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Joyee Deb
Aniko Oery
Kevin R. Williams

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ABSTRACT

We study reward-based crowdfunding campaigns, a new class of dynamic contribution games where consumption is exclusive. Two types of backers participate: buyers want to consume the product while donors just want the campaign to succeed. The key tension is one of coordination between buyers, instead of free-riding. Donors can alleviate this coordination risk. We analyze a dynamic model of crowdfunding and demonstrate that its predictions are consistent with high-frequency data collected from Kickstarter. We compare the Kickstarter mechanism to alternative platform designs and evaluate the value of dynamically arriving information. We extend the model to incorporate social learning about quality.

Joyee Deb
Yale School of Management
165 Whitney Ave
New Haven, CT 06520
joyee.deb@yale.edu

Aniko Oery
Yale School of Management
165 Whitney Ave
New Haven, CT 06520
aniko.oery@yale.edu

Kevin R. Williams
Yale School of Management
165 Whitney Avenue
New Haven, CT 06520
and NBER
kevin.williams@yale.edu

1 Introduction

Reward-based crowdfunding platforms allow entrepreneurs to raise funds directly from a wide base of supporters prior to the launch of a product. A typical campaign specifies a product or reward, a price, a fund-raising goal, and a deadline. Backers only receive the product and pay the price if the goal amount is raised by the deadline. However, potential buyers of the product are not the only contributors to crowdfunding campaigns. Crowdfunding platforms also allow supporters to donate to campaigns and receive nothing in return. In fact, we find that donations constitute 26 percent of all funds raised on Kickstarter, the largest crowdfunding platform.[1]

Motivated by this, we study a new class of dynamic contribution games with two significant features. First, we allow contributors to have different preferences: buyers only care about obtaining the product and the donor simply cares about success of the campaign. Second, unlike the earlier literature on dynamic contribution games, such as Admati and Perry (1987) and Fershtman and Nitzan (1991), we do not study a public good—a good that is non-rivalrous and non-exclusive—but rather, we study a non-rivalrous but exclusive good. The product is exclusive in the sense that a buyer can enjoy the product only if she buys it. These features are economically substantive. With exclusive goods, buyers cannot free ride on other buyers' contributions.[2] Instead, the key trade-off in our setting is one of coordination since it is costly for buyers to commit to purchasing if they think the product is unlikely to succeed. For example, buyers might forgo an opportunity to buy a substitute good or to use the pledged money for other consumption. The donor who solely cares about the success of the campaign can alleviate some of this coordination risk of buyers by donating funds over time.

We examine how buyers and donors dynamically interact in such a contribution game both theoretically and empirically. The equilibrium behavior predicted by our continuous-time model closely matches our empirical findings using new, high-frequency data collected from Kickstarter.

[1] Kickstarter says the following on its website: "Many backers are rallying around their friends' projects. Some are supporting a new effort from someone they've long admired. Some are just inspired by a new idea, while others are motivated to pledge by a project's rewards...." (Source: https://help.kickstarter.com/hc/en-us/articles/115005047933-Why-do-people-back-projects-. Accessed on December 4, 2018.)

[2] Incentives to free-ride arise in contribution games (e.g., Varian (1994), Marx and Matthews (2000), Campbell et al. (2014) and Cvitanić and Georgiadis (2016)), as well as in other dynamic games with externalities (e.g., Bonatti and Hörner (2011)) where agents may free-ride on each other's effort rather than contributions. An exclusive good eliminates such incentives in our set-up. Further, by assuming a representative donor, we also abstract away from potential free-riding among donors

The theory allows us to make welfare comparisons between the current design of crowdfunding platforms, like Kickstarter, and other counterfactual mechanisms that highlight the importance of dynamic donations and the role of dynamic information revelation. Finally, we incorporate social learning to analyze its effects on contribution dynamics. We find that uncertainty about product quality reduces the effectiveness of donations in alleviating the coordination risk of buyers.

While our model is tailored to some of the institutional details of crowdfunding, the insights are applicable to other dynamic contribution settings. For instance, consider public schools that run campaigns targeting both direct beneficiaries, such as parents, and donors, such as local businesses, to raise money to fund programs not covered by their regular budgets. More broadly, our framework can be adapted to study the dynamics of charitable giving or the design of fundraising capital campaigns.[3]

The starting point for this paper is a set of new stylized facts that we establish using novel data collected from the largest crowdfunding platform, Kickstarter. We track all campaigns, every twelve hours, through one week past the deadline. Our data cover all campaigns between March 2017 and June 2018. Unlike all prior empirical work on crowdfunding, our data collection approach allows us to distinguish between buyers and donors on Kickstarter.

First, we show that donations are important. On average, 26 percent of campaign revenues come from donations, ranging from 17 to 38 percent for the four largest categories on the platform – Design, Film and Video, Games, and Technology. Second, we show that the distribution of completion times is U-shaped, and this shape is driven by different types of backers. Projects that finish close to the deadline are primarily funded by donations whereas projects that finish early are primarily driven by buyers. Late-finishing campaigns represent 28 percent of successful projects, and donations exceed purchases four-to-one in terms of revenue for these projects. Moreover, there is significant bunching of funds raised at the goal amount, especially for late-finishing campaigns. That is, donations just make the project succeed. On the contrary, projects that finish early tend to receive many donations and purchases, and they tend to raise significantly more funds than the goal amount. Projects that fail often fail with only a small amount raised.[4] Third, we find that donations

[3]The literature on charitable giving also considers, theoretically and with experiments, the effect of offering refunds in real fund-raising campaigns in case the fundraising goal is not met. See, for instance, List and Lucking-Reiley (2002) and Andreoni (1998).

[4]Kickstarter states on its website that 78% of projects that raised more than 20% of their goal were successfully funded. Source: https://www.kickstarter.com/help/stats. Accessed on November 10, 2018

tend to drop off sharply after a campaign meets its funding goal, while purchases continue. This is consistent with donors caring about the success of a campaign, but not necessarily the total amount of revenue raised. Finally, donations and purchases occur throughout time, which motivates our study of their dynamics.

We model crowdfunding as a finite-horizon contribution game in continuous time. Buyers arrive randomly over time and there is a single representative long-lived donor. A campaign is characterized in terms of the funding goal, a price, a deadline, and parameters that govern buyer preferences and donor wealth. A buyer, conditional on arriving, can either pledge to purchase the product at the posted price or choose an outside option. The long-lived donor, representing the network of family and friends, can choose to donate any amount, at any time, until the deadline. Crowdfunding platforms do not allow the entrepreneur to donate funds directly; this motivates our assumption on the preferences and payoffs of the donor. The donor receives a fixed benefit if the campaign is successful. The benefit is independent of the amount raised.[5]

Both types of agents face uncertainty. Arriving buyers are uncertain about both future arrivals of other buyers and of the donor's wealth. The donor is uncertain about future arrivals of buyers. If the campaign meets the funding goal, all transactions are realized, and if the campaign fails, then the donor and buyers get their pledges back, but buyers bear the opportunity cost of pledging and not exercising their outside option.

The role of the donor is to help coordinate the actions of potential buyers. He wants to maximize the probability of reaching the goal while minimizing the amount that he donates. As buyers make their pledging decision based on their assessment of the probability of success of the campaign, the donor has the ability to influence these expectations throughout time by signaling about his wealth via his dynamic donation strategy.

We analyze perfect Bayesian equilibria of this dynamic contribution game when buyers can observe the number of buyers who have pledged so far, total donations so far, and time. Our central focus is on the equilibrium that maximizes the probability of success of a campaign. We call this the "platform-optimal equilibrium." We show that the platform-optimal equilibrium is also optimal for the donor. We also characterize the buyer-optimal equilibrium and describe the entire set of

[5]While our modeling choices are inspired by Kickstarter, we make some simplifying assumptions. We focus on within-campaign dynamics and abstract away from competition on the platform and optimal campaign design. We also assume that there is a single product version offered at a fixed price with no capacity constraints, as our data indicates that typically capacity constraints are not binding.

equilibria.

In any equilibrium, the donor donates over time to ensure that the next arriving buyer is willing to purchase until donor wealth is exhausted. At the deadline, he donates the remaining amount needed to meet the goal, if he has any wealth left to do so. Buyers buy as long as they arrive "early enough;" we show that for any equilibrium, there exist threshold times such that a buyer purchases if and only if she arrives before the threshold time. These times depend on the number of additional buyer arrivals required for success and time remaining,

The equilibrium construction is not straightforward due to the richness of the payoff-relevant state space. Equilibrium behavior of buyers and the donor depend on total purchases, donations, and time remaining. We transform the problem and use an induction argument on the number of additional buyers necessary for the campaign to succeed, given current donations and time remaining, and then embed an induction with respect to time within each of these induction steps. We derive a recursive formula for the buyers' beliefs about the probability of success. The probability depends on current donations, the belief about the wealth of the donor, the random arrival time of the next buyer, and probability of success from the perspective of the next buyer. At each induction step, we also define the minimum level of total donation necessary to ensure that the next arriving buyer will purchase. This in turn allows us to implicitly define cutoff times after which buyers cannot be incentivized to pledge because the donor has run out of wealth.

Next, we derive some intuitive comparative statics. We show, unsurprisingly, that the probability of success of a campaign is increasing in the duration of the campaign and decreasing in the number of buyers required. However, and more surprisingly, we show that the completion time is neither monotonic in campaign duration nor monotonic in the number of buyers required.

The platform-optimal equilibrium is consistent with the data. Contributions can spike only at the start of the campaign and again at the deadline. They increase smoothly in the interim. The reason for the spike at the start is that if buyers believe that donor wealth is low, the donor may initially donate to signal that he has enough wealth to keep future buyers active.[6] Also, at the deadline, if the donor has not exhausted his wealth, he would donate the remainder required. During the interim, the equilibrium strategy prescribes continuous donations. This matches the U-shaped distribution of revenue over the duration of campaigns. Moreover, if arrivals do not occur soon enough, the

[6]The literature on charitable giving talks about the importance of "seed money" at the start of a campaign. In a sense, our equilibrium provides a theoretical justification for this. See Footnote 26 for a discussion.

donor may run out of funds early, i.e., reaching a sufficient number of buyers early on is crucial for success. Late-finishing campaigns attract a large amount of donations close to the deadline and just meet the goal, while early finishing campaigns attract more purchases. These campaigns continue to raise revenue after success. Finally, donations drop to zero once the campaign succeeds, while buying continues.

Our framework also allows us to ask how the existing mechanism on Kickstarter performs relative to alternative mechanisms. While optimal mechanism design is beyond the scope of this paper, we analyze natural conterfactual settings that highlight the role of dynamic donations and of dynamic information revelation.

First, we consider a setting with no donations, a setting in which the donor has only one opportunity to donate, at the start of the campaign, and a setting in which the donor has only one opportunity to donate, at the end of the campaign. βetabrand is an example of a platform that does not allow donations. It can also be considered a platform with a single donation at the start of the campaign, through a reduction in goal amount. We find that in all three counterfactual settings, the probability of success is lower and welfare is lower for both donor and buyers compared to any equilibrium of the Kickstarter mechanism. Note that the donor's and buyers' incentives are not aligned, as the buyer does not want to buy if the campaign is not sufficiently likely to succeed, and buyers' opportunity costs are not internalized by the donor. Thus, the Kickstarter mechanism coordinates the actions of different players better than those alternative mechanisms.

Second, since both the donor and buyers use the information about current total donations and the current number of buyers to optimize their decisions, we ask whether the dynamically arriving information unambiguously improves the probability of success of a campaign. We analyze a benchmark setting in which buyers and the donor do not observe the number of buyers or the donations made so far. We show, somewhat surprisingly, that dynamic information revelation can increase or decrease the probability of success of a campaign depending on its properties.

Finally, we extend the model to allow for learning about product quality. A perceived benefit of crowdfunding is that it allows for social learning. We consider an environment in which each buyer privately receives a binary signal about product quality and updates based on current pledges. The model is a "bad news" model in which a high quality project only generates positive signals. Buyers now care about the quality of the product conditional on the event that the campaign is successful.

We characterize an equilibrium analogous to that in the setting without learning and highlight a new trade-off. As before, if the probability of success is too low, buyers do not want to incur the opportunity cost of pledging. Thus, donations can help make buyers more optimistic. However, if the probability of success of the campaign is too high, then success is not a strong signal for quality and buyers' willingness to pay is low. Therefore, donations increase the probability of success but make learning less effective: a dollar of donation is less effective in encouraging buyers, especially if there is large uncertainty about the quality of the project.

1.1 Literature on Crowdfunding

Our paper is related to a new but growing literature on crowdfunding. Strausz (2017) solves a mechanism design problem when there is uncertainty about the number of interested buyers in the market - but not their purchasing decision - and when the entrepreneur suffers from a moral hazard problem. He shows that the all-or-nothing mechanism, where transactions only take place if a funding goal is reached, is optimal from a social perspective because the good is only produced if sufficiently many buyers are present to cover the fixed cost of production. If the entrepreneur faces a moral hazard problem—if she can run away with the money raised without delivering the product—the optimal mechanism entails an inefficiently high funding goal. Deferred payments, e.g. in the form of an "after-market," to the entrepreneur can alleviate this problem. We do not model the moral hazard problem, but we do empirically investigate its presence using unsupervised machine learning on platform comments and updates. We find that moral hazard is not widely prevalent, and most negative sentiment is related to delays, but not nondelivery. This is consistent with Strausz (2017) who argues that the Kickstarter mechanism optimally addresses moral hazard.

Bagnoli and Lipman (1989) show in a richer environment, but with complete information, that an equivalent mechanism even implements the first best.[7] Sahm (2016) and Chakraborty and Swinney (2016) consider a reduced form of Strausz (2017), but include a second stage of selling after the crowdfunding campaign. Ellman and Hurkens (2016) also consider a mechanism design approach, absent moral hazard, but where buyers can have idiosyncratic high or low valuations for the product.[8] Their focus is on the possibility to price discriminate by allowing high-valuation buyers to bid

[7] Agrawal et al. (2014) call this a "provision point mechanisms" in their survey paper.

[8] Chang (2016) analyzes the entrepreneur's choice of a funding mechanism in a similar setup.

a higher price to increase the probability of success. All the above papers abstract away from within-campaign dynamics, which is the focus of our work. We are interested in a complementary question of how strategic donors and buyers interact dynamically to affect the success of a campaign.

A closely related model in which consumers arrive sequentially and make strategic purchasing decisions is considered by Alaei et al. (2016). Our models share some basic features, but Alaei et al. (2016) do not allow for donations and characterize the equilibrium outcome using anticipating random walks which allows for nice simulations. We instead offer a parsimonious recursive characterization of the equilibrium that highlights the economic trade-offs. This allows us to capture the dynamic strategic interaction between buyers and donors, to conduct counterfactual analyses, and incorporate social learning. Liu (2018) allows for learning with strategic buyers; however, she also does not consider donations.[9]

To the best of our knowledge, this paper is first to examine empirically the dynamics of reward-based crowdfunding with both buyers and donors, though the literature has studied donors in different contexts.[10] Van de Rijt et al. (2014) design an experiment that shows donations affect key outcomes, like success probability, on Kickstarter. Agrawal et al. (2015) analyze the role of friends and family in equity-based crowdfunding campaigns for artists, where contributors buy shares of the company instead of receiving a product or just donating. They show that local contributors (friends and family) are less responsive to the cumulative amount raised than distant contributors. The authors conclude that this is because friends and family have superior information about the project and they invest based on their private information. For reward-based crowdfunding, we assume that friends and family do not directly care about the quality of the product because they do not receive any direct return like in equity crowdfunding. Lee and Persson (2016) also highlights the importance of informal financing and donations in a more classic investment setup. Other recent equity crowdfunding work include Grüner and Siemroth (2017), Abrams (2017), and Li (2017). Equity crowdfunding is closer to an investment game rather than a contribution game.

[9] Chemla and Tinn (2016) and Chang (2016) also allow for aggregate uncertainty about demand by considering a two and three period game, respectively.

[10] Kim et al. (2017) empirically investigates contributions, but they abstract from donations.

2 Data and Empirical Evidence

2.1 Background

Crowdfunding is a recent phenomenon.[11] Kickstarter is the largest platform in this space and has helped fund over 155,000 projects, raising over 4 billion dollars from 16 million people.[12] Entrepreneurs post projects with a funding goal, a deadline, prices for rewards, and if desired, capacity limits for each reward. Rewards are typically the product offered by the entrepreneur. Kickstarter limits the length of a campaign to at most two months. Individuals can pledge for a particular reward level or they can pledge to donate any amount and receive no reward in return. The platform uses an "all-or-nothing" model which means that transactions are realized if and only if the funding goal is reached by the deadline.

Several of the key features of Kickstarter campaigns can be seen in Figure 7 in Appendix A that shows an active campaign for a heated shirt. A basic summary of the campaign is included at the top of the page. It shows that the campaign has 16 days remaining, but it has already raised over $145,000 on a goal of just $10,000. The next part of the web page shows information about the project and how to contribute. The first bucket shown is the pure donation option. This disappears after the deadline. The next option is a $129 bucket that includes the main reward. Note that this reward states, "Pledge $129 or more," so one way to donate on the platform is to specify an amount greater than the listed price. Currently there are 542 backers with a capacity of 560. In total, the campaign has six different reward levels that all include one or more of the new heated shirt.

Campaigns on Kickstarter are diverse, with products ranging from documentary films to new high tech products. The largest categories in terms of revenue are Games, Design, and Technology. Individuals and startups are common creators of campaigns. For example, the Pebble Smartwatch is one of the most successful Kickstarter campaigns to date. The campaign raised over $20 million from a $500 thousand goal. Large companies have recently started participating in reward-based crowdfunding campaigns as well.[13]

[11]The two largest platforms are Kickstarter and Indiegogo, which were founded in 2009 and 2008, respectively. With reward-based crowdfunding, individuals pledge to either buy the reward offered or pledge a pure donation. These services differ from platforms, such as Gofundme and Fundly, that focus exclusively on donation-based campaigns. Donation-based platforms do not offer rewards; their aim is to raise money for causes. Other types of crowdfunding also exist, such as debt-based crowdfunding (e.g., Prosper.com) and equity-based crowdfunding.

[12]Source: https://www.kickstarter.com/help/stats. Accessed on December 5, 2018.

[13]Indiegogo Enterprise caters to established companies and has hosted campaigns for Bose, Sony, P&G and Heineken,

2.2 Data and Summary Statistics

We create a new data set on all campaigns posted on Kickstarter from March 2017 to September 2018 using a web scraper. We track the progress of each campaign, every twelve hours, through one week after a campaign's specified ending time. Whereas prior papers on Kickstarter use data reporting an aggregate measure of contributions, our data collection scripts capture some information on buyers and donors separately.[14] By text processing the campaign web pages, we recover the total number of donors every twelve hours. We also capture the total flow of revenue and revenue coming from reward purchases. The difference comprises donor contributions—coming from either the donate option or from entering a price greater than the bucket price—plus any incremental shipping revenue from purchases. This is because shipping costs are included in the progress towards the goal but are not included in the base prices listed on the platform, i.e., we see both left-hand-side variables in the equation below, but only the sum of the right-hand-side variables.

$$\text{Total Revenue}_t - \text{Backer Revenue}_t = \text{Donor Revenue}_t + \text{Shipping Costs}_t.$$

Shipping costs are not known exactly as we do not have individual purchase data and shipping costs vary across geographies. We subtract off shipping cost estimates by gathering reward-level shipping costs for every project, if they exist, and assigning a shipping cost to each purchase in the data. The most frequently seen shipping options are free shipping, single-rate shipping, or worldwide shipping with region-specific or country-specific prices. We complete all analyses under three assignments: (i) least-expensive shipping, (ii) all buyers are located in the United States, and (iii) most-expensive shipping.[15] These specifications are important because they provide lower and upper bounds of donations. In our baseline results, we use (ii); the other specifications appear in the robustness section (Section 2.4). In our calculations, we also incorporate the bound

$$\text{Shipping Costs}_t \leq \text{Total Revenue}_t - \text{Backer Revenue}_t,$$

among others. Source: http://enterprise.indiegogo.com/. Accessed on December 5, 2018.

