LOW INTEREST RATES, MARKET POWER, AND PRODUCTIVITY GROWTH

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## ABSTRACT

How does the production side of the economy respond to a low interest rate environment? This study provides a new theoretical result that low interest rates encourage market concentration by giving industry leaders a strategic advantage over followers, and this effect strengthens as the interest rate approaches zero. The model provides a unified explanation for why the fall in long-term interest rates has been associated with rising market concentration, reduced dynamism, a widening productivity-gap between industry leaders and followers, and slower productivity growth. Support for the model's key mechanism is established by showing that a decline in the ten year Treasury yield generates positive excess returns for industry leaders, and the magnitude of the excess returns rises as the Treasury yield approaches zero.

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# 1   Introduction

Long term interest rates have fallen globally since the 1980s, reaching their lowest levels in recent years. A large body of research explores both the causes and consequences of low long-term interest rates (e.g., Summers (2014)). This study analyzes the consequences of long-term interest rates for the production side of the economy. The question addressed can be put as follows: suppose long-term interest rates fall due to consumer-side issues, how does the production side of the economy respond?

Traditionally, a lower interest rate is viewed as expansionary for the production side of the economy. Consider a typical firm making an investment decision. A decline in the interest rate, all else equal, increases the net present value of future cash flows leading the firm to increase immediate investment. This mechanism explains why the production-side relationship between economic growth and interest rates is typically negative in endogenous growth models.[1] However, these models do not take strategic competition and market structure into account. Is it reasonable to assume that a significant reduction in the long-term interest rate would have no impact on the competitiveness of an industry?

This study examines this question through a model in which competition is introduced between firms and market structure is endogenous. The model is rooted in the dynamic competition literature (e.g., Aghion, Harris, Howitt and Vickers (2001)) where two firms compete in an industry for market share by investing in productivity-enhancing technology. Firms decide whether to invest at each point in time. Investment increases the probability that a firm improves its productivity position relative to its competitor. Such a relative improvement occurs gradually; a follower incrementally reduces the size of the productivity gap between the leader and himself. The decision to invest in the model is a function of the current productivity gap between the leader and the follower, which is the key state variable of the model. A larger productivity gap gives the leader a larger share of industry profits.

The solution to the model reveals two regions of market structure. If the productivity gap be-

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[1]See e.g. Aghion and Howitt (1992), Klette and Kortum (2004), Grossman and Helpman (1991) and Romer (1990). The production-side relationship between growth and the interest rate is flat in exogenous growth models such as the Solow and Ramsey models.

tween the leader and the follower is small, then the industry is in a "competitive region" in which both firms invest in an effort to escape competition. If the productivity gap becomes large, the industry enters a "monopolistic region" in which the follower does not invest due to a "discouragement effect": the prospect of overtaking the leader in the future is too small relative to the cost of investment. If the productivity gap becomes large enough, even the leader stops investing in productivity enhancement as the perceived threat of being overtaken becomes too small. The model includes a continuum of industries, all of which feature the dynamic game between the leader and follower. The state variable of each market is random and is governed by the stochastic process induced by investment decisions. The model shows that aggregate productivity growth, in a steady-state, declines as the fraction of markets that are in the monopolistic region increases.

The key comparative static explored by the model is the effect of a lower interest rate on aggregate productivity growth. In any given industry, a decline in the interest rate has a *traditional effect* of inducing both the leader and the follower to increase investment in productivity enhancement. However, the investment response to a lower interest rate is stronger for the leader relative to the follower. Intuitively, both leaders and followers invest in order to raise productivity, thereby acquiring market power and achieving higher payoffs in the future. The leader is closer to high-payoff states than the follower is; hence, not only is the leader's incentive to invest stronger than that of the follower, but so is the leader's marginal increase in incentive to invest following a decline in the interest rate. A lower interest rate induces firms to be more patient, and more patience leads to a stronger investment response only if the firm can ultimately achieve the high payoffs associated with market leadership. Such high payoffs are more achievable for the leader.

The stronger investment response of the leader to a lower interest rate leads to a *strategic effect* of a decline in the interest rate. In particular, the steady-state average productivity gap between the leader and the follower increases when the interest rate falls due to the unequal investment responses. The increase in the average productivity gap in turn discourages the follower from investing. Due to the strategic effect, the expected time that an industry spends in the monopolistic region increases when the interest rate declines.

The key theoretical result of the model is that the strategic effect dominates the traditional effect as the interest rate approaches zero; as a result, a given industry spends almost all of the

3

time in the monopolistic region at a low enough interest rate. This implies that as the interest rate declines, the fraction of industries in the monopolistic region of the state space expands and aggregate productivity growth falls.

This induces an inverted-U shaped production-side relationship between economic growth and the interest rate. Starting from a high level of the interest rate, growth increases as the interest rate declines because the traditional effect dominates the strategic effect. However, as the interest rate declines further, the endogenous investment response of the leader and follower causes the strategic effect to dominate, and economic growth begins to fall. The key theoretical result shows that this positive relationship between the interest rate and economic growth must happen before the interest rate hits zero.

Is the mechanism behind the model empirically plausible? A unique prediction of the model is that the value of industry leaders increases more than the value of industry followers in response to a decline in the interest rate, and, importantly, the magnitude of the relative increase in value of the leaders versus followers when the interest rate declines is *larger* at a lower initial level of the interest rate.

The empirical analysis tests this hypothesis using CRSP-Compustat merged data from 1962 onward. A "leader portfolio" is constructed that goes long industry leaders and shorts industry followers, and the analysis examines the portfolio's performance in response to changes in the ten year Treasury rate. The model's prediction is confirmed in the data. The leader portfolio exhibits higher returns in response to a decline in interest rates, and this response becomes stronger at a lower initial level of interest rates. The estimated effect is large in magnitude and robust to a number of tests.

The model provides a unified explanation for why the decline in long-term interest rates has been associated with rising market concentration, reduced dynamism, a widening productivity-gap between leaders and followers, and slower productivity growth. The fall in long term rates has been associated with a rise in industry concentration, higher markups and corporate profit

share, and a decline in business dynamism.[2] The dynamics of the empirical evidence are also consistent with the model. More specifically, in both the model and the data, the rise in market concentration precedes a decline in productivity growth.[3] This is not to deny the myriad factors discussed in the literature that may also be able to explain these aggregate patterns; but it is to say that the decline in long-term interest rates may be in important and relatively unexplored partial explanation.

The model also explains cross-sectional patterns in the productivity slowdown found in the literature. In the model, the slowdown in productivity growth is associated with a larger average productivity gap between the industry leader and followers. Using firm-level data from the OECD, Berlingieri and Criscuolo (2017) and Andrews et al. (2016) show that the productivity gap between the 90th versus 10th percentile firms within industries has been increasing since 2000. Moreover, the productivity gap between leaders and followers has risen most in industries where productivity growth has slowed down the most.[4]

The model proposes an alternative explanation for "secular stagnation," or the observation that growth has slowed down as long-term interest rates have fallen toward zero. Current explanations of secular stagnation (e.g., Summers (2014)) focus primarily on the consumer side; in these explanations, frictions such as the zero lower bound on nominal interest rates or nominal rigidity generate a long and persistent slowdown in growth.[5] In the framework presented here, the initial decline in the interest rate may come from a shock to the consumer side that raises the aggregate propensity to save. However, the key point of departure is the insight that if the interest rate falls to a low enough level, then the rise in market concentration may itself constrain growth. In such a framework, one does not need to rely on financial frictions, a liquidity trap, nominal rigidities, or a zero lower bound to explain the persistent growth slowdown.

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[2]See De Loecker and Eeckhout (2017), Barkai (2018), Autor, Dorn, Katz, Patterson and Van Reenen (2017), Gutiérrez and Philippon (2016, 2017), Grullon, Larkin and Michaely (2016), Davis and Haltiwanger (2014), Decker, Haltiwanger, Jarmin and Miranda (2016), Haltiwanger (2015), Hathaway and Litan (2015), and Andrews, Criscuolo and Gal (2016).

[3]The productivity growth slowdown started in 2005, well before the Great Recession, which suggests that structural as opposed to cyclical factors are behind the decline. This is consistent with the model presented here.

[4]Relatedly Gutiérrez and Philippon (2016, 2017) and Lee, Shin and Stulz (2016) show a sharp decline of investment relative to operating surplus and that the investment gap is especially pronounced in concentrated industries. Furthermore, Cette, Fernald and Mojon (2016) show in a two-variable VAR that a negative shock to long-term interest rates leads to a decline in productivity growth.

[5]See e.g., Krugman (1998), Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), Benigno and Fornaro (2019) and Eggertsson, Mehrotra and Robbins (2017).

There has been work in the past that links interest rates to the *level* of productivity (e.g. Caballero, Hoshi and Kashyap (2008); and Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2017)). This paper differs in its explicit modeling of the production side to investigate the relationship between interest rates and the *growth rate* of productivity.

This study also contributes to a large literature on endogenous growth.[6] Our model of within-industry competition as a leader-follower stochastic game builds on the seminal work of Aghion et al. (2001) but differs in that catch-up innovation by the follower happens step-by-step. The follower cannot leap-frog the leader instantly. This is a key assumption of the model, and it helps to explain many of the model's novel predictions relative to the existing literature. While leap-frogging does happen at times, especially during episodes of large technological disruptions, the model is closer to a more typical process of innovation which is gradual.

The derivation of the central theoretical result—that as the interest rate converges to zero, aggregate innovation must decline because the average distance between the leader and follower diverges—provides a technical contribution to the literature on dynamic patent races. Models of dynamic competition as stochastic games are difficult to analyze, and even seminal contributions in the literature either rely on numerical methods (e.g. Budd, Harris and Vickers (1993), Acemoglu and Akcigit (2012)) or impose significant restrictions on the state space to keep the analysis tractable (e.g. Aghion et al. (2001) and Aghion, Bloom, Blundell, Griffith and Howitt (2005)). By deriving first-order approximations of the recursive value functions when the discount rate is small, we are able to provide sharp, analytic characterizations of the asymptotic equilibrium in the limiting case when discounting tends to zero, even as the ergodic subset of the state space becomes infinitely large. The techniques should be applicable to other stochastic games of strategic interactions with a large state space and low discounting.

All proofs are in the appendix.

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[6]Recent contributions to this literature include Acemoglu and Akcigit (2012), Akcigit, Alp and Peters (2015), Akcigit and Kerr (2018), Garcia-Macia, Hsieh and Klenow (2018), Acemoglu, Akcigit, Bloom and Kerr (2019), Aghion, Bergeaud, Boppart and Li (2019), and Atkeson and Burstein (2019), among others.

# 2 Production-side model with investment and strategic competition

## 2.1 Setup

**Consumer**

The consumer side of the model is intentionally simplistic. Time is continuous. There is a representative consumer who, at each instance, chooses consumption $Y(t)$ and supplies labor $L(t)$ according to the within-period utility function $U(t) = \ln Y(t) - L(t)$. The consumption good is aggregated from differentiated goods according to:

$$\ln Y(t) \equiv \int_0^1 \ln y(t; v)\, dv,$$

where $v$ is an index for markets, and $y(t; v)$ is aggregator of each duopoly market:

$$y(t; v) = \left[ y_1(t; v)^{\frac{\sigma-1}{\sigma}} + y_2(t; v)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

$y_i(t; v)$ is the quantity produced by firm $i$ of market $v$.

Let $P(t) \equiv \exp\left( \int_0^1 \ln p(t; v) dv \right)$ be the aggregate price index and $p(t; v) = \left[ p_1(t; v)^{1-\sigma} + p_2(t; v)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$ be the price index for market $v$. We normalize the wage rate to one. This normalization, together with the preference structure, implies that the value of aggregate output as well as the total revenue in each market are always equal to one: $P(t)Y(t) = p(t; v)y(t; v) = 1$.

**Within-market competition**

We now discuss the within-market dynamic game between duopolists. For expositional simplicity, we drop the market index $v$ and describe the game for a generic market.

**Static block**   Over each time instance, the duopolists compete compete à la Bertrand in the product market. Let $z_1(t), z_2(t) \in \mathbb{Z}_{>0}$ denote the (log-)productivity levels of the two market participants; the marginal cost of a firm with productivity $z$ is $\lambda^{-z}$ with $\lambda > 1$.

The CES within-market demand structure in equation (1) and Bertrand competition implies that profits in each market is homogeneous of degree zero in both firms' marginal costs and can therefore be written as functions of their productivity gap rather than the productivity levels of both firms. Specifically, let $s(t) = |z_1(t) - z_2(t)| \in \mathbb{Z}_{\geq 0}$ be the state variable that captures the productivity gap of the two firms. When $s = 0$, the two participants are said to be neck-to-neck; when $s > 0$, one of the firm is a temporary leader ($L$) while the other is a follower ($F$). Let $\pi_s$ denote the profit of the leader in a market with productivity gap $s$, and likewise let $\pi_{-s}$ be the profit of the follower in the market.[7] Conditioning on the state variable, $\pi_s$ and $\pi_{-s}$ no longer depend on the time index or individual productivities $(z_1(t), z_2(t))$ and have the following properties.

**Lemma 1.** *Follower's flow profits $\pi_{-s}$ are non-negative, weakly decreasing, and convex; leader's and joint profits, $\pi_s$ and $(\pi_s + \pi_{-s})$, are bounded, weakly increasing, and eventually concave in $s$ (a sequence $\{a_s\}$ is eventually concave iff there exists $\bar{s}$ such that $a_s$ is concave in $s$ for all $s \geq \bar{s}$).*

It can be easily verfied that $\lim_{s \to \infty} \pi_{-s} = 0$ and $\lim_{s \to \infty} \pi_s = 1$. Nevertheless, our theoretical results apply to any sequence of profits that satisfy the technical properties in Lemma 1, including alternative market structures including Cournot competition and limit pricing (see Appendix A). Hence, we let $\pi \equiv \lim_{s \to \infty} \pi_s$ denote the limiting total profits in each market as $s \to \infty$, and we derive our theory using the notation $\pi$.

A higher productivity gap $s$ is associated with higher joint profits and more unequal profits between the leader and the follower. We interpret state $s$ to be more competitive than state $s'$ if $s < s'$ and more concentrated if $s > s'$. As an example of the market structure, the case of perfect substitutes within market ($\sigma = \infty$) under Bertrand competition generates profit $\pi_s = 1 - e^{-\lambda s}$ for leaders and $\pi_{-s} = 0$ for followers (e.g., see Peters (2016)).

**Dynamic block**   Each firm can invest to improve its productivity, which evolves in step-increments. Investment $\eta_s \in [0, \eta]$ in each state $s$ is bounded above by $\eta$ and carries a marginal cost $c$. Specifically, the firm can choose to pay a cost $c\eta_s$ in exchange for a Poisson rate $\eta_s$ with which the firm's

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[7]These profit functions $\pi_s$ and $\pi_{-s}$ can be written as $\pi_s = \frac{\rho_s^{1-\sigma}}{\sigma + \rho_s^{1-\sigma}}$ and $\pi_{-s} = \frac{1}{\sigma \rho_s^{1-\sigma} + 1}$, where $\rho_s$ is implicitly defined by $\rho_s = \lambda^{-s} \frac{(\sigma + \rho_s^{1-\sigma})\rho_s^{1-\sigma}}{\sigma \rho_s^{1-\sigma} + 1}$. These expressions are derived in Appendix A; also see Aghion et al. (2001).

productivity improves by one step, i.e. cost of production declines proportionally by $\lambda^{-1}$.[8] Given investment decisions $\{\eta_s, \eta_{-s}\}$ over interval $\Delta$ at time $t$, state $s$ transitions according to

$$
s(t + \Delta) = \begin{cases} s(t) + 1 & \text{with probability } \Delta \cdot \eta_s \\ s(t) - 1 & \text{with probability } \Delta \cdot (\kappa + \eta_{-s}) \\ s(t) & \text{otherwise.} \end{cases}
$$

The technology diffusion parameter $\kappa$ is the exogenous Poisson rate that the follower catches up by one step; it can also be seen as the rate of patent expiration.

In the model, firms are forward looking: they invest not only for gains in the flow profits in higher states, but, more importantly, they invest in order to also enhance market positions, thereby enabling them to reach for even higher profits in the future. For the follower, closing the productivity gap by one step enables him to further close the gap in the future and eventually catch up with the leader. For the leader, widening the productivity gap brings higher profits, the option value to further increase the lead in the future, as well as the improved expected duration of market leadership, because it would now take the follower additional steps to catch up.

Firms discount future payoffs at interest rate $r$. We take $r$ to be exogenous for now. Section 2.6 endogenizes $r$ by closing the model in general equilibrium. Each firm's value $v_s(t)$ in state $s$ at time $t$ can be expressed as the expected present-discount-value of future profits net of investment costs:

$$
v_s(t) = \mathbb{E}\left[\int_0^\infty e^{-r\tau}\left\{\pi(t + \tau) - c(t + \tau)\right\} \big| s\right].
$$

We look for a stationary symmetric Markov-perfect equilibrium such that the value functions and investment decisions depend on the state but not the time index. The HJB equations for firms in state $s \geq 1$ are

$$
rv_s \quad = \quad \pi_s + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \max_{\eta_s \in [0, \eta]} \{0, \eta_s(v_{s+1} - v_s - c)\} \tag{2}
$$

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[8]The central results of the model are not dependent on the assumption that investment intensity is bounded with a constant marginal cost. We show in a numerical example in section B of the appendix that our central results are similar when investment is modeled as unbounded with convex marginal costs. The bounded investment with a constant marginal cost allows for an analytical characterization of the equilibrium as the interest rate approaches zero.

$$rv_{-s} = \pi_{-s} + \eta_s \left( v_{-(s+1)} - v_{-s} \right) + \kappa \left( v_{-(s-1)} - v_{-s} \right)$$

$$+ \max_{\eta_{-s} \in [0,\eta]} \left\{ 0, \eta_{-s} \left( v_{-(s-1)} - v_{-s} - c \right) \right\}. \tag{3}$$

In state zero, the HJB equation for either market participants is

$$rv_0 = \pi_0 + \eta_0 \left( v_{-1} - v_0 \right) + \max_{\eta_0 \in [0,\eta]} \left\{ 0, \eta_0 \left( v_1 - v_0 - c \right) \right\}.$$

**Definition 1. (Equilibrium)** Given interest rate $r$, a symmetric Markov-perfect equilibrium is a collection of value functions and investment decisions $\{\eta_s, \eta_{-s}, v_s, v_{-s}\}_{s=0}^{\infty}$ that satisfy the infinite collection of equations in (2) and (3). The collection of flow profits $\{\pi_s, \pi_{-s}\}_{s=0}^{\infty}$ are generated by duopolistic competition in the static block.

The key assumption embodied in the investment technology is that catching up is a gradual process: the productivity gap has to be closed step-by-step, and the follower cannot "leapfrog" the leader by overtaking leadership with one successful innovation. This assumption plays an important role in the results and is the key difference between the model presented here and the setup in Aghion et al. (2001). On the other hand, that technology diffusion parameter $\kappa$ and investment cost $c$ are both state-independent constants is not a crucial assumption for the model presented here, as we discuss later.

**Aggregation: Steady-state and productivity growth**

**Steady-state** In each market, firms engage in both static competition—by maximizing flow profits, taking the productivity gap as given—and dynamic competition—by strategically choosing investment in order to raise their own productivity and maximize the present discounted value of future payoffs. The state variable in each market follows an endogenous Markov process with transition rates governed by the investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$ of market participants. We define a steady-state equilibrium as one in which the distribution of productivity gaps in the entire economy, $\{\mu_s\}_{s=0}^{\infty}$, is time invariant. The steady-state distribution of productivity gap must satisfy the property that, over each time instance, the density of markets leaving and entering each state

must be equal. This implies the following equations:

$$\underbrace{2\mu_0\eta_0}_{\substack{\text{density of markets} \\ \text{going from state 0 to 1}}} = \underbrace{(\eta_{-1} + \kappa)\,\mu_1}_{\substack{\text{density of markets} \\ \text{going from state 1 to 0}}}, \tag{4}$$

$$\underbrace{\mu_s\eta_s}_{\substack{\text{density of markets} \\ \text{going from state s to s+1}}} = \underbrace{\left(\eta_{-(s+1)} + \kappa\right)\mu_{s+1}}_{\substack{\text{density of markets} \\ \text{going from state s+1 to s}}} \quad \text{for all } s > 0, \tag{5}$$

(the number "2" on the left-hand-side of the first equation reflects the fact that a market leaves state zero if either participant makes a successful innovation).

**Definition 2.** (**Steady-State**) Given equilibrium investment $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$, a steady-state is the distribution $\{\mu_s\}_{s=0}^{\infty}$ ($\sum \mu_s = 1$) over state space that satisfies equations (4) and (5).

**Productivity growth**  The aggregate productivity is defined as the total cost of production relative to total value of output. Because the wage rate is normalized to one, aggregate productivity is inversely proportional to the aggregate price index $P(t)$. The aggregate productivity growth rate at time $t$, defined as $g \equiv -d\ln P(t)/dt$, can be written as an average of the productivity growth rate of each market—aggregated from firm-level investment decisions—weighted by the distribution over the productivity gap:

$$g = \ln \lambda \cdot \sum_{s=0}^{\infty} \mu_s \mathbb{E}[g_s].,$$

recalling that $\lambda$ is the proportional productivity increment for each successful investment.

**Lemma 2.** *In a steady state, the aggregate productivity growth rate is*

$$g = \ln \lambda \left( \sum_{s=0}^{\infty} \mu_s\eta_s + \mu_0\eta_0 \right).$$

The lemma shows that aggregate productivity growth can be simplified as the average productivity growth rate of market leaders, weighted by the fraction of markets in each state. Given that productivity improvements by followers also contribute to the growth of aggregate productivity, it might appear puzzling that follower investment decisions ($\eta_{-s}$) are absent from equa-

11

tion (2). The apparent omission is a direct consequence of the fact that, in a steady-state, the productivity growth rate of market leaders is, on average, the same as that of market followers. In fact, as we prove Lemma 2 in Appendix A, we show that aggregate productivity growth rate can also be written as a weighted average productivity growth rate of market followers,

$$g = \ln \lambda \cdot \sum_{s=1}^{\infty} \mu_s(\eta_{-s} + \kappa).$$

## 2.2 Analysis of the equilibrium and steady state

We first analyze the equilibrium structure of the two-firm dynamic game in a generic market, again dropping the market index $\nu$. We then aggregate market equilibrium to the economy and study aggregate comparative statics with respect to the interest rate $r$.

**Equilibrium in each market**

We impose the following regularity conditions.

**Assumption 1.** *1. The upper bound of investment, $\eta$, is sufficiently high: $\eta > \kappa$ and $2c\eta > \pi$. 2. $\pi_1 - \pi_0 > c\kappa > \pi_0 - \pi_{-1}$.*

The first assumption ensures that firms can scale up investment $\eta_s$ to a sufficiently large amount if they choose to. The condition ($\eta > \kappa$) means that, if the follower does not invest and the leader invests as much as possible, then the productivity gap tends to widen on average. The condition ($2c\eta > \pi$) means that if both firms choose to invest as much as possible, the total flow payoff ($\pi_s + \pi_{-s} - 2c\eta$) is negative in any state.

The second parametric assumption rules out a trivial equilibrium in which firms do not invest even when they are state zero, resulting in a degenerate steady-state distribution with zero growth.

Because investment costs are linear in investment intensities, firms generically invest at either the upper or lower bound in any state. Investment effectively becomes a binary decision, and any interior investment decisions can be interpreted as firms playing mixed strategies. For exposition purposes, we focus on pure-strategy equilibria in which $\eta_s \in \{0, \eta\}$, but all of our results hold in mixed-strategy equilibria as well. Also note that even though the dynamic duopoly game does

not always emit an unique equilibrium—because of the discreteness of the state space—we present results that hold across all equilibria.

Let $n + 1$ be the first state in which the market leader chooses not to invest, $n + 1 \equiv \min \{s | \eta_s < \eta\}$; likewise, let $k + 1$ be the first state in which the market follower chooses not to invest, $k + 1 \equiv \min \{s | \eta_{-s} < \eta\}$.

**Lemma 3.** *The leader invests in more states than the follower, $n \geq k$. Moreover, the follower does not invest in states $s = k + 2, ..., n + 1$.*

The lemma establishes that in any equilibrium, the leader must maintain investments in more states than the follower does. To understand this, note that the productivity gap closes at a slow rate $\kappa$ if the follower does not invest in the state and at a faster rate $\eta + \kappa$ if the follower does. Firms are motivated to invest because of the high future flow payoffs after consecutive successful investments. The leader is motivated to invest in all states $s \leq n$ in order to reach state $n + 1$, so that he can enjoy the flow payoff $\pi_{n+1}$ without having to pay the investment costs in that state. The state $(n + 1)$ is especially attractive if the follower does not invest in that state, because the leader can enjoy the payoff for a longer expected duration before the state stochastically transitions down to $n$, after which he has to incur investment cost again.

The follower, on the other hand, is also motivated by future payoffs. He incurs investment costs in exchange for the possibility of closing the gap and catching up with the leader, and for the possibility of eventually becoming the leader himself in the future so that he can enjoy the high flow payoffs. In other words, investment decisions for both forward-looking firms are motivated by high flow profits in the high states, and the incentive to reach these states is stronger for the leader because the leader is closer to those high-payoff states.

