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ABSTRACT

We investigate the bank lending channel of negative nominal policy rates from an empirical and theoretical perspective. We find that retail household deposit rates are subject to a lower bound (DLB). Empirically, once the DLB is met, the pass-through to lending rates and credit volumes is substantially lower and bank equity values decline in response to further policy rate cuts. We construct a banking sector model and use our estimate of the pass-through of negative policy rates to lending rates as an identified moment to parameterize the model and assess the impact of negative policy rates in general equilibrium. Using the theoretical framework, we derive a sufficient statistic for when negative policy rates are expansionary and when they are not.

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1 Introduction

Between 2012 and 2016, a handful of central banks reduced their policy rates below zero for the first time in history. While *real* interest rates have been negative on several occasions, *nominal* rates have not. The recent experience suggests that negative policy rates have become part of the central banker's toolbox, and calls into question the relevance of the zero lower bound (ZLB) as a constraint on monetary policy. However, the question of whether and when negative policy rates are an *effective* tool for macroeconomic stabilization remains largely unresolved.

Understanding the impact of negative policy rates on the macroeconomy is of urgent current interest. Policy rates have been declining steadily since the early 1980s, resulting in worries about secular stagnation (see e.g. Summers 2014, Eggertsson and Mehrotra 2014 and Caballero and Farhi 2017). Kiley and Roberts (2017) estimate that the ZLB will bind 30-40 percent of the time going forward, and the recent outbreak of COVID-19 has again brought interest rates close to zero in several economies.[1]

In this paper we investigate the impact of negative policy rates on the macroeconomy through the bank lending channel, both from an empirical and a theoretical perspective. We use a combination of aggregate and bank level data to empirically examine the pass-through of negative policy rates to deposit and lending rates, credit volumes and bank equity, using Sweden as a case study.[2] We then construct a theoretical model with a banking sector that can rationalize our empirical findings and parameterize it using, among other things, our empirical pass-through estimate as an identified moment. The model allows us to derive a sufficient statistic - discussed below - for when negative policy rates are expansionary and when they are not. We conclude by providing estimates for the quantitative impact of the bank lending channel of negative policy rates on key macroeconomic aggregates in the context of Sweden.

In the empirical section we investigate the pass-through of negative policy rates to banks assets and liabilities. We start by documenting virtually full pass-through of negative policy rates to money market rates, that is, interbank lending rates. This is not surprising, as the interbank rate is effectively equivalent to the policy rate (the "repo-rate") itself. The transmission of negative policy rates to other highly liquid assets such as short-term government bonds is also strong. A high degree of pass-through to interbank rates can be expansionary by lowering bank funding and opportunity costs. Yet, since interbank lending measures lending from one bank to another, rather than lending to the bank sector as a whole, its impact on aggregate funding costs is limited. Moreover, and as highlighted in our model, since banks invest in liquid assets to insurance against liquidity risk, lower returns on these assets directly *reduce* bank profitability.

We document a collapse in the pass-through of the policy rate to retail household deposit rates once the

[1] An alternative to negative interest rates is unconventional monetary policy measures. There are several reasons, however, why it is important to consider policy measures beyond these tools. Some of the credit policies used by the the Federal Reserve, the FDIC and the Treasury were severely constrained by Congress following the financial crisis, as stressed by Bernanke et al. (2018). Moreover, there remains little, if any, consensus among economists on how effective quantitative easing and forward guidance is. Plausible estimates range from considerable effects to none (see e.g. Greenlaw et al. (2018) for a somewhat skeptical review, Swanson (2017) for a more upbeat assessment, and Greenwood et al. (2014) for a discussion of debt management at the zero lower bound).

[2] The Swedish case is useful to study for a number of reasons, most importantly because we have access to high-frequent microdata on bank-level mortgage rates. In addition, the Swedish central bank - the Riksbank - cut interest rates four times in to negative territory, providing relatively rich variation in the data which can be used to estimate the pass-through of negative rates and how it depends on the lower bound on the deposit rate.

policy rate turns sufficiently negative. This is in sharp contrast to regular circumstances, in which the two move closely together. Because deposits are the main source of bank financing (approximately 50 % in our data), the transmission of policy rates to banks marginal funding costs is impaired once this point is reached. An important finding is that this collapse in pass-through does not need to happen at exactly zero. In our empirical setting, the deposit rate remained responsive to policy rate cuts, albeit less so, until the policy rate reached -0.25 percent.[3]

We proceed by considering the pass-through of negative policy rates to lending rates. To do so, we conduct an event study around days of policy rate cuts using daily bank level mortgage rates. Under regular circumstances, we show that the pass-through of policy rate cuts is around 80 percent within 30 days. Once policy rates reach a level at which deposit rates are not longer responsive however, the pass-through collapses. In fact, taking a weighted average over the different mortgage contracts offered across all banks in our sample, the average pass-through to aggregate mortgage rates becomes slightly *negative*, i.e. the empirical evidence suggests a modest increase in lending rates once the policy rate falls sufficiently below zero.

In addition to a collapse in the pass-through to lending rates, we also document an increase in the dispersion of lending rates as the policy rate turns negative. The rise in dispersion is linked to banks financing structures. Banks which rely more heavily on deposit financing are less likely to reduce their lending rates once the deposit rate has stopped responding. Moving to bank-level lending *volumes*, we show that Swedish banks that rely more heavily on deposit financing also have lower credit growth once the deposit rate has reached its lower bound.

A key question in the debate surrounding negative interest rates is whether they have detrimental effects on banks net worth. We conclude the empirical section by conducting an event study using equity values for publicly listed Swedish banks. In contrast to policy rate cuts in normal times, policy rate cuts in negative territory are found to negatively impact bank equity values.

Motivated by the empirical results, we construct a bank model which rationalizes the empirical findings, and provides additional theoretical predictions for when negative policy rates are effective. The model is also used to produce quantitative predictions on the impact of negative rates and to discuss policy interventions which can make negative rates more effective once retail deposit rates reaches a lower bound.

Negative nominal interest rates can arise in our model because banks and households are potentially willing to pay for storage and liquidity services provided by reserves and deposits. Taking this into account, our model derives in place of the conventional zero lower bound two new lower bounds: a lower bound on the policy rate (the "PLB") and a lower bound on the deposit rate banks can offer (the "DLB"). Both arise due the the presence of paper currency, although evidence on banks cash holdings suggests that the PLB has remained non-binding. Unlike the PLB, the available empirical evidence indicates the DLB is close to zero for small depositors and only modestly negative for larger depositors.

In our model, policy rate cuts are perfectly transmitted to deposit rates and lending rates when the lower bounds are non-binding, thus stimulating lending and aggregate demand. Once the DLB is reached, however, the transmission of policy rate to the main financing source of banks breaks down – as in the data. In this

[3]In countries where the spread between deposit and policy rates are higher, for instance in some Eurozone countries, deposit rates could fall by more, and hence the policy rate can go lower, before deposit rates reaches a DLB.

case, we show that the impact of negative policy rates on bank profits is a sufficient statistic for the qualitative impact of negative policy rates on bank lending.

Whether negative policy rates reduce or increase bank profits depends on the balance sheet composition of banks. In addition to lending (B) and reserves (R) on the asset side of the balance sheet, we assume that banks hold liquid assets (A), such as government bonds, to insure against liquidity shocks. On the liability side, in addition to deposits (D), banks finance themselves via external financing (F), which refers to all non-deposit financing which is not subject to the DLB (e.g. bank bond issuance and larger wholesale depositors). The sufficient statistic implies that negative policy rates expand bank lending if

$$\rho^a A + R < \rho^f F \tag{1}$$

where ρ^a and ρ^f measure the effective pass-through of the policy rate to liquid assets and external financing respectively. Intuitively, bank profits increase (and hence lending expands) if the effective pass-through of negative policy rates is larger to bank liabilities ($\rho^f F$) compared to bank assets ($\rho^a A + R$). Looking at Swedish data, we find that this condition is *not* satisfied, suggesting that further policy rate cuts at the DLB was ultimately counterproductive through the bank lending channel. This is consistent with the negative impact on bank equity values found in the data, and the lack of pass-through to lending rates. We discuss how this sufficient statistic can explain why different empirical studies provide seemingly conflicting evidence on the bank lending channel of negative policy rates.

After discussing our partial equilibrium results, we embed the banking model into a dynamics stochastic general equilibrium model, which nests the standard New Keynesian model as a special case. We use our empirical estimates of the pass-through of negative policy rates to lending rates, combined with existing empirical evidence[4] on the link between bank net worth and bank lending, as identified moments to pin down the key parameters of the model. The model can then be used to quantitatively evaluate the effect of negative rates on output and inflation depending on whether the DLB is binding or not, and on the initial balance sheet composition of the bank sector. Overall, we find that the effect of policy rate cuts on output at the DLB can be both positive and negative. In a benchmark parameterization where the balance sheet composition is set to match the Swedish banking sector, the impact is negative and implies that a 100 basis points reduction in the policy rate once the DLB is binding results in a 17.7 basis points contraction in output. Our quantitative exercise suggests that for negative policy rates to have a non-trivial positive impact on the economy, other channels – such as an asset price channel or an exchange rate channel – would have to be important.

Literature review Our paper relates to a growing empirical literature on the effects of negative interest rates on bank outcomes. The empirical literature has focused primarily on three questions, namely the impact of negative policy rates on deposit rates, lending rates and bank equity values.

Heider et al. (2016) document that median overnight deposit rates in the Eurozone did not fall below zero following the introduction of negative policy rates, while Basten and Mariathasan (2018) and Hong and Kandrac (2018) show similar findings for Switzerland and Japan, respectively. The zero lower bound on deposit rates in the Eurozone appears to be most prevalent for household and other retail deposits. Boucinha and Burlon (2020), Eisenshmidt and Smets (2019) and Altavilla et al. (2019) document that an increasing

[4] See the macroeconomic assessment group MAG (2010).

fraction of *corporate* depositors are charged a negative interest rate, although the pass-through is substantially weakened relative to when rates are positive.

In terms of bank lending, Bech and Malkhozov (2016) document how lending rates in Switzerland increased after the introduction of negative policy rates. Basten and Mariathasan (2018) further show that the lending margin for Swiss banks increased following negative rates, and that banks with larger reserve holdings increased interest rates by more. Amzallag et al. (2019) find that Italian banks with higher deposit ratios charged higher rates on fixed-rate mortgages in response to policy rates turning negative. Hong and Kandrac (2018) document lending rate increases in Japan. Eisenshmidt and Smets (2019) on the other hand, find no evidence that high-deposit banks in Germany increased loan rates relative to low-deposit banks. Bottero et al. (2019) show that Italian banks with more liquid balance sheets expanded lending in response to negative policy rates. Adolfsen and Spange (2020) document a reduction in pass-through to lending rates in Denmark but, in contrast to our findings, show that it is relatively uniform across bank funding structures.

Ampudia and Van den Heuvel (2018) show that while policy rate cuts normally expand bank equity values in the Eurozone, policy rate cuts in negative territory *lowers* bank equity values, which is consistent with our evidence from Sweden. Moreover, the effect is larger for banks with higher deposit shares.[5] Hong and Kandrac (2018) show related evidence from Japan, documenting that Japanese banks' equity values declined by five percent within an hour of the announcement of negative rates by the Bank of Japan.

To summarize: Our findings on the impact of negative policy rates on deposit rates and bank equity values are largely consistent with the existing literature for other countries. Our main contribution to the empirical literature is to use daily interest rate data from Sweden in an event study to estimate the pass-through of negative policy rates to lending rates. In contrast to much of the existing literature, our empirical estimates is informative about the *absolute* effect of negative policy rates on lending rates, rather than a *relative* difference between treated and control banks. For instance, Basten and Mariathasan (2018) study the impact on lending margins in Switzerland using a difference in difference analysis and find that banks with higher reserve holdings have lower pass-through *relative* to other banks. In contrast, we use an event study setup to estimate the *absolute* pass-through of negative policy rates to lending rates. This estimate is not only of particular policy interest, it also serves as an identified moment to match in our banking model.

The theoretical literature on negative interest rates is perhaps surprisingly somewhat smaller, given the high stakes in the policy debate.[6] Three studies are especially relevant for the theoretical analysis in this paper.

Rognlie (2015) provided a first analysis of the normative aspects of negative policy rates. A key distinction between his paper and ours is that in his model households face only one interest rate, and the central bank can control this interest rate directly. Since we are interested in theoretically evaluating the pass-through of negative policy rates to other interest rates, our model differs from his by having multiple interest rates and explicitly modeling the transmission mechanism of monetary policy through the bank sector.

Brunnermeier and Koby (2017) propose a theoretical model in which there is a reversal rate at which point further interest rate cuts become contractionary. There are three main differences between their model

[5] Heider et al. (2016) provides similar findings.

[6] There is however a large literature on the effects of the zero lower bound. See for example Krugman (1998) and Eggertsson and Woodford (2006) for two early contributions.

and ours. First, the main mechanism in their paper is not motivated by the DLB, which in our model is derived theoretically from the households portfolio allocation problem due to the existence of money as a nominal asset. Thus our focus is primarily on the problem of negative policy rates in the presence of paper currency, while the reversal rate arises independently of paper currency, and can occur at either positive or negative policy rates. Second, the reversal rate arises mainly due to maturity mismatch on the banks balance sheet, which gives rise to capital gains/losses due to unanticipated interest rate cuts. We abstract from banks balance sheet mismatch, as we aim to evaluate whether negative policy rates rates can substitute for regular policy rate cuts. This consideration arises independently of whether interest rate cuts are anticipated or not. Third, banks in our model have access to not only deposit financing and net worth, but also rely on market funding – which is potentially subject to a different pass-through. This gives rise to the second key mechanism for understanding the effectiveness of negative policy rates in our model, namely the banks exposure to negative policy rates on the asset side *relative* to the liability side, which is the key condition for determining if negative policy rates are effective or not in our model.

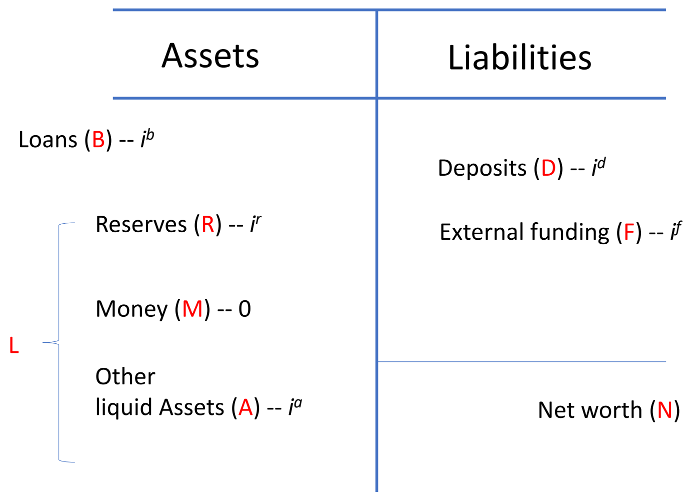
Finally, Ulate (2019) embeds a monopolistically competitive banking sector into an otherwise standard New Keynesian model and investigates the extent to which negative policy rates are expansionary. Our theoretical analysis differs from Ulate (2019) in that we model a richer bank balance sheet, allowing banks to finance themselves with non-deposit funding, as well as specifying an explicit role for liquid assets. The presence of external financing allows for substantial pass-through to banks overall funding costs, even when the DLB is binding. Our model highlights the importance of considering the net exposure of banks to negative policy rates, based on *both* assets and liabilities, rather than focusing on deposit shares alone. In Ulate (2019), negative policy rates always reduce bank profits. In our setting, negative policy rates can both increase and decrease bank profits depending on banks' net exposure to negative rates. Our model can therefore reconcile the sometimes contradicting findings of different empirical studies.

Although our results indicate that the transmission of negative interest rates through the bank sector can have diminishing returns depending on i) the distance to the DLB and ii) banks net exposure to negative rates, there are ways in which negative policy rates can have further expansionary effects which we do not capture in our model. Most importantly, we do not study other transmission mechanisms such as exchange rate effects or asset price effects. Second, if the DLB is overcome, our model predicts that negative policy rates should be an effective way to stimulate the economy. This could happen if banks over time become more willing to experiment with negative deposit rates, and depositors do not substitute to cash, or if there are institutional changes which affect the DLB. In Section 4 we consider under which conditions this could happen. An example of such policies is a direct tax on paper currency, as proposed first by Gesell (Gesell, 1916) and discussed in detail by Goodfriend (2000) and Buiter and Panigirtzoglou (2003) or actions that increase the storage cost of money, such as eliminating high denomination bills. Another possibility is abolishing paper currency altogether. These policies are discussed in, among others, Agarwal and Kimball (2015), Rogoff (2017a) and Rogoff (2017c), who also suggest more elaborate policy regimes to circumvent the zero lower bound. The results presented here do not contradict these ideas. Rather, they suggest that given the current institutional framework and our empirical findings, negative interest rates will not be an effective way to stimulate aggregate demand via the bank lending channel.

2 Empirical analysis

Both the empirical and the theoretical section are organized around the simplified bank balance sheet illustrated in Figure 1.

Figure 1: Bank balance sheet.



The policy instrument is the interest rate the central bank pays on reserve accounts that commercial banks hold to execute interbank transactions, i_t^r.[7] We denote the pass-through of the policy rate to an interest rate i by ρ^i, where i refers to either liquid assets (A), deposits (D), external financing (F) or bank lending (B). The main objective of this section is to empirically account for the pass-through of the policy rate to these other interest rates.

We first illustrate how the policy rate is transmitted to the interest rate on liquid assets before moving on to the liability side of the balance sheet. This part of the analysis is relatively straight forward and largely descriptive. The main focus, however, is on the impact of negative policy rates on lending rates and volumes. We conclude the empirical section by estimating the impact of negative policy rates on the price of equity (N). The objective of Section 3 is to provide a theoretical framework which outlines how each component of the balance sheet is determined under negative policy rates, using the empirical evidence to discipline the model.

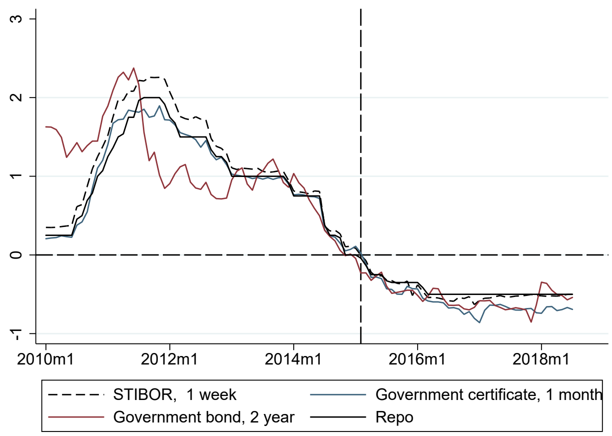
[7] The exact implementation of negative policy rates differ across the countries that have implemented them, see Bech and Malkhozov (2016) for a detailed overview across countries. In the case of Sweden, which we focus on, the Riksbank operates a corridor system and the policy rate is referred to as the repo-rate. The repo-rate is essentially the interest rate banks receive for holding transaction balances at the Riksbank. Banks can borrow from the Riksbank at 75 basis points above the policy rate and central bank reserves earn an interest rate 75 basis points below the policy rate. Consider for example a policy rate of - 0.5 percent. In order to implement this rate, the Riksbank sells certificates in repo transactions that pay - 0.5. As the banks are obtaining -1.25 on their reserves, they will use the reserves to purchase these certificates. In this sense the repo-rate is essentially equivalent to the Riksbank directly paying - 0.5 on bank reserves.

2.1 Liquid assets (L=M+R+A)

We start by investigating the pass-through of negative policy rates to banks' liquid assets. Figure 2 depicts the evolution of the repo-rate over the past 10 years, along with money market rates (STIBOR) and the interest rate on government bonds.

As the figure reveals, there has been approximately full pass-through from the policy rate to money market rates. The money market rate measures the rate at which commercial banks lend their excess reserves to other banks. It is not surprising that the money market rate follows the policy rate into negative territory, as long as the supply of reserves is sufficiently large.

Figure 2: Interest on liquid assets.



Notes: This figure shows the repo-rate together with the interest rates on 1-week interbank rate (STIBOR, 1 week), 1 month government certificates (Government certificate, 1 month) and 2 year government bonds (Government bond, 2 year). Source: Riksbank.

What can banks do with their excess reserves? Apart from lending them to other commercial banks, they can also buy liquid assets, such as government bonds. Viewed in this light, it is easy to see why there is also relatively strong pass-through to liquid asset rates. According to the figure, the short term risk-free interest rate on government bonds falls virtually one-to-one with the repo-rate and the money market rate.

2.2 Bank financing

2.2.1 Deposits (D)

Deposit financing accounts for about half of bank liabilities for large Swedish banks.[8] For smaller banks the deposit share is larger. The deposit rate is therefore especially important for evaluating banks' marginal costs of funds and overall profitability.

The left panel of Figure 3 depicts aggregate deposit rates in Sweden.[9] Prior to the policy rate turning negative, the aggregate deposit rate is below the policy rate and moves closely with it. As the policy rate turns

[8]Illustrated in Figure 12 in Appendix B

[9]The aggregate deposit rate is a weighted average of the interest rate on different deposit accounts. It thus includes both highly liquid checking accounts, as well as less liquid fixed deposit accounts with minimum deposit amounts.

negative this relationship breaks down. Instead of following the policy rate into negative territory, the deposit rate appears bounded at a level close to zero.

