is for clarity not tractability. Appendix A shows that it is very straightforward to generalize the rest of the results in the paper to non-nested-CES economies.

Nested CES economies can be written in many different equivalent ways. We adopt a standardized representation, which we call the *standard-form* representation. We treat every CES aggregator as a separate producer and rewrite the input-output matrix accordingly, so that each producer has a single elasticity of substitution associated with it; the representative household in each country c consumes a single specialized good which, with some abuse of notation, we also denote by c. Importantly, note that this procedure changes the set of producers, which, with some abuse of notation we still denote by N.[16] In other

[16] See Baqaee and Farhi (2017a) for a more detailed discussion of the standard-form representation.

words, every $k \in C + N$ has an associated cost function

$$p_k = \frac{\mu_k}{A_k}\left(\sum_{j\in N+F_c}\tilde{\Omega}_{kj}p_j^{1-\theta_k}\right)^{\frac{1}{1-\theta_k}},$$

where θ_k is the elasticity of substitution.[17]

For nested-CES economies, the input-output covariance turns out to be a central object.

Input-Output Covariance. We use the following matrix notation throughout. For a matrix X, we define $X^{(i)}$ to be its ith row and $X_{(j)}$ to be its jth column. We define the *input-output covariance operator* to be

$$Cov_{\tilde{\Omega}^{(k)}}(\Psi_{(i)}, \Psi_{(j)}) = \sum_{l\in N+F}\tilde{\Omega}_{kl}\Psi_{li}\Psi_{lj} - \left(\sum_{l\in N+F}\tilde{\Omega}_{kl}\Psi_{li}\right)\left(\sum_{l\in N+F}\tilde{\Omega}_{kl}\Psi_{lj}\right).$$

This is the covariance between the ith and jth columns of the Leontief inverse using the kth row of $\tilde{\Omega}$ as the probability distribution. We make extensive use of the input-output covariance operator throughout the rest of the paper.

4.2 Comparative Statics

Sales Shares and Prices. The following characterizes how prices and sales shares, including factor income shares, respond to perturbations in an open-economy.[18]

Theorem 3 (Prices and Sales Shares)**.** *For a vector of perturbations to productivity* $\mathrm{d}\log A$ *and wedges* $\mathrm{d}\log\mu$*, the change in the price of a good or factor* $i \in N + F$ *is*

$$\mathrm{d}\log p_i = \sum_{k\in N}\tilde{\Psi}_{ik}\left(\mathrm{d}\log\mu_k - \mathrm{d}\log A_k\right) + \sum_{f\in F}\tilde{\Psi}_{if}\,\mathrm{d}\log\Lambda_f. \tag{5}$$

The change in the sales share of a good or factor $i \in N + F$ *is*

$$\begin{aligned}\mathrm{d}\log\lambda_i = &\sum_{k\in N+F}\left(\mathbf{1}_{\{i=k\}} - \frac{\lambda_k}{\lambda_i}\Psi_{ki}\right)\mathrm{d}\log\mu_k + \sum_{k\in N}\frac{\lambda_k}{\lambda_i}\mu_k^{-1}(1-\theta_k)Cov_{\tilde{\Omega}^{(k)}}(\Psi_{(i)}, \mathrm{d}\log p)\\ &+ \sum_{g\in F^*}\sum_{c\in C}\frac{\lambda_i^{W_c} - \lambda_i}{\lambda_i}\Phi_{cg}\Lambda_g\,\mathrm{d}\log\Lambda_g,\end{aligned} \tag{6}$$

[17] See the second example in Section 7 for an example of an economy written in standard-form.

[18] Theorem 3 generalizes Propositions 2 and 3 from Baqaee and Farhi (2017b) to open-economies.

where $\mathrm{d}\log p$ *is the* $(N+F)\times 1$ *vector of price changes in* (5). *The change in wedge income accruing to household c (represented by a fictitious factor) is*

$$d\log\Lambda_c=\sum_i \frac{\Phi_{ci}\lambda_i}{\Lambda_c}\left(\mu_i^{-1}d\log\mu_i+(1-\mu_i^{-1})d\log\lambda_i\right). \tag{7}$$

Recall that for every fictitious or real factor $i\in F^*$, we interchangeably use λ_i or Λ_i to denote its Domar weight. This means that (6) pins down the change in primary factor income shares and (7) pins down changes in "fictitious" factor income shares. Therefore, substituting the vector of price changes (5) into (6) results in an $F^*\times F^*$ linear system in factor income shares $\mathrm{d}\log\Lambda$. The solution to this linear system gives the equilibrium changes in factor shares, which can be plugged back into equations (5) and (6) to get the change in the sales shares and prices for every (non-factor) good.

We discuss the intuition in detail below, but at a high level, equation (5) captures *forward propagation* of shocks — shocks to suppliers change the prices of their downstream consumers. On the other hand, equation (6) captures *backward propagation* of shocks — shocks to consumers change the sales of their upstream suppliers. Each term in these equations has a clear interpretation.

To see this intuition, start by considering the forward propagation equations (5): the first set of summands show that a change in the price of k, caused either by wedges $\mathrm{d}\log\mu_k$ or productivity $\mathrm{d}\log A_k$, affect the price of i via its direct and indirect exposures $\tilde{\Psi}_{ik}$ through supply chains. The second set of summands capture how changes in factor prices, which are measured by changes in factor income shares, also propagate through supply chains to affect the price of i. These expressions use the cost-based HAIO matrix $\tilde{\Omega}$, instead of the revenue-based HAIO matrix Ω, because Shephard's lemma implies that the elasticity of the price of i to the price of one of its inputs k is given by $\tilde{\Omega}_{ik}$ and not Ω_{ik}.

For the intuition of backward propagation equations (6), we proceed term by the term. The first term captures how an increase in the wedge $\mathrm{d}\log\mu_k$ reduces expenditures on suppliers i. If μ_k increases, then for each dollar k earns, relatively less of it makes it to i, and this reduces the sales of i.

The second term captures the fact that when relative prices change $\mathrm{d}\log p\neq 0$, then every producer k will substitute across its inputs in response to this change. Suppose that $\theta_k>1$, so that producer k substitutes (in expenditure shares) *towards* those inputs that have become cheaper. If those inputs that became cheap are also heavily reliant on i, then $Cov_{\tilde{\Omega}^{(k)}}(\Psi_{(i)},\mathrm{d}\log p)<0$. Hence, substitution by k towards cheaper inputs will increase demand for i. These substitutions, which happen at the level of each producer k, must be summed across all producers.

The last set of summands, on the second line of (6), capture the fact that changes in factor prices change the distribution of income across households in different countries. This affects the demand for i if the different households are differently exposed, directly and indirectly, to i. The overall effect can be found by summing over countries c the increase in c's share of aggregate income $\sum_{g \in F^*} \Phi_{cg} \Lambda_g \, d \log \Lambda_g$ multiplied by the relative welfare exposure $(\lambda_i^{W_c} - \lambda_i)/\lambda_i$ to i. If every household has the same consumption basket, the last term disappears.

Quantities. Theorem 3 can be used to characterize the response of quantities to shocks.[19]

Corollary 1. *(Quantities) The changes in the quantity of a good or factor i in response to a productivity shock to i is given by:*

$$d \log y_i = d \log \lambda_i - d \log p_i,$$

where $d \log \lambda$ *and* $d \log p$ *are given in Theorem 3.*

These results on the responses of prices and quantities to perturbations generalize classic results of Stolper-Samuelson and Rybczynski.

Real GDP and Welfare. Theorem 3 gives the response of factor shares to shocks as a function of microeconomic primitives. These were left implicit in Theorem 2. Furthermore, Theorem 3 and Corollary 1 also pin down changes in the sales and quantities of imported intermediate inputs, which were left implicit in Theorem 1. Hence, Theorem 3 used in conjunction with Theorems 1 and 2 characterizes the response of real GDP and welfare to shocks as a function of microeconomic primitives, up to the first order.

