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ABSTRACT

We model the widespread failure of contracts to share risk using available indices. A borrower and lender can share risk by conditioning repayments on an index. The lender has private information about the ability of this index to measure the true state the borrower would like to hedge. The lender is risk averse, and thus requires a premium to insure the borrower. The borrower, however, might be paying something for nothing, if the index is a poor measure of the true state. We provide sufficient conditions for this effect to cause the borrower to choose a non-indexed contract instead.

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1 Introduction

A central implication of the literature on financial contracting is that agents should structure contracts to share risk as efficiently as possible. In many financial markets, standard contracts are simple and do not include risk-sharing arrangements that condition payments on publicly available indices. A leading example of this phenomenon is the mortgage market. In this market, homeowners are exposed to the risk that their homes will decline in value. Lenders are arguably better equipped to bear this risk and could insulate homeowners against declines in house prices by making mortgage repayment terms contingent on a house-price index. These types of mortgage contracts have been widely proposed as a solution to problems facing the mortgage market, such as the subprime default crises of 2007,[1] but have failed to supplant the standard mortgage. Two common explanations for this type of market failure are that either the space of feasible contracts is incomplete (Hart and Moore [1988]) or that implementing risk-sharing contracts entails high transaction costs. Neither explanation applies when there are indices available that would allow agents to share risk efficiently and appear almost costless to contract upon.

In this paper, we develop a model in which the failure to condition on indices and thus efficiently share risk is an equilibrium outcome resulting from asymmetric information. In our model, an agent, whom we call the borrower, seeks financing from a set of lenders. This financial contract must be written in view of potential conflicts of interest between the lender and the borrower related to an "internal", or idiosyncratic, state. For example, this internal state could represent the hidden ability of a mortgage borrower to make payments to the lender. At the same time, there are potential risk sharing benefits between lenders and the borrower over some imperfectly measured state (e.g. local area house prices). We call this state "external" to indicate that it is unaffected by the actions of the lenders and the borrower. The external state is not directly observable, and to realize any risk sharing benefits, contracts must condition on some potentially imperfect measurement of the state which we call an index (e.g. a house price index). Lenders know the true joint distribution of the index and the external state, i.e. the quality of the index, while the borrower does not. In effect, the borrower faces an adverse selection problem over basis risk when lenders offer an indexed contract.

At least two equilibria can arise in the model. In the first type of equilibrium, which we call the

[1] See, for example the "Shared Responsibility Mortgage" proposed in Mian and Sufi [2015] in which interest and principal payments are contingent upon local house price indices or the "Shared-Equity Mortgage" proposed by Caplin et al. [2007] in which a borrower receives a second mortgage where payment is only due upon sale of the house and is contingent on house value.

full-information optimal contracts equilibrium, all lenders offer a contract featuring the optimal amount of insurance conditional on the true quality of the index. The full-information optimal contracts equilibrium always exists when there is competition between lenders, and features no loss in efficiency due to asymmetric information about the index. In the second type of equilibrium, which we call the non-contingent contracts equilibrium, all lenders offer a contract that does not condition on the index. To see why such an equilibrium can arise, consider the borrower's response when a single lender deviates and offers a contingent contract. To at least break even on such a contract, the lender must charge the borrower an insurance premium. At the same time, the borrower will be concerned that the index is in fact uncorrelated with the risk she is aiming to insure, i.e., the basis risk for the contract is too high to justify the premium. Lenders that know the basis risk is high are happy to offer insurance and charge a high premium since the insurance is cheap for them to provide (precisely because the basis risk is high). As a result, the borrower will reject the indexed contract in favor of a standard non-contingent contract. We note that the non-contingent-contracts equilibrium exists even though the contracting space allows the use of an index, there are no transactions costs, and lenders make competing offers.

To illustrate the intuition behind these two equilibria, suppose there are just two equally likely external states, "good" and "bad." Now suppose a borrower receives offers of 1 dollar of financing from several competing lenders. The borrower is risk-averse with respect to the external state, meaning that her expected marginal value of a dollar is $1/2$ in the good state and $3/2$ in the bad state. The lenders are also risk averse, but less so than the borrower. Their expected marginal value of a dollar is $3/4$ in the good state and $5/4$ in the bad state. Lenders can offer contracts that are contingent upon some index but not upon the true external state directly. The index can be "high quality," in which case it is perfectly correlated with the true underlying state, or "low quality," in which case it is entirely independent of the true state and hence unrelated to the either the borrower's or lender's preferences. The borrower believes these two cases are equally likely, but the lenders observe the quality of the index before making their offers. Finally, lenders cannot offer contracts that specify positive transfers from the lender to the borrower.

Suppose that lenders make the following offers, depending on the quality of the index. If the index is high quality, they offer a contract that calls for the borrower to repay $8/3$ dollars if the realization of the index indicates the good state and nothing otherwise. If the index is low quality, they offer a contract that calls for the borrower to repay 1 dollar regardless of the realization of the index. These offers constitute what we call a full-information optimal contracts equilibrium. To

see why they can constitute an equilibrium, note that all lenders are earning weakly positive profits and could not possibly earn more by making different offers. Moreover, given that all lenders have common information, the borrower can perfectly infer the quality of the index by observing the contract that the lenders offer. In other words, it is not possible for a single lender to convince the borrower the index is high quality if all the other lenders offer a non-contingent contract. This same intuition carries over to the second type of equilibrium we describe above.

Now suppose that all lenders offer a contract that calls for the borrower to repay 1 dollar regardless of whether the index is high-quality or low-quality. These offers constitute what we call the non-contingent-contracts equilibrium. Can a single lender gain by deviating and offering the best contingent-contract? Again the answer is no. If a single lender deviates by offering a contingent contract, then she will have to charge a premium for it to at least break even. In the case of the best contingent contract, that premium is $1/6$, or the difference between the net present value of the payment $(4/3)$ and the amount financed (1). If the index is low quality, this premium is pure profit because in that case, the realization of the index is unrelated to the lender's preferences and the lender is thus risk neutral with respect to the index. As a result, the lender would be at least as willing to make such an offer given a low quality index as given a high quality one, and as such, standard belief refinements imply that the borrower can believe that the index is low quality after observing this deviation. Given these beliefs, the borrower is strictly better off accepting one of the offers of a non-contingent contract. The failure of the agents to share risk in this case is closely related to the classic lemons market breakdown of Akerlof [1970].

Two elements are essential to the existence of the non-contingent contracts equilibrium in the simple example above. First, the lenders are risk averse with respect to expected payoffs across external states, and second, the borrowers are even more risk averse, meaning that it is efficient for the lender to insure the borrower. The first element means that deviating from the non-contingent contracts equilibrium requires that a lender charge a premium for a contingent contract, which makes such a deviation more attractive when the index is low quality. We discuss the example and these two key conditions in section §2.

Our general model encompasses settings in which there is an additional security design problem concerning payoffs given idiosyncratic states. These security design problems are important for our results in that they determine the borrower's and lenders' indirect utility over securities and external states and thus the potential gains from indexation. In our mortgage example (section §3), the borrower needs incentives to repay the lender across idiosyncratic states. In that example, con-

ditional on a particular external state, standard debt contracts are optimal. In principle, these debt contracts could allow for risk sharing over the external states by having a higher face value in a good external state than a bad one. However, the face value of a debt contract is not equivalent its expected payoff; put differently, promises are not payoffs. A lender can prefer a higher debt level in a good external state simply because the debt is more likely to be repaid in good external states. At the same time, the lender has a lower marginal utility in the good external state. The key condition to generate a non-contingent contracts equilibrium becomes a tradeoff between the lender's decreasing marginal utility and the increasing value of promises as the external state improves. If the latter force dominates, then the lender will not need to charge a premium to insure the borrower against bad external states, and the non-contingent contracts equilibrium does not exist. A key condition for the existence of the non-contingent contracts equilibrium is that the lender be sufficiently "risk averse over promises," a notion we formalize in our general model (sections §4, §5, and §6).

There is an important distinction between the type of adverse selection problem we consider and one in which lenders have information about the external state itself. In our model, lenders do not have better information about the distribution of the external state, only about the relationship between the index and the external state. In contrast, much of the literature on adverse selection (following Akerlof [1970]) assumes there is asymmetric information about something that is directly relevant to payoffs. For example, in the context of mortgages, lenders might know that local house prices are more likely to appreciate in the future than the borrower expects. In an extension of our model (section §7), we show that under our assumptions, the non-contingent contracts equilibrium does not exist if the index is known to be perfectly correlated with the external state, and the adverse selection is only about the distribution of the external state itself.

Our work is related to the literature on incomplete contracts, surveyed by Tirole [1999]. Papers focusing on incomplete contracts and asymmetric information include Spier [1992], Allen and Gale [1992], and Aghion and Hermalin [1990], among others.[2] Our model differs from most of this literature in several respects. First, our model emphasizes competitive markets, rather than bilateral negotiation. Second, our model is focused on asymmetric information about the quality of the index, rather than the "fundamentals." This second difference is what allows us to generate non-contingent contracts in equilibrium without relying on transaction costs of using the index or

[2]Papers that endogenize contractual incompleteness, but do not emphasize asymmetric information, include Anderlini and Felli [1994], Battigalli and Maggi [2002], Bernheim and Whinston [1998], Dewatripont and Maskin [1995], Kvaløy and Olsen [2009], Tirole [2009].

arguing that the index is manipulable. Like some, but not all, of the incomplete contracts literature, we focus on equilibria with no indexation at all (as opposed to explaining why agents might use the index but not achieve perfect risk-sharing).

More significantly, our model differs from the incomplete contracts literature in its assumptions about what is contractible and what is observable. In the risk-sharing extension of Hart and Moore [1988], the agents can renegotiate after observing a non-verifiable state. A subsequent literature (Green and Laffont [1992], Dewatripont and Maskin [1995], Segal and Whinston [2002]) has found that, by altering the outside options or other aspects of the renegotiation process, the agents can share risks and perhaps even achieve first-best risk sharing despite their inability to contract on the state. In contrast, in our model, the index is both observable and verifiable, whereas the true external state is not observed by the agents until the end of the game, when renegotiation is no longer possible.[3]

Formally, our model is similar in some respects to Allen and Gale [1992], although the focus of that paper is the manipulability of the index. One can also view our model as related to models of insurance, in the vein of Rothschild and Stiglitz [1976] or more recently Hendren [2013]. The key difference between our model and those models is that our model places the information advantage and the competition on the same side of the market (with lenders), rather than on opposite sides of the market. Loosely speaking, the key intuition in our model is that the insurance itself might be a "lemon," in the sense of Akerlof [1970].

A closely related paper to ours is Spier [1992]. Spier [1992] shows that asymmetric information can amplify the effect of transaction costs on the ability of agents to write contracts that condition on relevant information. In Spier [1992], an informed and risk-averse principal contracts with an uninformed and risk-neutral agent. If the principal offers a contract that insulates herself from risk, she must also signal her private information, which in turn reduces the benefits of risk sharing. This effect lowers the level of transaction costs needed to destroy risk sharing in equilibrium. However, in Spier [1992], if transaction costs are close enough to zero, asymmetric information alone does not eliminate risk sharing. In contrast, in our model, asymmetric information can lead to zero risk sharing without transaction costs. Another related paper is Asriyan [2015]. He shows that concern for future liquidity and dispersed private information can lead market participants to write very simple contracts. This intuition is that if the holder of a contract must liquidate at some future

[3]Relatedly, the Maskin and Tirole [1999] critique of the incomplete contracts literature applies when the agents are aware of the payoff-relevant states before actions are taken, and for this and other reasons is not directly applicable to our model.

date, she will want hold a contract that is as informationally insensitive as possible. In contrast, we emphasize situations in which there are risk-sharing failures associated with simple contracts. In other words, the value of simple contracts is informationally sensitive in our model, and only by using the index could the agents minimize information sensitivity.

We also employ a general space of states and contracts. As a result, there is a great deal of scope for signaling, in contrast with the previous literature (in Spier [1992] and Aghion and Hermalin [1990], the contract space has one or two dimensions). As a consequence of this ability to signal, to generate our results, borrowers must be somewhat "suspicious," in the sense that they place non-zero probability on the index being irrelevant. Belief in this possibility, however unlikely, creates at least some chance that the index is not useful (and in this sense is reminiscent of the conditions of the Myerson and Satterthwaite [1983] theorem).

The failure of risk-sharing in our model can be thought of as a coordination failure, in the sense that there are multiple, Pareto-ranked equilibria. In the context of mortgages, we view this multiplicity as a feature. Mortgage contracts differ substantially across countries in ways that are difficult to explain with "fundamentals." Relatedly, our model considers only a single index, but could naturally be extended to consider multiple indices (interest rates and house prices, for example). In this case, we expect that "partial indexing" equilibria (e.g. indexing to interest rates but not home prices, like an adjustable rate mortgage) exist. As a result of this multiplicity, there is the potential for policy to improve welfare in our model by ruling out undesirable equilibria. Our model does not feature any externalities as a result of this risk-sharing failure; the existence of such externalities (which are emphasized by Campbell et al. [2011], among others) would provide an additional motivation for policy interventions.

