NBER WORKING PAPER SERIES

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Working Paper 25769
http://www.nber.org/papers/w25769

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2019, Revised April 2021

This paper supplants an earlier paper entitled "Origins of Stock Market Fluctuations." We are grateful to Simcha Barkai, John Y. Campbell, Andrea Eisfeldt, Valentin Haddad, Ralph Koijen, Edward Nelson, Annette Vissing-Jorgensen, and Mindy Xiaolan for helpful comments, and to seminar participants at the October 2020 NBER EF&G meeting, 2020 Women in Macro conference, the 2021 American Finance Association meetings, the January 2021 NBER Long Term Asset Management conference, the Federal Reserve Board, the Harvard University economics department, the HEC Paris Finance department, the Ohio State University Fisher College of Business, the University of California Berkeley Haas School of Business, the University of Chicago Booth School of Business, the University of Michigan Ross School of Business, and the University of Minnesota Carlson School for helpful comments.The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 25769
April 2019, Revised April 2021
JEL No. G0,G12,G17

ABSTRACT

Why do stocks rise and fall? From the beginning of 1989 to the end of 2017, $34 trillion of real equity wealth (2017:Q4 dollars) was created by the U.S. corporate sector. We estimate that 44% of this increase was attributable to a reallocation of rewards to shareholders in a decelerating economy, virtually all of which came at the expense of labor compensation. Economic growth accounted for just 25%, followed by a lower risk price (18%), and lower interest rates (14%). The period 1952 to 1988 experienced less than one third of the growth in market equity, but economic growth accounted for more than 100%of it.

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1 Introduction

Why do stocks rise and fall? Surprisingly little academic research has focused directly on this question.[1] While much of the literature has concentrated on explaining expected quarterly or annual returns, this paper takes a longer view and considers the economic forces that have driven the total value of the market over the post-war era. According to textbook economic theories, the stock market and the broader economy should share a common trend, implying that the same factors that boost economic growth are also the key to rising equity values over longer periods of time.[2] In this paper, we directly test this paradigm.

Some basic empirical facts serve to motivate the investigation. While the U.S. equity market has done exceptionally well in the post-war period, this performance has been highly uneven over time, even at long horizons. For example, real market equity of the U.S. corporate sector grew at an average rate of 7.5% per annum over the last 29 years of our sample (1989 to 2017), compared to an average of merely 1.6% over the previous 29 years (1966 to 1988). At the same time, growth in the value of what was actually produced by the corporate sector has displayed a strikingly different temporal pattern. While real corporate net value added grew at a robust average rate of 3.9% per annum from 1966 to 1988 amid anemic stock returns, it averaged much lower growth of only 2.6% from 1989 to 2017 even as the stock market was booming. This multi-decade disconnect between growth in market equity and output presents a difficult challenge to theories in which economic growth is the key long-run determinant of market returns.

One potential resolution of this puzzle is to posit that economic fundamentals such as cash flows may be relatively unimportant for the value of market equity, with discount rates driving the bulk of growth even at long horizons. In this paper we entertain an alternative hypothesis motivated by an additional set of empirical facts. Within the total pool of net value added produced by the corporate sector, only a relatively small share — averaging 12.3% in our sample — accrues to the shareholder in the form of after-tax profits. Importantly, however, this share varies widely and persistently over time, fluctuating from less than 8% to nearly 20% over our sample. This suggests that swings in the profit share are strong enough to cause large and long-lasting deviations between cash flows and output. If so, growth in market equity could diverge from economic growth for an extended period of time, even when valuations are largely driven by fundamental cash flows. Indeed, while the

[1] We refer here to the question of what determines the level of equity values, as opposed to studying determinants of the price-dividend ratio or expected returns.

[2] This tenet goes back to at least Klein and Kosobud (1961), followed by a vast literature in macroeconomic theory that presumes balanced growth among economic aggregates over long periods of time. For a more recent variant, see Farhi and Gourio (2018).

1989-2017 period lagged the 1966-1988 period in economic growth, it exhibited growth in corporate *earnings* of 5.1% per annum that far outpaced the average 1.8% earnings growth of the previous period. Behind these trends are movements in the after-tax profit share of output, which fell from 15.3% in 1966 to 8.9% in 1988, before rising again to 17.4% by the end of 2017. These shifts are in turn made possible by a reverse pattern in labor's share of corporate output, which rises from 67.0% in 1966:Q1 to 72.4% in 1988:Q4, before reverting to 67.7% by 2017:Q4.

The upshot of these trends is a widening chasm between the stock market and the broader economy. This phenomenon is displayed in Figure 1, which plots the ratio of market equity for the corporate sector to three different measures of aggregate economic activity: gross domestic product, personal consumption expenditures, and net value added of the corporate sector. Despite substantial volatility in these ratios, each is at or near a post-war high by the end of 2017. Notably, however, the ratio of market equity to after-tax profits (earnings) for the corporate sector is far below its post-war high.

Figure 1: Stock Market Ratios (Scale: 1989:Q1 = 1)

Notes: To make the units comparable, each series has been normalized to unity in 1989:Q1. The sample spans the period 1952:Q1-2018:Q2. **ME:** Corporate Sector Stock Value. **E:** Corporate Sector After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Gross Value Added of Corporate Sector - Consumption of Fixed Capital.

What role, if any, might these trends have played in the evolution of the post-war stock market? To translate these empirical facts into a quantitative decomposition of the post-war growth in market equity, we construct and estimate a model of the U.S. equity market.

Although the specification of a model necessarily imposes some structure, our approach is intended to let the data speak as much as possible. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent factors, including not only factors driving productivity and profit shares, but also independent factors driving risk premia and risk-free interest rates.

Equity in our model is priced, not by a representative household, but by a representative shareholder, akin in the data to a wealthy household or large institutional investor. The remaining agents supply labor, but play no role in asset pricing. Shareholder preferences are subject to shocks that alter their patience and appetite for risk, driving variation in both the equity risk premium and in risk-free interest rates. Our representative shareholder consumes cash flows from firms, the variation of which is driven by shocks to the total rewards generated by productive activity, but also by shocks to how those rewards are divided between shareholders and other claimants. Our model is able to account for operating leverage effects due to capital investment, implying that the cash flow share of output moves more than one-for-one with the earnings share (the *leverage effect*), and that cash flow growth is more volatile when the earnings share is low (the *leverage risk effect*).

We estimate the full dynamic model using state space methods, allowing us to precisely decompose the market's observed growth into these distinct component sources. The model is flexible enough to explain the entirety of the change in equity values over our sample and at each point in time. To capture the influence of our primitive shocks at different horizons, we model each as a mixture of multiple stochastic processes driven by low and high frequency variation. Because our log-linear model is computationally tractable, we are able to account for uncertainty in both latent states and parameters using millions of Markov Chain Monte Carlo draws. We apply and estimate our model using data on the U.S. corporate sector over the period 1952:Q1-2017:Q4.

Our main results may be summarized as follows. First, we find that neither economic growth, risk premia, nor risk-free interest rates has been the foremost driving force behind the market's sharp gains over the last several decades. Instead, the single most important contributor has been a string of *factor share* shocks that reallocated the rewards of production without affecting the size of those rewards. Our estimates imply that the realizations of these shocks persistently reallocated rewards to shareholders, to such an extent that they account for 44% of the market increase since 1989. Decomposing the components of corporate earnings reveals that the vast majority of this increase in the profit share came at the expense of labor compensation.

Second, while equity values were also boosted since 1989 by persistent declines in the market price of risk, and in the real risk-free rate, these factors played smaller roles quantitatively, contributing 18% and 14%, respectively, to the increase in the stock market over this period.

Third, growth in the real value of corporate sector output contributed just 25% to the increase in equity values since 1989 and 54% over the full sample. By contrast, while economic growth accounted for more than 100% of the rise in equity values from 1952 to 1988, this 37 year period created less than a third of the growth in equity wealth generated over the 29 years from 1989 to the end of 2017.

Fourth, the considerable gains to holding equity over the post-war period can be in large part attributed to an unpredictable sequence of shocks, largely factor share shocks that reallocated rewards to shareholders. We estimate that roughly 2.1 percentage points of the post-war average annual log return on equity in excess of a short-term interest rate is attributable to this string of favorable shocks, rather than to genuine ex-ante compensation for bearing risk. These results imply that the common practice of averaging return, dividend, or payout data over the post-war sample to estimate an equity risk premium is likely to overstate the true risk premium by 43%.

Fifth, our model produces estimate of the conditional equity risk premium over time — a central input for theories of intangible capital and other macro-finance trends.[3] Our estimate is capable of simultaneously accounting for both the high frequency variation in the equity premium implied by options data (Martin (2017)), as well as the low frequency variation suggested by fluctuations in stock market valuation ratios. With the exception of an extreme spike upward during the financial crisis, we find that the equity premium has been declining for decades. By the end of 2017, our estimates imply that the equity premium had reached historic lows attained previously only two times: at the culminations of the tech boom in 2000 and the twin housing/equity booms in 2006.

Related Literature. The empirical asset pricing literature has traditionally focused on explaining stock market expected returns, typically measured over monthly, quarterly or annual horizons.[4] But as noted in Summers (1985), and still true today, surprisingly little attention has been given to understanding what drives the real level of the stock market over time. Previous studies have noted an apparent disconnect between economic growth and the rate of return on stocks over long periods of time, both domestically and internationally (see e.g., Estrada (2012); Ritter (2012); Siegel (2014)). But they have not provided a model and

[3] See e.g., Crouzet and Eberly (2020); Farhi and Gourio (2018)

[4] A body of research has addressed the question of whether expected returns or expected dividend growth drive valuation ratios, e.g., the price-dividend ratio, but this analysis is silent on the the primitive economic shocks that drive expected returns or dividend growth. For reviews of empirical asset pricing literature, see Campbell, Lo, and MacKinlay (1997), Cochrane (2005), and Ludvigson (2012).

evidence on the economic foundations of this disconnect or on the alternative forces that have driven the market in post-war U.S. data, a gap our study is intended to fill.[5]

In this regard, the two papers closest to this one are Lettau and Ludvigson (2013) and our previous work entitled "Origins of Stock Market Fluctuations," (Greenwald, Lettau, and Ludvigson (2014), hereafter GLL), which this paper supplants. Lettau and Ludvigson (2013) was a purely empirical exercise that showed under a natural rotation scheme, shocks from a VAR that push labor income and asset prices in opposite directions explain much of the long-term trend in stock wealth. GLL expanded on this analysis by demonstrating that a calibrated model could reproduce many of these VAR results. At the same time, neither paper undertook a complete structural estimation of an equity pricing model, and thus could not directly decompose movements in market valuations into fundamental structural forces. Compared to GLL, the model in this paper is both richer and more flexible in terms of its state variables and its cash flow process, is directly estimated on the time series rather than calibrated, and produces a period-by-period accounting of the drivers of market equity.[6]

Like GLL and Lettau, Ludvigson, and Ma (2018), the model of this paper adopts a heterogeneous agent perspective characterized by "shareholders," who hold the economy's financial wealth and consume capital income, and "workers" who finance consumption out of wages and salaries. This choice is motivated by the empirical observation: the top 5% of the stock wealth distribution owns 76% of the stock market value (and earns a relatively small fraction of income from labor compensation), while around half of households have no direct or indirect ownership of stocks at all.[7] In this sense our model relates to a classic older literature emphasizing the importance for stock pricing of limited stock market participation and heterogeneity.[8] We add to this literature by demonstrating the relevance of frameworks in which investors are concerned about shocks that have opposite effects on labor and capital.

Besides Lettau and Ludvigson (2013), GLL, and Lettau et al. (2018), a growing body of

[5]One exception is Lansing (2021), a paper subsequent to the initial draft of our work, who also estimates a model to exactly match and decompose macroeconomic and financial time series data, and emphasizes the role of sentiment.

[6]The older GLL paper solves a fully nonlinear model in place of an approximate log-linear model, demonstrating that the results in this paper are robust to allowing for these nonlinearities.

[7]Source: 2016 Survey of Consumer Finances (SCF). In the 2016 SCF, 52% of households report owning stock either directly or indirectly. Stockowners in the top 5% of the net worth distribution had a median wage-to-capital income ratio of 27%, where capital income is defined as the sum of income from dividends, capital gains, pensions, net rents, trusts, royalties, and/or sole proprietorship or farm. Even this low number likely overstates traditional worker income for this group, since the SCF and the IRS count income paid in the form of restricted stock and stock options as "wages and salaries." Executives who receive substantial sums of this form would be better categorized as "shareholders" in the model below, rather than as "workers" who own no (or very few) assets.

[8]See e.g., Mankiw (1986), Mankiw and Zeldes (1991), Constantinides and Duffie (1996), Vissing-Jorgensen (2002), Ait-Sahalia, Parker, and Yogo (2004), Guvenen (2009), and Malloy, Moskowitz, and Vissing-Jorgensen (2009).

literature considers the role of redistributive shocks in asset pricing or macro models, most in representative agent settings.[9] In addition to our main distinguishing contribution that we pursue a quantitative decomposition of the drivers of equity values over time using an estimated structural model, we differ from this literature in our treatment of equity risk and pricing. In this literature, labor compensation is a charge to claimants on the firm and therefore a source of cash-flow variation in stock and bond markets, but typically imply that a variant of the consumption CAPM using aggregate consumption still prices equity returns, implying that these frameworks cannot not account for the evidence in Lettau et al. (2018) that the capital (i.e., nonlabor) share of aggregate income is a strongly priced risk factor. In contrast, our framework allows these redistributive shocks to influence not only cash flows but also the quantity of risk faced by investors.[10]

Our work is also closely related to papers studying the sources of macroeconomic and financial transitions over time. Farhi and Gourio (2018) extend a representative agent neoclassical growth model to allow for time varying risk premia, and find a large role for rising market power in the high returns to equity over the last 30 years, similar to our findings regarding the importance of the factor share shock. Corhay, Kung, and Schmid (2018) find a similar result that they likewise attribute to market power using a rich model of the firm investment margin. An appealing feature of these approaches is that they specify a structural model of production that takes a firm stand on the sources of variation in the earnings share. In contrast, our modeling and estimation approach is designed to quantify what role the earnings share has played in stock market fluctuations, without requiring us to take a stand on the structural model that may have produced those equilibrium observations. As a result, we are able to explain the full transition dynamics of the data period-by-period, while Farhi and Gourio (2018) and Corhay et al. (2018) compare their richer production models only across different steady states. We view this work as complementary, but discuss the important implications of these differing methodological approaches further below.

Our work also relates to the literature estimating log-affine SDFs in reduced form.[11] These works describe the evolution of the state variables and the SDF in purely statistical terms, for example using a freely estimated vector autoregression (VAR) for state dynamics.

[9] See e.g., Danthine and Donaldson (2002), Favilukis and Lin (2016, 2013, 2015), Gomez (2016), Marfe (2016), Farhi and Gourio (2018).

[10] The factors share element of our paper is also related to a separate macroeconomic literature that examines the long-run variation in the labor share (e.g., Karabarbounis and Neiman (2013), and the theoretical study of Lansing (2014)). The factors share findings in this paper also echo those from previous studies that use very different methodologies but find that returns to human capital are negatively correlated with those to stock market wealth (Lustig and Van Nieuwerburgh (2008); Lettau and Ludvigson (2009); Chen, Favilukis, and Ludvigson (2014))).

[11] See e.g., Ang and Piazzesi (2003), Bekaert, Engstrom, and Xing (2009), Dai and Singleton (2002), Duffie and Kan (1996), Lustig, Van Nieuwerburgh, and Verdelhan (2013).

While less statistically flexible, our work features more economic structure, using separate and mutually uncorrelated fundamental components, as well as parametric restrictions on the SDF exposures obtained from theory, such as the leverage risk effect. This structure allows a much clearer interpretation of the drivers of asset prices. For example, unlike VAR-based models, which face the difficult task of transforming reduced-form residuals into identified structural shocks, our model allows us to directly read off the contribution of each latent state. We thus complement this literature by providing economic insight on the economic sources of market fluctuations, particularly the role of factor shares.

Our study connects with a large body on work contrasting the role of expected dividend growth vs. discount rates in driving valuation ratios.[12] Our emphasis on determinants of cash flows (i.e., the earnings share) as a key driver of valuations differs from these papers, which emphasize the role of discount rates, largely because we ask a different question. While this literature finds that cash flows have little impact on the value of equity relative to *dividends*, we focus on the value of equity relative to *output*. Because shifts in the earnings share work precisely by driving a wedge between the earnings/payouts of the corporate sector and its output, we do not view any contradiction between our results.

Last, we link to the broad literature that endogenizes equity risk premia based on the consumption processes of its investors.[13] Our work adds to this literature by providing a new mechanism through the leverage risk effect. This mechanism shares a deep similarity to the habit mechanism of Campbell and Cochrane (1999), but is driven by variation in earnings relative to an external target (reinvestment), rather than variation in aggregate consumption relative to an external target (habit). This approach thus offers novel quantitative and empirical implications for variation in risk premia over time that differ from consumption-based mechanisms.

Overview. The rest of this paper is organized as follows. Section 2 describes the theoretical model. Section 3 presents the data. Section 4 describes our estimation procedure. Section 5 presents our findings. Section 6 considers robustness and extensions. Section 7 concludes.

2 The Model

This section presents our structural model of the equity market. Throughout this exposition, lowercase letters denote variables in logs, while bolded symbols represent vectors or matrices,

[12] See e.g., Campbell and Shiller (1989), Cochrane (2011).

[13] See e.g., Bansal and Yaron (2004), Barro (2009), Campbell and Cochrane (1999), Campbell, Pflueger, and Viceira (2014), Constantinides and Duffie (1996), Wachter (2013).

in contrast to unbolded scalars.

Demographics The economy is populated by a representative firm that produces aggregate output, and two types of households. The first type are investors who typify owners of most equity wealth in the U.S. (i.e., wealthy households or large institutional investors). They may borrow and lend among themselves in the risk-free bond market. We refer to these investors simply as "shareholders." The second type are hand-to-mouth "workers" who finance consumption out of wages and salaries. The model is stylized, as we suppose that workers own no assets and consume their labor earnings. These coarsely different groups are an obvious abstraction from the real world, but represent a reasonable first approximation of the data given the high concentration of wealth at the top, the evidence that the wealthiest earn the overwhelming majority of their income from ownership of assets or firms, and that households outside of the top 5% of the stock wealth distribution own far less financial wealth of any kind.[14]

Productive Technology Aggregate output is governed by a constant returns to scale process:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}, \tag{1}$$

where A_t is a mean zero factor neutral total factor productivity (TFP) shock, N_t is the aggregate labor endowment (hours times a productivity factor) and K_t is input of capital, respectively. Workers inelastically supply labor to produce output. We seek a solution in which capital grows deterministically at a gross rate $G = \exp(g)$, while labor productivity grows deterministically at the same rate. Hours of labor supplied are fixed and normalized to unity, so $N_t = G^t$. Taken together, these assumptions imply that

$$Y_t = A_t (G^t K_0)^{\alpha} (G^t)^{1-\alpha} = A_t G^t K_0^{\alpha} \tag{2}$$

where K_0 is the fixed initial value of the capital stock.

Factor Shares Once output is produced, it is divided among the various factors of production and other entities. We define earnings (after-tax profits) as $E_t = S_t Y_t$, where the *earnings share* S_t represents the fraction of total output that accrues to shareholders in the

[14]See discussion above. In the 2016 SCF, the median household in the top 5% of the stock wealth distribution had \$2.97 million in nonstock financial wealth. By comparison, households with no equity holdings had median nonstock financial wealth of \$1,800, while all households (including equity owners) in the bottom 95% of the stock wealth distribution had median nonstock financial wealth of \$17,480. Additional evidence is presented in Lettau, Ludvigson, and Ma (2019).

form of earnings, arising from both foreign and domestic operations. The remaining fraction $1 - S_t$ of output accrues to workers in the form of labor compensation, to the government in the form of tax payments, and to debtholders in the form of interest payments. In our estimation, we assume an exogenous process for S_t that does not directly distinguish between shifts in these components, but provide details on how we map these variables to the data in Section 3, return to analyze their separate roles in Section 5.5. For now, we note that most variation in S_t is driven by movements in the labor share of domestic value added.