Another way to understand the intuition is to consider the contradiction brought by $n < k$. Suppose the leader stops investing before the follower does. In this case, the high flow payoff $\pi_{n+1}$ is transient for the leader and market leadership is fleeting because of the high rate of downward state transition; this implies that the value for being a leader in state $n + 1$ is low. However, because firms are forward-looking and their value functions depend on future payoffs, the low value in state $n + 1$ "trickles down" to affect value functions in all states, meaning the incentive for the

13

follower to invest—motivated by the dynamic prospect of eventually becoming the leader in state $n + 1$—is low. This generates a contradiction to the presumption that follower invests more than the leader does.

Under the lemma, the structure of an equilibrium can be represented by the following diagram. States are represented by circles, going from state $0$ on the very left to state $(n + 1)$ on the very right. The coloring of a circle represents investment decisions: states in which the firm invests are represented by dark circles, while white ones represent those in which the firm does not invest. The top row represents the leader's investment decisions while the bottom row represents the follower's investment decisions.

**Leader invests in the first $n$ states**



**Follower invests in the first $k$ states**
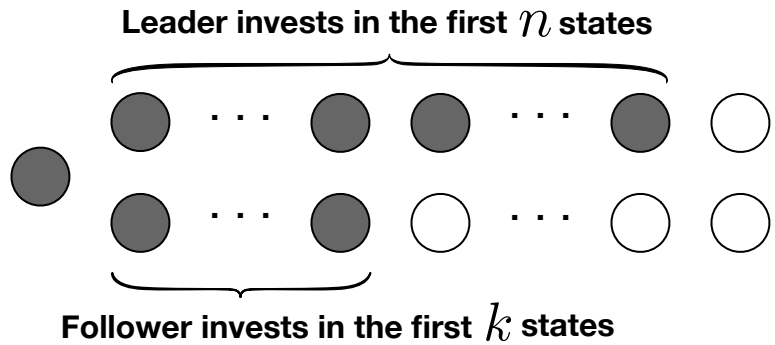
In the diagram, investment decisions are monotone for both firms: starting from state zero, they invest in consecutive states before reaching the respective cutoff state, $k$ and $n$, and then cease investment from there on. This is a manifestation of two effects. First, when the follower is too far behind, the firm value is low and the marginal value of catching up by one step is not worth the investment cost. This is also known as the "discouragement effect" in the dynamic contest literature (Konrad (2012)). Second, the leader's strategy is monotone due to a "lazy monopolist" effect: when the leader is far ahead of the follower, he ceases investment because the marginal gain in value brought by advancing market position is no longer worth the investment cost.

Technically, because leader profits $\{\pi_s\}$ are not always concave, investment decisions are not necessarily monotone in the state, and firms might resume investment after state $n + 1$. That being said, we focus on monotone equilibria in the paper for two reasons. First, given that market leaders do not invest in state $n + 1$, the steady-state distribution of market structure never exceeds $n + 1$, and investment decisions beyond state $n + 1$ are irrelevant for characterizing the steady-

state equilibrium. Second and more importantly, all equilibria follow the monotone structure when interest rate $r$ is small (because $\{\pi_s\}$ is eventually concave in $s$), and our main result concerns the comparative statics of the economy as we take the interest rate $r$ close to zero.

**Analysis of the steady-state**

The fact that the leader invests in more states than the follower enables us to partition the set of non-neck-to-neck states $\{1, \ldots, n+1\}$ into two regions: one in which the follower invests ($\{1, \ldots, k\}$) and the other in which the follower does not ($\{k+1, \ldots, n+1\}$). In the first region, the state transitions up with Poisson rate $\eta$ and transitions down with rate $(\eta + \kappa)$. In expectation, the state $s$ decreases over time in this region, and the market structure tends to move towards being more competitive. For this reason, we refer to this as the *competitive* region. Note that this label is not a reflection of the static profits, which can be very high for leaders in this region. Instead, the label reflects the fact that joint profits tend to decrease *dynamically*. In the second region, the downward transition happens at a lower rate ($\kappa$), and the market structure tends stay monopolistic and concentrated. We refer to this as the *monopolistic* region.
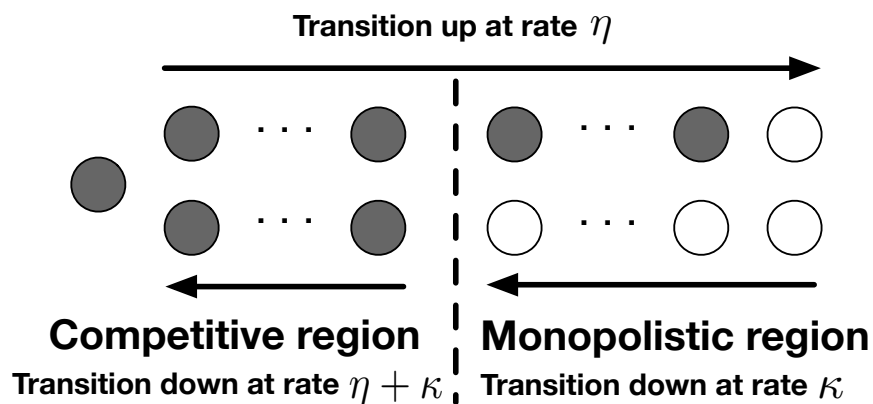


The aggregate productivity growth rate in the economy is a weighted average of the productivity growth in each market; hence, aggregate growth depends on both the investment decisions in each market structure as well as the distribution of market structure, which in turn is a function of the investment decisions. The following lemma shows that the aggregate growth rate can be characterized by the fraction of markets in each region.

**Lemma 4.** *In a steady-state induced by investment cut-off states $(n, k)$, the aggregate productivity growth rate is*

$$g = \ln \lambda \left( \mu^C \cdot (\eta + \kappa) + \mu^M \cdot \kappa \right),$$

*where $\mu^C \equiv \sum_{s=1}^{k} \mu_s$ is the fraction of markets in the competitive region and $\mu^M \equiv \sum_{s=k+1}^{n+1} \mu_s$ is the fraction of markets in the monopolistic region. The fraction of markets in each region satisfies*

$$\mu_0 + \mu^C + \mu^M = 1, \quad \mu_0 \propto (\kappa/\eta)^{n-k+1}(1 + \kappa/\eta)^k,$$

$$\mu^C \propto (\kappa/\eta)^{n-k} \left( (1 + \kappa/\eta)^k - 1 \right), \quad \mu^M \propto \frac{1 - (\kappa/\eta)^{n-k+1}}{1 - \kappa/\eta}.$$

The lemma shows that fractions of markets in the competitive and monopolistic regions are sufficient statistics for steady-state growth, and that markets in the competitive region contributes more to aggregate growth than those in the monopolistic region. This is intuitive: in the competitive region, both firms invest, and, consequently, productivity improvements are rapid, state transition rate is high, dynamic competition is fierce, leadership is contentious, and market power tends to decrease over time. On the other hand, the follower ceases to invest in the monopolistic region, and, once markets are in this region, they tend to become more monopolistic over time. This monopolistic region also includes markets in state $n + 1$, where even the leader stops investing. On average, this region features low rate of state transition and low productivity growth.

The lemma further implies that, conditioning on the follower investing at all ($k \geq 1$), steady-state growth is strictly increasing in the mass of markets in the competitive region and decreasing in the mass of markets in the monopolistic region. Stated in terms of the investment decisions, $g$ is increasing in $k$ decreasing in $(n - k)$. Higher $k$ implies follower investing in more states, thereby raising the steady-state fraction of markets in the competitive region. By contrast, higher $(n - k)$ expands the monopolistic region and reduces the fraction of markets in the competitive region and the neck-to-neck state, with a negative net effect on aggregate productivity growth when $k \geq 1$.

The last result of this section provides a lower bound of steady-state growth rate.

**Lemma 5.** *If followers invest at all ($k \geq 1$), then the steady-state aggregate productivity growth is bounded below by $\ln \lambda \cdot \kappa$, the step-size of productivity increments times the rate of technology diffusion.*

## 2.3 Comparative statics: declining interest rate toward zero

Our key theoretical results concern the limiting behavior of aggregate steady-state variables as the interest rate declines toward zero. Conventional intuition suggests that, *ceteris paribus*, when firms discount future profits at a lower rate, the incentive to invest should increase because the cost of investment is lower relative to future benefits. This intuition holds in our model, and we formalize it into the following lemma.

**Lemma 6.** $\lim_{r\to 0} k = \lim_{r\to 0} (n - k) = \infty.$

The result suggests that, as the interest rate declines toward zero, firms in all states tend to raise investment. In the limit, as firms become arbitrarily patient, they sustain investment even when arbitrarily far behind or ahead: followers are less easily discouraged, and leaders are less lazy.

However, the fact that firms raise investment in all states does not translate into high aggregate investment and growth. These aggregate economic variables are averages of the investment and growth rate in each market, weighted by the steady-state distribution of market structure. A decline in the interest rate not only affects state-dependent investment decisions but also shifts the steady-state distribution of market structures. As Lemma 4 shows, a decline in the interest rate can boost aggregate productivity growth if and only if it expands the fraction of markets in the competitive region; conversely, if more markets are in the monopolistic region—for instance if $n$ increases at a "faster" rate than $k$—aggregate productivity growth rate could slow down.

Our main result establishes that, as $r \to 0$, a slow down in aggregate productivity growth is inevitable and is accompanied by a decline in investment and a rise in market power.

**Proposition 1.** *As $r \to 0$,*

1. *The fraction of markets in the competitive region vanishes, and the monopoly region becomes absorbing:*

$$\lim_{r\to 0} \mu^C = 0; \qquad \lim_{r\to 0} \mu^M = 1.$$

2. *The productivity gap between leaders and followers diverges:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} \mu_s s = \infty.$$

3. Aggregate investment to GDP ratio declines:

$$\lim_{r \to 0} c \cdot \sum_{s=0}^{\infty} \mu_s \left( \eta_s + \eta_{-s} \right) = c\kappa.$$

4. Aggregate productivity growth slows down:

$$\lim_{r \to 0} g = \kappa \cdot \ln \lambda.$$

5. Industry leaders take over the whole market, with high profit shares and markups:

$$\lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s \frac{\pi_s}{p_s y_s} = 1,$$

where $p_s y_s$ is the total revenue of market $s$.

6. Market dynamism declines, and leadership becomes permanently persistent:

$$\lim_{r \to 0} \sum_{s=0}^{\infty} M_s \mu_s = \infty,$$

where $M_s$ is the expected duration before a leader in state $s$ reaches state zero.

7. Relative market valuation of leaders and followers diverges:

$$\lim_{r \to 0} \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.$$

The proposition states that, as $r \to 0$, all markets are in the monopolistic region in a steady-state, and leaders almost surely stay permanently as leaders. Followers cease to invest, and leaders invest only to counteract the exogenous technology diffusion. As a result, aggregate investment and productivity growth decline and converge to their respective lower bounds governed by the parameter $\kappa$.

The proposition highlights two competing forces of low interest rates on steady-state growth. As standard intuition goes, lower rates are expansionary as firms in all states tend to invest more. On the other hand, low rates are also anti-competitive, as leader's investment response to low $r$

is stronger than follower's. Proposition 1 shows that this second, anti-competitive force always dominates when the level of interest rate $r$ is sufficiently low.

The result therefore implies an inverted-U relationship between steady-state growth and the interest rate, as depicted in Figure 1. In a high-$r$ steady-state (i.e. $r > r_g^*$), few firms invest in any markets; consequently, a significant mass of markets are in the neck-to-neck state, and aggregate productivity growth is low. A marginally lower $r$ raises all firms' investments, and the expansionary effect dominates by drawing mass of markets out of the neck-to-neck state into the competitive region. When $r < r_g^*$, however, most markets are in the monopolistic region, in which followers cease to invest, and aggregate productivity growth is again low. The anti-competitive effect of low interest rates also generates other implications: a rising leader-follower productivity gap, diverging relative market valuation of leaders, a rising profit share of the leader, rising average markups, and declining business dynamism.

Figure 1: Steady-state growth: inverted-U
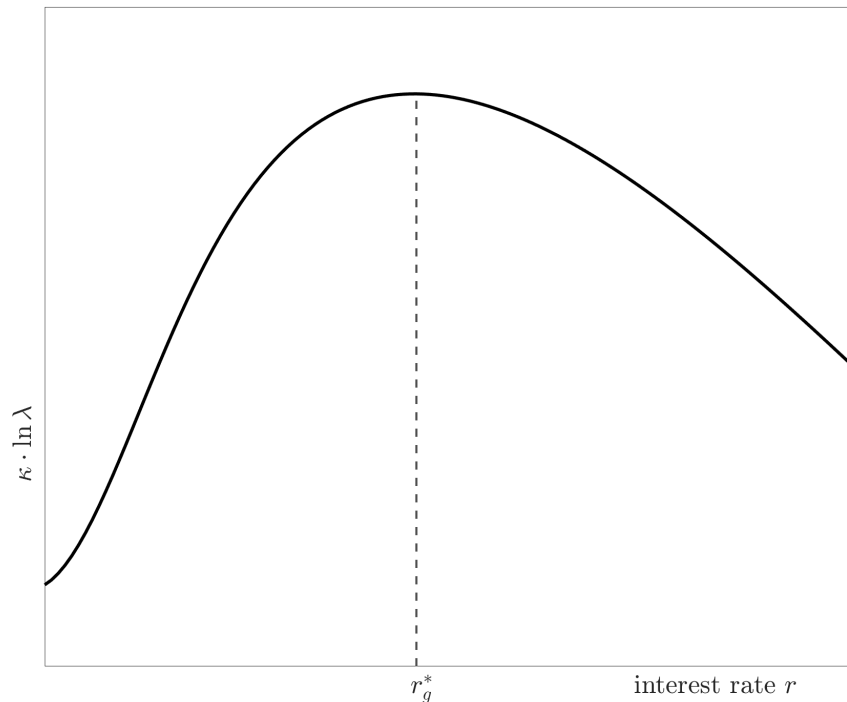


To gain intuitions for why leaders' investments are more responsive to low $r$ than followers',

it is useful to first demonstrate the firm value functions, as shown in the figure below. The solid black curve represents the value function of the leader, whereas the dotted black curve represents the value function of the follower. The two dashed and gray vertical lines respectively represent $k$ and $n$, the last states in which the follower and the leader invest, and together they separate the state space into the competitive and monopolistic regions.

Figure 2: Value functions
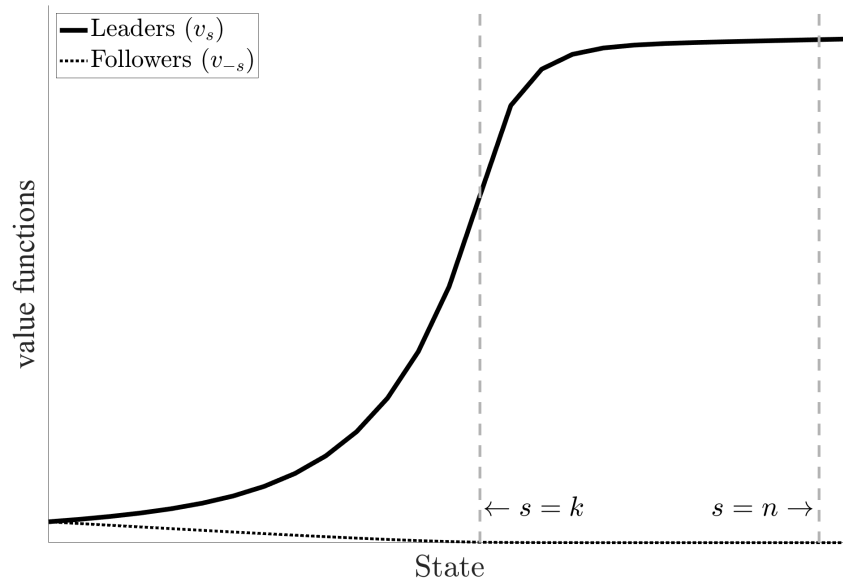


Lemma 6 shows that, as $r \rightarrow 0$, the *number* of states in both competitive and monopolistic regions diverge to infinity. Proposition 1 shows that the *fraction* of markets in the monopolistic region converges to one, which can happen if and only if the *number* of states in the monopolistic region asymptotically dominates that in the competitive region, i.e., the leader raises investment at a "faster rate" in response to lower interest rates than the follower does.

Intuitively, as the figure demonstrates, the firm value of leaders in the competitive region is small relative to the value towards the end of the monopolistic region, close to state $n + 1$. Therefore, a leader would experience a sharp decline in firm value once he falls back from the monopolistic region into the competitive region. A patient leader sustains investment even far into the monopolistic region to avoid the future prospect of falling back; he stops investing if and only the expected duration of staying in the monopolistic region is sufficient long. As a leader becomes

infinitely patient, even the distant threat of losing market power is perceived to be imminent; consequently, leaders scale back investment only if they expect to never leave the monopolistic region, causing market leadership to become endogenously permanent.

Why does a symmetric argument not apply to the follower, i.e., why does a patient follower stop investing once he falls more than $k$ steps behind (even though $k$ could be large)? The reason is that patience motivates the follower to invest *if and only if future leadership is attainable*. Consider the follower's decision in state $k$. As $r \to 0$, $k$ expands, meaning the follower in this marginal state is falling further behind, and—because the leader invests consistently in all states 0 through $k$ to retain leadership—the follower's prospect of catching up—from being $k$ steps behind—and becoming a future leader diminishes. In other words, once sufficiently behind, the prospect of becoming a future leader is perceived to be too low even for a patient follower, who then gives up endogenously.

Proposition 1 is an aggregate result that builds on sharp analytic characterizations of the dynamic game between duopolists in each market. The duopolist game is rooted in models of dynamic patent races and is notoriously difficulty to analyze: the state variable follows an endogenous stochastic process, and firms' value functions are recursively defined hence depend on flow payoffs and investment decisions in every state of the ergodic steady-state distribution $\{\mu_s\}_{s=0}^{n+1}$. Even seminal papers in the literature rely on numerical methods (e.g. Budd et al. (1993), Acemoglu and Akcigit (2012)) or restrictive simplifications[9] to make the analysis tractable. Relative to the literature, our analysis of an economy in a low-rate environment is further complicated by the fact that, as $r$ declines, the ergodic state space $\{0, 1, \cdots, n+1\}$ expands indefinitely.

In order to obtain Proposition 1, we derive a set of results that sharply characterize the asymptotic equilibrium as $r \to 0$, effectively enabling us to analytically solve for the value functions as a first-order approximation around $r = 0$ and analytically characterize the rate at which equilibrium investments ($k$ and $n$) diverge to infinity as $r \to 0$. We relegate the formal proof to the appendix. In what follows, we provide a sketch of the proof, in four steps. Each step aims to explain a specific feature in the shape of value functions. We use $x \to y$ to denote $\lim_{r\to 0} x = y$. Note that

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[9]For instance, Aghion et al. (2001) and Aghion et al. (2005) assume leaders do not invest in all $s \geq 1$, effectively restricting the ergodic state space as $\{0, 1\}$.

flow profits are bounded above by 1 (recall 1 the flow profit earned by a monopolist who charges infinite markup), hence $rv_s \leq 1$ for all $s$.

**Step 1: the value of leader in state $n + 1$ is asymptotically large.** Formally, $\lim_{r \to 0} rv_{n+1} > 0$. To see this, note the leader stops investing in state $n + 1$ if and only if the marginal investment cost is higher than the change in value function, implying

$$c \geq v_{n+2} - v_{n+1} \geq \frac{\pi_{n+2} - rv_{n+1}}{r + \kappa}, \tag{6}$$

where the last inequality follows from rearranging the HJB equation (2) for state $n + 2$. This in turn generates a lower bound for $rv_{n+1}$:

$$rv_{n+1} \geq \pi_{n+2} - c(r + \kappa) \to \pi - c\kappa.$$

**Step 2: the value of follower in state $k + 1$ is asymptotically small.** Formally, $rv_{-(k+1)} = 0$. This is because even a patient follower finds the marginal change in value function $v_{-k} - v_{-(k+1)}$ not worth the investment cost, despite knowing that, if he gives up, the market structure tends to move in the leader's favor indefinitely, as the leader will continue to invest in many states beyond $k + 1$. As $(n - k)$ grows large, $v_{-k} - v_{-(k+1)}$ is small if and only if $v_{-k}$ is low.

**Step 3: the value of being in the neck-to-neck state is asymptotically small.** Formally, $rv_0 \to 0$. This is because as $k \to \infty$, the market in state zero could expect to spend a significant fraction of time in the competitive region (states $s = 1, \ldots, k$), in which both firms invest at the upperbound and the joint flow payoff is negative ($\pi_s + \pi_{-s} < 2c\eta$ under Assumption 1). In an equilibrium, $k$ must diverge at a rate exactly consistent with an asymptotically small $v_0$ ($rv_0 \to 0$). Because firms are forward-looking, an asymptotically large $v_0$ can only be consistent with a slowly rising $k$ as $r \to 0$, but this in turn implies that $v_{-k}$ must be large which contradicts the earlier statement $rv_{-k} \to 0$. Conversely, the fact that $v_0$ must be non-negative (as firms can always guarantee at least zero payoff over every instance) imposes an upper bound on the rate at which $k$ diverges.

**Step 4: a leader experiences an asymptotically large decline in value as he falls from the monopolistic region into the competitive region.** Formally, $\lim_{r \to 0} r(v_{k+1} - v_k) > 0$. This follows from the fact that $v_{n+1}$ is asymptotically large (step 3) and $v_0$ is asymptotically small (step 1).

Step 4 implies that falling back into the competitive region is costly for the leader. Hence, starting from state $k + 1$, the leader keeps investing in additional states to consolidate market power and to diminish the prospect of falling back. The firm value increases as the productivity gap widens, and the leader stops only when the value function is sufficiently high, as characterized by (6). As a leader becomes infinitely patient, he must invest in sufficiently many states beyond $k$ until the prospect of falling back into the competitive region vanishes, thereby perpetuating market leadership and causing the monopolistic region to become endogenously absorbing.

## 2.4   Transitional dynamics: productivity and market power

Proposition 1 implies that, starting from a high level of the interest rate, a declining interest rate is at first expansionary—measured by *steady-state* growth—and only becomes contractionary when $r$ falls sufficiently low; yet, *steady-state* market power tends to rise when $r$ declines, starting from any level.

Something parallel is also true for transitional dynamics after an unanticipated, permanent change in interest rates, though for different reasons. Starting from a steady-state, a permanent decline in the interest rate immediately moves market participants to a new equilibrium, featuring higher investments and productivity growth given any productivity gap (top panel of figure 3). The equilibrium distribution of productivity gaps (markups) starts to rise, although it moves slowly, as depicted in the lower panel.

Over time, as the distribution of state variable converges to the new steady-state and as average productivity gap (markup) increases, the equilibrium growth rate and investment eventually decline to the new steady-state level. Whether productivity growth is higher or lower in the new steady-state, relative to the initial one, depends on the level of the starting interest rate before the interest rate falls. If the starting interest rate is sufficiently low, then the growth rate would be lower in the new steady-state. Productivity growth along the transitional path in such a case is

depicted in the top panel of Figure 3.

Figure 3: Time-path of markup and growth rate following a shock to $r$

**Productivity growth after a one-time, permanent decline in interest rate**



**Average markup after a one-time, permanent decline in interest rate**
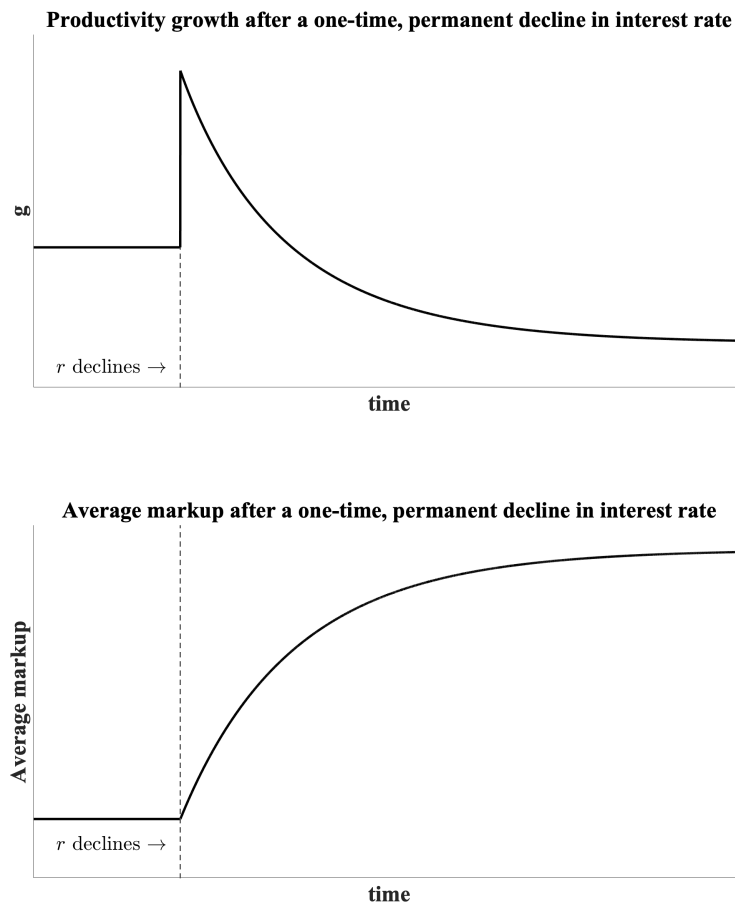


## 2.5 Asymmetric on-impact valuation responses to interest rate shocks

A unique implication of our model is that, starting from a steady-state with low interest rate, a decline in $r$ raises the expected duration of market leadership, thereby immediately raising the firm value of market leaders relative to followers. Moreover, the asymmetric effect is especially pronounced when the initial, pre-shock interest rate is low. We now formalize this result and explain its intuition; in Section 3 we further conduct empirical tests based on this result.