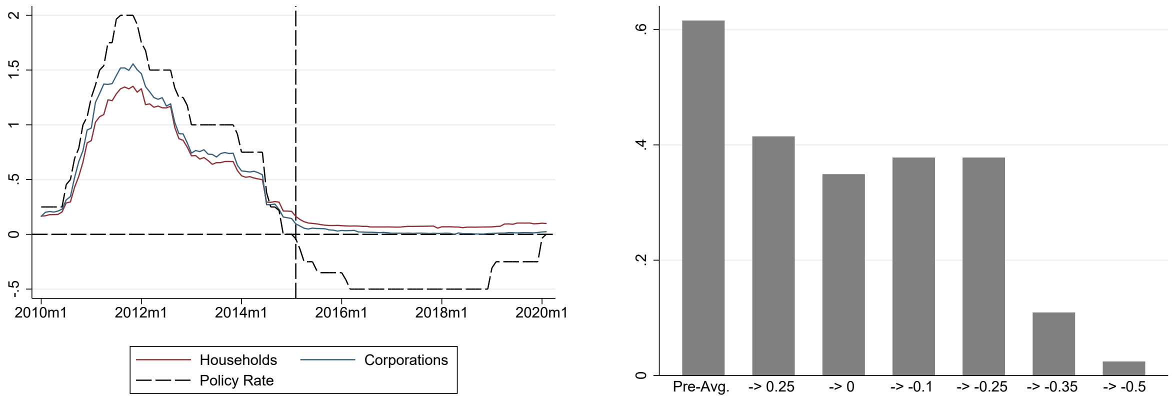


Figure 3: Deposit rates (left panel) and the relative change in deposit rates (right panel).

Notes: This figure shows the policy rate, household deposit rate and corporate deposit rate (left panel) and the pass-through of policy rate cuts to deposit rates (right panel). The policy rate is defined as the repo-rate. In the right panel, we show the change in the deposit rate relative to the change in the policy rate for the last six policy rate reductions. The numbers on the x-axis denotes the policy rate level to which the repo-rate was reduced. Source: The Riksbank, Statistics Sweden.

It is useful to take a closer look at the last six policy rate reductions. The change in the deposit rate relative to the change in the repo-rate is illustrated in the right panel of Figure 3. The first bar captures the average relative change in deposit rates prior to 2014. In this case, the pass-through to the aggregate deposit rate was approximately 60 percent. For the post-2014 data, the pass-through is lower. For policy rate cuts in positive territory, the pass-through is approximately 40 percent. For the first two cuts in negative territory, i.e. to -0.1 percent and to -0.25 percent, the pass-through remains relatively unchanged. For the final two interest rate cuts, however, the pass-through collapses to roughly zero. As the deposit rate reaches its lower bound (DLB), further reductions in the policy rate do not lead to continued reductions in the deposit rate. We refer to the period after the deposit rate has stopped responding, i.e. the last two policy rate reductions, as the period in which the deposit bound (DLB) is binding.

Even with the DLB binding, an increase in fees could decrease the effective deposit rate. In Appendix B, we document that the observed evolution of commission income is not consistent with fees increasing sufficiently to compensate for the DLB on the nominal deposit rate.

2.2.2 Other financing sources (F)

For larger Swedish banks, almost half of liabilities are non-deposits. The largest remaining component is covered bonds. The left panel of Figure 17 in Appendix B compares the interest rate on covered bonds and other interest rates to the policy rate. As with deposit rates, the correlation between the policy rate and covered bond rates is somewhat weaker once the policy rate turns negative.

Even if the pass-through to covered bond rates is weaker, we see from Figure 17 that the interest rate on covered bonds with shorter maturities eventually becomes negative, suggesting a stronger pass-through than for deposit rates. If banks respond to negative policy rates by shifting away from deposit financing, they would therefore reduce their marginal financing costs. However, as we show in Figure 18 in Appendix B, this is not the case. There is no increase in the issuance of covered bonds for Swedish banks, and the deposit share in fact *increases*.

There are at least four possible explanations for why banks did not shift away from deposit financing despite a cost advantage associated with non-deposit funding: i) maintaining a base of depositors creates some synergies which other financing sources do not, ii) other funding sources are associated with higher liquidity risk, iii) the room for new issuances of covered bonds may be limited by the availability of bank assets to use for collateral, and iv) Basel III regulation makes deposit financing more attractive in terms of satisfying new requirements. In section 3, these consideration are taken into account by assuming that banks insurance against refinancing risk by investing in liquid assets, and that the refinancing risk associated with external financing is larger than insured deposit-financing. Since the rate on liquid assets follows the policy rate into negative territory, an implication of this is that external financing does not necessarily become cheaper, which can explain the patterns observed in Figure 18.

2.3 Bank loans (B)

2.3.1 Bank lending rates

Official bank lending rates are only available at a monthly frequency, and not at the bank level. Here, we therefore use daily bank level data based on listed mortgage rates.[10]

Figure 4 plots the raw data for daily mortgage rates since 2014, with each line corresponding to one of the eleven banks in our sample. The upper left panel depicts lending rates for mortgages with three month fixed interest rate periods, referred to as floating rates. This is the most common mortgage contract in the Swedish market, accounting for roughly 70 percent of the market.[11] Consider first the four interest rate cuts which occur prior to the deposit rate reaching its lower bound, i.e. the policy rate reductions all the way down to -0.25 percent. This is the pre-DLB policy rate cuts. For these policy changes, banks respond to policy rate cuts by consistently reducing their lending rates. This stands in stark contrast to the last two policy rate cuts, i.e. to -0.35 and to -0.5 percent, in which there is virtually no reduction in bank lending rates. While one bank cuts its lending rate in response to the policy rate being reduced to -0.35, the lending rate is increased again shortly thereafter. Overall, the pass-through to lending rates appears severely weakened.

While floating rate mortgages are the most common mortgage contracts, Figure 4 also reports mortgage contracts with fixed rate periods of one year, three years and five years. Again, there is a collapse in pass-through for the two last policy rate reductions. In addition, the dispersion in bank lending rates increases.

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[10] The mortgage rates are the interest rates listed by banks on mortgages with floating rates or fixed rate periods from one to five years. Some banks also list lending rates on mortgages with fixed rate periods of ten years. However, we focus on the mortgage contracts used by most of the banks in our sample in order to increase power. Our sample consists of the eleven largest Swedish banks or financial institutions in terms of mortgage lending. According to The Swedish Bankers Association, the six largest banks account for 91 percent of the mortgage market – all of which are in our sample. The largest banks as of 2016 in terms of mortgage lending are Swedbank (24 %), Handelsbanken (23 %), Nordea (15 %), SEB, (15 %), SBAB (8 %), Lansforsakringar (6 %). The remaining 9 percent are attributed to among others Danske Bank and Skandiabanken. See the 2016 report: https://www.swedishbankers.se/media/1310/1611bolaanemarknaden_eng.pdf. All of the banks mentioned are in the sample.

[11] See the 2018 report on the mortgage market in Sweden: https://www.swedishbankers.se/ media/3906/1809_bolaanemarknad-2018_en.pdf

Below, we investigate the underlying sources for this increase in dispersion.

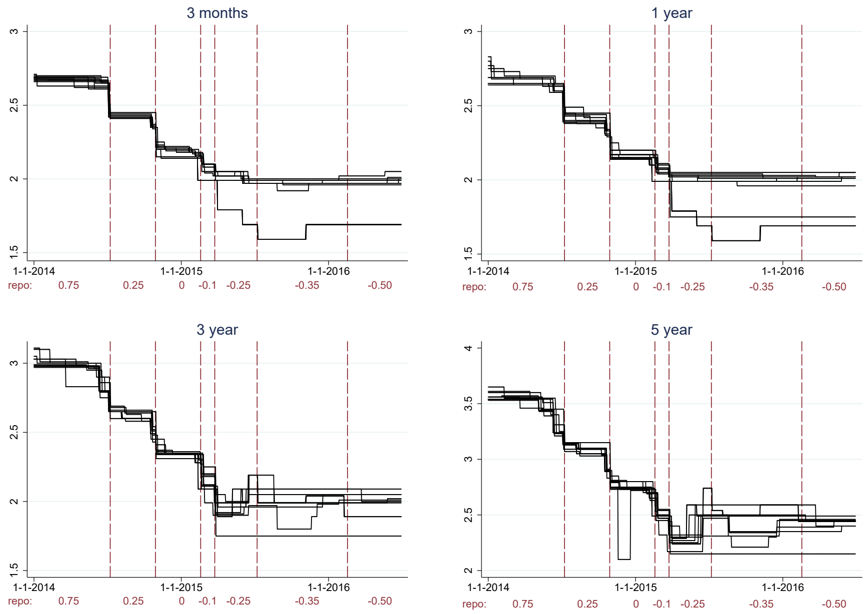

Figure 4: Bank-level mortgage rates for different maturities

Notes: This figure shows bank level mortgage rates for different maturities – 3 months, 1 year, 3 years and 5 years. 01.01.2014 - 31.12.2018. Dashed vertical lines indicate days of repo rate reductions. Source: compricer.se.

Event study

To formalize the pass-through of policy rate cuts to lending rates and investigate whether it changes at the DLB, we estimate the regression

$$\bar{i}^l_t = \alpha + \sum_{k=-30}^{30} \beta_k I_k + I^{DLB}_t \sum_{k=-30}^{30} \beta^{DLB}_k I_k + \epsilon_t \tag{2}$$

where i^l_t is the average, daily aggregate bank lending rate. We construct this variable by aggregating the lending rate of each bank, using their total assets as weight. Furthermore, each bank's lending rate has been constructed by weighting each of their lending contracts by their respective market share[12].

The fourteen interest rate cuts since 2009 were of different size. To account for this, we scale the lending rate by the change in the repo-rate. Thus a value of -100 implies full pass-through of the policy rate.

The dummy variables I_k indicates the number of days k since the repo-rate cut occurred.[13] We consider 30 days prior to and following each policy rate cut. Because the lending variable is scaled, the dummy coefficients, (the β_k's), have the interpretation of reflecting the degree of pass-through. For instance, a value of $\beta_{10} = -75$) means there has been 75 percent pass-through of the policy rate to the lending rate 10 days after

[12]The floating mortgage rate receives a weight of 0.7 to reflect its large market share, whereas the mortgage contracts with fixed interest rate periods of one year, three years and five years receives a weight of $0.3/3$ each.

[13]Note that we could also have used the bank level lending rates, and instead run the regression $i^l_{i,t} = \alpha + \sum_{k=-30}^{30} \beta_k I_k + \epsilon_{i,t}$ (with standard errors clustered at the bank level and using weights to capture bank i's market share). This leads to the same coefficient estimates, but results in smaller confidence intervals. We therefore use the more conservative approach of not using the panel dimension of the data in the regression.

the cut in the policy rate.

The third term in the regression addresses the question of whether the DLP changes the pass-through of the policy rate to the lending rate. This term interacts the daily indicator variable I_k with a dummy variable capturing whether the DLB is binding or not, I_k^{DLP}. The coefficients β_k^{DLB}'s thus measure if there has been a change in pass-through at the DLB. The sum $\beta_k^{DLB} + \beta_k$ measures the total pass-through of policy rate cuts once the DLB becomes binding.

The regression results are reported in Table 1. To conserve space we only report the coefficient estimates for every five days. Consider first the second column, which reports estimated values of β_k. In normal times, i.e. from 2009 and until the deposit rate reaches its lower bound in late 2015, the lending rate starts falling five days prior to the policy rate reduction. At the day of the policy rate change, however, the average lending rate has fallen by 40 percent as much as the policy rate. Thirty days after the policy change, the pass-through has reached 78 percent.

The third and fourth columns contain the main empirical result of the paper. The third column reports the effect of the DLB on the pass-through of policy rate cuts. The estimated coefficient is positive and statistically significant from the day of the policy rate cut and throughout the event window. This suggest that the DLB has a statistically significant negative effect on the pass-through of policy rates to lending rates. Moreover, the fourth column indicates that the point estimate of the effect of the DLB is strongly economically significant. Policy rate cuts, once the DLB is binding, have no significant pass-through to lending rates and the point estimates suggest that, if anything, further policy rate cuts increase lending rates.

Event time	β_k	β_k^{DLB}	Pass-through post-bound: $\beta_k + \beta_k^{DLB}$
- 30	0.00	0.00	0.00
- 25	- 2.94	3.32	0.38
- 20	- 0.45	8.93	8.48
- 15	0.06	8.43	8.48
- 10	- 4.89	13.6	8.72
- 5	- 23.2*	31.5	8.33
0	- 40.3***	48.7*	8.33
5	- 62.4***	68.3***	5.92
10	- 72.4***	78.4***	5.92
15	- 73.6***	79.5***	5.92
20	- 67.4***	73.3***	5.92
25	- 71.4***	77.3***	5.92
30	- 77.6***	83.4***	5.82

Table 1: Regression results from estimating equation (2).

Notes: This table reports the regression results from estimating equation (2). * denotes p< 0.1 and *** denotes p<0.01.

Figure 5 shows the observed interest rates for the policy rate cuts at the DLB compared to the average pass-through (and its associated confidence interval) when the policy rate is in positive territory. Each black dot captures the average pass-through k days after the repo-rate cut, and corresponds to the numbers in the second column of Table 1. The gray area captures a 95 percent confidence interval around the estimates. The red and blue lines depict average bank lending rates for the two final policy rate cuts, i.e. the policy changes which occur in the post-bound period. In both cases, there is a slight increase in bank lending rates, in stark contrast to the normal pass-though.[14]

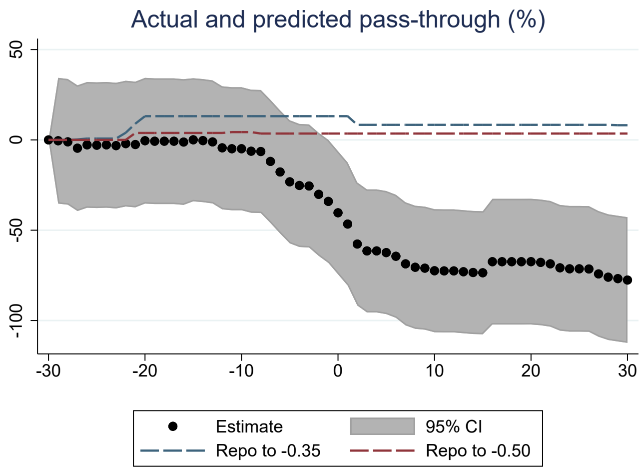

Figure 5: Event study on bank level mortgage rates.

Notes: This figure compares the actual and predicted pass-through for the two last policy rate cuts in our sample (repo rare reduced to -0.35 and repo reduced to -0.5). The interest rates are weighted average of 3 months, 1 year, 3 year and 5 year fixed interest rate period contracts. All twelve interest rate cuts in the period 2009 - 2015m4 are used to estimate the average pass-through and the corresponding confidence interval.

One important question is whether the evolution of the listed rates used here provides an accurate picture of the evolution of the *actual* rates borrowers face. For instance, Erikson and Vestin (2019) makes the point that observed lending rates - in contrast to listed rates - fell in 2017 and 2018 and attribute this to policy rate cuts in 2015 and 2016. In Appendix A, we aggregate the data and show that there are minimal differences between the implied aggregate listed rate and the observed actual rates for the sample period. Moreover, we show that the reductions in observed lending rates in 2017 and 2018 and the associated divergence between listed and actual rates is most likely due to macroprudential regulation ultimately affecting the pool of borrowers, rather than a (much delayed) response to policy rate cuts.

Cross-sectional correlation with deposit shares

In addition to a decline in the average pass-through, there is an increase in dispersion once the deposit rate has reached its lower bound. Moreover, this dispersion appears connected with the differential reliance of banks on deposit financing. To illustrate this, we estimate the following regression

$$\text{Pass through}_{ir} = \alpha + \text{Deposit share}_i + \epsilon_{ir} \tag{3}$$

where pass-through is defined as before, i denotes the bank and r indexes one of the last two repo-rate cuts. Data on deposit shares are from Statistics Sweden. We use deposit shares as of 2014, in order to not capture any changes in deposit shares in response to the policy rate cuts.[15] Observations are weighted by bank

[14] In Appendix Figure 19 we include plots similar to Figure 5 for mortgage contracts with fixed interest rate periods of 3 months, 1 year, 3 years and 5 years separately. They all show a collapse in pass-through once the deposit rate has reached its lower bound.

[15] This data is available for nine of the banks in the sample.

size. In the baseline regression, the deposit share is a continuous variable. As an alternative approach, we also include a specification where the treatment variable is an indicator variable for whether bank i has a deposit share above the sample median.

The regression results are reported in Table 2. They indicate a statistically significant negative correlation between deposit shares and pass-through, despite the low number of observations. From the first column, we see that a ten percentage point increase in the deposit share is associated with a decrease in pass-through of ten percentage points.[16]

	(1) Pass-through	(2) Pass-through
Deposit share	-0.957**	-12.49
	(-2.15)	(-1.68)
Deposit share variable:	continuous	high or low
N	18	18

Table 2: Pass-through and deposit shares.

Notes: This table shows regression results from estimating equation (3). t statistics in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure 6 illustrates the negative correlation between pass-through and deposit share, which suggests that banks which are more reliant on deposit financing are less willing to reduce their lending rates once the DLB is reached. Because the pass-through to deposit rates has been smaller than the pass-through to other financing sources, banks which rely heavily on deposit financing are likely to experience smaller reductions in their total funding costs once rates turn negative, a key feature of the model we present in section 2.

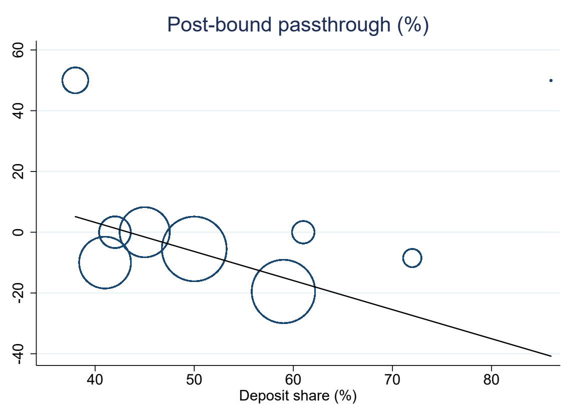

Figure 6: Pass-through and deposit shares.

Notes: This figure shows the average pass-through in the post-bound period as a function of 2014 deposit shares. Size of circles represent bank size. The line is a linear regression of bank-level pass-through on deposit share, where each bank is weighted by size.

[16] When using only an indicator variable for having low or high deposits in the second column, the regression coefficient becomes borderline insignificant. However, the coefficient estimate suggests that on average, banks with high deposit reliance have twelve percentage points lower pass-through than banks with low deposit reliance.

Identification We interpret the above event study to capture the impact of policy rate cuts on lending rates. However, one might worry about potential confounding factors. As we use daily data it seems unlikely that macroeconomic developments should influence our pass-through estimates. However, other monetary policy announcements, such as quantitative easing (QE) or forward guidance, could potentially affect our findings.

The Riksbank adopted QE and negative rates simultaneously in February 2015. Quantitative easing was subsequently increased in March 2015 and October 2015, when the policy rate was reduced further to -0.25 and -0.35 percent respectively. There was no further QE announcements related to the last policy rate cut to -0.50 percent in February 2016. Could quantitative easing explain the breakdown in pass-through? First, we note that the intention behind QE would be to lower lending rates. Hence, this policy would tend to work against our results. Second, the timing of QE in Sweden does not coincide with the DLP, and hence with the collapse in pass-through to lending rates. While quantitative easing was in place for three of the four policy rate cuts in negative territory, the DLP was binding for the two last policy rate cuts only. The collapse in pass-through to lending rates therefore, does not coincide with any changes to QE policies.

Another potential confounding factor is forward guidance and/or information about underlying economic conditions. We find it plausible that the introduction of negative rates may have had an element of forward guidance, as well as some signaling value about economic conditions. However, in order to explain the breakdown in pass-through to lending rates, this (negative) effect must have materialized not when negative rates were initially implemented, but rather for the two last policy rate cuts only. That is, reducing the interest rate to -0.35 and -0.50 percent must have entailed a substantially worse signal about economic conditions than reducing the policy rate to -0.10 and -0.25 percent. Moreover, it is unclear why this would generate a correlation between pass-through and deposit shares. Hence, it seems more plausible that the breakdown in pass-through is linked to the DLP becoming binding for the last two policy rate cuts.

To summarize, our daily data and the lack of confounding factors which occurred simultaneously with the DLP becoming binding makes us confident that the event study identifies the collapse in pass-through resulting from the deposit rate no longer responding to further policy rate cuts.