[14] Existing empirical work on Kickstarter has utilized data from Kicktraq, Kaggle or constructed data with similar key features. These papers include: Kuppuswamy and Bayus (2018), Grüner and Siemroth (2017) and Liu (2018). The empirical results in these papers are based on the sum of revenue coming from both buyers and donors, divided by the number of buyers and donors combined, each period.

[15] With USA shipping, we assume all products are delivered to the United States. This is motivated by the fact that most campaigns originate from the United States.

since donations are positive contributions to campaigns.

We do not have access to individual-specific donations and cannot separate pure donations from donations occurring from contributions above the reward price. However, the median number of donors per period is one. So we can usually capture the actual donation amount after subtracting shipping costs. We define buyers to be individuals who pledge for any reward; we see contributions and prices for each reward. Some rewards may better be classified as a donation, such as a thank you card, so we also conduct robustness to this definition in Section 2.4.

In addition to the information on buyers and donors, we also collect all the information presented on the main campaign page for nearly every successful project ever posted on Kickstarter. This includes all updates by the entrepreneur as well as all feedback from the community (comments) throughout the campaign. We use this data to evaluate the presence of moral hazard by the entrepreneur using textual analysis. A summary of our findings appears at the end of this section, with a full description placed in the Appendix.

Table 1: Summary Statistics for the Data Sample

Variable	Mean			Median	5th %	95th %
	(All)	(0)	(1)			
Project Length	32.4	33.9	30.5	30.0	15.0	60.0
Goal ($)	20,028.4	26,573.2	11,182.5	7,650.0	500.0	79,052.6
Number of Rewards	7.6	6.3	9.8	6.0	1.0	18.0
Donor Revenue (per period)	75.2	3.8	181.4	0.0	0.0	43.0
Buyer Revenue (per period)	131.5	13.4	307.1	0.0	0.0	400.0
Percent Donations at Deadline	26.1	29.8	21.9	12.9	0.0	100.0
Number of Projects	53,212	30,726	22,486	–	–	–

Note: Statistics are calculated for the 53,212 campaigns included in the sample after data cleaning. A period is twelve hours. Mean is computed for all projects (All) - unsuccessful projects (0) and successful projects (1).

Table 1 shows summary statistics for the 53,212 campaigns included in sample.[16] The table shows overall sample means and means for unsuccessful (0) and successful (1) projects, respectively. For example, most creators specify campaigns to last for 30 days, however, up to 60 days are

[16]We truncate the sample by dropping the bottom one percent and the top one percent of campaigns in terms of the goal amount. This removes campaigns with low $1 goals and campaigns with several million dollar goals (one in the billions). Repeating the analysis with the full data yields nearly identical results. In addition, we drop campaigns that were removed by the creator and campaigns under copyright dispute.

possible. There is a positive correlation between campaign length and goal ($\rho = .13$). The average goal amount is \$20.0 thousand. Unsuccessful projects tend to have higher goals than successful projects. Also, note that most campaigns fail—only 42 percent of campaigns are successful. This number is slightly higher than the statistic reported by Kickstarter (35 percent) for all campaigns since the platform's inception, suggesting the probability of success has increased over time.

All projects have a donation option, and most projects offer several different reward levels. The average number of buckets (rewards levels) in any campaign is 7.6. Successful campaigns have more rewards. Interestingly, 67 percent of buckets do not have a capacity limit and when capacity limits are used by entrepreneurs, only 18 percent of buckets ever reach their limit. This suggests that these buckets are likely not very important, and our model abstracts from capacity constraints. We will abstract from multiple versions of the product (multiple rewards) in the model.

Table 1 also presents basic information on the two types of players in campaigns: buyers and donors. Donors are important. In terms of total revenue, donations make up 26 percent of all contributions made on Kickstarter. We calculate average revenues of \$131 and \$75 per twelve hours for buyers and donors, respectively.[17] However, both distributions are heavily skewed to the right—small amounts are common. Unsuccessful campaigns receive lower revenues from both sources.

There is rich heterogeneity in the types of projects posted on Kickstarter. Table 2 presents the same summary statistics reported in Table 1, but separately for the top four categories, as measured by the number of campaigns. The top four categories are: Design, Film and Video, Games, and Technology. We see that the percent of successful projects vary a lot by category. For example, game campaigns are twice as likely to succeed as technology campaigns, but games also have less than half the average goal amount of technology campaigns. Donor revenue is higher for games, and buyer revenue is not substantially lower. The table also shows that donations are at least 17.3 percent for all four categories. The probability of success is negatively correlated with the amount of revenue coming from donations. The games and design categories have the lowest percent of revenue from donations across all categories in the data. The categories with the highest values are Dance and Journalism—categories with donations between 40-50 percent of total revenue. In the modeling section, we will focus on a single project; however, one could capture the heterogeneity in the data through project-specific parameters.

[17]Note that the time horizon of products differ; we report percent of donations averaged over products and do not weight for time horizon.

Table 2: Top Category Summary Statistics

	Design	Film & Video	Games	Technology
Project Length	33.8	32.8	30.4	35.4
	(10.7)	(12.3)	(10.1)	(11.9)
Goal ($)	23,549.1	20,479.1	19,509.5	39,428.2
	(34,938.3)	(42,273.1)	(36,644.5)	(53,129.6)
Number of Rewards	8.4	7.6	7.6	6.4
	(5.4)	(5.8)	(5.2)	(4.8)
Donor Revenue	156.0	26.2	182.8	100.0
(per period)	(4,121.7)	(660.0)	(4,566.4)	(4,403.1)
Buyer Revenue	327.9	54.8	177.4	218.1
(per period)	(7,308.4)	(593.7)	(1,678.1)	(3,890.6)
Percent Donations	17.5	38.2	17.3	28.4
at Deadline	(25.4)	(31.7)	(24.8)	(35.7)
Percent Successful	47.4	39.2	50.6	23.4
Number of Projects	5,714	5,759	6,919	5,622

Note: Summary statistics for the top four Kickstarter categories, based on the number of projects within category. Standard deviation reported in parentheses.

2.3 Empirical Analysis: Stylized Facts

We establish a series of stylized facts with our data on buyer and donor contributions over time. We return to these facts after discussing the model and equilibrium characterization.

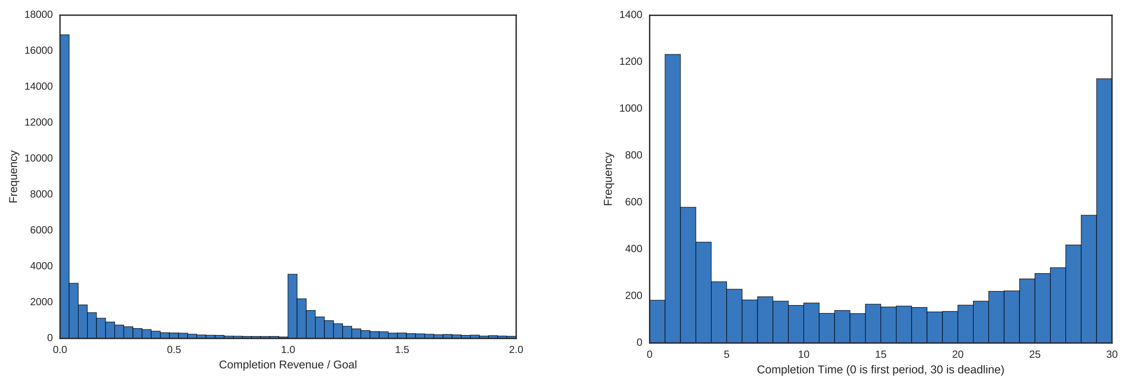
Bimodal revenue and completion times.

Figure 1 contains histograms of two metrics: (a) revenue relative to goal (R/G) at the end of the campaign and (b) the period in which successful campaigns reach their goal. The left panel (a) shows that most campaigns are either unsuccessful with very little money raised or they just reach their goal. That is, there is considerable bunching at zero and one. There is a thin tail beyond two that is not shown. The right panel (b) shows the histogram of success times for 30-day campaigns. The time index $t = 0$ corresponds to the first day the project is offered and $t = 30$ corresponds to the end time. This horizon will also be used with all subsequent time graphs. The graph shows success times are bimodal with many campaigns succeeding close to the start and close to the ending time.

These facts are consistent with the findings in existing work, such as in Kuppuswamy and Bayus (2018).

Figure 1: Frequency Histograms of Final Revenue and Success Time

(a) Final Revenue (b) Success Time



Note: (a) Total campaign revenue is the sum of donations and purchases. The fraction is defined as total revenue divided by the campaign goal at the deadline. (b) $t = 0$ corresponds to the first day of the campaign. $t = 30$ corresponds to the time at which the campaign ends.

Different roles of buyers and donors

Our data allows us to explain a key feature of the bimodal pattern of success times. The data suggests that buyers and donors play a very different role depending on the time until the deadline. We partition campaigns into three groups: campaigns that succeed in the first three days (early), those that succeed in days 3-27 (middle), and those that succeed in the last three days before the deadline (late). With this partition, there are 2,180, 4,427, and 2,729 campaigns in each group, respectively.

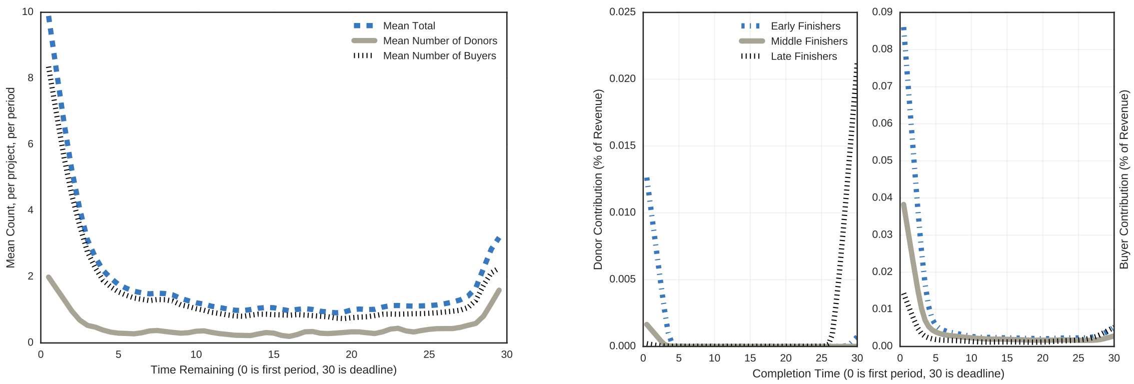
The graphs in Figure 2 show there is a relationship between completion time and the type of contributions being made. Together, the graphs show that for campaigns that succeed early, both donations and purchases are high, but for campaigns that succeed close to the deadline, donations overwhelmingly exceed purchases. That is, there is a deadline effect that is driven by donors.

The left panel (a) in Figure 2 shows the average number of contributions coming from buyers and donors over time. The combined curves present a U-shape pattern of contributions, established in earlier papers. However, this aggregate measure does not capture the selection of campaigns that appear over time. The right panel (b) segments campaigns into the three bins described above and plots contributions from buyers and donors separately.[18] This panel shows that the U-shape of the left plot is the result of different individuals (buyers versus donors) contributing over time.

Figure 2: Average Contributions of Buyers and Donors over Time

(a) Contribution Counts over Time

(b) Contribution Revenue, by Type, over Time



Note: (a) The mean count of donations, purchases, and the sum of donations and purchases for 30-day campaigns. $t = 0$ corresponds to the first day of the campaign; $t = 30$ corresponds to the ending time. (b) Revenue share is defined as the the amount of period donations (purchases), divided by the total amount of revenue (donations plus purchases) at the project end time. There are three lines, corresponding to: 1) campaigns that are successful within the first three days of posting, 2) campaigns that are successful in the last three days before the ending time, 3) all campaigns in between. Graphed are 30-day campaigns.

Figure 2(b) shows that for campaigns that finish early, both donations and purchases are high: nearly 17 percent (median, 14 percent) of total cumulative revenue is gathered within the first day on average. On the other hand, for campaigns that finish close to the deadline, donations exceed purchases by a four to one margin. The large mass of middle finishers see lower donations and lower purchases both in early periods as well as in later periods.[19]

Table 3 contains the descriptive statistics for the three groupings of campaigns and shows that early finishing campaigns are quite different from late finishing ones. Campaigns that finish close to the start time tend to raise much more money relative to the goal, as compared to late finishers (with mean $R/G = 7$ for early finishers versus $R/G = 1$ for late finishers, and medians of 3.5 and 1, respectively). The early- and late-finishing campaigns belong to different categories: Film and Video, Music, and Theater are the top three categories for late-finishing campaigns, whereas, for early finishers, these campaigns belong to Games, Theater, and Design. This again suggests that the

[18] Instead of number of contributions, we plot the percent of period revenue attributed to buyers and donors, divided by the total amount of revenue per project. This controls for the fact that some campaigns raise much more than the goal while others just reach the goal.

[19] One anomalous finding from this analysis is that for campaigns that finish early, there is a small increase in donations at the deadline. This is perhaps due to advertising or that donors also care about total amount raised.

within-campaign dynamics depend on campaign-specific details.

Donations and the time of success

Buyers and donors show different patterns around the success time of campaigns. Figure 3 plots average buyer and donor revenue for 30-day campaigns three days before and three days after the completion time (-3 corresponds to three days before success, +3 is three days after success). Only campaigns that are middle finishers appear in the plot. The plot establishes an important fact that we verify in the model: there is a spike in donations and purchases just before success, but once a campaign reaches its goal, donations drop to a level lower than prior to success and buyers remain active.

Table 3: Descriptive Statistics for Early, Middle and Late Finishing Campaigns

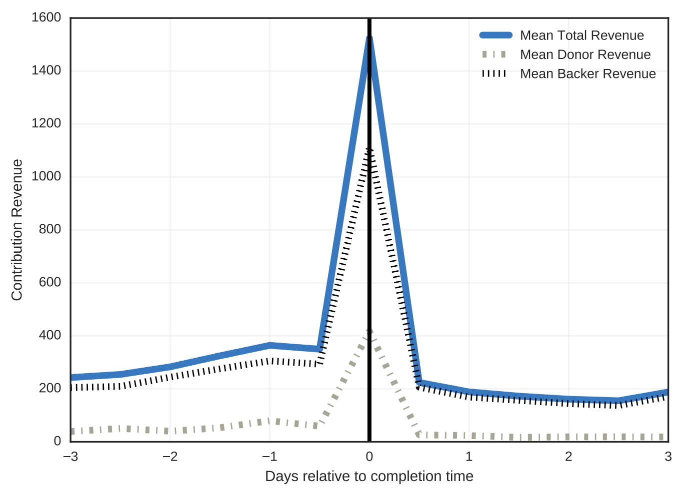
Variable	Early	Middle	Late
Goal ($)	13,664.83	9,890.58	11,398.10
	(32383.30)	(16268.93)	(16543.66)
Number of Rewards	10.25	9.73	10.00
	(7.09)	(6.47)	(6.59)
Average Price	365.49	292.96	325.00
	(3604.98)	(1396.30)	(433.12)
R/G	6.97	1.72	1.08
	(13.98)	(3.35)	(0.11)
$D/R(\%)$	12.26	19.42	29.68
	(18.75)	(20.85)	(22.37)
Number of Projects	2238	4543	2794
Top Categories	Games	Theater	Theater
	Theater	Games	Music
	Design	Music	Film & Video

Note: Summary statistics for successful campaigns partitioned by completion time. Only 30-day projects are included. Early finishers complete within three days. Late finishers complete in the last three days. All other projects are included in the middle category. Standard deviation reported in parentheses.

In levels, average donation revenue decreases by more than half, dropping from $54 to $20. This result can be interpreted as donors caring mainly about the campaign reaching its goal and is reasonable since donors do not receive any rewards. Donations should taper off after the campaign succeeds. In the data, there is still some, albeit lower, donor activity after success. While we cannot reject the null hypothesis that mean donations are equal to zero in all periods after success, we can in terms of donor counts. One possibility as to why donations do not go exactingly to zero is that donors also care about total revenues; however, our theoretical model will abstract from this.

The figure shows that buyer contributions also drop slightly. If buyers are interested in the product offered, then we would expect purchases to not decline after campaigns succeed. However, a drop in purchases is plausible if success and arrivals before success are correlated events. Advertising activity, both on and off the platform, likely declines after completion—Kickstarter does advertise campaigns close to the goal. Also, some buyers may be purchasing buckets that should be classified as donations, which we explore in the robustness section. Finally, another reason purchases decline may be related to increasing average prices after campaigns succeed.[20] Our model can accommodate decreased purchases after completion, however, we do not formally model this as we concentrate on prior-to-success/deadline dynamics.

Figure 3: Average Donations and Purchasers Relative to Success Time



Note: The success time is closest time after the campaign reaches its funding goal for 30 day campaigns. Positive 3 means 3 days after the project is funded; negative 3 means 3 days prior to the project being funded. This plot includes the subset of campaigns in which the success time is greater than three and less than 27 (N=4,996).

[20]While some entrepreneurs close the highest price buckets, they also decrease the capacity of lower buckets. We find that the percent of filled buckets decreases substantially from 2 percent to 11 percent immediately after success. As a result, conditional on looking at buckets with capacity, the mean bucket price increases by $15.40. For buckets with positive sales, the mean price increase is substantially higher at $64.63 (median, $10.63). As an aside, this suggests that entrepreneurs give up to a 10-20 percent discount prior to the success of the campaign, calculated at average campaign prices.

Moral hazard and the nondelivery of products

Finally, we examine the presence of moral hazard using natural language processing techniques. Strausz (2017) shows that the all-or-nothing mechanism implemented by Kickstarter is optimal to alleviate the moral hazard problem, albeit absent donors and strategic buyers. Thus, we do not expect moral hazard to be a first-order concern. Our data supports this hypothesis.

We create a second data set that contain all updates from the creator and all comments from backers, for nearly every campaign ever posted on Kickstarter. In total, we obtain over 1.3 million updates and over 10.4 million comments. We use this commentary to gauge the prevalence of moral hazard. We present the main findings below. (See Appendix B for the full data analysis and results.)

We find that more updates are positive than negative. Since updates are added by the creator, it is unsurprising that updates are more positive. But we also find that in the comments posted by backers positive words occur more frequently within and across projects than negative words. Most projects contain at least one update regarding delivery and the most frequently occurring negative terms involve delays. There is a positive correlation among the negative terms, with the strongest being "apologize" and "delay." Apologizing for issues or problems is much more common than apologizing for failure or refund. We identify 200 topic clusters using natural language processing. The results are consistent with the textual analysis—most clusters do not reflect any moral hazard. Instances of non-delivery or consumer demands for refunds are rare. Thus, in our model of crowdfunding, we abstract away from any moral hazard on the part of the entrepreneur.

