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REFLEXIVITY IN CREDIT MARKETS

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ABSTRACT

Reflexivity is the idea that investors' biased beliefs affect market outcomes and that market outcomes in turn affect investors' future biases. We develop a dynamic behavioral model of the credit cycle featuring this two-way feedback loop. Investors form beliefs about the likelihood of future defaults by extrapolating past defaults. Investor beliefs influence a firm's actual creditworthiness because the firm is less likely to default in the short run when it can issue debt on favorable terms. Our model matches many features of the credit cycle, including its imperfect synchronization with the real economy and the "calm before the storm" phenomenon.

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I. Introduction

Over the past decade, researchers have documented a number of new facts about the credit cycle. High credit growth is associated with both a higher probability of a future financial crisis and lower future GDP growth (Schularick and Taylor, 2012; López-Salido, Stein, and Zakrajšek, 2017; Mian, Sufi, and Verner, 2017). Credit market returns are also predictable, suggesting a role for investor sentiment in the credit cycle. Greenwood and Hanson (2013) document that periods of elevated corporate credit growth and low average borrower credit quality forecast low returns to credit. In a large panel of countries, Baron and Xiong (2017) find that high bank credit growth forecasts low returns to bank stocks. Moreover, Greenwood, Hanson, Shleifer, and Sørensen (2022) show that the combination of large credit expansions and asset price booms predicts financial crises.

An underappreciated feature of the credit cycle is how disconnected it can be from the stock market or the broader macroeconomy in the short run. In post-war U.S. history, credit expansions and contractions have often followed a similar pattern. Credit grows slowly as the economy emerges from a recession, picks up steam, but continues to expand even as the overall economy cools. For example, in the upswing preceding the 2008 financial crisis, GDP growth peaked in March 2005, but credit growth peaked two years later in March 2007, a period when credit spreads were near historical lows. Put simply, at short horizons, the credit cycle seems to have a life of its own. However, these disconnects point to a limitation of many well-known models of the credit cycle—e.g., Bernanke and Gertler (1989), Holmström and Tirole (1997), Bernanke, Gertler, and Gilchrist (1999)—and even for more recent behavioral models like Bordalo, Gennaioli, and Shleifer (2018). Each of these models involves a single state variable. As a result, credit market frictions or belief biases only amplify business cycle fluctuations in these models, and the business cycle and the credit cycle are essentially one and the same.

In this paper, we present a new behavioral model of the credit cycle that is consistent with much of the accumulating evidence on credit cycles, but also speaks to periods of disconnect between credit markets and economic fundamentals. A key feature of our model is "reflexivity," the idea that there is a dynamic two-way feedback loop between investors' *biased* beliefs and market outcomes. In finance, the idea of reflexivity is most prominently associated with the investor George Soros, who argued that "distorted views can influence the situation to which they relate because false views lead to inappropriate actions" (*Financial Times*, October 26, 2009).[1] In credit markets, reflexivity arises because investors who overestimate the creditworthiness of a borrower refinance maturing debt on more favorable terms, thereby making the borrower less likely to default, at least in the short run. In the long run, such a borrower may take on additional leverage, ultimately becoming more vulnerable to default.

Our model builds on prior dynamic models of firm capital structure, including Leland (1994), He and Xiong (2012), and DeMarzo and He (2021), by adding a behavioral component. In our model, debt financing is provided by a set of biased bond investors whose beliefs about the future likelihood of firm default depend only on the firm's recent default history. Following periods of low defaults, these bond investors—who are "default extrapolators"—believe that debt is safe and hence refinance maturing debt on attractive terms. We study the interaction between these behavioral bond investors and a representative rational firm who maximizes its equity value, thus adopting the "rational managers with irrational investors" approach to behavioral corporate finance. The core assumption in this literature and in our paper, which is supported by the evidence reviewed below, is that firm managers can detect and respond to market mispricing.[2] Specifically, the rational firm exploits bond investor sentiment by optimally issuing more (less) debt when bond investors are overly optimistic (pessimistic) about the likelihood of future defaults. The firm also optimally chooses when to default and this decision depends on the level of bond investor sentiment.[3]

Because bond investors hold extrapolative beliefs based on defaults—which are endogenously determined in the model—and not on the firm's exogenous fundamentals, this leads to a dynamic feedback loop between biased investor beliefs and firm defaults. Current investor beliefs affect future defaults via the terms on which investors are willing to refinance debt today. Figure 1 illustrates this two-way feedback loop. During credit booms, default rates are low, so investors believe that future default rates will continue to be low. In the near term, these beliefs can be self-fulfilling:

[1]See Soros (1987, 2013) for an extensive discussion of reflexivity.

[2]This approach to behavioral corporate finance dates to Stein (1996). Baker and Wurgler (2013) and Malmendier (2018) provide recent surveys of this literature. While the sort of mispricing we have in mind stems from the biased beliefs of some investors, the assumption that firms respond to mispricing does not mean that firms are more savvy than sophisticated investors. Firm managers might simply follow useful rules of thumb, issuing more debt when credit spreads are low. Similarly, firms may face fewer arbitrage constraints than sophisticated investors. While both firms and sophisticated investors might be capable of detecting mispricing, a variety of arbitrage constraints may make it difficult for sophisticated investors to aggressively sell (buy) bonds when credit spreads are low (high).

[3]Thus, in the absence of mispricing, debt issuance and the firm's leverage dynamics reflect dynamic trade-off theory considerations as in DeMarzo and He (2021): issuing debt is beneficial because interest is tax-deductible, but higher leverage comes at a cost because default entails dead-weight costs.

the perception of low future defaults leads to elevated bond prices and low credit spreads, which in turn makes it easier for firms to refinance their maturing debt or issue new debt. Holding constant the firm's cash flow fundamentals, cheaper debt financing has two effects. On the one hand, it lowers the cost of debt issuance, which leads the firm to optimally delay default, further reinforcing investor beliefs and keeping credit spreads low. On the other hand, cheaper debt financing causes the firm to lever up more aggressively, which in turn makes it more likely for the firm to eventually default over the longer run. Together, these two effects imply that a firm with weak fundamentals is able to skate by for some time when investors are bullish, a phenomenon that we refer to as the "calm before the storm." However, when the reality of poor firm fundamentals finally catches up with the firm, defaults escalate.

[Place Figure 1 about here]

Conversely, suppose that the economy has just experienced an initial default. Since investors over-extrapolate these recent outcomes, they believe that the likelihood of future defaults is high, even if the firm restructures and reduces its leverage following the default. Investor beliefs turn out to be partially self-fulfilling in the short run: these bearish beliefs make it more expensive for the firm to issue debt. As a result, the firm lowers the leverage threshold at which it defaults, raising the true probability of default in the short run. In some circumstances, our model generates "default spirals" in which an initial default leads to an extended spell of further defaults, much like the spells that have been observed with sovereign debt restructurings (Das, Papaioannou, and Trebesch, 2012). The economy's vulnerability to default spirals depends not only on investor beliefs, but also on the firm's pre-default balance sheet. Intuitively, default spirals are more likely after a longer calm period when the firm has built up considerable leverage. In this case, the firm may need to go through multiple rounds of default and restructuring in order to deleverage.

In our model, transitions between credit booms and credit busts are ultimately caused by changes in firm fundamentals. However, because investors extrapolate past defaults and not the firm's cash flow fundamentals, these transitions are not fully synchronized with changes in fundamentals and can be highly path-dependent. For example, our model generates "calm before the storm" periods in which the cash flow fundamentals of the economy have turned, but credit markets are still buoyant. These episodes are consistent with Krishnamurthy and Muir (2020), who show that credit spreads are typically too low in the years preceding financial crises.

What happens if bond investors exogenously become more optimistic, revising down their perceived likelihood of future defaults? We show that such a shock to beliefs can be self-fulfilling in our model. Rising investor optimism means that the firm can issue debt at more attractive credit spreads. As a result, the firm optimally delays default, leading investors to become even more optimistic and further reducing credit spreads. This mechanism may help make sense of the 2020-2021 period in U.S. credit markets, where improvements in investor sentiment may have helped save the economy from a wave of defaults stemming from poor fundamentals, as argued by Hanson, Stein, Sunderam, and Zwick (2020). There is a limit on the extent to which investor optimism can be self-fulfilling, however, because under-priced credit eventually induces firms to become highly leveraged, making them more vulnerable to adverse fundamental shocks and thus raising the likelihood of default over the longer run. Although the feedback loop between biased investor expectations and market outcomes is always present, there are times when it is stronger. We use the model to characterize the conditions under which changes in investors' biased expectations have the most powerful impact on market outcomes.

Our model matches many of the facts about credit cycles that researchers have documented in recent years. First, rapid credit growth appears to be quite useful for predicting future financial crises and business cycle downturns (Schularick and Taylor, 2012; Mian et al., 2017; López-Salido et al., 2017), a result that is consistent with our model because outstanding credit grows rapidly when sentiment is high but cash flow fundamentals are poor. Relatedly, economies that have experienced high credit growth are more fragile, in the sense that they are vulnerable to shocks (Krishnamurthy and Muir, 2020). Second, high credit growth predicts low future returns on risky bonds in a univariate forecasting regression (Greenwood and Hanson, 2013; Baron and Xiong, 2017; Muir, 2019), a result that obtains in our model because the firm optimally issues large quantities of debt when investors underestimate the likelihood of future defaults. In our model, credit spreads are typically too low just before the economy experiences a wave of defaults, consistent with the evidence in Krishnamurthy and Muir (2020). Third, when credit markets become highly overheated, our model generates conditional expected excess returns on risky bonds that are significantly *negative*. This result, which is consistent with prior empirical evidence, is difficult to square with rational, risk-based models of credit cycles and therefore motivates models like ours which prominently feature biased investor beliefs.

After developing the main model, we contrast our findings with those of a benchmark rational model, in which both equity and debt investors are fully rational and forward-looking. In this rational model, deteriorating firm fundamentals cause forward-looking bond investors to perceive a higher likelihood of future firm defaults, leading bond prices to fall and credit spreads to rise. Conversely, following a default, the post-restructuring decline in firm leverage (appropriately) makes rational bond investors less concerned about the prospect of default, reducing the likelihood of another default in the near future. In other words, this fully rational model does not generate the "calm before the storm" and "default spiral" phenomena captured by our behavioral model. Moreover, by construction, bond returns are not predictable in this benchmark rational model.

Finally, we discuss a key difference between our model and an alternative behavioral model that features a rational firm and a set of behavioral bond investors who extrapolate the firm's past fundamental growth (rather than extrapolating based on past defaults). Consider the case when the firm's fundamentals have been deteriorating but no default has occurred yet. In our main model, the firm optimally chooses to delay default because overly optimistic bond investors keep the cost of debt issuance low. In the alternative model in which investors extrapolate based on past fundamentals, however, the firm would optimally choose to lower the leverage threshold at which it defaults—this tends to accelerate default—because overly pessimistic bond investors make it more costly for the firm to issue debt.

Our paper has much in common with Austrian theories of the credit cycle, including Mises (1924) and Hayek (1925), as well as the accounts of booms, panics, and crashes by Minsky (1986) and Kindleberger (1978). More recently, the idea that investors may neglect tail risk in credit markets was developed theoretically by Greenwood and Hanson (2013), Gennaioli, Shleifer, and Vishny (2012, 2015), and Bordalo et al. (2018). We also draw on growing evidence that investors extrapolate cash flows, past returns, or past crash occurrences (Barberis, Shleifer, and Vishny, 1998; Greenwood and Shleifer, 2014; Barberis, Greenwood, Jin, and Shleifer, 2015, 2018; Jin, 2015; Greenwood and Hanson, 2015). Most related here is Jin (2015), who presents a model in which investors' perceptions of crash risk depend on recent experience. Bordalo et al. (2018) provide a model of credit cycles in which extrapolative investor expectations play an important role and in which bond returns are predictable. Their model is similar to ours in several respects, but extrapolative expectations in their model are perfectly tied to cash flow fundamentals, rather than to endogenous credit market outcomes; in our model, fundamentals and beliefs can become quite disconnected.[4] Also related is Krishnamurthy and Li (2021), who analyze to what degree a behavioral model of credit cycles, such as the one presented here, can quantitatively match the historical evidence.

In Section II, we briefly summarize a number of stylized facts about the credit cycle, drawing on the papers cited above but also presenting some novel observations about the synchronicity of the credit cycle and the business cycle. Section III develops a dynamic continuous-time model of the credit market with a rational firm and a set of behavioral bond investors who form beliefs about future firm defaults by extrapolating past defaults. We explain the two-way feedback mechanism that is at the heart of our model. In Section IV, we formally define reflexivity and explain its properties. We discuss how the model can match a number of features of credit cycles that researchers have documented in recent years, such as the predictability of returns and low credit spreads before crises. Section V contrasts our model with two other models: a rational model in which forward-looking bond investors hold rational beliefs, and an alternative behavioral model in which bond investors extrapolate growth in firm fundamentals. Section VI concludes.

II. Motivating facts about the credit cycle

We begin by summarizing a set of stylized facts about credit cycles. The first four facts are drawn from previous work but we show that they continue to hold using more recent data; the fifth is based on some new empirical work of our own.

Observation 1. *Rapid credit growth predicts financial crises and business cycle downturns.*

In a panel of 14 countries dating back to 1870, Schularick and Taylor (2012) show that rapid credit growth predicts financial crises. Schularick and Taylor (2012) interpret their evidence as suggesting that financial crises are episodes of "credit booms gone bust." Mian et al. (2017) show that rapid credit growth—especially growth in household credit—predicts future declines in GDP growth in an panel of 30 countries from 1960 to 2012. López-Salido et al. (2017) show that frothy credit market conditions—proxied using declines in the credit quality of corporate borrowers and

[4] See also Coval, Pan, and Stafford (2014) who suggest that in derivatives markets, model misspecification only reveals itself in extreme circumstances, by which time it is too late. Bebchuk and Goldstein (2011) present a model in which self-fulfilling credit market freezes can arise because of interdependence between firms.

low credit spreads—predict low GDP growth in U.S. data from 1929 to 2015. López-Salido et al. (2017) attribute their findings to predictable reversals in credit market sentiment. Consistent with this view, using an international panel of 38 countries, Kirti (2018) shows that rapid credit growth that is accompanied by a deterioration in lending standards—i.e., by declining borrower credit quality—is associated with low future GDP growth. By contrast, when rapid credit growth is accompanied by stable lending standards, there is no such decline in future GDP growth. More recently, Greenwood et al. (2022) show that the probability of a future crisis is particularly elevated when rapid credit growth is accompanied by a boom in asset price.

A corollary of Observation 1—i.e., that credit growth predicts financial crises—is that economies that have experienced high credit growth are more fragile. Krishnamurthy and Muir (2020) argue that a natural way to interpret the findings in Schularick and Taylor (2012) is that rapid credit growth creates financial fragility. When a more leveraged economy is exogenously hit by a negative fundamental shock, such as a large decline in house prices, this results in a financial crisis. Alternately, crises may be triggered by predictable reversals in credit market sentiment as argued by López-Salido et al. (2017). Consistent with this view, Krishnamurthy and Muir (2020) show that credit spreads are typically "too low" in the years preceding financial crises. The model we develop reflects these ideas: in our model, credit booms feature low credit spreads, rapid credit growth, an endogenous rise in firm leverage, and the associated built-up in financial fragility. As a result, credit booms tend to "go bust" in our model: following a boom, negative shocks to fundamentals are more likely to trigger a crisis, which takes the form of a sequence of consecutive firm defaults.

Observation 2. *Credit market overheating—signaled either by (i) a rapid growth in debt outstanding or (ii) by a decline in the credit quality of debt issuers set against the backdrop of relatively low credit spreads—predicts low future returns on risky bonds.*

A growing literature has demonstrated that credit market overheating predicts low future returns on risky bonds. Greenwood and Hanson (2013) find that rapid growth in outstanding corporate credit predicts low returns on risky bonds in U.S. data. Muir (2019) finds the same pattern in an panel of 17 developed economies from 1870 to 2016. Relatedly, Baron and Xiong (2017) show that bank credit expansion also predicts low bank equity returns—which are naturally tied to the returns on risky debt—in a panel of 20 developed economies from 1920 to 2012.

Greenwood and Hanson (2013) develop a measure of credit market overheating based on the credit quality of corporate debt issuers. Their "high yield share" measure—the share of all corporate bond issuance in a given year that is from high-yield-rated firms—captures the intuition that when credit markets are overheated, low quality firms issue more debt to take advantage. Greenwood and Hanson (2013) show that declines in issuer credit quality predict low future corporate bond returns in a univariate sense. Furthermore, as emphasized by Greenwood and Hanson (2013) and López-Salido et al. (2017), issuer quality contains information about future bond returns beyond that contained in credit spreads. Specifically, in a multivariate regression specification, low-quality issuance negatively predicts future bond returns and credit spreads positively predict future returns.[5]

[Place Table 1 about here]

Table 1 updates the data from Greenwood and Hanson (2013) and considers a set of additional proxies for credit market overheating. The table shows return forecasting regressions of the form:

$$rx^{HY}_{t\to t+k} = a + b \cdot \mathit{Overheating}_t + \varepsilon_{t\to t+k}, \tag{1}$$

where $rx^{HY}_{t\to t+k}$ denotes the log return on high yield bonds in excess of the log returns on like-maturity Treasuries over a $k = 2$- or 3-year horizon beginning in year t. Here, $\mathit{Overheating}_t$ is a proxy for credit market overheating, measured using data through the end of year t. All of our data begin in 1983 and run through 2020, predicting returns through 2022.[6]

Columns (1) and (5) show that the log high yield share ($\log(HYS_t)$) predicts low future excess bond returns. A one standard deviation increase in $\log(HYS_t)$ is associated with a 6.3 percentage point reduction in log excess bond returns over the next two years, and a 7.8 percentage point reduction over the next three years.

Columns (2) and (6) of Table 1 show that the same forecasting results hold when credit market overheating is measured using the growth in aggregate nonfinancial corporate credit outstanding ($\mathit{Credit\ Growth}_t$). Aggregate nonfinancial corporate credit is the sum of corporate debt securities

[5] Relatedly, Sørensen (2021) develops a measure of overheating based on the notion that credit conditions are "loose" when the bonds of higher default-risk-firms offer little additional spread relative to those of lower-risk firms. He shows that loose credit conditions in this sense forecast low excess returns on risky corporate bonds.

[6] For results over different time horizons and with additional controls, see Greenwood and Hanson (2013) who compute other proxies for issuer quality that extend back as far as 1926.

and loans from Table L103 of the Federal Reserve's Financial Accounts of the U.S. A one standard deviation increase in *Credit Growth*$_t$ forecasts a 4.8 percentage point reduction in excess bond returns over the next two years, and a 5.8 percentage point reduction over the next three years.

Table 1 shows results for two additional measures of credit market overheating. The first, *Easy Credit*$_t$, is based on the Federal Reserve's Senior Loan Officer Opinion Survey (SLOOS), and the second, $-1 \times EBP_t$, is negative one times the Excess Bond Premium (EBP_t) from Gilchrist and Zakrajšek (2012).[7,8] Table 1 shows that both of these additional measures of credit market overheating forecast low future returns on corporate bonds. To summarize, Table 1 confirms that periods of credit market overheating—periods featuring low credit quality debt issuance, rapid growth in outstanding credit, loose credit standards, and tight credit spreads compared to fundamentals—are on average followed by low subsequent returns on risky corporate bonds.

Observation 3. *Significant credit market overheating is associated with* ***negative*** *expected excess returns on risky bonds.*

The fact that corporate bond returns are predictable does not imply that corporate bonds are occasionally *mispriced*. If the rationally-required returns on risky corporate bonds fluctuate over time—e.g., due to movements in investor risk aversion (Campbell and Cochrane, 1999) or in the quantity of aggregate risk (Bansal and Yaron, 2004; Gabaix, 2012; Wachter, 2013)—then the level of credit spreads might forecast future returns on corporate bonds. And, combining such fluctuations in rationally-required returns with the neoclassical q-theory of investment, one might expect recent credit growth and declines in debt issuer quality to forecast low returns on risky corporate bonds (Greenwood and Hanson, 2013; Gomes, Grotteria, and Wachter, 2019; Santos and Veronesi, 2022).

However, Greenwood and Hanson (2013) and Baron and Xiong (2017) present evidence that conditional expected excess returns on risky corporate bonds and bank stocks become reliably *negative* when credit markets appear to be significantly overheated—i.e., when many low quality

[7] Each quarter, the Federal Reserve asks senior loan officers at major U.S. banks about their lending standards. Loan officers report whether they have eased or tightened standards in the past quarter. We construct a measure of credit market overheating, *Easy Credit*$_t$, by taking the three-year average of the percentage of banks that reported easing credit standards to firms. The idea behind this averaging procedure is that we want to capture the *level* of bankers' beliefs about future creditworthiness, whereas the quarterly survey tracks quarterly changes. The SLOOS begins in the first quarter of 1990, so this measure of overheating begins in December 1992. *Easy Credit*$_t$ is 24% correlated with the high yield share (HYS_t) and 71% correlated with *Credit Growth*$_t$.

[8] The EBP_t variable from Gilchrist and Zakrajšek (2012) equals average corporate credit spreads after deducting an estimate of each bond's expected credit losses and can be interpreted as a proxy for expected future credit returns.

borrowers are able to obtain credit and when credit growth is rapid. Furthermore, these same authors find that future risk is high when credit markets appear to be most overheated (see Muir, 2019 for further evidence on this point). These negative expected excess returns and the negative conditional relationship between expected future risk and return are quite difficult to square with rational risk-based models—even rational models with intermediation frictions—and are powerful motivations for the behavioral approach we adopt in this paper.[9]

Observation 4. *Episodes of credit market overheating tend to follow periods of tranquility in credit markets, namely periods when defaults are low and when the returns on risky bonds are high. This suggests that credit market investors extrapolate past defaults.*

What outcomes are credit-market investors over-extrapolating? One view is that investors over-extrapolate some underlying set of economic fundamentals—e.g., firm cash flows or the state of broader macroeconomy. This view leads to behavioral versions of the q-theory of investment (Greenwood and Hanson, 2015; Gennaioli, Ma, and Shleifer, 2016; Bordalo et al., 2018). However, an alternative view is that credit market investors tend to over-extrapolate recent credit market outcomes. Consistent with this view, Greenwood and Hanson (2013) show that past defaults and credit returns play a dominant role in shaping credit market sentiment. They find that debt issuer quality tends to deteriorate following periods with low realized corporate defaults and high realized returns on risky corporate bonds. However, after controlling for these recent credit market outcomes, recent equity returns and macro variables have relatively little impact on debt issuer quality. These findings motivate our model where credit investors extrapolate past bond defaults, which themselves are not perfectly tied to firm fundamentals.[10]

[Place Table 2 about here]

Table 2 presents additional evidence that periods of credit market overheating follow times

[9] In models with intermediation frictions, changes in the health of intermediary balance sheets and the resulting shifts in risk appetite play an important role in determining asset prices. See, for example, He and Krishnamurthy (2013), Tobias, Etula, and Muir (2014), Brunnermeier and Sannikov (2014), and He, Kelly, and Manela (2017). However, since risky corporate bonds experience low returns in bad times for financial intermediaries, the expected excess returns on risky bonds must always be positive in these models.