2.3.2 Bank lending volumes

As we have documented a negative relationship between lending rate pass-through and deposit shares, we would expect banks with higher deposit shares to also have lower lending growth once the DLB is binding. Monthly lending volumes for household lending are available at the bank-level from Statistics Sweden, and we focus on the period from January 2014 to December 2017. We restrict our sample to only include the banks used in the previous analysis, and run the following regression

$$\Delta \log(\text{lending}_{it}) = \alpha_i + \delta_t + \beta_t \text{ Deposit share}_i \times I_t^{DLB} + \epsilon_{it} \tag{4}$$

where $\Delta \log(\text{lending}_{it})$ is the 3-month percentage growth in lending rates for bank i at the monthly date t, α_i captures bank fixed effects to control for bank specific factors and δ_t captures time fixed effects to control for factors common to all banks. Standard errors are clustered at the bank level and observations are weighted by bank size.

The regression results, reported in Table 3, suggest that banks with higher deposit shares had lower lending growth once the DLB became binding. The first column says that a ten percentage points increase in the deposit share, say from 50 to 60 percent, reduces lending growth by on average 2.1 percentage points. This compares to a mean lending growth of 2.2 percent in 2014. The second column compares banks with above or below median deposit shares, and says that banks with above median deposit shares on average have 3.2 percentage points lower lending growth than banks with below median deposit shares in the post-bound period. Hence, we conclude that not only are banks which rely more heavily on deposit financing likely to increase mortgage rates in response to policy rate cuts once the DLB is reached, they also experience lower growth in household lending volumes.

	(1) Δ log(loans)	(2) Δ log(loans)
Post × deposit share	-0.217**	
	(0.0955)	
Post × deposit share		-3.235**
		(1.184)
N	361	361
No. of clusters	10	10
Mean of dependent variable	2.231	2.231
SD of dependent variable	7.052	7.052
Bank FE	Yes	Yes
Time FE	Yes	Yes
measure	Cont.	High or low

Table 3: Regression results from estimating equation (4).

Notes: * p<0.1, ** p<0.05, ***p<0.01. Post = 1 for the period when the repo-rate is below -0.25, and zero otherwise. Standard errors clustered at the bank level. Summary statistics taken over whole sample period (2014 - 2017).

A potential concern is that the estimated coefficients in Table 3 reflect structural differences in lending growth across banks with different deposit shares, and as such is not related to policy rate cuts at the DLB. To ensure that this is not the case, we conduct two falsification tests where we investigate the lending response of banks for earlier periods with policy rate reductions, including the initial cut into negative territory. The results are reported in Table 6 in Appendix B. Reassuringly, the deposit share is only informative about the response of bank lending to policy rate cuts once the DLB is binding.

2.4 Bank profits

Finally, we investigate how negative policy rates affect bank net worth. We perform a simple event study using data on stock prices in order to investigate whether market participants consider negative policy rates as good or bad news for bank profitability. Specifically, we use daily stock market data on four publicly listed Swedish banks.[17] We compare the excess return on these stocks relative to the return on the main index (OMX 30) in a window around policy rate announcement days.

[17]The four banks are Nordea, SEB, Handelsbanken and Swedbank, and the stock market data can be found at the Nordic Nasdaq. There are three other publicly listed banks or credit companies on the Swedish stock exchange (Arion Banki, Avanza and TF Bank), but the combined market share of these three banks is virtually zero, and so we drop them from our sample.

To formalize this we run the regression specified in equation (5), in which we define the excess return on bank stock i as $\text{Excess Return}_{it} = [\log(\text{Stock price})_{i,t} - \log(\text{Stock price})_{i,t-1}] \times 100 - [\log(\text{Stock price})_t^{\text{index}} - \log(\text{Stock price})_{t-1}^{\text{index}}] \times 100$.

$$\text{Excess Return}_{it} = \alpha + \gamma_k \text{ Event Day}_k + \beta_k \text{ Event Day}_k \times I_t^{\text{post-zero}} + \epsilon_{it} \tag{5}$$

The $\hat{\beta}_k$ coefficients capture the differential effect on bank stock excess returns in the post-zero period and are plotted in Figure 7. On the announcement day, bank stocks on average have an excess return of almost -1.5 percentage points in the post-zero period relative to normal times.[18] This negative effect is statistically significant at the five percent level, despite our small sample size. We thus conclude that policy rate cuts into negative territory have a detrimental impact on the excess return on Swedish bank stocks, suggesting that market participants view them as bad news for the profitability of Swedish banks.

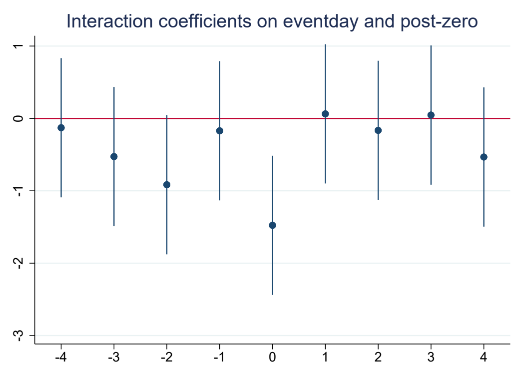

Figure 7: Excess stock return and negative policy rates.

Notes: This figure shows coefficient estimates $\hat{\beta}_k$ from estimating equation (5). The coefficient estimates can be interpreted as the relative excess return of bank stocks in the post-zero period in a window around policy rate cut announcement days ($k = 0$). Vertical bars correspond to 95 % confidence intervals.

3 Theoretical analysis

We now move on to construct a model of bank lending with negative policy rates based on the preceding empirical analysis. We first consider a partial equilibrium model where all interest rates are exogenous. This setting allows us to clarify a number of issues, including the existence of a *policy rate bound* (PLB) and how the presence of a deposit rate bound (DLB) affects the transmission of monetary policy also in absence of a lower bound on the policy rate. We then embed the partial equilibrium model into a general equilibrium framework in order to close the model and endogenize all interest rates. To conserve space, much of the derivation of the model is relegated to Appendix C.

[18] The significant negative effect on the announcement day is robust to replacing $I^{post-zero}$ with I^{DLB}, but the results become more noisy as we then only have 4 banks and 2 policy rate reductions in the post-period. Moreover, simply looking at the excess returns for every policy rate cut seems to suggest that the negative excess returns materializes once the policy rate becomes negative.

3.1 Bank lending and negative policy rates in partial equilibrium

Consider a perfectly competitive risk-neutral bank that maximizes the discounted present value of dividends

$$\max E_t \sum_{t=0}^{\infty} \delta^t DIV_t \tag{6}$$

where DIV_t is dividends and $0 < \delta < 1$ is a discount factor. The bank maximizes the path of dividends subject to the following flow budget constraint:

$$\begin{aligned} & DIV_t + B_t + R_t + A_t + M_t - D_t - F_t \\ = {} & (1 + i^b_{t-1})B_{t-1} + (1 + i^r_{t-1})R_{t-1} + (1 + i^a_{t-1})A_{t-1} \\ & + M_{t-1} - (1 + i^d_{t-1})D_{t-1} - (1 + i^f_{t-1})F_{t-1} \\ & - C(F_t, L_t, N_t) - \Psi(R_t, M_t) - \Gamma(B_t, N_t) - S(M_t) \end{aligned} \tag{7}$$

following the notation in Figure 1, i.e. B_t is loans the bank extends with interest i^b_t, R_t is reserves with interest i^r_t, M_t is paper currency which pays a zero return and A_t is "other" liquid assets with interest i^a_t. To finance its asset holdings, the bank raises funds via deposit-like financing D_t at an interest cost of i^d_t, and via other funds F_t at an interest cost of i^f_t. Throughout the analysis, D_t will capture funding that is subject to a lower bound, whereas F_t captures all other funding which in principle is not subject to a (strict) lower bound.

The net worth of the bank, N_t, is defined as:

$$N_t \equiv (1 + i^b_t)B_t + (1 + i^r_t)R_t + (1 + i^a_t)A_t + M_t - (1 + i^d_t)D_t - (1 + i^f_t)F_t \tag{8}$$

while liquid assets are defined as

$$L_t \equiv R_t + M_t + A_t \tag{9}$$

The bank can hold non-loan assets only in positive quantities, i.e. we impose the non-negativity constraints

$$A_t \geq 0, R_t \geq 0, M_t \geq 0. \tag{10}$$

Bank intermediation costs are captured by the four functions $C(.)$,$\Psi(.)$, $\Gamma(.)$ and $S(.)$ corresponding to different aspects of the bank's intermediation process. Since the model is solved using a log-linear approximation, only assumptions about the elasticity of each function with respect to their arguments are needed to characterize the dynamics.

In line with the banking literature, e.g. Freixas and Rochet (2008), we assume that banks hold liquid assets to insure against liquidity risk. Liquidity risk arises because loans B_t are illiquid while external financing F_t is subject to refinancing risk. The cost of liquidity risk is captured by the function $C(L_t, F_t, N_t)$.[19] To capture the costs associated with liquidity risk and banks' insurance motive in a reduced-form way, we assume that $\frac{\partial C}{\partial F} > 0$, $\frac{\partial C}{\partial N} < 0$ and $\frac{\partial C}{\partial L} \leq 0$. Liquid assets decreases the costs associated with liquidity risk up until a satiation point $L^*(N_t, F_t)$ and has zero impact for $L > L^*$. Away from this satiation point, the elasticity of C with respect to F is $\gamma_f \geq 0$ and the negative of the elasticities of C with respect to L and N are denoted by

[19]See Freixas and Rochet (2008) for micro foundations of a cost function that captures liquidity risk.

$\gamma_l \geq 0$ and $\gamma_n \geq 0$.[20] We assume that the cost associated with liquidity risk is decreasing in net worth.[21]

While all liquid assets reduce liquidity risk, reserves and paper currency also contribute to reducing banks operational costs due their special "money role," e.g. in settling inter-bank transactions and servicing the public's demand for cash. We capture this by the function $\Psi(.)$. For simplicity, money and reserves are perfect substitutes for the purpose of liquidity provision.[22] Define the monetary base $MB_t \equiv R_t + M_t$. We assume that $\Psi'(MB_t) \leq 0$ up to a satiation point - denoted MB^* - and $\Psi'(MB_t) = 0$ for $MB_t > MB^*$. The negative of the elasticity of the banks intermediation function with respect to MB_t, when $MB_t < MB^*$, is denoted $\gamma_{mb} \geq 0$.[23]

Holding money involves a cost captured by $S(M_t)$, with $S'(M_t) > 0$. A simple interpretation of this cost is that it involves the storage costs of money. More generally it should be thought of as the additional cost of using paper currency relative to reserves in financial intermediation. The elasticity of the storage costs with respect to M is denoted γ_{ms}.[24]. As holding currency is costly, while holding reserves is not, banks choose not to hold currency whenever the interest on reserves is positive. It may, however, choose to increase its currency holdings once the reserve rate becomes negative. In this case the bank needs to consider storage costs of holding cash as discussed in section 1.[25]

The function $\Gamma(B_t, N_t)$ captures the cost of lending. We assume that $\frac{\partial \Gamma}{B} > 0$, $\frac{\partial \Gamma}{\partial N} < 0$ and $\frac{\partial^2 \Gamma}{\partial B \partial N} < 0$. There is a variety of ways to micro-found these assumptions. The simplest is through regulatory capital requirements. [26] One simple interpretation of why intermediation costs are strictly increasing in lending is that they arise due to borrower default, see Curdia and Woodford (2011), e.g. due to capacity constraints in loan monitoring. The elasticity of this cost function with respect to lending is denoted by ν, while the negative of the elasticity of lending with respect to net worth is ι.[27]

We assume that the bank pays out a fixed fraction ω of its $t-1$ net worth in dividends.[28] The flow

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[20]Thus $\gamma_f \equiv \frac{\partial C}{\partial F}\frac{F}{C}$, $\gamma_N \equiv -\frac{\partial C}{\partial N}\frac{N}{C}$,$\gamma_L \equiv -\frac{\partial C}{\partial L}\frac{L}{C}$.

[21]This assumption is only important for Proposition 5 in the appendix.

[22]This simplification is not as restrictive as it may seem, as any relative disadvantage of cash to reserves can be captured by a storage cost function S().

[23]Thus $\gamma_{mb} \equiv \frac{\partial \Psi}{\partial MB}\frac{MB}{\Psi}$.

[24]Thus $\gamma_{ms} = \frac{\partial S}{\partial M}\frac{\bar{M}}{\bar{S}}$.

[25]Assuming that banks do not hold cash is a harmless abstraction as vault cash is a trivial component of banks balances. As currency is an asset with a zero return, however, explicitly accounting for it is critical when thinking about the central banks ability to charge negative interest rates on reserve balances since banks can always substitute reserves for cash – giving rise to the PLB

[26]The simplest interpretation of the role of net worth is that it comes about due to a capital requirement, often modeled as

$$\frac{N_t}{B_t} \geq \kappa$$

where κ is a parameter. A constraint of this form would be a limiting case of the smooth function assumed here, one in which the cost of intermediation goes to infinity if the constraint is breached. The dependence of lending costs on net worth is micro-founded in Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010), as well as documented empirically, for example, in Jimenez, Ongena, Peydro, and Saurina (2012).

[27]Thus, $\nu \equiv \frac{\partial \Gamma}{\partial B}\frac{B}{\Gamma}$ and
$\iota \equiv -\frac{\partial \Gamma}{\partial N}\frac{N}{\Gamma}$.

[28]A common specification in the literature, see e.g. Curdia and Woodford (2011), assumes that dividends are fully paid out within each period. In that case, net worth is always zero. The generalization considered here is important for our application since negative

constraint can then be simplified to[29]

$$\frac{1}{1+i_t^d}N_t = (1-\omega)N_{t-1} + Z_t \tag{11}$$

where Z_t is the profit of the bank, i.e.

$$Z_t \equiv \frac{i_t^b - i_t^d}{1+i_t^d}B_t + \frac{i_t^r - i_t^d}{1+i_t^d}R_t + \frac{i_t^a - i_t^d}{1+i_t^d}A_t - \frac{i_t^d}{1+i_t^d}M_t - \frac{i_t^f - i_t^d}{1+i_t^d}F_t - C(F_t, R_t, A_t, N_t) - \Gamma(B_t, N_t) - \Psi(R_t, M_t) - S(M_t) \tag{12}$$

Equation (11) says that net worth today depends on the retained earnings from last period plus the current profits of the bank.

Denoting the value of the bank at time t by $V(N_{t-1})$, the bank's problem is

$$V(N_{t-1}) = \max_{B_t, R_t, A_t, M_t, F_t} (\omega N_{t-1} + \gamma V(N_t))$$

s.t. (10) and (11). The full details of the solution to the bank's problem is included in Appendix C. In what follows, we characterize the key components.

The first order condition that determines optimal bank lending is:

$$\underbrace{\frac{i_t^b - i_t^d}{1+i_t^d}}_{\text{Marginal benefit of lending}} = \underbrace{\Gamma_B(B_t, N_t)}_{\text{Marginal cost of lending}} \tag{13}$$

This condition is the key equation of the model because it determines the lending activities of the banking sector. The left hand side denotes the marginal benefit of lending, given by the spread between the rate the bank obtains from extending loans (which it takes as given in partial equilibrium), and the interest on deposits, i_t^d.

Importantly, despite the presence of another source of external financing, the deposit rate captures the marginal cost of funding for banks. The reason is that external financing carries refinancing risk. In equilibrium, at any interior solution, the bank is indifferent between D_t and F_t. Put differently, the bank equates the marginal cost of deposits i_t^d with the marginal cost of external financing, i.e. $i_t^f + \frac{\partial C}{\partial F}$. Hence, a reduction in i_t^f relative to i_t^d leads to a shift in financing from deposits D_t to external financing F_t, while keeping the marginal cost of funding fixed at i_t^d.[30] The right hand side reflects the marginal cost of lending which is increasing in B.[31]

policy rates can have a negative effect on net worth, and via this channel, lending.

[29]To obtain this characterization use the expression for net worth 8 and solve for D_t and substitute for D_t in the flow constraint.

[30]Note that, even though marginal costs of the two funding sources are equalized, this does not mean that the presence of external financing does not affect bank outcomes. If banks shift from D to F, for instance when the policy rate is negative and the rate on D is at the DLB, the overall profitability of the bank is affected.

[31]An alternative to the set-up considered here would be to assume that banks engage in monopolistic competition in lending markets, as in for instance Ulate (2019). In such a setting, loans would not be priced according to marginal costs but rather set as a mark-up over the interest rate on liquid assets plus any interaction between lending and net worth, as for instance captured by a capital constraint. If the feedback from net worth to lending is small, this would imply that lending rates could fall when the policy rate is reduced, even though the deposit rate is at the DLB. Note, however, that such a scenario is not consistent with the empirical evidence in section 2. The empirical evidence in section 2 is, however, consistent with monopolistic competition and a *strong* feedback from net worth to lending rates. For lending and ultimately aggregate demand, such a case is isomorphic to what we consider here.

Before moving on to analyzing the effect of changes to the policy rate, it is worth asking: Is there a bound on how low the policy rate can go? The next section addresses this question.

3.1.1 The policy rate bound

The policy rate bound ("PLB") arises in the model because the banks have the option of exchanging reserves for paper currency. The PLB thus depends heavily on the cost for the banks of holding paper currency relatively to reserves, captured by the function $S(.)$. To understand how it emerges, it is helpful to consider the following specification

$$S(M_t) = \alpha^m M_t. \tag{14}$$

Suppose that the DLB is zero. In this case Proposition 1 defines the policy rate bound.

Proposition 1. *(The Policy Rate Bound) If the cost of using paper currency is given by equation* (14)*, the DLB is zero, then the lower bound on the policy rate is*

$$i_t^r \geq i^{PLB} \equiv -\alpha^m \tag{15}$$

The proof of this proposition follows directly from the first order conditions of the banking problem, and shown in Appendix C.2. Since banks value the services central bank reserves provide, the central bank can charge a negative interest rate on reserves. However, money also provides these services, but at a higher relative cost captured by $S(M_t) = \alpha^m M_t$. If the central bank charges a too high price for the service provided by reserves, the banks withdraw their reserves in favor of paper currency, using it to insure against liquidity risk and settle interbank transactions outside of the central bank. The cost function in equation (14) is proportional, so that if the interest rate charged on reserves (e.g. -2 percent) is lower than the negative of the marginal cost of storing cash captured by $-\alpha^m$ (e.g. -1.5 percent), the bank will withdraw all reserves. If the marginal storage cost is increasing, then as the central bank lowers the reserve rate, banks convert reserves into cash. If the cost of holding cash increases without a bound, there is in principle no PLB.

Empirically, there has been somewhat limited substitution from reserves to cash, see Figure 20 in the appendix. It is therefore to assume that the PLB is below rates seen so far. We therefore proceed under the assumption that the PLB does is non-binding.

3.1.2 The effect of policy rate cuts

We now consider the effect of policy rate cuts, under the assumption that the PLB is not binding, and keeping the lending rate, i_t^b fixed. Let us suppose that i_t^d, i_t^a and i_t^f , are functions of the policy rate i_t^r, with reduced form pass-through coefficients ρ^d, ρ^a and ρ^f respectively. If $\rho^i = 1$ there is full pass-through to the interest rate i, while $\rho^i = 0$ indicates zero pass-through. For instance, if the deposit rate is not responding to further policy rate cuts due to the DLB, then $\rho^d = 0$. In general equilibrium, the pass-through of the policy rate to all rates is determined endogenously. Yet, most of the insights can be illustrated in partial equilibrium.

We use a log-linear approximation of the model around a steady state in which there is full pass-through so that $\bar{\imath}^r = \bar{\imath}^d = \bar{\imath}^f = \bar{\imath}^a$, using bars to denote steady state values. The full set of log-linearized equations are listed in Appendix C. The following proposition highlights a useful observation.

Proposition 2. *(Full pass-through and liquidity satiation) If there is full pass-through in steady state, i.e.* $\bar{\imath}^r = \bar{\imath}^d = \bar{\imath}^f = \bar{\imath}^a$*, then the bank is satiated in liquid assets* ($L = L^*$) *and holds no paper currency* ($M = 0$).

The proof of this proposition follows directly from the first order condition of the banks problem, i.e. equations (54)-(56) in Appendix C. Together, these equations imply that if $i^f = i^a = i^d = i^r$, then $C_L = C_F = \Psi_R = 0$, i.e. the bank is satiated in liquid assets. The steady state values for external financing $\overline{F}$, other liquid assets $\overline{A}$ and central bank reserves $\overline{R}$ are then implicitly defined by equations (54)-(56) in Appendix C. Because paper currency and reserves serve the same role, but money generates storage costs, banks choose to hold no paper currency.

We now move on to analyze the impact of negative policy rates on bank lending in partial equilibrium, and how it depends on the transmission of the policy rate i_t^r to other interest rates, i.e. the pass-through coefficients ρ^i for $i \in \{d, a, f\}$.

