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COALITION FORMATION IN LEGISLATIVE BARGAINING

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ABSTRACT

We propose a new model of legislative bargaining in which coalitions have different values, reflecting the fact that the policies they can pursue are constrained by the identity of the coalition members. In the model, a formateur picks a coalition and negotiates for the allocation of the surplus it is expected to generate. The formateur is free to change coalitions to seek better deals with other coalitions, but she may lose her status if bargaining breaks down, in which case a new formateur is chosen. We show that as the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of a Nash Bargaining Solution in which–in contrast to the standard solution–the coalition is endogenous and determined by the relative coalitional values. A form of the hold-up problem specific to these bargaining games may lead to significant inefficiencies in the selection of the equilibrium coalition. We use the equilibrium characterization of the distortions to study the role of the head of state in avoiding (or containing) distortions. We also show that the model helps rationalizing well known empirical facts that are in conflict with the predictions of standard non-cooperative models of bargaining: the absence of significant (or even positive) premia in ministerial allocations for formateurs and their parties; the occurrence of supermajorities; and delays in reaching agreements.

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An online appendix is available at http://www.nber.org/data-appendix/w25664

1 Introduction

In most parliamentary democracies, public policies are not decided in elections, but are instead the outcome of elaborate bargaining processes in the Parliament. Elections often do not even determine the identity of the governing coalition, which may indeed be difficult to predict on the basis of the electoral outcome alone. After the 2017 German election, a grand coalition of the Christian Democrats (CDU/CSU) with the Social Democrats (SPD) was formed only after a failed attempt to form a coalition with the CDU, the Free Democratic Party (FDP) and the Greens; in Italy, the 5 Star movement contemplated the formation of a coalition on the left with the Democratic Party (DP), before converging to the Northern League on the right.[1] Predicting the outcome of legislative bargaining is generally hard because it is not just about dividing surplus within some minimal winning coalition. On the one hand, for the CDU/CSU forming a government with the SPD is more than just the number of ministers that need to be conceded to the SPD compared to the FDP or the Greens: it is also about what can be achieved in each coalition. On the other hand, while there is often a coalition that is more natural than others, the identities of the coalition partners is not exclusively determined by some ex ante measure of policy fitness. Understanding how these two often conflicting goals interact is clearly central in understanding how parliamentary democracies work.

In this paper we present a theory of legislative bargaining in which formateurs need to reconcile the need to form the most productive coalition with the desire to maximize the share of output that they capture. The key assumption underlying our analysis is that coalitions are heterogeneous in terms of the surplus that they are expected to generate for the legislators. Are there general lessons to learn on which types of coalitions will form? Will the coalition generating the highest surplus emerge in equilibrium? If this is not the case, will at least the bargaining process avoid that the worst coalition emerge? We propose a theoretical framework for bargaining that attempts to address these questions. Besides providing a new perspective on an old problem, our model may help explaining empirical evidence that has been seen in conflict with standard non cooperative models of legislative bargaining. It also helps studying the role of the head of state, a figure generally ignored in formal models of bargaining, but often playing an important role.

In our model, bargaining starts with the appointment of a formateur in charge of selecting a majority and allocating within its members the surplus it is expected to generate (for example by selecting ministerial appointments). Coalitions are heterogeneous because their feasible policy space, and thus the surplus that they generate, depends on their members. Each possible coalition C generates a value $V(C)$ to be distributed; once a coalition is selected the formateur negotiates

[1] For more detailed discussions of the feasible coalitions contemplated in Germany and Italy after the 2018 elections see Brauninger [2019] for Germany and Mailer [2018] and Valbruzzi [2018] for Italy.

with their members on how to allocate it with a process of alternating proposals. A key feature of this process is that, even after the start of internal negotiations, the formateur is not bound to a coalition, s/he can always turn to a different coalition if optimal. This capture the specific role played by a formateur: s/he is not just bargaining on an allocation within a coalition, but primarily in *search* of a coalition, thus free to halt negotiations with a stubborn coalitional partner and turn to another. We assume that protracting negotiation is costly because at every round there is a probability of bargaining breakdown as in Binmore, Rubinstein and Wolinsky [1986]. A bargaining breakdown leads to either a new election or to the nomination of a new formateur (perhaps after a new election). If a new formateur is selected, the process restart with the new formateur. Bargaining ends when a coalition reaches an agreement or (if it is a possibility) there is a bargaining breakdown that leads to a new election with exogenous status quo. The equilibrium of this game naturally depends on the order of formateurs. The goal is to generate general lessons that hold for all equilibria and all possible orders when the order is exogenous, and then to endogenize the order of formateurs by explicitly modeling the role of the head of state.

We first characterize the equilibrium of the bargaining game between formateur and the coalitions assuming exogenous continuation values in case of breakdown, as traditionally assumed in the literature.[2] We show that bargaining leads to a unique stationary equilibrium in which an inefficient coalition is generally selected. The equilibrium choice of coalition depends on the probability of bargaining breakdowns, the size of the coalition and its associated surplus. The inefficiency is due to a form of the *hold-up problem* that limits the formateur's bargaining power, due to the fact that s/he can not credibly commit to switch to any other coalition. The threat of being held up does not lead to underinvestment as in the standard hold-up problem, but to an inefficient choice of coalition (in terms of net total surplus). Remarkably, as the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of the Nash bargaining solution in which each coalition member receives its outside option plus an equal share of surplus net of reservation utilities.[3] This is the same allocation as in the classic Nash bargaining solution: the difference is that in the n-person Nash bargaining solution the coalition is assumed to be comprised by all players (or some other exogenous coalition), while in our model it is *endogenously* (and inefficiently) determined.

To understand the type of coalitions that may emerge in equilibrium (minimal winning or

[2] See for instance Osborne and Rubinstein [1990, ch.4] for a survey of models with the possibility of bargaining breakdown.

[3] As we will show, the exact weights in the Nash baragining solution may depend on the exact internal bargaining protocol. When bargaining within a coalition is done with random recognitions as in Baron and Ferejohn [1989], the equilibrium allocation coincides with a generalization of the weighted Nash Bargaining solution with weights that coincide with the recognition probabilities. In this case too the selected coalition is endogenous and generally inefficient (with the inefficiency depending on the recognition probabilities).

super majorities, for example) and how surplus is allocated within them, we fully endogenize the reservation utilities by assuming that a bargaining breakdown by some formateur is followed by an attempt by some other formateur. We show that equilibria of this fully recursive version of the game are very different from the equilibria emerging in the existing non cooperative models a' la Baron and Ferejohn [1989]. First of all, multiple stationary equilibria with different welfare properties and different payoff allocations typically exist. This is in conflict with the finding of Baron and Ferejohn [1989], where multiple stationary equilibria are possible but they all lead to a unique value for the players. Multiplicity reflects the complexity of the strategic interaction in the model in which both the identity of the coalition and the allocation are part of the outcome. Secondly, however, we show that multiplicity does not lead to indeterminate behavior, but to a characterization that is tight enough for welfare and positive analysis. This allows us to study the condition under which an efficient equilibrium is feasible. We show that, depending on the parameters, inefficient equilibria can coexist with efficient equilibria or be the unique outcome. Under some conditions the inefficiency can be so bad that the least efficient coalition is chosen in equilibrium: such inefficient equilibria always exist if the value of the feasible coalitions are sufficiently similar.

Three specific positive predictions of the model stand out because they help explaining a number of classic empirical shortcomings of standard non-cooperative models of bargaining. The first has to do with the size of the share of the pie captured by the formateur (the so called "formateur's premium"). Existing models of legislative bargaining predict a very large formateur's premium in terms of captured surplus (i.e. ministerial cabinets), in the order of three quarters of surplus.[4] Such large formateur premia are however not observed in the data, where indeed there is even evidence of negative formateurs's premia relative to their size. These findings have been interpreted as major failures of noncooperative models (Laver [1998]).[5] Our model provides a new natural explanation of why the formateur may be willing to accept no premium (or even a negative one) that does not depend on specific assumption on the order of formateurs or other details. In our model, while the formateur "calls the shots" by choosing the coalition and is always free to switch to a different coalition if the equilibrium demands are too high, s/he is constrained in doing this by the "credibility" of the threat to switch to an inferior coalition. This significantly limits

[4] See, for instance, Morelli [1999] for a discussion of the empirical implications on the formateur's premium of standard models of non-cooperative multilateral bargaining.

[5] The first paper to study surplus allocation in legislative bargaining and to note the lack of a formateur advantage is Browne and Franklin [1973]. They highlighted the presence of a "Relative Weakness Effect" according to which the largest party (who more often than not expresses the formateur) is underrepresented relative to its electoral size. More recent work is Laver and Schofield [1985], Warwick and Druckman [2001, 2006] among others. Warwick and Druckman [2006] explicitly show that being a formateur has a negative impact on the marginal effect of size on the appropriated surplus.

the formateur's ability to extract rents from the other parties. As we will discuss, an important lesson from this analysis is that the real benefit of being a formateur is not in the *share* s/he appropriates within a coalition, but in the *choice* of coalition.

The second prediction has to do the size of coalitions. One of the earliest and most robust theoretical prediction of formal models of legislative bargaining is the emergence of strict minimal winning coalitions.[6] It is however the case that since World War II European parliamentary democracies have formed more supermajorities than minimal winning coalitions.[7] Explaining supermajorities by assuming that they are inherently vastly superior (in terms of total surplus) would clearly provide a quick explanation to the phenomenon. Such an explanation may even be plausible in some environments, but it would certainly not contribute to explain their diffusion in the relative calmness of post World War II Europe. With our model we show that supermajorities can emerge as equilibrium phenomena even if they are only *marginally* superior than minimal winning coalitions. This is surprising because for any coalition a party needs unanimity of participating parties and bargaining leads to a classic *hold-up problem* in which all coalition members capture a fixed share of net surplus over their reservation values. We show conditions under which the hold-up problem is attenuated in equilibrium by endogenous "self regulating" reservation utilities.

The third prediction concerns whether legislative bargaining is characterized by an immediate agreement, a prediction that characterizes most noncooperative model but that is also not supported in the data.[8] There are of course many factor that seem important in explaining this phenomenon, such as incomplete and asymmetric information about the payoffs and behavioral biases. Our theory shows that even without these additional factors we can have equilibria with delay. Interestingly, the phenomenon is exquisitely strategic depending on equilibrium expectations: indeed under some conditions, equilibria with delay coexist with equilibria with no delay.

We use our model to study the role of the head of state in legislative bargaining. Many parliamentary democracies (including Austria, Belgium, Italy, Poland and others) empower their heads of state with significant discretion in the government formation process, in large part through the

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[6] The natural emergence of minimal winning coalitions in formal models of bargaining was first noted by Riker [1962] in the context of a general cooperative model. Norman [2002] shows that proposer models a' la Baron and Ferejohn [1989] cannot explain supra-minimal coalitions for a generic choice of preferences.

[7] Looking at parliamentary democracies in post Ward War II Europe, Druckmand and Thies (2002) note that there have been over 80 cases of supermajorities and only 74 minimum winning coalitions over the postwar period. See also Laver and Schoffield (1998) and Volden and Carrubba [2004].

[8] Using data from European democracies in the post World War II period, Golder [2008] reports that post-election government take about a month to form, though there is considerable differences in length of negotiations depending on the countries: it takes less than a week on average in the United Kingdom, France, Norway and Sweden, for instance. See also Diermeier and van Roozendaal [1998] on this issue.

choice of formateurs. Can the head of state influence the selection of the equilibrium coalition? In practice, we do observe that heads of states's role differ in quite significant ways even with the same country over time or between countries with similar constitutional rules.[9] What explains these changes in the role of the head of state? We show that the role of the head of state critically depends on the relative values of the coalitions: changes in these values changes its role independently of the constitutional context. When the relative values of coalitions are sufficiently large, the head of state is irrelevant: legislative bargaining will achieve the same equilibrium irrespective of the choice of formateur; a "wrong " choice of formateur would be followed by a failed attempt and delayed agreement until the "right" formateur is selected. Interestingly, the conditions do not only depend on how good the best coalition is, but also in a complementary way by how bad the worst coalition is too. When relative coalitional values are small, the president has partial control: s/he can make sure that the worst equilibrium is not selected, but s/he can not induce the most efficient equilibrium. The head of state is at the peak of his/her influence for intermediate levels of relative coalitional values: in this case it can induce a welfare maximizing coalition by selecting the appropriate order of formateurs. This dependence of the role of heads of state on the relative values of coalitions can explain why their role may change over time or similar countries.

The organization of the remainder of the paper is as follows. We discuss related literature in the next subsection. Section 2 outlines the model. Section 3 presents the characterization of the equilibrium with exogenous outside options, highlighting the connection with the Nash Bargaining Solution. The characterization in Section 3 is used in Section 4 to endogenize the outside options in a fully recursive model in which a bargaining breakdown is followed by the appointment of a new formateur. Section 5 presents the positive analysis of the model: the size of the formateur's premium, the emergence of super majorities, and strategic delays. Section 7 extends the normative analysis by studying the role of the head of state. Section 8 presents three extensions of the basic analysis. Section 9 concludes.

1.1 Related literature

In the literature on legislative bargaining and its many applications, the standard model is provided by Baron and Ferejohn [1989], who have proposed an elegant and practical extension of Rubinstein's model of bilateral bargaining to multilateral settings. Our work differs from Baron and Ferejohn [1989] in two main interrelated ways. First, Baron and Ferejohn is a purely distributive model in which n players allocate a fixed pie of seize one. We move beyond this framework by allowing coalitions to be heterogeneous in terms of the surplus they can generate. Second, we provide a different formalization of the bargaining protocol that, besides seeming in line with

[9] See for example Vassallo [1994], Cacace [2008] and Poguntke T. and P. Webb [2005].

legislative practice, highlights aspects of the strategic interaction that have not yet been explored in the literature. In Baron and Ferejohn [1989] the selection of the coalition and the allocation of the rents are collapsed in one step: a take it or leave it offer made by a proposer; if the offer is not accepted, a new randomly selected proposer is selected. We explicitly model the negotiation within the coalition and we allow the formateur to switch coalitions in the midst of negotiations. This has important theoretical implications. In a world in which a take it or leave it offer is followed by the random selection of another formateur, reservation utilities depend on the recognition probabilities and the associated random coalitions, but they do not allow the formateur to react to a no agreement decision. When negotiations are allowed to continue, reservation utilities depends on whether a threat to switch coalition is credible and whether negotiations are expected to lead to an agreement or not. At every step, the formateur is faced with a dilemma to keep negotiating with a coalition or to switch to some other coalition, perhaps less "ideal" in terms of surplus. Understanding the connection between these two elements, the heterogeneity of coalitions and the formateur's choice of coalitions, is key to understanding the logic behind our model and our results.

In modeling legislative bargaining with formateurs we build on two seminal contributions by Austen-Smith and Banks [1988] and Diermeier and Baron [2001], who consider models of bargaining with a formateur in spatial environments in which players have different ideal points.[10]

As in our model, in these papers there is a formateur that selects a coalition and who then negotiates with its members. Bargaining within the formateur's coalition, however, is limited to a take-it-or-leave it offer before the formateur looses power or a status quo is selected.[11] Because of this, the formateur cannot decide to switch coalition once bargaining has started, as discussed above. This leads to very different predictions. For instance, these models allow the formateur to capture a large share of surplus. This ability to capture surplus depends on the take-it-or-leave it nature of the formateur's offer and indeed play an important role in the selection of the coalition as we discuss below. The focus of these papers is in having tractable models of bargaining in which the parties' ideal points or their size in congress can be endogeneized by modeling elections before the bargaining takes place.

[10] Seidmann et al. [2007] and Bassi [2013] present alternative but related models of legislative bargaining with a formateur. Baron et al. [2012] present a repeated version of Baron and Diermeier [2001] in which the status quo may change over time. Important models of legislative bargaining in a spatial model but following Baron and Ferejohn's legislative protocol are also presented by Baron [1991, 1993] and Banks and Duggan [2000], among others.

[11] This is similar but different than in Baron and Ferejohn [1989] because in these models it is assumed that once the coalition is selected by the formateur, the offer must be approved by unanimity in the coalition. If the formateur selects a supermajority, then unanimity of the supermajority is required (in Baron and Ferejohn [1989] approval by any subset that is a minimal winning coalition is enough). The identity and number of parties that are included in the coalition is therefore important.

To develop our theory with heterogeneous coalitions and endogeneous coalition selection, we also build on an idea first presented in Osborne and Rubinstein [1990]. They consider a model with 3 players, one seller and 2 potential buyers with different valuations for the good sold by the seller.[12] In their model, the only feasible coalitions consist in the seller and one of the buyers. As in our model, the seller can switch partners after an offer is rejected and before making a new offer. Their game has a unique equilibrium in which the efficient allocation is always reached (i.e. the good is sold to the buyer with the highest valuation). We instead attempt to model a setting of multilateral bargaining with n players and more general coalition structures. While it is natural in Osborne and Rubinstein [1990] to assume that the player who can choose the partner is constant and exogenously given (in their model the seller), it is natural in our model to allow the formateur's identity to change. The key differences with their work, therefore, are that we allow for more general coalitional structure, and for the formateur to be replaced by another formateur if s/he fails to form a coalition. These differences also have important implications for the strategic analysis. For example, while Osborne and Rubinstein's [1990] model has a unique equilibrium in which the efficient coalition always form, in our model the equilibrium is generically inefficient if reservation utilities are exogenous and with endogenous reservation utilities, multiple equilibria are possible.[13]

As we noted, our model contributes explaining some empirical "anomalies" from the standard model whose study has characterized the empirical literature on legislative bargaining. A number of important works have attempted to provide theories to explain them. In the context of a purely distributive model, Morelli [1999] has provided a demand theory of legislative bargaining that can explain why the proposer does not receive large premia.[14] Baron and Diermeier [2000], Seidmann et al. [2007] and Baron [2019] has provided bargaining theories with a formateur in which supermajorities can emerge in equilibrium.[15] Acharia and Ortner [2013] and Ortner [2012] have recently provided models with strategic delay with two parties and unanimous voting rule. While all these papers contribute to understand different specific aspects of strategic bargaining, an advantage of our theory is that it provides a coherent and unified potential explanation of all

[12] See Osborne and Rubinstein [1990, ch. 9.4].