Investment and Payout Technology. The firm makes cash payments to shareholders, equal to earnings net of new investment. We assume that attaining balanced growth in capital requires the firm to invest a fixed fraction ω of its output beyond replacing depreciated capital. We view this as a parsimonious approximation to a richer model with time-varying investment, which allows us to solve the model in closed form without tracking the capital stock or solving the optimal investment problem.[15] Cash flows to shareholders consist of the remaining portion of earnings net of this reinvestment:

$$C_t = E_t - \omega Y_t = (S_t - \omega) Y_t. \tag{3}$$

The variable C_t is net payout, defined as net dividend payments minus net equity issuance. It encompasses any cash distribution to shareholders including share repurchases, which have become the dominant means of returning cash to shareholders in the U.S. For brevity, we shall refer to these payments simply as "cash flows."

Importantly, (3) implies that, since the cash flow share of output is equal to the earnings share minus a constant reinvestment share, the volatility of cash flow growth is amplified relative to earnings share growth — a form of operating leverage. For a numerical example, if $\omega = 6\%$, then an increase in the earnings share S_t from 12% to 18% increases the cash flow share from 6% to 12%. As a result, proportional growth in the cash flow share is twice as large as proportional growth in the earnings share, a phenomenon that we call the *leverage effect*. We note that this leverage effect should hold on average even if the reinvestment share is not exactly constant, under the natural assumption that in the long run investment is proportional to output rather than earnings. We present further support for this mechanism in Section 5.7.

[15] Jermann (1998) demonstrates that generating realistic asset pricing moments in production economies requires very large investment adjustment costs. Our investment process can be seen as a limiting case in which any deviation from this investment plan is infinitely costly.

Preferences. Let C_{it}^s denote the consumption of an individual stockholder indexed by i at time t. Identical shareholders maximize the function

$$U_0 = \mathbb{E} \sum_{t=0}^{\infty} \prod_{k=0}^{t} \beta_k u\left(C_{it}^s\right), \qquad u\left(C_{it}^s\right) = \frac{\left(C_{it}^s\right)^{1-x_{t-1}}}{1 - x_{t-1}} \tag{4}$$

where $\mathbb{E}$ denotes the expectation operator. This specification effectively corresponds to power utility preferences with a time-varying price of risk x_t, and a time-varying time discount factor β_t. Since shareholders perfectly insure idiosyncratic risk, shareholder consumption C_{it} is identically equal to aggregate cash flows C_t.[16] At the same time, because firm cash flows are only a subset of total economy-wide consumption, redistributive shocks to s_t that shift the share of income between labor and capital shift shareholder consumption are a source of systematic risk for asset owners. This implication has been explored by Lettau et al. (2019) who study risk pricing in a large number of cross-sections of return premia.

Aggregating over shareholders, equities are priced by the stochastic discount factor of a representative shareholder, taking the form

$$M_{t+1} = \beta_t \left(\frac{C_{t+1}}{C_t}\right)^{-x_t} \tag{5}$$

This specification is a generalization of the SDFs considered in previous work, (e.g., Campbell and Cochrane (1999) and Lettau and Wachter (2007)). As in these models, the preference shifters (x_t, β_t) are taken as exogenous processes (akin to an external habit) that are the same for each shareholder. We now discuss each of these items in turn.

Beginning with the risk price x_t, we allow this variable to fluctuate stochastically over time. Since an SDF always reflects both preferences and beliefs, an increase in x_t may be thought of as either an increase in effective risk aversion or an increase in pessimism about shareholder consumption. Thus, x_t may occasionally go negative, reflecting the possibility that investors sometimes behave in a confident or risk tolerant manner.[17]

Shareholder preferences are also subject to an exogenous shifts in the subjective discount factor β_t. As is well known, applying a realistic level of variation in the risk price while holding the time discount factor fixed would generate counterfactually high volatility in the

[16]This need not imply that individual shareholders are hand-to-mouth households. They may trade an arbitrary set of assets with each other, including a complete set of state contingent contracts. Because they perfectly share any identical idiosyncratic risk with other shareholders they each consume per capita aggregate shareholder cash flows C_t at equilibrium. See the Appendix for a stylized model.

[17]This does not imply a negative unconditional equity risk premium. Investors in the model occasionally behave in a risk tolerant manner while still being averse to risk on average. Indeed, our estimates reported below imply a substantial positive mean equity premium.

risk-free rate. Instead, following Ang and Piazzesi (2003), we specify β_t as

$$\beta_t = \frac{\exp(-\delta_t)}{\mathbb{E}_t \exp\left(-x_t \Delta c_{t+1}\right)}.$$

where Δc_{t+1} represents log cash flow growth. This specification implies

$$\mathbb{E}_t M_{t+1} = \exp(-\delta_t) \tag{6}$$

ensuring that the log risk-free rate exactly follows an exogenous stochastic process δ_t at all times, regardless of the values for the other state variables of the economy.

2.1 Model Solution and Parameterization

Exogenous Processes. Our model has four sets of exogenous processes that drive a_t, s_t, x_t, and δ_t, respectively. We specify TFP as a random walk in logs

$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \overset{\text{iid}}{\sim} N(0, \sigma_a^2).$$

Substituting into (2), we obtain

$$\Delta y_{t+1} = g + \varepsilon_{a,t+1}.$$

For the remaining latent states, we specify each as the sum of two components, each of which are in turn specified as an independent AR(1) process:

$$\begin{aligned}
s_t &= \bar{s} + \mathbf{1}'\tilde{\mathbf{s}}_t, & \tilde{\mathbf{s}}_{t+1} &= \mathbf{\Phi}_s \tilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, & \boldsymbol{\varepsilon}_{s,t+1} &\overset{\text{iid}}{\sim} N(0, \mathbf{\Sigma}_s), \\
x_t &= \bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t, & \tilde{\mathbf{x}}_{t+1} &= \mathbf{\Phi}_x \tilde{\mathbf{x}}_t + \boldsymbol{\varepsilon}_{x,t+1}, & \boldsymbol{\varepsilon}_{x,t+1} &\overset{\text{iid}}{\sim} N(0, \mathbf{\Sigma}_x), \\
\delta_t &= \bar{\delta} + \mathbf{1}'\tilde{\boldsymbol{\delta}}_t, & \tilde{\boldsymbol{\delta}}_{t+1} &= \mathbf{\Phi}_\delta \tilde{\boldsymbol{\delta}}_t + \boldsymbol{\varepsilon}_{\delta,t+1}, & \boldsymbol{\varepsilon}_{\delta,t+1} &\overset{\text{iid}}{\sim} N(0, \mathbf{\Sigma}_\delta)
\end{aligned}$$

where $\tilde{\mathbf{s}}_t, \tilde{\mathbf{x}}_t$, and $\tilde{\boldsymbol{\delta}}_t$ are 2×1 vectors, $\mathbf{\Phi}_s$, $\mathbf{\Phi}_x$, and $\mathbf{\Phi}_\delta$ are 2×2 diagonal matrices, and tildes indicate that the latent state vectors are demeaned.

We choose this two-component mixture specification for each process to allow the model to flexibly capture both high and low frequency variation in the latent states. Since equity gives its owners access to profits for the lifetime of the firm, it is a heavily forward looking asset that is much more influenced by persistent rather than transitory fluctuations. As a result, our specification allows the model to accurately capture both low frequency movements that have greater impact on equity prices, as well as higher frequency movements that have a smaller impact on equity prices but may nonetheless drive much of the variation in the observable series. Correspondingly, we refer to the components of each latent state vector as the high or low frequency component, so that e.g., $\text{diag}(\mathbf{\Phi_s}) = (\phi_{s,LF}, \phi_{s,HF})$ with $\phi_{s,LF} > \phi_{s,HF}$.

Stacking this system yields the transition equation

$$\mathbf{z}_{t+1} = \mathbf{\Phi}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}, \qquad \boldsymbol{\varepsilon} \overset{\text{iid}}{\sim} N(0, \mathbf{\Sigma}) \tag{7}$$

for

$$\mathbf{z}_t = \begin{bmatrix} \tilde{\mathbf{s}}_t \\ \tilde{\mathbf{x}}_t \\ \tilde{\boldsymbol{\delta}}_t \\ \Delta a_t \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_s & 0 & 0 & 0 \\ 0 & \mathbf{\Phi}_x & 0 & 0 \\ 0 & 0 & \mathbf{\Phi}_\delta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \boldsymbol{\varepsilon}_{s,t} \\ \boldsymbol{\varepsilon}_{x,t} \\ \boldsymbol{\varepsilon}_{\delta,t} \\ \varepsilon_{a,t} \end{bmatrix}, \quad \mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_s & 0 & 0 & 0 \\ 0 & \mathbf{\Sigma}_x & 0 & 0 \\ 0 & 0 & \mathbf{\Sigma}_\delta & 0 \\ 0 & 0 & 0 & \sigma_a^2 \end{bmatrix}, \tag{8}$$

where $\mathbf{z}_t$ is the state vector for this economy.[18]

Log-Linearization. We seek a specification that allows an analytical, log-linear solution for the price-dividend ratio. This solution requires three approximations: (i) a log-linear approximation of the equity return, (ii) a log-linear approximation of cash flow growth, and (iii) a second-order perturbation of the log SDF that allows for linear terms in the states and shocks, as well as interactions between the states and shocks. We summarize these approximations below, with full detail relegated to the appendix.

First, we approximate the return on equity

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$

where P_t denotes total market equity, i.e., price per share times shares outstanding. Following Campbell and Shiller (1989), we approximate the log return as

$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{9}$$

where $\kappa_1 = \exp\left(\overline{pc}\right) / \left(1 + \exp\left(\overline{pc}\right)\right)$, and $\kappa_0 = \ln(\exp\left(\overline{pc}\right) + 1) - \kappa_1 \overline{pc}$.

[18]The i.i.d. shock $\varepsilon_{a,t}$ is included the "state equation" (7) even though it is exactly pinned down by the observable series Δy_t so that we can estimate its mean and variance, since these parameters influence our asset pricing equations.

Second, we log-linearize the log cash flow to output ratio $c_t - y_t = \log(S_t - \omega)$ to obtain

$$c_t - y_t \simeq \overline{cy} + \xi(s_t - \bar{s}), \qquad \xi = \frac{\bar{S}}{\bar{S} - \omega}$$

where $\overline{cy} = \log(\bar{S} - \omega)$, and $\bar{S}$ is the average value of S_t. Differencing this relation and rearranging yields

$$\Delta c_t = \xi \Delta s_t + \Delta y_t. \tag{10}$$

Importantly, for $\omega > 0$ we have $\xi > 1$, so that changes in profit share map more than one-for-one into cash flows, preserving the leverage effect discussed above.

Finally, approximate our nonlinear SDF (5) using a perturbation around the steady state that includes terms linear in $\mathbf{z}_t$ and $\boldsymbol{\varepsilon}_{t+1}$, as well as interactions between $\mathbf{z}_t$ and $\boldsymbol{\varepsilon}_{t+1}$. While the complete solution and derivation can be found in the appendix, we present here the more intuitive form

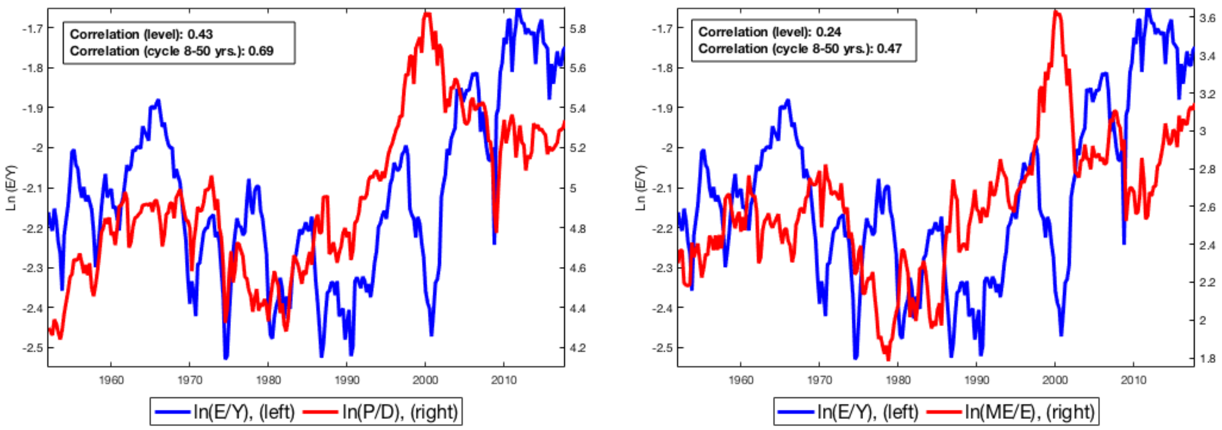
$$\log M_{t+1} \simeq -\delta_t - \mu_t - \underbrace{x_t \left(\xi \Delta s_{t+1} + \Delta y_{t+1}\right)}_{\text{baseline cash flow risk}} + \underbrace{\bar{x}\xi(\xi - 1)\left(\mathbb{E}_t[s_{t+1}] - \bar{s}\right)\Delta s_{t+1}}_{\text{leverage risk effect}} \tag{11}$$

where μ_t is implicitly set to ensure a log risk-free rate of δ_t. The "baseline cash flow risk" term represents the price of risk x_t times the change in cash flows under the approximation (10). The final *leverage risk effect* term is a second-order interaction, representing the fact that the same shock to the earnings share $\varepsilon_{s,t+1}$ has a larger proportional impact on cash flows when the current (and thus expected) earnings share is low. For example, if we again assume $\omega = 6\%$, then increasing the earnings share from 8% to 10% would increase the cash flow share of output by 100% (from 2% to 4%), while the same proportional increase in the earnings share from 16% to 20% would only increase the cash flow share of output by 40% (from 10% to 14%). The leverage risk effect captures this phenomenon, causing the SDF to load more negatively on changes in profit shares when the expected profit share is low. This leverage risk effect is very similar to the external habit mechanism of Campbell and Cochrane (1999), but applied to earnings in place of consumption.

The specification (11) implies that changes in the profit share influence market valuations by affecting both cash flows and risk premia. We view this risk premium channel as strongly supported by the data. Figure 2 displays the time-series variation in the corporate sector log earnings share of output, $e_t - y_t$, alongside either the corporate sector log price earnings ratio $p_t - e_t$, or the Center for Research in Securities Prices (CRSP) log price-dividend ratio $p_t - d_t$, showing that these variables are positively correlated, particularly at lower frequencies. For example, the correlation between $e_t - y_t$ and $p_t - d_t$ is 69% for band-pass filtered components of the raw series that retain fluctuations with cycles between 8 and 50 years.

Figure 2: Earnings Share and Valuations

(a) vs. Price-Dividend Ratio **(b)** vs. Price-Earnings Ratio



Notes: ln(E/Y) denotes the logarithm of the after-tax profit (earnings) share of output for the corporate sector. ln(ME/E) is the log of the market equity-to-earnings ratio. ln(PD) is the log of the CRSP price-dividend. Each plot present the correlation between the series (levels) and the correlation of the cycle of each series obtained using a band pass filter that isolates cycles between 8 and 50 years. The sample spans the period 1952:Q1-2017:Q4.

A model with no correlation between the the earnings share and the price or quantity of risk would instead unambiguously predict that $p_t - c_t$ and $p_t - e_t$ should be negatively correlated with the profit share. Because shocks to the profit share revert over time, they influence prices (discounted forward-looking cash flows) less than current cash flows. Prices in such a model will therefore rise less than proportionally with earnings in anticipation of their eventual mean reversion, thereby resulting in a *negative* correlation between the earnings share and valuation ratios — an intuition we formalize in our discussion of the equilibrium conditions below. The positive correlation observed in the data instead implies that persistently high earnings shares must coincide with a decline in expected future returns, so that valuation ratios still rise even as earnings and shareholder payouts are rationally expected to decline in the future.[19]

[19]If shocks to the earnings share improved shareholder fundamentals permanently, the model would imply that such shocks drive prices up proportionally with earnings, leaving valuation ratios unaffected and the correlation zero.

Equilibrium Stock Market Values. The first-order-condition for optimal shareholder consumption implies the following Euler equation:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right]. \tag{12}$$

The relevant state variables for the equilibrium pricing of equity are the vectors $\tilde{\mathbf{s}}_t$, $\tilde{\mathbf{x}}_t$, $\tilde{\boldsymbol{\delta}}_t$. We conjecture a solution to (12) taking the form

$$pc_t = A_0 + \mathbf{A}_s'\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x'\widetilde{\mathbf{x}}_t. \tag{13}$$

where $pc_t = \log(P_t/C_t)$ is the log price to cash-flow ratio. The solution, derived in Appendix A.3, implies that the coefficients on these state variables take the form

$$\begin{aligned}
\mathbf{A}_s' &= -\xi\Big[\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) - (\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}'\Big]\Big[(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_s) - \kappa_1\xi\boldsymbol{\Sigma}_s\mathbf{1}\boldsymbol{\Gamma}'\Big]^{-1} \\
\mathbf{A}_x' &= -\Big[\Big(\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + \sigma_a^2 + \kappa_1\xi(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1})\Big)\mathbf{1}'\Big](\mathbf{I} - \kappa_1\boldsymbol{\Phi}_x)^{-1} \\
\mathbf{A}_\delta' &= -\mathbf{1}'(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_\delta)^{-1}
\end{aligned}$$

where the term

$$\boldsymbol{\Gamma}' = \bar{x}\xi(\xi - 1)\mathbf{1}'\Phi$$

captures the influence of the leverage risk effect.

The coefficients $\mathbf{A}_x$, and $\mathbf{A}_\delta$ are all negative, while the sign of the coefficients for $\mathbf{A}_s$ depends on the value of $\boldsymbol{\Gamma}$. The signs of the coefficients $\mathbf{A}_\delta'$ and $\mathbf{A}_x'$ imply that an increase in the risk-free rate or an increase in the price of risk x_t originating from either component reduces the price-cash flow ratio because either event increases the rate at which future payouts are discounted. The size of these effects depend on the persistence of the movements in the risk-free rate and the price of risk, as captured by $\boldsymbol{\Phi}_\delta$ and $\boldsymbol{\Phi}_x$, with more persistent shocks translating into larger effects.

The signs of the elements of $\mathbf{A}_s'$ depend on the values of $\boldsymbol{\Gamma}$. As discussed above, were $\boldsymbol{\Gamma} = 0$, the elements of $\mathbf{A}_s$ would also be negative, yielding a counterfactual negative correlation between the earnings share and pc_t, with the correlation approaching zero as the persistence of the earnings share components approach unity. In contrast, when the leverage risk effect is active, we have $\boldsymbol{\Gamma} > 0$, reducing or reversing this counterfactual negative correlation.

As shown in Appendix A.3, the model's log equity premium is given by

$$\log \mathbb{E}_t \left[R_{t+1}/R_{f,t} \right] = \left(\Psi + \sigma_a^2 \right) x_t - \Psi \mathbf{\Gamma}' \tilde{\mathbf{s}}_t \tag{14}$$

where

$$\Psi = \xi(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}) + \kappa_1 \xi(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1})$$

is a measure of average earnings share risk, which arises directly through cash flows (first term), as well as the covariance of the earnings share with the *pc* ratio (second term). Equation (14) shows that the equity premium is the sum of two terms: (i) product of the price of risk and the average total cash flow risk, including both earnings share and output risk, and (ii) time variation in the risk premium due to e.g., higher earnings share risk when $\tilde{\mathbf{s}}_t$ is low, via the leverage risk effect.

3 Data

We next describe our data sources, with full details available in Appendix A.1.