Our motivating example is the mortgage market, although our most general model is abstract and could easily apply to other settings. In the context of home ownership, as noted by Sinai and Souleles [2005], purchasing a house hedges a homeowner against changes in future rents. Nevertheless, homeowners are exposed to both price and rent risks, and these could be hedged through the mortgage contract. Of course, as discussed by Case et al. [1995] and Shiller [2008], homeowners could also hedge these risks through other financial markets, although this almost never occurs in practice. This failure to hedge might be explained by limited access to such markets, or by the sophistication required to hedge in this manner. However, these arguments suggest that it would be profitable for a financial intermediary to provide hedging services, and mortgage lenders appear to be ideally situated to do this as part of mortgage contracts. Campbell et al. [2018] pro-

pose that mortgages that provide optional payment reductions during recessions increase financial stability. Mortgages that have a more equity-like claim on house value have been proposed (see, for example, Caplin et al. [2007]). Some of these early proposals made mortgage payments contingent on the sale price of the house, which clearly induces moral hazard for the borrower. More recent studies have pointed out that conditioning mortgage payments on an index of house prices avoids this problem. Piskorski and Tchistyi [2017] develop an equilibrium model of housing and mortgage markets and show that under many circumstances, the optimal mortgage design hedges the borrower against house price risk. Greenwald et al. [2018] provides a quantitative analysis of the general equilibrium effects of house priced indexed mortgage and show that using a local house price index improves financial stability. Proposals for mortgage reform after the recent financial crisis (e.g. Mian and Sufi [2015]) have advocated this approach. Although rare, shared appreciation mortgages are legal in the United States and used, for example, by Stanford University faculty who borrow from Stanford to purchase a house.[4] We develop a stylized model of mortgage borrowing, building on Hart and Moore [1998], and show that the conditions of our general theorem apply in this model and thus can explain the lack of prevalence of shared appreciation mortgages by appealing to asymmetric information over the quality of house price indices.

We begin in section §2 by using the example given above to illustrate the two key assumptions required for non-contingent contracts to be an equilibrium. Next, in section §3, we discuss a more complicated example, focused on mortgages, and show how these key assumptions must be adapted in a setting in which default is possible. We then being describing our general model, discussing the market for loans, the asymmetric information problem, and the equilibrium concept in section §4. In section §5, we discuss the zero-profit condition that arises from competition in our model, and characterize the "best" equilibria, which features contingent contracts. In section §6, we discuss our most general results, which describe assumptions under which risk-sharing fails and non-contingent contracts arise in equilibrium. In section §7, we describe a number of variations and extensions to our basic framework. We conclude in section §8.

[4]Stanford mortgages are indexed to an appraisal, rather than a local house price index, and involve renegotiation when the homeowner makes major investments.

2 Non-Contingent Equilibrium: A Simple Example

We begin by elaborating on the example in our introduction. In this example, there are two possible "external" states, bad and good. We call these states external to emphasize that they are outside the control of the borrower and lenders. Let $A = \{a_b, a_g\}$ be the set of possible external states. There are also two possible index realizations, high and low. Let $Z = \{z_l, z_h\}$ be the set of possible index realizations. The two external states are equally likely, $P(a = a_b) = P(a = a_g) = 1/2$, as are the two index values, $P(z = z_l) = P(z = z_h) = 1/2$. The index might be "perfect," in which case $z = z_h$ if and only if $a = a_g$, and $z = z_l$ if and only if $a = a_b$. The index might also be "uninformative," in which case the realizations of $z \in Z$ and $a \in A$ are independent.

Recall in our example that the borrowers' marginal value of a dollar is $1/2$ if $a = a_g$ and $3/2$ if $a = a_b$, whereas the lender's marginal value of a dollar is $3/4$ if $a = a_g$ and $5/4$ if $a = a_b$. We considered two contracts, a contingent (on $z \in Z$) contract and a non-contingent contract. The contingent contract required that the borrower pay $d = 8/3$ if $z = z_h$ and $d = 0$ if $z = z_l$, whereas the non-contingent contract required that the borrower pay $d = 1$ regardless of the value of z.

The initial investment required by the lender is $K = 1$. As a result, the non-contingent contract is break-even for the lender regardless of whether the index is perfect or uninformative, and the contingent contract is break-even for the lender if the index is perfect. However, the contingent contract is positive net present value for the lender if the index is uninformative. As a result, the lender has the greatest incentive to deviate from a non-contingent equilibrium when the index is uninformative.

We concluded that using the non-contingent contract is an equilibrium. If the borrower was expecting to be offered the non-contingent contract, and was offered the contingent contract instead, she could (and perhaps should) assume that the index is uninformative, because a lender with an uninformative index has the most to gain by deviating to the contingent contract. In this case, the lender's gain is the borrower's loss, since the lender is not providing insurance that is useful to the borrower, and consequently the borrower should reject this deviation.

This conclusion depended on two key assumptions. First, the borrower and lender agree on which states have high/low marginal values of a dollar. If instead the lender's marginal value of a dollar were $5/4$ if $a = a_g$ and $3/4$ if $a = a_b$ (reversing the order for the lender), the contingent contract that breaks even for the lender with a perfect index is $d = 8/5$ if $z = z_h$ and $d = 0$ if $z = z_l$. A lender with an uninformative index would not offer this contract, since it has a net present value less than one with an uninformative index, and as a result the non-contingent equilibrium could

not exist. We conclude that agreement about which states have high/low marginal values is critical for the existence of a non-contingent equilibrium.

The second key assumption is that the borrower should be buying insurance from the lender, and not vice-versa. Suppose that we switch the marginal values between the borrower and lender, so that the borrowers' marginal value of a dollar is $3/4$ if $a = a_g$ and $5/4$ if $a = a_b$, and the lender's marginal value of a dollar is $1/2$ if $a = a_g$ and $3/2$ if $a = a_b$. In this case, the best contingent contract that breaks even for the lender with a perfect index is $d = 0$ if $z = z_h$ and $d = 4/3$ if $z = z_l$. In other words, the lender is buying insurance from the borrower using this contingent contract. We find again that a lender with an uninformative index would not offer this contract, since it has a net present value less than one with an uninformative index, and as a result the non-contingent equilibrium could not exist. We conclude that the borrower being "more sensitive" to the external state a than the lender, in the sense that the lender should be insuring the borrower and not vice-versa, is critical for the existence of a non-contingent equilibrium.

Looking ahead, these two assumptions correspond exactly to the key assumptions in our general model. Intuitively, if the lender can insure the borrower at a negative insurance premium, or if the lender should be buying insurance from the borrower and can offer a high price for that insurance, the lender can prove that the index is perfect. But if the lender should be insuring the buyer, and requires a (weakly) positive insurance premium to do so, the non-contingent equilibrium can exist. Moreover, the non-contingent equilibrium exists even though, if the index is in fact perfect, both the lender and borrower can be made better-off by using the index.

This example is constructed to clearly illustrate the key requirements for the existence of a non-contingent equilibrium. In particular, the example is simple because both the borrower and lender have constant marginal utility within each external state a. While this might make sense for a lender (for example, if the stochastic discount factor of the lender is a function of a, and this transaction is small relative to the size of the lender), it often does not make sense as a model of borrowers. In the context of mortgages, we would prefer to assume that borrowers have concave utility. We would also like to incorporate the possibility that the borrower defaults instead of repaying the promised value d. For this reason, before describing our general model, we introduce a simple model of a mortgage borrower and lender. We will show how the two key assumptions of "agreement on marginal values" and "lender should insure the borrower" can be understood in this context.

3 Non-Contingent Equilibrium: A Mortgage Example

In this section, we discuss a simple model of mortgage lending that illustrates the key conditions necessary for a non-contingent contracts equilibrium.

3.1 The setup

There are two dates. At date zero, a borrower receives take-it-or-leave-it offers from $|L|$ mortgage lenders to finance the purchase of a house for K dollars. The borrower promises repayment at date one, collateralized by the house. The borrower can accept one offer and occupies the house until the start of date one, at which point she liquidates it and consumes her final wealth. The value of the house at date one is $x \in \{0, x_h\}$ with $K < x_h$.[5] As in the previous example, there are two external states, $A = \{a_b, a_g\}$, and two possible index values, $Z = \{z_l, z_h\}$. The external state $a \in A$ affects the likelihood of a high house price and both agents' other sources of income. The external state could represent the aggregate component of house prices in a local area containing the borrower's house or a broader economic variable that affects house prices and the agent's other sources of income. The index $z \in Z$ can therefore be thought of as either a house price index or a broader economic index.[6]

The realization of the index z is publicly observable, while the realizations of the house price x and external state a are not. The marginal distributions of a and z are common knowledge, and for simplicity we assume as in the previous example that $P(a = a_g) = P(z = z_h) = 1/2$. The joint distribution of x and a, $P(x = x_h|a) = \pi(a)$, is also common knowledge, with high house prices being more likely in good states, $\pi(a_b) < \pi(a_g)$.

The lenders all privately observe the joint distribution of a and z, given by $\theta(a,z)$ where

$$\theta(a_g, z_h) = \theta(a_b, z_l) = \frac{1}{4} + \rho,$$
$$\theta(a_b, z_l) = \theta(a_g, z_h) = \frac{1}{4} - \rho,$$

for some $\rho \in [0, \bar{\rho}]$.[7] We refer to ρ as the quality of the index. The borrower does not know ρ, and has a prior with full support on $\rho \in [0, \bar{\rho}]$. After the borrower observes the contracts offered by the

[5] The assumption that the liquidation value is either high or zero simplifies the exposition considerably.

[6] This example is too simple to draw a distinction between these two types of indices.

[7] If $\bar{\rho} = \frac{1}{4}$, a perfect index is possible. However, our assumptions require only that $\bar{\rho} > 0$.

various lenders, she may update her beliefs about ρ.

To motivate the use of debt (which can be non-contingent or contingent on the index z), we assume that the borrower privately observes the value of her house at date $t = 1$ and can make a report $\tilde{x}$ to her lender. While the lender cannot observe the value of the house, she can implement a foreclosure rule conditional on the borrower's report. For simplicity, we assume that foreclosure results in liquidation value of $x = 0$ and that the lender cannot randomize foreclosure.[8] As a result, any feasible outcome can be implemented by debt with face value $d(z)$ that depends on the index realization z, such that if the borrower fails to repay the face value, the lender forecloses on the house. Assuming $d(z) \leq x_h$, the borrower will default if $x = 0$ and repay if $x = x_h$.

In addition to her position in the house, the borrower also has a non-pledgeable endowment $y_B(a)$, with $y_B(a_b) < y_B(a_g)$, available at date one. Consequently, the borrower's final wealth is $W = y_B(a) + x_h - d$ if $x = x_h$ and the borrower owes d to the lender, and $W = y_B(a)$ otherwise. The borrower has CRRA expected utility $E[u_B(W)]$ over final wealth, where

$$u_B(W) = \frac{W^{1-\gamma_B} - 1}{1 - \gamma_B}.$$

We summarize the payoffs for a borrower that owes d in external state a using the indirect utility function

$$\phi_B(d,a) = \pi(a)u_B(y_B(a) + x_h - d) + (1 - \pi(a))u_B(y_B(a)).$$

If the borrower does not purchase the house, she receives $u_B(y_B(a))$ in external state a. We have assumed, for simplicity, that defaulting and never purchasing the house are identical from the borrower's perspective, so the borrower's participation constraint will never bind.

All lenders have an existing portfolio of assets (e.g., other mortgages) with value $y_L(a)$, again with $y_L(a_b) < y_L(a_g)$. Conditional on making the loan, a lender's total asset value is $R = y_L(a) + d - K$ if the borrower repays d, and $R = y_L(a) - K$ otherwise. Lenders derive CRRA expected utility $E[u_L(R)]$ of their total payoff R, where

$$u_L(R) = \frac{R^{1-\gamma_L} - 1}{1 - \gamma_L}.$$

We assume that this particular borrower is small relative to the lender, meaning that d and K

[8] This is a simplified costly state verification model (Townsend [1979], Gale and Hellwig [1985]).

are small relative to $y_L(a)$. We also normalize the lender's indirect utility function, $\phi_L(d,a)$, by subtracting the lender's expected utility if the lender the does not make a loan to this borrower. As a result, using a first-order Taylor expansion, if a lender is owed d in external state a, the lender has indirect utility

$$\phi_L(d,a) = \pi(a)u_L'(y_L(a))(d-K) - (1-\pi(a))u_L'(y_L(a))K.$$

Note that $u_L'(y_L(a))$ can be interpreted as a stochastic discount factor.