A log-linear approximation of (11) and (13) around the steady state yields

$$\hat{B}_t = \frac{1}{\nu - 1}\left(\frac{1 + \bar{\imath}^b}{\bar{\imath}^b - \bar{\imath}^d}\right)(\hat{\imath}_t^b - \hat{\imath}_t^d) + \frac{\iota}{\nu - 1}\hat{N}_t \tag{16}$$

$$\hat{N}_t = \frac{1 + \bar{\imath}^d}{\Theta}\{(1 - \omega)\hat{N}_{t-1} + \frac{\bar{B}}{\bar{N}}\hat{\imath}_t^b + \Omega_\rho \hat{\imath}_t^r\} \tag{17}$$

where $\hat{N}_t \equiv \log N_t / \log \bar{N}$, $\hat{B}_t \equiv \log B_t / \log \bar{B}$, $\hat{\imath}_t^d \equiv \log(1 + i_t^d)/(1 + \bar{\imath}^d)$, $\hat{\imath}_t^b \equiv \log(1 + i_t^b)/(1 + \bar{\imath}^b)$, and $\Theta \equiv 1 - \frac{\iota}{\nu}(\bar{\imath}^b - \bar{\imath}^d)\frac{\bar{B}}{\bar{N}} > 0$. The key coefficient is Ω_ρ, discussed in detail below.

In the remaining analysis, we make the following assumption

$$\frac{\iota}{\nu} < \frac{(1 - \omega)(1 + \bar{\imath}^d)}{\bar{\imath}^b - \bar{\imath}^d}\frac{\bar{N}}{\bar{B}} \tag{18}$$

which says that the feedback effect from changes to net worth to lending is not too strong, which is a necessary condition for the approximate solution to have a unique bounded solution. Observe that it implies that $\Theta > 0$.

An approximated partial equilibrium can be defined as a collection of processes for $\{\hat{N}_t, \hat{B}_t\}$ which solve equations (16) and (17) given an exogenous sequence for $\{\hat{\imath}_t^r, \hat{\imath}_t^d, \hat{\imath}_t^f, \hat{\imath}_t^a, \hat{\imath}_t^b\}$. An important observation is that there is no first-order impact of changes in non-loan asset holdings on the bank's lending decision determined by equation (16). Yet, as will be clear below, the steady state values of these variables are important via their impact on net worth through Ω_ρ.

The first term in equation (16) captures that bank lending is an increasing function of the spread between borrowing and deposit rates. Since $i_t^d = \rho^d i_t^r$, a lower policy rate translates into lower funding costs for the banks in proportion to the coefficient ρ^d. If $\rho^d = 0$, lower policy rates are not translated into lower deposit rates and thus the major financing cost channel of bank lending is unaffected by the decline in the policy rate. The second term shows that lending is increasing in net worth. This follows directly from the assumption that intermediation costs depend negatively on net worth. The strength of this force depends on both elasticities ι and ν .i.e. by how much intermediation costs change with higher net worth and increased lending. To understand the total effect of a policy rate cut on bank lending, we thus need to understand the effect on net worth.

Equation (17) summarizes the evolution of net worth as a function of past net worth, the lending rate and the policy rate. Net worth depends positively on its own lagged value as well as the lending rate. What about the effect of the policy rate on net worth? The key coefficient is Ω_ρ, defined as

$$\Omega_\rho \equiv \frac{\bar{A}}{\bar{N}}(\rho^a - \rho^d) + \frac{\bar{R}}{\bar{N}}(1 - \rho^d) - (\rho^f - \rho^d)\frac{\bar{F}}{\bar{N}} - \rho^d\left(\frac{1 + \bar{\iota}^b}{1 + \bar{\iota}^d}\frac{\bar{B}}{\bar{N}} - \frac{1}{1 + \bar{\iota}^d}\right) \quad (19)$$

This coefficient summarizes how changes in policy rates are translated into net worth through each component of the bank's balance sheet. Consider the term $\frac{\bar{A}}{\bar{N}}(\rho^a - \rho^d)$, for instance. If the policy rate cut changes the interest on deposits by more than the interest on liquid assets, i.e. $\rho^a < \rho^d$, then a policy rate cut increases the net interest margin on liquid asset holdings, which in turn contributes to increasing the net worth of the bank. The second term reflects this balance sheet effect due to reserves, the third term due to external financing and the final term due to bank lending.

Using equations (16) and (17), it is straightforward to show that the effect of a policy rate cut on lending at time t is

$$\frac{\partial \hat{B}_t}{\partial \hat{\iota}_t^r} = \underbrace{-\rho^d \frac{1}{\nu - 1}\left(\frac{1 + \bar{\iota}^b}{\bar{\iota}^b - \bar{\iota}^d}\right)}_{\text{Marginal funding cost channel}} + \underbrace{\frac{1 + \bar{\iota}^d}{\Theta}\frac{\iota}{\nu - 1}\Omega_\rho}_{\text{Bank capital channel}} \quad (20)$$

which then continues to affect future borrowing at time $t + j$ via the effect on net worth in future periods. In our model, policy rate cuts can therefore stimulate lending via two channels. The first channel arises because lower policy rates lower the marginal funding cost of banks. This impact relies on the pass-through to deposit rates ρ^d. Once there is no pass-through to deposit rates, this mechanism shuts down, as long as the bank remains indifferent between deposits and other sources of financing. However, policy rate cuts can still increase lending via a bank capital channel captured by Ω_ρ. If lower policy rates expand bank profitability, higher net worth will induce banks to lend more even if the deposit rate is unchanged.

The next proposition considers the case when there is full pass-through to all interest rates.

Proposition 3. *(Policy rate changes with full pass-through). Suppose that there is full pass-through to all interest rates so that $\rho^a = \rho^d = \rho^f = 1$. Then a reduction in $\hat{\iota}_t^r$ increases bank lending, i.e.* $\frac{\partial \hat{B}_t}{\partial \hat{\iota}_t^r} < 0$

To prove this proposition, note that for this special case, $\Omega_\rho = \frac{1}{1+\bar{\iota}^d} - \frac{1+\bar{\iota}^b}{1+\bar{\iota}^d}\frac{\bar{B}}{\bar{N}} < 0$. Hence, when there is full pass through, the effect of policy rate cuts on bank lending is unambiguous.

This conclusion of the last proposition changes when there is no pass-through to the deposit rate, i.e. $\rho^d = 0$. This corresponds to the case in which the DLB is binding, i.e. once policy rate reductions move far enough into negative territory. In this case the marginal financing cost of the bank is no longer affected. Whether or not a policy rate cut stimulates lending then depends on how net worth is affected.

Proposition 4. *(Policy rate cut when DLB is binding). Suppose there is no pass-through to deposit rates so that $\rho^d = 0$. Then a reduction in $\hat{\iota}_t^r$ increases lending if*

$$\rho^a \bar{A} + \bar{R} < \rho^f \bar{F} \quad (21)$$

and decreases lending otherwise.

To prove this note that in this case (19) becomes

$$\Omega_\rho = \frac{\bar{A}}{\bar{N}}\rho^a + \frac{\bar{R}}{\bar{N}} - \rho^f \frac{\bar{F}}{\bar{N}}$$

The condition for when interest rate cuts at the DLB have a positive effect on net worth is then given by equation (21). An important feature of Proposition 4, is that equation (21) is a sufficient statistic for the sign of the bank lending channel when deposit rates hits the DLB. Put differently, whether policy rate cuts are expansionary or not does not depend on the numerical values for all of the parameters of the banking model, but only on the sign of Ω_ρ. The quantitative effect, however, will in general depend upon all of the structural parameters.

3.1.3 Implications for external financing and liquid assets

To conserve space, we relegate to Appendix C a detailed discussion of the implication of negative policy rates once the DLB is binding on liquid assets and external financing. Below we summarize the key conclusions.

One implication is that for the equilibrium to be consistent with the banks maximization problem, there has to be full pass-through of the policy rate to liquid assets. The reason is that banks can use reserves to buy liquid assets from other banks. Accordingly, in equilibrium they must be indifferent between the two. This is consistent with the evidence in Figure 2, which indicates that the rates on highly liquid assets, such as government bonds, follow the policy rate essentially one to one even as the policy rate turns negative. Because the banks are indifferent between their holding of liquid assets and reserves, the actual quantity of reserves in equilibrium is indeterminate. The model also clarifies, why, even if the rates on external financing follow the policy rate into negative territory, banks may choose not to rely more heavily on it. As discussed above, the reason is that the interest rate, i_t^f, does not reflect the true marginal cost of financing. Instead, this form of financing is costly beyond the interest rate i_t^f, due to liquidity risk against which the bank needs to insure itself by holding liquid assets, which may carry negative returns. Finally, the model can be used to derive the demand for M_t as a function of the policy rate and the elasticities γ_m ad γ_R.

3.2 General equilibrium model of banking with negative policy rates

In this section, we outline the remaining blocks needed to close the model. We then provide an analytical characterization of the model.

3.2.1 Model set-up

In this section, we outline the remaining components of the model. We assume that there are "patient" households which save and "impatient" households which borrow, intermediated by banks. Nominal frictions enter via staggered price setting by firms as in Calvo (1983). The way households and firms are modeled follows closely Benigno et al. (2014). Accordingly, the exposition of this aspect of the model is brief. Most derivations, such as first order conditions, are relegated to the appendix.

Households

Borrowers (b) make up a fraction χ of households, while savers (s) make up the remaining share $1-\chi$.[32] Households have at time 0 a utility function of the form

$$\mathcal{U}_t^j = \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta^j)^t \left[U(C_t^j) + m\left(\frac{M_t^j}{P_t}\right) - \frac{(N_t^j)^{1+\eta}}{1+\eta} \right] \zeta_t \text{ with } j = s \text{ or } b \tag{22}$$

where E_t is the expectation operator, β^j is the discount factor with $0 \leq \beta^b \leq \beta^s < 1$, ζ_t is a preference shock and C_t^j is aggregate consumption

$$C_t^j \equiv \left[\int_0^1 C_t^j(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

where $C_t^j(i)$ is the consumption of good of variety i, $\theta > 1$ is the intratemporal elasticity of substitution between goods, N_t^j is hours worked and $\eta \geq 0$ is a parameter. To facilitate aggregation, the utility function for consumption is assumed to be $U(C_t^j) = 1 - \exp(-qC_t^j)$ where q is a parameter. The function $m\left(\frac{M_t^j}{P_t}\right)$ represents the utility of holding real money balances, which increases in real money balances up to a satiation point $\frac{M_t}{P_t} = m^*$, after which $m' = 0$.

The households' budget constraint is:

$$\begin{aligned} M_t^j - B_t^j &= W_t^j N_t^j - B_{t-1}^j \left(1 + i_{t-1}^j\right) + M_{t-1}^j \\ &\quad - P_t S_h\left(\frac{M_t^j}{P_t}\right) - P_t C_t^j + \Psi_t^j + \psi_t^j - T_t^j - F_t^j \end{aligned} \tag{23}$$

where B_t^j denotes a one period banking contract j. $B_t^b > 0$ is debt with interest rate i_t^b, while $B_t^s < 0$ is bank deposits with interest rate i_t^d. $S^h(\frac{M_{t-1}}{P_t})$ captures the storage costs of holding money, measured in units of the consumption good, Ψ_t^j is firms profits, and ψ_t^j is bank dividends. Firm profits are distributed to both household types based on their population shares while only savers receive bank dividends. F_t^j represents exogenous pension payments of each agent. This pension is the source of external funds on the bank's balance sheet.

The household maximize (22) subject to (23). This leads to the standard consumption Euler equations for each household type, an optimal labor supply condition and transversality conditions, all of which are relatively standard and reported in the appendix. In addition, the households rule for optimal holdings of money gives rise to the lower bound on the deposit rate.

The deposit lower bound Consider the optimal money holdings of the saver. If $m_t \leq \bar{m}$ then money demand is given by:

$$\frac{m'\left(\frac{M_t^s}{P_t}\right)}{U'(C_t^s)} = \frac{i_t^d}{1+i_t^d} - S'\left(\frac{M_t^s}{P_t}\right) \tag{24}$$

[32] We normalize the number of agents to 1.

while if $m_t \geq \bar{m}$ the household is satiated in money and

$$S'(\frac{M_t^s}{P_t}) = \frac{i_t^d}{1 + i_t^d} \tag{25}$$

Equation (25) generates the bound on deposit rates.

The simplest formulation of a storage costs is that it is proportional to money holdings, i.e. $S(\frac{M_t^s}{P_t}) = \alpha^H \frac{M_t}{P_t}$ for some $\alpha^H > 0$. In this case condition (25) implies

$$i_t^d = \frac{\alpha^H}{1 - \alpha^H} \equiv i^{DLB}$$

which represents the effective lower bound on deposit rates for small depositors.

A straight forward interpretation of why banks have not been willing to impose negative rates on deposits is that the marginal storage cost α^H is small for small depositors, i.e. $\alpha^H \approx 0$, and that for regular depositors cash provides a close substitute to deposits. We maintain this assumption for the remainder of the paper, and therefore set $\alpha^H = 0$ which implies a DLB for the deposit rate at zero, i.e. $i_t^d \geq i^{DLB} = 0$. Towards the end of the paper we discuss policies that may relax this bound.

Firms

Each good i is produced by a firm i. Production is linear in labor, i.e. $Y_t(i) = N_t(i)$, where $N_t(i)$ is a Cobb-Douglas composite $N_t(i) = \left(N_t^b(i)\right)^{\chi} \left(N_t^s(i)\right)^{1-\chi}$ as in Benigno et al. (2014). This implies that the two household types receive a fixed share of income. The preference specification implies that firms face a downward-sloping demand function $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t$. In each period, a fraction α of firms are not able to reset their price as in Calvo (1983). Thus, the likelihood that a price set in period t applies in period $T > t$ is α^{T-t} and in the absence of price adjustments, prices are assumed to be indexed to the inflation target Π. A firm that resets its price chooses the price that maximizes the present value of discounted profits in the event that the price remains fixed. That is, each firm i choose $P_t(i)$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\alpha\beta)^t \lambda_t \left[\Pi^t \frac{P_0(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} Y_t(i)\right] \tag{26}$$

where $\lambda_T \equiv q\left(\chi \exp\left\{-qC_t^b\right\} + (1-\chi)\exp\left\{-qC_t^s\right\}\right)$, which is the weighted marginal utility of consumption and $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$.[33] This maximization problem leads to the standard New Keynesian Phillips Curve.

Banks and pension funds

The banks problem has already been outlined in the partial equilibrium section and the first order conditions (53) - (58) from the appendix has to hold in equilibrium. In general equilibrium, however, the interest rates and the price level are endogenously determined.

In general equilibrium, the source of external funding for the banks are the pension funds that invest F_t and receive an interest rate i_t^f as well as government bonds A_t that pay an interest i_t^a. These pension funds are financed by a lump sum pension fee, F_t^j, in the households budget constraint. F_t should be interpreted as a stand-in for large investors, like pension funds, or large retail depositors which can possibly be charged a

[33]Recall that the firm is owned by both types of households according to their respective population shares.

negative interest rate, since storing assets in terms of paper currency might be prohibitively costly for this group. Liquid assets in the form of government bonds, A_t, are held by the banks as well as the households through the pension funds.[34]

The government

The government sets monetary policy according to the Taylor type policy rule

$$i_t = r_t^n (\frac{\Pi_t}{\bar{\Pi}})^{\phi_\pi} (\frac{y_t}{\bar{y}})^{\phi_\pi} \tag{27}$$

where r_t^n is the natural rate of interest which corresponds to the interest rate in the case of flexible prices, and ϕ_π and ϕ_y are coefficients. $\bar{\Pi}$ is the inflation target of the central bank and $\bar{y}$ is the natural level of output which is constant in the model and equal to steady state output.

We assume that when interest on reserves is above the deposit bound, then $i_t^r = i_t^d$. However, once the interest on reserves is below the DLB, then $\mathrm{i}_t^d = i^{DLB}$. Thus we impose the following constraint

$$i_t^d = \max\{i^{DLB}, i_t^r\} \tag{28}$$

which implies full pass-through to deposit rates, unless the effective lower bound is binding.

The government budget constraint is

$$A_t^g + M_t + R_t = (1 + i_t^g)A_{t-1}^g + M_{t-1} + (1 + i_{t-1}^r)R_{t-1} + G - T_t$$

Total government liabilities LB_t are assumed to be fixed and follow an exogenous process, i.e,

$$LB_t = A_t^g + M_t + R_t \tag{29}$$

Taxes adjust so that the government budget constraint is satisfied, and are equally distributed across the two agents in steady state. For simplicity, in response to shocks, only the tax on the saver is varied, so that

$$T_t = \bar{T}^b + T_t^s \tag{30}$$

Equilibrium and solution method

An equilibrium is defined as a collection of stochastic processes for the endogenous variables that solve the household problem, the firm problem, the bank problem, and monetary and fiscal policy follow the policy regimes specified in the previous section. The equilibrium is defined in the appendix. The model is solved via a log-linear approximation around the steady state, while explicitly taking of the DLB. From now on, we rewrite the model in real terms. Lower case letters refer to the real value of their nominal (big case letters) counterparts.

3.2.2 Analytic characterization

Before moving on to the numerical simulations, we characterize the log-linear approximated model briefly. We show that our model can be written in the form of the standard three-equation New Keynesian model, with two important differences: First, it is the deposit rate which enters the dynamic IS equation rather

[34] Observe that if the interest on government bonds goes negative, then individual household will substitute out of government bonds in favor of deposits that carry zero interest.

than the policy rate. Second, the natural rate of interest is now endogenous and depends on credit market outcomes.

The household and firm problems, together with the monetary policy rule, can be summarized as follows:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \sigma\{\hat{\imath}_t^d - E_t \hat{\pi}_{t+1} - \hat{r}_t^n\} \tag{31}$$

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} \tag{32}$$

$$\hat{\imath}_t^r = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \tag{33}$$

$$\hat{\imath}_t^d = \max(\bar{\imath}^{elb}, \hat{\imath}_t^r) \tag{34}$$

where $\sigma \equiv \frac{1}{q y}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(\eta+\sigma^{-1})}$, $\hat{y}_t \equiv \log \frac{y_t}{\bar{y}}$, $\hat{\imath}_t^d \equiv \log \frac{1+i_t^d}{1+\bar{\imath}^d}$, $\hat{\pi}_t \equiv \log \frac{\Pi_t}{\bar{\Pi}}$, $\bar{\imath}^{elb} \equiv -\log(1+\bar{\imath}^d)$ and $\hat{r}_t^n$ is the natural rate of interest, defined below.

Equation (31) is identical to the standard IS equation typically derived in the representative household framework. Here it is obtained by combining the consumption Euler equation of the two agents with the aggregate resource constraint. Equation (32) is the standard New Keynesian Phillips curve which is identical to the standard model, while equation (33) is the central bank's policy rule. Equation (34) specifies the deposit rate, at the lower bound and otherwise.

A major insight from the New Keynesian literature is that if the central bank tracks the natural rate of interest it stabilizes inflation and (under certain conditions) output. This remains the case in this model. A key difference, however, is that the natural rate of interest is not exogenous as in the standard model, but instead given by

$$\hat{r}_t^n \equiv -\chi\{\hat{\imath}_t^b - \hat{\imath}_t^d\} + \hat{\zeta}_t - E_t \hat{\zeta}_{t+1} \tag{35}$$

where $\hat{\zeta}_t \equiv \log \frac{\zeta_t}{\bar{\zeta}}$, and $\hat{\imath}_t^b \equiv \log \frac{1+i_t^b}{1+\bar{\imath}^b}$. While the natural rate of interest depends on the exogenous process for the household's preferences, more importantly it also depends on the endogenous credit market spread. Thus in response to a shock that increases the spread, the central bank needs to cut interest rates to stabilize inflation.

The spread between borrowing and lending rates is derived through the optimal lending decision of the bank yielding:

$$\hat{\imath}_t^b - \hat{\imath}_t^d = (\nu - 1)\frac{\beta^s - \beta^b}{\beta^s}\hat{b}_t - \iota\frac{\beta^s - \beta^b}{\beta^s}\hat{n}_t \tag{36}$$

A consequence, which is novel in our set-up relative to the existing literature, is that bank's net worth, via the impact on credit spreads, is an important variable for determining the natural rate of interest.

The dynamics of net worth is

$$\beta^s \hat{n}_t = \beta^s \hat{\imath}_t^d + \hat{z}_t + \frac{1-\omega}{\bar{\Pi}}\hat{n}_{t-1} - \frac{\beta^b}{\beta^s}\frac{1-\omega}{\bar{\Pi}}\hat{\pi}_t \tag{37}$$

where $\hat{z}_t$ is bank's profits given by

$$\frac{\overline{n}}{\bar{\Lambda}}\hat{z}_t = -\{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}} + \frac{\bar{r}}{\bar{\Lambda}} + \frac{\bar{a}}{\bar{\Lambda}} - \frac{\bar{f}}{\bar{\Lambda}}\}\hat{\imath}_t^d + \frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}\hat{\imath}_t^b + \frac{\bar{r}}{\bar{\Lambda}}\hat{\imath}_t^r + \frac{\bar{a}}{\bar{\Lambda}}\hat{\imath}_t^a - \frac{\bar{f}}{\bar{\Lambda}}\hat{\imath}_t^f + \iota\frac{\bar{\Gamma}}{\bar{\Lambda}}\hat{n}_t \tag{38}$$

$\hat{\imath}_t^a \equiv \log\frac{1+i_t^a}{1+\bar{\imath}^a}$, $\hat{\imath}_t^f \equiv \log\frac{1+i_t^f}{1+\bar{\imath}^f}$, $\hat{z}_t \equiv \frac{z_t-\bar{z}}{\bar{n}}$ and $\hat{n}_t \equiv \log\frac{n_t}{\bar{n}}$. The profit equation is expressed in terms of $\bar{\Lambda} \equiv \bar{a} + \bar{r} + \bar{b}$ which is the total assets of the bank in steady state.