For an efficient model, without wedges, real GDP in Theorem 1 does not depend on changes in factor shares to a first-order. In that, case, Theorem 3 gives the response of real GDP to shocks to the second order instead:

$$\frac{d \log Y_c}{d \log A_j} = \lambda_j^{Y_c}, \quad \frac{d^2 \log Y_c}{d \log A_j \, d \log A_i} = \frac{d \lambda_j^{Y_c}}{d \log A_i} = \lambda_j^{Y_c} \left(\frac{d \log \lambda_j}{d \log A_i} - \sum_{f \in N_c} \Lambda_f^{Y_c} \frac{d \log \Lambda_f}{d \log A_i} \right), \quad (8)$$

where $d \lambda_j / d \log A_i$ and $d \log \Lambda_f / d \log A_i$ are given by Theorem 3.[20] For world real GDP, suppress the c subscript.

[19] Recall that prices are expressed in the numeraire where $GDP = GNE = 1$ at the world level.

[20] The expression for $d^2 \log Y_c / (d \log A_j \, d \log A_i)$ abuses notation and must be handled with care. Technically, the change in real GDP from one allocation to another in general depends on the *path* taken. Hulten's theorem guarantees that changes in real GDP are a path integral of the vector field defined by the local Domar weights along a path of productivity changes. Hence, the expression $d^2 \log Y_c / (d \log A_j \, d \log A_i)$ is really the

Non-infinitesimal Shocks. Theorem 3, which is a generalization of hat-algebra (Jones, 1965), is useful for studying small shocks and gaining intuition. For large shocks, the trade literature instead relies on exact-hat algebra (e.g. Dekle et al., 2008; Costinot and Rodriguez-Clare, 2014), which involves solving the non-linear system of supply and demand relationships. Theorem 3 provides an alternative way to make hat-algebra exact by "chaining" together local effects. This amounts to viewing Theorem 3 as a system of differential equations that can be solved by iterative means (e.g. Euler's method or Runge-Kutta). In our quantitative exercises in Section 8, we find that the differential approach is ten times faster than using state-of-the-art nonlinear solvers to perform exact hat-algebra.[21] Furthermore, the non-parametric generalization of Theorem 3 in Appendix A can be used to feed estimates of the elasticity of substitution directly into the differential equation to compute global comparative statics without specifying a closed-form expression for production or cost functions. See Appendix C for more details about global comparative statics.

Other Uses of Theorem 3. Theorem 3 can also be used to characterize other statistics of interest. Appendix E provides the elasticity of the international factor demand system with respect to factor prices and iceberg shocks as a linear combination of microeconomic elasticities of substitution with weights that depend on the input-output table. Figure 4 in Appendix E quantifies these elasticities using input-output data. This relates to insights from Adao et al. (2017), who show that the factor demand system is sufficient for performing certain counterfactuals. As another application, Appendix F writes trade elasticities at any level of aggregation as a linear combination of underlying microeconomic elasticities of substitution with weights that depend on the input-output table.[22]

5 Losses from Tariffs and Other Distortions

Section 4 shows how changes in wedges affect output and welfare to a first-order. However, starting at an efficient allocation, the response of real GDP and aggregate welfare to changes in wedges is zero to a first-order (due to the envelope theorem). Losses are not zero to a second-order, and in this section, we characterize these losses. We show that losses are

derivative of the vector field defined by the local Domar weights. Conditional on the path taken from one allocation to the next, it can be used to compute the second derivative of the change in GDP at any point along that path.

[21] This type of approach is also used in the CGE literature, for example Dixon et al. (1982), to solve high-dimensional models because exact-hat algebra is computationally impracticable for very large models.

[22] Appendix F.3 shows that the effect of supply chains on the trade elasticity, emphasized by Yi (2003), are formally identical to the issues of reswitching and capital reversing identified in the Cambridge Capital Controversy of the 1950s and 60s.

approximately equal to a Domar-weighted sum of deadweight-loss triangles. This connects our results to the large, but mostly closed-economy, literature on misallocation. As usual, we present this result in two ways, using ex-post and ex-ante sufficient statistics.

5.1 Losses: Ex-Post Sufficient Statistics

Starting at the efficient point, consider introducing some tariffs or other distortions as $\exp(\Delta \log \mu_i)$. We provide approximations for small wedges $\Delta \log \mu_i$ around the efficient equilibrium, $\log \mu = 0$, for both real GDP and welfare.

Losses in Real GDP. We start by characterizing changes in real output.

Theorem 4 (Real GDP)**.** *Starting at an efficient equilibrium, up to the second order, in response to the introduction of small tariffs or other distortions, changes in the real GDP of country c are given by*

$$\Delta \log Y_c \approx \frac{1}{2} \sum_{i \in N_c} \lambda_i^{Y_c} \Delta \log y_i \Delta \log \mu_i.$$

Changes in world real GDP (and real GNE) are given by suppressing the country subscript.

Hence, for both the world and for each country, the reduction in real GDP from tariffs and other distortions is given by the sum of all the deadweight-loss triangles $1/2\Delta \log y_i \Delta \log \mu_i$ weighted by their corresponding local Domar weights.[23,24]

Theorem 4 shows that we only need to track changes in those quantities which are subject to a wedge — if a good is untaxed, or taxed but not included in real GDP (like a tax on imported consumption), then changes in that quantity are not directly relevant for real GDP.

To give some intuition for Theorem 4, we focus on the country level result for simplicity. Starting at an efficient equilibrium, the introduction of tariffs or other distortions leads to changes $\Delta \log y_i$ in the quantities of goods $i \in N_c$ in country c and to changes in the wedges $\Delta \log \mu_i$ between prices and marginal costs. The price-cost margin $p_i \Delta \log \mu_i$ measures the wedge between the marginal contribution to country real GDP and the marginal

[23] Theorem 4 holds in general equilibrium, but it has a more familiar partial equilibrium counterpart (Feenstra, 2015). For a small open economy operating in a perfectly competitive world market, import tariffs reduce the welfare by $\Delta W \approx (1/2) \sum_i \lambda_i \Delta \log y_i \Delta \log \mu_i$, where μ_i is the ith gross tariff (no tariff is $\mu_i = 1$), y_i is the quantity of the ith import, and λ_i is the corresponding Domar weight (see Appendix I for details). Theorem 4 shows that this type of intuition can be applied in general equilibrium as well.

[24] Harberger (1964) argues that an equation like the one in Theorem 4 can be used to measure welfare as long as there are compensating transfers to keep the distribution of income across households fixed. Theorem 4 shows that, in fact, a similar formula can be used for changes in real GDP, even in the absence of compensating transfers. Proposition 5 shows how Harberger's formula must be altered for aggregate welfare in the absence of compensating transfers.

cost to real GDP of increasing the quantity of good i by one unit. Hence, $\lambda_i^{Y_c}\Delta\log\mu_i$ is the marginal proportional increase in real GDP from a proportional increase in the output of good i. Integrating from the initial efficient point to the final distorted point, we find that $(1/2)\lambda_i^{Y_c}\Delta\log y_i\Delta\log\mu_i$ is the contribution of good i to the change in real GDP.

This formula helps explain why accounting for global value chains matters a great deal for the quantitative effects of tariffs. Intuitively, this is because the triangles $1/2\Delta\log y_i\Delta\log\mu_i$ are larger, and they are weighted more heavily λ_i and $\lambda_i^{Y_c}$, when there are input-output linkages.

Tariffs vs. Iceberg Trade Costs. It is instructive to compare the costs of tariffs to the costs of an increase in iceberg costs. At the world level, in response to a change $\Delta\log(1/A_i)$ in iceberg trade costs, following equation (8), the change in real GDP or real GNE is given up to a second-order by the sum of trapezoids rather than triangles:

$$\Delta\log Y = \Delta\log W \approx -\sum_{i\in N}\lambda_i\left(1+\frac{1}{2}\Delta\log\lambda_i\right)\Delta\log(1/A_i).$$

In contrast to equivalent shocks to tariffs, shocks to iceberg trade costs have nonzero first-order effects. This is a way to see why iceberg shocks are typically much more costly than tariffs.