[10] Greenwood and Shleifer (2014) show that past equity returns play an outsized role in shaping equity market sentiment, motivating the model in Barberis et al. (2015) where equity investors extrapolate past equity returns.

when corporate defaults are low. We estimate time-series regressions of the form:

$$Overheating_t = a + b \cdot Def_t + c \cdot Def_{t-1} + \varepsilon_t, \tag{2}$$

where Def_t denotes the default rate on high yield bonds in year t. We estimate this regression using the measures of credit market overheating from Table 1. Table 2 shows that there is a strong negative relationship between recent default rates and current credit market overheating. Some measures ($\log(HYS_t)$ and $-1 \times EBP_t$) are more highly correlated with most recent default rates, while others are also strongly correlated with lagged default rates (*Credit Growth*$_t$ and *Easy Credit*$_t$).

Observation 5. *The credit cycle and the business cycle can be quite disconnected in the short run.*

Consistent with the market-specific extrapolation view discussed above, the credit cycle can be quite disconnected from both the broader business cycle as well as equity markets in the short run. Figure 2 plots the annual growth in log U.S. GDP alongside the annual growth in log outstanding debt at nonfinancial corporations, both expressed in real terms. In the upswing proceeding the 2008 financial crisis, GDP growth peaked in March 2005, but credit growth peaked two years later. This pattern of credit expansion at the end of an economic expansion is also apparent in the late 1990s, with credit growth rising only at the end of the business cycle. During downturns, the economy often recovers well before credit growth returns to normal rates. In the post-2008 recovery, real credit growth first reached 3% in 2012, several years after the economy began its recovery. Overall, the correlation between real credit growth and real GDP growth is only 26%.

[Place Figures 2 and 3 about here]

Figure 3 further illustrates the disconnect between the credit cycle and the business cycle in U.S. data. Here, we provide additional perspective on the lack of synchronicity between the credit cycle and the business cycle. In particular, we show that credit growth tends to increase towards the end of a business cycle boom. In Panel A of Figure 3, we plot the annual growth in log real GDP from trough to peak of the business cycle, by business cycle expansion quarter (i.e., the first quarter after a recession ends is labeled as 1, and so on). As can be seen, GDP growth tends to be high in the beginning of business cycle expansions, but it stabilizes and, if anything, declines slightly in later quarters. By contrast, Panel B shows credit growth over the same periods. As the figure makes clear, credit expansion is particularly high in the later part of the business cycle.

III. A model of credit market sentiment

In this section, we develop a behavioral model of the credit cycle. The model has two key features. First, corporate bond investors form beliefs about the likelihood of future firm defaults by extrapolating past defaults and they set bond prices accordingly. Second, facing these behavioral bond investors, a representative firm maximizes its equity value by choosing the amount of debt to issue as well as whether to exercise its option to default. We begin by describing the model setup. We then discuss the model solution and make several observations about the model's key properties. We provide a detailed discussion of the model's implications in Section IV.

III.A. Model setup

Model setup. The model setup follows DeMarzo and He (2021) and is set in continuous time. We consider a representative firm that generates a pretax operating cash flow of δV_t at time t. We refer to V_t as the firm's fundamental and assume it follows a geometric Brownian motion

$$\frac{dV_t}{V_t} = \mu dt + \sigma d\omega_t, \tag{3}$$

where $\mu > 0$ is the average growth rate of the firm's cash flows and ω_t is a Brownian motion. All investors are risk neutral and have a constant discount rate of $r > \mu$.[11] We assume a constant corporate tax rate of $\tau > 0$. Thus, the unlevered value of the firm at time t is $\delta(1-\tau)V_t/(r-\mu)$.

The firm can issue bonds and we let F_t denote the total face value of outstanding bonds at time t. Bonds have a constant coupon rate of $c > 0$ and are exponentially amortizing at rate $\xi > 0$. Thus, over the period $[t, t+dt]$, bond holders are entitled to coupon payments of $cF_t dt$ and principal payments of $\xi F_t dt$. If investors thought default was not possible, then the bond price (per unit face value) would be $\int_0^\infty e^{-(r+\xi)t}(c+\xi)dt = (c+\xi)/(r+\xi)$. However, default is possible: if the firm's equity holders choose not to make these required payments, the firm defaults and we assume that bondholders recover a fixed fraction η of their outstanding principal, where $0 < \eta < (c+\xi)/(r+\xi)$.

[11] The assumption that investors are risk-neutral is without loss of generality. Specifically, we can always reinterpret the model as being written under a risk-neutral probability measure that is independent of the firm's decisions.

We assume that the firm's managers and equity investors are rational. Managers maximize the firm's equity value by choosing the amount of debt to issue as well as whether to default at each instant. Thus, in the version of our model with rational bond investors, debt issuance and the firm's leverage dynamics reflect dynamic trade-off theory considerations as in DeMarzo and He (2021). Specifically, issuing debt is beneficial because interest is tax-deductible, but higher leverage comes at a cost because default entails dead-weight costs; the interplay between these two frictions pins down the firm's optimal capital structure.

However, since we allow bond investors to have biased beliefs about the likelihood of future defaults, debt can be mispriced and firm debt issuance is partially driven a market-timing motive. Specifically, all else equal, firms choose to issue more (less) debt when bond investors are overly optimistic (pessimistic) about the likelihood of future default.

Bond investor beliefs. Motivated by the empirical evidence presented in Section II, we assume that bond investors form beliefs about the likelihood of future firm defaults by extrapolating past defaults. A formal way to model default extrapolation is through a regime-switching learning structure with misspecified regimes.[12] Specifically, we suppose that bond investors *incorrectly* believe that the instantaneous intensity of future default arrivals, $\tilde{\lambda}_t$, is a latent variable that switches between a low default-intensity regime where $\tilde{\lambda}_t = \lambda_l$ and a high default-intensity regime where $\tilde{\lambda}_t = \lambda_h > \lambda_l$ according to the following transition matrix

$$\begin{array}{cc} & \begin{array}{cc} \tilde{\lambda}_{t+dt} = \lambda_h & \tilde{\lambda}_{t+dt} = \lambda_l \end{array} \\ \begin{array}{c} \tilde{\lambda}_t = \lambda_h \\ \tilde{\lambda}_t = \lambda_l \end{array} & \begin{pmatrix} 1 - q \cdot dt & q \cdot dt \\ q \cdot dt & 1 - q \cdot dt \end{pmatrix} \end{array}, \tag{4}$$

where $q > 0$ is the regime transition intensity perceived by biased investors.[13]

Given this regime-switching learning structure, if the credit market has experienced no firm defaults for a long time, bond investors believe it is likely that $\tilde{\lambda}_t = \lambda_l$. Conversely, if the credit

[12] Barberis et al. (1998), Jin (2015), and Jin and Sui (2022) also model extrapolation using a regime-switching learning structure with misspecified regimes. One benefit of this modeling approach is that it allows us to more easily characterize our model's boundary conditions, making it easier to solve.

[13] This structure leads investors to have biased beliefs because, as we will see below, the true instantaneous default intensity is either zero when the firm is away from the default boundary or infinity when the firm hits the boundary.

market has recently experienced a wave of firm defaults, bond investors believe it is likely that $\tilde{\lambda}_t = \lambda_h$. Formally, at each point in time t, bond investors use past default occurrences up to time t as their information set to form beliefs about π_t, the time-t probability that $\tilde{\lambda}_t$ equals λ_h; their perceived default intensity is therefore $\lambda_t \equiv \mathbb{E}_t[\tilde{\lambda}_t] = \pi_t \lambda_h + (1-\pi_t)\lambda_l$. Solving this filtering problem leads to the following law of motion for investor beliefs λ_t:

$$d\lambda_t = \underbrace{[q(\lambda_h - \lambda_t) - q(\lambda_t - \lambda_l) - (\lambda_h - \lambda_t)(\lambda_t - \lambda_l)]}_{a(\lambda_t)} dt + \underbrace{\lambda_t^{-1}(\lambda_h - \lambda_t)(\lambda_t - \lambda_l)}_{b(\lambda_t)} dN_t, \tag{5}$$

where $dN_t = 1$ when a default occurs and $dN_t = 0$ in the absence of default.[14]

Equation (5) implies that λ_t stays between its upper bound of λ_h and its lower bound $\lambda_m > \lambda_l$. This lower bound λ_m is only reached in the limit as the time since the last default grows large and obtains when the drift term in equation (5) equals zero—i.e., $a(\lambda_m) = 0$. Solving for λ_m, we obtain

$$\lambda_m \equiv \frac{(\lambda_h + \lambda_l + 2q) - \sqrt{(\lambda_h - \lambda_l)^2 + 4q^2}}{2} > \lambda_l. \tag{6}$$

Importantly, the evolution of λ_t in (5) captures default extrapolation: in the absence of default, λ_t decreases deterministically towards λ_m. However, when there is a default, λ_t jumps up, and the size of the jump depends on the pre-jump level of λ_t. Also note that extrapolative investors' beliefs about the likelihood of future defaults λ_t depend *only* on the past history of defaults. Bond investors are not building a forward-looking "structural" model of default which would involve separately keeping track of firm fundamentals V_t and debt outstanding F_t. As such, these bond investors' beliefs are biased when compared with rational beliefs; we discuss this further in Section IV.A.

Bond prices. Given their beliefs, the bond investors set the bond price per unit of face value, $p(\lambda_t)$, so that

$$r \cdot p(\lambda_t) = c + \xi(1 - p(\lambda_t)) + a(\lambda_t) \cdot p'(\lambda_t) + \lambda_t(\eta - p(\lambda_t)), \tag{7}$$

where the expression for $a(\lambda_t)$ is from equation (5). Equation (7) says that bond prices are set so that behavioral investors expect to earn an instantaneous return of r over each instantaneous

[14] See Appendix A of Jin (2015) for a derivation of equation (5).

time increment. This return comes from the sum of coupon payments c, principal amortization $\xi(1 - p(\lambda_t))$, capital appreciation in the absence of default $a(\lambda_t) \cdot p'(\lambda_t)$ which arises because λ_t is then declining, and expected default losses $\lambda_t(\eta - p(\lambda_t))$.

In Appendix A, we show that the bond price has the following closed-form solution

$$p(\lambda_t) = m_1 + m_2 \cdot \lambda_t, \tag{8}$$

where

$$\begin{aligned} m_1 &= \frac{(c+\xi)(2q+\lambda_h+\lambda_l+r+\xi)+\eta q(\lambda_h+\lambda_l)+\eta\lambda_h\lambda_l}{(r+\xi)(2q+\lambda_h+\lambda_l+r+\xi)+q(\lambda_h+\lambda_l)+\lambda_h\lambda_l} > 0, \\ m_2 &= -\frac{(c+\xi)-(r+\xi)\eta}{(r+\xi)(2q+\lambda_h+\lambda_l+r+\xi)+q(\lambda_h+\lambda_l)+\lambda_h\lambda_l} < 0. \end{aligned} \tag{9}$$

Naturally, the bond price is decreasing in λ_t: when λ_t is higher, bond investors are more pessimistic about the likelihood of future firm defaults and therefore set a lower bond price.

Using the set of baseline parameter values that we specify in Section IV, we obtain $m_1 = 0.984$ and $m_2 = -0.154$; and λ ranges from $\lambda_m = 0.054$ to $\lambda_h = 2$. As λ decreases towards λ_m, the bond price increases towards 0.975, a level close to the par value of 1. As λ increases towards λ_h, the bond price decreases towards 0.676.

To gain further intuition about the bond price equation in (8), we consider a few limiting cases. First, if both λ_l and λ_h go to zero, the bond price goes to $p = m_1 = (c+\xi)/(r+\xi)$; this is the value of a default-free bond that pays coupons at rate c and amortizes at rate ξ. Second, if ξ goes to infinity, the bond price goes to $p = m_1 = 1$. This is the value of a bond that immediately matures at its par value. Finally, as λ_t approaches its lower bound of λ_m, the bond price converges to its upper bound of $p(\lambda_m) = (c+\xi+\eta\lambda_m)/(r+\xi+\lambda_m)$. This is the value of a bond that pays coupons at rate c, amortizes at rate ξ, and defaults at a *constant* intensity of λ_m.

Firm behavior and equity valuation. The firm's managers and equity investors are rational. Managers maximize the value of the firm's equity by choosing the amount of debt to issue as well as whether to default at each instant. Equity holders receive nothing in default, so the value of equity at time t is simply the discounted expected value of future net cash flows to equity from time t until the unknown time of default. As noted above, the firm's decisions are shaped by dynamic trade-off theory considerations as in DeMarzo and He (2021) as well as market-timing considerations that arise because debt can be mispriced.

Following DeMarzo and He (2021), the firm chooses the rate G_t at which it issues new debt. Since debt matures at rate ξ, the law of motion for the outstanding face value of debt F_t is:

$$dF_t = (G_t - \xi F_t)dt. \tag{10}$$

However, departing from DeMarzo and He (2021), we assume that debt issuance entails adjustment-like costs of the following form

$$\frac{1}{2}\psi F_t\left(\frac{G_t - \xi F_t}{F_t}\right)^2, \tag{11}$$

where the parameter $\psi > 0$ controls the magnitude of these costs. These issuance costs should be seen as a stand-in for a variety of frictional costs that make it costly for firms to issue additional debt. These frictional costs could include: (i) direct transaction costs from underwriting, legal, and auditing fees; and (ii) indirect transaction costs arising from the price-impact of issuance as in Stein (1996) and Baker and Wurgler (2013). At the same time, we assume that it is not costly for firms to refinance their maturing debt as it comes due. This corresponds to the model of He and Xiong (2012) in which G_t is fixed at ξF_t and there are no issuance costs.[15]

At time t, the firm's equity value E_t is a function of the firm's fundamental V_t, the firm's outstanding debt F_t, and the investor belief λ_t about the likelihood of future defaults. The evolution of $E_t = E(V_t, F_t, \lambda_t)$ is governed by the following Hamilton-Jacobi-Bellman (HJB) equation:

$$rE(V,F,\lambda) = \max_G \left\{ \begin{array}{c} \underbrace{\left[\delta V - (c+\xi)F - \tau(\delta V - cF) + Gp(\lambda) - \frac{1}{2}\psi F\left(\frac{G-\xi F}{F}\right)^2\right]}_{\text{equity cash flow}} \\ + \underbrace{\left[\ \mu V E_V + \frac{1}{2}\sigma^2 V^2 E_{VV} + a(\lambda)E_\lambda + (G-\xi F)E_F\ \right]}_{\text{expected equity capital gain}} \end{array} \right\}, \tag{12}$$

[15] Thus, the rational version of our model, which we spell out in Section V, simply adds debt issuance costs to DeMarzo and He (2021). Although we do not discuss these issues in detail since they are beyond the scope of the paper, the addition of debt issuance costs weakens some of the strongest implications of DeMarzo and He (2021) where $\psi = 0$. Specifically, introducing debt issuance costs is akin to giving firms partial commitment over their future leverage choices, weakening the leverage ratchet effect and allowing firms to capture some of the tax benefits of debt.

where the bond price $p(\lambda)$ is given by (8) and the subscripts denote partial derivatives—e.g., $E_V \equiv \partial E(V, F, \lambda)/\partial V$. The cash flows to the firm's equity holders are given by the firm's operating cash flows of δV, minus coupon and principal payments on its debt of $(c + \xi)F$, minus taxes of $\tau(\delta V - cF)$, plus debt issuance proceeds of $Gp(\lambda)$, and minus debt issuance costs from (11).[16]

The optimal rate of debt issuance is given by

$$G^* = \xi F + \frac{p(\lambda) + E_F}{\psi} F. \tag{13}$$

Equation (13) says that the firm is timing the bond market by exploiting bond investors' biased beliefs: the firm's optimal rate of debt issuance G^* depends on the difference between the bond price $p(\lambda)$ that is set by behavioral investors and $-E_F$, namely the reduction in equity value associated with an increase in debt outstanding. For instance, if $p(\lambda_t) > -E_F(V_t, F_t, \lambda_t)$, then equations (13) and (10) imply that $dF_t = (G_t^* - \xi F_t)dt > 0$—i.e., debt outstanding is growing. Substituting (13) back into (12), we obtain the following partial differential equation:

$$rE(V, F, \lambda) = \left[\begin{array}{l} \delta V - (c + \xi)F - \tau(\delta V - cF) + \xi F p(\lambda) + \dfrac{F}{2\psi}(p(\lambda) + E_F)^2 \\ + \mu V E_V + \frac{1}{2}\sigma^2 V^2 E_{VV} + a(\lambda) E_\lambda \end{array} \right]. \tag{14}$$

The firm also optimally chooses when to default. The firm defaults when its fundamental V reaches the default boundary $V_b(\lambda, F)$, but continues making the required interest and principal payments on its debt so long as $V > V_b(\lambda, F)$. This implies two standard boundary conditions for the partial differential equation in (14). Specifically, for any $F > 0$ and any $\lambda \in [\lambda_m, \lambda_h]$, we have the following value-matching and smooth-pasting conditions:

$$E(V_b(\lambda, F), F, \lambda) = 0, \quad E_V(V_b(\lambda, F), F, \lambda) = 0. \tag{15}$$

III.B. Model solution

To solve the model, we first exploit the model's homogeneity properties to reduce the number of state variables from three to two. After characterizing the reduced partial differential equations

[16]If equity cash flows are positive, shareholders are receiving dividends. If equity cash flows are negative, the firm is issuing additional equity and, for simplicity, we assume that such equity issuance is frictionless.

and boundary conditions, we solve the model numerically and discuss the properties of the solution.

Reduction of state variables. Our model has three state variables: the firm's fundamental V, its debt outstanding F, and the investor belief λ. Note that the belief structure in (4) is imposed on a unit-free quantity λ, the V process in equation (3) exhibits constant stochastic returns to scale, and the debt issuance cost in (11) is homogeneous of degree one in G and F. These assumptions imply that the equity value E is homogeneous of degree one in V and F and that the default boundary V_b is homogeneous of degree one in F. Thus, without loss of generality, we can write

$$E(V,F,\lambda) = e(\frac{V}{F},\lambda)F, \quad V_b(\lambda,F) = v_b(\lambda)F. \tag{16}$$

We define $v \equiv V/F$, which is the firm's fundamental normalized by its debt outstanding, and will sometimes refer to v as the firm's reduced fundamental. Note that v is an inverse measure of the firm's leverage. Namely, v is proportional to the firm's interest coverage ratio of $(\delta V)/(cF) = (\delta/c)v$.

Substituting (16) back into (14) leads to the following reduced partial differential equation

$$re(v,\lambda) = \begin{bmatrix} \delta v - (c+\xi) - \tau(\delta v - c) + \xi p(\lambda) + \dfrac{1}{2\psi}(p(\lambda) + e(v,\lambda) - ve_v)^2 \\ +\mu v e_v + \frac{1}{2}\sigma^2 v^2 e_{vv} + a(\lambda)e_\lambda \end{bmatrix}, \tag{17}$$

where the subscripts continue to denote partial derivatives—e.g., $e_v \equiv \partial e(v,\lambda)/\partial v$. The optimal rate of debt issuance relative to debt outstanding is

$$g^*(v,\lambda) \equiv \frac{G^*}{F} = \xi + \frac{p(\lambda) + e(v,\lambda) - ve_v}{\psi}. \tag{18}$$

The default boundary is now $v_b(\lambda)$ and the reduced boundary conditions are

$$e(v_b(\lambda),\lambda) = 0, \quad e_v(v_b(\lambda),\lambda) = 0. \tag{19}$$

Given that the partial differential equation in (17) involves the first derivative of e with respect to λ and the first and second derivatives of e with respect to v, solving it requires three boundary conditions. However, equation (19) only contains two boundary conditions. To obtain a third boundary condition, we exploit the fact that $a(\lambda_m) = 0$, implying that equation (17) becomes an ordinary differential equation when $\lambda = \lambda_m$. When solving for $e(v, \lambda)$, we first numerically solve for $e(v, \lambda_m)$ and then use it as an additional boundary condition alongside the two conditions in (19).[17]

In summary, equations (17), (19), and $e(v, \lambda_m)$ characterize a partial differential equation in two state variables (v and λ) and its boundary conditions. Formally, the system we are solving is a "free boundary problem": the partial differential equation in (17) needs to be solved for both an unknown function $e(v, \lambda)$ and an unknown boundary $v_b(\lambda)$. In other words, solving the partial differential equation is intricately linked with the problem of finding the domain over which the partial differential equation applies. This is a technically challenging problem and we tackle it using a numerical projection method that is detailed in Appendix B.

Below we present a numerical solution of the model with the following parameters: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. We provide more discussion of these parameter values in Section IV.

First, we examine the firm's optimal debt issuance policy. Specifically, we examine $g^*(v, \lambda) - \xi$. Since $dF_t/F_t = (g^*(v, \lambda) - \xi)dt$, this quantity is just the optimal instantaneous growth rate of debt outstanding. Panel A of Figure 4 plots the firm's optimal debt growth rate, $g^*(v, \lambda) - \xi$, as a function of v and λ within the no-default region. Panel B of Figure 4 plots $g^*(v, \lambda) - \xi$ as a function of v for different levels of λ and as a function of λ for different levels of v.

[Place Figure 4 about here]

Panel A of Figure 4 shows that the firm's debt growth rate $g^*(v, \lambda) - \xi$ is a decreasing function of the reduced fundamental v in our model. When v decreases, the firm's equity value falls and the cost to equity holders of having additional debt, $-E_F$, also falls. Since the bond price $p(\lambda)$ reflects default extrapolation, it is not a function of v. As such, the firm responds to the decline in v by raising the growth rate of debt. How does the debt growth rate respond to changes in investor beliefs λ holding v fixed? Panel B of Figure 4 shows that the relationship between the debt growth rate and λ depends on two opposing forces. On the one hand, a decline in λ pushes up the bond

[17] Solving our model using $y_t \equiv \delta v_t$ as the state variable describing fundamentals as in DeMarzo and He (2021) is equivalent to solving the model using v_t as the state variable. We choose the latter because its larger magnitude allows our numerical algorithm to more effectively search for the default boundary and thus solve for the equity value.

price $p(\lambda)$ which leads the firm to issue more debt. On the other hand, a decline in λ pushes the firm further away from the default boundary $v_b(\lambda)$, causing $-E_F$ to rise and hence leading the firm to issue less debt. Since the strength of the first force is independent of v, while the second force is strongest near the default boundary $v_b(\lambda)$ and recedes further from it, $g^*(v, \lambda) - \xi$ is monotonically decreasing in λ when v is large and is a U-shaped function of λ when v is small. Overall, Figure 4 shows that the debt growth rate is highest when λ is low and v is near the default boundary.

