THE FISCAL ROOTS OF INFLATION

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NBER Working Paper No. 25811
May 2019
JEL No. E31,E63

ABSTRACT

Unexpected inflation devalues nominal government bonds. This change in value must correspond to a change in expected future surpluses, a change in their discount rates, or a contemporaneous change in nominal bond returns. I develop a linearized version of the government debt valuation equation, and I measure each component via a vector autoregression. I find that discount rate variation is important. Unexpected inflation corresponds entirely to a rise in discount rates, with no change in the sum of expected future surpluses. A recession shock, which lowers inflation and output, signals persistent deficits, but also lower interest rates, which raise the value of debt and account fully for the lower inflation. A monetary policy shock, defined here as a rise in interest rates with no change in expected future surpluses, raises inflation immediately and persistently. Nominal rates rise more than real rates, raising the discount factor and thus accounting for the inflation. In these calculations, the present value of surpluses changes by more than current inflation. Persistently higher inflation and nominal interest rates cause current long term bonds to fall in value, soaking up variation in the present value of surpluses. By this mechanism monetary policy spreads fiscal shocks to persistent inflation rather than price level jumps. I also decompose the value of government debt. Half of the value of debt corresponds to forecasts of future primary surpluses, and half to discount rates, driven by variation in bond expected returns.

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1. Introduction

This paper measures the fiscal foundations of inflation, and of the value of US federal debt. Unexpected inflation devalues nominal government bonds. This change in value must correspond to a change in expected surpluses, a change in discount rates, or a contemporaneous change in nominal bond returns. I develop this linearized identity, and I use a vector autoregression (VAR) to measure each component.

I estimate that a 1% unexpected inflation corresponds entirely to a rise in discount rates, and no change in the sum of expected future surpluses. The change in discount rates produces a change in present value of the surplus even larger than 1%, which is met by a decline in nominal bond prices coincident with inflation.

The latter mechanism is important throughout the calculations. A persistent rise in inflation and nominal interest rates lowers long-term bond prices. Via this mechanism, monetary policy after a fiscal shock can spread inflationary consequences over time.

Studying the fiscal roots of inflation leads to a natural puzzle – why do we see less inflation during recessions, with larger deficits, and more inflation during booms, with smaller deficits? Well, larger deficits may come with expectations of larger subsequent surpluses, to pay off the accumulated debts. The present value of all future surpluses does not necessarily decline when deficits rise in a recession. Moreover, the larger current deficits in recessions also come with lower real interest rates, a lower discount rate for government debt. Lower discount rates make the stream of future surpluses more valuable, a deflationary force.

Do we see these effects, and what is their size? I estimate that a 1% recession-related disinflation comes with large deficits, that are not completely paid off by subsequent surpluses. However, the expected returns on bonds fall so much that the 1% disinflation is associated with a 3% rise in the present value of surpluses. 2% of that rise corresponds to the large ex-post return on long term bonds, which reflects the persistence of the decline in interest rates in the recession. The contrary pattern occurs in booms. Discount rate variation produces a fiscal Phillips curve – lower inflation with low output (and low interest rates and big deficits) in recessions, higher inflation with high output (and high interest rates and surpluses) in booms.

I examine the inflationary consequences of monetary and fiscal policy shocks, and the fiscal roots of such inflation. I define a monetary policy shock as an unexpected 1 percentage point movement in the nominal interest rate that does not move the sum of expected future surpluses, and I define a fiscal shock as an unexpected one percentage point movement in the sum of future surpluses that does not move the interest rate. Central banks control interest rates,

but they do not change taxes and spending; fiscal authorities change taxes and spending but cede control of the nominal rate to central banks. Conventional descriptions of monetary policy shocks mix both monetary and fiscal responses.

I find that such monetary policy shocks are super-Fisherian – they raise inflation immediately and persistently. The rise in inflation comes entirely from a rise in discount rates, which lowers the value of government debt. Nominal rates rise more than inflation, raising real rates and thus discount rates. The decline in present value of debt is again larger than the change in inflation, and long-term bonds soak up much of the fiscal shock, consistent with a drawn-out inflation response.

Fiscal shocks also result in a drawn out inflation. Again, long-term debt smooths the inflationary consequences of the fiscal shock over time. Discount rates offset about half of the fiscal shock.

The linearized government debt valuation equation also allows us to ask how high debt to GDP ratios have resolved historically – by high surpluses, by low real returns, expected or unexpected, or by large growth? I confirm that growth paid off about 2/3 of the WWII debts, but primary surpluses paid off the rest. After about 1975 we entered a new regime in which variation in debts corresponds to variation in surpluses.

I then examine the forward-looking sources of variation in the value of government debt – on average, do high values of debt come from larger future surpluses, or lower future growth-adjusted discount rates? I find that about half of the value of debt corresponds to variation in expected surpluses, and half from discount rate variation. Of that, the vast majority comes form variation in expected returns, not variation in expected growth. Filtering debt to emphasize business cycle frequencies, 70% of the business-cycle frequency variation in the value of debt comes from variation in expected surpluses. Innovations in the value of debt, however, are dominated by expected return news.

1.1. Literature

Much of the technique in this paper is imported from asset pricing. The general approach to linearizing the valuation identity follows the idea of the linearized present value identity for dividend yields in Campbell and Shiller (1988). I likewise linearize a return formula and iterate forward. I linearize in the level of the surplus to GDP ratio rather than its logarithm, as surpluses are often negative. My accounting of the variance of unexpected inflation in a VAR context is similar to the return variance decomposition in Campbell and Ammer (1993). However, to avoid covariance terms, I focus on an extension of the decomposition of variance in Cochrane (1992)

to a multivariate context. With $x = y + z$, I explore $var(x) = cov(x, y) + cov(x, z)$ rather than $var(x) = var(y) + var(z) + 2cov(y, z)$. The summary in Cochrane (2011b) and treatment of identities in Cochrane (2007) are obvious precursors. The uniting theme in the former is that discount rates matter. This paper is in many ways an arbitrage of that insight to questions of government debt and inflation, where discount rate variation is largely ignored.

The analysis of government finances, how debt is paid off, grown out of, or inflated away, is a huge literature. Hall and Sargent (1997), Hall and Sargent (2011) are the most important precursors. Hall and Sargent emphasize the importance of the market value of debt, not the face value reported by Treasury, and consequent proper accounting for interest costs. I use data provided by Hall, Payne, and Sargent (2018). Bohn (2008) examines a long history of US debt, notes its value is stationary arguing that present values are finite, and shows that primary surpluses are higher following large debts, though growth also brings down debt to GDP ratios. This paper adds asset pricing variance decompositions to this exercise.

1.2. Theories

The calculations in this paper are based on the government debt valuation equation, which states that the real value of nominal government debt equals the present value of primary surpluses. This equation is part of a wide variety of models. (The valuation equation requires that the present value of surpluses is finite, loosely that $r > g$, so it does not hold in dynamically inefficient models. I presume that this is the case without further comment.)

I am motivated to make the calculations in the context of the fiscal theory of the price level, in which case changes in the present value of surpluses are thought to cause changes in inflation. However, the calculations are just as valid if assumes "passive" fiscal policy (Leeper (1991)), that changes in inflation cause change in the present value of surpluses. A well-specified active-money, passive-fiscal regime must spell out a realistic passive-fiscal policy, and this paper can be read as providing a measurement of the latter element. The fact that discount rates do much of the adjusting, rather than the painless ex-post lump-sum taxes alluded to in theoretical footnotes, changes the fiscal underpinnings of such models substantially.

Similarly, I do not test the valuation equation or anything else. This valuation equation holds, in equilibrium, in both active-fiscal and passive-fiscal models, so such a test does not distinguish the two views and is uninteresting. Moreover, I discount surpluses at the ex-post return on the government debt portfolio, in which case the valuation equation is an identity, which has no testable content at all. But which element in an identity moves is still an interesting measurement: The identity tells us that inflation must correspond to surpluses or discount rates.

The identity does not tell us which one it is, or how quickly the surpluses and discount rates appear. Let us find out.

However, the calculations are most interesting as an empirical investigation of the fiscal theory of monetary policy, and a set of facts to guide construction of such theories. I interpret the results through that lens, with this acknowledgement that other interpretations are possible and the results are a measurement not a test. By this term, I mean a model that adopts the standard new-Keynesian ingredients – intertemporal optimization, market clearing, and pricing frictions, monetary policy that targets interest rates – but a model that substitutes an active fiscal policy for active monetary policy to select equilibria. Innovations to fiscal events are then thought to cause, rather than follow from, inflation.

As the simplest example, consider a frictionless model composed of only the Fisher equation (linearized intertemporal first-order condition in a constant-endowment economy)

$$i_t = r + E_t \pi_{t+1} \tag{1}$$

and a government debt valuation equation,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \tag{2}$$

where i_t is the nominal interest rate, r is a constant real rate, π_t is inflation, B_{t-1} is one-period nominal government debt, P_t is the price level, $\beta = 1/(1+r)$, and s_t are real primary surpluses.

Moving (2) one period forward, and taking expectations,

$$\frac{B_t}{P_t} \Delta E_{t+1} \left(\frac{P_t}{P_{t+1}} \right) = \Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}, \tag{3}$$

where

$$\Delta E_{t+1} \equiv E_{t+1} - E_t$$

denotes an innovation. Linearizing, unexpected inflation is equal to innovation to the present value of surpluses.

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j} / v_t, \tag{4}$$

where $v_t \equiv B_t / P_t$ is the real value of debt. This equation, generalized to long term debt, time-varying discount factors, and stationary ratios to GDP (equation (21)), is the centerpiece of this paper's empirical analysis.

The model consists, then, of two equations, (1) and (4).

If the central bank sets a (potentially time-varying or state-contingent) interest rate target i_t, and with a passive fiscal policy in which s_t reacts ex-post to make (2) hold for any inflation rate, this model determines expected inflation but not unexpected inflation. There are multiple equilibria corresponding to any value of $\Delta E_{t+1}\pi_{t+1} = \pi_{t+1} - E_t\pi_{t+1}$.

The standard new-Keynesian approach solves this multiplicity with an active monetary policy

$$(i_t - i_t^*) = \phi\left(\pi_t - \pi_t^*\right), \phi > 1, \tag{5}$$

where π_t^*, i_t^* are equilibrium values the central bank wishes to select from the multiple equilibria of (1). Modelers add a rule against nominal explosions, and retain the passive fiscal policy assumption. Now only one value of unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ remains. (Woodford (2003), Cochrane (2011a).)

For example, suppose the central bank wishes to produce an AR(1) inflation process,

$$\pi_{t+1}^* = \theta\pi_t^* + \varepsilon_{t+1}. \tag{6}$$

The central bank wishes to fully determine inflation, including its unexpected component ε_{t+1}. Then, by (1), the equilibrium interest rate must follow

$$i_t^* = r + \theta\pi_t^*. \tag{7}$$

This provision alone is insufficient, as it does not determine unexpected inflation ε_{t+1}. The central bank also specifies $\phi > 1$ and announces (5), that should another inflation $\pi_{t+1} \neq \pi_{t+1}^*$ emerge, the central bank will lead the economy to hyperinflation or deflation. The latter provision and the rule against nominally explosive equilibria selects (6) as the unique equilibrium.

Equation (5) is more commonly written $i_t = r + \phi\pi_t + v_t$, with $v_t = i_t^* - \phi\pi_t^*$. I write it in the slightly more pedantic form (5) to emphasize that "monetary policy," the the interest rate rule (7) that we observe in equilibrium, is a completely separate object from "equilibrium selection policy," the threat (5) that the central bank uses to select one of multiple equilibria.

A fiscal theory of monetary policy specifies a "passive" $\phi < 1$ equilibrium-selection policy (5) and turns off the passive fiscal policy assumption. Surpluses are not "exogenous." They may react endogenously to all sorts of variables, but they do not react one-for-one to multiple-equilibrium unexpected inflation, in such a way that (4) holds for any unexpected inflation. Now the combination (1) and (4) determines both expected and unexpected inflation.

Central banks still set interest rate targets, and expected inflation is still determined by the central bank's interest rate target. Interest rate targets may also react endogenously as in (7). (The interest rate target can be achieved by adjusting nominal debt B_t with no change in surpluses in this frictionless model.) Thus, monetary policy still sets interest rates and expected inflation in this model, while fiscal events only determine unexpected inflation. Monetary policy is still crucial for understanding economic dynamics. In particular, the path of *expected* inflation and interest rates following a shock is important in understanding the dynamics below, and with this sort of model in mind, I interpret those responses as monetary policy responses to events.

One can pursue the same idea with any new-Keynesian or DSGE model. Having thrown out the government valuation equation with the passive-fiscal assumption, such models have one degree of indeterminacy, one too few explosive eigenvalues, one undetermined linear combination of unexpected variables. Rather than introduce that explosive eigenvalue via active monetary policy, an off-equilibrium threat of the form (5), introduce it via active fiscal policy. Most simply, add the debt accumulation equation

$$\frac{B_t}{P_t} = (1 + i_t) \left[\frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} - s_t \right]$$

and impose that this real value of debt cannot explode, following the consumer's transversality condition. This condition produces the extra explosive eigenvalue and gives a globally unique determinate equilibrium.

Sims (2011), Cochrane (2017), and Cochrane (2019) present medium-scale models of this sort, with sticky prices, habit persistence preferences that induce hump-shaped responses, long term debt, and both surpluses and an interest rate target that react to economic conditions (generalized versions of (7)).

The medium-scale model produces interesting dynamics, beyond those of the simple frictionless model outlined here, and can serve as an explicit example of the kind of theory for which I view the calculations here as enlightening. In particular, the simple model here is Fisherian: In (1) - (4), a rise in the interest rate target i_t leads to a rise in expected inflation $E_t \pi_{t+1}$. Without a contemporaneous fiscal shock, there can be no decline in current inflation π_t either. The medium-scale models can, however, generate an instantaneous and protracted negative inflation response to a rise in the interest rate target, with no change in fiscal policy. Long term debt is a crucial ingredient. These models can also generate a protracted inflation response to a fiscal shock, where (3) suggests that a fiscal shock can only create a one-time price level jump.

2.   Linearized identities

The analysis is based on linearized flow and present value identities for government debt. Following the flow of money in a period, we have the linearized flow identity

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = s_{t+1} + v_{t+1}. \tag{8}$$

The log debt to GDP ratio at the end of period $t + 1$, v_{t+1}, is its value at the beginning of the period, v_t, increased by the real return on debt $r^n_{t+1} - \pi_{t+1}$ less GDP growth g_{t+1}, and decreased by a positive real primary surplus s_{t+1}.

