NBER WORKING PAPER SERIES

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Working Paper 25769
http://www.nber.org/papers/w25769

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2019, Revised April 2020

This paper supplants our earlier working paper "Origins of Stock Market Fluctuations." We are grateful to John Y. Campbell, Ralph Koijen, Edward Nelson, and Mindy Xiaolan for helpful comments, and to seminar participants at the Federal Reserve Board, the University of California Berkeley Haas School of Business, the University of Chicago Booth School of Business, and the University of Michigan Ross School of business for helpful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 25769
April 2019, Revised April 2020
JEL No. G0,G12,G17

ABSTRACT

Why do stocks rise and fall? From the beginning of 1989 to the end of 2017, $34 trillion of real equity wealth (2017:Q4 dollars) was created by the U.S. corporate sector. We estimate that 43% of this increase was attributable to a reallocation of rewards to shareholders in a decelerating economy, virtually all of which came at the expense of labor compensation. Economic growth accounted for just 25%, followed by a lower risk premium (24%), and lower interest rates (8%). From 1952 to 1988 less than half as much wealth was created, but economic growth accounted for more than 100% of it.

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1 Introduction

Why do stocks rise and fall? Surprisingly little academic research has focused directly on this question.[1] While much of the literature has concentrated on explaining expected quarterly or annual returns, this paper takes a longer view and considers the economic forces that have driven the total value of the market over the post-war era. According to textbook economic theories, the stock market and the broader economy should share a common trend, implying that the same factors that boost economic growth are also the key to rising equity values over longer periods of time.[2] In this paper, we directly test this paradigm.

Some basic empirical facts serve to motivate the investigation. While the U.S. equity market has done exceptionally well in the post-war period, this performance has been highly uneven over time, even at long horizons. For example, real market equity of the U.S. corporate sector grew at an average rate of 7.5% per annum over the last 29 years of our sample (1989 to 2017), compared to an average of merely 1.6% over the previous 22 years (1966 to 1988). At the same time, growth in the value of what was actually produced by the corporate sector has displayed a strikingly different temporal pattern. While real corporate net value added grew at a robust average rate of 3.9% per annum from 1966 to 1988 amid anemic stock returns, it averaged much lower growth of only 2.6% from 1989 to 2017 even as the stock market was booming. This multi-decade disconnect between growth in market equity and output presents a difficult challenge to theories in which economic growth is the key long-run determinant of market returns.

One potential resolution of this puzzle is to posit that economic fundamentals such as cash flows may be relatively unimportant for the value of market equity, with discount rates driving the bulk of growth even at long horizons. In this paper we entertain an alternative hypothesis motivated by an additional set of empirical facts. Within the total pool of net

[1] See the literature review below.

[2] This tenet goes back to at least Klein and Kosobud (1961), followed by a vast literature in macroeconomic theory that presumes balanced growth among economic aggregates over long periods of time. For a more recent variant, see Farhi and Gourio (2018).

value added produced by the corporate sector, only a relatively small share — averaging 12.3% in our sample — accrues to the shareholder in the form of after-tax profits. Importantly, however, this share varies widely and persistently over time, fluctuating from less than 8% to nearly 20% over our sample. This suggests that swings in the profit share are strong enough to cause large and long-lasting deviations between cash flows and output. If so, growth in market equity could diverge from economic growth for an extended period of time, even when valuations are largely driven by fundamental cash flows. Indeed, while the 1989-2017 period lagged the 1966-1988 period in economic growth, it exhibited growth in corporate *earnings* of 5.1% per annum that far outpaced the average 1.8% earnings growth of the previous period. Behind these trends are movements in the after-tax profit share of output, which fell from 15.3% in 1966 to 8.9% in 1988, before rising again to 17.4% by the end of 2017. These shifts are in turn made possible by a reverse pattern in labor's share of corporate output, which rises from 67.0% in 1966:Q1 to 72.4% in 1988:Q4, before reverting to 67.7% by 2017:Q4.

The upshot of these trends is a widening chasm between the stock market and the broader economy, a phenomenon displayed in Figure 1, which plots the ratio of market equity for the corporate sector to three different measures of aggregate economic activity: gross domestic product, personal consumption expenditures, and net value added of the corporate sector. (To make the units comparable, each series has been normalized to unity in 1989:Q1.) Despite substantial volatility in these ratios, each is at or near a post-war high by the end of 2017. Notably, however, the ratio of market equity to after-tax profits (earnings) for the corporate sector is far below its post-war high.

What role, if any, might these trends have played in the evolution of the post-war stock market? To translate these empirical facts into a quantitative decomposition of the post-war growth in market equity, we construct and estimate a model of the U.S. equity market. Although the specification of a model necessarily imposes some structure, our approach is intended to let the data speak as much as possible. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent factors, including not only factors driving productivity and profit shares, but also independent factors driving risk premia and risk-free interest rates.

Equity in our model is priced, not by a representative household, but by a representative shareholder, akin in the data to a wealthy household or large institutional investor. The remaining agents supply labor, but play no role in asset pricing. Shareholder preferences are subject to shocks that alter their patience and appetite for risk, driving variation in both the equity risk premium and in risk-free interest rates. Shareholders understand the laws of motion for these shocks and internalize them when forming expectations. Our representative shareholder consumes cash flows from firms, the variation of which is driven by shocks to the total rewards generated by productive activity, but also by shocks to how those rewards are divided between shareholders and other claimants.

We estimate the full dynamic model using state space methods, allowing us to precisely decompose the market's observed growth into these distinct component sources. The model is flexible enough to explain 100% of the change in equity values over our sample and at each point in time. To capture the influence of our primitive shocks at different horizons, we model each as a mixture of multiple stochastic processes driven by low- and high-frequency variation. Because our log-linear model is computationally tractable, we are able to account for uncertainty in both latent states and parameters using millions of Markov Chain Monte Carlo draws. We apply and estimate our model using data on the U.S. corporate sector over the period 1952:Q1-2017:Q4.

Our main results may be summarized as follows. First, we find that neither economic growth, risk premia, nor risk-free interest rates has been the foremost driving force behind the market's sharp gains over the last several decades. Instead, the single most important contributor has been a string of *factor share* shocks that reallocated the rewards of production without affecting the size of those rewards. Our estimates imply that the realizations of these shocks persistently reallocated rewards to shareholders, to such an extent that they account for 43% of the market increase since 1989. Decomposing the components of corporate output reveals that virtually all of these increases in the profit share came at the expense of labor compensation.

Second, while equity values were also boosted since 1989 by persistent declines in the equity risk premium and risk-free interest rate, these factors played smaller roles quantitatively, contributing 24% and 8.5%, respectively, to the increase in the stock market over this period.

Third, growth in the real value of corporate sector output contributed just 25% to the increase in equity values since 1989 and 54% over the full sample. By contrast, while economic growth accounted for more than 100% of the rise in equity values from 1952 to 1988 this 37 year period created less than half the equity wealth generated over the 29 years from 1989 to the end of 2017.

An implication of these findings is that the considerable gains to holding equity over the post-war period can be in large part attributed to an unpredictable sequence of factors share shocks that reallocated rewards to shareholders. We estimate that roughly 2.9 percentage points of the post-war average annual log return on equity in excess of a short-term interest rate is attributable to this string of favorable shocks, rather than to genuine ex-ante compensation for bearing risk. These results imply that the common practice of averaging return, dividend, or payout data over the post-war sample to estimate an equity risk premium is likely to overstate the true risk premium by about 44%.

As a by-product of our empirical implementation, we obtain an estimate of the conditional equity risk premium over time, a variable that should be of independent interest given the importance of this latent factor for theories of intangible capital and other determinants of macro-finance trends (e.g., Crouzet and Eberly (2020); Farhi and Gourio (2018)). By flexibly specifying the equity premium to be a mixture of processes with different components, our estimate is capable of simultaneously accounting for both the high frequency variation in the equity premium implied by options data (Martin (2017)), as well as the low frequency variation suggested by fluctuations in stock market valuation ratios. With the exception of an extreme spike upward during the financial crisis, we find that the equity premium has been declining for decades. By the end of 2017, our estimates imply that the equity premium had reached the record low levels attained previously in only two times: at the culminations of the tech boom in 2000 and the twin housing/equity booms in 2006.

The rest of this paper is organized as follows. The next section discusses related literature. Section 3 describes the theoretical model. Section 4 describes the econometric procedure and data. Section 5 presents our findings. Section 6 concludes.

2 Related Literature

The empirical asset pricing literature has traditionally focused on explaining stock market expected returns, typically measured over monthly, quarterly or annual horizons.[3] But as noted in Summers (1985), and still true today, surprisingly little attention has been given to understanding what drives the real level of the stock market over time. Previous studies have noted an apparent disconnect between economic growth and the rate of return on stocks over long periods of time, both domestically and internationally (see e.g., Estrada (2012); Ritter (2012); Siegel (2014)). But they have not provided a model and evidence on the economic foundations of this disconnect or on the alternative forces that have driven the market in post-war U.S. data, a gap our study is intended to fill.

In this regard, the two papers closest to this one are Lettau and Ludvigson (2013) and our previous work entitled "Origins of Stock Market Fluctuations," (Greenwald, Lettau, and Ludvigson (2014), GLL). These papers emphasized the relevance of factor share shocks in the data for explaining stock market values, but they differ in a number of substantive ways from the present study. Lettau and Ludvigson (2013) was a purely empirical exercise that investigated three shocks from a VAR, while GLL presented a model of the stochastic discount factor (SDF) to interpret these VAR shocks. But neither paper undertook a complete estimation of an equity pricing model, and the theoretical framework in GLL was less general and less flexible than that of this paper. In further contrast to GLL, which used a calibrated model to match simulated and data moments, we use state space methods to directly estimate the model on the time series data, allowing us to obtain maximum likelihood estimates of the parameters, as well as recover the latent state variables that have driven the actual equity prices over our sample. The model of the SDF in this paper also adds additional state variables not present in the model of GLL that allow for time variation in risk free interest rates, as well as separate low- and high-frequency components driving equity premia and

[3] A body of research has addressed the question of whether expected returns or expected dividend growth drive valuation ratios, e.g., the price-dividend ratio, but this analysis is silent on the the primitive economic shocks that drive expected returns or dividend growth. For reviews of empirical asset pricing literature, see Campbell, Lo, and MacKinlay (1997), Cochrane (2005), and Ludvigson (2012).

the share of rewards accruing to shareholders. Finally, the model in this paper also does away with a commonplace but implausible assumption that cash payments to shareholders are equal to earnings, by allowing for reinvestment.

Like GLL and Lettau, Ludvigson, and Ma (2018), the model of this paper adopts a heterogeneous agent perspective characterized by two types of agents and imperfect risk sharing between them: wealth is concentrated in the hands of a few investors, or "shareholders," while most households are "workers" who finance consumption primarily out of wages and salaries. This aspect adds an important element of realism to the model, since only about half of households report owning stocks either directly or indirectly in 2016. More importantly, even among those households that own equity, most own very little: the top 5% of the stock wealth distribution owns 76% of the stock market value and earns a relatively small fraction of income as labor compensation.[4] In this sense our model relates to a classic older literature emphasizing the importance for stock pricing of limited stock market participation and heterogeneity (Mankiw (1986), Mankiw and Zeldes (1991), Constantinides and Duffie (1996), Vissing-Jorgensen (2002), Ait-Sahalia, Parker, and Yogo (2004), Guvenen (2009), and Malloy, Moskowitz, and Vissing-Jorgensen (2009)). In contrast to this literature, our results suggest the relevance of frameworks in which investors are concerned about shocks that have opposite effects on labor and capital. Such redistributive shocks play no role in the traditional limited participation literature.

Besides Lettau and Ludvigson (2013), GLL, and Lettau, Ludvigson, and Ma (2018), a growing body of literature considers the role of redistributive shocks in asset pricing or macro models, most in representative agent settings (Danthine and Donaldson (2002); Favilukis and Lin (2013a, 2013b, 2015), Gomez (2016), Marfe (2016), Farhi and Gourio (2018)).

[4]Source: 2016 Survey of Consumer Finances (SCF). In the 2016 SCF, 52% of households report owning stock either directly or indirectly. Stockowners in the top 5% of the net worth distribution had a median wage-to-capital income ratio of 27%, where capital income is defined as the sum of income from dividends, capital gains, pensions, net rents, trusts, royalties, and/or sole proprietorship or farm. Even this low number likely overstates traditional worker income for this group, since the SCF and the IRS count income paid in the form of restricted stock and stock options as "wages and salaries." Executives who receive substantial sums of this form would be better categorized as "shareholders" in the model below, rather than as "workers" who own no (or very few) assets.

In this literature, labor compensation is a charge to claimants on the firm and therefore a source of cash-flow variation in stock and bond markets. In contrast to the limited participation/heterogeneous agent paradigm pursued here, representative agent models imply that a variant of the consumption CAPM using aggregate consumption still prices equity returns, so those frameworks cannot not account for the evidence in Lettau, Ludvigson, and Ma (2018) that the capital (i.e., nonlabor) share of aggregate income exhibits significant explanatory power for expected returns across a range of equity characteristic portfolios and non-equity asset classes.

The factors share element of our paper is related to a separate macroeconomic literature that examines the long-run variation in the labor share (e.g., Karabarbounis and Neiman (2013), and the theoretical study of Lansing (2014)). The factors share findings in this paper also echo those from previous studies that use very different methodologies but find that returns to human capital are negatively correlated with those to stock market wealth (Lustig and Van Nieuwerburgh (2008); Lettau and Ludvigson (2009); Chen, Favilukis, and Ludvigson (2014))).

Farhi and Gourio (2018) extend a representative agent neoclassical growth model to allow for time varying risk premia and study the sources of macro-finance trends in recent data. They find a large role for rising market power in the high returns to equity, similar to our findings regarding the importance of the factor share shock for driving equity values over the post-war period. An appealing feature of their approach is that it specifies a structural model of production that takes a firm stand on the sources of variation in the earnings share. Corhay, Kung, and Schmid (2018) find a similar result that they likewise attribute to rising market power in a rich model of the firm investment margin. But it is fair to say that the literature has not yet reached a consensus on the key structural features of the economy that drive variation over time in the earnings/labor share (e.g., see the differing explanations in Autor, Dorn, Katz, Patterson, and Van Reenen (2017), Hartman-Glaser, Lustig, and Xiaolan (2016), and Kehrig and Vincent (2018)).

Our modeling and estimation approach is designed to quantify what role the earnings share has played in stock market fluctuations, without requiring us to take a stand on the structural model that may have produced those equilibrium observations. We do this with estimates that match the observed earnings share exactly (i.e., without error) over the sample and at each point in time, and by estimating full transition dynamics for each factor that drives asset returns. This approach contrasts with that of Farhi and Gourio (2018) and Corhay, Kung, and Schmid (2018), who make inferences by estimating the time-invariant deep parameters of their structural model on different subsamples of the data and then making comparisons across subsamples. We discuss the implications of these differing methodological approaches further below.

3 The Model

The economy is populated by a representative firm that produces aggregate output, and two types of households. The first type are investors who typify those that own the majority of equity wealth in the U.S. These could be wealthy households or large institutional investors. They may borrow and lend amongst themselves in the risk-free bond market. We refer to these investors simply as "shareholders." The second type are hand-to-mouth "workers" who finance consumption out of wages and salaries. The model is stylized, as we suppose that workers own no assets and consume their labor earnings. Taken literally, these coarsely different groups are an obvious abstraction from the real world. But we argue that they are a reasonable first approximation of the data given the high concentration of wealth at the top, the evidence that the wealthiest earn the overwhelming majority of their income from ownership of assets or firms, and that households outside of the top 5% of the stock wealth distribution own far less financial wealth of any kind.[5]

Aggregate output is governed by a constant returns to scale process:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}, \tag{1}$$

where A_t is a mean zero factor neutral total factor productivity (TFP) shock, N_t is the

[5] See discussion above. In the 2016 SCF, the median household in the top 5% of the stock wealth distribution had $2.97 million in nonstock financial wealth. By comparison, households with no equity holdings had median nonstock financial wealth of $1,800, while all households (including equity owners) in the bottom 95% of the stock wealth distribution had median nonstock financial wealth of $17,480. Additional evidence is presented in Lettau, Ludvigson, and Ma (2019).

aggregate labor endowment (hours times a productivity factor) and K_t is input of capital, respectively. Workers inelastically supply labor to produce output. Capital grows deterministically at a gross rate $G = \exp(g)$, while labor productivity grows deterministically at the same rate. Hours of labor supplied are fixed and normalized to unity. Taken together, these assumptions imply that $Y_t = A_t G^{t\alpha} K_0 G^{t(1-\alpha)} = A_t K_0 G^t$, where K_0 is the fixed initial value of the capital stock.