2.4 Robustness Checks

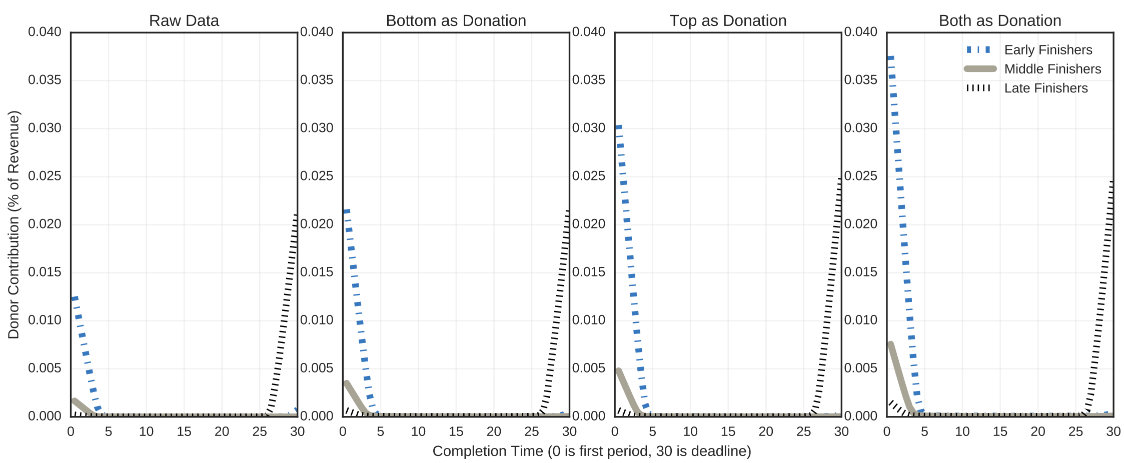
Our definition of a donation comes from backers entering an amount in the donation box, or from backers paying more than the reward price. However, some rewards themselves may be better interpreted as donations. Examples include a lower priced reward that approximates a thank you, or a more expensive reward that includes the product but also includes special recognition. The bias is in only one direction: we are possibly understating the magnitude of donations on the platform. This is not a problem, per se, but we would like to investigate what role this plays in our results.

Given the number of projects and buckets per project, manually assigning a reward or part of a reward as a donation is infeasible. There are over 500,000 rewards in the data. Instead, we perform the following analyses. First, we assume the least expensive bucket represents a pure donation.

Next, we assume the most expensive bucket represents a pure donation. Finally, we assume both the least and most expensive buckets constitute donations.

We reprocess the data and repeat the analyses in the previous subsections. For brevity, we only show one result and describe the others. Figure 4 shows a comparison of the raw data with the three robustness exercises, replicating the analysis of Figure 2(b)—donation revenue, as percentage, over time for early, middle, and late completing campaigns. The figure shows an intuitive result: as reward purchases are assigned as donations, the amount of revenue attributed to donations increases. However, the figure also shows that qualitatively, our key finding remains—donations spike at the deadline for late-completing campaigns. There are no noticeable spikes in donations for early- or middle-finishing campaigns.

Figure 4: Robustness to Donor Contributions over Time for Early-Middle-Late Campaigns



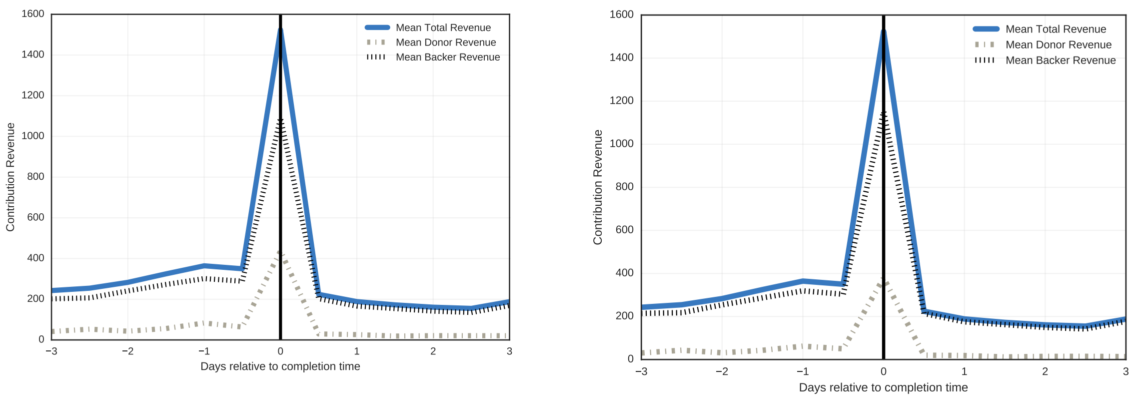
Note: Replication of Figure 2(b) using different definitions of donation. The left panel donates the origin version. Next, we assign the lowest priced bucket as donation. The following assigns the highest priced bucket to donations. Finally, the last panel moves both the lowest- and highest-priced buckets to donations.

Our other empirical results are also qualitatively similar. For example, recreating Figure 3, or buyer and donor revenue around the success time, yields similar results. Both purchases and donations spike—in the case of moving both the lowest and highest priced buckets to donations, donations spike to the level of purchases—but both again decline after success. Donations drop by more than half in all scenarios. Thus, we are confident that the empirical patterns uncovered in Section 2.3 are not sensitive to potential measurement error.

We also conduct robustness to our calculation of shipping costs. This is also important because donations are determined after subtracting off shipping costs. If we understate shipping costs, we overstate donatations. We reprocess all the data assuming all purchases are made from the country with the lowest, and then most expensive, shipping costs. Across all reward buckets, the average shipping cost in our baseline analysis is \$16; this lowers to \$10 under min-cost shipping and increases to \$25 in max-cost shipping. Moreover, these statistics are simply the overall mean; after assigning shipping costs to observed purchases, the average shipping cost drops by nearly 50%. That is, most transactions on the platform involve rewards with lower shipping costs.

Figure 5: Robustness Analysis to Shipping Costs

(a) Min-Cost Shipping (b) Max-Cost Shipping



Note: (a) We process the data and assign the least expensive shipping cost to each reward in the data. (b) We process the data and assign the most expensive shipping cost to each reward in the data.

All our results are robust to the various estimates of shipping costs. Utilizing the lower and upper bounds on shipping costs provides us with lower and upper bounds on donation amounts; we find all our results hold. For example, Figure 5 recreates Figure 3 under min- and max-cost shipping. The results are nearly identical with the mean backer revenue line in (b) being slightly closer to the mean overall revenue line as compared to (a). These two figures are qualitatively similar to our results using USA shipping costs. All our results are quantitatively similar as well. We bound donations at the deadline to be between 23.5% and 27.4% under min- and max-cost shipping. In our baseline results, we estimate this to be 26.1%.

3 A Model of Reward-Based Contribution Games

A firm (entrepreneur) would like to raise a goal amount $G > 0$ in time $T > 0$. It offers its product at a price $p > 0$ and it also accepts donations. Time is divided into periods of length Δ. Most results focus on the limiting outcomes as $\Delta \to 0$. The firm can only receive the funds if it raises the goal amount G by the deadline T.

Players and payoffs. There are two different types of contributors. In every period $t \in \mathbb{T}^{\Delta} := \{0, \Delta, 2\Delta, \ldots, T-\Delta\}$, a *buyer* (she) arrives with probability $\Delta\lambda$. As $\Delta \to 0$, the process N_t^{Δ} counting the number of arrivals up to t converges in distribution to a Poisson process N_t, with arrival rate $\lambda > 0$. We refer to the $N-$th buyer as buyer N. A buyer can either pledge to pay p to buy the product, or choose an outside option. All buyers have the same valuation $v > 0$ for the product. Accordingly, if a buyer pledges and the campaign is successful, she pays to receive the product and gets utility of $v - p > 0$. If the campaign is unsuccessful, she pays nothing and receives utility 0. Buyers value the outside option at $v_0 > 0$. We can interpret v_0 as the utility a buyer gets from purchasing an outside good right away, or simply, the opportunity cost of pledging money for the rest of the campaign duration. v_0 can also be interpreted as a cost of disappointment when the campaign is not successful. We assume that $v - p \geq v_0$.

There is also a *long-lived donor* (he) who values a successful project at W. We refer to W interchangeably as the donor's wealth or the donor's value. The donor's payoff is $W - D$ if the project is successful and total donations by time T are D. If the campaign is not successful, he receives a utility of 0.[21] Buyers do not observe the realization of W and only know that that it is drawn from a distribution on $(0, \overline{W})$ with a differentiable, strictly increasing cdf F, $f := F'$, $\overline{W} \in [0, \infty]$.[22] We refer to a donor with valuation W as donor W.[23]

Timing of the game. In every period, if a buyer arrives, she first decides whether to pledge p or not. Then, the donor decides whether and how much to donate. Thus, we can describe a

[21] If the donor is identical to the entrepreneur, one might argue that the donor can take back any money that he has donated in excess of G after a successful campaign. It turns out that the equilibria we construct are also equilibria in a game with a donor with such preferences but limited budget W.

[22] We do not allow buyers to donate: they can only pay exactly a price p. If buyers were able to donate, which they might rationally want to do because they care about the success of the project, then their belief over W can be thought to be heterogeneous across buyers as they know their own social value. For simplicity we abstract away from this.

[23] Allowing for multiple donors would introduce the familiar free-riding issue seen in standard contribution games. A single representative donor gives us a more tractable model and allows us to focus on the new element of coordination and provide a clean characterization of the dynamic interaction between donors and buyers.

general history faced by a buyer in period t by $h_t^B = (b_t, d_t)^{\frac{t}{\Delta}} \in \mathscr{H}_t^{B,\Delta} \equiv \left(\{0,1\} \times [0, \overline{W}]\right)^{\frac{t}{\Delta}}$, where d_t denotes the donation amount at time t and b_t the purchasing decision. Let $b_t = 1$ if a purchase occurred in period t and 0 otherwise. Analogously, a general history of the donor at time t is given by $h_t^D \in \mathscr{H}_t^{D,\Delta} \equiv \mathscr{H}_t^{B,\Delta} \times \{0,1\}$.

Observability and Strategies. We assume that a buyer in period $t + \Delta$ only observes total donations D_t at the end of time t, the number of buyers N_t who have arrived by the end of period t, and time t. We denote the time left until the deadline by $u := T - t$. Thus, we assume that a buyer only observes a reduced history of the form

$$\tilde{h}^B = (D, N, u) \in \tilde{\mathscr{H}} := [0, \overline{W}] \times \mathbb{N} \times [0, T].$$

We focus on strategies of the buyer that only depend on this reduced history rather than on the general history of all donations and purchases. This assumption eliminates unrealistic equilibria and is consistent with what is observed by buyers on the Kickstarter platform.

The strategy of a buyer in period t is a mapping $b^\Delta : \tilde{\mathscr{H}} \to \{0,1\}$ where

$$b^\Delta\left(\tilde{h}^B\right) = \begin{cases} 1 & \text{if a buyer in period } t \text{ buys after history } \tilde{h}^B \\ 0 & \text{otherwise} \end{cases}.$$

The donor's strategy at time t is a mapping $d^\Delta : \mathscr{H}_t^{D,\Delta} \times [0, \overline{W}] \to \mathbb{R}_+$, where $d^\Delta(\cdot; W)$ is the strategy of donor W. Given a history $\tilde{h}^B$, we denote the distribution of an equilibrium public belief of the buyer about W at time t by $\tilde{F}^\Delta(\cdot; \tilde{h}^B)$.

Equilibrium notion. A Perfect Bayesian Equilibrium (PBE) is given by a tuple $(d^\Delta, b^\Delta, \tilde{F}^\Delta)$ such that, (i) after any donor history h_t^D, strategy d^Δ maximizes the donor's expected payoff given buyers' strategies and beliefs, (ii) after any buyer history $\tilde{h}^B \in \tilde{\mathscr{H}}$, strategy b^Δ maximizes the expected payoff of the buyer given her beliefs, and (iii) beliefs $\tilde{F}^\Delta(\cdot; \tilde{h}^B)$ are formed according to Bayes' rule whenever possible. Given the realization of arrivals represented by arrival times $\tau_1^\Delta, \tau_2^\Delta, \dots \in \mathbb{T}^\Delta$ and donor wealth W, the outcome of the game is given by a function representing total donation path, $\mathscr{D}^\Delta((\tau_i^\Delta)_i, W) : \mathbb{T}^\Delta \to [0, \infty), t \mapsto D_t^\Delta$ where D_t^Δ is the total donation made by period t, and a function representing the purchase path $\mathscr{N}^\Delta((\tau_i^\Delta)_i, W) : \mathbb{T}^\Delta \to \mathbb{N}, t \mapsto N_t^\Delta$, where N_t^Δ is the total number of buyer arrivals by period t.

We are interested in limiting outcomes as $\Delta \to 0$ of PBE of the game, i.e., total donation and purchase paths $\mathscr{D}((t_i^\Delta)_i, W) : [0,T] \to [0,\overline{W}]$, $\mathscr{N}((t_i^\Delta)_i, W) : [0,T] \to \mathbb{N}$ given arrival times $\tau_1 < \tau_2 < \dots$ of the limiting Poisson process and W, such that there exists a sequence of $\Delta \to 0$ and equilibrium outcomes $(\mathscr{D}^\Delta((\tau_i^\Delta)_i, W), \mathscr{N}^\Delta((\tau_i^\Delta)_i, W))$, such that $\mathscr{D}^\Delta((\tau_i^\Delta)_i, W) \to \mathscr{D}((\tau_i)_i, W)$ and $\mathscr{N}^\Delta((\tau_i^\Delta)_i, W) \to \mathscr{N}((\tau_i)_i, W)$ uniformly for $\tau_i^\Delta \to \tau_i$.

Our main focus is to characterize the equilibria that maximize the probability of success of a campaign. We call such an equilibrium "platform-optimal." Later we describe the full set of PBE, and discuss analogously the "donor-optimal" and "buyer-optimal" equilibria.

Remark 1. If we interpret v_0 as an opportunity cost of pledging money for the campaign duration, we may want to allow v_0 to vary over time. For example, one can imagine that the opportunity cost of pledging is smaller close to the deadline than at the beginning of the campaign. We can do this without qualitatively changing the results. In the proof of the main equilibrium characterization, which we present in Appendix 4.2, we allow for such general $v_0(u)$ but focus on constant v_0 in the main text of the paper. We impose the following assumptions on $v_0(u)$ in order to maintain continuity of the problem:[24] $v_0(u) > 0$ for all $u \in [0,T]$, $v_0(\cdot)$ is continuous, $v - p \geq v_0(u)$ for all $u \in [0,T]$ and

$$\frac{v_0'(u)}{v-p} < \lambda e^{-\lambda u} \left(\frac{v_0(u)}{v-p} \right)^{G/p} F(p). \tag{1}$$

4 Dynamics of Reward-based Contribution Games

All proofs are located in the Appendix.

4.1 Preliminaries

For any Δ, given primitives $p, G, T, \lambda, v_0 > 0$, we construct a PBE. Given that the buyers' strategies only depend on the payoff-relevant state in $\tilde{\mathscr{H}}$, it is without loss to consider strategies such that the donors' best response after any history also only depends on the state $\tilde{\mathscr{H}}$. Henceforth, we denote the donor's strategy by $d^\Delta : \tilde{\mathscr{H}} \to \mathbb{R}$. Also, instead of characterizing donations in every period, we

[24] We do not think this assumption is necessary for the main qualitative insights of the paper though.

consider the total donation strategy,

$$D_{\Delta}^{+}(D,N,u;W) := D + d_{\Delta}(D,N,u;W),$$

which uniquely pins down $d^{\Delta}(D,N,u;W)$.

The challenge for the construction of equilibria is that the payoff-relevant state space $\tilde{\mathscr{H}}$ is rich which makes standard induction arguments over time impossible. At the same time, the presence of donations does not allow us to use inductive arguments in the number of contributors as is sometimes done in contribution games. Instead, we combine the two approaches. We first analyze the last period. Then, we recast the problem and use an inductive argument on the number of additional buyers needed for success, given current total donations and time. Within each of the induction steps, we use an induction with respect to time.

To proceed, we must introduce a few more pieces of notation to be used in the transformed problem. Let $\underline{M}(D)$ denote the number of buyers needed for completion given total donations D, if no further donations are made. That is, $\underline{M}(D)$ is the unique integer satisfying

$$(\underline{M}(D)-1)\cdot p < G - D \leq \underline{M}(D)\cdot p.$$

In particular, let M_0 denote the number of buyers needed for completion if there were no donations.

$$M_0 := \underline{M}(0).$$

Given total donations D and the total number of buyers N, we need $j = \underline{M}(D) - N$ more buyers to arrive and purchase in order to reach the goal. The characterization of the equilibrium proceeds by an induction argument on j.

Another useful piece of notion is the probability of reaching the goal from the perspective of buyer N+1 if she buys in state (D,N,u). Denote this $\pi_{\Delta}(D,N,u)$. Importantly, this probability is a function of the equilibrium strategies being played. At state (D,N,u), arriving buyer $N+1$ buys, that is, $b_{\Delta}(D,N,u) = 1$, if and only if

$$\pi_{\Delta}(D,N,u)\cdot(v-p) \geq \quad v_0. \tag{2}$$

Two sources of uncertainty and the equilibrium strategies affect this probability. First, buyers are uncertain about the wealth of the donor and they update their beliefs based on the amount donated D over time. Second, both types of agents face uncertainty about future arrivals. Since we mainly focus on the limiting outcomes, let us define, $\pi(D,N,u) := \lim_{\Delta\to 0}\pi_\Delta(D,N,u)$, whenever it exists. It follows from the proof that this limit is well-defined.

4.2 Equilibrium Characterization

We start with constructing a particular PBE of the game (Proposition 1). We show later that this equilibrium not only maximizes the probability of success of the campaign (platform-optimal), but also maximizes donor welfare. In this equilibrium, the donor always donates just enough to induce the next potential buyer to buy. Given the number of buyers so far and the time remaining (N,u), with $N < \underline{M}(D)$, let $\underline{D}(N,u)$ denote the minimum level of total donations required to guarantee that buyer $N+1$ buys. We show that $\underline{D}(N,u)$ makes buyer $N+1$ indifferent between buying and not buying and it is uniquely given by

$$\pi(\underline{D}(N,u),N,u) = \frac{v_0}{v-p}. \tag{3}$$

Moreover, we define the time when a donor with value W "runs out of funds" if no other potential buyer arrives. This happens when the minimum amount of total donations required for the next arriving buyer to buy exceeds W. That is, $\underline{D}(N,u) \geq W$. At this point, the donor does not want to donate any more, and the next arrival will surely not buy. Thus, for any $j = 2,3,\ldots,$ and W such that $\underline{M}(W) \geq j$, we define $\xi_j(W)$ by

$$\pi(W, \underline{M}(W) - j, \xi_j(W)) = \frac{v_0}{v-p}. \tag{4}$$

$\xi_j(W)$ represents the time remaining until the deadline T at which the donor with wealth W runs out of funds if j more buyers are needed to reach the goal. Moreover, let $\xi_i(W) = 0$ for all $i < 2$ and $W \geq 0$.[25] After $\xi_j(W)$ periods from the deadline, the probability of success drops to zero.