The symbol s_t can stand for the real primary surplus sp_t, divided by GDP Y_t and adjusted for the steady state value of the debt to GDP ratio V/Y,

$$s_t = \frac{sy_t}{e^v} = \frac{sp_t}{Y_t} \frac{1}{V/(PY)},$$

or it can stand for the real primary surplus to GDP ratio divided by the previous period's debt to GDP ratio,

$$s_t = sv_t = \frac{sp_t/Y_t}{V_{t-1}/(P_{t-1}Y_{t-1})} = \frac{sy_t}{e^{v_{t-1}}}.$$

Both definitions lead to the same linearized identity (8). I will refer to s_{t+1} calculated from (8) as just the "surplus."

Iterating (8) forward, we have the linearized present value identity,

$$v_t = \sum_{j=1}^{T} s_{t+j} - \sum_{j=1}^{T} \left(r^n_{t+j} - \pi_{t+j} - g_{t+j} \right) + v_{t+T}. \tag{9}$$

The value of government debt, divided by GDP, is the present value of future surpluses, discounted at the ex-post real return adjusted by GDP growth. In general, I take the $T \to \infty$ limit and show that the last term vanishes in expectation. One can linearize in such a way that terms β^j appear on the forward-looking terms, but it is not necessary to do so here.

Seigniorage and liquidity premiums for government debt are included in the possibility that the return on government bonds r^n is lower than returns of other similar securities. The effects of maturity structure are all captured in the rate of return r^n as well. If there were only one-period debt, for example, we would have $r^n_{t+1} = i_t$. A theoretical model must spell out maturity structure and bond pricing to derive the rate of return r^n_{t+1}, so that term empirically captures a lot of information. Relationship (9) holds ex-post, and therefore also ex-ante, with E_t in front of the sum and any information set that includes the value of the debt v_t.

2.1.   Derivation

The symbols are as follows:

$$V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}$$

is the nominal end-of-period market value of debt, M_t is non-interest-bearing money, $B_t^{(t+j)}$ is zero-coupon nominal debt outstanding at the end of period t and due at the beginning of period $t + j$, $Q_t^{(t+j)}$ is the time t price of that bond, with $Q_t^{(t)} = 1$. Y_t is real GDP or another stationarity-inducing divisor such as consumption, potential GDP, population, etc. I use consumption times the average GDP to consumption ratio in the empirical work, but I will call Y and ratios to Y "GDP" for simplicity. P_t is the price level, so

$$v_t \equiv \log\left(\frac{V_t}{Y_t P_t}\right)$$

is log market value of the debt divided by GDP.

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_t^{(t+1+j)}}{M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}}$$

is the nominal return on the portfolio of government debt, i.e. overnight from the end of t to the beginning of $t + 1$, and

$$r_{t+1}^n \equiv \log(R_{t+1}^n)$$

is the log nominal return on that portfolio.

$$\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right), \; g_t \equiv \log\left(\frac{Y_t}{Y_{t-1}}\right)$$

are log inflation and GDP growth rate.

Now, I establish the nonlinear flow and present value identities. In period t, we have

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} + M_{t-1} = P_t sp_t + \sum_{j=0}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} + M_t, \tag{10}$$

where sp_t denotes the real primary (not including interest payments) surplus or deficit. Money M_t at the end of period t is money brought in from the previous period M_{t-1} plus the effects of bond sales or purchases at price $Q_t^{(t+j)}$, less money soaked up by primary surpluses.

The left hand side of (10) is the beginning of period market value of debt, i.e. before debt sales or repurchases $B_t^{(t+j)} - B_{t-1}^{(t+j)}$ have taken place. It turns out to be more convenient here to

express equations in terms of the end-of-period market value of debt. To that end, shift the time index forward one period and rearrange to write

$$\frac{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}{P_t Y_t} \frac{R_{t+1}^n}{G_{t+1}} \frac{P_t}{P_{t+1}} = \frac{sp_{t+1}}{Y_{t+1}} + \frac{M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}}{P_{t+1} Y_{t+1}}. \tag{11}$$

We can iterate this flow identity (11) forward to express the nonlinear government debt valuation identity as

$$\frac{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}{P_t Y_t} = \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}/G_{t+k}} \right) \frac{sp_{t+1}}{Y_{t+1}}. \tag{12}$$

The market value of government debt at the end of period t, as a fraction of GDP, equals the present value of primary surplus to GDP ratios, discounted at the government debt rate of return less the GDP growth rate. (I assume here that the right hand side converges. Otherwise, keep the limiting debt term.)

The nonlinear identities (11) and (12) are cumbersome. I linearize the flow equation (11) and then iterate forward to obtain a linearized version of (12). Write (11) as

$$\frac{V_t}{P_t Y_t} R_{t+1}^n \frac{P_t}{P_{t+1}} \frac{Y_t}{Y_{t+1}} = \frac{sp_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}.$$

Taking logs,

$$v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} = \log \left(\frac{sp_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right) \tag{13}$$

To linearize in terms of the surplus/GDP ratio, Taylor expand the last term,

$$v_t + r_{t+1}^n - \pi_{t+1} + g_{t+1} = \log(e^v + sy) + \frac{e^v}{e^v + sy}(v_{t+1} - v) + \frac{1}{e^v + sy}(sy_{t+1} - sy)$$

where

$$sy_{t+1} \equiv \frac{sp_{t+1}}{Y_{t+1}}, \tag{14}$$

and variables without subscripts denote a steady state of (13). With $r \equiv r^n - \pi$,

$$r - g = \log \frac{e^v + sy}{e^v}.$$

Then,

$$v_t + r_{t+1}^n - \pi_{t+1} + g_{t+1} = \left[\log(e^v + sy) - \frac{e^v}{e^v + sy} \left(v + \frac{sy}{e^v} \right) \right] + \frac{e^v}{e^v + sy} v_{t+1} + \frac{e^v}{e^v + sy} \frac{sy_{t+1}}{e^v}$$

$$v_t + r^n_{t+1} - \pi_{t+1} + g_{t+1} = \left[v + r - g - \frac{e^v}{e^v + sy} \left(v + \frac{e^v + sy}{e^v} - 1 \right) \right] + \beta v_{t+1} + \beta \frac{sy_{t+1}}{e^v}$$

$$v_t + r^n_{t+1} - \pi_{t+1} + g_{t+1} = [r - g + (1 - \beta)(v - 1)] + \beta v_{t+1} + \beta \frac{sy_{t+1}}{e^v} \tag{15}$$

where

$$\beta \equiv e^{-(r-g)}. \tag{16}$$

Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is

$$v_t + r^n_{t+1} - \pi_{t+1} + g_{t+1} = \beta \frac{sy_{t+1}}{e^v} + \beta v_{t+1} \tag{17}$$

Iterating forward, the present value identity is

$$v_t = \sum_{j=1}^{T} \beta^{j-1} \left[\frac{sy_{t+j}}{e^v} - \left(r^n_{t+j} - \pi_{t+j} + g_{t+j} \right) \right] + \beta^T v_T. \tag{18}$$

If we linearize around $r - g = 0$, then the constant in (17) is zero ($sy = 0$), and we obtain the linearized flow and present value identities (8) and (9), with the symbol s_t representing sy_t/e^v. (The point of expansion need not be the sample mean.)

To approximate in terms of the surplus to value ratio, write (13) as

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(\frac{V_t}{P_t Y_t} \frac{\frac{sp_{t+1}}{Y_{t+1}}}{\frac{V_t}{P_t Y_t}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right)$$

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(\frac{\frac{sp_{t+1}}{Y_{t+1}}}{\frac{V_t}{P_t Y_t}} + \frac{\frac{V_{t+1}}{P_{t+1} Y_{t+1}}}{\frac{V_t}{P_t Y_t}} \right)$$

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(sv_{t+1} + e^{v_{t+1} - v_t} \right).$$

At a steady state

$$r - g = \log(1 + sv). \tag{19}$$

Taylor expanding around a steady state,

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log(1 + sv) + \frac{1}{(1 + sv)} (sv_{t+1} - sv + v_{t+1} - v_t)$$

$$v_t + (1 + sv) \left[r^n_{t+1} - \pi_{t+1} - g_{t+1} \right] = [(1 + sv) \log(1 + sv) - sv] + sv_{t+1} + v_{t+1} \tag{20}$$

The linearized flow identity (8) follows, with the symbol s_t representing the surplus to value ratio

$s_t = sv_t$, if we suppress the constant, using deviations from means in the analysis, or if we use $r = g$ or $sv = 0$, as a point of expansion.

The linearizations in terms of the surplus to value ratio sv_t are more accurate. The units of the flow identities (8), (17) are rates of return. Dividing the surplus by the previous period's value gives a better approximation to the growth in value, when the value of debt is far from the steady state. Suppose debt to GDP is 100%, the real rate of return is 0, and the surplus is -20% of GDP, and -20% of debt. Debt to GDP rises to 120%. If the steady state debt to GDP ratio is 50%, the linearization in surplus to GDP scaled by the steady state debt to GDP ratio predicts a 40% rise, resulting in a 20% error. Outside of WWII, most values are not this extreme, but the principle still holds.

The term v_{t+T} does not vanish in (9), where the term $\beta^T v_{t+T}$ can vanish in (18). In this paper, the presence of the v_{t+T} term is not a difficulty. I study expectations $E_t v_{t+T}$ and v_t is stationary, so $E_t v_{t+T} \to 0$. For other purposes, one may wish to use the surplus to GDP linearization and $r > g$ steady state, so that the limiting term vanishes automatically.

More deeply, a constant ratio of surplus to market value of debt for any price level path leads to a passive fiscal policy. An unexpected deflation raises the real value of debt. If surpluses always rise in response, they validate the lower price level. Thus, although on the equilibrium path one can describe dynamics via either linearization, if one wants to think about how fiscal-theory equilibria are formed, it is better to describe a surplus that does not react to price level changes, so only one value v_t emerges, as is the case in (18). It's also better to use the nonlinear versions of the identities for determinacy issues. The analysis of this paper is about what happens in equilibrium, and does not require an active-fiscal assumption, so the difference is irrelevant here.

I infer the surplus from the linearized flow identity (8) so which concept the surplus corresponds to makes no difference to the analysis. The difference is only the accuracy of approximation, how close the surplus recovered from the linearized flow identity corresponds to a surplus recovered from the nonlinear exact identity (13). I evaluate the accuracy of approximation below.

2.2. The inflation identity

To focus on the fiscal roots of inflation, I take time-$t + 1$ innovations $\Delta E_{t+1} = E_{t+1} - E_t$ of the present value identity (9), and rearrange, leaving

$$\Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1}\left(r^n_{t+1} - g_{t+1}\right) \tag{21}$$
$$= -\sum_{j=0}^{\infty} \Delta E_{t+1}s_{t+1+j} + \sum_{j=1}^{\infty} \Delta E_{t+1}\left(r^n_{t+1+j} - \pi_{t+1+j} - g_{t+1+j}\right).$$

I forecast variables using a VAR. Since we start with an identity, the result holds for any information set. I do not thereby assume that agents observe only the variables in the VAR. However, "innovation" refers to the VAR variables only, and not the larger set that agents observe.

I put unexpected inflation $\Delta E_{t+1}\pi_{t+1}$ first in (21), as it is the variable we focus on.

The second term $\Delta E_{t+1}\left(r^n_{t+1}\right)$ is the unexpected return on the portfolio of government bonds. With one-period debt, this term is zero, as $r^n_{t+1} = i_t$ is known ahead of time. With long-term debt, this term measures how much news about the present value of surplus shows up in current bond prices rather than current inflation. Higher future inflation leads to lower bond prices, so this term can measure how much a fiscal shock is spread out via long-term bonds to future inflation rather than immediate inflation.

The first term on the right hand side is the revision in expected future surpluses. Inflationary devaluation of government bonds comes from all future surpluses, not just current surpluses. If big deficits today are followed by large surpluses to pay off the debt, there need be no inflation today. The identity (21) lets us see if it happens.

The second term on the right hand side captures the discount rate effect: A rise in the expected return of government bonds lowers the value of future surpluses. This is an important and underappreciated effect. Most discussion of the valuation equation uses a constant discount rate for simplicity, and therefore ties variation in the value of government debt entirely to variation in prospective surpluses. Since the variables in the estimated VAR are stationary, a potential last term $\lim_{T \to \infty} \Delta E_{t+1}v_{t+T}$ is zero.

The different treatment of the time-$t+1$ term involving inflation, return, and growth, and the future terms looks strained at first glance. One is tempted to treat all the π_t, r^n_t and g_t terms together, and write the model as a model of long-term average of inflation $\sum_{j=0}^{\infty} \pi_{t+j}$. But the first and subsequent terms capture fundamentally different mechanisms. As an analogy, when bond yields rise unexpectedly, the current return is negative, but then all future expected returns rise.

2.3.   A variance decomposition

My central calculation is an analysis unexpected inflation based on the identity (21). We can think of this calculation just in terms of the impulse-response function.   When there is a 1% shock to inflation at time 1, $\Delta E_1 \pi_1$, how large are the innovations ΔE_1 of the other terms?

We can interpret the same calculation as an analysis of the variance of unexpected inflation. Multiply both sides of (21) by $\Delta E_{t+1} \pi_{t+1}$ and take expectations, giving

$$var\left(\Delta E_{t+1}\pi_{t+1}\right) - cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}\left(r^n_{t+1} - g_{t+1}\right)\right] \tag{22}$$

$$= -\sum_{j=0}^{\infty} cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}s_{t+1+j}\right] + \sum_{j=1}^{\infty} cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}\left(r^n_{t+1+j} - \pi_{t+1+j} - g_{t+1+j}\right)\right].$$

Unexpected inflation may only vary to the extent that it covaries with current bond returns, or if it forecasts surpluses or real discount rate. We should see those covariances in the data, and we can measure which ones are important.

Dividing by $var\left(\Delta E_{t+1}\pi_{t+1}\right)$, we can express each term as a fraction of the variance of unexpected inflation coming from that term. This decomposition adds up to 100%, within the accuracy of approximation, but it is not an orthogonal decomposition, nor are all the elements necessarily positive. Each term is also a regression coefficient of the other terms on unexpected inflation.

The two approaches give exactly the same result – the terms of (22) are exactly the terms of the impulse-response function, to an inflation shock orthogonalized last, i.e. a shock that moves all variables at time 1 including $\Delta E_1 \pi_1$.