In the accounting framework of the data, a fraction τ_t of Y_t is devoted to taxes and interest (and a catchall of "other" charges against earnings). We refer to τ_t simply as the "tax and interest" share for brevity. The remaining $1 - \tau_t$ is divided between labor compensation and domestic after-tax profits (domestic earnings, E_t^D). Labor compensation in the model is equal to $W_t N_t$, where W_t is an aggregate wage per unit of productivity. Define $Z_t \equiv 1 - \tau_t$ and let S_t^D denote the domestic after-tax profit share of combined after-tax profit and domestic labor compensation. We refer to S_t^D as the *domestic profit share* for short. Last, total earnings of the firm includes retained earnings from firms' foreign subsidiaries $E_t^F \equiv F_t Y_t$, where F_t is the ratio of foreign earnings to domestic output Y_t. Total earnings, $E_t = E_t^D + E_t^F$, are identically equal to[6]

$$E_t \equiv S_t Y_t = \left(S_t^D Z_t + F_t\right) Y_t.$$

Labor compensation is defined by

$$W_t N_t \equiv \left(1 - S_t^D\right) Z_t Y_t,$$

where $\left(1 - S_t^D\right)$ is the labor share of combined after-tax profits and labor compensation. The ratio $E_t / Y_t = S_t \equiv \left(S_t^D Z_t + F_t\right)$ is the total earnings share of domestic output, referred to hereafter as the *earnings share* for short. The variable $\left(1 - S_t^D\right)$ is the domestic labor share of after-tax domestic output, referred to hereafter as the *labor share* for short.

The variable S_t is modeled as an exogenous stochastic process with an innovation that we refer to as a *factor share shock*. This aggregate shock captures changes that may occur for any reason in the ratio of total earnings to domestic output. One component of that is the domestic after-tax profit share S_t^D, which moves inversely with the domestic labor

[6]This identity follows NIPA accounting for the corporate sector.

share. This component captures changes in the allocation of rewards between firms and workers in an imperfectly competitive environment, while holding fixed the size of those rewards. Possible sources of variation in S_t^D could include changes in industry concentration structure that alter the labor intensivity of aggregate production, changes in the bargaining power of U.S. workers due to international competition the prevalence of worker unions, changes in employment practices such as offshoring and outsourcing, or technological factors that alter how substitutable are labor and technology or capital inputs.[7] Earnings from overseas affiliates and taxes and interest make up the remaining components of the factor share process S_t.

In summary, the firm's total earnings can vary over time for four reasons. First, the level of output and economic growth may change, which affects the size of the domestic economic pie Y_t. Second, shifts in S_t^D alter the allocation of rewards between shareholders and domestic workers independently of Y_t. Third, the foreign retained earnings share of domestic output F_t can change over time. Fourth, the tax and interest wedge Z_t may vary over time.

We model the total earnings share S_t as an aggregate process, which makes it convenient to handle correlated components. For example, offshoring could lead to both an increase in the foreign earnings share F_t and in the domestic profit share S_t^D, if offshoring is partly motivated by a desire to replace domestic labor with less expensive foreign labor. To get a sense of the individual roles of S_t^D, Z_t, and F_t in driving the corporate earnings share, we report decompositions of S_t below that hold fixed one more components over time.

The firm makes cash payments to shareholders, which differs from domestic earnings by net new investment. Net new investment is required to attain long term (steady state) growth in output at the rate g. Specifically, the firm is modeled as reinvesting a fixed fraction ωY_t of current output each period. Thus, cash flows to shareholders, denoted C_t, are defined

[7] A literature has evolved to explain the decline in the aggregate labor share in the past 30 years. Some have argued that the aggregate labor share has fallen due to a reallocation of value added toward a few "superstar" firms with low labor shares (e.g., Autor, Dorn, Katz, Patterson, and Van Reenen (2017) Hartman-Glaser, Lustig, and Xiaolan (2016); Kehrig and Vincent (2018)). In our model this would show up as a decline in $1 - S_t^D$.

by

$$C_t = E_t - \omega Y_t = (S_t - \omega) Y_t \tag{2}$$

The variable C_t is net payout, defined as the sum of net dividend payments and net equity issuance. It encompasses any cash distribution to shareholders including share repurchases, which have become the dominant means of returning cash to shareholders in the U.S. For brevity, we shall refer to these payments simply as "cashflows."

Embedded in this model of reinvestment is the implicit presumption that the firm has access to a technology or market that allows it to swap the stochastic stream ωY_t for a fixed increment g to the net growth rate of Y_t forever. More generally, it is a simple way to capture the empirical fact that firms in aggregate retain part of their revenue for reinvestment, so that C_t is invariably less than E_t on average, and that this required reinvestment depends on output rather than the profit share.

Let C_{it}^s denote the consumption of an individual stockholder indexed by i at time t. Identical shareholders maximize the function

$$U = E \sum_{t=0}^{\infty} \prod_{k=0}^{t} \beta_k u\left(C_{it}^s\right) \tag{3}$$

with

$$u\left(C_{it}^s\right) = \frac{\left(C_{it}^s\right)^{1-x_{t-1}}}{1 - x_{t-1}}, \tag{4}$$

where β_t is a time-varying subjective time discount factor. The parameter x_t is not constant but instead varies stochastically over time. This variable, which can be thought of as a time-varying sentiment or preference shifter that shareholders take as given, is a latent state that drives the price of risk in the stochastic discount factor (SDF). Since an SDF always reflects both preferences and beliefs, an increase in x_t may be thought of as either an increase in effective risk aversion or an increase in pessimism about shareholder consumption. Thus, x_t may occasionally go negative, reflecting the possibility that investors sometimes behave in a confident or risk tolerant manner.[8]

[8] This does not imply a negative unconditional equity risk premium, however, since investors in the model can occasionally behave in a risk tolerant manner while still being averse to risk on average. Indeed, our estimates reported below imply a substantial positive mean equity premium.

Shareholder preferences are also subject to an externality in the subjective discount factor β_t. A time-varying specification for the subjective time discount factor is essential for obtaining a stable risk-free rate along with a volatile equity premium. If instead the subjective time discount factor were itself a constant, shocks to x_t and cashflow growth would generate counterfactual volatility in the risk-free rate.

Worker preferences play no role in asset pricing since they hold no assets. We assume that equities are priced by a representative *shareholder* who owns the entire corporate sector. In equilibrium, identical shareholders will have identical consumption, equal to per capita aggregate cashflows.[9] We therefore drop the i subscript and simply denote the consumption of a representative shareholder C_t from now on. This specification should be distinguished from the more common approach of modeling a representative *household* in which aggregate (average) consumption is the relevant source of systematic risk. In the model of this paper, it is not aggregate consumption but instead aggregate shareholder cashflows that are the appropriate source of systematic risk. Moreover, redistributive shocks such as those considered here that shift the share of income between labor and capital shift shareholder consumption are a source of systematic risk for asset owners. This implication has been explored by (Lettau, Ludvigson, and Ma (2019)) who study risk pricing in a large number of cross-sections of return premia.

The intertemporal marginal rate of substitution of shareholder consumption is the SDF and takes the form

$$\begin{aligned} M_{t+1} &= \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-x_t} \\ \ln M_{t+1} &= -\mathbf{1}'\delta_t - d_t - x_t \Delta \ln C_{t+1} \end{aligned} \tag{5}$$

where the subjective time discount factor $\beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)}$ is specified below. This SDF is a more general version of the lognormal models considered in previous work, (e.g., Campbell and

[9]This need not imply that individual shareholders are hand-to-mouth households. They may borrow and save in the risk-free bond and could have idiosyncratic investment income drawn from an identical distribution. But they are assumed to be able to perfectly share any identical idiosyncratic risk with other shareholders. This implies that in equilibrium, they each consume per capita aggregate shareholder cashflows C_t. See the Appendix for a simplified model.

Cochrane (1999) and Lettau and Wachter (2007)). As in these models, the preference shifter x_t is taken as an externality (akin to an external habit) that is the same for each stockowner and represents the market's willingness to bear risk.

3.1 A Loglinear Model

We work with a loglinear approximation of the model that can be solved analytically. A loglinear model facilitates estimation by permitting the resulting system of equations to be written in state space form so that the Kalman filter may be used to infer unobserved states. Such an approximation implies that the earnings share could, in principle, go above unity in the loglinear model. It never does so in practice, however, because the estimated earnings share process is over 14 standard deviations away from unity in steady state. It follows that such an event happens with effectively zero probability in a very long simulation of the model.

The model we explore has seven latent state variables and seven latent shocks whose evolutions are described below. These state variables include a factors share process s_t, a subjective time-discount factor process δ_t, and a latent price of risk process x_t, each of which are modeled as influenced by low- and high frequency components. In addition to shocks to each of these latent state components, a seventh latent shock is the i.i.d. innovation to TFP growth. For notation, we use lower case letters to denote log variables, e.g., $\ln(Y_t) = y_t$.

All shocks are modeled as Gaussian, independent over time, and mutually uncorrelated. This allows us to decompose the market's growth into distinct component sources. The econometric procedure is, however, free to estimate a small or even zero variance parameter for any of the mutually uncorrelated shocks. Thus the estimates give a sense of the quantitative importance of the mutually uncorrelated components of the latent series.

3.1.1 Earnings and Cashflow Growth

TFP is a random walk in logs, implying that the log difference is independently and identically distributed (i.i.d.):

$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \sim N\, i.i.d. \left(0, \sigma_a^2\right). \tag{6}$$

Since $Y_t = A_t K_0 G$, we have $y_t = a_t + k_0 + g$ and log output growth is

$$\Delta y_{t+1} = g + \varepsilon_{a,t+1}. \tag{7}$$

With earnings $E_t = S_t Y_t$, log earnings growth is

$$\Delta e_t = \Delta s_t + \Delta y_t.$$

Cash payments to shareholders are $C_t = (S_t - \omega) Y_t$. We may log-linearize this equation around $c_t - y_t = \overline{cy}$ to obtain the approximate relation

$$c_t - y_t = \overline{cy} + \xi s_t + y_t \tag{8}$$

where

$$\xi = \frac{\overline{S}}{\overline{S} - \omega}, \tag{9}$$

and $\overline{S}$ is the average value of S_t.

For convenience, we generalize the cash flow growth equation to the more abstract expression

$$\Delta c_t = \boldsymbol{\xi}' \Delta \mathbf{s}_t + \Delta y_t.$$

Using the vector $\mathbf{s}_t$ flexibly allows us to model any of the components s_t as a mixture of multiple stochastic processes. This is particularly useful for the factor share process s_t, since an inspection of its time series plainly suggests that it contains both lower and higher frequency sources of variation (as shown below in several figures). We accommodate this in the model by specifying the mixture of low- and higher-frequency components:

$$s_t = s_{LF,t} + s_{HF,t}$$

where one log component, $s_{LF,t}$, will be subject to lower frequency variation compared to the other, less persistent, component, denoted $s_{HF,t}$. Thus we have $\mathbf{s}_t' = (s_{LF,t}, s_{HF,t})'$ and $\boldsymbol{\xi}' = (\xi, \xi)$.

The dynamics of cashflow and earnings share growth are specified as independent first-order autoregressive processes with the following equations:

$$\Delta c_{t+1} = \boldsymbol{\xi}' \Delta \mathbf{s}_{t+1} + \Delta y_{t+1} \tag{10}$$

$$\mathbf{s}_{t+1} = (\mathbf{I} - \boldsymbol{\Phi}_s)\overline{\mathbf{s}} + \boldsymbol{\Phi}_s \mathbf{s}_t + \boldsymbol{\varepsilon}_{s,t+1}, \qquad \boldsymbol{\varepsilon}_{s,t+1} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_s) \tag{11}$$

$$\Delta \mathbf{s}_{t+1} = -(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, \qquad \widetilde{\mathbf{s}}_t \equiv \mathbf{s}_t - \overline{\mathbf{s}} \tag{12}$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{\Phi}_s$ is a diagonal matrix with the first-order autocorrelation coefficient of each $\mathbf{s}_t$ element in the diagonal entries, $\bar{\mathbf{s}}$ is a vector containing the means of the components, and $\mathbf{\Sigma}_s$ is a diagonal covariance matrix. Entries with "tildes" over them indicate demeaned variables.

3.1.2 Stochastic Discount Factor and Risk Free Rate

The log of the SDF is an affine function of log cashflow growth times the price of risk, but also depends on the subjective time discount factor $\beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)}$. Variation in the subjective time discount factor affects the return on the risk-free asset whose value is known with certainty at time t and given in gross units by

$$R_{f,t+1} \equiv \left(\mathbb{E}_t\left[M_{t+1}\right]\right)^{-1}.$$

An inspection of the data on short rates of interest, e.g., the three-month Treasury bill (T-bill) rate, suggest that short rates also contain both lower and higher frequency sources of variation. As above, we accommodate this in the empirical model by allowing δ_t to contain two components that are multiplicative in levels or linear in logs, i.e., $\delta_t = \mathbf{1}'\boldsymbol{\delta}_t$, where $\boldsymbol{\delta}_t = (\delta_{LFt}, \delta_{HFt})'$. One component, δ_{LFt}, will be subject to lower frequency variation compared to the other less persistent component, denoted δ_{HFt}. We assume $\boldsymbol{\delta}_{t+1}$ follow a multivariate Gaussian process so that the log SDF becomes

$$\begin{aligned} m_{t+1} &\equiv \ln M_{t+1} = -\mathbf{1}'\boldsymbol{\delta}_t - d_t - x_t \Delta c_{t+1} \\ \boldsymbol{\delta}_{t+1} &= (\mathbf{I} - \mathbf{\Phi}_\delta)\bar{\boldsymbol{\delta}} + \mathbf{\Phi}_\delta \boldsymbol{\delta}_t + \boldsymbol{\varepsilon}_{\delta,t+1},\ \boldsymbol{\varepsilon}_{\delta,t+1} \sim N(0, \mathbf{\Sigma}_\delta), \end{aligned} \tag{13}$$

where $\mathbf{\Phi}_\delta$ is a 2×2 diagonal matrix with the first-order autocorrelation coefficient of each component in the diagonal entries, $\bar{\boldsymbol{\delta}}$ is the vector of means of the two components of $\boldsymbol{\delta}_t$ and $\mathbf{\Sigma}_\delta$ is a diagonal covariance matrix. The stochastic process $\boldsymbol{\delta}_t$ is a shock to the subjective time-discount factor that moves the risk-free rate independently of cashflow growth variation. The parameter d_t is a compensating factor chosen to ensure that the log risk-free rate $r_{f,t} = -\ln \mathbb{E}_t \exp\left(m_{t+1}\right)$ obeys the process

$$r_{f,t} = \mathbf{1}'\boldsymbol{\delta}_t.$$

With Gaussian shocks, the SDF is conditionally lognormal, which implies that the log risk-free rate takes the form

$$r_{f,t+1} = \mathbf{1}'\boldsymbol{\delta}_t + d_t + x_t \left[g - \boldsymbol{\xi}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t \right] - \frac{1}{2} x_t^2 \left(\sigma_a^2 + \xi' \boldsymbol{\Sigma}_s \xi \right).$$

It follows that

$$d_t = -x_t \left[g - \boldsymbol{\xi}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t \right] + \frac{1}{2} x_t^2 \left(\sigma_a^2 + \xi' \boldsymbol{\Sigma}_s \xi \right).$$

The SDF depends on the state variable x_t, which we also specify as the sum of two components to account for both high and low frequency variation:

$$x_t = \underbrace{\mathbf{1}'\mathbf{x}_t}_{x_{\perp,t}} + \boldsymbol{\lambda}'\widetilde{\mathbf{s}}_t \tag{14}$$

$$\mathbf{x}_{t+1} = \left(\mathbf{I} - \boldsymbol{\Phi}_x \right) \overline{\mathbf{x}} + \Phi_x \mathbf{x}_t + \varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \sim N\, i.i.d. \left(0, \Sigma_x \right).$$

where Φ_x is a diagonal matrix, and $\mathbf{x}_t' = (x_{LF,t}, x_{HF,t})'$. The two components in $\mathbf{x}_{t+1}$ capture variation in the price of risk that is orthogonal to the other components of the aggregate economic state. Denote this component $x_{\perp,t} \equiv \mathbf{1}'\mathbf{x}_t = x_{LF,t} + x_{HF,t}$. We refer to $x_{\perp,t}$ and its components as *orthogonal risk price* factors. The second term on the right-hand-side of (14) allows the price of risk to vary systematically with the profit share, potentially because the willingness to bear risk rises as the earnings share increases. The parameter $\boldsymbol{\lambda}'$ is a vector of constants with the same value for both entries: $\boldsymbol{\lambda}' = (\lambda, \lambda)$ and determines the extent to which the risk price varies with the profit share. Because $\boldsymbol{\lambda}$ is freely estimated under flat priors and the procedure is free to recover $\boldsymbol{\lambda} = \mathbf{0}$, we can assess the extent to which risk premia in fact vary with the earnings share empirically.