Note that there is asymmetric information about $\xi_j(W)$: the donor knows W but buyers must form expectations over W. Buyers only know $\xi_j(W)$ is reached when no donations are made even

[25] We allow j to be negative.

though they should be on the equilibrium path.

Proposition 1. ***Platform-optimal PBE***

For a given p, G, T, λ and Δ, there exists an equilibrium $(d^{\Delta}, b^{\Delta}, \tilde{F}^{\Delta})$ that satisfies the following properties:

i) At the deadline, i.e., if $u = 0$, total donations satisfy

$$D^{+}(D, N, u; W) := \lim_{\Delta \to 0} D^{+}_{\Delta}(D, N, u; W) = \min\{\max\{D, G - Np\}, W\}; \tag{5}$$

ii) for all $u > 0$, donations satisfy

$$D^{+}(D, N, u; W) := \lim_{\Delta \to 0} D^{+}_{\Delta}(D, N, u; W) = \min\{\max\{D, \underline{D}(N, u)\}, W\}; \tag{6}$$

iii) buyers' beliefs are are given by

$$\tilde{F}_{\Delta}(w; (D, N, u)) = \frac{F(w) - F(D)}{1 - F(D)} \cdot 1(w \geq D); \tag{7}$$

iv) the limiting probability of success (as $\Delta \to 0$) from the perspective of buyer $N+1$ for $N < \underline{M}(D) - 1$ satisfies

$$\pi(D, N, u) = \\ \mathbb{E}_{W}\left[\int_{0}^{\max\{u - \xi_{\underline{M}(W) - N - 1}(W), 0\}} \lambda e^{-\lambda s} \pi(\max\{D, \underline{D}(N+1, u-s)\},\ N+1,\ u-s) ds \,\middle|\, W \geq D \right] \tag{8}$$

as long as $D \geq \underline{D}(N, u)$. If donations are less than $\underline{D}(N, u)$, then $\pi(D, \underline{M}(D) - j, u) = 0$. Finally, $\pi(D, N, u) = 1$ for all $N \geq \underline{M}(D) - 1$;

v) a buyer buys if and only if $\pi(D, N, u) \cdot (v - p) \geq v_0$. In particular, on equilibrium path, buyer $\underline{M}(D) - j + 1$ buys as long as the donor donates, that is, $u \geq \xi_{j}(\underline{M}(W))$.

This limiting equilibrium outcome has a clean structure. The donor donates a positive amount if and only if $D < \underline{D}(N, u)$. If he donates, he donates such that total donations equal $\underline{D}(N, u)$ until donations exhaust his wealth. Buyers continue to purchase as long as the donor's wealth is not depleted, which are characterized by the cutoff times $\xi_j(W)$s from the deadline. Total donations are always such that the next buyer who arrives is willing to buy. Hence, the probability of success can be calculated recursively. Only in the last period does the donor contribute the remaining amount needed for success, if possible.

The equilibrium in Proposition 1 has several nice properties. It implies that donations can have a discrete spike only at the start and at the deadline. A spike of $\underline{D}(0,T)$ can occur at $t=0$ in order to ensure that buyers assign a sufficiently high probability to the campaign being successful.[26] At $t=T$, donation spikes can occur to ensure success. In between, total donations increase smoothly, with the flow of donations dropping to zero for an interval straight after a buyer pledges. This pattern is consistent with what we observe in Figure 2.[27]

However, the contribution game supports a large set of other equilibria. Next, we characterize the full set of equilibria and show that the equilibrium described in Proposition 1 is platform-optimal and donor-optimal. To this end, it is useful to consider a special case of Proposition 1 where $F(W)=1$ for all W, i.e., $W=0$. Without donations, the payoff-relevant state of any project can be reduced to (N,u). Let us denote the probability of success in state (N,u) by $\pi^{\text{no}}(N,u)$ and define ξ_j^{no} by $\pi^{\text{no}}(M_0-j,\xi_j^{\text{no}})=\frac{v_0}{v-p}$ for $j=2,\dots,M$ and $\xi_i^{\text{no}}=0$ for $i<2$. This implies that

$$\pi^{\text{no}}(M-j,u)=\int_0^{\max\{u-\xi_{j-1}^{\text{no}},0\}}\lambda e^{-\lambda s}\pi^{\text{no}}(M-j+1,u-s)ds.$$

For $u\in(\xi_i^{\text{no}},\xi_{i-1}^{\text{no}}]$, define

$$\overline{D}(N,u):=\begin{cases} G-(N+i-1)p & \text{if } i\leq M_0-N \\ 0 & \text{if } i>M_0-N, \end{cases}$$

which is the amount to be donated so that the next buyer is willing to buy even if no more donations are made. Note that the donor never wants to donate more than this amount in any equilibrium. We

[26] This spike at the start of the campaign is reminiscent of the role played by "seed money" at the start of charitable fund-raising campaigns. See for instance, Andreoni (1998) for a discussion of increasing returns to initial donations, and the importance of seed money or "leadership gifts."

[27] If the buyer arrival rate changes over time, qualitative results do not change. However, instead of a constant inflow of buyers, there can be times at which purchases increase or decrease on average which is anticipated by the donor and buyers. The donor's strategy remains unchanged: he donates just enough to incentivize the next buyer to buy. A higher arrival rate at the beginning of the campaign driven by increased advertising can explain the spike in purchases early on in panel (b) of Figure 2. Note that this spike is alleviated when we also assign a few buckets to donations in Section 2.4.

use these definitions in the proposition below.

Proposition 2. ***Characterization of set of PBE***

i) In every PBE, $D^{+}(D,N,0;W)=\min\{\max\{D,G-Np\},W\}$ *and for* $u>0$

$$\min\{\max\{\underline{D}(N,u),D\},W\}\leq D^{+}(D,N,u;W)\leq\min\{\max\{\overline{D}(N,u),D\},W\},$$

ii) the equilibrium in Proposition 1 is both platform-optimal and donor-optimal.

Although we focus on the platform-optimal equilibrium, we also construct a buyer-optimal equilibrium. There are important differences between the equilibria. In the donor-optimal or platform-optimal equilibrium, buyers have the most optimistic beliefs about the donor's wealth. In contrast, in the buyer-optimal equilibrium buyer beliefs are the most pessimistic.

Example 1. Buyer-optimal Equilibrium The following constitutes an equilibrium for general F. The donor's strategy is given by

$$D^{+}(D,N,u;W)=\begin{cases}\min\{\max\{D,G-Np\},W\} & \text{for } u=0\\ \min\{\max\{D,\overline{D}(N,u)\},W\} & \text{for } u>0\end{cases}.$$

Buyers pledge if and only if their belief $\pi(D,N,w)\geq\frac{v_0}{v-p}$, and equilibrium beliefs of buyers are given by

$$\tilde{F}(w;(D,N,u))=\begin{cases}\frac{F(w)-F(D)}{1-F(D)}\cdot 1(w\geq D) & \text{if } D\geq\overline{D}(N,u)\\ 1(w\geq D) & \text{otherwise}\end{cases}.$$

This means that if buyers observe a total donation that is less than the equilibrium expectation of $\overline{D}(N,u)$, then they believe that the donor has exhausted his wealth.

Unlike in the equilibrium in Proposition 1, the equilibrium in this example has bursts of donations at ξ_i^{no} and no donations otherwise. However, it shares the feature that many donations occur in period 0 and period T. Note that in a game in which buyers know W, they would buy in state (D,N,u) only if the realized wealth satisfies $W\geq\overline{D}(N,u)$. Thus, this equilibrium implements the outcome in which there is no asymmetric information about W. In particular, this implies that total buyer surplus is maximized in this equilibrium.[28] ◊

[28]Note that a particular buyer might still prefer that other buyers not to know W, so they keep buying in the future.

4.3 Equilibrium Properties

We focus on properties of the platform-optimal equilibrium. Most properties, however, carry over to any other equilibria.

4.3.1 Basic properties and definitions

First, we show in Lemma 1 that we can assume without loss of generality that $\lambda = 1$ because λ simply scales time in the model. Thus, any comparative statics with respect to λ is equivalent to the comparative statics with respect to T.

Lemma 1. *Fix a goal amount G, price p, and distribution of donor wealth F. Consider two different campaigns $k = 1,2$ with arrival rates λ_k and time horizon T_k, where $\frac{T_1}{\lambda_1} = \frac{T_2}{\lambda_2}$. Then, the buyer's limiting equilibrium strategies b_k, the donor's equilibrium strategies $D_k^+(D,N,u)$, the probabilities of success $\pi_k(D,N,u)$, and cutoffs ξ_j^k satisfy: $b_2(D,N,u) = b_1\left(D,N,u\frac{T_1}{T_2}\right)$, $D_2^+(D,N,u) = D_1^+\left(D,N,u\frac{T_1}{T_2}\right)$, $\pi_2(D,N,u) = \pi_1\left(D,N,u\frac{T_1}{T_2}\right)$, and $\xi_j^2(W) = \xi_j^1(W)\cdot\frac{T_1}{T_2}$.*

Thus, G and T are the the only payoff-relevant characteristics of a campaign given $\frac{v_0}{v-p}$ and F. Henceforth, we focus on comparative statics with respect to the primitives G, T, p, and $\frac{v_0}{v-p}$.

One important implication of Proposition 1 is that a project has a non-zero probability of success from a buyer's (and researcher's) perspective if and only if $u \geq \xi_j(D)$ for observed donations D and $N = \underline{M}(D) - j$.

Definition 1. It is useful to introduce the following definitions for limiting outcomes:

i) We call a campaign *alive* in states (D,N,u) such that $u > 0$ and $u \geq \xi_{\underline{M}(D)-N}(D)$. We call a campaign *dead* in states (D,N,u) such that $0 < u < \xi_{\underline{M}(D)-N}(D)$.

ii) We call a campaign *successful* if and only if $T \geq \tau_{\underline{M}(W)}$ and $T - \xi_{\underline{M}(W)-j}(D) \geq \tau_j$ for all $j = 1, \ldots \underline{M}(W) - 1$. Denote the event of a campaign being successful by $\mathscr{S}(G,T)$. Denote the time of success by $\tau^{(G,T)} \leq T$, where we set $\tau = \infty$ if the project never gets completed.

So, a campaign is successful only if the the first buyer arrives before time $T - \xi_{\underline{M}(W)}(W)$, at which point a donor of wealth W runs out of funds. Further, we show in the Appendix that $\pi(0,0,u)$

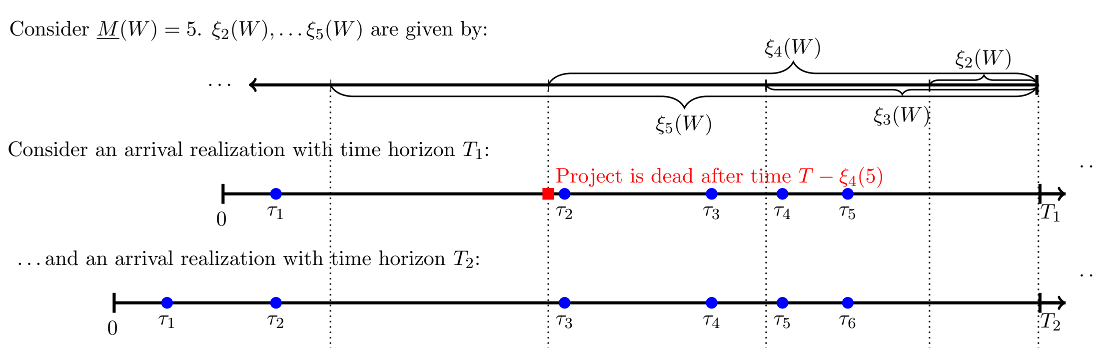
This hints at the complexities of this coordination game.

is increasing in u. This means that there exists a unique $\underline{u}$ such that $\pi(0,0,\underline{u}) = \frac{v_0}{v-p}$. If $T > \underline{u}$, then no donations are being made up to time $T - \underline{u}$. For $T < \underline{u}$, the donor donates just enough so that the probability of success is exactly $\frac{v_0}{v-p}$ by construction. This allows us to write down the ex-ante probability of success of a campaign. We denote this as $\Pi(G,T)$ as it is defined by

$$\Pi(G,T) = \begin{cases} \frac{v_0}{v-p} \int_0^\infty \left(1 - e^{-\max\{T - \xi_{\underline{M}(w)}(w), 0\}}\right) f(w) dw & \text{if } T \leq \underline{u} \\ \sum_{k=0}^\infty \frac{(T-\underline{u})^k \cdot e^{-(T-\underline{u})}}{k!} \Pi(G - kp, \underline{u}) & \text{if } T > \underline{u}. \end{cases} \tag{9}$$

Figure 6 provides a pictorial representation of sample paths of arrivals and shows the success or death of a campaign. Consider realizations of buyer arrivals in the second row. The deadline is T_1. In this case, the second arrival does not occur "in time," i.e., before time $T_1 - \xi_4(W)$. The donor runs out of funds and the probability of success drops to zero. This is consistent with the observation that if a project is unsuccessful, it fails with few total number of buyers as illustrated in panel (a) of Figure 1. The reason is that if buyers do not arrive sufficiently early, later buyers will not buy. Figure 6 also illustrates the outcome if the deadline is extended to $T_2 > T_1$. This is shown in the last row. For this realization, the project succeeds at some time $\tau^{(G,T)} \leq \tau_5$ where the exact timing of success depends on the distribution of donor wealth $F(W)$.

Figure 6: Realization of buyer arrival and W such that $\underline{M}(W) = 5$ for $T_1 < T_2$



4.3.2 Key Comparative Statics

Proposition 3 establishes some comparative statics in our model. We prove the result formally within the class of platform-optimal equilibria—for any set of parameters of the model. However, analogous comparative statics are also valid for the class of buyer-optimal equilibria.[29]

Proposition 3. *For the platform-optimal equilibria:*

i) The ex-ante probability of success $\Pi(G,T)$ is increasing in T and decreasing in G;

ii) donations $\underline{D}(0,T)$ (at time $t=0$) are decreasing in T, and increasing in G and $\frac{v_0}{v-p}$;

iii) on equilibrium path, the average total donations $\mathbb{E}^{W,(\tau_j)_{j=1}^T}[D_t]$ are decreasing in T, and increasing in G and $\frac{v_0}{v-p}$;

iv) for all $u>0$ and $N<M_0$, $\underline{D}(N,u)<G-Np$. Thus, if a campaign succeeds before the deadline, then it must be that it succeeds due to an arrival of a buyer at the time of success τ and zero donations between the previous arrival and $\tau^{(G,T)}$.

We discuss how this result matches our empirical facts. First, it is intuitive that the probability of success is higher if there is more time, or if the goal is lower. Indeed, higher goals and shorter time horizons are associated with lower success rates both in Table 1 and 2. It is also true that the donor needs to contribute more at the onset of a campaign if buyers alone are unlikely to be able to complete the project alone. This occurs if T is small, if G is large, or if the outside option is relatively attractive. These features are robust across equilibria and will also carry over to the model with social learning. Part iii) generalizes this observation to total donations at any point in time. The donation spike at the beginning of a campaign is a clear empirical pattern that we observe in the data.

Part iv) of Proposition 3 implies that campaigns that are completed strictly before the deadline must do so because the number of arriving buyers "exceeds expectations." This is consistent with the empirical observation in Figure 2(b) that shows that the revenue share from purchases is high for projects that complete early. This can also explain the spike in contributions at the completion time in Figure 3. The spike in donation revenues at the time of completion cannot be explained by our

[29]For the class of buyer-optimal PBE, statements (i), (ii) and (iii) are unaltered, while in statement (iv) $\underline{D}(N,u)$ is replaced by the analogous $\bar{D}(n,u)$.

model. One possible explanation is that advertising on the Kickstarter platform creates incentives for donors to push projects to the goal.

Finally, it is useful to understand how the realization of buyer arrivals affects the contribution dynamics. The total donation level can be written as

$$D_t \quad = \quad \max\left\{\underline{D}(n, T-\tau_n) \middle| n \in \{0,1,\dots\} \text{ and } \tau_n \leq t\right\}. \tag{10}$$

As a result, fewer buyers arriving early on results in higher total donations D_t. In particular, D_t is bounded above by $\underline{D}(0, T-t)$. The following lemma is immediate.

Lemma 2. *Consider two realizations of arrival times $\tau_1 < \tau_2 < \dots$ and $\tau'_1 < \tau'_2 < \dots$ and a time t so that $\tau_N \leq t < \tau_{N+1}$ and $\tau_{N'} \leq t < \tau_{N'+1}$, resulting in total donations D_t and D'_t, respectively. If $N \leq N'$ and $(\tau_1, \dots, \tau_N, t, \dots, t) > (\tau'_1, \dots, \tau_{N'})$, i.e., $\tau_i \geq \tau'_i$ for all $i \leq N$ and $\tau_i > \tau'_i$ for at least one $i \leq N$, then $D_t > D'_t$.*

While early arrivals result in lower donation levels, if some arrivals occur earlier and some later, one cannot clearly order the donation level without taking into account details of the parameters of the campaign. If for example two arrivals occur with $\tau_1 < \tau'_1$ but $\tau_2 > \tau'_2$, it is ambiguous which campaign has higher donations at the deadline without taking into account the exact timing, campaign length, goal size, cost, and price structure.

Surprisingly, the expected time of success does not change monotonically in G and T. For example, reducing M_0 has two opposing effects. Consider a realization of arrivals in $\mathscr{S}(G,T)$. If success occurs before the deadline, an arrival at time zero must increase the time of success by Proposition 3 iii). If the campaign is completed at the deadline, then an additional arrival at time zero results in success strictly before or at the deadline as well. Next, consider a realization of arrivals that is not in $\mathscr{S}(G,T)$, but results in success if an arrival is added at time zero. In that case, the time of success is on average high. The overall effect can be negative or positive depending on the value of M_0.

5 Alternative Platform Designs

Our assumptions about the mechanism and the information environment are motivated by Kickstarter. While optimal dynamic mechanism design is outside the scope of this paper, we examine two features of platform design. First, we analyze the optimal timing of donations. Next, we consider an alternate information environment to understand the value of dynamic information revelation about donations and the number of buyers.

5.1 Timing of Donations

As a benchmark, we consider a setting in which donations are not allowed as in Example 1. In that setting the probability of success is lower than when donations are allowed. The equilibrium is unique and the ex-ante probability of the project being successful in equilibrium can be written as

$$\Pi^{\text{no}}(G,T) := \int_0^{\max\{T-\xi^{\text{no}},0\}} \lambda e^{-\lambda s} \pi^{\text{no}}(0, T-s) ds.$$

This is per se unsurprising, but leads to the natural question of how to optimize the timing of donations. We analyze a setting in which the donor has only one opportunity to donate at the start of the campaign (i.e., $u = T$) and a setting in which the donor has only one opportunity to donate at the end of the campaign (i.e., $u = 0$).

If the donor has only one opportunity to donate at the start, then given any donation level D, the game is identical to one with no donations, but with $\underline{M}(D)$ buyers required instead of $\underline{M}(0)$. There is a unique PBE in which the donor chooses an optimal donation level $D_0^* < W$ so that

$$\Pi^0(G,T) := \mathbb{E}^W \left[\max_{D \le W} \Pi^{\text{no}}(G-D,T) \cdot (W-D) \right].$$

Finally, consider the setting in which the donor can only donate at the deadline. There exists a unique equilibrium that can be constructed analogously to Proposition 1 with $D = 0$ up to period T.