To calculate these quantities, we can just simulate the impulse response function out a long way and add up its terms. More elegantly, write the VAR

$$x_{t+1} = Ax_t + \varepsilon_{t+1} \tag{23}$$

so

$$\Delta E_{t+1}\sum_{j=1}^{\infty} x_{t+j} = (I - A)^{-1}\varepsilon_{t+1}.$$

Let a denote vectors which pull out each variable, i.e.

$$\pi_t = a'_\pi x_t, \ s_t = a'_s x_t, \tag{24}$$

etc. Then the present value identity (21) reads and may be calculated as

$$a_\pi' \varepsilon_{t+1} - (a_{r^n} - a_g)' \varepsilon_{t+1} = -a_s'(I-A)^{-1}\varepsilon_{t+1} + a_{rg}'(I-A)^{-1}A\varepsilon_{t+1} \tag{25}$$

where

$$a_{rg} \equiv a_{r^n} - a_\pi - a_g.$$

We can calculate the variance decomposition (22) by

$$a_\pi' \Sigma a_\pi - (a_{r^n} - a_g)' \Sigma a_\pi = -a_s'(I-A)^{-1}\Sigma a_\pi + a_{rg}'(I-A)^{-1}A\Sigma a_\pi$$

where $\Sigma = cov(\varepsilon_{t+1}, \varepsilon_{t+1}')$, and then divide by $a_\pi' \Sigma a_\pi$ to express the result as a fraction,

$$1 - (a_{r^n} - a_g)' \frac{\Sigma a_\pi}{a_\pi' \Sigma a_\pi} = -a_s'(I-A)^{-1}\frac{\Sigma a_\pi}{a_\pi' \Sigma a_\pi} + a_{rg}'(I-A)^{-1}A\frac{\Sigma a_\pi}{a_\pi' \Sigma a_\pi}. \tag{26}$$

To show that this variance decomposition is the same as the elements and sum of elements of the impulse-response function to an inflation shock, orthogonalized last, note that the regression coefficient of any other shock ε^z on the inflation shock is

$$b_{\varepsilon^z, \varepsilon^\pi} = \frac{cov(\varepsilon_{t+1}^z, \varepsilon_{t+1}^\pi)}{var(\varepsilon_{t+1}^\pi)} = \frac{a_z' \Sigma a_\pi}{a_\pi' \Sigma a_\pi},$$

so the VAR shock, consisting of a unit movement in inflation $\varepsilon_1^\pi = 1$ and movements $\varepsilon_1^z = b_{\varepsilon^z, \varepsilon^\pi}$ in each of the other variables is given by

$$\varepsilon_1 = \frac{\Sigma a_\pi}{a_\pi' \Sigma a_\pi}.$$

We recognize in (26) the responses and sums of responses to this shock. Dividing (22) by the variance of unexpected inflation, or examining the terms of (26), we recognize that each term is also the coefficient in a single regression of each quantity on unexpected inflation.

3. Estimates

3.1. Data

I use data on the market value of government debt held by the public, and the nominal rate of return of the government debt portfolio from Hall, Payne, and Sargent (2018). (That rate of return includes Fed holdings of treasurys, and not reserves and currency.)

I infer the primary surplus from the flow identities. This calculation measures how much money the government actually borrows. NIPA surplus data, though broadly similar, does not obey the identity. I infer the surplus from the linearized identity (8), to produce a series that exactly matches that identity. It is clearer to bundle the approximation error in the surplus, producing data that obey the approximate identity, than it is to produce numbers that should add up but do not due to approximation error.

To measure the accuracy of approximation, I also infer the real primary surplus from the exact nonlinear identity (10). I calculate the surplus in each month from the nonlinear flow identity (10). I then carry the surplus to the end of the year using the government bond return. This procedure produces an annual series for which the identity (10) continues to hold in annual data. I use standard BEA data for GDP and consumption. I use the GDP deflator as my measure of inflation. I use CRSP data for the three-month Treasury rate. I use the 10-year constant maturity government bond yield from 1953 on and the yield on long term United States bonds before that date.

I measure the debt to GDP and surplus to GDP ratios by the ratios of debt and surplus to personal consumption expenditures, times the average GDP to consumption ratio. Debt to GDP ratios are common measures across countries, but in our time-series application they introduce cyclical variation in GDP. We want only a detrending divisor, and an indicator of the economy's long-run level of tax revenue and spending. Potential GDP has a severe look-ahead bias. Consumption is a decent stochastic trend for GDP.

I use a data sample 1947-2014. The immense deficits of WWII would otherwise dominate the analysis, and one may well suspect that financing that war during an economic expansion follows a different pattern than the subsequent decades of largely cyclical deficits.

Figure 1 presents the surplus and compares three measures. The "Linear, s_t" line imputes the surplus from the linearized flow identity (8) directly at the one-year horizon, which is the measure I use in the following analysis. This series produces estimates in which all the implications of the approximate identity hold exactly. The "sv_t" and "sy_t/e^v" lines infer the surplus from the exact nonlinear flow identity (10), as above. The "sv_t" line presents the exact surplus to value ratio. The "sy_t/e^v" line presents the surplus to GDP ratio (14) – actually, the ratio of surplus to consumption, times the average consumption to GDP ratio – scaled by the average value to GDP ratio $e^{E(v_t)}$. The vertical dashed line indicates the post-1947 sample that I use in VAR analysis below.

The first piece of news is that there *are* primary surpluses. One's impression of endless deficits comes from the full deficit including interest payments on the debt. Even NIPA measures
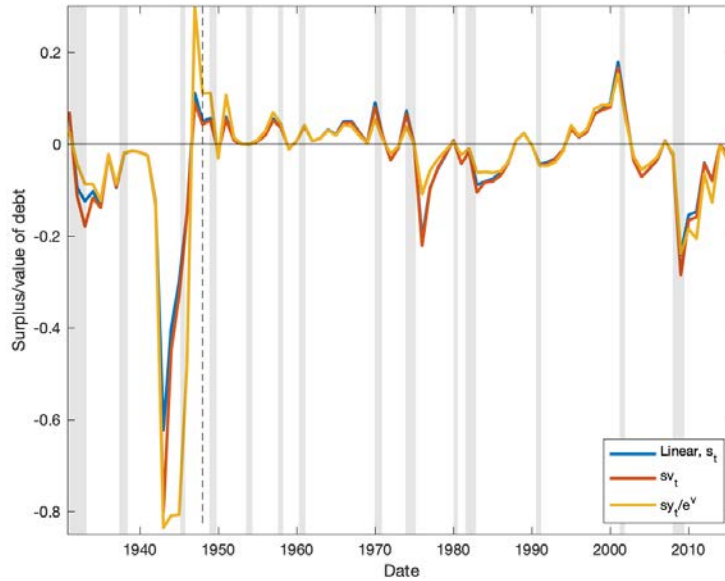
Figure 1: Surplus. "Linear" is inferred from the linearized flow identity. "sv " is the ratio of the primary surplus to the previous year's market value of the debt. "sy" is the ratio of surplus to consumption, scaled by the average value of debt. Vertical shading denotes NBER recessions.

show regular positive primary surpluses. Steady primary surpluses from 1947 to 1975 helped to pay off WWII debt. From 1975 there is a new era of primary deficits, but also interrupted by the strong surpluses of the late 1990s. On top of these trends, and outside of WWII, primary surpluses have a clear cyclical pattern. The primary surplus correlates very well with the unemployment rate (not shown).

The three measures in Figure 1 are close, and that is the point – the graph is a measure of the accuracy of the linearized identity (8), after aggregation to annual data. The linearized identity is a closer approximation to the surplus to value ratio sv. Those two lines are nearly indistinguishable in the post WWII data. It is a somewhat less good approximation to the surplus to GDP ratio sy. The difference is largest when the value of debt is far from its mean, both in WWII and in the 1970s.

3.2. Vector autoregression

Table 1 presents OLS estimates of the VAR coefficients. Each column is a separate regression. The VAR includes the central variables for the inflation identity – nominal return on the government bond portfolio r^n, consumption growth rate g, inflation π, surplus s and value v. I include the three-month interest rate i and the 10 year bond yield y as they are important forecasting

	r^n_{t+1}	g_{t+1}	π_{t+1}	s_{t+1}	v_{t+1}	i_{t+1}	y_{t+1}
r^n_t	-0.17**	-0.02	-0.10**	-0.30*	0.24	-0.08*	0.05*
g_t	-0.25*	0.20*	0.16*	1.34**	-1.95**	0.28**	0.06
π_t	-0.13	-0.14*	0.53**	-0.22	-0.30	0.09	0.04
s_t	0.11**	0.03	-0.03	0.39**	-0.28*	-0.04*	-0.04**
v_t	0.01	-0.00	-0.02**	0.05*	0.98**	-0.01	-0.00
i_t	-0.33*	-0.39*	0.30*	0.57	-0.81	0.74**	0.36**
y_t	1.96**	0.52**	-0.17	-0.17	1.77*	0.11	0.45**
$100 \times std(\epsilon_{t+1})$	2.17	1.58	1.15	4.49	6.51	1.30	0.84
R^2	0.72*	0.16*	0.72*	0.55*	0.97*	0.80*	0.90*
$100 \times std(x)$	4.07	1.72	2.18	6.72	36.04	2.95	2.61

Table 1: OLS VAR estimate. One (two) stars means the estimate is one (two) Monte Carlo standard errors away from zero.

variables for growth, inflation, and long-term bond returns.

It is important to include the value of debt v_t in the VAR, even if we are calculating terms of the innovation identity (21) that does not reference that value. When we deduce from (9) expressions $v_t = E_t(\cdot)$, we must include v_t in the information set that takes the expectation. Related, it is tempting to exclude v_t from the VAR, compute the right hand side of (9), and test whether the value of the debt comes out equal to this present value of discounted expected surpluses. The fact that the identity is an identity ought to warn us against trying to test it. Identities hold, and anything that purports to test them must be a mistake. (Cochrane (2001), Cochrane (2019) give examples of what goes wrong on omitting v from the VAR.)

I compute standard errors from a Monte Carlo. The stars in Table 1 represent one or two standard errors above zero. Since we aren't testing anything, stars are just a visual way to show standard errors without another table.

In the first column, the long term bond yield y_t forecasts the return on government bonds r^n_{t+1}. The negative coefficient on the three month rate i_t means that the long-short spread also forecasts those returns. Since the y_t and i_t coefficients are not repeated in forecasting inflation and growth, the long rate and long-short spread forecast real, growth-adjusted, and excess returns on government bonds, as we expect from the long literature in which yield spreads forecast bond risk premia (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005)). The long rate y_t is thus an important state variable for measuring expected bond returns, the relevant discount rate for our present value computations.

Growth g_t is slightly persistent (0.20). The term spread $y_t - i_t$ also predicts economic

growth, a common finding, and reinforcing the importance of the interest rates as state variables.

Inflation π_t is persistent, with a substantial own coefficient (0.53). The interest rate also helps to predict inflation.

The surplus is somewhat persistent, with an own coefficient of 0.39. Growth g_t predicts higher surpluses, an important and realistic feedback mechanism. Growth is also contemporaneously correlated with surpluses ($\rho = 0.45$, Table 7 below.) The surplus responds to the value of the debt, (0.05). This coefficient can be misinterpreted to measure a passive-fiscal regime. The active vs. passive fiscal question is how surpluses respond to multiple-equilibrium variation in the value of debt. We do not measure off-equilibrium responses from data drawn from equilibrium. Even a completely exogenous surplus process, in which a government borrows, then raises surpluses as promised to pay off the resulting debt, will show this coefficient.

The value of the debt is very persistent, with an 0.98 own coefficient. It thus becomes the most important state variable for long-run calculations. A larger surplus s_t results in less market value of debt, v_{t+1}, (-0.28), as one would expect. The long-run yield y_t forecasts a rise in the value of debt v_{t+1}, almost entirely through its effect on the rate of return r_{t+1}^n.

The short rate i_{t+1} is also highly autocorrelated with an 0.74 own coefficient. The long-run yield y_t does not forecast the short rate, again reflecting time-varying real returns. The long rate y is also autocorrelated, again reflecting standard yield curve dynamics.

3.3. Response to an inflation shock

To calculate the response to an inflation shock, I orthogonalize the inflation shock last – all other variables respond contemporaneously to whatever the shock is that moves inflation. This choice also means that any other orthogonalized shock will not impact unexpected inflation at the time of the shock, so we can ignore them and the order of their orthogonalization.

I specify $\varepsilon_1^\pi = 1$. Then I fill in shocks to the other variables by running regressions of their shocks on the inflation shock. For each variable z, I run

$$\varepsilon_{t+1}^z = b_{z,\pi}\varepsilon_{t+1}^\pi + \delta_{t+1}.$$

Then I start the VAR at

$$\varepsilon_1 = -\begin{bmatrix} b_{r^n,\pi} & b_{g,\pi} & \varepsilon_1^\pi = 1 & b_{s,\pi} & ... \end{bmatrix}'.$$

This procedure is equivalent to the usual orthogonalization of the shock covariance matrix, but it is more transparent and it generalizes more easily later. I refer to the VAR innovations as the

Component	Fraction
Inflation π_1	1.00
Bond return $-(r_1^n - g_1)$	0.23
of which $-r_1^n$	0.55
of which g_1	-0.32
Total current $\pi_1 - (r_1^n - g_1)$	1.23
-Future Σs	0.04
Future $\Sigma r - g$	1.20
Total future	1.23
Std dev $\pi \times 100$	1.15

Table 2: Decomposition of unexpected inflation variance

change in expectations at time 1, i.e. ΔE_1, and thus the response of variable x j periods in the future is $\Delta E_1 x_j$.

Figure 2 plots the responses to the inflation shock. Table 2 presents the terms of the variance decomposition (21), i.e.

$$\Delta E_1 \pi_1 - \Delta E_1 \left(r_1^n - g_1\right) = -\sum_{j=0}^{\infty} \Delta E_1 s_{1+j} + \sum_{j=1}^{\infty} \Delta E_1 \left(r_{1+j}^n - \pi_{1+j} - g_{1+j}\right). \tag{27}$$

The top panel of Figure 2 presents the main terms in the identity (27), and includes the sums of the future surplus and adjusted discount rate responses, labeled $r - g$, from this identity.

A positive inflation shock coincides with a negative shock to the surplus s, which builds with a hump shape. However, surpluses eventually do rise to pay back some of the incurred debt. In fact, the sum of all surplus responses is -0.04, so essentially all of the deficits associated with the inflation shock are offset by subsequent positive surpluses. Table 2 reports as 0.04% the fraction of the variance of inflation shocks accounted for by the sum of future surpluses.

The line marked $r - g$ plots the response of the real growth-adjusted discount rate, $\Delta E_1(r_{1+j}^n - \pi_{1+j} - g_{1+j})$. These are plotted at the time of the ex-post return, $1 + j$, so they are the expected return one period earlier at time j. For this reason, the line starts at time 2, where the terms of the sum in (27) start. After two periods of no movement, this discount rate rises. The sum of all discount rate terms is 1.20%. When inflation $\Delta E_1 \pi_1$ rises 1%, more than all of the corresponding decline in the value of government debt comes from a rise in discount rates.