Losses in Welfare. Theorem 4 shows how real GDP responds to changes in tariffs or other distortions. These results do not apply to welfare. At the country level, changes in tariffs and other distortions typically lead to first-order changes (due to reallocation effects). But even at the world level, where these effects wash out, changes in real expenditure no longer coincide with changes in welfare, since changes in world real expenditures $\mathrm{d}\log W$ cannot be integrated to arrive at a well-defined social welfare function.[25]

To measure world welfare, we introduce a homothetic social welfare function

$$W^{BS}(W_1,\dots,W_C) = \sum_c \overline{\chi}_c^W \log W_c,$$

where $\overline{\chi}_c^W$ is the initial income share of country c at the efficient equilibrium. These welfare weights are chosen so that there is no incentive to redistribute across agents at the initial equilibrium. Starting at an efficient allocation, to a first-order approximation, the response of world welfare to the introduction of wedges is zero because of the envelope theorem. Therefore, we consider the reduction in world welfare from the introduction of wedges to a second-order approximation.

[25]This has to do with the fact that individual household preferences across all countries are non-aggregable.

We measure the change in welfare by asking what fraction of consumption would society be prepared to give up to avoid the imposition of the tariffs. Formally, we measure changes in welfare by $\Delta \log \delta$, where δ solves the equation

$$W^{BS}(\delta\overline{W}_1, \dots, \delta\overline{W}_C) = W^{BS}(W_1, \dots, W_C),$$

where $\overline{W}_c$ and W_c are the values at the initial and final equilibrium. We use a similar definition for country level welfare δ_c.

Define $\chi_c^W = GNE_c/GNE$ to be country c's share of income. Then changes in country income shares are given up to the first order by

$$\Delta \log \chi_c^W \approx \sum_{g \in F} \Phi_{cg} \Lambda_g \Delta \log \Lambda_g + \sum_{i \in N} \Phi_{ci} \lambda_i \Delta \log \mu_i,$$

where the first set of summands show how c's income changes due to changes in factor prices, and the second set of summands capture revenues earned by the wedges accruing to c. Changes in the consumption price index of country c are given up to the first order by

$$\Delta \log P_{W_c} \approx \sum_{i \in N} \lambda_i^{W_c} \Delta \log \mu_i + \sum_{g \in F} \Lambda_g^{W_c} \Delta \log \Lambda_g,$$

where the first term captures changes in consumer prices due to the wedges and the second term captures changes in consumer prices due to changes in factor prices (the log change in the factor price is the same as the log change in the factor income share).

Proposition 5 (Welfare)**.** *Starting at an efficient equilibrium in response to the introduction of small tariffs or other distortions:*

(i) changes in world welfare are given up to the second order by

$$\Delta \log \delta \approx \Delta \log Y + Cov_{\Omega_{\chi^W}} \left(\Delta \log \chi_c^W, \Delta \log P_{W_c} \right);$$

(ii) changes in country real expenditure or welfare are given up to the first order by

$$\Delta \log \delta_c \approx \Delta \log W_c \approx \Delta \log \chi_c^W - \Delta \log P_{W_c}.$$

The change in world welfare is the sum of the change in world real expenditure (output) and a redistributive term. The redistributive term is positive whenever the covariance between the changes in household income shares and the changes in consumption price deflators is positive. It captures a familiar deviation from perfect risk sharing. It would be zero if households could engage in perfect risk sharing before the introduction of the tariffs or other distortions. In our applications, this redistributive effect is quantitatively small and so changes in world welfare are approximately equal to changes in world real GDP.

5.2 Losses: Ex-Ante Sufficient Statistics

Theorem 4 and Proposition 5 express the effects of tariffs and other distortions in terms of endogenous individual output changes. In this subsection, we provide formulas for these individual output changes, and hence for the effects of tariffs and other distortions, in terms of primitives: microeconomic elasticities of substitution and the HAIO matrix. To do this, we combine Theorem 4 with Theorem 3 and Corollary 1.[26]

Theorem 6 (Real GDP)**.** *Around an efficient equilibrium, changes in world real GDP/GNE in response to changes in tariffs or other distortions are given, up to the second order, by*

$$\begin{aligned}\Delta \log Y \approx -\frac{1}{2}\sum_{l\in N}\sum_{k\in N}\Delta \log \mu_k \Delta \log \mu_l \sum_{j\in N}\lambda_j\theta_j Cov_{\Omega^{(j)}}(\Psi_{(k)},\Psi_{(l)}) \\ -\frac{1}{2}\sum_{l\in N}\sum_{g\in F}\Delta \log \Lambda_g \Delta \log \mu_l \sum_{j\in N}\lambda_j\theta_j Cov_{\Omega^{(j)}}(\Psi_{(g)},\Psi_{(l)}) \\ +\frac{1}{2}\sum_{l\in N}\sum_{c\in C}\chi_c^W \Delta \log \chi_c^W \Delta \log \mu_l (\lambda_l^{W_c}-\lambda_l).\end{aligned}$$

Changes $\Delta \log Y_c$ in the real GDP of country c are similar and in Appendix N.

First, all the terms scale with the square of the tariffs or other distortions $\Delta \log \mu$. There is therefore a sense in which misallocation increases with the tariffs and other distortions. Second, all the terms scale with the elasticities of substitution θ of the different producers. There is therefore a sense in which elasticities of substitution magnify the costs of these tariffs and other distortions. Third, all the terms also scale with the sales shares λ of the different producers and with the square of the Leontief inverse matrix Ψ. There is therefore also a sense in which accounting for intermediate inputs magnifies the costs of tariffs and other distortions. Fourth, all the terms mix the wedges, the elasticities of substitution, and of properties of the network.

For a given producer $l \in N$, there are terms in $\Delta \log \mu_l$ on the three lines. Taken together, these terms sum up to the Harberger triangle $(1/2)\lambda_l \Delta \log \mu_l \Delta \log y_l$ corresponding to good l in terms of microeconomic primitives. The three lines break it down into

[26] Theorem 6 generalizes Proposition 5 from Baqaee and Farhi (2017b) to open-economies.

three components, corresponding to three different effects responsible for the change in the quantity $\Delta \log y_l$ of good l.

The term $-\sum_{k\in N} \Delta \log \mu_k \sum_{j\in N} \lambda_j \theta_j Cov_{\Omega^{(j)}}(\Psi_{(k)}, \Psi_{(l)})$ on the first line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *substitutions* by all producers j in response to changes in all tariffs and other distortions $\Delta \log \mu_k$, holding factor wages constant.

The term $\sum_{g\in F} \Delta \log \Lambda_g \sum_{j\in N} \lambda_j \theta_j Cov_{\Omega^{(j)}}(\Psi_{(g)}, \Psi_{(l)})$ on the second line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *substitutions* by all producers j in response to the endogenous changes in factor wages $\Delta \log w_g = \Delta \log \Lambda_g$ brought about by all the changes in tariffs and other distortions.

The term $\sum_{c\in C} \chi_c^W \Delta \log \chi_c^W (\lambda_l^{W_c} - \lambda_l)$ on the third line corresponds to the change $\Delta \log y_l$ in the quantity of good l coming from *redistribution* across agents with different spending patterns, in response to the endogenous changes in factor wages brought about by all the changes in tariffs and other distortions.

It is straightforward to combine Theorem 6 with Proposition 5 to arrive at ex-ante sufficient statistics for the change in welfare.

Corollary 2 (Welfare)**.** *Starting at an efficient equilibrium, changes in world and country welfare* $\Delta \log \delta$ *and* $\Delta \log \delta_c \approx \Delta \log W_c$ *are given via Proposition 5, respectively up to the second order (world) and up to the first order (country).*

6 The Gains from Trade

In this section, we characterize the change in welfare caused by trade shocks, for example, the gains relative to autarky. To reach autarky, we would have to raise iceberg trade costs to infinity, at which point our local approximations in Sections 3, 4, and 5 become unusable. In this section, we study the effect of large trade shocks on domestic welfare by relying on a dual representation of trade shocks. Formally, we show that the effects of foreign shocks on welfare are globally equivalent to the effects of productivity shocks on real GDP in a "dual" closed economy.[27] This allows us study the gains from trade by using characterizations of the linear and nonlinear effects of productivity shocks in closed economies provided respectively in Hulten (1978) and Baqaee and Farhi (2017a).

