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THE GLOBAL BUSINESS CYCLE: MEASUREMENT AND TRANSMISSION

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ABSTRACT

This paper uses sector-level data for 30 countries and up to 28 years to provide a quantitative account of the sources of international GDP comovement. We propose an accounting framework to decompose comovement into the components due to correlated shocks, and to the cross-country transmission of shocks. We apply this decomposition in a multi-country multi-sector DSGE model. We provide an analytical solution to the global influence matrix that characterizes every country's general equilibrium GDP elasticities with respect to shocks anywhere in the world. We then provide novel estimates of country-sector-level technology and non-technology shocks to assess their correlation and quantify their contribution to comovement. TFP shocks are virtually uncorrelated across countries, whereas non-technology shocks are positively correlated. These positively correlated shocks account for two thirds of the observed GDP comovement, with international transmission through trade accounting for the remaining one third. However, trade opening does not necessarily increase GDP correlations relative to autarky, because the contribution of trade openness to comovement depends on whether sectors with more or less correlated shocks grow in influence as countries increase input linkages. Finally, while the dynamic model features rich intertemporal propagation, quantitatively these components contribute little to GDP comovement as impact effects dominate.

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1 Introduction

Real GDP growth is positively correlated across countries. In spite of a large amount of research into the causes of international comovement, we still lack a comprehensive account of this phenomenon. Two related themes cut through the literature. First, does comovement occur because shocks are transmitted across countries via propagation mechanisms such as trade linkages (e.g. Frankel and Rose, 1998; di Giovanni, Levchenko, and Mejean, 2018), or because the shocks themselves are correlated across countries (Imbs, 2004)? Second, is international comovement driven predominantly by technology (Backus, Kehoe, and Kydland, 1992) or non-technology (Stockman and Tesar, 1995) shocks?

This paper provides a general and unified framework to answer both of these questions. To clarify the mechanisms at play and objects of interest for measurement, we start by setting up a simple accounting framework that extends the standard input network propagation model (e.g. Acemoglu et al., 2012) to an international setting. The GDP covariance between two countries can be expressed as a function of the covariances between primitive shocks and a global influence matrix. The latter collects the general equilibrium elasticities of GDP in each country with respect to all sector-country-specific shocks worldwide, and thus translates the variances and covariances of the primitive shocks into comovements of GDP. In particular, two countries can experience positive comovement if influential sectors in the two economies have correlated shocks. Comovement also arises if shocks in one country influence another country's GDP through trade and production linkages. We show that the GDP covariance between two countries can be written as a sum of two terms, respectively capturing correlated shocks and transmission.

The accounting framework provides a road map for the measurement and quantification exercises that follow. First, we must measure underlying shocks to determine the extent of their correlation across countries. Second, we must impose sufficient structure and bring sufficient data on international trade linkages to recover the global influence matrix. This will allow us to establish both how the matrix interacts with the shock correlation, and how it produces transmission.

The quantification combines sector-level data for 30 countries and up to 28 years with a multi-country, multi-sector, multi-factor DSGE model of world production and trade. Countries trade both intermediate and final goods. Each sector uses labor, capital, and intermediate inputs that can come from any sector and country in the world, and is subject to sector-specific TFP shocks. Between periods, capital and employees can be accumulated in each sector. However, within a period, labor and capital supply to each sector and country are upward-sloping in the real prices of labor and capital, respectively, and subject to sector-specific factor supply shocks. The model features standard international transmission mechanisms. A positive foreign shock lowers the prices of intermediate inputs coming from that country, stimulating demand in countries and sectors that use those inputs in production. At the same time, a positive shock in a foreign country makes final goods supplied by that country cheaper, reducing demand for final goods produced by countries competing with it in final goods markets.

We implement two versions of the model. The first is a static setting akin to the network literature (e.g. Acemoglu et al., 2012; Baqaee and Farhi, 2018). Closed-economy frameworks of shock propagation through a network write the change in real GDP as an inner product of the vector of sectoral shocks and the influence vector. We extend this approach to an international setting, and write the change in GDP of a single country as an inner product of the vector of shocks to all countries and sectors in the world and the country-specific influence vector that collects the elasticities of that country's GDP to every sectoral shock in the world. A unique feature of our analysis is that we provide an analytical solution for the first-order approximation to this influence vector in a multi-country general equilibrium setting. This analytical solution expresses the influence matrix in terms of observables that can be measured and structural elasticities.

The network propagation approach captures intra-temporal comovement but shuts down dynamic factor accumulation responses to shocks. An important feature of our theoretical framework is that the static and dynamic responses of the world economy to shocks are separable. That is, the analytical influence matrix characterizes the contemporaneous response of the world economy to shocks even in the fully dynamic model. Our framework thus bridges the network propagation and the dynamic international business cycle literatures. Our second set of exercises implements the dynamic version of the model, in which sectoral capital and labor can respond to both foreign and domestic shocks, subject to adjustment costs.

The quantitative framework provides a theoretical foundation for shock measurement. We begin by estimating utilization-adjusted TFP growth rates in our sample of countries, sectors, and years. When unobserved factor utilization responds to shocks, conventional Solow residuals are a misleading measure of technology shocks. Our model captures the notion of variable factor utilization: even conditional on the observed number of installed machines and employee-hours, the utilization rate of those machines and the employees' effort can vary within a period in response to shocks. When true factor usage is not perfectly observed, it must be accounted for in the estimation of shocks. Our approach uses the insights of Basu, Fernald, and Kimball (2006, henceforth BFK), who estimate TFP shocks for the United States controlling for unobserved input utilization and industry-level variable returns to scale. Importantly, they show that doing so produces a TFP series with substantially different properties than the traditional Solow residual. We bring this insight into the international context by estimating utilization-adjusted TFP series for a large sample of countries, and analyzing the international correlations in these series.

Next, we extract a non-technology shock. In the model, the non-technology shock is a sector-specific shift in the within-period factor supply curve. This shock can be viewed as a generalization of the "labor wedge" (e.g. Chari, Kehoe, and McGrattan, 2007). Though reduced-form, this shock has a variety of microfoundations, such as sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015), monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010). Our procedure infers it as the shock that rationalizes the observed growth in real value added, conditional on the global vector of TFP shocks, predetermined factors, and the input linkages in the data. Because all the sectors are connected through domestic and international trade, the entire global vector of non-technology shocks is inferred jointly.

Our first main finding is about the properties of the shocks themselves. We show that TFP growth is virtually uncorrelated across countries. In contrast, the aggregated non-technology shocks are quite correlated among the G7 countries, with correlation coefficients about one half of the observed correlation in real GDPs. Correspondingly, when we feed these measured shocks back into the model, the non-technology shocks are much more successful at generating GDP correlations than TFP shocks in the G7.

We next decompose the overall comovement into the correlated shocks and transmission components, as suggested by the accounting framework. Our second main finding is that correlated shocks account for about two thirds of the total GDP correlation, with the transmission component responsible for the remaining one third.

To further explore the role of the input network in generating comovement, we compare the baseline economy to counterfactuals in which countries are in autarky. This exercise reveals an underappreciated mechanism through which trade opening affects GDP comovement: it changes the relative influence of domestic sectors. Whether trade opening increases or lowers GDP comovement depends in part on whether it leads to the expansion or contraction of sectors with more correlated shocks. Our third main finding is that among the G7 countries autarky GDP correlations can actually be quite a bit *higher* than the corresponding correlations under trade.

To better understand this paradoxical result, we write the difference in GDP comovement between the trade and autarky equilibria as a sum of two terms: the international transmission of shocks, and the changes in the influence of the domestic shocks times the covariances of those shocks. While international transmission is positive in the trade equilibrium and increases comovement relative to autarky, it turns out that in the G7 sample the second term is negative. Moving from autarky to trade increases the relative influence of sectors whose shocks are less correlated, offsetting the positive international transmission and producing the outcome that autarky correlations are higher than the trade ones. These results reveal the unexpected role of input linkages in cross-border comovement:

they can lead to diversification away from the most internationally correlated sectors.[1]

Finally, we implement the full dynamic model to evaluate how delayed propagation of past shocks contributes to comovement. The model features non-trivial propagation over time, with the peak impact of a foreign shock occurring with a delay of several periods. Nonetheless, simulated model correlations with dynamics are not appreciably higher than the correlations in the static model. The overall GDP covariance can be written as the sum of the covariance of the instantaneous change in GDPs due to a shock innovation, and the infinite sum of responses to all the past innovations. The component capturing the impact effects of shocks predominates. Quantitatively, the intertemporal transmission is much less important for comovement than these impact effects.

Our paper draws from, and contributes to two literatures. The first is the effort to understand international business cycle comovement. A large literature builds models in which fluctuations are driven by productivity shocks, and asks under what conditions those models can generate observed international comovement (see, among many others, Backus, Kehoe, and Kydland, 1992; Heathcote and Perri, 2002). A smaller set of contributions adds non-technology shocks (Stockman and Tesar, 1995; Wen, 2007). In these analyses, productivity shocks are proxied by the Solow residual, and non-technology shocks are not typically measured based on data. Our quantitative assessment benefits from improved measurement of both types of shocks. While all papers on international business cycle comovement must take a stand on the relative importance of correlated shocks vs. transmission, we provide a way of cleanly separating these two potential sources of comovement. A number of papers are dedicated to documenting international correlations in productivity shocks and inputs (e.g. Imbs, 1999; Kose, Otrok, and Whiteman, 2003; Ambler, Cardia, and Zimmermann, 2004). Also related is the body of work that identifies technology and demand shocks in a VAR setting and examines their international propagation (e.g. Canova, 2005; Corsetti, Dedola, and Leduc, 2014; Levchenko and Pandalai-Nayar, 2018). Relative to these papers, we use sector-level data to provide novel estimates of both utilization-adjusted TFP and non-technology shocks, and expand the sample of countries.

The second is the active recent literature on shock propagation in production networks (e.g. Carvalho, 2010; Acemoglu et al., 2012; Barrot and Sauvagnat, 2016; Carvalho et al., 2016; Atalay, 2017; Baqaee, 2018; Baqaee and Farhi, 2018; Boehm, Flaaen, and Pandalai-Nayar, 2019). We contribute to this literature in two ways. First, we extend its insights to an international setting, building on the earlier contributions such as Kose and Yi (2006), Bems, Johnson, and Yi (2010), Johnson (2014), and Eaton et al. (2016) among others. And second, we measure the primitive shocks and explore the consequences of correlated shocks in an input network setting.

[1] This is a purely quantitative result, arising from the particular correlation properties of the estimated shocks and input coefficients. Nonetheless, it is a counterexample to the effect often invoked in the optimum currency area literature, whereby trade integration is expected to increase comovement by making aggregate shocks more correlated (e.g. Frankel and Rose, 1998). We reveal an alternative mechanism, through which countries become less correlated with trade integration despite the same underlying sectoral shocks.

The rest of the paper is organized as follows. Section 2 lays out a basic GDP accounting framework and presents the decompositions of the sources of comovement. Section 3 introduces the dynamic multi-country, multi-sector quantitative framework of production and trade. Section 4 describes the procedures for measuring the shocks, and the properties of these shocks. Section 5 uses the model to perform static counterfactuals and illustrate the role of the input network in international comovement. Section 6 presents the dynamic counterfactuals. Section 7 concludes. The appendices collect additional details of the estimation and theoretical framework as well as robustness checks.

2 Accounting Framework

Consider an international version of the standard static network propagation model (e.g. Acemoglu et al., 2012). There are J sectors indexed by j and i, and N countries indexed by n and m. Gross output in sector j country n aggregates a generic primary factor input bundle $\mathcal{I}_{nj}$ (for instance, capital and labor) and materials inputs X_{nj}:

$$Y_{nj} = F\left(\mathcal{I}_{nj}(\boldsymbol{\theta}), X_{nj}(\boldsymbol{\theta}); \boldsymbol{\theta}\right). \tag{2.1}$$

The bundle of inputs X_{nj} can include foreign imported intermediates. The sectoral output is affected by a generic matrix of shocks $\boldsymbol{\theta}$. For concreteness, one can think of productivity shocks. A productivity shock θ_{nj} to sector j in country n will directly affect output in that sector. Because the economy is interconnected through trade, output in every sector and country is in principle a function of all the shocks anywhere in the world, hence the dependence of Y_{nj} on the full world vector $\boldsymbol{\theta}$. The matrix $\boldsymbol{\theta}$ can include multiple types of shocks (such as technology and non-technology). The next section completely specifies the shocks, and the nature of output's dependence on those shocks in the context of a particular model.

Real GDP is defined as value added evaluated at base prices b:

$$\mathcal{Y}_n = \sum_{j=1}^{J} \left(P_{nj,b} Y_{nj}(\boldsymbol{\theta}) - P^X_{nj,b} X_{nj}(\boldsymbol{\theta})\right), \tag{2.2}$$

where $P_{nj,b}$ is the gross output base price, and $P^X_{nj,b}$ is the base price of inputs in that sector-country.

Let θ_{mi} be a scalar-valued shock affecting sector i in country m.[2] A first order approximation to the log change in real GDP of country n can be written as:

$$d \ln \mathcal{Y}_n \approx \sum_m \sum_i s_{mni} \theta_{mi}, \tag{2.3}$$

[2]The extension to vector-valued θ_{mi} is straightforward, i.e. each sector can experience multiple shocks simultaneously.

where s_{mni} are the elements of the global influence matrix, that give the elasticity of the GDP of country n with respect to shocks in sector i, country m. Notice that these elasticities capture the full impact of a shock through direct and indirect input-output links and general equilibrium effects.[3]

To highlight the sources of international GDP comovement, write real GDP growth as

$$d\ln Y_n = \underbrace{\sum_j s_{nnj}\theta_{nj}}_{\mathcal{D}_n} + \underbrace{\sum_j s_{mnj}\theta_{mj}}_{\mathcal{P}_n} + \underbrace{\sum_{n'\neq n,m}\sum_j s_{n'nj}\theta_{n'j}}_{\mathcal{T}_n}. \tag{2.4}$$

This equation simply breaks out the double sum in (2.3) into the component due to country n's own shocks ($\mathcal{D}_n$), the component due to a particular trading partner m's shocks ($\mathcal{P}_n$), and the impact of "third" countries that are neither n nor m ($\mathcal{T}_n$).

Then, the GDP covariance between country n and country m is:

$$\begin{aligned} \text{Cov}(d\ln Y_{nt}, d\ln Y_{mt}) &= \underbrace{\text{Cov}(\mathcal{D}_n, \mathcal{D}_m)}_{\text{Shock Correlation}} \\ &+ \underbrace{\text{Cov}(\mathcal{D}_n, \mathcal{P}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{D}_m) + \text{Cov}(\mathcal{P}_n, \mathcal{P}_m)}_{\text{Bilateral Transmission}} \\ &+ \underbrace{\text{Cov}(\mathcal{D}_n + \mathcal{P}_n + \mathcal{T}_n, \mathcal{T}_m) + \text{Cov}(\mathcal{T}_n, \mathcal{D}_m + \mathcal{P}_m)}_{\text{Multilateral Transmission}}. \end{aligned} \tag{2.5}$$

This expression underscores the sources of international comovement. The first term, $\text{Cov}(\mathcal{D}_n, \mathcal{D}_m)$, captures the fact that economies might be correlated even in the absence of trade if the underlying shocks themselves are correlated, especially in sectors influential in the two economies. The shock correlation term can be written as:

$$\text{Cov}(\mathcal{D}_n, \mathcal{D}_m) = \sum_j \sum_i s_{nnj} s_{mmi} \text{Cov}(\theta_{nj}, \theta_{mi}).$$

Thus, a full account of international comovement would have to start with a reliable estimation of the shock processes hitting the economies.

The second term captures bilateral or direct transmission. If the GDP of country n has an elasticity with respect to the shocks occurring in country m ($s_{mni} > 0$), that would contribute to comovement

[3]The form of s_{mni} is known for some simple economies. For instance, if country n is in autarky, factors of production are supplied inelastically, and returns to scale are constant, $s_{nni} = P_{ni}Y_{ni}/P_nY_n$ are the Domar weights (Hulten, 1978; Acemoglu et al., 2012), and $s_{mni} = 0\ \forall m \neq n$. We derive a first-order closed-form solution to the influence matrix in our model economy with international trade in Section 3.1.

as well. Taking one of the terms of the Bilateral Transmission component:

$$\begin{aligned} \mathrm{Cov}(\mathcal{D}_n, \mathcal{P}_m) &= \sum_j \sum_i s_{nnj} s_{nmi} \mathrm{Cov}(\theta_{nj}, \theta_{ni}) \\ &= \mathbf{s}'_{\mathbf{nn}} \boldsymbol{\Sigma}_n \mathbf{s}_{\mathbf{nm}}, \end{aligned} \tag{2.6}$$

where $\boldsymbol{\Sigma}_n$ is the $J \times J$ covariance matrix of shocks in country n, and $\mathbf{s}_{\mathbf{nm}}$ is the $J \times 1$ influence vector collecting the impact of shocks in n on GDP in m. This expression underscores that one source of comovement is that under trade, both country n and country m will be affected by shocks in n.

Finally, the Multilateral Transmission term collects all the other sources of comovement between n and m that do not come from shocks to either n or m, such as shocks in other countries.

Comovement in Autarky We can now write the difference in covariances between autarky and trade as a sum of two terms:

$$\begin{aligned} \Delta \mathrm{Cov}(d \ln Y_n, d \ln Y_m) &= \underbrace{\sum_j \sum_i \left(s_{nnj} s_{mmi} - s_{nj}^{AUT} s_{mi}^{AUT} \right) \mathrm{Cov}(\theta_{nj}, \theta_{mi})}_{\Delta \text{Shock Correlation}} \\ &\quad + \text{Bilateral Transmission} + \text{Multilateral Transmission}, \end{aligned} \tag{2.7}$$

where s_{mi}^{AUT} are the elements of the influence vectors in autarky. This expression shows that trade opening can affect GDP covariance in two ways. First, it can make countries sensitive to foreign shocks, as captured by the bilateral and multilateral transmission terms. Second, and more subtly, opening to trade can re-weight sectors in the two economies either towards, or away, from sectors with more correlated fundamental shocks. This is captured by the first line of the equation above.

Dynamic Decomposition These decompositions generalize to a dynamic environment in which shocks can have prolonged effects on output. In that case, GDP in period t, Y_{nt}, is potentially a function of all the history of shocks $\{\boldsymbol{\theta}_{t-k}\}_{k=0}^{\infty}$:

$$d \ln Y_{nt} \approx \sum_{k=0}^{\infty} \sum_m \sum_i s_{mni,k} \theta_{mi,t-k}, \tag{2.8}$$

where $\theta_{mi,t}$ is now interpreted as the time-t innovation to the shock process. All the results above are generalized simply by adding a summation over k.[4]

We can decompose overall comovement into the static (contemporaneous) and dynamic components.

[4] That is, (2.6) is unchanged, while $\mathcal{D}_n$ becomes $\mathcal{D}_n = \sum_{k=0}^{\infty} \sum_j s_{nnj,k} \theta_{nj,t-k}$, for example.

The covariance between countries n and m can be written as:

$$\text{Cov}(d\ln Y_{nt}, d\ln Y_{mt}) = \sum_{k=0}^{\infty} \boldsymbol{s}'_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}_{m,k}, \tag{2.9}$$

where $\boldsymbol{s}_{n,k}$ is the $NJ \times 1$ influence vector collecting the impact of all worldwide innovations k periods ago on country n, and $\boldsymbol{\Sigma}$ is the covariance matrix of innovations. Thus, the overall GDP covariance is additive in the component due to the contemporaneous innovations $\boldsymbol{s}'_{n,0}\boldsymbol{\Sigma}\boldsymbol{s}_{m,0}$ and the dynamic propagation of past shocks. The contemporaneous component is also notable because in the quantitative framework below, the contemporaneous influence vector $\boldsymbol{s}_{n,0}$ in the fully-specified dynamic model coincides with the influence vector in a static model that only features instantaneous propagation of shocks.

To summarize, in order to provide an account of international comovement, we must (i) measure shocks in order to understand their comovement properties; and (ii) assess how sectoral composition (the distribution of s_{nnj}'s) translates sectoral comovement of the primitive shocks into GDP comovement. Further, in order to understand the contribution of international trade to international comovement, we must (iii) capture not only the cross-border elements of the influence vectors (the s_{nmj}'s), but also how going from autarky to trade changes the sectoral composition of the economy (the differences between s_{nnj} and s_{nj}^{AUT}). Finally, (iv) we must discipline the persistence of both the shocks and equilibrium adjustments over time in order to quantify the relative importance of contemporaneous vs. intertemporal correlation.

3 Quantitative Framework

The decomposition above is general and would apply in any production economy. However, any measurement of shocks and of the elements of the influence matrix requires additional theoretical structure. We now provide one such theoretical framework and use it to quantify the role of correlated shocks and transmission through networks.

Preliminaries Each country n is populated by a representative household. The household consumes the final good available in country n and supplies labor and capital to firms. Trade is subject to iceberg costs τ_{mnj} to ship good j from country m to country n (throughout, we adopt the convention that the first subscript denotes source, and the second destination).

Our benchmark model assumes financial autarky. There are two reasons behind this assumption. First, as highlighted by Heathcote and Perri (2002), models featuring financial autarky outperform complete and incomplete markets models in accounting for business cycle comovement. Second, we will use the model to derive the influence matrix of GDP elasticities to shocks everywhere in the world. Under financial autarky, this influence matrix can be constructed using only observed export and import shares, the elasticity of substitution among intermediate goods, and the Frisch elasticity. Alternative financial market structures would require additional assumptions on the preferences and technology to derive this matrix. We therefore assume that there are only goods flows across countries, and further, trade is balanced period by period.[5]

Households We assume that there is a continuum of workers in a representative household who share the same consumption. The problem of the household is

$$\max_{\substack{\{M_{njt}\},\{N_{njt}\},\\ \{H_{njt}\},\{E_{njt}\},\{U_{njt}\}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ \Psi \left(C_{nt} - \sum_j \xi_{njt} N_{njt} G(H_{njt}, E_{njt}, U_{njt}) - \sum_j \Xi\left(N_{njt}\right) \right) \tag{3.1}$$

subject to

$$P_{nt} \left(C_{nt} + \sum_j I_{njt} \right) = \sum_j W_{njt} N_{njt} H_{njt} E_{njt} + \sum_j R_{njt} U_{njt} M_{njt}$$
$$M_{njt+1} = (1 - \delta_j) M_{njt} + I_{njt}$$

where C_{nt} is consumption, I_{njt} is investment, N_{njt} is the number of workers employed in sector j, H_{njt} is the number of hours per worker, E_{njt} is the amount of effort per worker, M_{njt} is the amount of machines (installed capital), and U_{njt} is the capital utilization rate. We denote the effective total efficiency units of labor supplied in a sector as $L_{njt} \equiv N_{njt} H_{njt} E_{njt}$, and the effective total efficiency units of capital supplied as $K_{njt} \equiv M_{njt} U_{njt}$. Labor collects a sector-specific wage W_{njt}, and capital is rented at the price R_{njt}.

To proceed to link the model with data, we assume the following functional form for $G\left(.\right)$:

$$G\left(H, E, U\right) = \left(\frac{H}{\psi_h}\right)^{\psi_h} + \left(\frac{E}{\psi_e}\right)^{\psi_e} + \left(\frac{U}{\psi_u}\right)^{\psi_u}. \tag{3.2}$$

We highlight three features of the household problem. First, labor and capital are differentiated by sector, as the household supplies factors to, and accumulates capital in, each sector separately. In this formulation, labor and capital are neither fixed to each sector nor fully flexible. As $\psi_\iota \to 1$, $\iota = h, e, u$, factor supply across sectors becomes more sensitive to factor price differentials, in the limit households supplying variable factors only to the sector offering the highest factor price. At the opposite extreme, as $\psi_\iota \to \infty$, the supply of hours, effort, and capital utilization is fixed in each

[5] We can incorporate deficits in a manner similar to Dekle, Eaton, and Kortum (2008), without much change in our results.

sector by the preference parameters.

