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ABSTRACT

Standard U.S. practice for public cost-benefit analysis is to bound the discount rate with the interest rate paid by capital investment and rate received by consumers. These bounding cases arise when future benefits accrue to consumers in either a two-period model or as a perpetuity. We generalize to consider benefits paid in any future period. We find that the appropriate discount rate converges to the consumption rate for benefits in the distant future. More generally, the range of rates depends on the temporal pattern. Applied to CO2 damages, we estimate the appropriate discount rates of between 2.6 and 3.4 percent.

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1. Introduction

Conducting cost-benefit analysis (CBA) for government policy requires a discount rate to compare costs and benefits over time in order to establish, on net, whether total benefits exceed total costs. For more than 15 years, federal guidelines for government CBA have indicated that two discount rates should be used, 7 and 3 percent (OMB 2003). When applied to government policies with costs today and benefits extending far into the future—our stylized arrangement throughout this paper[1]—the different outcomes associated with these rates can be striking. Recent government estimates of climate change benefits, for example, are six to nine times higher using 3 percent rather than 7 percent (see Appendix A).

The rationale for this range of rates is based on whether the costs of regulation today fall primarily on the allocation of private capital or instead directly affect household consumption. These rates reflect the pre-tax return paid by private capital and the return received by consumers, respectively, with the difference owing largely to taxes. For shorthand and consistency, we refer to these two rates as the *investment* and *consumer* rates. Using these two rates to bound the correct rate has a long history; it can also be derived under two alternative, specific assumptions. Either the economy exists in only two periods (Harberger 1972; Sandmo and Drèze 1971) or the pattern of benefits over time is a perpetuity (Marglin 1963a; Drèze 1974; Sjaastad and Wisecarver 1977).

[1] While costs and benefits could occur in any period, we have in mind problems where costs today are being measured against benefits in future periods. Most public policy problems break down this way, as investments are made today that then yield benefits. Even when there may be operating costs in the future, these can be subtracted from the future benefit flows.

At the same time, practical deviations from these assumptions have been shown to yield dramatically different results (Bradford 1975). The solution is to convert all costs and benefits to their consumption equivalents and then to discount at the consumer rate (Marglin 1963a, 1963b; Dasgupta, Marglin, and Sen 1972; Bradford 1975). Yet, such an approach is generally viewed as impractical because the shadow price to convert capital goods into consumption equivalents, as well as the distribution of costs and benefits onto capital versus consumption goods, are not always well-known. [2]

The main point of this paper is to show how the shadow price approach can be used to derive more general bounds on the discount rate when benefits are no longer assumed to be a perpetuity. In particular, we show that these bounds converge to the consumer interest rate when valuing benefits far into the future. This convergence result does not depend on the exact value of the shadow price itself or the distribution of costs and benefits on capital versus consumption goods. Intuitively, the effect of shadow pricing is at most a bounded multiplicative factor applied on top of ordinary discounting at the consumer rate. Mechanically, we can incorporate the bounding multiplicative factor into an upward and downward adjustment to the consumer discount rate. Looking far into the future, smaller and smaller adjustments to the consumer rate will be needed to compensate for that factor.

More generally, this range depends on the pattern of future benefits over time and the shadow price of capital. Applied to the pattern of climate mitigation benefits arising in a recent National Academy of Science report (NAS 2017), the

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[2] As stated on page 33 of Circular A-4 (OMB 2003), "Any agency that wishes to tackle this challenging analytical task should check with OMB before proceeding."

appropriate range is quite narrow. The estimated discount rate would be between 2.3 and 4.0 percent if we use the government's investment and consumer rates to derive a bound on the shadow price. Adding additional assumptions based on a Ramsey growth model, we can derive a particular shadow price of capital and an even narrower range of 2.6 to 3.4 percent for the discount rate. On the other hand, the discount rate for a policy with benefits entirely in the very near term could vary from -50 to +50 percent depending on the distribution of impacts on capital versus consumption goods.

At a practical level, our results provide an argument for focusing the social discount rate[3] discussion on an interval centered on the consumer discount rate and based on each application's particular pattern of benefits, particularly over long horizons. This need not require any more information than current government CBA, with the investment and consumer rates implying a bound on the shadow price.