Start from a steady-state economy with interest rate $r$ and consider an unexpected and perma-

nent decline in interest rate $dr$. Lower discounting for future cash flows raises market values of all firms, and we study the immediate, on-impact effect of this shock on the *relative* firm value between market leaders and followers. To this end, let $v_s$ and $\hat{v}_s$ respectively denote the pre- and post-shock value function in state $s$. Define $\frac{\hat{V}^L}{V^L} \equiv \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s \hat{v}_s}$. Note that the denominator evaluates leaders' market value in the new equilibrium using the productivity gap distribution from the pre-shock steady-state; therefore, $\frac{\hat{V}^L}{V^L}$ captures the *on-impact* effect of the interest rate shock $dr$ on total value of market leaders, before the economy starts transitioning to the new, post-shock steady-state. We define $\frac{\hat{V}^F}{V^F}$ analogously for followers.

**Proposition 2.** *Starting from a steady-state with interest rate $r$, a permanent decline in interest rate to $(r - dr)$ has the following on-impact, proportional effect on the valuation of leaders and followers:*

$$\frac{\hat{V}^L}{V^L} = \frac{r}{r - dr} + O(r) \quad \text{and} \quad \frac{\hat{V}^F}{V^F} = \frac{\ln(r - dr)}{\ln r} + O(r) \quad \text{as } r \to 0.$$

The proposition states that, starting from steady-state with low $r$—so that terms represented by $O(r)$ become negligible—a decline $dr$ in the interest rate immediately raises leaders' market value by a proportion of $r/(r - dr)$ and raises followers' value by a proportion of $\ln(r - dr)/\ln r$.

Because $\frac{\hat{V}^L/V^L}{\hat{V}^F/V^F} \approx \frac{r/(r-dr)}{\ln(r-dr)/\ln r}$ increases and diverges to infinity as $r \to 0$, Proposition 2 implies a strong and empirically-testable prediction about the on-impact asymmetric response in market value to declines in interest rate. Starting from a low-$r$ steady-state and following an unexpected further decline in interest rate, market leaders at the time of the shock should experience immediate valuation gains relative to market followers. Furthermore, the asymmetric valuation effects should be more pronounced when the pre-shock interest rate is lower. This last prediction is a strong test of the model's dynamics. In Section 3, we implement a triple difference-in-differences specification to test this key prediction.

We now explain the intuition behind Proposition 2. As the interest rate declines, all firms respond by raising investments. Followers' investment response is especially damaging to the firm value of leaders with small productivity leads—those within or close to the competitive region—as these leaders tend to lose future rents in the new equilibrium. On the other hand, higher $k$ poses little threat for leaders that are far ahead, i.e., those close to the end of the monopolistic region.

These leaders tend to enjoy near-permanent market power, and, consequently, their market value tends to be inversely related to the interest rate.

Proposition 2 aggregates these state-by-state valuation effects to the entire economy. If the initial interest rate is high, the steady-state features significant mass of markets in the competitive region and small mass of markets toward the right-end of the monopolistic region. Consequently, the average leader in the economy experiences a valuation loss relative to the average follower. On the other hand, starting from a low-$r$ steady-state, a large fraction of markets are deep into the monopolistic region, and, therefore, the average leader experiences a valuation gain relative to the average follower in the economy. The lower is the initial interest rate, the stronger is the asymmetry.

Figure A1 in the appendix demonstrates the time path of the aggregate valuation effects. Consider a decline in $r$ that hits a steady-state economy at time $t_0$, and define

$$\chi(t) \equiv \frac{\sum_{s=0}^{\infty} \mu_s(t) v_s(t)}{\sum_{s=0}^{\infty} \mu_s(t) v_{-s}(t)} \bigg/ \frac{\sum_{s=0}^{\infty} \mu_s(0) v_s(0)}{\sum_{s=0}^{\infty} \mu_s(0) v_{-s}(0)}.$$

$\chi(t)$ tracks the over-time proportional changes in the *total* market value of all leaders relative to all followers, as compared to the relative valuation in the pre-shock steady-state. $\chi(t)$ is equal to one before $t_0$. Following a decline in interest rate at $t_0$, market valuation changes immediately, resulting in a discrete jump in $\chi(t_0)$, the magnitude of which is an average of the on-impact valuation response in each individual state, weighted by the distribution over productivity gaps in the initial steady-state. As Proposition 2 suggests, a low interest rate shock benefits market leaders on-impact, especially when the initial interest rate is low.

Figure 4 demonstrates Proposition 2, that a decline in the interest rate has asymmetric on-impact effects on the market value of leaders and followers. Starting from a high-level of interest rate ($r > r_{val}^*$), a decline in $r$ hurts leaders on average; yet, starting from a low-level of $r$ ($r < r_{val}^*$), a further decline in $r$ unambiguously causes leaders' market value to appreciate relative to followers', and the asymmetry becomes stronger when the initial pre-shock level of interest rate is lower.

Figure 4: Asymmetric valuation effects
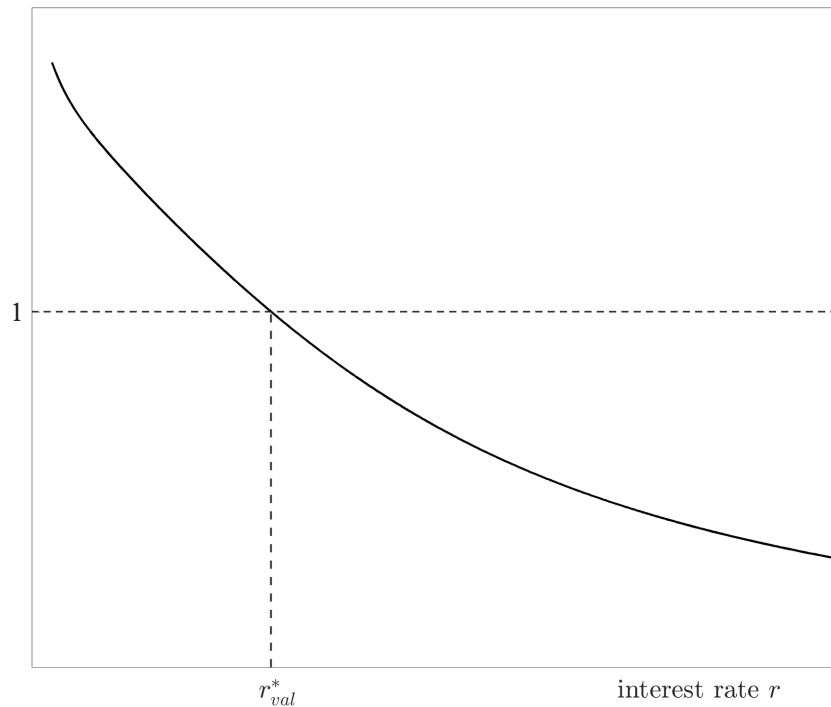


## 2.6 Robustness and general equilibrium

As highlighted earlier, the incremental nature of catch up is important for our results and is a key distinction between our model and the seminal work of Aghion et al. (2001). Our model thus applies to situations where innovation is incremental rather than revolutionary. A large fraction of innovation is incremental in practice, and Bloom, Jones, Reenen and Webb (2017) suggest that innovation is becoming more incremental in nature over time. However, there are of course times, such as the industrial revolution, when new technologies allow industry entrants to leapfrog industry leaders. To the extent such a technological wave hits the economy, it can overcome the negative competitive effects of low interest rates and boost productivity growth.

The baseline model makes the additional assumption that the technologies for productivity improvements—the exogenous catch-up rate $\kappa$ and the marginal cost of investment $c$—are independent of the productivity gap between the two firms. This assumption can be relaxed however without materially changing any of our conclusions. For example, consider an alternative envi-

ronment in which the both $\kappa_s$ and $c_s$ are increasing and bounded in the productivity gap $s$ for the leader, with $\lim_{s \to \infty} c_s = \bar{c}$ and $\lim_{s \to \infty} \kappa_s = \bar{\kappa}$. It is easy to show that Propositions 1 and 2 remain to hold as long as the upper bounds $\bar{\kappa}$ and $\bar{c}$ satisfy assumption 1. Intuitively, our results characterize the asymptotic equilibrium as $r \to 0$, and, consequently, the finite state-to-state variations in $\kappa_s$ and $c_s$ do not affect firms behavior when they are sufficiently patient.

The model also assumes that the marginal cost of investment $c$ is independent of the investment intensity $\eta_s$, and we assume $\eta_s$ is bounded above by $\eta$. These assumptions are there for analytic tractability, and are not critical. In the appendix we relax these assumptions and solve the model numuerically to show that our main propositions survive.

Finally, the focus has been on modelling the production side of the economy. It is relatively straightforward to close the model in general equilibrium by adding an Euler equation from the consumer side, following Aghion et al. (2001) and Benigno and Fornaro (2019). Doing so explicitly identifies where movement in interest rate might come from, i.e. shifts in the Euler equation, and solves for equilibrium interest rate and growth rate as well. Specifically, the inter-temporal preferences

$$\int_0^\infty U(t)^{\frac{\theta-1}{\theta}} e^{-\rho t} dt$$

generate an Euler equation $g(t) = \dot{Y}(t) = \frac{1}{\theta}(r(t) - \rho)$, which is an upward-sloping relationship between aggregate growth rate $g$ and interest rate $r$. Coupled with thr production side inverted-U relationship between $g$ and $r$, the two curves pin down the level of growth rate and interest rate on a balanced growth path, along with a stationary distribution of productivity gaps across markets.

Interest rate shocks that we refer to in the main text can be simply seen as shocks to the consumer discount rate $\rho$ (as in Krugman (1998)). An issue with this interpretation is that the interest rate is bounded below by the discount rate $\rho$, and any positive growth rate is incompatible with the interest rate being close to zero even as $\rho \to 0$. This issue is an artifact of the consumer side being frictionless; any incomplete markets and idiosyncratic risk would generate additional terms in the Euler equation that push down the real interest rate for any level of the growth rate. For instance, Benigno and Fornaro (2019) use idiosyncratic, uninsurable unemployment risk to micro-

found the following Euler equation

$$g(t) = \frac{1}{\theta}(r(t) - \rho + b).$$

The term $b \geq 0$ measures the severity of the unemployment risk under their specific microfoundation model, but it can be more broadly seen as a catch-all term for any shock on the consumer side that pushes consumers towards saving more and consuming less, including changes in preferences, tightened borrowing constraints (e.g. Eggertsson and Krugman (2012)), or structural shifts such as an aging population and rising inequality (e.g. Summers (2014)).

The figure below shows the consumer-side Euler equation as an upward sloping line. An inward shift in the consumer-side curve lowers the interest rate and increases concentration. If the prevailing interest rates are low, i.e. economy is in the upward sloping region of the production-side curve, then a fall in interest rate is also contractionary as productivity growth slows.

Figure 5: Growth and interest rates in general equilibrium
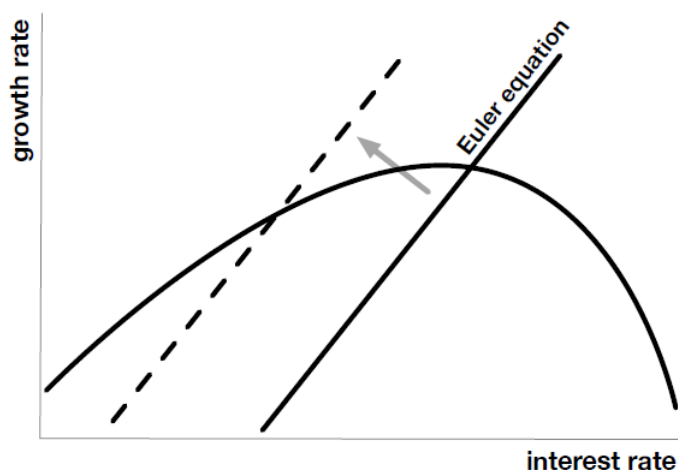


Hence, the model presents an alternative interpretation of "secular stagnation." As in traditional secular stagnation explanations, an initial inward shift in the consumer-side curve can lower equilibrium interest rates to very low levels. However, "stagnation" is not due to monetary constraints such as the zero lower bound or nominal rigidities. Instead, a large fall in interest rates can make the economy more monopolistic for reasons laid out in the model, thereby lowering innovation and productivity growth.

# 3 Empirical evidence

## 3.1 Testing the asymmetric effects of interest rate shocks on firm value

Proposition 2 implies that, starting from a low level of the interest rate, a decline in the interest rate immediately raises the relative market value of leaders versus followers, and this effect is especially pronounced when the initial level of the interest rate is low. The empirical analysis presented here implements a triple difference-in-difference specification to test this prediction.

The data set for the analysis is the CRSP-Compustat merged data set from 1980 onward, which is used to compute excess returns for industry leaders versus followers in response to a change in interest rates. The 10-year Treasury yield is used as the default measure of the long-run interest rate, and robustness tests using the real interest rate and alternative definitions of the interest rate yield are also shown.[10]

The analysis focuses on 1980 onward as the default time period since this is the period over which the most consistent time series (e.g., for the real interest rate) is available. Nonetheless, robustness tests are shown for the earliest available CRSP-Compustat data set from 1960 onward. The 10-year yield is used because it is the longest available historical time series. The Fama-French definition is the default classification for industries, and results using alternative definitions of industries are shown as robustness tests.

The baseline definition of industry "leaders" is size as measured by market value. A firm is classified as an industry leader if it is in the top 5 percent of firms in the industry based on market value at the beginning of the period when excess returns are computed. We also use the top five firms in an industry for robustness. Robustness results in the appendix show similar results when sorting firms based on EBITDA and sales.

The key empirical test implied by the model at the firm level can be written as,

$$R_{i,j,t} = \alpha_{j,t} + \beta_0 D_{i,j,t-1} + \beta_1 D_{i,j,t-1} * \Delta i_t + \beta_2 D_{i,j,t-1} * i_{t-1} + \beta_3 D_{i,j,t-1} * \Delta i_t * i_{t-1} + \varepsilon_{i,j,t} \quad (7)$$

---

[10]We prefer using the nominal interest rate given the measurement error introduced in attempting to measure the real interest rate. Inflation expectations have been relatively well-anchored during the time period analyzed.

where $R_{i,j,t}$ is the dividend and split-adjusted stock return of firm $i$ in industry $j$ from date $t - 91$ days to $t$ (i.e., one quarter growth), and $D_{i,j,t-1}$ is an indicator variable equal to 1 if firm $i$ is in the top 5% of market capitalization in its industry $j$ at date $t - 91$. Firms with $D_{i,j,t-1}$=1 are called leaders while the rest are called followers. The variable $i_t$ is the 10-year interest rate, with $i_{t-1}$ being the interest rate 91 days prior and $\Delta i_t$ being the change in the interest rate from date $t - 91$ to $t$. All regressions are value-weighted and standard errors are dually clustered by industry and date. The parameters $\alpha_{j,t}$ are industry-time period fixed effects.

The key coefficients of interest are $\beta_1$ and $\beta_3$. A negative estimate of $\beta_1$ implies that a decline in the interest rate leads to a larger increase in the stock return of industry leaders. A positive estimate of $\beta_3$ implies that this effect is stronger when the level of interest rates is lower. In other words, a negative estimate of $\beta_1$ and a positive estimate of $\beta_3$ signify that industry leaders experience higher excess returns when interest rates fall, and this effect is amplified when interest rates start from a low level. This is the key prediction of the model.

Table 1 shows the results of estimating (7) on our merged CRSP-Compustat data set from 1980 onwards. Only the relevant coefficients are displayed in the tables, but the actual regression includes all variables specified in the equation (7). Column (1) estimates equation (7) without interactions with the level of interest rate. The coefficient $\beta_1$ is negative and significant; leaders earn positive excess return when the interest rate falls.

Column (2) presents estimates from the full specification (7). The coefficient $\beta_3$ is positive and significant as predicted by the model. Excess returns for leaders are *higher* in response to a fall in the interest rate when the level of the interest rate is lower. This is succinctly captured by $\beta_1$ which reflects the increase in excess returns when interest rates fall near the zero lower bound (i.e., when $i_{t-1} \approx 0$). The excess return near the zero lower bound in column (2) (3.88) is three times the average excess return of 1.19 in column (1).

One concern with these results is that the measure of industry leaders is spuriously correlated with balance sheet factors that are more sensitive to interest rate movements. For example, perhaps leaders are more levered and a fall in the interest rate helps lower the interest burden. To test for this, and other related concerns, we include a number of firm level characteristics as controls

Table 1: Differential Interest Rate Responses of Leaders vs. Followers: Top 5 Percent

| | Stock Return | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Top 5 Percent=1 x $\Delta i$ | -1.187*** | -3.881** | -4.415*** | -3.582** | -4.005*** | -4.182*** |
| | (0.260) | (1.113) | (0.893) | (1.104) | (0.907) | (0.529) |
| | | | | | | |
| Top 5 Percent=1 x $\Delta i$ x Lagged $i$ | | 0.293** | 0.346*** | | | 0.301*** |
| | | (0.095) | (0.079) | | | (0.045) |
| | | | | | | |
| Top 5 Percent=1 x $\Delta i$ x Lagged real $i$ (Clev) | | | | 0.540** | 0.943*** | |
| | | | | (0.197) | (0.195) | |
| | | | | | | |
| Firm $\beta$ x $\Delta i$ | | | | | | 14.10*** |
| | | | | | | (0.795) |
| | | | | | | |
| Firm $\beta$ x $\Delta i$ x Lagged $i$ | | | | | | -1.260*** |
| | | | | | | (0.082) |
| Sample | All | All | All | All | All | All |
| Controls | N | N | Y | N | Y | |
| Industry-Date FE | Y | Y | Y | Y | Y | Y |
| N | 61,313,604 | 61,313,604 | 44,104,181 | 59,305,323 | 43,279,694 | 61,299,546 |
| R-sq | 0.403 | 0.403 | 0.415 | 0.400 | 0.413 | 0.409 |

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t}\Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t}\Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm $i$ in industry $j$ at date $t$. $\Delta \ln\left(P_{i,j,t}\right)$ is defined here as the log change in the stock price for firm $i$ in industry $j$ from date $t-91$ to $t$ (one quarter growth). $D_{i,j,t}$ is defined here as an indicator equal to 1 at date $t$ when a firm $i$ is in the top 5% of market capitalization in its industry $j$ on date $t-91$. Firms with $D_{i,j,t}=1$ are called leaders while the rest are called followers. $i_t$ is defined as the nominal 10-year Treasury yield, with $i_{t-1}$ being the interest rate 91 days prior and $\Delta i_t$ being the change in the interest rate from date $t-91$ to $t$. Controls $X$ include a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and percent of pre-tax income that goes to taxes. Industry classifications are the Fama-French industry classifications (FF). Lagged real rates were built using monthly 10-year inflation expectations from the Cleveland Fed and the daily 10-year Treasury yield at the beginning of each month (post-1982). Standard errors are dually clustered by industry and date.

by including all the interaction of the firm level characteristic with the change in interest rate as well as the level of the interest rate. We include the following firm-level characteristics, a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and the percent of pre-tax income that goes to taxes. The number of observations decreases because we have to limit the sample to Compustat firms with the available data on firm financials. Column (3) shows that the inclusion of this extensive list of firm-level controls does not change the coefficients of interest materially.

Columns (4) and (5) use ten year real interest rate for the *level* of lagged interest rate in equation (7). The change in the interest rate continues to be measured by the change in the 10-year nominal bond yield given that there are no reasonable estimates of the change in the real yield over short

time intervals. Furthermore, the change in nominal and real yields over short horizon is likely to be dominated by the change in the nominal interest rate. The real interest rate is calculated by subtracting 10-year inflation expectations published by the Cleveland Federal Reserve. Using the real interest rate, the coefficient ($\beta_3$) on the interaction term increases significantly.

Column (6) controls for another potentially spurious firm-level attribute. What if industry leaders are spuriously more cyclical? If a fall in the interest rate represents changing economic expectations, industry leaders might generally be more responsive to changing market conditions irrespective of the level of interest rate. To test for this possibility, the market beta of each firm is estimated using historical data as of $t-1$ and then it is interacted with both the change in the interest rate and the level of the interest rate in column (6). As before, the main coefficients of interest are not materially affected.

Table 2 performs a time-series version of the excess return test implemented in Table 1. In particular, the results are based on the following specification,

$$R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t * i_{t-1} + \varepsilon_t \tag{8}$$

where $R_t$ is the market-capitalization weighted average of returns for a stock portfolio that goes long industry-leader stocks and goes short industry-follower stocks from date $t-91$ to $t$. We refer to this portfolio as the "leader portfolio." Given that observations have overlapping differences, we compute standard errors using a Newey-West procedure with a maximum lag length of 60 days to account for built-in correlation. A negative estimate of coefficient $\beta_1$ would signify that a decline in interest rates boosts the return on the leader portfolio, while a positive estimate of $\beta_2$ would signify that the positive response of the return on the leader portfolio to a decline in interest rates is larger when the level of the interest rate is lower.

The estimates reported in columns (1) and (2) confirm earlier results. A decline in the interest rate is associated with positive returns for the leader portfolio, and this positive return response to a decline in the interest rate is larger in magnitude when the interest rate is lower. Column (3) uses the 10-year real interest rate level as before. The coefficient on the interaction between the real rate and the change in interest rate is even stronger than in Table 1 (0.54 versus 0.30).

Table 2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent

| | Portfolio Return | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $\Delta i_t$ | -1.150*** | -3.819*** | -3.515*** | -2.268*** | -3.657*** | -3.115*** |
| | (0.309) | (0.641) | (0.591) | (0.602) | (0.949) | (0.768) |
| $i_{t-1}$ | | 0.0842 | | 0.0336 | 0.160* | 0.148* |
| | | (0.050) | | (0.044) | (0.071) | (0.070) |
| $\Delta i_t \times i_{t-1}$ | | 0.294*** | | 0.117* | 0.328*** | 0.255* |
| | | (0.059) | | (0.056) | (0.081) | (0.100) |
| real $i_{t-1}$ (Clev) | | | 0.179* | | | |
| | | | (0.079) | | | |
| $\Delta i_t \times$ real $i_{t-1}$ (Clev) | | | 0.535*** | | | |
| | | | (0.120) | | | |
| Excess Market Return | | | | -0.168*** | | |
| | | | | (0.023) | | |
| High Minus Low | | | | 0.0371 | | |
| | | | | (0.044) | | |
| $(\Delta i_t > 0)$=1 $\times \Delta i_t$ | | | | | 0.341 | |
| | | | | | (1.717) | |
| $(\Delta i_t > 0)$=1 $\times \Delta i_t \times i_{t-1}$ | | | | | -0.102 | |
| | | | | | (0.170) | |
| PE Portfolio Return | | | | | | -0.293*** |
| | | | | | | (0.074) |
| N | 9,016 | 9,016 | 8,597 | 9,016 | 9,016 | 7,402 |
| R-sq | 0.044 | 0.089 | 0.087 | 0.228 | 0.092 | 0.219 |

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date $t$. $R_t$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to $t$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-91$. $i_t$ is defined as the nominal 10-year Treasury yield, with $i_{t-1}$ being the interest rate 91 days prior and $\Delta i_t$ being the change in the interest rate from date $t-91$ to $t$. Standard errors are Newey-West with a maximum lag length of 60 days prior. Real rates were built using monthly 10-year inflation expectations from the Cleveland Fed and the daily 10-year Treasury yield (post-1982). In column 5, the terms $(\Delta i_t > 0) = 1$ and $(\Delta i_t > 0) = 1$ x $i_{t-1}$ were suppressed from the table. Their coefficients are 0.0222 (0.602) and -0.0616 (0.086), respectively.

Column (4) shows that the results are not driven by the excess market return of the HML factor.[11] Column (5) shows that the results are driven by both positive and negative changes in interest rates. In particular, the excess return results are materially unchanged whether only positive changes in interest rates or only negative changes in interest rate are used.

There may be a concern that industry leaders tend to have a high price-to-earnings ratio and that such growth firms benefit disproportionately from a decline in long-term interest rates be-

[11]We cannot control for the small minus big size factor because it is very highly correlated with the leader portfolio.

cause more of their earnings are in the future. A test for this concern is presented in the specification reported in column (6), which controls for a "PE portfolio" that is long the top 5% of firms by PE in an industry and short the rest. Inclusion of the PE portfolio return does not change the coefficients of interest, and the leader-minus-follower portfolio is itself negatively corrected with the PE portfolio.[12]

Tables A1 and A2 in appendix show the robustness of results in Tables 1 and 2 to using CRSP-Compustat data from 1960 onward.[13] Table A3 in the appendix shows robustness to alternative definitions: top 5 instead of top 5 percent for industry leadership, SIC instead of Fama French industry classification, and sorting on EBITDA and sales instead of market value for defining leadership.