[13] The problem of studying bargaining with three players and heterogeneous coalitional values ("the three-player/three-cakes eating problem") was previously introduced by Binmore [1985], who focused on an axiomatic solution. See also Bennett and Houba [1992] and Bennett [1997]. Binmore also describes a non-cooperative approach, but he solves it only in a special case.

[14] In Morelli's [1999] demand bargaining model, parties sequentially make demands for themselves rather than offers to others. If the demands are not consistent with the feasible set, a status quo policy is selected. See also Morelli [1999] and Montero and Vidal-Puga [2011].

[15] Supermajorities can also be explained in the Baron and Ferejohn [1989] model under the assumption of deliberations under an "open rule." Even in this case however, the size of the coalition converges to the size of a minimal winning coalitions as the number of legislators is sufficiently large.

these empirical "anomalies." Finally, there is a significant literature studying the role of the head of state in parliamentary democracies, but for the most part it is empirical.[16]

2 Model

We consider a model in which n parties bargain over the formation of a government. The set of parties is denoted $N = \{1, ..., n\}$. A government is formed if a qualified majority approves it. The set of qualified majorities is denoted $\mathcal{C}$. We say that a coalition C is a minimal winning coalition if $C \in \mathcal{C}$ and for any $C' \subset C$, then $C' \notin \mathcal{C}$. The set of minimal winning coalitions is denoted $\mathcal{M}$, its complement in $\mathcal{C}$, $\mathcal{S}$, is the set of supermajorities. The set of qualified majorities and minimal winning coalitions to which party i belongs are denoted, respectively, $\mathcal{C}_i$ and $\mathcal{M}_i$.

A government is defined by the supporting winning coalition and an internal allocation of the surplus generated by the government. Coalitions are not necessarily equivalent in terms of generated surplus. We assume that the surplus generated by a coalition C is $V(C) \geq 0$ bounded above by a finite $\overline{V} = \max_C V(C)$ and below by nonnegative $\underline{V} = \min_C V(C)$. We assume that the members of the coalition can share the surplus in any way the want. A feasible allocation in C is $\mathbf{x} \in X(C)$, where:

$$X(C) := \left\{ \mathbf{x} \in R^{n(C)} \middle| x_i \geq 0,\ \sum\nolimits_{i \in C} x_i \leq V(C) \right\}.$$

where $n(C)$ is the number of parties in C. Parties evaluate the governments according to the surplus they receive, so $\{C, \mathbf{x}\} \succeq_i \{C', \mathbf{x}'\}$ if and only if $x_i \geq x_i'$.

In the baseline model bargaining is as follows. We assume that there is a set $\mathcal{T} = \{0, ..., T\} \subseteq N$ of potential formateurs who are ordered by a priority list. This list may depends on the result of the election: indeed, typically parties are recognized as formateurs in order of their electoral size. Since we do not model the electoral stage, we take this order as exogenous. We will endogenize the order of formateurs in Section 6, where we study the role of the head of state in selecting them.

A time $t = 0$ the first formateur f^0 is recognized and makes a proposal $\mathbf{x} \in X(C)$ to a coalition $C \in \mathcal{C}$ with $f^0 \in C$. If the proposal is accepted by all members of C, the game stops and the government is $\{C, \mathbf{x}\}$. If the proposal is not unanimously accepted by the parties in C, then bargaining within the coalition continues. At period $t+\Delta$ a different member i of C is recognized to make an offer to the coalition: again the proposal is accepted by all members of C, the game stops;

[16] For empirical studies, see for instance Amorim Neto and Strom [2008], Tavits [2009], Cox and Carroll [2011], Akirav and Cox [2018]. Akirav and Cox [2018] show that the formateur's bonus is positively correlated to the power of the monarch in European constitutional monarchies in the 20th century. They rationalize their empirical evidence with a model that incorporates the role of the head of state in the Baron and Ferejohn's model by allowing the monarch to choose the proposer with some probability p.

otherwise the process continues. Parties in C are ordered according to some permutation $\iota(k, C)$, so the proposer following at the kth stage for $k \leq n(C) - 1$ is identified as $\iota(k, C)$. The formateur is always the first proposer ($\iota(1, C) = f$) and the order is periodic ($\iota(n(C) + i, C) = \iota(i, C)$), so after all members of the coalition have a chance to make a proposal, proposal power returns to the formateur.[17] Every time that the formateur is recognized again, s/he can continue with the same coalition as in the previous rounds or move to a different coalition C'. This reflects the fact that the formateur is not bound to a specific coalition and thus can strategically choose to change "partners."

At any point in time after which a proposal is rejected and before a new proposal is made we have a negotiation breakdown with probability $p = 1 - e^{-r\Delta}$. In case the negotiation is interrupted, formateur f loses the status of formateur and a new formateur in $\mathcal{T}$ is appointed. If bargaining breaks down at stage $\tau < T$, formateur $f^{\tau+1}$ is selected according to an exogenous acyclical rule $f^{\tau+1} = \Psi(f^{\tau})$.

In Section 3 we first assume that if the last formateur f^T fails to form a government, new elections are held, yielding a status quo utilities $\mathbf{u} = (u_1, ..., u_n)$. In Section 4 we endogenize the reservation utilities assuming that in case f^T fails to form a government, the process repeats itself starting from f^0, thus $f^0 = \Psi(f^T)$. The case with exogenous outside options corresponds to a case in which failure to form the government leads to a new regime: for example a caretaker government or new elections with uncertain outcome. The case in which the process restarts with f^0 corresponds to a situation in which even if there are new elections, the bargaining positions in congress are not expected to change in a significant way, thus leading to the same strategic situation at the government formation stage. Both in the case in which bargaining terminates after f^T fails to form a government, or if it continues, the parties that are left outside a government receive their reservation utilities u_i. We assume that there is at least a coalition C with $C \cap \mathcal{T} \neq \emptyset$ and $\sum_{i \in C} x_i > \sum_{i \in C} u_i$.[18]

In the following we will be interested in the welfare properties of the equilibrium bargaining outcomes We will focus on the utilitarian criterion, according to which an equilibrium outcome $\{C, \mathbf{x}\}$ is efficient if it maximizes the sum of utilities. Since we have $W(\{C, \mathbf{x}\}) = V(C) + \sum_{j \notin C} u_j$

[17] These assumptions on the bargaining protocol within a coalition are not especially important for our results. In Section 7.1 we extend the analysis to assume that each coalition member is randomly recognized as proposer with a probability q_i as in Baron and Ferejohn [1989].

[18] Naturally, the reservation utilities received in the last stage of the game with finite horizon and the reservation utilities received by the parties left out of the winning coalition in the infinite horizon game may be different. We use the same notation $(u_j)_{j=1}^n$ for simplicity since it will not generate confusion.

that can be written as $V(C) - \sum_{j \in C} u_j + \sum_{j \in N} u_j$, the efficient coalition is:

$$C^* = \arg\max_{C \in \mathcal{C}} \left\{ V(C) - \sum_{j \in C} u_j \right\}.$$

A stationary equilibrium in pure strategies of the previous game is defined as follows. For a non formateur party j when the formateur is i and i selects coalition C with $j \in C$, a strategy is a function $\sigma_j(C, i) \to X(C) \times [0, V(C)]^{n(C)}$ that maps the identity i of the formateur and the coalition C chosen by i to a proposal $\mathbf{x}_j = \{x_{j,1}(C,i), ..., x_{j,n(C)}(C,i)\} \in X(C)$ when j is selected as proposer in C, and an acceptance threshold $\mathbf{a}_j(C,i) = \{a_{l,j}(C,i)\}_{l \in C} \in [0, V(C)]^{n(C)}$ when l is proposer and j has to vote.[19] The identity of the formateur, the coalition chosen by the formateur and the identity of the proposer in the coalitional bargaining are the state variables on which the strategies depend. For formateur $i \in \mathcal{T}$ a strategy is similarly defined by an allocation strategy $\sigma_i(C, i) \to X(C) \times [0, V(C)]^{n(C)}$ defined as above and by a government proposal C_i, that selects the coalition in $\mathcal{C}_i$ chosen by i whenever i becomes formateur and when, during coalitional bargaining s/he is recognized as proposer. A stationary equilibrium is a Nash equilibrium in stationary strategies. In the following we focus on stationary equilibria and for simplicity we refer to them simply as equilibria.

3 Bargaining with exogenous outside options

To study the equilibrium with finite rounds of formateurs and exogenous outside options, we start from the simple case in which there is only one formateur, i.e. $|\mathcal{T}| = 1$: in this case if bargaining breaks down, parties receive their outside options $\mathbf{u}$. The characterization of this case will be key for the characterization of the more complicated cases and it will allow to better understand the logic behind the equilibria.

Assume an equilibrium exists and, in this equilibrium, a coalition C_f is chosen by formateur f with payoffs equal to $\mathbf{x}_f = \{x^*_{f,1}, ..., x^*_{f,n(C_f)}\}$ with $\mathbf{x}_f \in X(C_f)$. We can now characterize the payoffs that would be achieved if f decides to choose a generic coalition C (that may or may not coincide with C_f)

To this goal let us extend $\mathbf{x}_f$ to comprise all players by defining $x^*_{f,i} = u_i$ for $i \notin C_f$. We can now define the acceptance threshold of a players i in C at stage $n(C)$ of bargaining as:

$$a_{\iota(n(C),C),i}(C, C_f) = p u_i + (1-p) x^*_{f,i} \tag{1}$$

[19] Proposal $x_{j,l}(C,i)$ is the surplus allocated to l when j is the proposer in a coalition C chosen by i. The threshold $a_{l,j}(C,i)$ is the minimal level of surplus acceptable by j when l is the proposer in coalition C chosen by i.

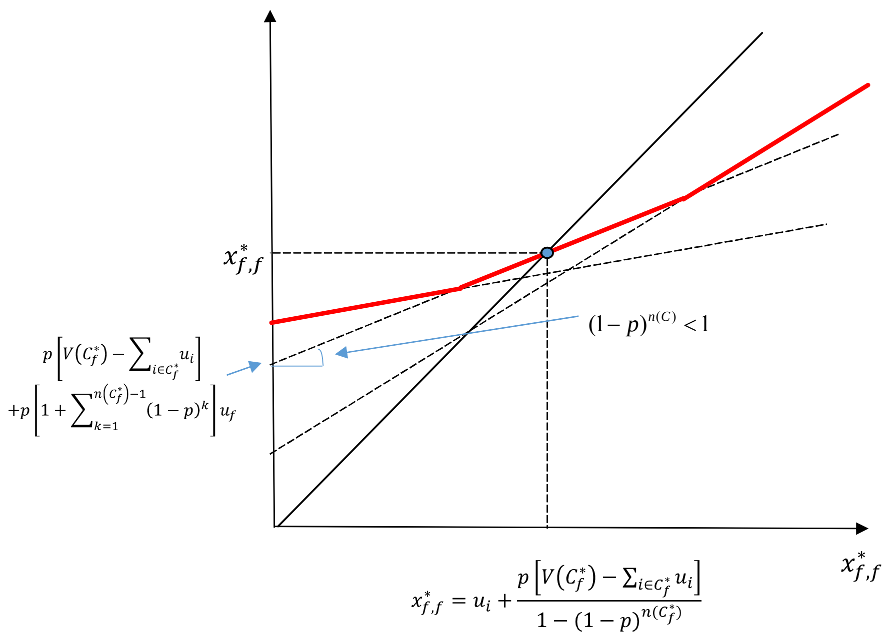

Figure 1: The selection of the equilibrium coalition

where we are using here the notation $a_{j,i}(C, C_f)$ to indicate the acceptance threshold of i when j proposes in coalition C and the expected equilibrium coalition is C_f to emphasize that the threshold depends on C_f. Player i knows that if s/he refuses the offer one of two events occurs: with probability p, s/he has a bargaining breakdown, in which case the utility is u_i; with probability $1-p$, formateur f is recognized after the $n(C)$-th proposer. At this stage, if C is equal to C_f, then the game continues recursively; if instead C is different than C_f, then the formateur is expected to return to C_f. This implies that party i expects to receive $x^*_{f,i}$. At this stage we can also easily define the proposal by proposer $\iota(n(C), C)$ as follows. If

$$V(C) - \sum_{i \in C \setminus \iota(n(C),C)} a_{\iota(n(C),C),i}(C, C_f) < a_{\iota(n(C),C),\iota(n(C),C)}(C, C_f), \tag{2}$$

then $\iota(n(C), C)$ can not make a proposal that is acceptable and that guarantees him/her the reservation utility, so the proposal at the $n(C)$ stage fails and the formateur is recognized again as proposer if there is not a bargaining breakdown. In this case the expected payoff at the beginning of stage $n(C)$ is $pu_i + (1-p)x^*_{f,i}$ for all players. If (2) is not satisfied, instead, we have $x_{\iota(n(C),C),i}(C, f) = a_{\iota(n(C),C),i}(C, C_f)$ for all $i \in C \setminus \iota(n(C), C)$ and $x_{\iota(n(C),C),\iota(n(C),C)}(C, f) = V(C) - \sum_{i \in C \setminus \iota(n(C),C)} a_{\iota(n(C),C),i}(C, C_f)$.

Proceeding in the same way by backward induction, we can uniquely define the acceptance threshold for all bargaining stages up to the first, when the formateur makes a proposal for the first time. At this stage, f proposes $a_{f,i}(C, C_f)$ to all other $i \in C \backslash f$, securing a payoff $V(C) - \sum_{i \in C \backslash f} a_{f,i}(C, C_f)$ if this is larger than $a_{f,f}(C, C_f)$, or a payoff $a_{f,f}(C, C_f)$ otherwise. The initial coalition C_f is chosen in equilibrium if and only if it is a fixed-point of the following correspondence that maps coalitions to coalitions:

$$C_f \in \arg\max_{C \in \mathcal{C}_f} \left\{ V(C) - \sum_{i \in C \backslash f} a_{f,i}(C, C_f) \right\} \tag{3}$$

When C_f satisfies (3), then it is indeed optimal for f to select it whenever s/he has proposal power. In this case, therefore, the expectation that after stage $n(C)$ the payoff will be $\mathbf{x}_f$ is correct. The following result tell us that a fixed-point of (3) exists and it is generically unique. Let

$$C_f^* = \arg\max_{C \in \mathcal{C}_f} \left\{ \frac{p\left[V(C) - \sum_{i \in C} u_i\right]}{1 - (1-p)^{n(C)}} \right\} \tag{4}$$

Naturally, since the set of coalitions is finite, C_f^* is well defined and, except for a non-generic choice of payoffs, unique. We have:

Lemma 1. *For a generic choice of payoffs, C_f^* is the unique fixed-point of (3).*

The idea behind Lemma 1 can be easily illustrated. In the appendix we explicitly solve for the acceptance thresholds $a_{\iota(\tau,C),i}(C; C^f)$ for all stages $\tau = 1, .., n(C)$ and all $i \in C$ and show that the equilibrium payoff obtained by f can be represented in a recursive way as:

$$x_{f,f}^* = \max_{C \in \mathcal{C}_f} \left\{ \begin{array}{c} p\left[V(C) - \sum_{i \in C} u_i\right] \\ +p\left[1 + \sum_{k=1}^{n(C)-1} (1-p)^k\right] u_f + (1-p)^{n(C)} \cdot x_{f,f}^* \end{array} \right\} \tag{5}$$

From (5) it can be seen that the choice of C has two effects on $x_{f,f}^*$. On the one hand, it affects the efficiency of the allocation, as represented by the first term in the brackets. Naturally the formateur would like to choose an efficient coalition, since this guarantees a larger pie to be divided. On the other hand, the choice of C affects the fraction of surplus that can be extracted by f: specifically, the larger is the coalition, the lower is the bargaining power of f. This is the hold-up problem and it is reflected in the slope of the curves, that are decreasing in the size of the coalition. Naturally the formateur's ability to extract surplus depends also on the probability of breakdown. When the probability is high, the formateur's offer is basically a take it or leave it offer, and all surplus can be extracted: in this case the lines are almost flat; and C_f^* converges to the surplus maximizing coalition. As p decreases, however, the formateur's commitment power and the share of surplus that s/he can appropriate is reduced.

Note that for all Cs, the term in the brackets is a contraction. It follows that the upper contour is a contraction as well, as illustrated by the thick line in Figure 1. We must therefore have a unique fixed-point $x^*_{f,f}$ and a unique associated optimal coalition for f that balances efficiency with bargaining power.

Given Lemma 1 we can now easily show that there is a unique equilibrium and it must be supported by parties in C^*_f. By construction, every time that the formateur has an opportunity of choosing C, s/he will chose C^*_f. It follows that in equilibrium C^*_f is chosen and we must satisfy for all $j \in C^*_f$

$$\begin{aligned} a^*_{i,j} &= pu_j + (1-p)x^*_{\iota(\iota^{-1}(i,C)+1,C),j} \text{ for all } i \in C^*_f \\ x^*_{j,j} &= V(C^*_f) - \sum_{i \in \mathcal{C}^*_f \backslash j} a^*_{j,i} \text{ and } x^*_{j,i} = a^*_{j,i} \text{ for all } i \in \mathcal{C}^*_f \backslash j \end{aligned}$$

In the appendix we show that this system is reduced to a system of $n(C^*_f) \times n(C^*_f)$ equations in $n(C^*_f) \times n(C^*_f)$ unknowns that has a unique solution. Using this solution we obtain the equilibrium payoffs as $x^*_{f,i}$ for all $i \in C^*_f$ and u_j for $j \in N \backslash C^*_f$; and the equilibrium strategies when C^*_f is chosen. Using these equilibrium values we can also uniquely define the strategies in all out of equilibrium subgames in which some other coalition C are chosen. We therefore have:[20]

Proposition 1. *The bargaining game has a unique stationary equilibrium in which coalition C^*_f is selected and payoffs are uniquely defined as:*

$$x^*_{f,f} = u_f + \frac{p\left[V(C^*_f) - \sum_{l \in C^*_f} u_l\right]}{1-(1-p)^{n(C^*_f)}} \tag{6}$$

$$x^*_{f,i} = u_i + \frac{p(1-p)^{\iota^{-1}(i,C^*_f)-1}\left[V(C^*_f) - \sum_{l \in C^*_f} u_l\right]}{1-(1-p)^{n(C^*_f)}} \text{ for } i \neq f \tag{7}$$

where $x^*_{f,f}$ is the payoff of the formateur, $x^*_{f,i}$ is the payoff of a party i different from the formateur who is in position $\iota^{-1}(i, C^*_f)$ in the bargaining queue. For all other parties we have u_j for $j \in N \backslash C^*_f$.