Our data consist of quarterly observations spanning the period 1952:Q1 to 2017:Q4. We construct all of our data series at the level of the U.S. corporate sector. This choice stands in contrast to previous work, which has compared trends in aggregate measures of output and the labor share against trends in the value of public equity.[20] A weakness of this existing approach is that the Bureau of Economic Analysis (BEA) data on output and labor share are not limited to the publicly traded sector and cover a far broader swath of the economy. This creates the potential for confounding compositional effects over time if, for example, publicly traded firms have experienced different shifts in their profit share or productivity compared to non-public firms. Moreover, Koh, Santaeulalia-Llopis, and Zheng (2016) find that versions of the profit and labor share based on these economic aggregates are highly sensitive to how "ambiguous" income is classified as either labor or capital income, posing additional measurement challenges.

In contrast, we construct all of our series at the same level — the U.S. corporate sector — to expressly avoid these compositional effects. Our data also have the additional advantage of "unambiguously" classifying labor and capital income, using the terminology of Koh et al. (2016), thereby avoiding any need to take classify the ambiguous portion of national income, a point we return to in Section 5.5. At the same time, a consequence of our approach is that our measure of equity values include equity in non-public firms. Although the vast majority

[20]See e.g., GLL, Farhi and Gourio (2018).

of equity value is accounted for by public firms, our equity measure is broader than is used in most existing work, and may exhibit slight differences as a result.[21]

For our estimation, we use observations on six data series: the log share of domestic output accruing to earnings (the earnings share), denoted $ey_t = e_t - y_t$; a measure of a short term real interest rate as a proxy for the log risk-free rate, denoted $r_{f,t}$; a forecast of average future real risk-free rates over the next 40Q, denoted $\bar{r}^{40}_{f,t}$; growth in output for the corporate sector as measured by growth in corporate net value added, denoted Δy_t; the log market equity to output ratio for the corporate sector, denoted $py_t = p_t - y_t$; and a proxy for the market risk premium, denoted rp_t.

Our motivation behind this choice of series is as follows. The series ey_t, $r_{f,t}$ and Δy_t pins down the values of $s_t, \delta_t, \Delta a_t$ in the model at each date. The series $\bar{r}^{40}_{f,t}$ ensures that the model correctly allocates between high and low frequency components of the risk free rate to match the expected persistence of the risk-free rate, in addition to its level. The series py_t ensures that the model is able to fully explain movements in market equity in each period, while the risk premium estimate rp_t provides additional discipline for the risk price process.

Turning to the data, our earnings share measure ey_t is equal to the ratio of total corporate earnings to domestic net value added. To compute total corporate earnings, we combine corporate domestic after-tax profits from the National Income and Product Accounts (NIPA) with data on corporate foreign direct investment income from the BEA's International Transaction Accounts. The domestic after-tax profits represent domestic net value added, net of domestic labor compensation, taxes, and interest payments. Foreign earnings represent equity income from directly held foreign subsidiaries. Because the foreign income data is only available from 1982 on, we impute this series over the full sample using NIPA data on total foreign income (direct and indirect) as a proxy, which provides an extremely close fit over the overlapping period (see Appendix A.1 for details).

For our other observables, our real risk-free rate measure $r_{f,t}$ is the 3-month Treasury Bill (T-Bill) yield net of expected inflation, computed using an ARMA(1, 1) model. Our real risk-free rate forecast $\bar{r}^{40}_{f,t}$ is the mean Survey of Professional Forecasters (SPF) expectation of the annual average 3-month T-Bill return over the next ten years (BILL10) less the mean SPF expectation of annual Consumer Price Index (CPI) inflation over the same period (CPI10).[22] Our equity-output ratio py_t is computed as the ratio of the market value of corporate equity from the Flow of Funds to corporate net value added from NIPA.

[21]The Flow of Funds separately tracks public and private ("closely held") equity for the U.S. corporate sector from 1996:Q4 onward. Over this subsample, public equity makes up 83% of total corporate equity on average. The remaining 17% is a mix of 11.5% of private S-corporation equity, and 6.5% of private C-corporation equity.

[22]We thank our discussant Annette Vissing-Jorgenson for this suggestion.

Finally, for our observable measure of the risk premium (rp_t), we use the SVIX-based estimate derived by Martin (2017). This variable is computed from options data and helps the model to discipline its estimates of the risk-premium process, especially its higher frequency variation. Martin (2017) uses option data to compute a lower bound on the equity risk premium, then argues that this lower bound is in fact tight and is therefore a good measure of the true risk premium on the stock market. That paper documents that a wide range of representative agent asset pricing theories fails to explain the high frequency variation in the risk premium implied by options data, even if they are broadly consistent with the lower-frequency variation suggested by variables like the price-dividend ratio or cay_t (Lettau and Ludvigson (2001)). Since our model allows for mixture processes, the risk premium we estimate is capable of accounting for both higher- and lower-frequency components of the risk premium.

4 Estimation

The model just described consists of a vector of primitive parameters

$$\boldsymbol{\theta} = \left(\omega, g, \sigma_a^2, \mathrm{diag}\left(\boldsymbol{\Phi}_s\right)', \mathrm{diag}\left(\boldsymbol{\Phi}_x\right)', \mathrm{diag}\left(\boldsymbol{\Phi}_\delta\right)', \mathrm{diag}\left(\boldsymbol{\Sigma}_s\right)', \mathrm{diag}\left(\boldsymbol{\Sigma}_x\right)', \mathrm{diag}\left(\boldsymbol{\Sigma}_\delta\right)', \bar{s}, \bar{\delta}, \bar{x}, \right)',$$

With the exception of a small group of parameters, discussed below, the primitive parameters are freely estimated using Bayesian methods with flat priors.

To construct our state space for estimation, we relate our observed series to the model's primitive parameters and latent state variables using the following system of equations:

$$\begin{aligned}
ey_t &= \mathbf{1}'\mathbf{s}_t \\
r_{f,t} &= \mathbf{1}'\boldsymbol{\delta}_t \\
\bar{r}_{f,t}^{40} &= \frac{1}{40}\sum_{j=0}^{40-1}\mathbb{E}_t\delta_{t+j} = \bar{\delta} + \frac{1}{40}\mathbf{1}'(I-\boldsymbol{\Phi}_\delta)^{-1}(I-\boldsymbol{\Phi}_\delta^{40})\tilde{\boldsymbol{\delta}}_t + \nu \\
py_t &= pc_t + cy_t \\
&= \overline{py} + (\mathbf{A}_s' + \xi')\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x'\widetilde{\mathbf{x}}_t \\
\Delta y_t &= g + \Delta a_t \\
rp_t &= \left(\Psi + \sigma_a^2\right)\left(\bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t\right) - \Psi\boldsymbol{\Gamma}'\tilde{\mathbf{s}}_t.
\end{aligned}$$

where $cy_t = c_t - y_t$, and $\overline{py} = A_0 + \bar{c} + \boldsymbol{\xi}'\bar{\mathbf{s}}$.[23] For the real risk-free rate forecast ($\bar{r}_{f,t}^{40}$), the

[23] We note that Δa_t is exactly pinned down by the observation equation for Δy_t.

parameter ν allows for an average difference between the model and data forecasts due to the different inflation series used (CPI for the forecasts vs. the GDP deflator for the model), as well as any average bias among the forecasts.[24]

The above equations can be stacked to form an observation equation

$$\mathcal{Y}_t = \mathbf{H}_t \mathbf{z}_t + \mathbf{b}_t \tag{15}$$

for the observation vector $\mathcal{Y}_t = \left(ey_t, r_{ft}, \bar{r}^{40}_{f,t}, py_t, \Delta y_t, rp_t\right)'$. Since our model has more shocks in $\boldsymbol{\varepsilon}$ than observable series in $\mathcal{Y}_t$, the model is able to explain all of the variation in our observable series, allowing us to estimate (15) without measurement error. We note that the coefficient matrix $\mathbf{H}_t$ and vector $\mathbf{b}_t$ depend on t because some of our observable data series are not available for the full sample. In particular, the sample for the real rate forecast $\bar{r}^{40}_{f,t}$ spans 1992:Q1 - 2017:Q1, with one observation every 4Q, while the SVIX risk premium rp_t spans 1996:Q1 - 2012:Q1 quarterly, both of which are shorter than our full sample period 1952:Q1 - 2017:Q4. As a result, the state-space estimation uses different measurement equations to include these equations for these respective series when the relevant data are available, and exclude them when they are missing (see Appendix A.4 for details). Combined, (7) and (15) describe the full state space system used for estimation.

We estimate the parameters of the model as follows. Given a vector of primitive parameters θ, we construct our state space system using (7) and (15). We then use the Kalman filter to compute the log likelihood $L(\theta)$, which is equivalent to the posterior under our flat priors, up to a restriction that ensures the correct ordering of our low and high frequency processes.[25] To sample draws of θ from the parameter space Θ we use a random walk Metropolis-Hastings (RWMH) algorithm. We generate ten independent chains, each containing 550,000 draws of $\boldsymbol{\theta}$. We discard the first 50,000 draws from each chain as burn-in, leaving 5,000,000 parameter draws. Since these draws are highly serially correlated, we increase computational efficiency in most applications by using every 500th draw, leaving a total of 10,000 draws over which our margins of parameter uncertainty are computed.

Given our parameter draws, we employ the simulation smoother of Durbin and Koopman (2002) to compute one draw of the latent states $\{\mathbf{z}^j_t\}$ for $t = 1, \ldots, T$ for each parameter draw $\{\theta^j\}$, yielding a distribution of latent state paths that can characterize our model's uncertainty over its latent state estimates. Given our lack of measurement error, each of these

[24] A full derivation of the formula for $\bar{r}^{40}_{f,t}$ can be found in Appendix A.3.3.

[25] The latent state space includes components that differ according to their degree of persistence. With flat priors, a penalty to the likelihood is required to ensure that the low frequency component has greater persistence than the higher frequency component. This is accomplished using a prior density that is equal to zero if $\phi_{(\cdot),LF} \leq \phi_{(\cdot),HF}$ for any relevant component of the state vector, and is constant elsewhere.

latent state paths are able to perfectly match the growth in market equity Δp_t over time and at each point in time, a property we exploit when calculating the growth decompositions discussed below.

Calibrated Parameters There are four parameters that are calibrated rather than estimated. The first three are the average growth rate of net value added g, the average log profit share $\bar{s}$, and the average real risk-free rate $\bar{\delta}$. Since these represent the means of our observable series, we take the conservative approach of fixing them equal to their sample means. We do this to avoid a potential estimation concern: because some of our series are very persistent, the estimation might otherwise have a wide degree of freedom in setting steady state values that are far from the observed sample means. At quarterly frequency, we obtain the values $g = 0.552\%$, $\bar{s} = -2.120$ (corresponding to a share in levels of 12.01%), and $\bar{\delta} = 0.281\%$.

The final calibrated parameter is $\xi = \frac{\overline{S}}{\overline{S}-\omega}$, which relates payout growth to earnings growth according to (10), and can also be pinned down directly by sample means. Since $C_t = (S_t - \omega)Y_t$, we can rearrange and take sample averages of both sides to obtain $\omega = \bar{S} - \overline{\frac{C}{Y}}$. Computing $\overline{S}$ as the mean of the total profit to domestic output ratio and $\overline{\frac{C}{Y}}$ as the mean of the payout to output ratio observed in the data yields the value $\xi = 2.002$. Appendix A.6 shows that this value of ξ yields an implied cash flow series that tracks corporate payouts very well at the low frequencies that are critical for moving asset valuations (see Figure A.4), while Section 5.7 confirms that the our implied series yields average growth and volatility of payouts that are close to, but do not exaggerate, those observed in the data.

A final comment on the data used for estimation is in order. Other than indirectly in the above calibration, we do not use NIPA payout data in estimation, for two reasons. First, payouts are a function of current and future earnings as well as transitory factors that affect the timing with which they are paid out, subject to an intertemporal budget constraint. These two sources of variation have very different implications for future payouts, and hence the value of market equity. Variation that is driven purely by the timing of payouts adds noise, rather than signal, for forecasting future payouts. This problem can be severe when estimating a model of equity pricing, since observed variation in the timing of payouts is large and subject to extreme swings due to temporary factors such as changes in tax law that are likely unrelated to economic fundamentals.[26] As a result, any serious use of these data

[26] For a recent example, see NIPA Table 4.1, which shows an unusually large increase in 2018:Q1 in net dividends received from the rest of the world by domestic businesses, which generated a very large decline in net payout. BEA has indicated that these unusual transactions reflect the effect of changes in the U.S. tax law attributable to the Tax Cut and Jobs Act of 2017 that eliminated taxes for U.S. multinationals on repatriated profits from their affiliates abroad.

Table 1: Parameter Estimates

Variable	Symbol	Median	5%	95%	Mode
Risk Price Mean	$\bar{x}$	6.0466	4.7683	7.6212	5.8676
Risk Price (HF) Pers.	$\phi_{x,HF}$	0.6943	0.5451	0.7986	0.6781
Risk Price (HF) Vol.	$\sigma_{x,HF}$	2.1854	1.6106	2.9669	2.1724
Risk Price (LF) Pers.	$\phi_{x,LF}$	0.9882	0.9809	0.9946	0.9886
Risk Price (LF) Vol.	$\sigma_{x,LF}$	0.6232	0.3736	0.9823	0.6295
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.8473	0.7704	0.8938	0.8413
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0017	0.0015	0.0019	0.0017
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9639	0.9514	0.9790	0.9630
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0010	0.0007	0.0013	0.0010
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.8846	0.8334	0.9194	0.9035
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0530	0.0472	0.0577	0.0527
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9930	0.9780	0.9988	0.9873
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0152	0.0084	0.0264	0.0168
Productivity Vol.	σ_a	0.0152	0.0142	0.0164	0.0154
Forecast Mean Adjustment	ν	0.0018	-0.0006	0.0042	0.0020

Notes: The table reports parameter estimates from the posterior distribution. All parameters are reported at quarterly frequency. The sample spans the period 1952:Q1-2017:Q4.

would require an extensive investigation to align what is being measured with the desired theoretical input. For these reasons, we consider earnings to be a better indicator of future payouts and the fundamental value of the firm than current payouts.

Second, a principle objective of our analysis is to investigate the role of trends in the U.S. corporate profit share and aggregate economic activity on the value of the stock market. This requires that we use data on the corporate profit share rather than the payout share, specifying a model of investment to stipulate how the two are linked. It is unclear how to use the payout data directly in estimation without breaking the theoretical linkage between the two and, along with it, our ability to study the role of the corporate earnings share in driving the stock market.

5 Results

5.1 Parameter Estimates

We begin with a discussion of the estimated parameter values and latent states. Table 1 presents the estimates of our primitive parameters based on the posterior distribution obtained with flat priors. A number of results are worth highlighting.

First, the persistence parameters of the low frequency components of the state variables are of immediate interest, since they determine the role of each latent variable in driving market equity values over longer periods of time. Table 1 shows that the earnings share and risk price contain highly persistent components, with median estimates of $\phi_{s,LF} = 0.993$ and $\phi_{x,LF} = 0.988$, respectively. In contrast, the low-frequency risk-free rate process is substantially less persistent, with median value $\phi_{\delta,LF} = 0.964$. Because more persistent processes have stronger influence on equity values in our model, this will lead us to find that the risk-free rate plays a smaller role in driving equity values than our other fundamental components.

For a more complete look at the estimated persistence of our stochastic processes, accounting for both low and high frequency components, Figure 3 compares the autocorrelations of the latent states in the model and data.[27] To account for small sample bias, the model autocorrelations are obtained from 10,000 simulations, each the length of the data sample, taken from 10,000 equally spaced parameter draws from our MCMC estimation.[28] The figure shows that the model implications for the autocorrelations of model-implied series are a good match for those of the corresponding observed series, especially at the longer lags that are more important for asset prices.

In both the model and the data, the autocorrelations of output growth hover around zero, consistent with our i.i.d. assumption, whereas the autocorrelations for the earnings share, the risk-free rate, and the log ME-to-output ratio start decline gradually as the lag order increases, consistent with persistent but stationary processes. Panel (b) shows that the model underestimates the persistence of the earnings share at long horizons. Since the effect of the earnings share on valuations is stronger for a more persistent change, this implies that our results on the earnings share are, if anything, conservative, a point we return to in Section 6.2 below.

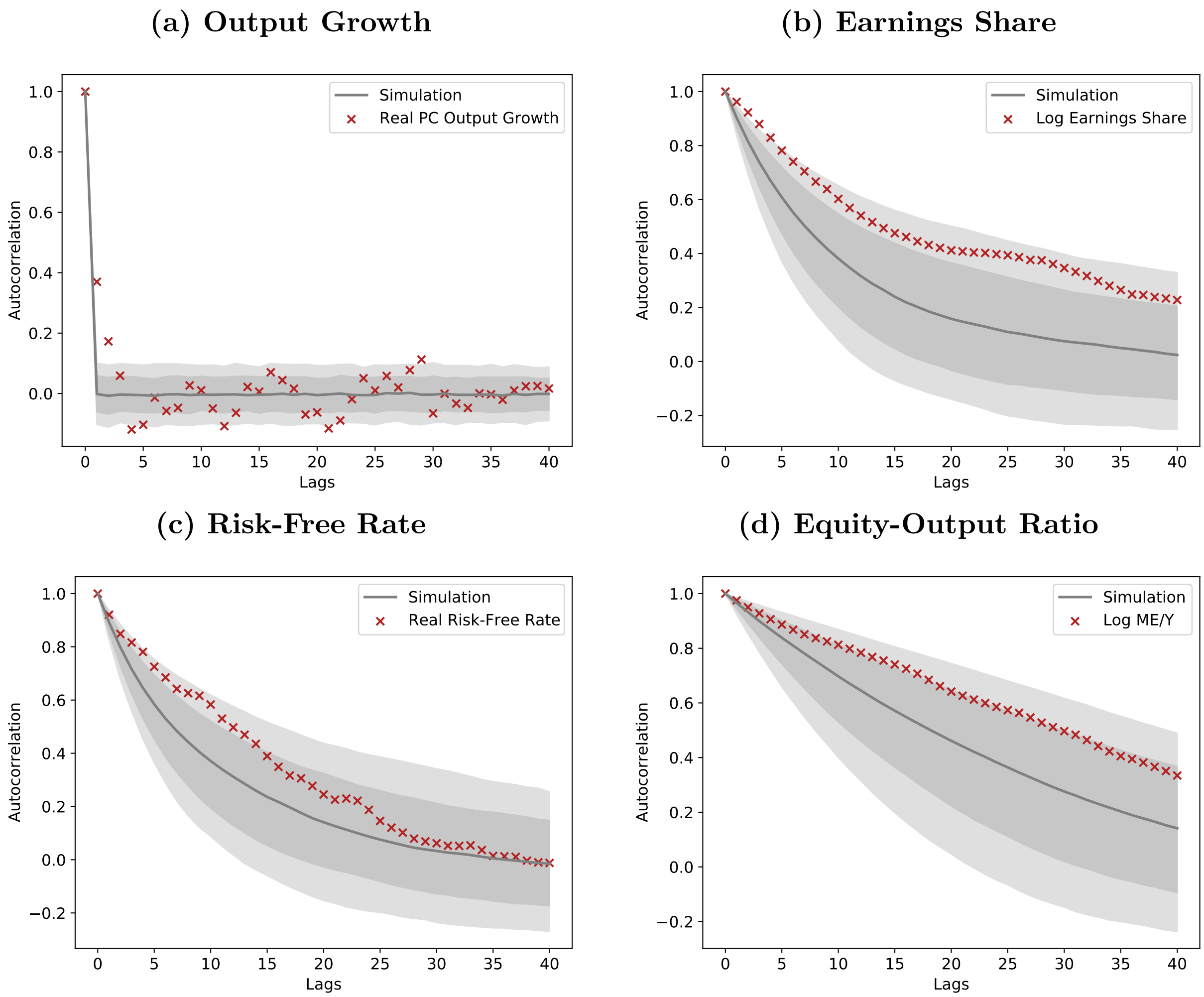
Panel (c) shows clearly that the autocorrelations of the risk-free rate converge to zero by quarterly lag 40 in both model and data, while Panel (d) shows that the autocorrelations for the log ME-to-output ratio remain substantially above zero. The shows that the risk-free rate process is not persistent enough to explain much of the considerably more persistent variation in the ME-to-output ratio observed in Figure 1.