What happens to the extra 20% decline in the present value of debt? The line $r^n - g$ shows the change in the first term of (27), $\Delta E_1 \left(r_1^n - g_1\right)$, which declines by 0.23%..
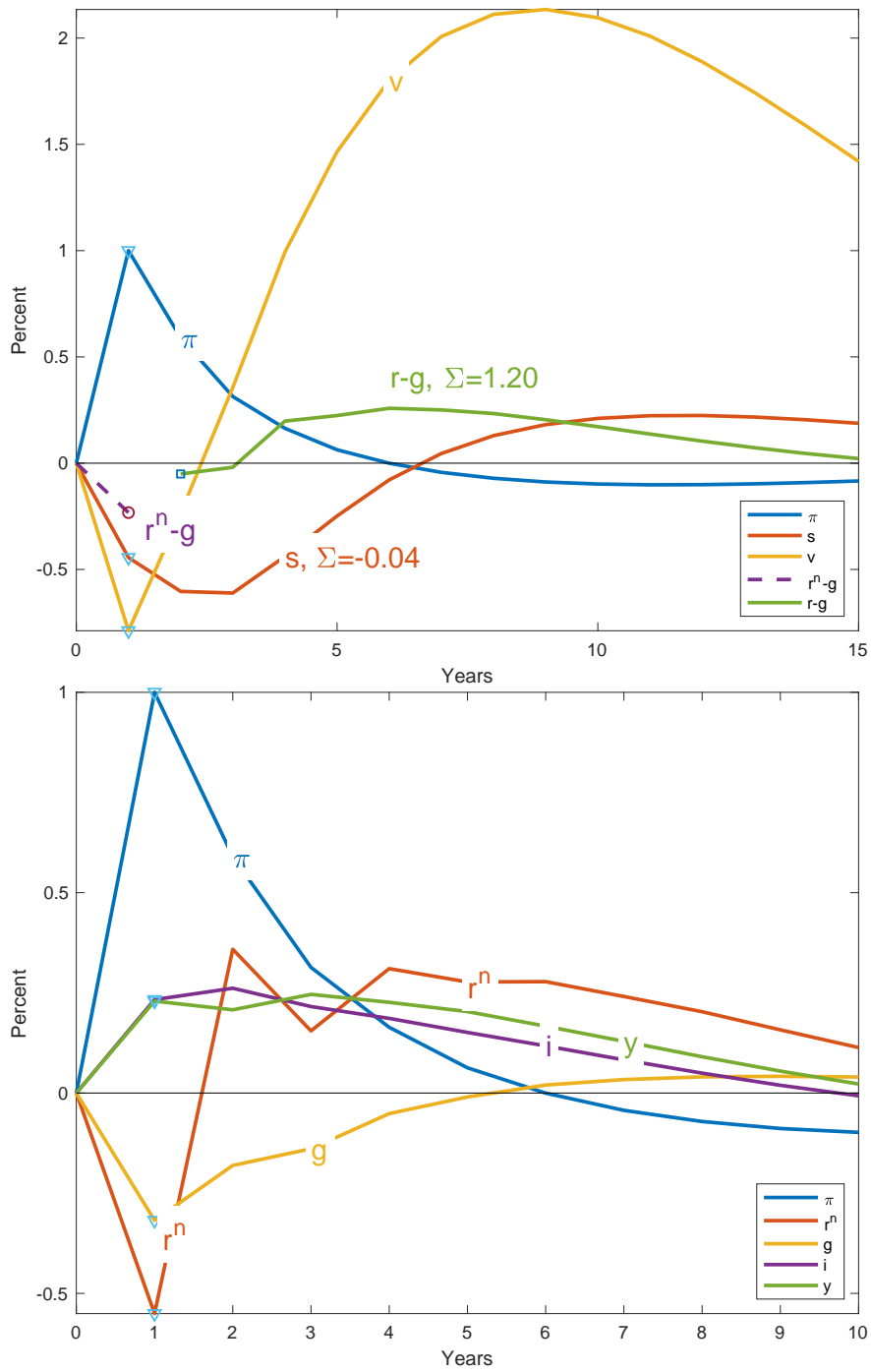
Figure 2: Response to a 1% inflation shock.

In sum,

- *The decline in present value of surplus corresponding to an inflation shock comes entirely from a rise in discount rate, and not from a change in expected surpluses.*

This is an important finding for matching the fiscal theory to data, or for understanding the fiscal side of passive-fiscal models. Both exercises have focused on the presence or absence of surpluses, not the discount rate.

- *A fifth of the decline in present value of surpluses associated with an inflation shock is soaked up by a decline in the adjusted value of long term bonds.*

The market value of the debt v_t initially declines, due to the combined effect of a negative return shock $r_1^n - g_1$, inflation π_1 and the deficit s_1 (The innovation version of the identity (8) is $\varepsilon_{t+1}^v = \varepsilon_{t+1}^{r^r} - \varepsilon_{t+1}^\pi - \varepsilon_{t+1}^g - \varepsilon_{t+1}^s$.) The negative surpluses (deficits) then soon pile up the debt, until surpluses reverse and the debt starts to decline. The rise in expected return adds a bit to the rise in market value of debt.

The lower panel of Figure 2 plots the response of rates of return in more detail, to give some intuition for the discount rate behavior of the upper panel, and Table 2 includes some of the relevant numerical values.

The response of growth g is negative and persistent. The inflation shock is, on average in this sample, stagflationary. Below, I isolate a shock in which unexpected inflation coincides with larger growth.

The return r_t^n takes a large one-period fall, but then rises. This is the picture of an unexpected rise in bond yields, which produces a one-period decline in bond prices but then a rise in bond expected return. (The sawtooth pattern comes from a slightly negative eigenvalue of the VAR, which is far below statistical significance.)

Both long and short bond yields rise throughout, largely an upwards level shift of the term structure. The expected return r^n rises a little more than both yields as the VAR assigns more power to forecast nominal returns to the long rate y than it subtracts from the short rate i. But most of the rise in expected return here is just tracked by the yield, and not a risk premium.

The rise in discount rate, labeled $r - g$ in the top panel, consisting of $\Delta E_1 \sum_{j=1}^{\infty} (r_{1+j}^n - g_{1+j} - \pi_{1+j})$, then comes mostly from the rise in nominal return with the contributions of growth and inflation largely offsetting past year 4.

The -0.23% growth-adjusted return $\Delta E_1(r_1^n - g_1)$ in the top panel, measuring how much of the fiscal shock is soaked up by bond prices, consists of a large -0.55% negative bond return, mitigated by the negative of the 0.32% decline in growth.

As a reminder, the calculations do not imply or require a causal structure. The termi-nology "impulse-response function" can carry a misleading causal implication that we read the "responses" as the "effects" of the shock. In fact, the fiscal theory interpretation goes exactly the other way: the inflation shock is caused by the change in expectations of other variables, or more precisely all are co-determined by underlying structural shocks. The statistical technique only measures a the correlation between unexpected inflation and the change in expectations of other variables.

3.4.   Long term debt and inflation smoothing

The mechanism by which long term debt can soak up fiscal shocks through the bond return $\Delta E_1 \left(r_1^n - g_1 \right)$ term of the inflation identity (27) bears a more detailed examination. (Cochrane (2001), Cochrane (2019) explore this mechanism in more detail.) Though it is quantitatively small here, it is larger below, and deeply important in the underlying theory.

It helps to look at a constant-discount rate version of the nonlinear present value relation, that the real market value of nominal government debt equals the present value of surpluses,

$$\frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

Moving the time index forward and taking innovations,

$$(E_{t+1} - E_t) \left(\frac{\sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} B_t^{(t+1+j)}}{P_t} \frac{P_t}{P_{t+1}} \right) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \tag{28}$$

This equation generalizes (3) to long-term debt, and is the nonlinear version of (21) for a con-stant discount rate and omitting growth. The point: A shock to the present value of surpluses on the right hand side may be met by a decline in the nominal value of long-term bonds, via changes in bond prices $Q_{t+1}^{(t+j)}$ as well as by inflation.

Bond prices, however, reflect future inflation. In a frictionless model,

$$Q_t^{(t+j)} = E_t \left(\beta^j \frac{P_t}{P_{t+j}} \right).$$

Thus, long term debt can spread a fiscal shock to future, rather than to current inflation. With only short term debt, $B_t^{(t+1+j)} = 0$ for $j > 1$, a fiscal shock on the right hand side of (28) would have to fully impact current inflation. With long term debt, it can now cause future rather than current inflation. The present value identity then holds because bond prices fall.

What determines whether a fiscal shock is met by future inflation and a bond price fall or by current inflation? Monetary policy. By changing the interest rate target, with no effect on surpluses, monetary policy sets the *expected* rate of inflation. In the frictionless model this control is immediate: $i_t = r + E_t\pi_{t+1}$. If the central bank raises the interest rate in response to the fiscal shock (or, in a more complex model, does what it takes to allow future inflation to emerge), then the fiscal shock will translate into a long lasting inflation. If not, there will be a one-period price level jump.

Now in the real world and the real model, real interest rates may vary as well, and sticky prices may entwine inflation with real interest rate variation. Figure 2 shows the inflation increase is persistent, which so far has been irrelevant to the analysis, and it shows nominal interest rates rising after the inflation shock. But are these movements quantitatively consistent? How much of the decline in bond portfolio value $\Delta E_1 r_1^n$ trace back to this simple inflation-smoothing mechanism, and how much will a model have to generate by time-varying real interest rates or risk premiums?

To make a rough calculation of the source of the bond return innovation, note the log price of an N year discount bond is

$$p_t^{(N)} = -\sum_{j=1}^{N} r_{t+j}^{n(N-j+1)} \tag{29}$$

where $r_{t+1}^{n(N)}$ denotes the nominal log return at time $t+1$ of a bond which has maturity N at time t. Moving the time index forward, taking innovations, and with $\Delta E_1 r_1^{n(N)} = \Delta E_1 p_1^{(N)}$,

$$\Delta E_1 \left(r_1^{n(N)} \right) = -\Delta E_1 \sum_{j=1}^{N} r_{1+j}^{n(N-j+1)}.$$

If expected returns are the same for all maturities, we can use the government bond portfolio return on both sides

$$\Delta E_1 \left(r_1^n \right) = -\Delta E_1 \sum_{j=1}^{N} r_{1+j}^n. \tag{30}$$

Under the expectations hypothesis $i_t = E_t r_{t+1}^n$ we can use the short-run interest rate on the right hand side

$$\Delta E_1 \left(r_1^n \right) = -\Delta E_1 \sum_{j=1}^{N} i_j. \tag{31}$$

Equations (30) and (31) give us an immediate source of the bond return. In particular, the extent to which (31) holds tells us the extent to which we can trace the decline in bond price to mone-

tary policy, i.e. the path of the short term rate following the shock, without worrying about risk premiums.

Adding and subtracting inflation, and taking innovations, we can break out the bond return to the separate effects of real interest rates and inflation,

$$\Delta E_1 \left(r_1^n \right) = -\Delta E_1 \sum_{j=1}^{N} \left[\left(r_{1+j}^n - \pi_{1+j} \right) + \pi_{1+j} \right]. \tag{32}$$

and

$$\Delta E_1 \left(r_1^n \right) = -\Delta E_1 \sum_{j=1}^{N} \left[\left(i_j - \pi_{1+j} \right) + \pi_{1+j} \right]. \tag{33}$$

N	Ex. ret, $\sum r_{1+j}^n$	Int. rate, $\sum i_j$	Real, $\sum r_{1+j}^n - \pi_{1+j}$	Real, $\sum i_j - \pi_{1+j}$	Inflation, $\sum \pi_{1+j}$
2	0.36	0.23	-0.23	-0.36	0.59
3	0.51	0.49	-0.39	-0.41	0.90
4	0.82	0.71	-0.24	-0.36	1.07
r_1^n	-0.55				

Table 3: Components of the decomposition of a bond return shock.

Table 3 presents the terms of this decomposition for a variety of N. The US maintains a relatively short maturity structure overall, so N = 2-4. The bottom entry is $\Delta E_1 \left(r_1^n \right) = -0.55$, the response of unexpected bond returns to the inflation shock that we wish to understand. The upper entries give, for each value of N, the terms of (30)-(33).

The expected return $\Delta E_1 \sum_{j=1}^{N} r_{1+j}^n$ and interest rate $\Delta E_1 \sum_{j=1}^{N} i_j$ in the first two columns account nicely for the bond return $\Delta E_1 r_1^n$. (Equations (30) and (31) have a negative sign – a rise in yield or expected return generates a negative ex-post return.) The former is essentially an identity, but it is nice to check that approximate identities hold. The latter verifies that for this slice of the data, the expectations hypothesis is a reasonable approximation, and the rise in bond expected returns stems from the rise in yields. Yes, the rise in bond return r_j^n from date 2 on and rise in interest rate i_j from date 1 on seen in the bottom panel of Figure 2 do account for the ex-post return shock $\Delta E_1 r_1^n$.

Breaking the return to real and nominal components in the next three columns, we see that inflation rises even more than nominal expected returns and yields rise, so real expected returns decline. Thus, real returns contribute in the wrong direction. Of the -0.55% change in ex-post return, real returns account for +0.2% to +0.4%. The rise in inflation correspondingly

explains more than all of the decline in bond prices, generating -0.59% to -1.07%.

On this back of the envelope basis, then, we can conclude that higher future inflation is the major driver of the lower bond return $\Delta E_1 r_1^n$, and the rise in interest rates. Thus,

- *By maintaining a maturity structure with about three years duration, and allowing interest rates and expected inflation to rise when there are shocks to the present value of surpluses, the US spreads the inflationary impact of changes in the present value of surpluses forward, absorbing shocks to the present value of surpluses in long term bond prices.*

This mechanism is not terribly important quantitatively in the point estimates so far. It is much more important in estimates that follow. Moreover, as one goes to higher frequency data, the bond return mechanism becomes more important and unexpected inflation less so. The continuous time models of Sims (2011) and Cochrane (2017) have long term debt, unexpected inflation but no price level jumps, so inflation does not directly devalue debt at all. All the effects of fiscal shocks show up in expected future inflation, and the identity holds entirely on the drop in long-term bond value.

3.5. Recession shocks

We can use the same procedure to understand the fiscal underpinnings of other shocks. For any interesting ε_1, we can compute impulse responses $A^j \varepsilon_1$, and thereby the terms of the decomposition (21). We can consider the calculation as a decomposition of the covariance of unexpected inflation with the shock ε_1, rather the decomposition of the variance of unexpected inflation.

I start with a recession shock. The response to an inflation shock, in Figure 2, is stagflationary, in that consumption (g) falls when inflation rises. Unexpected inflation is, in this sample, negatively correlated with unexpected consumption (and also GDP) growth. The stagflationary episodes outweigh the simple Phillips curve episodes. However, I started with a puzzle about events in which inflation falls during a recession such as 2008, though there are contemporaneous deficits. What are the fiscal underpinnings of such episodes – of the times when the data do move along a Phillips curve?