To gain insight on the equilibrium allocation, it is useful to consider the case in which the interaction between the parties is very frequent, that is when $\Delta \to 0$. This is important for two reasons. First, because it will give us a simple characterization of the payoffs that will be useful in the generalization studied in the next section. Second, because it highlights an interesting connection between the model of the previous section and the *Nash Bargaining Solution* (henceforth NBS). In the special case in which $N = 2$, the bargaining game considered in the

[20] A complete characterization of the equilibrium strategies is presented in the Proof of Proposition 1 in the appendix.

previous section coincides with Rubinstein's model with the risk of breakdown (see Binmore, Rubinstein and Wolinsky [1986]). It is well known that in this case the solution of Rubinstein's model coincides with the NBS. While the NBS formula can be extended to n players, there are two basic reasons why a mechanical extension is not advisable. First, because there is not a univocal way to rationalize the n-person NBS as the limit of a noncooperative game. Secondly, because even if one were to accept the specific bargaining protocol that rationalizes the n-person bargaining solution, then the solution would implicitly imply that the coalition of all players is formed and all players share the surplus. In the political context studied in this paper, such a scenario is highly unlikely. Ideally, a n-person generalization of NBS would provide indication of *which* coalition is selected and *how* surplus is divided in that coalition. A simple application of the NBS to a n-person environment gives a solution of how surplus should be divided, but it assumes that a n-person coalition is chosen.

The following result shows that the limit equilibria as the bargaining interval converge to zero of the bargaining game described in the previous provide a alternative generalization of the Nash solution to then n-person case. Define the *C-Nash Bargaining solution* as:

$$\mathcal{N}(C, \mathbf{u}) = \arg \max_{\mathbf{x} \in X(C)} \prod\nolimits_{l \in C} [x_l - u_l] \tag{8}$$

where $\mathcal{N}(C, \mathbf{u}) = \{\mathcal{N}_1(C, \mathbf{u}), ..., \mathcal{N}_{|C|}(C, \mathbf{u})\}$. Let, moreover, $\overline{C}_f$ be the coalition the coalition with the largest per capita surplus such that $f \in \overline{C}_f$, i.e. that maximizes $\left[V(C) - \sum_{l \in C} u_l\right] / n(C)$ for $C \in \mathcal{C}_f$. This is the Nash Bargaining solution when coalition C is chosen. We have:

Proposition 2. *As $\Delta \to 0$, the equilibrium of the bargaining problem converges to $x_i^* = \mathcal{N}_i(\overline{C}_f, \mathbf{u})$ for $i \in \overline{C}_f$ and $x_i^* = u_i$ for $i \notin \overline{C}_f$.*

A surprising feature of the equilibrium characterized above is that the formateur does not receive a share of the pie that is much larger than any of the other members of the winning coalition. Unsurprisingly, payoff depend on the order of proposal power in the coalition, since x_i^* is decreasing in i's position in the bargaining queue. When the bargaining process is fast, however, the difference in payoffs net of reservation utilities converges to zero. Define the formateur advantage with respect to a legislator i as the net benefit the formateur with respect to i net of the respective reservation utilities:

$$\mathcal{A}_i(\Delta) = x_f^*(\Delta) - x_i^*(\Delta) - (u_f - u_i)$$

where we have expressed the equilibrium payoffs $(x_i^*(\Delta))_{i \in N}$ as a function of Δ.

Proposition 3. *As $\Delta \to 0$, the formateur advantage with respect to any other party converges to zero: $\lim_{\Delta \to 0} \mathcal{A}_i(\Delta) = 0$. Moreover, the share of output captured by party i is nondecreasing in its electoral size if and only if u_i is non decreasing party i's electoral size.*

There are two reasons for why this result is interesting. First, the presence of a significant advantage for the formateur is one of the key characteristics of existing non-cooperative models of legislative bargaining and it has criticized by the empirical literature, since a formateur advantage has not been detected in many empirical studies. Second, and most importantly, the result seems surprising because in the bargaining model described above the proposer has significant bargaining power since proposal power always return to the formateur, who can always change coalition. The reason this happens is that coalitions are not all equal. As discussed above, given the continuation equilibrium, there is a unique optimal coalition to select for the formateur: this ties the hands of the formateur, since it makes deviations to other coalitions non credible. This allows the other coalition members in C_f^* to credibly insist on high reservation values. The inability of the formateur to capture a larger share of the surplus is the reason for the hold-up problem in bargaining and it leads to the selection of an inefficient coalition.

4 Endogenizing the outside options

In the previous section we have assumed that if bargaining with the formateur fails, then there are new elections and the parties receive exogenously specified expected utilities $(u_i)_{i \in N}$ from the elections. It is however common in legislative processes that if there is a bargaining breakdown, then a new party is selected as formateur and the process restarts. The theory of the previous section can be easily generalized to environments with multiple formateurs. As described in Section 2, we can imagine a set of $\mathcal{T} = \{0, ..., T\} \subseteq N$ formateurs such that after breakdown of bargaining with formateur $\tau \in \mathcal{T}$, formateur $\tau + 1$ is called to the job. In a finite horizon extension after $\tau = T$, the players receive exogenous utilities $\mathbf{u}_{T+1} = \{u_1, .., u_n\}$ as in the previous section; in an infinite horizon extension, after $\tau = T$, formateur $\tau = 0$ is called again to the job and the process repeats. This case represents an environment in which, after bargaining with the last formateur fails, there are new elections that, however, repropose the same strategic interactions between the parties. For simplicity, in the following we focus on the infinite horizon extension and the case in which $\Delta \to 0$. A very similar analysis can be done for the case with $\Delta > 0$.

Define $F_i(\mathbf{u}, C)$ as party i's payoff if coalition C is chosen and the continuation value functions are $\mathbf{u} = \{u_1, ..., u_n\}$. From the previous analysis we have:

$$F_i(\mathbf{u}, C) = \begin{cases} \mathcal{N}_i(C, \mathbf{u}) & if\ i \in C \text{ and } V(C) - \sum_{l \in C} u_l > 0 \\ u_i & else \end{cases}$$

and $F(\mathbf{u}, C) = \{F_1(\mathbf{u}, C), ..., F_n(\mathbf{u}, C)\}$. Define, moreover, $\mathcal{C}_\tau^*(\mathbf{u})$ as the family of coalitions in $\mathcal{C}_\tau$

with maximal average surplus when the outside option is $\mathbf{u}$:

$$\mathcal{C}_\tau^*(\mathbf{u}) = \left\{ C' \in \mathcal{C}_\tau \text{ s.t. } C' \in \arg\max_{C \in \mathcal{C}_\tau} \left\{ \left[V(C) - \sum_{l \in C} u_l \right] / n(C) \right\} \right\}$$

At stage τ the coalition either belongs to $\mathcal{C}_\tau^*(\mathbf{u})$, or formateur τ is unable to form a coalition. We can now define the following operator. At the last stage when T is formateur:

$$F_T^*(\mathbf{u}) := \{\mathbf{v} \text{ s.t. } \mathbf{v} = F(\mathbf{u}, C) \text{ for some } C \in \mathcal{C}_T^*(\mathbf{u})\}$$

The operator $F_T^*(\mathbf{u})$ maps the expected reservation utilities $\mathbf{u}$ if there is a bargaining breakdown with the last formateur T, to the equilibrium utilities that can be reached by if the stage in which formateur T is reached. For the previous stages, we define the payoffs recursively as:

$$F_{\tau-1}^*(\mathbf{u}) := \left\{\mathbf{v} \text{ s.t. } \mathbf{v} = F(\mathbf{u}^\tau, C) \text{ for some } C \in \mathcal{C}_{\tau-1}^*(\mathbf{u}^\tau) \text{ and } \mathbf{u}^\tau \in F_\tau^*(\mathbf{u})\right\}$$

The operator $F_{\tau-1}^*(\mathbf{u})$ maps the reservation utilities $\mathbf{u}$ to the utilities reached in equilibrium if formateur $\tau - 1$ is reached. For a generic choice of $\mathbf{u}$, $\mathcal{C}_{\tau-1}^*(\mathbf{u})$ is a singleton and thus either bargaining with $\tau - 1$ fails to form a government and we move to τ, or there is a unique optimal coalition for $\tau - 1$, thus $F_{\tau-1}^*(\mathbf{u})$ is a function. It is however convenient to allow $F_{\tau-1}^*(\mathbf{u})$ to be a correspondence since when $\mathbf{u}$ is endogenous, the formateur may be indifferent among different coalitions.

In this environment it is very natural to focus the analysis on pure strategy equilibria. As we mentioned, the optimal choice of coalition at stage τ, $\mathcal{C}_\tau^*(\mathbf{u})$ and the feasible payoffs $F_\tau^*(\mathbf{u})$ are uniquely defined for a generic choice of $\mathbf{u}$. The same is true in a finitely repeated version of the game (in which the complete loop of formateurs from 0 to T is repeated for a finite number of times, after which a generic $\mathbf{u}$ is assigned to the players if there is no agreement). Any finite horizon version of the model, therefore has generically a unique pure strategy equilibrium. This implies that an equilibrium of the infinite horizon game can be seen as the limit of equilibria of finite horizon games if and only if it is a pure strategy equilibrium. We have:

Definition 1. *An equilibrium outcome in the extended bargaining game is a vector of utilities* $\mathbf{u}^*$ *and a coalition* C^* *such that* $\mathbf{u}^* \in F_0^*(\mathbf{u}^*)$ *and* $C^* \in \mathcal{C}_0^*(\mathbf{u}^*)$.

To study the equilibria of the extended game, we now specialize the model assuming that there are three parties and each party has a chance to be formateur in some order, so $N = \{1, 2, 3\}$ and $N = \mathcal{T}$. Without loss of generality we can assume that parties are formateurs in order of their index and the value of the coalitions are $V(\{1, 2\}) = a - d$, $V(\{1, 3\}) = a + e$ and $V(\{2, 3\}) = a$, where $a > 0$; either d and e are nonnegative, or d and e are non-positive; and $a - d \geq 0$ even

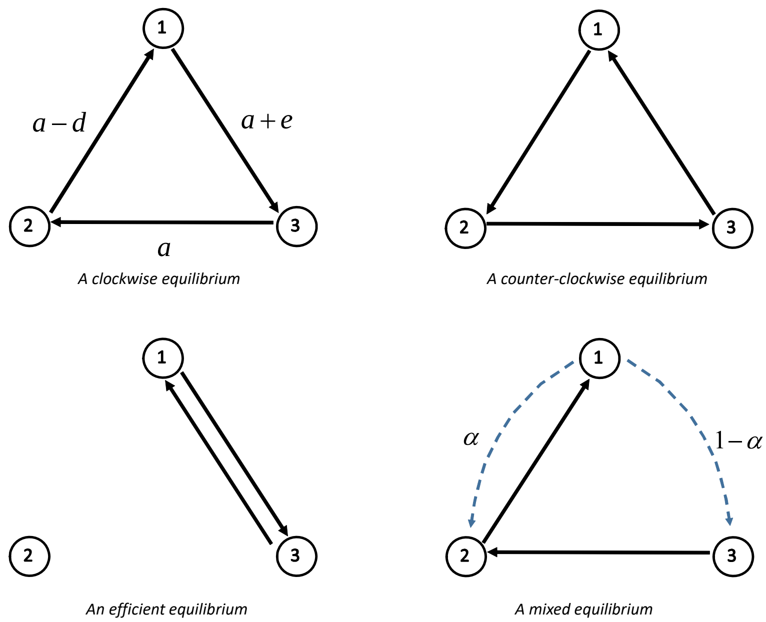

Figure 2: Classification of the possible equilibria

if $d \geq 0$ and $a + e \geq 0$ even if $e < 0$.[21] Here for simplicity, we assume that larger winning coalitions do not make policy agreement easier to reach and implement at the government stage, so we assume $C, C' \in \mathcal{C}$, $V(C) \geq V(C')$ if $C \subseteq C'$.[22] We relax this assumption in Section 5 where we study when supermajorities can arise in equilibrium.

The assumption of three parties is an assumption that has been previously adopted by Austen-Smith and Banks [1988], Baron [1991], Baron and Diermeier [2001] and many others: it allows us to keep the key strategic feature of the problem, minimizing the analytical complications. It is also realistic since most political systems have this feature.

Our first result characterizes the strategies that are feasible in a pure strategy equilibrium. Note first that there is essentially only one way to achieve an efficient allocation: if $d, e > 0$, coalition $\{1, 3\}$ should form, no matter what is the order of proposals; if $d, e < 0$, coalition $\{1, 2\}$

[21] To see that there is no loss, note that, for example, the game with $(d, e) = (f, g) > 0$ and order of proposal is $1, 3, 2$ is equivalent to the game with order $1, 2, 3$ and $(d, e) = (-f, -g)$. The game with, say, $d > 0$ and $e < 0$ is equivalent to a game with a larger a and both d and $e < 0$. Moreover, since the Markov equilibrium is memoryless, once we have characterized the equilibrium with some order starting from 1, we have also characterized any game with the same order starting from 2 or 3. Finally, note that under the assumption that $C, C' \in \mathcal{C}$, $V(C) \geq V(C')$ if $C \subseteq C'$, the grand coalition with all parties is never selected in equilibrium. We will relax this assumption in Section 6.2 where we focus on the possibility of supermajorities in equilibrium.

[22] The idea is that if a winning coalition can agree on a policy that generates a surplus $V(C)$, then additional member who can veto it in a larger coalition C' can only reduce the attainable value.

should form, no matter what is the order of proposals. This happens if the party who is not included in the efficient coalition is unable to form a coalition and no other party includes it in a coalition.

Definition 2. *If* $d, e > 0$, *an equilibrium is efficient if formateur 1 forms an equilibrium with 3, 3 forms an equilibrium with 1 and 2 is unable to form an equilibrium. If* $d, e < 0$, *an equilibrium is efficient if formateur 1 forms an equilibrium with 2, 2 forms an equilibrium with 1 and 3 is unable to form an equilibrium.*

On the other hand, inefficient coalition formation strategies come of different types and shapes. Two forms of inefficient strategies are particularly salient.

Definition 3. *A clockwise equilibrium is an equilibrium in which, when given a chance, party* 1 *selects* 3*, party* 3 *selects* 2 *and party* 2 *selects* 1.

Definition 4. *A counter-clockwise equilibrium is an equilibrium in which, when given a chance, party* 1 *selects* 2*, party* 2 *selects* 3 *and party* 3 *selects* 1.

The top left panel of Figure 2 illustrates the clockwise equilibrium, where the choice of coalition is illustrated by a pointed arrow (from the chooser to the chosed). The top right and bottom left panels illustrate the counter-clockwise and efficient equilibria respectively.[23] This classification is important because, as the following result shows, it exhausts all the possible cases that can occur in equilibrium. In the appendix we prove that:

Proposition 4. *A pure strategy equilibrium is either clockwise, counter-clockwise or efficient.*

The intuition behind Proposition 4 is simple. Assume, for the sake of the discussion here, that $d, e > 0$. Suppose that 1 chooses 2 to be in a coalition with her.[24] Is it possible that 3 also find it optimal to select 2 as her coalition partner if given a chance to be formateur? The problem with such a scenario is that if this were the case, then 2 would always be in a coalition, thus making her equilibrium outside option high (and making the equilibrium outside option of 3 very low); this would reduce the incentive to include 2 in a coalition, and make 3 more appealing. But then it is natural to expect that 1 will choose 3, since a coalition {1,3} is more valuable and 3 is "cheap." Proposition 4 shows that indeed 3 will not find it optimal to include 2, thus leaving only three other scenarios: either offers are "spread out" following the order of the formateurs (the counter-clockwise equilibrium); or offers are spread out following the opposite order (the clockwise equilibrium); or only the efficient coalition is feasible in which case 1 would not offer to 2 if $d, e > 0$

[23] The bottom right panel illustrates a mixed equilibrium that will be discussed below.

[24] As we will see, this may occur in equilibrium even if $d, e > 0$.

and the party that does not belong to the efficient coalition will never be included in the winning coalition.

We can now turn to the characterization of the equilibria. Note that the game can be seen as a classic stochastic game in which the only state variable at any point in time is the identity of the formateur. Proposal strategies are easily represented by functions x_j^i, describing the proposal that i makes to j when i is the formateur.[25]

Assume we are in a counter-clockwise equilibrium and consider the problem of formateur 1. She selects party 3 and makes a payment that depends on 2's expected reservation utility, that is the utility that 2 expects to achieve if bargaining with 1 breakdowns. The reservation utilities at this stage correspond to x_j^2, the payment that formateur 2 in equilibrium offers to j. From Proposition 2, x_j^1 must satisfy as $\Delta \to 0$:[26]

$$x_1^1 = x_1^2 + \frac{a - d - x_1^2 - x_2^2}{2},\ x_2^1 = x_2^2 + \frac{a - d - x_1^2 - x_2^2}{2},\ x_3^1 = 0. \tag{9}$$

Compared to the analysis of Section 3, now the parties' reservation utilities are endogenous and they themselves depend on what it is expected to happen if bargaining with formateur 2 breakdowns. Following the same logic as in (9), we obtain the allocations when 2 and 3 are formateurs:

$$\begin{aligned} x_1^2 &= 0,\ x_2^2 = \frac{a}{2} - \frac{x_3^3}{2},\ x_3^2 = \frac{a}{2} + \frac{x_3^3}{2} \\ x_1^3 &= \frac{a+e}{2} + \frac{x_1^1}{2},\ x_2^3 = 0,\ x_3^3 = \frac{a+e}{2} - \frac{x_1^1}{2}, \end{aligned} \tag{10}$$

Equations (9)-(10) define a system of nine equations in nine unknowns that gives us the following unique solution:

$$\begin{aligned} x_1^1 &= \frac{3a - 4d + e}{9},\ x_2^1 = \frac{6a - 5d - e}{9},\ x_3^1 = 0 \\ x_1^2 &= 0,\ x_2^2 = \frac{3a - d - 2e}{9},\ x_3^2 = \frac{6a + d + 2e}{9} \\ x_1^3 &= \frac{6a - 2d + 5e}{9},\ x_2^3 = 0,\ x_3^3 = \frac{3a + 2d + 4e}{9}, \end{aligned} \tag{11}$$

These are the equilibrium allocations under the assumption that we are in a counter-clockwise equilibrium. To complete the characterization we need to make sure that the coalition chosen

[25] We adopt the simplified notation here since it will not lead to confusion. In terms of the more general notation of Section 2, we have $x_j^i = x_j(\{i, j\}, i)$.