Corollary 1. *If the donor can only donate at the deadline T, there is a unique equilibrium that is described as follows. As $\Delta \to 0$, given a project in state (N, u),*

i) If $N = M_0 - 1$, buyer M_0 buys, and assigns probability 1 to the project being completed. The probability of success for $N = M_0 - 1$ is hence $\pi^T(M_0 - 1, u) = 1 - e^{-\lambda u} F(p)$ to the project *being completed if he buys and zero otherwise.*

ii) *If $N < M_0 - 1$, the probability of success for buyer $N+1$ is given by*

$$\pi^T(N,u) = \int_0^{\max\{u-\xi^T_{M_0-N-1}(W),0\}} \lambda e^{-\lambda s}\pi^T(N+1,u-s)ds + e^{-\lambda(u-\xi^T_{j-1}(W))}(1-F(jp)).$$

where $\xi^T_j(W)$ is defined by $\pi^T(M_0-j,\xi^T_j(W)) = \frac{v_0}{v-p}$, for $j \geq 2$ and $\xi^T_1(W) \equiv 0$.

Let the ex-ante probability of success in this equilibrium be denoted by $\Pi^T(G,T)$.

Proposition 4. *Consider campaigns such that the ex-ante probability of success is strictly positive in some equilibrium of the baseline model with continuous donations.*

i) *The probability of success in all counterfactual settings is strictly lower than that in any equilibrium of the baseline model.*

ii) *Donor utility in all counterfactual settings is strictly lower than that in any equilibrium in the baseline model.*

iii) *The expected buyer utility in all counterfactual settings is lower than that in any equilibrium in the baseline model. Some buyers are strictly worse off.*

This result illustrates that allowing continuous donations benefits players as it enables the donor to signal his wealth and thereby help buyers optimize their purchasing decisions, at a minimal cost for the donor. A first guess might have been that allowing donations only at the deadline may be better for the donor, because when continuous donations are allowed, buyers may learn about low realizations of donor wealth. However, since the donor signals his wealth by donations only if buyers would not buy otherwise, he reveals his low wealth only if arrivals of buyers were so low that the project would have died in the counterfactual anyway.

The comparison of the ex ante probability of success between the two counterfactuals (with donations only at $t=0$ or only at $t=T$) is ambiguous. Below, we present an example with $p < G \leq 2p$ in which the ex-ante probability of success is higher in the case of donations only at $t=T$ for sufficiently $\frac{v_0}{v-p}$ is high enough.

We also analyze a relaxed problem in which the donor is committed to donate all his wealth W at the start since this allows us to explicitly compute the ex-ante probability of success:

$$\overline{\Pi}^0(G,T) = F(\gamma)\Pi^{\text{no}}(G,T) + \sum_{i=1}^{M_0-1} \alpha_i \pi^{\text{no}}(G-ip,T) + 1 - F(G)$$

where $\gamma = G - p(M_0-1)$ and $\alpha_i = F(pi+\gamma) - F(p(i-1)+\gamma)$ for $i \in \{1,\dots M-1\}$.

Example 2. Consider a campaign with $M=2$, a time horizon T and $W \sim F$. Then,

$$\begin{aligned}
\Pi^{\text{no}}(2p,T) &= 1-(1+\lambda)e^{-\lambda T} \\
\overline{\Pi}^0(2p,T) &= 1-F(G-p)e^{-\lambda(T-\xi_2^{\text{no}})} - e^{-\lambda T}(F(G-p)\lambda(T-\xi_2^{\text{no}}) + F(G) - F(G-p)) \\
\Pi^T(2p,T) &= 1-F(G-p)\lambda(T-\xi_2^T)e^{-\lambda T} - F(G)e^{-\lambda(T-\xi_2^T)}
\end{aligned}$$

By definition of ξ_2^{no} and ξ_2^T, $e^{-\lambda\xi_2^{\text{no}}} = 1 - \frac{v_0}{v-p} = F(G-p)e^{-\lambda\xi_2^T}$ and hence, $\xi_2^{\text{no}} = \xi_2^T - \frac{\log(F(G-p))}{\lambda}$, which allows us to simplify

$$\begin{aligned}
\overline{\Pi}^0(2p,T) &= 1-e^{-\lambda T}\cdot F(G-p)\left(\lambda(T-\xi_2^{\text{no}}) + \frac{F(G)}{F(G-p)} - \frac{v_0}{v-p-v_0}\right) \\
\Pi^T(2p,T) &= 1-e^{-\lambda T}\cdot F(G-p)\left(\lambda(T-\xi_2^T) + F(G)\frac{v-p}{v-p-v_0}\right).
\end{aligned}$$

It follows immediately that $\overline{\Pi}^0(2,1) > \Pi^T(2,1)$ if and only if

$$\underbrace{\frac{1-F(G-p)}{F(G-p)}\frac{F(G)}{1-F(G)}}_{\geq 1} + \underbrace{\frac{\log(F(G-p))}{1-F(G)}}_{<0} > \frac{1}{\frac{v-p}{v_0}-1}.$$

It follows immediately that if $\frac{v-p}{v_0}$ is small or $F(G)$ is large, $\overline{\Pi}^0(2p,T) > \Pi^T(2p,T)$ and if $\frac{v-p}{v_0}$ is large or $F(G)$ is small, $\overline{\Pi}^0(2p,T) < \Pi^T(2p,T)$. $\Diamond$

The first best in this game, which maximizes the utility of donors and buyers, is to produce whenever $W + pN(T) > G$. This coincides with the entrepreneur's incentive. Thus, whenever the project gets funded in all the above mechanisms, it is also optimal to do so. However, the project may die due to miscoordination of buyers and the donor. More precisely, buyers might stop buying too early because they think the project is unlikely to be successful.

5.2 No information environment

In our equilibrium construction, information about D and N are used by both the donor and buyers: Information about total donations and the number of buyers helps buyers optimize their purchasing decisions. This raises the question of whether the dynamically arriving information unambiguously improves the probability of success of a campaign.[30]

Consider a "no-information" benchmark. Suppose that both buyers and the donor only observe the primitives G, p, T and u. In particular, they do not observe D or N over time. We show below that this does not necessarily imply a lower probability of success for a campaign.

In the absence of information about D and N, the belief about the probability of success for all buyers must be the same, regardless of when they arrive. Denote this probability by $\overline{\pi}$. Any buyer pledges if and only if $\overline{\pi} \geq \frac{v_0}{v-p}$. Therefore, an equilibrium in this environment is described by a belief threshold $\bar{\pi}$ and a donor strategy $\tilde{W}(W)$ given donor wealth W, such that

$$\tilde{W}(W) = \arg\max_{D} \sum_{k=M(D)}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!}(W - D) \qquad \text{and} \qquad \overline{\pi} = \int \sum_{k=M(\tilde{W}(W))}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} dF(W).$$

In the example below, we show that the induced probability of success in this equilibrium can be higher or lower than that in the the Kickstarter mechanism.

Example 3. Consider a campaign in which no buyer contributes if players do not observe any dynamic information, i.e., $\overline{\pi} = 0$. In that case, the Kickstarter mechanism trivially performs better and makes all players better off.

Next, consider a campaign for which $\overline{W} \approx 0$ but $\overline{\pi} > 0$. In that case, the Kickstarter mechanism results in a lower probability of success as not all buyers will buy and donations are not relevant.

This example highlights that if donations are not crucial for the campaign to succeed, then providing more information to buyers can hinder coordination. This is because, absent information revelation about N, a success requires enough buyer arrivals, while a success in the Kickstarter mechanism requires not just that enough buyers arrive, but also that they arrive early enough. $\lozenge$

Alternatively, if the donor could observe N over time but buyers could not, we would get a similar result. In this case, the donor can condition on the number of arrivals. This means that the

[30]We thank Heski Bar-Isaac and Alessandro Bonatti for raising this question.

belief of any buyer is given by $\bar{\bar{\pi}} = \int \sum_{k=M(W)}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} dF(W) > \overline{\pi}$. A buyer pledges if and only if $\bar{\bar{\pi}} > \frac{v_0}{v-p}$. In this case, the probability of success is higher than that in the Kickstarter mechanism for campaign primitives such that $\bar{\bar{\pi}} > \frac{v_0}{v-p}$. However, if $\bar{\bar{\pi}} = 0$, then revealing information as in the Kickstarter mechanism can yield a positive probability of success.

6 Learning about Quality

In practice, a widely mentioned benefit of reward-based crowdfunding is that it allows entrepreneurs to learn about demand, and enables potential buyers to learn about quality from the behavior of other buyers. In this section, we incorporate social learning into the baseline model to study how it interacts with the dynamic incentives of donor and buyers.

Let $q \in \{0,1\}$ denote the quality of the product. We can view q as the inherent quality of the product or an unknown common value component of demand. Buyers value a product of quality q at $v(q) = v \cdot q$ with $v > c$. All players share a public prior that the quality is $q = 1$ with probability μ_0 and 0 otherwise. On arrival, every buyer privately observes a signal $s \in \{0,1\}$ before she makes her pledge decision. We assume that $\mathbb{P}(s = 1|q = 1) = 1$ and $\mathbb{P}(s = 1|q = 0) = \alpha \in (0,1)$. Hence, a buyer who receives a bad signal $s = 0$ knows with certainty that the quality is low ($q = 0$).

Now, upon arrival, buyers not only form beliefs about the donor's value W, but also about the quality of the product. Let μ_t denote the public belief about quality at time t prior to observing the private signal. The payoff relevant state for a buyer in this game can then be described by

$$X_t^{\Delta} \equiv (\tilde{h}^B, \mu_t) \in [0, \overline{W}] \times \mathbb{N} \times [0,T] \times [0,1].$$

Analogous to the base model, denote the limiting strategy (as $\Delta \to 0$) of buyers by $b(X_t, s)$ where $X_t \equiv (D_t, N_t, u, \mu_t)$. As in the main model, we are interested in the limiting outcome of the PBE of the game. In this section, we will entirely focus on the "platform-optimal" equilibrium in which buyers' beliefs about W off- and on-path are given by the analog to (7)

$$\tilde{F}(w;(D,N,u,\mu)) = \frac{F(w) - F(D)}{1 - F(D)} \cdot 1(w \geq D). \tag{11}$$

We show below that the belief about W and μ_t are formed independently, so that Proposition 2 remains valid and the equilibrium considered below is indeed the platform-optimal equilibrium.

The updated public belief in every period depends on whether a purchase was observed or not. Let $\mu^B_{t+\Delta}$ and $\mu^N_{t+\Delta}$ denote the updated public belief after a purchase and after no purchase in period t, respectively. The evolution of public beliefs if the buyers' equilibrium strategy is $b(X_t,1) = b(X_t,0) = 1$, is given by $\mu^B_{t+\Delta} = \mu_t$ and $\mu^N_{t+\Delta}$ is arbitrary as Bayes' rule cannot be applied. If the buyers' equilibrium strategy is $b(X_t,1) = b(X_t,0) = 0$, then the public belief updates in $t+\Delta$ is given by $\mu^N_{t+\Delta} = \mu_t$ and $\mu^B_{t+\Delta}$ is arbitrary as Bayes' rule cannot be applied. If the buyers' equilibrium strategy is $b(X_t,1) = 1$ and $b(X_t,0) = 0$, the public beliefs are

$$\mu^B_{t+\Delta} = \frac{\mu_t}{\mu_t + (1-\mu_t)\alpha} \quad \text{and} \quad \mu^N_{t+\Delta} = \frac{\mu_t e^{-\lambda\Delta}}{\mu_t e^{-\lambda\Delta} + (1-\mu_t)e^{-\lambda\alpha\Delta}}$$

Then, in the limit as $\Delta \to 0$, if $b(X_t,1) = 1$ and $b(X_t,0) = 0$ on $t \in (s_1, s_2)$, then conditional on no observed purchase

$$\dot{\mu}_t = -\lambda(1-\alpha)\mu_t(1-\mu_t). \tag{12}$$

Let μ^s_t, $s \in \{0,1\}$ be the private posterior belief of a buyer who enters when public belief is μ_t and receives a signal s. The posterior is computed using Bayes Rule, i.e., $\mu^1_t = \frac{\mu_t}{\mu_t + (1-\mu_t)\alpha}$ and $\mu^0_t = 0$.

Finally, let us denote the probability of success as $\Delta \to 0$, conditional on quality q by

$$\beta_q(X_t) := \mathbb{P}(\mathscr{S}(G,T)|X_t, q).$$

Then, in the limit $\Delta \to 0$, a buyer with signal $s = 1$ buys if and only if

$$B(X_t) \equiv (v-p) \cdot \mu^1_t \cdot \beta_1(X_t) - p \cdot (1-\mu^1_t) \cdot \beta_0(X_t) \quad \geq \quad v_0. \tag{13}$$

If buyers with signal $s = 1$ do not buy given the current donation level, then the project will not be funded. A project stays alive if and only if either at the current donation level D (13) is satisfied or additional donations cannot incentivize buyers to buy. We show in the proof of Proposition 5 that $B(X_t)$ is increasing in D. Thus, we can define $\underline{D}(N,u,\mu_t)$ implicitly by

$$B(\underline{D}(N,u,\mu_t), N, u, \mu_t) = v_0. \tag{14}$$

The proposition below characterizes an equilibrium analogous to the one in Proposition 1 in this setting with learning. The interested reader can refer to the Appendix for the proof.

Proposition 5. *In the setting with learning, there exists an equilibrium analogous to the one in Proposition 1 with the following properties:*

i) If $\mu_t \leq \bar{\mu}^1 := \frac{(v_0+p)(1-\alpha)}{(v-(v_0+p))\alpha-(v_0+p)(1-\alpha)}$*, a buyer does not buy, regardless of her private signal. The campaign succeeds if and only if* $W \geq G - Np$*, where* N *is the number of purchases so far.*

ii) If $\mu_t > \bar{\mu}^1$*, then buyers with* $s = 1$ *buy until the donor runs out of funds.*

iii) When $\mu_t > \bar{\mu}_1$*, until the donor runs out of funds, the public belief evolves according to* (12).

In order to understand the relationship to the baseline model, it is useful to write an inequality analogous to (2) that describes when a buyer pledges. Denote the event in which the project succeeds by $\mathscr{S}(G,T)$. A buyer with private belief μ_t^s buys in period t if and only if

$$\mathbb{E}\big[(vq-p)\cdot \mathbf{1}(\mathscr{S}(G,T))|X_t,s\big] \geq v_0.$$

Rewriting the inequality yields an analogous expression to (2):

$$\mathbb{P}(\mathscr{S}(G,T)|X_t,s)\left(\frac{\mathbb{E}\big[vq\cdot \mathbf{1}(\mathscr{S}(G,T))|X_t,s\big]}{\mathbb{P}(\mathscr{S}(G,T)|X_t,s)}-p\right) \geq v_0.$$

Instead of the fixed valuation v, with learning, a buyer cares about the quality *conditional on success* $\frac{\mathbb{E}[vq\cdot \mathbf{1}(\mathscr{S}(G,T))|X_t,s]}{\mathbb{P}(\mathscr{S}(M,T)|X_t,s)}$. This valuation is decreasing in total donations D because less learning about quality can take place if a higher fraction of the goal is funded through donations which makes success a weaker signal of quality. Thus, one dollar of donations is less effective in incentivizing a buyer to buy. This also highlights why the comparative statics with respect to G, T, $\frac{v_0}{v-p}$ changes relative to Proposition 3. The relationship can become non-monotonic as a very low goal G, a very high T, or a very low v_0 makes success a weak signal which can reduce the probability of success.

7 Conclusion

We study reward-based crowdfunding campaigns as a new type of dynamic contribution game with private benefits and positive externalities. We view this paper as a necessary first step in understanding crowdfunding and alternative contribution games. Our results give rise to a number of follow-up research questions. For example, we abstract from versioning through the offering of different rewards. We also abstract away from competition on the platform. Our model also naturally gives rise to the key question of optimal mechanism design when donors and buyers are present.

References

Abrams, Eliot, "Securities Crowdfunding: More than Family, Friends, and Fools?," *working paper*, 2017.

Admati, Anat R and Motty Perry, "Strategic delay in bargaining," *The Review of Economic Studies*, 1987, *54* (3), 345–364.

Agrawal, Ajay, Christian Catalini, and Avi Goldfarb, "Some simple economics of crowdfunding," *Innovation Policy and the Economy*, 2014, *14* (1), 63–97.

_ , _ , and _ , "Crowdfunding: Geography, social networks, and the timing of investment decisions," *Journal of Economics & Management Strategy*, 2015, *24* (2), 253–274.

Alaei, Saeed, Azarakhsh Malekian, and Mohamed Mostagir, "A dynamic model of crowdfunding," *working paper*, 2016.

Andreoni, James, "Toward a theory of charitable fund-raising," *Journal of Political Economy*, 1998, *106* (6), 1186–1213.

Bagnoli, Mark and Barton L. Lipman, "Provision of Public Goods: Fully Implementing the Core through Private Contributions," *The Review of Economic Studies*, 1989, *56* (4), 583–601.

Bonatti, Alessandro and Johannes Hörner, "Collaborating," *American Economic Review*, 2011, *101* (2), 632–63.

Campbell, Arthur, Florian Ederer, and Johannes Spinnewijn, "Delay and deadlines: Freeriding and information revelation in partnerships," *American Economic Journal: Microeconomics*, 2014, *6* (2), 163–204.

Chakraborty, Soudipta and Robert Swinney, "Signaling to the crowd: Private quality information and rewards-based crowdfunding," *working paper*, 2016.

Chang, Jen-Wen, "The economics of crowdfunding," *working paper*, 2016.

Chemla, Gilles and Katrin Tinn, "Learning through crowdfunding," *working paper*, 2016.

Cvitanić, Jakša and George Georgiadis, "Achieving efficiency in dynamic contribution games," *American Economic Journal: Microeconomics*, 2016, *8* (4), 309–42.

de Rijt, Arnout Van, Soong Moon Kang, Michael Restivo, and Akshay Patil, "Field experiments of success-breeds-success dynamics," *Proceedings of the National Academy of Sciences*, 2014, p. 201316836.

Ellman, Matthew and Sjaak Hurkens, "Optimal crowdfunding design," *working paper*, 2016.

Fershtman, Chaim and Shmuel Nitzan, "Dynamic voluntary provision of public goods," *European Economic Review*, 1991, *35* (5), 1057–1067.

Grüner, Hans Peter and Christoph Siemroth, "Crowdfunding, Efficiency, and Inequality," *working paper*, 2017.

Kim, Jin-Hyuk, Peter Newberry, and Calvin Qiu, "The role of information signals in determining crowdfunding outcomes," *working paper*, 2017.

Kuppuswamy, Venkat and Barry L Bayus, "Crowdfunding creative ideas: The dynamics of project backers," in "The Economics of Crowdfunding," Springer, 2018, pp. 151–182.

Lee, Samuel and Petra Persson, "Financing from family and friends," *The Review of Financial Studies*, 2016, *29* (9), 2341–2386.

Li, Jiasun, "Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd," *working paper*, 2017.

List, John A and David Lucking-Reiley, "The effects of seed money and refunds on charitable giving: Experimental evidence from a university capital campaign," *Journal of political Economy*, 2002, *110* (1), 215–233.