To answer that question, we want to study a shock in which inflation and GDP go in the same direction. I simply create such as shock – I specify $\varepsilon_1^\pi = -1, \varepsilon_1^g = -1$. (The model is linear, so the sign doesn't matter, but the story is clearer for a recession.) Again, we want shocks to other variables to have whatever value they have, on average, conditional on the inflation and output

Component	Recession	Monetary	Fiscal
Inflation π_1	-1.00	1.94	0.48
Bond return $(r_1^n - g_1)$	2.13	-0.80	0.08
of which r_1^n	1.13	-0.80	0.16
of which g_1	-1.00	-0.00	0.09
Total current $\pi_1 - (r_1^n - g_1)$	-3.13	2.74	0.40
Future $\sum s$	-1.37	0.00	-1.00
Future $\sum(r - g)$	-4.50	2.74	-0.60
Total future $-\sum s + \sum(r - g)$	-3.13	2.74	0.40

Table 4: Inflation identity terms for recession, monetary, and fiscal shocks

shock. To fill out the other shocks, then, I run a multiple regression

$$\varepsilon_{t+1}^z = b_{z,\pi}\varepsilon_{t+1}^\pi + b_{z,g}\varepsilon_{t+1}^g + \delta_{t+1}$$

and then I fill in the other shocks at time 1 from their predicted variables given $\varepsilon_1^\pi = -1$ and $\varepsilon_1^g = -1$. I then start the VAR at

$$\varepsilon_1 = -\left[\begin{array}{ccccc} b_{r^n,\pi} + b_{r^n,g} & \varepsilon_1^g = 1 & \varepsilon_1^\pi = 1 & b_{s,\pi} + b_{s,g} & ... \end{array}\right]'.$$

This procedure is again equivalent to an orthogonalization of the shock covariance matrix.

Figure 3 presents responses to this recession shock, and Table 4 collects the inflation decomposition elements.

In the bottom panel, both inflation π and growth g responses start at -1%, by construction. Consumption growth returns rapidly, but does not much overshoot zero, so the level of consumption does not recover much at all. Consumption is roughly a random walk in response to this shock. The nominal interest rate i falls in the recession, and recovers slowly, in parallel with inflation. Long term bond yields y also fall, but not as much as the short term rate, for about 4 years. The persistent fall in interest rate, inflation and and the smaller fall in bond yield correspond to a large positive ex-post bond return $\Delta E_1 r_1^n$. In short, we see a standard picture of a recession.

In the top panel, the recession includes a deficit s, which continues for three years. These deficits, reinforced by the positive return shock $r^n - g$ imply a large rise in the value of debt, v. Surpluses subsequently turn positive paying down some of the debt. But the total surplus is still -1.37%. Left to their own devices, surpluses would produce a 1.37% *inflation* during the
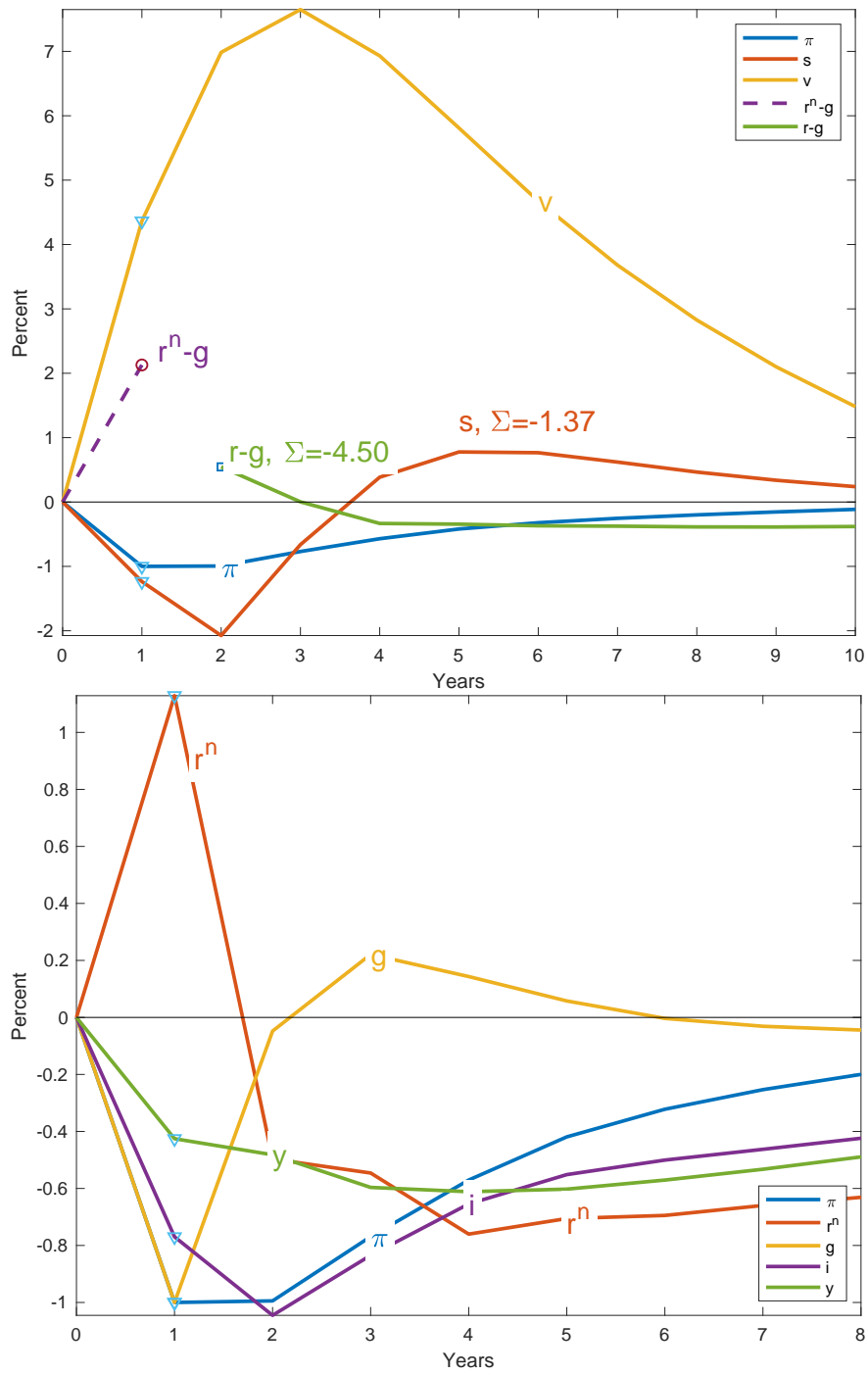
Figure 3: Response to a recession shock, $\varepsilon_1^\pi = \varepsilon_1^g = -1$.

recession. A potential story that disinflation results from future surpluses more than matching today's deficits is wrong.

Discount rates are thus the central story. After one period, expected real returns $r - g$ decline persistently in the top panel, raising the value of debt by a total of 4.50 percentage points. We can see the underlying forces in the bottom panel: At year 3, which are expected values at year 2, the nominal return $\Delta E_1 r_j^n$ falls more than inflation $\Delta E_1 \pi_j$.

In sum, rounding the numbers,

- *Disinflation in a recession is driven by a higher discount rate. For each 1% disinflation shock, the expected return on bonds falls so much that the present value of debt rises by 4.5%. This discount rate shock overcomes a 1.4% percent inflationary shock coming from persistent deficits. 2.1% of the change in present value raises ex-post bond returns, adjusted for growth, which rise reflects the persistence of the disinflation and interest rate responses.*

The opposite conclusions hold of inflationary shocks in a boom.

3.6.   Monetary and fiscal shocks

Central banks move interest rates, but cannot tax or spend – monetary policy cannot on its own move the primary surplus. Therefore, I define a monetary policy shock as one that moves interest rates $\Delta E_1 i_1$, but does not affect the sum of current and future surpluses, $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = 0$.

The main object of interest here is unexpected inflation $\Delta E_1 \pi_1$ coincident with the monetary policy shock $\Delta E_1 i_1$. Monetary policy can change expected inflation without changing surpluses – in the simple frictionless model, $i_t = E_t \pi_{t+1}$ can change $E_t \pi_{t+1}$, by changing debt B_t without changing surpluses. But, as captured by the unexpected inflation identity, a monetary policy shock at time $t + 1$ can only affect unexpected inflation $\Delta E_{t+1} \pi_{t+1}$ if it affects the present value of surpluses. With a constant sum of future surpluses, such monetary policy must produce unexpected inflation by changing discount rates. With sticky prices or other non-neutralities, monetary policy can affect real rates, so it is interesting to measure if and how this mechanism operates.

I further orthogonalize the monetary policy shock so that it does not move the growth rate g_t, conservatively ascribing contemporaneous correlation between growth and interest rate shocks as a Taylor-rule reaction of the Fed to growth and not the other way around. A longer discussion of specification follows the results.

Conversely, I define a fiscal shock as a movement in current and expected primary surpluses $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j}$ that comes with no movement in the short-run interest rate $\Delta E_1 i_1 = 0$.

Here there is no reason to orthogonalize with respect to growth g_t.

The response of the sum of future surpluses to a shock ε_1 is

$$\Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = a_s'(I - A)^{-1}\varepsilon_1.$$

To calculate how other shocks respond instantaneously to a monetary shock, then, I run for each variable z a multiple regression

$$\varepsilon_{t+1}^z = b_{z,i}\varepsilon_{t+1}^i + b_{z,pv}a_s'(I - A)^{-1}\varepsilon_{t+1} + b_{z,g}\varepsilon_{t+1}^g + \delta_{t+1}. \tag{34}$$

The monetary policy shock wants

$$\varepsilon_1^i = 1, \ a_s'(I - A)^{-1}\varepsilon_1 = 0, \ \varepsilon_1^g = 0.$$

Thus, we start the monetary policy impulse-response function with

$$\varepsilon_1 = \left[\begin{array}{ccccc} b_{r^n,i} & b_{g,i} = 0 & b_{\pi,i} & ... & b_{\iota,i} = 1 & ... \end{array} \right]',$$

For the fiscal shock I run the same regression without g_t,

$$\varepsilon_{t+1}^z = b_{z,i}\varepsilon_{t+1}^i + b_{z,pv}a_s'(I - A)^{-1}\varepsilon_{t+1} + \delta_{t+1},$$

And then I start the fiscal impulse-response function with

$$\varepsilon_1 = \left[\begin{array}{cccc} b_{r^n,pv} & b_{g,pv} & b_{\pi,pv} & ... \end{array} \right]'.$$

Figure 4 presents the responses to the monetary policy shock. Table 4 collects relevant magnitudes of the responses. The instantaneous response of the nominal interest rate $\Delta E_1 i_1$ is 1% by construction, as is the zero response of growth $\Delta E_1 g_1 = 0$ and the zero response of surpluses $\Delta E_1 \sum_{j=0}^{\infty} s_{1+j} = 0$. The response of inflation, like that of other variables, is now a result of the regression (34), not an assumption.

The lower panel of Figure 4 shows that

- *The response of inflation to this monetary policy shock, with no change in fiscal policy, is super-Fisherian, with inflation rising immediately.*

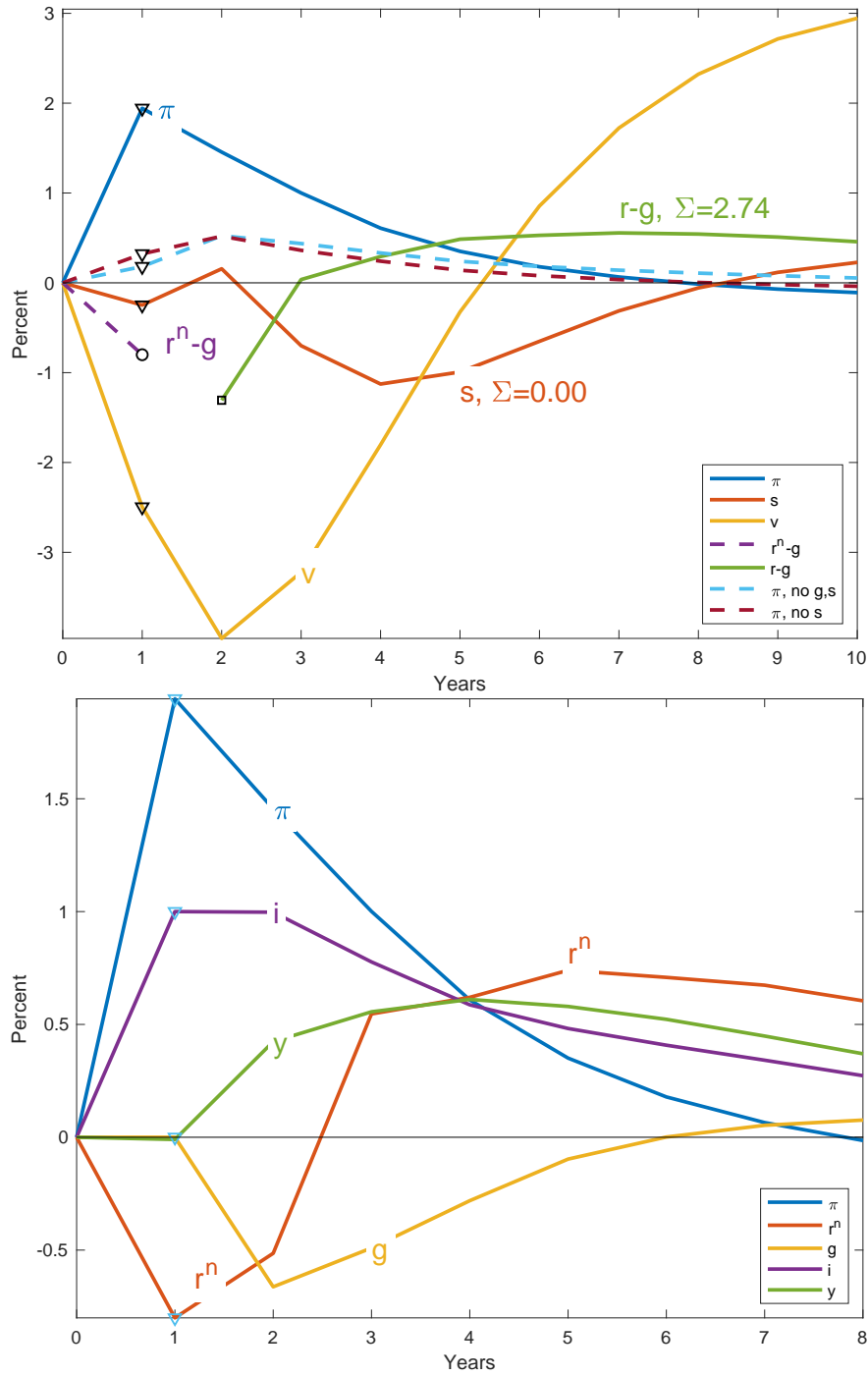A "Fisherian" response has come to mean that if the central bank raises the interest rate i_t,

Figure 4: Response to a monetary policy shock – a movement in the interest rate y_1 with no movement in the sum of future surpluses $\sum_1 s v_{1+j}$ or growth g. The dashed lines labeled "no g,s" show the inflation response with movement in growth and surplus.

then inflation follows with a one period lag, fulfilling the simple Fisher relation $i_t = r + E_t \pi_{t+1}$. A "super-Fisherian" response is one in which raising the interest rate i_t raises inflation π_t contemporaneously. That is the pattern shown in Figure 4.