Turning to the risk price, Table 1 shows that our median estimate of the average risk price is only $\bar{x} = 6.047$. Because shareholders in our model consume corporate cash flows

[27] We include all four observable series that are available over the full sample. We omit the SPF real rate forecast and the SVIX risk premium, both of which are available only on a much shorter sample, and are therefore unsuitable for computing longer autocorrelations.

[28] "Equally spaced" means sampled at regular intervals from the Markov Chain. Because the parameter draws in the original Markov Chain are highly serially correlated, this resampling dramatically speeds up computation time with little loss of fidelity in characterizing the distribution of parameters.

Figure 3: Observable Autocorrelations



Notes: The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the model. For the model equivalents, we use 10,000 evenly spaced parameter draws from our MCMC chain, and for each compute the autocorrelations from a simulation the same length as the data. The center line corresponds to the mean of these autocorrelations, while the dark and light gray bands represent 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

that are much more volatile than aggregate consumption, our model is able to reproduce a large equity risk premium without high levels of aversion to risk or ambiguity.

Finally, returning to the discussion in Section 2.1, the correlation between the *pc* ratio and the earnings share components depends on the strength of the leverage risk effect, which is in turn determined by our model parameters. Our median estimate is $A_s' = (-0.07, -1.80)$, with the entries corresponding to the low and high frequency components, respectively. Thus, while the high frequency component of the earnings share is still negatively correlated with the *pc* ratio, the correlation becomes effectively zero for the low frequency component. This correlation is closer to the data pattern displayed in Figure 2, but again implies a conservative estimate of the influence of the earnings share on risk premia.

5.2 Latent State Estimates

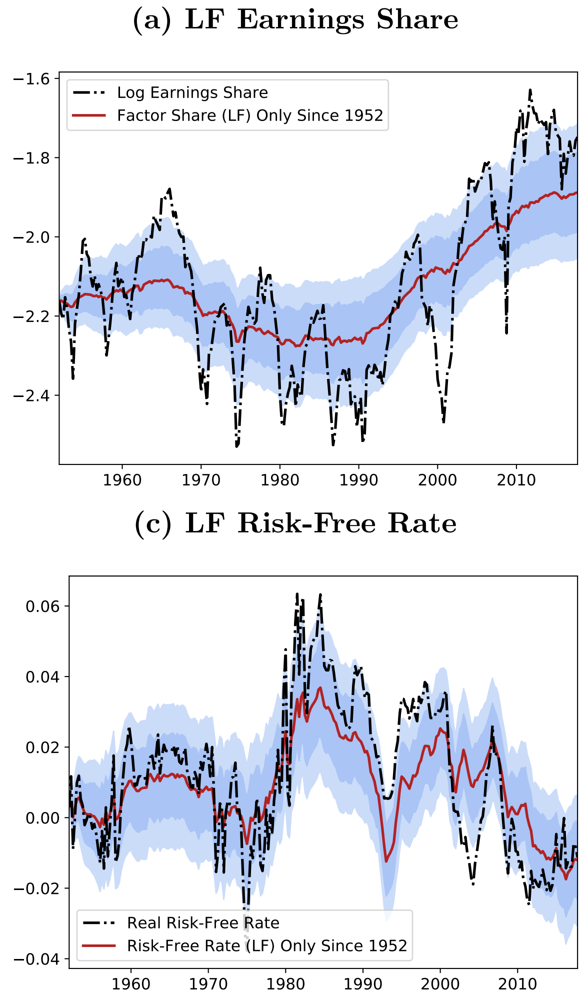
Figure 4 displays our model's decomposition of the earnings share s_t and real risk-free rate δ_t into their low and high frequency components. Each panel plots the observable series, alongside the variation attributable to a single high or low frequency component in isolation. In all of our decomposition figures throughout the paper, the red (solid) line shows the median outcome over our parameter and latent state estimates, while the shaded areas are 66.7% and 90% credible sets that take into account both parameter and latent state uncertainty.

Panels (a) and (b) show the time-variation in the log earnings share ey_t over our sample, along with the portion of this variation attributable to each estimated factors share component $s_{LF,t}$ and $s_{HF,t}$. Beginning with the data series itself, we observe that the earnings share undergoes major variation over the sample period. From a starting point in levels of 11.5% in 1952:Q1, the earning share rises to 15.3% in 1966:Q1, before falling to a low of 8.0% in 1974:Q3. After remaining low for nearly 15 years, the earnings share undergoes a dramatic rise from in the last three decades in the sample, from 8.9% in 1989:Q1 to a high of 19.6% in 2011:Q4, before ending at 17.4% in 2017:Q4.

Our model decomposes this overall series into a low and high frequency component. This decomposition is central to our asset pricing results, since forward looking equity prices in our model respond strongly to movements in the low frequency component, while movements in the high frequency component are too transitory to have a large effect. As a result, it is critical that this series accurately tracks the true data series, and is not distorted by the model's need to match asset prices. Reassuringly, Figure (a) shows that the estimated low frequency component accurately tracks the slow-moving trend in the earnings share series.

Panels (c) and (d) similarly show the evolution of the risk-free rate over time, along with the portion of this variation attributable to the estimated low and high frequency components. From the data series, it is clear that, although real rates are low today, they are not unusually so by historical standards, with real rates at similarly low levels at several points in the 1950s and late 1970s. Further, the series appears far from a unit root, with even these swings reverting relatively quickly. This stands in sharp contrast to the time series for nominal interest rates, which feature a much more persistent trend in inflation, demonstrating the importance of using real rates. The low frequency component $\delta_{LF,t}$ in Panel (c) faithfully captures the underlying trend, as well as driving most of the variation in 10-year risk-free rate forecasts to match our SPF data. The high frequency component $\delta_{HF,t}$

Figure 4: Latent State Components



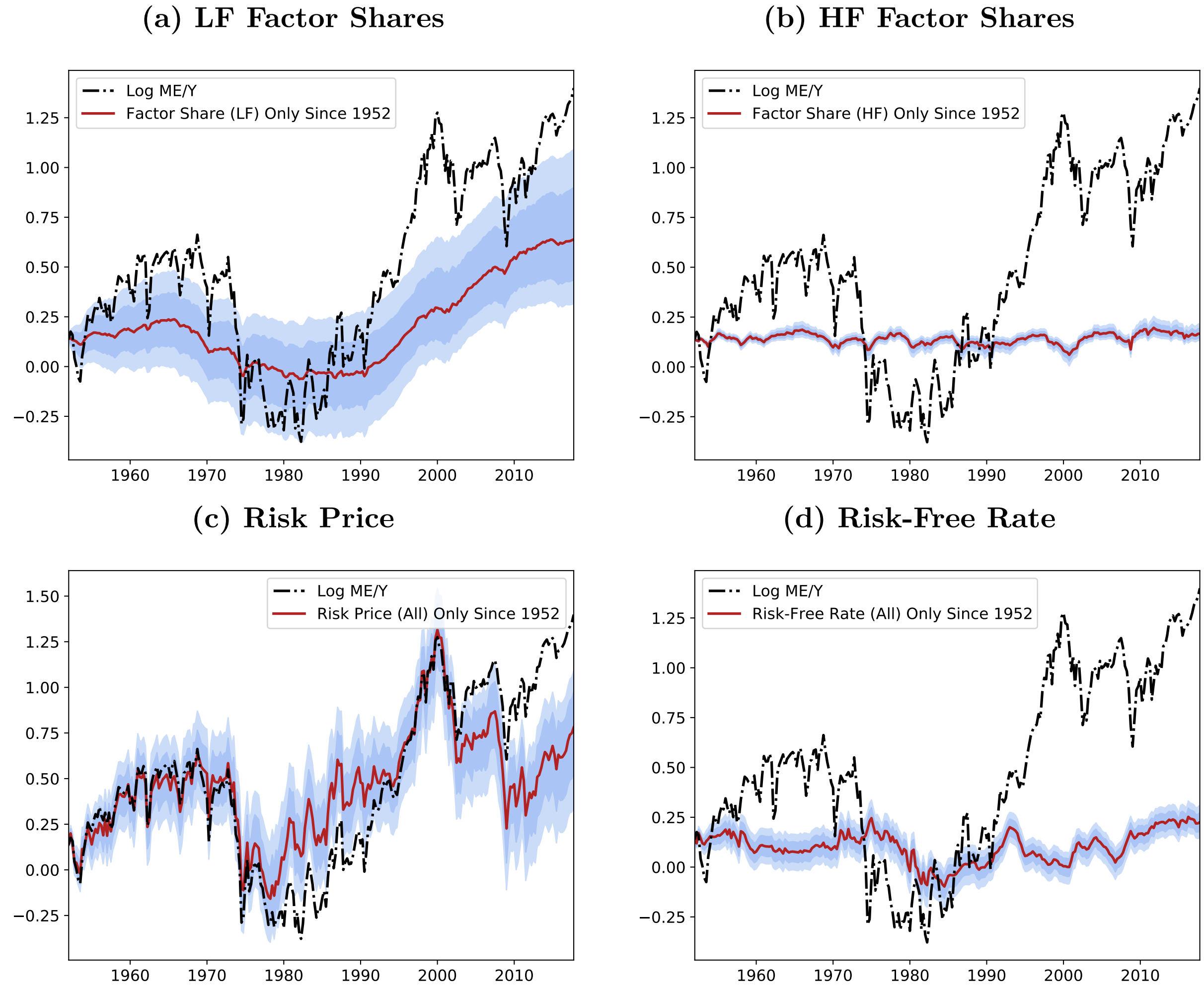
Notes: The figure exhibits the observed earnings share and real risk-free rate series along with the model-implied variation in the series attributable to their low and high frequency components. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

shown in Panel (d) largely captures transitory fluctuations, as well as some sharp movements in the early 1980s and 2000s.

5.3 Dynamics of Equity Values

With the estimation of the latent states complete, we next present their contribution to the evolution of market equity over our sample, displayed in Figure 5. Each panel displays the log equity-to-output ratio py_t, alongside the variation in py_t attributable to an individual latent component, holding the others fixed at their initial value in 1952:Q1. We note that this decomposition is additive, so that the contributions in the four panels, if demeaned by the average value of the data, would add exactly to the true demeaned data series.

Figure 5: Market Equity-Output Ratio Decomposition



Notes: This figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components. The top row displays the contribution of the low and high frequency components of the earnings share $s_{LF,t}$ and $s_{HF,t}$, while the bottom row displays the total contribution of the orthogonal risk price x_t and the real risk-free rate δ_t. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.

Beginning with the top row, Panel (a) shows that the low frequency factor shares component $s_{LF,t}$ explains much of the low frequency trend in the *py* ratio, particularly over the last three decades of the sample. In contrast, the high frequency component $s_{HF,t}$ explains much less of the variation in equity values. Importantly, this weak effect is not due to a lack of variability of the $s_{HF,t}$ series, which Figure 4 Panel (b) shows is highly volatile, and explains a large portion of earnings share dynamics. Instead, it occurs because more transitory movements in profits have weaker effects on forward-looking asset prices, demonstrating the importance of estimating our latent state processes at multiple frequencies.

Moving to the bottom row, Panel (c) displays the combined contribution of the risk price components $x_{LF,t}$ and $x_{HF,t}$. These secular variations in the risk price drive most variation in valuations at at high and medium frequencies, as well as much of the lower-frequency trend in the first half of the sample. In particular, this component explains nearly all of the large short-run swings in equity values over our sample, including the technology boom/bust, and the crash following financial crisis. At the same time, movements in the risk price fail to explain much of the rise in equity valuations over the last three decades, with little upward trend in this series between the late 1980s and the end of the sample.

Last, Panel (d) shows the combined contribution of the risk-free rate components $\delta_{LF,t}$ and $\delta_{HF,t}$. Our estimates attribute a minimal role to risk-free rate variation in explaining equity valuations over our sample. This finding, which stands in sharp contrast to alternative works in the literature, is due to our relatively low estimated persistence for the risk-free rate processes. We return to this discussion in Section 5.8.

To clarify the contributions since 1989 — the portion of the sample with extremely high growth in both equity valuations and earnings shares — Figure 6 plots the contributions of factor shares and the risk price, respectively, for this subsample. This figure shows that the prime driver of valuations over this period is the factor shares process. Movements in the risk price explain much of the cyclical variation, but fail to capture the overall upward trend.

5.4 Growth Decompositions

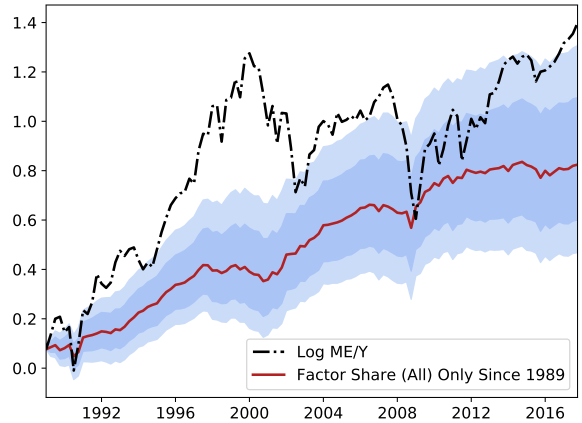
In this section, we summarize the contributions of the different components over our sample. As for Figure 5, we compute the contribution of each component by the counterfactual growth in equity values that would have occurred allowing that component to vary while holding all others fixed at their initial values. By construction, these components sum to 100% of the observed variation in equity values, since our latent state estimates perfectly match at each point in time the observed log market equity-to-output ratio, py_t, as well as output growth Δy_t.

Table 2 presents the decompositions for the total change in the log of real market equity p_t, either over the whole sample or over the period before or since 1989.[29] Our estimates

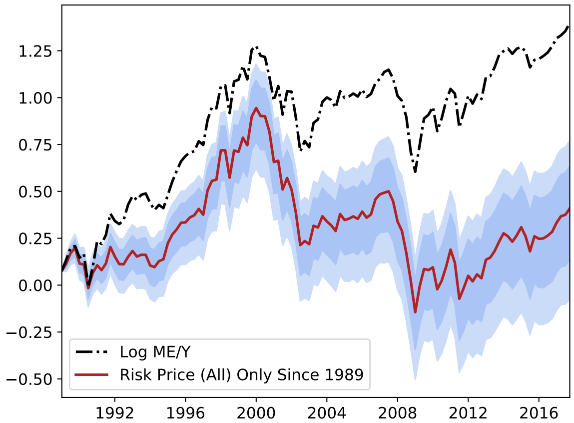
[29]The growth decompositions for the log level of real market equity p_t are computed by adding back the growth Δy_t in real output (net value added) to the growth Δpy_t. Since Δy_t is deflated by the implicit price deflator for net value added, the decomposition for p_t pertains to the value of market equity deflated by the implicit NVA price deflator.

Figure 6: Market Equity-Output Ratio Decomposition: 1989 - 2017 Subsample

(a) Factor Shares



(b) Risk Price



Notes: This figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components over the subsample 1989:Q1 - 2017:Q4. The left panel displays the combined contribution of the earnings share $\tilde{\mathbf{s}}_t$ while the right panel displays the combined contribution of the risk price components $\tilde{\mathbf{x}}$. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively.

indicate that about 44% of the market increase since 1989 and 21% over the full sample is attributable to the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$, with the vast majority of this coming from the low frequency component. In contrast to factor share movements, we find that growth in the real value of corporate output has been a far less important driver of equity values since 1989, explaining just 25% of the increase in equity values since 1989. We note that this number represents the *total* contribution of output growth, including its deterministic trend. The roles of the other components are smaller over the post-1989 period, with the decline in the risk price contributing 18%, and declining interest rates contributing 14%.

These patterns may be contrasted with the previous subsample, from 1952 to 1988, where economic growth accounted for 111% of the rise in the stock market, while factor share movements contributed negatively to the market's rise. underscore a striking aspect of post-war equity markets: in the longer 37 year subsample for which equity values grew comparatively slowly, economic growth propelled the market, while factor shares played a negative role. But the market experienced more than three times greater growth in value in the much shorter period from 1989 to 2017 day, when factor share shocks reallocated rewards to shareholders even as economic growth slowed.

Combining these periods, we find that output growth explains only 54% of total log

Table 2: Growth Decomposition

Contribution	1952-2017	1952-1988	1989-2017
Total	**1405.81%**	**151.23%**	**477.34%**
Factor Share s_t	20.50%	-21.09%	43.96%
Orth. Risk Price x_t	22.72%	25.33%	17.68%
Risk-Free Rate δ_t	3.24%	-15.65%	13.80%
Real PC Output Growth	53.54%	111.41%	24.57%

Notes: The table presents the growth decompositions for the real per-capita value of market equity. The row "Total" displays the total growth in market equity over this period, in levels. The remaining rows report the share of this overall growth explained by each component, obtained by measuring the difference in implied growth between the data and a counterfactual path in which that variable is held fixed at its initial value for the relevant subsample. To ensure an additive decomposition, we measure the share of total growth explained in logs. The reported statistics are means over shares computed from 10,000 equally spaced parameter draws from our MCMC chain. The sample spans the period 1952:Q1-2017:Q4.

growth in market equity, despite being the only non-stationary variable in our model, demonstrating the importance of non-output factors even over a horizon as long as sixty five years. Factor shares explained 21% of the full sample rise, while the declining risk price contributed 23%, and real interest rates contributed a much smaller 3%.

5.5 Sources of Earnings Share Variation

In this section, we investigate the drivers of the earnings share itself.

To begin, we briefly present the accounting breakdown of corporate output. Starting with gross value added, the BEA removes depreciation to yield net value added, which we use as our measure of output Y_t. Next, a fraction τ_t of Y_t is devoted to taxes and interest (and a catchall of "other" charges against earnings). We refer to τ_t simply as the "tax and interest" share for brevity. The remaining $1 - \tau_t$ is divided between labor compensation and domestic after-tax profits (domestic earnings, E_t^D).[30] We denote labor's share of domestic value added net of taxes and interest as L_t^D, so that $E_t^D = (1 - \tau_t)(1 - L_t^D)Y_t$. Finally, firms receive some earnings from their foreign subsidiaries $E_t^F = F_t Y_t$, where F_t is the ratio of

[30] We use the BEA corporate sector labor compensation data to measure $L_t^D\,(1 - \tau_t)\,Y_t$. Some researchers have questioned whether the BEA adequately accounts for all of employee compensation in the form of restricted stock or stock options (e.g., Koh et al. (2016), Eisfeldt, Falato, and Xiaolan (2018)). If no equity-based compensation were actually captured by the BEA labor compensation data, then S_t^D should be interpreted as the as the traditional cash compensation share, and fluctuations in S_t potentially influenced by any factor that drives the traditional cash compensation share. The precise interpretation of why S_t^D or S_t varies, although important and interesting in its own right, is not central to our investigation. Whatever the reason for a changing S_t, our empirical methodology can investigate the extent to such fluctuations have added to the rapid growth in the market value of corporate equity over the post-war period.

foreign earnings to domestic output, yielding the total earnings decomposition.

$$E_t = E_t^D + E_t^F = \underbrace{\Big((1-\tau_t)(1-L_t^D) + F_t\Big)}_{S_t} Y_t.$$

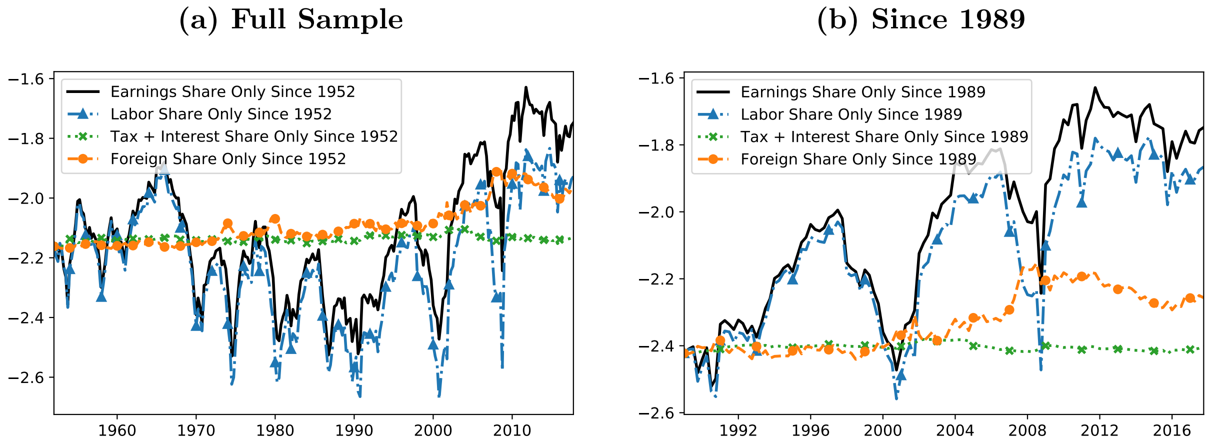
Before separately analyzing these series, it is important to connect our work to an existing controversy over measures of the labor share. In an influential recent paper, Koh et al. (2016) note that the measured decline in the labor share over recent decades depends heavily on the assumptions made. In particular, beyond clearly defined "unambiguous" sources of labor or capital income, lie various "ambiguous" components of the national accounts, particularly those related to intellectual property products. Koh et al. (2016) show that trends in the labor share largely hinge on whether these ambiguous components are classified as labor income or capital income. While we view this as a valuable line of research, we note that our earnings share uses only series classified by Koh et al. (2016) as "unambiguous" labor or capital income, and therefore sidesteps this interesting debate.