Second, we assume that the number of employed workers N_{njt} and machines M_{njt} in a sector is predetermined. While this approach is standard for machines, it is less common for employment, where it is usually assumed that hours and employment move in parallel. Specifically, in our model the number of workers in a particular sector has to be chosen before observing the current shocks as in Burnside, Eichenbaum, and Rebelo (1993), reflecting the fact that it takes time to adjust the labor force. Increasing the number of employed workers incurs additional costs $\Xi(N_{njt})$ where

$$\Xi(N) = \left(\frac{N}{\psi_n}\right)^{\psi_n}, \tag{3.3}$$

which is similar to Kydland and Prescott (1991) and Osuna and Rios-Rull (2003). This cost can be interpreted literally as a commuting cost, but it should be viewed more broadly as a stand in for frictions that limit the substitutability between employment and the workweek. The parameter ψ_n controls the volatility of employment relative to hours. On the other hand, within a period households can choose the hours H_{njt} and effort E_{njt} that change the effective amount of labor supply, and utilization rates U_{njt} that change the effective amount of capital supply. These margins capture the idea that utilization rates of factor inputs typically vary over the business cycle. Our framework thus implies that within a period, labor and capital supply to each sector are upward-sloping (e.g. Christiano, Motto, and Rostagno, 2014).

Third, our formulation of the disutility of the variable factor supply (3.2) is based on the Greenwood, Hercowitz, and Huffman (1988) preferences for labor and a similar isoelastic formulation of the utilization cost of capital. The GHH preferences mute the interest rate effects and income effects on the choice of hours, effort, and utilization rates, which helps to study the properties of the static equilibrium where the number of machines and employees are treated as predetermined.

The final use in the economy, denoted $\mathcal{F}_{nt} \equiv C_{nt} + \sum_j I_{njt}$, is a Cobb-Douglas aggregate across sectors. The functional form and its associated price index are given by

$$\mathcal{F}_{nt} = \prod_j \mathcal{F}_{njt}^{\omega_{jn}}, \qquad P_{nt} = \prod_j \left(\frac{P^f_{njt}}{\omega_{jn}}\right)^{\omega_{jn}},$$

where $\mathcal{F}_{njt}$ is the final use of sector j in country n, and P^f_{njt} is the final use price index in sector j and country n. Within each sector, aggregation across source countries is Armington, and the sector price index is defined in a straightforward way:

$$\mathcal{F}_{njt} = \left[\sum_m \vartheta_{mnj}^{\frac{1}{\rho}} \mathcal{F}_{mnjt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \qquad P^f_{njt} = \left[\sum_m \vartheta_{mnj} P_{mnjt}^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where $\mathcal{F}_{mnjt}$ is final use in n of sector j goods coming from country m, and P_{mnjt} is the price of $\mathcal{F}_{mnjt}$. For goods j, the expenditure share for final goods imported from country m is given by

$$\pi^f_{mnjt} = \frac{\vartheta_{mnj} P^{1-\rho}_{mnjt}}{\sum_k \vartheta_{knj} P^{1-\rho}_{knjt}}. \tag{3.4}$$

Static Decision Within a period, the supply curves are isoelastic in the factor prices relative to the consumption price index. The log of supply of hours, up to a normalization constant, is given by:

$$\left(\psi_h - 1 - \frac{\psi_h}{\psi_e}\right) \ln H_{njt} = -\ln \xi_{njt} + \ln\left(\frac{W_{njt}}{P_{nt}}\right).$$

Notice that the households' intra-temporal optimization problem leads to

$$H_{njt} G_h(H_{njt}, E_{njt}, U_{njt}) = E_{njt} G_e(H_{njt}, E_{njt}, U_{njt}).$$

Under the functional form adopted for $G(.)$, this condition implies that the choice of effort is a function of the choice of hours:

$$\ln E_{njt} = \frac{\psi_h}{\psi_e} \ln H_{njt}, \tag{3.5}$$

again up to a normalization constant.

A similar expression can be derived for the relationship between the optimal choice of capital utilization and the optimal choice of hours:

$$\frac{H_{njt} G_h(H_{njt}, E_{njt}, U_{njt})}{U_{njt} G_u(H_{njt}, E_{njt}, U_{njt})} = \frac{W_{njt} L_{njt}}{R_{njt} K_{njt}}.$$

As we will see from the firms' problem, the right-hand side of the equation above is equal to the ratio of output elasticities $\alpha_j/(1-\alpha_j)$, which is a constant. As a result, the utilization rate also has a log-linear relationship with hours worked:

$$\ln U_{njt} = \frac{\psi_h}{\psi_u} \ln H_{njt}, \tag{3.6}$$

up to a normalization constant. These properties capture the idea that flexible inputs tend to move jointly in the same direction, and facilitate the estimation of the utilization-adjusted TFP process in Section 4. Our setup provides a micro-foundation for the more reduced-form formula used by Basu, Fernald, and Kimball (2006). It also helps avoid the issue of whether to attribute the costs of variable factor utilization to labor income or capital income.

Dynamic Decision As discussed above, households also face intertemporal decisions determining capital accumulation and labor allocation over time. The first-order condition with respect to capital accumulation is

$$\Psi'_{nt} = \beta \mathbb{E}_t \left[\Psi'_{nt+1} \left(\frac{R_{njt+1}}{P_{nt+1}} U_{njt+1} + 1 - \delta_j \right) \right], \tag{3.7}$$

where Ψ'_{nt} stands for the marginal utility of final goods consumption in country n period t. This condition is similar to the standard Euler equation but is sector-specific and adjusted by the utilization rate.

The optimality condition with respect to N_{njt+1} is

$$\mathbb{E}_t \left[\Psi'_{nt+1} \left(\xi_{njt+1} G(H_{njt+1}, E_{njt+1}, U_{njt+1}) + \left(\frac{N_{njt+1}}{\psi_n} \right)^{\psi_n - 1} \right) \right] = \mathbb{E}_t \left[\Psi'_{nt+1} \frac{W_{njt+1}}{P_{nt+1}} H_{njt+1} E_{njt+1} \right].$$

Note that N_{njt+1} is chosen in period t before observing shocks in period $t+1$. The left hand-side is the expected marginal disutility of a unit increase in sector j employment, while the right-hand side is the corresponding marginal utility gain due to higher labor income.

Firms A representative firm in sector j in country n operates a CRS production function

$$Y_{njt} = Z_{njt} \Theta_{njt} \left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j}, \tag{3.8}$$

taking as given the total factor productivity $Z_{njt}\Theta_{njt}$. The intermediate input usage X_{njt} is an aggregate of inputs from potentially all countries and sectors:

$$X_{njt} \equiv \left(\sum_{m,i} \mu_{mi,nj}^{\frac{1}{\varepsilon}} X_{mi,njt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $X_{mi,njt}$ is the usage of inputs coming from sector i in country m in production of sector j in country n, and $\mu_{mi,nj}$ is the input coefficient.

The total factor productivity consists of two parts: the exogenous shock Z_{njt} and the endogenous component:

$$\Theta_{njt} = \left(\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right)^{\gamma_j - 1}, \tag{3.9}$$

where γ_j controls possible congestion or agglomeration effects. As a result, the sectoral aggregate production function is then

$$Y_{njt} = Z_{njt} \left[\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right]^{\gamma_j}. \tag{3.10}$$

Let P_{njt} denote the price of output produced by sector j in country n,[6] and let $P_{mi,njt}$ be the price paid in sector n, j for inputs from m, i. No arbitrage in shipping implies that the prices "at the factory gate" and the price at the time of final or intermediate usage are related by:

$$P_{mi,njt} = P_{mnit} = \tau_{mni} P_{mit},$$

where τ_{mni} is the iceberg trade cost.

Cost minimization implies that the payments to primary factors and intermediate inputs are:

$$\begin{aligned}
R_{njt} K_{njt} &= \alpha_j \eta_j P_{njt} Y_{njt} \\
W_{njt} L_{njt} &= (1 - \alpha_j) \eta_j P_{njt} Y_{njt} \\
P_{mi,njt} X_{mi,njt} &= \pi^x_{mi,njt} (1 - \eta_j) P_{njt} Y_{njt},
\end{aligned} \tag{3.11}$$

where $\pi^x_{mi,njt}$ is the share of intermediates from country m sector i in total intermediate spending by n, j, given by:

$$\pi^x_{mi,njt} = \frac{\mu_{mi,nj} (\tau_{mni} P_{mit})^{1-\varepsilon}}{\sum_{k,l} \mu_{kl,nj} (\tau_{knl} P_{klt})^{1-\varepsilon}}.$$

Shocks The economy experiences two types of shocks: the conventional TFP shock Z_{njt} in each sector j and country n, and the non-technology shock ξ_{njt} that enters the household problem in (3.1). Our framework conceives of ξ_{njt} as a (sector-specific) within-period shift in the variable supply of both primary factors. This specification follows in the tradition of modeling and measuring business cycle shocks that are distinct from contemporaneous productivity.[7] These can have a literal interpretation as exogenous shifts in intra-temporal factor supply curves. Alternatively, news shocks (e.g. Beaudry and Portier, 2006), or sentiment shocks (e.g. Angeletos and La'O, 2013; Huo and Takayama, 2015) would manifest themselves as shocks to ξ_{njt}, as agents react to a positive innovation in sentiment by supplying more factors. Straightforward manipulation shows that ξ_{njt} can also be viewed as a shifter in the optimality condition for factor usage. The literature has explored the aggregate labor version of this shifter, labeling it alternatively a "preference shifter" (Hall, 1997), "inefficiency gap" (Galí, Gertler, and López-Salido, 2007), or "labor wedge" (Chari, Kehoe, and McGrattan, 2007). While this object is treated as a reduced-form residual in much of this literature, we know that monetary policy shocks under sticky wages (Galí, Gertler, and López-Salido, 2007; Chari, Kehoe, and McGrattan, 2007), or shocks to working capital constraints (e.g. Neumeyer and Perri, 2005; Mendoza, 2010)

[6] Note this is not the same as the ideal price index P^f_{njt} of sector j final consumption in n, which aggregates imports from the other countries.

[7] A variety of measurement exercises attribute a large share of business cycle fluctuations to non-technology shocks. See, among many others, Blanchard and Quah (1989), Galí (1999), Basu, Fernald, and Kimball (2006), Angeletos, Collard, and Dellas (2018), and Levchenko and Pandalai-Nayar (2018).

manifest themselves as shocks to ξ_{njt}.

Our analysis does not cover all possible shocks. In particular, we restrict attention to within-period shocks, namely productivity and factor supply. An appealing feature of these shocks is that recovering them requires comparatively little structure and few assumptions. These shocks can be inferred from within-period relationships, and would thus be recovered correctly under a wide variety of assumptions, including both static and dynamic models. Relatedly, focusing on these shocks connects us seamlessly to the network propagation literature following Acemoglu et al. (2012). As in that class of models, in our framework the impact response of the economy to shocks is captured by the influence matrix.

Intertemporal shocks that enter the Euler equation have also been considered in the literature. Extracting these shocks requires significantly more assumptions, most importantly specifying an asset market structure of the world economy. Existing open-economy implementations use smaller-scale models and assume complete asset markets (Eaton et al., 2016; Ohanian, Restrepo-Echavarria, and Wright, 2017). Assessments of Euler equation shocks in closed- and open-economy settings find them to be the least important in accounting for macro fluctuations (Chari, Kehoe, and McGrattan, 2007; Ohanian, Restrepo-Echavarria, and Wright, 2017). To preserve the relative parsimony in the specification of shocks, we thus limit our analysis to the two shocks specified above.

Equilibrium An equilibrium in this economy is a set of goods and factor prices $\{P_{njt}, W_{njt}, R_{njt}\}$, factor allocations $\{M_{njt}, N_{njt}, H_{njt}, E_{njt}, U_{njt}\}$, and goods allocations $\{Y_{njt}\}$, $\{C_{nt}, I_{njt}, X_{mi,njt}\}$ for all countries and sectors such that (i) households maximize utility; (ii) firms maximize profits; and (iii) all markets clear.

At the sectoral level, the following market clearing condition has to hold for each country n sector j:

$$P_{njt}Y_{njt} = \sum_m P_{mt}\mathcal{F}_{mt}\omega_{mj}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i)P_{mit}Y_{mit}\pi^x_{nj,mit}. \tag{3.12}$$

Meanwhile, a direct implication of financial autarky is that each country's expenditure equals the sum of value added across domestic sectors

$$P_{mt}\mathcal{F}_{mt} = \sum_i \eta_i P_{mit}Y_{mit}. \tag{3.13}$$

Combining (3.12) and (3.13):

$$P_{njt}Y_{njt} = \sum_m \sum_i \eta_i P_{mit}Y_{mit}\omega_{mj}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i)P_{mit}Y_{mit}\pi^x_{nj,mit}. \tag{3.14}$$

Note that once we know the share of value added in production η_j, the expenditure shares ω_{mj}, π^f_{nmjt}, and $\pi^x_{nj,mit}$ for all n, m, i, j, we can compute the nominal output $P_{njt}Y_{njt}$ for all country-sectors (n, j) after choosing a numeraire good. There is no need to specify further details of the model, and we will utilize this property to derive the influence matrix.

3.1 Analytical Influence Matrix

We now provide an analytical expression for the global influence matrix. In general, closed-form solutions for the exact influence vectors cannot be obtained in multi-country multi-sector models such as ours. However, we can solve for the first-order approximation of the influence vector in our setting. Appendix B provides a more detailed derivation of the influence vector, and evaluates the fit of the first-order approximation relative to the full nonlinear model solution. The first-order approximation performs quite well.

The vectors $\ln \mathbf{P}_t$ and $\ln \mathbf{Y}_t$ of length NJ collect sector-country prices and quantities at time t. Linearizing the market clearing conditions (3.14), we obtain

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \underbrace{\left(\mathbf{\Psi}^f + \mathbf{\Psi}^x\right)(\ln \mathbf{P}_t + \ln \mathbf{Y}_t)}_{\text{destination country output variation}} + \underbrace{(1-\rho)\left(\text{diag}(\mathbf{\Psi}^f\mathbf{\Pi}^f\mathbf{1}) - \mathbf{\Psi}^f\mathbf{\Pi}^f\right)\ln \mathbf{P}_t}_{\text{consumption goods relative price variation}} + \underbrace{(1-\varepsilon)\left(\text{diag}(\mathbf{\Psi}^x\mathbf{\Pi}^x\mathbf{1}) - \mathbf{\Psi}^x\mathbf{\Pi}^x\right)\ln \mathbf{P}_t}_{\text{intermediate goods relative price variation}} \tag{3.15}$$

where $\mathbf{\Pi}^x$ and $\mathbf{\Pi}^f$ are matrices containing the steady-state import shares of intermediate and final goods, and $\mathbf{\Psi}^x$ and $\mathbf{\Psi}^f$ are matrices containing the steady-state export shares of intermediate and final goods.[8]

Then, equation (3.15) implies that we can express the vector of country-sector price changes in terms of output changes and known parameters: $\ln \mathbf{P}_t = \mathcal{P} \ln \mathbf{Y}_t$.

Let further the hours output elasticity and the intermediates output elasticity adjusted for utilization and returns-to-scale be $\mathcal{E}^h$ and $\mathcal{E}^x$. Combining equation (3.15) with linearized versions of the production function (3.10), labor market clearing and the demand for intermediate goods, the influence matrix is:[9]

$$\ln \mathbf{Y}_t = \left\{\mathbf{I} - \left(\mathcal{E}^h + \mathcal{E}^x\right)(\mathbf{I} + \mathcal{P}) + \left(\mathcal{E}^h\widetilde{\mathbf{\Pi}}^f + \mathcal{E}^x\mathbf{\Pi}^x\right)\mathcal{P}\right\}^{-1}\left(\ln \mathbf{Z}_t - \mathcal{E}^h \ln \boldsymbol{\xi}_t\right). \tag{3.16}$$

[8] See equations (B.8) and (B.9) for the construction of $\mathbf{\Pi}^x$, $\mathbf{\Pi}^f$, $\mathbf{\Psi}^x$ and $\mathbf{\Psi}^f$.

[9] See equation (B.11) for the construction of $\mathcal{E}^h$, $\mathcal{E}^x$, and $\widetilde{\mathbf{\Pi}}^f$.

Equation (3.16) illustrates that all we need to understand the GDP elasticity to various sector-country shocks in this quantitative framework are measures of steady state final goods consumption and production shares, as well as model elasticities. The influence matrix encodes the general equilibrium response of sectoral output in a country to shocks in any sector-country, taking into account the full model structure and all direct and indirect links between the countries and sectors. This is particularly evident in equation (3.15), which pins down the matrix $\mathcal{P}$ relating changes in quantities to changes in prices. The first term contains the response of GDP that arises from output changes in every country and sector in response to a shock in a sector-country. The second term contains the relative price changes of final goods and the final term the relative price changes of intermediate inputs.

Two aspects of the influence matrix are worth noting. The first is a resemblance of (3.16) to the typical solution to a network model, that writes the equilibrium change in output as a product of the Leontief inverse and the vector of shocks. Our expression also features a vector of shocks, and an inverse of a matrix that is more complicated due to the multi-country structure of our model combined with elastic factor supply and the departure from unitary elasticities of substitution.

Second, the response of output in a static model (fixing M_{njt} and N_{njt} in each sector) coincides with the *impact* response in the fully dynamic DSGE model. Both are given by (3.16). Our analysis thus integrates the static network propagation literature that follows Acemoglu et al. (2012) and the dynamic international business cycle literature. We can cleanly separate the instantaneous propagation analyzed in the former and the delayed responses to shocks emphasized by the latter. In later periods the response of GDP will depend on the persistence of shocks and the capital and labor accumulation decisions, which are not encoded in this vector (but can be evaluated numerically).

GDP Change It is straightforward to go from sector-level output changes in (3.16) to GDP changes. To do that, we need to aggregate the changes of sector-country real value added, as in (2.2).[10] Denote by $\mathcal{D}$ the matrix of Domar weights for sector j in country n, $\frac{P_{nj}Y_{nj}}{P_n\mathcal{F}_n}$. Also define the matrix $\boldsymbol{\eta}$ of sector value-added ratios, η_j.[11]

The real GDP changes are given by

$$\ln \mathbf{GDP}_t = \boldsymbol{\eta}\mathcal{D}\ln \mathbf{Y}_t - (\mathbf{I}-\boldsymbol{\eta})\mathcal{D}(\mathbf{I}-\mathbf{\Pi}^x)\mathcal{P}\ln \mathbf{Y}_t, \tag{3.17}$$

[10] Evaluated at base prices, the log-deviation of country n's real GDP in period t can be expressed as

$$\ln Y_{nt} = \sum_j \left(\frac{P_{nj}Y_{nj}}{P_n\mathcal{F}_n} \ln Y_{njt} - \sum_{m,i} \frac{P_{mi}X_{mi,nj}}{P_n\mathcal{F}_n} \ln X_{mi,njt} \right).$$

[11] See equation (B.12) for the construction of $\mathcal{D}$ and $\boldsymbol{\eta}$.

where $\ln \mathbf{GDP}_t$ is the $N \times 1$ vector of log changes of GDP Y_{nt}. The first term in equation (3.17) captures the changes in quantity which is aggregated with Domar weights. The second term captures the relative changes between the prices of domestically produced goods and the prices of imported intermediate goods.

4 Measurement

4.1 Estimating TFP Shocks

Unobserved Factor Utilization: As emphasized by BFK, measuring TFP innovations is difficult because the intensity with which factors are used in production varies over the business cycle, and cannot be directly observed by the econometrician. As unobserved factor utilization will respond to TFP innovations, it is especially important to account for it in estimation, otherwise factor usage will appear in estimated TFP. BFK develop an approach to control for unobserved factor utilization which leads to a TFP series in the United States that has very different properties than the Solow residual. Our approach is similar in spirit.

In the model above, the true factor inputs are $K_{njt} \equiv U_{njt}M_{njt}$ and $L_{njt} \equiv E_{njt}H_{njt}N_{njt}$. The true capital input is the product of the quantity of capital input ("machines") M_{njt} that can be measured in the data, and capital utilization U_{njt} that is not directly observable. Similarly, the true labor input is the product of the number of workers N_{njt}, hours per worker H_{njt}, and labor effort E_{njt}. While N_{njt} and H_{njt} can be obtained from existing datasets, E_{njt} is unobservable.

Log-differencing (3.10), and writing input usage breaking up the observed and the unobserved components yields:

$$\begin{aligned} d\ln Y_{njt} \quad = \quad & \gamma_j \underbrace{\left(\alpha_j \eta_j d\ln M_{njt} + (1-\alpha_j)\eta_j d\ln\left(H_{njt}N_{njt}\right) + (1-\eta_j) d\ln X_{njt}\right)}_{\text{Observed Inputs}} \\ & + \gamma_j \underbrace{\left(\alpha_j \eta_j d\ln U_{njt} + (1-\alpha_j)\eta_j d\ln E_{njt}\right) + d\ln Z_{njt}}_{\text{Unobserved Inputs}}. \end{aligned} \tag{4.1}$$

We rely on the theoretical framework to derive an estimable equation that takes into account the unobserved components of (4.1) and thus allows us to recover estimates of the true TFP $d\ln Z_{njt}$. The first-order conditions of the profit maximization problem of the firm with the production function (3.8) imply that the cost shares of the composite labor and capital inputs are $(1-\alpha_j)\,\eta_j$ and $\alpha_j\eta_j$ respectively. Given a wage W_{njt} or a rental rate R_{njt}, the firm is indifferent between increasing effort/hours or employees holding other inputs constant, and similarly between utilization and machines. However, we assumed that the household faces increasing disutility from supplying more on any individual margin (effort, hours, or utilization of capital), and N_{mjt} and M_{mjt} are predetermined within a period. The market-clearing wages and rental rates therefore pin down the equilibrium choices of effort, hours, and utilization in a period. The household's optimal choices of unobserved utilization and effort will be proportional to its choice of observed hours (see 3.5 and 3.6). The household intra-temporal first-order conditions therefore allow us express unobserved effort and capital utilization as a log-linear function of observed hours:

$$\gamma_j \left(\alpha_j \eta_j d \ln U_{njt} + (1-\alpha_j)\eta_j d \ln E_{njt}\right) = \zeta_j d \ln H_{njt}, \tag{4.2}$$

where $\zeta_j = \gamma_j \eta_j \left(\alpha_j \frac{\psi^h}{\psi^u} + (1-\alpha_j)\frac{\psi^h}{\psi^e}\right)$.

Plugging these relationships into (4.1) yields the following estimating equation:

$$\begin{aligned} d\ln Y_{njt} \quad = \quad & \delta_j^1 \left(\alpha_j \eta_j d \ln M_{njt} + (1-\alpha_j)\eta_j d \ln \left(H_{njt} N_{njt}\right) + (1-\eta_j) d \ln X_{njt}\right) \\ & + \delta_j^2 d \ln H_{njt} + \delta_{nj} + d \ln Z_{njt}, \end{aligned} \tag{4.3}$$

where we also added country×sector fixed effects δ_{nj} to allow for country-sector specific trend output growth rates. The estimation proceeds to regress real output growth on the growth of the composite observed input bundle and the change in hours per worker. The coefficient δ_j^1 is clearly an estimate of returns-to-scale γ_j. Equation (4.2) provides a structural interpretation for the constant $\delta_j^2 = \zeta_j$.[12]

We use a strategy similar to BFK to estimate (4.3). First, input usage will move with TFP shocks $d \ln Z_{njt}$, and thus the regressors in this equation are correlated with the residual. To overcome this endogeneity problem, our baseline approach uses two instruments. The first is oil shocks, defined as the difference between the log oil price and the maximum log oil price in the preceding four quarters. This oil price shock is either zero, or is positive when this difference is positive, reflecting the notion that oil prices have an asymmetric effect on output. The annualized oil shock is the sum over the four quarters of the preceding year. The second instrument is the growth rate in real government defense spending, lagged by one year.[13] Our baseline production function estimation sample is confined to the G7 countries. This tends to lead to the strongest instruments and most precisely estimated coefficients. Finally, following BFK, to reduce the number of parameters to be estimated, we restrict δ_j^2 to take only three values, according to a broad grouping of sectors: durable

[12]BFK derive the same estimating equation by assuming instead that firms face an upward-sloping cost schedule for increasing effort, hours, or utilization holding other factors constant. While our framework is somewhat less general, an advantage is that we do not have to assume ad-hoc convex cost functions for firm choices. The structural interpretation of the estimated parameters in our framework differs slightly from BFK, but we can still recover estimates of returns-to-scale and adjust for unobserved utilization.