We now impose assumptions on the endowments and preferences of the lenders and the borrower that will ensure agreement on marginal values and that the lender should insure the borrower. First, we assume that lenders' endowment and risk aversion satisfy

$$\gamma_L \log\left(\frac{y_L(a_g)}{y_L(a_b)}\right) \geq \log\left(\frac{\pi(a_g)}{\pi(a_b)}\right), \tag{1}$$

and the borrower's endowment and risk aversion satisfy

$$\gamma_B \log\left(\frac{y_B(a_g)+x_h}{y_B(a_b)+x_h}\right) \geq \log\left(\frac{\pi(a_g)}{\pi(a_b)}\right). \tag{2}$$

The conditions in equations (1) and (2) state that the percentage change in marginal utility between a_b and a_g is greater than the percentage change in the default probability. As a result, both the borrower and the lenders having decreasing (between a_b and a_g) marginal utilities with respect to payments that only occur if the borrower does not default. That is, they agree that there is a high marginal value of such payments in external state a_b and a low value in a_g.

Next, we assume that the borrower faces a greater cost of bearing the risk of her endowment than the lender, conditional on no-default. That is,

$$\gamma_B \log\left(\frac{y_B(a_g)+x_h}{y_B(a_b)+x_h}\right) > \gamma_L \log\left(\frac{y_L(a_g)}{y_L(a_b)}\right). \tag{3}$$

Equation (3) implies that under full information, it is efficient for the lender to insure the borrower.[9]

Briefly, an equilibrium of this market is given by a set of offers $d_l(z)$, for each lender l, such that lenders maximize their expected utility, a borrower belief function that maps the set of possible offers to a posterior belief about the quality of the index and is consistent with Bayes' rule where

[9]Note that equations (3) and (1) imply equation (2). An analogous result appears in our general model.

possible, and an acceptance rule that that maximizes the borrowers utility conditional on her beliefs about the quality of the index. We describe the market structure and equilibrium definition more thoroughly in our general model.

3.2 Analysis

Equations (1) and (2) ensure that both the borrower and lender agree that the marginal value of a promise (a payment conditional on no-default) is higher in the bad external state. In other words, they are both risk averse with respect to promises. This kind of risk-aversion is related to, but not identical to, having a risk-averse utility function. For example, because the loan under consideration is small relative to the lender's other income, the lender is effectively risk-neutral with respect to the borrower's idiosyncratic outcome, but the lender can still be risk-averse with respect to promises, because the lender's stochastic discount factor is a function of the external state $a \in A$.

This type of risk aversion for the lender can arise for at least three reasons. First, the lender can be thought of as another agent in the economy, with her own CRRA preferences and other sources of income. Second, suppose that the lenders are intermediaries subject to regulatory or financial constraints and that y_L is the cash flow on the lender's portfolio of other mortgages.[10] When the external state is good (bad) because the aggregate component of house prices is high (low), the cash flow from the lender's mortgage portfolio is higher (lower), and repayment of one individual loan has a smaller (larger) effect on the health of lender's balance sheet. Third, suppose the lenders are integrated with financial markets, in which case y_L is the representative agent's consumption and γ_L is the representative agent's relative risk aversion. In this case, the external state affects broader economic conditions. When the economy is good (bad), the lender's stochastic discount factor is lower (higher).

There are two competing forces that will determine whether the lender is risk averse with respect to promises. First, the lender is risk averse with respect to the external state, meaning that the lender has higher marginal utility in the bad external state. Second, promises are more likely to be paid in the good external state ($\pi(a_h) > \pi(a_l)$). Equation (1) implies that the first of these forces weakly dominates the second, so that the lender has a higher marginal value of promises in bad states, meaning

$$\frac{\partial}{\partial d}\phi_L(d,a) = \pi(a)u_L'(y_L(a)) \tag{4}$$

[10] In this case, the CRRA preferences should be understood as proxy for the curvature induced by the financial or regulatory constraints.

is decreasing in a. Note that Equation (1) is equivalent to assuming that the lender's stochastic discount factor is more volatile than mortgage default probabilities.

As with the lender, there are two competing forces that determine whether the borrower is risk averse with respect to promises. The borrower is risk averse, and hence will have higher marginal utility in the bad state because of her non-pledgeable income. At the same time, in the bad state promises are less likely to be repaid, and hence are less costly to make. Equation (2) implies that the first of these forces dominates the second, so that

$$\frac{\partial}{\partial d}\phi_B(d,a) = -\pi(a)u_B'(y_B(a)+x_h-d) \tag{5}$$

is increasing in a. Note that, unlike the lender, the marginal cost of a promise for the borrower depends on the size of the promise. If the borrower is risk averse with respect to promises ($\frac{\partial}{\partial d}\phi_B(d,a)$, which is negative, is increasing in a) at $d=0$, she will be risk averse with respect to promises at all higher debt levels.[11]

As we discussed in section §2, that the borrower and lender agree on which external state has a higher marginal benefit/cost of promises is not sufficient to guarantee the existence of a non-contingent equilibrium. We also need to ensure that the lender should be insuring the borrower, and not vice-versa. Intuitively, which agent should be providing insurance and which agent should be receiving insurance depends on the ratio of the marginal values of promises. Equation (3) implies the borrower's marginal value of a promise is more sensitive to the external state than the lenders', and in particular that this property to holds at $d=0$,

$$\frac{\frac{\partial}{\partial d}\phi_B(d,a_g)|_{d=0}}{\frac{\partial}{\partial d}\phi_B(d,a_b)|_{d=0}} < \frac{\frac{\partial}{\partial d}\phi_L(d,a_g)|_{d=0}}{\frac{\partial}{\partial d}\phi_L(d,a_b)|_{d=0}}. \tag{6}$$

Equation (6) also implies that if the index is related to the external state ($\rho > 0$), then the first-best contract features promised face value payments that increase with the index, $d(z_l) < d(z_h)$.

We now give a heuristic argument that the asymmetric information between the lenders and borrower over the quality of the index can lead to the use of non-contingent contracts. Suppose that $L-1$ lenders offer the same non-contingent contract d^*. Can the the L-th lender offer a contingent contract d', with $d'(z_l) < d'(z_h)$, to exploit the risk sharing benefits that such contracts offer? If the borrower accepts the offer of d', the lower the index quality (lower ρ), the higher the

[11] This follows from $u_B'''(\cdot) > 0$, and we prove it in the proof of proposition 1.

profit for the lender. That is,

$$\frac{\partial}{\partial \rho} E[\phi_L(d'(z), a)] = (\phi_L(d'(z_h), a_g) - (\phi_L(d'(z_l), a_g)) - ((\phi_L(d'(z_h), a_b) - (\phi_L(d'(z_l), a_b)) < 0,$$

by the definition of ρ and the lenders' risk-aversion with respect to promises. Intuitively, the higher the index quality, the more variation in $d'(z)$ is correlated with lenders' endowment, and consequently the more costly it is for the lender to offer this insurance. Therefore, if the borrower is offered d', it is reasonable for the borrower to believe that the index has the lowest quality. Given this belief, the borrower will reject any d' that the L-th lender would be willing to offer. What if the L-th lender offered a contingent contract that was decreasing in z, $d'(z_l) > d'(z_h)$? In this case, the lender is purchasing insurance from the borrower, which is inefficient. As a result, if the lender is willing to offer a decreasing contract, the borrower is not willing to accept it. This logic leads to the following proposition:

Proposition 1. *There exists a $\bar{K} > 0$ such that, for all $K < \bar{K}$, there exists an equilibrium in which non-contingent contracts are used regardless of the type θ.*

Proof. See the appendix, section B.1. □

The requirement that $K < \bar{K}$ is needed to, among other things, guarantee that is possible to finance the house with a non-contingent contract.[12] It is vacuous if the level of the debt d^* that breaks even for the lender with loan size $\bar{K}$ is greater than the maximum possible debt level, x_h. However, to keep the analysis simple, we do not analyze whether this is indeed the case.

In this mortgage example, we have illustrated the distinction between promises and payments, and what it means to be risk-averse with respect to promises. We now turn to the general version of the model, which treats the indirect utility functions ϕ_B and ϕ_L as primitives. We provide a set of assumptions (assumptions 2 and 3 below) that generalize equations (1) and (3). These assumptions ensure that both lenders and the borrower are risk averse with respect to promises, and that it is efficient for the lender to insure the borrower. We then prove a general result, proposition 3, that shows that so long as these assumptions are satisfied, there exists an equilibrium with non-contingent contracts. The model of mortgage lending we have described in this section satisfies

[12] K must be sufficiently small to ensure that the mortgage example satisfies the assumptions of our general model. However, the assumptions of our general model are sufficient but not necessary; in the mortgage example, it is possible to show that the non-contingent equilibrium exists regardless of the value of K, as long as a non-contingent contract can satisfy both participation constraints.

the assumptions of our general model, meaning that proposition 1 is consequence of our general result, proposition 3.

4 The General Model

In this section, we setup our general model. At date zero, a borrower wishes to raise $K > 0$ dollars to pursue a project (e.g. purchasing a home). If the borrower accepts a contract offer by a lender, the borrower will initiate the project, and then at date one, payoffs will be determined.

We now describe each component of the model in more detail. First, we introduce the external states $a \in A$ and index $z \in Z$ that are the key exogenous random variables in the model. Second, we describe the contracting environment and the indirect utility functions that summarize the payoffs for the borrower and lender from using a particular contract. Third, we introduce the "types" in our model, which describe the index quality– that is, the relationship between the external state and the index. Fourth, we discuss the market structure, and lastly define equilibrium in the context of our model.

4.1 The States and the Index

After the borrower and lender agree to a contract and initiate the project, at date one an index $z \in Z$ is determined. This index is observable and verifiable, and related to the true external state $a \in A$. The true external state a is what enters the agents' indirect utility functions; they have no particular concern for the value of the index.

The index $z \in Z$ should be thought of an index based on the external state $a \in A$. For simplicity, we will assume that both A and Z are totally ordered sets. We will write $a \succ a'$ to denote the idea that the external state $a \in A$ is "better than" the external state $a' \in A$, and use the same notation for the index values. In the context of mortgages, the external state $a \in A$ might influence house prices, borrower income, and/or the cost of capital for lenders. The index $z \in Z$ is an index that, perhaps imperfectly, measures these things, such as a local area house price index, a wage index, or an interest rate. We assume that A and Z are finite sets.

The external state $a \in A$ influences the distribution of the borrower's idiosyncratic outcomes, $i \in I$. For a mortgage borrower, idiosyncratic outcomes could include the borrower's particular house price or income. The idiosyncratic outcomes may or may not be observable or contractible, and might be influenced by the borrower's behavior.

4.2 The Indirect Utility Functions and Contract Space

A contract is a function $s : I \times Z \to \mathbb{R}^+$ that takes the idiosyncratic outcome i and index z and determines a payment from the borrower to the lender. We use the notation $s_z : I \to \mathbb{R}^+$ to refer to the "conditional contract," which is the contract for a particular value of the index.

We require that conditional contracts are "ex-post efficient," appealing to notions of renegotiation-proofness after the index $z \in Z$ has been revealed. We further assume that the set of ex-post efficient contracts, which we denote as S_D, can be indexed by a number, $d \in D \subset \mathbb{R}$. We assume that D is a convex subset of the real line whose minimum is $d = 0$.[13] For example, if the ex-post efficient contract is a debt contract, as in many optimal contracting models (e.g. Hart and Moore [1998], Innes [1990], Townsend [1979], Hébert [2018]), d is the face value of the debt claim. For this reason, we refer to the parameter d as a "promise." We use the notation $s_d \in S_D$ to indicate an ex-post efficient contract with a promise of d. The set of feasible contracts S is the set of contracts such that, for each $z \in Z$, $s_z \in S_D$.

The "primitives" of our model are the indirect utility functions of the borrower and lender. Given a particular state $a \in A$ and ex-post efficient contract s_d,[14] the borrower's indirect utility function is $\phi_B(s_d, a)$. We refer to this as an indirect utility function because it summarizes the borrower's payoff, given some underlying relationship between the external state a, contract s_d, and the distribution of idiosyncratic outcomes. Similarly, we denote the lender's payoff (including the cost of the initial investment K) as $\phi_L(s_d, a)$. In both cases, these functions should be understood as expected utilities conditional on a, and do not necessarily imply that the borrower or lender perfectly knows a.

We treat the indirect utility functions as primitives that satisfy several properties. First, we assume that $\phi_L(s_d, a)$ is lower semi-continuous on $d \in D$ for all $a \in A$.[15] Second, we assume that $\phi_L(s_d, a)$ is negative for $d = 0$. We set the lender's outside option to zero, so this property implies that the lender is not willing to make the loan in exchange for a promise of zero. Third, the borrower's utility function satisfies a monotonicity property: if $d' > d$ for some $d, d' \in D$, then $\phi_B(s_{d'}, a) \leq \phi_B(s_d, a)$ for all $a \in A$. Intuitively, if the borrower makes a larger promise to the

[13] In our mortgage example (section §3), d is the face value of the debt claim, and it is natural to restrict attention to $D \in [0, x_h]$.