The approach in this section builds on Feenstra (1994) and Arkolakis et al. (2012). We use changes in domestic shares, and the elasticity of substitution between domestic and foreign varieties, to back out changes in the price of imports. We then show that changes

[27] Our results are related in spirit, but different, to those of Deardorff and Staiger (1988).

in the price of imports affect welfare in a way that is isomorphic to productivity shocks in a fictitious closed economy.

To facilitate exposition, we restrict attention to nested-CES economies where the country of interest has only one primary factor, which we call labor. We also assume that there are no domestic wedges. We discuss how the results may be extended beyond the CES functional form in Appendix A.[28]

6.1 Duality Mapping

Consider an open nested-CES economy c written in standard form. Each producer $i \in N_c$ in the domestic economy has a unit cost-function

$$p_i = \frac{1}{A_i} \left(\sum_{j \in N+F_c} \Omega_{ij} p_j^{1-\theta_i} \right)^{\frac{1}{1-\theta_i}},$$

where θ_i is the elasticity of substitution for i and A_i is a productivity shifter. Since there are no wedges $\tilde{\Omega}_{ij} = \Omega_{ij}$.

Construct a dual closed economy with the same set of producers $i \in N_c$ with CES production functions with the same set of elasticities θ_i and a HAIO matrix $\check{\Omega}$ given by $\check{\Omega}_{ij} = \Omega_{ij}/\Omega_{ic}$, where $\Omega_{ic} = \sum_{j \in N_c} \Omega_{ij}$ is the *domestic input share* of i.[29] The unit-cost function of producer i in the dual closed economy is given by

$$\check{p}_i = \frac{1}{\check{A}_i} \left(\sum_{j \in N_c+F_c} \check{\Omega}_{ij} \check{p}_j^{1-\theta_i} \right)^{\frac{1}{1-\theta_i}}.$$

In words, the closed dual economy has the same set of producers as the open economy with the same elasticities, except the expenditure shares of each producer on foreign goods has been set to zero, and domestic expenditures have been rescaled so they sum to one. Variables with "inverted-hats" are the closed-economy counterparts of the original variable. The shifter $\check{A}_i$ is the productivity shifter in the closed economy, to be defined below.

Denote the set of producers that directly use imports in their production function by $M_c \subseteq N_c$. If i is an importer $i \in M_c$, we sometimes use the notation $\epsilon_i = \theta_i - 1$ since this

[28] We also extend duality to the case with multiple domestic factors and tariffs in Appendix J. In Appendix L, we also show that duality can even be extended to Roy models with endogenous factor supply, along the lines of Galle et al. (2017).

[29] This means that the domestic input share of every producer must be greater than zero. If the domestic input share of some producer i is zero, then we treat i as a foreign producer and exclude it from the domestic economy. We can do this because if i's domestic input share is zero, then i generates no value-added for the domestic economy.

corresponds to the partial equilibrium trade elasticity for producer i.

6.2 Duality Results

Denote by $\check{W}_c$ the welfare of the dual closed economy. Since the "inverted-hat" economy is closed, welfare is equal to real output $\Delta \log \check{W}_c = \Delta \log \check{Y}_c$.

Theorem 7 (Exact Duality)**.** *The discrete change in welfare $\Delta \log W_c$ of the original open economy in response to discrete shocks to iceberg trade costs or productivities outside of country c is equal to the discrete change in real output $\Delta \log \check{Y}_c$ of the dual closed economy in response to discrete shocks to productivities $\Delta \log \check{A}_i = -(1/\varepsilon_i)\Delta \log \Omega_{ic}$.*

Recall that $\Omega_{ic} = \sum_{j \in N_c} \Omega_{ij}$ is the *domestic input share* of i. In words, when productivity shocks in the closed economy are the negative log change in domestic input shares divided by the trade elasticity, changes in welfare in the closed economy mirror changes in welfare in the open economy. Therefore, we can leverage results from the literature on the real GDP effects of productivity shocks in closed-economies to characterize the welfare effects of trade shocks in open economies.

Corollary 3 (First-Order Duality)**.** *A first-order approximation to the change in welfare of the original open economy is:*

$$\Delta \log W_c = \Delta \log \check{Y}_c \approx \sum_{i \in M_c} \check{\lambda}_i \Delta \log \check{A}_i,$$

where, applying Hulten's theorem, $\check{\lambda}_i$ is the sales share or Domar weight of producer i in the dual closed economy.

Conditional on the size of the associated productivity shocks $\Delta \log \check{A}$, intermediate inputs amplify the gains from trade shocks much in the same way that they amplify productivity shocks in closed economies. This is because sales shares are greater than value-added shares, reflecting an intermediate-input multiplier that magnifies the effect of productivity shocks. This observation is behind the findings of Costinot and Rodriguez-Clare (2014) that allowing for intermediate inputs significantly increases gains from trade.

An easily-missed subtlety is that the sales shares in the closed dual economy $\check{\lambda}$ are *not* the same as the sales shares λ^{Y_c} in the original open economy. To see this, imagine an economy with a representative domestic firm ι. Suppose that the household spends all its income on the domestic firm. Suppose ι is part of a global value chain and its sales are much greater than its value-added, so $\lambda_\iota^{Y_c} > 1$. In this example, the closed economy sales share of this firm is just $\check{\lambda}_\iota = 1 < \lambda_\iota^{Y_c}$, and so the gains from trade in this model are identical to the one-sector model in Arkolakis et al. (2012). This is because input-output linkages outside of a country's borders, although they increase Domar weights, do not amplify trade shocks given changes in observed domestic shares.

We can also use Theorem 7, and the closed-economy results in Baqaee and Farhi (2017a), to provide a second-order approximation of the effect of trade shocks.

Corollary 4 (Second-Order Duality)**.** *The second-derivative of welfare to trade shocks is*

$$\frac{d^2 \log \check{Y}_c}{\mathrm{d}\log \check{A}_j \,\mathrm{d}\log \check{A}_i} = \frac{\mathrm{d}\,\check{\lambda}_i}{\mathrm{d}\log \check{A}_j} = \sum_{k \in N_c} (\theta_k - 1)\check{\lambda}_k Cov_{\check{\Omega}^{(k)}}\left(\check{\Psi}_{(i)}, \check{\Psi}_{(j)}\right).$$

We can re-express the change in welfare in the original open economy as

$$\Delta \log W_c = \Delta \log \check{Y}_c \approx \sum_{i \in M_c} \check{\lambda}_i \Delta \log \check{A}_i + \frac{1}{2}\sum_{k \in N_c} (\theta_k - 1)\check{\lambda}_k Var_{\check{\Omega}^{(k)}}\left(\sum_{i \in M_c} \check{\Psi}_{(i)} \Delta \log \check{A}_i\right).$$

We start by discussing the first equation. It follows from Hulten's theorem that $\mathrm{d}\log \check{Y}_c / \mathrm{d}\log \check{A}_i = \check{\lambda}_i$. This immediately implies that $\mathrm{d}^2 \log \check{Y}_c / (\mathrm{d}\log \check{A}_j \,\mathrm{d}\log \check{A}_i) = \mathrm{d}\,\check{\lambda}_i / \mathrm{d}\log \check{A}_j$. Hence, the nonlinear effect depends on how Domar weights in the closed economy change. The Domar weight of each i changes due to substitution. In response to a shock to j, substitution by k changes the sales of i by $(\theta_k - 1)\check{\lambda}_k Cov_{\check{\Omega}^{(k)}}(\check{\Psi}_{(i)}, \check{\Psi}_{(j)})$. These substitution effects must be weighted by the size $\check{\lambda}_k$ of each k and summed over all k. This equation has a similar intuition, and structure, to the backward propagation equations (6) in Theorem 3.