The baseline specification constructs returns and interest rate changes at a quarterly frequency. This reflects our view that this is the appropriate frequency because it captures interest rate movements that are deemed more permanent. Figure 6 plots the histograms of interest rate changes in the sample, from daily to annual frequency. On average interest rates went down during this time period. However, there is substantial variation with the change in the interest rate being positive on a high fraction of days. As already shown, the key findings are symmetric to whether the change in the interest rate is positive or negative.

As one moves from daily to annual frequency, the range of interest rate changes increases. This is another reason to focus on longer term differences; investors need sufficient time to incorporate a large change in interest rates when forming expectations. Table A4 in the appendix repeats the core specification for interest rate changes at frequencies ranging from daily to annual. For reasons discussed above, the effect tends to be stronger when the interest rate change is computed over longer horizons.

Another robustness test concerns the exact interest rate used in the specification. For example, do the excess return results depend on whether the change in the interest rate is at the short versus the long end of the yield curve? Statistically this is a somewhat hard test to perform because interest rate movements along the yield curve tend to be highly correlated. Table A5 in the appendix

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[12]The number of observations declines because earnings data are missing on certain dates.
[13]Real interest rate prior to 1980 is computed by subtracting realized inflation from the nominal rate.

Figure 6: Distribution of Interest Rate Changes at Varying Frequencies
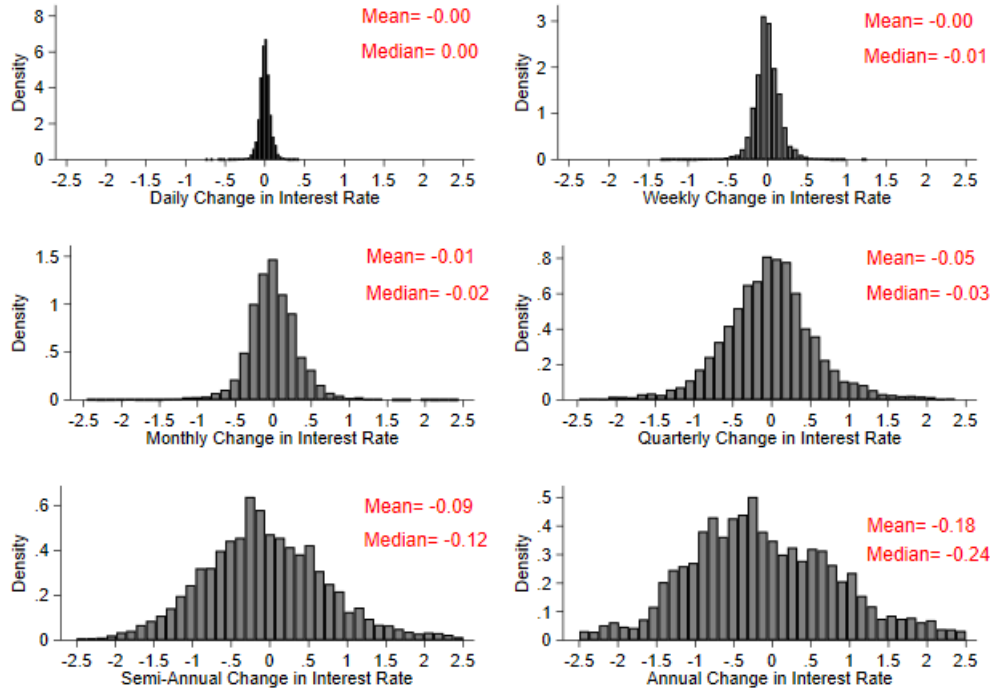
The panels plots the histograms of interest rate changes in our sample, from daily to annually.

shows the correlation matrix of quarterly changes in forward rates of varying non-overlapping durations. The correlations are generally quite high, leading to problems of collinearity in joint testing. The lowest correlation is in the range of 0.7 to 0.75 between change in 0-2 forward rate and longer term forward rates (e.g. 10-30).

Table 3 presents estimates of equation (8) using the forward rate of varying duration. The main takeaway is that the results shown above are similar for interest rate changes throughout the yield curve (columns (1) through (6)). When both the 0-2 and 10-30 forward rates are put in the specification together (columns (7) and (8)), both ends of the yield curve appear to be independently important, with some evidence that the longer end of the yield curve is more important.

Figure 7 plots the coefficients $\{\beta_{0,j}\}$ of the following specification:

$$R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t * i_{t-1} + \varepsilon_t \tag{9}$$

Table 3: Portfolio Returns Response to Interest Rate Changes: Along the Yield Curve

| | 30-Year | | 2-Year | | 10-30 Forward | | 2-Year & 10-30 Fwd. | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $\Delta i_t$ | -1.129** | -4.537*** | | | | | | |
| | (0.348) | (0.826) | | | | | | |
| $\Delta i_t \times i_{t-1}$ | | 0.362*** | | | | | | |
| | | (0.077) | | | | | | |
| $\Delta i_{t,0,2}$ | | | -0.584* | -3.535*** | | | -0.126 | -2.066* |
| | | | (0.244) | (0.833) | | | (0.349) | (0.970) |
| $\Delta i_{t,0,2} \times i_{t-1}$ | | | | 0.280*** | | | | 0.145 |
| | | | | (0.069) | | | | (0.080) |
| $\Delta i_{t,10,30}$ | | | | | -1.084** | -4.165*** | -0.938 | -3.138** |
| | | | | | (0.354) | (0.835) | (0.523) | (1.043) |
| $\Delta i_{t,10,30} \times i_{t-1}$ | | | | | | 0.334*** | | 0.289** |
| | | | | | | (0.080) | | (0.107) |
| N | 8,006 | 8,006 | 8,065 | 8,065 | 8,006 | 8,006 | 8,006 | 8,006 |
| R-sq | 0.036 | 0.078 | 0.021 | 0.063 | 0.030 | 0.066 | 0.031 | 0.084 |

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date $t$ in columns 1-2 and $R_t = \alpha + \beta_0 i_{t-1} + \beta_{1,1} \Delta i_{t,0,2} + \beta_{1,2} \Delta i_{t,10,30} + \beta_{1,3} \Delta i_{t,0,10} + \beta_{2,1} \Delta i_{t,0,2} i_{t-1} + \beta_{2,2} \Delta i_{t,10,30} i_{t-1} + \beta_{2,3} \Delta i_{t,0,10} i_{t-1} + \varepsilon_t$ at date $t$ in columns 3-8. $R_t$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ to $t$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - J$. $i_t$ is defined as the nominal 30-year Treasury yield, with $i_{t-1}$ being the interest rate $J$ days prior and $\Delta i_t$ being the change in the interest rate from date $t - 91$ to $t$. $i_{t,0,2}$, $i_{t,0,10}$ and $i_{t,10,30}$ are the 2-year and 10-year Treasury yield and 10 to 30 forward Treasury yield, respectively. Standard errors are Newey-West with a maximum lag length of 60 days prior. We cannot reject that the main and interaction coefficients in columns 7 and 8 are not equal.

$R_{t+j}$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t$ to $t + j$. In this specification, $\Delta i_t$ is defined as the change in the interest rate from date $t$ to $t + 91$. The coefficients $\beta_{0,j}$ can be interpreted as the effect of a change in interest rates from $t - 91$ to $t$ on the returns of the leader portfolio from time $t$ to time $t + j$ when the level of interest rates at $t - 1$ is equal to zero. In other words, the figure represents the impulse response function at a daily frequency of the leader portfolio return to a change in interest rate over one quarter.

As the figure shows, the effect of a change in interest rates starts quickly but the full effect is not realized until about 90 days. Further, there is no evidence of reversal over the following quarter. The increase in the value of the leader portfolio is persistent.

Figure 7: Impulse Response of Changes in Interest Rate when Rate is Zero
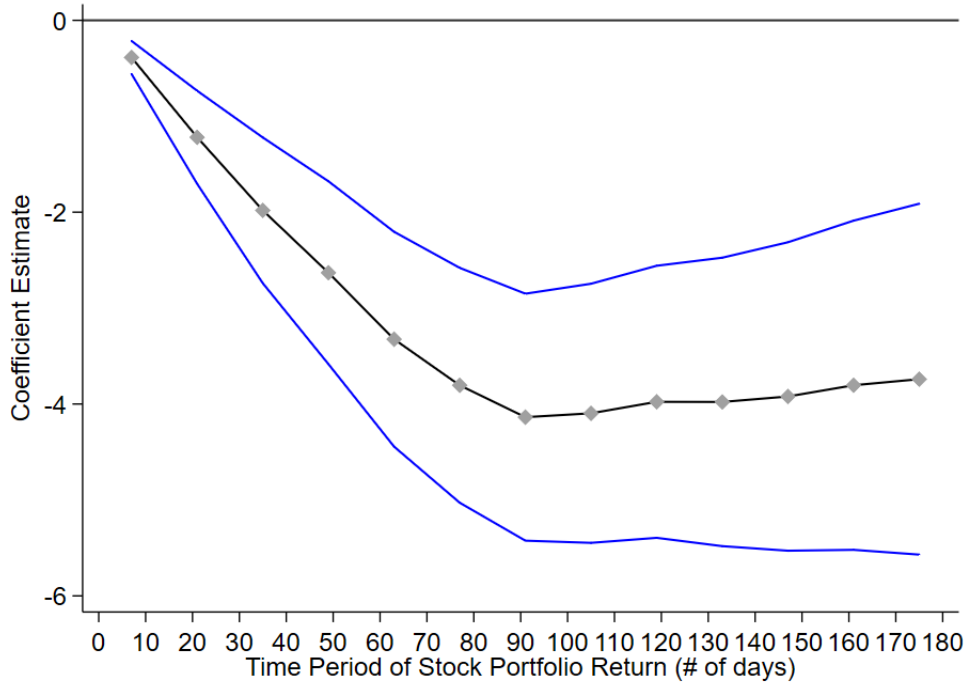


The figue plots the coefficients $\{\beta_{0,j}\}$ of the specification $R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t * i_{t-1} + \varepsilon_t$ at date $t$. $R_{t+j}$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t$ to $t + j$. $\Delta i_t$ is defined here as the change in the interest rate from date $t$ to $t + 91$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t$. Standard errors are Newey-West with a maximum lag length of 60 days prior.

## 3.2 Profits, business dynamism, and productivity

The empirical evidence above supports the key mechanism behind the model: a decline in long-term rates raises the leader's valuation more than the follower's, and this effect is stronger at low interest rates. As the model explains, a natural aggregate implication of this force is that falling interest rates make market structure more monopolistic. This is also confirmed in the data.

The left panel of Figure A4 in the appendix plots the profit share of GDP against the 10-year nominal U.S. Treasury rate over time and it shows a negative correlation. The right panel shows a negative correlation between the share of market value that goes to the top-5 firms within an industry and the 10-year treasury rate. Market concentration increases as the long-term interest rate declines.

The model also shows that as interest rates decline, "business dynamism," defined as the like-

lihood of a follower overtaking the leader, declines. One proxy for firm dynamism uses establishment entry and exit information for the United States from 1985 to 2014.[14] Figure A5 in the appendix shows that lower interest rates are associated with both a decline in the entry and exit rates of establishments in the United States. From the model's perspective, this decline in business dynamism reflects higher market power of industry leaders and the reduced incentives to enter new markets by followers.

The model also makes predictions on productivity growth. In particular, Proposition 1 shows that as interest rates decline toward zero, the steady state average gap in productivity growth between leaders and followers widens. Berlingieri and Criscuolo (2017) use firm level productivity data from OECD countries and estimate productivity separately for "leaders", defined as firms in the 90th percentile of the labor productivity distribution for a given 2-digit industry, and "followers" defined as firms in the 10th percentile of the distribution.

The authors show that, consistent with Proposition 1, the gap between leaders and followers increased steadily from 2000 to 2014 as long term interest rates fell. Figure A6 in the appendix shows their main result. The left panel shows that as interest rates fell from 2000 to 2015, the gap in labor productivity between leaders and followers (within an industry) grew for both manufacturing and services firms. The right panel shows that the larger gap in productivity between leaders and followers is robust to using alternative multi-factor productivity measures. A similar result is found in the study by Andrews et al. (2016), which shows a widening gap in labor productivity of frontier versus laggard firms in both manufacturing and services for OECD firms.

The study by Andrews et al. (2016) also shows additional cross-sectional evidence in support of the model. In particular, the study shows that industries in which the productivity gap between the leader and the follower is *rising* the most are the same industries where sector-aggregate productivity is *falling* the most. This is a subtle prediction of the model that is borne out in the data. Both the model and the data suggest that low interest rates lead to a large gap between the leader and follower in productivity investment; as a result, total investment in productivity of the industry falls.

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[14]The establishment entry and exit rates time series for the United States comes from the US Census' Business Dynamics Statistics database.

Aggregate evidence on productivity growth is also consistent with proposition 1. As is well-known in the productivity literature (e.g., Cette et al. (2016)), there is a marked slowdown in productivity growth in many advanced economies around 2005, well before the Great Recession. This suggests that there is a common global factor, not related to the Great Recession, that may be responsible for the growth slowdown. The model suggests the low level of long-term rates is a possible factor.[15]

## 3.3 Using empirical results to calibrate $r_g^*$ and $r_{val}^*$

One of the key results of the model is that the impact of low interest rates on market competition can be strong enough to have a negative overall effect on productivity growth. The negative effect of low interest rates starts to dominate in the *upward-sloping* region of Figure 1 to the left of $r_g^*$. The parameter $r_g^*$ is therefore important to calibrate because it tells us the likelihood of being in the region where low interest rates are harmful for growth.

What is $r_g^*$? One possible way to calibrate $r_g^*$ is by picking exact values for the parameters of the model, such as the marginal cost for investment $c$ and the exogenous catchup rate $\kappa$, and then solving for $r_g^*$. However, such a method would be speculative at best given that the model parameters are not directly observable.

An alternative is a "sufficient statistic" approach to calibrating $r_g^*$. The basic idea exploits the close relationship between $r_g^*$ and $r_{val}^*$ in Figure 4. The parameter $r_{val}^*$ is the level of the interest rate below which the marginal impact of a decline in the interest rate is stronger for the leader's valuation. The parameter $r_g^*$ is unobserved but the parameter $r_{val}^*$ can be estimated in the data. As discussed below, $r_g^* > r_{val}^*$ in all the cases that we numerically investigate. Therefore, $r_{val}^*$ gives us a lower bound on $r_g^*$.

The intuition for why $r_g^* > r_{val}^*$ is as follows. Lemma 4 implies that $r_g^*$ is the critical level of the interest rate at which the average market starts to leave the competitive region and enter the monopolistic region as $r$ declines. On the other hand, Section 2.5 shows that only leaders who

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[15]As in the model, investment as a share of GDP has also declined with lower interest rates. In related papers, Jones and Philippon (2016) show that increase in industry concentration is associated with lower firm investment and Gutiérrez and Philippon (2016) show that investment is not correlated with market valuation and profitability after 2000s.

are close to the end of the monopolistic region—those close to state $n + 1$—benefit from lower $r$ relative to the average followers. For the average leader in the market to experience relative valuation gains from negative interest rate shocks, the initial, pre-shock interest rate $r(< r^*_{val})$ must be sufficiently low so that the average market is deep into the monopolistic region and far away from the competitive region. This can be true only if $r^*_{val}$ is significantly lower than $r^*_g$, consistent with the numerical finding. While it is not possible to show this in closed form, we have numerically explored an extensive range of parameter values for the model, and found that $r^*_g > r^*_{val}$ in each case.

Let $\hat{r}^*_{t-1}$ be the empirically observed real interest rate below which $\beta = \frac{\partial R_t}{\partial \Delta i_t}$ is negative, where as before $R_t$ is the excess return on the leader portfolio and $\Delta i_t$ is the interest rate shock. The estimate $\hat{r}^*_{t-1}$ is the empirical analog to $r^*_{val}$ from the discussion above: it is the real interest rate below which the relative performance of the leader portfolio increases with a decline in the interest rate.

Figure 8: Estimating $\beta(i^*) = \frac{\partial R_t}{\partial \Delta i_t}\big|_{i_{t-1}=i^*}$
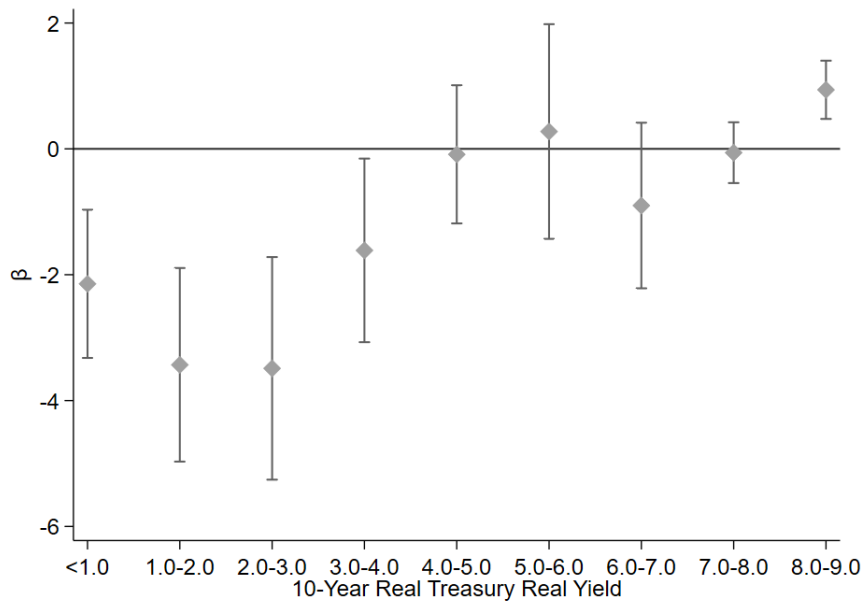


The figure plots coefficient $\beta$ from the specification $R_t = \alpha + \beta \Delta i_t + \varepsilon_t$, when the real interest rate at $t - 91$ days is within the range on the x-axis. $R_t$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ days to $t$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - 91$. $\Delta i_t$ is the change in the 10-year treasury interest rate from date $t - 91$ to $t$. Standard errors are Newey-West with a maximum lag length of 60 days prior. Real rates are calculated using monthly 10-year inflation expectations from the Cleveland Fed and the daily 10-year Treasury yield post-1982.

Figure 8 reports estimates of $\beta$ for different samples based on the lagged 10-year real Treasury rate. Moving from right to left in the figure, the estimate of $\beta$ first becomes statistically significantly negative when $r_{t-1}$ is between 3 and 4%, and the estimate of $\beta$ becomes even more negative for $r_{t-1}$ in the range of 1 to 3%. According this estimation, $\hat{r}^*_{t-1}$, the empirical analog to $r^*_{val}$, is between 3 and 4%. This result suggests and the economy has operated in the upward-sloping region of Figure 1 in recent years.

# 4   Conclusion

The focus of this paper is on understanding how the production side of the economy responds to a reduction in long-term interest rates driven by consumer-side forces. The existing literature in growth either assumes no production-side response to declining interest rates, or a positive response driven by an increased incentive to invest in the face of a higher discounted present value of future profits. The point of departure from this literature lies in explicitly modeling competition within an industry and analyzing how lower interest rates effect the nature of competition. The model builds on the dynamic contests literature to show that in a fairly general set up and without relying on any financial or other forms of frictions, the effect of lower interest rates on growth in a low interest rate regime can be negative.

A reduction in long-term interest rates tends to make market structure less competitive within an industry. The reason is that while both the leader and follower within an industry increase their investment in response to a reduction in interest rates, the increase in investment is always stronger for the leader. As a result, the gap between the leader and follower increases as interest rates decline, making an industry less competitive and more concentrated. When interest rates are already low, this negative effect of lower interest rates on industry competition tends to lower growth and overwhelms the traditional positive effect of lower interest rates on growth. This produces a hump-shaped inverted-U production-side relationship between growth and interest rates.

The model delivers a distinctive *upward* sloping production-side curve in a low interest rate regime. We believe that this insight is empirically relevant and useful in understanding the slow-

down in productivity growth in recent decades and the broader discussion regarding "secular stagnation." The slowdown in productivity growth is global as it shows up in almost all advanced economies. The slowdown started well before the Great Recession, suggesting that cyclical forces related to the crisis are unlikely to be the trigger. And the slowdown in productivity is highly persistent, lasting well over a decade. The long-run pattern suggests that explanations relying on price stickiness or the zero lower bound on nominal interest rates are unlikely to be the complete explanation.

This paper introduces the possibility of low interest rates as the common global "factor" that drives the slowdown in productivity growth. The mechanism that the theory postulates delivers a number of important predictions that are supported by empirical evidence. A reduction in long term interest rates increases market concentration and market power in the model. A fall in the interest rate also makes industry leadership and monopoly power more persistent. There is empirical support for these predictions in the data, both in aggregate time series as well as in firm-level panel data sets.

# References

**Acemoglu, Daron and Ufuk Akcigit**, "Intellectual Property Rights Policy, Competition and Innovation," *Journal of the European Economic Association*, 2012.

_ , _ , **Nicholas Bloom, and William Kerr**, "Innovation, Reallocation and Growth," *American Economic Review, forthcoming*, 2019.

**Aghion, Philippe and Peter Howitt**, "A Model of Growth Through Creative Destruction," *Econometrica*, 1992, *60* (2), 323–351.

_ , **Antonin Bergeaud, Timo Boppart, and Huiyi Li**, "Missing Growth from Creative Destruction," *American Economic Review, forthcoming*, 2019.

_ , **Christopher Harris, Peter Howitt, and John Vickers**, "Competition, Imitation and Growth with Step-by-Step Innovation," *The Review of Economic Studies*, 2001, *68*, 467–492.

_ , **Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt**, "Competition and Innovation: An Inverted-U Relationship," *The Review of Economic Studies*, 2005.

**Akcigit, Ufuk and William Kerr**, "Growth through Heterogeneous Innovations," *Journal of Political Economy*, 2018.

_ , **Harun Alp, and Michael Peters**, "Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries," *Working Paper*, 2015.

**Andrews, Dan, Chiara Criscuolo, and Peter N. Gal**, "The Best versus the Rest: The Global Productivity Slowdown, Divergence across Firms and the Role of Public Policy," Technical Report, The Organisation for Economic Co-operation and Development 2016.

**Atkeson, Andrew and Ariel Burstein**, "Aggregate Implications of Innovation Policy," *Journal of Political Economy, forthcoming*, 2019.

**Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen**, "Concentrating on the Fall of the Labor Share," *American Economic Review, Papers and Proceedings*, May 2017, *107* (5), 180–85.

**Barkai, Simcha**, "Declining Labor and Capital Shares," Technical Report, Woking Paper 2018.

**Benigno, Gianluca and Luca Fornaro**, "Stagnation Traps," *The Review of Economic Studies, forthcoming*, 2019.

**Berlingieri, Giuseppe and Chiara Criscuolo**, "The Great Divergene(s)," Technical Report, The Organisation for Economic Co-operation and Development 2017.

**Bloom, Nicholas, Charles I. Jones, John Van Reenen, and Michael Webb**, "Are Ideas Getting Harder to Find?," Working Paper 23782, National Bureau of Economic Research September 2017.

**Budd, Christopher, Christopher Harris, and John Vickers**, "A Model of the Evolution of Duopoly: Does the Asymmetry between Firms Tend to Increase or Decrease?," *The Quarterly Journal of Economics*, 1993.

**Caballero, Ricardo J, Takeo Hoshi, and Anil K Kashyap**, "Zombie lending and depressed restructuring in Japan," *The American Economic Review*, 2008, *98* (5), 1943–1977.

**Cette, Gilbert, John Fernald, and Benoît Mojon**, "The pre-Great Recession slowdown in productivity," *European Economic Review*, 2016, *88*, 3–20.

**Davis, Stephen J. and John Haltiwanger**, "Labor Market Fluidity and Economic Perfomance," Technical Report, Woking Paper 2014.

**Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda**, "Where has all the skewness gone? The decline in high-growth (young) firms in the US," *European Economic Review*, 2016, *86*, 4–23.

**Eggertsson, Gauti B. and Paul Krugman**, "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 2012, *127* (3), 1469–1513.

__ **, Neil R. Mehrotra, and Jacob A. Robbins**, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," Technical Report, National Bureau of Economic Research 2017.

**Garcia-Macia, Daniel, Chang-Tai Hsieh, and Pete Klenow**, "How Destructive is Innovation?," *Working Paper*, 2018.

**Gopinath, Gita, Sebnem Kalemli-Ozcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez**, "Capital Allocation and Productivity in South Europe," *Quarterly Journal of Economics*, 2017, *132* (4), 1915–1967.

**Grossman, Gene M. and Elhanan Helpman**, "Quality Ladders in the Theory of Growth," *The Review of Economic Studies*, 1991, *58*, 43–61.

**Grullon, Gustavo, Yelena Larkin, and Roni Michaely**, "Are US Industries Becoming More Concentrated?," Technical Report, Woking Paper 2016.

**Guerrieri, Veronica and Guido Lorenzoni**, "Credit crises, precautionary savings, and the liquidity trap," *The Quarterly Journal of Economics*, 2017, *132* (3), 1427–1467.

**Gutiérrez, Germán and Thomas Philippon**, "Investment-less growth: An empirical investigation," Technical Report, National Bureau of Economic Research 2016.

__ **and** __ , "Declining Competition and Investment in the U.S.," Technical Report, National Bureau of Economic Research 2017.

**Haltiwanger, John**, "Top Ten Signs of Declining Business Dynamism and Entrepreneurship in the U.S.," Technical Report, Woking Paper 2015.