We note that restricting $\boldsymbol{\lambda} = \mathbf{0}$ would be inappropriate given the properties of the data. Figure 2 presents two plots. One displays the time-series variation in the corporate sector log earnings share of output, $e_t - y_t$, along with the corporate sector log price-earnings ratio, $p_t - e_t$. The other displays $e_t - y_t$ along with the Center for Research in Securities Prices (CRSP) log price-dividend ratio $p_t - d_t$. The plots show that both $p_t - e_t$ and $p_t - d_t$ are positively correlated with the earnings share $e_t - y_t$. This is especially true at lower frequencies. For example, the correlations between $e_t - y_t$ and $p_t - e_t$ and between $e_t - y_t$ and $p_t - d_t$ are 47% and 69%, respectively, for components of the raw series that retain fluctuations with cycles between 8 and 50 years. Were there is no correlation between the the earnings share and the rate at which shareholder's discount future payouts, these large positive correlations would be difficult to rationalize. To see why, suppose that shocks to the earnings share have no influence on discount rates. If shocks to the earnings share result in a rise in earnings that is persistent but transitory, as we find, prices in the model will rise less than proportionally with earnings in anticipation of their eventual mean reversion, thereby resulting in a *negative* correlation between the earnings share and valuation ratios. (See the equilibrium equations below.) That the data supports a positive correlation, implies that persistently high earnings shares must coincide with a decline in the expected stock market return, so that valuation ratios still rise even as earnings and shareholder payouts are rationally expected to decline in the future.[10]

3.1.3 Equilibrium Stock Market Values

Let P_t denote total market equity, i.e., price per share times shares outstanding. Then with C_t equal to total equity payout, we write the return on equity from the end of t to the end of $t+1$ as

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$

Define $pc_t \equiv \ln\left(\frac{P_t}{C_t}\right)$. The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{15}$$

where $\kappa_1 = \exp(\overline{pc}) / (1 + \exp(\overline{pc}))$, and $\kappa_0 = \exp(\overline{pc}) + 1 - \kappa_1 \overline{pc}$.

The first-order-condition for optimal shareholder consumption implies the following Euler equation:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right]. \tag{16}$$

[10] If shocks to the earnings share improved shareholder fundamentals permanently, the model would imply that such shocks drive prices up proportionally with earnings, leaving valuation ratios unaffected and the correlation zero.

The relevant state variables for the equilibrium pricing of equity are the two components of s_t, δ_t, and x_t (low and high frequency). We conjecture a solution to (16) taking the form

$$pc_t = A_0 + \mathbf{A}_s'\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x'\widetilde{\mathbf{x}}_t, \tag{17}$$

where "tildes" indicate deviations from the mean. The solution verified in the Appendix implies that the coefficients on these state variables take the form

$$\mathbf{A}_s' = -\left[\boldsymbol{\xi}'(\mathbf{I} - \ \boldsymbol{\Phi}_s) + \Big(\big(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\big)\Big)\boldsymbol{\lambda}'\right]\left[(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_s) + \kappa_1\boldsymbol{\Sigma}_s\boldsymbol{\xi}\lambda'\right]^{-1}$$

$$\mathbf{A}_x' = -\left[\Big(\big(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\big) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big)\mathbf{1}'\right](\mathbf{I} - \kappa_1\boldsymbol{\Phi}_x)^{-1}$$

$$\mathbf{A}_\delta' = -\mathbf{1}'(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_\delta)^{-1}$$

The coefficients $\mathbf{A}_x$ and $\mathbf{A}_\delta$ are all negative, while the sign of the coefficients for $\mathbf{A}_s$ depend on the value of $\boldsymbol{\lambda}$. The signs of the coefficients $\mathbf{A}_\delta'$ and $\mathbf{A}_x'$ imply that an increase in the risk-free rate or an increase in the price of risk x_t originating from either component reduces the price-cashflow ratio because either event increases the rate at which future payouts are discounted. The size of these effects depend on the persistence of the movements in the risk-free rate and the price of risk, as captured by $\boldsymbol{\Phi}_\delta$ and $\boldsymbol{\Phi}_x$. The more persistent the shocks, the larger the effects.

The sign of the elements of $\mathbf{A}_s'$ depends on the sign of $\boldsymbol{\lambda}$. For $\boldsymbol{\lambda} = 0$, the elements of $\mathbf{A}_s'$ are also negative, since the elements of $\boldsymbol{\Phi}_s$ are both less than unity. In this case, any increase in the earnings share, while possibly highly persistent, is ultimately transitory and therefore delivers a transitory increase in cashflows to shareholders. Thus, with $\boldsymbol{\lambda} = \mathbf{0}$, shocks to either component of s_t cause equity values to rise proportionally less than current cashflows in anticipation of eventual mean reversion in payout, reducing pc_t and leading it to be negatively related to changes in $\widetilde{\mathbf{s}}_t$. The size of these effects depend on the persistence of the $\widetilde{\mathbf{s}}_t$ process, captured by the elements of $\boldsymbol{\Phi}_s$, with more persistent affects translating into smaller movements in pc_t, where the effects approach zero as the elements of $\boldsymbol{\Phi}_s$ approach unity. By contrast, if $\boldsymbol{\lambda} < \mathbf{0}$, implying that the price of risk falls when the earnings share rises, positive shocks to s_t may drive up pc_t, depending on the magnitude of $\boldsymbol{\lambda}$.

As shown in the Appendix, the model solution implies that the log equity premium is given by

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = \left[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2\right) + \kappa_1'\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s\right]\left(\mathbf{1}'\widetilde{\mathbf{x}}_t + \boldsymbol{\lambda}'\widetilde{\mathbf{s}}_t\right) - \frac{1}{2}\mathbb{V}_t(r_{t+1}),$$

$$\begin{aligned}\mathbb{V}_t(r_{t+1}) = \kappa_1^2 & \left(\mathbf{A}_s'\boldsymbol{\Sigma}_s\mathbf{A}_s + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\mathbf{A}_\delta\right) \\ & + \boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2 + 2\kappa_1'\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s,\end{aligned}$$

where $\mathbb{V}_t(\cdot)$ denotes variance conditional on time t information. The conditional variance is constant due to homoskedasticity of the shocks, but the equity premium varies over time with the price of risk x_t and, if $\boldsymbol{\lambda}$ is non-zero, the factor shares components in $\mathbf{s}_t$.

4 Estimation and Data

The model just described consists of a vector of primitive parameters

$$\boldsymbol{\theta} = \left(\boldsymbol{\xi}, g, \sigma_a^2, \operatorname{diag}\left(\boldsymbol{\Phi}_s\right)', \operatorname{diag}\left(\boldsymbol{\Phi}_x\right)', \operatorname{diag}\left(\boldsymbol{\Phi}_\delta\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_s\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_x\right)', \operatorname{diag}\left(\boldsymbol{\Sigma}_\delta\right)', \bar{s}, \bar{\delta}, \bar{x}, \right)',$$

where vec $(\cdot)$ denotes the vectorization of a matrix and variables with "bars" indicate means.

With the exception of a small group of parameters, discussed below, the primitive parameters are freely estimated.[11] We estimate these parameters using Bayesian methods with flat priors. Since the model is linear in logs, the latent states are recovered using the Kalman filter and inferred jointly with the primitive parameters of the model.

We use observations on five data series: the log share of domestic output accruing to earnings (the earnings share), denoted $e_t - y_t \equiv ey_t$,[12] a measure of a short term real interest rate as a proxy for the log risk-free rate, denoted $r_{f,t+1}$, growth in output for the corporate sector as measured by growth in corporate net value added, denoted Δy_t, and the log market equity market to output ratio for the corporate sector, denoted $p_t - y_t \equiv py_t$.

The final observable variable we use is a measure of the risk premium for the equity market taken as the SVIX variable of Martin (2017), denoted rp_t. This variable is computed

[11] Although $\mathbf{s}$ and $\boldsymbol{\delta}$ have multiple components, we assume without loss of generality that all but the first component of each series has zero mean, and therefore only estimate a single parameter for the mean of each.

[12] Observations on $ey_t = \ln(S_t) = \ln\left(S_t^D Z_t + F_t\right)$ are constructed from observations on the components S_t^D, Z_t, and F_t, as indicated in the Online Appendix.

from options data and we use it to help discipline the estimate of the risk-premium process, especially its higher frequency variation. Martin (2017) uses option data to compute a lower bound on the equity risk premium, then argues that this lower bound is in fact tight and is therefore a good measure of the true risk premium on the stock market. That paper documents that a wide range of representative agent asset pricing fail to explain the high frequency variation in the risk premium implied by options data, even if they are broadly consistent with the lower-frequency variation suggested by variables like the price-dividend ratio or cay_t (Lettau and Ludvigson (2001)). Since our model allows for mixture processes, the risk premium we estimate is capable of accounting for both higher- and lower-frequency components of the risk premium.

The model implies that these observed series are related to the primitive parameters and latent state variables according to the following system of equations:

$$\begin{aligned}
ey_t &= \mathbf{1}'\mathbf{s}_t \\
r_{ft} &= \mathbf{1}'\boldsymbol{\delta}_t \\
py_t &= pc_t + cy_t \\
&= \overline{py} + (\mathbf{A}_s' + \boldsymbol{\xi}')\,\widetilde{\mathbf{s}}_t + \mathbf{A}_\delta'\widetilde{\boldsymbol{\delta}}_t + \mathbf{A}_x\widetilde{\mathbf{x}}_t \qquad (18) \\
\Delta y_t &= g + \Delta\tilde{y}_t \\
rp_t &= \left[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_a^2\right) + \kappa_1\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s\right](\mathbf{1}'\mathbf{x}_t + \lambda'\widetilde{\mathbf{s}}_t) - \frac{1}{2}\mathbb{V}_t(r_{t+1})
\end{aligned}$$

where e.g., ι_s is an indicator vector that picks out the relevant entries of $\mathbf{s}_t$ (under the baseline, $\iota_s'\mathbf{s}_t = s_{LF,t} + s_{HF,t}$), rp_t is a measure of the risk premium, where we define $cy_t \equiv c_t - y_t$, and where $\overline{py} \equiv A_0 + \bar{c} + \boldsymbol{\xi}'\overline{\mathbf{s}}$. Note that $\Delta\tilde{y}_t$ is exactly pinned down by the observation equation for Δy_t.

Let K denote the number of latent state variables and let N denote the number of observation variables. The above equations may be written in state space form as follows:

$$\mathcal{Y}_t = \mathbf{H}_t'\boldsymbol{\beta}_t + \mathbf{b}_t \qquad (19)$$

$$\boldsymbol{\beta}_t = \mathbf{F}\boldsymbol{\beta}_{t-1} + \varepsilon_t, \qquad (20)$$

where the observation vector $\mathcal{Y}_t \equiv (ey_t, r_{ft}, py_t, rp_t, \Delta y_t)'$. The latent state vector is $\boldsymbol{\beta}_t \equiv \left(\tilde{\mathbf{s}}', \tilde{\boldsymbol{\delta}}_t', \tilde{\mathbf{x}}', \Delta\tilde{y}_t\right)'$, while the shock vector is $\varepsilon_t = \left(\varepsilon_{s,t}', \varepsilon_{\delta,t}', \varepsilon_{x,t}', \varepsilon_{a,t}\right)'$. In total, there are seven latent states, since $\tilde{\mathbf{s}}', \tilde{\delta}_t'$, and $\tilde{\mathbf{x}}'$ each have a low- and high frequency component. The i.i.d. shock $\varepsilon_{a,t}$ is included the "state equation" (20) even though it is exactly pinned down by the observable series Δy_t so that we can estimate its mean and variance, since these parameters influence our asset pricing equations. The coefficient matrix $\mathbf{H}_t'$ and vector $\mathbf{b}_t$ depend on t because the sample for the SVIX variable rp_t spans 1996:Q1 to 2012:Q1, which is shorter than our full sample period. Thus the state-space estimation uses two measurement equations to accommodate the missing data. (See the Online Appendix for details.)

The model is estimated as follows. The matrices, $\mathbf{F}$, $\mathbf{H}_t'$, and $\mathbf{b}_t'$ contain primitive parameters of dimensions $(K \times K)$, $(N \times K)$, and $(N \times 1)$, respectively. Given values for the primitive parameters $\boldsymbol{\theta}$ in the combinations $\mathbf{F}$, $\mathbf{H}_t'$, and $\mathbf{G}'$, we use the Kalman filter to obtain smoothed estimates of the state vector $\boldsymbol{\beta}_t$, which we denote $\boldsymbol{\beta}_{t|T}$. Since our model has more latent states than observable series in $\mathcal{Y}_t$, we explain all of the variation in our observable series and there is no measurement error in equation (19).Moreover, because we perfectly match $p_t - y_t$ and Δy_t, we also perfectly match the growth in market equity Δp_t over time and at each point in time, a property we exploit when calculating the growth decompositions discussed below.

The posterior distribution of $\boldsymbol{\theta}$ is obtained by computing the likelihood using the Kalman filter and combining it with priors. Since we use flat priors, the posterior coincides with the likelihood and the posterior mode estimate of $\boldsymbol{\theta}$ coincides with the maximum likelihood estimate (MLE). Uncertainty about the parameters $\boldsymbol{\theta}$ is characterized using a random walk Metropolis-Hastings (RWMH) algorithm, while uncertainty about the latent state $\boldsymbol{\beta}_t$ is characterized using the simulation smoother of Durbin and Koopman (2002). We use the RWMH algorithm to generate ten independent chains, each containing 550,000 draws of $\boldsymbol{\theta}$. We discard the first 50,000 draws from each chain as burn-in, leaving 5,000,000 parameter draws. Since these draws are highly serially correlated, we increase computational efficiency by using every 50th draw, leaving a total of 100,000 draws over which our margins of parameter uncertainty are computed. For every one of these 100,000 draws of parameters, we simulate one draw of the latent states using the simulation smoother. The plots below therefore reflect both parameter and latent state uncertainty.[13]

[13]The latent state space includes components that differ according to their degree of persistence. With

Our data consist of quarterly observations spanning the period 1952:Q1 to 2017:Q4. We focus on our analysis on the U.S. corporate sector. Previous research has examined joint trends in financial markets and aggregate economic quantities by combining data on the stock market with data from the Bureau of Economic Analysis (BEA) on aggregate measures of output and the labor share.[14] A weakness of this approach is that the stock market only covers publicly traded firms, while the BEA data on output and labor share are not limited to the publicly traded sector and cover a far broader swath of the economy. This creates the potential for confounding compositional effects over time. For example, if publicly traded firms have experienced larger shifts over time in their labor shares and/or output compared to non-public firms, movements in the aggregate quantities for output and labor compensation would not correctly describe the firms for which market equity is measured. Since it is important for our study that earnings, output, labor compensation, and the market value of equity all pertain to the same sector of the economy, we focus on the U.S. corporate sector (CS), where all of these variables can be directly measured. To the importance of this, consider again Figure (1), which plots the ratio of CS market equity (ME) to several different measures of macroeconomic activity over time. The one ratio for which market equity and aggregate activity are measured for the same sector is the CS ME-to-output, which has risen the most over time. The other ratios give a more distorted measure of this increase because the numerator and denominator are not in comparable units. The use of the corporate sector also has the advantage that the labor share, $1 - S_t$, is not affected by the statistical imputation of labor income from total income reported by sole proprietors and unincorporated businesses.

We use total CS market equity to measure p_t. Total output y_t, measured in real, per capita terms, is observed as net value added for the sector. Labor compensation, earnings, taxes and interest, and foreign retained earnings are also directly observed for this sector. The real risk-free rate is measured as the three-month T-bill rate less the fitted value from

flat priors, a penalty to the likelihood is required to ensure that the low frequency component has greater persistence than the higher frequency component. This is accomplished setting a penalty that forces the likelihood to negative infinity if the parameter search wanders into a space where $\phi_{j,LF} \leq \phi_{j,HF}$.