Consumption g starts at zero by construction. Then it declines, as one might expect from a monetary policy shock, and as a new-Keynesian Phillips curve would predict with declining inflation.

The top panel of Figure 4 shows that, although the sum $\Delta E_1 \sum_{j=1}^{\infty} s_j$ does not change by construction, near-term surpluses increase, and long-term surpluses decrease, roughly paralleling the path of consumption. The surplus is strongly procyclical.

With no change to future surpluses $\Delta E_1 \sum_{j=1}^{\infty} s_{1+j}$, the fiscal backing for inflation rise $\Delta E_1 \pi_1$ comes all from discount rates. Figure 4 shows that after one period, future $r - g$ rises uniformly and persistently, lowering the present value of the debt. The sum of the future $r - g$ terms, 2.74%, is even greater than the inflation shock, $\Delta E_1 \pi_1 = 1.94\%$. (Numbers are collected in Table 4.) This extra discount rate effect shows up in a current bond loss, $\Delta E_1 (r_1^n - g_1) = -0.80\%$, which again reflects the persistence of interest rate and inflation changes.

- *A monetary shock, defined as an interest rate rise with no change in surpluses and growth, leads to an immediate and persistent increase in inflation. Unexpected inflation all comes from a rise in discount rates, which lowers the value of government debt. For every 3 percentage points of decline in present value of surpluses, 2 percentage points results in immediate inflation, and 1 percentage points in lower bond prices, which reflect the drawn out interest rate and inflation response.*

Figure 5 presents responses to fiscal policy shocks. I specify a negative shock to produce positive inflation, which is a clearer story. In the bottom panel, the contemporaneous interest rate response to the fiscal shock is therefore $\Delta E_1 i_1 = 0$, by construction. The interest rate then rises slightly. The fiscal shock gives rise to a positive and persistent inflation.

In the top panel, though the sum of all s terms is -1.00% by construction, near-term surpluses rise, and long term surpluses fall even more. There are three periods of negative discount rate movement, adding up to -0.60%, and thus offsetting more than half of the surplus shock. This discount rate movement comes from the dynamics shown in the bottom panel. The long-term rate y declines for one period, which gives rise to a large one-period expected return r^n.

The remaining -0.40% shock to the present value of surplus results in 0.48% inflation, and a small 0.08% bond return term.

- *A fiscal shock sets off a protracted inflation. Discount rate variation seems to offset about*
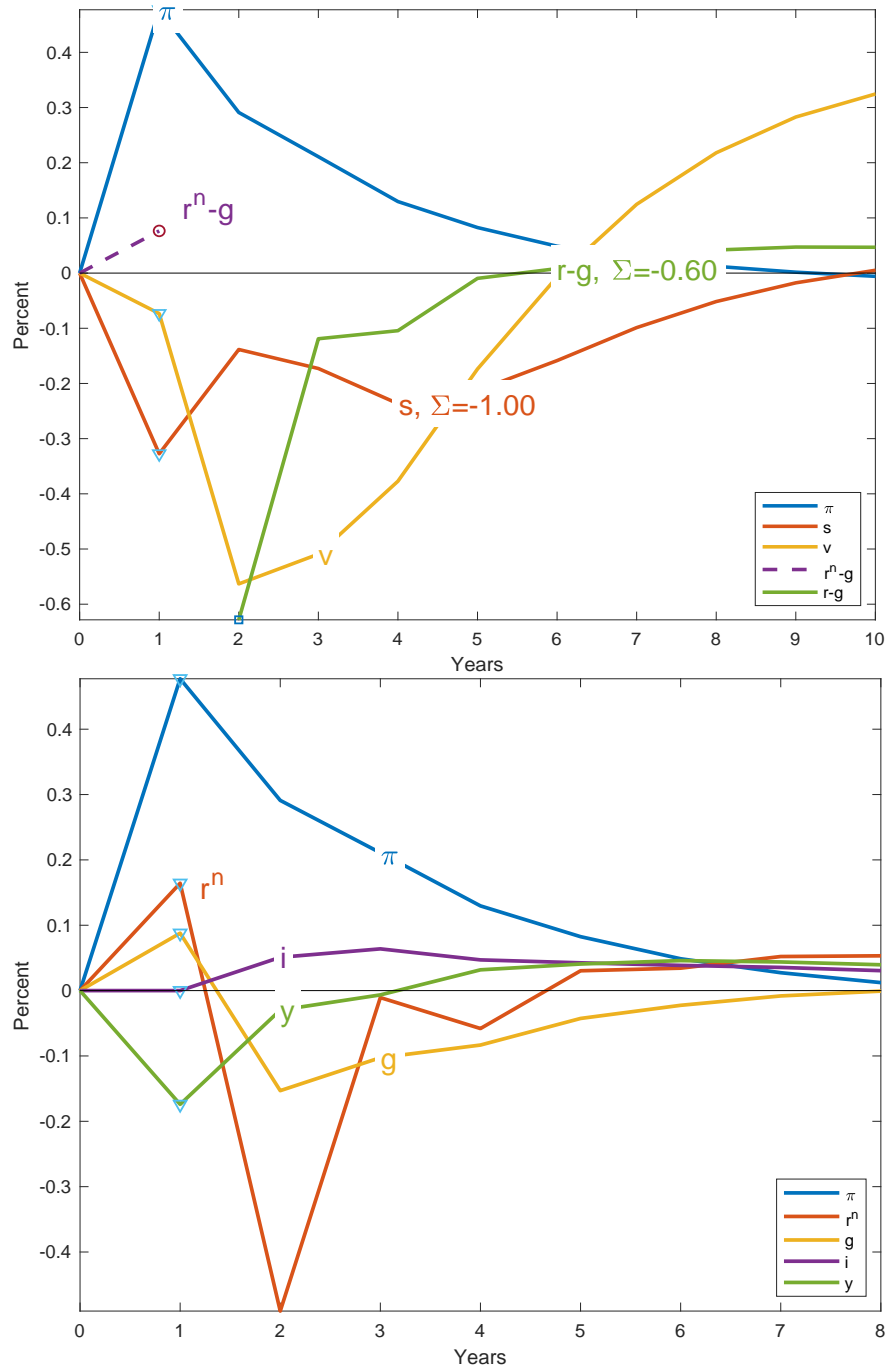
Figure 5: Fiscal policy response. Response to a shock to expected surpluses $\Delta E_{t+1} \sum_{j=1}^{\infty} sv_{t+j} = 1$, with no interest rate shock $v_{t+1}^y = 0$.

half of the fiscal shock.

Why does inflation rise in response to the monetary policy shock, contrary to the usual presumption? Most importantly, standard estimates do not measure fiscal variables or try to keep surpluses constant. Monetary and fiscal shocks are correlated. In historical episodes, both monetary and fiscal authorities react to the same events. On their own, VARs will find monetary policy shocks that also move fiscal variables. New-Keynesian models construct a fiscal response to a monetary policy shock by assumption of passive fiscal policy.

The main innovation of this calculation, then, is try to measure the effects of conceptually separate monetary policy and fiscal shocks, each holding the other constant.

The dashed lines in the top panel of Figure 4 present the inflation response without restriction on surpluses. One of the lines also removes the growth orthogonalization. The inflation response with a free fiscal response is much smaller, consistent with the view that defining the shock to hold surpluses constant is a big part of the story for a positive inflation response.

These definitions still produce a rise in inflation, though much smaller, so one cannot say that the fiscal orthogonalization changes a negative estimate to a positive one. But a small rise in inflation is also a common finding of standard approaches. For example, the comprehensive review in Ramey (2016) finds the "price puzzle" of rising inflation, especially in the short run, following a monetary policy tightening is pervasive in the VAR literature. A robust negative contemporaneous inflation effect is not there in the first place.

This calculation is also unsophisticated regarding exogeneity and orthogonalization. The contemporaneous correlation between interest rate shocks and shocks to inflation, GDP, or other variables can result from the Fed reacting within the period to those variables, as described for example by a Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_x x_t + v_t, \tag{35}$$

instead of the reaction of the economic variables to the Fed's shock, an innovation to v_t. Exogeneity poses an even harder conundrum: Perhaps even the apparent disturbance v_t is taken in response to news about future inflation or output, or other variables such as financial conditions that forecast inflation or output, news not captured by VAR variables.

I defined the shock orthogonalzed before the growth shock, conservatively assigning all the correlation to reverse causality from consumption to the interest rate. I can't do that for inflation, or there is by definition no inflation shock $\Delta E_1 \pi_1 = 0$, and nothing to analyze.

Reality of course lies somewhere in between, and thus requires something more sophisticated than a recursive identification, i.e. assuming either that the Fed does not react within a

year to specific economic variables, or that some variables not react within a year to the Fed, and that innovations to the VAR are orthogonal to all other news about future inflation and output. Very high frequency data, narrative approaches, or additional theory may help. However, Ramey (2016) leaves the clear taste that a nearly half-century of effort has still not led to a clear answer or procedure, on to which one can layer a fiscal policy assumption. The deepest problem is that the Fed never explains a movement as a random innovation. Rather, the Fed always describes every movement as a response to something. The only hope is to find a Fed response to something orthogonal to inflation, output, or employment, or forecasts of those, but given those are the Fed's mandate it's hard to think what that object could be.

My separation of monetary and fiscal policy, and definition of fiscal shock, is also crude. Fiscal policy also has a clearly endogenous component. At a minimum one wants an analogous rule or model such as

$$s_t = \theta_x x_t + u_t \tag{36}$$

where again x_t is output or a vector of related variables. Such a rule would capture the clear association of fiscal surpluses to output, via the standard effects of proportional income taxation, and both automatic and discretionary fiscal stabilizers. More deeply, one wants a model paralleling the on- and off-equilibrium distinctions of (5), in which surpluses rise to pay off debts accumulated from past deficits under the selected equilibrium path, but do not respond to multiple equilibrium inflation and especially deflation outbreaks, something like.

$$s_t = \theta_x x_t + \alpha(V_t/P_t^*) + u_t \tag{37}$$

where P^* is the desired equilibrium price level. Such a model is not unrealistic. A gold clause and gold standard operates in this way, for example: If gold becomes more valuable than dollars, the government is obliged to pay the larger value, and if dollars become more valuable than the gold standard prescribes, the government is only required to pay back the gold value. Our implicit fiscal commitments can operate the same way.

With something like (35) and (37) in mind, then, one would want to define a monetary policy shock as an innovation to v_t in (35) uncorrelated with current and future $\{u_t\}$ in (36) or (37), i.e. not holding constant surpluses themselves or their sum as I have done. That specification would allow and consider as part of "monetary policy" the endogenous fiscal responses to interest rate changes. The medium-scale fiscal theory of monetary policy model in Sims (2011) and Cochrane (2017) make such calculations.

Likewise, one might want to define a fiscal policy shock as an innovation to u_t or to ex-

pected future $\{u_t\}$ (u_t is particularly unlikely to be an AR(1), and to include promises of at least partial reversal), holding the monetary policy disturbance v_t fixed, not current and especially not future short-term interest rates. One may want to ask the question, "what happens there is a fiscal shock, and the Fed accommodates following a Taylor rule," not "what happens if there is a fiscal shock, and the Fed holds interest rates constant" either for a period as here, or forever. Cochrane (2017) makes this theoretical calculation as well.

But now we add the identification headache of measuring parameters of the rules, ϕ_π, ϕ_x, θ_x, and α, to our shock identification headache. So I stop here, about as far as one can get with recursive orthogonalization in my annual data set, acknowledging that the contribution is more to show that such calculations can be made than it is to provide a definitive answer.

At a minimum, impulse responses always answer the question, "if we see a surprise change in interest rate, with no change in the sum of surpluses or current consumption, how does that surprise change our forecasts of inflation, output and other variables?" and "if we see a surprise change in expected surpluses, with no change in the short-term interest rate, how does that surprise change our forecasts of inflation output and other variables?" without causal consequence.

The calculation does point in the Fisherian direction. Sims (2011), Cochrane (2018), and Cochrane (2017) went through a lot of trouble to show that with long-term debt, monetary policy shocks defined as here with no surplus response can produce a temporary decline in inflation. If the positive finding result is not the effect of reverse causality or endogeneity, the effort, though useful theoretically, is not needed empirically at least for the whole sample. (Sims created the model for the 1970s, so it may still hold for one episode.) Among other calibration and specification issues, the models assume a very long maturity structures to produce the negative effect. US debt may not have been of long enough maturity to generate the negative inflation response.

4.   Debt

Using the value identity (9), we can characterize the forces behind the value of debt. I then create a VAR-based decomposition of the variance of the value of debt.

4.1.   The history of debt

Figure 6 presents a backwards value decomposition. At each date it plots the terms of

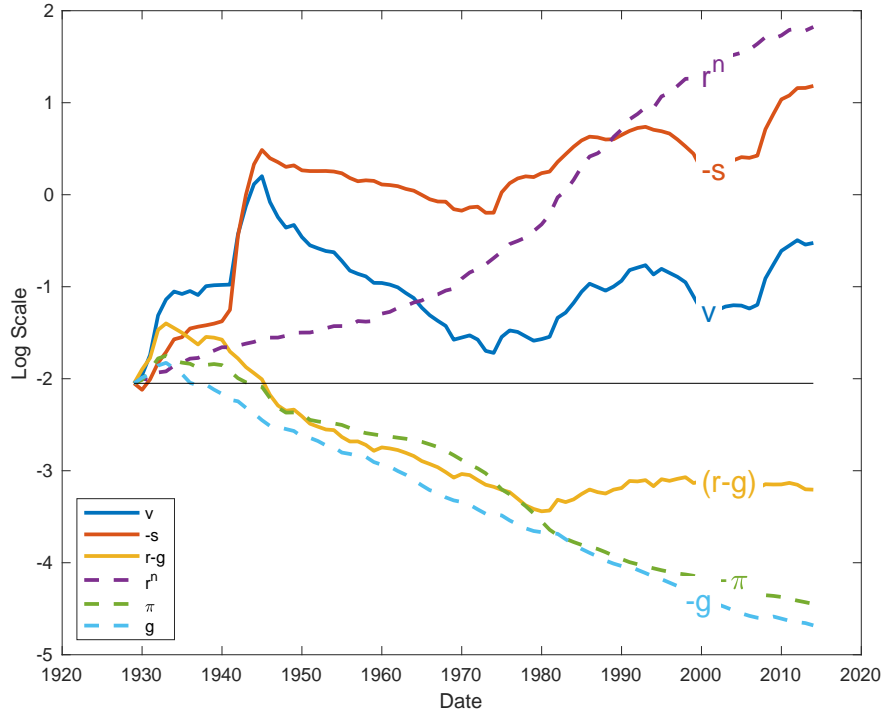$$v_t = v_0 - \sum_{j=1}^{t} s_j + \sum_{j=1}^{t} (r_t^n - \pi_t - g_t)$$

Figure 6: Backwards decomposition of the value of debt.

