NBER WORKING PAPER SERIES

INTERNATIONAL COMOVEMENT IN THE GLOBAL PRODUCTION NETWORK

Zhen Huo
Andrei A. Levchenko
Nitya Pandalai-Nayar

Working Paper 25978
http://www.nber.org/papers/w25978

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2019, Revised February 2020

We are grateful to David Baqaee, Chris Boehm, Lorenzo Caliendo, Yongsung Chang, Gabe Chodorow-Reich, Olivier Coibion, Javier Cravino, Emmanuel Farhi, Jesus Fernandez-Villaverde, Simon Gilchrist, Felipe Saffie, Kang Shi, Alireza Tahbaz-Salehi, Linda Tesar and seminar participants at various institutions for helpful comments, and to Barthélémy Bonadio for superb research assistance. This paper supersedes “The Global Business Cycle: Measurement and Transmission.” The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

International Comovement in the Global Production Network
Zhen Huo, Andrei A. Levchenko, and Nitya Pandalai-Nayar
NBER Working Paper No. 25978
June 2019, Revised February 2020
JEL No. F41,F44

ABSTRACT

This paper provides a general and unified framework to study the role of production networks in international GDP comovement. We first derive an additive decomposition of bilateral GDP comovement into components capturing shock transmission and shock correlation. We quantify this decomposition in a parsimonious multi-country, sector and factor network propagation model featuring a single composite supply shock, with data for 29 countries and up to 30 years. We find that the network transmission of shocks can only account for a minority of observed comovement under a range of standard values of structural elasticities. To assess the role of delayed propagation and intertemporal shocks– features absent in the standard static framework– we extend both the accounting decomposition and the model to a dynamic setting and enrich the space of shocks. Quantitatively, delayed propagation contributes relatively little to the overall GDP comovement compared to the impact effects captured by the static production networks model. Models featuring two intratemporal shocks (TFP and labor supply) strike a good balance between parsimony and fit to the data.

Zhen Huo
Department of Economics
Yale University
New Haven, CT 06520
zhen.huo@yale.edu

Andrei A. Levchenko
Department of Economics
University of Michigan
611 Tappan Street
Ann Arbor, MI 48109
and CEPR
and also NBER
alev@umich.edu

Nitya Pandalai-Nayar
Department of Economics
University of Texas at Austin
3.142 BRB, 2225 Speedway
Austin, TX 78713
and NBER
npnayar@utexas.edu

1 Introduction

Production networks play an important role in transmitting shocks within countries and amplifying aggregate fluctuations.[1] Since input networks increasingly cross national borders, a natural conjecture is that they may also help explain the well known positive correlation of real GDP growth across countries. However, we still lack a comprehensive theoretical and quantitative account of the role of production networks in international GDP synchronization. This paper provides a general and unified framework to address this question.

We proceed in three stages. First, we derive an additive decomposition of bilateral GDP comovement into components capturing shock transmission and shock correlation. Second, we quantify this decomposition in a parsimonious multi-country, sector and factor network propagation model featuring a single composite supply shock. We find that transmission of shocks through the network can only account for a minority of observed comovement under a range of standard values of structural elasticities. Third, we extend both the accounting decomposition and the model to a dynamic setting and enrich the space of shocks, to assess the role of delayed propagation and intertemporal shocks. Quantitatively, delayed propagation contributes relatively little to the overall GDP comovement compared to the impact effects captured by the static model. Models featuring two intratemporal shocks (TFP and labor supply) provide a reasonably good fit to the data.

Section 2 sets up a simple accounting framework that clarifies the mechanisms at play and objects of interest for measurement. Two countries can experience positive comovement if shocks in one country influence the other country's GDP through trade and production linkages. Comovement also arises if influential sectors in the two economies have correlated shocks. The GDP covariance between two countries can be expressed as a function of the primitive shock covariances and a global influence matrix. The latter collects the general equilibrium elasticities of GDP in each country with respect to all sector-country-specific shocks worldwide, and thus translates the variances and covariances of the primitive shocks into comovements of GDP. We show that the GDP covariance between two countries can be written as a sum of two terms, respectively capturing correlated shocks and transmission.

The accounting framework provides a road map for quantification and measurement. First, as the global influence matrix will always be model-dependent, we must impose sufficient structure and bring sufficient data on international production network linkages to recover the global influence matrix. Second, we must measure some underlying shocks to determine the extent of their correlation across countries. This will allow us to establish both how the influence matrix interacts with the shock

[1] The closed-economy literature on the micro origins of aggregate fluctuations and on shock propagation via input linkages goes back to Long and Plosser (1983), and has been modernized recently following the seminal contributions by Gabaix (2011) and Acemoglu et al. (2012). Jones (2011, 2013) shows that amplification of distortions in the input-output network can also help account for the large cross-country per capita income differences.

correlation, and how it produces transmission.

Section 3 sets up a multi-country, multi-sector, multi-factor model of the world economy, and implements it on sector-level data for 29 countries and up to 30 years. Countries trade both intermediate and final goods. Each sector uses labor, capital, and intermediate inputs that can come from any sector and country in the world. Sector-specific supply shocks propagate through the production network both domestically and internationally. Closed-economy models of network propagation (e.g. Acemoglu et al., 2012; Baqaee and Farhi, 2019a) write the change in real GDP as an inner product of the vector of sectoral shocks and the influence vector. We extend this approach to an international setting, and write the change in GDP of a single country as an inner product of the vector of shocks to all countries and sectors in the world and the country-specific influence vector that collects the elasticities of that country's GDP to every sectoral shock in the world. A key feature of our analysis is that we provide a first-order analytical solution for this influence vector in a multi-country general equilibrium setting. This analytical solution expresses the influence matrix in terms of observables that can be measured and a small number of structural elasticities.

The quantitative framework provides a means for shock measurement. As the global influence matrix translates sector-country specific shocks to equilibrium changes in output, it can also be inverted to infer supply shocks that rationalize the observed output changes. By construction, when these shocks are fed back into the model, they reproduce each country's GDP, and hence observed international GDP correlations exactly. This exercise is an open-economy version of Foerster, Sarte, and Watson (2011).[2]

The influence matrix and the recovered shocks are the two ingredients required to implement the decomposition of the overall comovement into the correlated shocks and transmission components from Section 2. Our first main finding is that the supply shocks required to rationalize output growth are correlated across countries, regardless of the choice of the elasticities needed to calibrate the influence matrix. In our preferred calibration the shock correlation accounts for about four-fifths of the total GDP correlation, with the transmission component responsible for the remaining one-fifth. Thus, while network linkages do propagate shocks across borders, the internal transmission mechanisms in global production network models cannot generate the bulk of observed comovement.

We perform two counterfactuals that further explore the role of the input network in generating comovement. First, we compare the baseline economy to one in which countries are in autarky. This exercise reveals an underappreciated mechanism through which trade opening affects GDP comovement: it changes the relative influence of domestic sectors. We write the difference in GDP

[2] In our framework, the TFP and factor supply shocks have the same effect on the global vector of output changes, up to a scaling factor. Thus, inverting the global influence matrix recovers a composite supply shock, which is sufficient to answer the main question posed in Section 3. This property is related to the point stressed by Baqaee and Farhi (2019a) that factor supply shocks can always be restated as productivity shocks, and thus are in that sense isomorphic to them. Section 4 unpacks the shocks.

comovement between the trade and autarky equilibria as a sum of two terms: the international transmission of shocks, and the changes in the influence of the domestic shocks times the covariances of those shocks. The second term captures whether trade opening leads to an expansion or contraction of sectors with more correlated shocks. International transmission is positive in the trade equilibrium and increases comovement relative to autarky. The second effect tends to be negative on average, and can be quantitatively large. For instance, in the G7 it reduces average GDP correlations by 0.05-0.1, depending on the calibration. This result reveals an unexpected role of input linkages in cross-border comovement: there are cases in which trade opening leads to a shift away from the most internationally correlated sectors.[3] Second, we calibrate a value added-only model in which there are no input linkages and all production and trade is in final goods. This framework is common in the international business cycle literature. The model that ignores the input network underpredicts comovement in all calibrations, and by about 8-10% in our preferred calibration.

Section 3 focuses on the role of the input network in amplifying or dampening the correlations of the sectoral shocks, but leaves several open questions. As is conventional in the literature on shock transmission in production networks, the model is static. Therefore it is silent on dynamic propagation and delayed responses of economies to shocks. The composite supply shock is by construction not especially informative on the underlying drivers of business cycles in general, and of international comovement in particular. Finally, connecting the two points above, intertemporal shocks cannot be extracted using the static influence matrix.

Section 4 introduces dynamics and enriches the model to allow for several distinct shocks. Capital can now be accumulated in each sector between periods, in response to both domestic and foreign shocks. In our theoretical framework the contemporaneous response of GDP in the dynamic model to intratemporal shocks is characterized by the global influence matrix, and thus coincides with the GDP response of the static world economy studied in Section 3. In addition, as illustrated in Section 2 the static and the dynamic components of the total covariance are simply additive. Thus the static and dynamic comovement are separable, and our framework bridges the network propagation and the international business cycle literatures. We consider 4 shocks: TFP, labor, capital/investment, and intermediate input. Together, these 4 shocks rationalize the data on output and labor, capital, and intermediate inputs.

Quantitatively, the intertemporal transmission through capital accumulation is much less important for comovement than the impact effects of correlated shocks amplified by the production network. We write overall GDP correlation as the sum of the correlations of the contemporaneous change

[3] This is a purely quantitative finding, arising from the particular correlation properties of the estimated shocks and input coefficients. Nonetheless, it is a counterexample to the effect often invoked in the optimum currency area literature, whereby trade integration is expected to increase comovement by making aggregate shocks more correlated (e.g. Frankel and Rose, 1998). We reveal an alternative mechanism, through which countries might become less correlated with trade integration despite the same underlying sectoral shocks.

in GDPs due to a shock innovation, and the infinite sum of responses to all the past innovations. The component capturing the contemporaneous effects of shocks accounts for 80% of the total GDP correlations.

Finally, we simulate the dynamic model conditional on subsets of shocks to understand which ones are most important for comovement. No single shock has a dominant role in international comovement. Individually, the labor and the TFP shocks appear most promising, but for different reasons.[4] The labor shock has the highest synchronizing impact, with the qualification that much of its overall effect appears to come from its correlation with other shocks rather than with itself. Taken alone, the TFP and labor shocks generate similar amounts of GDP comovement, but the TFP shock also produces GDP series closer to the data.

A model that combines labor and TFP shocks strikes a good balance between parsimony and fit to the data. The two shocks together generate more than two-thirds of the observed international correlation, and produce behavior of GDP similar to the data. This specification is parsimonious both in the sense that it relies on only two shocks, as well as in the sense that these shocks themselves are relatively simple, and would work in the same way in both static and dynamic models. The next shock in order of importance is the investment shock. Adding it to TFP and labor essentially reproduces the data. However, adding this shock comes at a cost of parsimony, especially because both extracting and using this shock requires solving and iterating on the full dynamic model. The intermediate input shock is the least important, either by itself or in conjunction with other shocks. A quantification is not missing much by omitting it.[5]

Related Literature Our paper draws from, and contributes to two literatures. The first is the active recent research agenda on shock propagation in production networks. A number of closed-economy papers following the seminal contributions of Carvalho (2010), Gabaix (2011) and Acemoglu et al. (2012) enrich the theory, provide econometric evidence, and estimate key structural parameters (see, among others, Foerster, Sarte, and Watson, 2011; Acemoglu, Akcigit, and Kerr, 2016; Barrot and Sauvagnat, 2016; Carvalho et al., 2016; Atalay, 2017; Grassi, 2017; Baqaee, 2018; Baqaee and Farhi, 2019a,b; Bigio and La'O, 2019; Boehm, Flaaen, and Pandalai-Nayar, 2019; Foerster et al., 2019). We apply the insights and tools developed by this body of work to the study of international GDP comovement.

[4] Our labor supply shock can be viewed as a generalization of the "labor wedge" (e.g. Chari, Kehoe, and McGrattan, 2007) to the sector level. Though reduced-form, this shock has a variety of microfoundations, such as sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015), monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010).

[5] One caveat with this analysis is that the 4 extracted shocks are not mutually uncorrelated across countries. So there is no additive decomposition of overall comovement into the components driven by a single shock. This is a well known feature of this type of business cycle accounting exercise (see, among others, Chari, Kehoe, and McGrattan, 2007; Eaton et al., 2016).

The second is the research program in international macro that studies business cycle comovement. A large literature builds models in which fluctuations are driven by productivity shocks, and asks under what conditions those models can generate observed international comovement (see, among many others, Backus, Kehoe, and Kydland, 1992; Heathcote and Perri, 2002). A smaller set of contributions adds non-technology shocks (Stockman and Tesar, 1995; Wen, 2007; Bai and Ríos-Rull, 2015). While all papers on international business cycle comovement must take a stand on the relative importance of correlated shocks vs. transmission, we provide a framework to cleanly separate these two potential sources of comovement that can be applied across models.[6]

The notion that international input trade is the key feature of the global economy goes back to Hummels, Ishii, and Yi (2001) and Yi (2003), and has more recently been documented and quantified in a series of contributions by Johnson and Noguera (2012, 2017) and Caliendo and Parro (2015). Burstein, Kurz, and Tesar (2008), Bems, Johnson, and Yi (2010), Johnson (2014), Eaton et al. (2016), and Eaton, Kortum, and Neiman (2016), among others, explore the role of input trade in shock transmission and business cycle comovement. We contribute to this research agenda by deriving a set of analytical results that help quantify the relative importance of transmission and correlated shocks, measuring the shocks, and expanding the scope of quantification to more countries and sectors.[7,8]

The theoretical component of our analysis is most closely related to ongoing work by Baqaee and Farhi (2019c), which provides a number of results in an international network model, including an analytical solution to the global influence matrix. While that paper focuses on long-run comparative statics such as gains from trade, our theoretical framework and quantitative application are designed to study business cycle comovement. One relevant difference is that in our framework factor supply is variable as is common in business cycle models, and thus our influence matrix is distinct from that in Baqaee and Farhi (2019c). In addition we measure the primitive shocks necessary for the model to match the data, explore the consequences of correlated shocks in an input network setting, and

[6] A number of papers are dedicated to documenting international correlations in productivity shocks and inputs (e.g. Imbs, 1999; Kose, Otrok, and Whiteman, 2003; Ambler, Cardia, and Zimmermann, 2004). Also related is the body of work that identifies technology and demand shocks in a VAR setting and examines their international propagation (e.g. Canova, 2005; Corsetti, Dedola, and Leduc, 2014; Levchenko and Pandalai-Nayar, 2018). Our work complements these papers by providing novel estimates of several types of shocks at sector level, and expanding the sample of countries.

[7] Also related is the large empirical and quantitative literature on the positive association between international trade and comovement (e.g. Frankel and Rose, 1998; Imbs, 2004; Kose and Yi, 2006; di Giovanni and Levchenko, 2010; Ng, 2010; Liao and Santacreu, 2015; di Giovanni, Levchenko, and Mejean, 2018; de Soyres and Gaillard, 2019; Drozd, Kolbin, and Nosal, 2020). While these papers focus on the slope of the trade-correlation relationship in a cross-section of countries, we broaden the scope to provide a complete treatment of international comovement. Appendix C.3 explores the connection between the "trade-comovement" regressions and our analysis.

[8] Following the network propagation literature, our analysis captures the shock transmission through the market for inputs. It leaves open the possibility that the presence of input trade endogenously leads to correlated shocks, for instance through coordination of monetary policy, flow of information/sentiments, or transmission of productivity shocks within multinationals, among others. Microfounding shock correlation is outside the scope of our analysis but remains a fruitful avenue for future research.

extend the analysis to dynamics.

2 Accounting Framework

Consider an economy comprised of J sectors indexed by j and i, and N countries indexed by n and m. Gross output in sector j country n aggregates a primary factor input bundle $\mathcal{I}_{nj}$ (for instance, capital and labor) and materials inputs X_{nj}:

$$Y_{nj} = F\left(\mathcal{I}_{nj}(\boldsymbol{\theta}), X_{nj}(\boldsymbol{\theta}); \boldsymbol{\theta}\right). \tag{2.1}$$

The bundle of inputs X_{nj} can include foreign imported intermediates. The sectoral output is affected by a generic matrix of shocks $\boldsymbol{\theta}$. For concreteness, one can think of productivity shocks. A productivity shock θ_{nj} to sector j in country n will directly affect output in that sector. Because the economy is interconnected through trade, output in every sector and country is in principle a function of all the shocks anywhere in the world, hence the dependence of Y_{nj} on the full world vector $\boldsymbol{\theta}$. The matrix $\boldsymbol{\theta}$ can include multiple types of shocks (such as technology and non-technology). The next section completely specifies the shocks, and the nature of output's dependence on those shocks in the context of a particular model.

Real GDP is defined as value added evaluated at base prices b:

$$V_n = \sum_{j=1}^{J} \left(P_{nj,b} Y_{nj}(\boldsymbol{\theta}) - P^X_{nj,b} X_{nj}(\boldsymbol{\theta}) \right), \tag{2.2}$$

where $P_{nj,b}$ is the gross output base price, and $P^X_{nj,b}$ is the base price of inputs in that sector-country. Let θ_{mi} be a scalar-valued shock affecting sector i in country m.[9] A first order approximation to the log change in real GDP of country n can be written as:

$$d \ln V_n \approx \sum_m \sum_i s_{mni} \theta_{mi}, \tag{2.3}$$

where s_{mni} are the elements of the global influence matrix, that give the elasticity of the GDP of country n with respect to shocks in sector i, country m. Notice that these elasticities capture the full impact of a shock through direct and indirect input-output links and general equilibrium effects.[10]

[9] The extension to vector-valued θ_{mi} is straightforward, i.e. each sector can experience multiple shocks simultaneously.

[10] The form of s_{mni} is known for some simple economies. For instance, if country n is in autarky, factors of production are supplied inelastically, and returns to scale are constant, $s_{nni} = P_{ni,b} Y_{ni,b} / V_{n,b}$ are the Domar weights (Hulten, 1978; Acemoglu et al., 2012), and $s_{mni} = 0 \ \forall m \neq n$. Baqaee and Farhi (2019c) describe first- and second-order analytical solutions for a class of non-parametric open economies with cross-border input linkages. We derive a first-order closed-form solution to the influence matrix in our model economy with international trade in Section 3.2.

To highlight the sources of international GDP comovement, write real GDP growth as

$$d \ln V_n = \underbrace{\sum_j s_{nnj}\theta_{nj}}_{\mathcal{D}_n} + \underbrace{\sum_j s_{mnj}\theta_{mj}}_{\mathcal{P}_n} + \underbrace{\sum_{n' \neq n,m} \sum_j s_{n'nj}\theta_{n'j}}_{\mathcal{T}_n}. \tag{2.4}$$

This equation simply breaks out the double sum in (2.3) into the component due to country n's own shocks ($\mathcal{D}_n$), the component due to a particular trading partner m's shocks ($\mathcal{P}_n$), and the impact of "third" countries that are neither n nor m ($\mathcal{T}_n$).

Then, the GDP covariance between country n and country m is:

$$\begin{aligned} \text{Cov}(d \ln V_n, d \ln V_m) &= \underbrace{\text{Cov}(\mathcal{D}_n, \mathcal{D}_m)}_{\text{Shock Correlation}} \\ &+ \underbrace{\text{Cov}(\mathcal{D}_n, \mathcal{P}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{D}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{P}_m)}_{\text{Bilateral Transmission}} \\ &+ \underbrace{\text{Cov}(\mathcal{D}_n + \mathcal{P}_n + \mathcal{T}_n, \mathcal{T}_m) + \text{Cov}(\mathcal{T}_n, \mathcal{D}_m + \mathcal{P}_m)}_{\text{Multilateral Transmission}}. \end{aligned} \tag{2.5}$$

This expression underscores the sources of international comovement. The first term, $\text{Cov}(\mathcal{D}_n, \mathcal{D}_m)$, captures the fact that economies might be correlated even in the absence of trade if the underlying shocks themselves are correlated, especially in sectors influential in the two economies. The shock correlation term can be written as:

$$\text{Cov}(\mathcal{D}_n, \mathcal{D}_m) = \sum_j \sum_i s_{nnj} s_{mmi} \text{Cov}(\theta_{nj}, \theta_{mi}).$$

The second term captures bilateral or direct transmission. If the GDP of country n has an elasticity with respect to the shocks occurring in country m ($s_{mni} > 0$), that would contribute to comovement as well. Taking one of the terms of the Bilateral Transmission component:

$$\begin{aligned} \text{Cov}(\mathcal{D}_n, \mathcal{P}_m) &= \sum_j \sum_i s_{nnj} s_{nmi} \text{Cov}(\theta_{nj}, \theta_{ni}) \\ &= \boldsymbol{s}'_{nn} \boldsymbol{\Sigma}_n \boldsymbol{s}_{nm}, \end{aligned} \tag{2.6}$$

where $\boldsymbol{\Sigma_n}$ is the $J \times J$ covariance matrix of shocks in country n, and $\boldsymbol{s}_{nm}$ is the $J \times 1$ influence vector collecting the impact of shocks in n on GDP in m. This expression underscores that one source of comovement is that under trade, both country n and country m will be affected by shocks in n.

Finally, the Multilateral Transmission term collects all the other sources of comovement between n

and m that do not come from shocks to either n or m, such as shocks in other countries.

Dynamics The decompositions above generalize to a dynamic environment in which shocks can have prolonged effects on output. In that case, GDP in period t, V_{nt}, is potentially a function of all the history of shocks $\{\boldsymbol{\theta}_{t-k}\}_{k=0}^{\infty}$:

$$d\ln V_{nt} \approx \sum_{k=0}^{\infty}\sum_{m}\sum_{i} s_{mni,k}\theta_{mi,t-k}, \tag{2.7}$$

where $\theta_{mi,t}$ is now interpreted as the time-t innovation to the shock process. All the results above are generalized simply by adding a summation over k.[11]

We can decompose overall comovement into the static (contemporaneous) and dynamic components. The covariance between countries n and m can be written as:

$$\text{Cov}(d\ln V_{nt}, d\ln V_{mt}) = \sum_{k=0}^{\infty} \boldsymbol{s}_{n,k}'\boldsymbol{\Sigma}\boldsymbol{s}_{m,k}, \tag{2.8}$$

where $\boldsymbol{s}_{n,k}$ is the $NJ \times 1$ influence vector collecting the impact of all worldwide innovations k periods ago on country n, and $\boldsymbol{\Sigma}$ is the covariance matrix of innovations. Thus, the overall GDP covariance is additive in the component due to the contemporaneous innovations $\boldsymbol{s}_{n,0}'\boldsymbol{\Sigma}\boldsymbol{s}_{m,0}$ and the dynamic propagation of past shocks. The contemporaneous component is also notable because in the quantitative framework below, the contemporaneous influence vector $\boldsymbol{s}_{n,0}$ in the fully-specified dynamic model coincides with the influence vector in a static model that only features instantaneous propagation of shocks.