[13]The instruments must be uncorrelated with the residual component of TFP growth. One may worry that in the presence of within-sector misallocation, an instrument that moves factor inputs will also change the degree of misallocation, thus violating the exclusion restriction. However, the instruments will change misallocation only when they affect *relative* factor inputs across firms. Since our instruments capture quite aggregate shocks, it seems less likely that they would lead to a large reshuffling across firms within a sector.

manufacturing, non-durable manufacturing, and all others.

Conditional on these estimates and the log changes in the observed inputs, we obtain the TFP shocks $d\ln Z_{njt}$ as residuals. We use the estimate of ζ_j in two places, as we need it to construct the $d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j}X_{njt}^{1-\eta_j}\right]$ term:

$$d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j}X_{njt}^{1-\eta_j}\right] \quad = \quad d\ln\left(M_{njt}^{\alpha_j\eta_j}N_{njt}^{(1-\alpha_j)\eta_j}H_{njt}^{(1-\alpha_j)\eta_j+\frac{\zeta_j}{\gamma_j}}X_{njt}^{1-\eta_j}\right),$$

where we substituted for unobserved inputs using (4.2).

4.2 Extracting Non-Technology Shocks

Conditional on the productivity shocks measured above, and on the pre-determined changes in employees and machines N_{njt} and M_{njt}, the non-technology shocks ξ_{njt} are recovered in such a way as to match actual value added growth in every country-sector (and therefore actual GDP growth in every country). Let $\ln V_{njt}$ denote value added in log deviations from steady state:

$$\ln V_{njt} = \frac{P_{nj}Y_{nj}}{V_{nj}}\ln Y_{njt} - \sum_{m,i}\frac{P_{mi}X_{mi,nj}}{V_{nj}}\ln X_{mi,njt}.$$

We have data on the $NJ\times 1$ vector of log changes in real value added $d\ln\mathbf{V}_t$ in each t. In the model, using the equilibrium relationships between gross output, price changes, and real input use, the vector of changes in value added can be written as[14]

$$d\ln\mathbf{V}_t = \mathcal{V}\, d\ln\mathbf{Y}_t. \tag{4.4}$$

Extending equation (3.16) to include the impacts of the predetermined changes in installed machines and employment, $d\ln\mathbf{M}_t$ and $d\ln\mathbf{N}_t$, the changes of gross output can be written as:[15]

$$d\ln\mathbf{Y}_t = \left\{\mathbf{I}-\left(\mathcal{E}^h+\mathcal{E}^x\right)(\mathbf{I}+\mathcal{P})+\left(\mathcal{E}^h\widetilde{\mathbf{\Pi}}^f+\mathcal{E}^x\mathbf{\Pi}^x\right)\mathcal{P}\right\}^{-1}\left(d\ln\mathbf{Z}_t-\mathcal{E}^h d\ln\boldsymbol{\xi}_t+\mathcal{E}^m d\ln\mathbf{M}_t+\mathcal{E}^n d\ln\mathbf{N}_t\right).$$

Combining the two equations above, the non-technology shocks can be recovered:

$$\begin{aligned} d\ln\boldsymbol{\xi}_t = (\mathcal{E}^h)^{-1}\Big\{ & d\ln\mathbf{Z}_t+\mathcal{E}^m d\ln\mathbf{M}_t+\mathcal{E}^n d\ln\mathbf{N}_t \\ & -\left(\mathbf{I}-\left(\mathcal{E}^h+\mathcal{E}^x\right)(\mathbf{I}+\mathcal{P})+\left(\mathcal{E}^h\widetilde{\mathbf{\Pi}}^f+\mathcal{E}^x\mathbf{\Pi}^x\right)\mathcal{P}\right)\mathcal{V}^{-1}d\ln\mathbf{V}_t\Big\}. \end{aligned} \tag{4.5}$$

[14] See equation (B.13) for the construction of $\mathcal{V}$.

[15] See equation (B.14) and (B.15) for the construction of $\mathcal{E}^m$ and $\mathcal{E}^n$.

In other words, the structure of the model world economy and the observed/measured objects can be used to infer a global vector of non-technology shocks $d\ln\boldsymbol{\xi}_t$ that rationalizes observed growth rates in real value added in each country-sector. Note that the interdependence between country-sectors through input linkages implies that the entire global vector $d\ln\boldsymbol{\xi}_t$ must be solved for jointly. Appendix B.3 describes in greater detail the procedure for extracting the non-technology shocks.

4.3 Data

The data requirements for estimating equation (4.3) is growth of real output and real inputs for a panel of countries, sectors, and years. The dataset with the broadest coverage of this information is KLEMS 2009 (O'Mahony and Timmer, 2009).[16] This database contains gross output, value added, labor and capital inputs, as well as output and input deflators. In a limited number of instances, we supplemented the information available in KLEMS with data from the WIOD Socioeconomic Accounts, which contains similar variables. After data quality checking and cleaning, we retain a sample of 30 countries, listed in Appendix Table A1. The database covers all sectors of the economy at a level slightly more aggregated than the 2-digit ISIC revision 3, yielding, after harmonization, 30 sectors listed in Appendix Table A2. In the best cases we have 28 years of data, 1970-2007, although the panel is not balanced and many emerging market countries do not appear in the data until the mid-1990s.

The oil price series is the West Texas Intermediate, obtained from the St. Louis Fed's FRED database. We have also alternatively used the Brent Crude oil price, obtained from the same source. Military expenditure comes from the Stockholm International Peace Research Institute (SIPRI).

The extraction of the non-technology shocks and the quantitative analysis require additional information on the input linkages at the country-sector-pair level, as well as on final goods trade. This information comes from the 2013 WIOD database (Timmer et al., 2015), which contains the global input-output matrix.

4.4 Empirical Results

TFP Estimation Table 1 reports the results of estimating equation (4.3). The returns to scale parameters vary from about 0.7 to 0.9 in durable manufacturing, from 0.3 to 1 in non-durable manufacturing, and from 0.4 to nearly 2 in the quite heterogeneous non-manufacturing sector. Thus, the estimates show departures from constant returns to scale in a number of industries, consistent with existing evidence. The coefficient on hours per worker ($d\ln H_{njt}$) is significantly different from

[16] This is not the latest vintage of KLEMS, as there is a version released in 2016. Unfortunately, the 2016 version has a shorter available time series, as the data start in 1995, and also has many fewer countries. A consistent concordance between the two vintages is challenging without substantial aggregation.

zero in two out of three industry groups, indicating that adjusting for unobserved utilization is important in the manufacturing industries.

Appendix A.1 reports a battery of robustness checks and additional results regarding the TFP estimation procedure, including: (i) the complete set of industry-specific production function estimates within each of these three broad groups; (ii) stability of estimates with respect to country sample, and to using BFK's coefficients; (iii) the correlation between our implied utilization series and survey-based utilization data.

TABLE 1: Summary of Production Function Parameter Estimates

Industry Group	Median Returns to Scale	Utilization Adjustment
Durables	0.806	1.420
	[0.701,0.895]	(0.389)
Non-durable manufacturing	0.753	2.929
	[0.291,0.926]	(1.771)
Non-durable non-manufacturing	1.244	0.260
	[0.451,1.864]	(0.643)

Notes: This table reports the range of estimates of γ_j in the three broad groups of sectors, and the estimates of ζ_j along with their standard errors clustered at the country-sector in parentheses.

Non-Technology Shocks Having estimated these production function parameters and TFP shocks, we simply back out the implied non-technology shocks using our data and equation (4.5). Doing so requires a small number of parameters in addition to data on the structure of world input and final goods production and trade. We defer the description of parameter estimation and calibration until Section 5.2.

Cross-Country Correlations With these estimates in hand, we are ready to examine cross-country correlations. The estimates of the TFP shocks alone deliver some insights about the direct effects of these shocks relative to the Solow residual (the traditional measure of TFP). We present results for two subsamples: the G7 countries and the full sample. The G7 countries have less variation among them, making patterns easier to detect. In addition, the production function coefficient estimates are most reliable for the G7 sample, and we use them as the baseline coefficients to be applied to all other countries, implying that TFP and inputs in other countries are likely measured with greater error.

In the first instance, we are interested in the proximate drivers of comovement between countries, and in particular whether aggregate comovement occurs because of correlated TFP or inputs. Appendix

B.1 shows that GDP growth can be written a sum of two components:

$$d \ln Y_{nt} \approx d \ln Z_{nt} + d \ln \mathcal{I}_{nt}, \tag{4.6}$$

where aggregate TFP is denoted by:

$$d \ln Z_{nt} = \sum_{j=1}^{J} D_{nj} d \ln Z_{njt}, \tag{4.7}$$

and $d \ln \mathcal{I}_{nt}$ is the log change in the scale-adjusted primary factor inputs (see equation B.4). According to (4.7), aggregate TFP growth is thus a weighted average of sectoral TFP growth rates, with D_{nj} being the "steady-state" Domar weights, proxied by the period average Domar weights.

Table 2 presents the basic summary statistics for the elements of the GDP decomposition in equation (4.6). While the non-technology shocks do not appear in this decomposition, these results are useful for highlighting the role of the TFP shocks and comparing them to the Solow residual. The top panel reports the correlations among the G7 countries. The average correlation of real GDP growth among these countries is 0.36. The second line summarizes correlations of the TFP shocks. Those are on average close to zero, if not negative. By contrast, input growth is positively correlated, with a mean of 0.26.

Appendix B.1 shows that the Solow residual can be written as a sum of the aggregate TFP growth and the aggregated variable utilization change $d \ln \mathcal{U}_{nt}$:

$$d \ln S_{nt} = d \ln Z_{nt} + d \ln \mathcal{U}_{nt}, \tag{4.8}$$

with the expression for $d \ln \mathcal{U}_{nt}$ provided in (B.6).

Thus, it is an empirical question to what degree correlations in the Solow residual reflect true technology shock correlation as opposed to endogenous input adjustments. Table 2 shows that the Solow residual has an average correlation of about 0.09 in this sample of countries. If Solow residual was taken to be a measure of TFP shocks, we would have concluded that TFP is positively correlated in this set of countries. As we can see, this conclusion would be misleading. Indeed, the correlation in the utilization term $\mathcal{U}_{nt}$, which is the difference between the TFP shock $d \ln Z_{nt}$ and the Solow residual, accounts for the all of the correlation in the Solow residual, on average. This indicates that the correlation in the Solow residual is in fact driven by unobserved input utilization and scale adjustments. The left panel of Figure 1 depicts the kernel densities of the correlations of real GDP, TFP, and inputs. There is a clear hierarchy, with the real GDP being most correlated, and the TFP being least correlated and centered on zero.

Table 2: Correlations Summary Statistics

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
$d\ln Y_{nt}$	0.358	0.337	0.242	0.565
$d\ln Z_{nt}$	0.013	-0.002	-0.079	0.152
$d\ln \mathcal{I}_{nt}$	0.257	0.207	0.074	0.449
$d\ln \xi_{nt}$	0.218	0.224	0.060	0.405
$d\ln S_{nt}$	0.085	0.111	-0.022	0.314
$d\ln \mathcal{U}_{nt}$	0.135	0.125	-0.018	0.245
	All countries (N. obs. = 406)			
$d\ln Y_{nt}$	0.190	0.231	-0.027	0.437
$d\ln Z_{nt}$	-0.004	0.007	-0.189	0.214
$d\ln \mathcal{I}_{nt}$	0.087	0.094	-0.150	0.339
$d\ln \xi_{nt}$	0.017	0.039	-0.205	0.240
$d\ln S_{nt}$	0.056	0.084	-0.151	0.303
$d\ln \mathcal{U}_{nt}$	0.025	0.038	-0.175	0.237

Notes: This table presents the summary statistics of the correlations in the sample of G7 countries (top panel) and full sample (bottom panel). Variable definitions and sources are described in detail in the text.

The bottom panel of Table 2 repeats the exercise in the full sample of countries. The basic message is the same as for the G7 but quantitatively the picture is not as stark and the variation is greater. It is still the case that $d\ln Z_{nt}$ has a zero average correlation. It is also still the case that the inputs $d\ln \mathcal{I}_{nt}$ have greater correlation, and that their correlation is on average about half of the average real GDP correlation. The Solow residuals are also more correlated than $d\ln Z_{nt}$, and part of the difference is accounted for by the fact that the unobserved inputs are positively correlated. The right panel of Figure 1 displays the kernel densities of the correlations in the full sample. Appendix Figure A2 also plots the estimated TFP series against the Solow residual for all the countries in the sample.

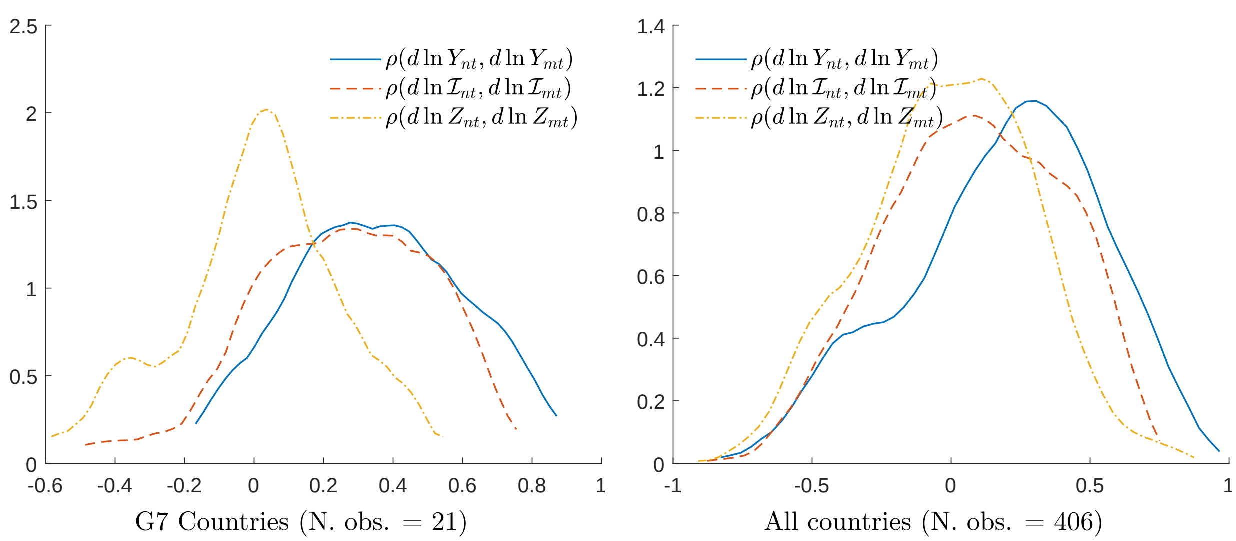
This is of course only an accounting decomposition. Factor usage will respond to TFP shocks at home and abroad. Since the growth in $\mathcal{I}_{nt}$ has not been cleaned of the impact of technology shocks, it cannot be attributed exclusively by non-technology shocks. We next turn to assessing the unconditional Domar-weighted correlation of non-technology shocks across countries as we did for TFP shocks. Then, in Section 5 we use our full model and the decompositions outlined in Section 2 to perform a number of exercises aimed at understanding the full role of these shocks in international comovement.

Patterns in Non-Technology Shocks Across Countries: Unlike the decomposition of GDP growth into TFP and inputs in (4.6), there is no decomposition that isolates the domestic non-technology shocks $d\ln\xi_{njt+1}$ as an additive component in the GDP growth rate. Nonetheless, to provide a simple illustration of the correlations of $d\ln\xi_{njt}$ across countries, we construct a Domar-weighted non-technology shock, to parallel the Domar-weighted TFP shock in (4.7):

$$d\ln\xi_{nt} = \sum_{j=1}^{J} D_{nj} d\ln\xi_{njt}. \tag{4.9}$$

Table 2 reports the correlations in $d\ln\xi_{nt}$ among the G7 and in the full sample. The non-technology shocks are positively correlated across countries, unlike TFP. The correlation of non-technology shocks is around 0.2 on average in the G7 countries, which is well short of the observed GDP correlation, but substantially higher than the zero average TFP correlation in this set of countries. In the full sample, aggregated non-technology shocks have about a 0.02-0.04 correlation on average, which is not very different from the TFP correlation. This suggests that, when considering the G7 group of countries alone, non-technology shocks have a better chance of producing positive output correlations observed in the data. Appendix Figure A3 plots the $d\ln\xi_{nt}$'s against our estimated TFP shocks for all countries. Appendix Table A8 shows that the pattern of $d\ln\xi_{nt}$ correlations remains very similar when using alternative calibrations.

FIGURE 1: Correlations: Kernel Densities



Notes: This figure displays the kernel densities of real GDP growth, the utilization-adjusted TFP, and input correlations in the sample of G7 countries (left panel) and full sample (right panel). Variable definitions and sources are described in detail in the text.

5 Quantitative Assessment

Shocks in our model can affect aggregate outcomes via a contemporaneous impact – their correlation and the intratemporal transmission through the network – as well as a dynamic impact driven by the response of capital accumulation and intertemporal labor adjustment to the shocks.

To understand and separate the mechanisms in the model that generate comovement, it is useful to first consider a static version of the model in the spirit of the network propagation literature following Acemoglu et al. (2012), in which both capital accumulation within a sector and sectoral employment adjustments are not permitted. This exercise emphasizes the role of the input-output structure of the model in amplifying or dampening the underlying contemporaneous correlations of the sectoral shocks.

Importantly, in our framework the contemporaneous response of the world economy to shocks is characterized by the global influence matrix and coincides with the impact response in the dynamic model. In addition, as emphasized in Section 2 the static and the dynamic components of the total covariance are simply additive. Thus the static and dynamic comovement are separable, and we begin by considering the static component.

5.1 Static Counterfactuals

The static counterfactual simulates output growth rates in a setting where machines M_{njt} and employees N_{njt} are held constant. The first-order analytical solution expressed as a global influence matrix is in Section 3.1. For the static model we can also obtain the exact solution using the hat algebra approach of Dekle, Eaton, and Kortum (2008). The details of the exact solution to the model are in Appendix B.4. The appendix also provides a comparison between the GDP growth rates implied by the first-order approach and the exact GDP growth rates. It turns out that in our setting, the exact and first-order approximation solutions are very close to each other, with a correlation between the two GDP growth rates of 0.999. Below, we will present the cross-country GDP correlations coming from the first-order analytical solution, as it permits the decomposition of the overall comovement into the additive shock correlation and transmission terms.

5.2 Calibration

In implementing this static approach, we only need to take a stand on the value of a small number of parameters, and use our data to provide the required quantities. Table 3 summarizes the parameter assumptions for the static model and data sources. Appendix A.2 undertakes the estimation of the substitution elasticities in final and intermediate use. Based on these estimation results, the final consumption Armington elasticity ρ is set to either 2.75 or 1, and the intermediate elasticity ε to

Table 3: Parameter Values

Param.	Value	Source	Related to
ρ	2.75 or 1	Our estimates	final substitution elasticity
ε	1	Our estimates	intermediate substitution elasticity
ψ_e, ψ_h	4	Chetty et al. (2013)	Frisch elasticity
ψ_u	4 or 1.01	Our estimates	capital supply elasticity
α_j, β_j		KLEMS	labor and capital shares
γ_j		Our estimates	returns to scale
π^f_{mnjt}		WIOD	final use trade shares
$\pi^x_{mi,njt}$		WIOD	intermediate use trade shares
ω_{nj}		WIOD	final consumption shares

Notes: This table summarizes the parameters and data targets used in the quantitative model, and their sources.

1. Two parameters ψ_e and ψ_h govern the elasticity of different margins of labor supply (hours and effort). As we lack evidence that the elasticity with respect to hours should differ from that for effort, we set them both to 4, implying the Frisch labor supply elasticity is 0.5 as advocated by Chetty et al. (2013). This value is conservative relative to the elasticity of 2 common in the business cycle literature. Raising the Frisch elasticity leads to greater transmission of shocks and higher GDP correlations in our model. We have less guidance to set the capital supply parameter ψ_u. Our TFP estimation procedure coupled with our choices of ψ_e and ψ_h provides an overidentification restriction for ψ_u, evident in (4.2). However, the range of values that satisfy this restriction is large, and includes values that imply very elastic and inelastic capital supply. We therefore choose a baseline value of 4, implying a relatively inelastic capital supply, but also assess the performance of the model for a value of 1.01 – a highly elastic capital supply.

All other parameters in the static model have close counterparts in basic data and thus we compute them directly. Capital shares in total output α_j come from KLEMS, and are averaged in each sector across countries and time. The scale parameters γ_j come from our own production function estimates reported in Appendix Table A3. We initialize both the static and dynamic models in the same steady state. Steady state input shares $\pi^x_{mi,nj}$ and final consumption shares π^f_{mnj} are computed from WIOD as time averages.

5.3 Impulse Responses

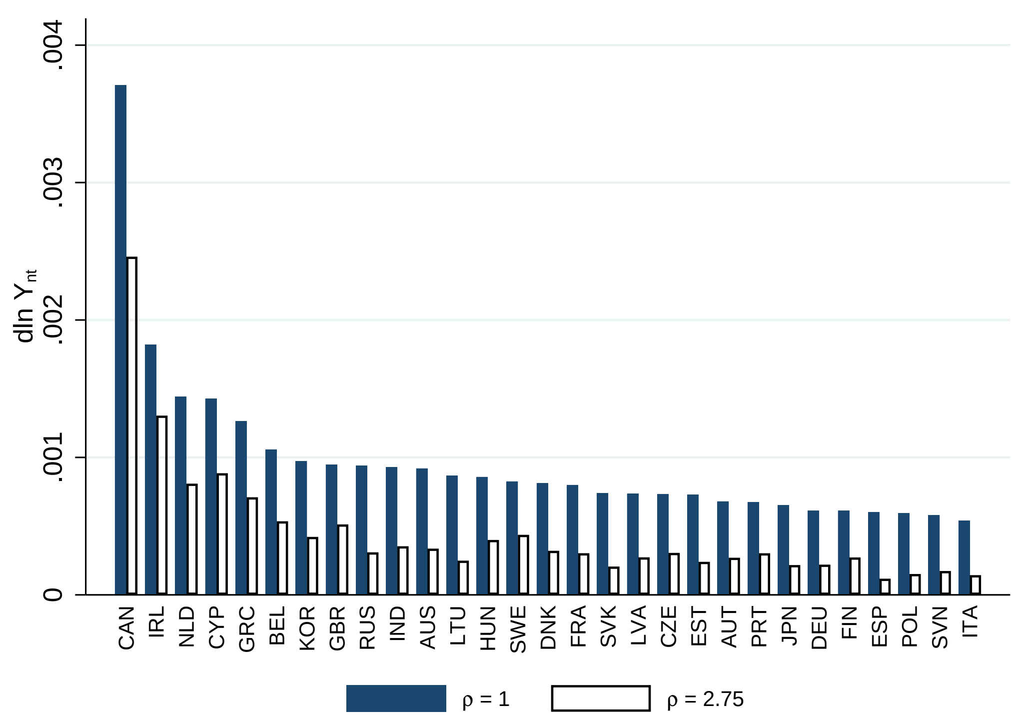
Prior to simulating the model with the observed shocks, we "test drive" the propagation mechanism by simulating some simple hypothetical shocks:

1. a 1% U.S. shock in all sectors,
2. a 1% rest-of-the-world shock in all sectors from the perspective of each country, and,
3. a symmetric shock in each sector in every country of the world.

The rest-of-the-world exercise assumes that the country in question is not shocked, but all other possible countries and sectors are, and thus has to be conducted country by country. In each exercise, we simulate a hypothetical technology shock. Examining the expression for the change in world output due to shocks in (3.16) reveals that up to a scaling parameter the technology and non-technology shocks do not have differential transmission properties in this model. Thus to conserve space we only report the impulse responses to TFP shocks.

Figure 2 displays the change in real GDP in every other country in the world following a 1% U.S. shock in each sector. The white bars depict the GDP responses under $\rho = 2.75$, while the dark bars depict the response under $\rho = 1$.