[Place Figure 5 about here]

Next, we examine the firm's optimal default decisions. Panel A of Figure 5 plots the default boundary $v_b(\lambda)$ as a function of λ, showing that $v_b(\lambda)$ is an increasing function of λ. Specifically, when $\lambda = \lambda_m = 0.054$, $v_b(\lambda) = 0.49$. However, when $\lambda = \lambda_h = 2$, $v_b(\lambda) = 1.41$. When λ declines, bond investors become more optimistic and push up bond prices, raising the proceeds the firm receives when issuing debt which makes the firm's equity more valuable. As a result, the firm chooses to delay default by lowering its default boundary $v_b(\lambda)$ (i.e., the leverage threshold at which firms default is high), making a near-term default less likely. Conversely, when λ rises, bond investors become more pessimistic and bond prices fall. In this case, the proceeds the firm receives from issuing debt fall and so does the value of the firm's equity. As a result, the firm chooses to accelerate default by increasing its default boundary $v_b(\lambda)$.

Together, Figure 4 and Figure 5 Panel A point to the the two-way feedback loop between biased investor beliefs and market outcomes. On the one hand, as investor's perceived likelihood of default λ declines, the firm lowers its default boundary $v_b(\lambda)$, thereby reducing the objective probability of near-term default. On the other hand, when there have not been many recent defaults, investor's perceived likelihood of default λ also declines, as shown by equation (5). Together these observations imply that, at least in the short-run, bond investors' biased beliefs can become self-fulfilling in equilibrium. This dynamic two-way feedback loop lies at the heart of our model.

Finally, we examine the value of the firm's equity. Panel B of Figure 5 plots the firm's equity value $e(v, \lambda)$ as a function of v and λ. Naturally, the equity value $e(v, \lambda)$ is an increasing and convex function of the firm's fundamental v, with the relationship becoming linear as v grows large. At the default boundary $v = v_b(\lambda)$, the equity value is $e(v_b(\lambda), \lambda) = 0$. Moreover, $e(v, \lambda)$ is decreasing in λ: when λ rises, bond investors' beliefs become more pessimistic, and the proceeds the firm receives from debt issuance decline which reduces the value of firm equity. The resulting relationships between the firm equity value e and credit market conditions v and λ are broadly consistent with the recent findings of Baron, Verner, and Xiong (2021), which suggest that equity market valuations depend on the level of credit market sentiment.

III.C. Post-default firm restructuring

To simulate our model over time, we must specify the rule for post-default firm restructuring. Suppose the firm defaults at time t. Denote the firm's pre-default fundamentals, its pre-default debt, and investors' pre-default beliefs as V_{t_-}, F_{t_-}, and λ_{t_-}, respectively. From equation (5), we know that upon default, investor belief jumps up to $\lambda_{t_+} = \lambda_{t_-} + \lambda_{t_-}^{-1}(\lambda_h - \lambda_{t_-})(\lambda_{t_-} - \lambda_l)$. We need to further specify the post-default firm fundamental V_{t_+} and the post-default firm debt F_{t_+}. For simplicity, we assume that $V_{t_+} = (1-\alpha)V_{t_-}$ where $\alpha \in (0,1)$ reflects the real deadweight costs of bankruptcy and that $F_{t_+} = (1-\kappa)F_{t_-}$ where $\kappa \in (\alpha, 1)$ reflects the reduction in debt associated with bankruptcy restructuring. Thus, letting $v_{t_+} = V_{t_+}/F_{t_+}$, we have

$$v_{t_+} = (1+\Theta)v_{t_-}, \tag{20}$$

where $(1+\Theta) = (1-\alpha)/(1-\kappa) > 1$ measures the net deleveraging associated with bankruptcy.

Note that specifying post-default value v_{t_+} is outside the model described in Section III.A. Solving equation (14) for the firm's equity value within the no-default boundary does not involve specifying what happens to the firm after default. Instead, equation (20) provides a simple model of post-default firm restructuring: the firm's existing equity holders are wiped out and the firm's existing debt holders are given a mix of debt and equity in the restructured firm.[18] Given these assumptions, it may take multiple rounds of restructuring for the firm to emerge from default: if $v_{t_+} < v_b(\lambda_{t_+})$, then at t_+, the firm remains in the default region. In this case, the firm defaults again and another round of restructuring takes place, so v_t again rises by the fraction Θ.[19]

[18]We assume that bonds recover a fixed fraction η of their face value in default and that the amount of deleveraging in bankruptcy Θ is constant. Therefore, to be fully consistent, we must assume that κ and α vary as a function of pre-default investor beliefs λ_{t_-}. Specifically, letting $\lambda_+(\lambda_{t_-}) = \lambda_{t_-} + \lambda_{t_-}^{-1}(\lambda_h - \lambda_{t_-})(\lambda_{t_-} - \lambda_l)$, these functions must satisfy $\eta = [e(v_b(\lambda_{t_-})(1+\Theta), \lambda_+(\lambda_{t_-})) + p(\lambda_+(\lambda_{t_-}))](1-\kappa(\lambda_{t_-}))$ and $(1-\alpha(\lambda_{t_-})) = (1+\Theta)(1-\kappa(\lambda_{t_-}))$. We believe this specification for post-default restructuring balances realism and model tractability. For instance, allowing the recovery rates on bonds to be state-contingent—e.g., due to optimal restructuring decisions in bankruptcy—would add additional modeling complexities that seem best left to future work.

[19]In Section IV, we discretize and simulate our model at a monthly frequency, setting $\Delta t = 1/12$. If the firm

IV. Model implications

Our model captures the idea that, in credit markets, there is a dynamic two-way feedback loop between investors' biased beliefs and financial market outcomes. Past defaults affect investors' biased beliefs about future defaults through equation (5). These biased beliefs have two effects. First, they affect bond prices through equation (8), which in turn affect firms' optimal debt issuance through equation (13) and their debt accumulation through equation (10). Second, biased beliefs lead firms to optimally adjust their default boundary $v_b(\lambda)$ as shown in Figure 5 Panel A. Together, these two effects allow biased investor beliefs to impact the true probability of future defaults. Specifically, in the short run, biased investor beliefs tend to be partially self-fulfilling and hence self-sustaining. However, over the longer run, biased investor beliefs can become self-defeating: optimistic investor beliefs eventually lead to a build-up of leverage and financial fragility that raises the longer-run probability of default.

In this section, we conduct comprehensive numerical analyses to illustrate the key implications of our model. We start by illustrating the model's dynamics using a baseline set of model parameters. We then lay out four key implications: defining and understanding reflexivity, the "calm before the storm" phenomenon, the "default spiral" phenomenon, and the predictability of bond returns.

We discretize our continuous-time model and simulate it at a monthly frequency. Figure 6 shows a typical sample path of simulated data for a length of twenty-five years. To generate this data, we use the following set of baseline parameters:

- **Discount rate:** $r = 0.04$.
- **Dynamics of firm fundamentals:** $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.
- **Debt coupon and amortization rates:** $c = 0.04$ and $\xi = 1$.
- **Debt issuance cost parameter:** $\psi = 20$.
- **Bond investor belief parameters:** $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$.
- **Post-default firm restructuring parameter:** $\Theta = 0.25$.
- **Default recovery parameter:** $\eta = 0.5$.
- **Tax rate:** $\tau = 0.3$.

defaults at time t, then $\lambda_{t_+} = \lambda_{t_-} + \lambda_{t_-}^{-1}(\lambda_h - \lambda_{t_-})(\lambda_{t_-} - \lambda_l)$ and $v_{t_+} = (1 + \Theta)v_{t_-}$. From t to $t + \Delta t$, v_t remains unchanged and λ_t decreases by a small amount of $[(\lambda_h - \lambda_{t_+})(\lambda_{t_+} - \lambda_l) - q(\lambda_h - \lambda_{t_+}) + q(\lambda_{t_+} - \lambda_l)]\Delta t$. At time $t + \Delta t$, we check and see whether $v_{t+\Delta t} \leq v_b(\lambda_{t+\Delta t})$. If so, the firm defaults again.

While these parameters are only illustrative, they can be justified as follows. First, many parameters, such as the volatility of firm fundamentals σ and the bond maturity rate ξ, are similar to those used in the dynamic capital structure literature (He and Xiong, 2012 and DeMarzo and He, 2021). Second, the belief parameters λ_l, λ_h, and q that govern default extrapolation are chosen to generate realistic dynamics for investor beliefs, bond prices, and firm defaults. Specifically, a long period without any defaults leads investor beliefs about the likelihood of future defaults λ_t to decline towards a lower bound λ_m that is near zero, causing bond prices $p(\lambda)$ to rise towards an upper bound that is near the fair value of $(c + \xi)/(r + \xi)$. Third, other parameters, such as μ and Θ, are chosen so that defaults are fairly rare but not extremely rare, as illustrated in Figure 6.[20]

We have further explored a variety of alternative parameter values and find that our model's key qualitative implications are robust to other parameter choices. Overall, we believe that our chosen parameter values are reasonable and generate illustrative dynamics. A full quantitative calibration of the model is beyond the scope of the paper.[21]

[Place Figure 6 about here]

The panels in Figure 6 show the path of investor beliefs λ_t, reduced firm fundamentals $v_t = V_t/F_t$, bond prices $p(\lambda_t)$, the endogenous default boundary $v_b(\lambda_t)$, a default indicator D_t that equals one if there is a default in month t, and the firm's optimal net debt issuance $g^*(v_t, \lambda_t) - \xi$.[22] Although it is just a single sample path, Figure 6 nicely illustrates some of the model's key features.

Consider the time period between month 173 and month 239 in Figure 6. During this stretch, firm's fundamentals v_t initially decline due to a series of adverse shocks. However, because the time since the last default is rising, bond investor beliefs become increasingly optimistic and bond

[20] Since we set $\delta = c$, the firm's interest coverage ratio in our numerical analysis is simply $(\delta V)/(cF) = V/F = v$.

[21] A full quantitative calibration of our default extrapolation model faces a challenge. This is because, like He and Xiong (2012), our model does not generate a stationary process for v so we cannot analyze its long-run, steady-state properties. To see why, note that bond investors who extrapolate past defaults do not take the firm's fundamental v into account. Thus, when v grows large and the time since the last default grows long, investor beliefs λ converge to $\lambda_m > 0$ and bond prices converge to $p(\lambda_m) = (c+\xi+\eta\lambda_m)/(r+\xi+\lambda_m)$, which is below the fair value of $(c+\xi)/(r+\xi)$. The resulting under-pricing deters firms from issuing debt, so the growth rate of debt outstanding remains negative and v diverges to infinity. Thus, like He and Xiong (2012), our model permits sample paths where the firm "outgrows" its initial level of debt. However, the rational benchmark model and the fundamental extrapolation model we study in Section V do not face this challenge because bond investors take the firm's fundamental v into account, so bond prices converge to their fair value as v grows large. As a result, debt issuance rises as firms lever up in an attempt to capture the tax-benefits of debt, leading to a stationary process for v.

[22] We only plot $g^*(v_t, \lambda_t) - \xi$ when there is no default in month t. In the default region, the firm is undergoing restructuring, so the optimal debt issuance derived within the no-default region no longer applies.

prices rise. As a result, the firm optimally lowers its default boundary and increases its rate of debt issuance, raising its leverage and further driving down $v = V/F$. As we will see, these firm responses to investor optimism tend to reduce the objective likelihood of default in the near-term, but can sometimes raise the likelihood of a major wave of defaults over the longer run.

Indeed, in month 240, v_t finally hits the default boundary $v_b(\lambda_t)$ and the firm chooses to default. Following this initial default, investor beliefs immediately become far more pessimistic and bond prices plummet. In response, the firm significantly raises its default boundary $v_b(\lambda_t)$. The combination of growing investor pessimism and a rising default boundary triggers a consecutive sequence of defaults from months 240 to 244. This waves of defaults is associated with a reduction of firm leverage from its high pre-default level towards a much lower post-default level. Eventually, the firm re-enters the no-default region as v_t rises above $v_b(\lambda_t)$. However, due to default extrapolation, it still takes many more months for investor beliefs and bond prices to recover.

IV.A. Defining and understanding reflexivity

Reflexivity is the idea that there is a dynamic two-way feedback loop between investors' *biased* beliefs and market outcomes. In this section, we focus on a particular market outcome, namely the *true* probability that the firm defaults over the next year. We let $\Pr^R(v, \lambda)$ denote this true probability of default and provide a formal definition of reflexivity.

As before, we discretize our continuous-time model at a monthly frequency. Then, for a given initial state (v, λ), $\Pr^R(v, \lambda)$ is computed as follows. We simulate the economy 25,000 times, all starting from this same initial state. $\Pr^R(v, \lambda)$ is the fraction of these simulations where the firm default within the next 12 months. With $\Pr^R(v, \lambda)$ in hand, we can measure the degree of investor bias by comparing this true default probability with investors' perceived probability of observing a default in the next 12 months, which is $\Pr^B(\lambda) = \int_0^1 e^{-\lambda s} ds = 1 - e^{-\lambda}$. Here, superscript "$B$" denotes the biased expectations of extrapolative investors. Thus, we define investors' bias as $\Pr^R(v, \lambda) - \Pr^B(\lambda)$. If this quantity is positive, investor beliefs are overly optimistic; if it is negative, investor beliefs are overly pessimistic.

[Place Figure 7 about here]

Figure 7 presents heatmaps of the true default probability $\Pr^R(v, \lambda)$ (left panel) and investors' belief bias $\Pr^R(v,\lambda) - \Pr^B(\lambda)$ (right panel) for different values of v and λ. The left panel shows that $\Pr^R(v,\lambda)$ is close to zero when v is significantly higher than the default boundary $v_b(\lambda)$. $\Pr^R(v,\lambda)$ only begins to rise when v approaches $v_b(\lambda)$. The right panel shows that for low values of λ and for low values of v that are near the default boundary $v_b(\lambda)$, investors are overly optimistic: their perceived probability of observing a default within 12 months is significantly lower than the true probability. However, for high values of λ, investors are overly pessimistic: their perceived probability of observing a default within 12 months is significantly higher than the true probability. Moreover, for most values of v and λ, $\Pr^R(v,\lambda) - \Pr^B(\lambda)$ is either significantly positive or significantly negative, indicating that at a given point in time, investor beliefs are often biased.

[Place Figure 8 about here]

We now study how investors' biased beliefs impact the true future probability of default. We propose $\partial \Pr^R(v,\lambda)/\partial\lambda$ as a formal measure of reflexivity—this is the sensitivity of the true default probability $\Pr^R(v,\lambda)$ with respect to changes in investor beliefs λ.[23] The left panel of Figure 8 shows that $\partial \Pr^R(v,\lambda)/\partial\lambda$ is positive for all values of v and λ: making investors more pessimistic (raising λ) always increases the true probability of a firm default within 12 months. Moreover, $\partial \Pr^R(v,\lambda)/\partial\lambda$ is particularly large for high values of λ and low values of v that are close to the default boundary $v_b(\lambda)$. When investors are very pessimistic, bond prices are low and firms set a high default boundary $v_b(\lambda)$. In this case, reducing λ will raise bond prices, leading firms to significantly lower their optimal default boundary and, hence, delaying or even preventing future defaults. Conversely, for low values of λ and low values of v near the default boundary $v_b(\lambda)$, $\partial \Pr^R(v,\lambda)/\partial\lambda$ remains positive but its magnitude is much smaller. As we will discuss in Section IV.C, this region with low values of both λ and v is associated with high financial fragility in credit markets.

[23] It is worth emphasizing that our model does not give rise to multiple equilibria, so our notion of "reflexivity" differs from the idea of switching between multiple forward-looking, rational expectations equilibria. To elaborate, in our default extrapolation model, investor beliefs λ_t at each point in time t about the future likelihood of default are backward-looking: they are uniquely determined by the firm's history of past defaults, but are not fully pinned down by the current level of the firm's fundamental V_t and debt outstanding F_t. Our notion of reflexivity is to examine how the firm's *future* market outcomes are affected when changing investor beliefs λ_t while holding V_t and F_t fixed. In contrast, in the rational model we later analyze in Section V.A, investor beliefs about the likelihood of a future firm default are rational and forward-looking: they are *fully* pinned down by the current level of the firm's fundamental V_t and debt outstanding F_t. As such, investor beliefs cannot change without varying either V_t or F_t. Thus, by construction, our notion of reflexivity does not directly apply in this rational model.

IV.B. The "calm before the storm" phenomenon

An elevated level of credit market sentiment—a lower level of λ—causes bond prices to rise and debt issuance to become less costly. As a result, firms optimally choose to lower the default boundary $v_b(\lambda)$, endogenously delaying or even preventing future defaults. We term this phenomenon the "calm before the storm." Below, we provide a detailed analysis of this phenomenon.

We start by comparing two sample paths of simulated monthly data. We consider two trajectories of (v_t, λ_t) that have *identical* Brownian shocks that govern the evolution of V_t in equation (3), but *slightly* different initial conditions. The first trajectory starts at $(v_0, \lambda_0) = (0.65, 0.34)$. The second trajectory starts at $(v_0, \lambda_0) = (0.65, 0.33)$. For these two very similar sets of initial conditions, Figure 9 then plots the time series of v_t and λ_t over the next 60 months (5 years). The dashed (red) line corresponds to the simulation starting from $(v_0, \lambda_0) = (0.65, 0.34)$. The solid (blue) line corresponds to the simulation starting from $(v_0, \lambda_0) = (0.65, 0.33)$. Figure 9 shows that when v takes low values that are near the default boundary $v_b(\lambda)$ as in this example, more optimistic credit market sentiment indeed lengthens the calm period. Specifically, reducing λ_0 from 0.34 to 0.33 pushes the first default further into the future, in this example, from month 1 to month 39.

[Place Figure 9 about here]

Next, we investigate the "calm before the storm" phenomenon more systematically by examining *many* sample paths of simulated data. We construct

$$\Phi(v_t, \lambda_t) \equiv \mathbb{E}^R[(\text{Duration of calm})_{t\to t+5} | v_t, \lambda_t], \tag{21}$$

which is the rationally-expected duration of the calm period within the next 5 years given the current state (v_t, λ_t). Specifically, for each initial state (v_t, λ_t), we run 10,000 simulations at a monthly frequency for a length of 5 years. For each simulation, if the first default takes place at month $t + k$ and if $k \leq 60$, we set $(\text{Duration of calm})_{t\to t+5} = k/12$ years. If no default takes place for 5 years after month t, we set $(\text{Duration of calm})_{t\to t+5} = 5$ years. $\Phi(v_t, \lambda_t)$ is simply the average value of $(\text{Duration of calm})_{t\to t+5}$ across these 10,000 simulations. Figure 10 presents a heatmap of $\Phi(v, \lambda)$, $\partial\Phi(v, \lambda)/\partial v$, and $\partial\Phi(v, \lambda)/\partial\lambda$ for different values of (v, λ).

[Place Figure 10 about here]

We focus on $\partial\Phi(v_t, \lambda_t)/\partial\lambda_t$: How does changing λ_t affect the duration of the calm period within the next 5 years? Figure 10 shows that $\partial\Phi(v, \lambda)/\partial\lambda$ is negative across all values of v and λ: making investors more optimistic (lowering λ) always increases the duration of the calm period. Moreover, $\partial\Phi(v, \lambda)/\partial\lambda$ is particularly negative for high values of λ and low values of v that are near the default boundary $v_b(\lambda)$. When investors are very pessimistic, reducing λ will raise bond prices and cause the firm to optimally lower the default boundary, delaying or even preventing future defaults.

This "calm before the storm" phenomenon is consistent with the findings in Krishnamurthy and Muir (2020). Krishnamurthy and Muir (2020) examine the behavior of credit spreads around a large sample of financial crises in developed countries and argue that spreads are typically "too low" in the years before crises. Consistent with this finding, credit spreads in our model are indeed too low in the run-up to a default: a long calm period leads to low values of λ. For low values of λ and for v that is near the default boundary $v_b(\lambda)$, the left panel of Figure 7 shows that the true probability of a future crisis is elevated. However, the right panel of Figure 7 shows that investors' perceived probability of observing a default over the next twelve months is significantly lower than the true probability, implying that bond prices are too high and credit spreads are too low.

The "calm before the storm" phenomenon also helps make sense of what Gennaioli and Shleifer (2018) have dubbed the "quiet period" of the 2008 global financial crisis—the period between the initial disruptions in housing and credit markets in the summer of 2007 and onset of a full-blown financial crises in the fall of 2008. Specifically, backward-looking default extrapolation helps create a self-fulfilling "quiet period." In our model, the absence of recent defaults leads to a lower λ, which, as Figure 10 shows, tends to further lengthen the calm period. Indeed, as Gennaioli and Shleifer (2018) argue, if investors were fully forward-looking, one should have expected a more rapid deterioration of financial conditions in late 2007 rather than the delayed onset that was witnessed.

IV.C. The "default spiral" phenomenon

Once the first default hits the credit market, default extrapolation can generate a "default spiral": extrapolative, backward-looking beliefs lead to a form of default persistence that is absent when beliefs are fully rational and forward-looking.[24] Specifically, following a default, equation (5)

[24] In the fully rational model we will describe in Section V, a default realization causes firm leverage to decline post-restructuring, which immediately makes debt investors' beliefs *less* pessimistic and hence reduces the likelihood of another default in the near future.

says that investor beliefs become more pessimistic. This pushes down bond prices, increasing the cost of debt issuance for firms and causing them to raise their default boundary. As a consequence of the rising default boundary, firms experience a series of defaults in rapid succession. Along the way, they gradually delever and eventually re-enter the no-default region.

Figure 9 from Section IV.B provides a numerical example of the "default spiral" phenomenon: in the sample path with $(v_0, \lambda_0) = (0.65, 0.33)$, the first default hits the credit market at month 39 after a long calm period; it is then followed by 4 more consecutive defaults. In this section, we provide further analysis of this phenomenon.