[14] These indirect utility functions are naturally defined over all conditional contracts, not just ex-post efficient ones, but ex-post inefficient contracts play no role in our analysis.

[15] Inefficient foreclosure/liquidation can generate downward jumps in the lender's payoff in certain examples, which is why we assume lower semi-continuity instead of continuity.

lender, she is worse off. For the lender, this property does not necessarily hold; promises will not necessarily be paid, and demanding excessive repayment can result in lower expected utility for the lender.

We use debt as our leading example, but these conditions can describe other families of securities as well. Examples include the set of fixed payments of varying size, the set of 100% equity claims less a fixed payment of varying size, and the set of equity shares of varying percentages. The first two of these examples could be motivated by risk-sharing type problems, and the third by security design problems resulting in equity as the optimal security design.

Our simple example (section §2) and mortgage example (section §3) provide examples of indirect utility functions ϕ_L and ϕ_B that satisfy our assumptions. We provide another example in the appendix, based on costly state verification models (section §A).

4.3 Types

We define $\theta(a,z)$ as the joint distribution of the external state and the index. This joint distribution is common knowledge amongst the lenders, but is not known to the borrower; it is the type in our adverse selection problem. The types θ are drawn from a set Θ, which we define as the set of all joint distributions that have the same marginal distributions for $a \in A$ and $z \in Z$, which we denote $p(a)$ and $q(z)$, respectively. Without loss of generality, we assume these marginal distributions have full support over A and Z, respectively. Let $\theta_0(a,z) = p(a)q(z)$ denote an "uninformative type" (the type with an index that is independent of the external state).

The borrower's prior belief over these types is μ_0. In effect, the borrower is uncertain about the relationship between the index and the external state. A homeowner, for example, might not be certain how the S&P Case-Shiller index for his metro area is related to the price of his particular house. We assume that the borrower is aware of the marginal distributions, to abstract from the problems generated by that type of asymmetric information and focus on the borrower's doubt about the relevance of the index (we revisit this in our extensions, section §7). We do not require that the beliefs μ_0 have full support on Θ, but will impose assumptions on the support, which we describe below. In particular, we do not require that the type space contains a perfectly informative index, although our results apply if such a type exists.

Having defined the type space, we next describe the market for loans.

4.4 The Market for Loans

Let L denote the set of lenders, with $|L| \geq 3$, each of whom can post a contract. After these lenders post contracts, the borrower can pick whichever one she prefers, or choose to forgo the investment opportunity. The outside options for the lenders are normalized to zero. Note that, from the borrower's perspective, lenders are perfect substitutes.

Let $S^L = (s^1, s^2, \ldots, s^{|L|})$ be the menu of contracts offered by the lenders at date zero. From lender l's perspective, the expected utility of offering a contract $s^l \in S$, when the other lenders offer contracts S^{-l}, the resulting menu is $S^L = (s^l, S^{-l})$, and the common type is θ, is

$$\sigma(s^l, S^L) \sum_{a \in A, z \in Z} \theta(a,z)\phi_L(s^l_z, a), \tag{7}$$

where $\sigma(s^l, S^L)$ is the probability that the buyer accepts the contract s^l, given the menu of contracts posted. This notation implicitly assumes that the buyer's decision does not depend on the identity of the lender, only on the contract that the lender offers. We will assume this in the equilibria we study, and note that it is consistent with the assumption that the borrower's utility does not depend on the lender she chooses, only on the design of the contract.

Assuming the borrower chooses to borrow, his expected payoff for contract s is (abusing summation notation)

$$\sum_{\theta' \in \Theta, a \in A, z \in Z} \mu(\theta'; S^L)\theta'(a,z)\phi_B(s_z, a), \tag{8}$$

where $\mu(\theta'; S^L)$ denotes the borrower's beliefs about the distribution of the lender's common type θ' after observing the menu S^L. The beliefs $\mu(\theta'; S^L)$ are central to our theory. The borrower does not observe the lender's common type θ; initially, she has prior μ_0 over the set of types Θ, but might refine these beliefs based on the menu of securities offered. It is important to note that, because the type θ is common across lenders, an optimal mechanism could allow the borrower to solicit this information and then negotiate a contract (Cremer and McLean [1988]). The market structure we impose, which we believe is realistic in many contexts, prevents the buyer from conducting this sort of auction.[16]

Having discussed the basic structure of the model, we next describe the equilibrium concept and the refinements for off-equilibrium beliefs that we employ.

[16]The mechanism of Cremer and McLean [1988] also requires commitment, and hence is inconsistent with our ex-post efficiency assumption.

4.5 Equilibrium Definition

The basic equilibrium concept we use is perfect Bayesian. Given the strategies of the other lenders (S^{-l*}) and the borrower (σ^*), and the common type θ, we require that lender l posts

$$s^l \in \arg\max_{s \in S} \sigma^*(s,(s,S^{-l*})) \sum_{a \in A, z \in Z} \theta(a,z)\phi_L(s_z^l,a), \tag{9}$$

if that strategy yields weakly positive expected utility, and otherwise does not participate. That is, each lender's choice of contract maximizes her utility, given the strategies of the other lenders and borrower.

If the borrower is offered any contracts, she must choose a strategy $\sigma(s^l,S^L)$ such that, given posterior beliefs $\mu(\cdot;S^L)$, if $\sigma(s^l,S^L) > 0$, then

$$s^l \in \arg\max_{s \in S^L} \sum_{\theta' \in \Theta, a \in A, z \in Z} \mu(\theta';S^L)\theta'(a,z)\phi_B(s_z,a), \tag{10}$$

and

$$\sum_{\theta' \in \Theta, a \in A, z \in Z} \mu(\theta';S^L)\theta'(a,z)\phi_B(s_z^l,a) \geq \bar{\phi}_B, \tag{11}$$

where $\bar{\phi}_B$ denotes the borrower's payoff if she does not accept any contract. In words, the borrower must maximize his utility given the menu of contracts being offered.

The equilibrium strategies of the lenders create a function $S^*(\theta)$ that describes the menu of securities that might be offered, given the common type. If the borrower observes a menu S^L for which there exists a type θ' such that $S^L = S^*(\theta')$, then she must update her beliefs according to Bayes' rule:

$$\mu(\theta;S^L) = \frac{\mu_0(\theta)1(S^L = S^*(\theta))}{\sum_{\theta' \in \Theta} \mu_0(\theta')1(S^L = S^*(\theta'))}. \tag{12}$$

This does not, of course, pin down what the borrower believes when he observes some menu S^L that could not have been generated from the equilibrium strategies $S^*(\theta)$, for any $\theta \in \Theta$ with $\mu_0(\theta) > 0$. For the purpose of determining if a conjectured set of strategies is an equilibrium, we only need to consider menus S^L that differ from a menu $S^*(\theta')$ for a single lender.

The result we are building towards is that there are many equilibria. This would be expected in the absence of refinements for off-equilibrium beliefs. Without refinements, the borrower can in effect dictate the contract by forming pessimistic beliefs when offered any other contract, jus-

tifying rejection. For this reason, we employ two refinements. The first refinement requires that the borrower believe the minimal number of lenders have deviated from equilibrium play. For concreteness, suppose the true common type is θ, and that all but one of the lenders offer an equilibrium contract for that type. The other lender deviates by offering another security that is not offered by type θ in equilibrium. Moreover, suppose the resulting menu could not have arisen from the equilibrium strategies of any type. Absent this refinement, the borrower could believe that multiple lenders have deviated. Imposing our refinement, and using the fact that there are at least three lenders, the borrower must instead correctly identify the deviating lender.

The second refinement we employ is the $D1$ equilibrium refinement (Banks and Sobel [1987]). This refinement captures the intuition that, if confronted with a "deviating" contract, the borrower should believe the lender is of a type that would benefit from this deviation. Under our first refinement, the borrower is able to identify the deviating lender (when there is only a single deviating lender), and it is to the security offered by this lender that we apply the $D1$ refinement. We believe our results are robust to using other refinements (aside from $D1$) that provide a similar intuition.

We use the standard definition of $D1$, and think of the borrower's "strategy" as consisting of an acceptance probability ξ. A lender of type θ offering contract s', instead of the equilibrium contract s, would benefit, given that the buyer accepts the deviating contract with probability ξ, if

$$\xi \sum_{a\in A, z\in Z} \theta(a,z)\phi_L(s_z^{'},a) \geq \sigma^*(s,S^*(\theta)) \sum_{a\in A, z\in Z} \theta(a,z)\phi_L(s_z,a). \tag{13}$$

The types for whom the set of $\xi \in [0,1]$ satisfying this condition is maximal are the types with positive support in the buyer's beliefs following this deviation.

Looking ahead, we will show that in equilibrium, lender expected utility is equal to their outside option of zero due to the effects of competition. As a result, the $D1$ refinement will simply state that the buyer must place the support of her beliefs on types that would weakly profit from offering the deviating contract, if that contract were accepted and such a type exists. The buyer cannot believe the deviating lender is of a type such that the lender would lose money if the buyer accepted the deviating contract, unless every lender type would lose money if the contract were accepted (and in this case, the deviating contract would never be offered).

Our analysis will focus on a particular set of equilibria, symmetric pure-strategy equilibria. These equilibria are pure strategy equilibria and symmetric in the sense, for all types $\theta \in \Theta$, either all of the lenders offer the same security with certainty, $s(\theta)$, or none of the lenders offer a security.

They are also symmetric in the sense that the borrower, faced with a menu of identical securities, chooses each lender with probability $|L|^{-1}$.

5 Preliminary Analysis

We begin our analysis by focusing on the effects of competition. Consider a symmetric pure-strategy equilibrium, and imagine that the lenders' profits from offering the contract $s(\theta)$ are strictly positive. Intuitively, this could not be an equilibrium. Suppose a lender offered a deviating contract $s' \in S$ such that, for each index value $z \in Z$, the associated promise d'_z was less than the promise associated with the original contract, d_z. The buyer would be better off regardless of her beliefs, and therefore accept the contract with probability one. The lender, by sacrificing some profit, would capture the entire market, and be better off. Because of the monotonicity property of the buyer's indirect utility function and the lower semi-continuity property of the lender's indirect utility function, standard Bertrand competition effects apply, and profits must be zero in equilibrium.

Lemma 1. *In any symmetric pure-strategy equilibrium, lender expected utility is zero.*

Proof. See the appendix, section B.2. □

We next introduce an assumption to ensure that there are contracts which can satisfy both the lender and borrower's participation constraints.

Assumption 1. *There exists a contract $s \in S_D$ that offers sufficient utility to the borrower, while satisfying the lender's participation constraint. That is, the problem*

$$\max_{s \in S_D} \sum_{a \in A, z \in Z} \theta_0(a,z)\phi_B(s,a)$$

subject to the constraint $\sum_{a \in A, z \in Z} \theta_0(a,z)\phi_L(s,a) = 0$ is feasible and has a solution weakly greater than the borrower's outside option $\bar{\phi}_B$.

Because we have assumed that the marginal distributions are the same for all types $\theta \in \Theta$, this assumption is sufficient to ensure that for any type, there is a contract that both the borrower and lender would be willing to accept under full information.

Next, we discuss the existence of a "best" equilibrium. Consider a symmetric, pure-strategy equilibrium, described by an offer of the contract $s(\theta)$. Suppose that the mapping between types θ and securities $s(\theta)$ is one-to-one. In this case, in equilibrium, the borrower knows the lenders' common type. Define a full-information optimal contract as

$$\bar{s}(\theta) \in \arg\max_{s \in S} \sum_{a \in A, z \in Z} \theta(a,z)\phi_B(s_z,a), \tag{14}$$

subject to the constraint that $\sum_{a \in A, z \in Z} \theta(a,z)\phi_L(s_z,a) = 0$. By assumption 1, the solution to the above maximization can offer the buyer a higher payoff than her outside option for all types $\theta \in \Theta$.

A set of full-information optimal contracts is on the Pareto frontier for all θ, and offers the lender zero expected utility. As a result, for any deviating contract a lender might be willing to offer, if the borrower correctly inferred the lenders' true type, the borrower would weakly prefer the full-information optimal contract being offered. The $D1$ refinement in our model allows the borrower to make this inference, and the presence of a competing lender allows the borrower to choose the equilibrium full-information optimal contract instead of the deviating contract. The following proposition summarizes this logic:

Proposition 2. *Under assumption 1, the pure-strategy symmetric equilibrium $s(\theta) = \bar{s}(\theta)$ exists.*

Proof. See the appendix, section B.3. □

The above proposition describes a "best" pure-strategy symmetric equilibrium, in which a full-information optimal contract is offered.[17] Our main results describe the conditions under which another type of pure-strategy symmetric equilibrium exists. This alternative equilibrium is notable because it uses a non-contingent contract, is a pooling equilibrium, and is Pareto-inferior to the "best" equilibrium, from an ex-ante perspective.