Liu, Shannon, "A theory of collective investment with application to venture funding," *working paper*, 2018.

Marx, Leslie M and Steven A Matthews, "Dynamic voluntary contribution to a public project," *The Review of Economic Studies*, 2000, *67* (2), 327–358.

Rehurek, Radim and Petr Sojka, "Software framework for topic modelling with large corpora," in "In Proceedings of the LREC 2010 Workshop on New Challenges for NLP Frameworks" Citeseer 2010.

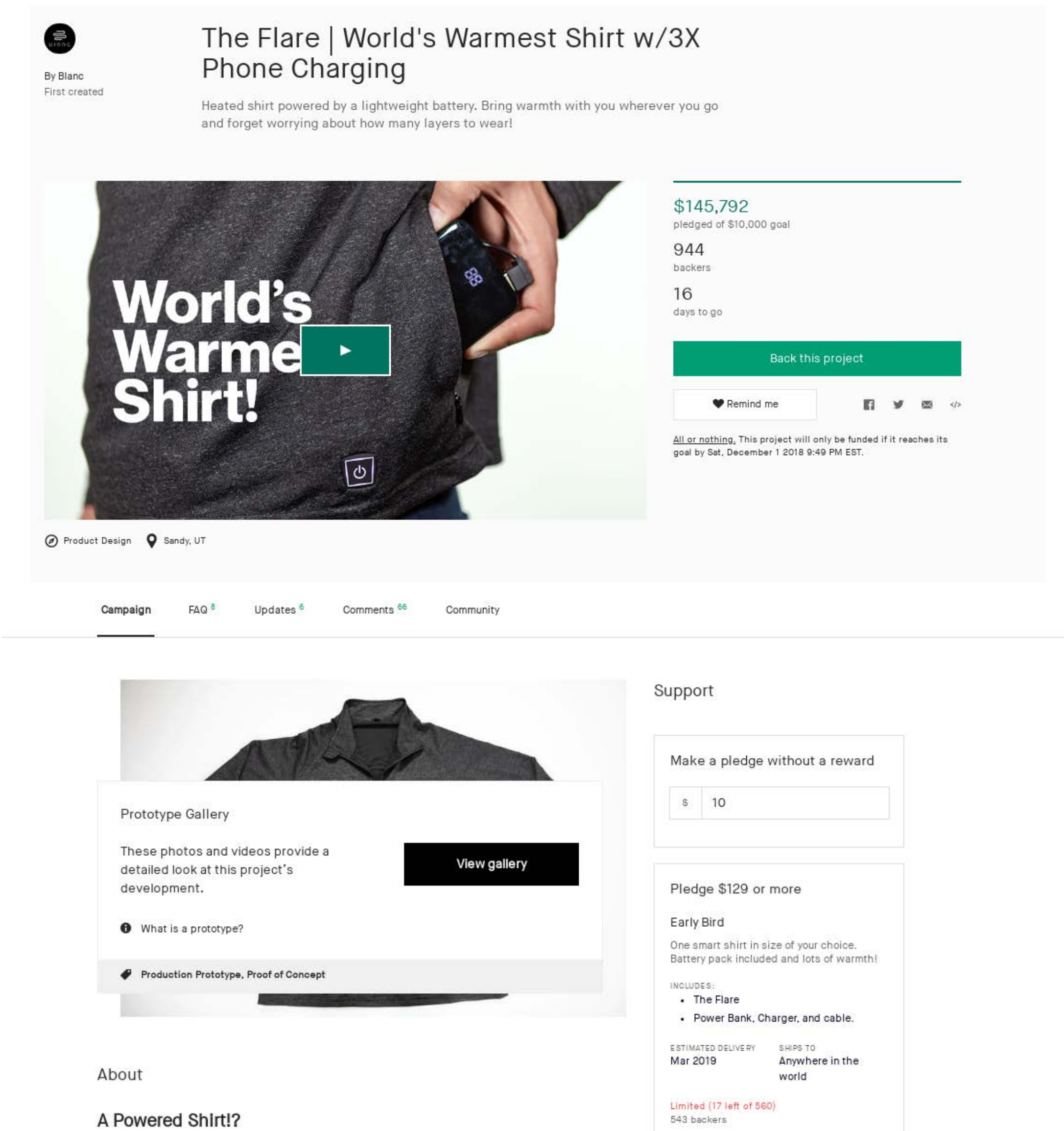
Sahm, Marco, "Advance-purchase financing of projects with few buyers," *working paper*, 2016.

Strausz, Roland, "A Theory of Crowdfunding: A Mechanism Design Approach with Demand Uncertainty and Moral Hazard," *American Economic Review*, June 2017, *107* (6), 1430–76.

Varian, Hal R, "Sequential contributions to public goods," *Journal of Public Economics*, 1994, *53* (2), 165–186.

A Sample Campaign

Figure 7: Example Kickstarter page



Note: Additional information on this campaign can be found at https://www.kickstarter.com/projects/wearblanc/the-worlds-warmest-shirt. Accessed on December 5, 2018

B Analysis of Moral Hazard

To investigate the existence of moral hazard, we create a second data set—a cross section of information on the near universe of Kickstarter campaigns. Included are all updates from the creator and all comments from backers for each campaign. We use targeted search queries and are able to collect information on 125,393 of the roughly 134,000 successful campaigns at the time of data collection. We cannot recover comments and updates that are protected as they can only be viewed by supporters of the campaign. These text data form the basis of our analysis. In total, we obtain over 1.3 million updates and over 10.4 million comments.

Comments and updates sometimes discuss the moral hazard issue, that is, successful or unsuccessful delivery, directly. Ironically, the last update posted to the first ever Kickstarter campaign is titled "sorry for no updates" and reads:[31]

> "I didn't head for the border with the cash. not yet at least...."

On the contrary, one update is titled "Rewards in the mail! Upwards and onwards!,"[32] and reads

> "We prevailed through it all to bring you your rewards! And we cannot thank you enough. We seriously would not have been able to do this without y'all."

We search all messages (comments/updates) posted after completion for key terms (T) that might be consistent or inconsistent with the presence of moral hazard. For updates, we search in both the title and text, and for comments, we search just within the text body. Our selected terms appear in Table 4. We concentrate on negative words only for comments and later confirm our findings using natural language processing on all the text.[33]

For each message i and project j, we calculate $\mathbb{I}\left[t_{ij}\right]$, where $\mathbb{I}[\cdot]$ is the indicator function for the presence of term t in the message. We aggregate over messages within project (N_j) and define $t_{\cdot j} = \max_{i \in N_j} \mathbb{I}\left[t_{ij}\right]$, or the presence of term t in any of the updates for a particular project.

[31]Source: https://www.kickstarter.com/projects/darkpony/drawing-for-dollars/posts/106. Accessed on December 5, 2018. We do not know for certain if the product was ever delivered.

[32]Source: https://www.kickstarter.com/projects/1329316091/tandem-ceramics/posts/1288618. Accessed on December 5, 2018.

[33]Performing analysis on positive and negative words reveals a similar pattern to updates: positive words occur more frequently within and across projects than negative words in the comments. However, we are concerned about inferring incorrect sentiment: "I received" is different than "When will I receive?" This is why we utilize natural language processing techniques. Also, we chose the word Terms of Use as several backers cite Kickstarter policy to return funds if the project cannot be completed.

Figure 8 plots the fraction of projects that contain each term t, or $\left(\sum_j t_{\cdot j}/J\right)$, separated by positive and negative terms. The total bars track the existence of any positive/negative term within project, or $\left(\sum_j \left[\max_{t\in T} t_{\cdot j}\right]/J\right)$. Most projects contain at least one update regarding delivery, and the most frequently occurring negative terms involve delays. There is a positive correlation among the negative terms, with the strongest being "apologize" and "delay." Apologizing for issues or problems is much more common than apologizing for failure or refund. Negative correlation occurs between some positive and negative terms, such as "in the mail" and "cancel."

More updates are positive than negative. Figure 9 presents a histogram of the magnitude of net positive updates across projects. We calculate

$$\text{NetPosCommentary}_j = \frac{\sum_{i\in N_j}\left(\max_{t\in T^{\text{pos}}} t_{i,j} - \max_{t\in T^{\text{neg}}} t_{i,j}\right)}{\#(N_j)},$$

a measure that is bounded between -1 and 1 for each project, with 1 meaning all comments only contain positive terms. There is a large mass at zero; 43 percent of this mass is due to updates that contain neither positive nor negative terms. The histogram shows that the distribution is heavily skewed towards positive values, that is, terms such as "sent" dominate the appearance of terms such as "delay" within projects.

It is perhaps unsurprising that creators tend to post more positive news. Figure 10 looks at comments, and plots the density of the fraction of comments within projects that contain negative terms. The plotted measure is

$$\text{FracNegComments}_j = \frac{\sum_{i\in N_j} \max_{t\in T^{\text{neg}}} t_{i,j}}{\#(N_j)},$$

Panel (a) shows the unweighted density and (b) plots the weighted density, where we weight by the number of comments within each project. The plot shows that less than a quarter of comments within a project are negative. Comparing (a) to (b), we see that there is positive correlation between the fraction of negative comments and the number of comments. That is, negative sentiment appears concentrated. For reference, Figure 11 shows the weighted distribution by each key term; the words delay, issue, and problem are the most prevalent.

Finally, we leverage all the data and use natural language processing techniques to decipher message topics. We perform the following data cleaning steps: (a) we convert the text to bag-of-words and remove short comments, (b) we remove commonly used words that are not useful for classification (or stop words, such as "a" and "the"), (c) we lemmatize the text (convert both nouns and verbs, such as running to run), (d) we create a frequency dictionary and remove infrequently used words, and (e) we add to the dictionary the results of analyzing contiguous words via an n-gram model with $n = 4$ (a computational linguistics model that decides if “not delivered” should be analyzed as a single term). This results in a corpus of 3.8 million comments with a dictionary of 26,000 words. An example comment is “have:the:cds:gone_out:yet:i:haven_t:seen:one:yet”.

After cleaning the data, we use Latent Dirichlet Allocation (LDA) and identify 200 topic clusters.[34] The results are consistent with the textual analysis above—most clusters do not reflect the moral hazard issue. For example, two clusters related to shipping discuss customs fees for EU backers.[35] Our analysis identifies five delivery clusters, three regarding timing of delivery (such as month), one expressing concern about some backers not receiving their reward (other backers is a key term), and another that includes the word scam and words likely relating to projects in the games category.

We investigate requests for refunds as the algorithm groups words like refund, cancel, and not respond together. Performing a second LDA analysis on these comments identifies clusters involving product issues but also identifies some projects where consumers demand refunds due to nondelivery. An example includes the Tiko printer, a project with 25,000 comments, thousands from backers asking for a refund (for years) due to nondelivery. In expectation, relatively few projects are subject to these topics.

[34] See Rehurek and Sojka (2010) for a description of the algorithm and the basics of implementation.

[35] An example cluster is: shipping:to_pay:custom:pay:u:not:fee:tax:eu:order.

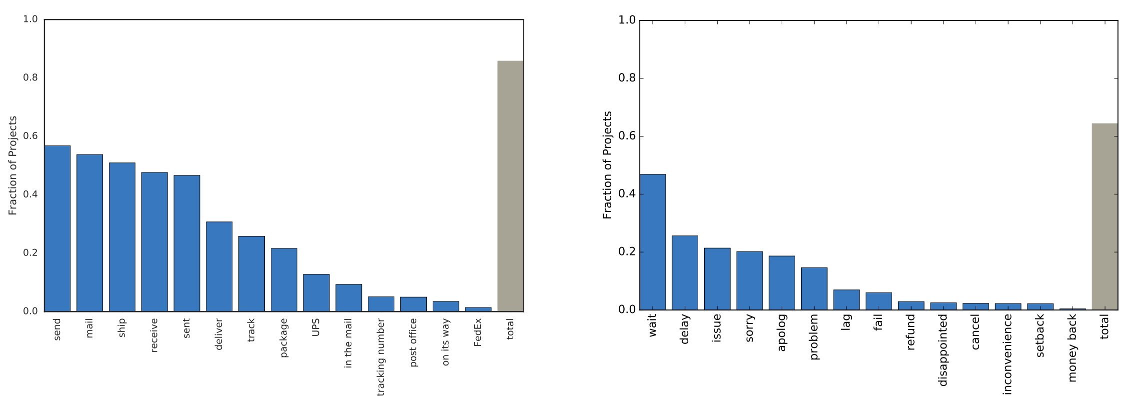
Table 4: Positive and Negative Sentiment Terms

Update - Positive	Update - Negative	Comment - Negative
delivered	apologies	cancel
FedEx	apologize	delay
in the mail	cancel	didn't deliver
mailed	delay	disappointed
on its way	disappointed	dissatisfied
package	fail	fail
post office	inconvenience	issue
received	issue	lag
send	lag	money back
sent	money back	no response
shipped	problem	not delivered
tracking	refund	not received
tracking number	setback	problem
UPS	sorry	refund
	wait	scam
		Terms of Use
		upset
		wait

Note: We search for all forms of many terms. For example, we capture deliver, delivered, will deliver, etc.

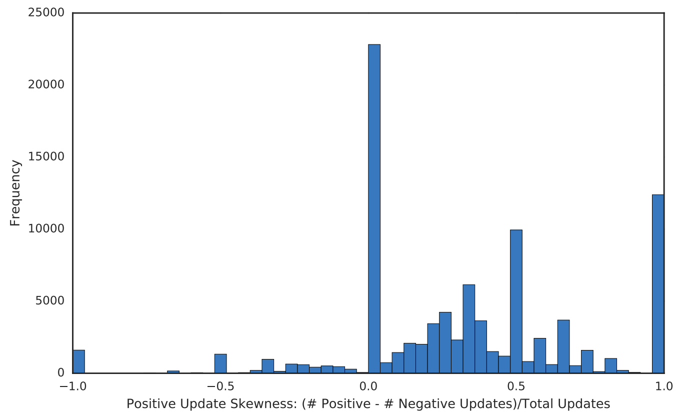
Figure 8: Fraction of Projects that Contain Positive and Negative Words

(a) Positive Words (b) Negative Words



Note: Fraction of projects that contain at least one update with each key term. Total denotes the existence of any term.

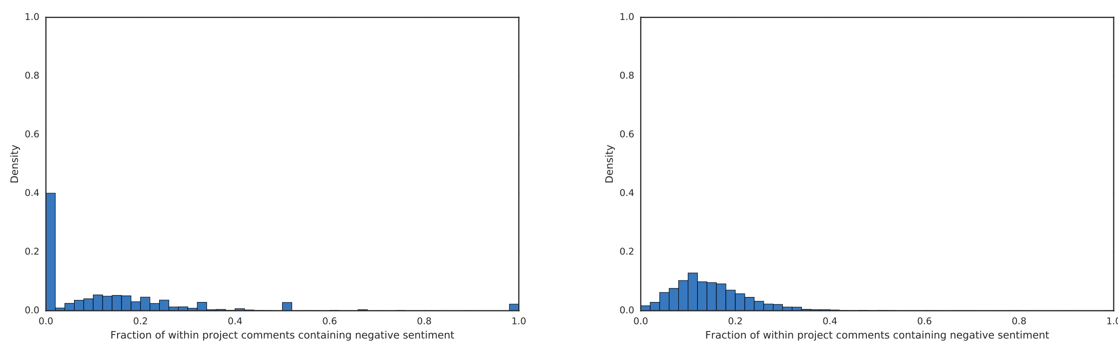
Figure 9: Histogram of Net Positive Updates from Creators



Note: The histogram plots $\text{NetPosCommentary}_j = \frac{\sum_{i \in N_j} \left(\max_{t \in T^{\text{pos}}} t_{i,j} - \max_{t \in T^{\text{neg}}} t_{i,j} \right)}{\#(N_j)}$, or the net fraction of updates within project that contain positive (vs negative) words. At -1, all updates are negative. At +1, all updates are positive.

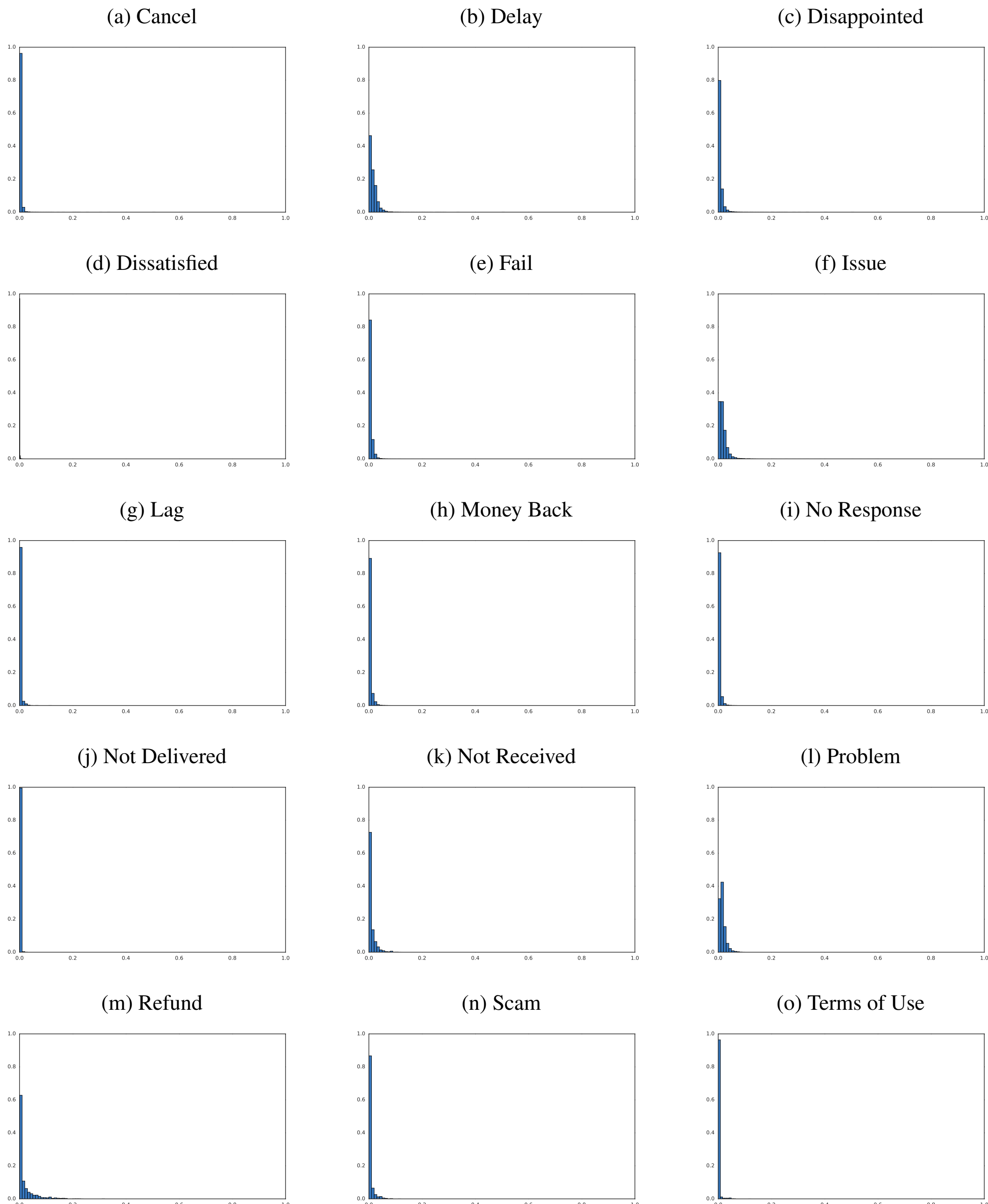
Figure 10: Density of Within Project Negative Sentiment from Comments.