[14] See e.g., GLL, Farhi and Gourio (2018).

a regression of inflation on lags of inflation. The Online Appendix provides a detailed description of these data and our sources.

There are four parameters that are calibrated rather than estimated. The first three are the average growth rate of net value added g, the average log profit share $\bar{s}$, and the average real risk-free rate $\bar{\delta}$. Since these represent the means of our observable series, we take a conservative approach by fixing them equal to their sample means. We do this to avoid a potential estimation concern: because some of our series are very persistent, the estimation might otherwise have a wide degree of freedom in setting steady state values that are far from the observed sample means. At quarterly frequency, we obtain the values $g = 0.552\%$, $\bar{s} = -2.120$ (corresponding to a share in levels of 12.01%), and $\bar{\delta} = 0.283\%$.

The final calibrated parameter is $\xi = \frac{\overline{S}}{\overline{S}-\omega}$, which relates payout growth to earnings growth according to (9). The National Income and Product Account (NIPA) estimate of net payout for the U.S. corporate sector is noisy and subject to large swings due to temporary factors such as changes in tax law that are likely to be unrelated to longer-term fundamentals.[15] As a result, we choose not to directly include payouts in our observable series, and instead calibrate ω directly. Since $C_t = (S_t - \omega)Y_t$, we can rearrange and take sample averages of both sides to obtain $\omega = \bar{S} - \overline{\frac{C}{Y}}$. Computing $\overline{S}$ as the mean of the total profit to domestic output ratio observed in the data yields the value $\xi = 2.19$. We confirm in our results that this yields average growth and volatility of payouts close to those observed in the data.

5 Results

5.1 Parameter and Latent State Estimates

We being with a discussion of the estimated parameter values and latent states.

Table 1 presents the estimates of our primitive parameters based on the posterior distri-

[15] For a recent example, see NIPA Table 4.1, which shows an unusually large increase in 2018:Q1 in net dividends received from the rest of the world by domestic businesses, which generated a very large decline in net payout. BEA has indicated that these unusual transactions reflect the effect of changes in the U.S. tax law attributable to the Tax Cut and Jobs Act of 2017 that eliminated taxes for U.S. multinationals on repatriated profits from their affiliates abroad.

bution obtained with flat priors. A number of results are worth highlighting.

First, the persistence parameters of the mixture series are of immediate interest, since they determine the role of each latent variable on market equity values over longer periods of time. Consider the risk-free rate processes. The mode estimate of the autoregressive coefficient of the high frequency component is $\widehat{\phi}_{\delta,HF} = 0.56$, whereas that of the low frequency component is $\widehat{\phi}_{\delta,LF} = 0.93$. While the latter is clearly persistent, it is not highly so. This explains why the large declines in real interest rates observed over the last several decades need not play a large role in explaining the boom in equity values over the same time period. Although rates have declined, it is not today's rate but the expected path of future rates that matters for equity values. This evidence implies that the low rates of recent years are unlikely to persist. We return to this point in Section 5.5.

Second, the estimates for the factor share and risk price autoregressive parameters suggest much more persistence, with modal values of $\widehat{\phi}_{s,HF} = 0.879$ and $\widehat{\phi}_{s,LF} = 0.985$, and $\widehat{\phi}_{x,HF} = 0.671$ and $\widehat{\phi}_{x,LF} = 0.986$, respectively. These estimates indicate that the low frequency components of the factor share and risk premium components are substantially more persistent than those of interest rate changes, though still not permanent, foreshadowing their larger role in the longer-term swings of the market that we document below. As a further indication that these estimates are plausible, Figure 10 shows that, at these estimated parameter values, the model implications for the autocorrelations of model-implied series closely match those of the corresponding observed series. In both the model and the data, the autocorrelations of output growth hover around zero, suggesting a near i.i.d. process, whereas the autocorrelations for the earnings share, the risk-free rate, and the log ME-to-output ratio start well above zero and converge toward zero as the lag order increases, suggestive of persistent but stationary processes. Panel (c) shows clearly that the autocorrelations of the risk-free rate converge to zero by quarterly lag 35, while panel (d) shows that the autocorrelations for the log ME-to-output ratio remain above 0.5 at that lag. The shows that the risk-free rate process is not nearly persistent enough to explain much of the considerably more persistent variation in the ME-to-output ratio observed in Figure 1.

Third, Table 1 shows that the mode estimate of the mean risk price parameter $\overline{x}$ is about 4.0, a modest value that reflects the volatility in cash payments to shareholders that are the source of systematic risk to shareholders in the model. Because of this, outsized aversion to risk or ambiguity is not needed to explain the high average equity return premium in the data.

Fourth, the parameter estimate $\widehat{\lambda} = -7.93$ implies that a one unit increase in the log earnings share s_t, approximately a one percent increase in the earnings share S_t around the mean, results in a 0.08 unit decrease in the price of risk x_t. This implies that persistent increases in the earnings share are associated with a decline in expected returns and the risk premium.

Fifth, the mode estimate of the volatility of the productivity shock (output growth) is 0.015, which is roughly equal to that of quarterly real personal consumption growth.

What about the latent states? The next three figures illustrate our estimates of the latent states over time. Figure 3 shows the low and high frequency components of the earnings share, s_t; Figure 4 shows the low and high frequency components of the risk-free rate, δ_t. The figures show the observable series, along with its component attributable to different sources of variation. This is accomplished by fixing one component or the other at its value at the beginning of the sample. The shaded areas around each estimated component source are 90% credible sets that take into account both parameter and latent state uncertainty. Credible sets are known to be wide in estimations with flat priors, as here. But its important to note that the *sum* of the high- and low frequency components add up to the observed series exactly, without error, both over the sample and at each point in time, since measurement error is effectively zero in the observation equations.

Figure 3 shows the time-variation in the log earnings share ey_t over our sample, along with the portion of this variation attributable to each estimated factors share component $s_{LF,t}$ and $s_{HF,t}$. The plot shows that the log earnings share was high in the 1950s and 1960s, low in the 1970s and 1980s, and then began an upward trajectory starting around 1990 that continues to the end of the sample, interrupted only temporarily by the tech bust in 2000-2001. Panel (a) juxtaposes observations on ey_t with the model's implications for this series if only the low frequency component $s_{HF,t}$ were varying over our sample. The low frequency $s_{LF,t}$ component captures the longer term swings in the earnings share, which increases over the sample, and has risen sharply since the year 1990. Panel (b) shows the high frequency component $s_{HF,t}$ captures all the transitory variation in the earnings share, but cannot account for the full rise in ey_t in recent decades.

Figure 4 shows the evolution of the risk-free rate over time, along with the portion of this variation attributable to the estimated low and high frequency components. From the series on the raw interest rate data, it is clear that, although rates are low today, they are not unusually so by historical standards. Real rates were very low at several points in the 1950s and late 1970s, then rose under the Volcker disinflation around 1979 and remained elevated for over a decade before declining to their current low values. The low frequency component $\delta_{LF,t}$ in panel (a) captures all of these longer-term movements. The high frequency component $\delta_{HF,t}$, shown in panel (b), picks up the most transitory wiggles in the series.

Figure 5 shows the estimate of the equity premium over time, but breaks the components out in a different way. Panel (a) plots our overall estimated risk premium, which is affected by both the orthogonal risk price component $x_{\perp,t}$ and, since $\widehat{\lambda} \neq 0$, movements in s_t. Panel (b) shows our estimate the equity premium variation that is attributable to only the high frequency component of the of the orthogonal risk price component, $x_{HF,t}$. Both panels superimpose the equity premium implied by the three-month SVIX over the subperiod for which the latter is available, from 1996:Q1-2012:Q1. Two points are worth noting. First, with the exception of the spike upward during the financial crisis of 2008-2009, panel (a) shows that the estimated equity premium has been declining steadily over the past several decades and is quite low by historical standards at the end of the sample. Specifically, by 2017:Q4, the estimates imply that the equity premium reached the record low values it had attained previously only in two episodes: at the end of the tech boom in 1999-2000, and at the end of the twin housing/equity booms in 2006. Second, panel (b) shows that the estimation assigns to the high frequency orthogonal risk price component, $x_{HF,t}$, virtually all of the variation in the risk premium implied by the options data, while the remaining variation is ascribed to the lower frequency component. The overall risk premium is therefore influenced by a trending low frequency component and a volatile high frequency component.

5.2 Asset Pricing Moments

Table 2 presents the model's implications for asset pricing moments and compares them to data for the corporate sector. The columns labeled "Model" are computed using the mode parameter estimates and then simulating the model 1,000 times using a sample length equal to that of our historical sample. The asset pricing moments in the "Model" columns are averages across the simulations. The columns labeled "Fitted," compute moments using the mode parameter estimates but combine these with the estimated latent states obtained by fitting the model to the observed historical sample. These fitted values therefore represent the model's implications for the asset pricing moments conditional on the observed sequence of shocks that actually generated the historical data, so they are directly comparable to the sample "Data" moments that are also reported in the table. Note that the "Fitted" and "Data" moments are identical by construction for the risk-free rate, earnings growth, and earnings share growth, because we use these series as observables and fit their behavior exactly over the sample with no measurement error.

For moments pertaining to other series, Table 2 shows that the model fitted moments do a reasonably good job of matching the sample moments for the excess return on equity and the log price-payout ratio. The fitted mean log excess return, which is 6.4% per annum, is lower than the 7.3% per annum for the average excess return in historical data because the model's implied series for payout share growth (which was not a target of our estimation) understates the observed sample mean. Despite this understatement, the fitted means are in the right ballpark of the actual payout data, which are highly volatile. Since no data on payout were used in our estimation, these results increase our confidence that the model is able to realistically account for the dynamics of payouts over the sample.

These estimates imply that much of the reward from holding equity in the post-war era has been attributable to a long sequence of distributional shocks that have fallen predominantly in one direction. Table 2 shows that the model mean log equity premium is 3.7% per annum. This number is an estimate of the mean risk premium implied by the parameter estimates; it reflects only compensation for bearing risk in the stock market, i.e., covariance with the SDF. By contrast, the estimated *fitted* equity premium, or mean excess stock market return, is 6.6%. This number is affected by covariance with the SDF but also reflects persistent movements in earnings and payout over the sample. The divergence between the two is attributable to the fact that the model stipulates stationary earnings and payout shares of output that can't grow in perpetuity (these shares are bounded below by 0 and above by unity), while the 65 year sample upon which the model is estimated exhibits strong upward growth in these shares. We can see this by observing that the fitted means for earnings growth and payout growth reported in Table 2 are both estimated to be substantially larger than the model means for these series. Moreover, since the earnings and payout shares have a constant steady state mean in the model, the model means of the steady state *growth* in the shares is zero, while the fitted means over our sample are estimated to be large and positive.

In short, our estimates imply that the high returns to holding equity in the post-war period have been driven, in large part, by a highly unusual sample, one characterized by a long string of factors share shocks that redistributed rewards from productive activity toward shareholders. Taken together, the estimates imply that roughly 2.9% of the post-war mean log return on stocks in excess of a T-bill rate is attributable to this string of factors share shocks, rather than to genuine compensation for bearing risk. The results here suggest that such sample averages for the excess return overstate the true equity risk premium by over 44%. These findings provide a cautionary tale for the common practice of using the sample mean excess return, or components of the sample mean return such as the dividend-price ratio or dividend-earnings ratio, to infer an equity risk premium, even over samples as long as that of the post-war period.

5.3 Dynamics of Equity Values

The results in Table 2 tell us about the full sample moments but are silent on the dynamic forces that have given rise to sharply increasing equity values over time in the sample. To consider these dynamic forces, it is instructive to study a graphical representation of our observed variables over time, juxtaposed by the estimated portion of variation attributable to each latent component in the model. This is accomplished by either fixing one component at its mean during a particular period, or by allowing only one component to vary. The shaded areas around each estimated component source are 90% credible sets that take into account both parameter and latent state uncertainty.

The next several figures decompose our target variable, the log market equity-to-output ratio, py_t, into sources of variation attributable to different latent components. This is accomplished, as above, by either fixing one component during a particular period, or by allowing only one component to vary. In all figures, the red (solid) line shows the model point estimate, based on the posterior mode parameter values.

Figure 6 shows the observed py_t series, which has a low frequency upward trend in it over our sample that is captured by the low frequency factors share component $s_{LF,t}$, as exhibited in panel (a). The high frequency component $s_{HF,t}$, produces some "wiggles" in the series but contributes nothing to the trend, as shown in panel (b). Panel (c) shows that if we fix both components of the factor share process at its value at the beginning of our sample in 1952:Q1, the model is unable to capture any of the upward trajectory in the price-output ratio since about 2000. Panel (d) exhibits a similar pattern by showing the actual series since 1989 against what the model would imply if s_t were fixed at its value in 1989: only a small part of the upward trend in the market equity-to-output ratio since 1989 can be explained if there is no role for a changing earnings share. Taken together the results show a large role for factors share shifts in driving upward the market value of equity relative to output.

Figure 7 shows the part of the observed equity value-to-output ratio variation that can be attributed to movements in the risk-free rate. Panels (a)-(d) show that the estimates attribute only a small role to risk-free rate variation in explaining the rise in equity values relative to output over our sample. The last two panels show that shutting down either component does little to the model's ability to match the trend movements in py_t, which are driven primarily by other variables. Panel (d) a small role for lower risk-free rates in the last 30 years, especially since around 2000: by the end of the sample, the price-output ratio would be about two-tenths of a log point lower had there been no change in risk-free rates since 1989.[16]

[16] These findings may be related to those in Bianchi, Lettau, and Ludvigson (2016), who find evidence for a monetary policy role in low-frequency variation of short term interest rates, which is linked to low-frequency

Figure 8 shows the part of the observed py_t variation that can be attributed to fluctuations in the component of the one-step-ahead equity risk premium, $\mathbb{E}_t[r_{t+1}] - r_{f,t}$, that is driven by the orthogonal component of the risk price, $x_{\perp,t}$. Panel (a) shows that this component of risk premium variation explains almost all of the transitory booms and busts in equity values relative to output over our sample, including the technology boom/bust, the boom in equity values leading up to the financial crisis of 2008-2009, and the sharp decline in those values during the financial crisis. In particular, Panel (a) shows that an estimated decline in the risk premium explained almost all the boom in equity values during the run-up to the tech bust in 2000. (Recall that a decline in $x_{\perp,t}$ translates into a *rise* in py_t.) Panel (b) shows that high frequency component of the risk price explains virtually none of the big swings in the price-output ratio. Panel (c), which fixes $x_{\perp,t}$ at its value in 1952, indicates that the lower-frequency component of the orthogonal risk price explains some, but not nearly all, of the rise in the price-output ratio over the full sample, since if $x_{\perp,t}$ is fixed at its 1952 value, the model does not match the full increase in py_t over the sample. The rest of the rise thus comes from other factors, namely the factor share component. Panel (d) shows what happens if $x_{\perp,t}$ is fixed at the value it took in 1989:Q1. The counterfactual in panel (d) suggests that only a small portion of the rise in py_t since 1989 is explained by a decline in $x_{\perp,t}$, based on the posterior mode parameter values. But the credible sets indicate that there is significant uncertainty over whether this component contributed at all to the rise in the market equity-to-output ratio over the last 30 years.

5.4 Growth Decompositions

In this section we quantify the importance of differing drivers of equity values over the post-war period by calculating a set of growth decompositions that decompose the total growth in equity values into distinct sources attributable to each latent state variable in the model. The contributions are computed by taking the total growth in the target variable and dividing it into parts attributable to only a single component (fixing all other components at their values

time-variation in the consumption-wealth variable cay_t. These findings are not directly comparable to those here since the monetary policy component they uncover is correlated with risk-premium variation, whereas we identify only the mutually uncorrelated components of risk-free rate and equity premium variation.

at the beginning of the sample). By construction, these components sum to 100% of the observed variation in equity values, since the model along with the fitted latent components perfectly matches at each point in time the observed log market equity-to-output ratio, py_t, as well as output growth Δy_t. Table 3 presents the decompositions for the total change in the log of real market equity p_t, either over the whole sample or over the period since 1989.[17]

The estimates indicate that about 43% of the market increase since 1989 and 19% over the full sample is attributable to the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$, with the vast majority of this coming from the low frequency component. Over the period since 1989, the roles of the other components are smaller. For example, persistently declining interest rates contributed 8.5%, while the decline in the equity risk premium driven by orthogonal movements in the price of risk contributed about 24%. Over the full sample, real interest rates contributed a much smaller 2.1%, while the declining risk price contributed 20%.