We say a contract is "non-contingent" if $s_z = s_{z'}$ for all $z, z' \in Z$; that is, the contract does not make use of the index. We will consider the existence of a non-contingent contract pooling

[17] Note that there is a tension between the existence of the best equilibrium, which uses indexed (contingent on $z \in Z$) contracts, and the assumption that the lender's indirect utility function depends only on the external state and not directly on the index. For example, if the lender made a number of other indexed loans, the lender's marginal utility might be a function of both $a \in A$ and $z \in Z$. Because the focus of our analysis is the existence of an equilibrium without indexation, we do not discuss this issue in more detail.

equilibrium, in which, for all $\theta \in \Theta$ with $\mu_0(\theta) > 0$,

$$s_z(\theta) = s^* \in \arg\max_{s \in S_D} \sum_{a \in A} p(a)\phi_B(s,a), \tag{15}$$
$$\text{subject to} \sum_{a \in A} p(a)\phi_L(s,a) = K.$$

By assumption 1, this contract can offer the buyer a higher payoff than her outside option for all types $\theta \in \Theta$. Note also that this equilibrium is at least weakly Pareto-inferior to the "best" equilibrium, and strictly inferior if the non-contingent contract s^* is sub-optimal for any type θ under full information.

6 Risk-Sharing Failure in Equilibrium

In this section, we provide sufficient conditions for the existence of a "non-contingent" equilibrium. This equilibrium will exist despite its ex-ante Pareto-inferiority to the "best" equilibrium discussed above.

Our second assumption states that the value of a larger promise to the lender is higher in bad external states than in good external states. In other words, the lender is risk-averse with respect to promises.

Assumption 2. *For all $d', d \in D$ with $d' > d$, $\phi_L(s_{d'}, a) - \phi_L(s_d, a)$ is weakly decreasing on $a \in A$.*

From the lender's perspective, it is preferable to receive larger promises in worse states. In other words, $\phi_L(s_d, a)$ is sub-modular in (d, a). This is what we mean by the idea that "lenders are risk-averse with respect to promises."

Our third assumption is defined using the variable λ^*, which is the Pareto-weight associated with the non-contingent contract s^*:

$$s^* \in \arg\max_{s \in S_D} \sum_{a \in A} p(a)U(s,a;\lambda^*). \tag{16}$$

The Pareto-weight $\lambda^* > 0$ is also the multiplier on the constraint in equation (15), and hence s^* causes the lender to receive zero expected utility.

Our assumption requires that the marginal social value of a promise to lender is lower in bad external states than in good external states. In other words, it is efficient to the lender to insure the

borrower.

Assumption 3. *For all $d, d' \in D$ with $d' > d$, $U(s_{d'}, a; \lambda^*) - U(s_d, a; \lambda^*)$ is weakly increasing on $a \in A$.*

The social welfare function is super-modular in (d, a), implying, among other things, that the borrower's indirect utility function, $\phi_B(s_d, a)$, is super-modular in (d, a). That is, this assumption implicitly embeds the assumption that the borrower is also "risk-averse with respect to promises."

These two assumptions can be understood as consisting of several claims. The first claim is that the "marginal benefit of debt" to the lender, $\phi_L(s_{d'}, a) - \phi_L(s_d, a)$, is monotone in the aggregate state, regardless of the levels of debt involved. The first part of this claim can be thought of as defining the order on the aggregate states– up to this point, nothing has depended on that order. The second part ("regardless of the level of debt") is the key point. The second claim is that the "marginal cost of debt" to the borrower, $\phi_B(s_d, a) - \phi_B(s_{d'}, a)$, is monotone and increases in the same direction as the marginal benefit of debt to the lender. In other words, states in which the lender would really like larger promises are also states in which the borrower would really prefer not to make larger promises. The third claim is that the borrower is "more risk averse" than the lender in this sense. That is, in states in which the lender would really like a large promise, and the borrower would really prefer a small promise, the latter effect dominates, and under the Pareto weight λ^*, it is more efficient to have smaller promises when both "marginal cost" and "marginal benefit" are high. In other words, the optimal contract would involve the lender insuring the borrower, and because preferences are aligned, this is costly for the lender.

As suggested by this description, our results do not really depend on the ordering over the external states. That is, the proof of proposition 3 below would hold almost unchanged if we imposed, instead these two assumptions, that $\phi_L(s_d, a)$ was super-modular and that $U(s_d, a; \lambda^*)$ was sub-modular.

Our last assumption requires that the set of possible types (the support of the prior μ_0) be sufficiently rich, in the sense that there is always a "less interrelated" type. There are a variety of ways of defining "less interrelated" in the context of joint probability distributions with identical marginal distributions. For two variables (i.e. $a \in A$ and $z \in Z$), many of these orders are equivalent (Meyer and Strulovici [2012]). One intuitive way of measuring interrelatedness is the greater weak association relation defined by Meyer and Strulovici [2012]. A definition, in our context, follows.

Definition 1. A type $\theta \in \Theta$ has greater weak association than a type $\theta \in \Theta$, $\theta \succeq_{GWA} \theta'$, if, for all

non-decreasing functions $h : A \to \mathbb{R}$ and $g : Z \to R$,

$$Cov^{\theta}(h(a), g(z)) \geq Cov^{\theta'}(h(a), g(z)).$$

Put another way, the type θ' has less correlation than the type θ, regardless of how the external states a and index values z are mapped to real numbers. One particular consequence of $\theta \succeq_{GWA} \theta'$ is that (again, from Meyer and Strulovici [2012]),

$$\sum_{a \in A, z \in Z} (\theta(a,z) - \theta'(a,z)) f(a,z) \geq 0$$

for all super-modular functions f. We can also relate greater weak association to the more familiar notion of first-order stochastic dominance. If $\theta \succeq_{GWA} \theta'$, then for all $z' \in Z$, the conditional distribution $\theta(a|z \succeq z')$ first-order stochastically dominates $\theta'(a|z \succeq z')$, and $\theta'(a|z \prec z')$ first-order stochastically dominates $\theta(a|z \prec z')$. That is, if $\theta \succeq_{GWA} \theta'$, higher values of the index are more strongly associated with better external states under θ than under θ'.

We assume that every type θ in the support of μ_0 has greater weak association than the uninformative type θ_0 (i.e. all $\theta \in \Theta$ are weakly associated). The key part of our assumption is that "it can always be worse." That is, for any type θ that is possible ($\mu_0(\theta) > 0$), every type that is less interrelated is also possible, including in particular the uninformative type.

Assumption 4. *For all $\theta \in \Theta$ in the support of μ_0, $\theta \succeq_{GWA} \theta_0$, and if $\mu(\theta) > 0$, then $\mu(\theta') > 0$ for all $\theta' \in \Theta$ such that $\theta \succeq_{GWA} \theta' \succeq_{GWA} \theta_0$.*

This assumption ensures that for any type, there is a rich set of less informative types, which limits the lender's ability to simultaneously signal her type and capture risk-sharing benefits.

Before proving our main result, we note that we have made assumptions 2 and 3 weak so that they are easy to satisfy. In this spirit, we have also imposed relatively little structure on the indirect utility functions ϕ_L and ϕ_B. As a consequence, although it is guaranteed that the full-information optimal contract is weakly better than the non-contingent contract, we have not assumed enough to show that it is strictly better. We provide the following lemma to show that stronger versions of our assumptions are sufficient, but not necessary, to ensure that the full-information optimal contract is strictly better.

Recall that both the full-information optimal contract $\bar{s}(\theta)$ and the non-contingent optimal contract s^* are designed to ensure the lender earns zero expected utility.

Lemma 2. *Let $d(s^*)$ denote the value of $d \in D$ associated with the non-contingent optimal contract s^*, and suppose it is in the interior of D. If $\phi_B(s_d, a)$ and $\phi_L(s_d, a)$ are both differentiable with respect to d at $d(s^*)$ and*

$$\frac{\partial}{\partial d} U(s_d, a; \lambda^*)|_{d=d^*(s)}$$

is strictly increasing on $a \in A$, then for all types $\theta \in \Theta$ such that $\theta \succeq_{GWA} \theta_0$, except θ_0 itself, the full-information optimal contract is strictly Pareto-superior to the non-contingent contract,

$$\sum_{a \in A, z \in Z} \theta(a,z) \phi_B(\bar{s}_z(\theta), a) > \sum_{a \in A, z \in Z} \theta(a,z) \phi_B(s^*, a).$$

Proof. See the appendix, section B.4. □

The stronger assumptions of this lemma rule out things like the possibility that the full-information optimal contract is non-contingent due to some kind of boundary or discontinuity, or that there are really no risk-sharing benefits. In the examples we have constructed, these issues do not arise except in pathological cases, and the full-information optimal contract is indeed better than the non-contingent contract for almost all θ in the support of μ_0. This sets up the "puzzle" in our model: can the non-contingent equilibrium exist even though the best equilibrium is ex-ante strictly Pareto-superior? Our main result answers this question in the affirmative:

Proposition 3. *Under assumptions 1, 2, 3, and 4, there exists a symmetric pure-strategy equilibrium in which $s(\theta) = s^*$.*

Proof. See the appendix, section B.5. The proof relies on results from Meyer and Strulovici [2015]. □

This proposition establishes that the assumptions given above are sufficient for the existence of a non-contingent equilibrium. Intuitively, if it is not efficient for the borrower to insure the lender, the deviations necessary to separate from the uninformative type are never welfare-improving. Our conditions are designed to ensure that this is the case.

Having presented the main result, we briefly comment on the importance of each of our assumptions. Assumption 1 ensures that both the full-information optimal contract and the non-contingent optimal contract are feasible from a participation constraint perspective. However, because the full-information optimal contract can strictly Pareto-dominate the non-contingent contract for many types, under an alternative assumption it is possible to have the full-information

optimal contract feasible for some types, while the non-contingent contract is infeasible. In this case, there could not be a non-contingent equilibrium. However, the proof of proposition 3 could be adapted to prove that a "no trade" equilibrium exists in this case, despite the possibility of gains from trade for some types.

Assumption 2, lender risk-aversion with respect to promises, is essential to the result. If the lender were risk-seeking with respect to promises while the borrower remained risk-averse with respect to promises, lenders with a more accurate index could separate from the uninformative type by paying higher prices to provide insurance. Assumption 3, which implies that it is efficient for the lender to insure the borrower and not vice versa, is essential for similar reasons. If it were instead optimal for the lender to purchase insurance from the borrower, a lender with a more accurate index could separate from the uninformative type by paying a high price for insurance.[18]

Assumption 4 is essential to rule out non-monotone (in z) security designs. If the security were required to be monotone in z (but allowed to be either increasing or decreasing), the possibility of the uninformative type θ_0 would be sufficient to generate the non-contingent equilibrium. Using non-monotone securities potentially allows a lender to purchase insurance over some subset of Z at a high price, separating from the uninformative type, while providing insurance over another subset of Z, perhaps generating enough gains from trade to make such a deviation worthwhile. The richness of the type space allows the borrower to be suspicious of such an offer, thinking that the index is likely to work well over the subset of Z for which the lender is buying insurance but poorly over the subset for which the borrower is buying insurance.

Assumptions 1, 2, and 3 are properties of the indirect utility functions ϕ_B and ϕ_L, and the required funds K. As a result, they can be checked in the context of a specific model, such as the mortgage model presented previously. For another example, using a costly state verification model, see appendix section §A.

Having presented our main result, we next turn to variations and extensions of the model.

7 Variations and Extensions

In this section, we discuss modifications and extensions to the model. We begin by discussing a model with positive profits for lenders, that nevertheless retains the competition between lenders.

[18] We do not mean to imply that our assumptions are necessary; they are only sufficient, and we speculate that our result could be proven under modified versions of these assumptions.

In this case, our results go through essentially unchanged. We then discuss what would happen with a single, monopoly lender. We will see that there is no "full-information optimal contracts" equilibrium with a monopoly lender, but there is still a non-contingent equilibrium. Finally, we will discuss how to extend our results to settings in which there is adverse selection about the marginal distribution of the index (q), or only about the distribution of the external states and not about the index quality.

7.1 Profitable Lending

In this extension, we describe a model in which lenders make positive profits, that is, recieve expected utility greater than their outside option, in equilibrium, but nevertheless face competition. We introduce profits into the economy by assuming that each lender faces a convex cost in the number of loans she makes, and that there is a unit mass of borrowers.[19] Let Q_l be the number of loans made by lender l. Suppose that a lender of type θ who makes Q loans using contract s receives utility

$$\Pi(s,Q,\theta) = Q\{\sum_{a\in A, z\in Z} \theta(a,z)\phi_L(s_z,a)\} - C(Q), \tag{17}$$

where $C(Q)$ is a convex, twice differentiable function with $C'(|L|^{-1}) = 0$.