To summarize, in order to provide an account of international comovement, we must (i) measure shocks in order to understand their comovement properties; and (ii) assess how sectoral composition (the distribution of s_{nnj}'s) translates sectoral comovement of the primitive shocks into GDP comovement. Finally, (iii) we must discipline the persistence of both the shocks and equilibrium adjustments over time in order to quantify the relative importance of contemporaneous vs. intertemporal correlation.

3 Global Network Model

The decomposition above is general and would apply in any production economy. However, any measurement of the elements of the influence matrix and of shocks requires additional theoretical structure. This section introduces and quantifies a parsimonious multi-country production network model in the spirit of Acemoglu et al. (2012) and Baqaee and Farhi (2019c), and uses it to quantify the contributions of correlated shocks and transmission in GDP comovement.

[11] That is, (2.5) is unchanged, while $\mathcal{D}_n$ becomes $\mathcal{D}_n = \sum_{k=0}^{\infty}\sum_j s_{nnj,k}\theta_{nj,t-k}$, for example.

3.1 Setup

Preliminaries Each country n is populated by a representative household. The household consumes the final good available in country n and supplies labor and capital to firms. Trade is subject to iceberg costs τ_{mnj} to ship good j from country m to country n (throughout, we adopt the convention that the first subscript denotes source, and the second destination).

Households There is a continuum of workers in a representative household who share the same consumption. The problem of the household is

$$\max_{\mathcal{F}_n, \{H_{nj}\}} U\left(\mathcal{F}_n - \sum_j H_{nj}^{1+\frac{1}{\psi}}\right) \tag{3.1}$$

subject to

$$P_n \mathcal{F}_n = \sum_j W_{nj} H_{nj} + \sum_j R_{nj} K_{nj}$$

where $\mathcal{F}_n$ is consumption of final goods, H_{nj} is the total labor hours supplied to sector j, and K_{nj} is the amount of installed capital, which for now is assumed to be exogenous. Labor collects a sector-specific wage W_{nj}, and capital is rented at the price R_{nj}.

We highlight two features of the household problem. First, our formulation of the disutility of the labor supply is based on the Greenwood, Hercowitz, and Huffman (1988) preferences. The GHH preferences mute the interest rate effects and income effects on the labor supply, which helps to study the properties of the static equilibrium where the amount of capital is treated as predetermined.

Second, labor and capital are differentiated by sector, as the household supplies factors to each sector separately. In this formulation, labor is neither fixed to each sector nor fully flexible, and its responsiveness is determined by the Frisch elasticity ψ. As $\psi \to \infty$, labor supply across sectors becomes more sensitive to wage differentials, in the limit households supplying labor only to the sector offering the highest wage. At the opposite extreme, as $\psi \to 0$, the supply of labor is fixed in each sector by the preference parameters.[12]

The final use in the economy, denoted $\mathcal{F}_n$, is an Armington aggregate across countries and sectors.

[12] The specification of labor supply bears an affinity to the "Roy-Frechet" models common in international trade (e.g. Galle, Rodríguez-Clare, and Yi, 2017), in the sense that the relative supply of hours to two different sectors is isoelastic in the relative wages in the two sectors. The difference is that in most existing Roy-Frechet implementations, aggregate labor supply is fixed and only sectoral shares vary, whereas in our analysis total economywide labor supply shifts as well.

The functional form and its associated price index are given by

$$\mathcal{F}_n = \left[\sum_j \sum_m \vartheta_{mnj}^{\frac{1}{\rho}} \mathcal{F}_{mnj}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \qquad P_n = \left[\sum_j \sum_m \vartheta_{mnj} P_{mnj}^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{3.2}$$

where $\mathcal{F}_{mnj}$ is final use in n of sector j goods coming from country m, and P_{mnj} is the price of $\mathcal{F}_{mnj}$. For goods j, the expenditure share for final goods imported from country m is given by

$$\pi_{mnj}^{f} = \frac{\vartheta_{mnj} P_{mnj}^{1-\rho}}{\sum_{k,\ell} \vartheta_{kn\ell} P_{kn\ell}^{1-\rho}}.$$

The labor supply curves are isoelastic in the wages relative to the consumption price index, and given by (up to a normalization constant):

$$H_{nj}^{\frac{1}{\psi}} = \frac{W_{nj}}{P_n}.$$

Firms A representative firm in sector j in country n operates a CRS production function

$$Y_{nj} = Z_{nj} \left(K_{nj}^{\alpha_j} H_{nj}^{1-\alpha_j}\right)^{\eta_j} X_{nj}^{1-\eta_j}, \tag{3.3}$$

where the total factor productivity is denoted by Z_{nj}, and the intermediate input usage X_{nj} is an aggregate of inputs from potentially all countries and sectors:

$$X_{nj} \equiv \left(\sum_i \sum_m \mu_{mi,nj}^{\frac{1}{\varepsilon}} X_{mi,nj}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $X_{mi,nj}$ is the usage of inputs coming from sector i in country m in production of sector j in country n, and $\mu_{mi,nj}$ is the taste shifter.

Let P_{nj} denote the price of output produced by sector j in country n, and let $P_{mi,nj}$ be the price paid in sector n, j for inputs from m, i. No arbitrage in shipping implies that the prices "at the factory gate" and the price at the time of final or intermediate usage are related by:

$$P_{mi,nj} = P_{mni} = \tau_{mni} P_{mi},$$

where τ_{mni} is the iceberg trade cost.

Cost minimization implies that the payments to primary factors and intermediate inputs are:

$$
\begin{aligned}
R_{nj}K_{nj} &= \alpha_j \eta_j P_{nj} Y_{nj} \\
W_{nj}H_{nj} &= (1-\alpha_j)\,\eta_j P_{nj} Y_{nj} && (3.4) \\
P_{mi,nj}X_{mi,nj} &= \pi^x_{mi,nj}\,(1-\eta_j)\,P_{nj}Y_{nj}, && (3.5)
\end{aligned}
$$

where $\pi^x_{mi,nj}$ is the share of intermediates from country m sector i in total intermediate spending by n, j, given by:

$$
\pi^x_{mi,nj} = \frac{\mu_{mi,nj}\left(\tau_{mni}P_{mi}\right)^{1-\varepsilon}}{\sum_{k,\ell}\mu_{k\ell,nj}\left(\tau_{knl}P_{k\ell}\right)^{1-\varepsilon}}.
$$

Equilibrium An equilibrium in this economy is a set of goods and factor prices $\{P_{nj}, W_{nj}, R_{nj}\}$, factor allocations $\{K_{nj}, H_{nj}\}$, and goods allocations $\{Y_{nj}\}$, $\{\mathcal{F}_{mnj}, X_{mi,nj}\}$ for all countries and sectors such that (i) households maximize utility; (ii) firms maximize profits; and (iii) all markets clear.

At the sectoral level, the following market clearing condition has to hold for each country n sector j:

$$
P_{nj}Y_{nj} = \sum_m P_m \mathcal{F}_m \pi^f_{nmj} + \sum_m \sum_i (1-\eta_i) P_{mi} Y_{mi} \pi^x_{nj,mi}. \tag{3.6}
$$

Meanwhile, trade balance implies that each country's final expenditure equals the sum of value added across domestic sectors[13]

$$
P_m \mathcal{F}_m = \sum_i \eta_{ni} P_{mi} Y_{mi}. \tag{3.7}
$$

Note that once we know the share of value added in production η_j, the expenditure shares π^f_{nmj} and $\pi^x_{nj,mi}$ for all n, m, i, j, we can compute the nominal output $P_{nj}Y_{nj}$ for all country-sectors (n, j) after choosing a numeraire good. There is no need to specify further details of the model, and we will utilize this property to derive the influence matrix.

Shocks At a formal level, the only shock in this world economy is the TFP shock Z_{nj}. From the perspective of the quantification below, Z_{nj} should be interpreted broadly as a composite supply shock, encompassing both technology and primary factors. In the model, labor supply is upward-sloping in real wages, but shifts in the labor supply curve are isomorphic to TFP shocks in their effect on the global vector of output changes, up to a scaling factor. The model is static, so changes in the capital stock from period to period in the data are trivially viewed by the model as shocks to capital supply, and also isomorphic to TFP. Thus, inverting the global influence matrix recovers a composite supply shock, which is sufficient to answer the main question posed in this section. This

[13] We can incorporate deficits in a manner similar to Dekle, Eaton, and Kortum (2008), without much change in our results.

property is related to the point stressed by Baqaee and Farhi (2019a) that factor supply shocks can always be restated as productivity shocks, and thus are in that sense isomorphic to them. Section 4 separates technology, labor supply, and capital supply shocks.

3.2 Analytical Solution

We now provide an analytical expression for the global influence matrix. In general, closed-form solutions for the exact influence vectors cannot be obtained in multi-country multi-sector models such as ours. However, we can solve for the first-order approximation of the influence vector in our setting. Denote by "ln" the log-deviation from steady state/pre-shock equilibrium. Let the vector $\ln \mathbf{Y}$ of length NJ collect the worldwide sectoral output changes.

Theorem 1. *The response of* $\ln \mathbf{Y}$ *to the global vector of supply shocks* $\ln \mathbf{Z}$ *is to a first order approximation given by*

$$\ln \mathbf{Y} = \mathbf{\Lambda} \ln \mathbf{Z}, \tag{3.8}$$

where

$$\mathbf{\Lambda} = \left(\mathbf{I} - \frac{\psi}{1+\psi}\boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right)\mathcal{P}\right) - (\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^x\right)\mathcal{P}\right)\right)^{-1}, \tag{3.9}$$

$\boldsymbol{\eta}$ *and* $\boldsymbol{\alpha}$ *are matrices of output elasticities,* $\mathbf{\Pi}^f$ *and* $\mathbf{\Pi}^x$ *are matrices of final consumption and intermediate shares, respectively, and* $\mathcal{P}$ *is a matrix that combines both structural elasticities and spending shares.*[14]

Proof. See Appendix A.1. □

Equations (3.8)-(3.9) illustrate that all we need to understand the response of worldwide output to various sector-country shocks in this quantitative framework are measures of steady state final goods consumption and production shares, as well as model elasticities. The matrix $\mathbf{\Lambda}$ is the influence matrix. It encodes the general equilibrium response of sectoral output in a country to shocks in any sector-country, taking into account the full model structure and all direct and indirect links between the countries and sectors. In equation (3.9), the term $\frac{\psi}{1+\psi}\boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right)\mathcal{P}\right)$ captures the movements in real wages and the resulting adjustments in labor supply, and the $(\mathbf{I}-\boldsymbol{\eta})\left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^x\right)\mathcal{P}\right)$ term reflects changes in relative prices of intermediate inputs.

Two aspects of the influence matrix are worth noting. The first is a resemblance of (3.8)-(3.9) to the typical solution to a network model, that writes the equilibrium change in output as a product of the Leontief inverse and the vector of shocks. Our expression also features a vector of shocks,

[14] The $NJ \times NJ$ diagonal matrices $\boldsymbol{\eta}$ and $\boldsymbol{\alpha}$ collect the η_j's and α_j's. A typical element of $\mathbf{\Pi}^f$ is π^f_{mnj} and a typical element of $\mathbf{\Pi}^x$ is $\pi^x_{mi,nj}$. All of these matrices are defined precisely in Appendix A.1.

and an inverse of a matrix that is more complicated due to the multi-country structure of our model combined with elastic factor supply and non-unitary elasticities of substitution.

Second, the response of output in a static model (fixing K_{nj} in each sector) coincides with the *impact* response in the fully dynamic DSGE model in Section 4. Both are given by (3.8)-(3.9). Our analysis thus integrates the static network propagation literature that follows Acemoglu et al. (2012) and the dynamic international business cycle literature. We can cleanly separate the instantaneous propagation analyzed in the former and the delayed responses to shocks emphasized by the latter. In later periods the response of GDP will depend on the persistence of shocks and the capital accumulation decisions, which are not encoded in this vector (but can be evaluated numerically).

The proof proceeds by manipulating the equilibrium conditions of the model. To build intuition on the nature of general equilibrium effects captured by the influence matrix, linearize the market clearing conditions (3.6) to obtain

$$\ln \mathbf{P} + \ln \mathbf{Y} = \underbrace{\left(\mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x\right)(\ln \mathbf{P} + \ln \mathbf{Y})}_{\text{destination country output variation}} + \underbrace{(1-\rho)\left(\text{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^f\right)\ln \mathbf{P}}_{\text{consumption goods relative price variation}} + \underbrace{(1-\varepsilon)\left(\text{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x\right)\ln \mathbf{P}}_{\text{intermediate goods relative price variation}} \tag{3.10}$$

where the vector $\ln \mathbf{P}$ collects sector-level log-deviations in prices, $\mathbf{\Psi}^x$ and $\mathbf{\Psi}^f$ are matrices containing the steady-state export shares of intermediate and final goods, and $\mathbf{\Upsilon}$ is a matrix of value added shares.[15] The first term contains the response of nominal output that arises from output changes in every country and sector following a shock. The second term contains the relative price changes of final goods and the final term the relative price changes of intermediate inputs. Equation (3.10) implies that we can solve for the vector of country-sector price changes as a function of output changes and a matrix $\mathcal{P}$ that depends only on spending shares and structural elasticities:

$$\ln \mathbf{P} = \mathcal{P} \ln \mathbf{Y}. \tag{3.11}$$

The matrix $\mathcal{P}$ is then an input into the influence matrix. Combining (3.11) with linearized versions of the production function (3.3), labor market clearing, and the demand for intermediate goods leads to the model solution (3.8).

To build intuition, we can write the change in output (3.8) as a sum of the partial equilibrium (PE)

[15] A typical element of $\mathbf{\Psi}^f$ is $\frac{P_m \mathcal{F}_m \pi^f_{nmj}}{P_{nj} Y_{nj}}$, a typical element of $\mathbf{\Psi}^x$ is $\frac{P_{mi} Y_{mi} \pi^x_{nj,mi}}{P_{nj} Y_{nj}}$, and a typical element of $\mathbf{\Upsilon}$ is $\frac{\eta_{mi} P_{mi} Y_{mi}}{P_m \mathcal{F}_m}$. See the proof of Theorem 1 in Appendix A.1 for the detailed definitions.

and general equilibrium (GE) effects in response to a change in TFP:

$$\ln \mathbf{Y} \;=\; \underbrace{\Big(\boldsymbol{\alpha\eta}\Big)^{-1}\ln \mathbf{Z}}_{\text{PE effects}} + \underbrace{\Big(\boldsymbol{\gamma}+\boldsymbol{\gamma}^2+\boldsymbol{\gamma}^3+\dots\Big)\Big(\boldsymbol{\eta\alpha}\Big)^{-1}\ln \mathbf{Z}}_{\text{GE effects}}, \tag{3.12}$$

where the matrices $\boldsymbol{\eta\alpha}$ and $\boldsymbol{\gamma}$ have the following relationship to the influence matrix: $(\mathbf{I}-\boldsymbol{\gamma})^{-1}(\boldsymbol{\alpha\eta})^{-1} = \boldsymbol{\Lambda}$. The PE effect is the change in output following a productivity shock when movements in prices, wage rates, and other aggregate variables are muted. The PE response of a sector output depends only on own productivity shock, and not on others', and is captured by the diagonal matrix $(\boldsymbol{\alpha\eta})^{-1}$. Note that even in partial equilibrium, a sector's output changes more than one-for-one with TFP, due to the endogenous response of firms' demand for labor and intermediate inputs. The GE effect is captured by the infinite summation of the powers of the $\boldsymbol{\gamma}$ matrix, reflecting first-, second-, etc. round effects propagating via relative price changes through the input and factor markets. Through the GE effects, each sector's output is generically a function of the entire global vector of productivity shocks.

Appendix A evaluates the fit of the first-order approximation relative to the full nonlinear model solution. The first-order approximation performs quite well.

GDP Change Theorem 1 states the change in gross output, whereas GDP is value added. The following corollary describes the GDP changes.

Corollary 1. *The real GDP change in any country n is given by*

$$\ln V_n = \sum_{j=1} \frac{P_{nj}Y_{nj}}{V_n}\ln Z_{nj} \;+\; \sum_{j=1}(1-\alpha_j)\eta_j \frac{P_{nj}Y_{nj}}{V_n}\ln H_{nj}. \tag{3.13}$$

The global vector of changes in hours is given by:

$$\ln \mathbf{H} = \frac{\psi}{1+\psi}\left(\mathbf{I} + \left(\mathbf{I} - \boldsymbol{\Pi}^f \otimes \mathbf{1}\right)\mathcal{P}\right)\boldsymbol{\Lambda}\ln \mathbf{Z} \equiv \boldsymbol{\mathcal{H}}\ln \mathbf{Z}. \tag{3.14}$$

Proof. See Appendix A.1. □

To construct GDP, we need to aggregate the changes of sector-country real value added, as in (2.2). The first term in equation (3.13) captures the impact of domestic TFP changes on GDP. Note that there is no direct dependence of country n's GDP on foreign TFP changes. The second term in (3.13) captures the changes in hours. Equation (3.14) underscores that hours in every country and sector depend on the entire vector of TFP changes worldwide.

Corollary 1 connects to the accounting decomposition of comovement into shock correlation and transmission components (2.5). The direct effect of own TFP changes (the first term in 3.13) plus the diagonal elements of $\boldsymbol{\mathcal{H}}$ together make up $\mathcal{D}_n$ – the influence of domestic shocks on GDP, and thus determine the shock correlation term. The off-diagonal elements in $\boldsymbol{\mathcal{H}}$ capture the influence of foreign shocks on a country's GDP, and thus make up the transmission terms.

The corollary also highlights the difference between our analytical results and Baqaee and Farhi (2019c). The main analysis in that paper considers the case of exogenous factor supplies. Corollary 1 under the assumption that $\ln H_{nj}$ is exogenous is essentially Theorem 1 in that paper (and the earlier result in Kehoe and Ruhl, 2008). One of our contributions is an analytical solution when factor supplies are endogenous to productivity shocks. The endogenous response of factor supplies leads to a departure from the result in Baqaee and Farhi (2019c) that a country's real GDP is invariant to foreign TFP shocks. The responses of GDP to foreign TFP shocks in our framework are encapsulated in (3.14).

Recovering Shocks We now describe how to recover the supply shocks Z_{nj} in such a way as to match actual value added growth in every country-sector (and therefore actual GDP growth in every country). The procedure is inspired by Foerster, Sarte, and Watson (2011), who perform a related exercise in a closed economy.

Let the vector $\ln \mathbf{V}$ of length NJ denote sectoral value added in log deviations from steady state. Similar to Corollary 1, sectoral value added can also be expressed as changes in primary inputs

$$\ln \mathbf{V} = \boldsymbol{\eta}^{-1} \ln \mathbf{Z} + \boldsymbol{\alpha} \ln \mathbf{H}.$$

We have data on the $NJ \times 1$ vector of log changes in real value added $\ln \mathbf{V}$ in each year, which allows us to recover the shocks:

$$\ln \mathbf{Z} = \left(\boldsymbol{\alpha}\boldsymbol{\mathcal{H}} + \boldsymbol{\eta}^{-1} \right)^{-1} \ln \mathbf{V}. \tag{3.15}$$

In other words, the structure of the model world economy and the observed/measured objects can be used to infer a global vector of supply shocks $\ln \mathbf{Z}$ that rationalizes observed growth rates in real value added in each country-sector. Note that the interdependence between country-sectors through input linkages implies that the entire global vector $\ln \mathbf{Z}$ must be solved for jointly. As stressed above, the recovered $\ln \mathbf{Z}$ should be viewed more broadly as a general sector-level supply shock.

3.3 Data and Calibration

The data requirements for recovering shocks using (3.15) and computing the instantaneous global response to shocks using (3.8) are: (i) growth of real value added for a panel of countries, sectors, and years and (ii) global input-output linkages. The dataset with the broadest coverage for real value added is KLEMS 2009 (O'Mahony and Timmer, 2009).[16] This database contains gross output, value added, labor and capital inputs, as well as output and input deflators. In a limited number of instances, we supplemented the information available in KLEMS with data from the WIOD Socioeconomic Accounts, which contains similar variables. After data quality checking and cleaning, we retain a sample of 29 countries, listed in Appendix Table A1. The database covers all sectors of the economy at a level slightly more aggregated than the 2-digit ISIC revision 3, yielding, after harmonization, 30 sectors listed in Appendix Table A2. In the best cases we have 38 years of data, 1970-2007, although the panel is not balanced and many emerging market countries do not appear in the data until the mid-1990s.

The data on input linkages at the country-sector-pair level, as well as on final goods trade come from the 2013 WIOD database (Timmer et al., 2015), which contains the global input-output matrix.

In implementing the network model, we only need to take a stand on the value of a small number of parameters, and use our data to provide the required quantities. Table 1 summarizes the parameter assumptions for the network model and data sources. We estimate the substitution elasticities in final and intermediate use. The estimation procedure, described in detail in Appendix B.1, uses the model-implied relationship between log changes in relative expenditure shares and in relative prices of different source countries. For our purposes, the two benefits of our elasticity estimation procedure are that (i) we perform it separately for intermediate and final goods; and (ii) we do it at the annual frequency, which corresponds to the business cycle focus of our quantification. Based on these estimation results, the final consumption Armington elasticity ρ is set to either 2.75 or 1, and the intermediate elasticity ε to 1.[17] The parameter ψ governs the labor supply elasticity. We set it to imply the Frisch labor supply elasticity of 2, common in the business cycle literature. Appendix C explores alternative Frisch elasticity values.

All other parameters in the static model have close counterparts in basic data and thus we compute them directly. Capital shares in total output α_j and value added shares in gross output η_j come from KLEMS, and are averaged in each sector across countries and time to reduce noise. We initialize both the static and dynamic models in the same steady state. Steady state input shares $\pi^x_{mi,nj}$ and final consumption shares π^f_{mnj} are computed from WIOD as time averages.

Road Map The rest of this section uses the static network model laid out and calibrated above to understand international GDP comovement. We start by subjecting the world economy to several

[16] This is not the latest vintage of KLEMS, as there is a version released in 2016. Unfortunately, the 2016 version has a shorter available time series, as the data start in 1995, and also has many fewer countries. A consistent concordance between the two vintages is challenging without substantial aggregation.