FIGURE 2: Impulse Responses to a US 1% Shock



Notes: This figure displays the change in log real GDP of every other country in the sample when the United States experiences a productivity shock of 0.01 in every sector.

The results show that the observed trade linkages do result in transmission. Smaller economies with large trade linkages to the U.S., such as Canada, are the most strongly affected by the U.S. shocks. Under the low elasticity, the mean response of foreign GDP is 0.21%, and the maximum response – Canada – is about 0.35%. On the other hand, the final substitution elasticity matters a great deal for the size of the effects: the response of foreign GDP to the US shocks is about twice as high for $\rho = 1$ than for $\rho = 2.75$.

Next, we simulate the real GDP responses of each country n in the sample when all other countries (excluding n) experience a 1% technology shock. The exercise answers the question, if there is a 1% world shock outside of the country, how much of that shock will manifest itself in the country's GDP? Figure 3 displays the results. In response to a 1% world TFP shock, under the low elasticity of substitution the mean country's GDP increases by 0.85%, with the impact ranging from around 0.3% in the U.S. and Japan to 1.1-1.4% in Latvia and Lithuania. Smaller countries are not surprisingly more affected by shocks in their trade partners. The magnitude of transmission is uniformly lower with the higher elasticity. In this case, the mean impact is about 0.4% for the 1% technology shock. All in all, these results suggest that world shocks have a significant impact on most countries.

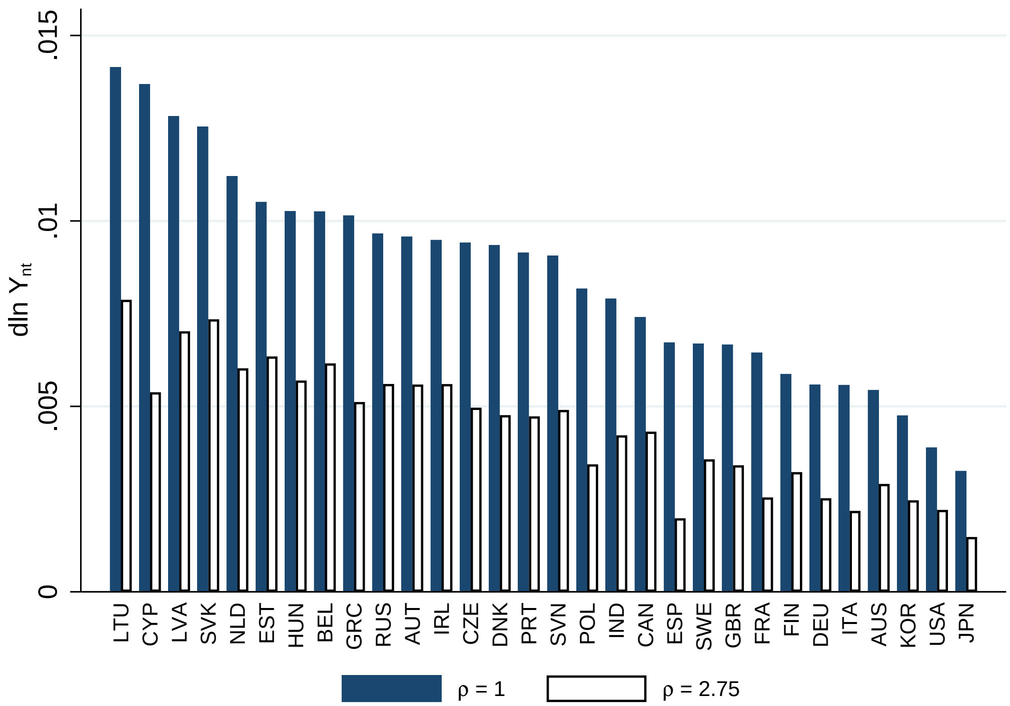
Figure 4 illustrates the results of our third impulse response exercise, a 1% productivity shock to every country and sector in the world, under $\rho = 2.75$. Here, we are most interested in the share of the total GDP change that comes from the shocks to the country's own productivity, and how much comes from foreign shocks. Thus, we use the linear approximation to a country's GDP growth (2.4), and separate the overall impact into the own term $\mathcal{D}_n$ and the rest. The figure highlights that for all countries, shocks to domestic sectors matter much more for GDP growth than foreign sector shocks. The mean and the median share of the foreign terms in the total GDP change is 11%. The impact is heterogeneous across countries, with the fraction of GDP change due to foreign impact ranging from 3 to 5% of the total for Japan, Spain, and the U.S., to nearly 17% of the total for Lithuania and Estonia.

5.4 GDP Correlations in the Model

We next simulate the full static model by feeding in the estimated shocks. Tables 4 and 5 report correlations in our model simulated with both technology and non-technology shocks, as well as counterfactual economies featuring only technology or non-technology shocks, under $\rho = 2.75$ and $\rho = 1$, respectively. Trade is balanced in every period.[17] The first two lines report the summary statistics for the real GDP correlations in the data and in the baseline model in which both shocks

[17] Appendix Table A15 reports the fit of the model and counterfactual exercises where deficits are allowed to evolve as in the data.

Figure 3: Impulse Responses to Rest of the World 1% Shocks



Notes: This figure displays the change in log real GDP of every country in the sample when the rest of the world excluding the country experiences a TFP shock of 0.01 in every sector.

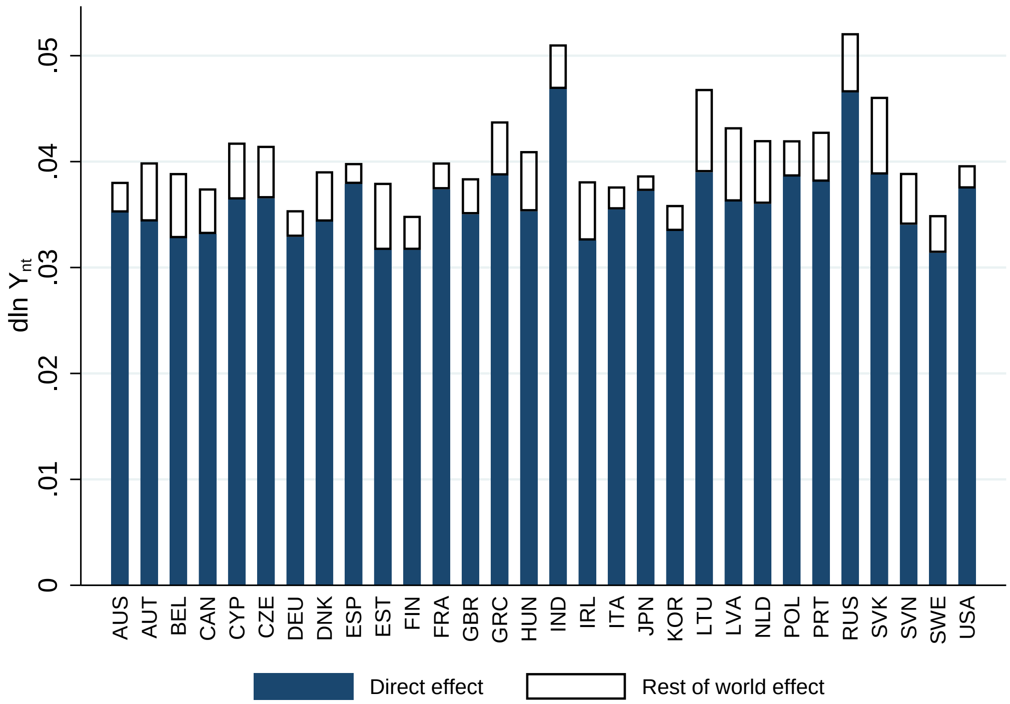
are as measured in the data. Our static model generates mean correlations that are about two-thirds of what is observed in the data, for both the G7 and the full sample.

Next, we simulate the model under only non-technology and only TFP shocks. It is immediately apparent that the non-technology shocks are responsible for much of the comovement in the model. For the G7 group, the model with only non-technology shocks generates 75% of the average correlations in the data, more than that implied by the model with both shocks. By contrast, the model with only technology shocks generates 28% of the comovement on average. The results for all countries are similar in terms of relative magnitudes, though even non-technology shocks account for less comovement: technology shocks generate a negligible fraction of the comovement of the full model at the mean, while the non-technology shocks generate about 15% of the comovement. These relative magnitudes are not sensitive to the two alternative values of ρ.[18] Appendix Table A10 reports results

[18] As we emphasized throughout, in the static framework technology and non-technology shocks can differ in relative importance only due to differences in their correlations. For the non-G7 countries, the non-technology shocks are less

FIGURE 4: Impulse Responses to a 1% Shock in Every Sector in Every Country

$d \ln Z_{mjt} = 0.01, \forall m, j$



Notes: This figure displays the change in log real GDP of every country in the sample, decomposed into a direct effect and a rest of world effect, when all sectors in every country experience a productivity shock of 0.01.

under a higher Frisch elasticity of 2. Predictably, the correlations generated by the model rise when the Frisch elasticity is higher, but the relative contributions of the two types of shocks do not change.

To assess the importance of correlated shocks relative to transmission in the model with the estimated shocks, we decompose bilateral correlations along the lines of equation (2.6), rewritten in correlations. That equation combined with the first-order solution to the model in (3.17) produces a breakdown of the overall comovement into additive shock correlation and transmission terms. Table 6 reports the results. For the G7 countries, the correlation of shocks is responsible for around two-thirds of the model correlations in the simulation with both shocks. Nonetheless, the bilateral and multilateral transmission terms have a non-negligible contribution to the overall correlation, accounting for the remaining one-third. The share accounted for by correlated shocks is similar in the full sample.

Note that the findings regarding the ability of TFP shocks to generate correlations, and the relative

correlated than for the G7 countries. So their smaller contribution to cross-country correlations is not surprising.

Table 4: Model Fit and Counterfactuals: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.236	0.363	-0.030	0.567
Non-Technology Shocks Only	0.299	0.348	0.122	0.453
Technology Shocks Only	0.111	0.163	-0.088	0.322
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.124	0.130	-0.104	0.412
Non-Technology Shocks Only	0.029	0.014	-0.188	0.272
Technology Shocks Only	0.005	0.023	-0.211	0.222

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks. Variable definitions and sources are described in detail in the text.

importance of transmission vs. correlated shocks do not hinge on our utilization-adjusted series. Appendix C simulates the model with Solow residuals instead of the utilization-adjusted TFP shocks, and shows that the main conclusions are unchanged if we use the Solow residuals instead of our estimates.

5.5 The Role of the Input Network

Another way to quantify the role of transmission in generating observed comovement is to compare the correlations in the baseline model to correlations that would obtain in an autarky counterfactual. As emphasized in Section 2, the difference between the autarky and trade influence vectors s_{nj}^{AUT} and s_{nnj} is the key input into this comparison. The assumptions put on the counterfactual autarky input-output matrix will determine s_{nj}^{AUT}. Theory does not offer clear guidance on the autarky input-output structure. We only observe the global input-output matrix under the current levels of trade costs, which in our analysis is taken as given in steady state. A natural way to construct autarky would be to set the trade costs to infinity. However, since the baseline production function is Cobb-Douglas in the factors and all materials inputs, sending trade costs to infinity would result in zero output in all country-sectors that source foreign intermediates directly or indirectly.

TABLE 5: Model Fit and Counterfactuals: Correlations of $d\ln Y_{nt}$, $\rho = 1$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.246	0.341	0.020	0.564
Non-Technology Shocks Only	0.322	0.369	0.148	0.469
Technology Shocks Only	0.126	0.169	-0.091	0.362
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.119	0.118	-0.128	0.402
Non-Technology Shocks Only	0.045	0.046	-0.186	0.294
Technology Shocks Only	0.016	0.019	-0.220	0.235

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks. Variable definitions and sources are described in detail in the text.

To better illustrate the role of various input linkages in comovement, we report results of 3 autarky counterfactuals. Appendix B.5 shows that these autarky counterfactuals can be thought of as limiting cases as trade costs go to infinity, and elasticities differ from 1 in different ways.

The first is a value added-only model: $\eta_j^{AUT1} = 1\ \forall j$. In this model, there are no input-output linkages, domestic or international.

The second is a model in which the domestic input coefficients are unchanged as a share of gross output, whereas the sum total of the observed foreign input coefficients is reapportioned to value added:

$$\pi^{x,AUT2}_{ni,nj} = \pi^{x}_{ni,nj} \tag{5.1}$$

$$\eta^{AUT2}_{nj} = \eta_j + \sum_{i;m\neq n} \pi^{x}_{mi,nj}. \tag{5.2}$$

In other words, the second autarky counterfactual assumes that in each sector and country, the intermediates that in the data are imported will be replaced by value added.[19] This counterfactual keeps the propagation of shocks through the domestic linkages unchanged.

[19]The input spending shares $\pi^x_{mi,nj,t}$ are not parameters when the aggregation is CES. However, the quantitative implementation uses a unitary elasticity, and thus the $\pi^x_{mi,nj,t}$ can be treated as parameters with no ambiguity.

TABLE 6: Correlated Shocks vs. Transmission Decomposition, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Baseline:	0.236	0.363	-0.030	0.567
Decomposition:				
Shock Correlation	0.158	0.269	-0.159	0.482
Bilateral Transmission	0.027	0.015	0.014	0.026
Multilateral Transmission	0.051	0.035	0.003	0.084
	All countries (N. obs. = 406)			
Baseline:	0.124	0.130	-0.104	0.412
Decomposition:				
Shock Correlation	0.078	0.083	-0.136	0.347
Bilateral Transmission	0.009	0.004	0.002	0.010
Multilateral Transmission	0.037	0.029	0.008	0.065

Notes: This table presents the decomposition of the GDP correlations generated by the model into the shock correlation, the direct transmission, and the multilateral transmission terms as in equation (2.6).

Finally, the third autarky counterfactual reassigns foreign input coefficients to the domestic inputs, while keeping the value added share of gross output the same as in the baseline:

$$\pi^{x,AUT3}_{ni,nj} = \pi^{x}_{ni,nj} + \sum_{m \neq n} \pi^{x}_{mi,nj} \tag{5.3}$$

$$\eta^{AUT3}_{j} = \eta_j. \tag{5.4}$$

As an example, suppose that the US Apparel sector spent 10 cents on US Textile inputs and 5 cents on Chinese Textile inputs per dollar of Apparel output, the remaining 85 cents being accounted for by value added. The second autarky counterfactual assumes that this sector continues to spend 10 cents on US Textile inputs, while its value added rises to 90 cents per dollar of output. The third autarky counterfactual assumes instead that value added continues to be 85 cents per dollar of gross output, but now the sector spends 15 cents on US Textile inputs. The third autarky counterfactual thus raises the domestic input coefficients for each sector by the amount of lost foreign input coefficients. As a result, it increases the scope for propagation of domestic shocks even as it

TABLE 7: Autarky Counterfactuals: Correlations of $d\ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
Data	0.358	0.337	0.242	0.565
Baseline	0.236	0.363	-0.030	0.567
Autarky Models:				
$AUT1$: VA Only	0.382	0.361	0.273	0.454
$AUT2$: Same Dom. Links	0.316	0.316	0.065	0.518
$AUT3$: Increased Dom. Links	0.195	0.307	-0.112	0.488
	All countries (N. obs. = 406)			
	Mean	Median	25th pctile	75th pctile
Data	0.190	0.231	-0.027	0.437
Baseline	0.124	0.130	-0.104	0.412
Autarky Models:				
$AUT1$: VA Only	0.017	0.028	-0.203	0.250
$AUT2$: Same Dom. Links	0.023	0.022	-0.230	0.257
$AUT3$: Increased Dom. Links	0.058	0.078	-0.189	0.328

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different assumptions on trade linkages. Variable definitions and sources are described in detail in the text.

rules out propagation of shocks from abroad. By construction, all autarky counterfactuals assume that there is no international input trade: $\pi^{x,AUT1}_{mi,nj} = \pi^{x,AUT2}_{mi,nj} = \pi^{x,AUT3}_{mi,nj} = 0\ \forall m \neq n$.

Using the input and final consumption shares implied by the three autarky counterfactuals, we can apply the first-order analytical influence vector from Section 3 to compute GDP growth rates in all countries and the resulting GDP correlations. The changes in GDP comovement between autarky and trade will depend on how these influence vectors differ across models, as emphasized by equation (2.7).

Tables 7-8 report the GDP correlations in the three autarky counterfactuals. The row labeled "VA Only" summarizes the correlations in the $AUT1$ model, with no domestic input linkages. In the G7 sample the autarky value-added-only model produces *higher* mean GDP correlations than the model with the full international input linkages. This model generates around 0.38 mean correlations in the G7 countries, compared to the 0.24 mean in the baseline with the high ρ. The lines labeled "Same

Table 8: Autarky Counterfactuals: Correlations of $d\ln Y_{nt}$, $\rho = 1$

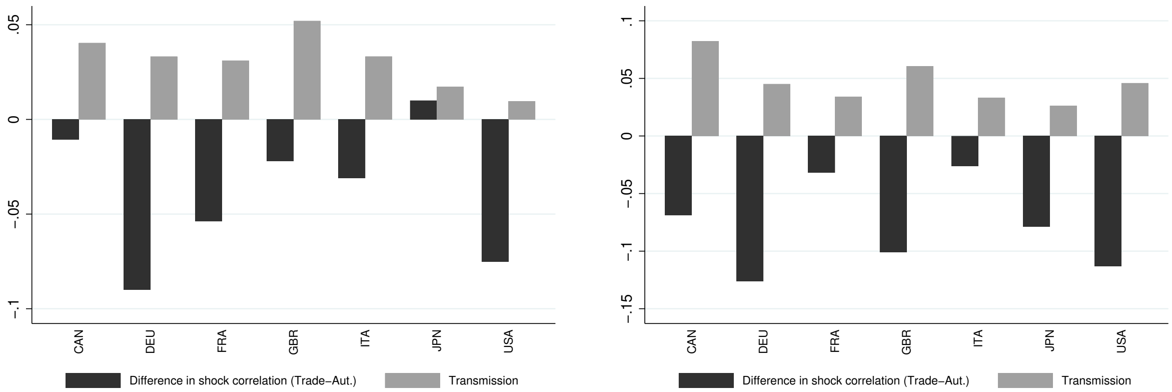
	Mean	Median	25th pctile	75th pctile
Data	0.358	0.337	0.242	0.565
Model	0.246	0.341	0.020	0.564
Autarky Models:				
$AUT1$: VA Only	0.320	0.328	0.281	0.436
$AUT2$: Same Dom. Links	0.228	0.244	0.043	0.470
$AUT3$: Increased Dom. Links	0.080	0.199	-0.301	0.343
		All countries (N. obs. = 406)		
	Mean	Median	25th pctile	75th pctile
Data	0.190	0.231	-0.027	0.437
Model	0.119	0.118	-0.128	0.402
Autarky Models:				
$AUT1$: VA Only	0.038	0.063	-0.190	0.269
$AUT2$: Same Dom. Links	0.037	0.029	-0.220	0.302
$AUT3$: Increased Dom. Links	0.036	0.036	-0.216	0.324

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different assumption on trade linkages. Variable definitions and sources are described in detail in the text.

Dom. Links" report the correlations under the $AUT2$ autarky counterfactuals. These correlations fall relative to the $AUT1$ scenario, but do not fall all the way to the baseline means for the G7. Finally, the $AUT3$ counterfactuals are reported under "Increased Dom. Links." This scenario generates averages that are lower than in the baseline with trade in the G7. This pattern holds for both the high and low ρ. Outside of the G7 sample, the comparison of the autarky correlations does not reveal a clear ranking, and the autarky correlations are all lower than the correlation under trade.

Equation (2.7) helps understand these results. The change in GDP comovement between autarky and trade is actually a sum of two terms: the re-weighting of sectors towards or away from those with more correlated shocks (ΔShock Correlation$_{mn}$), and the international transmission terms. The simulated impulse responses above suggest that the international transmission should generally be positive. Thus, to observe the lower average correlation under trade, it must be that the change in the shock correlation term is negative. Figure 5 illustrates this by plotting the average ΔShock Correlation$_{mn}$ and the transmission terms for the G7. There is non-negligible positive transmission of shocks in the model with trade, but it is more than offset by the negative ΔShock Correlation$_{mn}$ terms.

Figure 5: Decomposing Changes in Correlations: Trade vs. Autarky



TFP shocks $\qquad$ ξ shocks

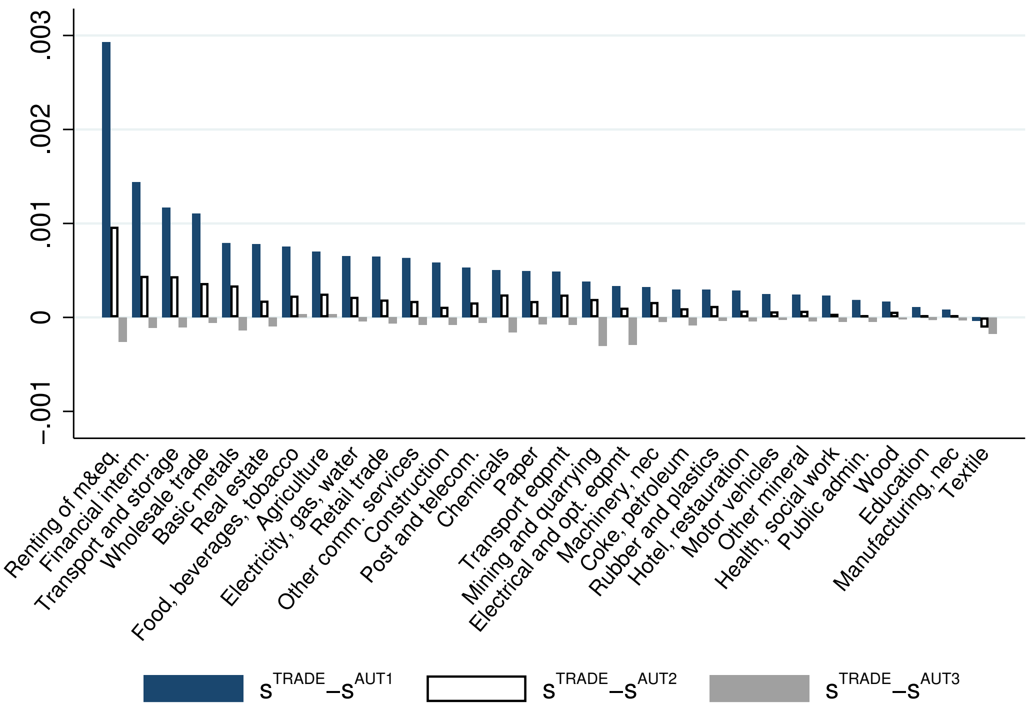
Notes: This figure illustrates how the pattern of correlations changes between the full model and the autarky value added only model for the G7. The gray bars illustrate the average bilateral and multilateral transmission terms and the dark bars illustrate the decreasing direct correlation effect.

The ΔShock Correlation$_{mn}$ term will be negative when the sectors with less correlated primitive shocks have a higher influence in the trade equilibrium than in autarky. Figure 6 plots the average changes in the domestic elements influence vectors s_{nnj} in the G7 sample, by sector. The figure reveals which sectors receive a higher influence in the full baseline model, compared to each of the autarky models. It is clear that the largest changes are for the non-tradeable sectors (Machinery and Equipment Rentals and Other Business Services; and Real Estate Activities). These sectors have a much larger influence in the trade equilibrium compared to the value-added only model ($AUT1$). By contrast, the influence vectors change much less between the trade model and the autarky model with increased domestic linkages ($AUT3$). The intermediate model ($AUT2$) is in-between those two extremes.

The reason that these services sectors have a much higher influence in the model with IO linkages relative to the value added-only model is that these sectors are important input suppliers to other sectors. The left panel of Figure 7 reports the scatterplot of the change in the influence of a sector against the intensity with which other sectors use it as inputs in the data. The correlation between the two is 0.75: sectors used as inputs experience an increase in influence as we move from a value added-only model to the full IO model.