We are not the first to suggest that the consumer rate is a more appropriate discount rate over long horizons, but we believe we are the first to provide particularly compelling arguments against the investment rate. OMB guidelines note that policies with intergenerational effects raise additional, ethical concerns. They suggest that policies with such effects might consider a lower, but positive rate, as a sensitivity analysis *in addition to* calculations based on 3 and 7 percent. They do not suggest the investment interest rate is incorrect. Government estimates of the social cost of carbon dioxide (SC-CO_2) under the Obama administration were based only on consumption interest rates. The underlying

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[3] We use the term social discount rate when we want to emphasize the use of discounting for public investments or government regulation that generates public goods.

analysis (IWG 2010) concluded the consumer rate was appropriate based on the "economics literature" but it is hard to find such a clear conclusion in the noted references. The NAS report argues against an investment interest rate because such a rate is correct only under very restrictive assumptions (NAS 2017). While true, the same could be said about the consumption interest rate so long as taxes create a wedge between consumer and investment rates and the impacts on capital and consumption are unclear.

A different but related line of research has argued, generally, for lower discount rates in the future based on uncertain states of nature (Weitzman 2001; Newell and Pizer 2003; Arrow et al. 2014; Gollier 2014). Those arguments follow from the observation that, as we look further into the future, states of nature with persistently high discount rates matter less in expected net-present-value (NPV) calculations. In this paper, the future discount rate is not uncertain. Rather, we are uncertain about whether costs and benefits affect capital investment or household consumption (and, to a lesser extent, the appropriate shadow price to convert between the two).

There is also a different but related debate over the appropriate investment and consumption interest rates for CBA, regardless of uncertainty about which one to use or uncertainty in the future. A recent government white paper, for example, attempted to distill recent evidence regarding both rates, arguing that the current consumer rate might be lower than 3 percent (CEA 2017). Another paper recently surveyed experts and arrived at a similar conclusion (Drupp et al. 2018).

For those concerned about CBA's application to regulating persistent environmental pollution, including carbon dioxide and other greenhouse gases, it is hard to overstate the importance of these questions about the appropriate discount rate. On October 16, 2017, the Trump administration issued a CBA of the Clean Power Plan to support its repeal (U.S. EPA 2017). In contrast to the

Obama administration's CBA that used a central benefit estimate based on a 3 percent discount rate (U.S. EPA 2015), the new CBA of the same regulation gave equal weight to estimates based on 3 and 7 percent. As noted above and described in more detail in Appendix A, climate benefits are 6 to 9 times lower using 7 rather than 3 percent. By focusing on the mid-point of benefit estimates based on 3 and 7 percent, the new central benefit estimate is brought down by more than 40 percent. This question of whether or not to use the investment rate has thus emerged as a particularly salient discounting policy question.

Our paper is organized as follows. In the next section, we review the literature supporting the use of consumer and investment rates as bounding values for the social discount rate, including Marglin (1963a, 1963b), Harberger (1972) and Sjaastad and Wisecarver (1977) among others. We also review the very different results obtained by Bradford (1975) when the assumptions underlying that result is relaxed. As we review these early papers, we develop a simple model to formalize ideas and define the range of social discount rates appropriate for an arbitrary pattern of benefits. We use this model to examine long-run behavior, where we show that the range of discount rates for benefits in the distant future converges to the consumption rate of interest. More generally, the range of appropriate rates depends on the temporal pattern of benefits being valued. The other key parameter that defines this range is the shadow price of capital, which we explore in more detail in section 4. We show that the ratio of investment and consumer rates is an upper bound on the shadow price under relatively weak assumptions. Based on the U.S. government rates, this would be 7/3. Using a Ramsey model along with additional assumptions about depreciation, the output-capital elasticity, and growth, we derive a shadow price of capital of roughly 1.5. This implies that the range of social discount rate for benefits several decades in the future has already converged to roughly the consumer rate. Finally, we turn to the climate change application. Using the temporal pattern of climate change damages used in the noted NAS report (2017), we show that the appropriate rate for discounting climate damages lies between 2.6 and 3.4 percent based on a consumer rate of 3 percent and a shadow price of 1.5. Using this range, we find that estimates for social cost of CO_2 differ by a factor of two. This is considerably less than estimates using 3 and 7 percent rates, where the SC-CO_2 varies by at least a factor of six.

2. History of discounting at consumption and investment interest rates

Discussions of discounting in CBA for public policy have recognized two main approaches to identifying appropriate discount rates. A *prescriptive* approach examines the ethical basis for discounting and focuses on some notion of how society ought to value future consequences. A *descriptive* approach instead focuses on the observed behavior of households and firms (Arrow et al., 1996).

Government policy in the United States has largely followed the descriptive approach. Circular A-94 (OMB 1992; BOB 1969) established 10 percent as the official discount rate for government CBA. This rate was based on the "average rate of return on private investment, before taxes and after inflation." It was subsequently revised to 7 percent (OMB 1992).