[26] The formulas look analogous, just a little more complicated when $\Delta > 0$. For example, we would have: and $x_1^1 = x_1^2 + \frac{p(1-p)}{1-(1-p)^2}\left[a - d - x_1^2 - x_2^2\right]$, $x_3^1 = 0$, $x_2^1 = x_2^2 + \frac{p(1-p)}{1-(1-p)^2}\left[a - d - x_1^2 - x_2^2\right]$. Analogous formula can be derived for the reservation values in (10) below.

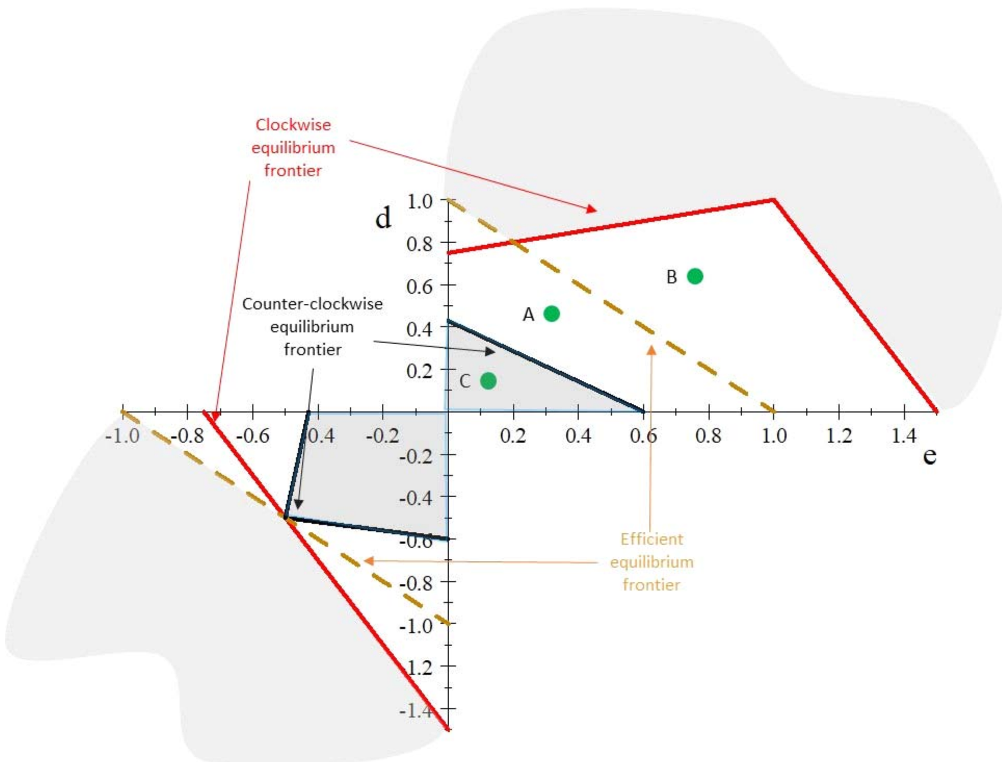

Figure 3: The equilibrium characterization

in this type of equilibrium correspond are the players' best responses. Let $S_\tau(C)$ be the average surplus in coalition C when τ is the formateur, that is

$$S_\tau(C) = \frac{1}{2}\left[V(C) - (\sum\nolimits_{j\in C} x_j^{\tau+1})\right].$$

For illustration, let us assume here that d and e are negative, so that the most efficient coalition is $\{1,2\}$ (the case in which d and e are positive is very similar and presented in the appendix). Consider first the case in which 2 is the formateur. From Proposition 2, formateur 2 selects coalition $\{2,3\}$ if $S_1(\{2,3\}) \geq S_1(\{1,2\})$. From (11), we have:

$$\begin{aligned} S_2(\{1,2\}) &= \frac{1}{2}\left(a - d - x_1^3\right) = \frac{3a-7d-5e}{18} \\ S_2(\{2,3\}) &= \frac{1}{2}\left(a - x_3^3\right) = \frac{6a-2d-4e}{18} \end{aligned}$$

so $S_2(\{2,3\}) \geq S_2(\{1,2\})$ if $d \geq -\frac{3}{5}a - \frac{1}{5}e$. Proceeding analogously we can verify that formateur 1 and 3 finds it optimal to select, respectively, coalitions $\{1,2\}$ and $\{1,3\}$ if and only if $d \leq 3a + 7e$. We conclude that a counterclockwise equilibrium exists only if $d \geq -\frac{3}{5}a - \frac{1}{5}e$ and

$d \le 3a + 7e$ (the darkly shaded region in the negative orthant in Figure 3). Using a similar logic we can characterize all the other feasible equilibria and obtain the following full characterization of equilibrium bargaining:

Proposition 5. *We have that:*

- *A clock-wise equilibrium exists if and only if* $d \le \min\left(\frac{3}{4}a + \frac{e}{4}, 3a - 2e\right)$ *when* $d, e > 0$*, and if* $d \ge -\frac{3}{2}a - 2e$ if $d, e < 0$
- *A counter-clockwise equilibrium exists if and only if:* $d \le \frac{3}{7}a - \frac{5}{7}e$ if $d, e > 0$; and $d \ge -\frac{3}{5}a - \frac{1}{5}e$ and $d \le 3a + 7e$ if $d, e < 0$.
- *An efficient equilibrium exists if an only if* $d \ge a - e$ *when* $d, e > 0$*, and if* $d \ge -(a+e)$if $d, e < 0$

The equilibrium structure is described in Figure 3. In the following two sections we will discuss some important welfare and positive implication of the equilibria. We conclude this section commenting on three aspects of the characterization that make the analysis distinctive from previous work on legislative bargaining: the possibility of inefficient equilibria; multiple equilibria that depend on the parties expectations of likely coalitions; and the possibility of strategic delays.

The issue of inefficient equilibria has been studied little in the previous literature because they focused on distributive politics.[27] Noncooperative models a' la Baron and Ferejohn [1989] assume that all coalition generate the same surplus, thus restricting the analysis to how surplus is allocated and making the choice of coalition irrelevant; cooperative models of bargaining, on the contrary, assume that the largest coalition is the most efficient and always selected. In the model studied above, instead, the focus is on how the equilibrium coalition is chosen and how this choice depends on the associated surplus. Proposition 1 and 2 showed that an inefficient coalition can be selected, though in that case the equilibrium coalition is always the one that maximizes the average surplus among the coalitions that are feasible for the formateur (as $\Delta \to 0$, at least). Proposition 5 makes a step further showing that indeed inefficient coalitions can be selected even if the formateur is a member of the efficient coalition, and the selected coalition may not even be the one that maximizes average surplus. This happens when the equilibrium "expectation" of the other member of the efficient coalition is too high, thus making convenient to choose a less efficient coalition. Surprisingly, this is not just one of the equilibrium outcomes, but under some condition the unique equilibrium outcome. For example, with $d, e < 0$ and $d > -(a+e)$, $d < -\frac{3}{5}a - \frac{1}{5}e$,

[27] Welfare in bargaining models has been explicitly studied in models that explicitly contemplate public goods (for instance, Battaglini and Coate [2007], Volden and Wiseman [2007]) and in model of public debt with distortionary taxation (Battaglini and Coate [2008]).

this is the unique equilibrium when 1 is the first formateur (who elects 3 instead of the efficient 2); with $d, e > 0$ and $d > \frac{3}{7}a - \frac{5}{7}e$, $d < \min\{(a-e), \frac{3}{4} + \frac{e}{4}\}$, this is the unique equilibrium when 3 is the first formateur (who selects 2 instead of the efficient 1).

Related to the expectations on continuation values that are feasible in equilibrium is also the issue of multiplicity. For the sake of the discussion here, assume $d, e > 0$. In the area below the counter-clockwise equilibrium threshold (the lower solid line) in Figure 3, both the counterclockwise and the clockwise equilibria exist (but no efficient equilibrium exists); in the area above the efficient equilibrium threshold (the dashed line) and below the clockwise equilibrium threshold (the higher solid line), we have both an efficient and an inefficient equilibrium. Contrary to what happens in Baron and Ferejohn [1989], this multiplicity of equilibria is payoff relevant.[28] For example in the area in which both the clockwise and the counterclockwise equilibria coexist, the counter-clockwise equilibrium is associated to the payoffs in (11); as we show in the proof of Proposition 5 in the appendix, the payoffs with a clockwise equilibrium are:

$$\begin{aligned} x_1^1 &= \frac{6a+5e-2d}{9},\ x_2^1 = 0,\ x_3^1 = \frac{3a+4e+2d}{9} \\ x_1^2 &= \frac{3a+e-4d}{9},\ x_2^2 = \frac{6a-e-5d}{9},\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \frac{3a-2e-d}{9},\ x_3^3 = \frac{6a+2e+d}{9} \end{aligned} \tag{12}$$

The payoff of party 1 when s/he is the formateur, for example, is much higher in the clockwise equilibrium than in the counterclockwise equilibrium. The multiplicity of equilibria capture the complexity of the strategic interaction in this model and it is natural in this environment, since it reflects the fact that reservation utilities depend in a non trivial way on endogenous expectations on future. As we will discuss in the following section, this multiplicity has important implications for understanding the limits of presidential power to control bargaining outcome by just controlling the order and identity of formateurs. Despite this, the model generates only few equilibria and thus allows to derive sharp predictions on behavior and welfare.

5 Positive analysis

We now apply the theory developed in the previous sections to discuss three important empirical facts about legislative majorities. First, we explain the absence of significant formateur's premia, and indeed the evidence of negative premia in the data. Second, we explain the fact that *grand coalitions* comprising more than 50% of the seats are very common in legislative bargaining. Third, we explain the fact that legislative bargaining often includes failed attempts before an agreement

[28] In Baron and Ferejohn [1989] we typically have multiple stationary equilibria, but they all lead to the same equilibrium payoffs.

is reached. These facts are important because they have often been seen as evidence against strategic models of legislative bargaining. We show that they emerge naturally in richer strategic environment such as the one studied here.

5.1 The formateur's premium.

As we mentioned in Section 3, a key prediction of standard noncooperative models a' la Baron and Ferejohn [1989] is that the party selecting the coalition receives a very significant premium in terms of surplus allocation. This reflects the fact that in these models the proposer has, at least temporary, monopoly power on the choice of the allocation and can exploit this advantage. A surprising but robust finding in the empirical literature is that not only such a premium does not exist, but that indeed formateurs suffers a proposer's penalty.[29] For instance, Warwick and Druckman [2001] show that the payoff formateurs receive falls short of their vote contributions to the coalition by 13.3%.[30]

The empirical phenomenon in which the proposer receives the same or less than the other coalition member can be explained in an intuitive way in the equilibrium of Proposition 5. To address these questions (and the questions in the next subsections), it is useful to express all payoffs in terms of how similar the values generated by the different coalitions are. Normalize $a = 1$ and let $e = \epsilon \cdot \theta$ and $d = \vartheta \cdot \theta$, where ϵ and ϑ are generic coefficients in $(0, 1)$ and θ can be positive or negative with $|\theta| \in (0, 1)$.[31] When $|\theta|$ is small, we are in an environment in which all coalitions generate a similar value (as in the Baron and Ferejohn's [1989] benchmark); as $|\theta|$ increases the differences between the different coalitions become more prominent.

Assume first that $|\theta|$ is small: this corresponds to an environment where the different coalitions generate similar valuations, so there is not an obviously superior or inferior coalition. In this case, we always have a counterclock equilibrium (the area in the dark shadow in Figure (3)). Consider party 1 when it is the proposer: can it extract a formateur's premium? It is easy to see that this is not possible and indeed it will be very willing to concede a bonus to the coalition partner. If negotiation fails 2 becomes proposer: 1 expects 2 to form a coalition with 3, leaving himself marginalized. This makes it rational for 1 to leave 2 more than 50% of the surplus generated in their coalition. Naturally, 1 can try with 3, but 3 would require an even higher surplus since he expects

[29] This literaure, stared by Browne and Franklin [1973] and [1980] typically uses ministerial portfolio allocations (often weighted by the importance of the cabinets) as a measure of surplus allocation. See, among many others, Schofield and Laver [1985], Warwick and Druckman [2001] and [2006].

[30] This finding does not depend on the fact that coalitions split surplus equally but the formaterur is systematically the largest party. Druckman and Warwick [2001] find that the coefficient of a variable interacting the size of a political party with a dummy equal to one when the party is formateur has a significantly negative sign.

[31] The normalization of $a = 1$ is without loss of generality. If a is not equal to one, we obtain the same results defining $e = \epsilon\theta a$ and $d = \vartheta\theta a$.

to be in a coalition with 2 in which indeed 2 will be willing to leave him more than 50% of the surplus (again, this because 2 fears that if proposal power movers to 3, then 3 will form a coalition with 1).[32] This is a manifestation of the *hold-up problem* in multilateral bargaining. Formateur's 1 can threaten 2 to switch to 3 while bargaining, but the threat would not be credible. Party 2 knows that 1 will either return to the table or fail as formateur. Party 2 then can comfortably hold 1 up and extract more than 50% of the surplus.

Assume now that $|\theta|$ is large: this corresponds to the case in which there is a clearly superior (and a clearly inferior) coalition. In this case (when $a \leq d + e$ so $\theta \geq a/(\vartheta + \epsilon)$), only the efficient coalition can form. In this case too party 1 can not credibly extract a positive proposer's bonus. If, for example, $\theta > 0$, then party 2 knows that party 1 really has no choice, since the coalition $\{1, 3\}$ is so inferior. In this case, 1 and 3 are basically stuck with each other, so 1 can expect to receive anything in $[a - d, e]$; and 2 can expect to receive the reminder $a + e - x_1$. Positive, zero or negative proposer's bonuses are now possible in equilibrium. Again, the hold-up problem is at work here: this time one in which 1 and 2 can hold each other up.

To formalize these considerations, recall that $e = \epsilon \cdot \theta$ and $d = \vartheta \cdot \theta$. define the thresholds $\theta^F_{**} = 1/(\vartheta + \epsilon)$ and:

$$\theta^F_* = \begin{cases} 3/(7\vartheta + 5\epsilon) & \theta > 0 \\ 3 \cdot \max\left(1/(7\epsilon - \vartheta), 1/(5\vartheta + \epsilon)\right) & \theta < 0 \end{cases}$$

The threshold θ^F_* defines the dark shaded area in Fig. 3; the threshold θ^F_{**}, instead, defines the area above, the dashed efficiency frontier for $d, e > 0$ (below for $d, e < 0$). We can now state:

Proposition 6. *A positive formateur's advantage is the unique equilibrium prediction only if* $|\theta| \in \left(\theta^F_*, \theta^F_{**}\right)$. *If* $|\theta| < \theta^F_*$ *or* $|\theta| > \theta^F_{**}$ *there is an equilibrium in which the formateur receives a negative bonus.*

Proposition 6 paints a more nuanced picture than other existing models of non cooperative bargaining. This result does not prove that we should necessary observe a small formateur's premium, or that payoffs should be proportional to the parties' vote shares. It however shows that both possibilities are consistent and even natural in our noncooperative bargaining model and that these phenomena are not features to very specific bargaining protocols. The feasible allocations now depends on the details of the environment in which bargaining takes place. The strength of Proposition 6 is that the conditions on θ only refer to the relative magnitude of the values generated by the possible coalitions: the result is true no matter what selection procedure

[32] The argument is indeed general and true for any party who may be proposer and indeed for any order of proposer.

is chosen for the first formateur (who does not need to be the largest party and indeed can be randomly selected), and no matter what is the sequence of formateurs in case of bargaining breakdown.[33]

Far from being a limitation, the multiplicity of equilibria may explain some puzzling empirical results, such as the fact that even in environments with three relevant political parties we observe that payoffs are distributed in a way that is seemingly proportional to the parties vote share.[34] This is puzzling because with 3 players and a minimal coalition of two, the precise share of votes of each player is irrelevant in any bargaining model.[35] Evidence from countries with three-party legislatures where only two parties can form a majority government however strongly suggests a positive and significant correlation between seat shares and portfolio allocations (see Warwick and Druckman [2001]).[36] Political environments in which bargaining is played with 3 relevant parties are cases in which it is natural to imagine there is a coalition that is significantly superior to the other. Proposition 5 and 6 predict that in this case the precise distribution of surplus is not fully determinate, thus making the vote share a natural rational focal point.

Proposition 6, however, suggests a more important point regarding how surplus is distributed in government formation. Most of the discussion has focused on how surplus is divided between parties in the *realized* coalition. Failure of the formateur to capture more than 50% has been interpreted as evidence that the formateur does not benefit from its proposal power. This is natural in a world in which all coalition have the same value (as in standard non cooperative models) and in which the "grand coalition" is the most valuable coalition (as implicitly assumed in all cooperative models, that have little to say on the choice of coalition). In a model in which coalitions have heterogeneous values and equilibria may be inefficient, the formateur's benefit of proposal power mostly comes from the choice of the coalition, rather than from the share of surplus that is obtained. For example, party 1 obtains less than 50% of $a - d$ when proposer in a counterclockwise equilibrium leading to a coalition $\{1, 2\}$, but even less than this if he attempts to form the more efficient coalition $\{1, 3\}$, and exactly zero if he loses proposal power (since $\{2, 3\}$ forms in this case). The real benefit of being formateur for party 1 is in selecting $\{1, 2\}$.

[33] As explained in Footnote 22, by considering $d, e > 0$ and $d, e < 0$, the characterization of Proposition 5 (and thus of Proposition 7) captures equilibrium behavior of all sequences of formateurs.

[34] Another puzzling empirical effect that can be explained by our theory is presented by Fujiwara and Sanz [2018] who show that in multilateral bargaining the party with slightly more votes is substantially more likely to appoint the mayor (thus extracts significantly more rents from negotiations). The social norm that "the most voted party should express the major" can indeed serve as a natural focal point that helps the parties pin down the appropriate reservation utilities in the presence of multiple equilibria.

[35] Standard measures of power in non cooperative game theory (such as the Shapley-Shubik index or the Banzhaf index) give all parties the same power of 0.33. Morelli [1999]'s demand bargaining model also predicts that surplus is split in the middle in any minimal winning coalition.