We now proceed by constructing a measure of the subsequent "default severity"

$$\Psi(v_t, \lambda_t) \equiv \mathbb{E}^R[(\text{Number of defaults})_{t \to t+5} | v_t, \lambda_t], \tag{22}$$

which is the rationally-expected number of firm defaults over the next 5 years given the current state (v_t, λ_t). Again, for each initial state (v_t, λ_t), we run 10,000 simulations at a monthly frequency for a length of 5 years. For each simulation, we compute $(\text{Number of defaults})_{t \to t+5}$, the number of defaults between month t and month $t + 60$. We then compute $\Psi(v_t, \lambda_t)$ by averaging the number of defaults across these 10,000 simulations.

[Place Figure 11 about here]

Panel A of Figure 11 presents a heatmap of $\Psi(v, \lambda)$ for different values of (v, λ). It shows that subsequent default severity is particularly high for low values of λ and low values of v that are near the default boundary $v_b(\lambda)$. In this region, a long calm period in credit markets has led the firm to accumulate a significant amount of leverage, making it vulnerable to adverse fundamental shocks. In other words, there has been an endogenous build-up of financial fragility.

What drives the endogenous build-up of fragility in credit markets? Following a long calm period without any defaults, investors believe that the likelihood of future defaults λ is low, so bond prices $p(\lambda)$ are high. Meanwhile, if firm fundamentals deteriorate so v is near the default boundary $v_b(\lambda)$, firms will optimally choose to issue large quantities of debt because bonds prices are high and additional debt is not costly from the perspective of equity holders—i.e., $-E_F$ is low. (Recall that Figure 4 showed that $g^*(v, \lambda) - \xi$ is highest when both v and λ take low values.)

Moreover, since bond investors are optimistic (λ is low), firms endogenously choose a low default boundary $v_b(\lambda)$, delaying the arrival of defaults. Together, the combination of rapid debt issuance and a low default boundary tends to keep v at low values, inducing financial fragility.

This heightened financial fragility in the credit market has two consequences in our model. First, the probability of an eventual default is high: with v hovering at low values, firms are likely to default at some point in the future. Second, once the first default hits the credit market, the subsequent "storm" is likely to be severe, featuring a long sequence of defaults that is accompanied by declining bond prices. Indeed, as shown in Figure 11, the "default spiral" phenomenon is most pronounced for low values of λ and for low values of v that are near the default boundary $v_b(\lambda)$.

Panel A of Figure 11 highlights another key implication of our model: a longer calm period tends to give rise to a bigger storm. With a longer calm period, investor beliefs λ tend to take lower values and firm fundamentals v tend to be near the default boundary $v_b(\lambda)$. In this region of low v and low λ, Figure 11 Panel A suggests that subsequent defaults tend to be much more severe: firms are likely to experience a succession of defaults before returning to the no-default region.

We further study how the default severity measure $\Psi(v, \lambda)$ is affected by changes in v and λ. First, we examine $\partial\Psi(v, \lambda)/\partial v$: How does changing v affect the expected number of firm defaults over the next 5 years? The left heatmap in Panel B of Figure 11 plots $\partial\Psi(v, \lambda)/\partial v$ for different values of (v, λ). It shows that $\partial\Psi(v, \lambda)/\partial v$ is negative for all values of v and λ: an increase in firm leverage (i.e., a lower level of v) always raises the expected number of future defaults. Moreover, $\partial\Psi(v, \lambda)/\partial v$ is particularly negative for low to moderate values of λ and low values of v that are near the default boundary $v_b(\lambda)$. In this case, $v_b(\lambda)$ is significantly lower than the default boundary that firms will optimally set once λ spikes after the first default. As a result, lowering v in this region raises the likelihood of a significant "default spiral" that will persist for multiple periods.

Next, we turn to $\partial\Psi(v, \lambda)/\partial\lambda$: How does changing investor beliefs λ affect the expected number of defaults over the next 5 years? The right heatmap in Panel B of Figure 11 plots $\partial\Psi(v, \lambda)/\partial\lambda$ for different values of (v, λ). It shows that for most values of v and λ, $\partial\Psi(v, \lambda)/\partial\lambda$ is positive—i.e., making bond investors more pessimistic raises the expected number of defaults. $\partial\Psi(v, \lambda)/\partial\lambda$ is particularly elevated for high values of λ and low values of v that are near $v_b(\lambda)$. In this region, investors are overly pessimistic and firms set up a high default boundary. If λ declines, firms find it less costly to issue debt and lower the default boundary $v_b(\lambda)$, reducing the number of future defaults.

However, for moderately low values of λ and for low values of v near the default boundary $v_b(\lambda)$, $\partial\Psi(v,\lambda)/\partial\lambda$ can sometimes be *negative*. In this region, making investors more optimistic (lowering λ) tends to raise the expected number of defaults—a bigger storm—in the future. This result can also be seen from the example in Figure 9: when λ_0 is reduced from 0.34 to 0.33 while v_0 is kept at 0.65, the onset of defaults is delayed from month 1 to month 39, but the total number of defaults increases from 4 to 5.

How can making debt investors more optimistic—i.e., lowering λ—sometimes lead to a bigger expected future storm? With a moderately low λ and deteriorating firm fundamentals, lowering λ tends to move the credit market towards the region of low v and low λ—i.e., the region where there is an endogenous built-up of fragility. In this region, firms set a low default boundary $v_b(\lambda)$ and issue large amounts of debt, becoming highly levered and thus more vulnerable to future adverse schools to fundamentals. When adverse fundamental shocks are eventually realized, the credit market subsequently experiences a more prolonged sequence of consecutive defaults.

The "calm before the storm" and the "default spiral" dynamics together highlight the potential disconnect between the endogenous credit cycle and the underlying business cycle that is at the heart of our model. The calm period is in part self-fulfilling: the absence of recent defaults leads to a lower λ, which allows the firm to lower the default boundary $v_b(\lambda)$ and hence temporarily avoid default even in the face of deteriorating fundamentals. Thus, towards the end of a long calm period, the credit cycle and the business cycle are often not synchronized: firm fundamentals are deteriorating while credit expansion continues. Once defaults hit the credit market, the extrapolative nature of investor beliefs then makes the financial recovery from a crisis slower and more protracted than it would be in a world with forward-looking, fully rational investors, again creating a disconnect between the credit cycle and the business cycle.

IV.D. Bond return predictability

In our behavioral model, bond returns are predictable. To see this, first note from the bond pricing equation (7) that biased bond investors always believe that

$$
\begin{aligned}
\mathbb{E}_t^B[\tilde{r}_{t\to t+dt}dt] &= \frac{\mathbb{E}_t^B[(1-\tilde{D}_{t+dt})(1-\xi dt)(c\cdot dt + p(\lambda_{t+dt})) + \tilde{D}_{t+dt}(1-\xi dt)\eta + \xi dt]}{p(\lambda_t)} - 1 \\
&= r\cdot dt, \qquad (23)
\end{aligned}
$$

where $\tilde{r}_{t\to t+dt}$ represents the (annualized) instantaneous bond return from time t to $t+dt$, $\tilde{D}_{t+dt}$ equals one if the firm defaults over the next dt period and equals zero otherwise, and the superscript "B" denotes the biased expectations of bond investors. In other words, the instantaneous return expected by biased investors always equals their required instantaneous return. Also note that at any time t, bond investors believe that $\mathbb{E}_t^B[\tilde{D}_{t+dt}] = \lambda_t dt$.

Now we consider a discretized version of the model and we examine the bond returns over a Δt period, from t to $t+\Delta t$, assuming that $\Delta t > 0$ is small but non-infinitesimal. Specifically, in our simulations, we will set $\Delta t = 1/12$ (i.e., one month). In this case, a rational econometrician believes that $\mathbb{E}_t^R[\tilde{D}_{t+\Delta t}] = \lambda^R(v_t, \lambda_t; \Delta t)\Delta t$, where $\lambda^R(v_t, \lambda_t; \Delta t)$ denotes the (annualized) true default intensity over the next Δt period given the current state (v_t, λ_t). Since biased investors believe that $\mathbb{E}_t^B[\tilde{D}_{t+\Delta t}] \approx \lambda_t \Delta t$, equation (23) then implies:[25]

$$
\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}] - r \approx -\frac{(p(\lambda_t)-\eta)(\lambda^R(v_t,\lambda_t;\Delta t) - \lambda_t)}{p(\lambda_t)}. \qquad (24)
$$

Since $p(\lambda_t) > \eta$ for all $\lambda_t \in [\lambda_m, \lambda_h]$, equation (24) shows that expected bond returns are less than the required return of r when investors are overly optimistic about the likelihood of default (i.e., when $\lambda^R(v_t, \lambda_t; \Delta t) > \lambda_t$). This is typically the case in a "calm before the storm" scenario, when firm fundamentals have deteriorated but extrapolative investors remain bullish because they have not recently witnessed a default. Furthermore, when $\lambda^R(v_t, \lambda_t; \Delta t)$ is significantly higher than λ_t, expected bond excess returns can become highly *negative*. For example, through simulations, we find that when $v_t = 0.65$ and $\lambda_t = 0.33$, the annualized expected bond excess return $\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}] - r$ is -38.3%.[26] Conversely, equation (24) shows that expected bond returns are higher than r when

[25] Equation (24) is only an approximation because we neglect the higher-order $o(\Delta t)$ terms.

[26] Specifically, we compute $\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}]$ as follows. We discretize the continuous-time model at a monthly frequency. We then simulate the economy 25,000 times, all starting from the same initial state (v_t, λ_t). For each simulation, we compute the annualized bond return from t to $t+\Delta t$ as

$$
\{\xi\Delta t\cdot(1/p(\lambda_t)) + (1-\xi\Delta t)[D_{t+\Delta t}\cdot(\eta/p(\lambda_t)) + (1-D_{t+\Delta t})\cdot((p(\lambda_{t+\Delta t}) + c\Delta t)/p(\lambda_t))] - 1\}/\Delta t.
$$

investors are overly pessimistic about the probability of default (i.e., when $\lambda^R(v_t, \lambda_t; \Delta t) < \lambda_t$). This is the case in a "default spiral" scenario where investors over-estimate the likelihood of future defaults because they have just witnessed a default.

Holding fixed λ_t, we can ask how an increase in v_t impacts expected bond returns. Since $\partial \lambda^R(v_t, \lambda_t; \Delta t)/\partial v_t < 0$, equation (24) implies

$$\frac{\partial \mathbb{E}_t^R[\tilde{r}_{t \to t+\Delta t}]}{\partial v_t} \approx -\frac{p(\lambda_t) - \eta}{p(\lambda_t)} \frac{\partial \lambda^R(v_t, \lambda_t; \Delta t)}{\partial v_t} > 0. \tag{25}$$

Interpreting v_t as a measure of issuing firms' creditworthiness, equation (25) is then consistent with Greenwood and Hanson (2013), who find that a deterioration in the creditworthiness of issuing firms negatively predicts the excess returns on corporate bonds. A decrease in v_t—either due to a decrease in the firm's fundamental V_t or an increase in its outstanding debt F_t—leads to an increase in $\lambda^R(v_t, \lambda_t; \Delta t)$, which leads to a decline in expected bond returns. Intuitively, when investors are extrapolative, holding fixed their beliefs λ_t, worse firm fundamentals and higher levels of leverage predict lower future bond returns.

What is more interesting and subtle is that, holding fixed v_t, changes in investor beliefs—i.e., movements in λ_t—have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets. When expected future debt repayments are constant, more bearish investor beliefs (higher values of λ_t) lower bond prices, raising expected bond returns. This is the intuition we have from standard settings where beliefs do not impact security payoffs. However, there is a competing effect that arises in our model because investor beliefs about future defaults are partially self-fulfilling. Specifically, more bearish investor sentiment makes it more costly for firms to issue debt, so they raise the default boundary $v_b(\lambda_t)$, causing an increase in the true probability of default and a decrease in expected future debt repayments. And, in highly reflexive states where investor beliefs have a large impact on the true likelihood of default—i.e., where $\partial \lambda^R(v_t, \lambda_t; \Delta t)/\partial \lambda_t$ is large—the latter effect can outweigh the former. As a result, the total impact of a shift in λ_t on expected returns is ambiguous: depending on which effect dominates, a small increase in λ_t can either lead $\mathbb{E}_t^R[\tilde{r}_{t \to t+\Delta t}]$ to rise or fall.[27]

Here, $D_{t+\Delta t} = 1$ if the firm defaults at $t+\Delta t$; otherwise, $D_{t+\Delta t} = 0$. The rationally-expected bond return $\mathbb{E}_t^R[\tilde{r}_{t \to t+\Delta t}]$ is computed as the average of these 25,000 realized returns.

[27] In Section IV.A, we define $\partial \Pr^R(v, \lambda)/\partial \lambda$ as a formal measure of reflexivity. Here, $\partial \lambda^R(v, \lambda; \Delta t)/\partial \lambda$ can be

We illustrate the potential ambiguous relationship between $\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}]$ and λ_t using numerical examples. When v_t is near the default boundary $v_b(\lambda_t)$, $\partial\lambda^R(v_t,\lambda_t;\Delta t)/\partial\lambda_t$ can be highly positive. For example, when $v_t = 0.65$ and $\lambda_t = 0.33$, $\partial\lambda^R(v_t,\lambda_t;\Delta t)/\partial\lambda_t = 13.34$. In this case, $\partial\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}]/\partial\lambda_t = -5.32 < 0$. However, when v_t is far from the default boundary $v_b(\lambda_t)$, $\partial\lambda^R(v_t,\lambda_t;\Delta t)/\partial\lambda_t$ is close to zero. For example, when $v_t = 1$ and $\lambda_t = 0.33$, $\partial\lambda^R(v_t,\lambda_t;\Delta t)/\partial\lambda_t = 0$. In this case, $\partial\mathbb{E}_t^R[\tilde{r}_{t\to t+\Delta t}]/\partial\lambda_t = 0.41 > 0$.

We now use numerical simulations to confirm that the model's implications fit the stylized facts about return predictability over the credit cycle. First, we consider two univariate regressions:

$$r_{t\to t+1} = a + b\cdot \underbrace{\lambda^R(v_t,\lambda_t;\Delta t)}_{\text{inverse measure of bond quality}} + \varepsilon_{t+1} \tag{26}$$

and

$$r_{t\to t+1} = a + b\cdot \underbrace{g^*(v_t,\lambda_t)}_{\text{debt issuance}} + \varepsilon_{t+1}. \tag{27}$$

We then consider two bivariate regressions:

$$r_{t\to t+1} = a + b_1\cdot \underbrace{(1-p(\lambda_t))}_{\text{credit spread}} + b_2\cdot \underbrace{\lambda^R(v_t,\lambda_t;\Delta t)}_{\text{inverse measure of bond quality}} + \varepsilon_{t+1} \tag{28}$$

and

$$r_{t\to t+1} = a + b_1\cdot \underbrace{(1-p(\lambda_t))}_{\text{credit spread}} + b_2\cdot \underbrace{g^*(v_t,\lambda_t)}_{\text{debt issuance}} + \varepsilon_{t+1}. \tag{29}$$

For each of these regressions, we simulate the economy 100 times, all starting from the same initial state (v_0,λ_0). For each simulation, we discretize the continuous-time model at a monthly frequency and simulate the model for a total of 100 years. At the beginning of each year t, we then compute the current inverse measure of bond quality $\lambda^R(v_t,\lambda_t;\Delta t)$ based on the firm's true default intensity from t to $t+\Delta t$, the current rate of debt issuance $g(v_t,\lambda_t)$, the current credit spread $1-p(\lambda_t)$, and the cumulative bond return over the next year $r_{t\to t+1}$ (which is obtained by compounding twelve monthly bond returns). We chain the data across 100 simulations, obtaining 10,000 years

interpreted as an alternative measure of reflexivity.

of simulated data, which we use to run the regressions specified in equations (26) to (29) above. Finally, we repeat this exercise for different values of (v_0, λ_0).[28]

All four regressions produce results that are consistent with the stylized facts about bond return predictability over the credit cycles. The signs of the regression coefficients do not change as we vary the initial conditions (v_0, λ_0). Furthermore, the magnitude of the coefficients is not sensitive to changes in (v_0, λ_0).[29] For example, when $v_0 = 1$ and $\lambda_0 = 0.5$, we obtain the following results:

- The univariate regression in (26) produces a coefficient of $b = -0.03$. In other words, low quality debt issuance predicts low future bond returns, consistent with the empirical findings of Greenwood and Hanson (2013).
- The univariate regression in (27) produces a coefficient b of –4.88. That is, high debt issuance predicts low future bond returns, consistent with the findings of Greenwood and Hanson (2013), Baron and Xiong (2017), and Muir (2019).
- The bivariate regression in (28) produces coefficients of $b_1 = 2.16$ and $b_2 = -0.08$, which is consistent with the findings of Greenwood and Hanson (2013) and López-Salido et al. (2017).
- Finally, the bivariate regression in (29) produces coefficients of $b_1 = 1.32$ and $b_2 = -3.18$.

V. Model comparisons

In the baseline model in Section III, we assumed that a rational firm faces bond investors who have biased beliefs. Specifically, we assumed that bond investors form beliefs about the likelihood of future firm defaults by extrapolating past defaults. In this section, we compare our baseline model with two alternative models of the credit market. These comparisons allow us to better understand the workings and distinctive predictions of our default extrapolation model. First, we analyze a benchmark model in which both equity and debt investors are fully rational. Second, we discuss an alternative behavioral model that features a rational firm and set of behavioral bond investors who extrapolate the firm's past fundamental growth rate.

[28]This simulation procedure is necessary because, as noted above, our default extrapolation does not have a well-defined steady state. Thus, if we simulate the model for a long enough period of time, v_t will eventually grow large and there will be no additional defaults.

[29]We examine many different values of λ between λ_m and λ_h and many different values of v between $v_b(\lambda)$ and 3.

V.A. A rational benchmark

We begin by comparing our baseline behavioral model with a rational benchmark. To facilitate this comparison, the two models are kept almost completely identical. The only difference is that, in the benchmark rational model, bond prices are set by investors who are fully rational. These rational bond investors keep track of firm fundamentals V and debt outstanding F and they price bonds based on how far away V is from the default boundary $V_b(F)$. Formally, the benchmark rational version of our model simply adds debt issuance costs to DeMarzo and He (2021).

The firm continues making coupon and principal payments so long as $V > V_b(F)$, but optimally chooses to default when $V = V_b(F)$. The evolution of the bond price $p(V, F)$ in the no-default region is governed by

$$r \cdot p(V, F) = c + \xi(1 - p(V, F)) + (G^*(V, F) - \xi F)p_F + \mu V p_V + \tfrac{1}{2}\sigma^2 V^2 p_{VV}, \tag{30}$$

where $G^*(V, F)$ is the firm's optimally chosen rate of debt issuance. In addition, the bond price must satisfy the following two boundary conditions:

$$p(V_b(F), F) = \eta, \quad \lim_{V \to \infty} p(V, F) = \frac{c + \xi}{r + \xi}, \tag{31}$$

which say that bonds recover η in default and that, holding F fixed, bond prices converge to their default-free value as V grows large.

Within the no-default region where $V > V_b(F)$, the evolution of the firm's equity value $E(V, F)$ is governed by the following HJB equation:

$$rE(V, F) = \max_G \left\{ \begin{array}{l} \underbrace{\left[\delta V - (c + \xi)F - \tau(\delta V - cF) + Gp(V, F) - \tfrac{1}{2}\psi F \left(\frac{G - \xi F}{F}\right)^2\right]}_{\text{equity cash flow}} \\ + \underbrace{\left[\mu V E_V + \tfrac{1}{2}\sigma^2 V^2 E_{VV} + (G - \xi F)E_F\right]}_{\text{expected equity capital gain}} \end{array} \right\}. \tag{32}$$

The optimal rate debt issuance is therefore

$$G^*(V, F) = \xi F + \frac{p(V, F) + E_F}{\psi} F. \tag{33}$$

Substituting (33) back into (32), we obtain the following partial differential equation:

$$rE(V,F) = \left[\begin{array}{l} \delta V - (c+\xi)F - \tau(\delta V - cF) + \xi F p(V,F) + \dfrac{F}{2\psi}(p(V,F) + E_F)^2 \\ +\mu V E_V + \frac{1}{2}\sigma^2 V^2 E_{VV} \end{array} \right]. \quad (34)$$

Since equity holders receive nothing in default and since the default boundary $V_b(F)$ is optimally chosen, we have the following value-matching and smooth-pasting conditions:

$$E(V_b(F), F) = 0, \quad E_V(V_b(F), F) = 0, \quad (35)$$

for any $F > 0$.[30]

Using numerical methods, we jointly solve the two partial differential equations (30) and (34) and the default boundary $V_b(F)$. In Appendix C, we first show that the equity value E is homogeneous of degree one in V and F, that the default boundary V_b is homogeneous of degree one in F, and that the bond price p is homogeneous of degree zero in V and F. Given these results, we write

$$E(V,F) = e(\frac{V}{F})F, \quad V_b(F) = v_b F, \quad p(V,F) = p(\frac{V}{F}). \quad (36)$$

And as in our main model, we define $v \equiv V/F$, namely the firm's fundamental normalized by its debt outstanding. We can then rewrite (30) and (34), reducing these two partial differential equations to two ordinary differential equations. Finally, in Appendix C, we discuss our numerical procedure for solving this benchmark rational model.

[Place Figure 12 about here]

The panels in Figure 12 show the path of reduced firm fundamentals $v_t = V_t/F_t$, bond prices

[30]Since we assume that $\psi > 0$, the benchmark rational version of our model simply adds debt issuance costs to DeMarzo and He (2021). The optimal rate of debt issuance $G^*(V,F)$ is given by (33). As $V \to \infty$ holding F fixed, the rate of debt issuance remains finite and, thus, the firm's debt becomes default-free in this limit as stated in (31). By contrast, when $\psi = 0$ as in DeMarzo and He (2021), the first-order condition for G implies that $p(V,F) = -E_F$ and, as shown in DeMarzo and He (2021), the optimal rate of debt issuance is given by $G^*(V,F) = -\tau c/p_F(V,F) = \tau c/E_{FF}(V,F)$. Thus, when $\psi = 0$, $G^*(V,F) \to \infty$ as $V \to \infty$ holding F fixed, so debt no longer becomes default-free in this limit—i.e., the second condition in (31) no longer holds. Intuitively, unlike in DeMarzo and He (2021) where the firm has no ability to make commitments about its future leverage choices, the existence of debt issuance costs ($\psi > 0$) gives the firm partial commitment. Thus, while there is still a leverage ratchet effect, it is not as strong, which allows the firm to capture some of the tax benefits of debt.

$p(v_t)$, the default indicator D_t, and the firm's optimal net debt issuance $g^*(v_t) - \xi$.[31] For these plots, we take the baseline parameters from Section IV: $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\Theta = 0.25$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.