[17] In the quantitative implementation, we introduce an additional nest in the final goods aggregation: the top layer is Cobb-Douglas across sectors, whereas the within-sector (bottom) layer is CES across source countries with elasticity ρ. This makes no difference when $\rho = 1$, but reduces substitutability across sectors when $\rho = 2.75$.

Table 1: Parameter Values

Param.	Value	Source	Related to
ρ	2.75 or 1	Appendix B.1	final substitution elasticity
ε	1	Appendix B.1	intermediate substitution elasticity
ψ	2		Frisch elasticity
α_j	[0.21, 0.60]	KLEMS	labor and capital shares
η_j	[0.31, 0.67]	KLEMS	intermediate input shares
π^f_{mnj}		WIOD	final use trade shares
$\pi^x_{mi,nj}$		WIOD	intermediate use trade shares

Notes: This table summarizes the parameters and data targets used in the quantitative model, and their sources. For α_j and η_j, the table reports the 10th and 90th percentiles of the range of these parameters.

hypothetical productivity shocks, to highlight the strength of the transmission forces embodied in the model. We then use actual data on value added growth to recover productivity shocks as in (3.15), and present the headline decomposition of GDP growth correlations into shock correlation and transmission components as in (2.5). We then illustrate the roles of transmission and correlated shocks by means of several counterfactual exercises.

3.4 Impulse Responses

Prior to recovering the underlying supply shocks and simulating the model with them, we "test drive" the propagation mechanism by computing the world economy's response to some simple hypothetical shocks:

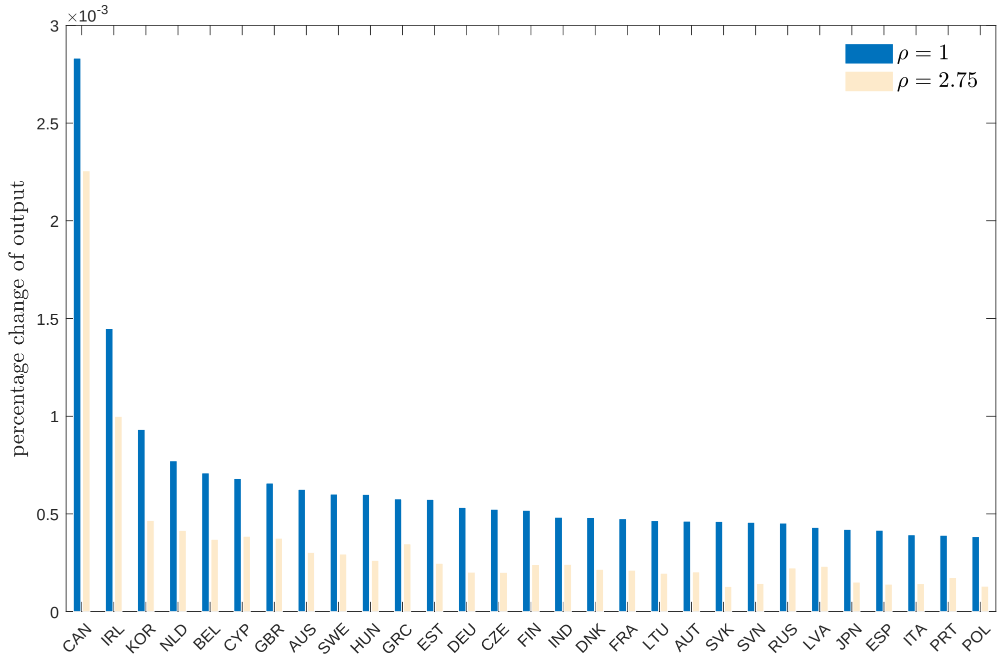
1. a 1% US shock in all sectors,
2. a 1% rest-of-the-world shock in all sectors from the perspective of each country, and,
3. a 1% symmetric shock in each sector in every country of the world.

The rest-of-the-world exercise assumes that the country in question is not shocked, but all other possible countries and sectors are, and thus has to be conducted country by country.

Figure 1 displays the change in real GDP in every other country in the world following a 1% US shock in each sector. The light bars depict the GDP responses under $\rho = 2.75$, while the dark bars depict the response under $\rho = 1$.

The results show that the observed trade linkages do result in transmission. Smaller economies with large trade linkages to the US, such as Canada, are the most strongly affected by the US shocks.

FIGURE 1: Impulse Responses to a US 1% Shock



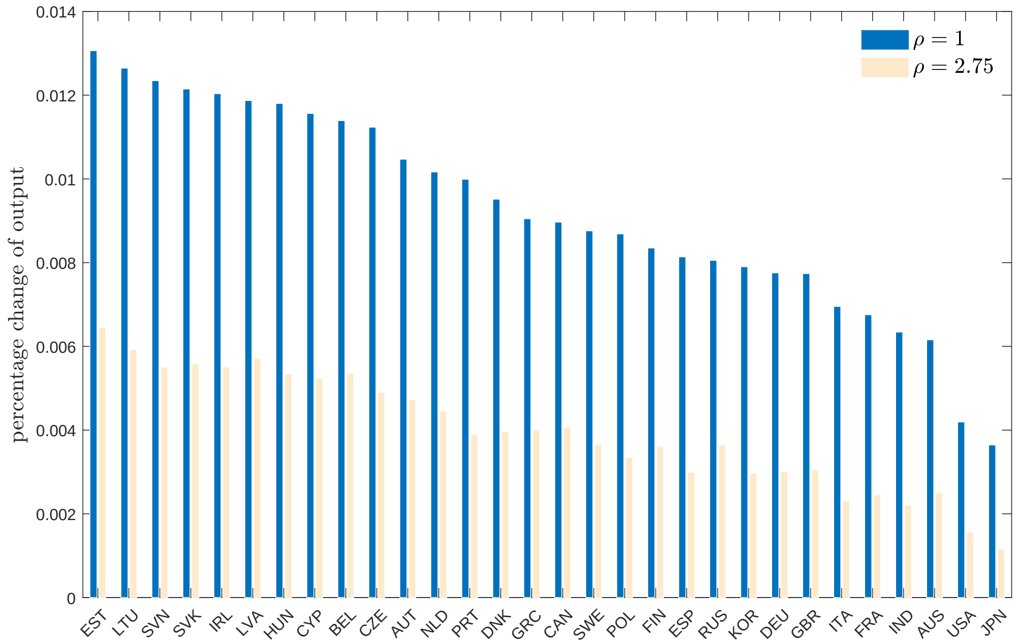
Notes: This figure displays the change in log real GDP of every other country in the sample when the United States experiences a supply shock of 0.01 in every sector for different values of ρ.

Under the low elasticity, the mean response of foreign GDP is 0.06%, and the maximum response – Canada – is about 0.28%. On the other hand, the final substitution elasticity matters a great deal for the size of the effects: the response of foreign GDP to the US shocks is about twice as high for $\rho = 1$ than for $\rho = 2.75$.

Next, we simulate the real GDP responses of each country n in the sample when all other countries (excluding n) experience a 1% technology shock. The exercise answers the question, if there is a 1% world shock outside of the country, how much of that shock will manifest itself in the country's GDP? Figure 2 displays the results. In response to a 1% outside world shock, under the low elasticity of substitution the mean country's GDP increases by 0.93%, with the impact ranging from around 0.3-0.4% in the US and Japan to 1.2-1.3% in Lithuania and Estonia. Not surprisingly, smaller countries are more affected by shocks in their trade partners. The magnitude of transmission is uniformly lower with the higher elasticity. In this case, the mean impact is about 0.4% for the 1% shock. All in all, these results suggest that outside world shocks have a significant impact on most countries.

Figure 3 illustrates the results of our third impulse response exercise, a 1% shock to every country and sector in the world, under $\rho = 2.75$. Here, we are most interested in the share of the total GDP change that comes from the shocks to the country's own productivity, and how much comes from

Figure 2: Impulse Responses to Rest of the World 1% Shocks



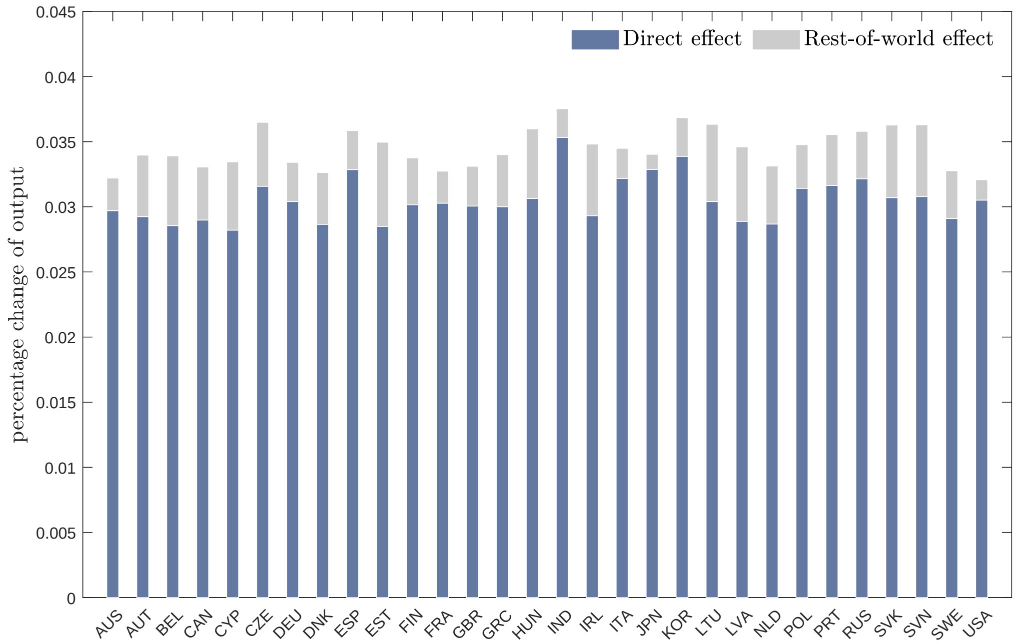
Notes: This figure displays the change in log real GDP of every country in the sample when the rest of the world excluding the country experiences a supply shock of 0.01 in every sector for different values of ρ.

foreign shocks. Thus, we use the linear approximation to a country's GDP growth (2.4), and separate the overall impact into the own term $\mathcal{D}_n$ and the rest. The figure highlights that for all countries, shocks to domestic sectors matter much more for GDP growth than foreign sector shocks. The mean and the median share of the foreign terms in the total GDP change are around 11%. The impact is heterogeneous across countries, with the fraction of GDP change due to foreign impact ranging from 3 to 6% of the total for Japan, the US and India to 18% of the total for Estonia.

3.5 GDP Correlations in the Model

We next use the calibrated model to recover the supply shock vector $\ln \mathbf{Z}$ as in (3.15), and simulate the full static model by feeding in the recovered shocks. Throughout, we use the first-order analytical solution expressed as a global influence matrix in Section 3.2. Alternatively, we can also obtain the exact solution using the hat algebra approach of Dekle, Eaton, and Kortum (2008). The details of the exact solution to the model are in Appendix A.3. The appendix also provides a comparison between the GDP growth rates implied by the first-order approach and the exact GDP growth rates. It turns out that in our setting, the exact and first-order approximation solutions are very close to each other, with a correlation between the two GDP growth rates of 0.999. Below, we work with the first-order analytical solution because it permits the decomposition of the overall comovement into

Figure 3: Impulse Responses to a 1% Shock in Every Sector in Every Country



Notes: This figure displays the change in log real GDP of every country in the sample, decomposed into a direct effect and a rest of world effect, when all sectors in every country experience a supply shock of 0.01, under $\rho = 2.75$.

the additive shock correlation and transmission terms.[18]

Table 2 presents the results. The first row of each panel reports the GDP growth correlations in the data, the second row in the model. Our procedure is designed to match the growth rates in the data. The difference is due to the fact that in the model, we use fixed values for all the expenditure weights, that are set to the time averages of those values in WIOD, whereas in the data all expenditure weights evolve over time. The discrepancy introduced by this divergence between the model in the data is small (see also Appendix C.1).

To assess the importance of correlated shocks relative to transmission, we decompose bilateral correlations along the lines of equation (2.5), rewritten in correlations. That equation combined with the first-order solution to the model in (3.13) produces a breakdown of the overall comovement into the shock correlation and transmission terms. For the G7 countries, the correlation of shocks is responsible for around four-fifths of the total comovement. Nonetheless, the bilateral and multilateral transmission terms have a non-negligible contribution to the overall correlation, accounting for the

[18]Throughout the quantitative analysis, we report correlations of growth rates. Since our data are annual and for many countries in our sample only start in 1995, we do not have sufficient time series to implement other detrending methods. Appendix Table A5 compares the properties of GDP growth rates to HP-filtered series for the G7 countries, where we have a longer time series of 30 annual observations. The properties of GDP growth rates and HP-filtered GDP series are quite similar.

remaining one-fifth. The share accounted for by transmission is slightly larger in the full sample, about one quarter.

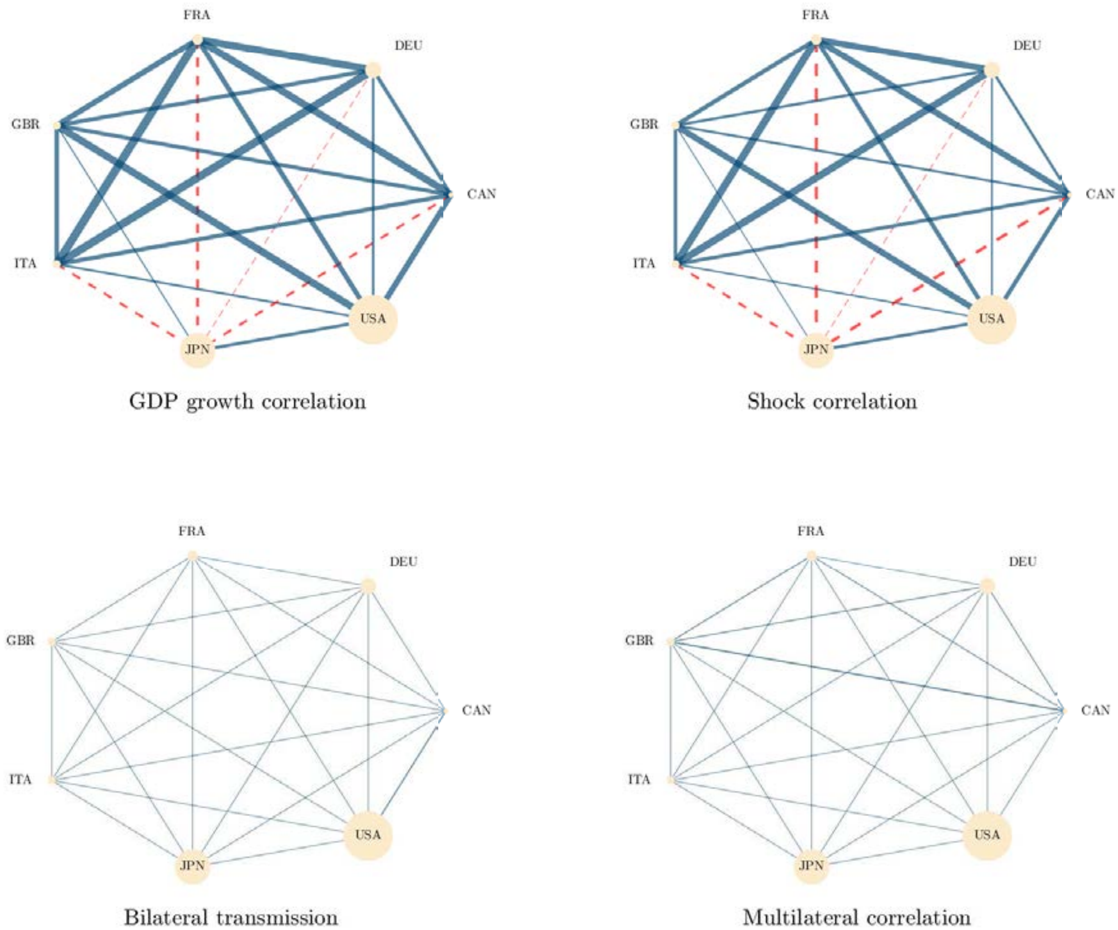
TABLE 2: Correlated Shocks vs. Transmission Decomposition, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Baseline:	0.381	0.435	0.232	0.610
Decomposition:				
Shock Correlation	0.301	0.328	0.183	0.459
Bilateral Transmission	0.019	0.015	0.007	0.019
Multilateral Transmission	0.060	0.058	0.032	0.092
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Baseline:	0.188	0.193	-0.110	0.498
Decomposition:				
Shock Correlation	0.139	0.164	-0.125	0.427
Bilateral Transmission	0.007	0.004	0.002	0.008
Multilateral Transmission	0.043	0.034	0.011	0.074

Notes: This table presents the decomposition of the GDP correlations into the shock correlation, the direct transmission, and the multilateral transmission terms as in equation (2.5).

Figure 4 displays the heterogeneity in the correlation and transmission terms in a network graph for the G7 countries. The upper left panel depicts all the bilateral correlations among those countries, with thicker lines denoting larger values, and blue (resp. red) depicting positive (resp. negative) correlations. The top right panel displays the same for the shock correlation component, and the bottom two panels the bilateral and multilateral components. The scale (thickness of the lines) is the same in all four panels. It is clear that the differences in the shock correlation component are responsible for both the bulk of the overall correlation, as well as the variation across countries. For instance, none of the transmission components are negative, and thus all the negative actual correlations are due to the negative correlations of the shocks.

Figure 4: Total Correlation, Shock Correlation, and Transmission in the G7



Notes: This figure displays the network of GDP correlations (top left), decomposing it into the shock correlation (top right) and transmission (lower panel) components. Thicker lines denote higher values. Blue displays positive values, red negative values. Larger nodes (countries) displayed with bigger dots.

3.6 Understanding Model Mechanisms

To better understand the headline result, we perform two exercises. First, we use heat maps of the trade network, the influence matrix, and shock correlations to visualize why the shock correlation has to play an important role in understanding comovement. Second, we use counterfactual exercises to isolate the role of the input network in international comovement.

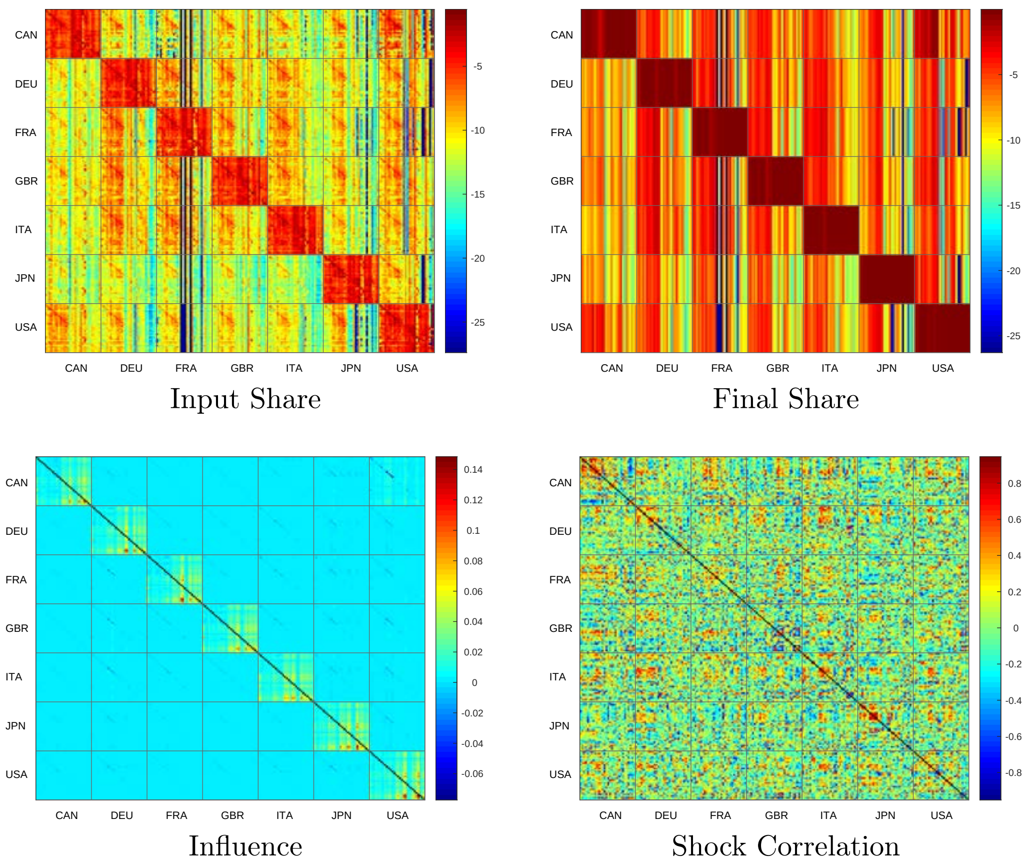
3.6.1 Influence Matrix and Shock Correlation

Figure 5 displays four heat maps for the G7 countries. In each, both rows and columns are broken into country-sectors, though due to space constraints sectors are too small to be labeled. In the top left is the usual heat map of log intermediate input shares in the WIOD, with the suppliers on the x-axis, and input users on the y-axis. Versions of this heat map have appeared in the literature (see, e.g., Jones, 2013). The most saturated reds, indicating greater input linkages, are in blocks on the diagonal, corresponding to countries' domestic linkages. In the top right panel is the heat map of log final expenditure shares instead. For final shares, there is no notion of a using (y-axis) sector, as final expenditure is undertaken by a representative consumer in each country. Nonetheless, we keep the format of the plot the same as the others. Once again, domestic final shares are the highest, but there is meaningful variation across sectors within country pairs. Both intermediate and final shares are inputs into the influence matrix. By construction, all the values displayed in the top two panels are positive.

The bottom left panel displays the influence matrix. It shares some clear similarities with the input-output and final shares matrices. Specifically, the largest positive entries tend to be domestic, and there are clear relationships between close trading partners like Canada and the US (upper right corner). However, there is one important difference: entries of the influence matrix are sometimes negative. Visible negative values in this heatmap are darker blue lines running parallel to the diagonal. These correspond to the same industries in different countries: in our influence matrix, a positive supply shock to foreign producers in the same industry tends to have a negative impact on sectoral output. This is in spite of the fact that often, the input shares in those sectors are also relatively high (the lines parallel to the diagonal are also evident in the input-output heat map). This discussion illustrates that the influence matrix conveys information distinct from the IO matrix itself.