[36] Warwick and Druckman [2001] report a correlation between portfolio shares and seat contributions of $r = 0.850$ with $p < 0.001$ in countries with 3 relevant political parties such as West Germany or Austria.

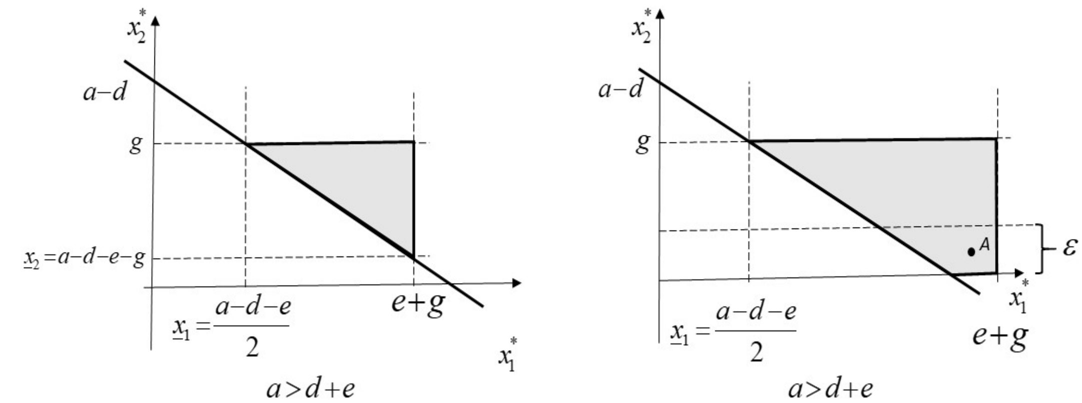

Figure 4: The existence of a Grand Coalition Equilibrium

5.2 Grand Coalitions

In Section 4 we assumed that there is no intrinsic benefit in the size of a coalition. If C is a winning coalition that can agree, say, on a policy p^*, enlarging the coalition to C' with $C \subset C'$ can only add veto players, thus reducing the surplus to be divided.[37] Formally, we assumed $V(C) \geq V(C')$ for $C \subset C'$. Under this assumption, supermajorities are never optimal in equilibrium. It is intuitive that the model presented above may explain the emergence of supermajorities if we drop this assumption and allow supermajorities to be more valuable than smaller coalitions. In a supermajority the formateur needs to compromise with more parties, but if the surplus is sufficiently large this may be worthwhile. It would however not be extremely surprising if we could only explain supermajorities by assuming a very large surplus advantage for a large coalition. The interesting question is: can we explain supermajorities even when the supermajorities are only *marginally* better than minimal winning coalitions? This would go a long way in explaining why we observe them so often.

To study these questions, let us focus without loss of generality that $d, e > 0$. We now assume that any party can choose the unanimous coalition and obtain $V(\{1,2,3\}) = a + e + g$. Coalition $N = \{1,2,3\}$ is now the most efficient coalition and $g \geq 0$ measures the supermajority premium: the larger is g the more efficient is N, as g converges to zero, this premium disappears.

When does an equilibrium in which all formateurs choose a supermajority emerge? We should

[37] In our formalization we have not specified policies, but they are implicit in the value function $V(C)$. The government chooses a policy p_C in coalition C that generates a surplus $s_i(p_C)$ to party $i \in C$. We can link p_C to C by letting $V(C) = \sum s_i(p_C)$. Assuming transferable utilities as in Austen-Smith and Banks [1988] and Baron and Diermeier [1999], each party can now achieve $u_i(p_C) = s_i(p_C) + t_i$, where naturally we need $\sum t_i \leq \sum s_i(p_C)$ and $t_i \in [-s_i(p_C), \sum s_i(p_C)]$.

first note that in such a situation a player j receives a payoff x_j^* independent from the identity of the formateur. To see this, assume i is the proposer who is followed by $i+1$ in case of bargaining breakdown, then j receives:

$$x_j^i = x_j^{i+1} + \frac{1}{3}\left[V(N) - \sum_j x_j^{i+1}\right] = x_j^{i+1} = x_j^*$$

where the second equality follows from the fact that $\sum_j x_j^{i+1} = V(N)$. For $(x_j^*)_{j\in N}$ to be an equilibrium, it must be that no party finds any other coalition more profitable. Party 1 chooses N if the average surplus $S_1(N) = \left(V(N) - \sum_j x_j^*\right)/3$ in equilibrium is larger than the surplus in the other two alternatives: $S_1(\{1,2\}), S_1(\{1,3\})$. This implies two conditions on payoffs must be verified: $x_1^* + x_2^* \geq a - d$ and $x_1^* + x_3^* \geq a + e$. When considering the constraints implied by 2 and 3 optimal response and the budget $\sum_j x_j^* = a + e + g$, we obtain that a supermajority equilibrium emerges if and only if:

$$x_1^* \leq e + g,\; x_2^* \leq g,\; x_2^* \geq a - d - x_1^* \tag{13}$$

The set of feasible equilibrium payoffs is illustrated by the shaded area in Figure 4. Two facts should be emphasized. First, while multiple equilibria are possible, the equilibrium restriction limit the set of payoffs that are feasible. Second and most importantly, a supermajority equilibrium is not always possible and (13) characterize its feasibility. We have:

Proposition 7. *An equilibrium in which all parties choose a grand coalition exists if and only if* $g \geq (a - d - e)/2$.

The most important implication of this result is that the grand coalition can emerge in equilibrium even if its welfare advantage on the minimal winning coalition is minimal or indeed zero: when $a - d - e$ is small (left panel of Figure 4 or negative (see point A in the left panel of Figure 4, for example). Note that for $g > (a - d - e)/2$ there is indeed an equilibrium in which *all* parties *strictly* prefer the grand coalition to a minimal winning coalition. This is surprising since it implies that, for example, even when g is arbitrarily small party 1 prefers to propose a grand coalition including 2 and 3 rather than a minimal winning coalition with 2 only, despite the fact that in bargaining with a grand coalition he will have to leave surplus to two other parties, while in a minimal coalition with 3 he can generate most of the surplus and safely ignore 2's demands.

The general lesson is that the hold-up problem in legislative bargaining is mitigated when reservation utilities are endogenous. The intuition of this that in this equilibrium party 2's value function self regulate to make it appealing for the other parties to bring it on the bargaining table. It is interesting to note that the possibility of supermajority critically depends on the heterogeneous value of the coalitions. As d and e converges to zero, the lowerbound on the bonus value required by the grand coalition to sustain a grand coalition equilibrium converges to 50% of the value of the minimal winning coalitions, a very high threshold.

5.3 Strategic delays

The third difference with previous work is the fact that we may have strategic delay in equilibrium. An equilibrium with strategic delay occurs when the efficient coalition is possible, say $\{1, 3\}$ with $d, e > 0$, and 2 has a chance to be formateur; delay is inevitable under the conditions of Proposition 6, when the efficient equilibrium is the unique equilibrium. Previous stories about strategic delay relied on the fact that players expected random shocks that may increase the size of the pie. These explanations rely on the assumption that the "size of the pie" may change frequently and unpredictably, in the relative short time in which negotiations occur, that is days or weeks. In our case the pie is not stochastic: delay occurs because one party is unable to convince the others not to wait for better opportunities. This, moreover, is not due to the fact that the opportunity is by assumption too large not to be waited for (in terms of expected surplus). In the case with $d, e > 0$, for example, when $d \leq \min\left(\frac{3}{4}a + \frac{e}{4}, 3a - 2e\right)$ and $d \geq a - e$ (the area marked "B" in Figure 3), *both* an efficient equilibrium with delay *and* an inefficient clockwise equilibrium coexist: party 2 is unable to form a government because of the players' (self-fulfilling) equilibrium beliefs. The delays in reaching an agreement, therefore, is an exquisitely strategic phenomenon.

6 Welfare and the role of the head of state in legislative bargaining

The characterization of the bargaining equilibrium presented in the previous section allows us to study the role of the head of state in affecting legislative bargaining and thus shaping governing coalitions by exerting his/her prerogatives in selecting the formateurs. Among the prerogative of a head of state in a parliamentary democracy, that of choosing the government formateur is one of the most important.[38] Many European constitutions empower their heads of state with significant discretion in the government formation process, including Austria, Croatia, the Czeck Republic, France, Iceland, Italy, Poland and constitutional monarchies such as Belgium, Denmark, Luxemburg, Norway, and the United Kingdom (Brunclick [2015]). The role of the head of state in this process is not generally formally regulated and it may change over time. Italy, for example, used to have relatively weak heads of state; but this tradition ended with President Scalfaro (1992-

[38] See Amorim Neto and Strom [2006], Tavits [2009], Carroll and Cox [2011] among others for a discussion of the role of the head of state. Other important prerogatives of the president include the timing of dissolution and the vetoing of legislation which are less relevant to our discussion.

1999) who leveraged his prerogatives in the face of a weakened party system after the corruption scandals of the 90s. In the United Kingdom the monarch has repeatedly showed its influence in the presence of a "hung parliament" (Kavanagh et al. [2005]). In some cases the power of the head of state is so strong that legislatures proceeded to curb it, as in the case of Sweden and the Netherlands in 1975 and 2012 respectively.

The equilibrium characterization of the previous section suggests 3 questions. As we have seen, the legislative process does not generally arrive to an efficient equilibrium in which the coalition that generates the most surplus is chosen (even when the cost of delay is small). Indeed, we have seen that it is even possible that the worst equilibrium, in which the least efficient of the coalitions forms in equilibrium, is achieved. First, is there room for a head of state to improve the outcome of the bargaining process, just by controlling the order of formateurs? Second, if this is the case, what are the limits of this power? Ideally the head of state would want to induce an efficient equilibrium; under what conditions is this possible? Finally, if full control of the outcome is impossible, can at least the head of state avoid that the worst equilibrium is achieved?

As done in Section 5, it is useful to normalize a to 1 and use the notation according to which $e = \epsilon \cdot \theta$ and $d = \vartheta \cdot \theta$, where ϵ and ϑ are generic coefficients in $(0, 1)$ and θ can be positive or negative with $|\theta| < 1$. It is easy to see that when $|\theta|$ is sufficiently large (the area in the light shade in Figure 3), the head of state is irrelevant: no matter what the order of formateur is, the efficient coalition is so much better than the others, that it will form anyway. Assume for example that $d, e > 0$. If 1 or 3 are selected, then $\{1, 3\}$ forms; if 2 is selected, then 2's attempt is destined to fail and $\{1, 3\}$ forms again. A similar phenomenon is true for $d, e < 0$, in which case $\{1, 2\}$ always forms if $|\theta|$ is sufficiently large. The area in the (lightly) shaded area in which this happens is easily characterized by $|\theta| > \overline{\theta}^P$ where:[39]

$$\overline{\theta}^P = \begin{cases} \max\left(1/(\vartheta + \epsilon), 3 \cdot \min\left(1/(4\vartheta - \epsilon), 1/(\vartheta + 2\epsilon)\right)\right) & \text{if } \theta \geq 0 \\ \max\left(1/(\vartheta + \epsilon), 3/(2\vartheta + 4\epsilon)\right) & \text{if } \theta < 0 \end{cases}$$

As we reduce $|\theta|$ below the area defined by $\overline{\theta}^P$, efficiency is not a forgone conclusion anymore. Define the threshold that defines the area in which both the cockwise and counter-clockwise equilibria exist (the darkly shaded area in Figure 3):

$$\underline{\theta}^P = \begin{cases} 3/(7\vartheta + 5\epsilon) & \text{if } \theta \geq 0 \\ 3 \cdot \max\left(1/(5\vartheta + \epsilon), 1/(7\epsilon - \vartheta)\right) & \text{if } \theta < 0 \end{cases}$$

[39] The threshold presented below follow directly from Proposition 5. For details see the proof of Proposition 8.

When $|\theta| < \underline{\theta}^P$, the head of state is faced with multiple inefficient equilibria, no matter the order of formateurs. Consider point C in Figure 3. Here the head of state can achieve an efficient equilibrium by choosing an order $3, 1, 2$ and aim at a clockwise equilibrium. If this happens, then 1 forms a coalition with 3 and an efficient outcome is achieved. If however, the parties play a counterclockwise equilibrium, the result is the opposite: 1 offers to 2 and the least efficient coalition forms in equilibrium. The head of state can attempt to achieve an efficient equilibrium by choosing an order $1, 2, 3$, letting 3 to go first and aim at a counter-clockwise equilibrium: in this case 3 proposes to 1, and again we have an efficient outcome. If however the players play a clockwise equilibrium, the outcome is again inefficient, though not the least efficient coalition. The natural choice for the head of state is to go for this second scenario since it makes sure that the least efficient equilibrium is not achieved: but it is impossible to guarantee an efficient equilibrium.

In the intermediate case $|\theta| \in \left(\underline{\theta}^P, \overline{\theta}^P\right]$, there are two possibilities. Define

$$\theta_*^P = \begin{cases} \min\left(1/(\vartheta+\epsilon), 3\cdot\min\left(1/(4\vartheta-\epsilon), 1/(\vartheta+3\epsilon)\right)\right) & \text{if } \theta \geq 0 \\ \min\left(1/(\vartheta+\epsilon), 3/(2\vartheta+4\epsilon)\right) & \text{if } \theta < 0 \end{cases}$$

which is in between $\underline{\theta}^P$ and $\overline{\theta}^P$. For $|\theta| \in \left(\underline{\theta}^P, \theta_*^P\right]$ the head of state can make sure that the efficient outcome will be achieved in all equilibria. Consider point A in Figure 3. Here the clockwise equilibrium is the unique equilibrium. If the order of formateurs is $1, 2, 3$, an efficient allocation is formed if 1 is selected as formateur; an inefficient equilibrium is instead obtained if 2 or 3 is selected as formateurs: indeed the *worst* equilibrium $\{1, 2\}$ is obtained if 2 is selected as formateur. In point B the head of state is faced with a different scenario. There is indeed an equilibrium in which the efficient equilibrium always form, the head of state would be irrelevant in this case; there is however also a clockwise equilibrium in which efficiency is achieved only if 1 is selected as first proposer (when the order is $1, 2, 3$). The equilibrium is not unique, but the efficient equilibrium can be implemented for sure by the head of state by an appropriate choice of order of formateurs.

In the area $|\theta| \in \left(\theta_*^P, \overline{\theta}^P\right]$, instead, a pure strategy equilibrium does not exist: these are the two triangles below the dashed efficiency line and above the solid clockwise equilibrium frontier with $\theta > 0$; and above the dashed efficiency line and below the solid clockwise equilibrium frontier with $\theta < 0$. The lack of existence of a pure equilibrium in this region is problematic for the head of state. As we will show in Section 7.2 a mixed equilibrium with which an efficient equilibrium can be achieved exists in this region. Still, mixed equilibria are fragile because they can not be seen as limits of finite horizon versions of the game: this implies that the outcome with a finite horizon can not be approximated by them and it is sensitive to details as the exact number of periods of the bargaining game in a finite horizon.

To summarize this discussion it is useful to have the following definitions. We say that the head of state has *full control* if s/he can select the order of formateurs in such a way that the efficient outcome is achieved in all possible equilibria. We say that the head of state is *irrelevant* if no matter what selection of the order of formateurs is chosen, the outcome is efficient. We say that the head of state has *incomplete control* if for any selection of the order of formateurs there is an inefficient equilibrium or there is no pure strategy equilibrium that achieves efficiency.

Proposition 8. *The head of state has full control if and only if* $|\theta| \in \left(\underline{\theta}^P, \theta_*^P\right]$*. S/he has limited power elsewhere: s/he is irrelevant if* $|\theta| > \overline{\theta}^P$*; and s/he has incomplete control if* $|\theta| \leq \underline{\theta}^P$ *and* $|\theta| \in \left(\theta_*^P, \overline{\theta}^P\right]$.

The region in which the head of state can induce the efficient outcome as a unique equilibrium just by selecting the formateurs is actually larger than the region characterized in Proposition 8, i.e. $|\theta| \in \left(\underline{\theta}^P, \theta_*^P\right]$. The reason is that in Proposition 8, we change payoffs, but we keep the order of formateur constant at $1 \to 2 \to 3$.[40] Of course, given payoffs, the head of state can use different orders, for example $1 \to 3 \to 2$. As we commented in Section 4, Proposition 5 characterizes the equilibria in these cases as well. For example, assume $V(\{1,2\}) = a - d$ and $V(\{1,3\}) = a + e$ with $d, e > 0$. It is easy to see that if there is a clockwise equilibrium in the game with order $1 \to 3 \to 2$,[41] if and only if there is a counterclock equilibrium in the game with order $1 \to 2 \to 3$ and $V(\{1,2\}) = a + e$ and $V(\{1,3\}) = a - d$ (to see this, just switch the labels of 2 and 3 and use the results of Proposition 5 in the case with negative d, e). When we take this into account, the set in which the head of state can induce an efficient outcome as unique equilibrium is given by the (lightly and darkly) shaded area between the solid lines in Figure 5.[42]

An implicit assumption in this analysis, however, is that the head of state can commit to an order of formateurs. This is however problematic. Consider point A in figure 5, where $d, e > 0$. If the head of state selects and order $1 \to 2 \to 3$ and this is credible, then 1 forms with 3 and the efficient equilibrium forms. But what if, out of equilibrium, 1 fails to form and the head of state needs to select a different formateur? In this case it would not be optimal to select 2, who would form a coalition $\{1,2\}$, the least efficient. The choice of an order $1 \to 2 \to 3$ would not be credible absent some form of commitment device. It turns out that the head of state can still induce the efficient equilibrium by inducing a clockwise equilibrium an order $1 \to 3 \to 2$, but in a

[40] With the notation $i \to j \to k$ we mean that j is formateur after i, k after j, and i either is first or comes after k.

[41] With order $1 \to 3 \to 2$, a clockwise equilibrium is still an equilibrium in which 1 forms, $\{1,3\}$, 3 forms $\{1,2\}$ and 2 forms $\{1,2\}$.

[42] The analytical characterization of this set is presented in the proof of Proposition 9 in the appendix.