Comparing the simulated paths in Figure 12 and Figure 6, we immediately see two important differences between the benchmark rational model and our behavioral model with default extrapolation. First, bond prices in the rational model decline as the firm's fundamentals deteriorate in the run-up to any default. As v_t approaches v_b, rational bond investors understand that default risk is rising, so bond prices fall. For example, in Figure 12, prior to the default at $t = 15$, the bond price at $t = 14$ is only 0.612, way below its par value of one. By contrast, bond prices in the default extrapolation model remain elevated so long as there have been no recent defaults. Here, behavioral bond investors underestimate bonds' default risk as they extrapolate past defaults when forming beliefs about the likelihood of future defaults. In Figure 6, prior to the firm default at $t = 61$, the bond price at $t = 60$ is 0.975, which is close to the par value of one.

Second, if the firm has just experienced a default, bond prices in the rational model rise and credit spreads decline. Forward-looking rational investors understand that the resulting restructuring raises v_t above v_b, reducing *future* default risk and leading to higher bond prices. For example, in Figure 12, after the firm defaults at $t = 15$, bond prices rise from 0.5 before the default to 0.827 right afterwards. In contrast, bond prices in the default extrapolation model fall sharply and credit spreads jump up following default. Here, behavioral investors who extrapolate past defaults tend to overestimate bonds' future default risk since they have just observed a default. In Figure 6, after the firm default at $t = 61$, bond prices drop from 0.975 before the default to 0.704 right afterwards.

Together, these two important differences imply that the rational model does not generate the "calm before the storm" and "default spiral" phenomena captured by our default extrapolation model. Moreover, by construction, bond returns are not predictable in the rational model.

V.B. Fundamental extrapolation

Our baseline behavioral model in Section III is a model of default extrapolation: bond investors do not directly attend to firm fundamentals; instead, they form beliefs about the firm's creditworthiness by extrapolating past defaults. As emphasized in Section IV, default extrapolation gives

[31] Following the discussion in Section III.C, if the firm defaults at time t, then $v_{t_+} = (1 + \Theta)v_{t_-}$.

rise to both the "calm before the storm" and the "default spiral" phenomena, which allow for a significant disconnect between the credit cycle and the business cycle.

It is interesting to contrast our model of default extrapolation with models of fundamental extrapolation models in credit markets such as Bordalo et al. (2018). Specifically, consider an alternative model that features a rational firm and behavioral bond investors who extrapolate the firm's past fundamental growth. A tractable way of modeling such fundamental extrapolation would be to assume that the true evolution of firm fundamentals follows (3), which has a constant growth rate of μ. However, bond investors would incorrectly believe that the growth rate of fundamentals $\tilde{\mu}_t$ is a latent variable that randomly switches between a high value of $\tilde{\mu}_t = \mu_h$ and a low value $\tilde{\mu}_t = \mu_l < \mu_h$. These bond investors would then use Bayesian inference to compute the expected growth rate $\mu_t = \mathbb{E}_t^B[\tilde{\mu}_t]$ of fundamental growth at each point in time t, leading them to incorrectly expect high (low) future fundamental growth when recent fundamental growth has been high (low).

Almost by construction, this alternative behavioral model would be consistent with some of the evidence on bond return predictability. However, since bond investors extrapolate from firm's exogenous fundamentals, this alternative model is not "reflexive": it does not feature a dynamic *two-way* feedback loop between biased investor beliefs and market outcomes. As a result, the credit cycle would remain more tightly linked with the business cycle—i.e., with the evolution of firm fundamentals. Specifically, while investors' biased beliefs would certainly affect market prices, the firm's debt issuance decisions, and the firm's default decisions, these endogenous market outcomes would not affect future investors biases: the evolution of biases would depend solely on the exogenous future evolution of fundamentals. As a result, this alternative model would not give rise to either the "calm before the storm" or the "default spiral" phenomenon.

For instance, consider a scenario when the firm's fundamentals have been deteriorating but no default has occurred yet. In this case, our default extrapolation model predicts that the firm will optimally delay default because overly optimistic investors pay high prices for bonds, keeping the cost of debt issuance low. However, the fundamental extrapolation model predicts that the firm will optimally accelerate default—by lowering the leverage threshold at which it defaults—because overly pessimistic bond investors make it more costly for the firm to issue debt. As a result, a model with fundamental extrapolation does not give rise to the "calm before the storm" phenomenon. Instead, we would see light rain long before it pours.

This alternative model would also not give rise to "default spirals." While bond investors have biased beliefs about the growth rate of fundamentals, bond investors in this alternative model are pricing bonds using a full "structural" model of default. Thus, as in our fully rational benchmark model, bond prices would rise following a default and the resulting restructuring. Specifically, bond investors would understand that the resulting restructuring event raises v_t above the default boundary, reducing *future* default risk and leading to higher bond prices post-restructuring.

VI. Conclusion

We develop a model of credit cycles in which investors extrapolate past defaults. Our key contribution is to model reflexivity in credit markets, a dynamic two-way feedback loop between biased investor beliefs and credit market outcomes. This two-way feedback mechanism is particularly germane in credit markets because firms must return to the market to refinance maturing debts, the terms on which debt can be refinanced impacts the likelihood of future defaults, and future default realizations drive future investor beliefs.

As we have shown, the combination of extrapolative beliefs and reflexive dynamics can lead to large short-run disconnects between cash flow fundamentals and credit market outcomes, including "calm before the storm" and "default spiral" episodes. Extrapolative beliefs also naturally lead to bond return predictability. But what is most striking here is that changes in investor sentiment can have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets. When investors become more bullish, in the short run this can predict positive returns, even if at longer horizons expected returns become more negative.

Our analysis leaves open at least three areas for future research. First, we have not allowed conditions in credit markets to explicitly affect firm investment or the underlying fundamentals of the economy.[32] However, as demonstrated by a growing macro-finance literature, the inability to access credit on reasonable terms following a credit market bust may exacerbate an incipient economic downturn. Indeed, López-Salido et al. (2017) and Mian et al. (2017) show that periods

[32]One could endogenize investment in our model as in DeMarzo and He (2021). Specifically, one could assume that a higher rate of current investment, which reduces the current cash flows to equity holders, raises the growth rate of firm fundamentals—i.e., the growth rate of pretax operating cash flows. Unless the true risk of a near-term default was extremely rare, bond investor sentiment would then affect the optimal level of firm investment in this setting.

of credit market overheating forecast low economic growth. Incorporating these features into our model would likely further strengthen the feedback loop between investor sentiment and credit market outcomes.[33] Recent papers by Krishnamurthy and Li (2021), Bordalo, Gennaioli, Shleifer, and Terry (2021), and Maxted (2023) have made progress in this direction.

Second, we have been silent on issues of welfare and optimal policy, even though our model suggests a potential role for policy. During credit booms, high sentiment can prevent defaults from occurring in the near future, which can be welfare-improving if fundamentals recover soon enough. Nonetheless, self-fulfilling beliefs during default spirals can be welfare-reducing, both because these deteriorating beliefs accelerate future default realizations and because they lead to a slow recovery in the presence of improving fundamentals. Accepting these take-aways at face value, our model suggests a role for policy in moderating investor beliefs.

Third, our model has relevance for the literature on sovereign debt crises, suggesting how one might incorporate extrapolative expectations into standard models of sovereign crises (Calvo, 1988; Cole and Kehoe, 2000). Specifically, the introduction of extrapolative expectations may help explain the kinds of "slow-moving debt crises" studied in Lorenzoni and Werning (2019). And, further extending our model to allow for multiple borrowers may help capture the idea of belief-driven market contagion across sovereign borrowers, which may prove useful in understanding events like the 1997 Asian financial crisis and the post-2010 European debt crisis.

[33] Reflexivity in our model might represent a channel of real effects of financial markets: changes in credit investors' biased belief affect market outcomes such as firm defaults, which in turn can affect the real economy. For a review of the real effects of financial markets, see Bond, Edmans, and Goldstein (2012).

REFERENCES

Baker, Malcolm, and Jeffrey Wurgler, 2013, Behavioral corporate finance: A current survey, in George M. Constantinides, Milton Harris, , and Rene M. Stulz, eds., *Handbook of the Economics of Finance*, volume 2A, 357–424 (Elsevier, New York).

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barberis, Nicholas, Robin Greenwood, Lawrence J. Jin, and Andrei Shleifer, 2015, X-CAPM: An extrapolative capital asset pricing model, *Journal of Financial Economics* 115, 1–24.

Barberis, Nicholas, Robin Greenwood, Lawrence J. Jin, and Andrei Shleifer, 2018, Extrapolation and bubbles, *Journal of Financial Economics* 129, 203–227.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Baron, Matthew, Emil Verner, and Wei Xiong, 2021, Banking crises without panics, *Quarterly Journal of Economics* 136, 51–113.

Baron, Matthew, and Wei Xiong, 2017, Credit expansion and neglected crash risk, *Quarterly Journal of Economics* 132, 713–764.

Bebchuk, Lucian A., and Itay Goldstein, 2011, Self-fulfilling credit market freezes, *Review of Financial Studies* 24, 3519–3555.

Bernanke, Ben, and Mark Gertler, 1989, Agency costs, net worth, and business fluctuations, *American Economic Review* 79, 14–31.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, The financial accelerator in a quantitative business cycle framework, in John B. Taylor, and Michael Woodford, eds., *Handbook of Macroeconomics*, volume 1, 1341–1393 (North Holland, Amsterdam).

Bond, Philip, Alex Edmans, and Itay Goldstein, 2012, The real effects of financial markets, *Annual Review of Financial Economics* 4, 339–360.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, 2018, Diagnostic expectations and credit cycles, *Journal of Finance* 73, 199–227.

Bordalo, Pedro, Nicola Gennaioli, Andrei Shleifer, and Stephen J. Terry, 2021, Real credit cycles, NBER working paper No. 28416.

Brunnermeier, Markus K., and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Calvo, Guillermo A., 1988, Servicing the public debt: The role of expectations, *American Economic Review* 78, 647–666.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Cole, Harold L., and Timothy Kehoe, 2000, Self-fulfilling debt crises, *Review of Economic Studies* 67, 91–116.

Coval, Joshua, Kevin Pan, and Erik Stafford, 2014, Capital market blind spots, Working paper.

Das, Udaibir S., Michael G. Papaioannou, and Christoph Trebesch, 2012, Sovereign debt restructurings 1950–2010: Literature survey, data, and stylized facts, International Monetary Fund Working paper No.12/203.

DeMarzo, Peter M., and Zhiguo He, 2021, Leverage dynamics without commitment, *Journal of Finance* 76, 1195–1250.

Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.

Gennaioli, Nicola, Yueran Ma, and Andrei Shleifer, 2016, Expectations and investment, in *NBER Macroeconomics Annual*, volume 30, 379–431 (University of Chicago Press).

Gennaioli, Nicola, and Andrei Shleifer, 2018, *A crisis of beliefs: Investor psychology and financial instability* (Princeton University Press, Princeton, New Jersey).

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2012, Neglected risks, financial innovation, and financial fragility, *Journal of Financial Economics* 104, 452–468.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2015, Neglected risks: The psychology of financial crises, *American Economic Review* 105, 310–314.

Gilchrist, Simon, and Egon Zakrajšek, 2012, Credit spreads and business cycle fluctuations, *American Economic Review* 102, 1692–1720.

Gomes, João F., Marco Grotteria, and Jessica A. Wachter, 2019, Cyclical dispersion in expected defaults, *Review of Financial Studies* 32, 1275–1308.

Greenwood, Robin, and Samuel G. Hanson, 2013, Issuer quality and corporate bond returns, *Review of Financial Studies* 26, 1483–1525.

Greenwood, Robin, and Samuel G. Hanson, 2015, Waves in ship prices and investment, *Quarterly Journal of Economics* 130, 55–109.

Greenwood, Robin, Samuel G. Hanson, Andrei Shleifer, and Jakob Ahm Sørensen, 2022, Predictable financial crises, *Journal of Finance* 77, 863–921.

Greenwood, Robin, and Andrei Shleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Hanson, Samuel G., Jeremy C. Stein, Adi Sunderam, and Eric Zwick, 2020, Business credit programs in the pandemic era, Brookings papers on economic activity.

Hayek, Friedrich A., 1925, Die währungspolitik der vereinigten staaten seit der überwindung der krise von 1920, in *Zeitschrift für Volkswirtschaft und Sozialpolitik N.S.*, volume 5, 25–65 and 254–317.

He, Zhiguo, Bryan Kelly, and Asaf Manela, 2017, Intermediary asset pricing: New evidence from many asset classes, *Journal of Financial Economics* 126, 1–35.

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary asset pricing, *American Economic Review* 103, 732–770.

He, Zhiguo, and Wei Xiong, 2012, Rollover risk and credit risk, *Journal of Finance* 67, 391–430.

Holmström, Bengt, and Jean Tirole, 1997, Financial intermediation, loanable funds, and the real sector, *Quarterly Journal of Economics* 112, 663–691.

Jin, Lawrence J., 2015, A speculative asset pricing model of financial instability, Working paper.

Jin, Lawrence J., and Pengfei Sui, 2022, Asset pricing with return extrapolation, *Journal of Financial Economics* 145, 273–295.

Kindleberger, Charles P., 1978, *Manias, panics and crashes* (Basic Books, New York).

Kirti, Divya, 2018, Lending standards and output growth, International Monetary Fund Working paper No.18/23.

Krishnamurthy, Arvind, and Wenhao Li, 2021, Dissecting mechanisms of financial crises: Intermediation and sentiment, NBER working paper No. 27088.

Krishnamurthy, Arvind, and Tyler Muir, 2020, How credit cycles across a financial crisis, NBER working paper No. 23850.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

López-Salido, David, Jeremy C. Stein, and Egon Zakrajšek, 2017, Credit-market sentiment and the business cycle, *Quarterly Journal of Economics* 132, 1373–1426.

Lorenzoni, Guido, and Iván Werning, 2019, Slowmoving debt crises, *American Economic Review* 109, 3229–3263.

Malmendier, Ulrike, 2018, Behavioral corporate finance, in Douglas Bernheim, Stefano DellaVigna, and David Laibson, eds., *Handbook of Behavioral Economics* (North Holland, Amsterdam).

Mason, John C., and David C. Handscomb, 2003, *Chebyshev Polynomials* (Chapman & Hall/CRC, New York).

Maxted, Peter, 2023, A macro-finance model with sentiment, *Review of Economic Studies* forthcoming.

Mian, Atif, Amir Sufi, and Emil Verner, 2017, Household debt and business cycles worldwide, *Quarterly Journal of Economics* 132, 1755–1817.

Minsky, Hyman, 1986, *Stabilizing an unstable economy* (Yale University Press, New Haven, CT).

Mises, Ludwig, 1924, *Theorie des geldes und der umlaufsmittel*, second edition (Munich: Duncker and Humblot).

Muir, Tyler, 2019, Is risk mispriced in a credit boom?, Paper prepared for the 2019 INET Private Debt Initiative Conference.

Santos, Tano, and Pietro Veronesi, 2022, Leverage, *Journal of Financial Economics* 145, 362–386.

Schularick, Moritz, and Alan M. Taylor, 2012, Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008, *American Economic Review* 102, 1029–1061.

Soros, George, 1987, *The alchemy of finance* (John Wiley & Sons, Inc., Hoboken, NJ).

Soros, George, 2013, Fallibility, reflexivity, and the human uncertainty principle, *Journal of Economic Methodology* 20, 309–329.

Stein, Jeremy C., 1996, Rational capital budgeting in an irrational world, *Journal of Business* 69, 429–455.

Sørensen, Jakob Ahm, 2021, Risk neglect in the corporate bond market, Working paper.

Tobias, Adrian, Erkko Etula, and Tyler Muir, 2014, Financial intermediaries and the cross-section of asset returns, *Journal of Finance* 69, 2557–2596.

Wachter, Jessica A., 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *Journal of Finance* 68, 987–1035.

Figure 1. The credit cycle.

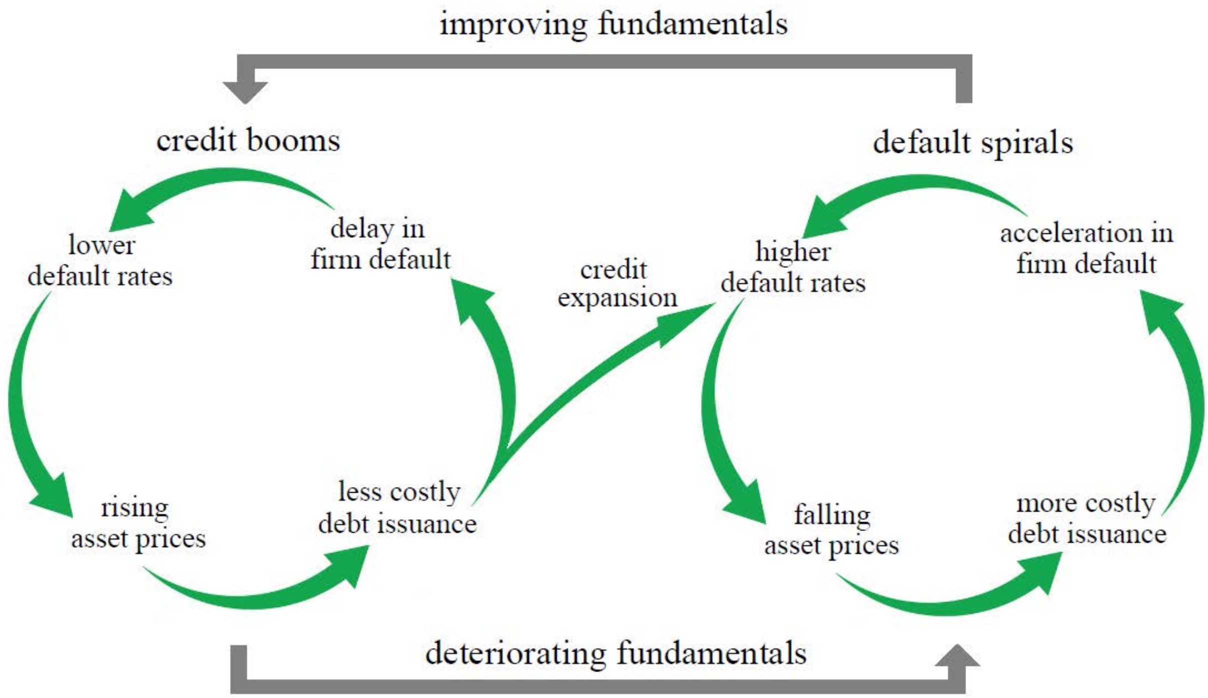


Figure 2. The credit market cycle. Panel A plots the year-over-year growth in log real GDP and the year-over-year growth in log real credit outstanding (defined as the sum of loans and bonds) to nonfinancial corporate businesses from the Federal Reserve's Financial Accounts of the United States. Panel B plots the year-over-year growth in log real credit versus the corporate credit spread, measured as the yield on Moody's seasoned Baa corporate bond yield minus the 10-year constant maturity Treasury yield. Shaded regions represent recessions, as defined by the NBER. The data begin in 1952 and end in 2022.

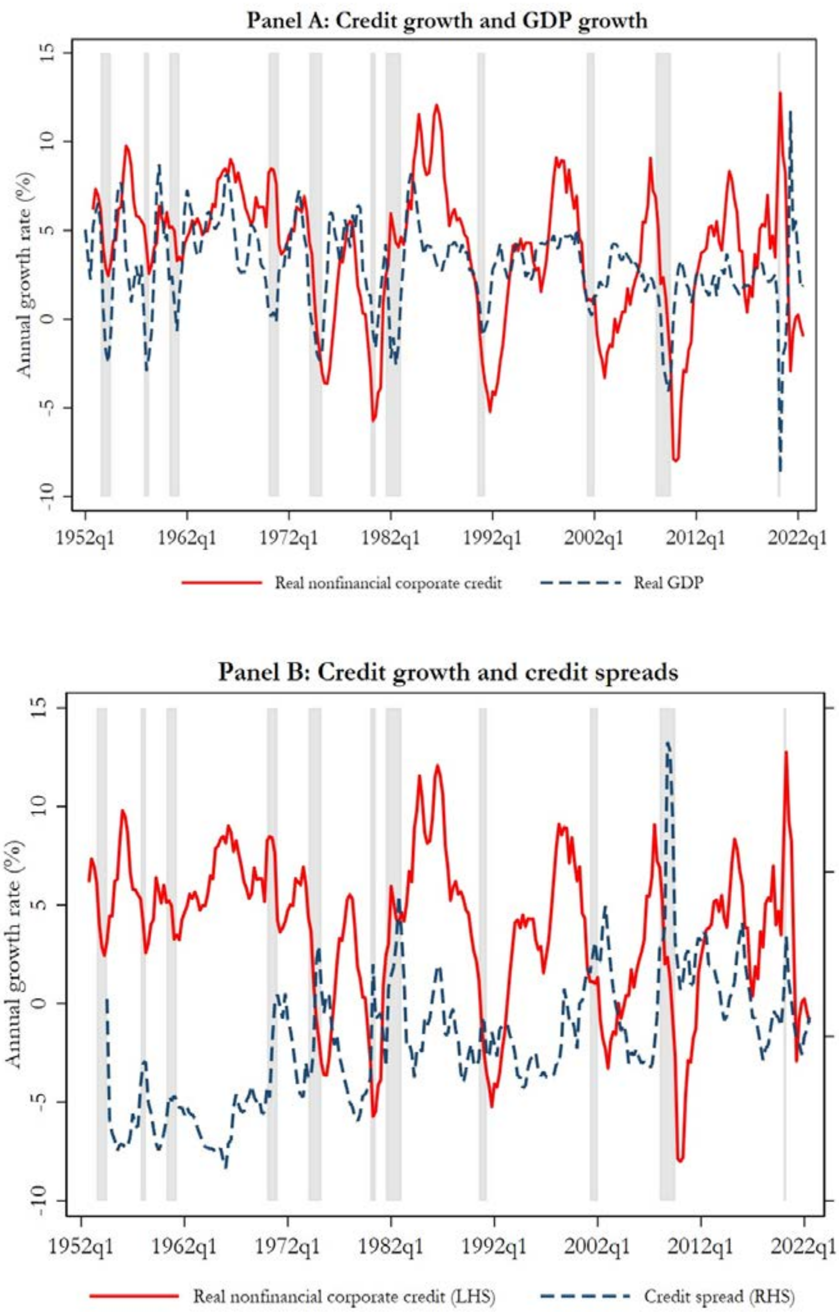


Figure 3. Real GDP growth and credit growth as a function of business cycle expansion quarter. Panel A and B, respectively, plot the year-over-year growth in log real GDP and log real credit outstanding (defined as the sum of loans and bonds) to nonfinancial corporate businesses from the Federal Reserve's Financial Accounts of the United States—as a function of NBER business cycle expansion quarter. The data begin in 1952 and end in 2022.