Returning to our measure of the earnings share, to decompose the contributions of the various components described above we once again compute counterfactual series allowing a single component (τ_t, L_t^D, or F_t) to vary, while holding the others fixed at their initial values. The resulting series are displayed in Figure 7. Panel (a) shows that, overall, movements in the domestic labor share L_t^D explain the vast majority of variation in the earnings share. At lower frequencies, an upward trend in the foreign earnings share has also contributed substantially to the rise in S_t. Combined, the tax and interest shares play close to zero role.[31] Panel (b) repeats this decomposition on the 1989 - 2017 subsample featuring rapid increases in equity valuations. This panel shows that the dominant driver of the earnings share over this period has been movements in the domestic labor share, with the foreign share playing a much smaller role, and the tax and interest share again playing effectively zero role.

Taken together, these results imply that the declining domestic labor share has played the largest role in the sustained rise in the corporate earnings share. Combined with our results earlier in the section showing that the earnings share has been the main driver of valuations since 1989, this suggests that much of stock market gains over this period have come at the expense of US labor compensation.

[31] This result is partially due to the separate impacts of the tax and interest shares, which are negatively correlated, largely canceling out.

Figure 7: Role of Components in Earnings Share



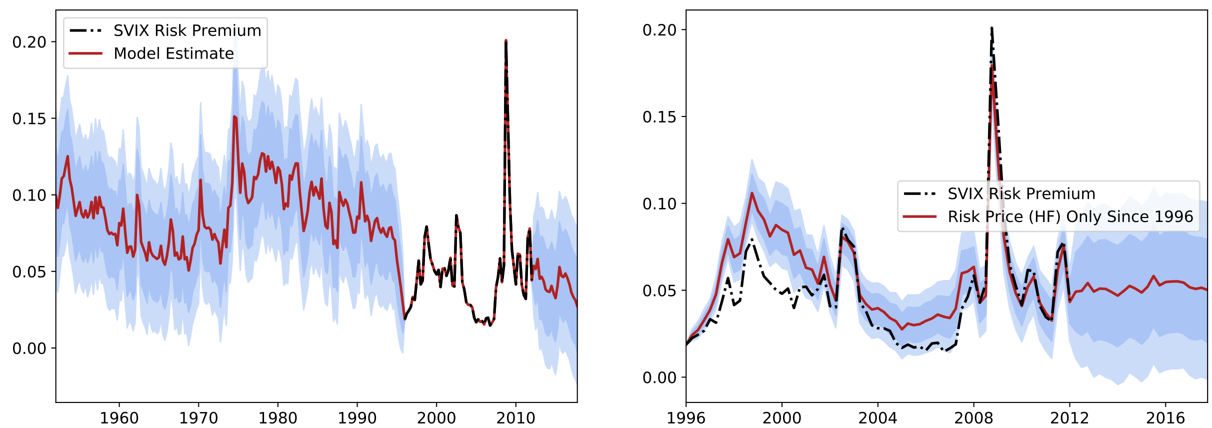
Notes: The figure decomposes the corporate earnings share S_t into contributions from changes in the domestic labor share S^D, the tax and interest share Z, and the foreign share F. Each series shows the result of allowing a single component to vary, while the others are held fixed at their initial values for that period (1952 or 1989). The sample spans the period 1952:Q1-2017:Q4.

5.6 Dynamics of the Equity Premium

In addition to decomposing the growth in market equity, our model also estimates a time series for the equity risk premium, shown in Figure 8. Panel (a) plots our overall estimated risk premium, which is affected by both the orthogonal risk price component x_t and by s_t through the leverage risk effect. Panel (b) shows our estimate of the equity premium variation that is attributable to only the high frequency component of the of the orthogonal risk price component, $x_{HF,t}$. Both panels superimpose the equity premium implied by the three-month SVIX over the subperiod for which the latter is available, from 1996:Q1-2012:Q1. Two points are worth noting. First, with the exception of the spike upward during the financial crisis of 2008-2009, Panel (a) shows that the estimated equity premium has been declining steadily over the past several decades and is quite low by historical standards at the end of the sample. By 2017:Q4, the estimates imply that the equity premium reached the record low values it had attained previously only in two episodes: at the end of the tech boom in 1999-2000, and at the end of the twin housing/equity booms in 2006. Second, Panel (b) shows that the estimation assigns to the high frequency orthogonal risk price component, $x_{HF,t}$, virtually all of the variation in the risk premium implied by the options data, while the remaining variation is ascribed to both the lower frequency component of the risk price, and to the earnings share via the leverage risk effect. The overall risk premium is therefore influenced by a trending low frequency component and a volatile high frequency component, consistent with the findings of Martin (2017).

Figure 8: Estimated Risk Premium and Risk Price Component

(a) Full Sample **(b) Contribution of HF Risk Price**



Notes: Panel (a) plots the estimated risk premium over the sample along with the risk premium implied by the SVIX, available for the subperiod 1996:Q1-2012Q1. Panel (b) plots the component of the risk-premium driven only by the high frequency orthogonal risk price along with the risk preimium implied by the 3-month SVIX. The label "Only Since" followed by a date describes a counterfactual path where a single component is allowed to vary, while all other components of the risk premium were held fixed from that date on. The red center line corresponds to the median of the distribution of outcomes, accounting for both parameter and latent state uncertainty, while the dark and light blue bands correspond to 66.7% and 90% credible sets, respectively. The period 1996:Q1-2012:Q1 in Panel (a) lacks bands because the our estimation procedure ensures that the risk premium matches the data exactly for each quarter of this subsample. The sample spans the period 1952:Q1-2017:Q4.

5.7 Asset Pricing Moments

Up to this point, our model estimates have decomposed the contributions of various forces over our data sample. At the same time, the observed sample is only a single realization of many possible paths for the data. In this section, we use the model to compare the observed asset pricing moments over our sample to the unconditional distributions of these moments implied by the model.

To this end, Table 3 presents the model's key asset pricing moments and compares them to data for the corporate sector. The columns labeled "Model" report averages across 10,000 unconditional simulations of the model, evaluated at 10,000 equally spaced parameter draws from our MCMC chains, each using a sample length equal to that of our historical sample. Next, the columns labeled "Fitted," compute moments using draws of the estimated latent states conditional on the observed historical sample produced by the disturbance smoother. These fitted values therefore represent the model's implications for the asset pricing moments conditional on the sequence of shocks that actually generated the historical data. Finally, the columns labeled "Data" report the actual sample moments of our observed data series.

Table 3: Asset Pricing Moments

	Model		Fitted		Data	
Variable	Mean	StD	Mean	StD	Mean	StD
Log Equity Return	5.920	17.866	7.994	16.906	8.856	15.844
Log Risk-Free Rate	1.130	1.729	1.124	1.946	1.124	1.946
Log Excess Return	4.791	17.969	6.871	16.893	7.389	16.562
Log Price-Payout Ratio	3.488	0.424	3.317	0.364	3.437	0.465
Log Earnings Growth	2.216	8.704	2.819	11.911	2.819	11.911
Log Payout Growth	2.205	16.867	3.444	22.017	4.115	33.459
Log Earnings Share Growth	-0.011	8.323	0.624	10.459	0.624	10.459
Log Payout Share Growth	-0.022	16.669	1.249	20.814	1.920	32.186

Notes: All statistics are computed for annual (continuously compounded) data and reported in units of percent. For annualization, returns, earnings, and payouts are summed over the year in levels. Log growth of earnings, payouts, the earnings share, the payout share, and the price-payout ratio are computed using these annual sums of earnings and payouts, as well as Q4 equity prices from each year. "Model" numbers are averages across 10,000 simulations of the model of the same size as our data sample. "Fitted" numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Note that the "Fitted" and "Data" moments are identical by construction for the risk-free rate, earnings growth, and earnings share growth, because we use these series as observables and fit their behavior exactly over the sample with no measurement error.

For series not matched by construction, Table 3 shows that the fitted moments are close to the data. Importantly, the model's operating leverage effect allows it to match the fact that both the mean and volatility of growth in the log payout share $c_t - y_t$ are substantially higher compared to the same moments for growth in the log earnings share $(e_t - y_t)$.[32] Since no data on payout were used in our estimation, these results increase our confidence that the model is able to realistically account for the dynamics of payouts over the sample. At the same time, the fact that our payout growth slightly understates the data suggests that our calibration for ξ is conservative, and that our model is not overstating the impact of the leverage effect. This slight understatement of cash flow growth leads the fitted log excess return (6.9%) to slightly understate its data counterpart (7.4%).

Turning to returns, these estimates imply that much of the reward from holding equity in the post-war era has been attributable to a long sequence of distributional shocks that redistributed rewards to shareholders. Table 3 shows that the model's unconditional average log excess equity return is 4.8% per annum. This number is an estimate of the mean risk premium implied by the parameter estimates, and reflects only compensation for bearing

[32]This volatility is scaled upward by exactly $\xi = 2.002$ in the model's quarterly simulations, but differ from this precise ratio in Table 3 due to annualization.

risk in the stock market (i.e., covariance with the SDF). By contrast, the mean *fitted* excess stock market return is 6.9%, which reflects not only ex-ante risk compensation, but also the effect of unexpected realizations of shocks over our sample.

This difference in our estimates of fitted and unconditional excess returns reflects that high returns to holding equity in the post-war period have been driven in large part by a highly unusual sample, one characterized by a long string of factors share shocks that redistributed rewards from productive activity toward shareholders. Taken together, the estimates imply that roughly 2.1 percentage points per annum of the post-war mean log return on stocks in excess of a T-Bill rate is attributable to these and other realized shocks, leading the excess return overstate the true equity risk premium by 43%. These findings provide a cautionary tale for the common practice of using the sample mean excess return as an estimate of the average equity risk premium, even over this 66-year post-war period.

5.8 Interest Rate Persistence

Our results differ in important ways from contemporaneous papers such as Farhi and Gourio (2018) and Corhay et al. (2018). While these papers find a crucial role for falling interest rates in driving the increase in asset prices over recent decades, we find that interest rates account for only 14% of stock market growth since 1989. Moreover, while these papers both conclude that risk premia have risen over this period, Panel (a) of Figure 8 shows that we estimate risk premia to have fallen to historically low levels by the end of our sample.

We believe these opposing results are mostly due to the different estimation approaches behind them. Importantly, while we estimate our model directly on the time series, allowing for shocks to enter with a variety of estimated persistences, Farhi and Gourio (2018) and Corhay et al. (2018) measure changes across steady states, in which parameters can change only permanently. As a result, these papers interpret the observed drop in risk-free rates as a permanent shift, causing major changes in how long-term cash flows are discounted, and leading to a huge increase in market value. Since the implied increase in market value from falling risk-free rates is even larger than the actual increase observed, these models infer that risk premia must have risen at the same time to match the realized growth in asset prices.

In contrast, our model views changes in interest rates as far from permanent, since our median estimate of the quarterly persistence of the low frequency component of interest rates is 0.964. As a result, investors in our model did not believe that interest rates would remain permanently high in the 1980s, nor do they expect them to remain permanently low today, strongly dampening the effect of the fall in rates on the value of market equity. This smaller direct effect from interest rates allows us to match the observed rise in asset prices in an environment with falling risk premia.

We view our approach, and therefore our findings, as strongly preferred by the data. Recall that, because we include the mean SPF expectation of the average real 3-Month T-Bill return over the next ten years as an observable, our estimates of the risk-free rate process match this forecast in each period it is available. Since we also match the current real short rate, this means that our model is able to reproduce the expected persistence of real interest rates, as perceived by actual investors, at each point in time. This strong discipline on our estimated persistence parameters and latent states allows us to avoid making assumptions about the perceived persistence of interest rate changes and instead compute precise estimates the risk-free rate persistence, which we can bound far from unity.

For more direct statistical evidence, we appeal to Figure 3, which compares the observed autocorrelations of our observable series to the implied autocorrelations generated from many short simulations of the model. Panel (c) shows that the autocorrelation of the real risk-free rate decreases rapidly with the lag order, falling by half within the first 15 quarters, and falling close to zero at the 10-year horizon — a pattern inconsistent with a process dominated by permanent changes. Our model is able to match this pattern well, and does not understate the autocorrelation at long horizons.[33]

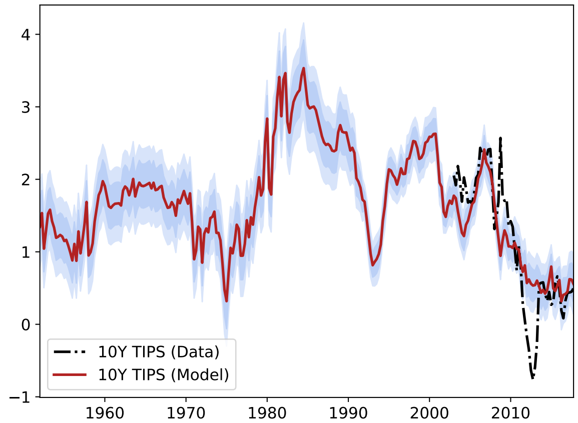
While our model directly targets the forecasted path of future real short rates, we do not target long bond yields. To check how closely our non-permanent process fits data on long bonds, we compare long real bond rates in the model and data. Figure 9 displays 10-year real bond yields in the model alongside 10-year TIPS yields in the data (details on this computation can be found in Appendix A.3.4). To allow for the fact that TIPS are computed using CPI inflation while our real rates are computed using the GDP deflator, as well as for the possibility that 10-year TIPS may include term or liquidity premia on average, we add a constant to our model-implied TIPS rate, equal to 0.61%, so that our model-implied and actual TIPS rates have the same mean over the subsample over which TIPS data are available (2003:Q1 - 2017:Q4). Panel (a) displays the full sample, while Panel (b) zooms in on the 2003:Q1 - 2017:Q4 TIPS subsample.

Comparing the series in Figure 9 shows that the general trajectory of the model-implied TIPS yields closely matches the data. Model and data yields display similar overall declines over the sample, equal to 1.22% and 1.55% from 2003:Q1 (the start of the TIPS sample) to 2017:Q3, respectively. Similarly, model and data yields exhibit falls of 1.65% and 1.85% from the 2006:Q3 (the TIPS peak, outside of a brief spike during the financial crisis) to

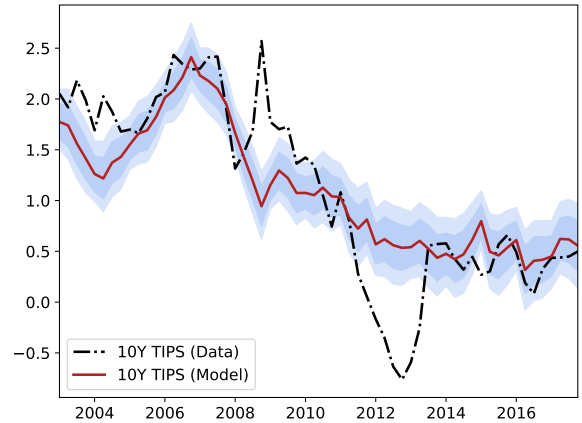
[33] These findings are not directly comparable to those in Bianchi, Lettau, and Ludvigson (2016), who find evidence of a low frequency component in interest rates driven by monetary policy, since the monetary policy component they uncover is correlated with risk-premium variation, whereas we identify only the mutually uncorrelated components of risk-free rate and equity premium variation.

Figure 9: 10-Year TIPS Yields, Model vs. Data

(a) Full Sample



(b) TIPS Sample



Notes: These plots compare the yields on ten-year real bonds in model and data. The data measure is obtained from the Federal Reserve Board of Governors (FRED code: FII10). The model value is obtained using the bond pricing formulas in Appendix A.3.4. The red line displays the mean estimate taken over 10,000 equally spaced parameter draws, while the light and dark blue bands represent 67% and 90% confidence intervals, respectively. The left panel displays the full sample period 1952:Q1-2017:Q4, while the right panel displays the 2003:Q1-2017:Q4 subsample on which the TIPS data is available.

2017:Q4, respectively. Although Panel (b) shows some deviations between model and data — in particular a failure of the model to capture a dip in rates between 2012 and 2013 — these results show that the model explains most movements in real long-term bonds at the 10-year horizon, despite the fact that these data are not a target of the estimation.

To close, we note that while we are confident in our evidence supporting a smaller role for risk-free rates in driving the market since 1989, this debate is largely orthogonal to our other core results. In particular, the risk-free rate considerations outlined above determine the relative contributions of the risk-free rate as opposed to the risk price, and are largely irrelevant to the estimated contribution of the earnings share.

6 Robustness

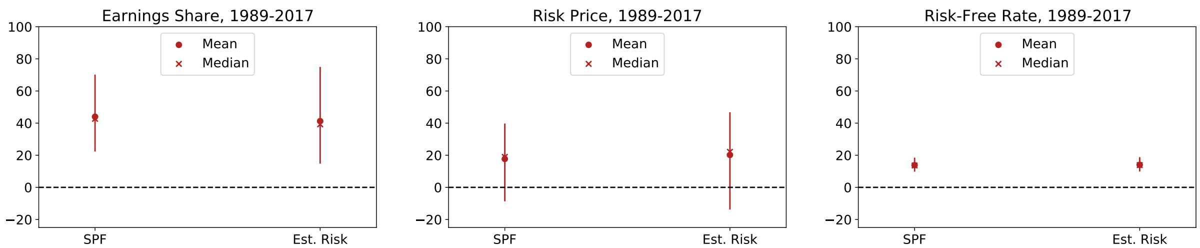
To close our results, we examine the robustness of our findings to our key modeling and calibration assumptions.

6.1 Leverage Risk Effect

For our first set of robustness checks, we consider an alternative to our specification that estimates the role of factor shares fluctuations on risk premia rather than imposing the

Figure 10: Model Comparison, Contributions to $p_t - y_t$.