The second equation in the corollary indicates that the ultimate impact of the shock depends on how heterogeneously exposed each producer k is to the average productivity shock via its different inputs as captured by the term $Var_{\check{\Omega}^{(k)}}\left(\sum_{i \in M_c} \check{\Psi}_{(i)} \Delta \log \check{A}_i\right)$, and on whether these different inputs are complements ($\theta_k < 1$), substitutes ($\theta_k > 1$) or neither ($\theta_k = 1$). It indicates that complementarities lead to negative second-order terms which amplify negative shocks and mitigate positive shocks. Conversely, substitutabilities lead to positive second-order terms which mitigate negative shocks and amplify positive shocks. Of course, there are no second-order terms in the Cobb-Douglas case.

Duality with an Industry Structure. To discuss these results further, we focus on economies with an *industry structure*: producers are grouped into industries and the goods produced in any given industry are aggregated with a CES production function; and all other agents only use aggregated industry goods. In this case, all domestic producers in a given industry are uniformly exposed to any other given domestic producer. This implies that in Corollary 4, only the elasticities of substitution across industries receive non-zero weights.

The elasticities of substitution across producers within industries receive a zero weight, and they only matter via their influence on the productivity shocks through the trade elasticities.

In fact, the matrix $\check{\Omega}$ of the dual closed economy can be specified entirely at the industry level where the different producers are the different industries $\iota \in \mathcal{N}_c$. Given the productivity shocks $\Delta \log \check{A}_\iota$ to the importing industries $\iota \in \mathcal{M}_c$, Theorem 7 and Corollaries 3 and 4 can then be applied at the industry level, with this industry level input-output matrix, and with only elasticities of substitution across industries.

Many cases considered in the literature have such an industry structure, and impose the additional assumption that all the elasticities of substitution across industries (and with the factor) in production and in consumption are unitary (but those within industries are above unity). This makes the dual closed economy Cobb-Douglas. Such assumptions are made for example by Arkolakis et al. (2012), Costinot and Rodriguez-Clare (2014), and Caliendo and Parro (2015). In this Cobb-Douglas case, the dual closed economy is exactly log-linear in the dual productivity shocks $\Delta \log \check{A}_\iota$. The effects of shocks to iceberg trade costs or to productivities outside of the country then coincide with the first-order effects of the dual shocks given by Corollary 3. Their second-order effects given by Corollary 4 are zero, and the same goes for their higher-order effects.

Our results therefore generalize some of the insights of Arkolakis et al. (2012) and of Costinot and Rodriguez-Clare (2014) to models with input-output linkages and where elasticities of substitution across industries (and with the factor) are not unitary.[30] In such models, the dual closed economy is no longer Cobb-Douglas. Deviations from Cobb Douglas generate nonlinearities, which can either mitigate or amplify the effects of the shocks depending on whether there are complementarities or substituabilities, and with an intensity which depends on how heterogeneously exposed the different producers are to the shocks.

Corollary 5 (Exact Duality and Nonlinearities with an Industry Structure)**.** *For country c with an industry structure, we have the following exact characterization of the nonlinearities in welfare changes of the original open economy.*

(i) (Industry Elasticities) Consider two economies with the same initial input-output matrix and industry structure, the same trade elasticities and changes in domestic input shares, but with

[30] Costinot and Rodriguez-Clare (2014) show that the gains from trade are higher in multi-sector economies without input-output linkages when sectors are complements in consumption. Corollary 5 generalizes these results to economies with input-output linkages. This matters quantitatively given that most empirical evidence points to the presence of much more important complementarities in production than in consumption. Furthermore, even if the production and consumption elasticities were the same, Corollary 4 shows that given the size of the trade shocks $\Delta \log \check{A}_i$, the nonlinear effects of non-unitary elasticities $\theta_k - 1$ scale with the size of the Leontief inverse and the Domar weights. Therefore, even if the elasticities are identical in consumption and production, input-output linkages amplify the gains from trade.

lower elasticities across industries for one than for the other so that $\theta_\kappa \leq \theta'_\kappa$ for all industries κ. Then $\Delta \log W_c = \Delta \log \check{Y}_c \leq \Delta \log W'_c = \Delta \log \check{Y}'_c$ so that negative (positive) shocks have larger negative (smaller positive) welfare effects in the economy with the lower elasticities.

(ii) *(Curvature) Suppose that all the elasticities of substitution θ_κ across industries are less than (greater than) unity, then $\Delta \log W_c = \Delta \log \check{Y}_c$ is concave (convex) in $\Delta \log \check{A}$. So nonlinearities amplify (mitigate) negative shocks and mitigate (amplify) positive shocks compared to a loglinear approximation.*

Since elasticities of substitution across industries are likely below one, Corollary 5 suggests that accounting for nonlinearities will amplify the gains relative to autarky, but mitigate the gains from opening up further (for fixed changes in import shares).

7 Analytical Examples

In this section, we consider some simple examples to hone intuition and illustrate the sorts of questions our results can be used to answer. For each example, the section with the relevant propositions is listed in parenthesis.

Two Countries with Arbitrary IO Linkages (Section 4). This example uses the forward and backward propagation equations in Theorem 3 to linearize a model with arbitrary input-output relationships and two single-factor countries.

Consider a two-country economy (home and foreign), with each country owning one primary factor. Hence, $C = F = 2$. Denote foreign variables by an asterisk and let L index the home factor and L_* the foreign factor. Assume that there are no wedges, and consider a productivity shock $d \log A_j$ to producer j. Applying Theorem 3, the change in the home factor's share of income is

$$\frac{d \log \Lambda_L}{d \log A_j} = \frac{\sum_k (\theta_k - 1) \lambda_k Cov_{\Omega^{(k)}} \left(\Psi_{(j)}, \frac{\Psi_{(L)}}{\Lambda_L} \right)}{1 + \frac{\Lambda_L}{(1-\Lambda_L)} \sum_k (\theta_k - 1) \lambda_k Var_{\Omega^{(k)}} \left(\frac{\Psi_{(L)}}{\Lambda_L} \right) - \left(\Lambda_L^W - \Lambda_L^{W_*} \right)}. \tag{9}$$

The numerator captures the fact that a shock to j directs demand towards the home factor L if inputs are substitutes $\theta_k > 1$ and exposure to j and L are positively correlated $Cov_{\Omega^{(k)}}(\Psi_{(j)}, \Psi_{(L)}) > 0$ (this is reversed if inputs are complements). In this case, as k substitutes to use inputs most heavily exposed to j, it boosts demand for the home factor L.

The denominator captures the general equilibrium effects of changes in factor prices. An increase in the price of L triggers its own substitution effects and redistributes income between home and foreign. The terms in the denominator reflect these two effects. If inputs are substitutes $\theta_k > 1$ and k is heterogeneously exposed $Var_{\Omega^{(k)}}(\Psi_{(L)}) > 0$ to L, then an increase in the price of L will cause k to substitute away from L and this mitigates the partial equilibrium effect in the numerator. The final term in brackets in the denominator accounts for the fact that an increase in the income share of L raises the income share of the domestic consumer and lowers the income share of the foreign consumer. If the domestic consumer is more heavily exposed to the domestic factor $\Lambda_L^W > \Lambda_L^{W_*}$, then this amplifies the partial equilibrium effect in the numerator.

Since the factor shares must sum to one, we know that $d\Lambda_L = -d\Lambda_{L_*}$. This gives us closed-form equations for changes in both factor shares. Hence, Theorem 3 can be used to obtain closed-form expressions for changes in the sales share and the price of every other producer in the economy.

Dutch Disease in a Cobb-Douglas Model (Section 4). To make the previous example more concrete, we apply Equation (9) to a Cobb-Douglas economy, and use it to characterize conditions under which the home economy experiences Dutch disease. The example below also shows how to map a specific nested-CES model into *standard-form* required by Theorem 3.

Suppose there are n industries at home and foreign. The utility function of home and foreign consumers is

$$W = \prod_{i=1}^{n} (x_{0i})^{\Omega_{0i}}, \qquad W_* = \prod_{i=1}^{n} (x_{0i}^*)^{\Omega_{0i}},$$

where x_{0i} and x_{0i}^* are home and foreign consumption of goods from industry i. The production function of industry i (at home or foreign) is a Cobb-Douglas aggregate of intermediates and the local factor

$$y_i = L_{ij}^{\Omega_{iL}} \prod_{i=1}^{n} x_{ij}^{\Omega_{ij}}.$$

Suppose that the intermediate good x_{ij} is a CES combination of domestic and foreign varieties of j, with initial home share Ω_j and foreign share $\Omega_j^* = 1 - \Omega_j$, and elasticity of substitution $\varepsilon_j + 1$. Since the market share of home and foreign in industry j does not vary by consumer i, this means there is no home-bias.