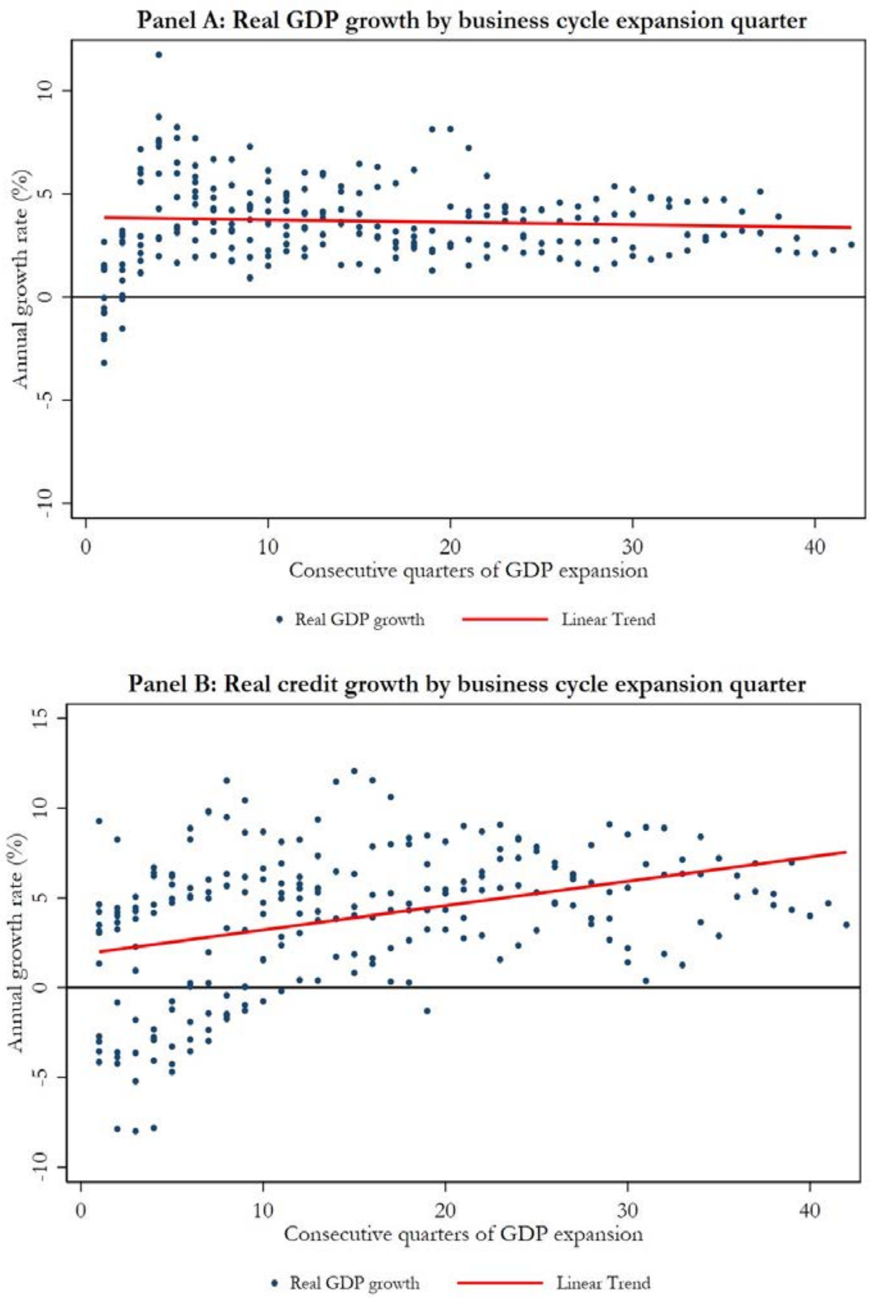


Figure 4. Optimal debt growth rate $g^*(v,\lambda)-\xi$ as a function of v and λ. Panel A plots the firm's optimal debt growth rate $g^*(v,\lambda)-\xi$ as a function of v and λ within the no-default region. Panel B plots $g^*(v,\lambda)-\xi$ as a function of v for $\lambda = 0.1$, 0.5, 1, 1.5, 1.75, and 1.95; it also plots $g^*(v,\lambda)-\xi$ as a function of λ for $v = 0.75$, 1, 1.5, 2, 3, and 5. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.

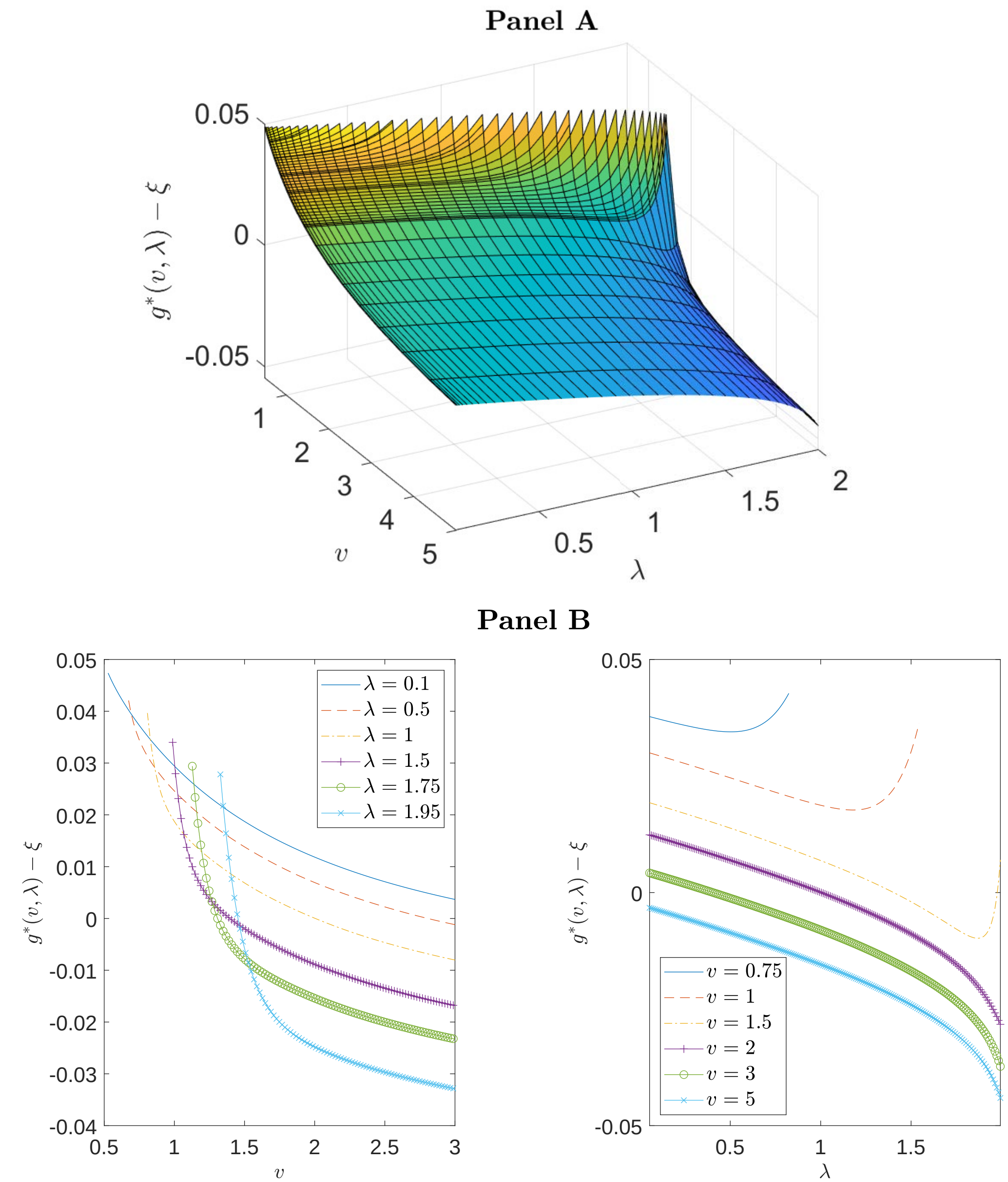
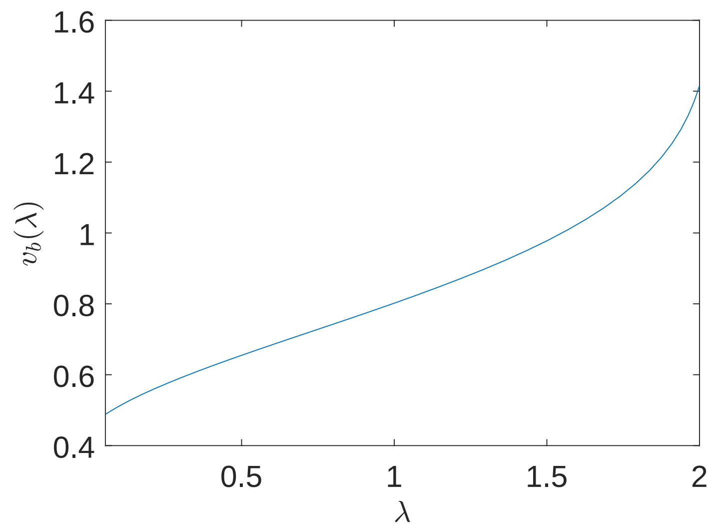


Figure 5. Optimal default boundary v_b and optimal equity value e. Panel A plots the default boundary $v_b(\lambda)$ as a function of λ. Panel B plots the firm's equity value $e(v, \lambda)$ as a function of v and λ within the no-default region. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.

Panel A



Panel B

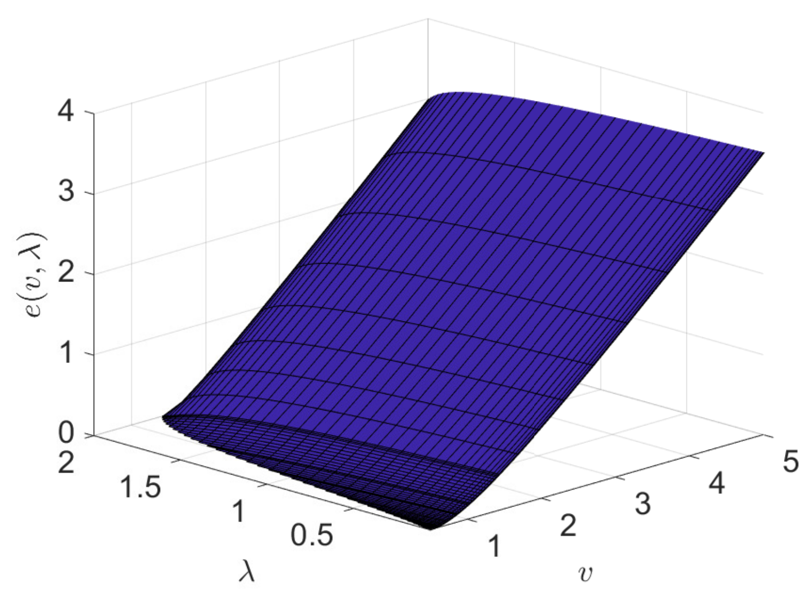


Figure 6. Default extrapolation model: Simulated data using baseline parameter values. This figure shows a typical path of simulated data using our baseline set of parameter values. Specifically, the parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. The simulation is at a monthly frequency for a length of 25 years. It starts with $(v_0, \lambda_0) = (0.8, 0.2)$. We plot the evolution of investor beliefs (λ_t), reduced firm fundamentals ($v_t = V_t/F_t$), bond prices ($p(\lambda_t)$), the endogenous default boundary ($v_b(\lambda_t)$), the default indicator (D_t), and the firm's optimal debt growth rate ($g^*(v_t, \lambda_t) - \xi$). For the evolution of λ_t and v_t, if there is a default in month t, then we plot both the pre-default level, v_{t-} and λ_{t-}, and the post-default level, v_{t+} and λ_{t+}, with an arrow symbol pointing from the former to the latter. For the default indicator, we set D_t to one if there is a default in month t; otherwise, we set D_t to zero. For the firm's optimal debt growth rate, we only plot $g^*(v_t, \lambda_t) - \xi$ when there is no default in month t.

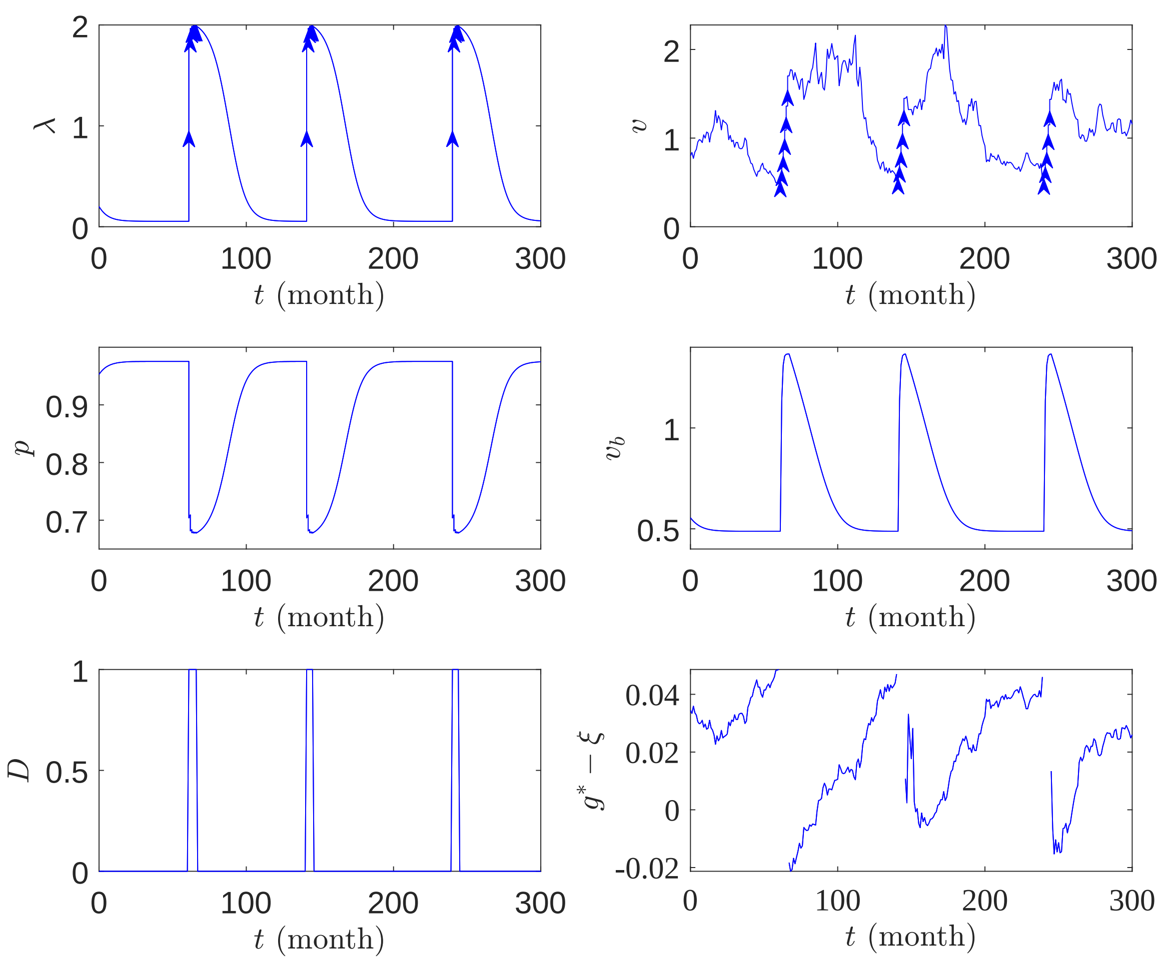


Figure 7. True default probability and investors' belief bias. The heatmap in the left panel shows $\Pr^R(v,\lambda)$, the true probability that the firm defaults within the next 12 months, as a function of v and λ. The heatmap in the right panel shows $\Pr^R(v,\lambda) - \Pr^B(\lambda)$, our measure of investors' belief bias, as a function of v and λ. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. We compute $\Pr^R(v,\lambda)$ as the fraction of 25,000 simulations where the firm defaults within 12 months. All simulations start from the same initial value of (v,λ). Each simulation is at a monthly frequency for a length of one year. We have $\Pr^B(\lambda) = \int_0^1 e^{-\lambda s} ds = 1 - e^{-\lambda}$.

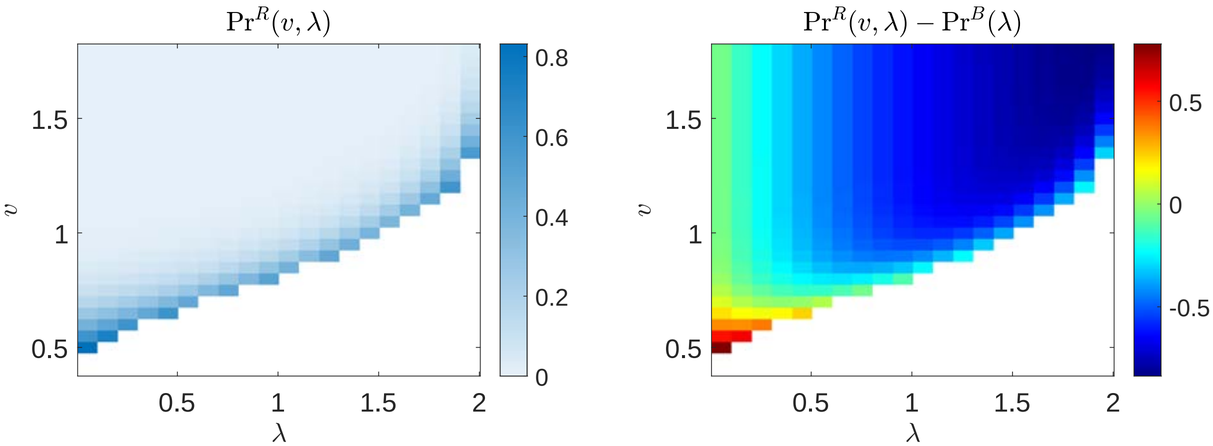


Figure 8. A reflexivity measure. The heatmap in the left panel shows $\partial \Pr^R(v,\lambda)/\partial\lambda$ as a function of v and λ. The figure in the right panel shows comparative statics: it plots $\partial \Pr^R(v,\lambda)/\partial\lambda$ as a function of v for different levels of λ. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. We compute $\Pr^R(v,\lambda)$ as the fraction of 25,000 simulations where the firm defaults within 12 months. All simulations start from the same initial value of (v,λ). Each simulation is at a monthly frequency for a length of one year.

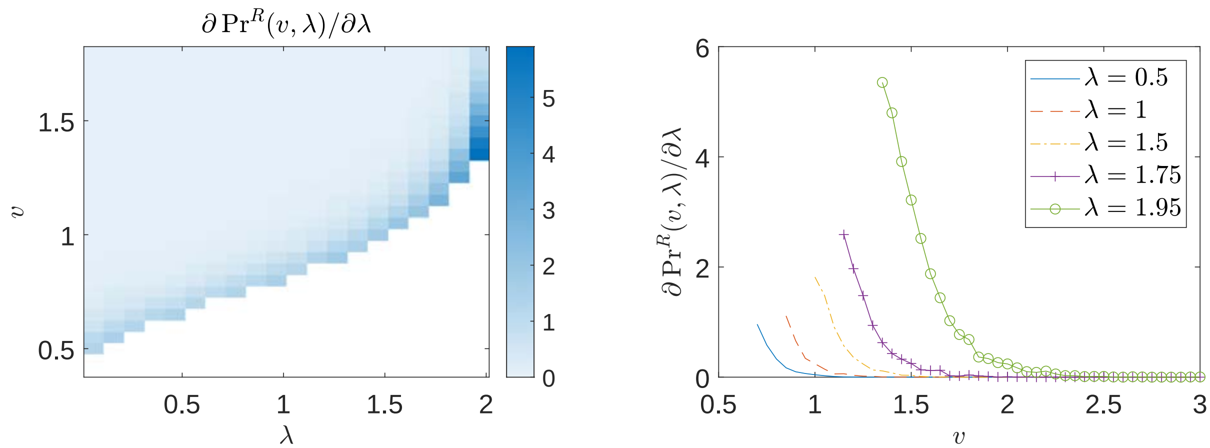


Figure 9. Calm before the storm: Two trajectories. The dashed (red) lines plot a sample trajectory of λ_t (left panel) and v_t (right panel) with initial values of $(v_0, \lambda_0) = (0.65, 0.34)$; the solid (blue) lines plot a sample trajectory of λ_t (left panel) and v_t (right panel) with initial values of $(v_0, \lambda_0) = (0.65, 0.33)$. The two trajectories share identical Brownian shocks that govern the evolution of V_t as specified in equation (3). If there is a default in month t, we plot both the pre-default level, v_{t-} and λ_{t-}, and the post-default level, v_{t+} and λ_{t+}, with an arrow symbol pointing from the former to the latter. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.

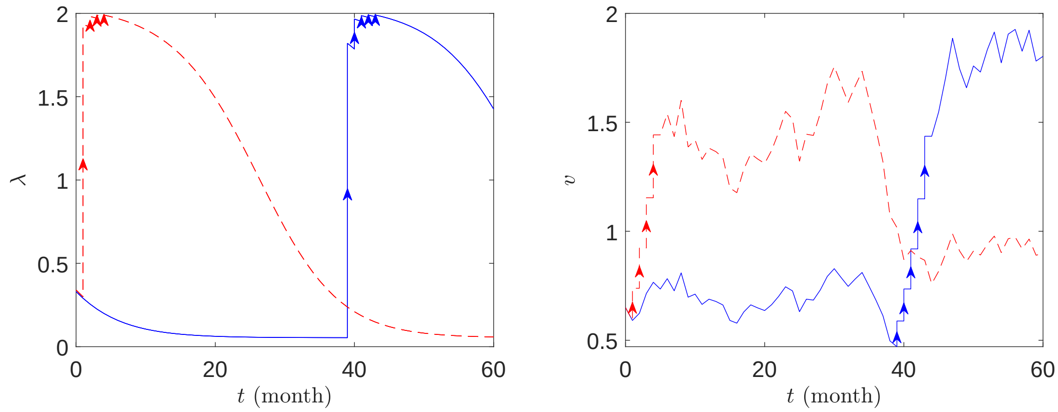
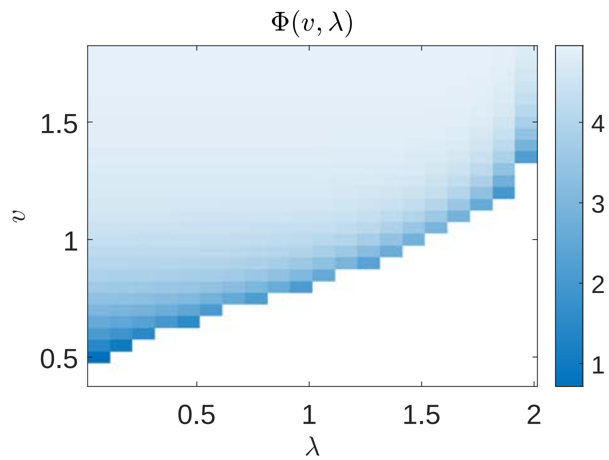


Figure 10. Duration of the calm period and its sensitivity with respect to changes in v or λ. The heatmap in Panel A shows $\Phi(v, \lambda)$ as a function of v and λ. The left heatmap in Panel B shows $\partial\Phi(v, \lambda)/\partial v$ as a function of v and λ. The right heatmap in Panel B shows $\partial\Phi(v, \lambda)/\partial\lambda$ as a function of v and λ. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. We construct $\Phi(v, \lambda)$ by averaging the duration of the calm period across 10,000 simulations. All simulations start from the same initial value of (v, λ). Each simulation is at a monthly frequency for a length of 5 years.