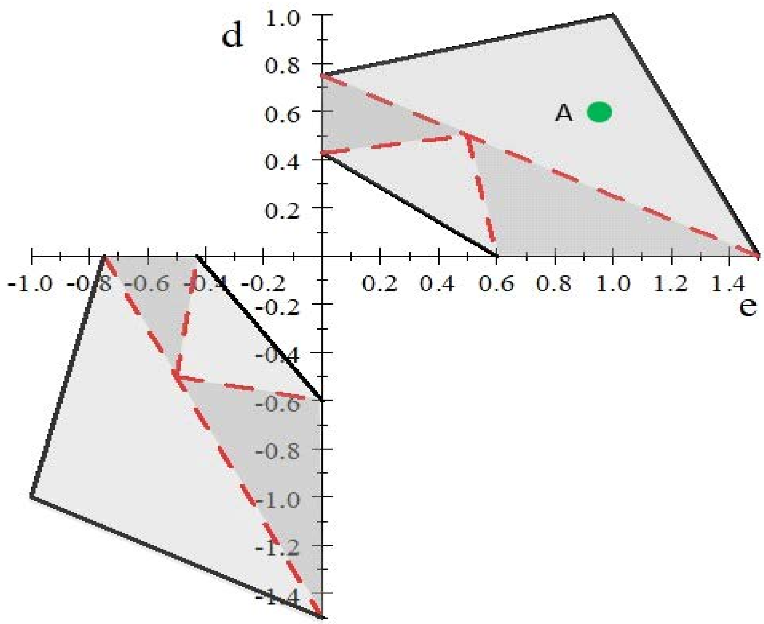

Figure 5: Dynamically consistent and inconsistent selections of formateurs by the head of state.

much smaller region. We have:

Proposition 9. *The head of state can induce the efficient outcome without commitment iff:*

$$\theta \leq 3/(4\vartheta + 2\epsilon) \text{ and either } \theta \geq 3/(\vartheta + 5\epsilon) \text{ or } \theta \leq 3/(7\vartheta - \epsilon) \text{ if } \theta > 0$$

$$\theta \leq 3 \cdot \max\{1/(\vartheta - 7\epsilon), -1/(2\vartheta + 4\epsilon)\} \text{ and } \theta \geq -3/(5\vartheta + \epsilon) \text{ if } \theta < 0.$$

The region defined by the inequalities in Proposition 9 is the darker section delimited by the dashed lines in Figure 5.[43] Proposition 8 and 9 presents two general lessons. First, the head of state can affect legislative bargaining, but his/her power is severely limited. This power is even more limited if the head of state lacks the reputational capital to commit to a course of action.

The second lesson is that the influence of the head of state depends on the relative coalitional values of the potential coalitions. As said, the relative importance of picking the right coalition can then be measured by θ. When θ is high there is a big bonus of choosing well (that is $\{1,3\}$, when $d, e > 0$) and a big cost of choosing badly ($\{1,2\}$, when $d, e > 0$). Proposition 8 says that when θ is sufficiently high, the bargaining process will necessarily gravitate toward the efficient equilibrium and the head of state role is minimal. When θ is sufficiently small, the head of state

[43] In the proof of Proposition 9 we present the formulas also in terms of d and e.

is not irrelevant, but s/he plays a limited role: s/he can not pick the best equilibrium, but can make sure that the worst equilibrium is not selected. It is only for intermediate levels of θ that the head of state is the most powerful in steering the bargaining process, where the selection of the order of formateur is most important and can lead to an efficient equilibrium when done properly, or to even the least efficient equilibrium if not. This dependency of the head of state's power can explain why heads of states may play dramatically different roles over time in the same institutional context. We have mentioned the case of President Scalfaro in Italy (1992-1999) who played a key role in shaping the Italian governments compared to previous presidents (Vassallo [1994]). Here the change in the environment was the fall of the Berlin wall and the end of the cold war, that made the traditional coalition formed by the Christian Democrats (CD) less compelling, and a coalition led by Democratic Party (DP, former Communist party) less of a taboo. The pre-1992 period corresponded to a period in which a coalition $\{1, 3\}$ (led by the CD) was the most viable coalition ($d + e > a$): with the end of the cold war, the stigma of a leftist coalition led by the DP decreased (fall in d), moving the equilibrium to the region with presidential full control (Case 2 in Proposition 8).

7 Extensions

7.1 Alternative bargaining procedures

In the previous analysis we assumed a specific bargaining procedure within a coalition C: its members have a chance of making a proposal in some order given by $\iota(k, C)$. We have also shown that as the interaction between parties become frequent ($\Delta \to 0$), the order is irrelevant and the allocation within the coalition converges to the C-Nash solution (8). An alternative specification of the bargaining inside the coalition is to assume that each member of the coalition $i \in C$ has a probability $q_i(C)$ of being recognized and thus make a proposal. As in the previous analysis, whenever the formateur is recognized, s/he can choose to continue bargaining in C or move to an alternative coalition. The analysis in this variant is qualitatively similar to the analysis presented above, but as it adds an additional source of information (the recognition probabilities $q_i(C)$).

In this case a stationary equilibrium in pure strategies is defined as follows. For a non formateur party j when the formateur is i and i selects coalition C with $j \in C$, a strategy is a function $\sigma_j(C, i) \to X(C) \times [0, V(C)]$ that maps the identity i of the formateur and the coalition C chosen by i to a proposal $\mathbf{x}_j(C, i) = \{x_{j,1}(C, i), ..., x_{j,n(C)}(C, i)\} \in X(C)$ when j is selected as proposer in C, and an acceptance threshold $\mathbf{a}_j(C, i) = \{a_j(C, i)\} \in [0, V(C)]$ when j has to vote.[44] For formateur

[44] Proposal $x_{j,l}(C, i)$ is the surplus allocated to l when j is the proposer in a coalition C chosen by i. The threshold $a_j(C, i)$ is the minimal level of surplus acceptable by j in coalition C chosen by i.

$i \in \mathcal{T}$ a strategy is similarly defined by an allocation strategy $\sigma_i(C,i) \to X(C) \times [0, V(C)]$ defined as above; and by a government proposal $C^f(i)$, that selects the coalition in $\mathcal{C}_i$ chosen by i whenever i becomes formateur and when, during coalitional bargaining s/he is recognized as proposer.

As in the analysis of Section 3, the equilibrium coalition must be optimal for the formateur given the requests of the coalition members, who expect the same coalition to be chosen by the formateur, thus being a fixed-point in (3). Similarly as in (5), the formateur's payoff can be characterized recursively as the fixed-point of a contraction that now takes the form:[45]

$$x_f^*(C, C_f) = \max_{C \in C_f} \left\{ p \left[V(C) - \sum_{l \in C \setminus f} u_l \right] + (1-p) \left[\frac{q_f(C) x_f^*(C, C_f) + (1 - q_f(C)) u_f p}{1 - (1-p)(1 - q_f(C))} \right] \right\} \tag{14}$$

where we use the notation $x_f^*(C, C_f)$ to indicate the formateur's payoff when C is selected and the equilibrium coalition is C_f, to emphasize how it depends on C_f. Naturally, now (14) depends on the recognition probabilities associated to each coalition C. In the online appendix, we show that there is (generically) a unique coalition that the formateur chooses in equilibrium and it is:

$$C_q^* = \arg\max_{C \in C_f} \left\{ (1 - (1-p)(1 - q_f(C))) \left[V(C) - \sum_{l \in C} u_l \right] \right\}$$

Given this, it is immediate to derive equilibrium payoffs for all players as in Proposition 1. It is however useful to see what happens as $\Delta \to 0$. Define the C_q-*Nash Bargaining solution* as:

$$\mathcal{Q}(C, \mathbf{u}) = \arg \max_{\mathbf{x} \in X(C)} \prod\nolimits_{l \in C} [x_l - u_l]^{q_l(C)} \tag{15}$$

where $\mathcal{Q}(C, \mathbf{u}) = \{\mathcal{Q}^1(C, \mathbf{u}), ..., \mathcal{Q}^n(C, \mathbf{u})\}$. This is the weighted Nash Bargaining solution when coalition C is chosen and legislators have weights $\mathbf{q}(\mathbf{C}) = \{q_1(C), .., q_n(C)\}$. We have:

Proposition 10. *As $\Delta \to 0$, the equilibrium of the bargaining problem in the model with recognition probabilities $\mathbf{q}(\mathbf{C})$ converges to $x_i^* = \mathcal{Q}^i(C_f(q), \mathbf{u})$ for $i \in C_f(q)$ where*

$$C_f(q) = \arg\max_{C \in C_f} \left\{ q_f(C) \cdot \left[V(C) - \sum_{l \in C} u_l \right] \right\}$$

and $x_i^* = u_i$ $i \notin C_f(q)$.

As Proposition 2, Proposition 10 can be seen as an institutionally based extension of the Nash Bargaining solution, specifically the weighted Nash Bargaining solution in which the weights are given by the recognition probabilities in the bargaining protocol.

[45] Details of the derivation of (14) are presented in the proof of Proposition 10 in the appendix.

Given Proposition 9, it is straightforward to extend the analysis of Section 4 with endogenous reservation values. The analysis of Section 4 remains completely unchanged if we assume that all parties have the same recognition probabilities, as generally assumed in the analysis of the Baron and Ferejohn's [1989] model. If we allow for heterogenous recognition probabilities, however, the thresholds for the existence of the different types of equilibria become dependent on the weights $\mathbf{q}(\mathbf{C})$, thus providing an additional channel through which the details of the environment may affect the allocation of resources.

7.2 Mixed strategies

The analysis presented above focused on pure strategy equilibria. As we said, there is a good reason for this: for a generic choice of payoffs, the limit of a finite horizon version of the bargaining game must be a pure strategy equilibrium. Any mixed strategy equilibrium can therefore not be seen as the limit of any finite horizon game, it literally requires an infinite horizon. It is however interesting to note that Proposition 5 implies that a pure strategy equilibrium does not always exist. When $d, e < 0$, no pure equilibrium exists if $d > -(a+e)$ and $d < -\frac{3}{2}a - 2e$, i.e. the triangle defined by the dashed efficiency frontier and the solid line of the clockwise equilibrium in the negative orthant of 3. Similarly, when $d, e > 0$, no pure strategy equilibrium exists if $d > \min\left(\frac{3}{4}a + \frac{e}{4}, 3a - 2e\right)$ and $d < a - e$, i.e. the triangle defined by the dashed efficiency frontier and the solid line of the counterclockwise equilibrium in the positive orthant of 3. These are situations in which, if we believe in a literally infinite horizon, the equilibrium can be searched among the mixed equilibria; but if we believe the model has a finite but perhaps long horizon, then the exact number of stages is important, without exact knowledge of this information the outcome is indeterminate.

For completeness in the online appendix, we have characterized a mixed equilibrium in these regions as well. In this equilibrium, party 1 randomizes between forming a coalition with 2 and 3; party 2 forms a coalition with party 1 with probability one, and party 3 forms a coalition with party 2 with probability one (see the lower right quadrant in Figure 3). As we show in the appendix, this equilibrium can be used to show that the head of state can implement the efficient outcome in an equilibrium in these regions in which no pure strategy equilibrium exists.

An interesting qualitative feature of this equilibrium is that it makes it clear how using the "formateur premium" as a way of measuring the advantage of being a formateur may lead to misleading and even puzzling conclusions. Let $\Delta_{i,j} = x_i^i - x_j^i$ be the premium of formateur i when forming a government with j. In the case in which $d, e < 0$, formateur 1 receives a premium $\Delta_{1,2} = -a - 2e < 0$ when $\{1, 2\}$ is chosen (with probability α); and a premium $\Delta_{1,3} = a + e > 0$ when $\{1, 3\}$ (with probability α): so the formateur's premium is simultaneously positive and negative. Naturally, this is irrelevant, since in both cases the payoff of the formateur is the same,

$x_1^1 = a + e$.

7.3 Endogenizing elections

In the preceding analysis we have abstracted from the voting stage that elects the parties to congress, by instead focusing on the bargaining stage for a given set of parties. Clearly the voting stage may play an important role, since it may affect the bargaining stage in many ways. It may, for example, tie the hands of the head of state when selecting the identity of the formateurs. This is the approach followed by Austen-Smith and Banks [1988] and Baron and Diermeier [1999] who assumed that the largest party is the unique designated formateur and focused on how this strategically affects voting to influence bargaining outcomes. The outcome of the elections may also affect bargaining in the proto-coalitioon (selected by the formateur) by affecting the order of proposal or, more significantly, the probability of proposal as studied in Section 7.1. Finally, the electoral outcome may affect the internal composition of parties, thus affecting the value $V(C)$ of a given coalition C. If we assumed that the options for the head of state are constrained (for example that the head of state is forced to choose the largest party), then we delegate the decision to the voters as in Austen-Smith and Banks [1988] and Baron and Diermeier [1999] and we have a theory that maps voting decisions to parliamentary coalitions. The bargaining model presented above may however constitute an important building block of an integrated theory of elections and subsequent coalition formation in congress because it allows us to endogenize the reservation utilities of the parties in congress. Both Austen-Smith and Banks [1988] and Baron and Diermeier [1999] assumed take it or leave it bargaining protocols in which if the formateur's proposal is refused by the proto-coalition, a default policy is selected. Allowing reservation values to be endogenous would drastically limit the ability of the formateur to extract rents from the other parties and change the voters' calculus when attempting to affect bargaining in congress. We leave a comprehensive study of such an integrated model to future research.

8 Conclusions

In this paper we have proposed a new model of multilateral bargaining to study how majorities are formed in legislatures when coalitions are heterogeneous in terms of the surplus they are expected to generate. In our model, a formateur picks a coalition and negotiates for the allocation of the surplus. The formateur is free to change coalition to seek better deals with other coalitions, but s/he may lose her status if bargaining breaks down, in which case a new formateur is chosen. In this context, a formateur needs to reconcile the need to form the most productive coalition with the desire to maximize the share of output that s/he captures. This seems an important feature that has characterized most legislative negotiations in parliamentary democracies in the post World War II period.

The model provides a new perspective on legislative bargaining and helps explain a number of well established empirical facts at odds with existing noncooperative models of multilateral bargaining. From a theoretical point of view, we have shown that, as the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of the Nash bargaining solution in which each coalition member receives its outside option plus a share of surplus net of reservation utilities. The difference with respect to the Nash's solution is that in the n-person Nash Bargaining Solution the coalition is assumed to be comprised by all players (or chosen exogenously), while in our model it is *endogenously* determined. A form of the hold-up problem specific to these bargaining games may lead to significant inefficiencies in the selection of the equilibrium coalition. When reservation utilities are endogeneized in a fully recursive model in which a bargaining breakdown is followed by the appointment of a new formateur, moreover, we may have multiple stationary equilibria with different welfare implications. The equilibrium characterization is however sufficiently tight for positive and normative analysis.

In terms of positive analysis, the model helps explaining three well known empirical facts that have been hard to reconcile with non-cooperative models of multilateral bargaining: the absence of significant (or even positive) premia in ministerial allocations for formateurs and their parties; the occurrence of supermajorities; and delays in reaching agreements. While a number of important previous works have attempted to explain these facts individually, our theory has the advantage of providing a unified and intuitive explanation for all them. Finally, in terms of normative analysis, the model provides a simple framework to study the role of the head of state in legislative bargaining processes, helping to understands the limits of its prerogatives in selecting the formateurs.

There are many directions in which the model presented here can be extended. A direction discussed in Section 7.3. concerns endogenizing elections as in Austen-Smith and Banks [1988] and Baron and Diermeier [1999]. As a starting point, we have assumed that the value $V(C)$ associated to a coalition C is exogenous. In a more general model with elections these values can be made dependent on the campaign platforms and thus endogeneized. The bargaining model presented here may therefore contribute to the formulation of a more complete theory of how policies are made in parliamentary democracies that integrates the determination of the platforms, the determination of the size of the parties and legislative bargaining over governing coalitions. Another direction, concerns dynamic models of debt and public good choice as in Battaglini and Coate [2007, 2008], in which models of legislative bargaining are integrated to dynamic models of accumulation of public goods or debt. We leave these extension for future research.