It answers, "where did each date's market value of debt come from?" Each term in the Figure adds initial debt v_0, so each line presents what debt v_t would be at date t, starting from v_0, if that were the only term.

In v_t we see the evolution of the market value of debt to GDP ratio since 1930. There are three distinct periods, with different behavior. Debt rises in the great depression and in WWII, to a debt-to-GDP ratio greater than one i.e. $v_t = 0 = \log(1)$. The s line here is the primary deficit, the negative of surplus, i.e. how surplus contributes to debt. The rise in debt largely parallels the surplus line, i.e. the rise in market value of debt is driven by the deficits of the 1930s and WWII. Returns matter as well, however. About half of the rise in market value of debt to GDP ratio in the great depression comes from the $r - g$ term, reflecting the fall in GDP and deflation of the early 1930s. Likewise, the rise in market value of debt to GDP ratio in WWII was lower than the cumulative deficits. Here low returns and the rise in GDP kept the debt to GDP ratio from rising as much as it otherwise would have done.

From the end of WWII to about 1975, the value of debt fell steadily. About a third of this fall comes from primary surpluses, as shown by the fall in the $-s$ line. The rest came from the sharp fall in $r - g$. Breaking the latter out to nominal returns, inflation, and growth, we see that nominal returns and inflation largely cancel, leading to the standard conclusion, that about 2/3 of the fall

in debt to GDP ratio came from rising GDP. In fact, ex-post, $g > r$ in this period. However, about 1/3 of the fall did come from the persistent primary surpluses of the first 30 postwar years, seen in Figure 1 . It wasn't all growth.

The inflation of the 1970s is visible, in a speeding up of the π line. But ex post returns in the r^n line rise as well, so the net effect $r = r^n - \pi$ is not as large as one would suppose. To devalue outstanding debt, inflation must either come swiftly, devaluing short term debt before it can be rolled over at higher interest rates, or the outstanding debt must have long maturity. The short maturity structure of debt in the 1970s means that even that large and unexpected inflation did not really do that much to lower the value of debt.

That even the inflation of the 1970s does not contribute in a big way to the value of the debt helps to explain why I focus the previous section, and this paper, on the inflation innovation accounting, rather than value of the debt accounting. Variation in the value of debt comes largely from surpluses, growth, and real returns. Inflation, in the US, in this sample, is a smaller effect.

A sharp break occurs between 1975 and 1980. First, there are two waves of large deficits, with an interlude of surpluses in the 1990s. (See also Figure 1.) The rise and fall of the value of debt largely reflects these surpluses and deficits – the s and v lines move in parallel. The cumulative effect of returns $r - g$ is small. However, that cumulative effect includes a change in behavior. The nominal r^n and real $r^n - \pi$ returns rise – bonds did well in the 1980s and 1990s. Growth does not bring down the debt to GDP ratio as it once did.

There is some feeling that the US has entered a period with $r < g$ so the debt to GDP ratio can withstand primary deficits. The graph does not yet show such behavior.

Figure 7 presents a forward-looking value decomposition,

$$v_t = \sum_{j=1}^{T} s_{t+j} - \sum_{j=1}^{T} \left(r_{t+j}^n - \pi_{t+j} - g_{t+j} \right) + v_{t+T}.$$

Regarding the value of the debt as the present value of future surpluses, this decomposition asks, "suppose people knew the future, and discounted the actual ex-post surpluses using actual ex-post real returns. How would those ex-post expectations account for the value of the debt at each date?"

Mirroring the break seen in the previous graph, we see that after 1975-1980, the value of the debt largely follows the value of future surpluses, with little contribution from discount rates $r - g$. Why, for example, is the value of debt low in 1980? A big part of that is foreknowledge that the value of the debt will be high at the end of the sample v_{t+T} – there will not be a default, an inflation, a growth disaster, or another big hit to $r - g$. Beyond that, the value of debt is low
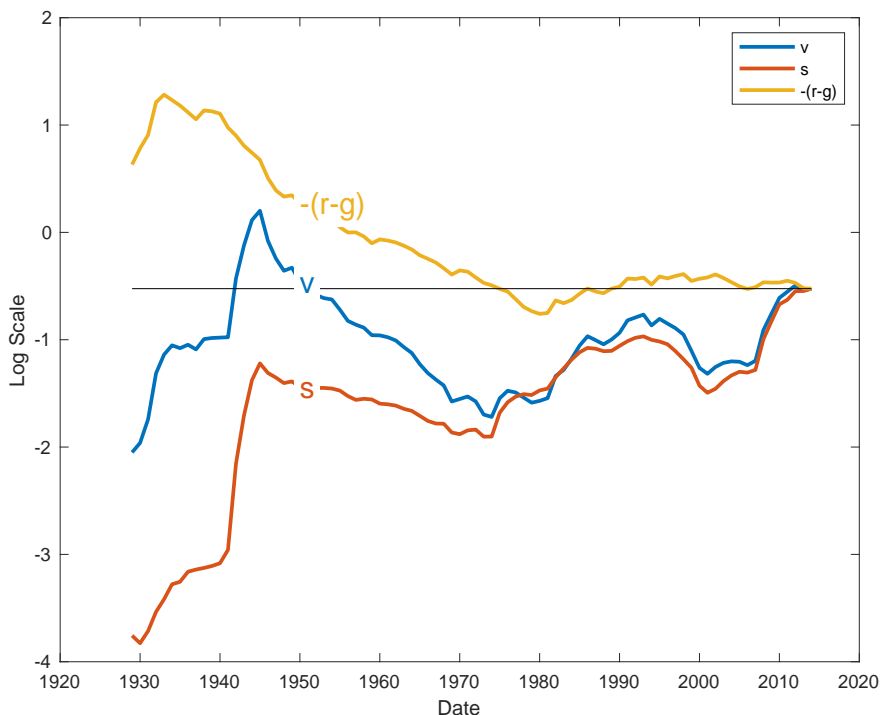
Figure 7: Forward decomposition of the market value of debt to GDP ratio.

because on net, there will be a string of primary deficits in the 1980s and after 2018, so the s line is negative throughout. Time series variation in the value of deficits to come drives time series variation in the value of debt.

From WWII to 1975, we see a different picture. Now, assuming again perfect foresight, we tell the same story backwards. Why was the value of debt so high at the end of WWII? About 2/3 of it was low growth-adjusted discount rates, with about 1/3 knowledge of primary surpluses to come.

Of course, it is quite possible that either data point contains events people did not expect. One may speculate that in 1947 people expected lower growth, positive real interest rates, and therefore a long period of larger primary surpluses. That low returns, some inflation, and high growth brought down the debt to GDP ratio once in the past does not imply that people will hold large debts expecting the same in the future.

4.2.   A variance decomposition for the value of the debt

We can construct a variance decomposition for the value of debt, to see how real values of the debt are resolved on average, not just with the ex-post luck of one history. Under rational expectations we can interpret that calculation to tell us why real values are what they are.

Let tildes stand for three possible transformations of variables,

$$\tilde{v}_t = v_t \text{ or } \theta(L)v_t \text{ or } (E_t - E_{t-1})\, v_t$$

and similarly for the other variables. Applying any of these transformations to the the present value identity (9), the same identify holds for the transformed versions. Multiplying by $\tilde{v}_t - E(\tilde{v}_t)$ and taking expectations, we obtain a variance decomposition for the value of debt,

$$var\,(\tilde{v}_t) = \sum_{j=1}^{\infty} cov(\tilde{v}_t, \tilde{s}_{t+j}) - \sum_{j=1}^{\infty} cov\left[\tilde{v}_t, \left(\tilde{r}^n_{t+j} - \tilde{\pi}_{t+j} - \tilde{g}_{t+j}\right)\right].$$

Dividing by $var\,(\tilde{v}_t)$, we can express the result as fractions of variance due to each component. The components are also the coefficients of single regressions by which $\tilde{v}_t$ forecasts the other terms.

Why transform the data? The quick look at the last two graphs reveals an obvious trouble: U. S. debt is dominated by the runup in WWII, its steady decline, and it so-far-unresolved second runup. Two to three data points are not much on which to take averages. It is also economically interesting to separate debt variation into components. Perhaps war debts are paid off one way, but business cycle debts are paid off another way. For example, perhaps business cycle variation in the value of debt corresponds to real interest rate variation, but secular debts are paid by surpluses or growth.

Filtering valuation equations is delicate. If we start with a valuation equation

$$p_t = E_t\,(m_{t+1}x_{t+1})\,,$$

we can write

$$(E_t - E_{t-1})\,p_t = (E_t - E_{t-1})\,(m_{t+1}x_{t+1})\,,$$

and

$$\theta(L)p_t = \theta(L)E_t\,(m_{t+1}x_{t+1})\,,$$

but we cannot write

$$\theta(L)p_t = E_t\left[\theta(L)\,(m_{t+1}x_{t+1})\right].$$

The lag operator must also apply to the expectation on the outside. For example, if x_t is i.i.d. with $0 = E_t(x_{t+1})$, then applying $(1 - L)$ yields $E_t x_{t+1} - E_{t-1}x_t = 0$ not $E_t(x_{t+1} - x_t) = -x_t$. But since we start here with an ex-post identity, we can first filter and then take expectations, so we

do not make this mistake. (Filtering is less useful for asset pricing applications, as the hypothesis that filtered returns are unpredictable is not particularly interesting. Studying the innovation in price-dividend ratios is still interesting, and can produce an interesting high frequency decomposition.)

To estimate the decomposition for plain and innovation cases, start with the VAR representation of the present value identity (9) as in (23),

$$v_t = E\left(\sum_{j=1}^{\infty} s_{t+j}|x_t\right) - E\left[\sum_{j=1}^{\infty}\left(r_{t+j}^n - \pi_{t+j} - g_{t+j}\right)|x_t\right], \tag{38}$$

$$v_t = a_s'(I-A)^{-1}Ax_t - a_{rg}'(I-A)^{-1}Ax_t.$$

The variance decomposition of the value of debt, with $\tilde{v}_t = v_t$, is then

$$a_v'Va_v = a_s'(I-A)^{-1}AVa_v - a_{rg}'(I-A)^{-1}AVa_v \tag{39}$$

where

$$V \equiv cov(x_t, x_t').$$

The time-t innovation of the present value identity (9) is

$$\Delta E_t v_t = \Delta E_t \sum_{j=1}^{\infty} s_{t+j} - \Delta E_t \sum_{j=1}^{\infty}\left(r_{t+j}^n - \pi_{t+j} - g_{t+j}\right)$$

$$a_v'\varepsilon_t = a_s'(I-A)^{-1}A\varepsilon_t - a_{rg}'(I-A)^{-1}A\varepsilon_t$$

so the variance decomposition of debt innovations, with $\tilde{v}_t = \Delta E_t v_t$, is

$$a_v'\Sigma a_v = a_s'(I-A)^{-1}A\Sigma a_v - a_{rg}'(I-A)^{-1}A\Sigma a_v$$

where

$$\Sigma \equiv cov(\varepsilon_t, \varepsilon_t').$$

The formula is the same as (39), with the innovation covariance matrix Σ in the place of the covariance matrix V. It will therefore de-emphasize a persistent variable such as v_t whose variance is much larger than its shock variance.

To analyze inflation above, I took the time $t + 1$ innovation ΔE_{t+1} of the present value identity, which eliminated the value of debt v_t on the left hand side. Here I take the time t inno-

vation ΔE_t of the present value identity, to focus on the innovation of the value of debt.

To estimate the filtered variance decomposition $\tilde{v}_t = \theta(L)v_t$, we can most simply rerun the VAR with filtered data,

$$\theta(L)x_{t+1} = \tilde{A}\theta(L)x_t + \eta_{t+1}$$

$$\tilde{x}_{t+1} = \tilde{A}\tilde{x}_t + \eta_{t+1}$$

and use the same formula (39).

4.3. Variance of value estimates

In addition to analyzing the log value of debt to GDP $\tilde{v}_t = v_t$, I filter by using the difference between the log value of debt and three lags, $\tilde{v}_t = v_t^f = v_t - \frac{1}{3}(v_{t-1} + v_{t-2} + v_{t-3})$, and I use the VAR innovation in the value of debt, $\tilde{v}_t = \Delta E_t(v_t) = v_t - E_{t-1}(v_t)$.
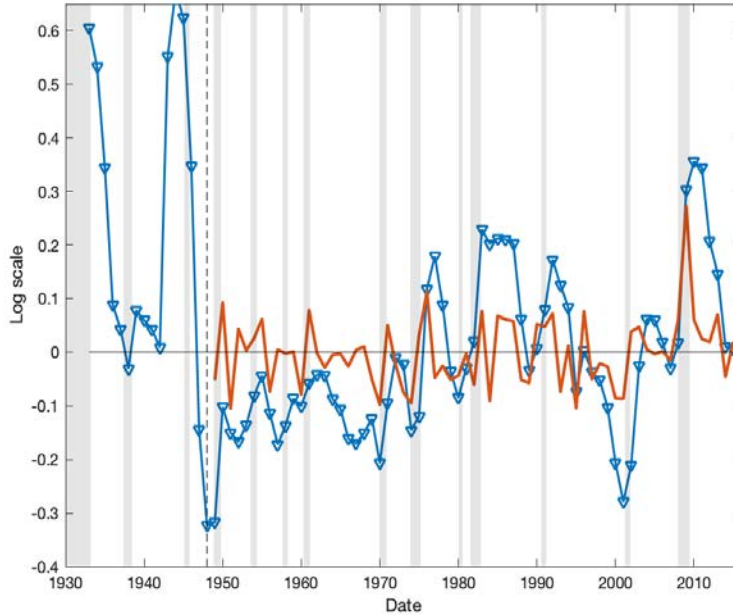

Figure 8: Filtered value of debt v_t^f (symbols) and the VAR innovation in the value of debt, $v_t - E_{t-1}(v_t)$. The filter is $v_t - \frac{1}{3}(v_{t-1} + v_{t-2} + v_{t-3})$. The vertical dashed line denotes the beginning of the sample used for VAR and statistical analysis. Shaded areas are NBER recessions.

Figure 8 presents the filtered value of debt and innovation to the value of debt. Both measures pick up familiar cyclical movements, and de-emphasize the large variation of long-term trends. For example, you can see the big increase in debt following the 2008 financial crisis, the

decrease of the 1990s, the buildup in the early 1980s, and variation in debt through the recessions of the 1970s.