**Hathaway, Ian and Robert Litan**, "Declining Business Dynamism: It's For Real," Technical Report, Brookings Institution 2015.

**Jones, Callum and Thomas Philippon**, "The Secular Stagnation of Investment?," 2016. Working Paper.

**Klette, Jakob and Samuel Kortum**, "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 2004, *112* (5), 986–1018.

**Konrad, Kai A.**, "Dynamic Contests and the Discouragement Effect," *Revue D'economie Politique*, 2012.

**Krugman, Paul**, "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Paper on Economic Activity*, 1998, *1998* (2), 137–205.

**Lee, Dong, Han Shin, and René M Stulz**, "Why Does Capital No Longer Flow More to the Industries with the Best Growth Opportunities?," Technical Report, National Bureau of Economic Research 2016.

**Loecker, Jan De and Jan Eeckhout**, "The Rise of Market Power and the Macroeconomic Implications," Technical Report, Woking Paper 2017.

**Romer, Paul M.**, "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98* (5), 71–102.

**Summers, Lawrence H**, "US economic prospects: Secular stagnation, hysteresis, and the zero lower bound," *Business Economics*, 2014, *49* (2), 65–73.

# A   Appendix: Proofs

## A.1   Properties of flow profits and steady-state growth rate

In this appendix section, we prove lemmas 1 and 2.

In the main text, we model market structure as Bertrand competition, generating a sequence of state-dependent flow profits $\{\pi_s, \pi_{-s}\}$ that satisfy properties outlined in Lemma 1, and that $\lim_{s\to\infty} \pi_s = 1$, $\lim_{s\to-\infty} \pi_s = 0$. Our theoretical results hold under any sequence of flow profits that satisfy Lemma 1; hence, our theory nests other market structures. We use $\pi \equiv \lim_{s\to 0} \pi_s$ to denote the limiting total profits in each market, and we exposit using the notation $\pi$.

**Lemma 1: Follower's flow profits $\pi_{-s}$ are non-negative, weakly decreasing, and convex; leader's and joint profits ($\pi_s$ and $(\pi_s + \pi_{-s})$) are bounded, weakly increasing, and eventually concave in $s$ (a sequence $\{a_s\}$ is eventually concave iff there exists $\bar{s}$ such that $a_s$ is concave in $s$ for all $s \geq \bar{s}$).**

**Proof.**   Let $\delta_i$ be the market share of firm $i$. The CES demand structure within each market implies that $\delta_i \equiv \frac{p_i y_i}{p_1 y_1 + p_2 y_2} = \frac{p_i^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}$. Under Bertrand competition, the price charged by a firm with productivity $z_i$ must solve $p_i = \frac{\sigma(1-\delta_i)+\delta_i}{(\sigma-1)(1-\delta_i)} \lambda^{-z_i}$ (recall we normalize wage rate to 1). Aghion et al. (2001) and Atkeson and Burstein (2008) provides detailed derivations of these expressions.

Define $\rho_s$ as an implicit function of the productivity gap: $\rho_s^\sigma = \lambda^{-s} \frac{(\sigma \rho_s^{\sigma-1}+1)}{\sigma + \rho_s^{\sigma-1}}$. It can be verified that flow profits satisfy $\pi_s = \frac{1}{\sigma \rho_s^{\sigma-1}+1}$ for any productivity gap $s$. The fact that follower's flow profits is convex in $s$ follows from algebra. Moreover, $\lim_{s\to\infty} \rho_s^\sigma \lambda^s = 1/\sigma$ and $\lim_{s\to-\infty} \rho_s^\sigma \lambda^s = \sigma$; hence, for large $s$, $\pi_s \approx \frac{1}{\sigma^{\frac{1}{\sigma}} \lambda^{-\frac{\sigma-1}{\sigma}s}+1}$ and $\pi_{-s} \approx \frac{1}{\sigma^{\frac{2\sigma-1}{\sigma}} \lambda^{\frac{\sigma-1}{\sigma}s}+1}$. The eventual concavity of $\pi_s$ and $(\pi_s + \pi_{-s})$ as $s \to \infty$ is immediate.

**Lemma 2: In a steady state, the aggregate productivity growth rate is $g \equiv \ln \lambda \left( \sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0 \right)$.**

**Proof**  The expression $\left(\sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0\right)$ tracks the weighted-average growth rate of the productivity frontier in the economy, i.e., the rate at which markets leave the current state $s$ and move to state $s+1$. In a steady-state, the growth rate of frontier must be the same as the rate at which states fall down by one step, from $s+1$ to $s$; hence, aggregate growth rate $g$ can also be written as $g = \ln \lambda \left(\sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa\right)\right)$.

To prove the expression formally, we proceed in two steps. First, we express aggregate productivity growth as a weighted average of productivity growth in each market. We then use the fact that, given homothetic within-market demand, if a follower in state $s$ improves productivity by one step (i.e. by a factor $\lambda$) and a leader in state $s-1$ improves also by one step, the net effect should be equivalent to one step improvement in the overall productivity of a single market.

Aggregate productivity growth is a weighted average of productivity growth in each market:

$$
\begin{aligned}
g = -\frac{d \ln P}{d \ln t} &= -\frac{d \int_0^1 \ln p\,(v)\, dv}{d \ln t} \\
&= -\sum_{s=0}^{\infty} \mu_s \times \frac{d \left[ \int_{z^F} \ln p\,\left(s, z^F\right) dF\,\left(z^F\right)\right]}{d \ln t},
\end{aligned}
$$

where we use $\left(s, z^F\right)$ to index for markets in the second line. Now recognize that productivity growth rate in each market, $-\frac{d \ln p\left(s, z^F\right)}{d \ln t}$, is a function of only the productivity gap $s$ and is invariant to the productivity of follower, $z^F$. Specifically, suppose the follower in market $\left(s, z^F\right)$ experiences an innovation, the market price index becomes $p\,\left(s-1, z^F+1\right)$. Similarly, if the leader experiences an innovation, the price index becomes $p\,\left(s+1, z^F\right)$. The corresponding log-changes in price indices are respectively

$$
\begin{aligned}
a_s^F &\equiv \ln p\,\left(s-1, z^F+1\right) - \ln p\,\left(s, z^F\right) \\
&= -\ln \lambda + \ln \left[\rho_{s-1}^{1-\sigma} + 1\right]^{\frac{1}{1-\sigma}} - \ln \left[\rho_s^{1-\sigma} + 1\right]^{\frac{1}{1-\sigma}},
\end{aligned}
$$

$$
\begin{aligned}
a_s^L &= \ln p\,\left(s+1, z^F\right) - \ln p\,\left(s, z^F\right) \\
&= \ln \left[\rho_{s+1}^{1-\sigma} + 1\right]^{\frac{1}{1-\sigma}} - \ln \left[\rho_s^{1-\sigma} + 1\right]^{\frac{1}{1-\sigma}},
\end{aligned}
$$

where $\rho_s$ is the implicit function defined in the proof for Lemma 1. The log-change in price index is independent of $z^F$ in either case. Hence, over time interval $[t, t + \Delta]$, the change in price index for markets with state variable $s$ at time $t$ follows

$$\Delta \ln p\left(s, z^F\right) = \begin{cases} a_s^L & \text{with probability } \eta_s \Delta, \\ a_s^F & \text{with probability } (\eta_{-s} + \kappa \cdot \mathbf{1}\,(s \neq 0))\, \Delta. \end{cases}$$

The aggregate productivity growth can therefore be written as

$$g = -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + (\eta_{-s} + \kappa)\, a_s^F\right).$$

Lastly, note that if both leader and follower in a market experiences productivity improvements, regardless of the order in which these events happen, the price index in the market changes by a factor of $\lambda^{-1}$:

$$a_s^F + a_{s-1}^L = a_s^L + a_{s+1}^F = -\ln \lambda \quad \text{for all } s \geq 1.$$

Hence,

$$
\begin{aligned}
g &= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + (\eta_{-s} + \kappa)\, a_s^F\right) \\
&= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + (\eta_{-s} + \kappa)\left(-\ln \lambda - a_{s-1}^L\right)\right) \\
&= \ln \lambda \cdot \sum_{s=1}^{\infty} \mu_s\,(\eta_{-s} + \kappa) - \left(\sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L - a_{s-1}^L\,(\eta_{-s} + \kappa)\right) + \mu_0 2\eta_0 a_0\right).
\end{aligned}
$$

Given that steady-state distribution $\{\mu_s\}$ must follow

$$\mu_s\,(\eta_{-s} + \kappa) = \begin{cases} \mu_{s-1} \eta_{s-1} & \text{if } s > 1, \\ 2\mu_0 \eta_0 & \text{if } s = 1, \end{cases} \tag{10}$$

we know

$$\sum_{s=1}^{\infty} \mu_s \times \left( \eta_s a_s^L - a_{s-1}^L (\eta_{-s} + \kappa) \right) + \mu_0 2\eta_0 a_0$$

$$= \sum_{s=1}^{\infty} \mu_s \eta_s a_s^L + \mu_0 2\eta_0 a_0 - \left( \sum_{s=1}^{\infty} \mu_s a_{s-1}^L (\eta_{-s} + \kappa) \right)$$

$$= 0.$$

Hence aggregate growth rate simplifies to $g = \ln \lambda \cdot \sum_{s=1}^{\infty} \mu_s (\eta_{-s} + \kappa)$, which traces the growth rate of productivity laggards. We can also apply substitutions in (10) again to express productivity growth as a weighted average of frontier growth:

$$g = \ln \lambda \cdot \left( \sum_{s=1}^{\infty} \mu_s \eta_s + 2\mu_0 \eta_0 \right).$$

## A.2    Structure of Equilibrium

It is useful to first understand the structure of value functions given any sequence of (potentially non-equilibrium) investment decisions $\{\eta_s\}_{s=-\infty}^{\infty}$. The fact that firms are forward-looking implies that value function in each state can be written as a weighted average of flow payoffs in all ergodic states induced by the investment decisions, i.e.

$$v_s = \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} \times PV_{s'}, \quad \text{where} \quad \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} = 1 \quad \text{for all } s. \tag{11}$$

The term $PV_{s'} \equiv \frac{\pi_{s'} - c\eta_{s'}}{r}$ represents the permanent value in state $s'$, i.e. the present-discounted value of flow payoff in state $s'$ if the firm stays in that state permanently; $s' > 0$ means the firm is a leader when the productivity gap is $s'$, and $s' < 0$ means the firm is a follower when the productivity gap is $-s'$. In equilibrium, the firm value in state $s$ can be written as a weighted average of the permanent value across all ergodic states. The weight $\lambda_{s'|s}$ can be interpreted as the present-discount fraction of time that the firm is going to be $s'$ steps ahead of his competitor, given that he is currently $s$ steps ahead. The weights $\{\lambda_{s'|s}\}_{s'=\infty}^{\infty}$ form a measure conditional on the current state $s$. When the current state $s$ is high, the firm is expected to spend more time in higher indexed states, and the conditional distribution $\{\lambda_{s'|s+1}\}_{s'=\infty}^{\infty}$ first-order stochastically dominates

$\{\lambda_{s'|s}\}_{s'=\infty}^{\infty}$ for all $s$.

Likewise, let $w_s \equiv v_s + v_{-s}$ be the joint value of leader and follower in state $s$. Following the same logic as in equation (11), we can rewrite $w_s$ as a weighted average of the sum of permanent values of leader and follower in every state:

$$w_s = \sum_{s'=0}^{\infty} \tilde{\lambda}_{s'|s} \cdot (PV_{s'} + PV_{-s'}), \quad \text{where} \sum_{s'=0}^{\infty} \tilde{\lambda}_{s'|s} = 1. \tag{12}$$

The weights $\tilde{\lambda}_{s'|s}$ can be interpreted as the present-discounted fraction of time that the state variable is $s'$, i.e. when either firm is $s'$ steps ahead of the other, conditioning on the current gap being $s$; hence, $\tilde{\lambda}_{s'|s} = \lambda_{s'|s} + \lambda_{s'|-s}$. It is easy to verify that $\{\tilde{\lambda}_{s'|s+1}\}$ first order stochastically dominate $\{\tilde{\lambda}_{s'|s}\}$.

To understand the role of interest rate, note that the firm value in state $s$ can be written as a weighted average of the permanent state payoff in state $s$ and the firm value in neighboring states $s-1$ and $s+1$:

$$v_s = \frac{r}{r + \kappa + \eta_{-s} + \eta_s} \cdot PV_s + \frac{\kappa + \eta_{-s}}{r + \kappa + \eta_{-s} + \eta_s} v_{s-1} + \frac{\eta_s}{r + \kappa + \eta_{-s} + \eta_s} v_{s+1}$$

Holding investment decisions constant, a fall in interest rate $r$ reduces the relative weight on the permanent value of state $s$, thereby reducing the difference in value functions across states. In fact, holding investment decisions fixed, if there is a state in which the leader chooses not to invest at all ($\eta_{\bar{s}} = 0$ for some $\bar{s}$), then $rv_s \to rv_0$ for all $s \leq \bar{s}$.

We now prove results about the structure of equilibria. For expositional purposes, we assume firms play pure strategies (i.e. they invest at either lower or upper bounds $\eta_s \in \{0, \eta\}$); all of our claims hold for mixed strategy equilibria (i.e. those involving interior investment intensities).

**Lemma 3. The leader invests in more states than the follower, $n \geq k$. Moreover, the follower does not invest in states $s = k+1, \cdots, n+1$.** Recall $n+1$ is the first state in which market leaders choose not to invest, and $k+1$ is the first state in which followers choose not to invest: $n+1 \equiv \min\{s|s \geq 0, \eta_s < \eta\}$ and $k+1 \equiv \min\{s|s \leq 0, \eta_s < \eta\}$.

**Proof**  Suppose $n < k$, i.e. leader invests in states 1 through $n$ whereas follower invests in states 1 through at least $n + 1$. We first show these investment decisions can only be supported by certain lower bounds on the value function of both leader and follower in state $n + 1$. We reach for a contradiction, showing that, if $n < k$, then market power is too transient to support these lower bounds on value functions.

The HJB equation for the leader in state $n + 2$ implies

$$
\begin{aligned}
rv_{n+2} &= \max_{\eta_{n+2}\in[0,\eta]} \pi_{n+2} + \eta_{n+2}\left(v_{n+3} - v_{n+2} - c\right) + \left(\eta_{-(n+2)} + \kappa\right)\left(v_{n+1} - v_{n+2}\right) \\
&\geq \pi_{n+2} + (\eta + \kappa)\left(v_{n+1} - v_{n+2}\right).
\end{aligned}
$$

The fact that leader does not invest in state $n + 1$ implies $c \geq v_{n+2} - v_{n+1}$; combining with the previous inequality, we obtain

$$
rv_{n+1} \geq \pi_{n+2} - c\left(\eta + \kappa + r\right).
$$

The HJB equation for the follower in state $n + 1$ implies

$$
\begin{aligned}
rv_{-(n+1)} &= \max_{\eta_{-(n+1)}\in[0,\eta]} \pi_{-(n+1)} + \left(\eta_{-(n+1)} + \kappa\right)\left(v_{-n} - v_{-(n+1)}\right) - c\eta_{-(n+1)} \\
&\geq \pi_{-(n+1)} + \kappa\left(v_{-n} - v_{-(n+1)}\right).
\end{aligned}
$$

The fact that follower chooses to invest in state $n + 1$ implies $c \leq v_{-n} - v_{-(n+1)}$; combining with the previous inequality, we obtain

$$
rv_{-(n+1)} \geq \pi_{-(n+1)} + c\kappa. \tag{13}
$$

Combining this with the earlier inequality involving $rv_{n+1}$, we obtain an inequality on the joint value in state $n + 1$:

$$
rw_{n+1} \geq \pi_{n+2} + \pi_{-(n+1)} - c\left(\eta + r\right) \tag{14}
$$

We now show that inequalities (13) and (14) cannot both be true. To do so, we construct al-

ternative economic environments with value functions that dominate $w_{n+1}$ and $v_{-(n+1)}$; we then show that even these dominating value functions cannot satisfy both inequalities.

First, fix $n$ and fix investment strategies (leader invests until state $n+1$ and follower invests at least through $n+1$); suppose for all states $1 \leq s \leq n+1$, follower's profits are equal to $\pi_{-(n+1)}$ and leader's profits are equal to $\pi_{n+2}$; two firms each earn $\frac{\pi_{-(n+1)}+\pi_{n+2}}{2}$ in state zero. The joint profits in this modified economic environment are independent of the state by construction; moreover, the joint flow profits always weakly dominate those in the original environment and strictly dominate in state zero ($\pi_{n+2} + \pi_{-(n+1)} \geq \pi_1 + \pi_{-1} > 2\pi_0$). Let $\hat{w}_s$ denote the value function in the modified environment; $\hat{w}_s > w_s$ for all $s \leq n+1$.

Consider the joint value in this modified environment but under alternative investment strategies. Let $\bar{n}$ index for investment strategies: leader invests in states $1$ through $\bar{n}$ whereas the follower invests at least through $\bar{n}+1$. Let $\hat{w}_s^{(\bar{n})}$ denote the joint value in state $s$ under investments indexed by $\bar{n}$; we argue that $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$. To see this, note that the joint flow payoffs in all states $0$ through $\bar{n}$ is constant by construction and is equal to $\left(\pi_{n+2} + \pi_{-(n+1)} - 2c\eta\right)$—total profits net of investment costs. The joint flow payoff in state $\bar{n}+1$ is $\left(\pi_{n+2} + \pi_{-(n+1)} - c\eta\right)$. Hence, the joint market value in state $\bar{n}+1$ under the investment strategies indexed by $\bar{n}$ is equal to

$$\hat{w}_{\bar{n}+1}^{(\bar{n})} = \frac{\pi_{n+2} + \pi_{-(n+1)} - 2c\eta\left(1 - \tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}/2\right)}{r},$$

where $\tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}$ is the present discount fraction of time that the market spends in state $\bar{n}+1$, conditioning on the current state is $\bar{n}+1$, and that firms follow investment strategies indexed by $\bar{n}$. The object $\tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$: the more states in which both firms invest, the less time that the market will spend in the state $\bar{n}+1$ in which only one firm (the follower) invests. Hence, $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$, and that $\hat{w}_1^{(0)} \geq \hat{w}_{n+1}^{(n)} > w_{n+1}$. The same logic also implies $\hat{v}_0^{(0)} = \frac{1}{2}\hat{w}_0^{(0)} > \frac{1}{2}w_0 = v_0$.

The follower's value $\hat{v}_{-1}^{(0)}$, in the alternative environment, when investment strategies are in-

dexed by zero (i.e. firms invest in states $0$ and $-1$ only), is higher than $v_{-(n+1)}$. This is because

$$
\begin{aligned}
\hat{v}_{-1}^{(0)} &= \frac{\pi_{-(n+1)} - c\eta + \kappa \hat{v}_0^{(0)}}{r + \kappa + \eta} \\
&> \frac{\pi_{-(n+1)} - c\eta + \kappa v_0}{r + \kappa + \eta} \\
&\geq \frac{\pi_{-(n+1)} - c\eta + \kappa v_{-n}}{r + \kappa + \eta} \\
&= v_{-(n+1)}.
\end{aligned}
$$

We now show that the inequalities $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ and $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c(\eta + r)$ cannot both hold. We can explicitly solve for the value functions from the HJB equations:

$$
\hat{w}_0^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - 2c\eta + 2\eta \hat{w}_1^{(0)}}{r + 2\eta}
$$

$$
\hat{w}_1^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - c\eta + (\eta + \kappa) \hat{w}_0^{(0)}}{r + \eta + \kappa}
$$

$$
\hat{v}_{-1}^{(0)} = \frac{\pi_{-(n+1)} - c\eta + (\eta + \kappa) \hat{w}_0^{(0)}/2}{r + \eta + \kappa}
$$

Solving for $\hat{w}_1^{(0)}$ and $\hat{v}_{-1}^{(0)}$, we obtain

$$
r\hat{w}_1^{(0)} = \pi_{n+2} + \pi_{-(n+1)} - c\eta \left( 1 + \frac{\eta + \kappa}{r + 3\eta + \kappa} \right)
$$

$$
(r + \eta + \kappa) r\hat{v}_{-1}^{(0)} = r\left( \pi_{-(n+1)} - c\eta \right) + (\eta + \kappa) \left( \frac{\pi_{n+2} + \pi_{-(n+1)}}{2} - c\eta \frac{r + 2\eta + \kappa}{r + 3\eta + \kappa} \right)
$$

That $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ implies

$$
\begin{aligned}
&(r + \eta + \kappa) r\hat{v}_{-1}^{(0)} \\
={}& r\left( \pi_{-(n+1)} - c\eta \right) + (\eta + \kappa) \left( \frac{\pi_{n+2} + \pi_{-(n+1)}}{2} - c\eta \frac{r + 2\eta + \kappa}{r + 3\eta + \kappa} \right) \\
\geq{}& (r + \eta + \kappa) \left( \pi_{-(n+1)} + c\kappa \right) \\[1em]
\implies{}& (\eta + \kappa) \left( \frac{\pi_{n+2} - \pi_{-(n+1)}}{2} - c\eta \frac{r + 2\eta + \kappa}{r + 3\eta + \kappa} \right) \geq (r + \eta + \kappa) c\kappa + c\eta r
\end{aligned}
$$

Since $\frac{\pi_{n+2}-\pi_{-(n+1)}}{2} \leq \frac{\pi_{n+2}}{2} < c\eta$, it must be the case that

$$(\eta + \kappa)\, c\eta > (r + \eta + \kappa)\, c\kappa + c\eta r + (\eta + \kappa)\, c\eta \frac{r + 2\eta + \kappa}{r + 3\eta + \kappa}.$$

On the other hand, that $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c(\eta + r)$ implies $r \geq \eta\frac{\eta+\kappa}{r+3\eta+\kappa}$; hence the previous inequality implies

$$
\begin{aligned}
(\eta + \kappa)\, c\eta \;>\;& (r + \eta + \kappa)\, c\kappa + (\eta + \kappa)\, c\eta \frac{\eta}{r + 3\eta + \kappa} + (\eta + \kappa)\, c\eta \frac{r + 2\eta + \kappa}{r + 3\eta + \kappa} \\
=\;& (r + \eta + \kappa)\, c\kappa + (\eta + \kappa)\, c\eta,
\end{aligned}
$$

which is impossible; hence $n \geq k$.

We now show that the follower does not invest in states $s \in \{k + 1, ..., n + 1\}$. First, note

$$
\begin{aligned}
(r + \eta + \kappa)\,(v_{-s} - v_{-s-1}) \;=\;& \pi_{-s} - \pi_{-s-1} + \kappa\,(v_{-s+1} - v_s) + \eta\,(v_{-s-1} - v_{-s-2}) \\
& + \max\{\eta\,(v_{-s+1} - v_{-s} - c), 0\} - \max\{\eta\,(v_{-s} - v_{-s-1} - c), 0\}.
\end{aligned}
$$

Suppose $v_{-s+1} - v_{-s} \geq (v_{-s} - v_{-s-1})$, then

$$(r + \eta + \kappa)\,(v_{-s} - v_{-s-1}) \geq \pi_{-s} - \pi_{-s-1} + \kappa\,(v_{-s+1} - v_{-s}) + \eta\,(v_{-s-1} - v_{-s-2})$$

$$\implies (r + \eta)\,(v_{-s} - v_{-s-1}) \geq \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2}). \tag{15}$$

If $v_{-s+1} - v_{-s} < (v_{-s} - v_{-s-1})$, then

$$
\begin{aligned}
(r + \eta)\,(v_{-s} - v_{-s-1}) \;<\;& \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2}) \\
& + \max\{\eta\,(v_{-s+1} - v_{-s} - c), 0\} \\
& - \max\{\eta\,(v_{-s} - v_{-s-1} - c), 0\} \\
\leq\;& \pi_{-s} - \pi_{-s-1} + \eta\,(v_{-s-1} - v_{-s-2}). \tag{16}
\end{aligned}
$$

Now suppose $\eta_{-k-1} = 0$ but $\eta_{-s'} = \eta$ for some $s' \in \{k+2, ..., n+1\}$. This implies

$$v_{-(k-1)} - v_{-k} \geq c > v_{-k} - v_{-k-1} < v_{-s'+1} - v_{-s'},$$

implying there must be at least one $s \in \{k+2, ..., n+1\}$ such that $v_{-s+1} - v_{-s} \geq v_{-s} - v_{-s-1} < v_{-s-1} - v_{s-2}$. Inequalities (15) and (16) implies

$$(r+\eta)(v_{-s} - v_{-s-1}) \geq \pi_{-s} - \pi_{-s-1} + \eta(v_{-s-1} - v_{-s-2}) \tag{17}$$

$$(r+\eta)(v_{-s-1} - v_{-s-2}) < \pi_{-s-1} - \pi_{-s-2} + \eta(v_{-s-2} - v_{-s-3})$$

Inequality (17) and $v_{-s} - v_{-s-1} < v_{-s-1} - v_{-s-2}$ implies $r(v_{-s} - v_{-s-1}) > \pi_{-s} - \pi_{-s-1}$; convexity in follower's profit functions further implies $r(v_{-s} - v_{-s-1}) > \pi_{-s-1} - \pi_{-s-2}$. Hence it must be the case that $(v_{-s-2} - v_{s-3}) > (v_{-s-1} - v_{-s-2})$. Applying inequality (16) again,

$$(r+\eta)(v_{-s-2} - v_{-s-3}) < \pi_{-s-2} - \pi_{-s-3} + \eta(v_{-s-3} - v_{-s-4}).$$

That $r(v_{-s-2} - v_{-s-3}) > \pi_{-s-2} - \pi_{-s-3}$ further implies $(v_{-s-3} - v_{-s-4}) > (v_{-s-2} - v_{-s-3})$. By induction, we can show

$$v_{s-1} - v_{s-2} < v_{s-2} - v_{s-3} < \cdots < v_{-n} - v_{-(n+1)}.$$

But

$$(r+\eta+\kappa)\left(v_{-n} - v_{-(n+1)}\right) \leq \pi_{-n} - \pi_{-(n+1)} + \kappa(v_{s+1} - v_s) + \eta(v_{-n+1} - v_{-n+1})$$

$$\implies (r+\eta)\left(v_{-n} - v_{-(n+1)}\right) \leq \pi_{-n} - \pi_{-(n+1)}$$

which is a contradiction, given convexity of the profit functions. Hence, we have shown $v_{-k} - v_{-(k+1)} \geq v_{-s} - v_{-s-1}$ for all $s \in \{k+1, ..., n+1\}$, establishing that follower cannot invest in these states.