9 Appendix

9.1 Proof of Lemma 1

Let $x^*_{f,f}(C, C_f)$ be the formateur's payoff when C is proposed, but the equilibrium coalition is C_f. We must have that $x^*_{f,f}(C, C_f) = V(C) - \sum_{i \in C \setminus f} a_{f,i}(C, C_f)$ where, as we did in Section 3, we are using the notation $a_{j,i}(C, C_f)$ to indicate the acceptance threshold of i when j proposes in coalition C and the expected equilibrium coalition is C_f to emphasize that the threshold depends on C_f. It follows that:

$$\begin{aligned} x^*_{f,f}(C, C_f) &= V(C) - p \sum_{l \in C \setminus f} u_l - (1-p) \begin{bmatrix} \sum_{l \in C \setminus \{f, \iota(2,C)\}} a_{\iota(2,C),l}(C, C_f) \\ +V(C) - \sum_{k \in C \setminus \{\iota(2,C)\}} a_{\iota(2,C),k}(C, C_f) \end{bmatrix} \\ &= p \left[V(C) - \sum_{l \in C \setminus f} u_l \right] + (1-p) \left[a_{\iota(2,C),f}(C, C_f) \right] \qquad (16) \end{aligned}$$

Note now that we must have: $a_{\iota(2,C),f}(C, C_f) = pu_f + (1-p) a_{\iota(3,C),f}(C, C_f)$. Iterating this formula $n(C) - 2$ times we have:

$$a_{\iota(2,C),f}(C, C_f) = p \sum_{k=0}^{n(C)-2} (1-p)^k u_f + (1-p)^{n(C)-1} x^*_{f,f}(C_f, C_f) \qquad (17)$$

Substituting (17) in (16), we conclude that in equilibrium we must have:

$$x^*_{f,f}(C_f, C_f) = \max_{C \in \mathcal{C}_f} \left\{ \begin{array}{c} p \left[V(C) - \sum_{i \in C} u_i \right] + p \left[1 + \sum_{k=1}^{n(C)-1} (1-p)^k \right] u_f \\ +(1-p)^{n(C)} \cdot x^*_{f,f}(C_f, C_f) \end{array} \right\} \qquad (18)$$

Recalling that C_f is a coalition that solves (18), from (18) we immediately have that:

$$x^*_{f,f}(C_f, C_f) = u_f + \frac{p \left[V(C_f) - \sum_{i \in C_f} u_i \right]}{1 - (1-p)^{n(C_f)}}. \qquad (19)$$

Assume now that we have an equilibrium in which a $C_f \neq C^*_f$, as defined in (4). Then can write:

$$\begin{aligned} x^*_{f,f}(C^*_f, C_f) &= p \left[V(C^*_f) - \sum_{i \in C^*_f} u_i \right] + p \left[\sum_{k=0}^{n(C^*_f)-1} (1-p)^k \right] u_f + (1-p)^{n(C^*_f)} \cdot x^*_{f,f}(C_f, C_f) \\ &= x^*_{f,f}(C_f, C_f) + \left[1 - (1-p)^{n(C^*_f)} \right] \cdot \left[\frac{p \left[V(C^*_f) - \sum_{i \in C^*_f} u_i \right]}{1 - (1-p)^{n(C^*_f)}} - \frac{p \left[V(C_f) - \sum_{i \in C_f} u_i \right]}{1 - (1-p)^{n(C_f)}} \right] \\ &> x^*_{f,f}(C_f, C_f) \end{aligned}$$

Implying that indeed C_f does not solve the problem in (18) if it does not solve (3), a contradiction. Similarly we have that $x^*_{f,f}(C_f, C^*_f) \leq x^*_{f,f}(C^*_f, C^*_f)$ for any $C_f \in \mathcal{C}^f$: we conclude that the unique fixed-point of (3) is C^*_f. ■

9.2 Proof of Proposition 1

From Lemma 1 we know that one and only one coalition is chosen by the formateur, C_f^* that is the unique fixpoint of (3). We now show that there is a unique distribution of surplus and we characterize it. Let $a_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*)$ be the acceptance threshold of the party who proposes at stage τ when the proposer is the party proposing at stage $\tau+j$. Moreover, $x_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*)$ is the payoff of τ when $\tau+j$ is the proposer. Following the same steps as in the derivation of (17) we have:

$$x^*_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)} = u_{\iota\left(\tau,C_f^*\right)} + \frac{p\left[V(C_f^*) - \sum_{l\in C_f^*} u_l\right]}{1-(1-p)^{n(C_f^*)}} \tag{20}$$

Moreover, we must have:

$$a^*_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)} = pu_{\iota\left(\tau,C_f^*\right)} + (1-p)a_{\iota\left(\tau+j+1,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*) \tag{21}$$

Iterating over (21), we can then write:

$$a^*_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)} = p\sum_{k=0}^{n(C_f^*)-j-1}(1-p)^k u_{\iota\left(\tau,C_f^*\right)} + (1-p)^{n(C_f^*)-j}x^*_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)} \tag{22}$$

for all $j = 1,...,n(C_f^*)-\tau$. Similarly we have:

$$a^*_{\iota\left(\tau-j,C_f^*\right),\iota\left(\tau,C_f^*\right)} = p\sum_{k=0}^{j-1}(1-p)^k u_{\iota\left(\tau,C_f^*\right)} + (1-p)^j x^*_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)} \tag{23}$$

for all $j = 1,...,\tau-1$. The system of equations (20), (22) and (23) gives a complete characterization of the optimal strategy for the agent proposing at stage τ in C_f^* (that is party $\iota(\tau, C_f^*)$).

Clearly, we must have $x^*_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*) = a^*_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*)$. The strategies and equilibrium payoffs are fully characterized by the system of $n(C_f^*)\times n(C_f^*)$ equations:

$$\begin{aligned}
x^*_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)} &= u_{\iota\left(\tau,C_f^*\right)} + \frac{p\left[V(C_f^*) - \sum_{l\in C_f^*} u_l\right]}{1-(1-p)^{n(C_f^*)}} \\
x^*_{\iota\left(\tau+j,C_f^*\right),\iota\left(\tau,C_f^*\right)} &= \begin{bmatrix} p\sum_{k=0}^{n(C_f^*)-j-1}(1-p)^k u_{\iota\left(\tau,C_f^*\right)} \\ +(1-p)^{n(C_f^*)-j}x_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*) \end{bmatrix} \text{ for } j = 1,...,n(C_f^*)-\tau \\
x^*_{\iota\left(\tau-j,C_f^*\right),\iota\left(\tau,C_f^*\right)} &= p\sum_{k=0}^{j-1}(1-p)^k u_{\iota\left(\tau,C_f^*\right)} + (1-p)^j x_{\iota\left(\tau,C_f^*\right),\iota\left(\tau,C_f^*\right)}(C_f^*) \text{ for } j = 1,...,\tau-1
\end{aligned} \tag{24}$$

for all $\tau \in \{1,...,n(C_f^*)\}$. It is immediate to verify that C_f^* and the strategies described in (24) are an equilibrium. To characterize the equilibrium payoffs for the players note that:

$$\begin{aligned} x^*_{f,\iota\left(\tau,C^*_f\right)} &= a^*_{f,\iota\left(\tau,C^*_f\right)} = p\sum_{k=0}^{\tau-2}(1-p)^k u_{\iota\left(\tau,C^*_f\right)} + (1-p)^{\tau-1} x_{\iota\left(\tau,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f) \qquad (25) \\ &= \left[1-(1-p^{\tau-1})\right] u_{\iota\left(\tau,C^*_f\right)} + (1-p)^{\tau-1} x_{\iota\left(\tau,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f) \\ &= u_{\iota\left(\tau,C^*_f\right)} + \frac{p(1-p)^{\tau-1}}{1-(1-p)^{n(C^*_f)}}\left[V(C^*_f) - \sum_{l\in C^*_f} u_l\right] \end{aligned}$$

for all $\tau \geq 2$. All other parties $j \in N\backslash C^*_f$ receive their reservation utility u_j. ■

9.3 Proof of Proposition 2

From Proposition 1, the limit of the formateur's payoff as $\Delta \to 0$ can be written as:

$$\lim_{\Delta\to 0} x^*_{f,f} = \lim_{\Delta\to 0}\left[u_f + \frac{1-e^{-r\Delta}}{1-e^{-rn(C^*_f)\Delta}}\left(V(C^*_f) - \sum_{i\in C^*_f} u_i\right)\right]$$

Applying *l'Hospital* rule, we obtain:

$$\begin{aligned} \lim_{\Delta\to 0} x^*_{f,f} &= \lim_{\Delta\to 0}\left[u_f + \frac{re^{-r\Delta}}{n(C^*_f)(1-p)^{n(C^*_f)-1}re^{-r\Delta}}\left(V(C^*_f) - \sum_{i\in C^*_f} u_i\right)\right] \\ &= u_f + \frac{1}{n(C^*_f)}\left[V(C^*_f) - \sum_{i\in C^*_f} u_i\right] \end{aligned}$$

It follows immediately from (24) and (25) that:

$$\lim_{\Delta\to 0}\left(\frac{x^*_i(\Delta)-u_i}{x^*_f(\Delta)-u_f}\right) = \lim_{\Delta\to 0}\left[(1-p)^{\iota^{-1}(i,C^*_f)-1}\right] = 1$$

It follows that $\lim_{\Delta\to 0}\left(x^*_f(C^*_f) - u_f\right) = \lim_{\Delta\to 0}\left(x^*_i(C^*_f) - u_i\right)$, proving the result. ■

9.4 Proof of Proposition 3

The proof of this result is presented in the online appendix. ■

9.5 Proof of Proposition 4

As in the main text, we adopt here and the proof of the remaining propositions a simplified notation, since it will not lead to confusion. We denote $x^i_j = x_j(\{i,j\}, i)$. We also denote $S_\tau(C)$ as the average surplus in coalition C when τ is the formateur, that is $S_\tau(C) = \frac{1}{2}\left[V(C) - (\sum_{j\in C} x^{\tau+1}_j)\right]$. There are two cases to consider: when d, e are non-negative, and when they are negative. We present here the argument in the first case with $d, e > 0$. The argument with $d, e < 0$ is analogous and presented in the online appendix. In the reminder of this section, we therefore assume $d, e > 0$.

We start by observing that some coalition must form in equilibrium. If this were not the case, then the reservation utilities at any τ would be equal to u_τ. However we are assuming that there is some coalition C such that $V(C) - \sum_{l \in C} u_l > 0$, so any member of C would be able to form a coalition C and profit from it. Given this, we first show that if 2 is unable to form a coalition, then no other party will choose to form a coalition with 2.

Lemma A.4.1. *If 2 is unable to form a coalition, then no other party chooses to form a coalition with 2.*

Proof. We proceed in two steps:

Step 1. Assume then that 3 forms a government with 2. If 1 is unable to form a government, then $x_1^1 = 0$, so $S_3(1,3) = \frac{1}{2}\left[a + e - x_3^1\right]$ and $S_3(2,3) = \frac{1}{2}\left[a - x_3^1 - x_2^1\right] < S_3(1,3)$. Assume then that 1 forms a government with 3. In this case we have:

$$\begin{aligned} x_1^1 &= \frac{a+e}{2} - \frac{x_3^3}{2},\, x_2^1 = 0,\, x_3^1 = \frac{a+e}{2} + \frac{x_3^3}{2} \\ x_1^3 &= 0,\, x_2^3 = \frac{a}{2} - \frac{x_3^1}{2},\, x_3^3 = \frac{a}{2} + \frac{x_3^1}{2} \end{aligned}$$

which implies $x_3^1 = a + \frac{2}{3}e$, $x_1^1 = \frac{1}{3}e$, and $x_3^3 = a + \frac{1}{3}e$. But then $S_3(\{1,3\}) = \frac{1}{2}\left[a + \frac{2}{3}e - x_3^1\right]$ and $S_3(\{2,3\}) = \frac{1}{2}\left[a - x_3^1 - x_3^2\right] < S_3(\{1,3\})$. Finally assume that 1 too forms a government with 2. In this case we must have:

$$\begin{aligned} x_1^1 &= \frac{a-d}{2} - \frac{x_2^3}{2}, x_2^1 = \frac{a-d}{2} + \frac{x_2^3}{2},\, x_3^1 = 0 \\ x_1^3 &= 0, x_2^3 = \frac{a}{2} + \frac{x_2^1}{2} \text{ and } x_3^3 = \frac{a}{2} - \frac{x_2^1}{2} \end{aligned}$$

This implies that we have $S_1(\{1,2\}) = \frac{1}{2}\left[a - d - x_2^3 - x_1^3\right] = \frac{1}{2}\left[a/2 - d - x_2^1/2\right]$ and $S_1(\{1,3\}) = \frac{1}{2}\left[a + e - x_3^3 - x_1^3\right] > S_1(\{1,2\})$, a contradiction.

Step 2. Assume now that 1 forms a government with 2. There are two possible sub-cases (besides the case in which 3 forms a coalition with 2, that was already considered in Step 1). First, the case in which 3 is unable to form a government. In this case $x_3^2 = 0$, so $S_1(\{1,3\}) = \frac{1}{2}\left[a + e - x_1^2\right] > \frac{1}{2}\left[a - d - x_1^2\right] \geq S_1(\{1,2\})$, a contradiction. Second, the case in which 3 forms a government with 1. In this case we have:

$$\begin{aligned} x_1^1 &= \frac{a-d}{2} + \frac{x_1^3}{2},\, x_2^1 = \frac{a-d}{2} - \frac{x_1^3}{2},\, x_3^1 = 0 \\ x_1^3 &= \frac{a+e}{2} + \frac{x_1^1}{2},\, x_2^3 = 0,\, x_3^3 = \frac{a+e}{2} - \frac{x_1^1}{2} \end{aligned}$$

This implies $x_1^1 = a - \frac{2d-e}{3}$ and $x_2^1 = -(d+e)/3 < 0$, a contradiction. ■

It follows from the previous lemma that if 2 fails to form a coalition with some other party, then the efficient coalition must form. To complete the proof of Proposition 3, we only need to show that if 2 is able to form a coalition, then we must be in either a clockwise or a counter-clockwise equilibrium. We first prove that if 2 forms a coalition with 1, then 1 forms a coalition with 3 and 3 with 2, thus we are in a clockwise equilibrium.

Lemma A.4.2. *If* 2 *forms a coalition* $\{2,3\}$*, then* 3 *forms* $\{1,3\}$ *and* 1 *forms* $\{1,2\}$*.*

Proof. We proceed in 4 steps.

Step 1. We first show that if $\{2,3\}$ is formed when 2 is formateur, then $\{3,2\}$ can not form when 3 is formateur. Assume not. Assume first that 1 is unable to form a coalition. In this case we have $S_1(\{1,3\}) = \frac{1}{2}\left[a+e-x_3^2\right]$. Note that $x_2^2 + x_3^2 \leq a$, therefore we have $S_1(\{1,3\}) > 0$: thus implying that 1 can forma profitable coalition with 3, a contradiction. Assume then that 1 forma a coalition with 3. In this case we have:

$$\begin{aligned}
x_1^1 &= \frac{a+e}{2} - \frac{x_3^2}{2},\ x_2^1 = 0,\ x_3^1 = \frac{a+e}{2} + \frac{x_3^2}{2} \\
x_1^2 &= 0,\ x_2^2 = \frac{a}{2} + \frac{x_2^3 - x_3^3}{2},\ x_3^2 = \frac{a}{2} - \frac{x_2^3 - x_3^3}{2} \\
x_1^3 &= 0,\ x_2^3 = \frac{a}{2} - \frac{x_3^1}{2},\ x_3^3 = \frac{a}{2} + \frac{x_3^1}{2}
\end{aligned}$$

This implies that $x_3^2 = a + \frac{e}{3}$, a contradiction since we must have $x_3^2 \leq a$. Finally assume that 1 forms a coalition with 2. In this case we have:

$$\begin{aligned}
x_1^1 &= \frac{a-d}{2} - \frac{x_2^2}{2},\ x_2^1 = \frac{a-d}{2} + \frac{x_2^2}{2},\ x_3^1 = 0 \\
x_1^2 &= 0,\ x_2^2 = \frac{a}{2} + \frac{x_2^3 - x_3^3}{2},\ x_3^2 = \frac{a}{2} - \frac{x_2^3 - x_3^3}{2} \\
x_1^3 &= 0,\ x_2^3 = \frac{a}{2} + \frac{x_2^1}{2},\ x_3^3 = \frac{a}{2} - \frac{x_2^1}{2}
\end{aligned}$$

In this case we have $x_2^2 = a - d/3$, $x_3^2 = d/3$, thus $S_1(\{1,2\}) = -\frac{1}{2}(2/3)d$, $S_1(\{1,3\}) = \frac{1}{2}\left[a+e-d/3\right] > S_1(\{1,2\})$, a contradiction.

Step 2. We now show that if $\{2,3\}$ is formed when 2 is formateur, then 3 must be able to form a coalition. Assume not and, first, that 1 also is unable to form a coalition. In this case, $S_1(\{1,3\}) = \frac{1}{2}\left[a+e-x_3^2\right] \geq \frac{e}{2} > 0$, where the first inequality follows from the fact that $x_3^2 \leq a$:

a contradiction. Assume then that 1 forms a coalition with 3. In this case:

$$\begin{aligned}
x_1^1 &= \frac{a+e}{2} - \frac{x_3^2}{2},\ x_2^1 = 0,\ x_3^1 = \frac{a+e}{2} + \frac{x_3^2}{2} \\
x_1^2 &= 0,\ x_2^2 = \frac{a}{2} - \frac{x_3^1}{2},\ x_3^2 = \frac{a}{2} + \frac{x_3^1}{2} \\
x_1^3 &= x_1^1,\ x_2^3 = 0,\ x_3^3 = x_3^1
\end{aligned}$$

This implies $x_3^1 = a + (2/3)\,e$, $x_1^1 = e/3$ and $x_2^1 = 0$. This implies $S_2(\{2,3\}) = -(1/3)e < 0$, a contradiction. Finally, assume 1 forms a coalition with 2. Note that we must have $x_1^1 \leq a - d$. It follows that $S_3(\{1,3\}) = \frac{1}{2}\left[a + e - x_1^1 - x_3^1\right] \geq \frac{1}{2}\left[e + d\right] > 0$, since $x_3^1 = 0$.

Step 3. We now show that if $\{2,3\}$ is formed when 2 is formateur and $\{1,3\}$ is formed by 3, then 1 must be able to form a coalition. If this were not the case, than we would have:

$$\begin{aligned}
x_1^2 &= 0,\ x_2^2 = \frac{a}{2} - \frac{x_3^3}{2},\ x_3^2 = \frac{a}{2} + \frac{x_3^3}{2} \\
x_1^3 &= \frac{a+e}{2} - \frac{x_3^2}{2},\ x_2^3 = 0,\ x_3^2 = \frac{a+e}{2} + \frac{x_3^2}{2}
\end{aligned}$$

These equations imply: $x_3^3 = (3a + 2e)\,/3$, $x_1^3 = e/3$. Consider now the net surplus when 2 is the formateur: $S_2(\{1,2\}) = (a - d - e/3)\,/2$ and $S_2(\{2,3\}) = -(2/6)e < 0$. This implies that 2 would rather choose $\{1,2\}$ (if $a - d - e/3 \geq 0$) or to form no government (if $a - d - e/3 < 0$), a contradiction.

Step 4. We finally show that if 2 forms $\{2,3\}$ and 3 forms $\{3,1\}$, then 1 can not be able or be willing to form $\{1,3\}$. Assume not, then we would have:

$$\begin{aligned}
x_1^1 &= \frac{a+e}{2} - \frac{x_3^2}{2},\ x_2^1 = 0,\ x_3^1 = \frac{a+e}{2} + \frac{x_3^2}{2} \\
x_1^2 &= 0,\ x_2^2 = \frac{a}{2} - \frac{x_3^3}{2},\ x_3^2 = \frac{a}{2} + \frac{x_3^3}{2} \\
x_1^3 &= \frac{a+e}{2} + \frac{x_1^1 - x_3^1}{2},\ x_2^3 = 0,\ x_3^3 = \frac{a+e}{2} - \frac{x_1^1 - x_3^1}{2},
\end{aligned}$$

The first and last equation in the first line imply $x_1^1 - x_3^1 = -x_3^2$. The last equation in the third line and the third in the second line imply: $x_3^2 = a + e/3$. But this is impossible since $x_3^2 \leq a$.

Steps 1-4 imply that if $\{2,3\}$ is formed when 2 is formateur, then 3 must be able to form a coalition $\{1,3\}$ and 1 must form $\{1,2\}$. ■

We now characterize the equilibria in the case complementary to the case of Lemma A.4.2.