The idea that public investments ought to provide the same return as private investments has a long history in economic thought (Harberger 1972; Lind 1982, 1990). Put simply, why invest in public projects that provide a lower return than private alternatives? Moreover, if the public project is displacing private investment, the notion is even more compelling as the opportunity cost of the forgone investment. Arguably, most public projects or government regulation involve upfront capital investment via either government spending or private dollars.

A countervailing view can arise when, instead, the project is viewed through the eyes of consumers. Economists generally look to households and revealed preference as the ultimate arbiters of welfare value. Household preferences should be revealed by the intertemporal prices that they face, so the relevant question is the rate of interest available for household savings. In particular, what kind of return does a household require to be better off? This would be particularly compelling for public projects that take away household consumption in one period and pay it back in another.

In a competitive capital market without distortions, a household's return to savings and the return on capital investment are both equal to the same rate. However, taxes on the income from capital drive a wedge between these two rates (Baumol 1968). This poses a puzzle for the descriptive approach: which is correct, the observed behavior of households or firms? Recognizing this ambiguity, government policy changed in 2000 to provide equal prominence to both a consumer and investment rate—3 and 7 percent, respectively (Lew 2000; OMB 2003).

The theoretical underpinnings of this practice date back to Marglin (1963a, 1963b), Harberger (1972), and Sjaastad and Wisecarver (1977), who all argue the correct discount rate lies between these two values. Here, we briefly review the Harberger and Sjaastad and Wisecarver approaches, which provide two different framings that yield the same result.

2.1 *Opportunity cost of capital for public projects*

The Harberger approach begins with a simple partial equilibrium that explains the distortion between consumption and investment interest rates. That is, there is a supply schedule of savings $S(r_c)$ net of taxes on capital income and a demand for investment (public and private) gross of taxes on capital return $I(r_i) = I_{private}(r_i) + I_{public}$. Here, we assume public investment is fixed. The investment and consumption interest rates are related by the tax rate τ, i.e. $r_c = r_i(1-\tau)$. An initial equilibrium is given by the intersection of the solid blue (investment) and red (savings) lines with $I = S = I_0$ in Figure 1.

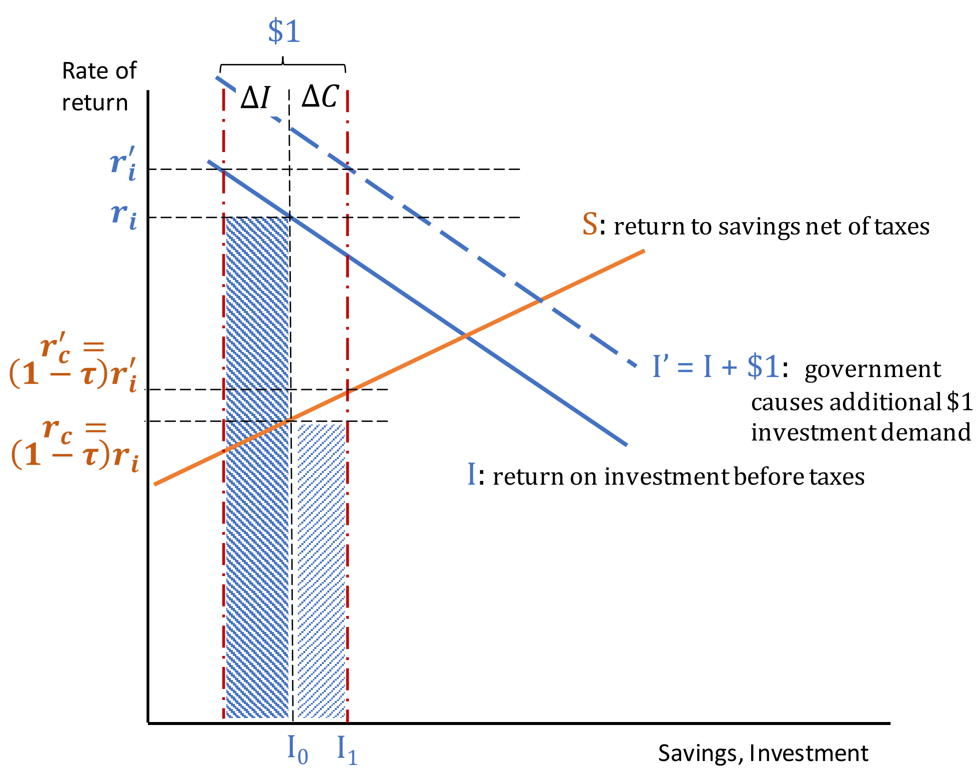

Figure 1 Partial equilibrium model of opportunity cost of capital for public projects.