**Lemma 4: In a steady-state induced by investment cutoffs $(n, k)$, the aggregate productivity growth rate is $g = \ln \lambda \cdot \left( \mu^C (\eta + \kappa) + \mu^M \kappa \right)$, where $\mu^C$ is the fraction of markets in the competitive region ($\mu^C = \sum_{s=1}^{k} \mu_s$) and $\mu^M$ is the fraction of markets in the monopolistic region ($\mu^M = \sum_{s=k+1}^{n+1} \mu_s$). The fraction of markets in each region satisfies**

$$\mu_0 + \mu^C + \mu^M = 1, \quad \mu_0 \propto (\kappa/\eta)^{n-k+1} (1 + \kappa/\eta)^k,$$

$$\mu^C \propto (\kappa/\eta)^{n-k} \left( (1 + \kappa/\eta)^k - 1 \right), \quad \mu^M \propto \frac{1 - (\kappa/\eta)^{n-k+1}}{1 - \kappa/\eta}.$$

**Proof.** Given the cutoff strategies $(n, k)$, aggregate productivity growth is (from Lemma 3)

$$g = \ln \lambda \cdot \left( \sum_{s=1}^{n} \mu_s \eta + 2\mu_0 \eta \right).$$

The steady-state distribution must follow

$$\mu_s \eta = \begin{cases} \mu_1 (\eta + \kappa) / 2 & \text{if } s = 0 \\ \mu_{s+1} (\eta + \kappa) & \text{if } 1 \leq s \leq k - 1 \\ \mu_{s+1} \kappa & \text{if } k \leq s \leq n + 1 \\ 0 & \text{if } s > n + 1 \end{cases}$$

Hence we can rewrite the aggregate growth rate as

$$\begin{aligned} g &= \ln \lambda \cdot \left( 2\mu_0 \eta + \sum_{s=1}^{k-1} \mu_s \eta + \sum_{s=k-1}^{n} \mu_s \eta \right) \\ &= \ln \lambda \cdot \left( \mu_1 (\eta + \kappa) + \sum_{s=2}^{k} \mu_s (\eta + \kappa) + \sum_{s=k}^{n+1} \mu_s \kappa \right) \\ &= \ln \lambda \cdot \left( \mu^C (\eta + \kappa) + \mu^M \kappa \right), \end{aligned}$$

as desired.

To solve for $\mu_0$, $\mu^C$, and $\mu^M$ as functions of $n$ and $k$, note that steady-state distribution follows:

$$\mu_s \eta = \begin{cases} \mu_1 \left( \eta + \kappa \right) /2 & \text{if } s = 0 \\[2mm] \mu_{s+1} \left( \eta + \kappa \right) & \text{if } 1 \leq s \leq k - 1 \\[2mm] \mu_{s+1} \kappa & \text{if } k \leq s \leq n + 1 \\[2mm] 0 & \text{if } s > n + 1. \end{cases}$$

We can rewrite $\mu_s$ as a function of $\mu_{n+1}$ for all $s$. Let $\alpha \equiv \kappa/\eta$, then

$$\mu_s = \begin{cases} \mu_{n+1} \alpha^{n+1-s} & \text{if } n + 1 \geq s \geq k \\[2mm] \mu_{n+1} \alpha^{n+1-k} \left( 1 + \alpha \right)^{k-s} & \text{if } k - 1 \geq s \geq 0 \end{cases}$$

Hence $\mu_0 = \mu_{n+1} \alpha^{n+1-k} \left( 1 + \alpha \right)^{k}$. The fraction of markets in the competitive and monopolistic regions can be written, respectively, as

$$\mu^M = \mu_{n+1} \sum_{s=k+1}^{n+1} \alpha^{n+1-s} = \mu_{n+1} \frac{1 - \alpha^{n-k+1}}{1 - \alpha}$$

$$\mu^C = \mu_{n+1} \alpha^{n+1-k} \sum_{s=1}^{k} \left( 1 + \alpha \right)^{k-s} = \mu_{n+1} \alpha^{n-k} \left( \left( 1 + \alpha \right)^{k} - 1 \right).$$

**Lemma 5: If follower invests in state 1, then the steady-state aggregate productivity growth is bounded below by $\ln \lambda \cdot \kappa$, the step-size of productivity increments times the rate of technology diffusion.**

**Proof.** Given $k \geq 1$, the fraction of markets in the competitive region can be written as

$$
\begin{aligned}
\mu^C &= \sum_{s=1}^{k} \mu_s \\
&= \mu_1 + \underbrace{\mu_1 (1+\alpha)^{-1}}_{=\mu_2} + \cdots + \underbrace{\mu_1 (1+\alpha)^{-(k-1)}}_{=\mu_k} \\
&= \underbrace{\mu_0 \frac{\kappa + \eta}{2\eta}}_{=\mu_1} \frac{1 - (1+\alpha)^{-k}}{1 - (1+\alpha)^{-1}} \\
&\geq \mu_0 \frac{\kappa + \eta}{2\eta}
\end{aligned}
$$

Aggregate growth rate can be re-written as

$$
\begin{aligned}
g &= \ln \lambda \cdot \left[ (1 - \mu_0) \kappa + \mu^C \eta \right] \\
&\geq \ln \lambda \cdot \left[ (1 - \mu_0) \kappa + \mu_0 \frac{\kappa + \eta}{2} \right] \\
&\geq \ln \lambda \cdot \left[ (1 - \mu_0) \kappa + \mu_0 \kappa \right] \\
&= \ln \lambda \cdot \kappa,
\end{aligned}
$$

as desired.

## A.3  Asymptotic Results as $r \to 0$

**Lemma A.1.** $\Delta w_0 \equiv w_1 - w_0 = \frac{r w_0 + 2c\eta - 2\pi_0}{2\eta}$; $\Delta w_0$ **is bounded away from zero.**

**Proof**  The equality from the HJB equation $r w_0 = 2\pi_0 - 2c\eta + 2\eta (w_1 - w_0)$. That $\Delta w_0$ is bounded away from zero follows from the fact that $r w_0 \geq 0$ and assumption 1 ($2c\eta > \pi \equiv \lim_{s \to 0} \pi_s + \pi_{-s} > 2\pi_0$). QED.

### A.3.1 Mathematical Preliminaries

Consider the following recursive formulation of value functions:

$$ru_{s+1} = \lambda + p\left(u_s - u_{s+1}\right) + q\left(u_{s+2} - u_{s+1}\right)$$

The HJB equation states that, starting from state $s$, there's a Poisson rate $p$ of moving up one state, and rate $q$ of moving down; the flow payoff is $\lambda$ and discount rate is $r$.

Fix a state $s$. Given $u_s$ and $\Delta u_s \equiv u_{s+1} - u_s$, we can solve for all $u_{s+t}$ with $t > 0$ as recursive functions of $u_s$ and $\Delta u_s$; for instance,

$$u_{s+2} - u_{s+1} = \frac{ru_s - \lambda}{q} + \left(\frac{p+r}{q}\right)\Delta u_s,$$

$$u_{s+3} - u_{s+3} = \frac{ru_s - \lambda}{q} + \left(\frac{p+r}{q}\right)\left(u_{s+2} - u_{s+1}\right) + \frac{r\Delta u_s}{q},$$

and so on. The recursive formulation generically does not have a nice closed-form representation, as the number of terms quickly explodes as we expand out the recursion. However, as $r \to 0$, the value functions do emit asymptotic closed form expressions, as Proposition A.1 shows. In what follows, let $\sim$ denote asymptotic equivalence as $r \to 0$, i.e. $x \sim y$ iff $\lim_{r\to 0}\left(x - y\right) = 0$.

**Proposition A.1.** Let $\delta \equiv \frac{ru_s - \lambda}{q}$, $a \equiv p/q$, $b \equiv r/q$, then for all $t > 0$,

$$
\begin{aligned}
u_{s+t} - u_s \quad \sim \quad & \left(\Delta u_s\right)\frac{1 - a^t}{1 - a} + \delta\frac{t - \frac{a - a^t}{1-a}}{1 - a} \\
& + \Delta u_s \cdot b\frac{\left(t - 1\right)\left(1 + a^t\right)\left(1 - a\right) - \left(2 - a\right)\left(a^t - a\right)}{\left(1 - a\right)^3} \\
& + \delta b\frac{1}{\left(1 - a\right)^3}\left(\frac{\left(t - 2\right)\left(t - 1\right)}{2}\left(1 - a\right) - \left(t - 3\right)a^t - a\left(2 - a\right)\left(t - 1\right) + 2a\left(1 - a\right)\right)
\end{aligned}
$$

$$u_{s+t} - u_{s+t-1} \sim \Delta u_s a^{t-1} + \delta \frac{1 - a^{t-1}}{1 - a}$$
$$+ \Delta u_s b \frac{\left((t-1)\left(1+a^t\right) - (t-2)\left(1+a^{t-1}\right)\right)(1-a) - (2-a)\left(a^t - a^{t-1}\right)}{(1-a)^3}$$
$$+ \delta b \frac{1}{(1-a)^3} \left( \frac{(t-2)(t-1) - (t-2)(t-3)}{2}(1-a) - (t-3)a^t + (t-4)a^{t-1} - a(2-a) \right)$$

The following simplifications of the formulas will be useful if $\lim_{r \to 0} t \to \infty$:

1. when $a < 1$:

$$u_{s+t} - u_{s+t-1} \sim \Delta u_s a^{t-1} + \frac{\delta}{1-a} + \frac{b \Delta u_s}{(1-a)^2};$$

   (a) if $r \Delta u_s \to 0$,

$$u_{s+t} - u_s \sim \Delta u_s \frac{1}{1-a} + \frac{t\delta}{1-a};$$

   (b) if $r \Delta u_s \not\to 0$,

$$r\left(u_{s+t} - u_s\right) \sim \frac{r \Delta u_s}{1-a}.$$

2. when $a > 1$, $r \Delta u_s \to 0$, and $\Delta u_s + \frac{\delta}{a-1} \not\sim 0$,

$$r\left(u_{s+t} - u_s\right) \sim \left(\Delta u_s + \frac{\delta}{a-1}\right) \frac{r a^t}{a-1},$$

$$r\left(u_{s+t} - u_{s+t-1}\right) \sim \left(\Delta u_s + \frac{\delta}{a-1}\right) r a^{t-1}.$$

If $\Delta u_s + \frac{\delta}{a-1} \sim 0$,

$$u_{s+t} - u_s \sim -\frac{b\delta}{(1-a)^4} \cdot a^{t+1}.$$

Suppose the flow payoffs are state-dependent $\{\lambda_s\}$, i.e.

$$r u_{s+1} = \lambda_{s+1} + p\left(u_s - u_{s+1}\right) + q\left(u_{s+2} - u_{s+1}\right)$$

If $\lambda$ is an upper bound for $\{\lambda_s\}$, then the formulas provide asymptotic lower bounds for $u_{s+t} - u_{s+t-1}$ and $u_{s+t}$ as functions of $u_s$ and $\Delta u_s$. Conversely, if $\lambda$ is a lower bound for $\{\lambda_s\}$, then the

formulas provide asymptotic upper bounds for $u_{s+t} - u_{s+t-1}$ and $u_{s+t}$.

One can analogously write $u_s$ and $\Delta u_s$ as asymptotic functions of $\Delta u_{s+t}$ and $u_{s+t}$.

**Proof of Proposition A.1.** The recursive formulation can be re-written as

$$u_{s+1} - u_s = \Delta u_s$$

$$
\begin{aligned}
u_{s+2} - u_{s+1} &= a \left( u_{s+1} - u_s \right) + \frac{r \left( u_{s+1} - u_s \right) + r u_s - v}{q} \\
&= a \Delta u_s + b \Delta u_s + \delta
\end{aligned}
$$

$$u_{s+2} - u_s = (1 + a) \Delta u_s + b \Delta u_s + \delta$$

Likewise,

$$u_{s+3} - u_{s+2} = a^2 \Delta u_s + (1 + 2a) b \Delta u_s + (1 + a) \delta + o \left( r^2 \right)$$

$$u_{s+3} - u_s = \left( 1 + a + a^2 \right) \Delta u_s + (1 + 1 + 2a) b \Delta u_s + (1 + 1 + a) \delta + b \delta + o \left( r^2 \right)$$

Applying the formula iteratively, one can show that

$$u_{s+t+1} - u_{s+t} = a^t \Delta u_s + \delta \sum_{z=0}^{t-1} a^z + b \Delta u_s \sum_{z=1}^{t} z a^{z-1} + b \delta \sum_{z=1}^{t-1} \sum_{m=1}^{z} m a^{m-1} + o \left( r^2 \right)$$

$$u_{s+t+1} - u_s = \Delta u_s \sum_{z=0}^{t} a^z + \delta \sum_{z=0}^{t} \sum_{m=0}^{z-1} a^m + b \Delta u_s \sum_{z=1}^{t} \sum_{m=1}^{z} m a^{m-1} + b \delta \sum_{x=1}^{t-1} \sum_{z=1}^{x} \sum_{m=1}^{z} m a^{m-1} + o \left( r^2 \right)$$

One obtains the Lemma by applying the following formulas for power series summation:

$$\sum_{z=0}^{t} a^z = \frac{1 - a^{t+1}}{1 - a}$$

$$\sum_{z=0}^{t} \sum_{m=0}^{z-1} a^m = \frac{t + 1 - \frac{a - a^{t+1}}{1-a}}{1 - a}$$

$$\sum_{z=1}^{t}\sum_{m=1}^{z} ma^{m-1} = \frac{t\left(1+a^{t+1}\right)(1-a)-(2-a)\left(a^{t+1}-a\right)}{(1-a)^3}$$

$$\sum_{x=1}^{t-1}\sum_{z=1}^{x}\sum_{m=1}^{z} ma^{m-1} = \frac{1}{(1-a)^3}\left(\frac{t(t-1)}{2}(1-a)-(t-2)a^{t+1}-a(2-a)t+2a(1-a)\right).$$

The third and fourth summations formulas follow because

$$\sum_{m=1}^{z} ma^{m-1} = \left(1+2a+3a^2+\cdots+za^{z-1}\right)$$

$$= \left(\frac{1-a^z}{1-a}+a\frac{1-a^{z-1}}{1-a}+\cdots+a^{z-1}\frac{1-a}{1-a}\right)$$

$$= \left(\frac{1+a+\cdots+a^{z-1}}{1-a}-\frac{za^z}{1-a}\right)$$

$$= \left(\frac{1-a^z}{(1-a)^2}-\frac{za^z}{1-a}\right)$$

$$\sum_{z=1}^{s}\sum_{m=1}^{z} ma^{m-1} = \sum_{z=1}^{s}\left(\frac{1-a^z}{(1-a)^2}-\frac{za^z}{1-a}\right)$$

$$= \frac{s}{(1-a)^2}-\frac{a-a^{s+1}}{(1-a)^2}-\frac{a}{1-a}\sum_{z=1}^{s} za^{z-1}$$

$$= \frac{s}{(1-a)^2}-\frac{a-a^{s+1}}{(1-a)^2}-\frac{a}{1-a}\left(\frac{1-a^s}{(1-a)^2}-\frac{sa^s}{1-a}\right)$$

$$= \frac{s(1-a)}{(1-a)^3}-\frac{a(1-a)-(1-a)a^{s+1}}{(1-a)^3}-\frac{a-a^{s+1}}{(1-a)^3}+\frac{sa^{s+1}(1-a)}{(1-a)^3}$$

$$= \frac{s\left(1+a^{s+1}\right)(1-a)-(2-a)\left(a^{s+1}-a\right)}{(1-a)^3}$$

$$\sum_{x=1}^{s-1}\sum_{z=1}^{x}\sum_{m=1}^{z} m a^{m-1} \;=\; \sum_{x=1}^{s-1} \frac{x\left(1+a^{x+1}\right)(1-a)-(2-a)\left(a^{x+1}-a\right)}{(1-a)^{3}}$$

$$= \frac{1}{(1-a)^{3}}\left(\sum_{x=1}^{s-1} x(1-a)+x a^{x+1}(1-a)-(2-a)\left(a^{x+1}-a\right)\right)$$

$$= \frac{1}{(1-a)^{3}}\left(\frac{s(s-1)}{2}(1-a)+a^{2}(1-a)\underbrace{\sum_{x=1}^{s-1} x a^{x-1}}_{=\frac{1-a^{s-1}}{(1-a)^{2}}-\frac{(s-1)a^{s-1}}{1-a}}\right.$$

$$\left. -a(2-a)(s-1)-(2-a)a^{2}\frac{1-a^{s-1}}{1-a}\right)$$

$$= \frac{1}{(1-a)^{3}}\left(\frac{s(s-1)}{2}(1-a)-(s-2)a^{s+1}-a(2-a)s+2a(1-a)\right)$$

### A.3.2 Proofs of Lemma 6: $\lim_{r\to 0} k = \lim_{r\to 0}(n-k)=\infty$

Recall $n$ and $k$ are the last states in which the leader and the follower, respectively, chooses to invest in an equilibrium. Both $n$ and $k$ are functions of the interest rate $r$. Also recall that we use $w_s \equiv v_s + v_{-s}$ to denote the total firm value of a market in state $s$.

**We first prove** $\lim_{r\to 0}(n-k)=\infty$.

Suppose $k$ and $(n-k)$ are both bounded as $r \to 0$; let $N$ be an upper bound for $n$, i.e. $N \geq n(r)$ for all $r$.

Consider the sequence of value functions $\hat{v}_s$ under alternative investment decisions: leader follows equilibrium strategies and invests in $n(r)$ states whereas follower does not invest at all. The sequence of value function dominates the equilibrium value functions ($\hat{v}_s \geq v_s$) for all $s \geq 0$, because:

1. The joint value is higher in every state $\hat{w}_s \geq w_s$, because flow payoffs are weakly higher and that the value functions are placing higher weights on higher states (which have higher flow payoffs). Hence the firm value in state zero is higher $\hat{v}_0 \geq v_0$.

2. The leader's value function can be written as a weighted average of flow payoffs in $s > 0$ and the value of being in state zero; the flow payoffs are the same for all $s > 0$, and $\hat{v}_0 \geq v_0$.

Furthermore when follower does not invest, the leader's value function always places higher weights in states with higher payoffs; hence $\hat{v}_s \geq v_s$ for all $s > 0$.

We now look for a contradiction. As $r \to 0$,

$$r\hat{v}_{N+1} = \frac{r\pi_{N+1} + \kappa r\hat{v}_N}{r + \kappa} \to r\hat{v}_N,$$

$$r\hat{v}_N = \frac{r\left(\pi_N - c\eta_N\right) + \kappa r\hat{v}_{N-1} + \eta_N r\hat{v}_{N+1}}{r + \kappa + \eta_N} \to r\hat{v}_{N-1},$$

and so on. By induction, $r\hat{v}_s \sim r\hat{v}_0$ for all $-N+1 \leq s \leq N+1$.

Also note that leader stops investing in state $n+1$ implies

$$\lim_{r\to 0} rv_{n+1} \geq \lim_{r\to 0} \pi_{n+2} - c\kappa,$$

thus $\lim_{r\to 0} r\hat{v}_0 \geq \lim_{r\to 0} \pi_{n+2} - c\kappa$.

Lastly, note $\Delta\hat{v}_0 \geq \Delta\hat{w}_0 = \frac{r\hat{w}_0 - (2\pi_0 - 2c\eta)}{2\eta} = \frac{r\hat{v}_0 - (\pi_0 - c\eta)}{\eta}$.

Putting these pieces together, we apply Proposition A.1 to compute a lower bound for $\Delta\hat{v}_n$ as a function of $\hat{v}_0$ and $\Delta\hat{v}_0$ (substituting $u_s = \hat{v}_0$, $u_{s+t} = \hat{v}_{n+1}$, $a = \kappa/\eta$, $b = r/\eta$, $\delta = \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta}$):

$$
\begin{aligned}
\lim_{r\to 0} \Delta\hat{v}_{n+1} &\geq \lim_{r\to 0}\left(\Delta\hat{v}_0\,(\kappa/\eta)^n + \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta}\frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta}\right) \\
&\geq \lim_{r\to 0}\frac{r\hat{v}_0 - (\pi_0 - c\eta)}{\eta}\,(\kappa/\eta)^n + \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta}\frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&\geq \lim_{r\to 0}\frac{\pi_{n+2} - c\kappa - (\pi_0 - c\eta)}{\eta}\,(\kappa/\eta)^n + \frac{\pi_{n+2} - c\kappa - (\pi_{n+2} - c\eta)}{\eta}\frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&> \lim_{r\to 0} c\,(\kappa/\eta)^n + \frac{c\,(\eta - \kappa)}{\eta}\frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&= c,
\end{aligned}
$$

where the last inequality follows from assumption 1, that $\pi_{n+2} - \pi_0 \geq \pi_1 - \pi_0 > c\kappa$. But this is a contradiction to the claim that leader stops investing in state $n+1$ (i.e. $\Delta\hat{v}_{n+1} \leq c$ for any $r$).

Next, suppose $\lim_{r\to 0} k = \infty$ but $(n-k)$ remain bounded. Let $\epsilon \equiv 2c\eta - \lim_{s\to\infty}(\pi_s + \pi_{-s})$; $\epsilon > 0$ under assumption 1. The joint flow payoff $\pi_s + \pi_{-s} - 2c\eta$ is negative and bounded above

by $-\epsilon$ in all states $s \leq k$. The joint market value in state 0 is

$$
\begin{aligned}
w_0 &= \sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) + \sum_{s'=k+1}^{n+1} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) \\
&\leq \frac{-\epsilon}{r} \cdot \left( \sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \right) + \sum_{s'=k+1}^{n+1} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) .
\end{aligned}
$$

As $k \to \infty$ while $n - k$ remain bounded, the present-discount fraction of time that the market spends in states $s \leq k$ converges to 1 ($\sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \to 1$), implying that $\lim_{r \to 0} r w_0$ is negative. Since firms can always ensure non-negative payoffs by not taking any investment, this cannot be an equilibrium, reaching a contradiction. Hence $\lim_{r \to 0} (n - k) = \infty$.

To show $\lim_{r \to 0} k = \infty$, we first establish a few additional asymptotic properties of the model.

**Lemma A.2.** The following statements are true:

1. $r v_n \sim \pi - c\kappa$, where $\pi \equiv \lim_{s \to \infty} \pi_s$.

2. $v_{n+1} - v_n \sim c$.

3. $r (n - k) \sim 0$.

4. $rk \sim 0$.

**Proof**

1. The claim follows from the fact that if firm invests in state $n$ but not in state $n + 1$, then

$$
v_{n+2} - v_{n+1} = \frac{\pi_{n+2} - r v_{n+1}}{r + \kappa} \leq c
$$

$$
v_{n+1} - v_n = \frac{\pi_{n+1} - r v_n}{r + \kappa} \geq c
$$

implying

$$
\pi - c\kappa = \lim_{r \to 0} (\pi_{n+2} - c\kappa) \geq \lim_{r \to 0} r v_n \geq \lim_{r \to 0} (\pi_{n+1} - c\kappa) = \pi - c\kappa,
$$

as desired.

67

2. The claim follows from the previous one: $v_{n+1} - v_n = \frac{\pi_{n+1}}{r+\kappa} - \frac{rv_n}{r+\kappa} \sim \frac{\pi - rv_n}{\kappa} \sim c.$

3. The previous claims show $rv_n \sim \pi - c\kappa$ and $\Delta v_n \sim c$. We apply Proposition A.1 to iterate backwards and obtain

$$
\begin{aligned}
\lim_{r\to\infty} r\,(v_k - v_n) \quad &\geq \quad \lim_{r\to\infty} -\frac{r^2}{\kappa^2} \frac{rv_n - (\pi - c\eta)}{(1 - \eta/\kappa)^4} \,(\eta/\kappa)^{n-k+1} \\
&\sim \quad -\frac{r^2}{\kappa^2} \frac{c\,(\eta - \kappa)}{(1 - \eta/\kappa)^4} \,(\eta/\kappa)^{n-k+1}
\end{aligned}
$$

Since $|\lim_{r\to 0} r\,(v_k - v_n)| \leq \pi$, it must be the case that $\lim_{r\to 0} r^2\,(\eta/\kappa)^{n-k+1}$ remain bounded; therefore $r\,(n-k) \sim 0$.