Panel A



Panel B

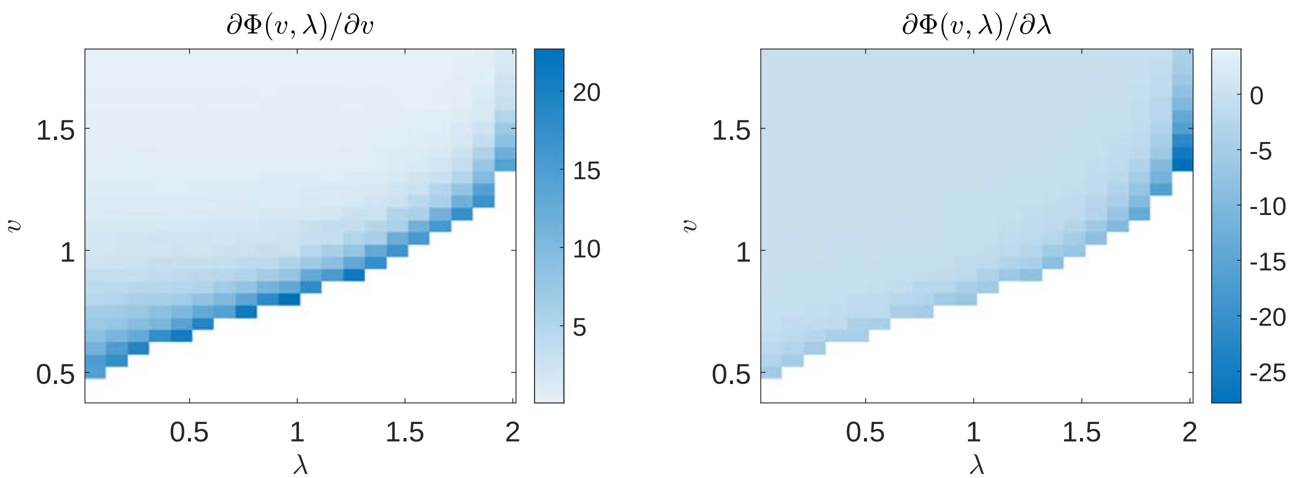
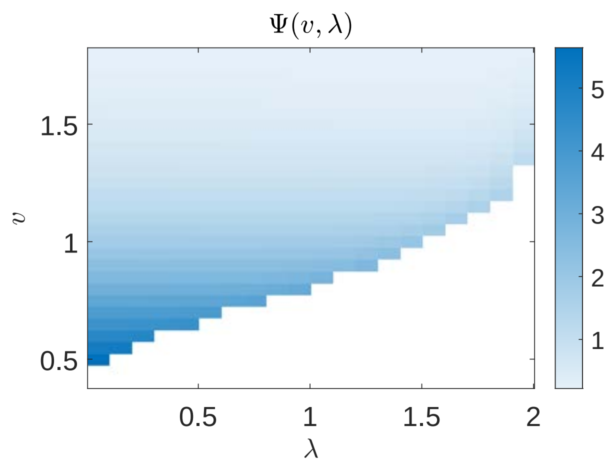


Figure 11. Default severity and its sensitivity with respect to changes in v or λ. The heatmap in Panel A shows $\Phi(v, \lambda)$ as a function of v and λ. The left heatmap in Panel B shows $\partial\Psi(v, \lambda)/\partial v$ as a function of v and λ. The right heatmap in Panel B shows $\partial\Psi(v, \lambda)/\partial\lambda$ as a function of v and λ. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. We construct $\Phi(v, \lambda)$ by averaging the number of defaults within the next 5 years across 10,000 simulations. All simulations start from the same initial value of (v, λ). Each simulation is at a monthly frequency for a length of 5 years.

Panel A



Panel B

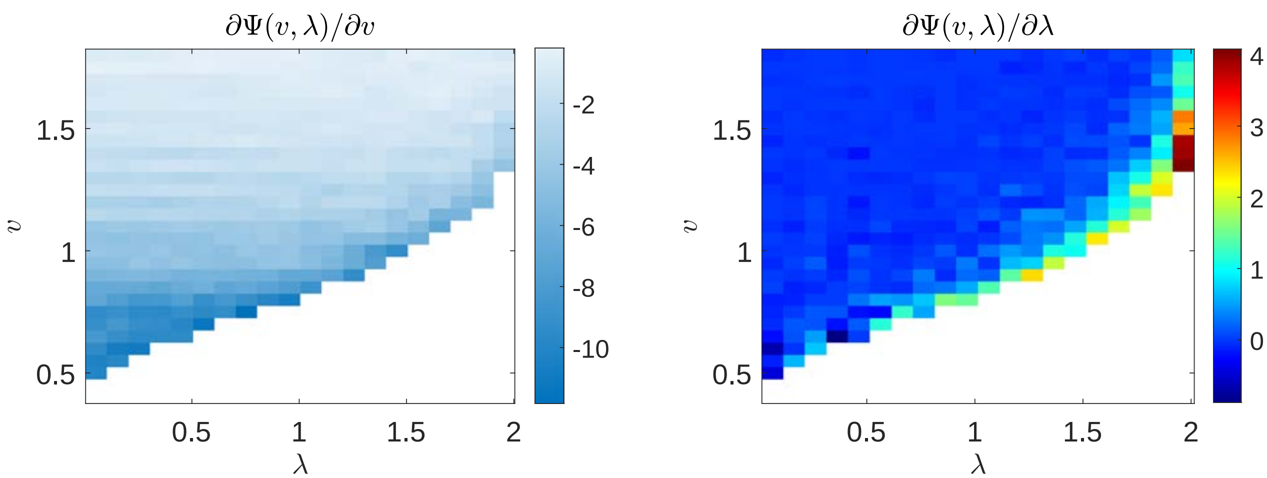


Figure 12. Rational model: Simulated data using baseline parameter values. This figure shows a typical path of simulated data under the rational model described in Section V.A. The parameter values are: $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. The simulation is at a monthly frequency for a length of 25 years. It starts with $v_0 = 1.5$. We plot the evolution of reduced firm fundamentals ($v_t = V_t/F_t$), bond prices ($p(v_t)$), the default indicator (D_t), and the firm's optimal debt growth rate ($g^*(v_t) - \xi$). For the evolution of v_t, if there is a default in month t, then we plot both the pre-default level v_{t-} and the post-default level v_{t+}, with an arrow symbol pointing from the former to the latter. For the default indicator, we set D_t to one if there is a default in month t; otherwise, we set D_t to zero. For the firm's optimal net debt issuance, we only plot $g^*(v_t) - \xi$ when there is no default in month t.

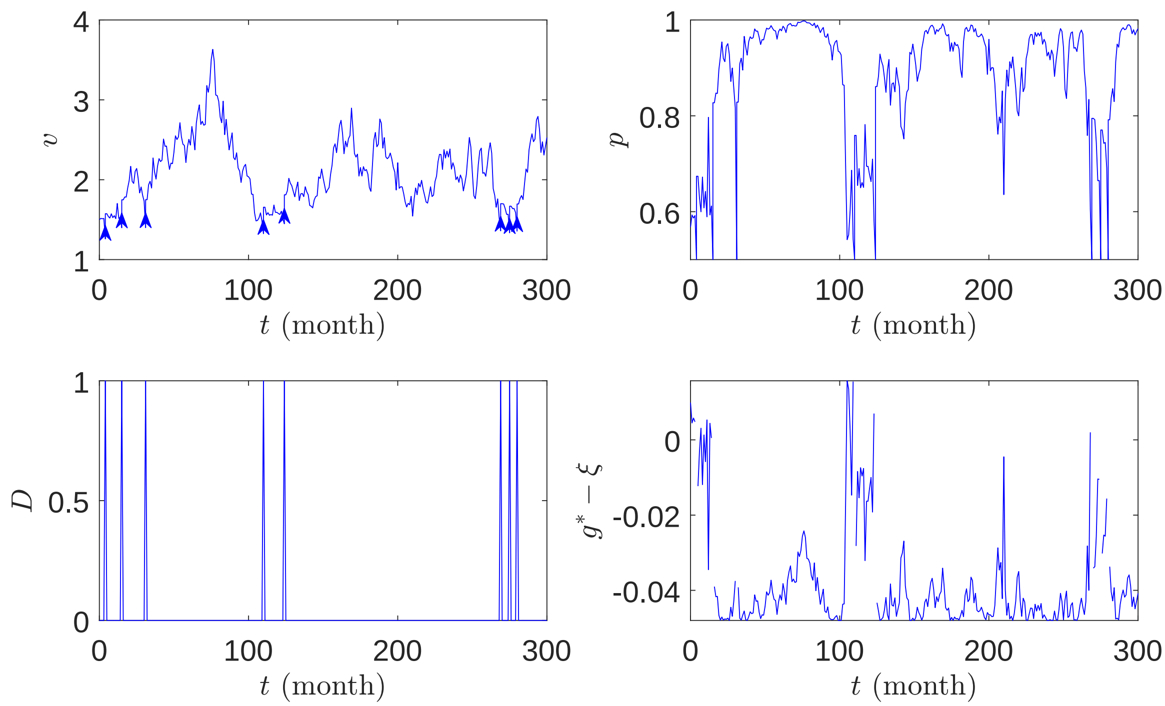


Table 1. Credit market overheating and future corporate bond returns. This table presents the results from estimating time-series regressions of the form

$$rx^{HK}_{t\to t+k} = a + b \cdot Overheating_t + \varepsilon_{t\to t+k},$$

where $Overheating_t$ is a proxy for credit market overheating in year t. The data begin in 1983 and run through 2020, predicting returns through 2022. The dependent variable is the cumulative k = 2- or 3-year excess return on high-yield bonds over like-maturity Treasuries. HYS_t is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's, as defined in Greenwood and Hanson (2013). $Credit\ Growth_t$ is the percentage change in outstanding corporate credit and is computed using Table L103 from the Flow of Funds. $Easy\ Credit_t$ is the three-year average of the percentage of bank loan officers reporting a loosening of commercial lending standards from the Federal Reserve's Senior Loan Office Opinion Survey. $-1 \times EBP_t$ is negative one times the excess bond premium from Gilchrist and Zakrajšek (2012). The t-statistics for k-period forecasting regressions (in parentheses) are based on Newey-West (1987) standard errors, allowing for serial correlation up to 3 lags for specifications (1)-(4) and 5 lags for specifications (5)-(8).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-year future excess return: $rx^{HY}_{t\to t+2}$				3-year future excess return: $rx^{HY}_{t\to t+3}$			
$\log(HYS_t)$	–13.33				–16.15			
	(–3.95)				(–3.47)			
$Credit\ Growth_t$		–110.02				–131.23		
		(–2.28)				(–2.20)		
$Easy\ Credit_t$			–0.42				–0.54	
			(–2.17)				(–3.37)	
$-1 \times EBP_t$				–15.94				–21.34
				(–2.66)				(–6.76)
Constant	–11.30	11.98	3.28	4.08	–12.09	15.82	5.10	6.21
	(–2.12)	(5.23)	(0.99)	(1.70)	(–1.71)	(6.82)	(1.19)	(2.17)
Observations	38	37	29	38	37	36	28	37
R-squared	0.15	0.08	0.14	0.20	0.20	0.11	0.21	0.34

Table 2. **Credit market overheating and current and past defaults.** This table presents the results from estimating time-series regressions of the form

$$Overheating_t = a + b \cdot Def_t + c \cdot Def_{t-1} + \varepsilon_t,$$

where *Def* denotes the default rate on speculative grade bonds and *Overheating* is a proxy for credit market overheating. The data begin in 1983 and end in 2022. HYS_t is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's, as defined in Greenwood and Hanson (2013). $Credit\ Growth_t$ is the percentage change in outstanding corporate credit and is computed using Table L103 from the Flow of Funds. $Easy\ Credit_t$ is the three-year average of the percentage of bank loan officers reporting a loosening of commercial lending standards from the Federal Reserve's Senior Loan Office Opinion Survey. $-1 \times EBP_t$ is negative one times the excess bond premium from Gilchrist and Zakrajšek (2012). The *t*-statistics (in parentheses) are based on Newey-West (1987) standard errors, allowing for serial correlation up to 3 lags.

	Dependent variable:			
	(1)	(2)	(3)	(4)
	$\log(HYS_t)$	$Credit\ Growth_t$	$Easy\ Credit_t$	$-1 \times EBP_t$
Def_t	–0.119	–0.004	–3.278	–0.145
	(–2.43)	(–1.21)	(–12.24)	(–5.32)
Def_{t-1}	0.010	–0.008	–3.250	0.036
	(0.43)	(–4.73)	(–6.75)	(1.54)
Constant	–0.765	0.111	21.487	0.409
	(–3.28)	(9.95)	(7.29)	(3.20)
Observations	37	39	31	39
R-squared	0.395	0.392	0.818	0.546

Internet Appendix

A. Derivation of the bond price

As stated in the main text, the bond price evolves according to the following ordinary differential equation:

$$r \cdot p(\lambda) = c + \xi(1 - p(\lambda)) + a(\lambda)p'(\lambda) + \lambda(\eta - p(\lambda)), \tag{A.1}$$

where

$$a(\lambda) \equiv \lambda^2 - (\lambda_h + \lambda_l + 2q)\lambda + q(\lambda_h + \lambda_l) + \lambda_h \lambda_l. \tag{A.2}$$

We conjecture that the bond price is a linear function of λ: $p(\lambda) = m_1 + m_2 \cdot \lambda$. Substituting this conjecture into equation (A.1), we obtain

$$\begin{aligned} r(m_1 + m_2\lambda) = \quad & c + m_2(\lambda^2 - (\lambda_h + \lambda_l + 2q)\lambda + q(\lambda_h + \lambda_l) + \lambda_h\lambda_l) \\ & +\lambda(\eta - m_1 - m_2\lambda) + \xi(1 - m_1 - m_2\lambda). \end{aligned} \tag{A.3}$$

Note that on the right hand side of (A.3), the two λ^2 terms, $m_2\lambda^2$ and $-m_2\lambda^2$, cancel each other out. Then, both sides of the equation are linear function of λ. Matching terms gives the expressions of m_1 and m_2 in equation (9) of the main text.

B. Numerical procedure for solving the default extrapolation model

B.1. Change of variables

For numerical considerations, we first define

$$W(X,\lambda) \equiv K \cdot e(\frac{X}{K},\lambda), \qquad \text{so } e(v,\lambda) = K^{-1}W(vK,\lambda), \tag{B.1}$$

where K is a large scaling factor (a constant). We substitute (B.1) into (17) of the main text and obtain

$$r \cdot W(X,\lambda) = \begin{bmatrix} \mu X W_X + \frac{1}{2}\sigma^2 X^2 W_{XX} + a(\lambda) W_\lambda \\ +\delta(1-\tau)\cdot X - [(c(1-\tau)+\xi) - \xi p(\lambda)]K \\ +\frac{1}{2\psi K}(p(\lambda)K + W - XW_X)^2 \end{bmatrix}. \tag{B.2}$$

The two boundary conditions in (19) of the main text now become

$$W(X_b(\lambda),\lambda) = 0, \quad W_X(X_b(\lambda),\lambda) = 0. \tag{B.3}$$

Our focus is to numerically solve the differential equation (B.2) as well as the default boundary $X_b(\lambda)$.

To do so, we first consider the limiting case when $\lambda = \lambda_m$. In this case, the partial differential equation in (B.2) becomes an ordinary differential equation

$$r \cdot W = \begin{bmatrix} \mu X W' + \frac{1}{2}\sigma^2 X^2 W'' \\ +\delta(1-\tau)\cdot X - [(c(1-\tau)+\xi) - \xi p(\lambda_m)]K \\ +\frac{1}{2\psi K}(p(\lambda_m)K + W - XW')^2 \end{bmatrix}, \tag{B.4}$$

where $W(X,\lambda_m)$ can be viewed as a univariate function of X. As X goes to infinity, we conjecture that

$$W(X,\lambda_m) \to \frac{(1-\tau)\delta}{r-\mu}X + W_2(\lambda_m); \tag{B.5}$$

this conjecture is easy to verify.

Substituting (B.5) back into (B.4), we obtain

$$r \cdot W_2 = \left[-[(c(1-\tau)+\xi) - \xi p(\lambda_m)]K + \frac{1}{2\psi K}(p(\lambda_m)K + W_2)^2\right]. \tag{B.6}$$

The two solutions are

$$\begin{aligned} W_{2,\pm}(\lambda_m) &= -(p(\lambda_m) - r\psi)K \\ &\quad \pm K\sqrt{(p(\lambda_m) - r\psi)^2 + (2\psi c(1-\tau) + 2\xi\psi(1 - p(\lambda_m)) - p^2(\lambda_m))}. \end{aligned} \tag{B.7}$$

Note that $-W_2$ can be interpreted as debt value, so the negative root $W_{2,-}(\lambda_m)$ is the relevant one.

We now define the market timing component of equity value as

$$H(X; \lambda_m) \equiv W(X, \lambda_m) - \frac{\delta(1-\tau)}{r-\mu}X. \tag{B.8}$$

Substituting (B.8) back into (B.4) gives

$$r \cdot H = \begin{bmatrix} \mu X H' + \frac{1}{2}\sigma^2 X^2 H'' \\ -[(c(1-\tau) + \xi) - \xi p(\lambda_m)]K + \frac{1}{2\psi K}(p(\lambda_m)K + H - XH')^2 \end{bmatrix}. \tag{B.9}$$

Further substituting (B.8) into the two boundary conditions in (B.3) gives

$$H(X_b(\lambda_m); \lambda_m) = -\frac{\delta(1-\tau)}{r-\mu}X_b(\lambda_m), \quad H_X(X_b(\lambda_m); \lambda_m) = -\frac{\delta(1-\tau)}{r-\mu}. \tag{B.10}$$

These two conditions, together with the following condition

$$\lim_{X\to\infty} H(X; \lambda_m) = W_2(\lambda_m), \tag{B.11}$$

allows us to solve for H and $X_b(\lambda_m)$, the default boundary evaluated at $\lambda = \lambda_m$.

B.2. Solving $H(X)$

To facilitate subsequent numerical analyses, we take the following change of variables

$$x = \frac{X-\zeta}{X+\zeta} \tag{B.12}$$

and define $l(x) \equiv H(X(x))$ with $X = \zeta(1+x)/(1-x)$. Now, equation (B.9) becomes

$$rl = \begin{bmatrix} \frac{1}{2}\mu(1-x^2)l' + \frac{1}{2}\sigma^2\left(\zeta\frac{1+x}{1-x}\right)^2\left(l''\frac{(1-x)^4}{4\zeta^2} - l'\frac{(1-x)^3}{2\zeta^2}\right) \\ -[(c(1-\tau)+\xi) - \xi p(\lambda_m)]K + \frac{1}{2\psi K}[p(\lambda_m)K + l - \frac{1}{2}(1-x^2)l']^2 \end{bmatrix}. \tag{B.13}$$

The two boundary conditions in (B.10) become

$$l(x_b) = -\frac{\zeta\delta(1-\tau)}{r-\mu}\cdot\frac{1+x_b}{1-x_b}, \quad l'(x_b)\frac{(1-x_b)^2}{2\zeta} = -\frac{\delta(1-\tau)}{r-\mu}, \tag{B.14}$$

where $x_b \equiv (X_b(\lambda_m) - \zeta)/(X_b(\lambda_m) + \zeta)$ is the transformed value of the default boundary $X_b(\lambda_m)$. The limiting condition in (B.11) is

$$l(1) = W_2(\lambda_m). \tag{B.15}$$

Next, we approximate the function l by

$$\hat{l}(x) = \sum\nolimits_{r=0}^{n} b_r T_r(x), \tag{B.16}$$

where $\{b_r\}_{0 \leq r \leq n}$ represents $n+1$ coefficients and $T_r(x)$ represents the r^{th} degree Chebyshev polynomial of the first kind.[34] For a given value of x_b, equations (B.14) and (B.15) allow us to express $\{b_0, b_1, b_2\}$ each as a combination of the remaining $n-2$ coefficients. Then, equation (B.13) allows us to solve for the remaining coefficients as well as x_b through minimizing a mean squared error.

Define the optimal debt issuance as $g^*(x) \equiv g^*(v(x), \lambda_m)$. Then, from (18) of the main text, (B.1), (B.8), and (B.12), we obtain

$$g^*(x) = \xi + \frac{p(\lambda_m)K + l - \frac{1}{2}l'(1-x^2)}{\psi K}. \tag{B.17}$$

It can be approximated by

$$\hat{g}(x) = \xi + \frac{p(\lambda_m)K + \hat{l} - \frac{1}{2}\hat{l}'(1-x^2)}{\psi K}. \tag{B.18}$$

[Place Figure B.1 about here]

We illustrate the numerical procedure by showing an example. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. The scaling factor K is set to 10. The parameter ζ for the non-linear transformation in (B.12) is set to 25. The optimal default boundary is $X_b(\lambda_m) = 4.88$. We also know from (B.7) that $W_{2,-}(\lambda_m) = -12.63$. With this parameterization, Figure B.1 plots $\hat{l}$ and $\hat{g} - \xi$ against x, the transformed value of X.