With this quasi-linear functional form and the normalization that $C'(|L|^{-1}) = 0$, a lender that considers a deviation in which the lender offers a single, marginal borrower a different contract faces a problem that is identical to the one considered in our general model. In this case, the $D1$ refinement is the same as in our main analysis, because (in equilibrium) the marginal profit of each lender is zero.

However, if the lender contemplates a deviation in which he offers a deviating contract to all borrowers, then substantial profits could be at stake, because the average profits of lenders are positive. In this case, the $D1$ refinement requires that the borrower place her beliefs on the lender type that would break-even under the smallest amount of the demand for the deviating contract. This is equivalent to saying that the borrower must believe the lender is of a type for whom the difference between the marginal profit of the deviating contract and the marginal profit of the equilibrium contract is maximal.

Surprisingly, perhaps, our non-contingent equilibrium exists under the same conditions in this model. The intuition comes from the proof description in section §6. When a lender with a

[19] Introducing profits in this way is an old idea, described in the textbook of Tirole [1988].

"good" index offers a contract that insures the borrower, the lender requires a higher expected value of repayments to be indifferent between the deviating contract and the non-contingent contract. However, a lender with an irrelevant index could offer the same deviating contract at a profit, and therefore (in the case of profitable lending) the borrower must believe that the lender is of this type, or of a type that is even worse from the perspective of the borrower.

7.2 Monopoly Lending

In this extension, we consider what type of equilibrium can exist when the lender has monopoly power. Specifically, we assume that a single lender can make a take it or leave it offer to the borrower and that if the borrower rejects this offer, she receives her outside option. Neither the full-information optimal contract nor the non-contingent contract defined in section §5 are equilibria, because both offer positive surplus to the borrower and zero surplus to the lender.

To study the monopoly case, we parameterize both the full-information optimal contract and the non-contingent contract by the required investment. Suppose that there exists a $\bar{K} > K$ such that the full-information optimal contract, $\bar{s}(\theta, \bar{K})$, results a payoff for the borrower equal to her outside option. Likewise, suppose that there exists a $K^* > K$ such that the non-contingent contract, $s^*(K^*)$, also results in a payoff for the borrower equal to her outside option. In this sub-section, we will ask whether there exist equilibria with the contracts $\bar{s}(\theta, \bar{K})$ and $s^*(K^*)$. We will continue to impose the $D1$ refinement on off-equilibrium contract offers.

The answer is no for the full-information contract, and yes for the non-contingent contract. The existence of the non-contingent contracts equilibria follows from the proof of proposition 3– nothing in that proof depended on a specific value for K. The only effect of competition was to allow the borrower to choose a contract from another lender. Although the type θ is common to all lenders, because the non-contingent contract's payoff for the borrower does not depend on θ, the borrower's inference about θ does not change the appeal of the non-contingent contract. It is as if the borrower had a fixed outside option instead, which is what is assumed in the monopoly case.

However, for the full-information contract, competition is essential. For the uninformative type (θ_0), the full-information contract is identical to a non-contingent contract. For many other types (by lemma 2), the full-information contract is contingent and, due to lender risk-aversion over promises (assumption 2), offers a higher payoff to the uninformative type than the non-contingent contract. As a result, the uninformative type is tempted to deviate. When there are other lenders, the borrower can use their offers to determine the common type, and avoid being "tricked" by this

deviation. With a monopoly lender, this is not possible, and as a result there is no full-information contract equilibrium. In summary, competition is necessary for the existence of the best equilibrium, but the non-contingent equilibrium always exists.

Note that this result offers a hysteresis-based explanation for why we might expect the non-contingent equilibrium to occur despite the presence of competition. If, in the beginning of the market, there was only one lender, the non-contingent contract would be used. This might anchor borrower expectations, so that as other competing lenders entered, the non-contingent contract would continue to be employed. Entry of lenders would still benefit the borrower, due to better pricing (the difference between K^* and K described above), but would not achieve the full benefit of allowing for contingent contracts.

7.3 Adverse Selection about Marginal Distributions

Throughout the paper, we have assumed that the set Θ contained only joint distributions of the external state and index with marginal distributions $p(a)$ and $q(z)$. Suppose we relax this, and require only that the marginal distribution over external states, $p(a)$, be the same for all types. Under this assumption, there is no adverse selection about the true external state, only about the index, as in the main part of the paper. Intuitively, adding additional dimensions of adverse selection cannot improve the situation, and should only reinforce the non-contingent contracts equilibrium.

Formally, let $q(z;\theta)$ denote the marginal distribution of the index associated with type θ, and let $\Theta(q)$ be the set of all joint distributions with marginals $q(z)$ and $p(a)$, satisfying the monotone likelihood ratio property for the conditional distribution of a given z. Let Q be the set of all marginal distributions for the index, and let Θ be the union of all $\Theta(q)$ for each $q \in Q$. Modify our condition 4 (rich type space) so that it applies to each $\Theta(q)$ such that $\mu(\theta) > 0$ for some $\theta \in \Theta(q)$. In other words, there is a "rich" type space and an opportunity for risk-sharing for each possible marginal distribution of the index. Under this condition, the proof of proposition 3 is essentially unchanged, and the result holds.

7.4 Adverse Selection on External States

In this extension, we modify the model of the main text to consider the case in which the index is known to be perfect, but there is adverse selection about the marginal distribution of the aggregate state. This sort of adverse selection is closer to the problems studied in the literature (e.g. Spier

[1992], Asriyan [2015]). We build on the notation used in the previous extension. We assume that the set Z is identical to the set A, and that each $\Theta(q)$ is a singleton, containing only the joint distribution

$$\theta(a,z) = \delta(a,z)q(z), \tag{18}$$

where $\delta(a,z)$ is one if $a = z$ and zero otherwise. Adverse selection occurs because, in this context, there are types in Θ with different values of $q(z;\theta)$.

In this setting, the "rich type space" condition (condition 4) is irrelevant. We continue to impose our feasibility assumption (assumption 1) for each $\Theta(q)$. Our result in this section does not depend on detailed assumptions about risk-sharing (like assumptions 2 and 3), and therefore we will not discuss how to adapt them to this setting. We will assume instead the result of lemma 2: that the full-information optimal contract is contingent for all θ in the support of μ_0 and generates strictly higher payoffs than any non-contingent contract. We discuss how to weaken these assumptions below. We will also assume that both ϕ_L and ϕ_B are continuous in d.

For technical reasons, we assume that the support of the prior beliefs $\mu_0(\cdot)$ is a closed set, which was not required in the main text. We also assume that all types $\theta \in \Theta$ for which $\mu_0(\theta) > 0$ are associated with marginal distributions that have full support. In other words, $q(z;\theta) > 0$ for all $z \in Z$ and $\theta \in \Theta$ such that $\mu_0(\theta) > 0$. This generalizes the full support assumption of the main text.

Define the mapping $\Theta^*(\theta)$ as a set-valued function

$$\Theta^*(\theta) = \{\theta' \in \arg \max_{\theta'' \in \Theta : \mu_0(\theta'') > 0} \sum_{a \in A, z \in Z} \theta''(a,z)\phi_L(\bar{s}_z(\theta), a)\}, \tag{19}$$

where $\bar{s}_z(\theta)$ is the full-information optimal contract associated with the type θ. The set $\Theta^*(\theta)$ is the set of types in the support of $\mu_0(\cdot)$ that would earn the highest payoff from offering the security $\bar{s}_z(\theta)$.

The following lemma (which is based on standard fixed-point arguments) states that there is a fixed point to this mapping.

Lemma 3. *There exists a θ^* such that $\theta^* \in \Theta^*(\theta^*)$. For any such θ^*, for all $\theta' \in \Theta^*(\theta^*)$, $\bar{s}(\theta') = \bar{s}(\theta^*)$.*

Proof. See the appendix, section B.6. □

This type, θ^*, can essentially "prove itself" by offering its full-information optimal contract.

When a lender of type θ^* offers the contract $\bar{s}(\theta^*)$, it breaks even. All other types either recover less than the initial investment K, or also break even and have an identical full-information optimal contract. Hence, under the $D1$ refinement, the borrower must believe that she is being offered a full-information optimal contract.

By assumption, every full-information optimal contract is not equal to a non-contingent contract. By the Pareto-optimality of the full-information optimal contract, the borrower must be willing to accept this contract. Therefore, there cannot be an equilibrium in which a non-contingent contract is employed.[20]

What makes this setting different than the one studied in the main text? The key is that there is no type for which the full-information optimal contract is equal to the non-contingent contract. When there is adverse selection about the distribution of external states, this makes sense; the only way a non-contingent contract could be optimal is if some type of lender knew with certainty what the ex-post "fair" level of debt was. In contrast, in the case emphasized in the main text, a non-contingent contract can be optimal so long as it is possible that the index is irrelevant; perfect foresight about the external state is not required for a non-contingent contracts equilibrium.

In some sense, this result can be viewed as pointing to the necessity of an assumption like condition 4 in the main text. If borrower knew the index was at least somewhat relevant, the type with the least relevant index could "prove herself" and eliminate the non-contingent equilibrium. In this case, an equilibrium with a minimal (and ex-ante sub-optimal) level of indexing would exist. Non-contingency could be restored in this case by, following Spier [1992], by introducing a fixed cost of using the index in addition to asymmetric information. In this case, a non-contingent equilibrium would exist so long as the "worst" type was sufficiently bad, relative to the fixed cost.

Note also that, consistent with the Hirshleifer effect (Hirshleifer [1971]), with adverse selection on external states, risk-sharing will generally be reduced relative to the case in which the lenders shared the borrower's prior. That is, although one type can "prove itself," most types will not be able to, and the equilibrium will likely involve less risk-sharing than if the lenders were uninformed and shared the borrower's prior. Our point is that this reduction in risk-sharing is, under our assumptions, never enough to generate a non-contingent contracts equilibrium. In contrast, adverse selection about the relationship of the index and the external state, as studied in the main part of

[20]This argument does not really depend on every full-information optimal contract being contingent, only that the type which can prove itself has a contingent full-information optimal contract. To formalize the argument, we would need to use continuity to show that a lender can deviate to a contract close to the full information contract, but with a tiny profit, and that the borrower cannot reject such a contract.

the paper, can generate a non-contingent contracts equilibrium.

8 Conclusion

We have introduced a theory to explain the widespread lack of indexation observed in contracts. Intuitively, when a borrower is offered a contract that includes insurance, she is concerned that the insurance is not actually relevant for the risks she faces. Under the conditions described in our model, this effect is strong enough to cause the borrower to reject that offer, and choose instead a contract without insurance from a different lender. As a result, equilibria that feature little or no risk-sharing can arise, even though they are ex-ante Pareto-dominated by equilibria that feature full risk-sharing.

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A Costly State Verification Example

In this appendix section, we provide a numerical example under which a version of the costly state verification model (Townsend [1979], Gale and Hellwig [1985], and others) satisfies the conditions of our main theorem, in particular assumptions 2 and 3. We introduce some functional forms in order to apply the results of our general model as simply as possible. The key modifications to the standard CSV model are the introduction of an external state, risk-aversion for the borrower, and the existence of non-verifiable income for borrower. Better external states induce a better (in a monotone-likelihood-ratio property sense) distribution of both verifiable and non-verifiable income. The presence of the non-verifiable income, when combined with risk-aversion, implies that the borrower has low expected marginal utility in good external states, as in our mortgage example (section §3).

We now describe the specific modifications we make to the standard CSV model. The idiosyncratic state (as opposed to external state) for the borrower is a triple (x, y, y'), where x is the borrower's non-verifiable income, y is the borrower's verifiable income, and y' represents the borrower's report of her verifiable income, all of which are weakly positive reals. The conditional contract $s(y, y')$ can depend on both the true verifiable income and the report.[21] The borrower's utility is

$$u(x + y - s(y, y')),$$

where $u(\cdot)$ is the borrower's strictly increasing, twice-differentiable, concave utility function. Our numerical example will use CARA utility for the borrower. We will not analyze the borrower's participation constraint– making an assumption that the project is sufficient valuable from the borrower's perspective is always sufficient to ensure that the participation constraint is satisfied.

We assume there are two external states, $A = \{\frac{9}{10}, \frac{11}{10}\}$, which are equally likely, and two equally likely index values, $Z = \{z_l, z_h\}$, as in our two examples in the main text. We assume that the quality of the index (ρ, see section §3) is at most $\frac{1}{8}$.[22]

Let $f(y|a)$ denote the distribution of y given a, and suppose it has the following functional form:

$$f(y|a) = q(y) \exp(a \ln(y) - \psi(a)),$$

where $q(y) = f(y|0)$ is a measure on the positive reals. In other words, the distributions $f(\cdot|a)$ are an exponential family whose sufficient statistic is the expected log verifiable income. The function $\psi(a)$ ensures that each $f(y|a)$ integrates to one. In our numerical calculations, we assume that $q(y)$ is Gamma-distributed with shape parameter 4 and scale parameter $\frac{1}{2}$. As a result, the distributions $f(y|a = \frac{9}{10})$ and $f(y|a = \frac{11}{10})$ are also Gamma-distributed, with the same scale parameter and shape parameters slightly below and above 5, respectively. We choose the Gamma distribution because it generates tractable expressions for the integrals that define marginal utility, and has a "hump" shape.