The bottom right panel depicts the heat map of shock correlations. This panel is quite different from all the others. In fact, there is little if any similarity between this panel and the input, final share, and influence heat maps. Visually, it does not even appear to be the case that within-country shock correlations are that much higher than the cross-country ones. As the overall GDP correlation is built from the shock correlation and the influence matrices, these two panels convey the sources of variation in these two components and the relative importance of the two. In particular, whereas the off-block-diagonal (cross-country) elements of the influence matrix by and large are both small and display limited variation, there is a great deal of variation in the cross-border shock correlation.

Figure 5: Input, Influence, and Correlation Heat Maps, G7



Notes: This figure displays the heat maps of the log intermediate input shares in WIOD (top left), log final spending shares (top right), the influence matrix (bottom left), and the bilateral shock correlations (bottom right) for the G7 countries.

3.6.2 The Role of the Input Network

Autarky Another way to quantify the role of transmission in generating observed comovement is to compare the correlations in the baseline model to correlations that would obtain in an autarky counterfactual. We can write the difference in covariances between autarky and trade as a sum of two terms:

$$\Delta \text{Cov}(d \ln V_n, d \ln V_m) = \underbrace{\sum_j \sum_i \left(s_{nnj} s_{mmi} - s_{nj}^{AUT} s_{mi}^{AUT} \right) \text{Cov}(\theta_{nj}, \theta_{mi})}_{\Delta \text{Shock Correlation}} + \text{Bilateral Transmission} + \text{Multilateral Transmission}, \tag{3.16}$$

where s_{mi}^{AUT} are the elements of the influence vectors in autarky. This expression shows that trade opening can affect GDP covariance in two ways. First, it will make countries sensitive to foreign shocks, as captured by the bilateral and multilateral transmission terms. Second, and more subtly, the differences in GDP comovement between autarky and trade will also depend on the changes in the *domestic* elements of the influence vectors. Opening to trade can re-weight sectors in the two economies either towards, or away, from sectors with more correlated fundamental shocks. This is captured by the ΔShock Correlation term in the equation above.

Thus, in order to understand the contribution of international trade to international comovement, we must capture how going from autarky to trade changes the sectoral composition of the economy (the differences between s_{nnj} and s_{nj}^{AUT}). A natural way to construct autarky would be to set the trade costs to infinity. Since the baseline production function is Cobb-Douglas in the factors and all materials inputs, we conceive of the autarky counterfactual as a limiting case as all $\tau_{mnj} \to \infty$ while the substitution elasticity between inputs approaches 1 from above: $\varepsilon \downarrow 1$. Using the input and final consumption shares that obtain as trade costs go to infinity, we build the autarky influence matrix according to the same formula (Section 3.2). We then apply the recovered shocks to the autarky influence matrix to compute GDP growth rates in all countries and the resulting GDP correlations.

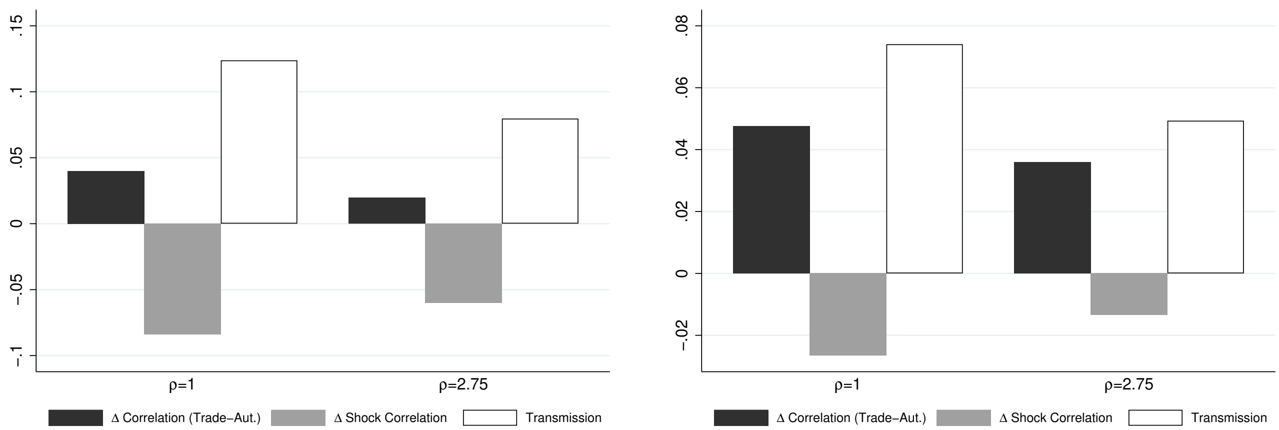
Figure 6 displays the mean difference between the trade and the autarky correlations, and decomposes them into the change into the ΔShock Correlation and transmission terms as in (3.16). Not surprisingly, moving from autarky to trade increases the overall correlations (the dark bars), and the transmission terms contribute positively to this increase in comovement (white bars). More unexpected is that the ΔShock Correlation$_{mn}$ is actually negative, in both the G7 and all countries.

The left panel of Figure 7 plots the average changes in the domestic elements influence vectors s_{nnj} in the G7 sample, by sector. On average, under trade s_{nnj}'s are lower than in autarky. This is sensible, as domestic shocks should have a smaller influence in an open economy compared to the closed one. The figure reveals which sectors fall in influence the most in the baseline model compared to autarky. The top 3 are Renting of Machinery & Equipment; Mining and Quarrying; and Electrical and Optical Equipment.

The fact that the ΔShock Correlation$_{mn}$ term is negative is already telling us that on average, sectors with more correlated shocks experience a relative decline in influence when the economy moves from autarky and trade. The right panel of Figure 7 presents the local polynomial fit between the two elements the ΔShock Correlation$_{mn}$: the shift in the combined influence of each sector pair $s_{nnj}s_{mmi} - s_{nj}^{AUT}s_{mi}^{AUT}$ and the correlation between the combined shocks in that pair, along with a 95% confidence band. The negative relationship is evident, as expected.

The autarky exercise emphasizes one channel through which changes in trade costs can affect inter-

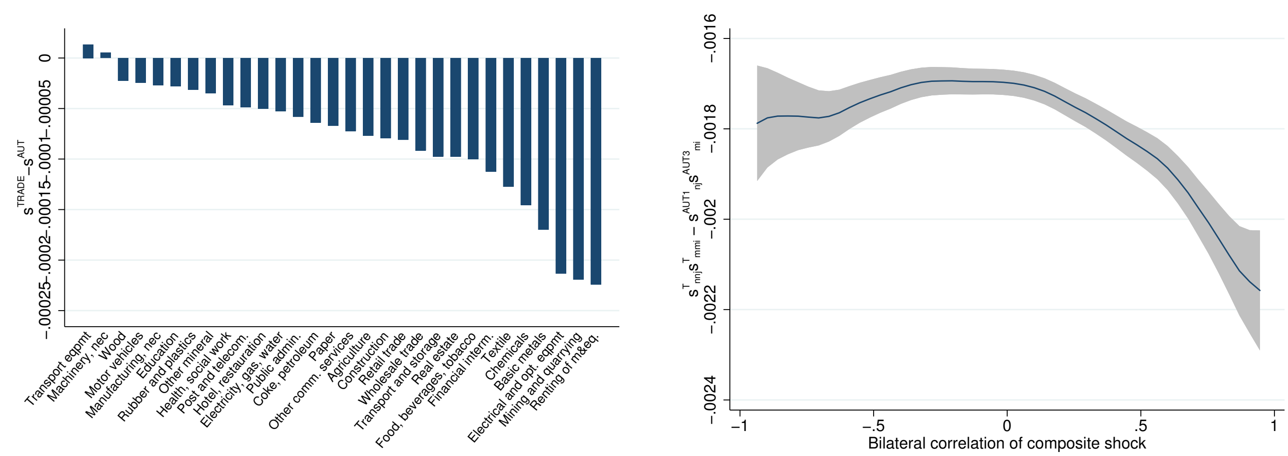
Figure 6: Change in GDP Correlation between Trade and Autarky: Decomposition



G7 | All Countries

Notes: This figure displays the mean change in the GDP correlation going from autarky to trade (dark bars), decomposing it into the mean ΔShock Correlation$_{mn}$ (gray bars) and Transmission (white bars) terms.

Figure 7: Changes in the Influence Vectors and Shock Correlation



Change in Influence | Change in Influence vs. Shock Correlation

Notes: This figure displays the mean change, by sector, in the domestic terms of the influence vector under trade relative to autarky (left panel), and the local polynomial fit between the bilateral sectoral correlation of shocks between pairs of home and foreign sectors and their change in influence from autarky to trade (right panel). The gray bands are the 95% confidence intervals.

national comovement: influence of the domestic sectors. Changes in trade costs can also affect the strength of transmission forces. Because the autarky case is too extreme on this point (transmission goes from positive to exactly zero), Appendix C.2 considers intermediate changes in trade costs. The main finding is that the impact of changes in trade costs on comovement depends on ε. When $\varepsilon = 1$ as in our baseline analysis, comovement is not sensitive to intermediate changes in trade costs, as the input revenue shares are constant. When $\varepsilon > 1$ we get the expected outcome that correlations fall in the level of trade costs. However, when $\varepsilon < 1$ correlations actually rise in trade costs. The explanation for this apparent paradox is that when $\varepsilon < 1$, a higher τ_{mnj} leads to higher foreign input spending shares. Because the influence of foreign shocks in the domestic economy increases in the foreign input shares, raising trade costs produces more transmission and higher correlations when inputs are complements.

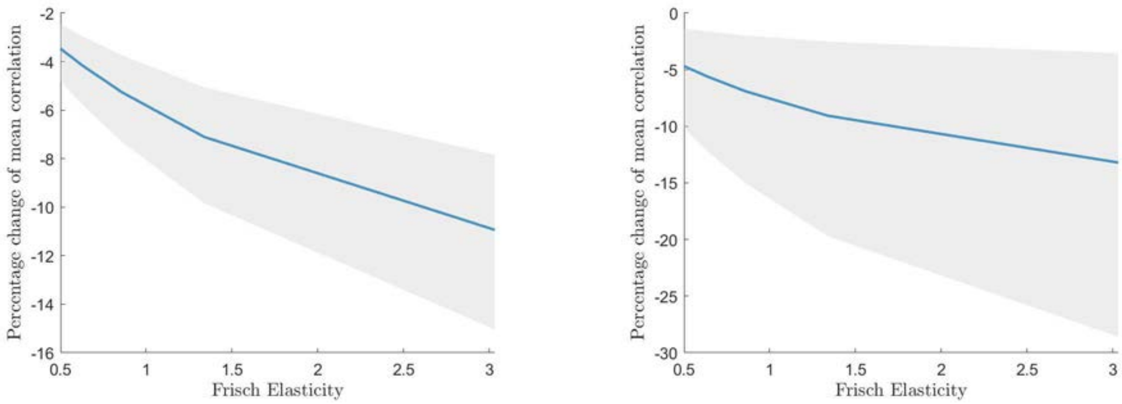
Input Trade We next turn to a counterfactual to understand the role of the input network in shaping GDP correlations. Our goal is to compare comovement in a model that does not feature input-output linkages, and all trade is in final goods only, to our benchmark model. Such value-added only economies are typically used to study comovement, but our benchmark model cannot easily be collapsed to such a structure.

As emphasized throughout, the forces shaping GDP comovement are both due to correlated sectoral shocks, and transmission (through production linkages). In our autarky comparative static, we highlighted that shutting down transmission can endogenously change sectoral sizes, which can impact aggregate comovement in unexpected ways. To assess whether the input network plays a crucial role *relative* to a simple value-added only final goods trade model, we must therefore ensure that the relative sector sizes remain the same in both models. We therefore implement a counterfactual with no production network that matches sectoral output in each country to the data, ensuring the ranking of relative sectoral sizes is constant. Further, we assume that the sectoral trade to output ratios are the same as in the data in both models. This counterfactual matches the volume of trade, but implies that the trade/GDP ratio will be lower in the model with no production networks than in our baseline.[19]

Figure 8 plots the change in the average correlation in the "no production networks" counterfactual relative to the baseline. Country-sectors in both models face the same underlying shocks. As detailed in Appendix C.1, the correlation of the measured sectoral shocks depends on the Frisch elasticity. Further, it is well-known that lower production elasticities increase comovement, all else equal. We therefore report changes in correlations while varying the Frisch elasticity as well as ρ and ε. In particular, the two panels in the figure correspond to our two estimates values of ρ of 2.75 and 1. On

[19]This is similar to the "low-trade" counterfactual in Baqaee and Farhi (2019c), except we match sectoral output instead of sectoral value-added as a size measure of sectors. We cannot implement their "high-trade" value-added counterfactual, as in our model it implies negative final consumption in some sectors in certain countries.

Figure 8: Change in Mean Correlation in the "No Production Networks" Model, G7



$\rho = 2.75$ $\rho = 1$

Notes: This figure displays the percent decrease in the average correlation in the counterfactual model with no production networks relative to our baseline model for the G7 countries. The solid line displays the change with the baseline $\varepsilon = 1$, while the shaded bands illustrate the changes as ε varies between 0.5 and 1.5.

the x-axes is the Frisch elasticity. The solid lines plot the change in correlation when $\varepsilon = 1$, whereas the grey bands depict the range of outcomes as ε varies from 0.5 to 1.5.

The average correlation in the model with no production networks is lower under *all* considered values of the three elasticities, highlighting that transmission through the input network is not easily approximated by final goods trade alone. The quantitative importance of the network varies. Under our baseline calibration of $\varepsilon = 1$ and Frisch $= 2$, comovement is about $8 - 10\%$ lower without production networks. Under alternative elasticities, the difference can be much larger– with very elastic labor supply, a Cobb-Douglas final goods aggregator within sectors and inputs that are complements ($\varepsilon = 0.5$), comovement is nearly 30% lower without production networks.

4 Dynamics and Multiple Shocks

4.1 Motivation

The analysis above explores in detail the contemporaneous impact of shocks on comovement. Those exercises emphasize the role of the input-output structure of the world economy in amplifying or dampening the underlying correlations of the sectoral shocks, but leave several unanswered questions:

Nature of Static Shocks The sectoral shock extracted by inversion of the influence matrix subsumes changes in productivity, labor, and capital inputs that the model cannot produce endogenously. This composite shock has two potential limitations. The first is that it is by construction not especially informative on the underlying drivers of business cycles in general, and of international comovement in particular. Second, at a more technical level, only the shocks to primary factor supplies can be made isomorphic to TFP, $\mathbf{Z}$. Shocks to frictions in the intermediate input market are not isomorphic to TFP. Thus, if shocks to the intermediate input market are non-trivial, this procedure would recover a supply shock that is in part a linear combination of all countries' and sectors' intermediate input market shocks.

Dynamics Shocks can affect aggregate outcomes via a contemporaneous impact – their correlation and the intratemporal transmission through the network – as well as a dynamic impact driven by the response of capital accumulation. The analysis above is silent on dynamic propagation and delayed responses of economies to shocks.

Intertemporal Shocks Finally, connecting the two points above, by construction intertemporal shocks cannot be extracted using the static influence matrix.

This section considers two extensions to the analysis in Section 3. First, we introduce dynamics and allow endogenous capital accumulation. And second, we enrich the model to allow for several distinct shocks. One benefit of these extensions is that we can now perfectly match multiple series: value added, labor, capital, and intermediate inputs. Another benefit is that a dynamic model can accommodate intertemporal as well as intratemporal shocks, and thus we can quantify their relative importance.

In our framework, the impact response of GDP in the dynamic model to intratemporal shocks is characterized by the global influence matrix, and thus coincides with the GDP response of the static world economy studied in Section 3.[20] In addition, as emphasized in Section 2 the static and the dynamic components of the total covariance are simply additive. Thus the static and dynamic comovement are separable, and adding dynamics does not discard the lessons learned in the static network model.

4.2 Model

Let t index time. Households are infinitely-lived, and choose consumption C_{nt}, investment I_{njt}, and labor supply H_{njt} to solve

$$\max_{C_{nt},\{I_{njt}\},\{H_{njt}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U \left(C_{nt} - \sum_j H_{njt}^{1+\frac{1}{\psi}} \right) \tag{4.1}$$

[20] This statement is of course conditional on the vector of capital stocks and all the expenditure shares that enter the influence matrix being the same.

subject to

$$P_{nt}\left(C_{nt}+\sum_j\left(1+\xi^I_{njt}\right)I_{njt}\right)=\sum_j\left(1-\xi^H_{njt}\right)W_{njt}H_{njt}+\sum_j R_{njt}K_{njt}+T_{nt}$$

$$K_{njt+1}=(1-\delta_j)K_{njt}+I_{njt}.$$

As is customary, it takes one period for investment to become capital, and thus the capital stock in each sector is predetermined by one period. Households now decide to split the final bundle into consumption and investment: $\mathcal{F}_{nt}\equiv C_{nt}+\left(1+\xi^I_{njt}\right)I_{njt}$, where $\mathcal{F}_{nt}$ is still an Armington aggregate of all sectors and source countries, as in (3.2).

The household decision problem is now subject to two shocks: labor ξ^H_{njt} and investment ξ^I_{njt}, which we interpret as distortive taxes following Chari, Kehoe, and McGrattan (2007). T_{nt} is a lump-sum transfer that rebates to the households all the within-country taxes. In particular, the labor supply and Euler equations now read:[21]

$$H_{njt}^{\frac{1}{\psi}}=\left(1-\xi^H_{njt}\right)\frac{W_{njt}}{P_{nt}},\tag{4.2}$$

and

$$U'_{nt}\left(1+\xi^I_{njt}\right)=\beta\mathbb{E}_t\left[U'_{nt+1}\left(\frac{R_{njt+1}}{P_{nt+1}}+(1-\delta)\left(1+\xi^I_{njt+1}\right)\right)\right].\tag{4.3}$$

Our benchmark model assumes financial autarky. There are two reasons behind this assumption. First, as highlighted by Heathcote and Perri (2002), models featuring financial autarky outperform complete and incomplete markets models in accounting for business cycle comovement.[22] Second, under financial autarky the contemporaneous response of output to shocks in the dynamic model is still given by the influence matrix (3.8), and can be constructed using only observed export and import shares, the elasticities of substitution among intermediate and final goods, and the Frisch elasticity. Alternative financial market structures would require additional assumptions on

[21]The labor shocks can have a literal interpretation as exogenous shifts in intra-temporal factor supply curves. Alternatively, news shocks (e.g. Beaudry and Portier, 2006), or sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015) would manifest themselves as shocks to ξ^H_{njt}, as agents react to a positive innovation in sentiment by supplying more labor. Straightforward manipulation shows that ξ^H_{njt} can also be viewed as a shifter in the optimality condition for factor usage. The literature has explored the aggregate labor version of this shifter, labeling it alternatively a "preference shifter" (Hall, 1997), "inefficiency gap" (Galí, Gertler, and López-Salido, 2007), or "labor wedge" (Chari, Kehoe, and McGrattan, 2007). While this object is treated as a reduced-form residual in much of this literature, we know that monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010) manifest themselves as shocks to ξ^H_{njt}.

[22]The financial autarky assumption is also adopted in Corsetti, Dedola, and Leduc (2008), Ruhl (2008), and many others. Kose and Yi (2006) show that when it comes to accounting for the trade-comovement relationship, the benchmarks of complete markets and financial autarky deliver similar results. We acknowledge that the financial autarky assumption excludes transmission mechanisms that operate through international capital flows. While this paper focuses on shock transmission through goods trade and production linkages, we leave the evaluation of other transmission mechanisms for future research.

the preferences and technology to derive this matrix. We therefore assume that there are only goods flows across countries, and further, trade is balanced period by period.

Production is subject to a TFP shock, which we relabel ξ^Z_{njt} in this section,[23] and a shock to the intermediate input market ξ^X_{njt}. The firm thus solves:

$$\max\left\{P_{njt}Y_{njt} - W_{njt}H_{njt} - R_{njt}K_{njt} - (1+\xi^X_{njt})P^x_{njt}X_{njt}\right\}$$

where

$$Y_{njt} = \xi^Z_{njt}\left(K^{\alpha_j}_{njt}H^{1-\alpha_j}_{njt}\right)^{\eta_j}X^{1-\eta_j}_{njt}. \tag{4.4}$$

The intermediate goods wedge affects the input choice decision:

$$(1-\eta_j)P_{njt}Y_{njt} = (1+\xi^X_{njt})P^X_{njt}X_{njt}. \tag{4.5}$$

The market clearing conditions are unchanged, and still given by (3.6). To examine the dynamic responses of the model and how it affects the international comovement, we proceed by solving the log-linearized model.

4.3 Calibration and Recovery of Shocks

As mentioned above, with 4 shocks instead of 1, we match multiple data series: value added, labor input, capital input, and intermediate input. Three of the 4 shocks can be recovered by applying data for Y_{njt}, H_{njt}, K_{njt}, and X_{njt} to intratemporal optimality conditions, and thus do not rely on the dynamic structure of the model. The TFP shock is read simply as the Solow residual from (4.4). The labor shock is recovered from the labor supply (4.2), after expressing for W_{njt} and P_{nt} as functions of Y_{njt} and model parameters. The intermediate wedge ξ^X_{njt} comes from (4.5), after expressing the prices as functions of Y_{njt}.

The TFP shock is the least reliant on model parameters, as in needs information only on factor shares α_j and η_j. The intermediate market shock requires in addition substitution elasticities ρ and ε, plus the global matrix of input and final good shares. Recovering the labor shocks relies on all of those plus the Frisch elasticity, that is, all the parameters of the static model.

The investment shock ξ^I_{njt} enters the Euler equation, and thus requires solving the full dynamic model. There is a small set of additional parameters relative to the static model, for which we adopt values standard in the business cycle literature. The model period is a year; we set the discount rate to $\beta = 0.96$. The period utility is $U(\cdot) = \ln(\cdot)$, and the depreciation rate is set to match sector

[23]The TFP shocks in this section match the measured Solow residual, and differ from the Z shock recovered in Section 3.

specific depreciation rates obtained from the BEA for 2001.[24] For the elasticity of substitution, we employ the baseline values from the static model, that is, $\rho = 2.75$ and $\varepsilon = 1$.[25]

The most demanding task in the calibration is choosing shock processes for different countries and sectors, as policy functions depend on the perceived stochastic processes of all shocks. In exercises similar to ours, Chari, Kehoe, and McGrattan (2007) and Ohanian, Restrepo-Echavarria, and Wright (2018) use maximum likelihood and Bayesian estimation respectively to obtain the shock processes. However, the number of parameters to be estimated in our exercise is an order of magnitude larger because all shocks are at the sectoral level, which makes either maximum likelihood or Bayesian estimation intractable.