At the same time, shocks in these sectors are on average less correlated with the foreign shocks. The right panel in Figure 7 presents the local polynomial fit between the two elements the ΔShock Correlation$_{mn}$: the shift in the combined influence of each sector pair $s_{nnj}s_{mmi} - s_{nj}^{AUT}s_{mi}^{AUT}$ and the correlation

Figure 6: Average Changes in the Influence Vectors: Trade vs. Autarky, $\rho = 2.75$



Notes: This figure displays the average change in the direct influence vectors between the baseline model and each of the autarky models.

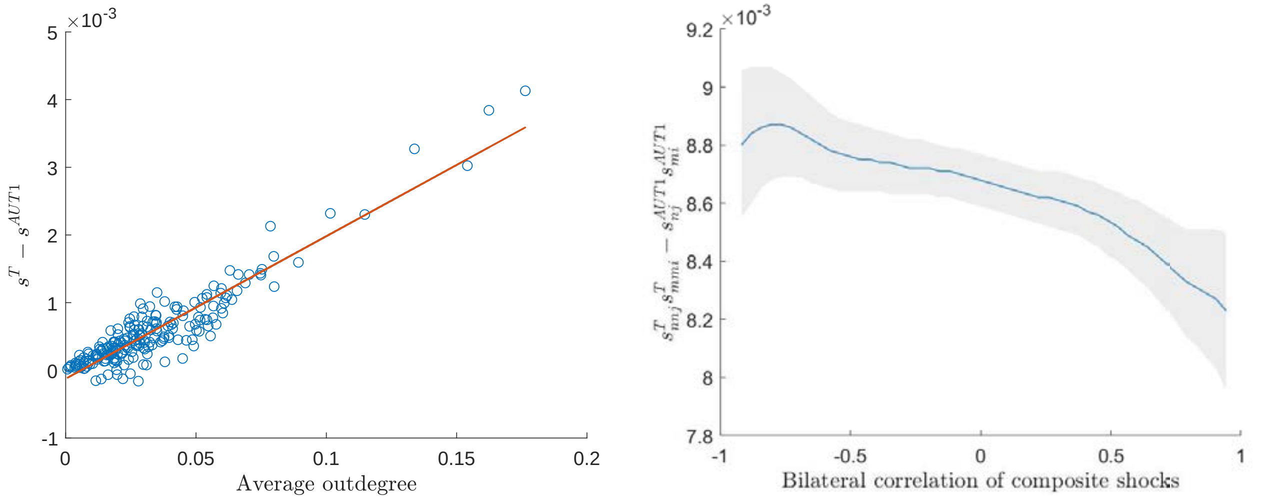
between the combined shocks in that pair, along with a 95% confidence intervals. The negative relationship is evident, as expected.

6 Dynamic Responses

The preceding section explored an environment in which machines M_{njt} and employment N_{njt} are kept constant. In that setting, the model is an international extension of the canonical static network propagation model. We could solve analytically for the global influence matrix, and study how output across countries and sectors responds to contemporaneous shocks. By construction, past shocks had no effect on current output correlations. In this section, we allow households to adjust machines and employment endogenously as in Section 3. Consequently, a shock to sector j in country n can have persistent effects on other countries and sectors, and the properties of output correlations also depend on the dynamic propagation of shocks over time and across regions.

To examine the dynamic responses of the model and how it affects the output correlation, we proceed

Figure 7: Changes in the Influence Vectors, Intensity of Use as an Input, and Shock Correlation



Intensity of Use as Input

Change in Sectoral Influence vs. Shock Correlation

Notes: This figure displays the change in the influence vectors against the intensity with which a sector is used as an intermediate input (left panel), and the correlation of shocks between pairs of home and foreign sectors with their change in influence from value-added only autarky to trade (right panel). The line through the data in the left panel is the OLS fit. The right panel displays the fit from a local polynomial regression, with 95% confidence intervals.

by solving the log-linearized model. In the linearized model, the taste parameters ϑ_{mnj} and $\mu_{mi,nj}$ and the trade cost τ_{mni} affect the dynamics only via the the final use and the intermediate use trade shares. Once we match the trade shares as in the data, there is no need to pin down the trade costs and taste parameters separately. The dynamic model requires a small set of additional parameters relative to the static model. We adopt values standard in the business cycle literature. The model period is a year; we set the discount rate to $\beta = 0.96$. The period utility is $\Psi(\cdot) = \log(\cdot)$, and the depreciation rate is $\delta_j = 0.10$. We set $\psi_n = \psi_h = 4$ as in the baseline specification, and vary the value of ψ_n. For the elasticity of substitution, we employ the baseline specification as in the static model, that is, $\rho = 2.75$ and $\varepsilon = 1$.

The most demanding task in the calibration is to choose shock processes for different countries and sectors, as these shocks are correlated with each other. The perceived shock processes matter for the intertemporal decisions of households. We estimate shock processes from the identified shocks recovered above. For non-G7 countries, the panel is too short to obtain reliable estimates of the shock processes. Therefore in this section we narrow the focus to the G7 countries. We assume that the country-sector technology and non-technology shocks follow a vector autoregressive process. However, due to the large number of countries and sectors, it is not feasible to estimate the fully unrestricted VAR. Thus, we impose a parsimonious structure on the shock process. Log technology and non-technology shocks are assumed to follow:

$$\ln Z_{njt} = \rho^{z}_{nj} \ln Z_{njt-1} + \zeta^{z}_{n} \mathbf{1} \left(m = n, k \neq j\right) \ln Z_{mkt-1} + \theta^{z}_{njt}, \tag{6.1}$$

$$\ln \xi_{njt} = \rho^{\xi}_{nj} \ln \ \xi_{njt-1} + \zeta^{\xi}_{n} \mathbf{1} \left(m = n, k \neq j\right) \ln \ \xi_{mkt-1} + \theta^{\xi}_{njt}. \tag{6.2}$$

That is, each sector's shock depends on its own lagged value, and lagged spillover terms from other sectors in its country as well as its own innovation.[20] We also allow the vector of innovations across countries and sectors and between the two types of shocks to be correlated, $\boldsymbol{\theta}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}\right)$, that is, there is a full covariance matrix. The residuals in the estimating equations (6.1) and (6.2) are used to construct this covariance matrix. Further details on the estimation and the results are in Appendix A.3.

Table 9: GDP Growth Correlations in the Dynamic Model

	Mean	Median	25th pctile	75th pctile
Data	0.358	0.337	0.242	0.565
			$\psi_n = 4$	
Model with both shocks	0.250	0.438	-0.057	0.526
Technology-shock only	0.088	0.144	-0.154	0.292
Non-technology-shock only	0.320	0.353	0.133	0.430
			$\psi_n = 2$	
Model with both shocks	0.288	0.434	-0.048	0.535
Technology-shock only	0.083	0.161	-0.177	0.280
Non-technology-shock only	0.347	0.353	0.228	0.460
			$\psi_n = 20$	
Model with both shocks	0.231	0.420	-0.054	0.522
Technology-shock only	0.091	0.133	-0.134	0.303
Non-technology-shock only	0.295	0.331	0.092	0.415

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries in various calibrations of the dynamic model and under the different shocks. Variable definitions and sources are described in detail in the text.

[20]We also experimented with including within-sector spillover terms and dependence on other past variables, but it turns out that most of these terms are not significant.

Table 9 displays the results of the dynamic models. In the baseline specification with $\psi_n = 4$, the output growth correlations are similar to the static model in Table 4. When $\psi_n = 20$, employment moves much less, and capital is the main input factor responsible for dynamic transmission. When $\psi_n = 2$, the employment is much more responsive. Though the correlations become larger when the employment is more responsive, they remain similar to the static model overall.

The results in Table 9 shows that the dynamic model does not substantially increase the GDP growth correlations relative to the static model. Recall from Section 2 that the GDP growth rate can be expressed as a function of current and past shocks via the global influence vector

$$d\ln Y_{nt} \approx \sum_{k=0}^{\infty}\sum_{m}\sum_{i} s_{mni,k}\theta_{mi,t-k}.$$

To assess the importance of the dynamic propagation of shocks in determining the comovements, we implement the following correlation decomposition:

$$\varrho(d\ln Y_{nt}, d\ln Y_{mt}) = \sum_{k=0}^{\infty}\omega_{nm,k}\ \varrho_{nm,k}, \tag{6.3}$$

where $\varrho_{nm,k}$ is the correlation between components $\boldsymbol{s}_{n,k}\boldsymbol{\theta}_{t-k}$ and $\boldsymbol{s}_{m,k}\boldsymbol{\theta}_{t-k}$ and $\omega_{nm,k}$ is its corresponding weight

$$\varrho_{nm,k} = \frac{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}{\sqrt{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,k}}\sqrt{\boldsymbol{s}_{m,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}} \qquad \text{with} \qquad \omega_{nm,k} = \frac{\sqrt{\boldsymbol{s}_{n,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,k}}\sqrt{\boldsymbol{s}_{m,k}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,k}}}{\sqrt{\sum_{i=0}^{\infty}\boldsymbol{s}_{n,i}\boldsymbol{\Sigma}\boldsymbol{s}'_{n,i}}\sqrt{\sum_{i=0}^{\infty}\boldsymbol{s}_{m,i}\boldsymbol{\Sigma}\boldsymbol{s}'_{m,i}}}.$$

It turns out that while the correlations across k are quite different from each other, the $k = 0$ component is dominant. Figure 8 shows the average $\varrho_{nm,k}\omega_{nm,0}$ across country pairs. The $k = 0$ component, which is the impact effect, accounts for over 80% of total correlation, which explains why adding dynamics does not significantly raise the GDP correlations. This is mainly due to the fact that the response to a country's own shock on impact dominates the response to other countries' shocks. To illustrate this pattern, Figure 9 compares the response of US output to its own shock and that to shocks to the rest of the world. Both the shape and the magnitude of the responses are different. The response to its own shock is high on impact, and gradually dies out. In contrast, the response to other countries' shocks peaks after over twenty periods, but at most horizons the magnitude of the US response to its own shock is much larger.

As hinted above, even though the dynamic model only modestly increases output growth correlation, it does not imply that there is no endogenous propagation over time. We now revisit the impulse response exercise in Section 5.3. The left panel of Figure 10 displays the response of other countries

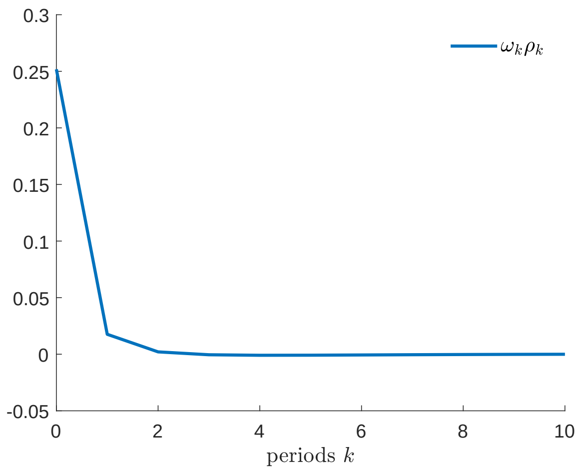


FIGURE 8: Correlation Decomposition

Notes: This figure displays the elements of the dynamic composition of the overall correlation into the components accounted for by elements at horizon k, as in (6.3).

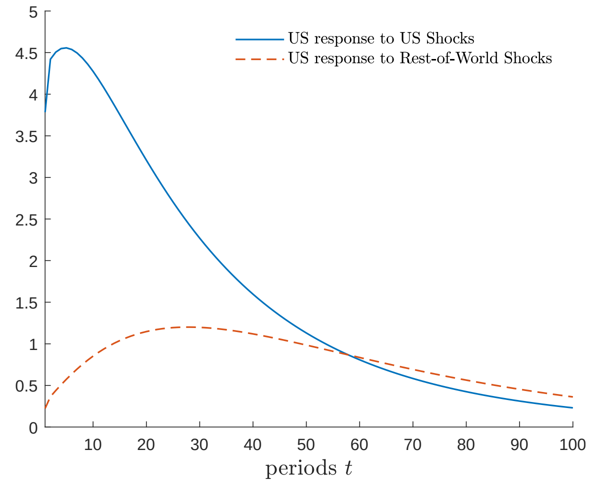


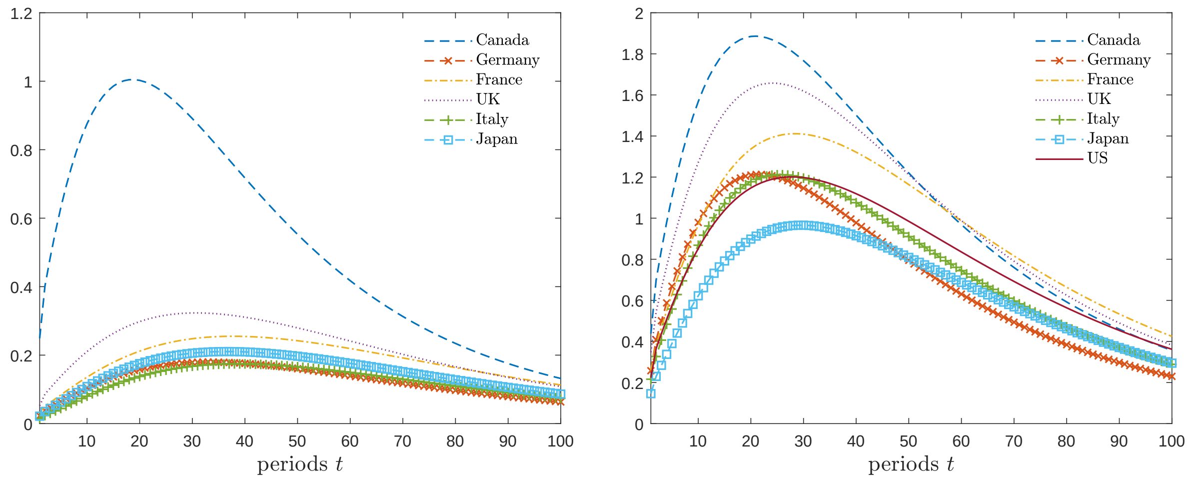
FIGURE 9: IRF of US to Technology Shock

Notes: This figure displays the impulse responses of US TFP following a 1% US TFP shock, and following a 1% TFP shock in the rest of the world excluding the US.

to a 1% US productivity shock. Similar to the static model, Canada experiences the largest response as it has the largest trade linkages to the US. Note that the responses of all the countries are quite persistent over time. The hump-shaped IRF indicates that there is nontrivial endogenous propagation. Meanwhile, all the countries co-move quite closely in response to the US shock. The right panel of Figure 10 displays the response to a hypothetical rest-of-the-world shock in all sectors from the perspective of each country. Again, we observe significant persistence. Similar to the static model, the responses are heterogeneous across countries. Japan, US, and Italy respond little, while Canada and UK are quite sensitive to foreign global shocks.

We conclude this section by looking at the decomposition of the growth correlation into shock correlation, bilateral transmission, and multilateral transmission terms in the dynamic model. Note that in the static model, we can use the unconditional covariance matrix of the shocks to compute the decomposition, which is no longer possible in the dynamic model as the impact of the shocks are correlated over time. We proceed by computing the theoretical decomposition based on the estimated processes in (6.1) and (6.2). Table 10 shows that around 70% of the overall comovement is due to the shock correlation, and 30% is due to transmission. These proportions are quite similar to those found the static model.

Figure 10: Impulse Responses to US and Rest of the World Shocks



Response to a US Technology Shock

Response to a Rest-of-the-World Technology Shock

Notes: This figure displays the impulse responses of log real GDP of each G7 country following a 1% US TFP shock (left panel), and following a 1% TFP shock in the rest of the world excluding the country.

Table 10: GDP Growth Correlations Decomposition, Dynamic Model

	Mean	Median	25th pctile	75th pctile
Total Correlation	0.272	0.104	-0.125	0.749
Decomposition				
Shock Correlation	0.192	0.051	-0.243	0.612
Bilateral Transmission	0.021	0.009	0.003	0.026
Multilateral Transmission	0.059	0.072	-0.002	0.103

Notes: This table presents the decomposition of the GDP correlations generated by the model into the shock correlation, the direct transmission, and the multilateral transmission terms in the dynamic model.

7 Conclusion

We set out to provide a comprehensive account of international comovement in real GDP. Using a simple accounting framework, we decomposed the GDP covariance into additive components representing correlated shocks and cross-border transmission. The relative importance of these two terms is determined jointly by the correlations of the primitive shocks and the strength of domestic and international input-output linkages. The accounting framework also clarifies the role of dynamic propagation: the total GDP covariance is the sum of the covariance due to the instantaneous responses to shock innovations, and dynamic terms that capture the lagged responses to shocks.

We measured two types of shocks in the data for a large sample of countries and sectors: utilization-adjusted TFP and a reduced-form non-technology shock that manifests itself as a sector-specific shift in factor supply. We then used the data on the world input-output linkages to discipline sectoral influence and cross-border transmission.

Our main findings are fourfold. First, non-technology shocks contribute more to international comovement than TFP shocks, because they are positively correlated, whereas TFP shocks are on average uncorrelated across countries. Second, most of the observed GDP comovement is accounted for by correlated shocks. Transmission of shocks has an economically meaningful, but smaller role in comovement. Third, autarky correlations can in some cases be higher than trade correlations. This happens when trade opening increases the domestic influence of sectors whose primitive shocks are relatively less correlated. And finally, the bulk of the observed overall correlation is due to the instantaneous response of the economy to shocks, rather than dynamic propagation of past shocks.

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Appendix A Estimation

A.1 TFP Estimation

Table A1 lists the countries and Table A2 the sectors in our sample. We require instruments orthogonal to the TFP shocks in our panel that have predictive power for movements in inputs. BFK use a monetary policy shock identified in a VAR, an oil price shock and the growth in real defense spending. We use instruments similar in spirit: the lagged growth in real defense spending in each country, an oil price shock constructed using the approach in Hamilton (1994) and a version of a monetary policy shock that relies on the exogenous movements in base-country interest rates affecting countries that are pegged to a base country. This last instrument cannot be used for large countries like the US, UK, or Germany.

Table A1: Country Sample

Australia	Germany	Netherlands
Austria	Greece	Poland
Belgium	Hungary	Portugal
Canada	India	Russian Federation
Cyprus	Ireland	Slovak Republic
Czech Republic	Italy	Slovenia
Denmark	Japan	Spain
Estonia	Republic of Korea	Sweden
Finland	Latvia	UK
France	Lithuania	USA

Baseline Estimates: Our baseline estimates for TFP rely only on the G7 sample of countries, as these estimates are the least noisy. For these countries, the modified monetary policy (exchange-rate based) instrument cannot be used. We therefore rely on the oil shock and defense spending instruments. Table A3 reports the full results. Notice that we have multiple instruments and multiple endogenous variables in our estimation. The standard first-stage F statistic diagnostic therefore does not capture whether we suffer from a weak instruments problem. To assess whether we suffer from this issue we calculate the Sanderson-Windmeijer F statistic, which explicitly tests for the presence of weak instruments in the context of multiple instruments and endogenous variables. The SW-F statistic is 7.97, suggesting that the instruments are not weak. In 2 sectors, Mining and Quarrying and Food, Beverages and Tobacco returns to scale coefficient point estimates are negative. We drop those sectors from the estimation sample, and set their returns to scale coefficient to 1 in the quantitative model. The utilization adjustment coefficient ($\widehat{\delta_j^2}$) for those sectors is set equal to the utilization adjustment coefficient estimated for the group of sectors to which they belong, non-durable manufacturing for Food, Beverages and Tobacco, and non-manufacturing for Mining and Quarrying.

Table A2: Sector Sample

agriculture hunting forestry and fishing	basic metals and fabricated metal	financial intermediation
mining and quarrying	machinery nec	real estate activities
food beverages and tobacco	electrical and optical equipment	renting of m&eq and other business activities
textiles textile leather and footwear	transport equipment	public admin and defense; compulsory social security
wood and of wood and cork	manufacturing nec; recycling	education
pulp paper paper printing and publishing	electricity gas and water supply	health and social work
coke refined petroleum and nuclear fuel	construction	other community social and personal services
chemicals and chemical products	hotels and restaurants	sale maintenance and repair of motor vehicles
rubber and plastics	transport and storage	wholesale trade and commission trade
other nonmetallic mineral	post and telecommunications	retail trade except of motor vehicles

Comparison to BFK's Estimates and Sensitivity: While the point estimates of both the returns to scale for our sectors and the coefficients on the utilization adjustment term naturally differ from those in BFK, they are not significantly different from the estimates in that paper in many cases. For instance, we estimate coefficients on the utilization adjustment term of 1.419(0.389), 2.939(1.767) and 0.245(0.649) for durables, non-durables and non-manufacturing respectively. The comparable estimates in BFK Table 1 are 1.34(0.22), 2.13(0.38) and 0.64(0.34) respectively.

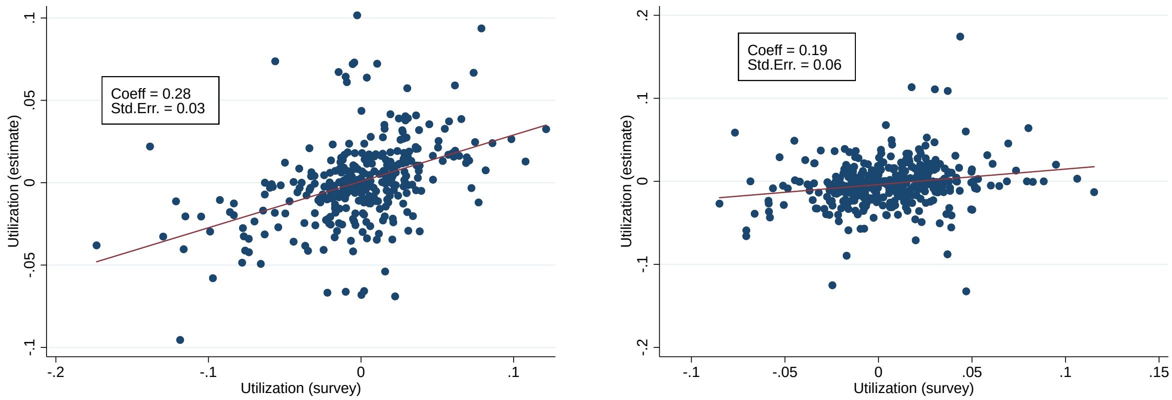
We construct TFP series directly using the coefficient estimates in BFK (applied to all countries), and correlate that series to our TFP series. Table A4 reports the results. The TFP series based on the BFK coefficients have an 86% correlation with ours. To assuage concerns that for some countries these instruments might individually be weak, we estimate the coefficients excluding each of the G7 countries one after another, and construct TFP series with those alternative coefficients. Table A4 presents the pairwise correlations between our baseline TFP series, and all TFP series dropping an individual country. With the partial exception of dropping Canada, excluding individual G7 countries produces TFP series quite correlated with our baseline. All in all, the TFP series are highly correlated across all approaches, suggesting our estimates are not driven by any country in particular.

Finally, we estimate the production function parameters using the full 30-country sample. In this sample, we introduce a third instrument, which is the foreign monetary policy shock interacted with the exchange rate regime. This instrument follows di Giovanni and Shambaugh (2008) and di Giovanni, McCrary, and von Wachter (2009), who show that major country interest rates have a significant effect on countries' output when they peg their currency to that major country. The assumption in specifications that use this instrument is that for many countries, interest rates in the US, Germany, or the UK are exogenous. We exclude the "base interest rate" countries themselves (the US, Germany, and the UK) from the sample. The exchange rate regime classification along with information on the base country comes from Shambaugh (2004), updated in 2015. Finally, base country interest rates are proxied by the Money Market interest rates in these economies, and obtained from the IMF International Financial Statistics. Table A4 correlates the resulting TFP estimates to with our baseline. This alternative TFP series is positively correlated with the baseline, with a coefficient of 0.6.

Comparing Estimates of Utilization with Other Available Measures: Our TFP estimation process also provides us with series for utilization rates by sector. In the U.S., the Federal Reserve Board (FRB) also publishes series of industry-level utilization for manufacturing industries only. These series are constructed by dividing an index of industrial production by an index of estimated industrial capacity. The FRB series are constructed using a number of sources including survey data from the U.S. Census Bureau. The FRB cautions that these series should not be compared across industries (in contrast to our estimates). See Boehm and Pandalai-Nayar (2019) for a discussion.