Because the lending of the banks is the debt of the borrower, the model is closed by the debt dynamics from the borrowers budget constraint and the borrowers consumption Euler equation below

$$\hat{b}_t = \frac{1}{\beta^b}\hat{b}_{t-1} - \frac{1}{\beta^b}\hat{\pi}_t + \frac{1}{\beta^b}\hat{\imath}_{t-1}^b - \chi\frac{\bar{y}}{\bar{b}}\hat{y}_t + \chi\frac{\bar{y}}{\bar{b}}\hat{c}_t^b \tag{39}$$

$$\hat{c}_t^b = E_t\hat{c}_{t+1}^b - \sigma(\hat{\imath}_t^b - E_t\pi_{t+1} - \hat{\zeta}_t + E_t\hat{\zeta}_{t+1}) \tag{40}$$

We assume full pass-through from the policy rate to liquid assets and external financing away from the DLB, but allow for partial pass-through at the DLB, i.e.:

$$\hat{\imath}_t^a = \rho^a\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^a = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > \bar{\imath}^{DLB} \tag{41}$$

$$\hat{\imath}_t^f = \rho^f\hat{\imath}_t^r \text{ if } \hat{\imath}_t^r < i^{DLB} \text{ and } \hat{\imath}_t^f = \hat{\imath}_t^r \text{ if } \hat{\imath}_t^r > \bar{\imath}^{DLB} \tag{42}$$

An approximate equilibrium is a collection of stochastic processes for prices $\{\hat{\pi}_t, \hat{\imath}_t^r, \hat{\imath}_t^d, \hat{\imath}_t^f, \hat{\imath}_t^a, \hat{\imath}_t^b, \hat{r}_t^n\}$ and quantities $\{\hat{y}_t, \hat{c}_t^b, \hat{b}_t, \hat{n}_t, \hat{z}_t\}$ that solve equations (31) - (40) given an exogenous process $\{\hat{\zeta}_t\}$.

Monetary policy affects aggregate demand in equation (31) through two channels. If the DLB is not binding, the most direct one is via a reduction in the deposit rate, stimulating spending. The second is that policy rate endogenously changes the natural rate of interest through the credit spread in equation (35).

To illustrate the key mechanism analytically, we make three simplifying assumptions to clarify a point that holds more generally, namely that condition (21) for the net balance sheet exposure to negative policy rates is once again the key determinant of whether negative policy rates are expansionary or not once the DLB is binding.

Consider the case in which $\omega = 1$, and $\hat{b}_t = 0$ for $t > 0$. [35] Finally, suppose that prices are fixed so that $\kappa = 0$. Consider now the effect of a one-time reduction in the policy rate at time 0, while the deposit rate is unchanged (with the pass-through to liquid assets and external financing given by ρ_a and ρ_f).

Solving (31) through (40) yields:

$$\hat{b}_0 = \frac{(\frac{\bar{r}}{\bar{n}} + \rho^a\frac{\bar{a}}{\bar{n}} - \rho^f\frac{\bar{f}}{\bar{n}})}{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{n}} + \Theta\{\frac{\nu-1}{\iota}\Upsilon + \frac{1}{\iota}\frac{\beta^s}{\beta^s-\beta^b}\}}\Upsilon\hat{\imath}_t^r \tag{43}$$

$$\hat{\imath}_0^b = -\frac{1}{\Upsilon}\hat{b}_0 \tag{44}$$

[35]The simplest way of thinking about the second assumption, is that in response to a shock in period 0 there are government lump sum redistributions to bring debt back to steady state in period 1.

$$\hat{y}_0 = -\sigma\chi\hat{\imath}_0^b \tag{45}$$

where $\Upsilon \equiv \sigma\chi\{(1-\chi)\frac{\bar{y}}{\bar{b}} > 0$ and the parameter $\Theta > 0$ is defined as in the partial equilibrium model. Thus, just as in the partial equilibrium model, policy rate cuts once the DLB is binding reduce lending according to equation (44) if the condition in equation (21) is violated. In general equilibrium, however, as shown in equation (44) this increases borrowing rates, in line with our empirical evidence from Sweden. Moreover, as shown in equation (45), the result is a contraction in aggregate demand. While these results can be shown more generally, the more fundamental question is what is the quantitative implication of negative policy rates.

An inspection of the equations above reveals that the model is fully parameterized via the assignment of the banks balance sheet parameters $(\frac{\bar{n}}{\bar{\Lambda}}, \frac{\bar{b}}{\bar{\Lambda}}, \frac{\bar{f}}{\bar{\Lambda}}, \frac{\bar{r}}{\bar{\Lambda}}, \frac{\bar{a}}{\bar{\Lambda}}, \frac{\bar{y}}{\bar{b}}, \frac{\bar{\Gamma}}{\bar{b}})$, the interest rate pass-through parameters ρ^a, ρ^f, the structural parameters, $(\sigma, \kappa, \beta^s, \beta^b, \chi, \nu, \iota, \omega)$, the parameters of the policy function $(\phi_\pi, \phi_y, \bar{\Pi})$, along with the stochastic process for ξ_t. We now turn to the parameterization and quantitative predictions of the model.

3.3 Quantitative evaluation

In this section, we take the model outlined above to the data. A natural starting point is to evaluate the empirical balance sheet exposure of banks to negative policy rates.

3.3.1 Balance sheet exposure to negative rates

The analysis so far suggests that a reduction in the policy rate, once the DLB is reached, contracts lending if the transmission of negative policy rates to the asset side is larger than to the liability side, or

$$\underbrace{\frac{\bar{r}}{\bar{\Lambda}}}_{0.05} + \underbrace{\rho^a \frac{\bar{a}}{\bar{\Lambda}}}_{0.42} > \underbrace{\rho^f \frac{\bar{f}}{\bar{\Lambda}}}_{0.21} \tag{46}$$

where the relevant components of the balance sheet are expressed as a ratio of total assets.

The values reported under the curly brackets in equation (46) come from Swedish data and suggest that – in the Swedish case – banks exposure to negative policy rates is such that policy rate cuts at the DLB are expected to reduce bank profits. The condition is calibrated using balance sheet data from 2014 to capture the state of the banking system just prior to negative rates being implemented. The empirical counterpart of $\bar{a}$ is total assets less reserves and bank lending, $\bar{f}$ is total liabilities less deposits and equity, $\bar{r}$ is reserve balances with the central bank and cash.

As shown in the previous section, the approximated model also requires assigning values to $\frac{\bar{n}}{\bar{\Lambda}}$ and $\frac{\bar{b}}{\bar{\Lambda}}$. The values assigned to these ratios come from the same data source and are reported, along with the other relevant balance sheet ratios in Table 4. In addition to these ratios, the ratio $\frac{\bar{\Gamma}}{\bar{\Lambda}}$ appears in the approximated model. From the optimal lending conditions, the steady state of the model implies that $\frac{\bar{\Gamma}}{\bar{\Lambda}} = \frac{\bar{b}}{\bar{\Lambda}} \frac{\frac{\beta^s}{\beta^b} - 1}{\nu}$.

The pass-through to other liquid assets at the DLB (ρ^a) is assumed to be one, reflecting the strong pass-through of negative rates to reserve-like assets such as money market instruments and short-run government

Banking Sector Moments	Value	Empirical Counterpart
Liquid asset ratio	$\frac{\overline{a}}{\overline{\Delta}} = 0.42$	Liquid assets to total assets of 42%
Reserve ratio	$\frac{\overline{r}}{\overline{\Delta}} = 0.05$	Reserves to total assets of 5 %
External financing ratio	$\frac{\overline{f}}{\overline{\Delta}} = 0.53$	External financing to total assets of 53 %
Loans to total assets	$\frac{\overline{b}}{\overline{\Delta}} = 0.53$	Loans to total assets of 53 %
Net worth to total assets	$\frac{\overline{n}}{\overline{\Delta}} = 0.05$	Net worth to total assets of 5 %
Profit to net worth	$\frac{\overline{z}}{\overline{n}} = 0.173$	Profit to net worth of 17.3 %
Loans to GDP	$\frac{\overline{b}}{\overline{y}} = 1.4$	Loans to GDP of 140 %
Pass-through coefficient to liquid assets	$\rho^a = 1$	Pass-through of negative rates of 100 %
Pass-through coefficient to external financing	$\rho^f = 0.4$	Pass-through of external financing of 40 %

Table 4: Banking sector parameters.

Notes: This table shows key banking sector moments and pass-through coefficients necessary to calibrate the log-linear approximated model, outlined in Appendix C. Banking sector moments are based on the 2014 balance sheet of the Swedish banking system. Pass-through coefficients are computed based on Figure 3 and Figure 17.

bonds documented in Figure 17. The pass-through to external financing at the DLB (ρ^f) is assumed to be 0.4. This number is obtained by first computing i_t^f as the weighted average of the money market rate, weighting the money market rates and covered bond rates with weights computed from the banks balance sheets.[36]

3.3.2 Assigning values to structural parameters

Numerical values are assigned to the remaining structural parameters in three steps. First, values are chosen from the existing literature in cases where they are relatively standard. Second, values for the novel parameters of our model are chosen by matching certain features of the data. Finally, given the other parameters, some structural values can be assigned exploiting steady state relationships.

We start by setting the relatively standard parameters equal to values used in the existing literature, following especially Benigno et al. (2014) closely. The values chosen for $\sigma, \kappa, \chi, \beta^s, \beta^b, \phi_\pi, \phi_y, \bar{\Pi}$ are reported in Table 5. The values for the policy rule are standard, see e.g. Gali (2008). Steady state saving and borrowing interest rates are pinned down by β^s, β^b and $\bar{\Pi}$. The risk-free rate is assumed to be 1.5 percent in steady state based on US data. The borrowing rate is pinned down based on Mehra and Prescott (2008), who report a

[36]In order to calculate this number, we first compute i^f as

$$i^f = \frac{1}{3} \times \text{Money Market Rates} + \frac{1}{3} \times \text{Covered bond, 2Y} + \frac{1}{3} \times \text{Covered bond, 5Y} \tag{47}$$

where the different interest rates used as inputs are taken from Figure 17. We then set

$$\rho^f = \frac{\Delta i_t^f}{\Delta i_t^r} \tag{48}$$

for the period after $i_t^d \approx 0$, i.e. the post-bound period.

Parameter	Value	Source/Target
Intertemporal elasticity of substitution	$\sigma = 0.66$	Smets and Wouters (2003)
Share of borrowers	$\chi = 0.61$	Justiniano et al. (2015)
Steady-state gross inflation rate	$\bar{\Pi} = 1.005$	Match annual inflation target of 2
Discount factor, saver	$\beta^s = 0.9963$	Annual real savings rate of 1.5
Discount factor, borrower	$\beta^b = 0.991$	Annual real borrowing rate of 3.5 (Mehra and Prescott, 2008)
Slope of AS equation	$\kappa = 0.02$	Eggertsson and Woodford (2003)
Taylor coefficient on inflation gap	$\phi_\Pi = 1.5$	Gali (2008)
Taylor coefficient on output gap	$\phi_Y = 0.5/4$	Gali (2008)
Marginal cost of lending	$\nu = 4.9$	Match pass-through of negative rates from Table 1.
Elasticity of lending costs wrt. net worth	$\iota = 2.55$	$\iota = (\nu - 1) \times 1.89/2.89$, based on MAG (2010) and model equations
Payout ratio	$\omega = 0.17$	Generate profit to net worth ratio of 17.3 %
Shock	**Value**	**Source/Target**
Preference shock	13.37 % temporary decrease in ζ_t	Generate a 4 drop in output on impact
Persistence of preference shock	$\rho = 0.88$	Duration of lower bound of 12 quarters

Table 5: Calibration

spread of 2 percent in US data. The values for κ, σ and χ reported in Table 5 are relatively conventional and values in this range can be found in a large number of sources, see for instance Benigno et al. (2014) or Justiniano et al. (2015).

The two key parameters that are specific to the model are ι and ν. We assign values to these parameters based on our own estimate of the pass-through of negative policy rates to borrowing rates at the DLB from Table 1 and empirical estimates from MAG (2010). MAG (2010) is a meta study released by the Bank of International Settlements that considers the effect of a reduction in bank equity on lending, using a large number of models. A summary finding is that a one percentage point increase in bank target capital leads to a 1.89 percentage point reduction in lending.

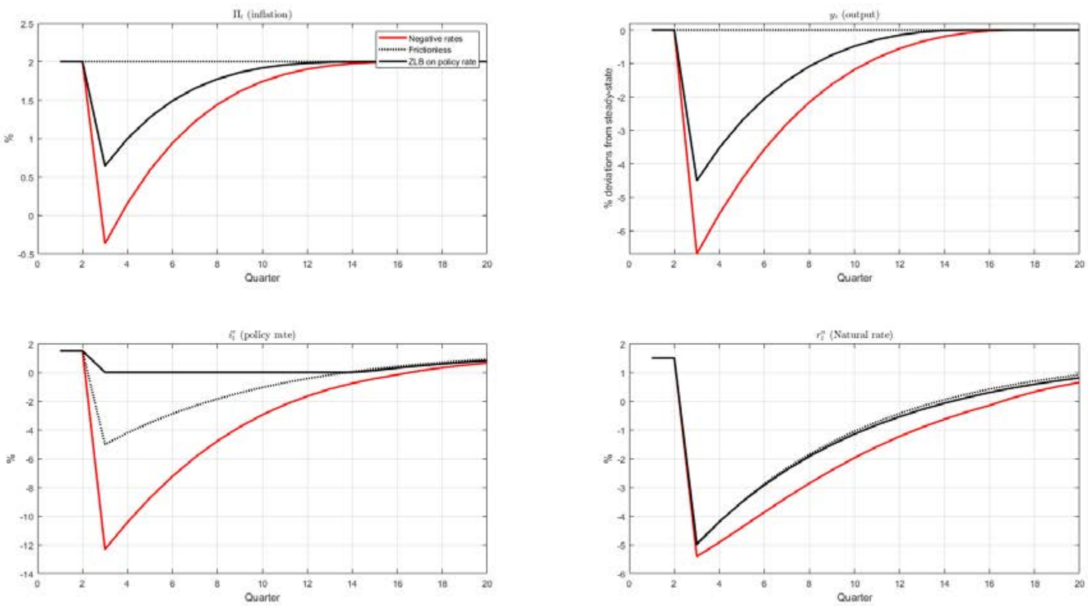
Using these two pieces of evidence, ι and ν are pinned down in two steps. First, defining the bank capital ratio as $\Upsilon_t = \frac{n_t}{b_t}$, and taking lending rates as given, condition (13) can be used to relate lending and bank capital to yield $\hat{b}_t = \dfrac{\iota}{\nu - \iota - 1}\hat{\Upsilon}_t$. MAG (2010) provides an empirical estimate for $\dfrac{d\hat{b}_t}{d\hat{\Psi}_t} = 1.89$ from which it follows that $\iota = \dfrac{1.89}{2.89} \times (\nu - 1)$. In the second step, we use the pass-through of policy rate cuts to borrowing rates at the DLB estimated in Table 1 as an identified moment (Nakamura and Steinsson, 2018) to set ν. Specifically, the full model is solved and ν is chosen so that in general equilibrium - if the DLB is binding - a 100 basis points reduction in the policy rate increases borrowing rates by 5.9 basis points on impact.

A remaining parameter is ω. To choose this parameter we exploit that in steady state $\bar{z}/\bar{n} = \beta^s + \omega - 1$. Using data on $\bar{z}/\bar{n}$ from Table 5 yields $\omega = 0.173$.

Finally, in order to conduct numerical experiments in the model, we further need to specify a shock process for ξ_t. In the numerical experiment considered next, we assume that ξ_t follows a first order autoregressive process with persistence ρ. At time 0 there is an initial shock. The size of the shock, and its persistence, is picked to generate a 4.5 percent drop in output and a duration of the lower bound of twelve quarters assuming that the PLB is binding at zero. The output drop is calibrated to match the average decline in real output in Switzerland, the Euro Area, Sweden and Denmark in the aftermath of the financial crisis.

3.4 Simulation results

Figure 8: Response of model under the baseline calibration to a preference shock.



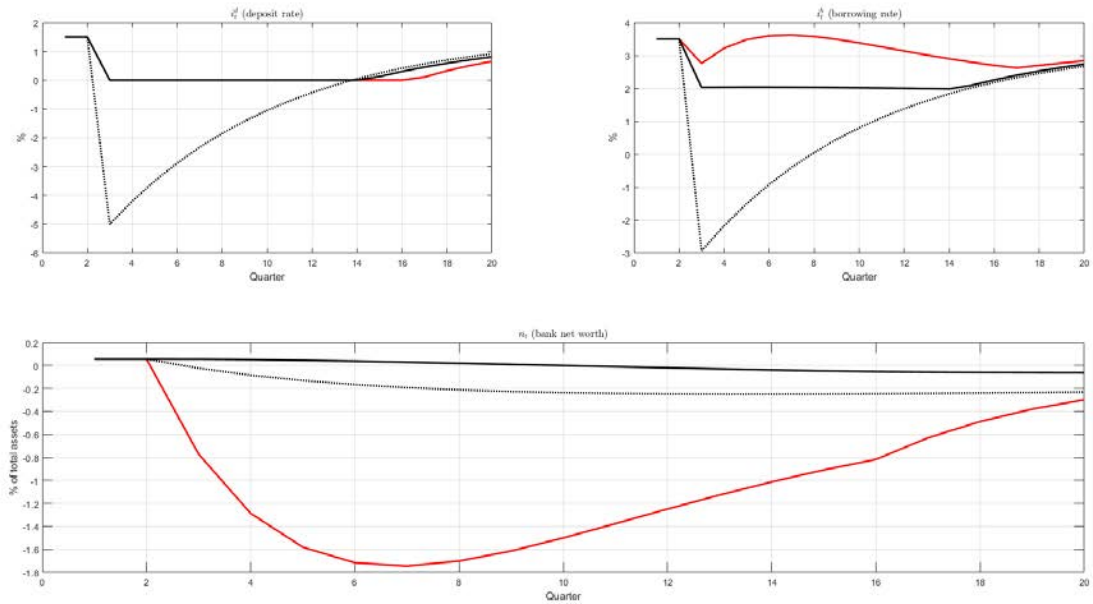
Notes: This figure shows the impulse response functions of inflation, output, the policy rate and the natural rate of interest in response to a preference shock. The red solid line corresponds to a model where the central bank follows the Taylor rule into negative territory, but in which the deposit rate is subject to a lower bound. The black solid line corresponds to a model where the central bank follows the Taylor rule in positive territory, but where the policy rate is subject to a zero lower bound. The dashed black line corresponds to a model in which there are no bounds on any interest rate.

Figure 8 plots the impulse responses of inflation, output, the natural rate and the policy rate to an innovation in $\hat{\xi}_0$. The solid black line shows the evolution of these variables in response to the shock, assuming the policy rate and the deposit rate are constrained at zero. The shock directly reduces the natural rate to approximately five percent. Since the policy rate does not follow the natural rate into negative territory, the result is an output contraction of -4.5 percent and a drop in inflation from the target rate of 2 percent to slightly above zero.

The dashed black line shows the response of the economy if there are no lower bounds. In this case, the central bank fully accommodates the shock by cutting interest rates to -5 percent. This policy rate reduction is sufficient to offset the effect of the initial shock, leading to no drop in inflation and output.

The red line illustrates the case when we impose a DLB on the deposit rate but not on the policy rate. In response to the shock, the central bank cuts the policy rate to roughly -12 %, which is implied by the Taylor rule. In this case, given the balance sheet structure of the banking sector, output contracts by by roughly two additional percentage points.

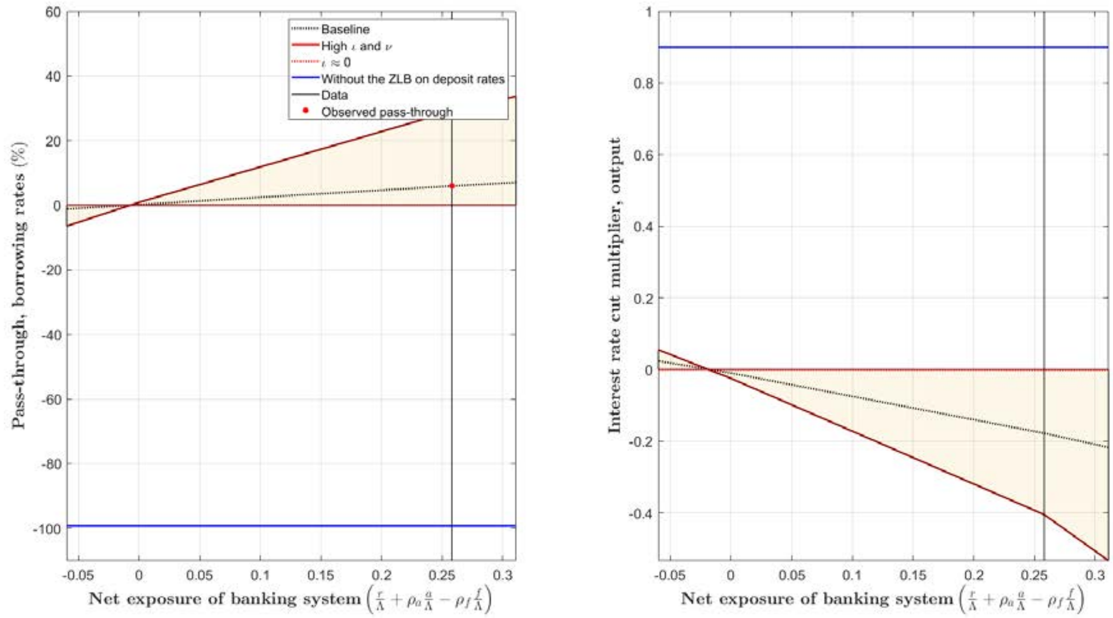
Figure 9: Response of model under the baseline calibration to a preference shock.