In standard-form, this economy has $N = 3n$ producers: the first n are industries at home, the second n are industries in foreign, and the last n are CES aggregates of domestic and foreign varieties that every other industry buys. The HAIO matrix for this economy, in standard-form, is $(2+3n+2) \times (2+3n+2)$:

$$\Omega = \left[\begin{array}{c|c|c|c|cc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \left[\begin{array}{c}\Omega_{0i}\end{array}\right]_{i=1}^{n} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \left[\begin{array}{c}\Omega_{0i}\end{array}\right]_{i=1}^{n} & \mathbf{0} & \mathbf{0} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \left[\begin{array}{c}\Omega_{ij}\end{array}\right]_{i,j=1}^{n} & \left[\begin{array}{c}\Omega_{iL}\end{array}\right]_{i=1}^{n} & \mathbf{0} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \left[\begin{array}{c}\Omega_{ij}\end{array}\right]_{i,j=1}^{n} & \mathbf{0} & \left[\begin{array}{c}\Omega_{iL}\end{array}\right]_{i=1}^{n} \\
\hline
\mathbf{0} & \begin{array}{ccc}\Omega_1 & \cdots & 0 \\ & \ddots & \\ 0 & & \Omega_n\end{array} & \begin{array}{ccc}\Omega_1^* & \cdots & 0 \\ & \ddots & \\ 0 & & \Omega_n^*\end{array} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right].$$

The first two rows and columns correspond to the households, the next $2n$ rows and columns correspond to home industries and foreign industries respectively. The next n rows and columns correspond to bundles of home and foreign varieties. The last two rows and columns correspond to the home and foreign factor. The vector elasticities of substitution θ for this economy is a vector with $2+3n$ elements $\theta = (1, \cdots, 1, \varepsilon_1 + 1, \cdots, \varepsilon_n + 1)$, where ε_i is the trade elasticity in industry i.

Using Equation (9), the change in home's share of income following a productivity shock $d \log A_j$ to some *domestic* producer j is

$$\frac{d \log \Lambda_L}{d \log A_j} = \frac{\lambda_j}{\Lambda_L} \frac{\varepsilon_j \Omega_j^* \Omega_{jL}}{1 + \sum_i \varepsilon_i \frac{\lambda_i \Omega_{iL}}{\Lambda_L} \frac{\Omega_{iL}}{1-\Lambda_L} \Omega_i^*} \geq 0,$$

which is positive as long as domestic and foreign varieties are substitutes $\varepsilon_j > 0$. The numerator captures the fact that a shock to j will increase demand for the home factor if j uses the home factor $\Omega_{jL} > 0$. The denominator captures the fact that an increase in the price of the home factor attenuates the increase in demand for the home factor by bidding up the price of home goods.

The positive productivity shock to j will therefore shrink the market share of every other domestic producer, a phenomenon known as Dutch disease. To see this, apply Theorem 3 to some domestic producer $i \neq j$ to get

$$\frac{d \log \lambda_i}{d \log A_j} = -\varepsilon_i \Omega_i^* \frac{\Omega_{iL}}{1 - \Lambda_L} \frac{d \log \Lambda_L}{d \log A_j} < 0.$$

In words, the shock to j boosts the price of the home factor, which makes i less competitive in the world market if i relies on the home factor $\Omega_{iL} > 0$. Of course, (9) can easily be used to write down the necessary and sufficient conditions for Dutch disease for the more general model as well.

Incidence of Tariffs with Global Value Chain (Section 4). Theorem 3 can also be used to compute the incidence of tariffs on different factors in the presence of input-output linkages. For example, consider the simple economy depicted in Figure 1. Country 1 is the home country and country 2 represents the rest of the world. The home country has two factors: L_1 representing manufacturing labor and L_3 representing services labor. Manufacturing labor participates in a global value-chain with the rest of the world, whereas services labor sells domestically only.

The rest of the world is kept simple, and foreign factors can either be used as part of the value-chain with home or they can be used directly to supply foreign consumers. To simplify the algebra, we make the stark assumption that the foreign market is perfectly competitive — that is, the elasticity of substitution for foreign consumers $\theta_{H_2} = \infty$.

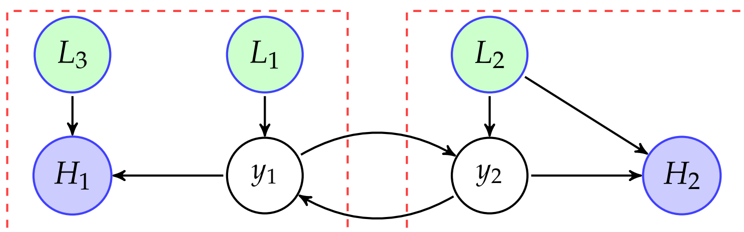

Figure 1: Solid lines show flow of goods. Green, purple, and white nodes are factors, households, and goods. Boundaries of countries are represented by dashed boxes.

Now, suppose that country 1 introduces a tariff $d \log \mu$ on foreign imports in an attempt to shield manufacturing workers from foreign competition. We can use Theorem 3 to calculate the change in the real wages of both types of workers. In this example, the policy backfires, since the real wage of manufacturing workers is

$$\lim_{\theta_{H_2} \to \infty} \frac{d \log \Lambda_{L_1}}{d \log \mu} - \frac{d \log p_{H_1}}{d \log \mu} = -\frac{\Omega}{1-\Omega} < 0,$$

where Ω is the intermediate input share of y_1. The losses increase in the intermediate input share. Intuitively, the tariff raises the marginal cost of y_1. Since the foreign market is perfectly competitive $\theta_{H_2} = \infty$, the price of the y_1 falls by exactly enough to offset the tariff. This comes about via a reduction in manufacturing workers's wages. Service workers are unaffected by the tariff $\lim_{\theta_{H_2} \to \infty} d \log \Lambda_{L_3} - d \log p_{H_1} = 0$. Welfare overall for the home country does not change $d \log W_{H_2} = 0$, because the reduction in the real wages of the manufacturing workers are precisely cancelled out by the real revenue raised by the tariffs.

Trade War with Global Value Chain (Section 5). To see how input-output connections can amplify the losses from protectionism, consider the example depicted in Figure 2. Assume the two countries are symmetric, let Ω be imports as a share of sales, and θ be the elasticity of substitution between intermediates and labor.

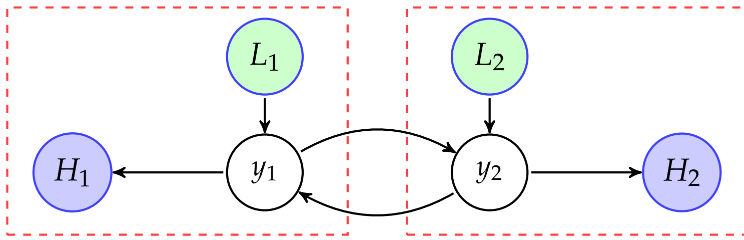

Figure 2: Solid lines show flow of goods. Green, purple, and white nodes are factors, households, and goods. Boundaries of countries are represented by dashed boxes.

Suppose that each country introduces a symmetric tax $\Delta \log \mu$ on its imports from the other country. By symmetry, changes in country real output, country welfare, world real output, and world welfare are all the same. Hence, using Theorem 4 and Theorem 6, up to a second order approximation, the reduction in real GDP and welfare are

$$\Delta \log W = \Delta \log Y \approx -\frac{1}{2}(\lambda_{12}\Delta \log y_{12}\Delta \log \mu + \lambda_{21}\Delta \log y_{21}\Delta \log \mu) \approx \theta\frac{\Omega}{2(1-\Omega)^2}(\Delta \log \mu)^2,$$

where y_{ij} is the quantity of imports from country j by country i, λ_{ij} is the corresponding sales share. By symmetry $y_{12} = y_{21}$ and $\lambda_{12} = \lambda_{21}$.