We suppose (for tractability) that the non-verifiable income x is equal to

$$x = \chi a y,$$

[21] It is without loss of generality to assume that it does not depend on a report of the borrower's non-verifiable income, since the borrower would also report the non-verifiable income level that minimized her repayment.

[22] The maximum value of ρ matters only when determining the largest debt level that must be considered.

where χ is a positive constant. In our numerical exercise, we set $\chi = 1$, meaning roughly half of the borrower's income is non-verifiable. We have in mind, for example, future labor income. Under our assumptions, the expected total pledgeable and non-pledgeable income is five, and the standard deviation of the total income is roughly half this value. The expected total income is about 15% higher the good external state than the bad external state. We set the cost of project, K, to one.

One consequence of our assumptions is that the borrower, observing x and y, can infer the true external state a. This implication is by no means necessary– we could add noise to the value of x and prevent the borrower from inferring a, at the cost of having a more complicated example.

We now turn to the lender. The lender is risk-neutral within each external state (i.e. with respect to the borrower's idiosyncratic state), but risk-averse with respect to the external state. Let $M(a)$ denote the lender's marginal utility given the external state. We use the notation $M(a)$ to emphasize that this can be interpreted as the lender's stochastic discount factor. We use the functional form $M(a) = \bar{M}a^{-\frac{1}{2}}$, setting $\bar{M}$ so that the expected value of $M(a)$ is equal to one. Interpreting $M(a)$ as an SDF, this is setting the risk-free rate to zero.

If the conditional (on $z \in Z$) contract differs depending on the true value, for a given value of the report, there is a verification cost paid by the lender. Let $c(y';s) = \bar{c} > 0$ if there exists a y_1, y_2 such that the conditional contract s offers different payments for (y_1, y') and (y_2, y'), and zero otherwise. In our numerical calculation, we use $\bar{c} = 10\%$, recalling that we have normalized the project size to one.

We next describe the general forms of the indirect utility functions. Let $\omega(y'|y,x)$ denote a (possibly mixed) reporting strategy by the borrower, and let $\omega^*(y'|y,x)$ be the optimal reporting strategy. The indirect utility functions are

$$\phi_B(s,a) = \max_{\omega(y'|y,x)\in\Omega(s)} \int_0^\infty \int_0^\infty u(\chi ay + y - s(y,y'))\omega(y'|y,\chi ay) f(y|a) dy' dy,$$

$$\phi_L(s,a) = M(a) \int_0^\infty \int_0^\infty (s(y,y') - c(y';s) - K)\omega^*(y'|y,\chi ay) f(y|a) dy' dy,$$

where $\Omega(s)$ denotes the constraints in the reporting strategy, which we describe next. We impose limited liability, meaning that, for all reports y', either $s(y_1,y') = s(y_2,y')$ for all y_1, y_2 and $0 \leq s(y',y') \leq y'$ (the non-verification case), or $0 \leq s(y,y') \leq y$ for all y (the verification case). We restrict the reporting strategies $\omega(y'|y,x)$ to place support only on y' for which the reports are

feasible, meaning that if $s(y_1,y') = s(y_2,y')$ for all y_1,y_2, then $\omega(y'|y,x) = 0$ if $y < s(y',y')$. In words, for reports that do not trigger verification, the borrower must have the funds to repay the loan.

Although this model is slightly different from Townsend [1979] and Gale and Hellwig [1985], the arguments for the optimality of a debt contract are essentially unchanged. Fixing some distribution over external states $p(a)$, and taking the expected value of the indirect utility functions, it is immediately apparent that the model is exactly that of Townsend [1979], except with non-verifiable income. However, the argument for the optimality of debt depends only on non-satiation and that utility at zero verifiable income net debt repayments is not infinite. Therefore, with either $u'(0) > -\infty$ or $x > 0$ with probability one, debt will be optimal for all $p(a)$. It follows that debt is "ex-post optimal" in the sense assumed in the main text, and therefore we restrict attention to contracts that are (possibly indexed on $z \in Z$) debt contracts.

The set of feasible debt levels is $D = [0,\bar{d}]$. The level of $\bar{d}$ is determined by the smallest promise such that, if the promise for the other value of $z \in Z$ is zero, all lender types at least break even. Any promise larger than this will necessarily generate profits for all lender types, and can therefore be rejected by the borrower. The value of $\bar{d}$ is determined by the ϕ_L function and our assumption on the set of possible index qualities; we omit the details for brevity.

Specializing the indirect utility functions to a debt contract, which induces truthful reporting,

$$\begin{aligned}\phi_B(s_d,a) = &\int_d^\infty u((1+\chi a)y - d)f(y|a)dy \\ &+ \int_0^d u(\chi a y)f(y|a)dy,\end{aligned}$$

and

$$\begin{aligned}\phi_L(s_d,a) = &M(a)\int_d^\infty (d-K)f(y|a)dy \\ &+ M(a)\int_0^d (y-K-\bar{c})f(y|a)dy.\end{aligned}$$

We note that these indirect utility functions satisfy the assumption we have imposed. In particular, they are both differentiable (and hence continuous) in d and the lender's indirect utility function is zero when the level of debt is zero. The derivative of the borrower's indirect utility function with

respect to the level of debt is

$$\phi_{B,d}(s_d,a) = -\int_d^\infty u'((1+\chi a)y-d)f(y|a)dy,$$

and hence is strictly negative, satisfying our monotonicity requirement. The derivative of the lender's indirect utility function is

$$\phi_{L,d}(s_d,a) = -M(a)f(d|a)\bar{c} + M(a)\int_d^\infty f(y|a)dy.$$

As in the mortgage example of section §3, there are several key forces that will determine whether assumptions 2 and 3 are satisfied. Loosely speaking, lender risk-aversion (decreasing $M(a)$) must outweigh the increasing likelihood of not being repaid in bad states, and borrower risk-aversion with respect to promises must sufficiently dominate lender risk-aversion with respect to promises.

We now turn to our numerical analysis. The table below summarizes our functional form and parameter assumptions. Under these functional forms and assumptions, we have verified using Mathematica that assumptions 3 and 4 are satisfied for all $a \in \{\frac{9}{10}, \frac{11}{10}\}$ and $d \in [0, \bar{d}]$.

Table 1: Parameters of CSV Example

Function/Parameter	Functional Form	Value
Utility $u(\cdot)$	CARA	1
Marginal Utility $\beta_L(a)$	$M(a) = \bar{M}a^{-\frac{1}{2}}$	$\bar{M} = [\frac{1}{2}(\frac{9}{10})^{-\frac{1}{2}} + \frac{1}{2}(\frac{11}{10})^{-\frac{1}{2}}]^{-1}$
PDF $q(y)$	Gamma(κ,θ)	$\kappa = 4, \theta = \frac{1}{2}$
Verification cost $\bar{c}$		$\frac{1}{10}$
N.V. Income Function $\mu(y,a)$	$\mu(y,a) = \chi ya$	$\chi = 1$
Required Funds K		1
External States A		$\{\frac{9}{10}, \frac{11}{10}\}$
Probabilities $p(a)$		$(\frac{1}{2}, \frac{1}{2})$

B Proofs

B.1 Proof of proposition 1

We first show that assumption 2, lender sub-modularity is satisfied. Sub-modularity, in a differentiable context, is

$$\phi_{L,d}(s_d, a) = \pi(a)u'_L(y_L(a))$$

decreasing on $a \in A$. With two states, and CRRA, this is

$$y_L(a_{good})^{-\gamma_L}\pi(a_{good}) \leq y_L(a_{bad})^{-\gamma_L}\pi(a_{bad})$$

and therefore by equation (1) lender sub-modularity holds.

Social welfare super-modularity (assumption 3) requires that

$$U_d(s_d, a; \lambda^*)$$

be increasing in a. We can write

$$U_d(s_d, a; \lambda^*) = \phi_{B,d}(s_d, a) + \lambda^*\phi_{L,d}(s_d, a).$$

The definition of λ^* is (by Equation (16), noting that an interior solution is guaranteed for K (and hence d^*) sufficiently small)

$$\sum_{a \in A} p(a)U_d(s^*, a; \lambda^*) = 0,$$

which is

$$\lambda^* = -\frac{\phi_{B,d}(s^*, a_g) + \phi_{B,d}(s^*, a_b)}{\phi_{L,d}(s^*, a_g) + \phi_{L,d}(s^*, a_b)}.$$

The denominator is positive by $\lambda^* > 0$. To have super-modularity, we must have

$$U_d(s_d, a; \lambda^*)(\phi_{L,d}(s^*, a_g) + \phi_{L,d}(s^*, a_b))$$

increasing in a, which is

$$\phi_{B,d}(s_d,a_g)[\phi_{L,d}(s^*,a_g)+\phi_{L,d}(s^*,a_b)]-\phi_{L,d}(s_d,a_g)[\phi_{B,d}(s^*,a_g)+\phi_{B,d}(s^*,a_b)]\geq$$
$$\phi_{B,d}(s_d,a_b)[\phi_{L,d}(s^*,a_g)+\phi_{L,d}(s^*,a_b)]-\phi_{L,d}(s_d,a_b)[\phi_{B,d}(s^*,a_g)+\phi_{B,d}(s^*,a_b)].$$

We can rewrite this as

$$[\phi_{B,d}(s_d,a_g)-\phi_{B,d}(s_d,a_b)][\phi_{L,d}(s^*,a_g)+\phi_{L,d}(s^*,a_b)]\geq$$
$$[\phi_{L,d}(s_d,a_g)-\phi_{L,d}(s_d,a_b)][\phi_{B,d}(s^*,a_g)+\phi_{B,d}(s^*,a_b)],$$

Observe that, for all $d,d'\in D$,

$$\phi_{L,d}(s_d,a)=\phi_{L,d}(s_{d'},a).$$

We have

$$\phi_{B,d}(s_d,a)=-\pi(a)u_B'(y_B(a)+x_h-d)$$

and therefore

$$\ln(\frac{-\phi_{B,d}(s_d,a_g)}{-\phi_{B,d}(s_d,a_b)})=\ln(\frac{\pi(a_g)}{\pi(a_b)})-\gamma_B\ln(\frac{y_B(a_g)+x_h-d}{y_B(a_b)+x_h-d}).$$

It follows that the bound is tightest at $d=0$,

$$[\phi_{B,d}(s_0,a_g)-\phi_{B,d}(s_0,a_b)][\phi_{L,d}(s_0,a_g)+\phi_{L,d}(s_0,a_b)]\geq$$
$$[\phi_{L,d}(s_0,a_g)-\phi_{L,d}(s_0,a_b)][\phi_{B,d}(s^*,a_g)+\phi_{B,d}(s^*,a_b)],$$

Observe now that $-\phi_{B,d}(s_d,a)$ is increasing in d and that, by sub-modularity, $\phi_{L,d}(s_0,a_g)\leq\phi_{L,d}(s_0,a_b)$. Consequently, the easiest version of the bound to satisfy is if $s^*=s_0$, the security associated with $d=0$.

The debt level associated with s^*, d^*, is determined by the lender's break-even condition,

$$0=\sum_{a\in A}p(a)\{\pi(a)u_L'(y_L(a))(d^*-K)-(1-\pi(a))u_L'(y_L(a))K\},$$

which is

$$d^*=K\frac{\sum_{a\in A}p(a)\pi(a)u_L'(y_L(a))}{\sum_{a\in A}p(a)\{\pi(a)u_L'(y_L(a))+(1-\pi(a))u_L'(y_L(a))\}}.$$

Consequently, holding all other parameters fixed,

$$\lim_{K\to 0} d^*(K) = 0.$$

It follows by the continuity of $\phi_{B,d}(s_d, a)$ in d that if

$$[\phi_{B,d}(s_0,a_g) - \phi_{B,d}(s_0,a_b)][\phi_{L,d}(s_0,a_g) + \phi_{L,d}(s_0,a_b)] >$$
$$[\phi_{L,d}(s_0,a_g) - \phi_{L,d}(s_0,a_b)][\phi_{B,d}(s_0,a_g) + \phi_{B,d}(s_0,a_b)],$$

there exists a $\bar{K}$ such that for all $K < \bar{K}$, the required inequality holds. This condition can be simplified to

$$\phi_{B,d}(s_0,a_g)\phi_{L,d}(s_0,a_b) - \phi_{B,d}(s_0,a_b)\phi_{L,d}(s_0,a_g) >$$
$$-\phi_{B,d}(s_0,a_g)\phi_{L,d}(s_0,a_b) + \phi_{B,d}(s_0,a_b)\phi_{L,d}(s_0,a_g),$$

which further simplifies to

$$\frac{-\phi_{B,d}(s_0,a_g)}{-\phi_{B,d}(s_0,a_b)} < \frac{\phi_{L,d}(s_0,a_g)}{\phi_{L,d}(s_0,a_b)}.$$

Plugging in functional forms, this is

$$\frac{(y_B(a_g)+x_h)^{-\gamma_B}}{(y_B(a_b)+x_h)^{-\gamma_B}} < \frac{(y_L(a_g))^{-\gamma_L}}{(y_L(a_b))^{-\gamma_L}},$$

or

$$\gamma_B \ln(\frac{y_B(a_g)+x_h}{y_B(a_b)+x_h}) > \gamma_L \ln(\frac{y_L(a_g)}{y_L(a_b)}),$$

as assumed by equation (3)

To verify assumption 1, we require

$$\sum_{a\in A} p(a)\{\pi(a)u_B(y_B(a)+x_h-d) + (1-\pi(a))u_B(y_B(a))\} \geq \sum_{a\in A} p(a)u_B(y_B(a)),$$

which is always be satisfied.