The left panel of Figure A1 compares our industry-level estimates to these public series. The two are positively correlated, despite the different underlying data sources and methodology

Figure A1: Comparison between Estimated Utilization and Survey Data



U.S. sectoral data

Country average

Notes: This figure compares our estimated utilization growth rate and the change in the survey measure of utilization of capacity. The left panel plots growth rates of the sector-level utilization series for the US based on our procedure against the FRB utilization survey. The right panel plots the growth rate of the country-level average utilization rate based on our procedure against utilization growth rates based on surveys by the FRB for the US and Eurostat for European countries. Both plots include the OLS fit, and report the coefficient point estimate and the standard error.

used for constructing them. The right panel of the figure compares our estimates for the country-level average utilization growth rate against the country-level utilization based on the FRB data for the US, and Eurostat data for some European countries. Again, we find a positive correlation.

Properties of the TFP series: Figure A2 contrasts the Solow residual with the utilization-adjusted TFP series for all the countries in our sample. While we do find that the utilization-adjusted TFP series is less volatile than the Solow residual for the U.S., as in BFK, for the large majority of other countries the adjusted TFP series is more volatile. In fact, the mean (median) variance of the TFP series is .0006 (0.0005), while for the Solow residual it is 0.0008 (0.0004).

TABLE A3: Production Function Estimation Results

Industry	Returns to Scale ($\widehat{\delta}_j^1$)	Utilization ($\widehat{\delta}_j^2$)	Industry	Returns to Scale ($\widehat{\delta}_j^1$)	Utilization ($\widehat{\delta}_j^2$)
Durables			**Non-durable non-manufacturing**		
wood and of wood and cork	0.750***		sale maintenance and repair of motor	1.652***	
	(0.133)		vehicles and motorcycles; retail sale of fuel	(0.316)	
basic metals and fabricated metal	0.701**		wholesale trade and commission trade	1.471***	
	(0.329)		except of motor vehicles and motorcycles	(0.167)	
machinery nec	0.791***		retail trade except of motor vehicles	0.870*	
	(0.241)	1.420***	and motorcycles; repair of household goods	(0.454)	
electrical and optical equipment	0.711**	(0.389)	transport and storage	1.079***	
	(0.300)			(0.163)	
transport equipment	0.843***		post and telecommunications	0.633***	
	(0.207)			(0.150)	
manufacturing nec; recycling	0.895***		real estate activities	0.451	
	(0.129)			(0.329)	
			renting of m&eq and other business activities	1.225***	
				(0.233)	0.260
			agriculture hunting forestry and fishing	1.714*	(0.643)
				(0.892)	
Non-durable manufacturing			electricity gas and water supply	1.825	
				(1.387)	
textiles textile leather and footwear	0.291		construction	1.041***	
	(0.523)			(0.225)	
pulp paper paper printing and publishing	0.507		hotels and restaurants	1.267***	
	(0.431)			(0.429)	
coke refined petroleum and nuclear fuel	0.832	2.929*	financial intermediation	1.335***	
	(1.022)	(1.771)		(0.368)	
chemicals and chemical products	0.808*		public admin and defense;	1.863	
	(0.427)		compulsory social security	(1.504)	
rubber and plastics	0.926***		education	0.674***	
	(0.279)			(0.246)	
other nonmetallic mineral	0.698		health and social work	1.374	
	(0.487)			(2.173)	
			other community social and personal services	0.752***	
				(0.198)	

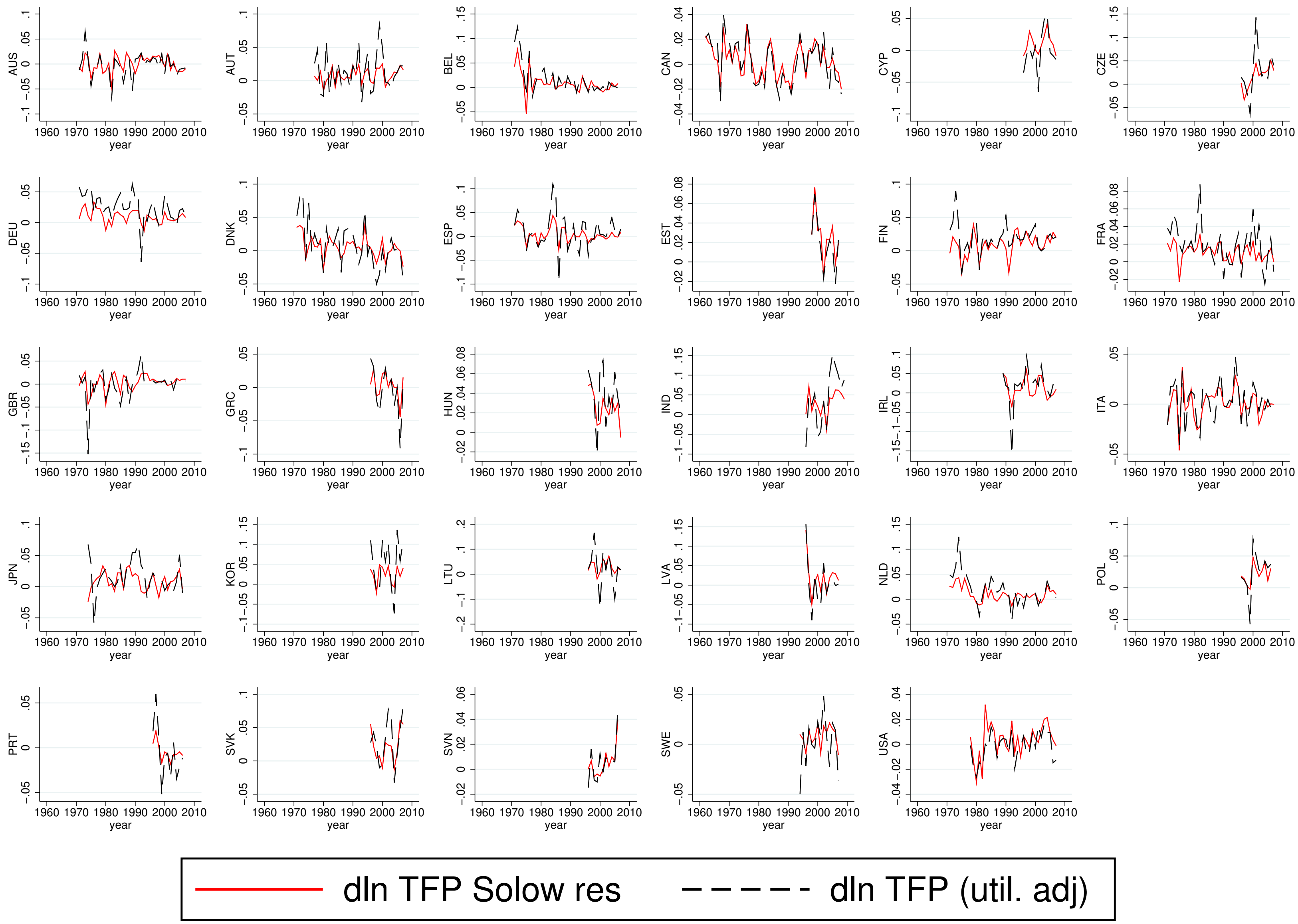
Notes: This table contains the results from the production function estimation described in Section 4. Standard errors in parentheses. Significance levels are indicated by *** p<0.01, ** p<0.05, * p<0.1. SW-F statistic for the first stage is 7.97.

Table A4: Correlation between Alternative TFP Estimates

	Baseline	BFK coefficients	ex-USA	ex-UK	ex-Canada	ex-Germany	ex-France	ex-Italy	ex-Japan	30-country estimation
Baseline	1.000									
BFK coefficients	0.861	1.000								
ex-USA	0.956	0.781	1.000							
ex-UK	0.889	0.751	0.788	1.000						
ex-Canada	0.472	0.415	0.307	0.591	1.000					
ex-Germany	0.965	0.869	0.914	0.843	0.579	1.000				
ex-France	0.951	0.820	0.969	0.738	0.248	0.910	1.000			
ex-Italy	0.878	0.793	0.798	0.865	0.415	0.814	0.796	1.000		
ex-Japan	0.794	0.728	0.692	0.676	0.572	0.813	0.687	0.645	1.000	
30-country est.	0.595	0.511	0.586	0.514	0.404	0.666	0.513	0.520	0.570	1.000

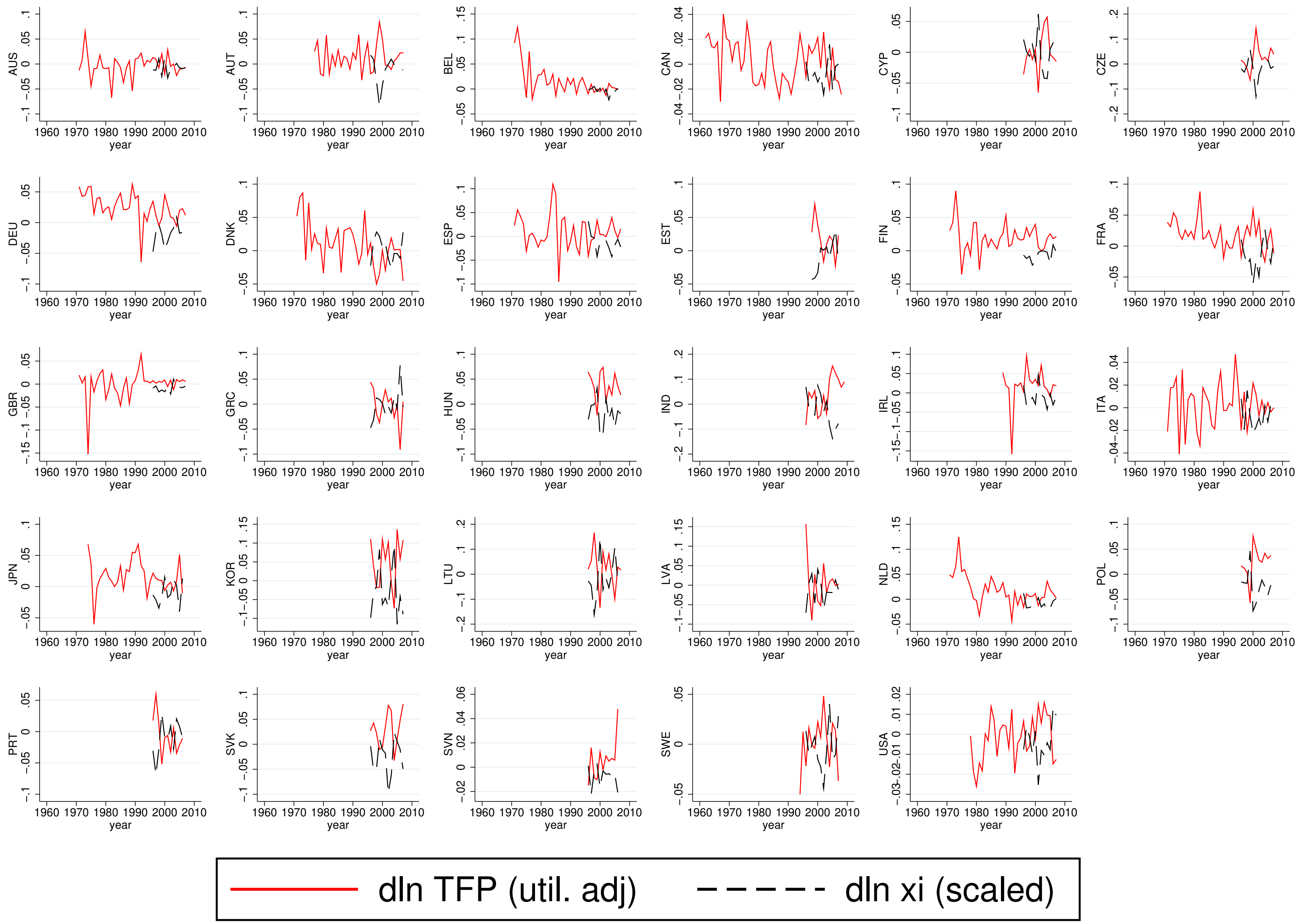
Notes: This table report the correlations of the estimated TFP series using a number of different approaches."BFK estimate" refers to TFP series for all countries using the coefficient estimates in Basu, Fernald, and Kimball (2006) and "ex-COUNTRY" refers to TFP series using the production function coefficient estimates from a sample that, excludes the G7 country in question. "30-country estimation" refers to the TFP series using the production function estimation based on 30 countries.

Figure A2: Comparison between Utilization-Adjusted TFP and the Solow Residual



Notes: This figure displays the log changes in the Solow residual and in the utilization-adjusted TFP series for every country in our sample.

Figure A3: Comparison between Utilization-Adjusted TFP and the Non-Technology Shocks



Notes: This figure displays log changes in the utilization-adjusted TFP and in the recovered non-technology shocks series for every country in our sample.

A.2 Estimating Model Elasticities

We use model-implied relationships to estimate ρ and ε. Denote by a "hat" the gross proportional change in any variable $\widehat{x}_t \equiv x_t/x_{t-1}$. To introduce an error term in the estimating equations, assume that iceberg trade costs, final consumer taste shocks, and input share shocks have a stochastic element, and denote their gross proportional changes by $\widehat{\tau}_{mnjt}$, $\widehat{\vartheta}_{mnjt}$, and $\widehat{\mu}_{mj,nit}$, respectively. Straightforward manipulation of CES consumption shares yields the following relationships between shares and prices:

$$\ln\left(\frac{\widehat{\pi}^f_{mnjt}}{\widehat{\pi}^f_{m'njt}}\right) = (1-\rho)\ln\left(\frac{\widehat{P}_{mjt}}{\widehat{P}_{m'jt}}\right) + \ln\left(\frac{\widehat{\vartheta}_{mnjt}\widehat{\tau}^{1-\rho}_{mnjt}}{\widehat{\vartheta}_{m'njt}\widehat{\tau}^{1-\rho}_{m'njt}}\right) \tag{A.1}$$

and

$$\ln\left(\frac{\widehat{\pi}^x_{mj,nit}}{\widehat{\pi}^x_{m'j,nit}}\right) = (1-\varepsilon)\ln\left(\frac{\widehat{P}_{mjt}}{\widehat{P}_{m'jt}}\right) + \ln\left(\frac{\widehat{\mu}_{mj,ni,t}\widehat{\tau}^{1-\varepsilon}_{mnjt}}{\widehat{\mu}_{m'j,ni,t}\widehat{\tau}^{1-\varepsilon}_{m'njt}}\right). \tag{A.2}$$

We express the final consumption share change $\widehat{\pi}^f_{mnjt}$ relative to the final consumption share change in a reference country m'. This reference country is chosen separately for each importing country-sector n, j as the country with the largest average expenditure share in that country-sector. (Thus, strictly speaking, the identity of the reference country m' is distinct for each importing country-sector, but we suppress the dependence of m' on n, j to streamline notation.) Furthermore, we drop the own expenditure shares $\widehat{\pi}^f_{nnjt}$ from the estimation sample, as those are computed as residuals in WIOD, whereas import shares from other countries are taken directly from the international trade data. Dropping the own expenditure shares has the added benefit of making the regressions less endogenous, as the domestic taste shocks are much more likely to affect domestic prices.

We use two estimation approaches for (A.1)-(A.2). We first show the results with OLS. To absorb as much of the error term as possible, we include source-destination-reference country-time ($n \times m \times m' \times t$) fixed effects. These absorb any common components occurring at the country 3-tuple-time level, such as exchange rate changes and other taste and transport cost changes, and thus the coefficient is estimated from the variation in the relative sectoral price indices and relative sectoral share movements within that cell. The identifying assumption is then that price change ratio $\widehat{P}_{mjt}/\widehat{P}_{m'jt}$ is uncorrelated with the residual net of the $n \times m \times m' \times t$ fixed effects. The remaining errors would be largely measurement error. If this measurement error is uncorrelated with the price change ratios, then the OLS estimates are unbiased, and if not, we would expect a bias towards zero. In the latter case, the IV estimates (described below) should be larger than the OLS estimates, assuming the measurement error in (A.1) and (A.2) is independent of the measurement error in the technology shock ratios.

The estimation amounts to regressing relative share changes on relative price changes. A threat to identification would be that relative price changes are affected by demand shocks (e.g. $\widehat{\vartheta}_{mnjt}$), and thus correlated with the residual. As a way to mitigate this concern, we also report estimates based on the subsample in which destination countries are all non-G7, and the source and reference countries are all G7 countries. In this sample it is less likely that taste shocks in the (smaller) destination countries will affect relative price changes in the larger G7 source countries. Finally, to reduce the impact of small shares on the estimates, we report results weighting by the size of the initial shares ($\pi^{f}_{mnj,t-1}$ and $\pi^{x}_{mj,ni,t-1}$).

We also implement IV estimation. We use the TFP shocks $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$ as instruments for changes in relative prices. The exclusion restriction is that the technology shocks are uncorrelated with taste and trade cost shocks, and thus only affect the share ratios through changing the prices. Even if the shock ratio $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$ is a valid instrument for observed prices, it does not include the general-equilibrium effects on prices in the model. To use all of the information –both the direct and indirect GE effects –incorporated in the model, we also use the model-optimal IV approach to construct the instrument. In our context this simply involves computing the model using only the estimated technology shocks, and solving for the sequence of equilibrium prices in all countries and sectors. The model-implied prices are then the optimal instrument for the prices observed in the data. See Chamberlain (1987) for a discussion of optimal instruments, and Adao, Arkolakis, and Esposito (2017) and Bartelme et al. (2018) for two recent applications of this approach. The results from the model-optimal IV are very similar to simply instrumenting with the TFP shock ratio, and we do not report them to conserve space.

Table A5 presents the results of estimating equations (A.1) and (A.2). Columns 1-3 report the OLS estimates of ρ (top panel) and ε (bottom panel). The OLS estimates of ρ are all significantly larger than zero, and we cannot rule out a Cobb-Douglas final demand elasticity. The OLS estimates for ρ are also not very sensitive to restricting the sample to non-G7 destinations and G7 sources, or to weighting by the initial share. The IV estimates in columns 4-6 are substantially larger than the OLS coefficients, ranging from 2.27 to 3.04, and significantly different from 1 in most cases. This difference between OLS and IV could suggest either measurement error in (A.1), or greater noise in the IV estimator (Young, 2017). Given the substantial disagreement between OLS and IV estimates of ρ, we report the results under two values: $\rho = 1$, corresponding to the OLS estimates, and $\rho = 2.75$ based on the IV.

The OLS and IV estimates of ε display somewhat greater consensus. The OLS point estimates are in the range 0.68, and not sensitive to the sample restriction or weighting. The IV estimates are less stable. While the full sample (column 4) yields an elasticity of 2.8, either restricting to the non-G7 destinations/G7 sources, or weighting by size reduces the coefficient dramatically and renders it not statistically different from 1. Such evidence for the low substitutability of intermediate inputs is consistent with the recent estimates by Atalay (2017) and Boehm, Flaaen, and Pandalai-Nayar (2019), who find even stronger complementarity. We therefore set $\varepsilon = 1$ for all implementations of the model.

A.3 Estimating Shock Processes

As discussed in Section 6, estimating an unrestricted process for the technology and non-technology shocks is not possible due to the short panel of measured shocks. We restrict the

Table A5: Elasticity Estimates

	(1) OLS	(2) OLS (G7 m, m', non-G7 n)	(3) OLS (weighted)	(4) IV	(5) IV (G7 m, m', non-G7 n)	(6) IV (weighted)
ρ	0.775	0.730	1.051	2.881	2.273	3.037
SE	(0.055)	(0.146)	(0.082)	(0.584)	(0.966)	(0.470)
First stage K-P F				92.117	30.539	89.669
FE	Yes	Yes	Yes	Yes	Yes	Yes
ε	0.698	0.686	0.682	2.838	0.382	1.322
SE	(0.051)	(0.120)	(0.143)	(0.578)	(0.872)	(0.856)
First stage K-P F				94.863	16.188	86.631
FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at the destination-source-reference country level in parentheses. This table presents results from the OLS and IV estimation of (A.1) and (A.2). The fixed effects used in each regression are $n \times m \times m' \times t$. The instruments are the relative productivity shocks $\widehat{Z}_{mjt}/\widehat{Z}_{m'jt}$, with the Kleibergen-Papp first stage F-statistic reported. The weights in columns 3 and 6 are lagged share ratios π^{f}_{mnjt-1} and $\pi^{x}_{mj,nit-1}$.

dynamic model to the G7 countries, for which we have the longest panel of shocks. While we still cannot estimate a completely unrestricted VAR, we impose minimal restrictions that allow the shocks to be correlated (as the measured shocks are), and further, allow for spillovers between country-sectors. Our specification allows for contemporaneous spillovers between country-sectors, but restricts the structure of lagged spillovers. We permit a country-sector specific lagged autoregressive parameter, so country-sector shocks can be persistent. We restrict lagged spillovers to be common within a country (across sectors), and zero otherwise. We allow for a full variance-covariance matrix of the error terms, which amounts to assuming completely unrestricted contemporaneous spillovers. The sample variance-covariance matrix of the residuals for the period 1995-2007 serves as an estimate of the covariance matrix of the error term. The technology shock process we estimate is:

$$\ln z_{njt} = \rho^{z}_{nj} \ln z_{njt-1} + \zeta^{z}_{n} \mathbf{1}\left(m = n, k \neq j\right) \ln z_{mkt-1} + \theta^{z}_{njt}. \tag{A.3}$$

$$\ln \xi_{njt} = \rho^{\xi}_{nj} \ln \xi_{njt-1} + \zeta^{\xi}_{n} \mathbf{1}\left(m = n, k \neq j\right) \ln \xi_{mkt-1} + \theta^{\xi}_{njt}. \tag{A.4}$$

with $\boldsymbol{\theta}_t \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}\right)$, that is, we permit a full covariance matrix.

The choice of restrictions strikes a balance between relative parsimony, which improves the precision of the parameters estimates, and sufficient flexibility to replicate the measured shock correlations in the data. We experimented with other processes using methods such as LASSO regressions without much change to the simulated shock correlations. In particular, we have modified the equations above to also include a sector-specific lagged spillover term, but these coefficients were all insignificant, and so we use the more parsimonious process in the baseline analysis. The processes (A.3)-(A.4) are estimated separately for each country-sector. Table A6 summarizes the estimation results.

Table A6: Shock Processes: Autoregressive Parameters

	Mean	Median	25th pctile	75th pctile
			$\ln z_{njt}$	
Own lag (ρ^z_{nj})	0.857	0.864	0.830	0.893
Spillover lag (δ^z_n)	0.000	-0.001	-0.002	0.001
			$\ln \xi_{njt}$	
Own lag (ρ^ξ_{nj})	0.698	0.750	0.628	0.824
Spillover lag (δ^ξ_n)	0.004	0.002	-0.001	0.006

Notes: This table presents results from estimating the shock stochastic processes (A.3)-(A.4). The measures are summary statistics of the coefficients in the sample of sectors and countries. The ξ_{njt} series is computed under the calibration of $\rho = 2.75$ and $\psi_u = 4$.

Appendix B Model and Quantitative Results

B.1 TFP, GDP, and the Solow Residual

This appendix presents the derivation of the decomposition of GDP growth into the movement in aggregate TFP and aggregate factor inputs, and of the Solow residual into the components due to TFP and unobserved factor utilization.