In contrast to factor share movements, growth in the real value of what was actually produced by the sector is a far less important driver of equity values since 1989. Indeed economic growth explains just 25% of the increase in equity values since 1989 and, as a result of this weak contribution, explains only 54% over the full post-war sample. This may be contrasted with the previous subsample, from 1952 to 1988, where economic growth account for 111% of the rise in the stock market, while factor share movements contributed negatively to the market's rise. But the 37 year subperiod from 1952 to the end of 1988 created less than half the wealth generated in the 29 years 1989:Q1-2017:Q4. These findings underscore a striking aspect of post-war equity markets: in the longer 37 year subsample for which equity values grew comparatively slowly, economic growth propelled the market, while factor shares played a negative role. But the market made far greater gains in much shorter time from 1989 to present day, when factor share shocks reallocated rewards to shareholders even as economic growth slowed.

[17] The growth decompositions for the log level of real market equity p_t are computed by adding back the growth Δy_t in real output (net value added) to the growth Δpy_t. Since Δy_t is deflated by the implicit price deflator for net value added, the decomposition for p_t pertains to the value of market equity deflated by the implicit NVA price deflator.

Since the corporate earnings share has been the most important driver of the price-output ratio in recent decades, we close this section by discussing the model's implications for what has been driving the corporate earnings share. As above, we do so by showing the observable series, along with the component attributable to different sources of variation. This is accomplished by fixing one component or the other at its value at the beginning of the sample. Recall that the total earnings share in levels $S_t = \left(S_t^D Z_t + F_t\right)$ is influenced by the domestic profit share S_t^D (one minus the labor share), the tax/interest/other share Z_t, and the foreign earnings share F_t.

Figure 9 shows four panels. In panel (a), the data on $\ln(S_t) = s_t$ are plotted over time along with counterfactuals that allow only one component to vary over time. All other components are held fixed at their values at the beginning of the sample in 1952. Panel (b) does the same but zeros in on the post 1989 period by fixing components at their values in that year. Both panels, but especially when zeroing in on the last 30 years in panel (b), show that a declining domestic labor share accounts for the bulk of the rise in the corporate earnings share. The tax and interest share plays virtually no role. The foreign earnings share played a modest role from 2000-2007, but has played a lesser role in the sample since the Great Recession. Next, panel (c) shows s_t along with counterfactuals that leave one component fixed at its value in 1952 while allowing all others to vary. Panel (d) shows the same but fixes the component at its value in 1989. A similar pattern emerges: fixing the labor share component, we explain very little of the sustained run-up in the corporate earnings share since 1989 nor can we explain most of its variation over the whole sample (panel (a)). The tax and interest share explains virtually none of the variation, and the foreign earnings component only a small part. Putting this all together, these results imply that the declining domestic labor share has played largest role in the sustained rise in the corporate earnings share.

5.5 Contrast to Existing Literature

Our results differ in important ways from contemporaneous papers such as Farhi and Gourio (2018) and Corhay, Kung, and Schmid (2018). While these papers find a crucial role for falling interest rates in driving the increase in asset prices over recent decades, we find that interest rates account for only 8% of stock market growth since 1989. Moreover, while these papers both conclude that risk premia have risen over this period, Panel (a) of Figure 5 shows that we estimate risk premia to have fallen to historically low levels.

We believe these opposing results are mostly due to the different estimation approaches behind them. Importantly, while we estimate our model directly on the time series, allowing for shocks to enter with a variety of estimated persistences, Farhi and Gourio (2018) and Corhay, Kung, and Schmid (2018) measure changes across steady states, in which parameters can change only permanently. As a result, these papers interpret the observed drop in risk-free rates as a permanent shift, causing major changes in how long-term cash flows are discounted, and leading a huge increase in market value. Since the implied increase in market value from falling risk-free rates would be even larger than the actual increase observed, these models infer that risk premia must have risen at the same time to match the realized growth in asset prices.

In contrast, our model views changes in interest rates as far from permanent, since we estimate the quarterly persistence of the low frequency component of interest rates to be 0.93. As a result, investors in our model did not believe that interest rates would remain permanently high in the 1980s, nor do they expect them to remain permanently low today, strongly dampening the effect of the fall in rates on the value of market equity. This smaller direct effect from interest rates allows us to match the observed rise in asset prices in an environment with falling risk premia.

We view our approach, and therefore our findings, as strongly preferred by the data. To support this claim, we appeal to Figure 10, which compares the observed autocorrelations of our observable series to the implied autocorrelations generated from a long simulation of the model.[18] Panel (c) shows that the autocorrelation of the real risk-free rate decreases rapidly with the lag order, falling below 50% after 12 quarters, and falling close to zero at the 10-year horizon. This pattern is deeply inconsistent with a process dominated by permanent

[18] We include all four observable series that are available over the full sample. We omit the SVIX risk premium, which is available only on a much shorter sample, and is therefore unsuitable for computing longer autocorrelations.

changes, which would instead imply autocorrelations close to unity at all horizons. Our model is able to match this pattern well, and does not understate the autocorrelation at long horizons. More broadly, the other panels of Figure 10 demonstrate that the earnings share and ME/Y ratio are also far from a unit root. Since the persistences of the underlying shocks are critical for pricing long-term assets, we believe that our estimation approach, which is able to closely match their autocorrelograms, is likely to generate more realistic asset behavior than approaches that assume permanent changes across regimes.

6 Conclusion

We investigate the reasons for rising equity values over the post-war period. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent components, while at the same time inferring what values those components must have taken over our sample to explain the data. The identification of mutually uncorrelated components and the specification of a log linear model allow us to precisely decompose the observed market growth into distinct component sources. The model is flexible enough to explain 100% of the rise in equity values over our sample and at each point in time.

We confront our model with data on equity values, output, the earnings share of output, interest rates, and a measure of the conditional equity premium implied by options data. We find that the high returns to holding equity over the post-war era have been attributable in large part to an unpredictable string of factor share shocks that reallocated rewards away from labor compensation and toward shareholders. Indeed, our estimates suggest that at least 2.9 percentage points of the post-war average annual log equity return in excess of a short-term interest rate is attributable to this string of reallocative shocks, rather than to genuine compensation for bearing risk. This estimate implies that the sample mean log excess equity return overstates the true risk premium by at least 44%.

Factors share shocks alone would have driven a 543% increase in the value of market equity since 1989, an increase equal to 43% of the actual rise. Equity values were modestly boosted since 1989 by persistently declining interest rates and a decline in risk premia, which contributed 8.5 and 24%, respectively, to rising equity values. But growth in the real value of aggregate output contributed just 25% since 1989 and just 54% over the full sample. By contrast, economic growth was overwhelmingly important for rising equity values from 1952 to 1988, where it explained over 100% of the market's rise. But that 37 year period generated less than half the wealth created in the 29 years since 1989. In this sense, factor shares, more than economic growth, have been the preponderant measure of fundamental value in the stock market over 60 years.

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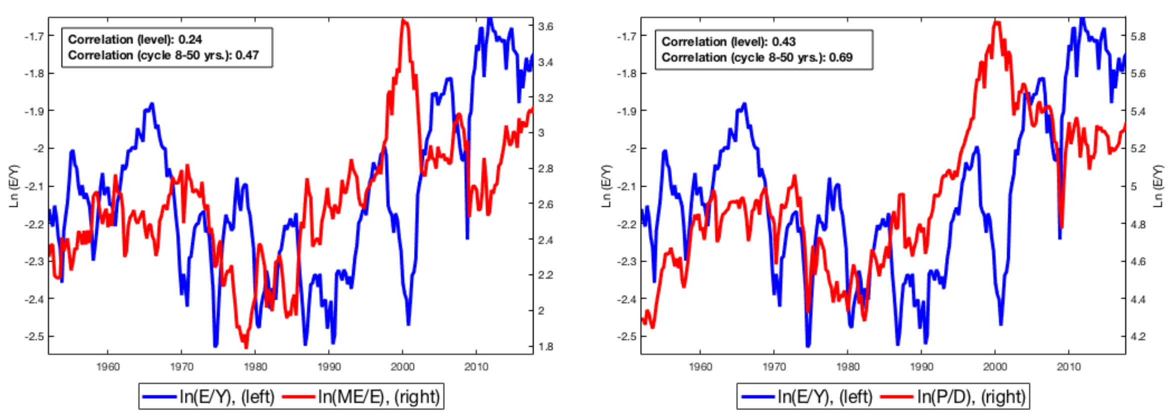
Figures and Tables

Figure 1: Stock Market Ratios

Notes: To make the units comparable, each series has been normalized to unity in 1989:Q1. The sample spans the period 1952:Q1-2018:Q2. **ME:** Corporate Sector Stock Value. **E:** Corporate Sector After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Gross Value Added of Corporate Sector - Consumption of Fixed Capital.

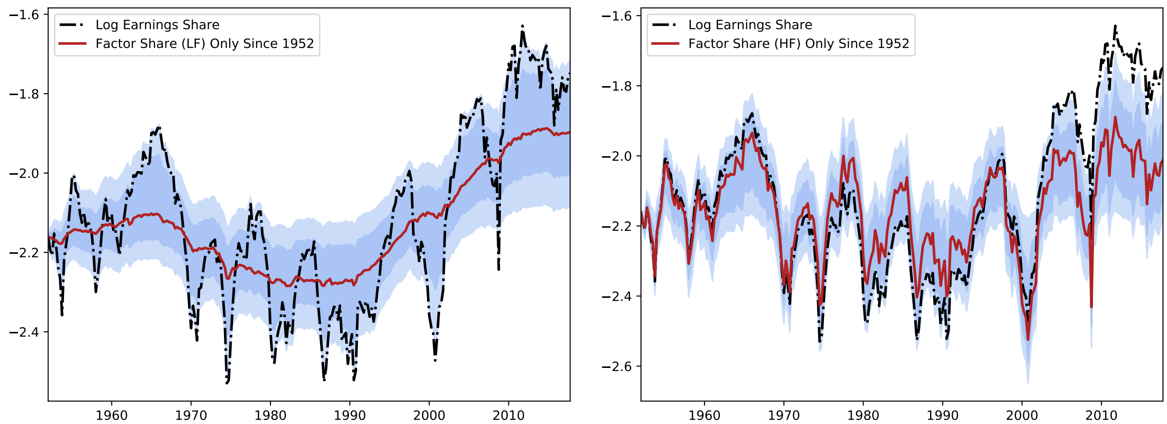
Figure 2: Earnings Share and Valuations



Notes: ln(E/Y) denotes the logarithm of the total profit share of the corporate sector. ln(ME/E) is the log of the stock wealth-profit ratio. ln(PD) is the log of the CRSP price-dividend. Each plot present the correlation between the series (levels) and the correlation of the cycle of each series obtained using a passband filter that isolates cycles between 8 and 50 years. The sample spans the period 1952:Q1-2017:Q4.

Figure 3: Earnings Share Components

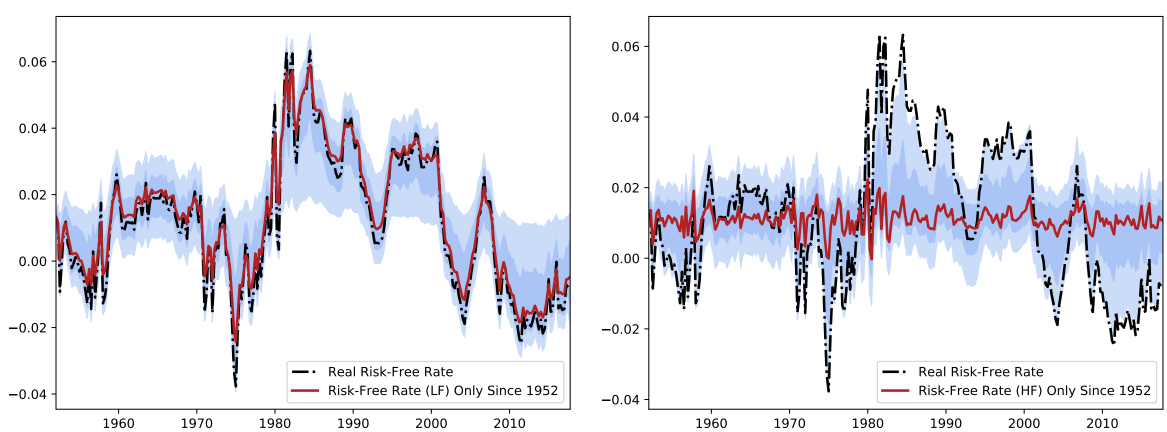
(a) (b)



Notes: The figure exhbits the observed earnings share series along with the model-implied variation in the series attributable to certain latent components. The label "Fixed Since" followed by a date describes a counterfactual path where this variable was held fixed from that date on. The notation "Only Since" followed by a date describes a counterfactual path where all other variables were held fixed from that date on. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

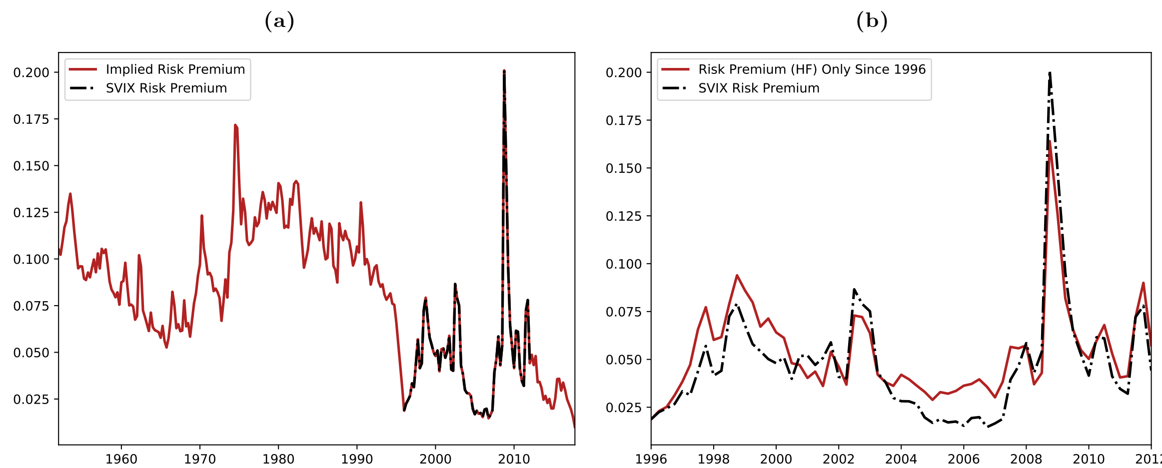
Figure 4: Risk-Free Rate Components

(a) (b)