The losses increase with the elasticity of substitution θ and with the intermediate input share Ω. This is both because the relevant sales shares $\lambda_{12} = \lambda_{21} = \Omega/[2(1-\Omega)]$ and the reductions in the quantities of imports $-\Delta \log y_{12} = -\Delta \log y_{21} = [\theta/(1-\Omega)]\Delta \log \mu$ are increasing in θ and Ω. The latter effect occurs because when Ω is higher, goods effectively cross borders more times, and hence get hit by the tariffs more times, which increases the relative price of imports more and leads to a larger reduction in their quantity.

The Gains from Trade with Critical Inputs (Section 6). The last example uses the duality results in Section 6 to give some intuition for how nonlinearities in the domestic production network affect the gains from trade. In particular, how complementarities in the domestic economy amplify the losses from moving towards autarky and mitigate the gains from further trade liberalization. In this example, we consider how the existence of a universal intermediate input, like foreign energy, can increase the losses of moving to autarky.

Consider country c depicted in Figure 3. The only traded good is energy E.[31] The

[31] This example is an open-economy version of an example in Baqaee and Farhi (2017a).

household consumes domestic goods 1 through to N with some elasticity of substitution θ_0 and equal sales shares $1/N$ at the initial point. Goods 1 through to M are made using labor L and energy E with an elasticity of substitution θ_1, with an initial energy share $(N/M)\check{\lambda}_E$. Energy is a CES aggregate of domestic and foreign energy with elasticity of substitution $\theta_E > 1$. Domestic energy E and consumption goods $M+1$ through to N are made using only domestic labor. Assume that the elasticity of substitution in production $\theta_1 < 1$, and that production has stronger complementarities than consumption $\theta_1 < \theta_0$.

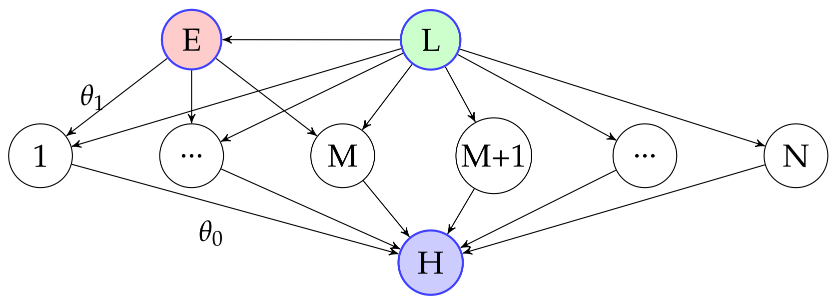

Figure 3: Industries substitutes across labor and energy with elasticity $\theta_1 < 1$. The household substitutes with elasticity of substitution $\theta_0 > \theta_1$. Energy is produced domestically and sourced from foreign with an elasticity of substitution $\theta_E > 1$.

Consider an increase in iceberg trade costs that increase the cost of importing foreign energy. The welfare effect of this trade shock is the same as that of a negative productivity shock to the energy producer of the dual closed economy

$$\Delta \log \check{A}_E = -\frac{1}{\varepsilon_E} \Delta \log \check{\Omega}_{Ec} < 0,$$

where $\varepsilon_E = \theta_E - 1$ is the trade elasticity of the energy composite good E and $\Delta \log \Omega_{Ec}$ is the change of its domestic expenditure share.

Corollary 4 shows that, to a second order, the change in welfare is[32]

$$\begin{aligned}\Delta \log W_c \approx& \check{\lambda}_E \Delta \log \check{A}_E + \frac{1}{2} \sum_{k \in N_c} (\theta_k - 1) \check{\lambda}_k Var_{\check{\Omega}^{(k)}} \left(\check{\Psi}_{(E)} \Delta \log \check{A}_E \right), \\ =& \check{\lambda}_E \Delta \log \check{A}_E + \frac{1}{2} \check{\lambda}_E \left((\theta_0 - 1) \check{\lambda}_E (\frac{N}{M} - 1) + (\theta_1 - 1)(1 - \frac{N}{M} \check{\lambda}_E) \right) (\Delta \log \check{A}_E)^2.\end{aligned}$$

When $M = N$, energy becomes a universal input, and the elasticity of substitution in consumption θ_0 drops out of the expression because $Var_{\check{\Omega}^{(0)}}(\check{\Psi}_E) = 0$. This is because all consumption goods are uniformly exposed to the trade shock, and so substitution by the

[32]Since the closed dual economy in this example is acyclic, we can actually write the output function in closed-form. See Appendix J for more details.

household is irrelevant. Since $\theta_1 < 1$, nonlinearities captured by the second-order term amplify the negative welfare effects of the trade shock. This is because complementarities between energy and labor imply that the sales share of energy $\check{\lambda}_E$ increases with the shock, thereby amplifying its negative effect.

When $M < N$, the elasticity of substitution in consumption θ_0 matters. Since $\theta_0 > \theta_1$, the nonlinear adverse effect of the trade shock is reduced compared to the case $M = N$ when we keep the initial sales share of energy $\check{\lambda}_E$ constant. This is true generally but the effect is easiest to see when $\theta_0 > 1$ since the household can now substitute away from energy-intensive goods, which mitigates the increase of the sales share of energy $\check{\lambda}_E$, and hence the negative welfare effects of the shock. These effects are stronger, the lower is M, i.e. the more heterogeneous are the exposures of the different goods to energy.

If θ_0 and θ_1 are both less than one, then Corollary 5 implies that domestic welfare is concave in the trade shock $\Delta \log \check{A}_E$. Hence, for the same magnitude change in import shares, complementarities magnify the losses from moving towards autarky and mitigate the benefits of further integration relative to when the domestic input-output network is Cobb-Douglas ($\theta_0 = \theta_1 = 1$).

8 Quantitative Examples

In this section, we use a multi-factor production network model calibrated to match world input-output data. We quantify the way increasing trade costs (tariffs or iceberg) affect output, welfare, and factor rewards, and use our analytical results to give intuition for our findings. We provide flexible Matlab code, detailed in Appendix K, that loglinearizes arbitrary general equilibrium models of the type studied in this paper and computes local and global comparative statics.

Calibration. The benchmark model has 40 countries as well as a "rest-of-the-world" composite country, each with four factors of production: high-skilled, medium-skilled, low-skilled labor, and capital. Each country has 30 industries each of which produces a single industry good. The model has a nested-CES structure. Each industry produces output by combining its value-added (consisting of the four domestic factors) with intermediate goods (consisting of the 30 goods). The elasticity of substitution across intermediates is θ_1, between factors and intermediate inputs is θ_2, across different primary factors is θ_3, and the elasticity of substitution of household consumption across industries is θ_0. When a producer or the household in country c purchases inputs from industry j, it consumes a CES aggregate of goods from this industry sourced from various countries with elasticity

Table 1: Change in welfare, in log points, for a subset of countries in response to a universal 10%, 5%, and 1% change in trade costs, and comparison to a loglinear approximation.