We now consider how a government project or policy that increases demand for investment goods by a marginal amount, say \$1, affects private investment and savings (with increases in savings reducing consumption by an equal amount).[4] The outcome can be depicted graphically in Figure 1 by the equilibrium $I' = S = I_1$. Savings rises (and consumption declines) by ΔC. Meanwhile, private investment declines by ΔI, remembering public investment demand (including the new additional dollar) is fixed. The opportunity cost of postponing consumption by one period is given by the light-shaded area $r_c \Delta C$ while the opportunity cost of postponing private investment is the dark-shaded area $r_i \Delta I$ – the foregone surplus in the figure.[5] So long as the additional dollar of government caused demand repays that amount (plus the \$1 principal) in the next period, the economy maintains its current level of welfare. This leads to a weighted cost of capital,[6]

$$\rho_h = \frac{\Delta C}{\Delta C + \Delta I} r_c + \frac{\Delta I}{\Delta C + \Delta I} r_i, \tag{1}$$

Here, we let ρ_h reflect the social discount rate defined by Harberger. That is, the required return on the public project is the weighted average of the consumer and investment interest rates that makes the economy whole—hence the notion of opportunity cost. Recalling our assumption that costs are up front and benefits flow in the future, the alternative use of the investment and consumer rate provides bounding values for the CBA without knowing exactly how

4 In this paper, dollars spent today on public investment are not narrowly defined as costs of a project generating financial returns, but broadly refer to any public policy or regulations that lead to government spending and/or alter private investment decisions.

5 Ignoring the slight difference in prices at the new equilibrium.

6 See page 99 of Harberger (1972).

consumption and investment are affected. The investment rate will maximally disfavor future benefits, while the consumer rate will maximally favor them.

Harberger's partial equilibrium analysis has been derived in a general equilibrium context by Sandmo and Drèze (1971) and Drèze (1974) for both the case of a two-period economy and the case of an infinite horizon where investments (public and private) yield a perpetual return. This approach views the rates of return as the relevant prices associated with investment and consumption displaced by a public investment. Implicit in this approach is that public and private investment have similar patterns of future returns over time. That is how opportunity cost of investment is defined. An alternative approach is to convert the pattern of future investment returns into an equivalent value today based on the consumer rate of interest.

2.2 Shadow price of capital approach

The shadow price of capital approach places consumer valuation at the center of the analysis (Marglin 1963a, 1963b; Dasgupta, Marglin, and Sen 1972; Bradford 1975). Impacts on different goods—including investment—in any period can be translated into equivalent consumption impacts in that period based on either actual market prices or *shadow prices*. Shadow prices are appropriate for either unpriced goods that affect household utility or, as in the case of capital, priced goods where the price is distorted by taxes, regulation, or market failures. The shadow price of a particular good in period t reflects the equivalent dollar change in period t consumption from a dollar change in that good at the established equilibrium. Once we have consumption-equivalent changes in each period, the net present value is computed using a consumption rate of interest.

We would expect these two approaches to deliver the same result when assumptions about the pattern of investment returns are the same. Sjaastad and

Wisecarver (1977) show precisely this result for both cases noted above, a two-period economy and a perpetual stream of constant benefits. We now develop those cases on our way to the most general possible model.

The former case essentially follows Harberger. Assuming the world exists in two periods, we can write the net present value (NPV) of a project that yields future benefits B for each dollar invested today as

$$NPV = -\theta_0 + \frac{B}{(1 + r_c)}, \tag{2}$$

where r_c is the consumption rate of interest and θ_0 is the social opportunity cost per government dollar spent today in terms of household consumption:

$$\theta_0 = \alpha \cdot v + (1 - \alpha) \cdot 1 \tag{3}$$

In this definition α is the share of today's cost displacing capital and $(1 - \alpha)$ is the share of displacing consumption. The shadow price of capital is denoted by v. The parameter θ_0 is therefore bounded by 1 and v. Without a future beyond the second period, it is easy to figure out the shadow price of a capital investment with return r_i as (4) in a two-period model,

$$v_{2p} = \frac{1 + r_i}{1 + r_c} \tag{4}$$

That is, the value of the investment today is the gross return, including return of principal, $1 + r_i$ valued in terms of today's consumption based on the consumer discount rate r_c. Here, we use the subscript "*2p*" to remind us this is the shadow price in a two-period model. If we rewrite the *NPV* in terms of "project dollars" by dividing by θ_0, we have

$$\begin{aligned} NPV' = -1 + \frac{B}{\theta_0(1 + r_c)} &= -1 + \frac{B}{1 + (\alpha r_i + (1 - \alpha) r_c)} \\ &= -1 + \frac{B}{1 + \rho_h}, \end{aligned} \tag{5}$$

This yields the Harberger result that the correct social discount rate is the weighted average of the consumer and investment rates.