B.2 Proof of lemma 1

First, note that, for any values of θ for the which the lenders do not offer a security, expected utility is zero.

Proof by contradiction: suppose that there exists a symmetric pure-strategy equilibrium such that, for some values of $\theta \in \Theta$, the security $s(\theta)$ is offered and equilibrium lender expected utility is strictly positive.

Let θ' and $s' = s(\theta')$ denote the equilibrium type and security for which lender expected utility is strictly positive. In this equilibrium, each lender earns

$$|L|^{-1} \sum_{a \in A, z \in Z} \theta'(a,z)\phi_L(s'_z, a) > 0.$$

Let $d'(z)$ be the function satisfying $s'_z = s_{d'(z)}$ for all $z \in Z$. Consider a deviation by some lender to the security $s''_z = s_{d''(z)}$, where $d''(z) = \alpha d(z)$ from some $\alpha \in (0,1)$. By assumption, $s'' \in S$. By the monotonicity property of the borrower's indirect utility function, $\phi_B(s_d, a)$, in d, we have

$$\sum_{a \in A, z \in Z} \theta(a,z)\phi_B(s''_z, a) > \sum_{a \in A, z \in Z} \theta(a,z)\phi_B(s'_z, a)$$

for all $\theta \in \Theta$. It follows that, regardless of the beliefs the borrower forms off-equilibrium, she will accept security s'' if offered, for any value of $\alpha \in [0,1)$.

The change in expected utility for the deviating lender is

$$\sum_{a \in A, z \in Z} \theta'(a,z)(\phi_L(s''_z, a) - |L|^{-1}\phi_L(s'_z, a)).$$

By the lower semi-continuity of ϕ_L in d and the fact that $|L| > 1$, there exists an $\alpha \in (0,1)$ such that this quantity is positive. It follows that an equilibrium where lenders earn strictly positive expect utility cannot exist.

B.3 Proof of proposition 2

By assumption 1, this equilibrium delivers sufficient utility for the borrower. Therefore, the borrower is willing to participate, and lenders earn zero profits (by the construction of $\bar{s}(\theta)$) and therefore are also willing to participate.

Now consider a deviation by a single lender: suppose some lender of type θ offers security s' instead of $\bar{s}(\theta)$, and would weakly profit from doing so if the security was accepted. Because the lender can weakly profit from offering this deviation, the borrower is free to place the full support of her beliefs on the lender's true type, if the security s' is not an offer of any type in equilibrium. If the security s' is offered in equilibrium by some type other than θ, by the existence at least three lenders, the borrower can infer the true common type. Because the security $\bar{s}(\theta)$ is on the Pareto-frontier, and offers zero profit to lenders, it follows that the borrower must be weakly worse off using security s', and therefore would prefer the security $\bar{s}(\theta)$. Because there are multiple lenders, the borrower can choose a non-deviating lender and reject the deviating security. Given that the security will be rejected, the lender does not profit from offering it, and therefore $s(\theta) = \bar{s}(\theta)$ is an equilibrium.

B.4 Proof of lemma 2

First, observe by the definition of the full-information optimal security that, for all $\theta \in \Theta$,

$$\sum_{a \in A, z \in Z} \theta(a,z)\phi_B(\bar{s}_z(\theta), a) \geq \sum_{a \in A, z \in Z} \theta(a,z)\phi_B(s^*, a).$$

Proof by contradiction: suppose the full-information optimal security is non-contingent. The full-information optimal security solves, for some value of $\lambda > 0$,

$$\max_{\{d(z) \in D\}_{z \in Z}} \sum_{a \in A, z \in Z} \theta(a,z) U(d(z), a; \lambda).$$

Suppose that the optimal $d'(z) = d(s^*)$ for all $z \in Z$, with $d^*(s)$ in the interior of D. In this case, $\lambda = \lambda^*$ and by the differentiability of ϕ_B and ϕ_L at that point, we must have, all $z \in Z$,

$$\sum_{a \in A} \theta(a,z) U_d(d(s^*), a; \lambda^*) = 0.$$

Because $\theta \succeq_{GWA} \theta_0$, it also dominates it in the super-modular stochastic order (Meyer and Strulovici [2012]). It follows by Meyer and Strulovici [2015] that θ can be expressed using those authors' "elementary transformations" $t \in \mathscr{T}$. That is,

$$\theta = \theta_0 + \sum_{t \in \mathscr{T}} \alpha_t t,$$

for some constants $\alpha_t \geq 0$. For all $\theta \neq \theta_0$, there must exist at least one $\alpha_t > 0$.

Consequently, there exists some $z' \in Z$ such that

$$\sum_{a \in A, z \in Z} (\theta(a,z) - \theta_0(a,z)) h(a) 1(z \succeq z') > 0$$

for any strictly increasing function $h(a)$, and in particular $U_d(d(s^*), a; \lambda^*)$, contradicting the requirement that

$$\sum_{a \in A} \theta(a,z) U_d(d(s^*), a; \lambda^*) = 0$$

for all $z \in Z$.

B.5 Proof of proposition 3

The non-contingent security $s^* = s_{d^*}$ has payoffs that do not depend on the index. As a result, it offers zero expected utility for the lender, regardless of the lender's type, by the assumption that all $\theta \in \Theta$ have the same marginal distribution with respect to the external state. By assumption 1, s_{d^*} can deliver sufficient utility to the borrower, and therefore the participation constraints are satisfied in this equilibrium. It is sufficient to rule out deviations in which a single lender offers security s' instead of s_{d^*}, when the common type is θ', to demonstrate that this is an equilibrium.

The security $s' \in S$, $s'_z = s_{d'(z)}$ with must offer strictly positive utility for the lender of type θ' (if accepted) to break the equilibrium:

$$\sum_{a \in A, z \in Z} \theta'(a,z) \phi_L(s_{d'(z)}, a) > 0.$$

Define $\bar{\theta}$ as a type that would profit the most from offering the security s':

$$\bar{\theta} \in \arg \max_{\theta \in \Theta : \mu_0(\theta) > 0} \sum_{a \in A, z \in Z} \theta(a,z) \phi_L(s_{d'(z)}, a)$$

Consider the set of "elementary transformations" $t \in \mathscr{T}$ defined by Meyer and Strulovici [2015], and suppose that for some $z', z'' \in Z$ that are adjacent in the order on Z (with $z'' \succ z'$), $d(z'') > d(z')$. By Meyer and Strulovici [2015], we can write

$$\bar{\theta} = \theta_0 + \sum_{t \in \mathscr{T}} \alpha_t t,$$

for some constants $\alpha_t \geq 0$. If there exists a $t \in \mathscr{T}$ with support on z' and z'' such that $\alpha_t > 0$, then by the richness of the type space (assumption 4), there exists a type $\hat{\theta} = \bar{\theta} - \beta t$, for some $\beta > 0$, such that $\tilde{\theta}$ is in the support of μ_0. By the sub-modularity of ϕ_L (assumption 2),

$$\sum_{a \in A, z \in Z} \hat{\theta}(a,z)\phi_L(s_{d'(z)}, a) \geq \sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_L(s_{d'(z)}, a).$$

Therefore, it is without loss of generality to assume that the security $d'(z)$ is weakly decreasing between adjacent pairs z', z'' such that $\alpha_t > 0$ for some elementary transformation with support on those pairs.

By this result and the super-modularity of the social welfare function with Pareto-weight λ^* (assumption 3), we must have

$$\sum_{a \in A, z \in Z} \bar{\theta}(a,z)U(s_{d'(z)}, a; \lambda^*) \leq \sum_{a \in A, z \in Z} \theta_0(a,z)U(s_{d'(z)}, a; \lambda^*).$$

By the Pareto-optimality of the non-contingent security s^* under θ_0,

$$\sum_{a \in A, z \in Z} \theta_0(a,z)U(s_{d'(z)}, a; \lambda^*) \leq \sum_{a \in A, z \in Z} \theta_0(a,z)U(s^*, a; \lambda^*).$$

Therefore,

$$\sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_B(s_{d'(z)}, a) + \lambda^* \sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_L(s_{d'(z)}, a) \leq$$
$$\sum_{a \in A, z \in Z} \theta_0(a,z)\phi_B(s^*, a) + \lambda^* \sum_{a \in A, z \in Z} \theta_0(a,z)\phi_L(s^*, a).$$

It follows by

$$\sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_L(s_{d'(z)}, a) > 0 = \sum_{a \in A, z \in Z} \theta_0(a,z)\phi_L(s^*, a)$$

that

$$\sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_B(s_{d'(z)}, a) < \sum_{a \in A, z \in Z} \theta_0(a,z)\phi_B(s^*, a),$$

and by the non-contingency of s^*,

$$\sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_B(s_{d'(z)}, a) < \sum_{a \in A, z \in Z} \bar{\theta}(a,z)\phi_B(s^*, a).$$

By the $D1$ refinement, the borrower can place the support of her beliefs entirely on the type $\bar{\theta}$. Consequently, if there exists a θ' for which the deviation is profitable, the borrower can believe she is worse off and reject the deviation.

B.6 Proof of lemma 3

By definition, the set Θ is bounded, by assumption it is closed, and therefore it is compact. It follows that $\Theta^*(\theta)$ is non-empty. By the linearity of

$$\sum_{a\in A, z\in Z} \theta''(a,z)\phi_L(\bar{s}_z(\theta),a)$$

in θ'', $\Theta^*(\theta)$ is convex.

By the assumption of ex-post efficiency, $\bar{s}_z(\theta) = s_{d(z,\theta)}$ for some $d(z,\theta) \in D$. By definition,

$$d(z,\theta) = \arg \max_{\{d(z)\in D\}_{z\in Z}} \sum_{a\in A, z\in Z} \theta(a,z)\phi_B(s_{d(z)},a)$$

subject to

$$\sum_{a\in A, z\in Z} \theta(a,z)\phi_L(s_{d(z)},a) \geq 0.$$

By assumption, $\phi_L(s_d,a)$ and $\phi_B(s_d,a)$ are continuous in d, and hence it follows that $d(z,\theta)$ is continuous in θ and that

$$\sum_{a\in A, z\in Z} \theta''(a,z)\phi_L(\bar{s}_z(\theta),a)$$

is jointly continuous in (θ,θ''). Therefore by Berge's theorem (the theorem of the maximum), $\Theta^*(\theta)$ is upper semi-continuous.

It follows that Kakutani's fixed point theorem holds, and therefore that there exists a θ^* such that

$$\theta^* \in \Theta^*(\theta^*),$$

as claimed.

Now suppose there is another $\theta' \in \Theta^*(\theta^*)$. We must have

$$\sum_{a\in A, z\in Z} \theta'(a,z)\phi_L(s_{d(z,\theta)},a) = 0.$$

Because the index and the external state are perfectly correlated, we can rewrite this as

$$\sum_{a\in A, z\in Z} q(z;\theta')\delta(a,z)\phi_L(s_{d(z,\theta)},a) = 0,$$

where $q(z;\theta')$ is the marginal distribution associated with θ'.

By the definition of $\bar{s}(\theta)$, for all $z \in Z$ (by the full support assumption, $q(z;\theta^*) > 0$),

$$\phi_B(s_{d(z;\theta^*)},a) + \lambda\phi_B(s_{d(z;\theta^*)},a) \geq \phi_B(s_{d'},a) + \lambda\phi_B(s_{d'},a)$$

for all $d' \in D$ and some multiplier $\lambda > 0$. It follows that the optimality would also hold if $d(z;\theta') = d(z;\theta^*)$, and feasibility is satisfied, and therefore

$$\bar{s}(\theta') = \bar{s}(\theta^*),$$

as required.