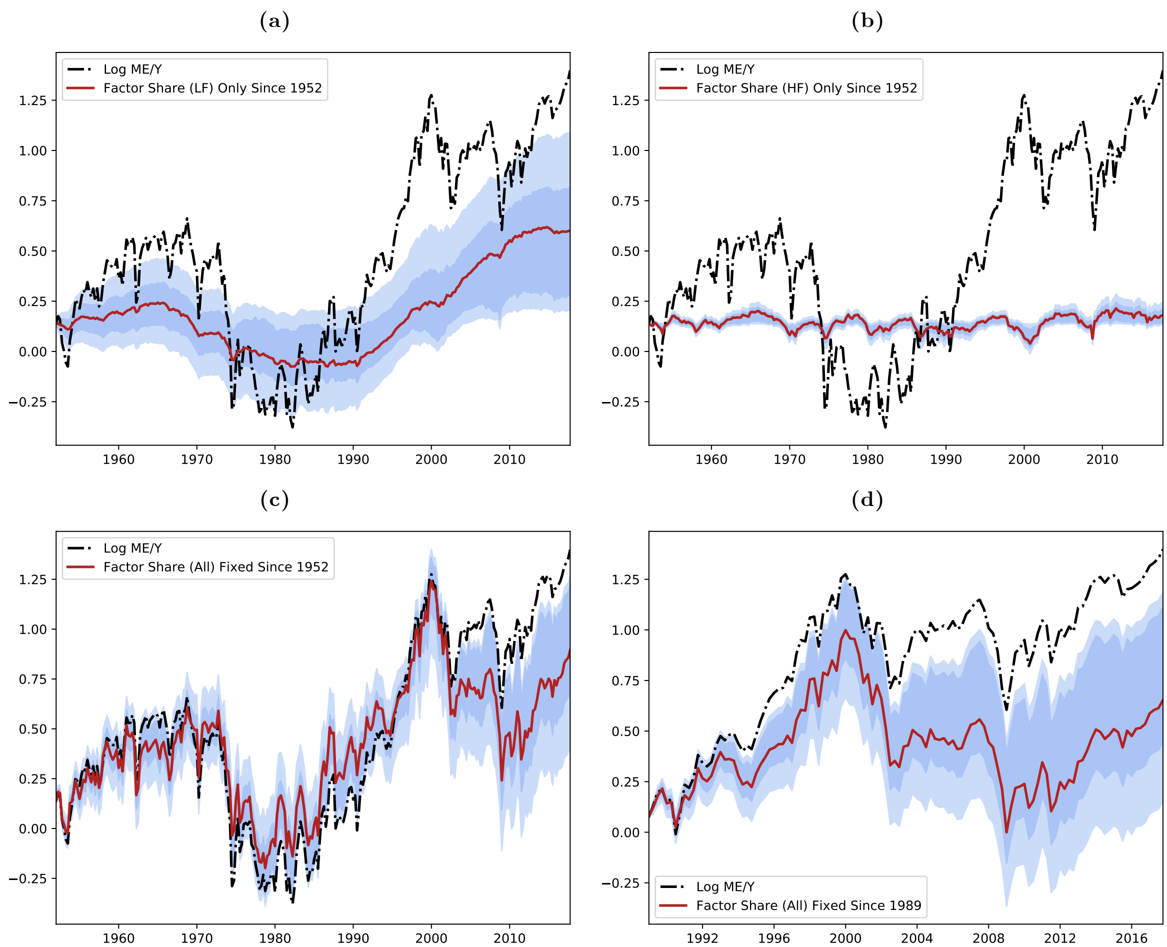
Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhbits the observed risk-free rate series along with the model-implied variation in the series attributable to certain latent components. The label "Fixed Since" followed by a date describes a counterfactual path where this variable was held fixed from that date on. The notation "Only Since" followed by a date describes a counterfactual path where all other variables were held fixed from that date on. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Figure 5: Implied Risk Premium and Risk Premium Component



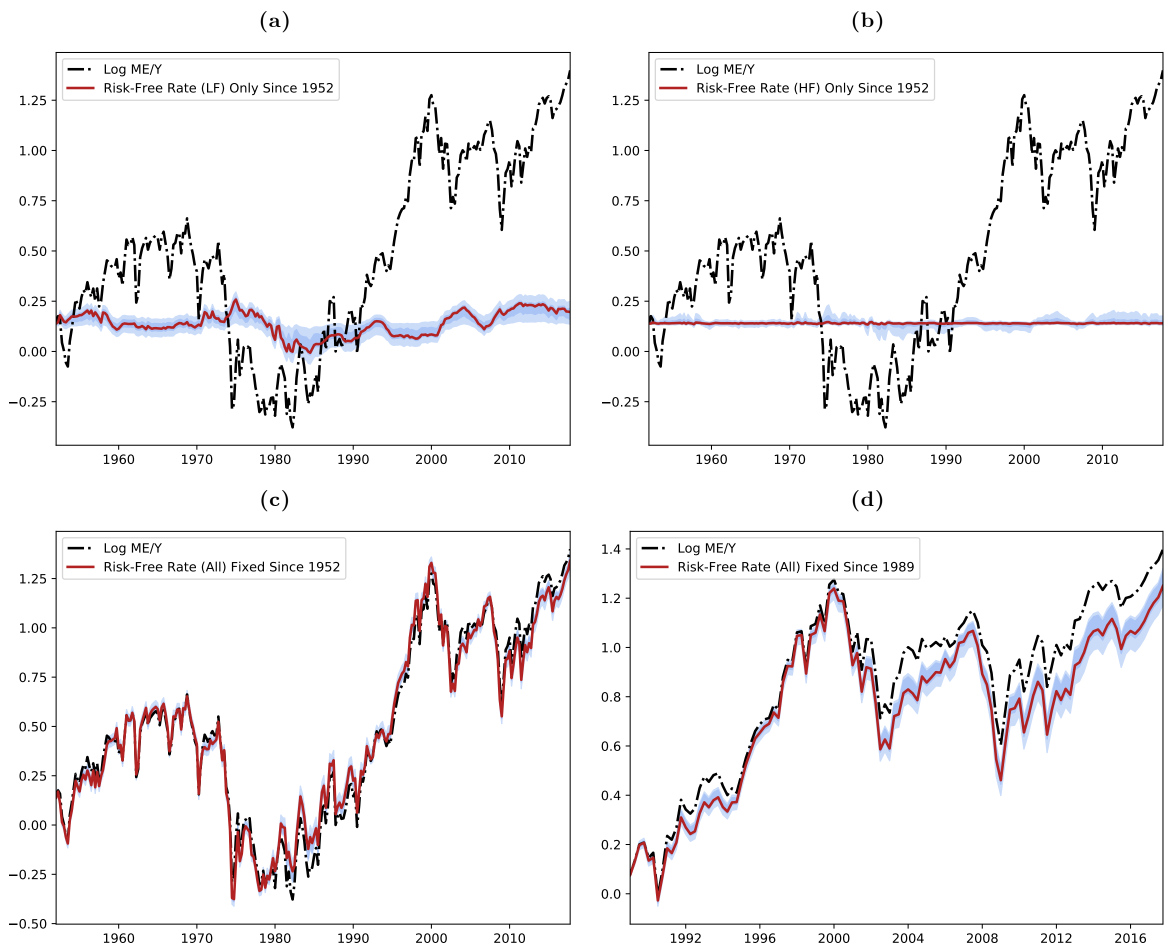
Notes: Panel (a) plots the estimated risk premium over time along with the risk premium implied by the SVIX, available for the subperiod 1996:Q1-2012Q1. Panel (b) plots the high-frequency component of the risk-premium along with the risk preimium implied by the 3-month SVIX. The label "Only Since" followed by a date describes a counterfactual path where all other variables were held fixed from that date on. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The sample spans the period 1952:Q1-2017:Q4.

Figure 6: Market Equity-Output Ratio and Factors Share Component



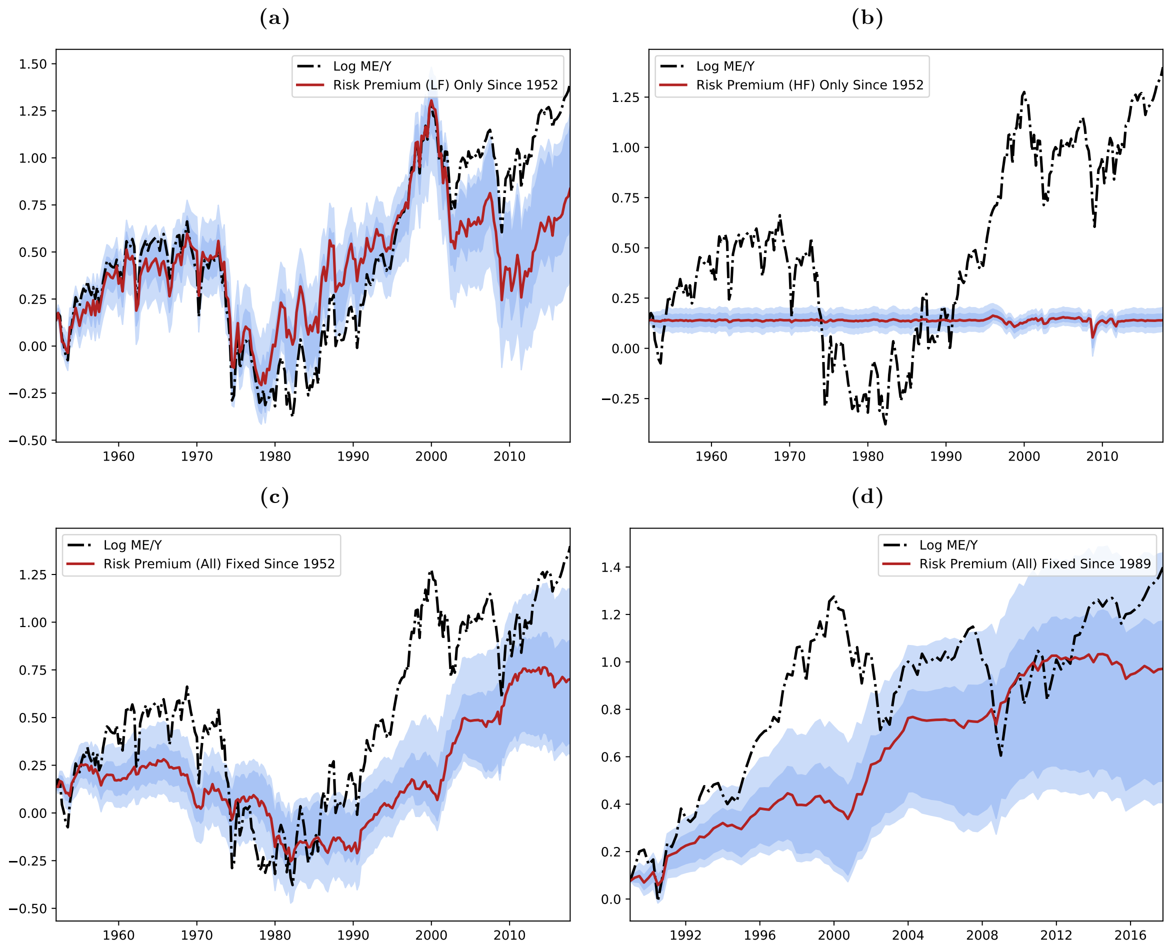
Notes: The figure exhbits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components. The label “Fixed Since” followed by a date describes a counterfactual path where this variable was held fixed from that date on. The notation “Only Since” followed by a date describes a counterfactual path where all other variables were held fixed from that date on. "(LF)" and "(HF)" refer to the low- and high-frequency components, while "(All)" refers to both components. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Figure 7: Market Equity-Output Ratio and Risk-Free Rate Component



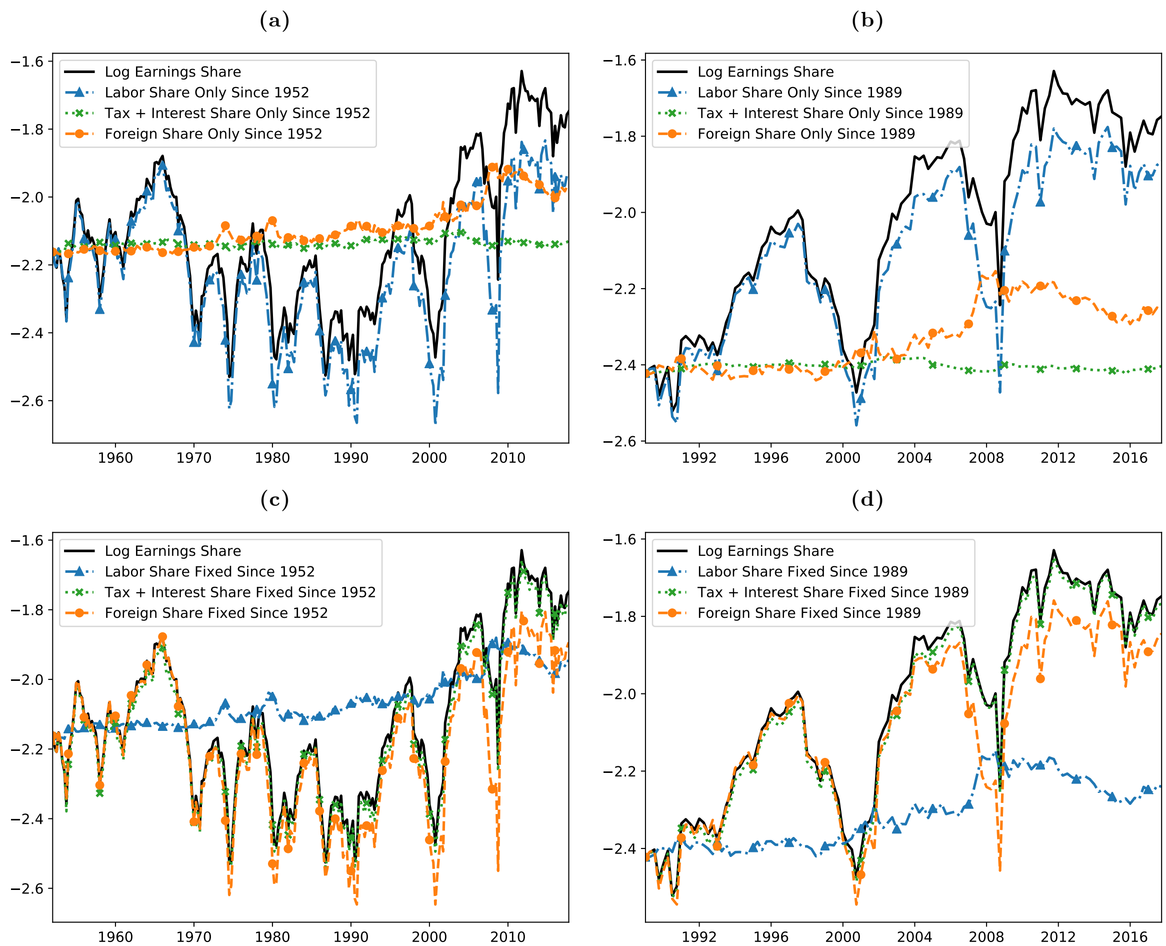
Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-free rate component. The label "Fixed Since" followed by a date describes a counterfactual path where this variable was held fixed from that date on. The notation "Only Since" followed by a date describes a counterfactual path where all other variables were held fixed from that date on. "(LF)" and "(HF)" refer to the low- and high-frequency components, while "(All)" refers to both components. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Figure 8: Market Equity-Output Ratio and Orthogonal Risk Price Component



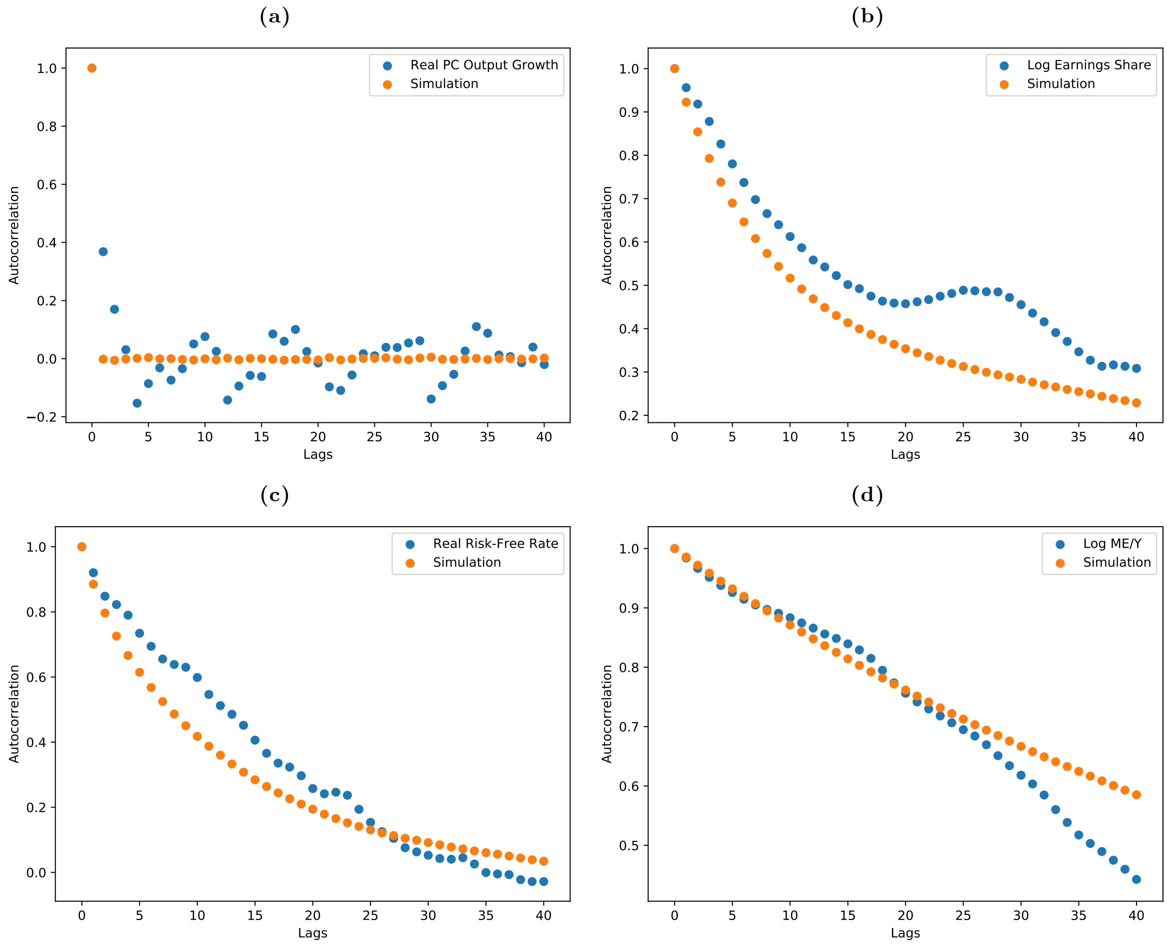
Notes: The figure exhibits the observed market equity-to-output series along with the model-implied variation in the series attributable to the component of the risk premium driven by shocks to the price of risk that are mutually uncorrelated with shocks to all other state variables in the model. The label "Fixed Since" followed by a date describes a counterfactual path where this variable was held fixed from that date on. The notation "Only Since" followed by a date describes a counterfactual path where all other variables were held fixed from that date on. "(LF)" and "(HF)" refer to the low- and high-frequency components, while "(All)" refers to both components. The red center line corresponds to point estimates, obtained from the Kalman smoother at the parameter mode. The shaded regions are 66.7% (darker bands) and 90% (lighter bands) credible sets. The sample spans the period 1952:Q1-2017:Q4.