4. We apply Proposition A.1 to find a lower bound for $w_k - w_0$:

$$
\begin{aligned}
\lim_{r\to 0} r\,(w_k - w_0) \quad &\geq \quad \lim_{r\to 0} \left( \Delta w_0 + \frac{rw_0 - (\pi - 2c\eta)}{a - 1} \right) \frac{ra^k}{a - 1} \\
&\geq \quad \lim_{r\to 0} \left( \frac{2c\eta - \pi}{a - 1} \right) \frac{ra^k}{a - 1}.
\end{aligned}
$$

Since $r\,(w_k - w_0)$ stays bounded, it must be the case that $ra^k$ is bounded; therefore $rk \sim 0$.

**Lemma A.3.** $rv_{-k} \sim r\Delta v_{-k} \sim rv_{-n} \sim \Delta v_{-n} \sim 0.$

**Proof.**  First, note that follower does not invest in state $k + 1$ implies $c \geq \Delta v_{-(k+1)}$. We apply Proposition A.1 to find an upper bound for $(v_{-n} - v_{-k})$ as a function of $rv_{-k}$ and $\Delta v_{-(k+1)}$:

$$
v_{-n} - v_{-k} \leq \lim_{r\to 0} \left( -\Delta v_{-(k+1)} \frac{\eta}{\eta - \kappa} + (n - k) \frac{rv_{-k}}{\eta - \kappa} \right).
$$

Hence, $v_{-n} - v_{-k} \leq -c\frac{\eta}{\eta - \kappa}$ and $r\,(v_{-n} - v_{-k}) \sim 0$.

Let $m = \text{floor}(k + \frac{n-k}{2})$. That the follower does not invest in state $m$ implies that $c \geq \Delta v_{-m}$.

Proposition A.1. provides a lower bound for $v_{-n} - v_{-(n-1)}$ as a function of $rv_{-m}$ and $\Delta v_{-(m+1)}$:

$$\lim_{r \to 0} \left( v_{-(n+1)} - v_{-n} \right) \geq \lim_{r \to 0} -\Delta v_{-(m+1)} \left( \kappa / \eta \right)^{n-m} + \frac{rv_{-m} - \pi_{-m}}{\eta - \kappa}$$

$$= \lim_{r \to 0} \frac{rv_{-m}}{\eta - \kappa},$$

where the equality follows from $\lim_{m \to \infty} \pi_{-m} \to 0$. Hence, since the LHS is non-positive, it must be the case that $\lim_{r \to 0} \Delta v_{-n} = \lim_{r \to 0} rv_{-m} = 0$. But since $rv_{-n} \leq rv_{-m}$, it must be that $rv_{-n} \sim rv_{-k} \sim 0$. That $r\Delta v_{-k} \sim 0$ follows directly from the HJB equation for state $k$.

**We now prove $\lim_{r \to 0} k = \infty$.**

We first show that, if $k$ is bounded, both $rw_k$ and $r\Delta w_k$ must be asymptotically zero in order to be consistent with $rv_{-k} \sim 0$. Specifically, we use the fact that $0 \leq \pi_{-s}$ for all $0 \leq s \leq k$ and apply Proposition A.1 (simplification 1a, substituting $u_s \equiv v_{-k+1}$, $u_{s+t} = v_0$, $t = k+1$, $\Delta u_s = \Delta v_{-k}$, $a = \frac{\eta}{\eta + \kappa}$, $b = \frac{r}{\eta + \kappa}$, $\delta = \frac{rv_{-(k+1)} - (-c\eta)}{\eta + \kappa}$) to find an asymptotic upper bound for $rv_0$:

$$\lim_{r \to 0} rv_0 = \lim_{r \to 0} r \left( v_0 - v_{-(k+1)} \right)$$

$$\leq \lim_{r \to 0} \frac{r}{1 - \kappa / \eta} \left( \Delta v_{-(k+1)} + k \frac{rv_{-(k+1)} + c\eta}{\eta} \right)$$

If $k$ is bounded, the last expression converges to zero, implying that $rv_0 \sim rw_0 \sim 0$. Lemma A.1 further implies that $\Delta w_0 \sim c$. Upper bounds for $rw_k$ and $r\Delta w_k$ can be found, as functions of $\Delta w_0$ and $rw_0$, using Proposition A.1 (simplification 2, substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta + \kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_0 - (-2c\eta)}{\eta}$):

$$\lim_{r \to 0} \left( rw_k - rw_0 \right) \leq \lim_{r \to 0} \left( \Delta w_0 + \frac{rw_0 + 2c\eta}{\kappa} \right) \frac{\eta}{\kappa} r \left( \frac{\eta + \kappa}{\eta} \right)^k$$

$$\lim_{r \to 0} \left( r\Delta w_k \right) \leq \lim_{r \to 0} \left( \Delta w_0 + \frac{rw_0 + 2c\eta}{\kappa} \right) r \left( \frac{\eta + \kappa}{\eta} \right)^{k-1}. \tag{18}$$

If $k$ is bounded, the RHS of both inequalities converge to zero, implying $rw_k \sim r\Delta w_k \sim 0$.

We now look for a contradiction. Suppose $rw_k \sim r\Delta w_k \sim 0$; we apply Proposition A.1 (sim-

69

plification 1a, substituting $u_s \equiv w_k$, $u_{s+t} = w_{n+1}$, $t = n+1-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_k - (\pi_k - c\eta)}{\eta})$ and obtain $\frac{rw_k - (\pi_k - c\eta)}{\eta - \kappa}$ as an asymptotic upper bound for $w_{n+1} - w_n$ (noting that $\pi_k$ is a lower bound for $\pi_s$ for all $n \geq s \geq k$). Lemma A.2 part 2 further implies that

$$\lim_{r \to 0} \frac{rw_k - (\pi_k - c\eta)}{\eta - \kappa} \geq c$$

$$\iff \lim_{r \to 0} rw_k \geq \pi - c\kappa > 0, \tag{19}$$

which contradicts the presumption that $rw_k \sim 0$. The last inequality follows from assumption 1 ($\pi_1 - \pi_0 \geq c\kappa$), that firms is state 0 has incentive to invest when sufficiently patient. QED.

The fact that $\lim_{r \to 0} r \Delta w_k > 0$, together with inequality (18), implies $\lim_{r \to 0} r \left( \frac{\eta + \kappa}{\eta} \right)^k > 0$. We summarize these statements into a lemma, which will be useful later.

**Lemma A.4.** $\lim_{r \to 0} r \left( \frac{\eta + \kappa}{\eta} \right)^k > 0$ **and** $\lim_{r \to 0} r \Delta w_k > 0$.

### A.3.3  Proof of Proposition 1.

Lemma 4 implies $g = \ln \lambda \times \left( \mu^C \cdot (\eta + \kappa) + \mu^M \cdot \kappa \right)$. We now show $\lim_{r \to 0} \alpha^{n-k} (1 + \alpha)^k = 0$, which, based on Lemma 4, is a sufficient condition for $\mu^M \to 1$, $\mu^C \to 0$, and $g \to \kappa \cdot \ln \lambda$.

To proceed, we first find a lower bound for $\Delta w_k$ by applying simplification 2 of Proposition A.1 (substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta + \kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_0 - (\pi - 2c\eta)}{\eta}$):

$$\lim_{r \to 0} r \Delta w_k \geq C_2 \equiv \lim_{r \to 0} \left( \Delta w_0 + \frac{rw_0 - (\pi - 2c\eta)}{\kappa} \right) r \left( \frac{\eta + \kappa}{\eta} \right)^k. \tag{20}$$

Simplification 1 of Proposition A.1 provides asymptotic bounds for $\Delta w_n$ (substituting $u_s \equiv w_k$, $u_{s+t} = w_n$, $t = n - k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; the upper bound is obtained using $\delta = \frac{rw_k - (\pi_k - c\eta)}{\eta}$ and the lower bound is obtained using $\delta = \frac{rw_k - (\pi - c\eta)}{\eta}$):

$$\lim_{r \to 0} \left[ \Delta w_k \left( (\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta - \kappa)^2} \right) + \frac{rw_k + c\eta - \pi_k}{\eta - \kappa} \right] \geq \lim_{r \to 0} \Delta w_n$$

and

$$\lim_{r \to 0} \Delta w_n \geq \lim_{r \to 0} \left[ \Delta w_k \left( (\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta - \kappa)^2} \right) + \frac{rw_k + c\eta - \pi}{\eta - \kappa} \right].$$

Since $\lim_{r \to 0} \pi_k = \pi$, the lower and upper bounds coincide asymptotically. Furthermore, Lemma A.2 shows $\Delta w_n \sim c$; hence,

$$c \quad \sim \quad \Delta w_k \left( (\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta - \kappa)^2} \right) + \frac{rw_k + c\eta - \pi}{\eta - \kappa}. \tag{21}$$

Next, we apply simplification 1b of Proposition A.1 to obtain (substituting $u_s \equiv w_k$, $u_{s+t} = w_n$, $t = n - k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; the simplification applies because $\lim_{r \to 0} r\Delta w_k > 0$, as stated in Lemma A.4):

$$r(w_n - w_k) \quad \sim \quad \frac{r\Delta w_k}{(\eta - \kappa)/\eta} \tag{22}$$

$$\implies \pi - c\kappa - rw_k \sim \frac{r\Delta w_k}{(\eta - \kappa)/\eta} \tag{23}$$

where equivalence (23) follows from part 1 of Lemma A2.

Substituting asymptotic equivalence (23) into (21), we obtain

$$c \quad \sim \quad c + \Delta w_k \left( (\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta - \kappa)^2} \right) - \frac{r\eta \Delta w_k}{(\eta - \kappa)^2}$$

$$\iff 0 \sim \Delta w_k (\kappa/\eta)^{n-k}$$

Inequality (20) implies

$$0 \geq \lim_{r \to 0} \left( \Delta w_0 + \frac{rw_0 - (\pi - 2c\eta)}{\kappa} \right) \left( \frac{\eta + \kappa}{\eta} \right)^k (\kappa/\eta)^{n-k}$$

Given $\Delta w_0 \geq 0$, $rw_0 \geq 0$, and $2c\eta - \pi > 0$, the inequality can hold if and only if

$$\lim_{r \to 0} \left( \frac{\eta + \kappa}{\eta} \right)^k (\kappa/\eta)^{n-k} = 0,$$

as desired.

71

Note also that the equivalence (20) implies $r(1+\alpha)^k$ converges to a non-negative constant; hence, $k$ grows at rate $\log r$.

### A.3.4 Proof of Proposition 2.

Let $(k, n)$ be the equilibrium investment decisions under interest rate $r$ and $(k_2, n_2)$ be the investments under $r - dr$. Proposition A.1 enables us to provide first-order approximations of value functions before and after the interest rate shock $dr$ (denoted by $\{v_s\}_{s=-\infty}^\infty$ and $\{\hat{v}_s\}_{s=-\infty}^\infty$ respectively). We then use these expressions to show

$$
\begin{aligned}
\frac{\hat{V}^F}{V^F} &= \frac{\sum_{s=1}^{n+1} \mu_s \hat{v}_{-s}}{\sum_{s=1}^{n+1} \mu_s v_{-s}} + O(r) \\
&= \frac{k_2}{k} + O(r).
\end{aligned}
$$

The fact that $r\left(\frac{\eta+\kappa}{\eta}\right)^k$ converges to a non-negative constant (c.f. Lemma A.4) implies

$$
\frac{\hat{V}^F}{V^F} = \frac{\log(r - dr)}{\log r} + O(r).
$$

The part about the on-impact, proportional change in the total market value of leaders formally follows from similar derivations, but it has a more straight-forward intuition. As $r \to 0$, market leadership becomes endogenous absorbing, and the total market value of leaders becomes inversely proportional to the interest rate: $\lim_{r \to 0} rV^L = C_3 > 0$. Hence, following a decline in interest rate, the value of leaders changes proportionally with the interest rate, i.e. $\hat{V}^L/V^L = \frac{r}{r-dr} + O(r)$.

Before we prove the claim, we first establish the following lemma.

**Lemma A.5.** $\Delta v_{-k} \sim c, v_{-k} \sim \frac{c}{1-\kappa/\eta}, v_{-(n+1)} \sim 0.$ Proof. Note that $v_{-(k-1)} - v_{-k} \geq c, v_{-(k-2)} - v_{-(k-1)} \geq c$, and $c \geq v_{-k} - v_{-(k+1)}$. The HJB equation for followers in state $k-1$ and $k$ respectively

imply

$$rv_{-(k-1)} = \pi_{-(k-1)} + \eta\left(v_{-(k-2)} - v_{-(k-1)} - c\right)$$
$$+\kappa\left(v_{-(k-2)} - v_{-(k-1)}\right) + \eta\left(v_{-k} - v_{-(k-1)}\right)$$

$$rv_{-k} = \pi_{-k} + \eta\left(v_{-(k-1)} - v_{-k} - c\right)$$
$$+\kappa\left(v_{-(k-1)} - v_{-k}\right) + \eta\left(v_{-(k+1)} - v_{-k}\right).$$

Substituting the previous inequalities, we get

$$rv_{-(k-1)} \leq \pi_{-(k-1)} + \kappa c + \eta\left(v_{-k} - v_{-(k-1)}\right)$$

$$rv_{-k} \geq \pi_{-k} + (\eta + \kappa)\left(v_{-(k-1)} - v_{-k}\right) - 2\eta c.$$

Hence,

$$rv_{-(k-1)} - rv_{-k} \leq \pi_{-(k-1)} - \pi_{-k} - (2\eta + \kappa)\left(v_{-(k-1)} - v_{-k}\right) + (2\eta + \kappa)\,c$$

$$\Longleftrightarrow \left(v_{-(k-1)} - v_{-k}\right) \leq \frac{\pi_{-(k-1)} - \pi_{-k}}{2\eta + \kappa + r} + \frac{2\eta + \kappa}{2\eta + \kappa + r}c,$$

which implies $\lim_{r\to 0}\left(v_{-(k-1)} - v_{-k}\right) \leq c$. Coupled with the fact that $v_{-(k-1)} - v_{-k} \geq c$, this establishes that $v_{-(k-1)} - v_{-k} \sim c$.

We can apply simplification 1a) of Proposition A1 to show $v_{-k} - v_{-(n+1)} \sim \frac{c}{1-\kappa/\eta}$; the lemma is thus complete once we show $v_{-(n+1)} \sim 0$. Note that we can write $v_{-(n+1)}$ as a weighted average of the flow payoffs in states $k+1$ through $n+1$ and the value function in state $-k$:

$$v_{-(n+1)} = \sum_{s=k+1}^{n+1} \epsilon_s \pi_{-s} + \epsilon_k v_{-k}, \quad \text{where } \sum_{s=k}^{n} \epsilon_k = 1.$$

The flow payoffs $\pi_{-k}$ approach zero as $r \to 0$; hence, $v_{-(n+1)} \sim \epsilon_k v_{-k}$. The term $\epsilon_k$ can be found

by solving the recursive relationship

$$
\begin{aligned}
v_{-(n+1)} &= \frac{\kappa}{r+\kappa} v_{-n} \\
v_{-n} &= \frac{\kappa}{r+\kappa+\eta} v_{-(n-1)} + \frac{\eta}{r+\kappa+\eta} v_{-(n+1)} \\
&\vdots \\
v_{-(k+1)} &= \frac{\kappa}{r+\kappa+\eta} v_{-k} + \frac{\eta}{r+\kappa+\eta} v_{-(k+2)}.
\end{aligned}
$$

It is easy to see that $\epsilon_k < (\kappa/\eta)^{n-k}$; hence, as $r \to 0$, $\frac{v_{-(n+1)}}{v_{-k}} \to 0$. This implies that $v_{-(n+1)} \sim 0$ and $v_{-k} \sim \frac{c}{1-\kappa/\eta}$, as desired. QED.

We now show $\frac{\hat{V}^F}{V^F} = \frac{k_2}{k} + O(r)$. The total market value of followers is

$$
\begin{aligned}
& \sum_{s=1}^{k} \mu_s v_{-s} + \sum_{s=k+1}^{n+1} \mu_s v_{-s} \\
=\; & \underbrace{2\mu_0 \left( a v_{-1} + a^2 v_{-2} + \cdots + a^k v_{-k} \right)}_{\text{total value of followers in the competitive region}} \\
& + \underbrace{\mu_{k+1} \left( v_{-(k+1)} + b v_{-(k+2)} + b^2 v_{-(k+3)} + \cdots + b^{n-k} v_{-(n+1)} \right)}_{\text{total value of followers in the monopolistic region}},
\end{aligned}
$$

where $a \equiv \frac{\eta}{\eta+\kappa}$ and $b \equiv \eta/\kappa$. We analyze the two terms separately.

First, by the fact that $\Delta v_{-k} \sim c$ and $v_{-k} \sim \frac{c}{1-a}$ we can apply Proposition A1 to show, for all $s \le k$, $v_{-s} \sim \frac{c}{1-a}\left(1 - a^{k-s}\right) + \frac{ca}{1-a}\left((k-s) - \frac{a-a^{k-s}}{1-a}\right)$. Hence,

$$
\begin{aligned}
\sum_{s=1}^{k} \mu_s v_{-s} &= 2\mu_0 \left( a v_{-1} + a^2 v_{-2} + \cdots + a^k v_{-k} \right) \\
&\sim 2\mu_0 \frac{c}{1-a} \sum_{s=1}^{k} \left( a^s - a^k + a\left( a^s(k-s) - \frac{a^s - a^k}{1-a} \right) \right) \\
&\sim 2\mu_0 c \left( \frac{a}{1-a} \right)^2 k,
\end{aligned}
$$

where the last line follows after applying the summation formula $\sum_{s=1}^{k}(k-s)\cdot a^s = \frac{a}{1-a}\left(k - \frac{1-a^k}{1-a}\right)$.

We now compute the market value of followers in the monopolistic region. Using Proposition

A1, we derive

$$v_{-(k+s)} \sim v_{-k} - \frac{c}{1 - \kappa/\eta} \left(1 - (\kappa/\eta)^s\right) \sim \frac{c}{1 - \kappa/\eta} (\kappa/\eta)^s,$$

thus

$$\sum_{s=k+1}^{n+1} \mu_s v_{-s}$$

$$= \mu_{k+1} \left(v_{-(k+1)} + (\eta/\kappa) v_{-(k+2)} + (\eta/\kappa)^2 v_{-(k+3)} + \cdots + (\eta/\kappa)^{n-k} v_{-(n+1)}\right)$$

$$\sim \mu_{k+1} \frac{\alpha c}{1 - \alpha} (n - k)$$

The total market value of followers is thus

$$V^F \equiv \sum_{s=1}^{k} \mu_s v_{-s} + \sum_{s=k+1}^{n+1} \mu_s v_{-s}$$

$$\sim 2\mu_0 c \left(\frac{a}{1 - a}\right)^2 k + \mu_{k+1} \frac{\alpha c}{1 - \alpha} (n - k)$$

$$= 2\mu_0 \left(c \left(\frac{a}{1 - a}\right)^2 k + \left(\frac{\eta}{\eta + \kappa}\right)^{k+1} \frac{\alpha c}{1 - \alpha} (n - k)\right)$$

$$\sim 2\mu_0 c \left(\frac{a}{1 - a}\right)^2 k.$$

Now consider the new equilibrium characterized $(k_2, n_2)$ under interest rate $r - dr$. Let value functions be denoted by $\hat{v}_s$ under the new equilibrium. The market value of followers, evaluated using the steady-state under $r$, is

$$\hat{V}^F \equiv \sum_{s=1}^{k} \mu_s \hat{v}_{-s} + \sum_{s=k+1}^{n+1} \mu_s \hat{v}_{-s}.$$

Following the same derivation as before, we can show

$$\hat{V}^F \sim 2\mu_0 c \left(\frac{a}{1 - a}\right)^2 k_2,$$

thus

$$\frac{\hat{V}^F}{V^F} = \frac{k_2}{k} + O(r),$$

as desired. That $\frac{\hat{V}^F}{V^F} = \frac{\log(r-dr)}{\log r} + O\left(r\right)$ follows from the convergence of $r\left(\frac{\eta+\kappa}{\eta}\right)^k$ to a non-negative constant (Lemma A.4.)

The on-impact, proportional change in the total market value of leaders can be derived analogously, as Proposition A.1 enables us to derive an asymptotic analytic approximation for the value functions. We omit the derivations here and instead provide a simpler intuition for the result. As interest rate converges to zero, the total market value of leaders becomes inversely proportional to the interest rate: $\lim_{r\to 0} rV^L = C_3 > 0$. Hence, following a small decline in interest rate, the value of leaders changes proportionally with the interest rate, i.e. $\hat{V}^L/V^L = \frac{r}{r-dr} + O\left(r\right)$.

# B  Appendix: A numerical illustration

In this numerical exercise, we relax the assumption that investments are bounded with a constant marginal cost; instead, we parametrize the investment cost as a quadratic function of the investment intensity: $c(\eta) = \delta\eta^2/2$ for $\eta \in [0, \infty)$, where $\delta$ is a cost parameter we calibrate. This is done for three reasons. First, we demonstrate numerically that Proposition 1 survive beyond the bounded and constant-marginal-cost specification. Second, a convex cost function implies that first-order conditions with respect to investments are sufficient for the model solution, reducing computational burdens. Third, the specification implies that changes in investment intensities are smoothed out across states, thereby getting around the discrete changes in investments in the "bang-bang" solution of the baseline model. All other ingredients remain unchanged from the baseline model.

The HJB equations of the numerical model follow

$$rv_s = \max_{\eta\geq 0} \pi_s - \eta^2/2 + (\kappa + \eta_{-s})\left(v_{s-1} - v_s\right) + \eta\left(v_{s+1} - v_s\right)$$

$$rv_{-s} = \max_{\eta\geq 0} \pi_{-s} - \eta^2/2 + (\kappa + \eta)\left(v_{-(s-1)} - v_{-s}\right) + \eta_s\left(v_{-(s+1)} - v_{-s}\right)$$

$$rv_0 = \max_{\eta\geq 0} \pi_0 - \eta^2/2 + \eta_0\left(v_{-1} - v_0\right) + \eta\left(v_1 - v_0\right).$$

We now provide demonstrations of the investment function $\{\eta_s\}$, the steady-state distribution

$\{\mu_s\}$, and value functions as well as how these functions change in response to lower interest rates. We also provide numerical illustrations of how steady-state levels of productivity growth vary with interest rates. In generating these numerical plots, we parametrize the within-market demand aggregator using $\sigma = \infty$, the case in which two firms produce perfect substitutes.

The top panel in Figure A2 shows the investment functions of the leader and follower across states for a high interest rate. The figure illustrates the leader dominance of Lemma 3; the leader invests more in all states beyond the neck-to-neck state. The dotted lines show the investment functions of the leader and follower for a lower interest rate. Both the leader and follower invest more in all states when the interest rate is lower, which represents the traditional effect of lower interest rates on investment.

However, as the bottom panel demonstrates, the leader's investment response to a lower interest rate is stronger than the follower's response for all states. The stronger response of the leader's investment to lower interest rates is the driving force behind the strategic effect through which lower interest rates boost market concentration.

The top panel of Figure A3 shows that, following a decline in $r$, the steady-state distribution of market structure shifts to the right, and aggregate market power increases.

Why does the leader's investment respond more to a lower interest rate? The bottom panel of Figure A3 shows the leader's and follower's value functions before and after a decline in the interest rate. The change in the leader's value is larger than the change in the follower's value; this is the key driver behind the leader's stronger investment response following a drop in $r$. Finally, Figure 1 numerically verifies the central result of the Proposition above. For a low enough interest rate, a further decline in the interest rate leads to lower growth. Figure 1 also verifies that $g \to \kappa \cdot \ln \lambda$ in the numerical exercise with variable investment intensity.

Figure 4 demonstrates Proposition 2, that declines in interest rate has asymmetric on-impact effects on the market value of leaders and followers. Starting from a high-level of interest rate, declines in $r$ hurts leaders on average; yet, starting from a low-level of $r$, further declines in $r$ unambiguously causes leaders' market value to appreciate relative to followers', and the asymmetry becomes stronger when the initial, pre-shock level of interest rate is lower.

## Appendix Tables and Figures

[These tables and figures are referenced in the main text.]