Aggregate GDP Growth Using the definition of real GDP (2.2), the change in real GDP at time t relative to steady state is:

$$\Delta Y_{nt} = \sum_{j=1}^{J} \left(P_{nj} \Delta Y_{njt} - P_{nj}^{X} \Delta X_{njt} \right),$$

where P_{nj} and P_{nj}^{X} are the steady state ("base") prices. The proportional change relative to steady state is:

$$\begin{aligned} \frac{\Delta Y_{nt}}{Y_n} &= \frac{\sum_{j=1}^{J} \left(P_{nj} \Delta Y_{njt} - P_{nj}^{X} \Delta X_{njt} \right)}{Y_n} \\ &= \sum_{j=1}^{J} D_{nj} \left(\frac{\Delta Y_{njt}}{Y_{nj}} - \frac{\Delta X_{njt}}{X_{nj}} \frac{P_{nj}^{X} X_{nj}}{P_{nj} Y_{nj}} \right), \end{aligned}$$

where the omission of time subscripts denotes steady state values, and $D_{nj} \equiv \frac{P_{nj} Y_{nj}}{Y_n}$ is the steady state Domar weight of sector j in country n, that is, the weight of the sector's gross sales in aggregate value added. Approximate the growth rate with log difference:

$$\begin{aligned} d \ln Y_{nt} &\approx \sum_{j=1}^{J} D_{nj} \left(d \ln Y_{njt} - \frac{P_{nj}^{X} X_{nj}}{P_{nj} Y_{nj}} d \ln X_{njt} \right) \qquad \text{(B.1)} \\ &= \sum_{j=1}^{J} D_{nj} \Bigg(d \ln Z_{njt} + \gamma_j \alpha_j \eta_j d \ln K_{njt} + \gamma_j (1-\alpha_j) \eta_j d \ln L_{njt} \\ &\qquad + \gamma_j \left(1 - \eta_j\right) d \ln X_{njt} - \frac{P_{nj}^{X} X_{nj}}{P_{nj} Y_{nj}} d \ln X_{njt} \Bigg). \end{aligned}$$

Under the assumption that the share of payments to inputs in total revenues is the same as in total costs, the growth in real GDP can be written as:[21]

$$d\ln Y_{nt} \approx \sum_{j=1}^{J} D_{nj} \left\{ \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1) d\ln \left[\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right]}_{\text{Scale Effect}} \right. \tag{B.2}$$

$$\left. + \underbrace{\alpha_j \eta_j d\ln K_{njt} + (1-\alpha_j)\eta_j d\ln L_{njt}}_{\text{Primary Inputs}} \right\}.$$

Then, the growth rate of GDP can be expressed in terms of observable and estimated values:

$$d\ln Y_{nt} \approx \sum_{j=1}^{J} D_{nj} \left\{ \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1) \left[d\ln \left(M_{njt}^{\alpha_j\eta_j} N_{njt}^{(1-\alpha_j)\eta_j} H_{njt}^{(1-\alpha_j)\eta_j + \xi_j} X_{njt}^{1-\eta_j} \right) \right]}_{\text{Scale Effect}} \right. \tag{B.3}$$

$$\left. + \underbrace{(\alpha_j \eta_j d\ln M_{njt} + (1-\alpha_j)\eta_j d\ln H_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}) + \zeta_j d\ln H_{njt}}_{\text{Utilization-adjusted Primary Inputs}} \right\},$$

leading to equations (4.6) and (4.7) in the main text, with the input-driven component of GDP growth defined as:

$$d\ln \mathcal{I}_{nt} \equiv \sum_{j=1}^{J} D_{nj} \left\{ \underbrace{(\gamma_j - 1) d\ln \left[\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right]}_{\text{Scale Effect}} \right. \tag{B.4}$$

$$\left. + \underbrace{\alpha_j \eta_j d\ln K_{njt} + (1-\alpha_j)\eta_j d\ln L_{njt}}_{\text{Primary Inputs}} \right\}.$$

Relationship to Solow residual The expression in equation (4.6) is useful to compare the estimated TFP series to the traditional measure of technology, the Solow residual. The Solow residual S_{njt} takes factor shares and nets out the observable factor uses. It has the following relationship to gross output and observed inputs:

$$d\ln Y_{njt} = d\ln S_{njt} + \alpha_j \eta_j d\ln M_{njt} + (1-\alpha_j)\eta_j d\ln H_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt} + (1-\eta_j)\, d\ln X_{njt}.$$

[21] Recall that, regardless of the nature of variable returns to scale or market structure, under cost minimization $\alpha_j \eta_j$ is the share of payments to capital in the total costs, while $(1-\alpha_j)\eta_j$ is the share of payments to labor. We do not observe total costs, only total revenues. We assume that $\alpha_j \eta_j$ also reflects the share of payments to capital in total revenues. Under our assumption that sector j is competitive and the variable returns to scale are external to the firm, this assumption is satisfied. In that case, these can be taken directly from the data as $\alpha_j \eta_j = R_{nj} K_{nj} / P_{nj} Y_{nj}$ and $(1-\alpha_j)\eta_j = W_{nj} L_{nj} / P_{nj} Y_{nj}$. In practice, we compute these steady state values as time averages in our data.

Plugging this way of writing output growth into the real GDP growth equation (B.1), we get the following expression:

$$
\begin{aligned}
d\ln Y_{nt} &\approx \sum_{j=1}^{J} D_{nj}\left(d\ln S_{njt} + \alpha_j\eta_j d\ln M_{njt} + (1-\alpha_j)\eta_j d\ln H_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}\right. \\
&\qquad \left. + (1-\eta_j)\, d\ln X_{njt} - d\ln X_{njt}\frac{p^X_{njt-1}X_{njt-1}}{p_{njt-1}Y_{njt-1}}\right) \qquad \text{(B.5)} \\
&= \sum_{j=1}^{J} D_{nj}\left(d\ln S_{njt} + \alpha_j\eta_j d\ln M_{njt} + (1-\alpha_j)\eta_j d\ln H_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}\right).
\end{aligned}
$$

Comparing (B.2) to (B.5), the Solow residual contains the following components:

$$
\begin{aligned}
d\ln S_{njt} &= \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1)d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j} X_{njt}^{1-\eta_j}\right]}_{\text{Scale Effect}} \\
&\quad + \underbrace{\alpha_j\eta_j d\ln U_{njt} + (1-\alpha_j)\eta_j d\ln E_{njt}}_{\text{Unobserved Utilization}}.
\end{aligned}
$$

This expression makes it transparent that in this setting, the Solow residual can diverge from the true TFP shock for two reasons: departures from constant returns to scale at the industry level, and unobserved utilization of inputs.

Let the aggregate Solow residual be denoted by:

$$
\begin{aligned}
d\ln S_{nt} &= \sum_{j=1}^{J} D_{nj} d\ln S_{njt} \\
&= d\ln Z_{nt} + d\ln \mathcal{U}_{nt},
\end{aligned}
$$

where in the second equality, $d\ln\mathcal{U}_{nt}$ is the aggregate utilization adjustment:

$$
\begin{aligned}
d\ln\mathcal{U}_{nt} &\equiv \sum_{j=1}^{J} D_{nj}\left\{(\gamma_j - 1)d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j} X_{njt}^{1-\eta_j}\right]\right. \qquad \text{(B.6)} \\
&\quad \left. + \alpha_j\eta_j d\ln U_{njt} + (1-\alpha_j)\eta_j d\ln E_{njt}\right\}.
\end{aligned}
$$

It is immediate that the observed Solow residual can be correlated across countries both due to correlated shocks to true TFP, and due to correlated unobserved input adjustments.

B.2 Derivation of the Analytical Influence Vector

The accounting equation for the sales in country n sector j is

$$P_{njt}Y_{njt} = \sum_m \omega_{mj} P_{mt}\mathcal{F}_{mt}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i) P_{mit}Y_{mit}\pi^x_{nj,mit}.$$

Note that with financial autarky, the total sales of final goods is the same as the value added across sectors

$$P_{mt}\mathcal{F}_{mt} = \sum_i \eta_i P_{mit}Y_{mit}.$$

The accounting equation can be rewritten as

$$P_{njt}Y_{njt} = \sum_m \sum_i \omega_{mj}\eta_i P_{mit}Y_{mit}\pi^f_{nmjt} + \sum_m \sum_i (1-\eta_i) P_{mit}Y_{mit}\pi^x_{nj,mit}.$$

Now we consider the log-linearized version. It follows that

$$\begin{aligned}\ln P_{njt} + \ln Y_{njt} = & \sum_m \sum_i \frac{\eta_i \omega_{mj}\pi^f_{nmj}P_{mi}Y_{mi}}{P_{nj}Y_{nj}} \left(\ln P_{mit} + \ln Y_{mit} + \ln \pi^f_{nmjt}\right) \\ & + \sum_m \sum_i \frac{(1-\eta_i)\pi^x_{nj,mi}P_{mi}Y_{mi}}{P_{nj}Y_{nj}} (\ln P_{mit} + \ln Y_{mit} + \ln \pi^x_{nj,mit}),\end{aligned} \tag{B.7}$$

and the log-deviation of import shares are given by

$$\begin{aligned}\ln \pi^f_{nmjt} = & (1-\rho)\sum_k \pi^f_{kmj}\left(\ln P_{njt} - \ln P_{kjt}\right) \\ \ln \pi^x_{nj,mit} = & (1-\varepsilon)\sum_{k,l} \pi^x_{klmi}\left(\ln P_{njt} - \ln P_{k\ell t}\right).\end{aligned}$$

where the variables without subscript t stand for their corresponding steady-state values.

Denote by $\mathbf{\Psi}^f$ and $\mathbf{\Psi}^x$ the matrices that collect export shares for final use and for intermediate use. The dimension of these matrices is $NJ \times NJ$, with typical elements being

$$\Psi^f_{nj,mi} \equiv \frac{\eta_i \omega_{mj}\pi^f_{nmj}P_{mi}Y_{mi}}{P_{nj}Y_{nj}}, \quad \text{and} \quad \Psi^x_{nj,mi} \equiv \frac{(1-\eta_i)P_{mi}Y_{mi}\pi^x_{nj,mi}}{P_{nj}Y_{nj}}. \tag{B.8}$$

Denote by $\mathbf{\Pi}^f$ and $\mathbf{\Pi}^x$ the matrices that collect import shares for final use and for intermediate use. The typical elements of these matrices are

$$\Pi^f_{nj,mi} \equiv \left\{ \begin{array}{ll} 0, & \text{if} \quad i \neq j \\ \pi^f_{mnj}, & \text{if} \quad i = j \end{array} \right., \qquad \text{and} \qquad \Pi^x_{nj,mi} \equiv \pi^x_{mi,nj}. \tag{B.9}$$

In matrix form, equation (B.7) can be written as

$$\ln \mathbf{P}_t + \ln \mathbf{Y}_t = \left(\mathbf{\Psi}^f + \mathbf{\Psi}^x \right) (\ln \mathbf{P}_t + \ln \mathbf{Y}_t) +$$
$$(1-\rho) \left(\operatorname{diag}\left(\mathbf{\Psi}^f \mathbf{\Pi}^f \mathbf{1} \right) - \mathbf{\Psi}^f \mathbf{\Pi}^f \right) \ln \mathbf{P}_t \; + \; (1-\varepsilon) \left(\operatorname{diag}\left(\mathbf{\Psi}^x \mathbf{\Pi}^x \mathbf{1} \right) - \mathbf{\Psi}^x \mathbf{\Pi}^x \right) \ln \mathbf{P}_t$$

Together with the choice of a numeraire good, we can express the change of prices as a function of changes in outputs:

$$\ln \mathbf{P}_t = \mathcal{P} \ln \mathbf{Y}_t$$

.

At the supply side, the optimality conditions on the labor supply are

$$\left(\psi_h - 1 - \frac{\psi_h}{\psi_e} \right) \ln H_{njt} = -\log \xi_{njt} + \ln \left(\frac{W_{njt}}{P_{nt}} \right),$$
$$\ln E_{njt} = \frac{\psi^h}{\psi^e} \ln H_{njt},$$
$$\ln U_{njt} = \frac{\psi^h}{\psi^u} \ln U_{njt}.$$

Combining these with the production function leads to:

$$\ln Y_{njt} = \ln Z_{njt} + \left(\gamma_j \eta_j \alpha_j \frac{\psi^h}{\psi^u} + \gamma_j \eta_j (1-\alpha_j) \frac{\psi^h}{\psi^e} + \gamma_j \eta_j (1-\alpha_j) \right) \ln H_{njt} + \gamma_j (1-\eta_j) \ln X_{njt}$$

. Note that here the variation of machines and employment are muted in a static model. The wage rate and the price for the intermediate goods equal to their marginal products

$$\ln W_{njt} - \ln P_{njt} = \ln Y_{njt} - \ln H_{njt} - \ln E_{njt},$$
$$\ln P^x_{njt} - \ln P_{njt} = \ln Y_{njt} - \ln X_{njt}.$$

The log-deviations of final goods prices and intermediate goods prices to their steady-state values are

$$\ln P_{nt} = \omega_{ni} \sum_m \pi^f_{mni} \ln P_{mit}, \qquad \text{and} \qquad \ln P^x_{njt} = \omega_{ni} \sum_m \pi^x_{mi,nj} \ln P_{mit}.$$

We can relate the outputs with the prices as

$$
\begin{aligned}
\ln Y_{njt} = & \ln Z_{njt} - \left(\frac{\gamma_j\eta_j\alpha_j}{\psi^u} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^e} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^h}\right)\left(\sum_i \omega_{ni}\sum_m \pi^f_{mni}\ln P_{mit} + \ln\xi_{njt}\right) \\
& + \left(\frac{\gamma_j\eta_j\alpha_j}{\psi^u} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^e} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^h} + \gamma_j(1-\eta_j)\right)(\ln P_{njt} + \ln Y_{njt}) \\
& - \gamma_j(1-\eta_j)\sum_{m,i}\pi^x_{mi,nj}\ln P_{mit}
\end{aligned}
\tag{B.10}
$$

By defining matrices $\mathcal{E}^h$, $\mathcal{E}^x$, and $\widetilde{\mathbf{\Pi}}^f$ as

$$
\begin{aligned}
\mathcal{E}^h_{nj,mi} &\equiv \begin{cases} \frac{\gamma_j\eta_j\alpha_j}{\psi^u} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^e} + \frac{\gamma_j\eta_j(1-\alpha_j)}{\psi^h}, & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases} \\
\mathcal{E}^x_{nj,mi} &\equiv \begin{cases} \gamma_j(1-\eta_j), & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases} \\
\widetilde{\Pi}^f_{nj,mi} &\equiv \omega_{ni}\pi^f_{mni},
\end{aligned}
\tag{B.11}
$$

it follows that

$$
\ln \mathbf{Y}_t = \ln \mathbf{Z}_t - \mathcal{E}^h\widetilde{\mathbf{\Pi}}^f \ln \mathbf{P}_t - \mathcal{E}^h\widetilde{\mathbf{\Pi}}^f \ln \boldsymbol{\xi}_t + (\mathcal{E}^h + \mathcal{E}^x)(\ln \mathbf{P}_t + \ln \mathbf{Y}_t) - \mathcal{E}^x\mathbf{\Pi}^x \ln \mathbf{P}_t.
$$

Combining with the previous result that $\ln \mathbf{P}_t = \mathcal{P}\ln \mathbf{Y}_t$, we obtain the solution:

$$
\ln \mathbf{Y}_t = \left\{\mathbf{I} - \left(\mathcal{E}^h + \mathcal{E}^x\right)(\mathbf{I} + \mathcal{P}) + \left(\mathcal{E}^h\widetilde{\mathbf{\Pi}}^f + \mathcal{E}^x\mathbf{\Pi}^x\right)\mathcal{P}\right\}^{-1}\left(\ln \mathbf{Z}_t - \mathcal{E}^h \ln \boldsymbol{\xi}_t\right).
$$

The expression for the Change in GDP Evaluated at base prices, the total real value added or GDP in country n is

$$
Y_{nt} = \sum_j \left(P_{nj}Y_{njt} - \sum_{m,i} P_{mi}X_{mi,njt}\right).
$$

Expressed in log-deviations, this becomes

$$
\ln Y_{nt} = \sum_j \left(\frac{P_{nj}Y_{nj}}{P_n\mathcal{F}_n}\ln Y_{njt} - \sum_{m,i}\frac{P_{mi}X_{mi,nj}}{P_n\mathcal{F}_n}\ln X_{mi,njt}\right).
$$

Note that

$$\begin{aligned}
\ln X_{mi,njt} &= \ln P_{njt} + \ln Y_{njt} + \ln \pi^x_{mi,njt} - \ln P_{mit} \\
&= \ln P_{njt} + \ln Y_{njt} + (1-\varepsilon)\sum_{k,l} \pi^x_{kl,nj}(\ln P_{mit} - \ln P_{k\ell t}) - \ln P_{mit} \\
&= \ln P_{njt} + \ln Y_{njt} - \varepsilon \ln P_{mit} + (\varepsilon - 1)\sum_{k,l} \pi^x_{kl,nj} \ln P_{k\ell t}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\ln Y_{nt} = &\sum_j \left(\frac{\eta_j P_{nj} Y_{nj}}{P_n \mathcal{F}_n} \ln Y_{njt} - \frac{(1-\eta_j) P_{nj} Y_{nj}}{P_n \mathcal{F}_n} \left(\ln P_{njt} + (\varepsilon - 1) \sum_{k,l} \pi^x_{kl,nj} \ln P_{k\ell t} \right) \right. \\
&\left. + \varepsilon \sum_{m,i} \frac{P_{mi} X_{mi,nj}}{P_n \mathcal{F}_n} \ln P_{mit} \right) \\
&= \sum_j \left(\frac{\eta_j P_{nj} Y_{nj}}{P_n \mathcal{F}_n} \ln Y_{njt} - \frac{(1-\eta_j) P_{nj} Y_{nj}}{P_n \mathcal{F}_n} \left(\ln P_{njt} - \sum_{k,l} \pi^x_{kl,nj} \ln P_{k\ell t} \right) \right).
\end{aligned}$$

The second term reflects the relative price changes between outputs and inputs. Define the matrices $\boldsymbol{\eta}$ and $\mathcal{D}$ as

$$\boldsymbol{\eta}_{nj,mi} \equiv \begin{cases} \eta_j, & \text{if } i = j \text{ and } n = m \\ 0, & \text{otherwise} \end{cases}, \text{ and } \mathcal{D}_{nj,mi} \equiv \begin{cases} \frac{P_{nj} Y_{nj}}{P_n \mathcal{F}_n}, & \text{if } \ i = j \text{ and } n = m \\ 0, & \text{otherwise} \end{cases} \tag{B.12}$$

The vector of total value added/GDP is

$$\ln \mathbf{GDP}_t = \boldsymbol{\eta} \mathcal{D} \ln \mathbf{Y}_t - (\mathbf{I} - \boldsymbol{\eta}) \mathcal{D} (\mathbf{I} - \mathbf{\Pi}^x) \mathcal{P} \ln \mathbf{Y}_t.$$

B.3 Extracting Non-Technology Shocks

Similar to the GDP measurement, the log-deviation of the value added in country n sector j is

$$\ln V_{njt} = \frac{\eta_j P_{nj} Y_{nj}}{V_{nj}} \ln Y_{njt} - \frac{(1-\eta_j) P_{nj} Y_{nj}}{V_{nj}} \left(\ln P_{njt} - \sum_{k,l} \pi^x_{kl,nj} \ln P_{k\ell t} \right).$$

Define $\widetilde{\mathbf{V}}^1$, $\widetilde{\mathbf{V}}^2$, and $\widetilde{\mathbf{V}}^3$ as

$$\widetilde{V}^1_{nj,mi} \equiv \begin{cases} \frac{\eta_j P_{nj} Y_{nj}}{V_{nj}}, & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases}$$

$$\widetilde{V}^2_{nj,mi} \equiv \begin{cases} -\frac{(1-\eta_j) P_{nj} Y_{nj}}{V_{nj}}, & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases}$$

$$\widetilde{V}^3_{nj,mi} \equiv \frac{(1-\eta_j) P_{nj} Y_{nj}}{V_{nj}} \pi^x_{mi,nj}$$

We have

$$\ln \mathbf{V}_t = \mathcal{V} \, \ln \mathbf{Y}_t,$$

where

$$\mathcal{V} = \widetilde{\mathbf{V}}^1 + (\widetilde{\mathbf{V}}^2 + \widetilde{\mathbf{V}}^3)\mathcal{P}. \tag{B.13}$$

With changes of machines and employment, equation (B.10) extends to

$$\begin{aligned} \ln Y_{njt} = \ln Z_{njt} &- \left(\frac{\gamma_j \eta_j \alpha_j}{\psi^u} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^e} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^h} \right) \left(\sum_i \omega_{ni} \sum_m \pi^f_{mni} \ln P_{mit} + \ln \xi_{njt} \right) \\ &+ \left(\frac{\gamma_j \eta_j \alpha_j}{\psi^u} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^e} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^h} + \gamma_j (1-\eta_j) \right) (\ln P_{njt} + \ln Y_{njt}) \\ &- \gamma_j (1-\eta_j) \sum_{m,i} \pi^x_{mi,nj} \ln P_{mit} \\ &+ \gamma_j \eta_j \alpha_j \ln M_{njt} - \left(\frac{\gamma_j \eta_j \alpha_j}{\psi^u} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^e} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^h} + \gamma_j \eta_j (1-\alpha_j) \right) \ln N_{njt}. \end{aligned}$$

Define

$$\mathcal{E}^m_{nj,mi} \equiv \begin{cases} \gamma_j \eta_j \alpha_j, & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases} \tag{B.14}$$

$$\mathcal{E}^n_{nj,mi} \equiv \begin{cases} \frac{\gamma_j \eta_j \alpha_j}{\psi^u} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^e} + \frac{\gamma_j \eta_j (1-\alpha_j)}{\psi^h} + \gamma_j \eta_j (1-\alpha_j), & \text{if} \quad i=j \text{ and } n=m \\ 0, & \text{otherwise} \end{cases} \tag{B.15}$$

It follows that

$$d \ln \mathbf{Y}_t = \left\{ \mathbf{I} - \left(\mathcal{E}^h + \mathcal{E}^x \right) (\mathbf{I} + \mathcal{P}) + \left(\mathcal{E}^h \widetilde{\mathbf{\Pi}}^f + \mathcal{E}^x \mathbf{\Pi}^x \right) \mathcal{P} \right\}^{-1} \left(d \ln \mathbf{Z}_t - \mathcal{E}^h d \ln \boldsymbol{\xi}_t + \mathcal{E}^m d \ln \mathbf{M}_t + \mathcal{E}^n d \ln \mathbf{N}_t \right).$$

B.4 Exact Solution to Static Counterfactuals

This section sets up the exact solution to the static model, in changes, following the methodology of Dekle, Eaton, and Kortum (2008). Denote by a "hat" the gross proportional change in any variable $\widehat{x}_t \equiv x_t/x_{t-1}$. To streamline notation, define $\Upsilon_{njt} \equiv P_{njt}Y_{njt}$ to be the gross revenue in sector j, country n. In response to TFP and non-TFP shocks, the price in sector j, country n experiences the change:

$$\begin{aligned}\widehat{P}_{njt} \quad = \quad & \widehat{Z}_{njt}^{-1}\widehat{\Upsilon}_{njt}^{1-\gamma_j+\alpha_j\eta_j\left(\frac{\psi^u-1}{\psi^u}\right)\gamma_j+\left(1-\tilde{\psi}\right)(1-\alpha_j)\eta_j\gamma_j}\widehat{M}_{njt}^{-\alpha_j\eta_j\gamma_j}\left(\widehat{\xi}_{njt}\widehat{P}_{nt}\right)^{\left(\frac{\alpha_j}{\psi^u}+\tilde{\psi}(1-\alpha_j)\right)\eta_j\gamma_j} \\ & \widehat{N}_{njt}^{\left(\tilde{\psi}-1\right)(1-\alpha_j)\eta_j\gamma_j+\frac{\alpha_j\eta_j}{\psi^u}\gamma_j}\left(\sum_{m,i}\pi_{mi,njt-1}\widehat{P}_{mit}^{1-\varepsilon}\right)^{\frac{1-\eta_j}{1-\varepsilon}\gamma_j}.\end{aligned} \tag{B.16}$$

This, together with the dependence of $\widehat{P}_{nt}$ on the constituent $\widehat{P}_{njt}$:

$$\widehat{P}_{nt} \quad = \quad \prod_j\left(\widehat{P}_{njt}^f\right)^{\omega_{jn}} \tag{B.17}$$

$$\widehat{P}_{njt}^f \quad = \quad \left[\sum_m \widehat{P}_{mjt}^{1-\rho}\pi_{mnjt-1}^f\right]^{\frac{1}{1-\rho}} \tag{B.18}$$

defines a system of $J \times N$ equations in prices, conditional on known initial-period data quantities (such as π_{mnjt-1}^f), a vector of $\widehat{\Upsilon}_{njt}$'s, and an assumption on $\widehat{M}_{njt}$ and $\widehat{N}_{njt}$. The price changes in turn determine next period's shares:

$$\pi_{nmjt}^f \quad = \quad \frac{\widehat{P}_{njt}^{1-\rho}\pi_{nmjt-1}^f}{\sum_k \widehat{P}_{kjt}^{1-\rho}\pi_{kmjt-1}^f}, \tag{B.19}$$

$$\pi_{nj,mit}^x = \frac{\widehat{P}_{njt}^{1-\varepsilon}\pi_{nj,mit-1}^x}{\sum_{k,l}\widehat{P}_{klt}^{1-\varepsilon}\pi_{kl,mit-1}^x}. \tag{B.20}$$

These trade shares have to be consistent with market clearing at the counterfactual t, expressed using proportional changes as:

$$\begin{aligned}\widehat{\Upsilon}_{njt}\Upsilon_{njt-1} \quad = \quad & \sum_m\left[\pi_{nmjt}^f\omega_{jm}\left(\sum_i \eta_i\widehat{\Upsilon}_{mit}\Upsilon_{mit-1}\right)\right. \\ & \left. +\sum_i \pi_{nj,mit}^x\left(1-\eta_i\right)\widehat{\Upsilon}_{mit}\Upsilon_{mit-1}\right].\end{aligned} \tag{B.21}$$

The sets of equations (B.16)-(B.21) represent a system of $2 \times N \times J + N^2 \times J + N^2 \times J^2$ unknowns, $\widehat{P}_{njt}\ \forall n,j$, $\widehat{\Upsilon}_{njt}\ \forall n,j$, $\pi_{nmjt}^f\ \forall n,m,j$, and $\pi_{nj,mit}^x\ \forall n,j,m,i$ that is solved under given parameter values and under a set of shocks $\widehat{Z}_{njt}$ and $\widehat{\xi}_{njt}$.