(a) Earnings Share (b) Risk Price (c) Risk-Free Rate



Notes: These figures plot the growth decompositions for the real value of market equity under alternative model specifications, with each panel corresponding to a different fundamental component, and the different lines in each panel corresponding to alternative models. Growth decompositions are obtained by measuring the difference in implied growth between the data and a counterfactual path in which that variable is held fixed at its initial value for the relevant subsample. To ensure that these shares add up to 100%, these rows measure the share of total growth explained in logs. Each model reports the mean and median in red, while the red error bars span from the 5th to 95th percentiles.

structure of the model that delivers the leverage risk effect endogenously. To do so, we solve an "Estimated Risk" model in which we set $\mathbf{\Gamma} = 0$, thereby shutting down the leverage risk effect, and instead allow the price of risk to load on the earnings share according to

$$x_t = \bar{x} + \mathbf{1}'\tilde{\mathbf{x}}_t + \lambda \mathbf{1}'\tilde{\mathbf{s}}_t.$$

We view this as a parsimonious way of capturing the effects of the earnings share on risk premia. The parameter λ controls the strength of the link between the earnings share and the risk premium, with $\lambda < 0$ delivering the negative correlation obtained in the model from the leverage risk effect. While the strength of the leverage risk effect is effectively pinned down by theory in the baseline model, our Estimated Risk specification is thus able to flexibly estimate the strength of the spillover from the earnings share to risk premia.

The results are displayed graphically in Figure 10, which compares the Benchmark and Estimated Risk models for our main results: the contributions to the rise in market equity valuations since 1989. The figure shows that this more flexible specification delivers a decomposition nearly identical to the Benchmark model. Although the error bars on the Estimated Risk model are slightly wider, likely due to the inclusion of an additional free parameter, the point estimates and general ranges are highly similar. To be more precise, the Estimated Risk model delivers average contributions of (41%, 20%, 14%) for the earnings share, risk price, and risk-free rate, respectively, compared to equivalent values of (44%, 18%, 14%) for the Benchmark model. In particular, our result that the decline in risk-free rates played a limited role is unchanged in the Estimated Risk specification.

Because the model was free to choose an arbitrary strength for the influence of the earnings share on risk premia, these nearly identical results provide strong quantitative support for our specification the leverage risk effect.

6.2 Decomposition: Cash Flows vs. Risk Premia

For our final set of results, we decompose the contribution of the earnings share into the components driven by the change in cash flows vs. the change in the risk premium, then consider robustness to alternative degrees of earnings share persistence.

We begin with the workhorse decomposition of Campbell and Shiller (1989) for the price-payout ratio:

$$pc_t = \text{const} + \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} - \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j r_{t+j+1}.$$

Since the log ME/Y ratio py_t is equal to the sum $pc_t + cy_t$, we can apply a single restriction — our loglinear approximation for the cash flow share of output (10) — to obtain[34]

$$py_t = \text{const} + \underbrace{\xi s_t + \xi \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta s_{t+j+1}}_{\text{direct cash flow component}=py_t^{CF}} + \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1} - \mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j r_{t+j+1}. \tag{16}$$

The term in braces represents the contribution of the earnings share process s_t to the log ratio of market equity to output, directly through cash flows, while ignoring any influence on risk premia. In our structural model, this component, which we denote py_t^{CF} is given by

$$py_t^{CF} = \xi \left\{ \bar{s} + \mathbf{1}' \Big[\mathbf{I} - \mathbf{1}'(\mathbf{I} - \Phi_s)(\mathbf{I} - \kappa^1 \Phi_s)^{-1} \Big] \tilde{\mathbf{s}}_t \right\}. \tag{17}$$

We can therefore compare growth of py_t^{CF} and py_t to compute the contribution of growth in the log ME/Y explained by the direct cash flow effect.[35]

The results are displayed in Table 4. Under our Benchmark model estimates, the direct change in cash flows explains 26.38% of the rise in the py ratio since 1989. Since the total contribution of earnings share changes is estimated at 67.75%, these results imply that around 39% of our overall earnings share contribution in the Benchmark model are due to

[34] We thank our discussant, Valentin Haddad, for this helpful suggestion.

[35] These numbers differ from those in Table 4 because they decompose growth in the ratio of market equity to output (py) instead of the growth in real per-capita market equity (p).

Table 4: Comparison, Share of ME/Y Explained (1989 - 2017)

		AR(1) Models (by ϕ_s)			
	Bench.	0.980	0.990	0.995	1.000
Cash Flow Contribution	26.38%	28.71%	45.08%	62.68%	102.15%

Notes: This table displays the share of the growth in the log market equity to output ratio explained by the implied contributions of the earnings share via cash flows, defined as $(py^{CF}_{2017:Q4} - py^{CF}_{1989:Q1})/(py_{2017:Q4} - py_{1989:Q1})$, where py is the log ratio of market equity to output, and py^{CF} is the direct cash-flow component.

the direct influence on cash flows, with the remaining share due to the influence on risk premia through the leverage risk effect.

While these results already imply that the direct cash flow component is an important driver of market equity over this period, with a magnitude similar to total economic growth, we believe these results understate the true cash flow contribution. This is due to the fact that the Benchmark model appears to understate the autocorrelation process in the data. This can be seen in Figure 3 Panel (b), which shows that small sample autocorrelations from our model simulations lie below the true sample autocorrelations on average at all lag orders, including the longest.

This downward bias in persistence estimates is common in statistical applications, and is not straightforward to correct in our baseline model.[36] Instead, we provide results from a simpler parametric specification to demonstrate the strength of the cash flow effect at plausible levels of bias-corrected persistence. In place of our full structural model, we approximate s_t by a simpler AR(1) process

$$s_{t+1} = (1 - \phi_s)\bar{s} + \phi_s s_t + \varepsilon_{s,t+1}$$

which implies $\mathbb{E}_t \Delta s_{t+j+1} = -(1 - \phi_s)\phi^j s_t$. Substituting, solving for the geometric sum, and omitting all terms not entering our "direct cash flow component" above, we obtain the following expression for py^{CF}_t under the AR(1) specification:

$$py^{CF}_t = \xi \left[1 + \left(\frac{1 - \phi_s}{1 - \kappa_1 \phi_s} \right) \right] s_t.$$

[36] Because the model has freedom to allocate between the low and high frequency components, manually increasing the persistence of the low frequency component, or even both components, results in the model assigning more of the underlying variation to the high frequency component, leaving the overall results unchanged.

For given parameter choices of ϕ_s, ξ, and κ_1, we can thus directly evaluate the contribution of the earnings share over time, without taking a stand on the remaining blocks of the model. We obtain ξ and κ_1 directly from the data, using $\xi = 2.002$, as explained above, and calibrating $\kappa_1 = \exp(\overline{pc})/\exp(\overline{pc}+1)$, where we obtain $\overline{pc} = 4.823$ as the average of our log price-to-payout ratio from our sample.[37] The implied contributions since 1989:Q1 are reported in Panel (b) of Table 4 for a range of possible persistences: $\phi_s \in \{0.98, 0.99, 0.995, 1.000\}$. Appendix Figure A.6 shows that the implied py_t^{CF} series are reasonable, and track the true py_t series well over time, even at the high persistence values, providing support for the specification. As can be seen from Table 4, the direct cash flow contribution is nontrivial even for a persistence of 0.980, while the assumption of a unit root would explain more than 100% of the rise in the log ME/Y ratio over this period.

The question then becomes which of these persistence values is most appropriate. A bootstrap bias corrected AR(1) estimate yields $\phi_s = 0.990$ (see Appendix Section A.5.1 for details), which corresponds to nearly half of the rise in the log ME/Y ratio being explained through the direct influence of the earnings share on cash flows. For equity values, however, the autocorrelation at the first lag is not as important as the autocorrelation at longer lags. Appendix Figure A.7 reproduces the simulated autocorrelation plots from Figure 3 for these AR(1) models, showing that even more persistent processes, such as $\phi = 0.995$, or even a unit root, provide a better fit of the observed longer autocorrelation pattern for s_t. Although the empirical autocorrelations in the data decay substantially with the lag order, our simulation results indicate that this is an endemic feature of sample autocorrelations given our sample size, even for extremely persistent processes. For more formal evidence, an augmented Dickey-Fuller test also fails to reject the presence of a unit root with p-value 0.164. From our results in Table 4, these values would imply direct cash flow contributions of 60% or more over the 1989 - 2017 period.

Overall, this analysis implies that under minimal assumptions — that investment is proportional to output, not earnings, and that the earnings share follows a simple AR(1) process — the data are consistent with a strong direct effect of the earnings share on the value of market equity over the last three decades. By contrast, the same bias correction would have little influence on the estimated contribution of the risk-free rate, since–as shown in Figure 3–there is no downward bias in those autocorrelations at long lag orders, which evidently converge to zero by quarterly lag 40 in both the model and the data. We believe these findings imply that our estimates are if anything a lower bound on the true contribution of the earnings share to the growth in market equity since 1989.

[37]This value of κ_1 differs slightly from our baseline results, in which κ_1 is pinned down by the equilibrium *pd* ratio in the model.

7 Conclusion

In this paper, we investigate the causes of rising equity values over the post-war period. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent components, while at the same time inferring what values those components must have taken over our sample to explain the data. The identification of mutually uncorrelated components and the specification of a log linear model allow us to precisely decompose the observed market growth into distinct component sources explaining 100% of the variation in equity values over our sample and at each point in time.

We confront our model with data on equity values, output, the earnings share of output, interest rates, and a measure of the conditional equity premium implied by options data. We find that the high returns to holding equity over the post-war era have been attributable in large part to an unpredictable string of factor share shocks that reallocated rewards away from labor compensation and toward shareholders. Indeed, our estimates suggest that at least 2.1 percentage points of the post-war average annual log equity return in excess of a short-term interest rate is attributable to this string of reallocative shocks, rather than to genuine compensation for bearing risk. This estimate implies that the sample mean log excess equity return overstates the true risk premium by 43%.

Factors share shocks alone would have driven a 116% increase in the value of real per-capita market equity since 1989, explaining 44% of actual log growth over this period. Equity values were modestly boosted since 1989 by persistently declining interest rates and a decline in the price of risk, which contributed 14% and 18%, respectively, to the rise in log equity values over this period. But growth in the real value of aggregate output contributed just 25% since 1989 and just 54% over the full sample. By contrast, economic growth was overwhelmingly important for rising equity values from 1952 to 1988, where it explained over 100% of the market's rise. Still, this 37 year period generated less than half the growth in equity wealth created in the 29 years since 1989. In this sense, factor shares, far more than economic growth, have been the preponderant measure of fundamental value in the stock market over the last three decades.

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Appendix: For Online Publication

A.1 Data Description

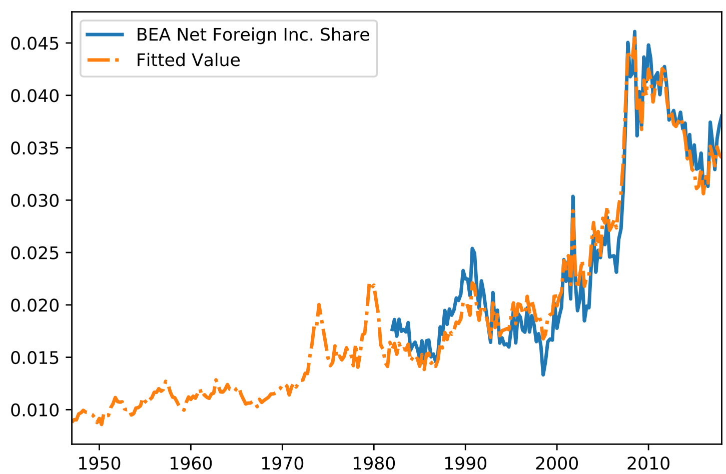
Corporate Equity. Corporate equity is obtained from the Flow of Funds Table B103, series code LM103164103, nonfinancial corporate business; corporate equities; liability. Unadjusted transactions estimated by Federal Reserve Board (Capital Markets and Flow of Funds Sections), using data from the following commercial sources: cash mergers and acquisitions data from Thompson Financial Services SDC database; public issuance and share repurchase data from Standard and Poor's Compustat database; and private equity issuance data from Dow Jones Private Equity Analyst and PriceWaterhouseCoopers Money tree report. Level at market value is obtained separately as the sum of the market value of the nonfinancial corporate business (FOF series LM103164103) and the financial corporate business (FOF series LM793164105). Source: Federal Reserve Board.

Foreign Earnings. Total earnings is the sum of domestic after-tax profits from NIPA and earnings of U.S. multinational enterprises on their overseas operations. Total earnings are defined as as share of domestic net-value-added for the corporate sector. (See the next subsection for the sources of domestic data.)

$$
\begin{aligned}
E_t &= S_t Y_t \\
&= \left(S_t^D Z_t + F_t\right) Y_t.
\end{aligned}
$$

In the above, F_t is the foreign profit share of domestic output. The measure of foreign profits in the numerator of F_t is based on data from Table 4.2 of the U.S. International Transactions in Primary Income on Direct Investment, obtained from BEA's International Data section. We refer to this simply as corporate "direct investment." Specifically, these data are from the "income on equity" row 2 of Direct investment income on assets, asset/liability basis. Note that U.S. direct investment abroad is ownership by a U.S. investor of at least 10 percent of a foreign business, and so excludes household portfolio investment. This series is available from 1982:Q1 to the present. To extend this series backward, we first take data on net foreign receipts from abroad (Corporate profits with IVA and CCAdj from BEA NIPA Table 1.12. (A051RC) or from Flow of Funds (FOF) Table F.3 (FA096060035.Q less corporate profits with IVA and CCAdj, domestic industries from BEA NIPA Table 1.14 (A445RC)), which is available from the post-war period onward. This series includes portfolio investment income of households as well as direct investment, but its share of domestic net-value-added for the corporate sector is highly correlated with the foreign direct investment share of net-value-added. We regress the direct investment share of net-value-added on the foreign receipts share of domestic net-value-added and then use the fitted value from this regression as the measure of F_t in data pre-1982. Because the portfolio income component is relatively small, the fit of this regression is high, as seen in Figure A.1, which compares the fitted series with the actual series over the post-1982 period.

Figure A.1: Net Foreign Income Share: Data vs. Fitted value



Notes: The sample spans the period 1952:Q1-2018:Q2.

Domestic Variables: Corp. Net Value Added, Corp. Labor Compensation, Corp. After-Tax Profits, Taxes, and Interest. Define domestic corporate earnings E_t^D as

$$E_t^D = S_t^D (1 - \tau_t) NVA_t,$$

which is equivalent to

$$E_t^D = \left[1 - \underbrace{\frac{LC_t}{ATP_t + LC_t}}_{\text{Labor share of labor+profit}} \right] (1 - \tau_t) NVA_t.$$

Data for the net value added (NVA) comes from NIPA Table 1.14 (corporate sector series codes A457RC1 and A438RC1). We use per capita real net value added, deflated by the implicit price deflator for net value added. After tax profits (ATP) for the domestic sector come from NIPA Table 1.14 (corporate sector series code: W273RC1). Corporate sector labor compensation (LC) for the domestic sector is from Table 1.14 (series code A442RC). The doestic after-tax profit share ($ATPS$) of NVA is identically equal to

$$
\begin{aligned}
ATPS &= \frac{ATP}{ATP+LC}\frac{ATP+LC}{NVA} = \underbrace{\frac{ATP}{ATP+LC}}_{=S_t^D}\frac{NVA-(\text{taxes and interest})}{NVA} \\
&= S_t^D\left[1-\underbrace{\left(\frac{\text{taxes and interest}}{NVA}\right)}_{=\tau_t}\right] \\
&= S_t^D Z_t,
\end{aligned}
$$

where S_t^D is the domestic after-tax profit share of combined profit plus labor compensation, "taxes and interest" is the sum of taxes on production and imports less subsidies (W325RC1), net interest and miscellaneous payments (B471RC1), business current transfer payments (Net) (W327RC1), and taxes on corporate income (B465RC1). Source: Bureau of Economic Analysis.

Net Dividends Plus Net Repurchases (Equity Payout). Net dividends minus net equity issuance is computed using flow of funds data. Net dividends ("netdiv") is the series named for corporate business; net dividends paid (FA096121073.Q). Net repurchases are repurchases net of share issuance, so net repurchases is the negative of net equity issuance. Net equity issuance ("netequi") is the sum of Equity Issuance for Nonfinancial corporate business; corporate equities; liability (Table F.103, series FA103164103) and Equity Issuance for domestic financial sectors; corporate equities; liability (Table F.108, series FA793164105). Since netdiv and netequi are annualized, the quarterly payout is computed as payout=(netdiv-netequi)/4. The units are in millions of dollars. Source: Federal Reserve Board.

Price Deflators. Implicit price deflator and GDP deflator. A chain-type price deflator for the nonfinancial corporate sector (NFCS) is obtained implicitly by dividing the net value added of nonfinancial corporate business by the chained real dollar net value added of nonfinancial corporate business from NIPA Table 1.14. This index is used to deflate net value added of the corporate sector. There is no implicit price deflator available for the whole corporate sector, so we use deflator for the non-financial corporate sector instead. The GDP deflator is used to construct a real returns and a real interest (see below). GDPDEF is retrieved from FRED. Our source is the Bureau of Economic Analysis.

Interest Rate. The nominal risk-free rate is measured by the 3-Month Treasury Bill rate, secondary market rate. We take the (average) quarterly 3-Month Treasury bill from FRED (code: TB3MS). A real rate is constructed by subtracting the fitted value from a regression of GDP deflator inflation onto lags of inflation from the nominal rate. Our source is the board of governors of the Federal Reserve System and the Bureau of Economic Analysis.

Risk Premium Measure. Our measure of the risk premium comes from Martin (2017). This paper uses option data to compute a lower bound on the equity risk premium, then argues that this lower bound is in fact tight, and a good measure of the true risk premium on the stock market. We obtain this series from the spreadsheet `epbound.xls` on Ian Martin's website, which corresponds to the value

$$EPBound_{t\to T} = 100 \times \left(R_{f,t} SVIX^2_{t\to T} - 1\right)$$

which is equivalent to the bound on the annualized net risk premium, in percent. To translate these measures to our model's quarterly frequency, we use the risk premium measure computed over the next three months. We then convert this variable into a log return, average it over the quarter, and label it rp_t.

Survey Data on Expected Average Risk-Free Rate For the average short-term expected nominal interest rate, we use the mean forecast from the Survey of Professional Forecasters for variable BILL10, which is the 10-year annual-average forecast for returns on 3-month Treasury bills. We subtract from this the mean forecast for variable CPI10, the 10-year annual-average forecast of inflation, to obtain a survey forecast for the average real rate over the next ten years.

A.2 A Stylized Model of Workers and Shareholders

We consider a stylized limited participation endowment economy in which wealth is concentrated in the hands of a few asset owners, or "shareholders," while most households are "workers" who finance consumption out of wages and salaries. The economy is closed. Workers own no risky asset shares and consume their labor earnings. There is no risk-sharing between workers and shareholders. A representative firm issues no new shares and buys back no shares. Cash flows are equal to output minus a wage bill,

$$C_t = Y_t - w_t N_t,$$

where w_t equals the wage and N_t is aggregate labor supply. The wage bill is equal to Y_t times a time-varying labor share α_t,

$$w_t N_t = \alpha_t Y_t \implies C_t = (1 - \alpha_t) Y_t. \tag{A.1}$$

We rule out short sales in the risky asset:

$$\theta_t^i \geq 0.$$

Asset owners not only purchase shares in the risky security, but also trade with one another in a one-period bond with price at time t denoted by q_t. The real quantity of bonds is denoted B_{t+1}, where $B_{t+1} < 0$ represents a borrowing position. The bond is in zero net supply among asset owners. Asset owners could have idiosyncratic investment income ζ_t^i, which is idependently and identically distributed across investors and time. The gross financial assets of investor i at time t are given by

$$A_t^i = \theta_t^i (V_t + C_t) + B_t^i.$$

The budget constraint for the ith investor is

$$\begin{aligned} C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i V_t &= A_t^i + \zeta_t^i \\ &= \theta_t^i (V_t + C_t) + B_t^i + \zeta_t^i, \end{aligned} \tag{A.2}$$

where C_t^i denotes the consumption of investor i.