Instead, we apply an iterative algorithm to recover the shock processes. Rational expectations impose the cross-equation restriction that perceived shock processes need to be consistent with actual shock processes. In our implementation, the estimation strategy is the following: given some perceived shock processes, the policy functions can be computed. Under these candidate policy functions, we can recover the time series for the realizations of shocks that rationalize the data. These candidate shock realizations can in turn be used to estimate the stochastic shock processes, which become the perceived processes in the next iteration. We iterate until the perceived and estimated shock processes coincide.

For non-G7 countries, the panel is too short to obtain reliable estimates of the shock processes. Therefore in this section we narrow the focus to the G7 countries. We assume that the country-sector shocks follow a vector autoregressive process. However, due to the large number of countries and sectors, it is not feasible to estimate the fully unrestricted VAR. We impose parsimonious functional forms on the shock processes due to the large number of countries and sectors in the model:

$$\ln \xi^x_{njt} = \rho_{nj} \ln \xi^x_{njt-1} + \zeta_n \mathbf{1}\left(m = n, k \neq j\right) \ln \xi^x_{mkt-1} + \epsilon^x_{njt} \tag{4.6}$$

for $x = Z, X, H, I$. These processes allow for own autocorrelation and a within-country lagged spillover of a sectoral shock. In addition, innovations ϵ^x_{njt} can have an arbitrary contemporaneous cross-border, cross-sector and cross-shock covariance structure. On the other hand, (4.6) does not allow for lagged cross-border spillovers.[26] Appendix Table A4 summarizes the parameter estimates for the processes. The estimated processes are persistent, with mean own-lag parameter estimates ranging from 0.85 for $\ln \xi^X_{njt}$ to 0.93 for $\ln \xi^I_{njt}$. Lagged cross-sector within-country spillovers are very

[24]The BEA provides depreciation rates for 1995-2007 that can be mapped to NAICS codes. We concord these to the sectors in the WIOD. As the depreciation rates are relatively stable over time, we use the mid-point year of 2001.

[25]See Appendix A.4 for more details on solving the dynamic model and recovering the shocks. In the linearized model, the taste parameters ϑ_{mnj} and $\mu_{mi,nj}$ and the trade costs τ_{mni} affect the dynamics only via the the final use and the intermediate use trade shares. Once we match the trade shares as in the data, there is no need to pin down the trade costs and taste parameters separately.

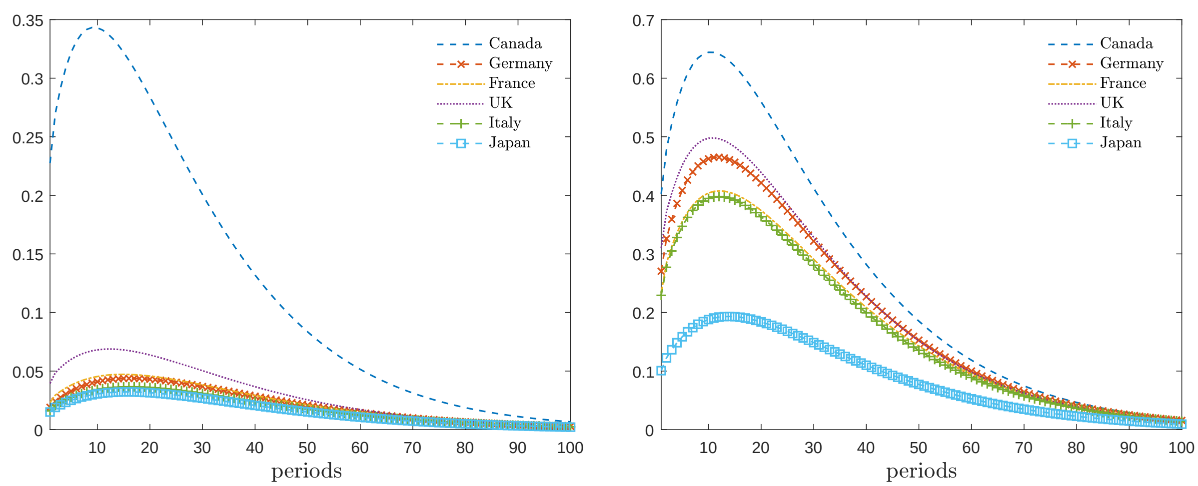
[26]We have also experimented with shock processes that include lagged within-sector cross-border spillovers, and the quantitative results remain similar. Further details on the estimation are in Appendix B.2.

small and close to zero on average for all series.

4.4 Impulse Responses

We begin by revisiting the impulse response exercise in Section 3.4. The left panel of Figure 9 displays the response of other countries to a 1% US productivity shock. Similar to the static model, Canada experiences the largest response as it has the largest trade linkages to the US. Note that the model features rich propagation, as the responses of all the countries are quite persistent over time. The hump-shaped IRF indicates that there is nontrivial endogenous propagation. Meanwhile, all the countries co-move quite closely in response to the US shock. The right panel of Figure 9 displays the response to a hypothetical rest-of-the-world shock in all sectors from the perspective of each country. Again, we observe significant persistence. Similar to the static model, the responses are heterogeneous across countries. Japan, US, and Italy respond little, while Canada and UK are quite sensitive to foreign global shocks.

FIGURE 9: Impulse Responses to US and Rest of the World Shocks



Response to a US Technology Shock | Response to a Rest-of-the-World Technology Shock

Notes: This figure displays the impulse responses of log real GDP of each G7 country following a 1% US TFP shock (left panel), and following a 1% TFP shock in the rest of the world excluding the country.

4.5 Dynamic Comovement Decomposition

We next explore the quantitative importance of the intertemporal propagation relative to the contemporaneous response for international comovement. As emphasized in Section 2, in the dynamic model the GDP growth rate can be expressed as a function of current and past shocks:

$$d \ln V_{nt} \approx \sum_{k=0}^{\infty} \sum_{m} \sum_{i} \boldsymbol{s}_{mni,k} \boldsymbol{\epsilon}_{mi,t-k}.$$

where $\boldsymbol{\epsilon}_{mi,t-k} \equiv \left\{\epsilon^{Z}_{mit}, \epsilon^{H}_{mit}, \epsilon^{X}_{mit}, \epsilon^{I}_{mit}\right\}$ is the vector of the innovations with covariance matrix $\boldsymbol{\Sigma}$. This leads to the following correlation decomposition:

$$\varrho(d \ln V_{nt}, d \ln V_{mt}) = \sum_{k=0}^{\infty} \omega_{nm,k} \; \varrho_{nm,k}, \tag{4.7}$$

where $\varrho_{nm,k}$ is the correlation between components $\boldsymbol{s}_{n,k}\boldsymbol{\epsilon}_{t-k}$ and $\boldsymbol{s}_{m,k}\boldsymbol{\epsilon}_{t-k}$ and $\omega_{nm,k}$ is its corresponding weight:

$$\varrho_{nm,k} = \frac{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}{\sqrt{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,k}}\sqrt{\boldsymbol{s}_{m,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}} \qquad \text{with} \qquad \omega_{nm,k} = \frac{\sqrt{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,k}}\sqrt{\boldsymbol{s}_{m,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}}{\sqrt{\sum_{i=0}^{\infty}\boldsymbol{s}_{n,i}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,i}}\sqrt{\sum_{i=0}^{\infty}\boldsymbol{s}_{m,i}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,i}}}.$$

Decomposition (4.7) is just (2.8) from Section 2, expressed in correlations.

Figure 10 plots $\varrho_{nm,k}\omega_{nm,k}$ across horizons k, averaging over country pairs. The correlation of contemporaneous responses corresponds to $k = 0$. It turns out that the contemporaneous component is dominant, accounting for over 80% of the total correlation. That is, adding dynamics does not substantially raise the GDP correlations.

The model features rich intertemporal propagation patterns, as evidenced by Figure 9. The small contribution of dynamics to overall comovement is mainly due to the fact that the timing of the response to the same shock is not sufficiently similar across countries to induce substantial delayed correlation. To illustrate this pattern, Figure 11 compares the response of US GDP to its own and rest-of-the-world shocks. While the country responds positively and persistently to positive foreign shocks, both the shape and the magnitude of the responses are different. The response to its own shock is high on impact, and gradually dies out. In contrast, the response to other countries' shocks continues to build slowly and peaks after over twenty periods. In addition, the response to a country's own shock is far larger than the response to other countries' shocks, both on impact and at most horizons.

Table 3 decomposes overall comovement into shock correlation and transmission terms in the dynamic model. The row labeled "Data" reports the correlations in the data. The row "Dynamic model with actual shocks" presents the correlations when the actual path of recovered shocks is fed into the dynamic model. The row "Dynamic model with simulated shocks" reports instead the correlations

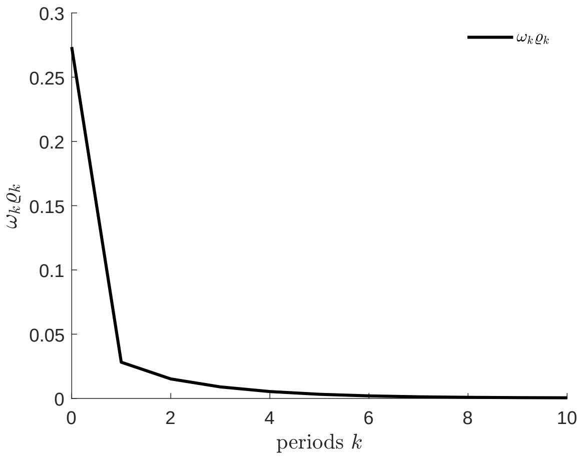


FIGURE 10: Correlation Decomposition

Notes: This figure displays the elements of the dynamic decomposition of the overall correlation into the components accounted for by elements at horizon k, as in (4.7).

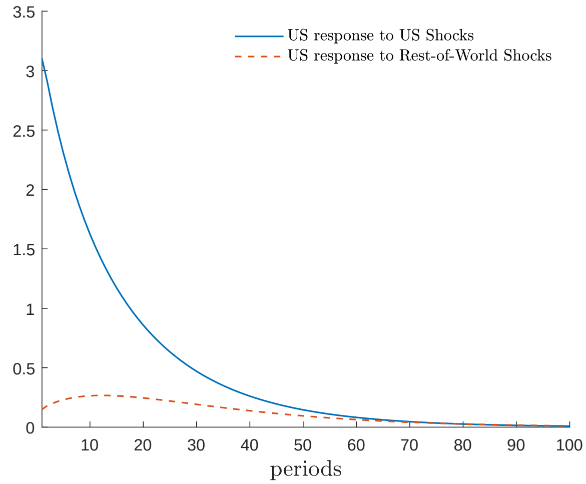


FIGURE 11: IRF of US to Technology Shock

Notes: This figure displays the impulse responses of US GDP following a 1% US TFP shock, and following a 1% TFP shock in the rest of the world excluding the US.

resulting from simulating the estimated shock process (4.6), and feeding it into the model.

In the dynamic model, the decomposition of GDP comovement into shock correlation and transmission must rely on the simulated model where innovations to shocks are well defined. As shown in the second and third rows of Table 3, the model produces quite similar GDP growth correlations when feeding the actual shock realizations and the simulated shocks, suggesting that the estimated shock processes capture the properties of the actual shocks relatively well. The decomposition into shock correlation and transmission terms in the dynamic model requires a straightforward combination of (2.5) and (2.8). The bottom panel of Table 3 reports the results. The shock correlation term is the most important component, accounting for more than half of the observed GDP growth correlation. Different from the static model, the multilateral transmission term now accounts for about 40% of overall comovement, a larger contribution compared to what we found in Section 3.

4.6 The Role of Individual Shocks

A well-known feature of this type of business cycle accounting exercise is that the 4 extracted shocks are not mutually uncorrelated (see, among others, Chari, Kehoe, and McGrattan, 2007; Eaton et al., 2016). Hence there is no additive decomposition of overall comovement into the components driven by each of the 4 shocks. Instead, part of the GDP correlation will come from cross-shock covariance terms, for instance comovement driven by correlation of TFP in country j with the labor shock

Table 3: GDP Growth Correlations in the Dynamic Model

	Mean	Median	25th pctile	75th pctile
Data	0.358	0.337	0.242	0.565
Dynamic model with actual shocks	0.350	0.356	0.124	0.558
Dynamic model with simulated shocks	0.349	0.411	0.099	0.544
Decomposition				
Shock correlation	0.185	0.239	-0.099	0.382
Bilateral transmission	0.018	0.011	0.006	0.024
Multilateral transmission	0.146	0.145	0.088	0.188

Notes: This table presents the summary statistics of the correlations of $d\ln V_{nt}$ in the sample of G7 countries, and the decomposition into the shock correlation and transmission components. Variable definitions and sources are described in detail in the text.

in country i.[27] In that sense, there is no unique answer to the question of which single shock is responsible for the most comovement.

We proceed by presenting two polar exercises. In the first, we take out one shock at a time, keeping the other 3. In terms of shock correlation, removing a shock gets rid of both the correlations of that shock with the same shock in other countries (e.g., TFP in country j with TFP in country i), and the correlation of that shock with other shocks in other countries (e.g., TFP in country j with labor in country i). In the second exercise, we feed in one shock at a time. This exercise generates comovement only through correlation of a shock with the same shock abroad. Needless to say, in both types of exercises, the transmission terms change as well. Note that shocks can have a negative correlation, and thus it is not necessarily the case that the first exercise leads to a larger contribution of a single shock to comovement compared to the second exercise.

Table 4 reports the resulting correlations. The first two rows present the data and the correlations conditional on all four shocks, which by definition match the data perfectly. The next 4 rows remove one shock at a time. The largest impact on correlation is due to the labor shock: removing it lowers the mean correlation by nearly 60%. The second-most important shock is TFP, whose removal

[27] As in other business cycle accounting analyses, different types shocks within the same country are correlated among themselves. With a small number of aggregate shocks in a single country, one can in principle orthogonalize them. Our object is international correlations, and there are 4 shocks in each sector in each country. There is no practical way to transform them in such a way that a given type of shocks in one country is only correlated with the same type of shocks in the other countries, but orthogonal to all other categories of foreign shocks.

Table 4: GDP Growth Correlations Conditional on Subsets of Shocks

	Cross-country Correlation			
	Mean	Median	St.dev.(GDP)	Corr w/data
Data	0.358	0.337	1.745	1.000
All shocks	0.350	0.356	1.663	1.000
No TFP shock	0.241	0.253	1.393	0.285
No labor shock	0.153	0.158	1.955	0.815
No intermediate input shock	0.328	0.336	1.763	0.945
No investment shock	0.311	0.317	0.938	0.408
Only TFP shock	0.108	0.119	1.908	0.672
Only labor shock	0.111	0.064	1.094	0.043
Only intermediate input shock	0.158	0.096	0.577	-0.021
Only investment shock	0.565	0.567	0.856	0.547
TFP and labor shock	0.235	0.307	1.579	0.818

Notes: This table presents the summary statistics of the correlations of $d\ln V_{nt}$ in the sample of G7 countries, for the model with all four shocks, and taking out one shock at a time. Variable definitions and sources are described in detail in the text.

Table 5: Correlations Among GDP Growth Rates Driven by Single Shocks

	TFP	Labor	Intermediate	Investment
TFP	0.108	-0.004	0.167	-0.045
Labor	0.080	0.111	-0.056	0.156
Intermediate	0.094	0.004	0.158	-0.123
Investment	-0.015	0.113	-0.104	0.565

Notes: This table reports the average GDP correlations that result when GDP growth in one country is driven by the shock in the row of the table, and the GDP growth in the other country is driven by the shock in the column of the table.

lowers the correlations by 30%. The intermediate inputs and investment shocks have much less impact, conditional on the other shocks operating.

The bottom four rows report instead the correlations conditional on a single shock. Here, the correlation generated by the labor shock, 0.11, is not that different from the correlations generated by the TFP and intermediate shocks. The outlier is the investment shock, which by itself generates the highest correlations. The two rightmost columns of Table 4 report two additional diagnostics on the alternative shock models: the standard deviation of GDP growth, and the correlation between GDP growth generated by each subset of shocks and the data GDP growth. The intermediate and investment shocks do not generate sufficient volatility. Alone, the intermediate input and investment shocks produce standard deviations of GDP growth that are one-third to one-half of the data values. GDP growth driven by intermediate, and more surprisingly by labor shocks, is uncorrelated with the data. By contrast, while the TFP shock by itself does not produce a lot of international comovement, the GDP growth rates generated by TFP are more correlated with the data than those generated by other shocks, as evidenced by the last column of Table 4.

It may appear puzzling that the labor shock comes out as the most important in the first exercise, but much less so in the second one. The discrepancy in the conclusions from the two exercises is resolved by the fact that the labor shock is positively correlated with other shocks abroad, whereas the investment shock is negatively correlated with other shocks abroad. Table 5 illustrates this by reporting the mean GDP correlations when GDP in country j is driven by a shock in the row of the table, and GDP in country i is driven by a shock in the column. The labor shock-driven GDP is positively correlated with the TFP-driven GDP (0.08), and with the investment shock-driven GDP (0.16). By contrast, the investment shock-driven GDP has negative correlations with TFP- and intermediate shock-driven GDP.

We synthesize these results as follows. First, no single shock has a dominant role in international comovement. Individually, the labor and the TFP shocks appear most promising, but for different reasons. The labor shock has the highest synchronizing impact, with the qualification that much of its overall effect appears to come from its correlation with other shocks rather than with itself. Taken alone, the TFP and labor shocks generate similar amounts of GDP comovement, but the TFP shock also produces GDP series closer to the data.

Second, a model that combines labor and TFP shocks strikes a good balance between parsimony and fit to the data. The bottom row of Table 4 reports the statistics for the model performance with these two shocks. The two shocks together generate more than two-thirds of the observed international correlation. At the same time, they reproduce the observed GDP volatility and generate a GDP series with an 0.82 correlation with the data. The model is parsimonious both in the sense that it relies on only two shocks, and in the sense that these shocks themselves are relatively simple, and would work in the same way in both static and dynamic models.

Third, the next shock in order of importance is the investment shock. Adding it to TFP and labor essentially reproduces the data (see row "No intermediate input shock" in Table 4). However, adding this shock comes at a cost of parsimony, especially because both extracting and using this shock requires solving and iterating on the full dynamic model. While the baseline model abstracts from modeling financial shocks explicitly, as shown by Chari, Kehoe, and McGrattan (2007) some types of financial frictions will manifest themselves as investment shocks. Our results can be viewed as suggestive that financial shocks may play a non-trivial role in international comovement, but perhaps a less important one than the TFP and labor shocks.

Finally, the intermediate input shock is the least important, either by itself or in conjunction with other shocks. A quantification is not missing much by omitting it.

4.7 Correlates of Recovered Shocks

We stress that the 4 shocks recovered in our procedure use no external information or exogenous variation. Thus, they are consistent with a variety of microfoundations. To understand whether these recovered shocks correspond closely to existing independently identified shocks, we collect a number of shocks from earlier studies, and correlate them with each of our 4 recovered shocks. We make use of the following standard shock series: (i) the Fernald (2014) utilization-adjusted TFP; (ii) the Barsky and Sims (2011) news of future TFP; (iii) the Levchenko and Pandalai-Nayar (2018) sentiment shocks; (iv) the Romer and Romer (2004) monetary policy shocks, updated by Coibion et al. (2017); (v) federal spending (Ramey, 2011) and tax (Romer and Romer, 2010) shocks; (vi) oil price increases (Hamilton, 2003) and identified oil supply shocks (Baumeister and Hamilton, 2019); (vii) financial shocks proxied by excess bond premia (Gilchrist and Zakrajšek, 2012), and (viii) uncertainty shocks in the form of innovations to VIX (Bloom, 2009) and to the Baker-Bloom-Davis index of policy uncertainty. Many of these shocks come at monthly or quarterly frequency, and we convert them to annual frequency to match with our data.

Unfortunately, most of these identified shocks are specific to the US. The ideal exercise here would collect all possible independently identified shocks for all countries to determine which shocks are the most promising for explaining the patterns of comovement. To our knowledge, however, collections of these shocks do not exist for multiple countries. Thus, we can only compare these identified shocks to the annual series of shocks for the US recovered from our model. We have 30 years of observations. While this exercise will not speak directly to sources of cross-country comovement, it can at least tell us whether our recovered shocks correlate closely with externally identified shocks for one country.

We begin with regressing each model-recovered shock on an individual category of identified shocks. Table 6 reports the resulting R^2's. For TFP and labor market shocks, the Barsky-Sims news shocks have the highest bivariate explanatory power. Least explainable is the intermediate input shock, with the lowest overall R^2 and no set of identified shocks having an R^2 of over 0.1. By contrast, the investment shock appears to be the most correlated with other identified shocks, with sentiment, oil, financial, and uncertainty shocks having an R^2 of 15% or more.

Of course, the externally identified shocks themselves can be mutually correlated. To see which identified shocks have the strongest conditional correlations with our recovered shocks, we regress the recovered shocks on all the identified shocks together. Note that with only 30 annual observations and 10 regressors, there are not that many degrees of freedom left. Nonetheless, columns labeled "Sig." report the level of significance of individual shocks when all are included in the same regression. There is some variation in which shocks are most important. Overall, news and sentiment shocks appear most correlated with our shocks, but their relative importance also varies across shocks. Fiscal, monetary, and oil shocks appear with varying levels of significance for individual series.

All in all, there is no clear pattern of correlation, whereby a recovered shock can be convincingly attributed predominantly to a particular externally identified shock. Nonetheless, this exercise suggests that at least for the US, our shocks bear some resemblance to prominent identified sources of business cycle fluctuations.

5 Conclusion

We set out to provide a comprehensive account of international comovement in real GDP. Using a simple accounting framework, we decomposed the GDP covariance into additive components representing correlated shocks and cross-border transmission. The relative importance of these two terms is determined jointly by the correlations of the primitive shocks and the strength of domestic and international input-output linkages. The accounting framework also clarifies the role of dynamic propagation: the total GDP covariance is the sum of the covariance due to the instantaneous

Table 6: Projecting Recovered Shocks on Identified Shocks

	TFP		Labor		Intermediate		Investment	
	R^2	Sig.	R^2	Sig.	R^2	Sig.	R^2	Sig.
Fernald TFP	0.08		0.00		0.02		0.01	
News	0.22	*	0.23	**	0.00		0.04	*
Sentiment	0.08		0.06		0.09	**	0.18	**
Monetary	0.04		0.03		0.04	**	0.06	
Fiscal	0.03		0.05	*	0.03		0.01	
Oil	0.08		0.00		0.01		0.15	***
Financial	0.05		0.06		0.05		0.26	
Uncertainty	0.20	*	0.08		0.07		0.19	
All together	0.54		0.43		0.36		0.60	

Notes: The columns labeled "R^2" report, for each recovered shock in the column, the R^2 from a bivariate regression of the growth in that shock on identified shocks in the row. The column labeled "Sig." reports the level of significance of the category of identified shocks when all the identified shocks are used as regressors together in a multivariate regression. ***: significant at the 1% level; **: significant at the 5% level; *: significant at the 10% level. Variable definitions and sources are described in detail in the text.

responses to shock innovations, and dynamic terms that capture the lagged responses to shocks.