Figure 9: Role of Components in Earnings Share



Notes: The figure decomposes the corporate earnings share S_t into contributions from changes in the domestic labor share S^D, the tax and interest share Z, and the foreign share F. Series denoted "only" show the result of allowing only that component to vary, while the others are held fixed at their initial values for that period (1952 or 1989). Series denoted "Fixed" show the result of leaving that one component fixed at the start of the period while allowing all of the other components to vary. The sample spans the period 1952:Q1-2017:Q4.

Figure 10: Observable Autocorrelations



Notes: The figure compares the data autocorrelations for the observable variables available over the full sample, compared to the same statistics from the model, obtained from a long simulation of 100,000 periods. The sample spans the period 1952:Q1-2017:Q4.

Table 1: Parameter Estimates

Variable	Symbol	Mode	5%	Median	95%
Risk Price Mean	$\bar{x}$	4.0460	3.3619	4.5315	6.5421
Risk Price (HF) Pers.	$\phi_{x,HF}$	0.6705	0.5337	0.6916	0.8074
Risk Price (HF) Vol.	$\sigma_{x,HF}$	1.5370	1.2031	1.8191	2.9421
Risk Price (LF) Pers.	$\phi_{x,LF}$	0.9864	0.9781	0.9855	0.9915
Risk Price (LF) Vol.	$\sigma_{x,LF}$	0.4933	0.3525	0.5841	0.9693
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.5639	0.1590	0.6630	0.8849
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0012	0.0002	0.0011	0.0019
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9267	0.8739	0.9147	0.9655
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0015	0.0004	0.0016	0.0020
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.8787	0.7917	0.8735	0.9176
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0534	0.0298	0.0520	0.0576
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9848	0.9363	0.9834	0.9966
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0162	0.0074	0.0175	0.0456
Productivity Vol.	σ_a	0.0152	0.0143	0.0153	0.0165
Risk Loading, LF Factor Share	λ	-7.9304	-10.5362	-7.1975	-3.4726

Notes: The table reports parameter estimates from the posterior distribution. The sample spans the period 1952:Q1-2017:Q4.

Table 2: Asset Pricing Moments

Variable	Model Mean	Model SD	Fitted Mean	Fitted SD	Data Mean	Data SD
Log Equity Return	4.852	17.423	7.681	16.791	8.852	15.724
Log Risk-Free Rate	1.114	1.450	1.126	1.932	1.129	1.929
Log Excess Return	3.738	17.499	6.560	16.785	7.389	16.436
Log Price-Payout Ratio	3.778	0.404	3.410	0.376	3.434	0.465
Log Earnings Growth	2.226	8.671	2.819	11.819	2.819	11.819
Log Payout Growth	2.226	18.369	3.790	23.845	4.045	33.455
Log Earnings Share Growth	0.000	8.310	0.624	10.379	0.624	10.379
Log Payout Share Growth	0.001	18.203	1.651	22.621	1.907	32.186

Notes: All statistics are computed for annual (continuously compounded) data. "Model"numbers are averages across 1000 simulations of the model of the same size as our data sample. "Fitted"numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Table 3: Growth Decomposition: Baseline

	Panel: Market Equity		
Contribution	1952-2017	1952-1988	1989-2017
Total	1405.81%	151.23%	477.34%
Factor Share s_t	18.57%	-23.34%	42.53%
$s_{LF,t}$	17.05%	-21.59%	37.88%
$s_{HF,t}$	1.52%	-1.75%	4.64%
Risk Price $x_{\perp,t}$	25.73%	20.46%	24.42%
$x_{LF,t}$	0.05%	-0.32%	24.32%
$x_{HF,t}$	25.68%	20.78%	0.10%
Risk-free Rate δ_t	2.16%	-8.52%	8.48%
$\delta_{LF,t}$	2.11%	-8.57%	8.35%
$\delta_{HF,t}$	0.05%	0.05%	0.13%
Real PC Output Growth	53.54%	111.41%	24.57%

Notes: The table presents the growth decompositions for the real value of market equity. The sample spans the period 1952:Q1-2017:Q4.

Appendix: For Online Publication

Data Description

CORPORATE EQUITY

Corporate equity is obtained from the Flow of Funds Table B103, series code LM103164103, nonfinancial corporate business; corporate equities; liability. Unadjusted transactions estimated by Federal Reserve Board (Capital Markets and Flow of Funds Sections), using data from the following commercial sources: cash mergers and acquisitions data from Thompson Financial Services SDC database; public issuance and share repurchase data from Standard and Poor's Compustat database; and private equity issuance data from Dow Jones Private Equity Analyst and PriceWaterhouseCoopers Money tree report. Level at market value is obtained separately as the sum of the market value of the nonfinancial corporate business (FOF series LM103164103) and the financial corporate business (FOF series LM793164105). Source: Federal Reserve Board.

FOREIGN EARNINGS

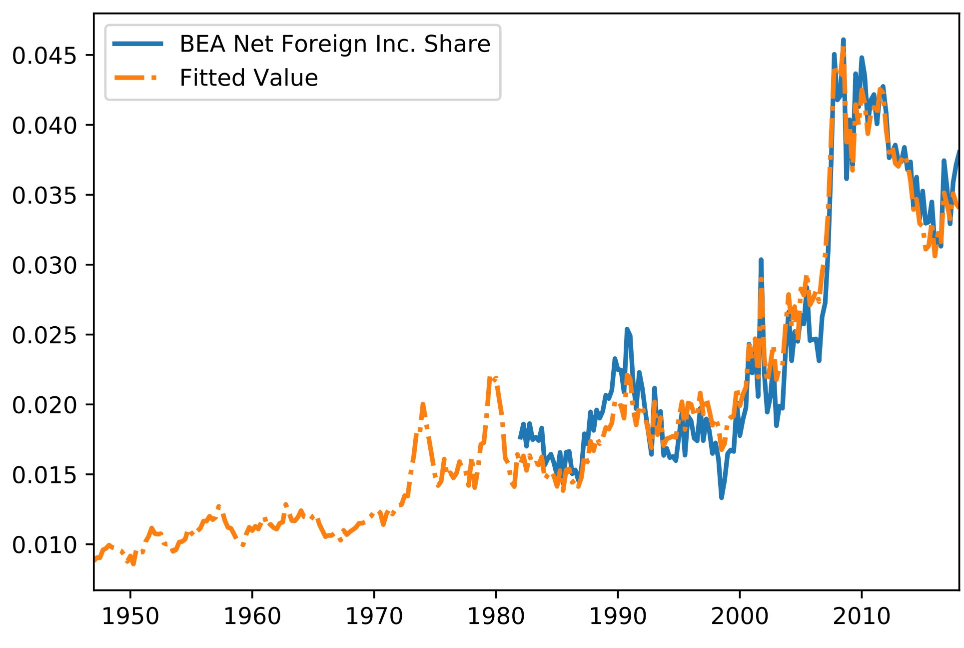
Total earnings is the sum of domestic after-tax profits from NIPA and earnings of U.S. multinational enterprises on their overseas operations. Total earnings are defined as as share of domestic net-value-added for the corporate sector. (See the next subsection for the sources of domestic data.)

$$\begin{aligned} E_t &\equiv S_t Y_t \\ &= \left(S_t^D Z_t + F_t\right) Y_t. \end{aligned}$$

In the above, F_t is the foreign profit share of domestic output. The measure of foreign profits in the numerator of F_t is based on data from Table 4.2 of the U.S. International Transactions in Primary Income on Direct Investment, obtained from BEA's International Data section. We refer to this simply as corporate "direct investment." Specifically, these data are from the "income on equity" row 2 of Direct investment income on assets, asset/liability basis. Note that U.S. direct investment abroad is ownership by a U.S. investor of at least 10 percent of a foreign business, and so excludes household portfolio investment.

This series is available from 1982:Q1 to the present. To extend this series backward, we first take data on net foreign receipts from abroad (Corporate profits with IVA and CCAdj from BEA NIPA Table 1.12. (A051RC) or from Flow of Funds (FOF) Table F.3 (FA096060035.Q less corporate profits with IVA and CCAdj, domestic industries from BEA NIPA Table 1.14 (A445RC)), which is available from the post-war period onward. This series includes portfolio investment income of households as well as direct investment, but its share of domestic net-value-added for the corporate sector is highly correlated with the foreign direct investment share of net-value-added. We regress the direct investment share of net-value-added on the foreign receipts share of domestic net-value-added and then use the fitted value from this regression as the measure of F_t in data pre-1982. Because the portfolio income component is relatively small, the fit of this regression is high, as seen in Figure A.1, which compares the fitted series with the actual series over the post-1982 period.

Figure A.1: Net Foreign Income Share: Data vs. Fitted value



Notes: The sample spans the period 1952:Q1-2018:Q2.

DOMESTIC VARIABLES: CORP. NET VALUE ADDED, CORP. LABOR COMPENSATION, CORP. AFTER-TAX PROFITS, TAXES AND INTEREST

Define domestic corporate earnings E_t as

$$E_t \equiv S_t^D \left(1 - \tau_t\right) NVA_t,$$

which is equivalent to

$$E_t = \left[1 - \underbrace{\frac{LC_t}{ATP_t + LC_t}}_{\text{Labor share of labor+profit}}\right]\left(1 - \tau_t\right) NVA_t.$$

Data for the net value added (NVA) comes from NIPA Table 1.14 (corporate sector series codes A457RC1 and A438RC1). We use per capita real net value added, deflated by the implicit price deflator for net value added. After tax profits (ATP) for the domestic sector come from NIPA Table 1.14 (corporate sector series code: W273RC1). Nonfinancial corporate sector labor compensation (LC) for the domestic sector is from Table 1.14 (series code A460RC). The doestic after-tax profit share ($ATPS$) of NVA is identically equal to

$$\begin{aligned} ATPS &= \frac{ATP}{ATP + LC}\frac{ATP + LC}{NVA} = \underbrace{\frac{ATP}{ATP + LC}}_{\equiv S_t}\frac{NVA - (\text{taxes and interest})}{NVA} \\ &= S_t^D\left[1 - \underbrace{\left(\frac{\text{taxes and interest}}{NVA}\right)}_{\equiv \tau_t}\right] \\ &= S_t^D Z_t, \end{aligned}$$

where S_t^D is the domestic after-tax profit share of combined profit plus labor compensation, "taxes and interest" is the sum of taxes on production and imports less subsidies (W325RC1), net interest and miscellaneous payments (B471RC1), business current transfer payments (Net) (W327RC1), and taxes on corporate income (B465RC1). Source: Bureau of Economic Analysis.

NET DIVIDENDS PLUS NET REPURCHASES (EQUITY PAYOUT)

Net dividends minus net equity issuance is computed using flow of funds data. Net dividends ("netdiv") is the series named for corporate business; net dividends paid (FA096121073.Q). Net repurchases are repurchases net of share issuance, so net repurchases is the negative of net equity issuance. Net equity issuance ("netequi") is the sum of Equity Issuance for Nonfinancial corporate business; corporate equities; liability (Table F.103, series FA103164103) and Equity Issuance for domestic financial sectors; corporate equities; liability (Table F.108, series FA793164105). Since netdiv and netequi are annualized, the quarterly payout is computed as payout=(netdiv-netequi)/4. The units are in millions of dollars. Source: Federal Reserve Board.

PRICE DEFLATORS

Implicit price deflator and GDP deflator. A chain-type price deflator for the nonfinancial corporate sector (NFCS) is obtained implicitly by dividing the net value added of nonfinancial corporate business by the chained real dollar net value added of nonfinancial corporate business from NIPA Table 1.14. This index is used to deflate net value added of the corporate sector. There is no implicit price deflator available for the whole corporate sector, so we use deflator for the non-financial corporate sector instead. The GDP deflator is used to construct a real returns and a real interest (see below). GDPDEF is retrieved from FRED. Our source is the Bureau of Economic Analysis.

INTEREST RATE

The nominal risk-free rate is measured by the 3-Month Treasury Bill rate, secondary market rate. We take the (average) quarterly 3-Month Treasury bill from FRED [TB3MS]. A real rate is constructed by subtracting the fitted value from a regression of GDP deflator inflation onto lags of inflation from the nominal rate. Our source is the board of governors of the Federal Reserve System and the Bureau of Economic Analysis.

RISK PREMIUM MEASURE

Our measure of the risk premium comes from Martin (2017). This paper uses option data to compute a lower bound on the equity risk premium, then argues that this lower bound is in fact tight, and a good measure of the true risk premium on the stock market. We obtain this series from the spreadsheet `epbound.xls` on Ian Martin's website, which corresponds to the value

$$EPBound_{t\to T} = 100 \times \left(R_{f,t} SVIX^2_{t\to T} - 1\right)$$

which is equivalent to the bound on the annualized net risk premium, in percent. To translate these measures to our model's quarterly frequency use the risk premium measure computed over the next three months. We then convert this variable into a log return, average it over the quarter, and label is rp_t. Since sample for this variable spans 1996:Q1 to 2012:Q1, which is shorter than our full sample period, our state-space estimation uses two measurement equations to accommodate the missing data. Specifically, we use the measurement equation

$$\begin{bmatrix} s_t & r_{ft} & py_t & \Delta y_t & rp_t \end{bmatrix}' = \mathbf{H}_1' \boldsymbol{\beta}_t + \mathbf{G}_1$$

for periods where this risk premium data is available, and

$$\begin{bmatrix} s_t & r_{ft} & py_t & \Delta y_t \end{bmatrix}' = \mathbf{H}_0 \boldsymbol{\beta}_t + \mathbf{G}_0$$

for periods where this data is not available, where $\mathbf{H}_0$ and $\mathbf{G}_0$ are identical to $\mathbf{H}_1$ and $\mathbf{G}_1$, except that they omit the rows corresponding to the missing data rp_t.

A Simple Model of Workers and Shareholders

We consider a stylized limited participation endowment economy in which wealth is concentrated in the hands of a few asset owners, or "shareholders," while most households are "workers" who finance consumption out of wages and salaries. The economy is closed. Workers own no risky asset shares and consume their labor earnings. There is no risk-sharing between workers and shareholders. A representative firm issues no new shares and buys back no shares. Cashflows are equal to output minus a wage bill,

$$C_t = Y_t - w_t N_t,$$

where w_t equals the wage and N_t is aggregate labor supply. The wage bill is equal to Y_t times a time-varying labor share α_t,

$$w_t N_t = \alpha_t Y_t => C_t = (1 - \alpha_t) Y_t. \tag{A. 1}$$

We rule out short sales in the risky asset:

$$\theta_t^i \geq 0.$$

Asset owners not only purchase shares in the risky security, but also trade with one another in a one-period bond with price at time t denoted by q_t. The real quantity of bonds is denoted B_{t+1}, where $B_{t+1} < 0$ represents a borrowing position. The bond is in zero net supply among asset owners. Asset owners could have idiosyncratic investment income ζ_t^i, which is idependently and identically distributed across investors and time. The gross financial assets of investor i at time t are given by

$$A_t^i \equiv \theta_t^i (V_t + C_t) + B_t^i.$$

The budget constraint for the ith investor is

$$\begin{aligned} C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i V_t &= A_t^i + \zeta_t^i \\ &= \theta_t^i (V_t + C_t) + B_t^i + \zeta_t^i, \end{aligned} \tag{A. 2}$$

where C_t^i denotes the consumption of investor i.