B.3. Solving the two-state variable partial differential equation

We now solve the full model. Recall that the partial differential equation we are solving is

$$r \cdot W(X, \lambda) = \begin{bmatrix} \mu X W_X + \frac{1}{2}\sigma^2 X^2 W_{XX} + a(\lambda) W_\lambda \\ +\delta(1-\tau) \cdot X - [(c(1-\tau) + \xi) - \xi p(\lambda)]K \\ +\dfrac{1}{2\psi K}(p(\lambda)K + W - XW_X)^2 \end{bmatrix}. \tag{B.19}$$

[34]See Mason and Handscomb (2003) for a detailed discussion of the properties of Chebyshev polynomials.

Optimal debt issuance is

$$g^*(X,\lambda) = \xi + \frac{p(\lambda)K + W - XW_X}{\psi K}. \tag{B.20}$$

We define the "residual" equity value as

$$H(X,\lambda) \equiv W(X,\lambda) - \frac{\delta(1-\tau)}{r-\mu}X. \tag{B.21}$$

The evolution of H is governed by

$$rH = \left[\begin{array}{l} \mu X H_X + \frac{1}{2}\sigma^2 X^2 H_{XX} + a(\lambda)H_\lambda \\ -[(c(1-\tau)+\xi) - \xi p(\lambda)]K + \frac{1}{2\psi K}(p(\lambda)K + H - XH_X)^2 \end{array}\right]. \tag{B.22}$$

Optimal debt issuance becomes

$$g^*(X,\lambda) \equiv \frac{G}{F} = \xi + \frac{p(\lambda)K + H - XH_X}{\psi K}. \tag{B.23}$$

The boundary conditions are

$$H(X_b(\lambda),\lambda) = -\frac{\delta(1-\tau)}{r-\mu}X_b(\lambda), \quad H_X(X_b(\lambda),\lambda) = -\frac{\delta(1-\tau)}{r-\mu}, \tag{B.24}$$

for $\lambda_m \leq \lambda \leq \lambda_h$.

Numerically, we have solved one other boundary condition

$$H(X,\lambda_m) = \hat{l}(x(X)), \tag{B.25}$$

which, as we see below, will be helpful when solving (B.22).

We then have the following changes of variables

$$x = \frac{X-\zeta}{X+\zeta}, \; z = \xi_1\lambda + \xi_2, \qquad \text{where } \xi_1 = \frac{2}{\lambda_h - \lambda_m}, \; \xi_2 = -\frac{\lambda_h + \lambda_m}{\lambda_h - \lambda_m}. \tag{B.26}$$

Given (B.26), define $q(x,z) \equiv H(X(x),\lambda(z))$. The default boundary is $k(z) \equiv X_b(\lambda(z))$. Equation (B.22) becomes

$$rq(x,z) = \left[\begin{array}{l} \mu\frac{1}{2}(1-x^2)q_x + \frac{1}{2}\sigma^2\left(\zeta\frac{1+x}{1-x}\right)^2\left(q_{xx}\frac{(1-x)^4}{4\zeta^2} - q_x\frac{(1-x)^3}{2\zeta^2}\right) \\ +a(\lambda(z))\xi_1 q_z - [(c(1-\tau)+\xi) - \xi p(\lambda(z))]K \\ +\frac{1}{2\psi K}\left(p(\lambda(z))K + q(x,z) - \frac{1}{2}(1-x^2)q_x\right)^2 \end{array}\right]. \tag{B.27}$$

The optimal debt issuance from (B.23) becomes

$$g^*(x,z) = \xi + \frac{p(\lambda(z))K + q(x,z) - \frac{1}{2}(1-x^2)q_x}{\psi K}. \tag{B.28}$$

The two boundary conditions in (B.24) are now written as

$$q(\tfrac{k(z)-\zeta}{k(z)+\zeta}, z) = -\frac{\delta(1-\tau)}{r-\mu}k(z), \quad q_x(\tfrac{k(z)-\zeta}{k(z)+\zeta}, z) = -\frac{2\delta(1-\tau)\zeta}{(r-\mu)(1-\frac{k(z)-\zeta}{k(z)+\zeta})^2}. \tag{B.29}$$

Given (B.25), we approximate the residual equity value $q(x,z)$ by the following

$$\hat{q}(x,z) = \hat{l}(x) + (1+z)\sum_{0\le i+j\le m} a(i,j)T_i(z)T_j(x), \tag{B.30}$$

where $\{a(i,j)\}_{0\le i+j\le m}$ represents $(m+1)(m+2)/2$ coefficients. Note that

$$\begin{aligned} \hat{l}(1) &= W_{2,-}(\lambda_m) \\ &= -(p(\lambda_m) - r\psi)K \\ &\quad -K\sqrt{(p(\lambda_m) - r\psi)^2 + (2\psi c(1-\tau) + 2\xi\psi(1-p(\lambda_m)) - p^2(\lambda_m))}. \end{aligned} \tag{B.31}$$

We approximate the default boundary k by

$$\hat{k}(z) = \sum\nolimits_{r=0}^{n} d_r T_r(z). \tag{B.32}$$

When numerically solving (B.13), we have obtained a numerical value for $X_b(\lambda_m)$. As such,

$$\lim_{z\to -1}\hat{k}(z) = \sum\nolimits_{r=0}^{n} d_r \cdot (-1)^r = X_b(\lambda_m). \tag{B.33}$$

That is, we can write d_n as a linear function of $\{d_r\}_{r=0}^{n-1}$.

Within the boundary—that is, when $(k(z)-\psi)/(k(z)+\psi)$ and when $-1\le z\le 1$—the partial differential equation in (B.27) is satisfied. We consider M grid points between -1 and 1 for the variable z; these are the M zeros of $T_M(z)$ and we denote them as $\{z_i\}_{i=1}^{M}$. These M zeros are given by

$$z_i = \cos\left(\frac{(i-0.5)\pi}{M}\right), \quad 1\le i\le M. \tag{B.34}$$

We assume the same M grid points for the variable x; we denote them as $\{x_j\}_{j=1}^{M}$. For each z_i, the following set of x_j corresponds to firm fundamentals that are above the default boundary

$$1 \le j \le \bar{j}(i) \equiv \left\lfloor \frac{M}{\pi}\arccos\left(\frac{\hat{k}(z_i)-\psi}{\hat{k}(z_i)+\psi}\right) + 0.5\right\rfloor. \tag{B.35}$$

Finally, we choose coefficients $\{a(i,j)\}_{0\le i+j\le m}$ and coefficients $\{d_r\}_{r=0}^{n-1}$ to minimize the following weighted sum of squared errors

$$\sum_{j=1}^{\bar{j}(i)}\sum_{i=1}^{M} w(j,i)\left[\begin{array}{l} \mu\frac{1}{2}(1-x_j^2)\hat{q}_x+\frac{1}{2}\sigma^2\left(\zeta\frac{1+x_j}{1-x_j}\right)^2\left(\hat{q}_{xx}\frac{(1-x_j)^4}{4\zeta^2}-\hat{q}_x\frac{(1-x_j)^3}{2\zeta^2}\right)\\ +a(\lambda(z_i))\xi_1\hat{q}_z\\ -[(c(1-\tau)+\xi)-\xi p(\lambda(z_i))]K+\frac{1}{2\psi K}\left(p(\lambda(z_i))K+q(x_j,z_i)-\frac{1}{2}(1-x_j^2)\hat{q}_x\right)^2\\ -r\hat{q}(x_j,z_i) \end{array}\right]^2 \tag{B.36}$$
$$+K_1\cdot\sum_{i=1}^{M}\frac{1}{\sqrt{1-z_i^2}}\left[\hat{q}\left(\tfrac{\hat{k}(z_i)-\zeta}{\hat{k}(z_i)+\zeta},z_i\right)+\frac{\delta(1-\tau)}{r-\mu}\hat{k}(z_i)\right]^2$$
$$+K_2\cdot\sum_{i=1}^{M}\frac{1}{\sqrt{1-z_i^2}}\left[\hat{q}_x\left(\tfrac{\hat{k}(z_i)-\zeta}{\hat{k}(z_i)+\zeta},z_i\right)+\frac{2\zeta\delta(1-\tau)}{(r-\mu)(1-\frac{\hat{k}(z_i)-\zeta}{\hat{k}(z_i)+\zeta})^2}\right]^2,$$

where $w(j,i)=[(1-x_j^2)(1-z_i^2)]^{-1/2}$ and K_1 and K_2 are large positive coefficients.

In (B.36), the expressions of $\hat{q}_x$, $\hat{q}_{xx}$, and $\hat{q}_z$ are given by

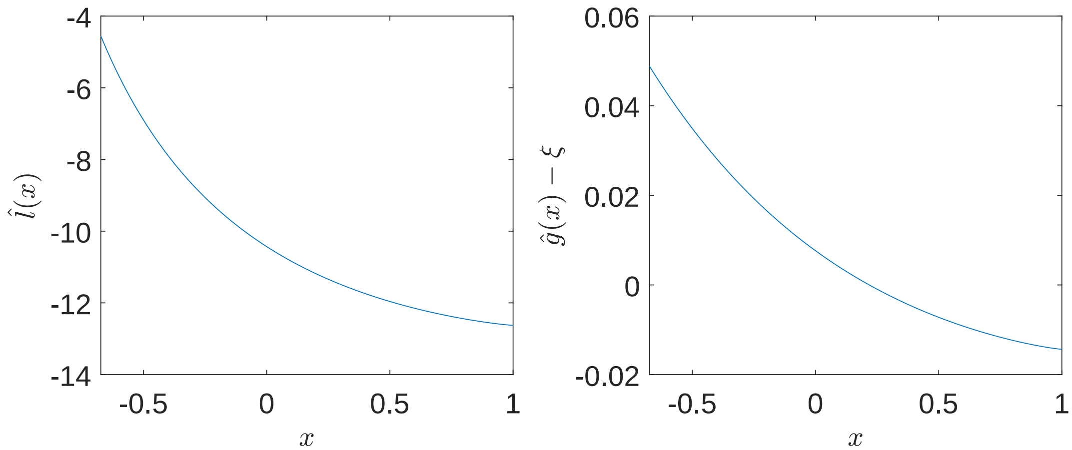
$$\begin{aligned}\hat{q}_x(x,z) &= \hat{l}_x(x)+(1+z)\sum_{0\le i+j\le m}a(i,j)T_i(z){T'}_j(x),\\ \hat{q}_{xx}(x,z) &= \hat{l}_{xx}(x)+(1+z)\sum_{0\le i+j\le m}a(i,j)T_i(z){T''}_j(x),\end{aligned} \tag{B.37}$$

and

$$\hat{q}_z(x,z)=\sum_{0\le i+j\le m}a(i,j){T'}_i(z)T_j(x)+(1+z)\sum_{0\le i+j\le m}a(i,j){T'}_i(z)T_j(x). \tag{B.38}$$

For the numerical results in the main text, we set $m=30$, $n=30$, $M=70$, $K_1=10^6$, $K_2=2,500$, $K=10$, and $\zeta=25$. We then apply the Levenberg-Marquardt algorithm. Recall that the parameter values are: $\lambda_l=0.005$, $\lambda_h=2$, $q=0.05$, $\eta=0.5$, $r=0.04$, $c=0.04$, $\xi=1$, $\tau=0.3$, $\psi=20$, $\mu=0.01$, $\delta=0.04$, and $\sigma=0.25$. For (B.36), we obtain a minimized sum of squared errors at 8.2×10^{-3}.

Figure B.1. Equity value $\hat{l}(x)$ and optimal net debt issuance $\hat{g}(x) - \xi$, each as a function of x. The parameter values are: $\lambda_l = 0.005$, $\lambda_h = 2$, $q = 0.05$, $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$.



C. Numerical procedure for solving the rational model

C.1. Reduction of state variables

Note that the V_t process in equation (3) exhibits constant stochastic returns to scale and that the adjustment cost in (11) is homogeneous of degree one in G_t and F_t. These assumptions imply that the equity value E is homogeneous of degree one in V_t and F_t, that the default boundary V_b is homogeneous of degree one in F_t, and that the bond price p is homogeneous of degree zero in V_t and F_t. Without loss of generality, we write

$$E(V,F) = e(\frac{V}{F})F, \quad V_b(F) = v_b F, \quad p(V,F) = p(\frac{V}{F}), \tag{C.1}$$

as in equation (36) of the main text.

Substituting (C.1) back into (30) and (34) gives

$$r \cdot e = \left[\begin{array}{l} \mu v e' + \frac{1}{2}\sigma^2 v^2 e'' \\ +\delta(1-\tau)\cdot v - (c(1-\tau)+\xi) + \xi p + \dfrac{1}{2\psi}(p+e-ve')^2 \end{array} \right] \tag{C.2}$$

and

$$r \cdot p = c + \xi(1-p) + (\mu + \xi - g^*)vp' + \frac{1}{2}\sigma^2 v^2 p''. \tag{C.3}$$

In (C.3), the optimal debt issuance is

$$g^*(v) \equiv \frac{G}{F} = \xi + \frac{p(v) + e - ve'}{\psi}. \tag{C.4}$$

The reduced boundary conditions are

$$e(v_b) = 0, \quad e'(v_b) = 0, \quad p(v_b) = \eta, \quad \lim_{v\to\infty} p(v) = \frac{c+\xi}{r+\xi}. \tag{C.5}$$

As v goes to infinity, we exclude any bubble component from $e(v)$—we set $e''(v)$ in (C.2) to zero—and we know from (C.5) that $p(v)$ goes to $(c+\xi)/(r+\xi)$. In this case, (C.2) implies

$$\lim_{v\to\infty} e(v) = A + \frac{\delta(1-\tau)}{r-\mu} v, \tag{C.6}$$

where

$$A = -(p-\psi r) - \sqrt{(p-\psi r)^2 - 2\left(\psi(\xi(p-1) - c(1-\tau)) + \tfrac{1}{2}p^2\right)} \tag{C.7}$$

and $p = (c+\xi)/(r+\xi)$. Equation (C.6) serves as another boundary condition.

We further define the "residual" equity value

$$h(v) \equiv e(v) - \frac{\delta(1-\tau)}{r-\mu} v, \tag{C.8}$$

which is the difference between the equity value and the present value of the firm's unlevered cash flows. From (C.6) we know that

$$\lim_{v \to \infty} h(v) = A. \tag{C.9}$$

Equations (C.2) to (C.6) characterize a system of two ordinary differential equations, with a single state variable v. We solve equations (C.2) to (C.6) and the value of v_b using numerical methods, which we elaborate next.

C.2. Change of variables

For numerical considerations, we first define

$$W(X) \equiv K \cdot e(\frac{X}{K}), \qquad \text{so } e(v) = K^{-1} W(vK), \tag{C.10}$$

where K is a large scaling factor (a constant). We substitute (C.10) into (C.2) and obtain

$$r \cdot W = \left[\begin{array}{l} \mu X W' + \frac{1}{2}\sigma^2 X^2 W'' \\ +\delta(1-\tau) \cdot X - [(c(1-\tau)+\xi) - \xi \cdot p]K \\ +\dfrac{1}{2\psi K}(p \cdot K + W - XW')^2 \end{array} \right]. \tag{C.11}$$

The two boundary conditions regarding $e(v)$ in (C.5) now become

$$W(X_b) = 0, \quad W_X(X_b) = 0. \tag{C.12}$$

We then define

$$P(X) \equiv p(\frac{X}{K}), \quad \text{so } p(v) = P(vK). \tag{C.13}$$

We substitute (C.11) into (C.3) and obtain

$$r \cdot P = c + \xi(1-P) + (\mu + \xi - g^*) X P' + \frac{1}{2}\sigma^2 X^2 P''. \tag{C.14}$$

The two boundary conditions regarding $p(v)$ in (C.5) now become

$$P(X_b) = \eta, \quad \lim_{X \to \infty} P(X) = \frac{c+\xi}{r+\xi}. \tag{C.15}$$

Our focus is to numerically solve the differential equations (C.11) and (C.14) as well as the default boundary X_b.

To do so, we now define the market timing component of equity value as

$$H(X) \equiv W(X) - \frac{\delta(1-\tau)}{r-\mu}X. \tag{C.16}$$

Substituting (C.16) back into (C.11) gives

$$r \cdot H = \left[\begin{array}{l} \mu X H' + \frac{1}{2}\sigma^2 X^2 H'' \\ -[(c(1-\tau)+\xi) - \xi \cdot P]K + \frac{1}{2\psi K}(P \cdot K + H - XH')^2 \end{array} \right]. \tag{C.17}$$

Further substituting (C.16) into the two boundary conditions in (C.12) gives

$$H(X_b) = -\frac{\delta(1-\tau)}{r-\mu}X_b, \quad H'(X_b) = -\frac{\delta(1-\tau)}{r-\mu}. \tag{C.18}$$

Moreover, from (C.6), we know that

$$\lim_{X\to\infty} H(X) = K \cdot A, \tag{C.19}$$

where coefficient A is given by (C.7).

C.3. Solving $H(X)$ and $P(X)$

To facilitate subsequent numerical analyses, we take the following change of variable

$$x = \frac{X-\zeta}{X+\zeta} \tag{C.20}$$

and define $l(x) \equiv H(X(x))$ and $f(x) \equiv P(X(x))$ with $X = \zeta(1+x)/(1-x)$. Now, equation (C.17) becomes

$$rl = \left[\begin{array}{l} \frac{1}{2}\mu(1-x^2)l' + \frac{1}{2}\sigma^2\left(\zeta\frac{1+x}{1-x}\right)^2\left(l''\frac{(1-x)^4}{4\zeta^2} - l'\frac{(1-x)^3}{2\zeta^2}\right) \\ -[(c(1-\tau)+\xi) - \xi \cdot f]K + \frac{1}{2\psi K}[f \cdot K + l - \frac{1}{2}(1-x^2)l']^2 \end{array} \right], \tag{C.21}$$

with the following optimal debt issuance

$$g^*(x) = \xi + \frac{f \cdot K + l - \frac{1}{2}(1-x^2)l'}{\psi K}. \tag{C.22}$$

The two boundary conditions in (C.18) become

$$l(x_b) = -\frac{\zeta\delta(1-\tau)}{r-\mu}\frac{1+x_b}{1-x_b}, \quad l'(x_b)\frac{(1-x_b)^2}{2\zeta} = -\frac{\delta(1-\tau)}{r-\mu}. \tag{C.23}$$

The boundary condition in (C.19) becomes

$$l(1) = K \cdot A. \tag{C.24}$$

Equation (C.14) becomes

$$\begin{aligned} r \cdot f = \quad & c + \xi(1-f) + (\mu + \xi - g^*)\tfrac{1}{2}(1-x^2)f' \\ & + \tfrac{1}{2}\sigma^2 \left(\zeta \frac{1+x}{1-x}\right)^2 \left(f'' \frac{(1-x)^4}{4\zeta^2} - f' \frac{(1-x)^3}{2\zeta^2}\right). \end{aligned} \tag{C.25}$$

The two boundary conditions in (C.15) become

$$f(x_b) = \eta, \quad f(1) = \frac{c+\xi}{r+\xi}. \tag{C.26}$$

We then approximate the two functions l and f by

$$\hat{l}(x) = \sum\nolimits_{r=0}^{n} b_r T_r(x), \qquad \hat{f}(x) = \sum\nolimits_{r=0}^{n} c_r T_r(x), \tag{C.27}$$

where $\{b_r\}_{0 \le r \le n}$ and $\{c_r\}_{0 \le r \le n}$ each represents $n+1$ coefficients, and $T_r(x)$ represents the r^{th} degree Chebyshev polynomial of the first kind. Define $x_b \equiv (X_b - \zeta)/(X_b + \zeta)$ as the transformed value of the default boundary X_b. For a given value of x_b, equations (C.23) and (C.24) allow us to express $\{b_0, b_1, b_2\}$ each as a combination of the remaining $n-2$ coefficients. Similarly, equation (C.26) allows us to express $\{c_0, c_1\}$ each as a combination of the remaining $n-1$ coefficients. We also search for x_b. Together, we have a total of $2n-2$ unknown coefficients.

Note that the optimal debt issuance in (C.22) is approximated by

$$\hat{g}(x) = \xi + \frac{\hat{f} \cdot K + \hat{l} - \frac{1}{2}\hat{l}'(1-x^2)}{\psi K}. \tag{C.28}$$

Finally, we choose coefficients $\{b_r\}_{0 \le r \le n}$, coefficients $\{c_r\}_{0 \le r \le n}$, as well as the default boundary x_b to minimize the following weighted sum of squared errors

$$\begin{aligned} & \sum\nolimits_{j=1}^{M} w(j) \mathbf{1}_{x_j \ge x_b} \left[\begin{array}{l} \frac{1}{2}\mu(1-x_j^2)\hat{l}' + \frac{1}{2}\sigma^2 \left(\zeta \frac{1+x_j}{1-x_j}\right)^2 \left(\hat{l}'' \frac{(1-x_j)^4}{4\zeta^2} - \hat{l}' \frac{(1-x_j)^3}{2\zeta^2}\right) \\ -[(c(1-\tau)+\xi) - \xi \cdot \hat{f}]K + \frac{1}{2\psi K}\left(\hat{f} \cdot K + \hat{l} - \frac{1}{2}(1-x_j^2)\hat{l}'\right)^2 - r\hat{l} \end{array} \right]^2 \\ & + K_1 \cdot \sum\nolimits_{j=1}^{M} w(j) \mathbf{1}_{x_j \ge x_b} \left[\begin{array}{l} c + \xi(1-\hat{f}) + (\mu + \xi - g^*)\frac{1}{2}(1-x_j^2)\hat{f}' \\ + \frac{1}{2}\sigma^2 \left(\zeta \frac{1+x_j}{1-x_j}\right)^2 \left(\hat{f}'' \frac{(1-x_j)^4}{4\zeta^2} - \hat{f}' \frac{(1-x_j)^3}{2\zeta^2}\right) - r\hat{f} \end{array} \right]^2, \end{aligned} \tag{C.29}$$

where $w(j) = (1-x_j^2)^{-1/2}$, and K_1 is a positive scaling coefficient. The indicator functions in (C.29) mean that the system of ordinary differential equations only hold in the region where v is above the default boundary.

For the numerical results in the main text, we set $n = 30$, $M = 300$, $\zeta = 25$, $K_1 = 4$, $K = 10$, and $\zeta = 25$. We then apply the Levenberg-Marquardt algorithm. The parameter values are: $\eta = 0.5$, $r = 0.04$, $c = 0.04$, $\xi = 1$, $\tau = 0.3$, $\psi = 20$, $\mu = 0.01$, $\delta = 0.04$, and $\sigma = 0.25$. For (C.29), we obtain a minimized sum of squared errors at 1.47×10^{-4}. The default boundary is solved to be $v_b = 1.46$.