Figure A1: market value of leaders respond more to decline in $r$, especially when initial $r$ is low



**Realtive changes in total market value of leaders over time following a small decline in interest rate**

— starting from low $r$
--- starting from moderate $r$
- - starting from high $r$

time
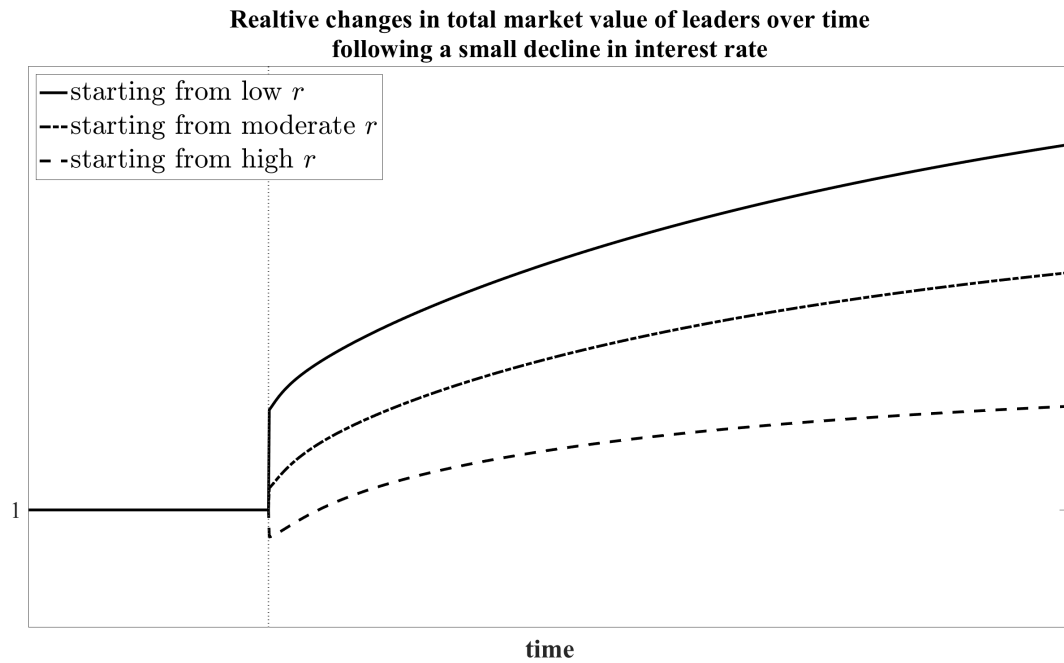
## Figure A2: Investment response to a decline in $r$

Investment by productivity gap: leader ($\eta_s$) and follower ($\eta_{-s}$)



Legend:
- leader (low $r$)
- follower (low $r$)
- leader (high $r$)
- follower (high $r$)

$\eta_s$

State $s$

Investment response to a drop in interest rate



Legend:
- leader
- follower

$\Delta\eta_s$
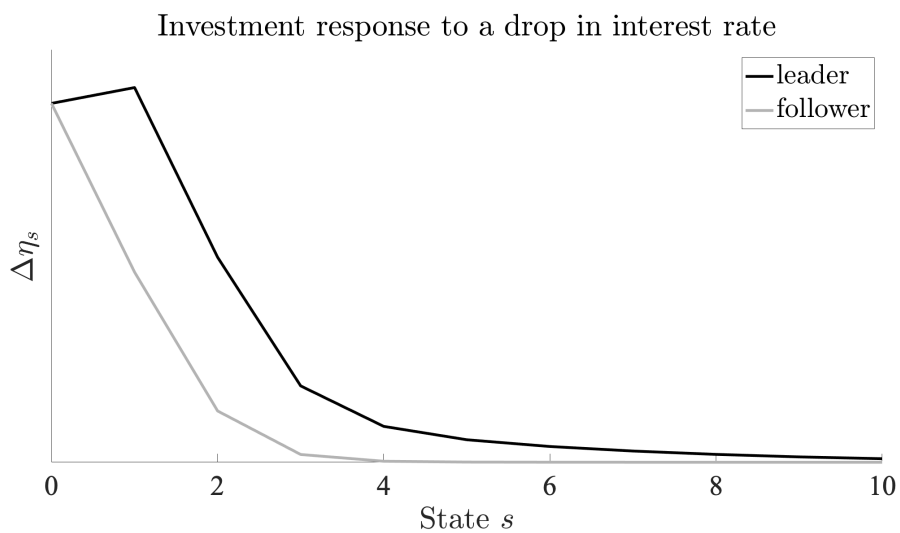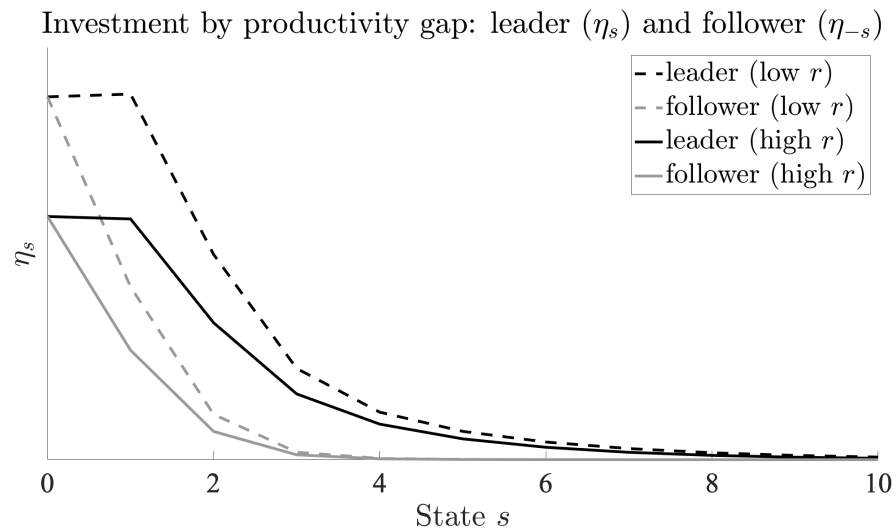
State $s$

Figure A3: Response of steady-state distribution and value functions to a decline in $r$

Steady-state distribution of productivity gap: $\mu_s$
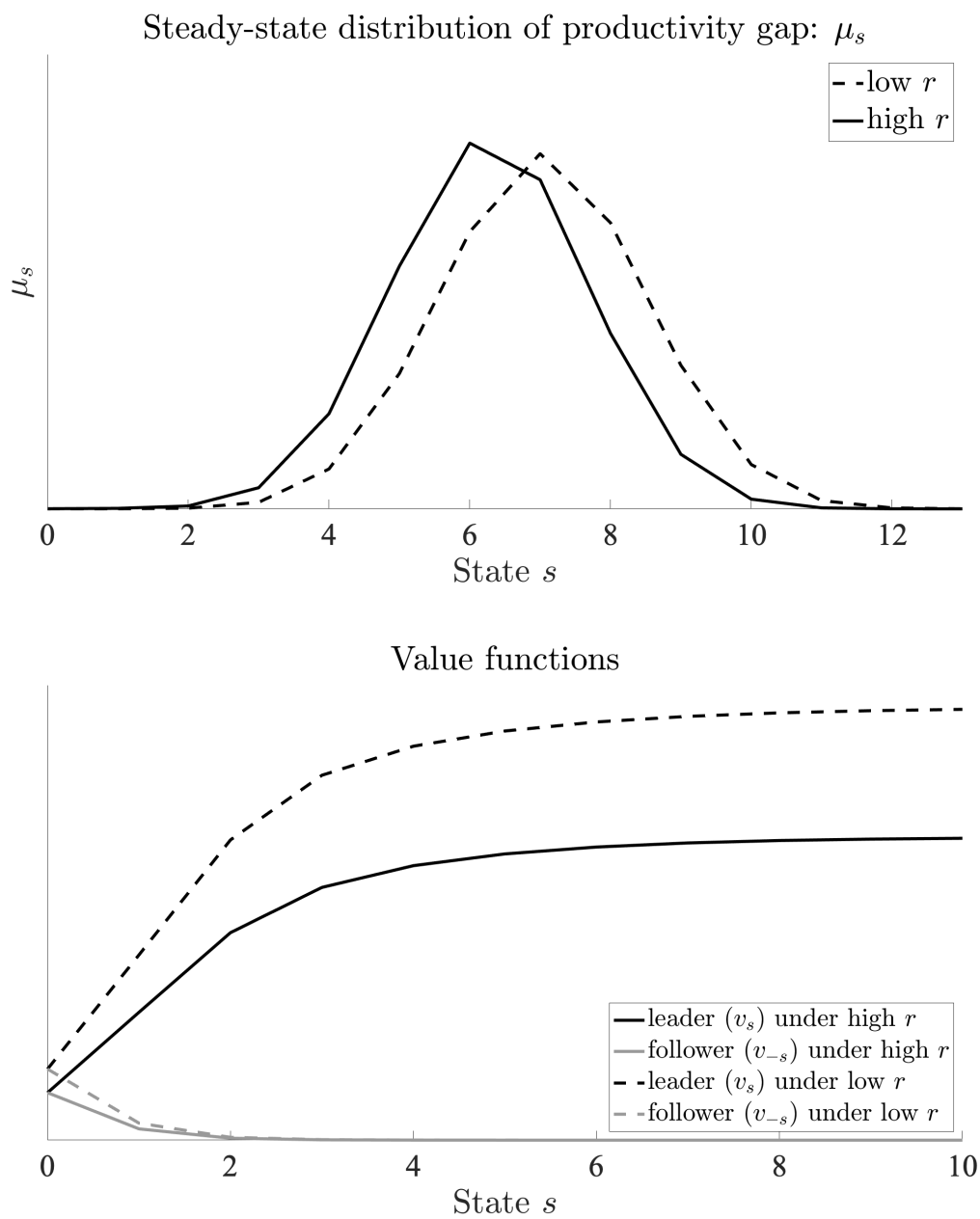


Value functions

Figure A4: Aggregate profit share, market concentration and interest rate
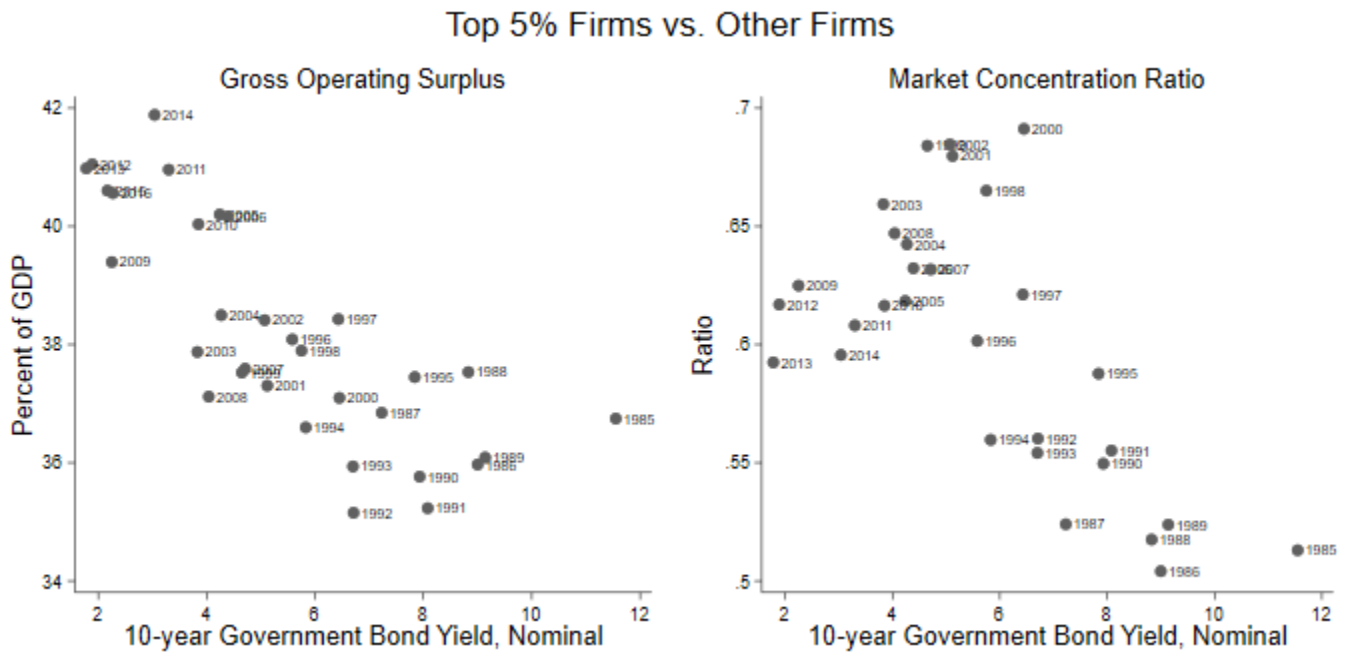
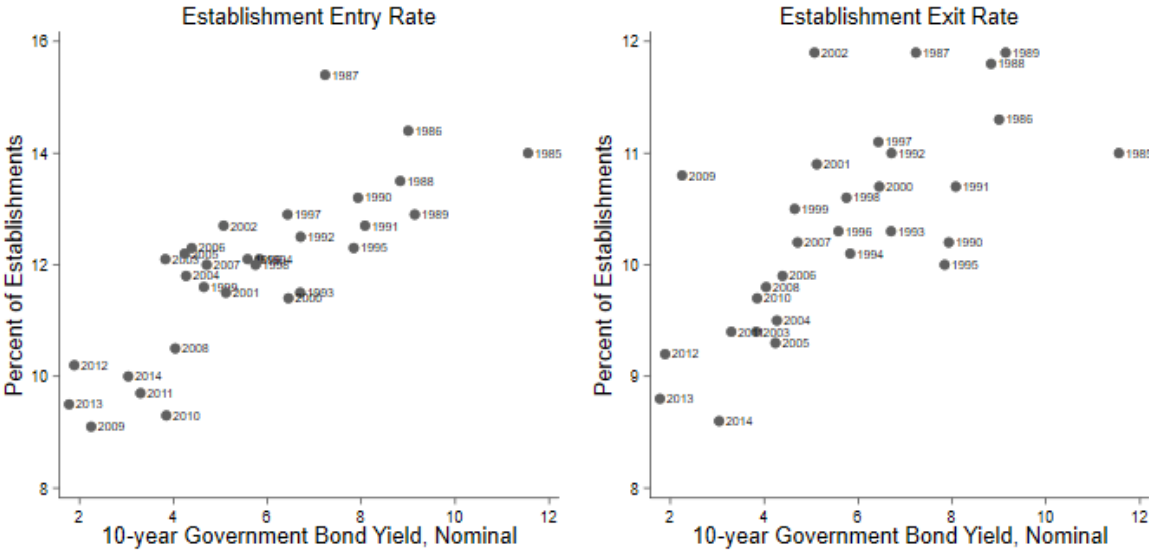# Figure A5: Business Dynamism

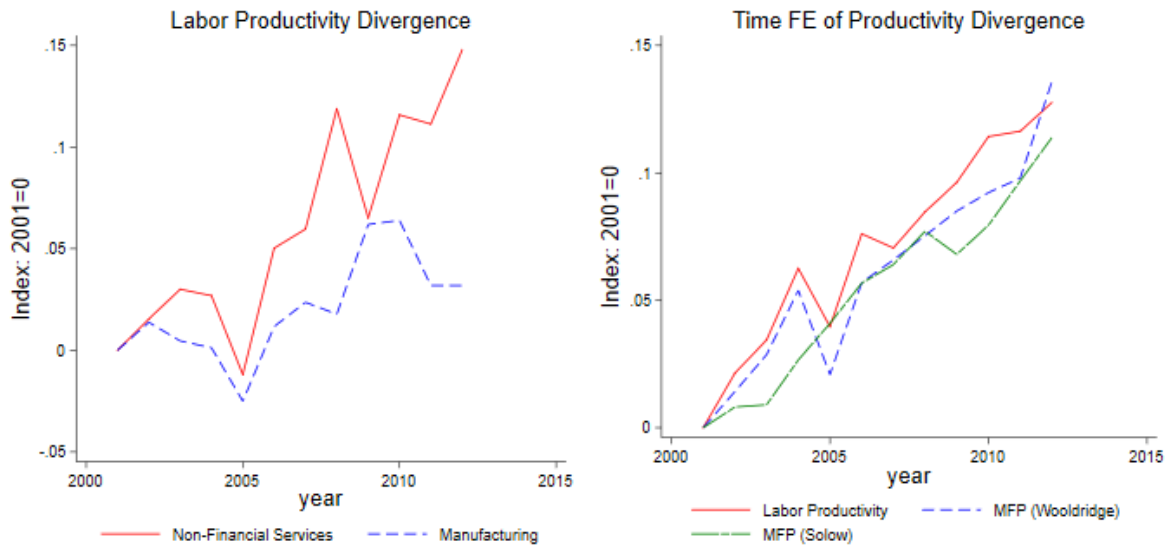Figure A6: Widening productivity gap between leaders and followers

Table A1: Differential Interest Rate Responses of Leaders vs. Followers: Top 5 Percent (Full Sample)

| | Stock Return | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Top 5 Percent=1 x $\Delta i$ | -1.019*** | -3.303** | -4.390*** | -2.183*** | -3.128*** | -3.493*** |
| | (0.219) | (0.943) | (0.883) | (0.595) | (0.661) | (0.517) |
| Top 5 Percent=1 x $\Delta i$ x Lagged $i$ | | 0.254** | 0.341*** | | | 0.259*** |
| | | (0.084) | (0.079) | | | (0.044) |
| Top 5 Percent=1 x $\Delta i$ x Lagged real $i$ (Clev and Fred) | | | | 0.330** | 0.783*** | |
| | | | | (0.116) | (0.165) | |
| Firm $\beta$ x $\Delta i$ | | | | | | 11.74*** |
| | | | | | | (0.919) |
| Firm $\beta$ x $\Delta i$ x Lagged $i$ | | | | | | -1.096*** |
| | | | | | | (0.098) |
| Sample | All | All | All | All | All | All |
| Controls | N | N | Y | N | Y | |
| Industry-Date FE | Y | Y | Y | Y | Y | Y |
| N | 74,103,576 | 74,103,576 | 46,832,612 | 74,103,576 | 46,832,612 | 73,745,550 |
| R-sq | 0.426 | 0.426 | 0.423 | 0.426 | 0.423 | 0.430 |

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t}\Delta i_t + \beta_2 D_{i,j,t}i_{t-1} + \beta_3 D_{i,j,t}\Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm $i$ in industry $j$ at date $t$. $\Delta \ln\left(P_{i,j,t}\right)$ is defined here as the log change in the stock price for firm $i$ in industry $j$ from date $t-91$ to $t$ (one quarter growth). $D_{i,j,t}$ is defined here as an indicator equal to 1 at date $t$ when a firm $i$ is in the top 5% of market capitalization in its industry $j$ on date $t-91$. Firms with $D_{i,j,t}$=1 are called leaders while the rest are called followers. $i_t$ is defined as the nominal 10-year Treasury yield, with $i_{t-1}$ being the interest rate 91 days prior and $\Delta i_t$ being the change in the interest rate from date $t-91$ to $t$. Controls $X$ include a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and percent of pre-tax income that goes to taxes. Industry classifications are the Fama-French industry classifications (FF). Lagged real rates were built using monthly 10-year inflation expectations from the Cleveland Fed and the daily 10-year Treasury yield at the beginning of each month (post-1982), and the CPI series from the FED (pre-1982). Standard errors are dually clustered by industry and date.

Table A2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent (Full Sample)

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | | | Portfolio Return | | | |
| $\Delta i_t$ | -0.985*** | -3.237*** | -2.210*** | -1.874*** | -3.176*** | -2.885*** |
| | (0.277) | (0.616) | (0.497) | (0.558) | (0.909) | (0.797) |
| $i_{t-1}$ | | 0.0597 | | 0.00316 | 0.0222 | 0.0927 |
| | | (0.048) | | (0.042) | (0.075) | (0.067) |
| $\Delta i_t \times i_{t-1}$ | | 0.255*** | | 0.0727 | 0.234** | 0.281** |
| | | (0.058) | | (0.053) | (0.081) | (0.106) |
| real $i_{t-1}$ (Clev and Fred) | | | 0.285*** | | | |
| | | | (0.074) | | | |
| $\Delta i_t \times$ real $i_{t-1}$ (Clev and Fred) | | | 0.344*** | | | |
| | | | (0.103) | | | |
| Excess Market Return | | | | -0.204*** | | |
| | | | | (0.019) | | |
| High Minus Low | | | | 0.0153 | | |
| | | | | (0.037) | | |
| $(\Delta i_t > 0)$=1 $\times \Delta i_t$ | | | | | -0.103 | |
| | | | | | (1.569) | |
| $(\Delta i_t > 0)$=1 $\times \Delta i_t \times i_{t-1}$ | | | | | 0.00546 | |
| | | | | | (0.163) | |
| PE Portfolio Return | | | | | | -0.272*** |
| | | | | | | (0.055) |
| N | 13,190 | 13,190 | 13,190 | 13,190 | 13,190 | 10,575 |
| R-sq | 0.025 | 0.049 | 0.058 | 0.243 | 0.049 | 0.151 |

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date $t$. $R_t$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ to $t$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - 91$. $i_t$ is defined as the nominal 10-year Treasury yield, with $i_{t-1}$ being the interest rate 91 days prior and $\Delta i_t$ being the change in the interest rate from date $t - 91$ to $t$. Standard errors are Newey-West with a maximum lag length of 60 days prior. Real rates were built using monthly 10-year inflation expectations from the Cleveland Fed and the daily 10-year Treasury yield (post-1982), and the CPI series from the FED (pre-1982). In column 5, the terms $(\Delta i_t > 0) = 1$ and $(\Delta i_t > 0) = 1 \times i_{t-1}$ were suppressed from the table. Their coefficients are 0.0222 (0.602) and -0.0616 (0.086), respectively.

Table A3: Differential Interest Rate Responses of Leaders vs. Followers: Robustness Checks

| | Top 5 | | SIC | | EBITDA | | SALES | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Top 5 Percent=1 x $\Delta i$ | -1.106*** | -3.847** | -1.204*** | -3.903*** | -1.501*** | -4.805*** | -1.205** | -3.684** |
| | (0.273) | (1.220) | (0.222) | (0.936) | (0.287) | (1.077) | (0.350) | (1.325) |
| Top 5 Percent=1 x $\Delta i$ x Lagged $i$ | | 0.303** | | 0.293*** | | 0.372*** | | 0.277* |
| | | (0.105) | | (0.081) | | (0.092) | | (0.112) |
| Sample | All | All | All | All | All | All | All | All |
| Industry-Date FE | Y | Y | Y | Y | Y | Y | Y | Y |
| N | 61,313,604 | 61,313,604 | 61,277,070 | 61,277,070 | 38,957,740 | 38,957,740 | 48,247,714 | 48,247,714 |
| R-sq | 0.403 | 0.403 | 0.403 | 0.404 | 0.427 | 0.428 | 0.411 | 0.412 |

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln \left( P_{i,j,t} \right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t} \Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t} \Delta i_t i_{t-1} + X_{i,j,t} \gamma + \varepsilon_{i,j,t}$ for firm $i$ in industry $j$ at date $t$. The definitions are the same as in Table 2 except for $D_{i,j,t}$. In columns 1 and 2, leaders are chosen by the top 5 number of firms by market capitalization within an industry and date. In columns 3 and 4, leaders are chosen by the top 5% of firms by market capitalization within an industry and date, where we change the definition of industry to be the 2-digit Standard Industry Classification (SIC) codes. In columns 5 and 6, leaders are chosen by the top 5% of firms by earnings before interest, taxes, depreciation, and amortization (EBITDA) within an industry and date. In columns 7 and 8, leaders are chosen by the top 5% of firms by sales within an industry and date. Standard errors are dually clustered by industry and date.

Table A4: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent, Different Frequencies

| | Yearly | | Semi-Yearly | | Monthly | | Weekly | | Daily | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| $\Delta i_t$ | -1.061** | -5.570*** | -1.188*** | -4.594*** | -1.000*** | -2.365*** | -0.964*** | -1.846*** | -0.839*** | -1.244*** |
| | (0.403) | (1.134) | (0.345) | (0.764) | (0.196) | (0.463) | (0.171) | (0.309) | (0.170) | (0.208) |
| $i_{t-1}$ | | 0.381** | | 0.149 | | 0.0273 | | 0.00928 | | 0.00327** |
| | | (0.134) | | (0.080) | | (0.019) | | (0.005) | | (0.001) |
| $\Delta i_t \times i_{t-1}$ | | 0.493*** | | 0.385*** | | 0.150*** | | 0.0984** | | 0.0470 |
| | | (0.106) | | (0.073) | | (0.040) | | (0.035) | | (0.027) |
| Sample | All | All | All | All | All | All | All | All | All | All |
| N | 9,037 | 9,037 | 8,962 | 8,962 | 9,081 | 9,081 | 9,099 | 9,099 | 9,080 | 9,080 |
| R-sq | 0.024 | 0.095 | 0.040 | 0.101 | 0.036 | 0.050 | 0.032 | 0.039 | 0.019 | 0.020 |

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date $t$. $R_t$ is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t - 91$ to $t$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t - J$. $i_t$ is defined as the nominal 10-year Treasury yield, with $i_{t-1}$ being the interest rate $J$ days prior and $\Delta i_t$ being the change in the interest rate from date $t - 91$ to $t$. For columns 1 and 2, $J = 364$; columns 3 and 4, $J = 28$; columns 5 and 6, $J = 7$; columns 7 and 8, $J = 1$, where 1 is one trading day. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Table A5: Correlation Table of Forward Rates

| Variables | 0-2 | 2-3 | 3-5 | 5-7 | 7-10 | 10-30 |
|-----------|-----|-----|-----|-----|------|-------|
| 0-2 | 1.00 | | | | | |
| 2-3 | 0.85 | 1.00 | | | | |
| 3-5 | 0.85 | 0.85 | 1.00 | | | |
| 5-7 | 0.80 | 0.76 | 0.67 | 1.00 | | |
| 7-10 | 0.70 | 0.65 | 0.47 | 0.53 | 1.00 | |
| 10-30 | 0.80 | 0.77 | 0.93 | 0.95 | 0.94 | 1.00 |

Correlation table of forward rates. P-values in parentheses.