A large number of identical nonrich workers, denoted by w, receive labor income and do not participate in asset markets. The budget constraint for the representative worker is therefore

$$C^w = \alpha_t Y_t. \tag{A. 3}$$

Equity market clearing requires

$$\sum_i \theta_t^i = 1.$$

Bond market clearing requires

$$\sum_i B_t^i = 0.$$

Aggregating (A. 2) and (A. 3) and imposing both market clearing and (A. 1) implies that aggregate (worker plus shareholder) consumption C_t^{Agg} is equal to total output Y_t. Aggregating over the budget constraint of shareholders shows that their consumption is equal to the capital share times aggregate consumption C_t^{Agg}:

$$C_t^S = C_t = \underbrace{(1 - \alpha_t)}_{KS_t} C_t^{AGG}.$$

A representative shareholder who owns the entire corporate sector will therefore have consumption equal to $C_t^{Agg} \cdot KS_t$. This reasoning goes through as an approximation if workers own a small fraction of the corporate sector even if there is some risk-sharing in the form of risk-free borrowing and lending between workers and shareholders, as long as any risk-sharing across these groups is imperfect. While individual shareholders can smooth out transitory fluctuations in income by buying and selling assets, shareholders as a whole are less able to do so since purchases and sales of any asset must net to zero across all asset owners.

Model Solution

This section derives the coefficients of the main asset pricing equation (17). To begin, define for convenience the variables

$$u_{t+1} = \log(PC_{t+1} + 1) - pc_t$$
$$q_{t+1} = m_{t+1} + \Delta c_{t+1}$$

so that $m_{t+1} + r_{t+1} = u_{t+1} + q_{t+1}$. Applying the log linear approximation to $\log(PC_{t+1} + 1)$ and substituting in our guessed functional form (17) yields

$$\begin{aligned} u_{t+1} &\equiv \log(PC_{t+1} + 1) - pc_t \\ &= \kappa_0 + \kappa_1 \Big(A_0 + \mathbf{A}_s' \; \tilde{\mathbf{v}}_{t+1} + \mathbf{A}_x' \tilde{\mathbf{x}}_{t+1} + \mathbf{A}_\delta' \tilde{\boldsymbol{\delta}}_t \Big) - \Big(A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_t + \mathbf{A}_x' \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' \; \tilde{\boldsymbol{\delta}}_t \Big) \\ &= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - I \right) \tilde{\mathbf{s}}_t + \mathbf{A}_x' (\kappa_1 \mathbf{\Phi}_x - I) \tilde{\mathbf{x}}_t + \mathbf{A}_\delta' (\kappa_1 \Phi_\delta - I) \; \tilde{\boldsymbol{\delta}}_t \\ &\qquad + \kappa_1 \mathbf{A}_s' \varepsilon_{s,t+1} + \kappa_1 \mathbf{A}_x' \varepsilon_{x,t+1} + \kappa_1 \mathbf{A}_\delta' \varepsilon_{\delta,t+1}. \end{aligned}$$

Now turning to q_{t+1}, we can expand the expression to yield

$$q_{t+1} = -\delta_t + x_t g - x_t \boldsymbol{\xi}' (\mathbf{I} - \; \mathbf{\Phi}_s) \tilde{\mathbf{s}}_t - \frac{1}{2} x_t^2 \Big(\boldsymbol{\xi}' \mathbf{\Sigma}_s \boldsymbol{\xi} + \sigma_y^2 \Big) + (1 - x_t) \Delta c_{t+1}.$$

Next, we apply our fundamental asset pricing equation $0 = \log E_t \left[q_{t+1} + u_{t+1} \right]$, which under lognormality implies

$$0 = E_t[q_{t+1}] + E_t[u_{t+1}] + \frac{1}{2} \text{Var}_t(q_{t+1}) + \frac{1}{2} \text{Var}_t(u_{t+1}) + \text{Cov}(q_{t+1}, u_{t+1}).$$

These moments can be calculated as

$$
\begin{aligned}
E_t[q_{t+1}] &= -\delta_t + g - \boldsymbol{\xi}'(\mathbf{I} - \ \boldsymbol{\Phi}_s)\tilde{\mathbf{s}}_t - \frac{1}{2}x_t^2\Big(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\Big) \\
E_t[u_{t+1}] &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s'\left(\kappa_1\boldsymbol{\Phi}_s - \mathbf{I}\right)\tilde{\mathbf{s}}_t + \mathbf{A}_x'(\kappa_1\boldsymbol{\Phi}_x - \mathbf{I})\tilde{\mathbf{x}}_t + \mathbf{A}_\delta'(\kappa_1\Phi_\delta - \mathbf{I})\tilde{\boldsymbol{\delta}}_t \\
\mathrm{Var}_t(q_{t+1}) &= (1 - x_t)^2\left(\boldsymbol{\xi}'\ \boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) \\
\mathrm{Var}_t(u_{t+1}) &= \kappa_1^2\Big(\mathbf{A}_s'\ \boldsymbol{\Sigma}_s\mathbf{A}_s + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\ \mathbf{A}_\delta\Big) \\
\mathrm{Cov}_t(q_{t+1}, u_{t+1}) &= \kappa_1(1 - x_t)(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\mathbf{A}_s).
\end{aligned}
$$

Substituting, we obtain

$$
\begin{aligned}
0 = g - \mathbf{1}'\bar{\delta} &+ \kappa_0 + (\kappa_1 - 1)A_0 + \frac{1}{2}\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\ \boldsymbol{\Sigma}_s\mathbf{A}_s + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\mathbf{A}_\delta\Big) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi}) \\
&- \Big[\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big]\mathbf{1}'\bar{x} \\
&- \Big[\boldsymbol{\xi}'(\mathbf{I} - \ \boldsymbol{\Phi}_s) + \mathbf{A}_s'\left(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_s\right) + \Big(\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big)\lambda'\Big]\tilde{\mathbf{s}}_t \\
&- \Big[\mathbf{A}_x'(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_x) + \Big(\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big)\mathbf{1}'\Big]\tilde{\mathbf{x}}_t \\
&- \Big[\mathbf{1}' + \mathbf{A}_\delta'(\mathbf{I} - \kappa_1\Phi_\delta)\Big]\tilde{\boldsymbol{\delta}}_t
\end{aligned}
$$

Applying the method of undetermined coefficients now yields the solutions

$$
\begin{aligned}
\mathbf{A}_s' &= -\Big[\boldsymbol{\xi}'(\mathbf{I} - \ \boldsymbol{\Phi}_s) + \Big(\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right)\Big)\lambda'\Big]\Big[(\mathbf{I} - \kappa_1\boldsymbol{\Phi}_s) + \kappa_1\boldsymbol{\Sigma}_s\boldsymbol{\xi}\lambda'\Big]^{-1} \\
\mathbf{A}_x' &= -\Big[\Big(\left(\boldsymbol{\xi}'\boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big)\mathbf{1}'\Big](\mathbf{I} - \kappa_1\boldsymbol{\Phi}_x)^{-1} \\
\mathbf{A}_r' &= -\mathbf{1}'(\mathbf{I} - \kappa_1\Phi_\delta)^{-1}
\end{aligned}
$$

while the constant term must solve

$$
\begin{aligned}
0 = g - \mathbf{1}'\bar{\delta} + \kappa_0 + (\kappa_1 - 1)A_0 &+ \frac{1}{2}(1 - 2\bar{x})\Big[\left(\boldsymbol{\xi}'\ \boldsymbol{\Sigma}_s\boldsymbol{\xi} + \sigma_y^2\right) + \kappa_1(\mathbf{A}_s'\boldsymbol{\Sigma}_s\boldsymbol{\xi})\Big] \\
&+ \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\ \boldsymbol{\Sigma}_s\mathbf{A}_s + \mathbf{A}_x'\boldsymbol{\Sigma}_x\mathbf{A}_x + \mathbf{A}_\delta'\boldsymbol{\Sigma}_\delta\ \mathbf{A}_\delta\Big)
\end{aligned} \tag{A. 4}
$$

Equilibrium Selection

The parameters κ_0 and κ_1 determine the steady state pc (price-payout ratio), which depends on A_0. But since κ_0 and κ_1 are both themselves nonlinear functions of A_0, the equilibrium condition (A. 4) is also nonlinear, leading to the possibility that multiple solutions, or no solution, exists. In fact, we confirm that both of these outcomes can occur in our numerical solutions. However, our numerical results indicate that, when there is more than one solution there are at most two, and one can be discarded because it is economically implausible. To see this, rewrite (A. 4) as

$$0 = \mathbb{E}[m] + \mathbb{E}[r] + \frac{1}{2}\mathrm{Var}(m) + \frac{1}{2}\mathrm{Var}(r) + \mathrm{Cov}(m, r)$$

where m and r are the log SDF and equity return. We are interested in the relationship between the steady state pc and the other terms that depend on it in equilibrium. The terms $\mathbb{E}[m]$ and $\mathrm{Var}(m)$ do not depend on the pc ratio, so we can ignore these and focus on the remaining terms. Alternatively, consider the log risk premium, given in equilibrium by

$$\mathbb{E}[r_{t+1}] - r_{f,t} = -\frac{1}{2}\mathrm{Var}(r_{t+1}) - \mathrm{Cov}(m_{t+1}, r_{t+1}).$$

In the case where there are two solutions, one solution typically has a plausible level for the steady state pc, and implies that higher pc ratios (which take different values depending on where in the posterior distribution of model parameters we evaluate the function) coincide with lower risk premia $\mathbb{E}[r_{t+1}] - r_{f,t}$ and a lower absolute covariance with the SDF (i.e., a less negative $\mathrm{Cov}(m_{t+1}, r_{t+1})$). This solution is economically reasonable. By contrast, when there is a second solution, it is always characterized by values for pc that are higher than the economically reasonable solution, and for typical parameter values delivers a value for pc that are extremely implausible (e.g., a value for $\exp(pc)$ of almost 3,000 at the posterior mode). In addition, this solution has the property that the higher pc ratios coincide with lower risk premia vis-a-vis the plausible solution, but also *higher* absolute covariances with the SDF (i.e., a more negative $\mathrm{Cov}(m_{t+1}, r_{t+1})$). Thus the higher pc ratios in this solution must be explained by a lower absolute covariance with the SDF and a Jensen's term $\frac{1}{2}\mathrm{Var}(r_{t+1})$ that in some cases converges to infinity. In summary, since the higher pc solution typically implies extreme values and unreasonable behavior of pc, we select between these solutions by enforcing that the equilibrium chosen always chooses the lower pc solution.

Expected Returns

Combining the relations

$$\begin{aligned} 0 &= \log E_t[M_{t+1}R_{t+1}] \\ &= E_t[m_{t+1}] + E_t[r_{t+1}] + \frac{1}{2}\mathrm{Var}_t(m_{t+1}) + \frac{1}{2}\mathrm{Var}_t(r_{t+1}) + \mathrm{Cov}_t(m_{t+1}, r_{t+1}) \\ -r_{f,t} &= \log E_t[M_{t+1}] \\ &= E_t[m_{t+1}] + \frac{1}{2}\mathrm{Var}_t(m_{t+1}) \end{aligned}$$

and rearranging, we obtain

$$E_t[r_{t+1}] = r_{f,t} - \frac{1}{2}\mathrm{Var}_t(r_{t+1}) - \mathrm{Cov}_t(m_{t+1}, r_{t+1})$$

which is the usual decomposition of the expected log return into the risk-free rate, a Jensen term, and the risk premium. Since

$$\begin{aligned} r_{t+1} &= \mathrm{const}_t + \underbrace{\kappa_1 \mathbf{A}_s' \varepsilon_{s,t+1} + \kappa_1 \mathbf{A}_x' \varepsilon_{x,t+1} + \kappa_1 \mathbf{A}_\delta' \varepsilon_{\delta,t+1}}_{pc \text{ growth}} + \underbrace{\boldsymbol{\xi}' \varepsilon_{s,t+1} + \varepsilon_{y,t+1}}_{\text{cash flow growth}} \\ m_{t+1} &= \mathrm{const}_t - x_t \boldsymbol{\xi}' \varepsilon_{s,t+1} - x_t \varepsilon_{y,t+1} \end{aligned}$$

we obtain

$$\begin{aligned} \mathrm{Var}_t(r_{t+1}) &= (\kappa_1 \mathbf{A}_s + \boldsymbol{\xi})' \, \boldsymbol{\Sigma}_s \, (\kappa_1 \mathbf{A}_s + \boldsymbol{\xi}) + \kappa_1^2 \mathbf{A}_x' \boldsymbol{\Sigma}_s \mathbf{A}_x + \kappa_1^2 \mathbf{A}_\delta' \boldsymbol{\Sigma}_r \mathbf{A}_\delta + \sigma_y^2 \\ \mathrm{Cov}_t(m_{t+1}, r_{t+1}) &= -x_t \Big[\boldsymbol{\xi}' \, \boldsymbol{\Sigma}_s \, (\kappa_1 \mathbf{A}_s + \boldsymbol{\xi}) + \sigma_y^2 \Big] \end{aligned}$$

which can be substituted to obtain

$$E_t[r_{t+1}] - r_{f,t} = x_t \Big[\boldsymbol{\xi}' \, \boldsymbol{\Sigma}_s \boldsymbol{\xi} + \kappa_1 \boldsymbol{\xi}' \, \boldsymbol{\Sigma}_s \mathbf{A}_s + \sigma_y^2 \Big] - \frac{1}{2}\mathrm{Var}_t(r_{t+1})$$

Estimation Details

This section describes the procedure used to obtain the parameter draws. First, because some of our variables are bounded by definition (e.g., volatilities cannot be negative), we define a set of parameter vectors satisfying these bounds denoted Θ. We exclude parameters outside of this set, which formally means that we apply a Bayesian prior

$$p(\boldsymbol{\theta}) = \begin{cases} \mathrm{const} & \text{for } \boldsymbol{\theta} \in \Theta \\ 0 & \text{for } \boldsymbol{\theta} \notin \Theta \end{cases}$$

Our restrictions on Θ are as follows: all volatilities (σ), the average risk price $\bar{x}$, the average growth rate g, and the average real risk-free rate $\bar{\delta}$ are bounded below at zero. All persistence parameters (ϕ) and the average profit share $\exp(\bar{s})$ are bounded between zero and unity.

With these bounds set, we can evaluate the posterior by

$$\pi(\boldsymbol{\theta}) = L(y|\boldsymbol{\theta})p(\boldsymbol{\theta}).$$

so that the posterior is simply proportional to the likelihood over Θ and is equal to zero outside of Θ.

To draw from this posterior, we use a Random Walk Metropolis Hastings algorithm. We initialize the first draw $\boldsymbol{\theta}_0$ at the mode, and then iterate on the following algorithm:

1. Given $\boldsymbol{\theta}_j$, draw a proposal $\boldsymbol{\theta}^*$ from the distribution $\mathcal{N}(\boldsymbol{\theta}_j, c\boldsymbol{\Sigma}_\theta)$ for some scalar c and matrix $\boldsymbol{\Sigma}_\theta$ defined below.

2. Compute the ratio
$$\alpha = \frac{\pi(\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}_j)}.$$

3. Draw u from a Uniform [0, 1] distribution.

4. If $u < \alpha$, we accept the proposed draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}^*$. Otherwise, we reject the draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j$.

For the covariance term, we initialize $\boldsymbol{\Sigma}_\theta$ to be the inverse Hessian of the log likelihood function at the mode. Once we have saved 10,000 draws, we begin updating $\boldsymbol{\Sigma}_\theta$ to be the sample covariance of the draws to date, following Haario, Saksman, Tamminen, et al. (2001), with the matrix re-computed after every 1,000 saved draws. For the scaling parameter c, we initialize it at $2.4/\text{length}(\boldsymbol{\theta})$ as recommended in Gelman, Stern, Carlin, Dunson, Vehtari, and Rubin (2013). To target an acceptance rate for our algorithm of 25%, we adapt the approach of Herbst and Schorfheide (2014) in updating

$$c_{new} = c_{old} \cdot \left(0.95 + 0.1\frac{\exp(16(x - 0.25))}{1 + \exp(16(x - 0.25))}\right)$$

after every 1,000 saved draws, where c_{old} is the pre-update value of c.