Notes: This figure shows the impulse response functions of the deposit rate, borrowing rate and bank net worth, in response to a preference shock. The red solid line corresponds to a model where the central bank follows the Taylor rule into negative territory, but in which the deposit rate is subject to a lower bound. The black solid line corresponds to a model where the central bank follows the Taylor rule in positive territory, but where the policy rate is subject to a zero lower bound. The dashed black line corresponds to a model in which there are no bounds on any interest rate.

Figure 9 sheds light on the underlying mechanism. Due to the DLB, the negative policy rate is not transmitted to deposit rates. As a result, there is limited pass-through to banks financing costs and so no major reduction in lending rates. Further, because banks in our economy are net holders of assets with a negative rate, the introduction of negative rates yields a reduction in bank profits. This translates into lower bank net worth and higher intermediation costs, thus increasing the interest rate spread – indeed the parameter ν was chosen so as match the estimated increase in spreads in the data. As a result, output falls by an additional amount when the central bank goes negative. Going into negative territory, once the DLB is binding, is thus contractionary. The overall effect of negative rates in the simulation is only illustrative, and is not meant to represent actual policy by any central bank in recent past, since it is assumed that the policy rate follows a Taylor rule, even once rate turn negative, which implies very negative rates – negative enough so that the PLB might be an issue. Subsection 3.4.2 gives a better sense for the likely quantitative effect of negative rates for more modest policy rate cuts into negative territory, and highlight how this effect depends on parameter values assumed.

Figure 10: Effect of negative policy rates and net exposure of the banking system



Notes: This figure shows the on-impact difference in borrowing rates (left) and output (right) between two models - one model where the central bank does not impose a negative policy rate and one where the central bank follows a standard Taylor rule also below zero. The output and borrowing rate difference is scaled by absolute value of the average policy rate below zero. On the x-axis, we redo the exercise for different values of equation (46). We fix R, A and F and solve the model for different values of $\rho^f \in (0.3, 1)$. The black dashed line is the results using our baseline calibration. The red solid line is a calibration where $\nu = 51.5$, which generates a pass-through at the DLB equal to the coefficient estimate in Table 1 plus 1 standard deviation. The blue horizontal lines show the effects in the case when the DLB is non-binding.

3.4.1 Transmission of negative policy rates to lending rates

In Table 1 we reported the pass-through of policy rate cuts to lending rates at the DLB. Pass-through can also be computed in the model. The horizontal blue line in the left panel of Figure 10 shows the pass-through of interest rate cuts when the DLB is not binding. As the figure reveals it is approximately -100, that is, borrowing rates fall approximately as much as the repo-rate in response to policy rate cuts in the quarter in which they occur.

The x axis captures the exposure of the banking sector to negative rates, according to Proposition 4. A simple way of changing the exposure is to vary ρ_f, the pass-through of negative rates to external financing. In the absence of the DLB, the exposure of the banking system to negative rates is irrelevant. The dashed black line shows the pass-through to borrowing rates once the DLB is binding. The red dot denotes a pass-through of 5.9, in line with our empirical estimates. The vertical line highlights the empirical exposure to negative rates using Swedish bank balance sheets. As ρ_f varies, the pass-through to borrowing rates varies. The shaded yellow area captures variations in the pass-through resulting from different values of $\{\nu, \iota\}$.

3.4.2 Output effect of negative policy rates

To quantitatively summarize how the DLB reduces the efficiency of policy rate cuts, and how sensitive the conclusions are to parameter values, it is useful to compute a interest rate cut multiplier, M_0, defined as

$$M_0 = \frac{\hat{y}_1^* - \hat{y}_1^{\text{PLB}=0}}{|\hat{\imath}_1^{r^*} - \hat{\imath}_1^{r\,\text{PLB}=0}|} = \frac{\hat{y}_1^* - \hat{y}_1^{\text{PLB}=0}}{|\hat{\imath}_1^{r^*}|} \tag{49}$$

where $\hat{y}_1^{\text{PLB}=0}$ denotes the output on impact if the policy rate is at a lower bound of 0 and $\hat{\imath}_1^{r\,\text{zero}} = 0$ is the corresponding policy rate at the bound. These numbers correspond to the initial point of the black line in Figure 10. $\imath_1^{r^*}$ is a policy intervention and the corresponding output response is $\hat{y}_1^*$ for a particular scenario. Consider, for example, the interest rate multiplier if there is no bound on any interest rate – the blue line in Figure 10. In that case, the multiplier is 0.9 and has the interpretation that a 100 basis point cut in the policy rate leads to a 0.9 percent increase in output. Moreover, it is independent of the balance sheet composition of the banking sector.

Once the DLB is reached, however, the balance sheet composition of banks is fundamental as illustrated by the condition in Proposition 4. The dashed line in Figure 10 shows the interest rate multiplier for different net exposure measures. The vertical line illustrates the benchmark parameterization, in which case the multiplier is - 0.177. Put differently, a 100 basis point interest rate cut reduces output by 0.177 percent. The figure also illustrates two extreme cases for the choice of ι and ν. The vertical horizontal line shows the case of $\iota = 0$, in which there is no output effect of negative rates, while the dark red line shows what we consider relatively high values of ν. The figure illustrates that the sign of the interest rate cut multiplier can change, depending on the banking sectors net exposure to negative rates.

3.5 Rationalizing different findings in the literature

We end this section by briefly discussing the existing empirical literature on the bank lending channel of negative policy rates in the context of our model. The literature on the bank lending channel of negative rates suggests, at first glance, that there are contrasting views as to whether negative policy rates expand bank credit. While our empirical results on credit growth volumes, consistent with Heider et al. (2016), indicate that the bank lending channel of monetary policy is weaker under negative policy rates, other studies arrive at opposite conclusions.[37] For instance, Bottero et al. (2019) and Basten and Mariathasan (2018) show evidence indicating that credit supplied increases in response to negative policy rates.

Our model can rationalize these different findings, as they are based on economies and banking systems operating under different circumstances. Bottero et al. (2019) analyze the lending response of primarily Italian banks, and show that the deposit share is not an important predictor for how banks respond to negative policy rates. This is consistent with average deposit rates in Italy at the time of the introduction of negative policy rates being above zero. Hence, as the ECB lowered the policy rate into negative territory, there was substantial scope for deposit rates to fall further, lowering bank financing costs and thereby boosting lending. This is the prediction of the model if the deposit rate had not been subject to a zero lower bound – or if the bound was not

[37] We note, however, that most existing studies focuses on the relative differences in bank lending under negative policy rates according to different bank characteristics, while our results are also informative about the *level* of the aggregate lending response.

yet binding.

The presence or absence of a zero lower bound on deposit rates is *not* sufficient to predict whether the bank lending channel will be fully operational. This is illustrated by comparing the findings in our paper to the findings in Basten and Mariathasan (2018). They investigate the impact on bank lending of negative policy rates in Switzerland. When the Swiss National Bank went below zero, deposit rates were already close to the zero lower bound. However, Swiss banks according to their summary statistics[38] had a relatively low share of liquid assets relative to total assets, and a higher share of debt and interbank funding, suggesting a negative net exposure to negative rates.[39] In our data, the average net exposure to negative interest rates is positive, potentially explaining why the overall impact on bank net worth in Sweden and Switzerland differs. The empirical findings in both this paper and in Basten and Mariathasan (2018) are therefore consistent with our model.

4 How can negative interest rates be more effective?

Our empirical analysis focuses on how the bank lending channel responds to negative interest rates given the current institutional setting. In this section, we discuss the broader question of whether negative interest rates can stimulate aggregate demand either through other transmission channels or with institutional changes.

4.1 Other transmission channels

One important omitted channel in our analysis is exchange rate movements. Both the Swiss and Danish central banks motivated their decision to implement negative central bank rates by a need to stabilize the exchange rate. The impact on the exchange rate would depend on which interest rates are most important for explaining movements in the exchange rate, which in turn can depend on several institutional details. The empirical literature on the impact of negative rates on exchange rates is somewhat scant, but some early evidence can not reject that negative interest rates have limited impact on exchange rates, especially over longer time-horizons (Hameed and Rose, 2016).

Another mechanism through which negative interest rates could affect aggregate demand is through signaling about future interest rates. De Groot and Haas (2020) build a model where negative interest rates can signal lower future deposit rates, which in equilibrium boosts aggregate demand and output.

Finally, it is also worth noting that government borrowing rates have fallen into negative territory. To the extent that this can stimulate fiscal expansions, that would be an additional way through which negative rates could have a positive effect.

Summing up, negative interest rates can affect the macroeconomy through other channels, even when the bank lending channel is relatively less efficient. Overall, however, there is limited conclusive evidence as of now with regards to the effectiveness of these other channels.

[38] See Table 1 in their draft.

[39] In fact, the profit margin of Swiss banks went up under negative interest rates, according to their paper (column 12 in Table A3).

4.2 Eliminating or lowering the lower bound

The critical friction in our model is the zero-lower bound on deposit rates. Without this friction, negative interest rates would be similar to conventional monetary policy. Consistent with our findings, Altavilla et al. (2019) find that banks which were able to impose negative deposit rates also expanded lending in response to interest rate cuts into negative territory. Unfortunately, however, so far only a minority of banks have been able to do so – at lest for a meaningful fraction of their deposit customers.

The critical question is, therefore, how to remove the zero lower bound on deposit rates more broadly. One way of doing so in the context of our model is if the government takes action to increase the cost of holding paper money. There are several ways to do this. The oldest example is a tax on currency, as outlined by Gesell (n.d.). Gesell's idea would show up as a direct reduction in the bound on the deposit rate in our model, thus giving the central bank more room to lower the interest rate on reserves - and the funding costs of banks.

Another possibility is to ban higher denomination bills, a proposal discussed in among others Rogoff (2017a,c). To the extent that this would increase the storage cost of money, this too should reduce the bound on the banks' deposit rate.

An even more radical idea, which would require some extensions to our model, is to let the reserve currency and the paper currency trade at different values, rather than on par as we have assumed. This proposal would imply an exchange rate between electronic money and paper money. Agarwal and Kimball (2015), Rogoff (2017c) and Rogoff (2017b) discuss a concrete proposal, where a key pillar – but perhaps also a challenge to implementability – is that the reserve currency is the economy-wide unit of account by which taxes are paid, and accordingly what matters for firms price setting. If such an institutional arrangement is achieved, then there is nothing that prevents a negative interest rate on the reserve currency while cash in circulation is traded at a different price, given by an arbitrage condition. We do not attempt to incorporate this extension to our model but note that it seems relatively straightforward, and has the potential of solving the DLB problem.

Indeed, the take-away from the paper should not be that negative nominal rates are always non-expansionary, simply that they are predicted to be less stimulative than normal interest rate cuts under the current institutional arrangement. The prevalence of the low interest rates going forward gives all the more reason to contemplate departures from the current framework, such as those mentioned briefly here and discussed in more detail by some of the authors cited above.

5 Conclusion

Since 2014, several countries have experimented with negative policy rates. In this paper, we have documented that negative central bank rates have not been transmitted to aggregate deposit rates, which remain stuck at levels close to zero. Using bank level data from Sweden, we documented a disconnect between the policy rate and lending rates, once the policy rate became sufficiently negative. We further showed that this disconnect was partially explained by reliance on deposit financing. Banks which rely more heavily on deposit financing were less likely to reduce their lending rates in response to policy rate cuts once the deposit rate had reached its lower bound. Consistent with this, we found that Swedish banks with high deposit shares experienced lower credit growth after the deposit rate had become unresponsive. Furthermore, we documented negative excess returns on Swedish bank stocks surrounding the announcement of a negative policy rate, which significantly differed from the response in positive territory.

Motivated by our empirical findings, we developed a New Keynesian model with savers, borrowers, and a bank sector. By including money storage costs and central bank reserves, we captured the disconnect between the policy rate and the deposit rate at the lower bound. In this framework, we highlighted how the strength of the bank lending channel of negative policy rates depends crucially on whether (part) of banks financing costs are subject to a binding lower bound and if so, the mix of assets and liabilities with different degrees of pass-through on the banks balance sheet.

Given the long-term decline in interest rates, the need for unconventional monetary policy is likely to remain high in the future. Our findings highlight conditions for when negative policy rates can work and when they are likely to be ineffective. The question remains, however, are there other unconventional monetary pools that can be more effective? While the existing literature has made some progress in evaluating these measures, the question of how monetary policy should optimally be implemented in a low interest rate environment remains a question which should be high on the research agenda.

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A Listed vs. transaction-based rates

An important question is whether the listed rates used in the main body of the text is a valid proxy for actual interest rates. One approach to gauge whether this is the case, is to compare the implied aggregate listed rates with observed aggregate transaction-based rates.

Figure 11 depicts a weighted average of the bank level floating mortgage rates aggregated to the monthly level, along with the official aggregate rates that are based upon transaction data. First note that the listed rates are somewhat higher than the transaction rates. This is not surprising, as bank customers can negotiate a lower rate based on loan characteristics. [40] The dashed vertical lines represent repo-rate reductions. Importantly, around these policy changes, the difference between listed rates and aggregate rates is stable, suggesting that the breakdown in pass-through documented in the event study is not an artifact of the listed rates.

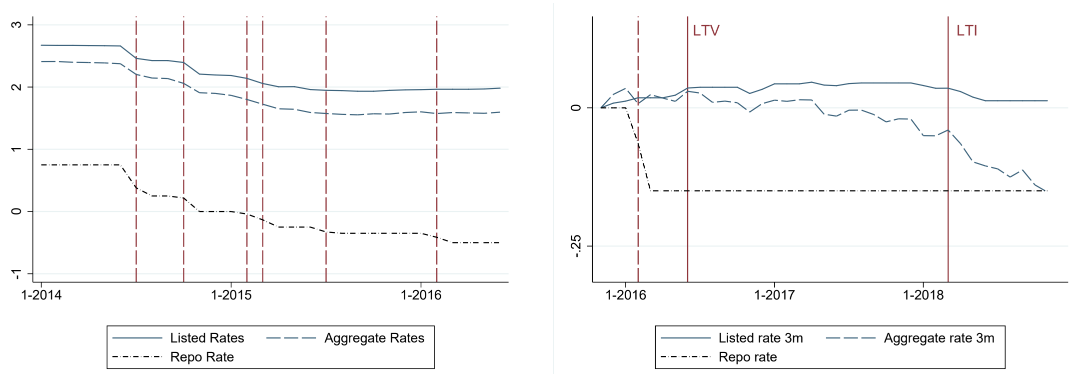

Figure 11: Listed rates and aggregate rates (left panel) and cumulative changes (right panel).

Notes: This figure shows listed and aggregate rates (left panel) and the cumulative changes in listed and aggregate rates (right panel). Cumulative change calculated since November 2015. Dashed vertical lines correspond to repo-rate reduction dates. Solid vertical lines in the right panel correspond to June 2016 and March 2018, when mortgage market regulation was introduced.

Erikson and Vestin (2019) point out that there was some reduction in aggregate rates in 2017 and 2018, and suggest that this might reflect a delayed pass-through from the last repo-rate reduction. While we cannot rule out that there was some long-term pass-through, we find it likely that the reduction in aggregate rates was connected to the introduction of new mortgage market regulation.

We follow Erikson and Vestin (2019) and plot the cumulative change in interest rates since November 2015 in the right panel of Figure 11. The last repo-rate reduction is captured by the dashed vertical line. Note that there is no pass-through to either listed rates or official transaction rates in the months following the repo-rate cut. Over time however, there is a divergence between listed rates and aggregate rates. As a result, by the end of 2018 – almost three years after the last repo-rate reduction – there appears to have been full pass-through aggregate rates, while listed rates remain unchanged. What explains the discrepancy?

As documented in the event study, normally full pass-through – or close to full pass-through – is achieved within a period of thirty days. Full pass-through within a period of two/three years would therefore entail a

[40] For example, the Swedish bank Nordea writes alongside their listed rate that "The mortgage rate we offer is an individual offer. Your mortgage rate can therefore turn out lower than the rate we list here." In Swedish, from Nordeas website 5/10/2019: *Bolanerantan vi erbjuder dig ar individuell. (...) Din bolaneranta kan darfor bli lagre an den ranta vi visar har.*

severe delay of the effectiveness of monetary policy. An alternative explanation for why aggregate transaction rates fell is unrelated to the interest rate policy and rather triggered by change in composition of Swedish borrowers. A possible explanation is that during this period the Swedish Financial Authority implemented new amortization requirement that encouraged encouraged lower loan-to-value (LTV) and loan-to-income (LTI) ratios, both of which would explain why transaction rates during this period started deviating more and more from listed rates. These regulation were implemented in June 2016 and March 2018, and led to a reduction in average LTV and LTI ratios for new borrowers, the dates are denoted by the two vertical lines in figure 11. As shown in Figure 11, the new policies introduced seem to coincide almost perfectly with the divergence between listed rates and aggregate rates. Moreover, we have studied the effect of a similar policy change in 2010 to evaluate whether the same pattern of divergence was observed then. As shown in Figure 15 in Appendix A, there was a quantitatively similar divergence following the new regulation in 2010. Hence, our assessment is that this empirical evidence is highly suggestive of that the reduction in aggregate lending rates two/three years after the last repo-rate cut was driven by regulatory measures, not by the repo-rate cuts themselves.

B Additional figures and tables

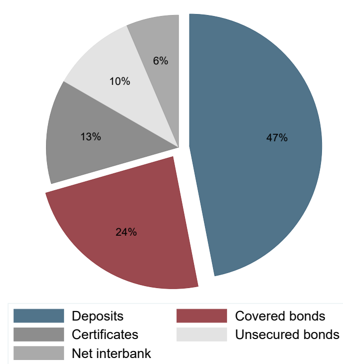

Figure 12: Decomposition of liabilities (as of September 2015) for large Swedish banks. Source: The Riksbank

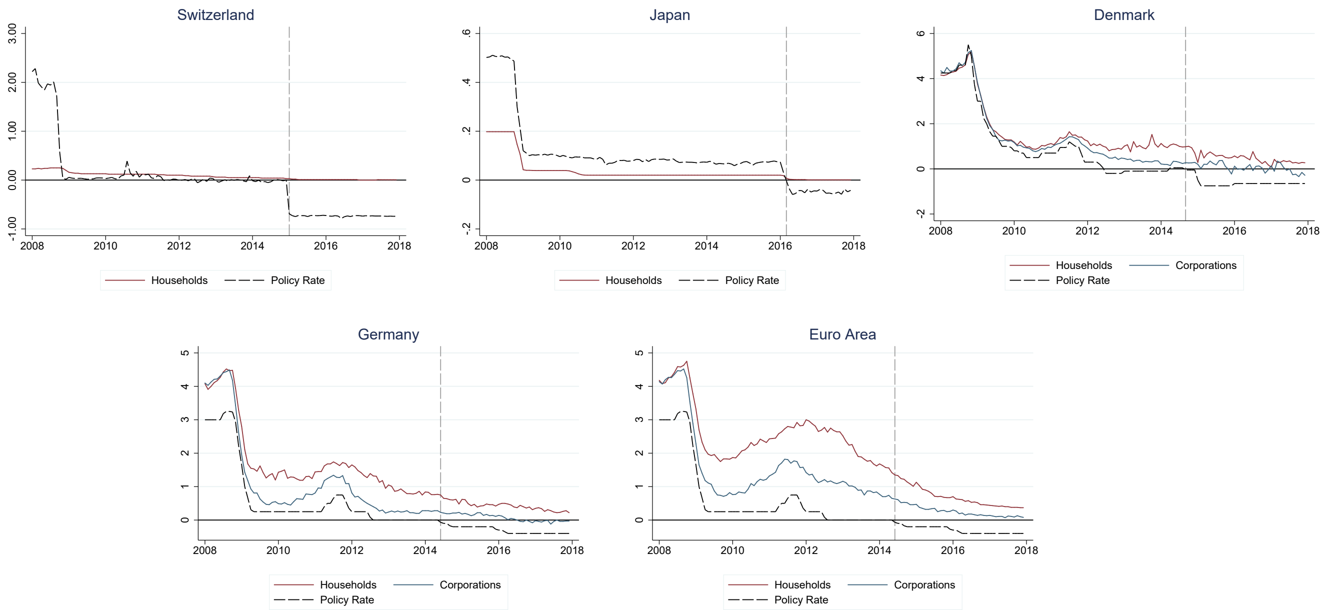

Figure 13: Aggregate deposit rates for Switzerland, Japan, Denmark, the Euro Area and Germany. The policy rates are defined as SARON (Switzerland), the Uncollaterized Overnight Call Rate (Japan), the Certificates of Deposit Rate (Denmark) and the Deposit Rate (Euro Area and Germany). The red vertical lines mark the month in which policy rates became negative. Source: the Swiss National Bank (SNB), Bank of Japan, the Danish National Bank (DNB), and the European Central Bank (ECB).