(a) Unweighted (b) Weighted



Note: The densities plot $\text{FracNegComments}_j = \frac{\sum_{i \in N_j} \max_{t \in T^{\text{neg}}} t_{i,j}}{\#(N_j)}$, or the fraction of comments within projects that contain negative sentiment. Panel (a) plots the unweighted density and panel (b) plots the weighted density, where the density is weighted by the number of comments per project.

Figure 11: Density of Within Project Negative Sentiment from Comments, by Term



Note: Weighted density plots of negative sentiment by key term. The vertical axis is density. The horizontal axis measures the fraction of comments within project that contain the key term. Observations are weighted by the number of comments within project. The total sample size for each plot is 5,097,240, aggregated up to the project level: $n = 53,384$.

C Proofs

C.1 Proof of Proposition 1

We prove a more general version of Proposition 1 where the outside option can depend on time as discussed in Remark 1. Thus, we allow v_0 to be a function $v_0(u)$ for all definitions in the main text.

While proving Proposition 1, we will simultaneously establish some properties of the functions $\pi(D,N,u)$, $\underline{D}(N,u)$, and $\xi_j(W)$ for any $u>0$ (before the deadline). These properties are stated in Lemma 3 below, and will be useful for the comparative statics results in Section 4.3.2.

Lemma 3. *Given $u \in (0,T]$, the following holds:*
(i) For $\pi(D,N,u)\in(0,1)$, $\pi(D,N,u)$ is increasing in D, N, T, and $\frac{1}{v-p}$, and it is decreasing in G. Moreover, π is continuous in D and $\pi(D,N,u)(v-p)-v_0(u)$ in strictly increasing in u.
(ii) $\underline{D}(N,u)$ is decreasing in N and decreasing in u for $Np<G$. It is increasing in G and $\frac{1}{v-p}$.
(iii) $\xi_j(W)$ is decreasing in j and increasing in W, G, and $\frac{1}{v-p}$ for $j\geq 2$.

Proof. (Proposition 1 and Lemma 3) The starting point of the equilibrium construction is to specify beliefs according to (7). The equilibrium constructed is then shown to be consistent with this belief system, i.e., the buyers update according to Bayes' rule whenever possible, given the equilibrium strategies.

Step 1 (Last period; $u=\Delta$): First, consider the donor in period $T-\Delta$. Suppose that total donations so far are D and the number of buyers is N. The campaign only succeeds if at the end of period T, total donations $D^+_\Delta(D,N,0)$ satisfy $Np+D^+_\Delta(D,N,0)\geq G$. Note that D^+_Δ is weakly increasing in W, D and G, and weakly decreasing in N and $\frac{v_0}{v-p}$. Further, since D^+_Δ is independent of the period length Δ, the limit as $\Delta\to 0$ is automatically well-defined and converges to (5).

Next, consider the buyer in period $T-\Delta$. Suppose that total donations so far are D and the number of buyers is N. Given beliefs in (7), this buyer assigns the following probability to the campaign being successful (if she buys):

$$\pi_\Delta(D,N,\Delta)=\begin{cases}\frac{1-F(G-p(N+1))}{1-F(D)} & \text{if } N+1<\underline{M}(D)\\ 1 & \text{otherwise.}\end{cases}.$$

She buys if and only if $\pi_\Delta(D,N,\Delta)\geq\frac{v_0(\Delta)}{v-p}$. Note that $\pi_\Delta(D,N,\Delta)$ is increasing in D, N, p, decreasing in G, and continuous in D. π_Δ is independent of the period length Δ, so the limit $\pi(D,N,0)$ is automatically well-defined and inherits these properties.

Step 2 (Earlier periods; $u > \Delta$): For periods before the deadline, i.e., for $t < T-\Delta$, we construct the best responses of buyers and the donor in state (D,N,u) if $N = \underline{M}(D) - j$ and construct $\pi(D,N,u)$, $\underline{D}(N,u)$, and $\xi_j(W)$, by induction on j.

For the induction step it is useful to also show that $D^+_\Delta(D,N,u;W)$ is weakly increasing in W, D and G, and weakly decreasing in u and N. Since the induction is not in time t, it is straightforward to see that the limit as $\Delta \to 0$ is well defined, so we oftentimes do not write the expressions for $\Delta > 0$ but only present the expressions for $\Delta \to 0$.

I) Suppose $N \geq \underline{M}(D) - 1$: An arriving buyer knows that if he buys, the campaign will be successful. Such a buyer will always buy and $\pi_\Delta(D,N,u) = 1$ for all $u > 0$. The donor does not donate, i.e., $D^+_\Delta(D,N,u;W) = D$.

Trivially, D^+_Δ is weakly increasing in W, D, and G, and weakly decreasing in u, N and p. Let $\xi_1(W) \equiv 0$ for all W, i.e., as long as a buyer arrives before T, she will always buy. Lemma 3 trivially holds.

II) Suppose $N = \underline{M}(D) - 2$:[36] From a buyer's perspective, if she buys, the project will be successful if and only if either at least one more buyer arrives in the time remaining or the donor donates the rest. Thus, given beliefs (7), for $u \in \mathbb{T}_\Delta$:

$$\begin{aligned} \pi_\Delta(D,N,u) = &\quad 1-(1-\Delta\lambda)^{\frac{u}{\Delta}} + (1-\Delta\lambda)^{\frac{u}{\Delta}} \tfrac{1-F(G-p(N+1))}{1-F(D)} \\ \Longrightarrow \quad \lim_{\Delta\to 0} \pi_\Delta(D,N,u) = &\quad 1-e^{-\lambda u} + e^{-\lambda u} \tfrac{1-F(G-p(N+1))}{1-F(D)} \qquad = \pi(D,N,u). \end{aligned}$$

All donor types want to donate just enough to make a potential buyer in the next period buy, i.e., so that $\pi_\Delta(D,N,u) \geq \frac{v_0(u)}{v-p}$.

Next, note that $\pi(D,N,u)(v-p) - v_0(u)$ is strictly increasing in u. This is because $F' > 0$ and by (1), we have

$$\frac{\partial}{\partial u}\pi(D,N,u) \geq \lambda e^{-\lambda u} F(p) > \frac{v_0'(u)}{v-p}.$$

The rest of Lemma 3 (i) follows immediately.

[36] This step is redundant for the proof of Proposition 1, but necessary for Lemma 3 and useful for the proofs of Subsection 4.3.2.

As $\pi_\Delta(D,N,u)$ is increasing in D, the donor's best response in state (D,N,u) as $\Delta \to 0$ is to donate just enough to assure that the next buyer buys. To this end, define $\underline{D}_\Delta(N,u)$ as

$$\pi_\Delta(\underline{D}_\Delta(N,u),N,u) = \frac{v_0(u)}{v-p},$$

so that $\lim_{\Delta\to 0} \underline{D}_\Delta(N,u) = \underline{D}(N,u)$ defined in (3). Thus, the limit of the donor's optimal strategy is given by (6). $\underline{D}(N,u)$ is decreasing in N because π is increasing in both D and N. Also, $\underline{D}(N,u)$ is decreasing in u because $\pi(D,N,u)(v-p)-v_0(u)$ is strictly increasing in u and D. Finally, note that $\underline{D}(N,u)$ is increasing in $\frac{1}{v-p}$ because $\pi(D,N,u)$ is increasing in D, and it is increasing in G because π is decreasing in G, but increasing in D. Thus, Lemma 3 (ii) holds. Therefore, D^+ is weakly increasing in W, D and weakly decreasing in u, N.

Define $\xi_2(W)$ implicitly by $\pi(W,\underline{M}(W)-2,\xi_2(W))(v-p)-v_0(u)=0$ for $\underline{M}(W)>2$ and $\xi_2(W)=0$ otherwise. Note that locally, as long as $\underline{M}(W)$ is unchanged, $\xi_2(W)$ is decreasing in W: This is because π is weakly increasing in its first argument, so locally, $\xi_2(W)$ must be increasing in W. Moreover, since π is increasing in N, $\xi_2(W)$ is decreasing in W even if an increase in W decreases $\underline{M}(W)$. This is the case if $\underline{M}(W) = \frac{G-W}{p}$. Finally, note that $\xi_2(W)$ is increasing in $\frac{1}{v-p}$, i.e., Lemma 3 (iii) holds.

Finally, if there are no more arrivals and the remaining time until the deadline is $\xi_2(W)$, then the project is bound to die as $\Delta \to 0$. Even if the donor donates up to his full valuation, buyers are not willing to purchase as $\pi(W,\underline{M}(W),u)=0$ for all $u<\xi_2(W)$.

III) For the induction step, assume that for each D and $i=2\dots j-1$, $D^+(D,\underline{M}(D)-i,u)$ is given by (6) where $\underline{D}(\underline{M}(D)-i,u)$ is given by (3), $\xi_i(W)$ given by (4) and $\pi(D,\underline{M}(D)-i,u)$ specifies the probability of success given equilibrium beliefs (7) and the specified equilibrium strategies where

- if $\pi(D,\underline{M}(D)-i,u) \geq \frac{v_0(u)}{v-p}$, for all $u \in (0,T]$

$$\frac{\partial}{\partial u}\pi(D,\underline{M}(D)-i,u) \geq \lambda e^{-\lambda u}\left(\frac{v_0(u)}{v-p}\right)^i F(p)$$

- Lemma 3 holds for all $i \leq j-1$ and $D^+(D,N,u)$ is weakly increasing in W, D and weakly decreasing in u, N.

Then, given any state (D,N,u) with $N=\underline{M}(D)-j$ and $u \geq \xi_{j-1}(D)$, a buyer faces the following probability of success, given beliefs (7), for $\Delta \to 0$:

$$\pi(D,N,u) = \int \int_0^{\max\{u-\xi_{\underline{M}(W)-N-1}(w),0\}} \lambda e^{-\lambda s} \pi(\max\{D, \underline{D}(N+1,u-2)\}, N+1, u-s) ds \frac{f(w)}{1-F(D)} dw$$

Note that $\underline{M}(D) \geq \underline{M}(D^+(D,\underline{M}(D)-j+1,s))$, i.e., $\underline{M}(D)-j+1 = \underline{M}(D^+(D,\underline{M}(D)-j+1,s))-i+1$ for $i \leq j$, i.e., the probability in the integral can be calculated using the induction assumption.

First, it follows immediately that π is continuous in D. $\pi(D,N,u)$ is locally increasing in D, i.e., for a fixed $\underline{M}(D)$, because $\frac{1}{1-F(D)}$ is increasing in D (as $F' > 0$), D^+ is weakly increasing in D, and $\pi(D,N,u)$ is increasing in D. Furthermore, $\underline{M}(D)$ is weakly decreasing in D. Thus, if D is increased to a D' such that $\underline{M}(D') < \underline{M}(D)$, then $\pi(D',N,u) = \pi(D',\underline{M}(D')-i,u)$ for $i < j$ which is by assumption increasing in D', and $\pi(D',N,u) > \pi(D,N,u)$ because $\xi_{j-1}(\cdot) > \xi_{i-1}(\cdot)$, $D^+(\cdot,N+1,u-s)$ is weakly increasing (by assumption), $\pi(\cdot,N+1,u-s)$ is strictly increasing (by assumption).

$\pi(D,N,u)$ is increasing in p, and decreasing in G as $\pi(D,N+1,u)$ has these properties by assumption. Finally, it follows immediately that $\pi(D,N,u) < \pi(D,N+1,u)$. Thus Lemma 3 (i) holds. The buyer buys if and only if $\pi(D,N,u) \geq \frac{v_0(u)}{v-p}$ and because $\pi(D,N,u)$ is increasing in D there is a unique $\underline{D}(N,u)$ such that $\pi(\underline{D}(N,u),N,u) = \frac{v_0(u)}{v-p}$. The donor's optimal strategy is then given by (6).

Next, note that for $\pi(D,N,u) \geq \frac{v_0(u)}{v-p}$, we have that

$$\begin{aligned}
\frac{\partial}{\partial u}\pi(D,N,u) &\geq \\
\left(\lambda e^{-\lambda u}\left(\frac{v_0(u)}{v-p}\right)^{j-1} F(p)\right)\cdot & \\
\int \int_0^{\max\{u-\xi_{j-1}(w),0\}} \lambda e^{-\lambda s}\cdot \mathbf{1}_{\{\pi(D^+(D,N+1,u-s),N+1,u-s)\geq \frac{v_0(u)}{v-p}\}} ds \frac{f(w)}{1-F(D)} dw &\geq \\
\left(\lambda e^{-\lambda u}\left(\frac{v_0(u)}{v-p}\right)^{j-1} F(p)\right)\cdot \frac{v_0(u)}{v-p} &
\end{aligned}$$

Thus, by (1), $u \mapsto \pi(D,N,u) - \frac{v_0(u)}{v-p}$ is strictly increasing. Hence, there exists $\xi_j(W)$, such that $\pi(W,\underline{M}(W)-j,u) - \frac{v_0(u)}{v-p} \geq 0$ for $u \leq \xi_j(W)$ and $\pi(W,\underline{M}(W)-j,u) - \frac{v_0(u)}{v-p} < 0$ for $u > \xi_j(W)$. Thus, in equilibrium for $u < \xi_{j-1}(D)$, $\pi(D,\underline{M}(D)-j,u) = 0$ and $\underline{D}(N,u) > W$. Note that $\xi_j(W) >$

$\xi_{j-1}(W)$ because $\pi(W, \underline{M} - i, u)$ is decreasing in i. The other properties of $\xi_j(W)$, $\underline{D}(N,u)$ and $D^+(D,N,u)$ follow analogously to II), i.e., Lemma 3 (ii), (iii) hold.

Finally, note that given the optimal strategies constructed, the buyers' beliefs on equilibrium path are indeed truncations of F given by (7) according to Bayes' rule because all donors with $W \geq D$ play the same strategy. Off-path, the beliefs given by (7) support the equilibrium constructed. ■

C.2 Proof of Proposition 2

Proof. (Proposition 2) Consider an arbitrary equilibirum and a realization of W. In the last period, the donor and buyers must play the same strategies as in Proposition 1. In the following we show that on-path beliefs must always be a truncation of the prior F.

Lemma 4. *Given a D that has positive mass on equlibrium path, given N, u,*

$$\tilde{F}(w;(D,N,u)) = \frac{F(w) - F(D)}{1 - F(D)} \cdot 1(w \geq D).$$

Proof. (Lemma 4) To this end, we need to show that all donor types who can afford it must always donate the same total amount D. This can be shown by backward induction in time for given D and N.

Consider the second but last period $u = 2\Delta$. Then, given a N and given that buyers have belief system $\tilde{F}(w;(D,N,\Delta))$ in the last period ($u = \Delta$), it is optimal for all donor types to donate just enough to guarantee that the last buyer buys, i.e., all donor types donate

$$D^*(N,2\Delta) := \min\left\{D' \geq D | 1 - \tilde{F}((M_0 - (N+1))p;(D',N,\Delta)) \geq \frac{v_0}{v-p}\right\}$$

whenever they can so that

$$D^+(D,N,2\Delta) = \min\left\{\max\{D^*(N,2\Delta), D\}, W\right\}.^{37}$$

Consequently, in order to be consistent with Bayes' rule, $\tilde{F}(w;(D,N,\Delta))$ – the belief system in the last period – must satisfy the following. Let $\tilde{F}_{2\Delta}$ be the distribution of donors that on path survive until period 2Δ if N buyers have arrived so far. Then, on path (in particular it must hold that

$D \geq D^*(N, 2\Delta)$) beliefs must be a truncation of $\tilde{F}_{2\Delta}$, so that

$$\tilde{F}(w;(D,N,\Delta)) = \frac{\tilde{F}_{2\Delta}(w) - \tilde{F}_{2\Delta}(D)}{1 - \tilde{F}_{2\Delta}(D)} \cdot 1(w \geq D).$$

Next, consider $u > 2\Delta$ and assume that in all periods $u' < u$ the buyer only truncates beliefs on path in every state (D, N, u'), i.e., that on equilibirum path

$$\tilde{F}(w;(D,N,u')) = \frac{\tilde{F}_{u'+\Delta}(w) - \tilde{F}_{u'+\Delta}(D)}{1 - \tilde{F}_{u'+\Delta}(D)} \cdot 1(w \geq D).$$

for some cdf $\tilde{F}_{u'+\Delta}$. Since all buyers only observe the current total donation level, their donation strategy in period u only affects the buying behavior in the next period $u-\Delta$. Thus, it is optimal for all donor types to donate just enough to guarantee that buyer $u-\Delta$ buys. Formally, given a belief system $\tilde{F}(w;(D,N,u))$, let the probability of success in the continuation game be $\pi^{\Delta}(D,N,u)$, all donor types donate

$$D^*(N,u) := \min\left\{D' \geq D | \pi^{\Delta}(D',N,u) \geq \frac{v_0}{v-p}\right\}$$

whenever they can so that

$$D^+(D,N,u) = \min\left\{\max\{D^*(N,u), D\}, W\right\}.$$

As at the onset of the campaign the prior distribution of W is F, on path beliefs must all be a truncation of F and

$$\tilde{F}(w;(D,N,u)) = \frac{F(w) - F(D)}{1 - F(D)} \cdot 1(w \geq D).$$

■

Having established that on-path beliefs are always a truncation of F, we next need to show that $D^*(D,N,u) \in [\underline{D}(N,u), \overline{D}(N,u)]$. Note that Lemma 4 does not pin down equilibrium beliefs as off path beliefs are arbitrary. Which donation levels are enforced on path depends on the off path beliefs.

We argue by contradiction. If $D^*(N,u) < \underline{D}(N,u)$], then the on-path equilibrium belief after

N arrivals of buyers and u periods form the deadline is smaller than $\frac{v_0}{v-p}$ by definition. Thus, $D^*(N,u) \geq \underline{D}(N,u)$]. Similarly, if $D^*(N,u) > \overline{D}(N,u)$], then the probability of success is sufficiently high for buyers do buy even if the donor does not ever donate anything at all. Thus, it is a strictly dominant strategy for the donor to not donate more than $\overline{D}(N,u)$. Consequently, $D^*(N,u) \leq \overline{D}(N,u)$].