Our main findings are fourfold. First, most of the observed GDP comovement is accounted for by correlated shocks. Transmission of shocks has an economically meaningful, but smaller role in comovement. Second, the structure of the global production and trade network matters of GDP comovement. Going from autarky to trade increases the domestic influence of sectors whose primitive shocks are relatively less correlated, which reduces GDP comovement all else equal. At the current levels of trade openness, a value added-only model that ignores input trade underpredicts GDP comovement. Third, the majority of the observed overall correlation is due to the instantaneous response of the economy to shocks, rather than dynamic propagation of past shocks. And finally, while no single shock is predominantly responsible for international comovement, a relatively parsimonious model with two shocks – TFP and labor – appears to generate the bulk of the observed GDP correlations. Our results suggest that when searching for correlated shocks that synchronize GDP, TFP shocks are not sufficient, and that we should instead focus on non-technology shocks that have a labor wedge representation in a prototype model. Structural estimates of various candidate shocks that might explain the correlation in these "labor wedges" across countries are not yet available, and identifying them is a promising avenue for future research.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2016. "Networks and the Macroeconomy: An Empirical Exploration." *NBER Macroeconomics Annual 2015* 30:276–335.

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica* 80 (5):1977–2016.

Adao, Rodrigo, Costas Arkolakis, and Federico Esposito. 2017. "Trade, Agglomeration, and Labor Markets: Theory and Evidence." Mimeo, University of Chicago Booth and Yale University and Tufts University.

Ambler, Steve, Emanuela Cardia, and Christian Zimmermann. 2004. "International business cycles: What are the facts?" *Journal of Monetary Economics* 51 (2):257–276.

Angeletos, George-Marios and Jennifer La'O. 2013. "Sentiments." *Econometrica* 81 (2):739–779.

Atalay, Enghin. 2017. "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics* 9 (4):254–280.

Backus, David K, Patrick J Kehoe, and Finn E Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy* 100 (4):745–75.

Bai, Yan and José-Víctor Ríos-Rull. 2015. "Demand shocks and open economy puzzles." *American Economic Review* 105 (5):644–49.

Baqaee, David Rezza. 2018. "Cascading Failures in Production Networks." *Econometrica* 86 (5):1819–1838.

Baqaee, David Rezza and Emmanuel Farhi. 2019a. "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem." *Econometrica* 87 (4):1155–1203.

———. 2019b. "Macroeconomics with heterogeneous agents and input-output networks." Mimeo, UCLA and Harvard.

———. 2019c. "Networks, Barriers, and Trade." Mimeo, UCLA and Harvard.

Barrot, Jean-Noël and Julien Sauvagnat. 2016. "Input Specificity and the Propagation of Idiosyncratic Shocks in Production Networks." *Quarterly Journal of Economics* 131 (3):1543–1592.

Barsky, Robert B. and Eric R. Sims. 2011. "News shocks and business cycles." *Journal of*

Monetary Economics 58 (3):273–289.

Bartelme, Dominick, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare. 2018. "External Economies of Scale and Industrial Policy: A View from Trade." Mimeo, University of Michigan, MIT, and UC Berkeley.

Baumeister, Christiane and James D. Hamilton. 2019. "Structural Interpretation of Vector Autoregressions with Incomplete Identification: Revisiting the Role of Oil Supply and Demand Shocks." *American Economic Review* 109 (5):1873–1910.

Beaudry, Paul and Franck Portier. 2006. "Stock Prices, News, and Economic Fluctuations." *American Economic Review* 96 (4):1293–1307.

Bems, Rudolfs, Robert C Johnson, and Kei-Mu Yi. 2010. "Demand Spillovers and the Collapse of Trade in the Global Recession." *IMF Economic Review* 58 (2):295–326.

Bigio, Saki and Jennifer La'O. 2019. "Distortions in Production Networks." Forthcoming, *Quarterly Journal of Economics*.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77 (3):623–685.

Boehm, Christoph E., Aaron Flaaen, and Nitya Pandalai-Nayar. 2019. "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake." *The Review of Economics and Statistics* 101 (1):60–75.

Burstein, Ariel and Javier Cravino. 2015. "Measured Aggregate Gains from International Trade." *American Economic Journal: Macroeconomics* 7 (2):181–218.

Burstein, Ariel, Christopher Kurz, and Linda L. Tesar. 2008. "Trade, Production Sharing, and the International Transmission of Business Cycles." *Journal of Monetary Economics* 55:775–795.

Caliendo, Lorenzo and Fernando Parro. 2015. "Estimates of the Trade and Welfare Effects of NAFTA." *The Review of Economic Studies* 82 (1):1–44.

Canova, Fabio. 2005. "The transmission of US shocks to Latin America." *Journal of Applied Econometrics* 20 (2):229–251.

Carvalho, Vasco M. 2010. "Aggregate Fluctuations and the Network Structure of Intersectoral Trade." Mimeo, CREi and Universitat Pompeu Fabra.

Carvalho, Vasco M., Makoto Nirei, Yukiko U. Saito, and Alireza Tahbaz-Salehi. 2016. "Supply

Chain Disruptions: Evidence from the Great East Japan Earthquake." Mimeo, University of Cambridge, Policy Research Institute, Ministry of Finance of Japan, RIETI, and Columbia GSB.

Chamberlain, Gary. 1987. "Asymptotic efficiency in estimation with conditional moment restrictions." *Journal of Econometrics* 34 (3):305–334.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75 (3):781–836.

Coibion, Olivier, Yuriy Gorodnichenko, Lorenz Kueng, and John Silvia. 2017. "Innocent Bystanders? Monetary policy and inequality." *Journal of Monetary Economics* 88 (Supplement C):70 – 89.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2008. "International risk sharing and the transmission of productivity shocks." *The Review of Economic Studies* 75 (2):443–473.

———. 2014. "The International Dimension of Productivity and Demand Shocks in the US Economy." *Journal of the European Economic Association* 12 (1):153–176.

de Soyres, François and Alexandre Gaillard. 2019. "Value Added and Productivity Linkages Across Countries." International Finance Discussion Papers 1266.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum. 2008. "Global Rebalancing with Gravity: Measuring the Burden of Adjustment." *IMF Staff Papers* 55 (3):511–540.

di Giovanni, Julian and Andrei A. Levchenko. 2010. "Putting the Parts Together: Trade, Vertical Linkages, and Business Cycle Comovement." *American Economic Journal: Macroeconomics* 2 (2):95–124.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean. 2018. "The Micro Origins of International Business Cycle Comovement." *American Economic Review* 108 (1):82–108.

Drozd, Lukasz A., Sergey Kolbin, and Jaromir B. Nosal. 2020. "The Trade-Comovement Puzzle." Forthcoming, *American Economic Journal: Macroeconomics*.

Eaton, Jonathan, Samuel Kortum, and Brent Neiman. 2016. "Obstfeld and RogoffŒşs international macro puzzles: a quantitative assessment." *Journal of Economic Dynamics and Control* 72 (C):5–23.

Eaton, Jonathan, Samuel S. Kortum, Brent Neiman, and John Romalis. 2016. "Trade and the

Global Recession." *American Economic Review* 106 (11):3401–3438.

Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." Federal Reserve Bank of San Francisco Working Paper 2012-19.

Foerster, Andrew T., Andreas Hornstein, Pierre-Daniel G. Sarte, and Mark W. Watson. 2019. "Aggregate Implications of Changing Sectoral Trends." Federal Reserve Bank of Richmond Working Paper 19-11R.

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson. 2011. "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy* 119 (1):1–38.

Frankel, Jeffrey A. and Andrew K. Rose. 1998. "The Endogeneity of the Optimum Currency Area Criteria." *Economic Journal* 108 (449):1009–25.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations." *Econometrica* 79 (3):733–772.

Galí, Jordi, Mark Gertler, and David López-Salido. 2007. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." *Review of Economics and Statistics* 89 (1):44–59.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi. 2017. "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade." NBER Working Paper 23737.

Gilchrist, Simon and Egon Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review* 102 (4):1692–1720.

Grassi, Basile. 2017. "IO in I-O: Size, Industrial Organization, and the Input-Output Network Make a Firm Structurally Important." Mimeo, Bocconi.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review* 78 (3):402–17.

Hall, Robert E. 1997. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics* 15 (1, pt. 2):s223–s250.

Hamilton, James D. 2003. "What is an oil shock?" *Journal of Econometrics* 113 (2):363 – 398.

Heathcote, Jonathan and Fabrizio Perri. 2002. "Financial autarky and international business cycles." *Journal of Monetary Economics* 49 (3):601–627.

Hulten, Charles R. 1978. "Growth Accounting with Intermediate Inputs." *The Review of Economic Studies* 45 (3):511–518.

Hummels, David, Jun Ishii, and Kei-Mu Yi. 2001. "The Nature and Growth of Vertical Specialization in World Trade." *Journal of International Economics* 54:75–96.

Huo, Zhen and Naoki Takayama. 2015. "Higher Order Beliefs, Confidence, and Business Cycles." Mimeo, University of Minnesota.

Imbs, Jean. 1999. "Technology, growth and the business cycle." *Journal of Monetary Economics* 44 (1):65 – 80.

———. 2004. "Trade, Finance, Specialization, and Synchronization." *Review of Economics and Statistics* 86 (3):723–34.

Johnson, Robert C. 2014. "Trade in Intermediate Inputs and Business Cycle Comovement." *American Economic Journal: Macroeconomics* 6 (4):39–83.

Johnson, Robert C. and Guillermo Noguera. 2012. "Accounting for intermediates: Production sharing and trade in value added." *Journal of International Economics* 86 (2):224 – 236.

———. 2017. "A Portrait of Trade in Value-Added over Four Decades." *The Review of Economics and Statistics* 99 (5):896–911.

Jones, Charles I. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics* 3 (2):1–28.

———. 2013. "Misallocation, Economic Growth, and Input-Output Economics." In *Advances in Economics and Econometrics*, vol. II, edited by D. Acemoglu, M. Arellano, and E. Dekel. Cambridge University Press, 419–454.

Kehoe, Timothy J. and Kim J. Ruhl. 2008. "Are Shocks to the Terms of Trade Shocks to Productivity?" *Review of Economic Dynamics* 11 (4):804–819.

Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman. 2003. "International Business Cycles: World, Region, and Country-Specific Factors." *American Economic Review* 93 (4):1216–1239.

Kose, M. Ayhan and Kei-Mu Yi. 2006. "Can the Standard International Business Cycle Model Explain the Relation Between Trade and Comovement." *Journal of International Economics* 68 (2):267–295.

Levchenko, Andrei A. and Nitya Pandalai-Nayar. 2018. "TFP, News, and "Sentiments": The International Transmission of Business Cycles." Forthcoming, *Journal of the European Economic Association.*

Liao, Wei and Ana Maria Santacreu. 2015. "The Trade Comovement Puzzle and the Margins of International Trade." *Journal of International Economics* 96 (2):266–288.

Long, John and Charles Plosser. 1983. "Real Business Cycles." *Journal of Political Economy* 91 (1):39–69.

Mendoza, Enrique G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review* 100 (5):1941–66.

Neumeyer, Pablo A. and Fabrizio Perri. 2005. "Business cycles in emerging economies: the role of interest rates." *Journal of Monetary Economics* 52 (2):345–380.

Ng, Eric C.Y. 2010. "Production fragmentation and business-cycle comovement." *Journal of International Economics* 82 (1):1 – 14.

Ohanian, Lee E., Paulina Restrepo-Echavarria, and Mark L. J. Wright. 2018. "Bad Investments and Missed Opportunities? Postwar Capital Flows to Asia and Latin America." *American Economic Review* 108 (12):3541–3582.

O'Mahony, Mary and Marcel P. Timmer. 2009. "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database." *The Economic Journal* 119 (538):F374–F403.

Ramey, Valerie A. 2011. "Identifying Government Spending Shocks: It's all in the Timing." *The Quarterly Journal of Economics* 126 (1):1–50.

Romer, Christina D. and David H. Romer. 2004. "A New Measure of Monetary Shocks: Derivation and Implications." *American Economic Review* 94 (4):1055–1084.

———. 2010. "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks." *American Economic Review* 100 (3):763–801.

Ruhl, Kim J. 2008. "The international elasticity puzzle." Working Paper, NYU.

Stockman, Alan C and Linda L Tesar. 1995. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review* 85 (1):168–85.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries. 2015. "An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production." *Review of International Economics* 23 (3):575–605.

Wen, Yi. 2007. "By force of demand: Explaining international comovements." *Journal of Economic Dynamics and Control* 31 (1):1–23.

Yi, Kei-Mu. 2003. "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy* 111 (1):52–102.

Young, Alwyn. 2017. "Consistency without Inference: Instrumental Variables in Practical Application." Mimeo, LSE.

ONLINE APPENDIX

(NOT FOR PUBLICATION)

Table A1: Country Sample

Australia	Germany	Netherlands
Austria	Greece	Poland
Belgium	Hungary	Portugal
Canada	India	Russian Federation
Cyprus	Ireland	Slovak Republic
Czech Republic	Italy	Slovenia
Denmark	Japan	Spain
Estonia	Republic of Korea	Sweden
Finland	Latvia	UK
France	Lithuania	USA

TABLE A2: Sector Sample

agriculture hunting forestry and fishing	basic metals and fabricated metal	financial intermediation
mining and quarrying	machinery nec	real estate activities
food beverages and tobacco	electrical and optical equipment	renting of m&eq and other business activities
textiles textile leather and footwear	transport equipment	public admin and defense; compulsory social security
wood and of wood and cork	manufacturing nec; recycling	education
pulp paper paper printing and publishing	electricity gas and water supply	health and social work
coke refined petroleum and nuclear fuel	construction	other community social and personal services
chemicals and chemical products	hotels and restaurants	sale maintenance and repair of motor vehicles
rubber and plastics	transport and storage	wholesale trade and commission trade
other nonmetallic mineral	post and telecommunications	retail trade except of motor vehicles

Appendix A Model and Quantitative Results

Throughout this appendix, variables without a t-subscript denote steady-state values, and variables with a t-subscript denote their realizations. We introduce the t-subscripts in the proofs of the network model results to streamline the transition to the dynamic model derivations.

A.1 Proofs

Proof of Theorem 1: The market clearing condition for the sales in country n sector j in levels is

$$P_{njt}Y_{njt} = \sum_m P_{mt}\mathcal{F}_{mt}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i)P_{mit}Y_{mit}\pi^x_{nj,mit}.$$

Note that with financial autarky, the total sales of final goods is the same as the value added across sectors

$$P_{mt}\mathcal{F}_{mt} = \sum_i \eta_i P_{mit}Y_{mit}.$$

The market clearing condition is then

$$P_{njt}Y_{njt} = \sum_m \sum_i \eta_i P_{mit}Y_{mit}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i)P_{mit}Y_{mit}\pi^x_{nj,mit}.$$

Log-linearizing:

$$\begin{aligned}\ln P_{njt} + \ln Y_{njt} = &\sum_m \sum_i \frac{\pi^f_{nmj}P_m\mathcal{F}_m}{P_{nj}Y_{nj}}\frac{\eta_i P_{mi}Y_{mi}}{P_m\mathcal{F}_m}\left(\ln P_{mit} + \ln Y_{mit} + \ln \pi^f_{nmjt}\right) \\ &+ \sum_m \sum_i \frac{(1-\eta_i)\pi^x_{nj,mi}P_{mi}Y_{mi}}{P_{nj}Y_{nj}}(\ln P_{mit} + \ln Y_{mit} + \ln \pi^x_{nj,mit}) \qquad \text{(A.1)}\end{aligned}$$

and the log-deviation of import shares are given by

$$\begin{aligned}\ln \pi^f_{nmjt} =&(1-\rho)\sum_{k,l}\pi^f_{km\ell}\left(\ln P_{njt} - \ln P_{k\ell t}\right) \\ \ln \pi^x_{nj,mit} =&(1-\varepsilon)\sum_{k,\ell}\pi^x_{kl,mi}\left(\ln P_{njt} - \ln P_{k\ell t}\right).\end{aligned}$$

Define the following share matrices:

1. $\mathbf{\Psi}^f$ is an $NJ \times N$ matrix whose (nj,m)th element is $\frac{\pi^f_{nmj}P_m\mathcal{F}_m}{P_{nj}Y_{nj}}$. That is, this matrix stores the share of total revenue in the country-sector in the row that comes from final spending in the country in the column.

2. $\mathbf{\Psi}^x$ is an $NJ \times NJ$ matrix whose (nj,mi)th element is $\frac{(1-\eta_i)\pi^x_{nj,mi}P_{mi}Y_{mi}}{P_{nj}Y_{nj}}$. That is, this matrix stores the share of total revenue in the country-sector in the row that comes from

intermediate spending in the country-sector in the column.

3. $\mathbf{\Upsilon}$ is an $N \times NJ$ matrix whose (n, mi)th element is $\frac{\eta_i P_{mi} Y_{mi}}{P_n \mathcal{F}_n}$. That is, this matrix stores the share of value added in the country-sector in the column in total GDP of the country in the row. Note that these are zero whenever $m \neq n$.

4. $\mathbf{\Pi}^f$ is an $N \times NJ$ matrix whose $(m, k\ell)$th element is $\pi^f_{km\ell}$. That is, this matrix stores the final expenditure share on goods coming from the column in the country in the row.

5. $\mathbf{\Pi}^x$ is an $NJ \times NJ$ matrix whose $(k\ell, mi)$th element is $\pi^x_{mi,k\ell}$. That is, this matrix stores the intermediate expenditure share on goods coming from the column in the country-sector in the row.

Then, equation (A.1) can be stated in matrix form:

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \left(\mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x \right) (\ln \mathbf{P}_t + \ln \mathbf{Y}_t) + (1-\rho) \left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^f \right) \ln \mathbf{P}_t$$
$$+ (1-\varepsilon) \left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x \right) \ln \mathbf{P}_t.$$

This allows us to express prices as a function of quantities as in (3.11), where[28]

$$\mathcal{P} = -\left(\mathbf{I} - \mathcal{M} \right)^+ \left(\mathbf{I} - \mathbf{\Psi}^f \mathbf{\Upsilon} - \mathbf{\Psi}^x \right)$$
$$\mathcal{M} = \mathbf{\Psi}^f \mathbf{\Upsilon} + \mathbf{\Psi}^x + (1-\rho) \left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{1}\right) - \mathbf{\Psi}^f \mathbf{\Pi}^f \right) + (1-\varepsilon) \left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{1}\right) - \mathbf{\Psi}^x \mathbf{\Pi}^x \right).$$

Turning to the supply side, the labor demand (3.4) stacked into an $NJ \times 1$ vector is:

$$\ln \mathbf{W}_t - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{H}_t.$$

The labor supply in vector notation is:

$$\ln \mathbf{H}_t = \psi(\ln \mathbf{W}_t - \ln \mathbf{P}^f_t),$$

where $\ln \mathbf{P}^f_t$ denotes the consumption price index (3.2) that prevails at each sector in log-deviations and matrix notation:

$$\ln \mathbf{P}^f_t = \left(\mathbf{\Pi}^f \otimes \mathbf{1} \right) \ln \mathbf{P}_t.$$

Here, $\mathbf{1}$ is a vector of one with length J and the Kronecker product transforms the dimension of the consumption price index at country level to the country-sector level.

[28] The + sign stands for the Moore-Penrose inverse as $\mathbf{I} - \mathcal{M}$ is not invertible. The non-invertibility is a consequence of the fact that the vector of prices is only defined up to a numeraire.