A large number of identical nonrich workers, denoted by w, receive labor income and do not participate in asset markets. The budget constraint for the representative worker is therefore

$$C^w = \alpha_t Y_t. \tag{A.3}$$

Equity market clearing requires

$$\sum_i \theta_t^i = 1.$$

Bond market clearing requires

$$\sum_i B_t^i = 0.$$

Aggregating (A.2) and (A.3) and imposing both market clearing and (A.1) implies that aggregate (worker plus shareholder) consumption C_t^{Agg} is equal to total output Y_t. Aggregating over the budget constraint of shareholders shows that their consumption is equal to the capital share times aggregate consumption C_t^{Agg}:

$$C_t^S = C_t = \underbrace{(1-\alpha_t)}_{KS_t} C_t^{AGG}.$$

A representative shareholder who owns the entire corporate sector will therefore have consumption equal to $C_t^{Agg} \cdot KS_t$. This reasoning goes through as an approximation if workers own a small fraction of the corporate sector even if there is some risk-sharing in the form of risk-free borrowing and lending between workers and shareholders, as long as any risk-sharing across these groups is imperfect. While individual shareholders can smooth out transitory fluctuations in income by buying and selling assets, shareholders as a whole are less able to do so since purchases and sales of any asset must net to zero across all asset owners.

A.3 Model Solution

Perturbation Details. The derivation of the perturbed stochastic discount factor (11) follows here. We seek a perturbation of the terms determining the risk exposure of the SDF, the nonlinear expression $\tilde{m}_{t+1} = -x_t \Delta c_{t+1}$. Our perturbation includes terms linear in both the state vector $\mathbf{z}_t$, the shock vector $\boldsymbol{\varepsilon}_{t+1}$, and interactions between the two, while omitting all other higher-order terms. While our solution could handle terms quadratic in $\mathbf{z}_t$, they would be irrelevant since the term μ_t would implicitly offset them in all states. On the other hand, terms quadratic in $\boldsymbol{\varepsilon}_{t+1}$ would influence the solution, but would break our solution methodology.

To derive the perturbed stochastic discount factor, we first express $\hat{m}_t$ in terms of the current period's states and next period's shocks:

$$\hat{m}_{t+1} = -x_t \left\{ \log\left[\exp\Big(\bar{s} + \mathbf{1}'\left(\boldsymbol{\Phi}_s \tilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}\right)\Big) - \omega\right] - \log\left[\exp\Big(\bar{s} + \mathbf{1}'\tilde{\mathbf{s}}_t\Big) - \omega\right] + g + \varepsilon_{y,t+1} \right\}.$$

Evaluating the derivatives of this expression with respect to shocks and states, we obtain

$$\begin{aligned}
\frac{\partial \hat{m}_{t+1}}{\partial \tilde{\mathbf{x}}_t} &= -\Delta c_{t+1}\mathbf{1}' \\
\frac{\partial \hat{m}_{t+1}}{\partial \tilde{\mathbf{s}}_t} &= -x_t \left\{ \left(\frac{S_{t+1}}{S_{t+1}-\omega} \right) \mathbf{1}'\Phi - \left(\frac{S_t}{S_t-\omega} \right) \mathbf{1}' \right\} \\
\frac{\partial \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1}} &= -x_t \left(\frac{S_{t+1}}{S_{t+1}-\omega} \right) \mathbf{1}' \\
\frac{\partial \hat{m}_{t+1}}{\partial \varepsilon_{y,t+1}} &= -x_t \\
\frac{\partial^2 \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1} \partial \tilde{\mathbf{x}}_t} &= -\mathbf{1} \left(\frac{S_{t+1}}{S_{t+1}-\omega} \right) \mathbf{1}' \\
\frac{\partial^2 \hat{m}_{t+1}}{\partial \boldsymbol{\varepsilon}_{s,t+1} \partial \tilde{\mathbf{s}}_t} &= x_t \mathbf{1} \left(\frac{S_{t+1}}{S_{t+1}-\omega} \right) \left(\frac{\omega}{S_{t+1}-\omega} \right) \mathbf{1}'\mathbf{\Phi}_s
\end{aligned}$$

We therefore approximate

$$\begin{aligned}
\hat{m}_{t+1} \simeq& -\bar{x}g - g\mathbf{1}'\tilde{\mathbf{x}}_t - \bar{x}\xi\mathbf{1}'(\mathbf{\Phi}_s - I)\tilde{\mathbf{s}}_t - \bar{x}\xi\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} - \bar{x}\varepsilon_{y,t+1} \\
& - \tilde{\mathbf{x}}_t'\mathbf{1}\xi\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} + \tilde{\mathbf{s}}_t'\mathbf{\Phi}_s'\bar{x}\xi(\xi-1)\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} - \tilde{\mathbf{x}}_t'\mathbf{1}\varepsilon_{y,t+1} \\
=& \cdots - \big(x_t\xi - \bar{x}\xi(\xi-1)(\mathbf{1}'\mathbf{\Phi}_s\tilde{\mathbf{s}}_t)\big)\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} - x_t\varepsilon_{y,t+1}
\end{aligned}$$

where the omitted terms are known at time t, and whose exact values are thus irrelevant to our solution as they will be directly offset by the implicitly defined μ_t. Combining this result with the identity

$$\mathbb{E}_t[s_{t+1}] = \bar{s} + \mathbf{1}'\mathbf{\Phi}_s\tilde{\mathbf{s}}_t$$

and rearranging yields (11).

Price-Payout Ratio This section derives the coefficients of the main asset pricing equation (13). To begin, define for convenience the variables

$$\begin{aligned}
u_{t+1} &= \log(PC_{t+1} + 1) - pc_t \\
q_{t+1} &= m_{t+1} + \Delta c_{t+1}
\end{aligned}$$

so that $m_{t+1} + r_{t+1} = u_{t+1} + q_{t+1}$. Applying the log linear approximation to $\log(PC_{t+1} + 1)$ and substituting in our guessed functional form (13) yields

$$
\begin{aligned}
u_{t+1} &= \log(PC_{t+1} + 1) - pd_t \\
&= \kappa_0 + \kappa_1 \Big(A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_{t+1} + \mathbf{A}_x' \tilde{\mathbf{x}}_{t+1} + \mathbf{A}_\delta' \tilde{\boldsymbol{\delta}}_t \Big) - \Big(A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_t + \mathbf{A}_x' \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' \tilde{\boldsymbol{\delta}}_t \Big) \\
&= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left(\kappa_1 \boldsymbol{\Phi}_s - \mathbf{I} \right) \tilde{\mathbf{s}}_t + \mathbf{A}_x' (\kappa_1 \Phi_x - \mathbf{I}) \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' (\kappa_1 \boldsymbol{\Phi}_\delta - \mathbf{I}) \tilde{\boldsymbol{\delta}}_t \\
&\qquad + \kappa_1 \mathbf{A}_s' \boldsymbol{\varepsilon}_{s,t+1} + \kappa_1 \mathbf{A}_x' \boldsymbol{\varepsilon}_{x,t+1} + \kappa_1 \mathbf{A}_\delta' \boldsymbol{\varepsilon}_{\delta,t+1}.
\end{aligned}
$$

Now turning to q_{t+1}, we can expand the expression to yield

$$
q_{t+1} = -\delta_t - \mu_t + g + \xi \mathbb{E}_t \Delta s_{t+1} + (1 - \gamma_{s,t}) \xi \mathbf{1}' \boldsymbol{\varepsilon}_{s,t+1} + (1 - x_t) \varepsilon_{y,t+1}
$$

where

$$
\begin{aligned}
\gamma_{s,t} &= x_t - \bar{x}(1 - \xi)(\mathbb{E}_t[s_{t+1}] - \bar{s}) \\
&= \bar{x} + \mathbf{1}' \tilde{\mathbf{x}}_t - \bar{x}(1 - \xi) \mathbf{1}' \Phi \tilde{\mathbf{s}}_t \\
&= \bar{x} + \mathbf{1}' \tilde{\mathbf{x}}_t + \boldsymbol{\Gamma}' \tilde{\mathbf{s}}_t.
\end{aligned}
$$

for

$$
\boldsymbol{\Gamma}' = -\bar{x}(1 - \xi) \mathbf{1}' \boldsymbol{\Phi}_s.
$$

Next, we apply our fundamental asset pricing equation $0 = \log \mathbb{E}_t \left[q_{t+1} + u_{t+1} \right]$, which under lognormality implies

$$
0 = \mathbb{E}_t[q_{t+1}] + \mathbb{E}_t[u_{t+1}] + \frac{1}{2} \mathrm{Var}_t(q_{t+1}) + \frac{1}{2} \mathrm{Var}_t(u_{t+1}) + \mathrm{Cov}(q_{t+1}, u_{t+1}).
$$

These moments can be calculated as

$$
\begin{aligned}
\mathbb{E}_t[q_{t+1}] &= -\delta_t - \mu_t + g - \xi \mathbf{1}' (\mathbf{I} - \boldsymbol{\Phi}_s) \tilde{\mathbf{s}}_t \\
\mathbb{E}_t[z_{t+1}] &= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left(\kappa_1 \boldsymbol{\Phi}_s - \mathbf{I} \right) \tilde{\mathbf{s}}_t + \mathbf{A}_x' (\kappa_1 \Phi_x - \mathbf{I}) \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' (\kappa_1 \boldsymbol{\Phi}_\delta - \mathbf{I}) \tilde{\boldsymbol{\delta}}_t \\
\mathrm{Var}_t(q_{t+1}) &= (1 - \gamma_{s,t})^2 \xi^2 (\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1}) + (1 - x_t)^2 \sigma_a^2 \\
\mathrm{Var}_t(z_{t+1}) &= \kappa_1^2 \Big(\mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{A}_s' + \mathbf{A}_x' \boldsymbol{\Sigma}_x \mathbf{A}_x + \mathbf{A}_\delta' \boldsymbol{\Sigma}_r \mathbf{A}_\delta \Big) \\
\mathrm{Cov}_t(q_{t+1}, z_{t+1}) &= \kappa_1 \xi (1 - \gamma_{s,t}) \mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{1}
\end{aligned}
$$

Substituting, we obtain

$$
\begin{aligned}
0 = -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 + \frac{1}{2}\Big((1-2\bar{x})\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + (1-2\bar{x})\sigma_a^2\Big) \\
+ \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s' + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_r\mathbf{A}_\delta\Big) + \kappa_1\xi(1-\bar{x})\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1} \\
+ \Big[-\xi\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) + \mathbf{A}_s'(\kappa_1\boldsymbol{\Phi}_s - \mathbf{I}) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}_s' - \kappa_1\xi\boldsymbol{\Gamma}_s'\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}\Big]\tilde{\mathbf{s}}_t \\
+ \Big[\mathbf{A}_x'(\kappa_1\Phi_x - \mathbf{I}) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\mathbf{1}' - \mathbf{1}'\sigma_a^2 - \kappa_1\xi\mathbf{1}'\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}\Big]\tilde{\mathbf{x}}_t \\
+ \Big[\mathbf{A}_\delta'(\kappa_1\boldsymbol{\Phi}_\delta - \mathbf{I}) - \mathbf{1}\Big]\tilde{\boldsymbol{\delta}}_t.
\end{aligned}
$$

Applying the method of undetermined coefficients now yields the solutions

$$
\begin{aligned}
\mathbf{A}_s' &= \Big[\xi\mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) - \xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1})\boldsymbol{\Gamma}_s'\Big]\Big[(\kappa_1\boldsymbol{\Phi}_s - \mathbf{I}) + \kappa_1\xi\boldsymbol{\Sigma}_s\mathbf{1}\boldsymbol{\Gamma}_s'\Big]^{-1} \\
\mathbf{A}_x' &= \Big[\Big(\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + \sigma_a^2 + \kappa_1\xi(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1})\Big)\mathbf{1}'\Big](\kappa_1\Phi_x - \mathbf{I})^{-1} \\
\mathbf{A}_\delta' &= \mathbf{1}(\kappa_1\boldsymbol{\Phi}_\delta - \mathbf{I})^{-1}
\end{aligned}
$$

while the constant term must solve

$$
\begin{aligned}
0 = -\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 + \frac{1}{2}\Big((1-2\bar{x})\xi^2(\mathbf{1}'\boldsymbol{\Sigma}_s\mathbf{1}) + (1-2\bar{x})\sigma_a^2\Big) \\
+ \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s' + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_r\mathbf{A}_\delta\Big) + \kappa_1\xi(1-\bar{x})\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{1}.
\end{aligned} \tag{A.4}
$$

A.3.1 Equilibrium Selection

The parameters κ_0 and κ_1 determine the steady state pc (price-payout ratio), which depends on A_0. But since κ_0 and κ_1 are both themselves nonlinear functions of A_0, the equilibrium condition (A.4) is also nonlinear, leading to the possibility that multiple solutions, or no solution, exists. In fact, we confirm that both of these outcomes can occur in our numerical solutions. However, our numerical results indicate that, when there is more than one solution there are at most two, and one can be discarded because it is economically implausible. To see this, rewrite (A.4) as

$$
0 = \mathbb{E}[m] + \mathbb{E}[r] + \frac{1}{2}\mathrm{Var}(m) + \frac{1}{2}\mathrm{Var}(r) + \mathrm{Cov}(m, r)
$$

where m and r are the log SDF and equity return. We are interested in the relationship between the steady state pc and the other terms that depend on it in equilibrium. The terms $\mathbb{E}[m]$ and $\mathrm{Var}(m)$ do not depend on the pc ratio, so we can ignore these and focus on the remaining terms. Alternatively, consider the log risk premium, given in equilibrium by

$$\mathbb{E}[r_{t+1}] - r_{f,t} = -\frac{1}{2}\text{Var}(r_{t+1}) - \text{Cov}(m_{t+1}, r_{t+1}).$$

In the case where there are two solutions, one solution typically has a plausible level for the steady state pc, and implies that higher pc ratios (which take different values depending on where in the posterior distribution of model parameters we evaluate the function) coincide with lower risk premia $\mathbb{E}[r_{t+1}] - r_{f,t}$ and a lower absolute covariance with the SDF (i.e., a less negative $\text{Cov}(m_{t+1}, r_{t+1})$). This solution is economically reasonable. By contrast, when there is a second solution, it is always characterized by values for pc that are higher than the economically reasonable solution, and for typical parameter values delivers a value for pc that are extremely implausible (e.g., a value for $\exp(pc)$ of almost 3,000 at the posterior mode). In addition, this solution has the property that the higher pc ratios coincide with lower risk premia vis-a-vis the plausible solution, but also *higher* absolute covariances with the SDF (i.e., a more negative $\text{Cov}(m_{t+1}, r_{t+1})$). Thus the higher pc ratios in this solution must be explained by a lower absolute covariance with the SDF and a Jensen's term $\frac{1}{2}\text{Var}(r_{t+1})$ that in some cases converges to infinity. In summary, since the higher pc solution typically implies extreme values and unreasonable behavior of pc, we select between these solutions by enforcing that the equilibrium chosen always chooses the lower pc solution.

A.3.2 Expected Returns

Combining the relations

$$\begin{aligned}
0 &= \log \mathbb{E}_t[M_{t+1}R_{t+1}] \\
&= \mathbb{E}_t[m_{t+1}] + \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\text{Var}_t(m_{t+1}) + \frac{1}{2}\text{Var}_t(r_{t+1}) + \text{Cov}_t(m_{t+1}, r_{t+1}) \\
-r_{f,t} &= \log \mathbb{E}_t[M_{t+1}] \\
&= \mathbb{E}_t[m_{t+1}] + \frac{1}{2}\text{Var}_t(m_{t+1})
\end{aligned}$$

and rearranging, we obtain

$$\begin{aligned}
\log \mathbb{E}_t[R_{t+1}/R_{f,t}] &= \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\text{Var}_t(r_{t+1}) - r_{f,t} \\
&= -\text{Cov}_t(m_{t+1}, r_{t+1}).
\end{aligned}$$

Since

$$
\begin{aligned}
r_{t+1} &= \text{const}_t + \underbrace{\kappa_1\Big(\mathbf{A}_s'\boldsymbol{\varepsilon}_{s,t+1} + \mathbf{A}_x'\boldsymbol{\varepsilon}_{x,t+1} + \mathbf{A}_\delta\boldsymbol{\varepsilon}_{\delta,t+1}\Big)}_{pc\text{ growth}} + \underbrace{\xi\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} + \varepsilon_{a,t+1}}_{\text{cash flow growth}} \\
m_{t+1} &= \text{const}_t - \gamma_{s,t}\mathbf{1}'\boldsymbol{\varepsilon}_{s,t+1} - x_t\varepsilon_{a,t+1}
\end{aligned}
$$

we obtain

$$
\text{Cov}_t(m_{t+1}, r_{t+1}) = -\gamma_{s,t}\big(\kappa_1\mathbf{A}_s' + \xi\mathbf{1}'\big)\boldsymbol{\Sigma}_s\mathbf{1} - x_t\sigma_a^2
$$

Substituting for $\gamma_{s,t}$ and rearranging yields (14).

A.3.3 Forecasting Real Rates

This section derives our 40Q average real rate forecast in the model. As an intermediate step, note that for a given matrix A, the geometric sum, assuming it converges, is equal to

$$
\sum_{j=0}^{\infty} A^j = I + A\sum_{j=0}^{\infty} A^j
$$

which implies

$$
\sum_{j=0}^{\infty} A^j = (I-A)^{-1}.
$$

Similarly the partial sum can be obtained as

$$
\begin{aligned}
\sum_{j=0}^{N-1} A^j &= \sum_{j=0}^{\infty} A^j - \sum_{j=N}^{\infty} A_j = \sum_{j=0}^{\infty} A^j - A^N\left(\sum_{j=0}^{\infty} A^j\right) = (I-A)^{-1} - (I-A)^{-1}A^N \\
&= (I-A)^{-1}(I-A^N).
\end{aligned}
$$

Applying this to our interest rate forecast, we have

$$
\bar{r}_{f,t}^N = \frac{1}{N}\sum_{j=0}^{N-1}\mathbb{E}_t\delta_{t+j} = \bar{\delta} + \frac{1}{N}\mathbf{1}'\sum_{j=0}^{N-1}\mathbb{E}_t\tilde{\boldsymbol{\delta}}_{t+j}.
$$

Since our law of motion for δ implies $\mathbb{E}_t\tilde{\boldsymbol{\delta}}_{t+j} = \boldsymbol{\Phi}_\delta^j\tilde{\boldsymbol{\delta}}_t$, we can substitute to obtain

$$\begin{aligned}
\bar{r}_{f,t}^N &= \bar{\delta} + \frac{1}{N}\mathbf{1}'\left(\sum_{j=0}^{N-1}\boldsymbol{\Phi}_\delta^j\right)\tilde{\boldsymbol{\delta}}_t \\
&= \bar{\delta} + \frac{1}{N}\mathbf{1}'(I - \boldsymbol{\Phi}_\delta)^{-1}(I - \boldsymbol{\Phi}_\delta^N)\tilde{\boldsymbol{\delta}}_t
\end{aligned}$$

where the last line follows from our partial geometric sum formula above.