Lemma A.4.3. *If party* 2 *forms a coalition* $\{1,2\}$*, then* 1 *forms* $\{1,3\}$ *and* 3 *forms* $\{3,2\}$*.*

Proof. Assume that formateur 2 forms a coalition $\{2,1\}$, but then 1 also forms a coalition $\{1,2\}$. Then we have $x_3^2 = 0$. This implies that

$$\begin{aligned} S_1(\{1,3\}) &= \frac{1}{2}\left(a+e-x_1^2-x_3^2\right) = \frac{1}{2}\left(a+e-x_1^2\right) \\ S_1(\{1,2\}) &= \frac{1}{2}\left(a-d-x_1^2-x_2^2\right) \leq \frac{1}{2}\left(a-x_1^2\right) \end{aligned}$$

It follows that $S_1(\{1,3\}) > S_1(\{1,2\})$, a contradiction with the assumption that 1 chooses to form a coalition with 2.

Assume that 2 forms with 1, and 1 is unable to form a coalition. Then, no matter what 3 does, we must have $x_3^2 = 0$ and $x_1^2 + x_2^2 \leq a - d$, so $x_1^2 \leq a - d$. It follows that $S_1(\{1,3\}) = \frac{1}{2}\left[a+e-x_1^2-x_3^2\right] \geq \frac{1}{2}[e+d] > 0$, a contradiction.

Assume then that 2 forms with 1, and 1 with 3. There are 2 possible cases to rule out: 3 is unable to form a coalition (Case 1); and that 3 forms with 1 (Case 2).

Case 1. In this case we have:

$$\begin{aligned} x_1^1 &= \frac{a+e}{2} + \frac{x_1^2}{2},\ x_2^1 = 0,\ x_3^1 = \frac{a+e}{2} - \frac{x_1^2}{2} \\ x_1^2 &= \frac{a-d}{2} + \frac{x_1^1}{2},\ x_2^2 = \frac{a-d}{2} - \frac{x_1^1}{2},\ x_3^2 = 0 \\ x_1^3 &= x_1^1,\ x_2^3 = 0,\ x_3^3 = x_3^1, \end{aligned}$$

Solving we find that $x_1^1 = a + \frac{2e-d}{3}$ and $x_1^2 = a + \frac{e-2d}{3}$. Note that $x_1^2 > a - d$, a contradiction with the hypothesis that 2 forms a coalition with 1.

Case 2. In this case we have:

$$\begin{aligned} x_1^1 &= \frac{a+e}{2} + \frac{x_1^2}{2},\ x_2^1 = 0,\ x_3^1 = \frac{a+e}{2} - \frac{x_1^2}{2} \\ x_1^2 &= \frac{a-d}{2} + \frac{x_1^3}{2},\ x_2^2 = \frac{a-d}{2} - \frac{x_1^3}{2},\ x_3^2 = 0 \\ x_1^3 &= \frac{a+e}{2} + \frac{x_1^1 - x_3^1}{2},\ x_2^3 = 0,\ x_3^3 = \frac{a+e}{2} - \frac{x_1^1 - x_3^1}{2}, \end{aligned}$$

We have that $x_1^1 - x_3^1 = x_1^2$, so:

$$\begin{aligned} x_1^3 &= a + \frac{2e-d}{3}, x_2^3 = 0, x_3^3 = \frac{e+d}{3} \\ x_1^2 &= a + \frac{e-2d}{3}, x_2^2 = -\frac{d+e}{3}, x_3^2 = 0 \end{aligned}$$

This implies $x_2^2 = -\frac{d+e}{3} < 0$, thus a contradiction. ■

From Lemmata A.4.1-A.4.3, we conclude that either 2 is able to form a coalition, in which case the equilibrium is clockwise or counter-clockwise; or 2 is unable to form a coalition, in which case the only coalition that can form is $\{1,3\}$, the efficient coalition. ■

9.6 Proof of Proposition 5

We now characterize the set of equilibria. We start from the efficient equilibrium. Lemma A.5.1. deals with the case in which d and e are nonnegative.

Lemma A.5.1. Assume $d, e > 0$. *If $d < a - e$, then 2 forms a coalition with 1 or 3, when given proposal power. If $d \geq a - e$ then there is an efficient equilibrium in which 2 is excluded from any coalition; and 1 and 3 receive a share x_1, x_3 for any x_1, x_3 such that $x_1 \geq a - d, x_3 \geq a, x_1 + x_3 \leq a + e$.*

Proof. We have two cases to consider:

Case 1. We first consider the case $d < a - e$. Assume that when given proposal power, 2 fails to form a government. Then we must have:

$$\begin{aligned} a - d - x_2^3 - x_1^3 &\leq 0 \\ a - x_2^3 - x_3^3 &\leq 0 \end{aligned} \tag{26}$$

else two would be able and find it profitable to form a government with either 1 or 3. By Lemma A.4.1 in Proposition 4 if 2 is unable to form a coalition, then no other party chooses to forms a government with 2, so $x_2^1 = 0, x_2^3 = 0$. It follows from the first inequality in (26) that $x_1^3 \geq a - d$, and from the second that $x_3^3 \geq a$. Note that $x_1^3 + x_3^3 \leq a + e$. We conclude that an equilibrium in which 2 is never in a coalition can occur only if $d \geq a - e$.

Case 2. Consider now the case $d \geq a - e$. Let

$$X^* = \{x_1, x_3 \,|x_1 \geq a - d, x_3 \geq a, x_1 + x_3 \leq a + e\}$$

It is easy to verify that there is an equilibrium in which 2 is unable to form a coalition and never included in any coalition by others; 1 proposes to 3 and 3 proposes to 1; $x_i^1 = x_i^3 = x_i$ for $i = 1, 3$ and $x_1, x_3 \in X^*$; and $x_2^j = 0$ $j = 1, 3$. ■

Lemma A.5.2. deals with the case in which d and e are nonpositive.

Lemma A.5.2. Assume d and e are nonpositive. *If $d > -(a + e)$, then 3 forms a coalition with 1 or 2, when given proposal power. If $d \leq -(a + e)$ then there is an efficient equilibrium in which 3 is excluded from any coalition; and 1 and 2 receive a share x_1, x_2 for any x_1, x_2 such that $x_1 \geq a + e, x_2 \geq a, x_1 + x_2 \leq a - d$.*

Proof. The proof of this result is analogous to the proof of Lemma A.5.1. It is presented for completeness in the online appendix. ■

We now turn to the inefficient equilibria.

Lemma A.5.3 *An equilibrium in which* 1 *forms a coalition with 3, 3 with 2 and 2 with* 1 *(clockwise equilibrium) exists if and only if:* $d \leq \min\left\{3a - 2e, \frac{3}{4}a + \frac{1}{4}e\right\}$ if $d, e > 0$; and $d \geq -\frac{3}{2}a - 2e$ if $d, e < 0$.

Proof. We proceed in three steps. We first characterize the value functions assuming a clockwise equilibrium exists; we then prove that the strategies are optimal responses of the players first assuming $d, e \geq 0$, and finally $d, e < 0$.

Step 1. Let x_j^i be the equilibrium surplus captured by j if i is the formateur. Starting with formateur 1, we must have:

$$x_1^1 = x_1^2 + \frac{a + e - x_1^2 - x_3^2}{2},\ x_2^1 = 0,\ x_3^1 = x_3^2 + \frac{a + e - x_1^2 - x_3^2}{2}. \tag{27}$$

These formula follows from (8) using as outside options the equilibrium values received if the attempt of 1 fails, so formateur 2 is selected (or analogously (6) and (7) as $\Delta \to 0$). Similarly as in (27) we have:

$$\begin{aligned} x_1^2 &= x_1^3 + \frac{a - d - x_1^3 - x_2^3}{2},\ x_2^2 = x_2^3 + \frac{a - d - x_1^3 - x_2^3}{2},\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = x_2^1 + \frac{a - x_2^1 - x_3^1}{2},\ x_3^3 = x_3^1 + \frac{a - x_2^1 - x_3^1}{2} \end{aligned}$$

Equations (27) and (28) give us a system of 3 equations in 3 unknowns that gives us the following solutions:

$$\begin{aligned} x_1^1 &= \frac{6a + 5e - 2d}{9},\ x_2^1 = 0,\ x_3^1 = \frac{3a + 4e + 2d}{9} \\ x_1^2 &= \frac{3a + e - 4d}{9},\ x_2^2 = \frac{6a - e - 5d}{9},\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \frac{3a - 2e - d}{9},\ x_3^3 = \frac{6a + 2e + d}{9} \end{aligned} \tag{28}$$

Step 2. We now verify that they constitute an equilibrium if $d, e > 0$. When 1 is the formateur, by Proposition 2, we have that $\{1, 3\}$ is formed if it maximizes the average surplus of the coalition when 1 is the formateur. Let $S_1(C)$ be the average surplus in coalition C when 1 is the formateur. We have that:

$$\begin{aligned} S_1(\{1,3\}) &= \frac{1}{2}\left(a + e - \sum\nolimits_{j=1,3} x_j^2\right) = \left(a + e - x_1^2\right)/2 \\ &= \frac{6a + 4d + 8e}{18} > 0 \\ S_1(\{1,2\}) &= \frac{1}{2}\left(a - d - \sum\nolimits_{l=1,2} x_l^2\right) = \left(a - d - x_1^2 - x_2^2\right)/2 \\ &= S_1(\{1,3\}) - \frac{1}{2}\left(e + d + x_2^2\right) \leq S_1(\{1,3\}). \end{aligned}$$

Thus we have $S_1(\{1,3\}) > S_1(\{1,2\})$ and $S_1(\{1,3\}) > 0$, implying that $\{1,3\}$ is formed in equilibrium.

When 2 is the formateur, $\{1,2\}$ is formed if $S_2(\{1,2\}) \geq S_2(\{2,3\})$. Since have:

$$\begin{aligned} S_2(\{2,3\}) &= \frac{1}{2}\left(a - \sum\nolimits_{j=2,3} x_j^3\right) = 0 \\ S_2(\{1,2\}) &= \frac{1}{2}\left(a - \sum\nolimits_{j=1,2} x_j^3\right) = \frac{6a - 8d + 2e}{18} \end{aligned}$$

We therefore have $S_2(\{1,2\}) > S_2(\{2,3\})$ if and only if $d \leq \frac{3}{4}a + \frac{e}{4}$.

When 3 is the formateur, $\{2,3\}$ is formed if $S_3(\{2,3\}) \geq S_3(\{1,3\})$. Since have:

$$\begin{aligned} S_3(\{2,3\}) &= a - \sum\nolimits_{j=2,3} x_j^1 = \frac{6a - 4e - 2d}{9} \\ S_3(\{1,3\}) &= a - \sum\nolimits_{j=1,3} x_j^1 = 0 \end{aligned}$$

It follows that the condition is satisfied if and only if $d \leq 3a - 2e$.

We conclude that a clockwise equilibrium exists when $d, e \geq 0$ if and only if $d \leq \min\left\{3a - 2e, \frac{3}{4}a + \frac{1}{4}e\right\}$.

Step 3. We now check when the payoffs in (28) describe an equilibrium if $d, e < 0$. Consider first the case in which 1 is the formateur.

$$S_1(\{1,3\}) = \frac{1}{2}\left(a + e - x_1^2\right) = \frac{6a + 8e + 4d}{18}$$

and $S_1(\{1,2\}) = 0$. It follows that $S_1(\{1,3\}) \geq S_1(\{1,2\})$ if and only if $d \geq -\frac{3}{2}a - 2e$.

Consider now Formateur 2. We have:

$$S_2(\{1,2\}) = \frac{1}{2}\left(a - d - x_2^3\right) = \frac{6a - 8d + 2e}{18}$$

and $S_2(\{2,3\}) = 0$, so $S_2(\{1,2\}) > S_2(\{2,3\})$ if $6a - 8d + 2e \geq 0$. Note that if the condition for formateur 1 is verified, i.e. $d \geq -\frac{3}{2}a - 2e$, then $2e \geq -\frac{3}{2}a - d$, so

$$6a - 8d + 2e \geq \frac{9}{2}a - 9d > 0$$

implying that formateur 2 always finds it optimal to follow the strategies of the clockwise equilibrium. Finally consider formateur 3. We have:

$$S_3(\{2,3\}) = \frac{1}{2}\left(a - x_2^1 - x_3^1\right) = \frac{6a - 4e - 2d}{18} > 0$$

and $S_3(\{1,3\}) = 0$, so $S_3(\{2,3\}) > S_3(\{1,3\})$ is always true. We conclude that a clockwise equilibrium exists when $d, e < 0$ if and only if and only if $d \geq -\frac{3}{2}a - 2e$. ■

We now prove the existence of a counterclockwise equilibrium.

Lemma A.5.4. *An equilibrium in which* 1 *forms a coalition with 2, 2 with 3 and 3 with* 1 *(counterclockwise equilibrium) exists if and only if:* $d \leq \frac{3}{7}a - \frac{5}{7}e$ if $d, e > 0$; and $d \geq -\frac{3}{5}a - \frac{1}{5}e$ and $d \leq 3a + 7e$ if $d, e < 0$.

Proof. The proof of this result is presented in the online appendix. ■

Lemmata A.5.1-A.5.4 define the thresholds presented in Proposition 5. ■

9.7 Proof of Proposition 6-9

The proofs of these results are presented in the online appendix. ■

9.8 Proof of Proposition 10

We must have that $x_f^*(C, C_f) = V(C) - \sum_{l \in C \setminus f} a_l(C, f)$ where, as in Section 7.1, we use the notation $x_f^*(C, C_f)$ to indicate the formateur's payoff when C is selected and the equilibrium coalition is C_f, to emphasize how it depends on C_f. It follows that:

$$\begin{aligned}
x_f^*(C, C_f) &= V(C) - p \sum_{l \in C \setminus f} u_l - (1-p) \sum_{l \in C \setminus f} \left[\begin{array}{c} q_l(C) \left[\begin{array}{c} V(C) \\ -\sum_{k \in C \setminus l} a_k(C, f) \end{array} \right] \\ +(1 - q_l(C)) a_l(C, f) \end{array} \right] \\
&= V(C) - p \sum_{l \in C \setminus f} u_l - (1-p) \left[\begin{array}{c} (1 - q_f(C) \left[V(C) - \sum_{k \in C} a_k(C, f) \right] \\ +V(C) - x_f^*(C, C_f) \end{array} \right] \\
&= p \left[V(C) - \sum_{l \in C \setminus f} u_l \right] + (1-p) \left[q_f(C) x_f^*(C, C_f) + (1 - q_f(C) a_f(C, f) \right]
\end{aligned}$$

Note that by definition, we must have

$$a_f(C, f) = p u_f + (1-p) \left[q_f(C) x_f^*(C, C_f) + (1 - q_f(C) a_f(C, f) \right]$$

Thus, $a_f(C, f) = \frac{p u_f + (1-p) q_f(C) x_f^*(C, C_f)}{1 - (1-p)(1 - q_f(C))}$. It follows that:

$$x_f^*(C, C_f) = p \left[V(C) - \sum_{l \in C \setminus f} u_l \right] + (1-p) \left[\frac{q_f(C) x_f^*(C, C_f) + (1 - q_f(C)) u_f p}{1 - (1-p)(1 - q_f(C))} \right]$$

So,

$$x_f^*(C, C_f) = p u_f + p \left[V(C) - \sum_{l \in C} u_l \right] + (1-p) \left[\begin{array}{c} \frac{(1 - q_f(C)) p}{1 - (1-p)(1 - q_f(C))} u_f \\ + \frac{q_f(C)}{1 - (1-p)(1 - q_f(C))} x_f^*(C, C_f) \end{array} \right]$$

This implies:

$$\left[1-\frac{q_f(C)(1-p)}{1-(1-p)(1-q_f(C))}\right]x_f^*(C,C_f)=pu_f+p\begin{bmatrix} V(C) \\ -\sum_{l\in C}u_l\end{bmatrix}+(1-p)\frac{(1-q_f(C))p}{1-(1-p)(1-q_f(C))}u_f$$

and:

$$\left[\frac{p}{1-(1-p)(1-q_f(C))}\right]x_f^*(C,C_f)=p\left[1+(1-p)\frac{(1-q_f(C))}{1-(1-p)(1-q_f(C))}\right]u_f+p\begin{bmatrix} V(C) \\ -\sum_{l\in C}u_l\end{bmatrix}$$

Implying:

$$\begin{aligned} x_f^*(C,C_f) &= \begin{bmatrix} 1-(1-p)(1-q_f(C)) \\ +(1-p)(1-q_f(C))p\end{bmatrix}u_f+(1-(1-p)(1-q_f(C)))\begin{bmatrix} V(C) \\ -\sum_{l\in C}u_l\end{bmatrix} \\ &= u_f+(1-(1-p)(1-q_f(C)))\left[V(C)-\sum\nolimits_{l\in C}u_l\right] \end{aligned}$$

As $p\to 0$, we obtain: $x_f^*(C,C_f)=u_f+q_f(C)\left[V(C)-\sum_{l\in C}u_l\right]$. We now show that the equilibrium coalition is:

$$C_q^*=\arg\max_{C\in C_f}\left\{(1-(1-p)(1-q_f(C)))\left[V(C)-\sum_{l\in C}u_l\right]\right\}.$$

Assume by contradiction that we have an equilibrium in which a $C_q\neq C_q^*$. Then we have:

$$\begin{aligned} x_f^*(C_q^*,C_q) &= \frac{p}{1-(1-p)(1-q_f(C_q^*))}u_f+p\begin{bmatrix} V(C_q^*) \\ -\sum_{l\in C_q^*}u_l\end{bmatrix}+\left[\frac{(1-p)q_f(C_q^*)\cdot x_f^*(C_q,C_q)}{1-(1-p)(1-q_f(C_q^*))}\right] \\ &= x_f^*(C_q,C_q)+\frac{p}{1-(1-p)(1-q_f(C_q^*))}\begin{bmatrix} \big(1-(1-p)(1-q_f(C_q^*))\big)\begin{bmatrix} V(C_q^*) \\ -\sum_{l\in C_q^*}u_l\end{bmatrix} \\ -(1-(1-p)(1-q_f(C_q)))\begin{bmatrix} V(C_q) \\ -\sum_{l\in C_q}u_l\end{bmatrix}\end{bmatrix} \\ &> x_f^*(C_q,C_q) \end{aligned}$$

Implying that indeed C_q is not optimal for f, a contradiction. Similarly we have that $x_f^*(C_q,C_q^*)\leq x_f^*(C_q^*,C_q^*)$ for any $C_q\in\mathcal{C}_f$. We conclude that in equilibrium C_q^* is chosen. ■

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