	10% Shock		5% Shock		1% Shock	
Universal Iceberg Shock	Nonlinear	Loglinear	Nonlinear	Loglinear	Nonlinear	Loglinear
China	-1.30	-1.40	-0.69	-0.72	-0.14	-0.15
Great Britain	-2.52	-3.16	-1.42	-1.62	-0.32	-0.33
Luxembourg	-16.86	-19.75	-9.31	-10.11	-2.02	-2.06
Russia	-2.97	-3.00	-1.52	-1.54	-0.31	-0.31
USA	-1.06	-1.32	-0.60	-0.68	-0.13	-0.14
World	-2.26	-2.75	-1.26	-1.41	-0.28	-0.28
Universal Tariff Shock						
China	-0.16	0.93	0.12	0.48	0.08	0.10
Great Britain	-0.65	-0.40	-0.29	-0.20	-0.05	-0.04
Luxembourg	-5.37	-3.05	-2.17	-1.56	-0.33	-0.32
Russia	-1.59	-1.17	-0.74	-0.60	-0.13	-0.12
USA	0.09	0.30	0.07	0.15	0.03	0.03
World	-0.43	-0.85*	-0.15	-0.22*	-0.01	-0.01*

$*$ denotes a second order approximation following Theorem 6, because the first-order effect would be zero.

of substitution $\varepsilon_j + 1$. We use data from the World Input-Output Database (WIOD) (see Timmer et al., 2015) to calibrate the CES share parameters to match expenditure shares in the year 2008.[33]

We use the estimates from Caliendo and Parro (2015) to calibrate the elasticities ε_i between traded and domestic varieties of each industry. We set the elasticity of substitution across industries $\theta_2 = 0.2$, the one between value-added and intermediates $\theta_1 = 0.5$, and the one in consumption $\theta_0 = 0.9$. These elasticities are broadly consistent with the estimates of Atalay (2017), Boehm et al. (2015), Herrendorf et al. (2013), and Oberfield and Raval (2014). Finally, we set the elasticity of substitution among primary factors $\theta_3 = 0.5$. Overall, the evidence suggests that these elasticities are all less than one (sometimes significantly so). Appendix D contains additional details about how the model is mapped to the data.

Effect of Trade Barriers. In Table 1, we report the impact on welfare for a few countries of a universal increase in either the iceberg costs of trade or import tariffs. We compare the nonlinear response of the benchmark economy to the loglinear approximation implied by Theorem 2.

Across the board, and as suggested by the discussion of trapezoids and triangles in Section 5.1, an increase in iceberg trade costs is significantly more costly than an increase

[33] Since most tariffs in 2008 are close to zero, for simplicity, we assume that tariffs are equal to zero at the initial equilibrium. In Appendix G, we show that recomputing the results using initial tariffs does not meaningfully alter the results.

in tariffs. For the world, a universal 10% increase in iceberg costs reduces output by 2.26%. A similar increase in tariffs only reduces output by 0.43%. In Appendix B, we show that abstracting from intermediate inputs reduces these estimates by a factor of two or three.

For 1% shocks, the loglinear approximation performs very well for both iceberg shocks and tariff shocks. The approximation performs less well as the shocks get larger. For example, for a 10% universal increase in iceberg costs, a loglinear approximation suggests that world output should fall by 2.75% instead of 2.26%.

To compute the nonlinear effect of the shock, we can either solve the nonlinear system of supply and demand relationships (i.e. exact hat algebra), or we can repeatedly compute first-order approximations and chain the results. For exact hat algebra, we use a state-of-the-art numerical solver (Artelys Knitro), and we provide the solver with analytical derivatives. To compute a new equilibrium, the solver takes around 12 hours on a standard desktop. The linear approximation, on the other hand, takes around four minutes.[34] Therefore, differential exact hat algebra, where we take the derivative 20 times and cumulate the results, is about 10 times faster than exact hat algebra.

Table 2: Decomposition of welfare changes following Theorem 2 for a 10% universal iceberg shock.

	Δ log Welfare	Δ Technology	Δ Reallocation
China	-1.30	-1.88	0.58
Great Britain	-2.52	-2.51	-0.01
Luxembourg	-16.86	-4.00	-12.86
Russia	-2.97	-2.41	-0.57
USA	-1.06	-1.43	0.37
World	-2.26	-2.25	-0.01

Table 2 uses the chained-derivatives to decompose welfare changes into technology and reallocation effects following Theorem 2 due to a uniform 10% increase in all iceberg costs. To compute the numbers in Table 2, we compute the reallocation and technology effect locally following Theorem 2 and cumulate the results.[35]

Recall that the technology effect is the direct effect of the iceberg shock on households, holding fixed the distribution of resources. The reallocation effect measures the change in welfare that results from the (endogenous) redistribution of resources. These reallocation effects are not the same as changes in the terms of trade. Instead, they are related to the

[34] Solving the linear system described in Theorem 3 takes seconds; the four minutes are almost entirely spent constructing the relevant matrix representation that needs to be inverted.

[35] As with all nonlinear decompositions, the order in which we decompose effects matters. For this nonlinear decomposition, we simultaneously increase all iceberg costs at the same time.

factoral terms of trade introduced by Viner (1937) (see Appendix H for more discussion).

Naturally, small and very open economies, like Luxembourg, Belgium, Ireland, and Taiwan are worst affected by such a shock. Partly, this reflects the fact that their domestic consumers are more exposed to foreign goods, and this effect is captured by the pure technology effect. However, for small open economies, there are also large negative reallocation effects, whereas for large economies, like China or the USA, reallocation effects are positive. Intuitively, as trade becomes more restricted, expenditures shift away from imports and towards domestic factors. For small open economies, this means that the share of income claimed by their domestic factors falls. For very small countries, these negative reallocation effects are as or more important than the direct effect of the technology shock from exposure to traded goods. Naturally, for the world as a whole, there are no reallocation effects.

Gains from Trade: Intermediate Inputs and Nonlinearities. Finally, we use the duality in Theorem 7, to calculate the welfare losses from moving different countries to autarky. For this exercise, we aggregate the factors in each country into a single representative factor. The "dual" productivity shocks corresponding to autarky are $\Delta \log \check{A}_i = -(1/\varepsilon_i) \log \Omega_{ic}$, since all domestic input shares must go to one in autarky.

Table 3: Gains from trade for a selection of countries.

$(\theta_0, \theta_1, \theta_2)$	VA $(1,1,1)$	$(1,1,1)$	$(1,0.5,0.6)$	$(0.9,0.5,0.2)$
France	9.8%	18.5%	24.7%	30.2%
Japan	2.4%	5.2%	5.5%	5.7%
Mexico	11.5%	16.2%	21.3%	44.5%
USA	4.5%	9.1%	10.3%	13.0%

The first column is a multi-sector economy with no intermediates and Cobb-Douglas production/consumption. The second column has intermediates but maintains Cobb-Douglas. The third column has intermediates and complementarities. The final column is our benchmark calibration. The micro trade elasticities are kept constant, so the size of the shock to each industry is the same across all columns.

The gains from trade are in Table 3 for different values of the elasticities of substitution $(\theta_0, \theta_1, \theta_2)$. The first column replicates the results of a multi-sector model without intermediate inputs and with the Cobb-Douglas assumption $(\theta_0, \theta_1, \theta_2) = (1, 1, 1)$, reported in Costinot and Rodriguez-Clare (2014).The second column replicates the results of an a model which allows for intermediate inputs but maintains the Cobb-Douglas assumption, also reported in Costinot and Rodriguez-Clare (2014). As expected, allowing for intermediate inputs increases gains from trade. This is because of the first-order or log-linear effect captured by Corollary 3: it reflects the fact that abstracting away from intermediate inputs reduces the volume of imports relative to GDP. The other columns continue to allow for intermediate inputs, but deviate from the Cobb-Douglas assumption, giving rise to nonlinearities. Moving across columns towards more complementarities increases the gains from trade. This is because of the nonlinear effect captured by Corollary 4: more complementarities magnify gains from trade by increasing nonlinearities. Our benchmark calibration is the one on the far right, but the second to last column shows that even with milder complementarities, which are probably more relevant for longer-run applications, the nonlinearities remain sizeable.

The magnitudes of these different effects are different across countries. The importance of accounting for intermediate inputs is largely independent of the degree of openness of the country. By contrast, the importance of accounting for nonlinearities does depend on the degree of openness: the more open the country, the larger are the dual productivity shocks, and hence, the more nonlinearities matter. Overall, it seems that nonlinearities are as important as intermediate goods to the study of gains from trade.

9 Conclusion

This paper establishes a unified framework for studying output and welfare in open and potentially distorted economies. We provide ex-post sufficient statistics for measurement and ex-ante sufficient statistics for conducting local and global counterfactuals. Our formulas bring together results from the open and closed-economy literatures, and provide new characterizations of the gains from trade and the losses from trade protectionism. As discussed in the appendix, these results also have implications for the aggregation of trade elasticities, and the distributional consequences of trade policy.

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