	$var(\cdot)/var(v)$		
Component	Plain	Innovation	Filtered
$var(v)$	1.00	1.00	1.00
$cov(v, \Sigma s)$	0.54	0.18	0.77
$-cov(v, \Sigma(r-g))$	0.46	0.82	0.23
$cov(v, \Sigma r^n)$	-0.82	-1.58	-0.13
$cov(v, \Sigma \pi)$	-0.30	-0.65	0.08
$cov(v, \Sigma(r^n - \pi))$	-0.52	-0.93	-0.21
$cov(v, \Sigma g)$	-0.06	-0.11	0.02
$\sigma(v)$	0.36	0.07	0.15

Table 5: Decomposition of the variance of the value of debt.

Table 5 presents the decompositions of the variance of each of these measures of the value of debt $\tilde{v}_t$. The table entries are fractions of the variance of each measure of debt.

The last row gives the standard deviation of each measure of the value of debt. "Plain," the standard deviation of the log market value of debt to GDP v_t is 0.36 – the debt to GDP ratio varies by 36 percentage points, a lot. The variance of the debt innovation and filtered debt are lower, as they eliminate the large low-frequency variation, leaving standard deviations of 7 (innovation) and 15 (filtered) percentage points respectively.

The second and third rows give the variance decompositions. Looking down the first column,

- *Half (54%) of the variance of debt $var(v_t)$ does in fact correspond to forecasts of future surpluses. Half (46%) comes from discount rate variation. Practically all of this discount rate variation comes from real returns, and none from variation in forecasted growth.*

The point of taking the average, of a variance decomposition, is to avoid ex-post luck. One particular debt episode may be resolved by poor ex-post returns (i.e. default or devaluation via inflation) or unpredictably large growth, but investors may not buy debt ex-ante anticipating such an event. So, if the sample is long enough, we obtain estimates of rationally expected discount rate variation rather than ex-post luck.

That growth g is not important, on average here, illustrates the important difference between this variance accounting and the previous ex-post history. As above, much of the WWII debt was repaid, ex-post, by a lot of growth. However, the subsequent debt episodes did not

resolve by a lot of ex-post growth, and the VAR does not find the value of the debt a particularly powerful forecaster of growth, so on expected growth is not an important driver of the value of debt from an ex-ante point of view. The postwar growth was, by this measure, unexpected, which is reasonable given pervasive fear at the time of a return to depression, and that the world had never before seen three decades of growth such as occurred following WWII.

The result that forecasted variation in primary surpluses contributes as much as half of the variation of the value of debt stands in contrast to the usual result in asset pricing that discount rates account for all variation in valuation ratios. One big difference is important however: Here I account for variation in the *total market value* of debt, including (and dominated by) issuances and redemptions. The standard asset pricing calculation accounts for the variation of the value of a particular security, how the value of one government bond varies over time, or how the value of one dollar invested in the government bond portfolio varies over time. The variation in individual bond nominal values is 100% due to variation in nominal discount rates and none to variation in nominal cash flows, since the US has not defaulted in this sample. The variation of the real value of individual bonds is still dominated by real interest rate variation and not by unexpected real values of cash flows, eroded by inflation. These characterizations remain. Variation in total value is a different object than variation in the value of one security, so there is no necessary puzzle reconciling these results with the usual asset pricing results.

Turning to the "Innovation" and "Filtered" columns of Table 5, we see small but important differences.

- *The filtered value of debt corresponds even more, 77%, to future surpluses, and 23% to discount rates. Debt innovations are much less, 18%, due to future surpluses and somewhat more, 82%, due to discount rates.*

The latter comparison makes sense, as at high frequencies revaluations due to changes in bond prices should make a larger difference to values than surpluses and deficits. This view is reinforced in the breakdown, showing the very large 158% contribution of expected nominal return variation. The former comparison tells us that recession-related, and somewhat longer (Vietnam, 1980s deficits, 1990s surpluses) variation in debt is driven by expected surpluses, even more than the secular movement in debts. Time-varying expected returns in finance have very long horizons, so that is perhaps not a surprising finding.

5.   Standard errors

I have delayed a discussion of standard errors because there is nothing important to test. Identities are identities. If $x = y + z$ and x moves, y or z must move, and all we can do is to measure which one. Testing addition is not interesting. In addition, no important economic hypothesis here rests on whether it is y or z, surpluses or discount rates, that move. (In asset pricing, whether discount rates or cashflows account for variation in values is a much more meaningful hypothesis.) Standard errors only give us a sense of how accurate the measurement is.

To evaluate sampling distributions I run a Monte Carlo. I simulate the errors of the VAR, and repeat the analysis 50,000 times. Most of the interesting statistics – variance decompositions, impulse response functions, $(I - A)^{-1}$, etc. – are nonlinear functions of the underlying data, and the near-unit root in the evolution of value also induces non-normal distributions. For these reasons, I largely characterize the sampling distribution by the interquartile range – the 25% and 75% points of the sampling distribution. This roughly corresponds to one-standard error bands.
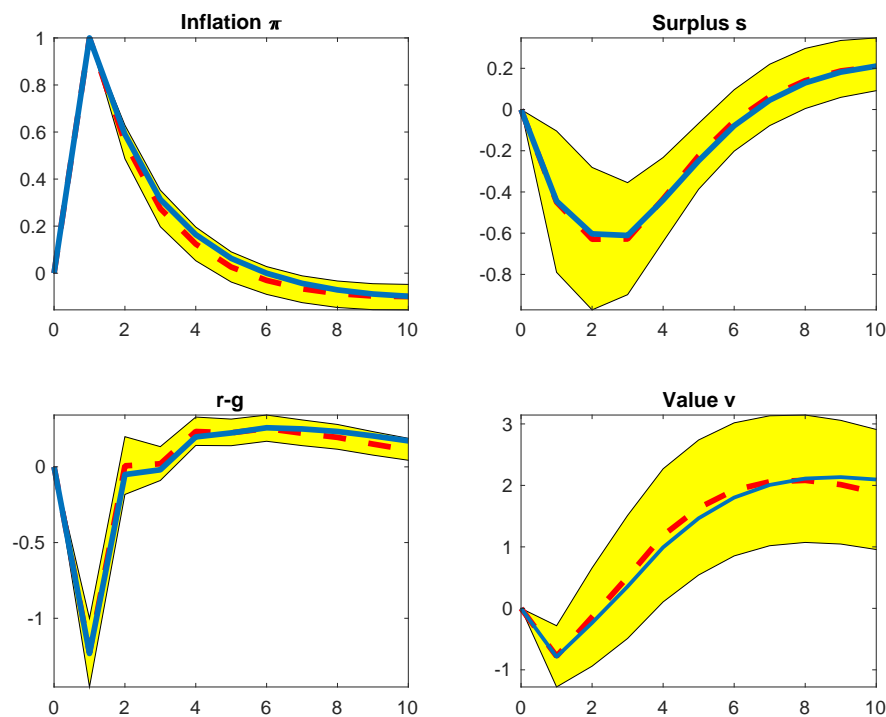

Figure 9: Distribution of the impulse response function, to an inflation shock. The bands are 25% and 75% points of the sampling distribution, the dashed line is the median, and the solid line is the estimate.

Table 6 collects the sampling quantiles for the variance decomposition of Table 2. Fig-

Component	Fraction Estimate	25%	75%	No b 25%	No b 75%	No A 25%	No A 75%
Inflation π_1	1.00						
Bond return $-(r_1^n - g_1)$	0.23	0.00	0.45	0.23	0.23	0.00	0.45
- Future Σs	0.04	-0.26	0.67	-0.16	0.57	-0.26	0.67
Future $\Sigma r - g$	1.20	0.43	1.62	0.66	1.39	0.43	1.62

Table 6: Decomposition of unexpected inflation variance – distribiution quantiles. No b holds the initial response constant across trials. No A holds the VAR regression coefficients constant across trials

	r^n	g	π	s	v	i	y
	Regression of other shocks on inflation shock						
Coefficient	-0.55	-0.32	1.00	-0.44	-0.79	0.23	0.23
Std. err.	(0.24)	(0.18)	(0.00)	(0.51)	(0.74)	(0.15)	(0.09)
	Correlation matrix of VAR shocks						
r^n	1.00	-0.23	-0.29	-0.32	0.66	-0.74	-0.93
g	-0.23	1.00	-0.23	0.45	-0.59	0.42	0.19
π	-0.29	-0.23	1.00	-0.11	-0.14	0.21	0.31
s	-0.32	0.45	-0.11	1.00	-0.89	0.44	0.31
v	0.66	-0.59	-0.14	-0.89	1.00	-0.69	-0.63
i	-0.74	0.42	0.21	0.44	-0.69	1.00	0.75
y	-0.93	0.19	0.31	0.31	-0.63	0.75	1.00

Table 7: Regression of other shocks on inflation shock, and correlation matrix of VAR shocks

ure 9 presents the main components of the impulse-response function relevant to the inflation variance decomposition. The bands are 25% and 75% points of the sampling distribution, the dashed line is the median, and the solid line is the estimate.

As shown in Table 6, the 0.04 future surplus contribution to the inflation decomposition has quartiles of -0.26 to 0.67. The 1.20 future return contribution has even larger quartiles of 0.43 and 1.62. Even the instantaneous return contribution of 0.23 has quartiles of 0.00 to 0.45. That discount rates matter is a pretty solid conclusion, but surpluses may matter more than the point estimate suggests.

There are several sources of this rather large sampling variation. First, the shocks are large. As shown in Table 1, the surplus innovation ε_t^s has a 4.49 percentage point standard deviation, and value 6.51 percentage points, compared to 1.15 for inflation. Our friend $\sigma/\sqrt{T}$ starts off badly.

	Recession		Monetary		Fiscal	
Component	25%	75%	25%	75%	25%	75%
Inflation π_1	-1.00	-1.00	0.88	1.54	0.13	0.41
Bond return $(r_1^n - g_1)$	1.91	2.33	-1.68	-0.82	-0.40	0.18
Future $\sum s$	-1.48	0.33	-1.16	2.38	-1.00	-1.00
Future $\sum(r - g)$	-4.70	-2.69	0.00	0.00	-0.91	-0.31

Table 8: Inflation identity quantiles for monetary, fiscal and recession shocks

Second, the shocks are poorly correlated. This matters, because in each case I find movements in other variables contemporaneous with the shock of interest by running a regression of the other shocks on the shock of interest. The sampling uncertainty of this orthogonalization adds to that of the VAR. Table 7 includes the regression of other shocks on inflation shock that starts off the main inflation decomposition, and thus determines the instantaneous response in Figures 2 and 9. The table also includes the correlation matrix of the shocks. The instantaneous response of the surplus to an inflation shock is clearly important to the calculation. Yet its value, -0.44, is measured with a 0.51 standard error, and the corresponding correlation is only -0.11. We see a correspondingly wide band around the initial surplus response in Figure 9.

There is hope in this observation, however. Higher frequency data can better identify shock correlations, at the cost that one must model the strong seasonal in primary surpluses. Moreover, other shock identifications may have better measured correlations.

Third, we are after the sum of future surpluses. The value of the debt is the main state variable, and uncertainty about its evolution adds to the uncertainty about the sum of surpluses. The coefficient of value v_t on its own lag is 0.98 in Table 1, so small variations in that value lead to large variation in $(I - A)^{-1}$ sums.

To measure the contribution of the latter two sources of variation, Table 6 includes two other sampling calculations. The "no b" columns resample data using the original regression of shocks ε_{t+1}^z on inflation shocks ε_{t+1}^π, the top row of Table 7, in each sample. The VAR coefficients still vary across samples, but the identification of the inflation shock does not. The "no A" columns likewise keep constant the VAR regression coefficients, but reestimate the shock regression in each sample. Turning off either source of sampling variation reduces that variation, but not as much as you might think. Sampling variation is still large in either case, and variances add, not standard deviations. Moreover the sampling variation associated with shock orthogonalization – the "no A" exercise – does not go away no matter how small the shocks. Both left and right hand sides of the shock on shock regressions get smaller at the same rate.

Table 8 presents quantiles of the inflation decomposition for recession, monetary, and fiscal shocks of Table 4. The -1.37 surplus response to a recession shock has large quantiles -1.48 to 0.33, spanning zero, while the -4.50 discount rate contribution has quantiles -4.70 to -2.69. The conclusion that discount rate variation is a central part of the story is supported, despite its large sampling error. Similarly, the positive inflation effect and strong discount rate effects of monetary policy shocks are well away from zero, as is the positive inflation effect and discount rate effect of the fiscal shock. The quantiles reveal asymmetric sampling distributions. In many cases the point estimate is well to the edge of the 25%-75% quantiles. The 1.94 inflation effect of monetary policy is even outside the 0.88-1.54 interquartile range.

Overall, the inflation decomposition is not well measured in this data set. However, remember that the terms are related by an identity that sums to one. Though we often cannot reject zero for each item individually, the hypothesis that all of the elements are zero is nonsense, and easily rejected. The cross-correlation of the individual estimates would quickly reveal that fact if one were to compute such a test. The uncertainty is over which component accounts for inflation, not whether the three components together do so.

6. Concluding comments

This analysis evidently just scratches the surface. Different definitions of inflation, and a parallel investigation of exchange rates beckons, following Jiang (2019a), Jiang (2019b). Quarterly data, which offer better measurement of correlations but require modeling seasonality, are attractive. Debt data go back centuries, allowing and requiring us to think what is the same and different across different periods of history. Inflation through wars and under the gold standard may well have different fiscal foundations than in the postwar environment. A narrative counterpart, just what happened in big episodes such as the 1970s and 1980s, awaits. Different countries under different monetary and exchange rate regimes and different fiscal constraints will behave differently. One could define shocks in many additional interesting ways.

Focusing on the decomposition of inflation and output variance, and the response to specific shocks such as inflation, recession, monetary and fiscal policy, I omitted analysis of the fiscal correlates of the remaining shocks in the VAR. These are large. A shock to any other variable, orthogonal to the inflation shock, can move all of the other terms of the inflation identity (21). Such movements must offset: If a shock does not move inflation, but does move the sum of future surpluses, then it must also move the sum of future discount rates or the current bond return, in such a way that inflation does not move in (21). Similarly, in asset pricing, variables that forecast

cashflows given the price-dividend ratio must also forecast returns, and the two marginal forecasts must exactly offset. These additional effects are large. The variation in $\Delta E_{t+1} \sum_{j=0}^{\infty} s_{t+1+j}$ when other shocks move is large; the corresponding movement in the discount rate term of (21) is also large, and the two movements are strongly negatively correlated. The meaning of such orthogonalized movements in expected surpluses, matched by movements in discount rates, in response to these other shocks needs to be understood.

Perhaps most of all, linking these theory-free characterizations to explicit fiscal theory of monetary policy models, such as the models and impulse-response functions in Cochrane (2019), or at least to explicit models of discount rates and long-term debt management policies, is an obviously important step.

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