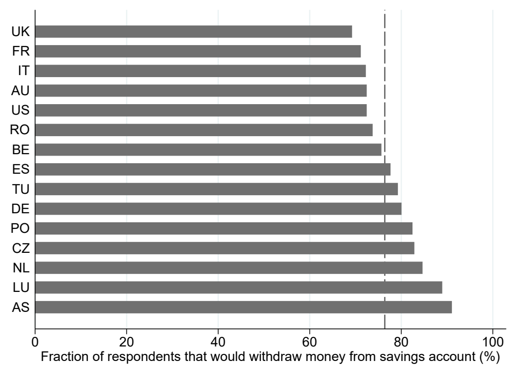

Figure 14: Fraction of households who would withdraw money from their savings account if they were levied a negative interest rate. Solid line represent unweighted average of 76.4 . Source: ING (2015)

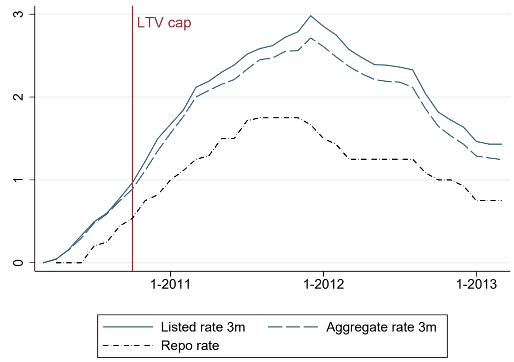

Figure 15: Listed rates and aggregate rates – 3 months. Cumulated change since March 2010.

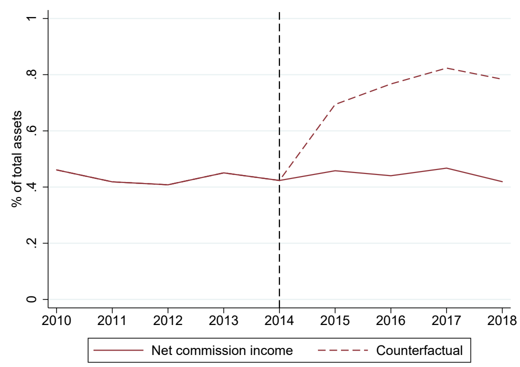

Figure 16: Actual and Counterfactual Commission Income as a Share of Assets (). The counterfactual commission income is calculated as the amount of commission income that would be necessary to make up for the bound on the nominal deposit rate, all else equal. The counterfactual commission income is given by actual commission income plus $\frac{Deposits_t}{Assets_t}\left(i_t - i_t^{cf}\right)$, where i_t is the average aggregate deposit rate and i_t^{cf} is a counterfactual deposit rate calculated under the assumption that the markdown to the repo-rate is constant and equal to the pre-zero markdown. Source: Statistics Sweden and own calculations.

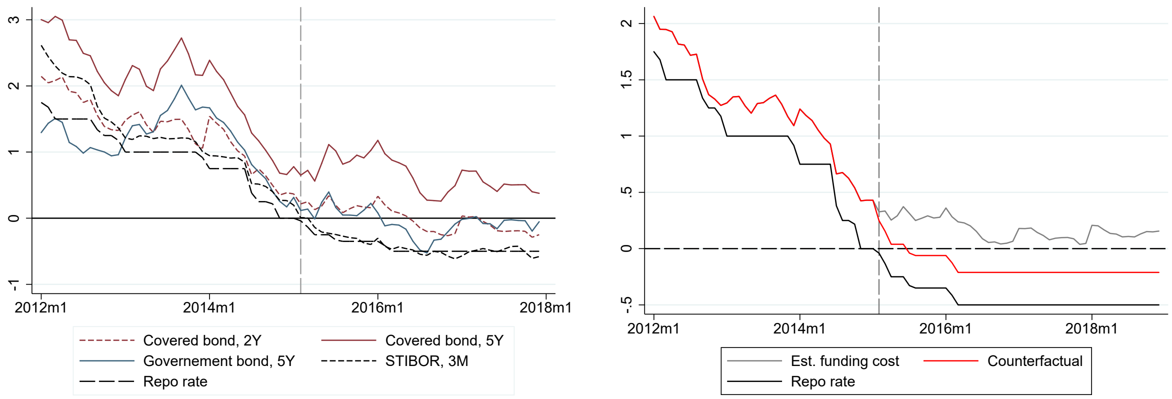

Figure 17: Other interest rates (left panel) and an estimate of average bank funding costs (right panel).

Notes: This figure shows other interest rates (left panel) and an estimate of the average bank funding cost (right panel). The estimated average funding cost is computed by taking the weighted average of the assumed interest rates of the different funding sources of the bank. Certificates are assumed to have the same interest rate as 2Y covered bonds, while unsecured debt are assumed to have the same interest rate as 2Y covered bonds plus a 2 percent constant risk-premium. The counterfactual series corresponds to the case when the spread between the repo-rate and the estimated funding cost remains fixed at pre-negative levels. Weights are based on the liability structure of large Swedish banks, see Figure 12. Source: The Riksbank

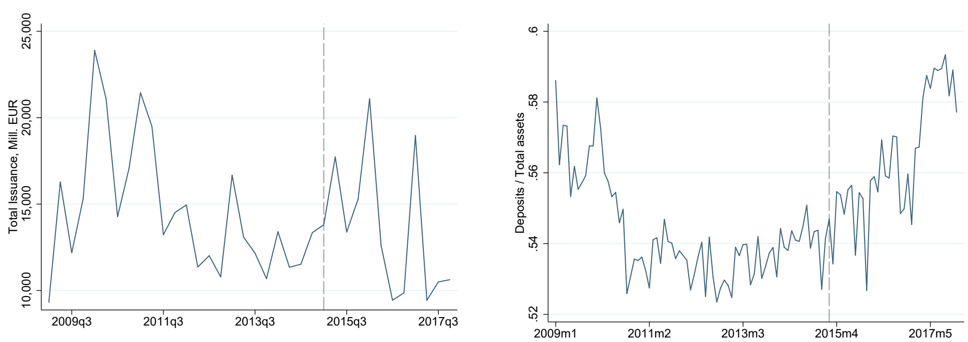

Figure 18: Left panel: Issuance of covered bonds, Swedish banks. Right panel: Deposit share, Swedish banks. Vertical lines correspond to the date negative interest rates were implemented. Source: Association of Swedish Covered Bond Issuers, The Riksbank and Statistics Sweden

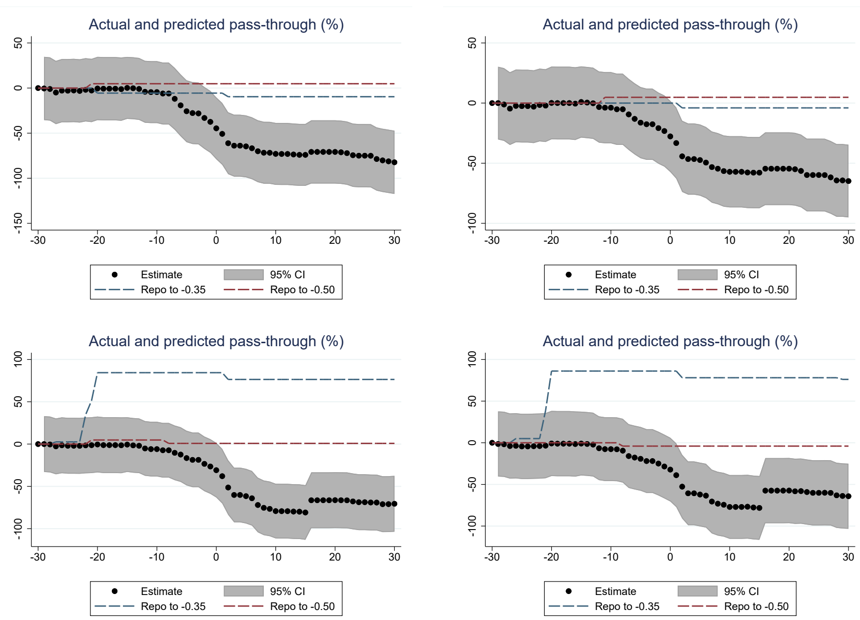

Figure 19: Event study different mortgage rates. Upper left: 3 months. Upper right: 1 year. Lower left: 3 years. Lower right: 5 years.

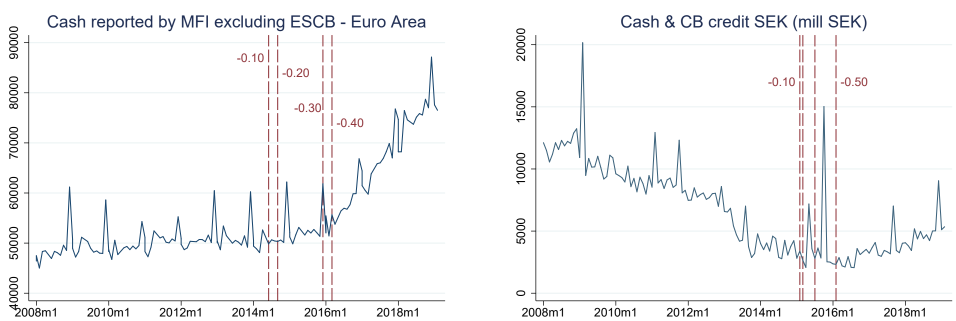

Figure 20: Left panel: Cash held by Euro Area Banks. Variable name: Cash reported by MFIs excluding ESCB - Euro Area. Millions of Euro. Source: ECB. Right panel: Cash held by Swedish banks. Variable name: Cash and credit balances at central banks in SEK. Million SEK. Source: Statistics Sweden. The dashed, red lines refer to the interest rate cuts in negative territory.

	(1) Δ log(loans)	(2) Δ log(loans)	(3) Δ log(loans)	(4) Δ log(loans)
Post × deposit share	0.0189			
	(0.0894)			
Post × deposit share		-0.0646		
		(0.0959)		
Post × deposit share			0.731	
			(1.476)	
Post × deposit share				-0.763
				(1.040)
N	1227	1227	1227	1227
No. of clusters	10	10	10	10
Mean of dependent variable	2.587	2.587	2.587	2.587
SD of dependent variable	7.713	7.713	7.713	7.713
Bank FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
measure	Cont.	Cont.	High or low	High or low
placeboperiod	repo-rate ∈ [1,0)	repo-rate ∈ (0,-0.25]	repo-rate ∈ [1,0)	repo-rate ∈ (0,-0.25]

Table 6: Falsification test of lending volume regression. We consider two different "post-periods", according to the level of the repo-rate. The first period is defined as the period from 18th of December 2012 to the 17th of February 2015, when the repo-rate is in the interval between 1 and 0 percent. The second period is the period from the 17th of February 2015 to the 7th of July 2015 when the repo-rate is in negative territory but the pass-through to deposit rates are above its ZLB.

C Additioanl details on the theoretical analysis

C.1 Partial equilibrium model

The bank holds liquid assets (L_t) due to liquidity risk. Liquidity risk arises because banks may be unable to liquidate their lending portfolio (B_t) in the event it's source of financing is disrupted, e.g. due to large scale withdrawals. The banking literature typically emphasizes that small depositors (D_t) are a stable form of financing and thus not subject to large liquidity risk. The key source of liquidity risk, instead, is external financing and large retail depositors (F_t). To cost of liquidity risk is captured in reduced form by the function $C(.)$ (see Freixas and Rochet (2008) for micro foundations of a cost function that captures liquidity risk). A bank has a larger motive to hold liquid assets the more it relies on F_t for financing, due to its higher liquidity risk. Moreover, liquidity risk is weakly decreasing in net worth. For given values of N_t and F_t, the bank is satiated in liquidity for some finite value of L_t, denoted L^*. The following functional form captures these assumptions:

$$C(F_t, L_t, N_t) = \begin{cases} \lambda_c F_t^{\gamma_f} L_t^{-\gamma_l} N_t^{-\gamma_n} \text{ if } L_t < L^*(N_t, F_t) \\ C^* \text{ if } L_t \geq L^*(N_t, F_t) \end{cases}$$

where $\gamma_f, \gamma_l, \gamma_n \geq 0$ are the elasticities of the banks liquidity costs with respect to external financing, liquid assets and net worth respectively. The following restriction on the parameters is imposed

$$\gamma_f > 1 + \gamma_l \tag{50}$$

which implies a lower bound on the elasticity of liquidity costs with respect to external financing. This condition ensures that the maximization problem of the bank is well behaved.

While all liquid asset reduce liquidity risk, reserves and paper currency contribute to reducing banks operational costs due their special "money role," e.g. in settling inter-bank transaction. Bank inter-mediation costs are decreasing in R and M up to a satiation point. The following functional form captures this assumption

$$\Psi(M_t, R_t) = \begin{cases} \lambda_R (M_t + R_t)^{-\gamma_R} \text{if} M_t + R_t < R^* \\ \bar{\Psi} \text{ if } M_t + R_t \geq R^* \end{cases}$$

where $\gamma_R > 0$ measures the elasticity of bank transaction costs with respect to R+M. For simplicity, cash and reserves play the same role in facilitating transactions. The difference between the two assets is that bank reserves pay an interest of i_t^r, while paper currency pays zero interest. This implies that the bank does not choose to hold currency while the interest on reserves is positive. It may, however, choose to do so once the reserve rate becomes negative. In this case the bank needs to consider storage costs of holding cash. The assumption that the bank does not hold cash at positive interest rates is a harmless abstraction as vault cash is a trivial component of banks balances. As money is an asset with a zero return, however, it requires careful consideration when thinking about the central banks ability to charge negative interest rates on reserve balances, as under the current institutional framework, banks can always substitute reserves for cash.

Holding money entails a storage cost, which is convex in money if $M_t > 0$:

$$S(M_t) = \lambda_{MS} (M_0 + M_t)^{\gamma_{MS}} \tag{51}$$

where $\gamma_{ms} > 1$. The parameter M_0 represents a fixed cost. Once rates turn negative, equation (51) determines how much cash the banks will hold.

The function $\Gamma(B_t, N_t)$ captures the cost of lending. It can be rationalized as being due to default, which becomes more pronounced the higher the lending due to finite monitoring resources, see e.g. Curdia and Woodford (2011) for a discussion of this interpretation. Both the total cost of lending and the marginal cost of lending are assumed to decrease in net worth. There is a variety of ways to underpin this assumption. The simplest is regulatory requirements, which stipulate that bank lending can only exceed a certain fraction of bank's net worth.[41]

The intermediation function takes the form

$$\Gamma(B, N) = \lambda_B B^{\nu} N^{-\iota}$$

where $1 \leq \nu < \bar{\nu}$ measures the elasticity of intermediation costs to lending, and $0 < \iota < \bar{\iota}$ measures the

[41] The simplest interpretation of the role of net worth is that it comes about due to a capital requirement, often modeled as

$$\frac{N_t}{B_t} \geq \kappa$$

where κ is a parameter. A constraint of this form would be a limiting case of the smooth function assumed here, one in which the cost of intermediation goes to infinity if the constraint is breached. More generally, tying deposits up in lending is typically considered costly due to the illiquidity of these assets – but illiquidity is of greater concern the lower the net worth of the bank. The dependence of lending costs on net worth is micro founded in Holmstrom and Tirole (1997) and Gertler and Kiyotaki (2010), as well as documented empirically, for example, in Jimenez, Ongena, Peydro, and Saurina (2012).

elasticity with respect to net worth, and

$$\bar{\nu} \equiv (\frac{i^b - i^d}{1+i^d})\frac{\bar{N}}{\bar{\Gamma}},\ \bar{\iota} \equiv \frac{1}{1+i^d}\frac{\bar{N}}{\bar{\Gamma}} \tag{52}$$

The upper bounds on $\bar{\nu}$ and $\bar{\iota}$ ensure the existence of a bounded solution to the banks problem.[42] Provided that $V'(N_t) > 0$ the first order conditions are:

$$\begin{aligned}
B_t : \frac{i_t^b - i_t^d}{1+i_t^d} &= \Gamma_B(B_t, N_t) && (53)\\
R_t : \frac{i_t^r - i_t^d}{1+i_t^d} &= C_L(F_t, L_t, N_t) + \Psi_R(R_t, M_t) && (54)\\
F_t : \frac{i_t^f - i_t^d}{1+i_t^d} &= -C_F(F_t, L_t, N_t) && (55)\\
A_t : \frac{i_t^a - i_t^d}{1+i_t^d} &= C_L(F_t, L_t, N_t) && (56)\\
N_t : \delta\omega - (1-\omega)\delta E_t(1+i_{t+1}^d)\phi_{t+1} + \phi_t + \Gamma_N(1+i_t^d) &= 0 && (57)\\
M_t : \frac{-i_t^d}{1+i_t^d} &= S_M + C_L + \Psi_M - \psi_t && (58)\\
M_t \geq 0, \psi_t \geq 0, \psi_t M_t &= 0 && (59)
\end{aligned}$$

where ϕ_t is the Lagrange multiplier of equation (11) and denotes the marginal value of net worth at time t to the bank, and the Envelope has been used to substitute out for the value function in the first order condition for N_t. Condition (58) is the money demand equation, and ψ_t is the Lagrange multiplier on $M_t \geq 0$. Condition (59) is a Kuhn-Tucker complementary slackness condition. A partial equilibrium is defined by an exogenous set of interest rates $\{i_t^d, i_t^b, i_t^r, i_t^f, i_t^a\}$ taken as given by the banks, and values for $\{B_t, M_t, N_t, F_t, R_t, A_t, \phi_t, \psi_t\}$ that solve equations (53)-(59), together with the flow budget constraint in equation (11).

C.2 Proof of the PLB

Proposition 2 (The Policy Rate Bound) If the cost of using paper currency is given by equation (14), the lower bound on the policy rate is

$$i_t^r \geq i^{PLB} \equiv -\alpha^m \tag{60}$$

Proof: Combine (54) and (58), noting that $\Psi_R = \Psi_M$, and assuming the DLB binding so that $i_d = 0$ to yield

$$\psi_t = S_M + i_t^r = \alpha^M + i_t^r 0 \tag{61}$$

where the last inequality follows from the complementary slackness condition (59).

C.3 Effect on Asset Holding and External Financing

The model has implications for how banks adjust their liquid asset holding and external financing in response to negative policy rates. A somewhat surprising empirical finding in Section 2 was that banks did

[42] The ratio $\frac{N}{\Gamma}$ is net worth as a fraction of the bank lending intermediation costs. Empirically this is a large number and this restriction does not impose a tight bound on the functional form assumed.

not increase their reliance on non-deposit financing despite substantially larger pass-through to these interest rates. Below we explain this in the context of the model. Let us first summarize a useful observation.

Proposition 5. *If $\rho^d = 0$ and $i_t^r < 0$, the bank is not satiated in liquidity and $L_t < L^*$.*

The proposition follows directly from equations (54)-(56). An immediate observation from inspecting (55) and (56) is that if there is greater pass-through of policy rates to the interest rate on assets and external financing, then there is a positive spread between the $i_t^a - i_t^d$ as well as $i_t^f - i_t^d$. This, in turn, implies that the bank will no longer be satiated in the liquid asset, $C_L > 0$ so that $L_t < L^*$.

The same is not true for reserves as seen from equation (54). It is possible that the bank is satiated in reserves to settle interbank transactions, i.e. $R_t \geq R^*$ so that $\chi_R = 0$, but that there is still a spread between i_t^r and i_t^d because reserves also serve a role to reduce liquidity risk via the function $C(.)$. Let us now discuss the implications of negative policy rates on external financing, reserves and other assets in the approximated equilibrium.

External financing Using the result from Proposition 5 that $L_t < L^*$, a log-linear approximation of the banks asset demand in equation (56), can be combined with a log-linear approximation of its demand for external financing in equation (55) to yield:

$$\hat{F}_t = \frac{(1+\gamma_l)(\rho^d - \rho^f)\frac{\bar{F}}{\bar{C}} + \gamma_f \frac{\bar{L}}{\bar{C}}(\rho^a - \rho^d)}{\gamma_f^2 - \gamma_f(1+\gamma_l)} \hat{i}_t^r + \frac{\gamma_f \gamma_n}{\gamma_f^2 - \gamma_f(1+\gamma_l)} \hat{N}_t \tag{62}$$

where the denominator is positive due to equation (50) and the solution for $\hat{N}_t$ is given by equation (17).

Equation (62) has a useful interpretation. If there is full pass-through, then external financing only increases and decreases with the bank's net worth. To the extent that negative rates have negative effects on banks equity, then, this reduces the banks reliance on external financing.