These three conditions imply the following equilibrium relationship for hours:

$$\ln \mathbf{H}_t = \frac{\psi}{1+\psi} \ln \mathbf{Y} + \frac{\psi}{1+\psi} \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right) \ln \mathbf{P}_t. \tag{A.2}$$

Similarly, market clearing for intermediate inputs is:

$$\ln \mathbf{P}^x_t - \ln \mathbf{P}_t = \ln \mathbf{Y}_t - \ln \mathbf{X}_t,$$

where $\ln \mathbf{P}^x_t$ is the vector of intermediate input price indices for all countries and sectors:

$$\ln \mathbf{P}^x_t \quad = \quad \mathbf{\Pi}^x \ln \mathbf{P}_t.$$

Jointly, these imply

$$\ln \mathbf{X}_t = \ln \mathbf{Y}_t + (\mathbf{I} - \mathbf{\Pi}^x) \ln \mathbf{P}_t.$$

Plugging these into the production function

$$\begin{aligned}
\ln \mathbf{Y}_t &= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) \ln \mathbf{H}_t + (\mathbf{I} - \boldsymbol{\eta}) \ln \mathbf{X}_t. \\
&= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) \left(\frac{\psi}{1+\psi} \ln \mathbf{Y} + \frac{\psi}{1+\psi} \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right) \ln \mathbf{P}_t \right) + (\mathbf{I} - \boldsymbol{\eta}) \left(\ln \mathbf{Y}_t + (\mathbf{I} - \mathbf{\Pi}^x) \ln \mathbf{P}_t\right). \\
&= \ln \mathbf{Z}_t + \left[\frac{\psi}{1+\psi} \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) \left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right) \mathcal{P}\right) + (\mathbf{I} - \boldsymbol{\eta}) \left(\mathbf{I} + (\mathbf{I} - \mathbf{\Pi}^x) \mathcal{P}\right) \right] \ln \mathbf{Y}_t.
\end{aligned}$$

where in the last step we use (3.11) for $\ln \mathbf{P}_t$. Inverting for $\ln \mathbf{Y}_t$ completes the proof:

$$\ln \mathbf{Y}_t = \left(\mathbf{I} - \frac{\psi}{1+\psi} \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) \left(\mathbf{I} + \left(\mathbf{I} - \mathbf{\Pi}^f \otimes \mathbf{1}\right) \mathcal{P}\right) - (\mathbf{I} - \boldsymbol{\eta}) \left(\mathbf{I} + (\mathbf{I} - \mathbf{\Pi}^x) \mathcal{P}\right) \right)^{-1} \ln \mathbf{Z}_t.$$

Derivation of (3.12): We can also write the production function as

$$\begin{aligned}
\ln \mathbf{Y}_t &= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) (\ln \mathbf{H}_t) + (\mathbf{I} - \boldsymbol{\eta}) (\ln \mathbf{X}_t), \\
&= \ln \mathbf{Z}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) (\ln \mathbf{Y}_t + \ln \mathbf{P}_t - \ln \mathbf{W}_t) + (\mathbf{I} - \boldsymbol{\eta}) (\ln \mathbf{Y}_t + \ln \mathbf{P}_t - \ln \mathbf{P}^x_t), \\
&= \ln \mathbf{Z}_t + (\mathbf{I} - \boldsymbol{\alpha\eta}) \ln \mathbf{Y}_t + \boldsymbol{\eta}(\mathbf{I} - \boldsymbol{\alpha}) (\ln \mathbf{P}_t - \ln \mathbf{W}_t) + (\mathbf{I} - \boldsymbol{\eta}) (\ln \mathbf{P}_t - \ln \mathbf{P}^x_t).
\end{aligned}$$

In partial equilibrium, prices and wages are fixed, and firms only respond to their own fundamental ($\ln \mathbf{Z}_t$) change. If this is the case, the output response is given by

$$\ln \mathbf{Y}_t = \ln \mathbf{Z}_t + (\mathbf{I} - \boldsymbol{\alpha\eta}) \ln \mathbf{Y}_t \quad \Rightarrow \quad \ln \mathbf{Y}_t = (\boldsymbol{\alpha\eta})^{-1} \ln \mathbf{Z}_t.$$

In general equilibrium, prices and wages also change. When these prices are expressed as a function of output, it follows that

$$\ln \mathbf{Y}_t = (\boldsymbol{\alpha\eta})^{-1} \ln \mathbf{Z}_t + \boldsymbol{\gamma} \ln \mathbf{Y}_t = \left(\boldsymbol{\eta\alpha}\right)^{-1} \ln \mathbf{Z}_t + \left(\boldsymbol{\gamma} + \boldsymbol{\gamma}^2 + \boldsymbol{\gamma}^3 + \ldots\right) \left(\boldsymbol{\eta\alpha}\right)^{-1} \ln \mathbf{Z}_t.$$

where $\mathbf{\Lambda} = (\mathbf{I} - \boldsymbol{\gamma})^{-1}(\boldsymbol{\alpha}\boldsymbol{\eta})^{-1}$.

Proof of Corollary 1: The log-deviation of country n's real GDP from steady state can be expressed as

$$\begin{aligned}
\ln V_{nt} &= \sum_j \left(\frac{P_{nj}Y_{nj}}{V_n} \ln Y_{njt} - \frac{P^x_{nj}X_{nj}}{V_n} \ln X_{njt} \right) \\
&= \sum_j \frac{P_{nj}Y_{nj}}{V_n} \left(\ln Y_{njt} - \frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} \ln X_{njt} \right) \\
&= \sum_j \frac{P_{nj}Y_{nj}}{V_n} \left(\ln Z_{njt} + \eta_j(1-\alpha_j) \ln H_{njt} + (1-\eta_j) \ln X_{njt} - \frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} \ln X_{njt} \right),
\end{aligned}$$

which leads directly to (3.13), since in equilibrium $\frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} = (1-\eta_j)$.

The derivation of (3.14) plugs (3.11) and (3.8)-(3.9) into (A.2).

□

A.2 Extracting Supply Shocks

The log-deviation of the value added in country n sector j from steady state can be expressed as

$$\begin{aligned}
\ln V_{njt} &= \frac{P_{nj}Y_{nj}}{V_{nj}} \ln Y_{njt} - \frac{P^x_{nj}X_{nj}}{V_{nj}} \ln X_{njt}, \\
&= \frac{1}{\eta_j}(\ln Z_{njt} + \alpha_j\eta_j \ln H_{njt} + (1-\eta_j) \ln X_{njt}) - \frac{1-\eta_j}{\eta_j} \ln X_{njt}, \\
&= \frac{1}{\eta_j} \ln Z_{njt} + \alpha_j \ln H_{njt},
\end{aligned}$$

where we have used the property that in equilibrium $\frac{P_{nj}Y_{nj}}{V_{nj}} = 1/\eta_j$ and $\frac{P^x_{nj}X_{nj}}{P_{nj}Y_{nj}} = (1-\eta_j)$. Stacking these into vectors and inverting for $\ln \mathbf{Z}$ gives (3.15).

A.3 Exact Solution to the Network Model

This section sets up the exact solution to the static network model, in changes, following the methodology of Dekle, Eaton, and Kortum (2008). In this section, denote by a "hat" the gross proportional change in any variable between the steady state x and a counterfactual x_t: $\widehat{x} \equiv x_t/x$. To streamline notation, define $\Upsilon_{njt} \equiv P_{njt}Y_{njt}$ to be the gross revenue in sector j, country n. Following a set of supply shocks $\widehat{Z}_{njt}$, the price in sector j, country n experiences the change:

$$\widehat{P}_{njt} = \widehat{Z}_{njt}^{-1}\widehat{\Upsilon}_{njt}^{\alpha_j\eta_j - \frac{1}{\psi+1}(1-\alpha_j)\eta_j}\widehat{P}_{nt}^{(1-\alpha_j)\eta_j\frac{\psi+1}{\psi}}\left(\sum_{m,i}\pi_{mi,nj}\widehat{P}_{mit}^{1-\varepsilon}\right)^{\frac{1-\eta_j}{1-\varepsilon}}. \tag{A.3}$$

This, together with the dependence of $\widehat{P}_{nt}$ on the constituent $\widehat{P}_{njt}$:

$$\widehat{P}_{nt} = \left[\sum_j\sum_m \widehat{P}_{mjt}^{1-\rho}\pi_{mnj}^f\right]^{\frac{1}{1-\rho}} \tag{A.4}$$

defines a system of $J \times N$ equations in prices, conditional on known steady-state data quantities (such as π_{mnj}^f), and a vector of $\widehat{\Upsilon}_{njt}$'s. The price changes in turn determine the counterfactual shares (denoted by a t-subscript):

$$\pi_{nmjt}^f = \frac{\widehat{P}_{njt}^{1-\rho}\pi_{nmj}^f}{\sum_k \widehat{P}_{kjt}^{1-\rho}\pi_{kmj}^f}, \tag{A.5}$$

$$\pi_{nj,mit}^x = \frac{\widehat{P}_{njt}^{1-\varepsilon}\pi_{nj,mi}^x}{\sum_{k,l}\widehat{P}_{klt}^{1-\varepsilon}\pi_{kl,mi}^x}. \tag{A.6}$$

These trade shares have to be consistent with market clearing at the counterfactual t, expressed using proportional changes as:

$$\widehat{\Upsilon}_{njt}\Upsilon_{nj} = \sum_m\left[\pi_{nmjt}^f\omega_{jm}\left(\sum_i \eta_i\widehat{\Upsilon}_{mit}\Upsilon_{mi}\right) + \sum_i \pi_{nj,mit}^x\left(1-\eta_i\right)\widehat{\Upsilon}_{mit}\Upsilon_{mi}\right]. \tag{A.7}$$

The sets of equations (A.3)-(A.7) represent a system of $2 \times N \times J + N^2 \times J + N^2 \times J^2$ unknowns, $\widehat{P}_{njt}$ $\forall n, j$, $\widehat{\Upsilon}_{njt}$ $\forall n, j$, π_{nmjt}^f $\forall n, m, j$, and $\pi_{nj,mit}^x$ $\forall n, j, m, i$ that is solved under given parameter values and under a set of shocks $\widehat{Z}_{njt}$.

A.3.1 Algorithm for Exact Solution to the Static Model

To solve the model, we use an initial guess for $\widehat{\Upsilon}_{njt}$ together with data on π_{mnj}^f and $\pi_{nj,mi}^x$. Given these variables, the algorithm is as follows:

1. Solve for $\widehat{P}_{njt}$ given the guess of $\widehat{\Upsilon}_{nj}$ and the data on π_{mnj}^f and $\pi_{nj,mi}^x$. This step uses equations (A.3) and (A.4) .

2. Update π_{mnjt}^f and $\pi_{nj,mit}^x$ given the solution to step 1 using equations (A.5) and (A.6).

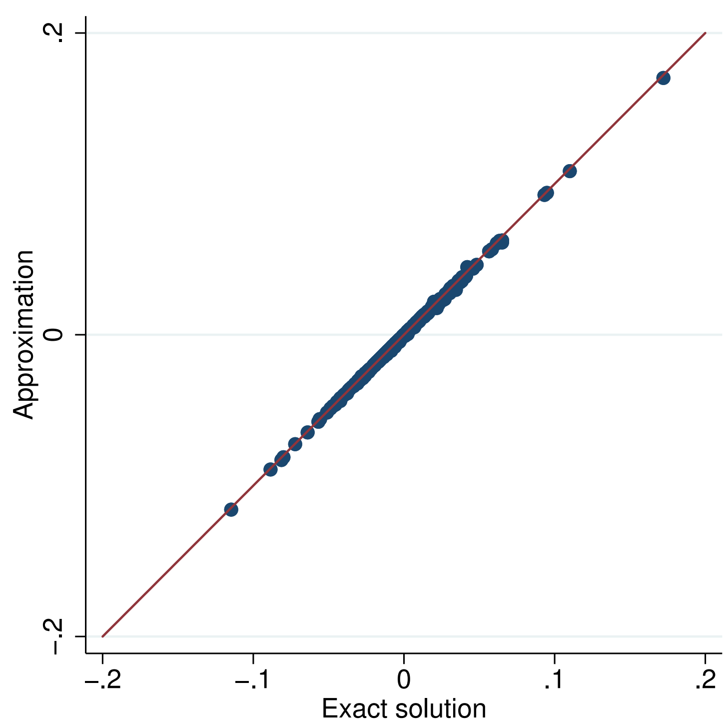
3. Solve for the new guess $\widehat{\Upsilon}'_{njt}$ using equation (A.7) given the prices $\widehat{P}_{njt}$ obtained in step 1 and the updated shares π^{f}_{mnjt} and $\pi^{x}_{nj,mit}$ from step 2.

4. Check if $\max|(\widehat{\Upsilon}'_{njt} - \widehat{\Upsilon}_{njt})| < \delta$, where δ is a tolerance parameter that is arbitrarily small. If not, update the guess of $\widehat{\Upsilon}_{njt}$ and repeat steps 1-4 until convergence.

A.3.2 Comparison of the Exact and First-Order Solutions

Figure A1 presents a scatterplot of GDP growth rates obtained under the first-order analytical solution to the global influence matrix in Section 3.2 against the exact solution computed as in this appendix. The line through the data is the 45-degree line. The GDP growth rates are computed under the observed shocks, and pooled across countries and years. It is clear that the first-order approximation is very good in the large majority of instances. The correlation between the two sets of growth rates is 0.999. Table A3 summarizes the GDP correlations obtained using GDP growth rates in the linear and exact solutions. The correlations are very close to each other.

FIGURE A1: Comparison of GDP Growth Rates between First-Order and Exact Solutions



Notes: This figure displays a scatterplot of the GDP growth rates obtained using the first-order approximation against the GDP growth rates in the exact solution to the model, pooling countries and years. The line through the data is the 45-degree line.

Table A3: First-Order and Exact Solutions: Correlations of $d\ln V_{nt}$, $\rho = 2.75$, Frisch= 2, $\varepsilon = 1$

	Mean	Median	25th pctile	75th pctile
		G-7 countries (N. obs. = 21)		
Baseline (approx.)	0.381	0.435	0.232	0.619
Exact solution	0.382	0.437	0.235	0.610
		All countries (N. obs. = 406)		
Baseline (approx.)	0.188	0.193	-0.110	0.498
Exact solution	0.181	0.192	-0.132	0.488

Notes: This table presents the summary statistics of the correlations of the model $d\ln V_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) computed using the linear approximation and the exact solution. Variable definitions and sources are described in detail in the text.

A.4 Dynamic Model Solution Details

With the labor shock, the optimality condition for the labor supply becomes

$$\frac{1}{\psi}\ln H_{njt} = -\ln \xi^H_{njt} + \ln W_{njt} - \ln P_{nt}.$$

With the intermediate input shock, the demand for intermediate input is

$$\ln P^x_{njt} + \ln \xi^X_{njt} + \ln X_{njt} = \ln Y_{njt} + \ln P_{njt}.$$

The output changes become

$$\ln \mathbf{Y}_t = \mathbf{\Lambda}\left(\ln \boldsymbol{\xi}^Z_t + \frac{\psi}{1+\psi}\boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\ln \boldsymbol{\xi}^H_t - (\mathbf{I}-\boldsymbol{\eta})\ln \boldsymbol{\xi}^X_t - \boldsymbol{\eta}(\mathbf{I}-\boldsymbol{\alpha})\ln \mathbf{K}_t\right).$$

Let $\ln m_{nt}$ denote the vector of log-deviation of marginal utility of consumption from its steady state in country n:

$$\ln m_{nt} = -\left(\frac{C_n \ln C_{nt} - \sum_j H_{nj}^{1+\frac{1}{\psi}}\left(1+\frac{1}{\psi}\right)\ln H_{njt}}{C_n - \sum_j H_{nj}^{1+\frac{1}{\psi}}}\right). \tag{A.8}$$

The linearized Euler equation for investment in country n sector j is

$$\ln m_{nt} + \ln \xi^I_{njt} = \mathbb{E}_t \Bigg[\ln m_{nt+1} + (1 - \beta + \beta\delta_j)(\ln P_{njt+1} - \ln P_{nt+1} + \ln Y_{njt+1} - \ln K_{njt+1}) + \beta(1 - \delta_j) \ln \xi^I_{njt+1} \Bigg].$$

The linearized resource constraint is

$$\ln C_{nt} + \sum_j \frac{\delta_j K_{nj}}{C_n} \left(\frac{1}{\delta} \ln K_{nt+1} - \frac{1-\delta}{\delta} \ln K_{njt} \right) = \sum_j \frac{\eta_j P_{nj} Y_{nj}}{P_n C_n} (\ln P_{njt} - \ln P_{nt} + \ln Y_{njt}).$$

Using the resource constraint and the static equilibrium, we can substitute $\ln \mathbf{C}_t$, $\ln \mathbf{H}_t$, and $\ln \mathbf{P}_t$ by $\ln \mathbf{K}_t, \ln \mathbf{K}_{t+1}$ and all the shocks $\ln \boldsymbol{\xi}_t = [\ln \boldsymbol{\xi}^Z_t, \ln \boldsymbol{\xi}^H_t, \ln \boldsymbol{\xi}^X_t, \ln \boldsymbol{\xi}^I_t]$. The Euler equation in matrix form becomes

$$\mathbf{A}_0 \ln \mathbf{K}_t + \mathbf{A}_1 \ln \mathbf{K}_{t+1} + \mathbf{B}_0 \ln \boldsymbol{\xi}_t = \mathbb{E}_t[\mathbf{A}_3 \ln \mathbf{K}_{t+2} + \mathbf{B}_1 \ln \boldsymbol{\xi}_{t+1}],$$

where the matrices only depend on parameters and steady-state values.

We proceed by a guess-and-verify approach. Assume that the policy function of capital is

$$\ln \mathbf{K}_{t+1} = \mathbf{M}_\xi \ln \boldsymbol{\xi}_t + \mathbf{M}_k \ln \mathbf{K}_t.$$

Also assume that we already know the process for the shocks

$$\ln \boldsymbol{\xi}_t = \mathbf{D} \ln \boldsymbol{\xi}_{t-1} + \boldsymbol{\epsilon}_t.$$

The Euler equation then becomes

$$(\mathbf{A}_0 + \mathbf{A}_1 \mathbf{M}_k) \ln \mathbf{K}_t + (\mathbf{A}_1 \mathbf{M}_\xi + \mathbf{B}_0) \ln \boldsymbol{\xi}_t = \mathbf{A}_3 \mathbf{M}_k^2 \ln \mathbf{K}_t + (\mathbf{A}_3(\mathbf{M}_\xi \mathbf{D} + \mathbf{M}_k \mathbf{M}_\xi) + \mathbf{B}_1 \mathbf{D}) \ln \boldsymbol{\xi}_t. \tag{A.9}$$

By method of undetermined coefficients, we can solve for $\mathbf{M}_k$ and $\mathbf{M}_\xi$.

Note that $\ln \boldsymbol{\xi}^Z_t, \ln \boldsymbol{\xi}^H_t$, and $\ln \boldsymbol{\xi}^X_t$ can be obtained from the static equilibrium conditions. To recover the investment shock, we can use equation (A.9) and the data on capital to calculate the implied $\ln \boldsymbol{\xi}^I_t$.

Appendix B Estimation

B.1 Estimating Model Elasticities

We use model-implied relationships to estimate ρ and ε. In this section, redefine a "hat" to mean the gross proportional change in any variable between time t and the previous year: $\widehat{x}_t \equiv x_t/x_{t-1}$. To introduce an error term in the estimating equations, assume that iceberg trade costs, final consumer taste shocks, and input share shocks have a stochastic element, and denote their gross proportional changes by $\widehat{\tau}_{mnjt}$, $\widehat{\vartheta}_{mnjt}$, and $\widehat{\mu}_{mj,nit}$, respectively. Straightforward manipulation of CES consumption shares yields the following relationships between shares and prices:

$$\ln\left(\frac{\widehat{\pi}^f_{mnjt}}{\widehat{\pi}^f_{m'njt}}\right) = (1-\rho)\ln\left(\frac{\widehat{P}_{mjt}}{\widehat{P}_{m'jt}}\right) + \ln\left(\frac{\widehat{\vartheta}_{mnjt}\widehat{\tau}^{1-\rho}_{mnjt}}{\widehat{\vartheta}_{m'njt}\widehat{\tau}^{1-\rho}_{m'njt}}\right) \tag{B.1}$$

and

$$\ln\left(\frac{\widehat{\pi}^x_{mj,nit}}{\widehat{\pi}^x_{m'j,nit}}\right) = (1-\varepsilon)\ln\left(\frac{\widehat{P}_{mjt}}{\widehat{P}_{m'jt}}\right) + \ln\left(\frac{\widehat{\mu}_{mj,ni,t}\widehat{\tau}^{1-\varepsilon}_{mnjt}}{\widehat{\mu}_{m'j,ni,t}\widehat{\tau}^{1-\varepsilon}_{m'njt}}\right). \tag{B.2}$$

We express the final consumption share change $\widehat{\pi}^f_{mnjt}$ relative to the final consumption share change in a reference country m'. This reference country is chosen separately for each importing country-sector n, j as the country with the largest average expenditure share in that country-sector. (Thus, strictly speaking, the identity of the reference country m' is distinct for each importing country-sector, but we suppress the dependence of m' on n, j to streamline notation.) Furthermore, we drop the own expenditure shares $\widehat{\pi}^f_{nnjt}$ from the estimation sample, as those are computed as residuals in WIOD, whereas import shares from other countries are taken directly from the international trade data. Dropping the own expenditure shares has the added benefit of making the regressions less endogenous, as the domestic taste shocks are much more likely to affect domestic prices.

We use two estimation approaches for (B.1)-(B.2). We first show the results with OLS. To absorb as much of the error term as possible, we include source-destination-reference country-time ($n \times m \times m' \times t$) fixed effects. These absorb any common components occurring at the country 3-tuple-time level, such as exchange rate changes and other taste and transport cost changes, and thus the coefficient is estimated from the variation in the relative sectoral price indices and relative sectoral share movements within that cell. The identifying assumption is then that price change ratio $\widehat{P}_{mjt}/\widehat{P}_{m'jt}$ is uncorrelated with the residual net of the $n \times m \times m' \times t$ fixed effects. The remaining errors would be largely measurement error. If this measurement error is uncorrelated with the price change ratios, then the OLS estimates are unbiased, and if not, we would expect a bias towards zero. In the latter case, the IV estimates (described below) should be larger than the OLS estimates, assuming the measurement error in (B.1) and (B.2) is independent of the measurement error in the technology shock ratios.

The estimation amounts to regressing relative share changes on relative price changes. A threat to identification would be that relative price changes are affected by demand shocks (e.g. $\widehat{\vartheta}_{mnjt}$), and thus correlated with the residual. As a way to mitigate this concern, we also report estimates based on the subsample in which destination countries are all non-G7, and the source and reference countries are all G7 countries. In this sample it is less likely that taste shocks in the (smaller) destination countries will affect relative price changes in the larger G7 source countries. Finally, to reduce the impact of small shares on the estimates, we report results weighting by the size of the initial shares ($\pi^f_{mnj,t-1}$ and $\pi^x_{mj,ni,t-1}$).

We also implement IV estimation. We use the TFP shocks $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$ as instruments for changes in relative prices. The exclusion restriction is that the technology shocks are uncorrelated with taste and trade cost shocks, and thus only affect the share ratios through changing the prices. Even if the shock ratio $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$ is a valid instrument for observed prices, it does not include the general-equilibrium effects on prices in the model. To use all of the information –both the direct and indirect GE effects –incorporated in the model, we also use the model-optimal IV approach to construct the instrument. In our context this simply involves computing the model using only the estimated technology shocks, and solving for the sequence of equilibrium prices in all countries and sectors. The model-implied prices are then the optimal instrument for the prices observed in the data. See Chamberlain (1987) for a discussion of optimal instruments, and Adao, Arkolakis, and Esposito (2017) and Bartelme et al. (2018) for two recent applications of this approach. The results from the model-optimal IV are very similar to simply instrumenting with the TFP shock ratio, and we do not report them to conserve space.

Table A3 presents the results of estimating equations (B.1) and (B.2). Columns 1-3 report the OLS estimates of ρ (top panel) and ε (bottom panel). The OLS estimates of ρ are all significantly larger than zero, and we cannot rule out a Cobb-Douglas final demand elasticity. The OLS estimates for ρ are also not very sensitive to restricting the sample to non-G7 destinations and G7 sources, or to weighting by the initial share. The IV estimates in columns 4-6 are substantially larger than the OLS coefficients, ranging from 1.77 to 2.74, and significantly different from 1 in most cases. This difference between OLS and IV could suggest either measurement error in (B.1), or greater noise in the IV estimator (Young, 2017). Given the substantial disagreement between OLS and IV estimates of ρ, we report the results under two values: $\rho = 1$, corresponding to the OLS estimates, and $\rho = 2.75$ based on the IV.