B.4.1 Algorithm for Exact Solution to the Static Model

To solve the model, we use an initial guess for $\widehat{\Upsilon}_{njt}$ together with data on π^{f}_{mnjt-1} and $\pi^{x}_{mj,nit-1}$. Given these variables, the algorithm is as follows:

- Solve for $\widehat{P}_{njt}$ given the guess of $\widehat{\Upsilon}_{nj,t}$ and the data on π^{f}_{mnjt-1} and $\pi^{x}_{mjnit-1}$. This step uses equations (B.18), (B.17) and (B.16).
- Update π^{f}_{mnjt} and $\pi^{x}_{mj,nit}$ given the solution to (1) and the guess of $\widehat{\Upsilon}_{njt}$ using equations (B.19) and (B.20).
- Solve for $\widehat{\Upsilon}'_{njt}$ using equation (B.21) given the prices $\widehat{P}_{njt}$ obtained in step (1) and the updated shares π^{f}_{mnjt} and $\pi_{mi,njt}$ from step (2).
- Check if $\max|(\widehat{\Upsilon}'_{njt}\text{-}\widehat{\Upsilon}_{njt})| < \delta$, where δ is a tolerance parameter that is arbitrarily small. If not, update the guess of $\widehat{\Upsilon}_{njt}$ and repeat steps (1)-(4) until convergence.

B.4.2 Comparison of the Exact and First-Order Solutions

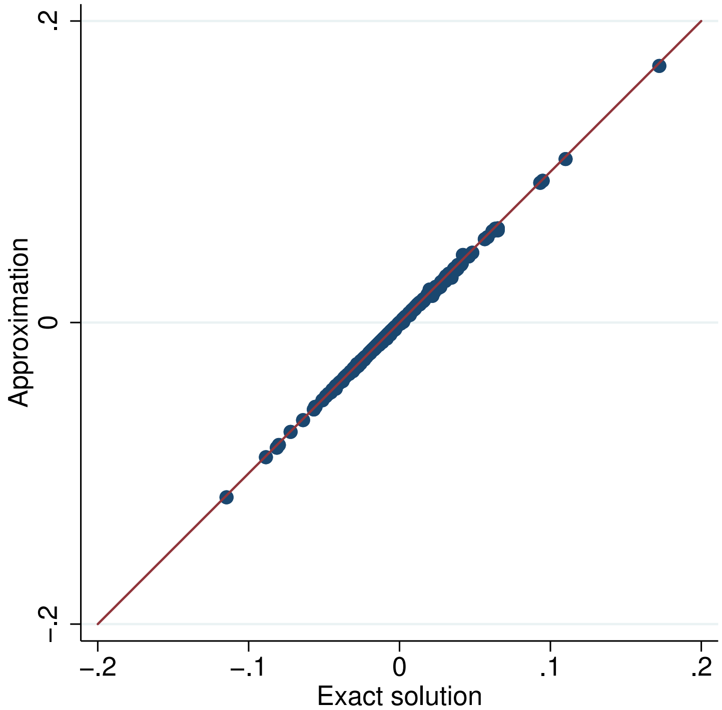
Figure A4 presents a scatterplot of GDP growth rates obtained under the first-order analytical solution to the global influence matrix in Section 3.1 against the exact solution computed as in this appendix. The line through the data is the 45-degree line. The GDP growth rates are computed under the observed shocks, and pooled across countries and years. It is clear that the first-order approximation is very good in the large majority of instances. The correlation between the two sets of growth rates is 0.999. Table A7 summarizes the GDP correlations obtained using GDP growth rates in the linear and exact solutions. The correlations are very close to each other.

B.5 Autarky Counterfactuals as Limiting Cases

This appendix shows that the three autarky counterfactuals in Section 5.5 can be thought of as limiting cases as trade costs go to infinity, and elasticities differ from 1 in different ways. Suppose the production function is CES with the elasticity of substitution between labor and materials σ, and the elasticity of substitution between intermediate inputs ε:

$$Y_{njt} = Z_{njt}\left[\nu_F^{\frac{1}{\sigma}}\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\frac{\sigma-1}{\sigma}} + \nu_X^{\frac{1}{\sigma}}\left(\sum_{m,i}\mu_{mi,nj}^{\frac{1}{\varepsilon}}X_{mi,njt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}\frac{\sigma-1}{\sigma}}\right]^{\gamma_j\frac{\sigma}{\sigma-1}}. \tag{B.22}$$

Figure A4: Comparison of GDP Growth Rates between First-Order and Exact Solutions



Notes: This figure displays a scatterplot of the GDP growth rates obtained using the first-order approximation against the GDP growth rates in the exact solution to the model, pooling countries and years. The line through the data is the 45-degree line.

Then the first and third autarky counterfactuals correspond to the following limiting cases:

$$\begin{aligned} AUT1 :& \tau_{mnj} \to \infty; \varepsilon \downarrow 0; \sigma \downarrow 1 \\ AUT3 :& \tau_{mnj} \to \infty; \varepsilon \downarrow 1; \sigma = 1. \end{aligned}$$

In other words, the first autarky counterfactual would obtain as a limiting case if domestic and foreign intermediates were strong complements, but value added and intermediates had a substitution elasticity greater than 1. The third counterfactual requires instead that the intermediate inputs and value added are Cobb-Douglas, whereas the foreign and domestic intermediates are substitutes.

The $AUT2$ counterfactual replaces foreign inputs with domestic value added in a sector, and thus requires domestic value added to be more substitutable with foreign inputs than with domestic inputs. Thus, the $AUT2$ scenario cannot be a limiting case of the production function

Table A7: First-Order and Exact Solutions: Correlations of $d\ln Y_{nt}$, $\rho = 2.75$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Baseline (approx.)	0.236	0.363	-0.030	0.567
Exact solution	0.240	0.371	-0.033	0.571
	All countries (N. obs. = 406)			
Baseline (approx.)	0.124	0.130	-0.104	0.412
Exact solution	0.103	0.107	-0.134	0.402

Notes: This table presents the summary statistics of the correlations of the model $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) computed using the linear approximation and the exact solution. Variable definitions and sources are described in detail in the text.

(B.22). Instead, we would need to posit the following production function:

$$Y_{njt} = Z_{njt}\left[\nu_F^{\frac{1}{\sigma}}\left(\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \sum_{\substack{m\neq n\\ i}}\mu_{mi,nj}^{\frac{1}{\varepsilon_1}}X_{mi,njt}^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right)^{\frac{\varepsilon_1}{\varepsilon_1-1}\frac{\sigma-1}{\sigma}} + \nu_X^{\frac{1}{\sigma}}\left(\sum_i \mu_{ni,nj}^{\frac{1}{\varepsilon_2}}X_{ni,njt}^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right)^{\frac{\varepsilon_2}{\varepsilon_2-1}\frac{\sigma-1}{\sigma}}\right]^{\gamma_j\frac{\sigma}{\sigma-1}}. \tag{B.23}$$

That is, domestic value added is bundled with foreign inputs, and then with domestic inputs with a possibly different elasticity. Note that it is still the case that as $\sigma \to 1, \varepsilon_1 \to 1, \varepsilon_2 \to 1$, we obtain the production function used in the baseline analysis. Then, the $AUT2$ counterfactual is the following limiting case:

$$AUT2 : \tau_{mnj} \to \infty; \varepsilon_1 \downarrow 1; \sigma = 1.$$

Appendix C Robustness and Additional Exercises

This appendix presents various robustness exercises for the results in our static model in Section 5.

Alternative Supply and Substitution Elasticities Table A8 reports the correlations of the aggregated non-technology shocks under different assumptions on ρ and ψ_u. Table A9 reports the correlations in the unweighted (rather than Domar-weighted) shocks under the different assumptions on ψ_u.

Table A10 computes the model with both shocks and each shock individually using a higher Frisch elasticity of 2 to calibrate ψ_h. This higher elasticity is commonly used in business cycle models as it increases the response of factor supply to shocks, improving model fit. We find that comovement is much higher for both G7 and all countries with the higher Frisch elasticity. The relative importance of technology and non-technology shocks does not change. Notice that our baseline choice of 0.5 for the Frisch elasticity is more consistent with evidence from microdata, and still delivers substantial comovement.

Increasing the elasticity of the capital supply curve has a similar effect to increasing the labor supply elasticity. Table A11 illustrates that a choice of a lower ψ_u, implying very elastic utilization, increases comovement. Finally, Table A12 illustrates the fit of the model with calibrations of ψ_u that vary by sector using the structural restriction on parameters implied by our TFP estimation in equation (4.3). While the structural estimates are very noisy (and not significantly different from our baseline choices of ψ_u in most cases), the results from the model and counterfactuals are not qualitatively different from the baseline.

Table A13 presents the results of the shock correlation-transmission decomposition (2.6) the G7 countries and $\rho = 1$. Transmission in the baseline model is higher under the lower elasticity, as would be expected. Figure A5 reports the change in the influence vectors by sector under $\rho = 1$.

Alternative Models Table A14 reports the results from a G7 only version of our model (using the time period 1978-2007, the longest available time period for these countries). Table A15 reports the results from our baseline model with trade deficits evolving as they do in the data, solved using the method in Dekle, Eaton, and Kortum (2008). In neither case do the conclusions regarding model fit or the relative importance of technology vs non-technology shocks change compared to the baseline.

TABLE A8: Correlations in $d\ln\xi_{nt}$ Summary Statistics

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. $=21$)			
$\rho = 2.75$				
$\psi_u = 4$	0.218	0.224	0.060	0.405
$\psi_u = 1.01$	0.238	0.303	0.080	0.426
ψ_u^j	0.164	0.144	0.002	0.311
$\rho = 1$				
$\psi_u = 4$	0.175	0.185	-0.013	0.430
$\psi_u = 1.01$	0.199	0.254	0.049	0.421
ψ_u^j	0.139	0.175	-0.017	0.264
	All countries (N. obs. $=406$)			
$\rho = 2.75$				
$\psi_u = 4$	0.017	0.039	-0.205	0.240
$\psi_u = 1.01$	0.020	0.050	-0.219	0.258
ψ_u^j	0.015	0.002	-0.181	0.221
$\rho = 1$				
$\psi_u = 4$	0.012	0.025	-0.207	0.242
$\psi_u = 1.01$	0.014	0.029	-0.217	0.255
ψ_u^j	0.020	-0.009	-0.181	0.220

Notes: This table presents the summary statistics of the correlations of $d\ln\xi_{nt}$ defined in (4.9) in the sample of G7 countries (top panel) and full sample (bottom panel), for alternative values of ψ_u. Rows labeled "ψ_u^j" report the results when ψ_u is inferred from the estimates of ζ_j using the structural relation implied by (4.2) and the other calibrated parameters. Variable definitions and sources are described in detail in the text.

Model correlations using the Solow residual Table A16 reports the model correlation using the Solow residual as technology shock, instead of the utilization-adjusted TFP. In this model, ψ_u and ψ_e are set to infinity, effectively shutting down the utilization and effort channel. Table A17 shows the decomposition of the correlation into direct effects, the direct transmission and the multilateral transmission.

Table A9: Correlations of unweighted shock country averages summary statistics, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
TFP	0.141	0.248	-0.146	0.323
Non-technology ($\psi_u = 4$)	0.199	0.244	0.097	0.366
Non-technology ($\psi_u = 1.01$)	0.192	0.202	0.033	0.441
Non-technology (ψ_u^j)	0.191	0.170	0.056	0.340
	All countries (N. obs. = 406)			
TFP	-0.004	0.017	-0.223	0.227
Non-technology ($\psi_u = 4$)	0.032	0.053	-0.190	0.260
Non-technology ($\psi_u = 1.01$)	0.036	0.055	-0.176	0.266
Non-technology (ψ_u^j)	0.014	0.030	-0.214	0.249

Notes: This table presents the summary statistics of the correlations of the unweighted average TFP and non-technology shocks in the sample of G7 countries (top panel) and full sample (bottom panel) for a Frisch elasticity of 0.5. Rows labeled "ψ_u^j" report the results when ψ_u is inferred from the estimates of ζ_j using the structural relation implied by (4.2) and the other calibrated parameters.

Table A10: Model Fit and Counterfactuals under Frisch elasticity=2: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.464	0.532	0.352	0.700
Non-Technology Shocks Only	0.317	0.371	0.165	0.469
Technology Shocks Only	0.149	0.187	-0.073	0.367
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.189	0.266	-0.095	0.513
Non-Technology Shocks Only	0.038	0.029	-0.190	0.263
Technology Shocks Only	0.018	0.033	-0.205	0.237

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks. Variable definitions and sources are described in detail in the text.

Table A11: Model Fit and Counterfactuals under $\psi_u = 1.01$: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.464	0.532	0.352	0.700
Non-Technology Shocks Only	0.317	0.371	0.165	0.469
Technology Shocks Only	0.149	0.187	-0.073	0.367
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.189	0.266	-0.095	0.513
Non-Technology Shocks Only	0.038	0.029	-0.190	0.263
Technology Shocks Only	0.018	0.033	-0.205	0.237

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks. Variable definitions and sources are described in detail in the text.

TABLE A12: Model Fit and Counterfactuals under Structural ψ_u^j: Correlations of $d\ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.146	0.141	-0.146	0.430
Non-Technology Shocks Only	0.255	0.246	0.064	0.461
Technology Shocks Only	0.123	0.157	-0.095	0.344
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.084	0.097	-0.152	0.346
Non-Technology Shocks Only	0.011	0.017	-0.196	0.233
Technology Shocks Only	0.001	0.004	-0.216	0.225

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks, when ψ_u^j is inferred from the estimates of ζ_j using the structural relation implied by (4.2) and the other calibrated parameters. Variable definitions and sources are described in detail in the text.

Table A13: Transmission of Shocks, $\rho = 1$

	G-7 countries (N. obs. = 21)			
Baseline:	0.246	0.341	0.020	0.564
Decomposition:				
Shock Correlation	0.108	0.184	-0.111	0.437
Bilateral Transmission	0.046	0.034	0.026	0.052
Multilateral Transmission	0.092	0.065	0.046	0.134
	All countries (N. obs. = 406)			
Baseline:	0.119	0.118	-0.128	0.402
Decomposition:				
Shock Correlation	0.048	0.054	-0.172	0.300
Bilateral Transmission	0.015	0.008	0.004	0.019
Multilateral Transmission	0.056	0.044	0.010	0.109

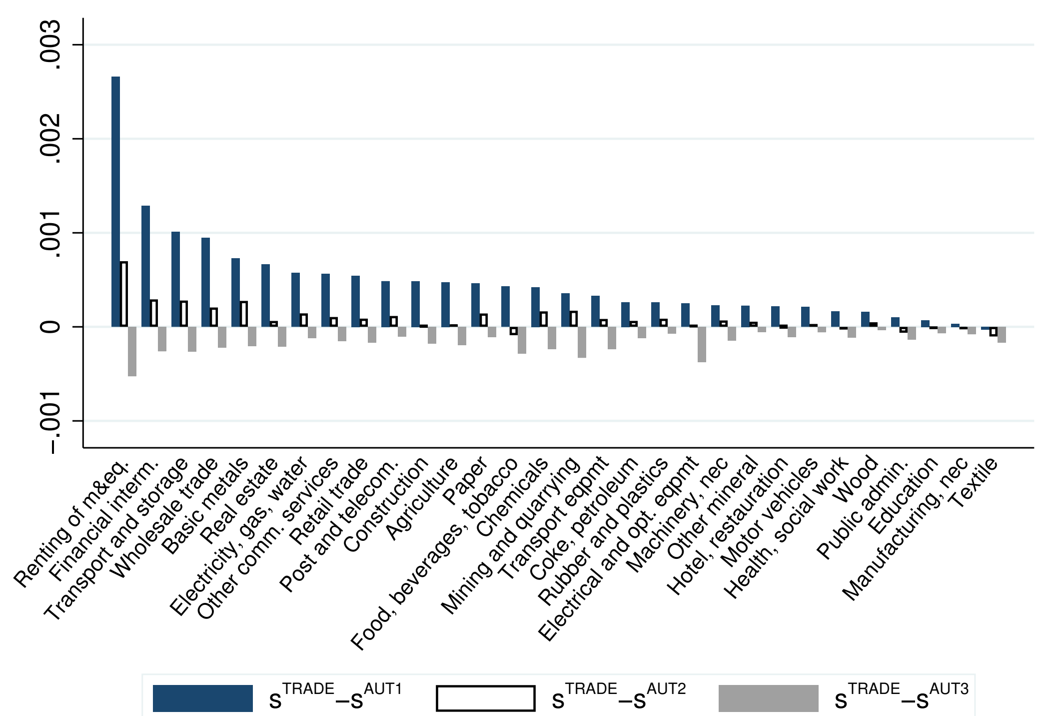
Notes: This table presents the decomposition of the transmission of observed shocks into direct effects, the direct transmission and the multilateral transmission based on the influence vector approximation.

Table A14: Model Fit and Counterfactuals with longer G7+RoW sample: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	Shocks correlations (N. obs. = 21)			
$d \ln Z_{nt}$	0.018	0.000	-0.078	0.151
$d \ln \xi_{nt}$	0.126	0.101	0.004	0.285
	$d \ln Y_{nt}$ correlations (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.291	0.345	0.124	0.423
Non-Technology Shocks Only	0.135	0.139	0.025	0.260
Technology Shocks Only	0.030	-0.001	-0.044	0.140

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries, for data from 1978 to 2007, in the data and the model under the different shocks, in a model with G7 countries only and a rest of the world composite. Variable definitions and sources are described in detail in the text.

Figure A5: Average Changes in the Influence Vectors: Trade vs. Autarky Models, $\rho = 1$



Notes: This figure displays the average change in the direct influence vectors between the baseline model and each of the autarky models.

Table A15: Model Fit and Counterfactuals with Deficits: $d\ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
Data	0.358	0.337	0.242	0.565
Model	0.238	0.369	-0.036	0.559
Non-Technology Shocks Only	0.295	0.332	0.070	0.449
Technology Shocks Only	0.112	0.157	-0.095	0.322
	All countries (N. obs. = 406)			
Data	0.190	0.231	-0.027	0.437
Model	0.103	0.109	-0.132	0.409
Non-Technology Shocks Only	0.021	0.011	-0.199	0.270
Technology Shocks Only	0.014	0.031	-0.209	0.226

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) in the data and the model under the different shocks, while allowing for the aggregate trade deficits to evolve as they do in the data. Variable definitions and sources are described in detail in the text.

Table A16: Counterfactual with Solow residuals only: $d\ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
Frisch elasticity=0.5	0.033	0.014	-0.144	0.205
Frisch elasticity=2	0.047	0.013	-0.138	0.223
	All countries (N. obs. = 406)			
Frisch elasticity=0.5	0.040	0.011	-0.189	0.298
Frisch elasticity=2	0.046	0.028	-0.199	0.307

Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel), when feeding the Solow residual as technology shock in our model, shutting down utilization and effort. Variable definitions and sources are described in detail in the text.

Table A17: Transmission of Shocks, for Solow residual only, $\rho = 2.75$, Frisch=0.5

	G-7 countries (N. obs. = 21)			
Total:	0.033	0.014	-0.144	0.205
Decomposition:				
Shock Correlation	0.019	0.002	-0.168	0.180
Bilateral Transmission	0.007	0.005	0.002	0.006
Multilateral Transmission	0.008	0.007	0.003	0.011
	All countries (N. obs. = 406)			
Total:	0.040	0.011	-0.189	0.298
Decomposition:				
Shock Correlation	0.031	0.012	-0.198	0.286
Bilateral Transmission	0.002	0.001	0.000	0.002
Multilateral Transmission	0.006	0.006	-0.001	0.014

Notes: This table presents the decomposition of the transmission of technology shocks measured as Solow residuals only into direct effects, the direct transmission and the multilateral transmission based on the influence vector approximation.

C.1 Static Counterfactuals: Other Business Cycle Moments

While our focus in this paper is on GDP comovement, we also report the standard deviations of key variables in our data and model in Table A18. The static model replicates about 30-40% of the standard deviation of real GDP and consumption and about 15% of the standard deviation of imports and exports. As we do not permit changes in m or n in this exercise, the difference in volatility between model and data is unsurprising.

TABLE A18: Business Cycle Moments: Volatility

	Mean	Median	Mean	Median
	G7		All countries	
		Data		
Real GDP	0.091	0.094	0.149	0.130
Consumption	0.095	0.107	0.151	0.134
Imports	0.194	0.186	0.246	0.243
Exports	0.159	0.132	0.241	0.235
		Model		
Real GDP	0.037	0.032	0.048	0.039
Consumption	0.037	0.032	0.048	0.039
Imports	0.033	0.025	0.042	0.035
Exports	0.035	0.030	0.046	0.038

Notes: This table presents the average standard deviation of the log of various variables for the data and the model for the static counterfactual.

C.2 The Trade-Comovement Relation

Table A19 reports the results of running the "standard" trade-comovement regression in our data and the static model. This is a regression of bilateral real GDP correlation on a measure of bilateral trade intensity. A long literature following Frankel and Rose (1998) tries to understand why economies that trade more display higher GDP comovement in the data. Vertical linkages have been suggested as an explanation for the trade-comovement puzzle in a number of papers (see for instance Kose and Yi (2006), di Giovanni and Levchenko (2010) and Johnson (2014)). Quantitatively, however, models have trouble generating even the same order of magnitude as the empirical relationship (model coefficients are often <10% of their empirical counterparts). Our model with just the static network linkages obtains the same order of magnitude of this relationship as in the data. The coefficients in the model are about 55% of their empirical counterparts.

Table A19: The Trade-Comovement Relation

	Dep. Var: Bilateral GDP growth correlation					
	Data		Model		Model with n and m	
Trade intensity (avg)	0.085***					
	(0.012)					
Trade intensity (1995)		0.086***				
		(0.011)				
Model trade intensity (avg)			0.047***		0.089***	
			(0.010)		(0.011)	
Model trade intensity (1995)				0.047***		0.089***
				(0.010)		(0.011)
N	406	406	406	406	406	406

Notes: This table presents the results of a regression of bilateral GDP growth correlation on trade intensity for the data (first panel), the baseline static model (second panel) and the static model with employment and capital growth from the data (third panel). Trade intensity is defined as the sum of bilateral flows over the sum of the two countries' GDPs. The first row uses the average trade intensity over the 1995-2007 period, while the second row uses the initial intensity.