Consider the implication of a collapse in pass-through to the deposit rate, i.e. $\rho^d = 0$. Then the first term says that banks will increase their reliance on external financing if

$$\frac{\gamma_f}{1+\gamma_l}\frac{\bar{L}}{\bar{F}} < \frac{\rho^f}{\rho^a}$$

Hence, even if the policy rate is fully passed through to the external financing rate, the bank might still choose not to rely more heavily on this funding source. To interpret this conditions, observe that the interest rate the bank pays for external financing is only one part of the cost to the bank. The other is that it needs to hold more liquid assets due to the higher liquidity risk which this source of funding generates. If there is incomplete pass-through to the external financing, while there is full pass-through to the liquid assets, then the bank may reduce its reliance on external financing once rates turn negative.

Since the pass-through in Sweden to liquid asset was stronger than to the bonds the banks could issue, this might rationalize why they did not rely more strongly on external financing in response to negative rates.

The determination of reserves Suppose that $R_t < \bar{R}$. Combining the log-linear approximation of the the banks demand for assets and its demand for reserves yields

$$\hat{R}_t = \frac{1-\rho^a}{\gamma_R\gamma_R + 1\frac{\bar{\chi}}{\bar{R}}}\hat{\imath}_t^r \tag{63}$$

This suggests that if there is incomplete pass-through to liquid assets, then banks reduces their reserves when the policy rate is lowered. The reason is that reserves pay a lower interest rate than the alternative liquid assets.

In partial equilibrium, $\hat{R}_t$ measures the reserve holdings of a single bank. Once the model is integrated into general equilibrium however, the government sets both the interest on reserves, $\hat{\imath}_t^r$, *and the quantity* $\hat{R}_t$. This deserves further discussion.

Reserves measure the money balances a given bank holds on an account at the central bank. What can it do with reserves? One option is to exchange them for paper currency from the central bank. We have so far made the assumption that the storage cost of money is large enough so that the banks choose not to hold any cash, but we will revisit this assumption shortly. Another option is to use reserves to buy a liquid assets. An important observation, however, is that while this reduces the reserves of the bank, *it leaves aggregate reserves unchanged*.[43]

Reserves and liquid assets given a negative reserve rate A natural question is what happens if the central bank chooses a negative interest on reserves when $\rho^a < 1$. As equation (63) suggests, this means that any single bank (in partial equilibrium) tries to reduce its reserve holdings. Yet, in general equilibrium, total reserves are pinned down by the government. How is an equilibrium ensured?

Equation (63) suggests that the only way $R_t \geq R^*$ can be consistent with general equilibrium is that there is full pass-through to the liquid assets, i.e. $\rho^a = 1$. In this case, all banks remain fully satiated in reserves for the purpose of interbank transactions once the rate turns negative. Yet, as suggested by Proposition 5, once rates turn negative, banks are not satiated in liquid assets. Since reserve and other liquid asset are perfect substitutes, the partial equilibrium model we have specified will only pin down the quantity of total liquid asset, not its composition. Solving the log-linear approximation of equations (54) and (56) together then yields

$$\hat{L}_t = \frac{\bar{C}}{\bar{L}}\frac{1}{(1+\gamma_l)\gamma_l}\hat{\imath}_t^r + \frac{\gamma_f}{1+\gamma_l}\hat{F}_t - \frac{\gamma_n}{1+\gamma_l}\hat{N}_t$$

where the solution for $\hat{F}_t$ is given by equation (62) and $\hat{N}_t$ by equation (17). This relationship suggests that negative rates do not have clear predictions for what happens to aggregate liquid assets, it depends on the effect it has on external financing and net worth. Meanwhile, from the perspective of the banks, the split of liquid assets between reserves and other assets is indeterminate, and ultimately depends on the choices made by the central bank which can choose both $\hat{\imath}_t^r$ and $\hat{R}_t$.

A key prediction of the model, however, is that for the equilibrium to be consistent with large excess reserves, there has to be full pass-through to liquid assets. Full pass-through of the policy rate to liquid assets

[43] Consider a bank A which uses 100 dollars of reserves to buy assets from a seller that holds an account at bank B. The payment takes place by bank A drawing down 100 dollars of its reserve balances at the central bank, and depositing them at bank B which in turn increases deposits for the seller of the liquid asset. Thus aggregate reserves are unchanged, the reduction of bank A's reserves is met by an increase by bank B. Thus, absent changing the reserves into paper currency, it represents a "closed system" in which total reserves are determined by the central bank.

is consistent with the experience in Sweden. There the negative policy rate was passed through to other liquid assets, such as interest on government debt. The pass-through to liquid assets was often used as evidence for the success of the policy. While this element of the policy intervention was successful in terms of creating more fiscal space for government finances, it is not an indication of success from the perspective of the bank lending channel. Since the negative rates show up on the asset side of the banks balances sheet, rather than on the liability side, they are a net negative for the banks.

Reserves and money So far we have not considered the implication of banks' option to convert reserves into paper currency. For simplicity, we assumed that the bank held no currency because it served the same role as reserves, which in contrast to cash paid an interest, and moreover had no storage cost.

Once interest on reserves turn negative, however, converting reserves into cash can be an attractive option for the bank. Consider an equilibrium in which interest rates on reserves are sufficiently negative so that $M_t > 0$ and assume that there is no pass-through to deposit rates, so that $\rho^d = 0$. In this case $\psi_t = 0$ in equation (59) and the log-linear approximation of the banks demand for money is

$$\hat{M}_t = -\frac{1}{\gamma_M(\gamma_M - 1)}\frac{M^*}{\bar{S}}\hat{\imath}_t^r + \gamma_R(\gamma_R + 1)\frac{\bar{\chi}}{\bar{R}}\frac{M^*}{\bar{S}}\hat{R}_t$$

which suggests that as the reserve rate turns negative, banks increase cash holdings. since cash offers a gross return of zero, although entailing a storage cost. Overall, the experience so far has been that banks have not to a significant extent moved into cash, indicative of a large value of γ_M.

An important assumption is the storage cost was convex, i.e. $\gamma_{ms} > 1$. One might argue, however, that there are constant or increasing returns to building storage facilities for money, at least when cash holding are sufficiently large. Consider, for example, the assumption of constant returns, in which $\gamma_M = 1$, when cash holding become sufficienly large, so that cost of storage is proportional to the supply of money the bank holds for any further cash holding, i.e. $\lambda_{MS} M_t$. In this case, provided that the bank holds money, the interest rate on reserves is bounded by $i_t^r \geq -\lambda_{MS}$ as in the example we considered in the main text, since if it charges more negative interest on reserves than λ_{MS}, banks will convert reserves into paper currency and interbank transactions will be settled outside of the central bank.

The assumption that money is as effective as reserves in settling interbank transactions might seem a bit extreme. Nevertheless, it seems reasonable to think that if the central bank charges very negative rates, banks will find it in their interest to exchange reserves for currency, and instead construct an alternative payment system that solves the same problem at a lower cost.

C.4 General equilibrium model

We write real variables as lower case letters of their nominal counterparts. The following definition defines the non-linear equilibrium of our model.

Definition C.1 (Non-linear equilibrium)**.** A non-linear equilibrium of our model is a sequence of 23 endogenous variables $\left\{y_t,\ \pi_t,\ c_t^b,\ c_t^s, m_t^b,\ m_t^s,\ b_t^b,\ b_t,\ r_t,\ a_t,\ m_t\ ,\psi_t, f_t, l_t, n_t\ , z_t, F_t,\ K_t,\ \lambda_t,\ \Delta_t, \overline{W}, T_t, \overline{f}\right\}_{t=0}^{\infty}$ and 5 endogenous prices $\left\{i_t^b,\ i_t^d, i_t^f\ , i_t^g\ i_t^r\right\}_{t=0}^{\infty}$ such that equations (64) - (91) holds.

From the household problems there is a consumption Euler equation for each the borrower (64) and the saver (65), respectively, the budget constraint of the borrower (66), the money demand equations for each type of agent (67) and (68). The aggregate resource constraint (82) implies all production is consumed. b_t^b is defined as aggregate borrowing per borrower, while $b_t \equiv \chi b_t^b$ is aggregate borrowing (81).

The banks problem give rise to (69)-(76). Relative to the text, the price level is now endogenous.

$$u'(c_t^b)\zeta_t = \beta_b(1+i_t^b)E_t u'(c_{t+1}^b)\Pi_{t+1}^{-1}\zeta_{t+1} \tag{64}$$

$$u'(C_t^s)\zeta_t = \beta_s(1+i_t^d)E_t u'(c_{t+1}^s)\Pi_{t+1}^{-1}\zeta_{t+1} \tag{65}$$

$$b_t^b = (1+i_{t-1}^b)\Pi_t^{-1}b_{t-1}^b - y_t + c_t^b - (1-\gamma^b)\Pi_t^{-1}m_{t-1}^s + m_t^s \tag{66}$$

$$\frac{\Omega'\left(m_t^s\right)}{u'(c_t^s)} = \frac{i_t^d}{1+i_t^d} \tag{67}$$

$$\frac{\Omega'\left(m_t^s\right)}{u'(c_t^b)} = \frac{i_t^b}{1+i_t^b} \tag{68}$$

$$\frac{i_t^b - i_t^d}{1+i_t^d} = \Gamma_b(b_t, n_t) \tag{69}$$

$$\frac{i_t^r - i_t^d}{1+i_t^d} = C_l(f_t, l_t, n_t) + \chi_m(r_t, m_t) \tag{70}$$

$$\frac{i_t^a - i_t^d}{1+i_t^d} = C_l(f_t, l_t, n_t) \tag{71}$$

$$\frac{i_t^f - i_d^r}{1+i_t^d} = -C_f\left(f_t, l_t, n_t\right) \tag{72}$$

$$-\frac{i_t^d}{1+i_t^d} = S_m(m_t) + C_l(ft, l_t, n_t) + \chi_m(r_t, m_t) + \psi_t \tag{73}$$

$$\psi_t M_t = 0 \tag{74}$$

$$n_t = \left(1+i_t^d\right)\left(z_t + (1-\omega)\pi_t^{-1}n_{t-1}\right) \tag{75}$$

$$z_t = \frac{i_t^b - i_t^d}{1+i_t^d}b_t + \frac{i_t^r - i_t^d}{1+i_t^d}r_t + \frac{i_t^g - i_t^d}{1+i_t^d}a_t - \frac{i_t^d}{1+i_t^d}m_t - \frac{i_t^f - i_t^d}{1+i_t^d}f_t - C(f_t, l_t, n_t) - \Gamma\left(b_t, n_t\right) - \chi(r_t, m_t) \tag{76}$$

$$i_t^r = r_t^n \Pi_t^{\phi_\Pi} y_t^{\phi_Y} \tag{77}$$

$$i_t^f = \rho^f i_t^r \tag{78}$$

$$i_t^g = \rho^a i_t^r \tag{79}$$

$$i_t^d = max\left\{-\gamma^s,\ i_t^r\right\} \tag{80}$$

$$\chi b_t^b = b_t \tag{81}$$

$$y_t = \chi c_t^b + (1-\chi)c_t^s \tag{82}$$

$$\tag{83}$$

$$a_t + m_t + r_t = (1 + i_t^a)a_{t-1} + m_{t-1} + (1 + i_{t-1}^r)r_{t-1} + G - T_t \tag{84}$$

$$a_t + m_t + r_t = \bar{W} \tag{85}$$

$$f_t = \bar{f} \tag{86}$$

$$\left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha} \right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{87}$$

$$F_t = \lambda_t y_t + \alpha\beta E_t \left[F_{t+1} \left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} \right] \tag{88}$$

$$K_t = \mu \frac{\lambda_t \Delta_t^{\eta} y_t^{1+\eta}}{q\ \exp\{-q y_t\}} + \alpha\beta E_t \left[K_{t+1} \left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} \right] \tag{89}$$

$$\lambda_t = q \left(\chi \exp\{-q c_t^b\} + (1-\chi) \exp\{-q c_t^s\} \right) \tag{90}$$

$$\Delta_t = \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta} \Delta_{t-1} + (1-\alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}} \tag{91}$$

C.5 Steady state

The model is approximated around a steady state when inflation is at the target level. We consider the cashless limit, in which neither the household or the banks hold money. However, we assume that they can hold money, which gives rise the the bound on deposits as explained in the text.

In steady-state, $\Delta = 1$ and $\bar{F} = \bar{K}$, which implies that production is pinned down by the AS curve.

$$\mu \times \frac{y^{\eta}}{q\,\exp\{-qY\}} = 1 \tag{92}$$

The consumption Euler equations (65) and (65) of the saver and borrowers imply that the the inverse of the discount factor of each agent is equal to the real borrowing and savings rate

$$(\beta^b)^{-1} = \frac{1 + \bar{\imath}^b}{\bar{\Pi}} \tag{93}$$

and

$$(\beta^s)^{-1} = \frac{1 + \bar{\imath}^s}{\bar{\Pi}} \tag{94}$$

In steady-state, we assume $\rho^f = 1$, $\rho^a = 1$ which implies that $\bar{\imath}^f = \bar{\imath}^g = \bar{\imath}^r$ where $i_t^r = r^n \Pi^{\phi_\Pi} y^{\phi_Y}$

The budget constraint (66), together with that in steady state implies

$$\frac{\bar{c}^b}{\bar{y}} = 1 - \frac{1-\beta^b}{\beta^b} \frac{1}{\chi} \bar{b} \tag{95}$$

which when combined with the resource constraint (82) implies the steady state consumption of the saver as

$$\frac{\bar{c}^s}{\bar{y}} = 1 + \frac{1}{1-\chi}\frac{1-\beta^b}{\beta^b}\bar{b} \tag{96}$$

We directly choose the steady-state output debt $\frac{\bar{b}}{\bar{y}}$ from the data, in which case the two equations above pin down $\frac{\bar{c}^b}{\bar{y}}$ and $\frac{\bar{c}^s}{\bar{y}}$ for a given χ. Neither ratio, however, enters the linear approximation of the model shown in next subsection.

The first-order condition for lending (53) implies that

$$\frac{\bar{i}^b - \bar{i}^d}{1+\bar{i}^b} = 1 - \frac{\beta^b}{\beta^s} = \nu\frac{\bar{\Gamma}}{\bar{b}} \tag{97}$$

which, given $\frac{\bar{b}}{\bar{y}}$ and ν pins down

$$\frac{\bar{\Gamma}}{\bar{y}} = \frac{1-\frac{\beta^b}{\beta^s}}{\nu\frac{\bar{b}}{\bar{y}}} \tag{98}$$

Steady state profit is then

$$\bar{z} = (1-\frac{\beta^b}{\beta^s})\bar{b} - \bar{C} - \bar{\Gamma} - \bar{\chi} \tag{99}$$

Steady state net worth is

$$\bar{n} = (1+i^d)(\bar{z} + (1-\omega)n\pi^{-1}) \tag{100}$$

or

$$\frac{\bar{z}}{\bar{n}} = \beta^s\pi^{-1} - (1-\omega)\pi^{-1} = \frac{\beta^s+\omega-1}{\pi} \tag{101}$$

Finally, we directly pick the steady state values of $\bar{r}$, $\bar{a}$ and $\bar{f}$ from the data. These do not affect the steady state values of the variables described above, but are important for dynamics.

C.6 Log-linear (approximated) equilibrium

The model is solved via log-linear approximation. Variables are defined as: $\hat{c}^b_t \equiv \frac{c^b_t - \bar{c}^b}{\bar{y}}$, $\hat{c}^s_t \equiv \frac{c^s_t - \bar{c}^s}{\bar{y}}$, $y_t \equiv \log\frac{y_t}{\bar{y}}$, $\hat{\pi}_t \equiv \log\frac{\Pi_t}{\Pi}$, $\hat{\imath}^d_t \equiv \log\frac{1+i^d_t}{1+\bar{\imath}^d}$, $\hat{\imath}^r_t \equiv \log\frac{1+i^r_t}{1+\bar{\imath}^r}$, $\hat{\imath}^b_t \equiv \log\frac{1+i^b_t}{1+\bar{\imath}^b}$, $\hat{b}_t \equiv \log\frac{b_t}{\bar{b}}$. Approximate equilibrium is a collection of stochastic processes for $\{\hat{c}^b_t, \hat{c}^s_t, \hat{\imath}^b_t, \hat{\imath}^d_t, \hat{\imath}^f_t, \hat{\imath}^r_t, \hat{\imath}^a_t, \hat{n}_t, \hat{y}_t, \hat{\pi}_t, \hat{b}_t, \hat{z}_t\}$ that solve equations (102)-(113)

Consumption Euler Equations (64) and the saver (65) yield

$$\hat{c}^b_t = E_t\hat{c}^b_{t+1} - \sigma(\hat{\imath}^b_t - E_t\hat{\pi}_{t+1} + \hat{\zeta}_t - E_t\hat{\zeta}_{t+1}) \tag{102}$$

$$\hat{c}^s_t = E_t\hat{c}^s_{t+1} - \sigma(\hat{\imath}^d_t - E_t\hat{\pi}_{t+1} + \hat{\zeta}_t - E_t\hat{\zeta}_{t+1}) \tag{103}$$

Budget constraint of borrower (66) yields

$$\hat{b}_t = \frac{1}{\beta^b}\hat{b}_{t-1} - \frac{1}{\beta^b}\hat{\pi}_t + \frac{1}{\beta^b}\hat{\imath}^b_{t-1} - \chi\frac{\bar{y}}{\bar{b}}\hat{y}_t + \chi\frac{\bar{y}}{\bar{b}}\hat{c}^b_t \tag{104}$$

Lending condition (69) yields

$$\hat{\imath}^b_t - \hat{\imath}^d_t = (\nu - 1)\frac{\beta^s - \beta^b}{\beta^s}\hat{b}_t - \iota\frac{\beta^s - \beta^b}{\beta^s}\hat{n}_t \tag{105}$$

Net worth condition yields

$$\beta^s\hat{n}_t = \beta^s\hat{\imath}^d_t + \hat{z}_t + \frac{1-\omega}{\bar{\Pi}}\hat{n}_{t-1} - \frac{\beta^b}{\beta^s}\frac{1-\omega}{\bar{\Pi}}\hat{\pi}_t \tag{106}$$

The definition of profits yields

$$\frac{\bar{n}}{\bar{\Lambda}}\hat{z}_t = -\{\frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}} + \frac{\bar{r}}{\bar{\Lambda}} + \frac{\bar{a}}{\bar{\Lambda}} - \frac{\bar{f}}{\bar{\Lambda}}\}\hat{\imath}^d_t + \frac{\beta^b}{\beta^s}\frac{\bar{b}}{\bar{\Lambda}}\hat{\imath}^b_t + \frac{\bar{r}}{\bar{\Lambda}}\hat{\imath}^r_t + \frac{\bar{a}}{\bar{\Lambda}}\hat{\imath}^a_t - \frac{\bar{f}}{\bar{\Lambda}}\hat{\imath}^f_t + \iota\frac{\bar{\Gamma}}{\bar{\Lambda}}\hat{n}_t \tag{107}$$

The resource constraint yields

$$\hat{y}_t = \chi\hat{c}^b_t + (1-\chi)\hat{c}^s_t \tag{108}$$

The supply side yields a Phillips Curve

$$\hat{\pi}_t = \kappa\hat{Y}_t + \beta E_t\hat{\pi}_{t+1} \tag{109}$$

Policy rule

$$\hat{\imath}^r_t = \hat{r}^n_t + \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t \tag{110}$$

Deposit bound

$$\hat{\imath}^d_t = \max(i^{elb}, \hat{\imath}^r_t) \tag{111}$$

Pass-through to other rates

$$\hat{\imath}^a_t = \rho^a\hat{\imath}^r_t \text{ if } \hat{\imath}^r_t < i^{elb} \text{ and } \hat{\imath}^a_t = \hat{\imath}^r_t \text{ if } \hat{\imath}^r_t > i^{elb} \tag{112}$$

$$\hat{\imath}^f_t = \rho^f\hat{\imath}^r_t \text{ if } \hat{\imath}^r_t < i^{elb} \text{ and } \hat{\imath}^f_t = \hat{\imath}^r_t \text{ if } \hat{\imath}^r_t > i^{elb} \tag{113}$$

Coefficient are defined as: $\sigma \equiv \frac{1}{\bar{q}\bar{y}}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(\eta+\sigma^{-1})}$, $\beta \equiv \chi\beta^b + (1-\chi)\beta^s$, $i^{elb} \equiv -\log(1+\bar{\imath}^d)$. Observe that once one chooses parameters for $(\sigma, \kappa, \beta.\beta^s, \beta^b, \chi, \nu, \iota)$ and pins from the data $(\frac{\bar{y}}{\bar{b}}, \frac{\bar{b}}{\bar{\Lambda}}, \frac{\bar{r}}{\bar{\Lambda}}, \frac{\bar{a}}{\bar{\Lambda}}, \frac{\bar{f}}{\bar{\Lambda}})$, then ω and $\frac{\bar{\Gamma}}{\bar{\Lambda}}$ are not free parameters but implied by the model equations.

The analytic characterization in the text for the IS equation is obtained by (1) combining the two consumption Euler equations with the resource constraint and (2) defining the natural rate of interest, $\hat{r}^n_t$, as the interest rate consistent with zero deviation of output from steady state and inflation on target in the absence of a lower bound.