The OLS and IV estimates of ε display somewhat greater consensus. The OLS point estimates are in the range 0.9, and not sensitive to the sample restriction or weighting. The IV estimates are less stable. While the full sample (column 4) yields an elasticity of 2.6, either restricting to the non-G7 destinations/G7 sources, or weighting by size reduces the coefficient dramatically and renders it not statistically different from 1. Such evidence for the low substitutability of intermediate inputs is consistent with the recent estimates by Atalay (2017) and Boehm, Flaaen, and Pandalai-Nayar (2019), who find even stronger complementarity. We therefore set $\varepsilon = 1$ for all implementations of the model.

Table A3: Elasticity Estimates

	(1) OLS	(2) OLS (G7 m, m', non-G7 n)	(3) OLS (weighted)	(4) IV	(5) IV (G7 m, m', non-G7 n)	(6) IV (weighted)
ρ	0.869	0.937	1.011	2.723	1.768	2.743
SE	(0.022)	(0.027)	(0.024)	(0.525)	(0.733)	(0.538)
First stage K-P F				76.33	35.37	86.72
FE	Yes	Yes	Yes	Yes	Yes	Yes
ε	0.884	0.941	0.917	2.613	0.283	1.896
SE	(0.019)	(0.022)	(0.039)	(0.381)	(0.847)	(0.495)
First stage K-P F				66.07	47.58	111.41
FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at the destination-source-reference country level in parentheses. This table presents results from the OLS and IV estimation of (B.1) and (B.2). The fixed effects used in each regression are $n \times m \times m' \times t$. The instruments are the relative productivity shocks $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$, with the Kleibergen-Papp first stage F-statistic reported. The weights in columns 3 and 6 are lagged share ratios π^f_{mnjt-1} and $\pi^x_{mj,nit-1}$.

B.2 Estimating Shock Processes

As discussed in Section 4, estimating an unrestricted process for shocks is not possible due to the short panel of measured shocks and the large number of parameters to be estimated. We restrict the estimation to the G7 countries, for which we have the longest panel of shocks. While we still cannot estimate a completely unrestricted VAR, we impose minimal restrictions that allow the shocks to be correlated (as the measured shocks are), and further, allow for spillovers between country-sectors.

In particular, our specification allows for contemporaneous shock correlations between country-sectors, but restricts the structure of lagged spillovers. We permit a country-sector specific lagged autoregressive parameter, so country-sector shocks can be persistent. We restrict lagged spillovers to be common within a country (across sectors), and zero otherwise. We allow for a full variance-covariance matrix of the error terms, which amounts to assuming completely unrestricted contemporaneous spillovers. The sample variance-covariance matrix of the residuals serves as an estimate of the covariance matrix of the error term. Equation (4.6) describes the estimating equation for the VAR process.

The choice of restrictions strikes a balance between relative parsimony, which improves the precision of the parameters estimated, and sufficient flexibility to replicate the measured shock correlations in the data. We experimented with other processes using methods such as LASSO regressions to estimate the process for the intertemporal shocks without much change to the simulated shock correlations. In particular, we have modified the estimating equations to also include a sector-specific lagged spillover term, but these coefficients were all insignificant, and so we use the more parsimonious process in the baseline analysis. Table A4 summarizes the estimation results.

Table A4: Shock Processes: Autoregressive Parameters

	Mean	Median	25th pctile	75th pctile
		$\ln \xi^Z_{njt}$		
Own lag (ρ_{nj})	0.877	0.876	0.856	0.882
Spillover lag (δ_n)	-0.002	-0.002	-0.003	-0.002
		$\ln \xi^X_{njt}$		
Own lag (ρ_{nj})	0.853	0.848	0.839	0.870
Spillover lag (δ_n)	0.002	0.002	0.001	0.002
		$\ln \xi^H_{njt}$		
Own lag (ρ_{nj})	0.896	0.894	0.882	0.916
Spillover lag (δ_n)	-0.000	-0.001	-0.001	-0.000
		$\ln \xi^I_{njt}$		
Own lag (ρ_{nj})	0.931	0.945	0.915	0.965
Spillover lag (δ_n)	-0.004	-0.005	-0.006	-0.002

Notes: This table presents results from estimating the shock stochastic processes (4.6). The measures are summary statistics of the coefficients in the sample of sectors and countries. The shock processes are estimated on shocks obtained from the baseline dynamic model in section 4.

Appendix C Sensitivity and Additional Exercises

C.1 Sensitivity

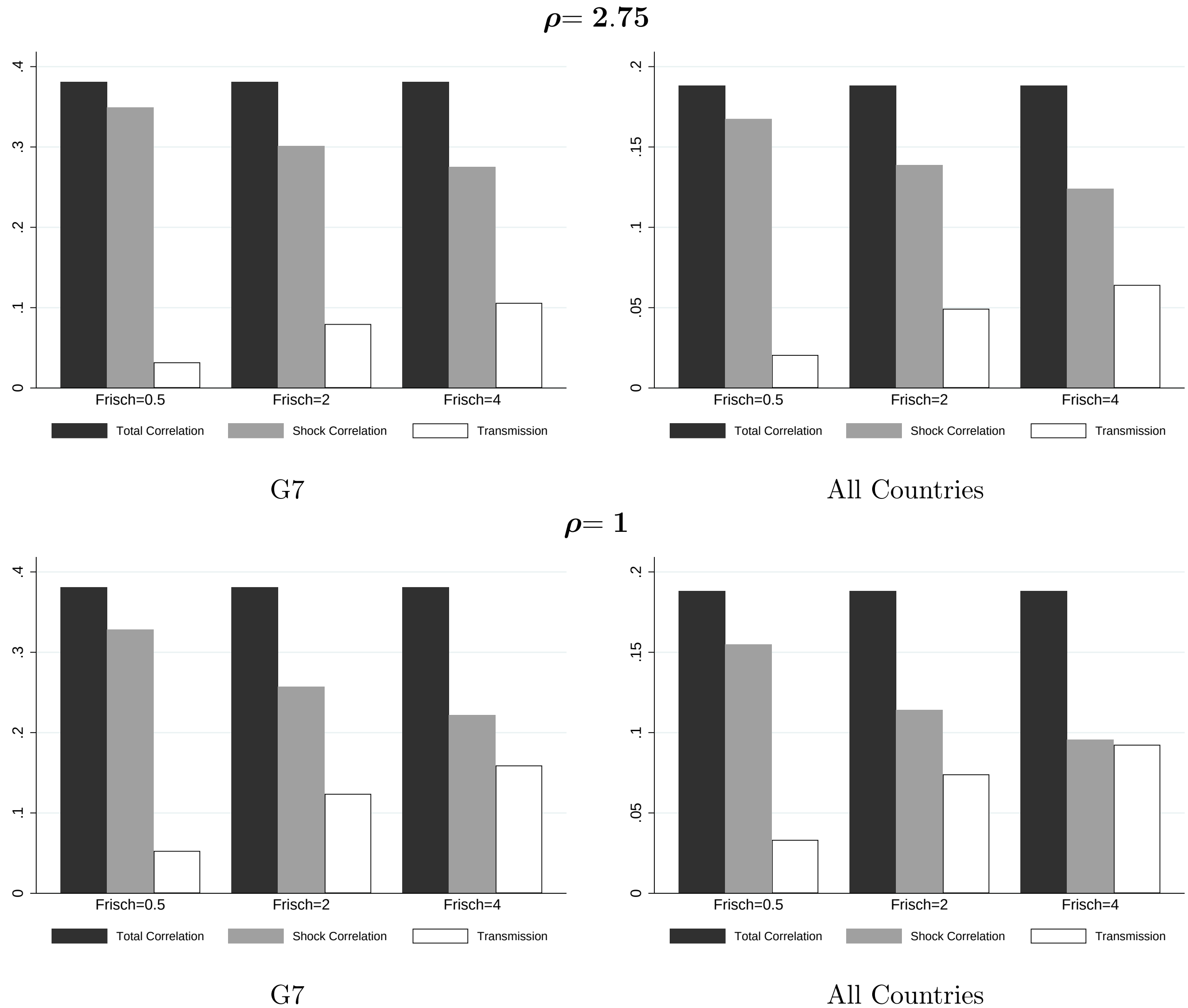
This subsection reports various sensitivity analyses for the network model results in Section 3.

Properties of Inferred Composite Shocks The procedure in Section 3 recovers the supply shocks by matching the global vector of sectoral value added changes conditional on a global influence matrix. Here, we show that when shock extraction uses only the internal structure of the model, quantitatively the relative importance of correlated shocks and transmission depends on the within-period factor supply elasticity. In our framework, labor is the only factor with an upward-sloping intra-temporal supply curve, whose shape is governed by the Frisch elasticity. Figure A2 compares the contributions of correlated shocks vs. transmission under our baseline of the Frisch elasticity of 2 to a model with a lower (0.5) and a higher one (4). The overall correlation generated by the models with alternative Frisch elasticities is by construction identical.

Even as the factor supply elasticity becomes implausibly large, the inferred shocks remain quite correlated, and the shock correlation component accounts for more than half of overall comovement. This suggests that candidate structural shocks to explain international GDP comovement must be at least somewhat correlated across countries. When the Frisch elasticity is high, the domestic GDP is more responsive to foreign shocks, and thus to rationalize the same observed data the implied shocks need not be as correlated. Conversely, when the elasticity is low, the economy will respond less to foreign shocks, and thus the procedure will make the shocks themselves more correlated to match the data. Indeed, the limiting case of zero intra-temporal factor supply elasticity is the well-known result that a country's real GDP does not change following a foreign shock (Kehoe and Ruhl, 2008; Burstein and Cravino, 2015; Baqaee and Farhi, 2019c), and thus the entirety of international comovement must be accounted for by correlated shocks in that case.

Alternative Elasticities, Models, and Samples Table A5 presents the GDP correlations and volatilities for GDP growth rates and HP-filtered series for the G7. The two detrending methods deliver similar results.

Figure A2: GDP Correlations and Decompositions into Correlated Shocks and Transmission



Notes: This figure displays the mean GDP correlation generated by the static model (dark bars), and its decomposition decomposing it into the mean Shock Correlation (gray bars) and Transmission (white bars) terms.

Table A5: GDP Growth and HP-Filtered GDP Correlation in G7 Countries

	Mean	Median	25th pctile	75th pctile	St.dev. (GDP)
GDP growth rate	0.358	0.337	0.242	0.565	1.745
HP-Filtered GDP	0.395	0.460	0.040	0.585	1.736

Notes: This table presents the summary statistics of the correlations of GDP growth rates and HP-filtered GDP in the sample of G7 countries. We use $\lambda = 100$ for the HP-filter parameter, as the data are annual.

Table A6 presents the results of the shock correlation-transmission decomposition (2.5) the G7 countries and $\rho = 1$. Transmission in the baseline model is higher under the lower elasticity, as would be expected.

Table A6: Correlated Shocks vs. Transmission Decomposition, $\rho = 1$

	G-7 countries (N. obs. = 21)			
Baseline:	0.381	0.435	0.232	0.610
Decomposition:				
Shock Correlation	0.257	0.298	0.131	0.413
Bilateral Transmission	0.033	0.027	0.017	0.037
Multilateral Transmission	0.091	0.095	0.047	0.132
	All countries (N. obs. = 406)			
Baseline:	0.188	0.193	-0.110	0.498
Decomposition:				
Shock Correlation	0.114	0.138	-0.131	0.396
Bilateral Transmission	0.011	0.006	0.003	0.014
Multilateral Transmission	0.064	0.049	0.016	0.113

Notes: This table presents the decomposition of the transmission of observed shocks into direct effects, the direct transmission and the multilateral transmission based on the influence vector approximation.

Table A7 reports the results from a G7 only version of our model (using the time period 1978-2007, the longest available time period for these countries). Table A8 reports the results from our baseline model with trade deficits evolving as they do in the data, solved using the method in Dekle, Eaton, and Kortum (2008). In both cases the results are similar to the baseline.

Comparison of the GDP Correlations Under Data and Model Domar Weights Because the model is simulated in log-deviations from steady state, it uses fixed Domar weights, set to period averages for each country and sector. In the data, Domar weights change from year to year. As a result, country GDP growth rates differ in the data and the model. Figure A3 plots the GDP growth correlations in the model under fixed Domar weights against those in the data, along with the 45-degree line. The two sets of correlations are quite similar.

TABLE A7: Transmission of Shocks with longer G7+RoW sample: Correlations of $d\ln V_{nt}$, $\rho = 2.75$, Frisch= 2

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Model correlation:	0.350	0.356	0.230	0.558
Decomposition:				
Shock Correlation	0.282	0.271	0.131	0.484
Bilateral Transmission	0.016	0.011	0.005	0.019
Multilateral Transmission	0.052	0.056	0.028	0.067

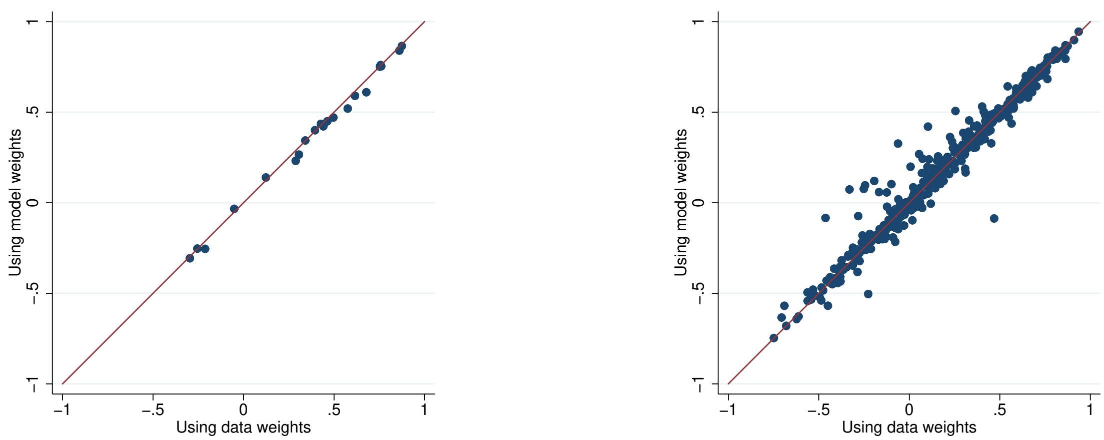
Notes: This table presents the summary statistics of the correlations of $d\ln V_{nt}$ in the sample of G7 countries, for data from 1978 to 2007, in the data and the model under the different shocks, in a model with G7 countries only and a rest of the world composite. Variable definitions and sources are described in detail in the text.

TABLE A8: Exact Solutions with Deficits: Correlations of $d\ln V_{nt}$, $\rho = 2.75$, Frisch=2, $\varepsilon = 1$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Baseline (approx. w/o deficits)	0.381	0.435	0.232	0.619
Exact solution with deficits	0.380	0.431	0.231	0.626
	All countries (N. obs. = 406)			
Baseline (approx. w/o deficits)	0.188	0.193	-0.110	0.498
Exact solution with deficits	0.181	0.192	-0.132	0.488

Notes: This table presents the summary statistics of the correlations of the model $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) computed using the linear approximation and the exact solution where aggregate trade deficits are allowed to move as in the data. Variable definitions and sources are described in detail in the text.

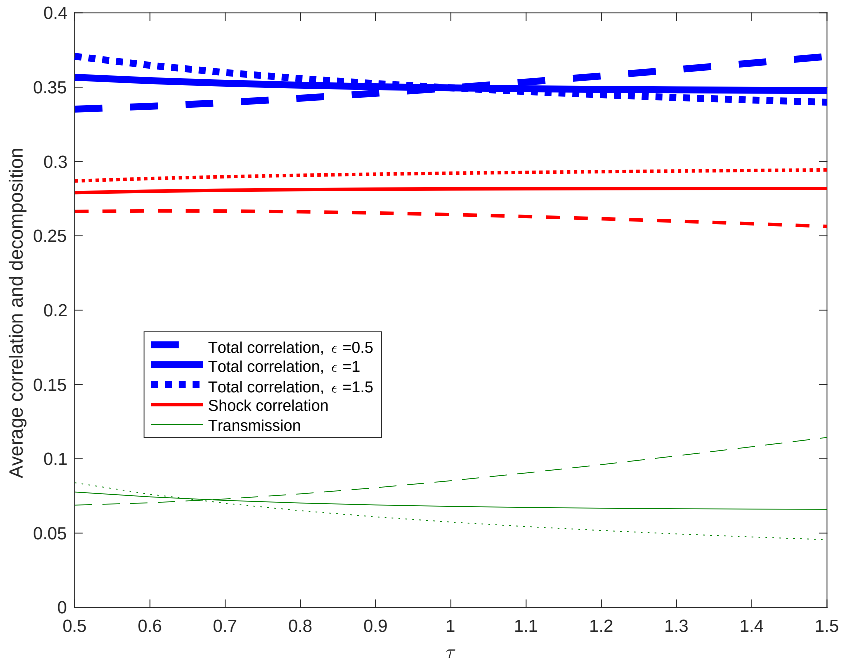
Figure A3: GDP Correlations Under Data and Model Domar Weights



G7 All Countries

Notes: This figure displays a scatter plot of the GDP growth correlation, when GDP growth is computed by aggregating sectoral growth using the data Domar weights against the model Domar weights for G7 country pairs (left panel) and all pairs (right panel).

Figure A4: Comparative Statics: Trade Costs



Notes: This figure displays the change in average correlation between the G7 countries, decomposed into shock correlation, direct and multilateral transmission, as trade costs increase/decrease relative to the baseline. The solid lines illustrate changes in the baseline model calibration with $\varepsilon = 1$. The dashed lines use an alternative calibration of $\varepsilon = 0.5$ and the dotted lines $\varepsilon = 1.5$.

C.2 The Role of Trade Costs

Starting from the observed world equilibrium, we simulate gross changes in iceberg costs $\widehat{\tau}_{mnj}, m \neq n$ ranging from 0.5 to 1.5. Because our baseline of $\varepsilon = 1$ keeps intermediate expenditure shares constant, we also report results with a higher ($\varepsilon = 1.5$) and a lower ($\varepsilon = 0.5$) value of input substitution elasticity.

The top-most (blue) lines in Figure A4 plot the average GDP correlation in the G7 sample for this range of trade cost changes. By construction, all three lines coincide at $\widehat{\tau}_{mnj} = 1$ (no change in trade costs), as the models reproduce the same GDP correlation at that point. Under $\varepsilon = 1$ the GDP correlation changes only imperceptibly as trade costs vary. This is because all input revenue shares are constant, and these input revenue shares are the ingredients of the influence matrix.[29]

Upward and downward deviations from $\varepsilon = 1$ produce the opposite results. When $\varepsilon = 1.5$ (short dashed line), we get the expected outcome that correlations fall in the level of trade

[29] The reason GDP correlation is not literally constant across all trade costs is that final goods trade is still governed by a non-unitary elasticity of $\rho = 2.75$. The Figure makes it clear that the final goods trade shares are not an important force in GDP correlations, at least for this range of trade cost changes.

costs, albeit modestly. However, when $\varepsilon = 0.5$ (long dashed line), correlations actually rise in trade costs. To understand this better, the middle (red) lines depict the shock correlation components, and the green lines at the bottom the transmission components. Though the level of the shock correlation components differs somewhat across ε's, there is little if any difference in the slopes with respect to trade costs. So the shock correlation components are not the ones responsible for the finding that different ε's produce the opposite sign relationship between trade costs and comovement.

Instead, this result is driven by the transmission terms. Transmission's contribution to comovement actually *rises* as trade costs increase for $\varepsilon = 0.5$, while the opposite is true for $\varepsilon = 1.5$. The explanation for this apparent paradox is that when $\varepsilon < 1$, a higher τ_{mnj} actually leads to higher foreign input spending shares. Because the influence of foreign shocks in the domestic economy increases in the foreign input shares, raising trade costs actually leads to more transmission and higher correlations when inputs are complements.

These results are informative for the debate regarding the role of the Armington elasticity in international business cycles. A number of papers point out that lowering this elasticity below 1 improves the ability of these models to generate positive correlations in the macro aggregates (Heathcote and Perri, 2002; Burstein, Kurz, and Tesar, 2008; Johnson, 2014). These papers do not typically perform the experiment of changing trade costs starting from the observed equilibrium. We show here that assuming complementarity in inputs comes along with the prediction that higher trade costs actually generate more comovement.

C.3 The Trade-Comovement Relation

Table A9 reports the results of running the "standard" trade-comovement regression in our data and the static model. This is a regression of bilateral real GDP correlation on a measure of bilateral trade intensity. A long literature following Frankel and Rose (1998) tries to understand why economies that trade more display higher GDP comovement in the data. Vertical linkages have been suggested as an explanation for the trade-comovement puzzle in a number of papers (see for instance Kose and Yi, 2006; di Giovanni and Levchenko, 2010; Johnson, 2014). Quantitatively, however, models have trouble generating even the same order of magnitude as the empirical relationship (model coefficients are often $<$10% of their empirical counterparts).

When it comes to GDP correlations, our model matches perfectly the trade-comovement relationship found in the data, by virtue of matching GDP growth rates for each country-year. Columns (1)-(2) run the trade-comovement regression in the data, while columns (3)-(4) do the same in the model. Since each GDP correlation can be additively decomposed into the Shock Correlation and Transmission components, we can also run the trade comovement regressions with those as dependent variables. This is done in columns (5)-(8). It turns out that trade intensity is correlated with both components of total GDP comovement. However, the bulk of the overall slope (0.7 out of 0.87) is accounted for by the positive relationship between trade intensity and shock correlation. This underscores the relevance of the Imbs (2004) critique of trade-comovement regressions: bilateral trade intensity can be a proxy of

Table A9: The Trade-Comovement Relation

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Data		Model					
Dep. Var:	Bilateral GDP growth correlation				Shock correlation		Transmission	
Trade intensity (avg)	0.085*** (0.012)							
Trade intensity (1995)		0.086*** (0.011)						
Trade intensity (model, avg)			0.087*** (0.012)		0.070*** (0.011)		0.016*** (0.001)	
Trade intensity (model, 1995)				0.087*** (0.012)		0.071*** (0.011)		0.016*** (0.001)
N	406	406	406	406	406	406	406	406

Notes: This table presents the results of a regression of bilateral GDP growth correlation on trade intensity for the data (first panel), the baseline static model (second panel) and the static model with employment and capital growth from the data (third panel). Trade intensity is defined as the sum of bilateral flows over the sum of the two countries' GDPs. The first row uses the average trade intensity over the 1995-2007 period, while the second row uses the initial intensity.

country similarity, and thus of correlated shocks.