A.3.4 Bond Pricing

We can represent our model in the form

$$\log M_{t+1} = -\delta_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\varepsilon_{t+1}$$

where

$$\begin{aligned}
\Lambda_t &= \Lambda_0 + \Lambda_1\mathbf{z}_t \\
&= \underbrace{\begin{bmatrix} \xi\bar{x} \\ 0 \\ 0 \\ \bar{x} \end{bmatrix}}_{\Lambda_0} + \underbrace{\begin{bmatrix} -\bar{x}\xi(\xi-1)\mathbf{1}'\boldsymbol{\Phi}_s & \xi\mathbf{1}' & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathbf{1}' & 0 & 0 \end{bmatrix}}_{\Lambda_1} \underbrace{\begin{bmatrix} \tilde{\mathbf{s}}_t \\ \tilde{\mathbf{x}}_t \\ \tilde{\boldsymbol{\delta}}_t \\ \Delta a_t \end{bmatrix}}_{\mathbf{z}_t}.
\end{aligned}$$

To price a zero-coupon bond of maturity n, we guess that the log bond price $p_{n,t}$ takes the functional form

$$p_{n,t} = A_n + B_n'\mathbf{z}_t.$$

This guess is trivially verified for $n = 0$, with $p_{0,t} = 0$ implying the initialization $A_0 = 0$, $B'_0 = 0$. To prove by induction, assume the claim holds for n. Then we have

$$\begin{aligned}
p_{n+1,t} &= \log E_t \exp\left\{-\delta_t - \frac{1}{2}\Lambda'_t\Sigma\Lambda_t - \Lambda'_t\varepsilon_{t+1} + A_n + B'_n\Phi_z z_t + B'_n\varepsilon_{t+1}\right\} \\
&= \log E_t \exp\left\{-\delta_t - \frac{1}{2}\Lambda'_t\Sigma\Lambda_t + (B'_n - \Lambda'_t)\varepsilon_{t+1} + A_n + B'_n\Phi_z z_t\right\} \\
&= \log E_t \exp\left\{-\delta_0 - \delta'_1 z_t - \frac{1}{2}\Lambda'_t\Sigma\Lambda_t + (B'_n - \Lambda'_t)\varepsilon_{t+1} + A_n + B'_n\Phi_z z_t\right\} \\
&= -\delta_0 - \delta'_1 z_t - \frac{1}{2}\Lambda'_t\Sigma\Lambda_t + \frac{1}{2}B'_n\Sigma B_n - B'_n\Sigma\Lambda_t + \frac{1}{2}\Lambda'_t\Sigma\Lambda_t + A_n + B'_n\Phi_z z_t \\
&= -\delta_0 - \delta'_1 z_t + \frac{1}{2}B'_n\Sigma B_n - B'_n\Sigma\Lambda_0 - B'_n\Sigma\Lambda_1 z_t + A_n + B'_n\Phi_z z_t \\
&= \left(-\delta_0 + \frac{1}{2}B'_n\Sigma B_n - B'_n\Sigma\Lambda_0 + A_n\right) + \left(-\delta'_1 - B'_n\Sigma\Lambda_1 + B'_n\Phi_z\right) z_t
\end{aligned}$$

which implies

$$\begin{aligned}
A_{n+1} &= -\delta_0 + \frac{1}{2}B'_n\Sigma B_n - B'_n\Sigma\Lambda_0 + A_n \\
B'_{n+1} &= -\delta'_1 - B'_n\Sigma\Lambda_1 + B'_n\Phi_z.
\end{aligned}$$

This both completes the proof and provides the recursion used to compute long-term real bond prices in our model.

A.4 Estimation Details

This section provides additional details on our state space specification and estimation procedure.

Time Variation in State Space. To begin, we provide additional details on time variation in our state space measurement equation (15). Because our risk premium measure rp_t is not available over the full sample, we use a time varying measurement equation to accommodate the missing values. In periods when our measure of the risk premium is available

(1996:Q1 to 2012:Q1), equation (15) takes the form

$$\begin{bmatrix} ey_t \\ r_{ft} \\ \bar{r}^{40}_{f,t} \\ py_t \\ \Delta y_t \\ rp_t \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \mathbf{z}_t + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

where $\mathbf{H}_2$ and $\mathbf{b}_2$ are the rows of the measurement matrix and constant vector that compute the implied value for rp_t, and $\mathbf{H}_1$ and $\mathbf{b}_1$ are the respective values for the other observables. In periods when data on rp_t is not available, equation (15) instead takes the form

$$\begin{bmatrix} ey_t \\ r_{ft} \\ \bar{r}^{40}_{f,t} \\ py_t \\ \Delta y_t \end{bmatrix} = \mathbf{H}_1 \mathbf{z}_t + \mathbf{b}_1.$$

A completely analogous procedure is used for the SPF forecast variable $\bar{r}^{40}_{f,t}$, since these forecasts are also not available over the entire sample. During periods when the forecast data are available, we expand the measurement equation (15) to include an additional row, while for periods when the data are not available, we omit this row.

MCMC Details. We next describe the procedure used to obtain the parameter draws. First, because some of our variables are bounded by definition (e.g., volatilities cannot be negative), we define a set of parameter vectors satisfying these bounds denoted Θ. We exclude parameters outside of this set, which formally means that we apply a Bayesian prior

$$p(\boldsymbol{\theta}) = \begin{cases} \text{const} & \text{for } \boldsymbol{\theta} \in \Theta \\ 0 & \text{for } \boldsymbol{\theta} \notin \Theta \end{cases}$$

Our restrictions on Θ are as follows: all volatilities (σ) and the average risk price $\bar{x}$ are bounded below at zero. All persistence parameters (ϕ) are bounded between zero and unity.

With these bounds set, we can evaluate the posterior by

$$\pi(\boldsymbol{\theta}) = L(y|\boldsymbol{\theta})p(\boldsymbol{\theta}).$$

so that the posterior is simply proportional to the likelihood over Θ and is equal to zero outside of Θ.

To draw from this posterior, we use a Random Walk Metropolis Hastings algorithm. We initialize the first draw $\boldsymbol{\theta}_0$ at the mode, and then iterate on the following algorithm:

1. Given $\boldsymbol{\theta}_j$, draw a proposal $\boldsymbol{\theta}^*$ from the distribution $\mathcal{N}(\boldsymbol{\theta}_j, c\boldsymbol{\Sigma}_\theta)$ for some scalar c and matrix $\boldsymbol{\Sigma}_\theta$ defined below.

2. Compute the ratio
$$\alpha = \frac{\pi(\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}_j)}.$$

3. Draw u from a Uniform [0, 1] distribution.

4. If $u < \alpha$, we accept the proposed draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}^*$. Otherwise, we reject the draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j$.

For the covariance term, we initialize $\boldsymbol{\Sigma}_\theta$ to be the inverse Hessian of the log likelihood function at the mode. Once we have saved 10,000 draws, we begin updating $\boldsymbol{\Sigma}_\theta$ to be the sample covariance of the draws to date, following Haario, Saksman, Tamminen et al. (2001), with the matrix re-computed after every 1,000 saved draws. For the scaling parameter c, we initialize it at 2.4/length($\boldsymbol{\theta}$) as recommended in Gelman, Stern, Carlin, Dunson, Vehtari, and Rubin (2013). To target an acceptance rate for our algorithm of 25%, we adapt the approach of Herbst and Schorfheide (2014) in updating

$$c_{new} = c_{old} \cdot \left(0.95 + 0.1 \frac{\exp(16(x - 0.25))}{1 + \exp(16(x - 0.25))}\right)$$

after every 1,000 saved draws, where c_{old} is the pre-update value of c.

A.5 Additional Details

A.5.1 Bootstrap Bias Correction

The bootstrap bias corrected estimate for the AR(1) model of the log earnings share is computed as follows. First, we run the regression

$$s_t = a + \phi s_{t-1} + \varepsilon_{s,t} \tag{A.5}$$

to obtain the estimates $\hat{a}^{OLS}, \hat{\phi}^{OLS}$. We then bootstrapping many samples from the data generating process

$$s_t^j = \hat{a}^{OLS} + \hat{\phi}^{OLS} + \tilde{\varepsilon}_{s,t}^j$$

where residuals $\tilde{\varepsilon}_{s,t}^{j}$ are drawn with replacement from $\{\varepsilon_{s,t}\}$. For each j in 100,000 simulations, we repeat the regression (A.5) to obtain estimates $\hat{a}^j, \hat{\phi}^j$, which we average to obtain the estimates $\hat{a}^{boot}, \hat{\phi}^{boot}$. The approximate bias is computed as $\hat{\phi}^{OLS} - \hat{\phi}^{boot}$, implying that the corrected persistence estimator is obtained as

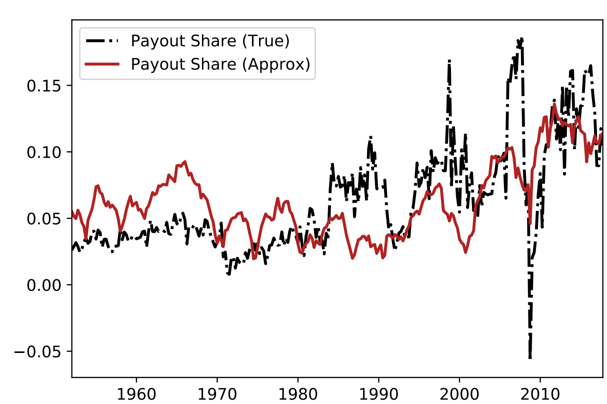
$$\hat{\phi}^* = \hat{\phi}^{OLS} + \left(\hat{\phi}^{OLS} - \hat{\phi}^{boot}\right).$$

A.6 Model-Implied Cash Flows

This section compares implied cash flows in the model to actual cash flows in the data. Because our model obtains implied cash flows from earnings, using (3), or its log-linear approximation (10), rather than using actual payout data, it is important to check that these series are sufficiently close.

Figure A.2 displays the model-implied cash flow share of output against the actual payout share from the data. The figure shows that the series line up moderately well, particularly toward the end of the sample, and that the model-implied series is able to reproduce a large rise in payouts over the second half of the sample. At the same time, the two series display nontrivial discrepancies at higher frequences.

Figure A.2: Cash Flow Share, Implied vs. Data

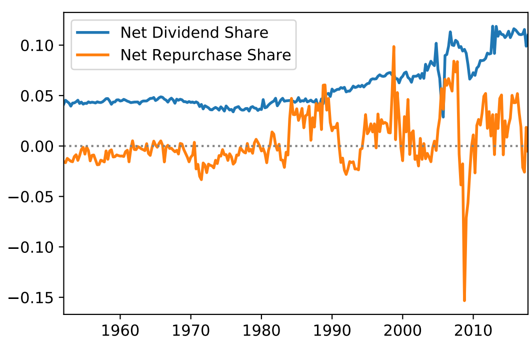


Notes: This figure compares the implied cash flow series used by our model $C_t = (S_t - \omega)E_t$, compared to the true corporate payout series in the data. The implied series takes S_t and E_t directly from the data, and uses the calibrated value $\omega = 0.0601$ consistent without our calibration of ξ.

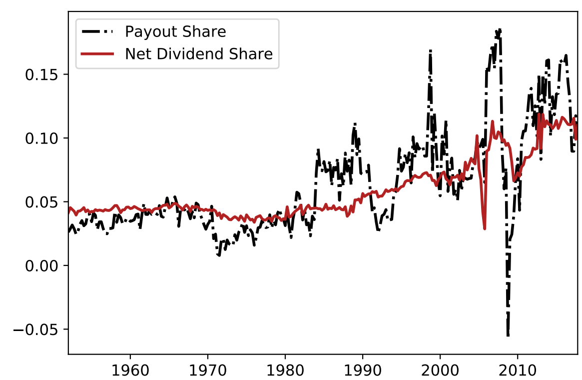
In our model, low frequency variation in cash flows are ultimately much more important for asset valuation. To focus in on the lower-frequency trend in cash flows, we split our payout data into its two component series: net dividends, and net repurchases (i.e., the negative of net equity issuance), displayed in Figure A.3. Figure A.3a display the raw series, showing that the net dividend share is always positive and exhibits slow and persistent dynamics, while the net repurchase share is close to zero on average and fluctuates wildly at high frequencies. Although repurchases have become an increasingly important form of payout, they largely cancel out with issuance of new equity, yielding a series without a major discernible trend. As a result, Panel (b) shows that we can effectively treat the net dividend share as a good measure of the low-frequency trend in payouts, with the exception of the transitory downward spike in net dividends during the financial crisis.

Figure A.3: Payout Data Components

(a) Net Dividend vs. Net Repurchase Shares



(b) Payout Share vs. Net Dividend Share



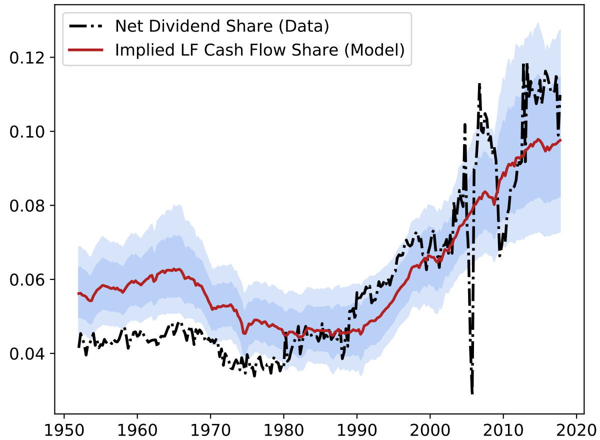
Notes: Panel (a) separately displays the two components of total payout: the net dividend share (equal to (net dividends) / (net value added) for the corporate sector) and the net repurchase share ($(-1)\times$ (net equity issuance) / (net value added) for the corporate sector). Panel (b) compares the net dividend share as just defined to the payout share (payouts / (net value added) for the corporate sector, where payouts are net dividends minus net equity issuance). Source: Flow of Funds.

With the net dividend share as a proxy for the low frequency trend in the payout share in the data, we can compare it to its counterpart in the model. This can be computed using (10) as

$$cy_{LF,t} = \overline{cy} + \xi \tilde{s}_{LF,t}. \tag{A.6}$$

The resulting series is displayed alongside the net dividend share in Figure A.4. Unlike the implied series for cy_t in Figure A.2, which can be computed directly from the data given ξ, computing $cy_{LF,t}$ depends on the decomposition of the earnings share into its low and high frequency components, and therefore on the model's parameter and latent state estimates. This leads to uncertainty in our estimate, characterized by the blue error bands, while the median is plotted in red.

Figure A.4: Low Frequency Cash Flow Share, Model vs. Data



Notes: Net Dividend Share is the ratio of net dividends to net value added for the corporate sector (source: Flow of Funds). Implied LF Cash Flow Share is equal to $cy_{LF,t}$ in equation (A.6). The red line represents the median outcome over our estimates, while the dark and light blue bands represent 66.7% and 90% confidence intervals, respectively.

Figure A.4 shows that the model's implied low frequency component cash flows delivers an excellent fit of the net dividend share in the data. The fit is particularly good over the subsample since 1989 on which our main results are based, and does not overstate the growth in payouts. The main discrepancy between the series is the transitory downward spike in the data during the financial crisis, which is not really representative of the low frequency trend, and in fact was more severe in the net dividends data than in the overall payout data, likely due to a slowdown in equity issuance at this time.

In summary, these results provide strong support for our model's approximate cash flow series (10). Although the implied and data series differ at high frequencies, these discrepancies largely reflect the timing of payouts, and should not play a huge role in equity pricing. At the same time, the model is highly effective at capturing the underlying trend in payouts through its low frequency component, providing an excellent fit for the data.

A.7 Additional Figures

Figure A.5: Model-Implied Risk-Free Rate Forecast

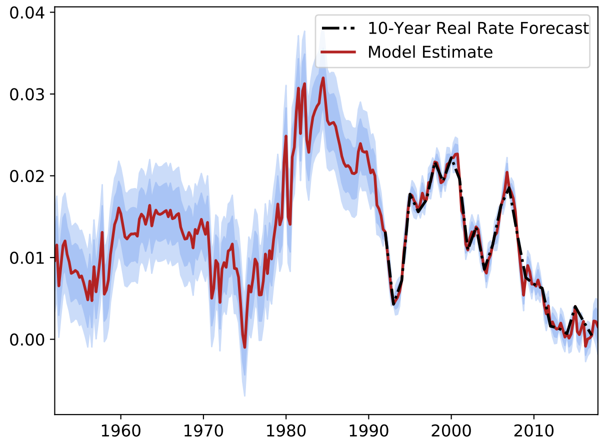
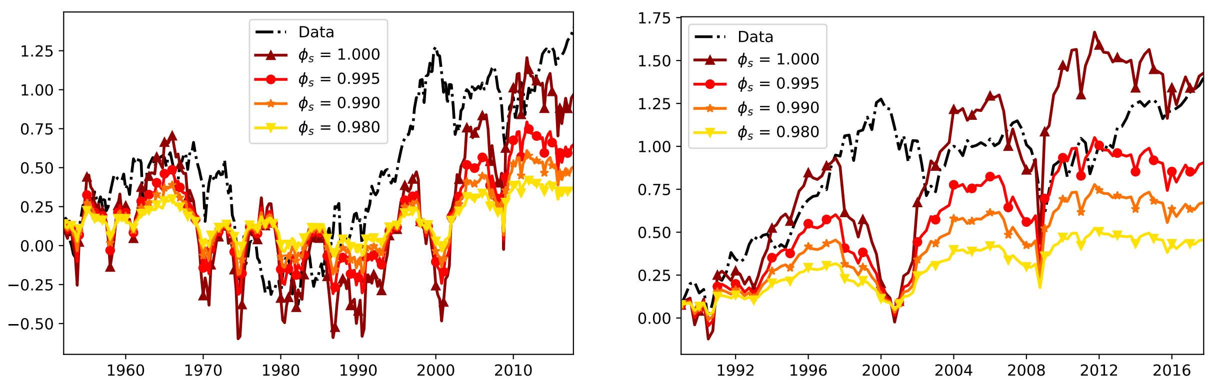


Figure A.6: Implied Contributions, AR(1) Models

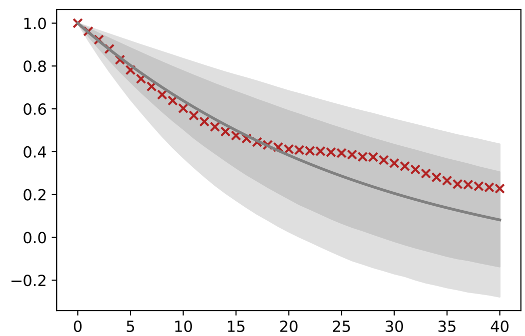
(a) Full Sample **(b) Since 1989**



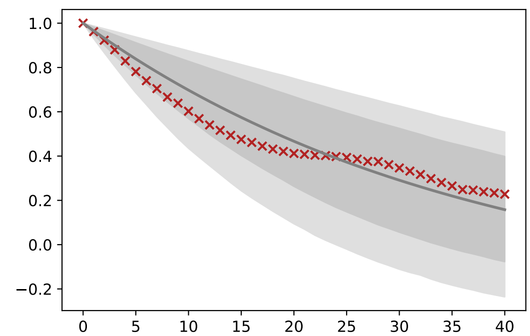
Notes: This figure superimposes the implied contributions of the earnings share to the log market equity to output ratio via cash flows measured as in equation (16), over the earnings share data. Each implied contribution is computed by adding the difference in the right hand side of (16) to the initial value of py_t at the start of the relevant subsample.

Figure A.7: Simulated Autocorrelations, AR(1) Models

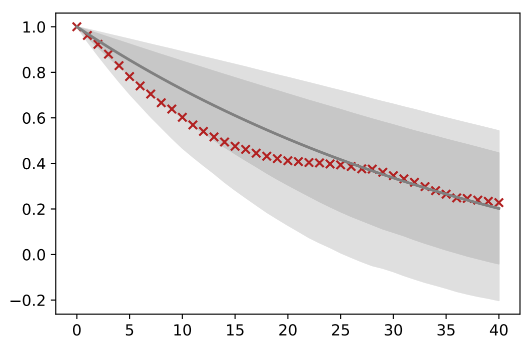
(a) $\phi_s = 0.98$



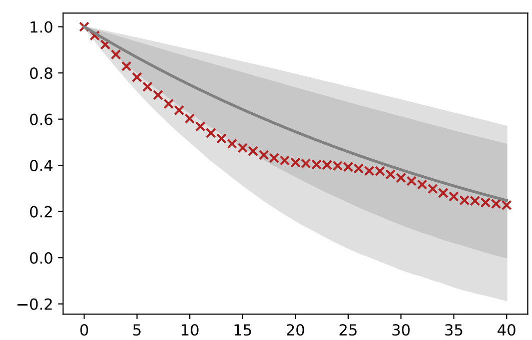
(b) $\phi_s = 0.99$



(c) $\phi_s = 0.995$



(d) $\phi_s = 1$



Notes: The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the simplified AR(1) models described in Section 6. For the model equivalents, we compute autocorrelations from each of 10,000 simulations the same length as the data, drawn with the persistence parameter found in that panel's title. The center line corresponds to the mean of these autocorrelations, while the dark and light gray bands represent 66.7% and 90% credible sets, respectively. The sample spans the period 1952:Q1-2017:Q4.