Given these bounds for donations it follows immediately that the equilibirum, that requires the lowest donations while still guaranteeing buyers in the next period to buy, must maximize the ex-ante probability of success and is optimal for the donor. ■

C.3 Proof of Lemma 1

Proof. (Lemma 1) Consider two campaigns $k = 1,2$ with arrival rates λ_k and time horizon T_k so that $\frac{T_1}{\lambda_1} = \frac{T_2}{\lambda_2}$, i.e., $\lambda_2 = \frac{T_2}{T_1}\lambda_1$, common goal G, price p and donor wealth distribution F. Let $N_k(t)$ denote the number of arrivals by time t in campaign k. Then, the probability of $N_1(t) = n$ is given by $\mathbb{P}(N_1(t) = n) = \frac{(\lambda_1 t)^n}{n!}e^{-\lambda_1 t}$. This is also equal to the probability of n arrivals by time $t\frac{T_1}{T_2}$ for campaign 2, since

$$\mathbb{P}\left(N_2\left(\frac{T_1}{T_2}t\right) = n\right) = \frac{\left(\lambda_2\left(\frac{T_1}{T_2}t\right)\right)^n}{n!}e^{-\lambda_2\left(\frac{T_1}{T_2}t\right)} = \frac{(\lambda_1\frac{T_2}{T_1}\frac{T_1}{T_2}t)^n}{n!}e^{-\lambda_1\frac{T_2}{T_1}\frac{T_1}{T_2}t} = \frac{(\lambda_1 t)^n}{n!}e^{-\lambda_1 t}.$$

Put differently, the arrival process of buyers for campaign 1 has the same distribution as the arrival process of campaign 2 if time is scaled by a factor $\frac{T_1}{T_2}$.

The donor and buyer's strategy in the last period when $u = 0$ is the same across campaigns. For $u > 0$, if the probability of success in campaign 1 is given by $\pi_1(D,N,u)$, then it follows by the construction in Proposition 1 and the observation about the distribution of arrivals of buyers that $\pi_2(D,N,u) = \pi_1\left(D,N,\frac{T_1}{T_2}u\right)$. All statements of Lemma 1 follow immediately. ■

C.4 Proof of Proposition 3

Proof. (Proposition 3)

i) $\Pi(G,T)$ is decreasing in G because by Lemma 3 (iii), $\xi_j(W)$ is increasing in G. It follows immediately from (9) that $\Pi(G,T)$ is increasing in T because $\underline{u}$ and $\xi_j(w)$ are independent of T.

ii) and iii) By Lemma 3 (ii) $\underline{D}(N,T-t)$ is decreasing in T. Thus, for any realization of arrivals

$(\tau_j)_{j=1}^{\infty}$, $\underline{D}(N, T-\tau_j)$ is decreasing in T, so the realized D_t is decreasing in T by (10). Taking expectations over realizations of buyer arrivals $(\tau_j)_{j=1}^{\infty}$ and donor wealth yields that $\mathbb{E}^{W,(\tau_j)_{j=1}^{\infty}}[D_t]$ is decreasing in T since the average of decreasing functions is decreasing.

Similarly, by Lemma 3 (ii), $\underline{D}(N, T-t)$ is increasing in G and $\frac{v_0}{v-p}$. Thus, by (10), $\mathbb{E}^{W,(\tau_j)_{j=1}^{\infty}}[D_t]$ is increasing in G and $\frac{v_0}{v-p}$.

(iv) Let $u>0$ and $N<M_0$. Then, by the definition of (3) if $\underline{D}(N,u)>G-Np$, then

$$1>\frac{v_0}{v-p}=\pi(\underline{D}(N,u),N,u)>\pi(G-Np,N,u)=1.$$

and since π is continuous in D by Lemma 3 (i), it can never be optimal for the donor to donate $G-Np$.

■

C.5 Proof of Proposition 4

Proof. (Proposition 4)

i) **Donations at the start of the campaign only:**

Consider the setting where donations are allowed only at the start of the campaign, and consider a realization of buyer arrivals and donor value W that leads to a success. We show that this event is also a success in any equilibrium of the game that allows continuous donations, i.e., it belongs to $\mathscr{S}(G,T)$.

Let D^0 denote the optimal donation made at the start of the campaign and by assumption the probability of success with that donation is positive. If continuous donations were allowed, the donor could replicate the strategy of donating D^0 at the start. Thus, if there have been N arrivals with u periods remaining, the probability of success is given by

$$\pi(D^0,N,u)\geq\pi^{\text{no}}(N,u)\geq\frac{v_0}{v-p}$$

given the equilibrium beliefs, where $\pi^{\text{no}}(N,u)$ denotes the probability of success of a game absent donations with goal $G-D_0$. The inequality follows from the fact that a current donation D^0 implies that $W\geq D^0$ in any equilibrium in the main model by Proposition 2, while absent further donations it is assumed that $W = D_0$. Consequently, this strategy would lead to a success even when continuous donations are allowed. The donor's optimal strategy must, therefore, also lead to a success in any PBE of the model with continuous donations, i.e., $\Pi(G,T) \geq \Pi^0(G,T)$ for all $G,T>0$. Furthermore, note that $D^0 < W$ for any realization of W, because donating all W gives the donor a payoff of zero while lowering it gives him a positive expected payoff. Thus, there is always a realization of arrivals that leads to a success with continuous donations, but not in the counterfactual, i.e., $\Pi(G,T) > \Pi^0(G,T)$ for all $G,T>0$. In all such events, the donor is strictly better off when continuous donations are allowed because he can induce a success with lower donations. For all other events, a success in the model with continuous donations must make him better off than no success.

Next, consider a realization of buyer arrivals. There are two cases: First, if a particular buyer bought in the setting in which donations are allowed only at the start, then she must also buy if donations are allowed throughout. For all such realizations, the probability of success is strictly higher if donations were allowed throughout, making the buyer strictly better off.

Second, if a buyer did not buy in the setting with donations allowed only at the start, then

- she buys if donations are allowed throughout if and only if $W \geq D^*(D,N,u)$ (where $D^*(D,N,u) = \max\{D, \underline{D}(N,u)\}$ in the platform optimal equilibrium and $D^*(D,N,u) = \max\{D, \overline{D}(N,u)\}$ in the platform optimal equilibrium) and
- she does not buy if and only if $W < D^*(D,N,u)$.

Thus, this buyer is better off in expectation upon arrival.

ii) **Donations at the deadline only:**

Next consider a realization of buyer arrivals τ_i and donor value W that leads to a success in the setting where donations are allowed only at the end of the campaign. We show that this is event is in $\mathscr{S}(G,T)$ for any PBE.

Buyers believe that $W \sim F$ throughout. (Their beliefs cannot be updated upward as in the case of continuous donations.) Thus, for this realization of arrivals, in a setting where continuous donations were allowed, buyers would buy even absent donations. Therefore, the optimal strategy of the donor would be to not donate until the deadline is reached. At the same time if there are few arrivals early on, the donor can induce a success with continuous donations given an equilibrium belief system, but the campaign would die if donations are only allowed at the deadline. Thus, $\Pi(G,T) > \Pi^T(G,T)$.

For all such realizations which end in success in a setting with donations only at T, the donor is indifferent between the model with continuous donations and the counterfactual, since he chooses the same strategy. Further, a success in the model with continuous donations must make him better off (and for some realizations when $D_T < W$ strictly better off) than no success.

Next, consider a realization of buyer arrivals. If a particular buyer bought in the setting with donations allowed only at T, then she must also buy if donations are allowed throughout. For all those realizations, the probability of success is higher if donations were allowed throughout, making the buyer strictly better off. If a buyer did not buy in the counterfactual, then

- she buys if donations are allowed throughout if and only if $W \geq D^*(D,N,u)$ and
- she does not buy if and only if $W < D^*(D,N,u)$.

Thus, this buyer is better off in expectation upon arrival.

■

C.6 Proof of Proposition 5

Proof. (Proposition 5) We follow the same steps as in the proof of Proposition 1. In the same vein, let us specify the belief system on- and off-path by (11). The construction will show how these beliefs are indeed consistent with Bayes' rule whenever possible.

First, define $\overline{t}(\mu,\mu')$ to be the time it takes to reach belief $\mu' < \mu$ starting from belief μ if a buyer with signal $s = 1$ buys while a buyer with signal $s = 0$ does not buy, i.e.,

$$\overline{t}(\mu,\mu') := \frac{1}{\lambda(1-\alpha)} \log\left(\frac{\mu}{1-\mu}\frac{1-\mu'}{\mu'}\right).$$

Moreover, the solution to (12) with initial condition $\mu_0 = \mu$ is given by

$$\hat{\mu}(\mu,t) = \frac{1}{1+e^{\lambda(1-\alpha)t+\log\frac{1-\mu}{\mu}}}.$$

Since a buyer who sees $s=0$ never buys, we do not need to discuss such a buyer's strategies.

Step 1 (Last period; $u=0$): First, consider the donor in period T, i.e., at $u=0$. Then, the donor's optimal strategy in state $X_T=(D,N,u,\mu_T)$ is given by

$$D^+(X_T)=\max\{D,\min\{G-Np,W\}\}.$$

Note that D^+ is weakly increasing in W, D and G, and weakly decreasing in N and p. Since D^+ is independent of the period length Δ, the limit as $\Delta\to 0$ is well-defined.

Next consider the buyer with $s=1$ in period T. The probability that this buyer assigns to the project being successful conditional on quality q in state $X_{T-\Delta}$ if she buys is given by

$$\beta_q^\Delta(X_{T-\Delta})\equiv\begin{cases}\frac{1-F(G-p(N+1))}{1-F(D)} & \text{if } N+1<\underline{M}(D)\\ 1 & \text{otherwise.}\end{cases}$$

If $s=0$, the buyer never buys. If $s=1$ she buys if and only if

$$(v-p)\cdot\mu_T^1\cdot\beta_1^\Delta(X_{T-\Delta})-p\cdot(1-\mu_T^1)\cdot\beta_0^\Delta(X_{T-\Delta}) \quad\geq\quad v_0,$$

i.e., $\mu_T^1\geq\frac{v_0+p\beta_q(X_{T-\Delta})}{v\beta_q^\Delta(X_{T-\Delta})}$. In the limit as $\Delta\to 0$, the buyer buys if and only if

$$\mu_T^1\geq\frac{v_0+p\beta_q(X_T)}{v\beta_q^\Delta(X_T)}.$$

Note that $\beta_q(X_T)=\lim_{\Delta\to 0}\beta_q^\Delta$ is increasing in D, N, p, decreasing in G, and continuous in D.

Step 2 (Earlier periods; $u>0$): For periods before the deadline, i.e., for $u>0$, we construct the best responses of buyers and the donor by induction in j, given that D donations have been made and $N=\underline{M}(D)-j$.

I) Suppose $N\geq\underline{M}(D)-1$: A buyer with $s=1$ buys if and only if $\mu_t^1\geq\frac{v_0+p}{v}$ and $\beta_1(X_t)=\beta_0(X_t)=1$. Trivially, D^+ is weakly increasing in W, D, and G, and weakly decreasing in u, N and p. Let $\xi_1(W)\equiv 0$ for all W, i.e., as long as a buyer arrives before T, she will always buy.

Define $\bar{\mu}^1$ to be such that

$$\frac{\bar{\mu}^1}{\bar{\mu}^1+(1-\bar{\mu}^1)\alpha}=\frac{v_0+p}{v}$$

i.e., $\bar{\mu}^1 = \frac{(v_0+p)(1-\alpha)}{(v-(v_0+p))\alpha-(v_0+p)(1-\alpha)}$. Thus for $\mu_t < \bar{\mu}^1$ no future buyer will buy and for $\mu_t \geq \bar{\mu}^1$ only buyers with $s = 1$ buy. This fully characterizes the buyers' strategies if $N \geq \underline{M}(D)-1$. Public beliefs remain unchanged if $\mu \in [0, \bar{\mu}^1)$ and they evolve (in the limit $\Delta \to 0$) according to (12) in $\mu_t \geq \bar{\mu}^1$. Let $\gamma_1(W,\mu) \equiv 1$. In the following construction, $\gamma_i(W,\mu)$ will be the analogon to $\xi_i(W)$ in the main model.

II) For the induction step assume that for each D and μ and $i = 2, \ldots, j-1$, we have constructed for $\Delta \to 0$, $\beta_q(D, \underline{M}(D)-i, u, \mu)$, $D^+(D, \underline{M}(D)-i, u, \mu)$, $\underline{D}(\underline{M}(D)-i, u, \mu)$ and $\gamma_i(W,\mu)$ satisfying

$$\begin{aligned} &(v-p)\cdot\hat{\mu}(\mu,\gamma_i(W,\mu))\cdot\beta_1(W,N,u,\hat{\mu}(\mu,\gamma_i(W,\mu))) \\ &-p\cdot(1-\hat{\mu}(\mu,\gamma_i(W,\mu)))\cdot\beta_0(W,N,u,\hat{\mu}(\mu,\gamma_i(W,\mu))) \quad = \quad c \end{aligned} \tag{15}$$

where $\gamma_i(W,\mu)$ is the time before which the next buyer must arrive for the project to stay alive, given donor wealth realization W, public belief μ and that $i+1$ buyers are still needed. Moreover, assume that D^+ is weakly increasing in W, D, and G and weakly decreasing in u, N, p. Also, assume that for all D, μ, and $i = 2, \ldots, j-1$ β_q is increasing in μ, $\frac{\partial \beta_1}{\partial \mu}(D, \underline{M}(D)-i, u, \mu) \geq \frac{\partial \beta_0}{\partial \mu}(D, \underline{M}(D)-i, u, \mu)$ and γ_i is decreasing in μ and W. As in the proof of Proposition 1, we focus on the limiting expressions as $\Delta \to 0$.

Given any state X_t where $N = \underline{M}(D)-j$ and $u \geq T - \gamma_{j-1}(D,\mu_t)$, first consider $\mu < \bar{\mu}^1$. In this case, no buyer buys, and the probability of success is given by $\beta_1(X_t) = \beta_0(X_t) = \frac{1-F(G-p(N+1))}{1-F(D)}$. Then, the donor's best response is to donate

$$D^+(X_t) = \max\{D, \min\{G - Np, W\}\}.$$

Next, consider $\mu \geq \bar{\mu}^1$. Then

$$\beta_1(\cdot) = \int \int_0^{\min\{\gamma_{j-1}(w,\mu),\bar{t}(\mu,\bar{\mu}^1),u\}} \lambda e^{-\lambda s} \beta_1(D^+(D, N+1, u-s, \hat{\mu}(\mu,s)), N+1, u-s, \hat{\mu}(\mu,s))\, ds \frac{f(w)}{1-F(D)} dw$$

$$\beta_0(\cdot) = \int \int_0^{\min\{\gamma_{j-1}(w,\mu),\bar{t}(\mu,\bar{\mu}^1),u\}} \alpha\lambda e^{-\alpha\lambda s} \beta_0(D^+(D, N+1, u-s, \hat{\mu}(\mu,s)), N+1, u-s, \hat{\mu}(\mu,s))\, ds \frac{f(w)}{1-F(D)} dw$$

The probability in the integral can be calculated using the induction assumption.

Given $X_t = (D, N, u, \mu_t)$, a buyer buys if and only if

$$B(X_t) \equiv (v-p) \cdot \mu_t^1 \cdot \beta_1(X_t) - p \cdot (1-\mu_t^1) \cdot \beta_0(X_t) \geq v_0.$$

Note that $B(X_t)$ is increasing in D and for any $\mu_t^1 > \bar{\mu}^1$ there exists a $D \leq G - p(N+1)$ such that the above inequality is violated, i.e., the buyer will purchase. Then as $\Delta \to 0$ it is optimal to donate just enough to make the next buyer with $s = 1$ indifferent between buying and not buying absent additional donations and the donor's limiting optimal strategy is given by

$$D^+(D, N, u, \mu_t) = \max\{D, \min\{\underline{D}(N, u, \mu_t), W\}\}.$$

Finally, we can define $\gamma_j(W, \mu)$ to be the time by which the $(M-j+1)$-th buyer must arrive for the project to stay alive, given donor wealth W and current belief μ to be

$$\begin{aligned}
&(v-p) \cdot \hat{\mu}(\mu, \gamma_j(W,\mu)) \cdot \beta_1(W, N, u, \hat{\mu}(\mu, \gamma_j(W,\mu))) \\
&-p \cdot (1-\hat{\mu}(\mu, \gamma_j(W,\mu))) \cdot \beta_0(W, N, u, \hat{\mu}(\mu, \gamma_j(W,\mu))) \quad = \quad v_0.
\end{aligned}$$

If the donor runs out of funds so that $\mu > \bar{\mu}^1$ and $B(D, N, u, \mu) < v_0$, the project dies forever, i.e., it suffices to show that for $t > \gamma_j(W, \mu)$, $B(D, N, u, \hat{\mu}(\mu, t - \gamma_2(W, \mu))) < v_0$. Note that for $\mu \geq \frac{v_0+p}{v}$

$$\begin{aligned}
\frac{\partial}{\partial \mu} B(D, N, u, \mu) \quad &> \quad (v-p)\mu \frac{\partial \beta_1(D, N, u, \mu)}{\partial \mu} - p(1-\mu) \frac{\beta_0(D, N, u, \mu)}{\partial \mu} \\
&\geq \quad (v - \mu v + c)\mu \frac{\partial \beta_1(D, N, u, \mu)}{\partial \mu} - (\mu v - c)(1-\mu) \frac{\beta_0(D, N, u, \mu)}{\partial \mu} \\
&= \quad \mu(1-\mu)v \left[\frac{\partial \beta_1(D, N, u, \mu)}{\partial \mu} - \frac{\beta_0(D, N, u, \mu)}{\partial \mu} \right] \\
&+ \quad \underbrace{\left[\mu \frac{\partial \beta_1(D, N, u, \mu)}{\partial \mu} + (1-\mu) \frac{\beta_0(D, N, u, \mu)}{\partial \mu} \right]}_{>0}
\end{aligned}$$

which is positive because for $y(\mu) := \min\{\gamma_{j-1}(w,\mu), \overline{t}(\mu,\bar{\mu}^1), u\}$

$$\frac{\partial \beta_1(D,N,u,\mu)}{\partial \mu} =$$

$$\underbrace{\left[\frac{\partial}{\partial \mu} y(\mu)\right]}_{>0} \cdot \int \lambda e^{-\lambda y(\mu)} \beta_1(D^+(D,N+1,u-s,\hat{\mu}(\mu,y(\mu))),N+1,u-y(\mu),\hat{\mu}(\mu,y(\mu))) \frac{f(w)}{1-F(D)} dw$$

$$+\int \int_0^{y(\mu)} \lambda e^{-\lambda s} \left(\frac{\partial \beta_1}{\partial \mu} + \frac{\partial \beta_1}{\partial D}\frac{\partial D^+}{\partial \mu}\right)\frac{\partial \hat{\mu}}{\partial \mu} ds \frac{f(w)}{1-F(D)} dw$$

$$> \underbrace{\left[\frac{\partial}{\partial \mu} y(\mu)\right]}_{>0} \cdot \int \alpha \lambda e^{-\lambda y(\mu)} \beta_0(D^+(D,N+1,u-s,\hat{\mu}(\mu,y(\mu))),N+1,u-w(\mu),\hat{\mu}(\mu,y(\mu))) \frac{f(w)}{1-F(D)} dw$$

$$+\int \int_0^{y(\mu)} \lambda e^{-\lambda s} \left(\frac{\partial \beta_0}{\partial \mu} + \frac{\partial \beta_0}{\partial D}\frac{\partial D^+}{\partial \mu}\right)\frac{\partial \hat{\mu}}{\partial \mu} ds \frac{f(w)}{1-F(D)} dw$$

$$= \frac{\partial \beta_0(D,N,u,\mu)}{\partial \mu}$$

by the induction assumption whenever the expressions are differentiable. Note that we have omitted the arguments for the partial derivatives for expositional clarity. If the expression is not differentiable at a point, we can write down the differences instead yielding the same inequality.

Note that this equilibrium behavior is indeed consistent with the imposed belief system on equilibrium path.

■