For the latter case, consider a public project or policy yielding a perpetual stream of constant benefits, B, every period in the future. We can again define NPV as:

$$NPV = -\theta_0 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c)^t}, \tag{6}$$

And we can again rewrite in terms of project dollars by slightly re-arranging,

$$\begin{aligned} NPV &= -\theta_0 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c)^t} = -\theta_0 + \frac{B}{r_C} = \theta_0 \left(-1 + \frac{B}{r_c\theta_0}\right) \\ &= \theta_0 \cdot NPV' \end{aligned} \tag{7}$$

where

$$NPV' = -1 + \sum_{t=1}^{\infty} \frac{B}{(1+r_c\theta_0)^t} \tag{8}$$

As in the two-period case, a key parameter is θ_0 and, in turn, the shadow price of capital v that defines θ_0. If we further assume each dollar of capital investment has a perpetual return of r_i dollars of household consumption in each future period, as first proposed by Marglin, it is straightforward to compute the shadow price:

$$v_m = \sum_{t=1}^{\infty} \frac{r_i}{(1+r_c)^t} = \frac{r_i}{r_c}. \tag{9}$$

Here, we use the subscript "*m*" to denote Marglin's shadow price (we will return to the more general question of shadow prices in section 4). We can substitute this shadow price of capital v_m into θ_0, and θ_0 into (8), to yield

$$
\begin{aligned}
NPV' &= -1 + \sum_{t=1}^{\infty} \frac{B}{\big(1 + (\alpha r_i + (1-\alpha) r_c)\big)^t} \\
&= -1 + \sum_{t=1}^{\infty} \frac{B}{(1+\rho_h)^t}.
\end{aligned} \tag{10}
$$

Again, we have the social discount rate equal to a weighted average of the investment and consumer rates.

This result, and the discussion in Sjaastad and Wisecarver more generally, emphasizes how the opportunity cost of capital and shadow price of capital approaches can yield the same result. Both can support the idea that the social discount rate equals a weighted average of the investment and consumption rates of interest. With the assumption of costs up front and benefits in the future, already implicit in the above derivations, CBA based on the investment and consumer rates provide bounding values.

We note that this result supports the recommended approach followed by the U.S. Government since 2000. It also highlights the rather strong assumptions that are required. In particular, benefits either are repaid in a second, final period,[7] or are constant and perpetual.

There is a second, subtle but equally strong assumption lurking in equations (2) and (6). While much attention focuses on whether costs in the first period fall on either consumption or investment, benefits are assumed to accrue entirely in the form of consumption. In the two-period model, there is no investment in the second, final period, and the assumption is unavoidable. This is

[7] As shown in Bradford (1975), the key assumption is that the distortion between the consumer and investment rates vanishes in the second period and the shadow price of capital equals one.

more clearly an assumption in the perpetuity models developed by Sjaastad and Wisecarver, Marglin, and Drèze. If we relax the assumption that benefits accrue entirely to consumers in the perpetuity model, treating costs and benefits symmetrically, it is easy to show that the lower interest rate bound would become r_c^2/r_i. We now turn to that case in the two-period model, developed by Bradford (1975).

2.3 Shadow price of capital, generalization over two periods

Following the idea in Bradford (1975), let β be the share of benefits accruing to private investment in the second period. The parameter $\theta_1 = \beta v + (1-\beta)$ is the consumption equivalent of one dollar of benefits, averaged across the share accruing to consumption and the share accruing to capital. Here, we assume v is also the shadow price of capital in the second period.[8] For a dollar of investment yielding total benefits B in the next period, we now generalize (2)

$$NPV = -\theta_0 + \frac{\theta_1 B}{(1+r_c)}, \tag{11}$$

As before, we can rewrite this NPV in terms of project dollars,

$$\begin{aligned} NPV' = \frac{NPV}{\theta_0} &= -1 + \frac{\left(\frac{\theta_1}{\theta_0}\right) B}{(1+r_c)} = -1 + \frac{B}{1 + \left((1+r_c)\frac{\theta_0}{\theta_1} - 1\right)} \\ &= -1 + \frac{B}{1+\rho_b} \end{aligned} \tag{12}$$

[8] Unlike Bradford, we assume the same shadow price in both periods. This is unimportant for our ultimate goal, which is to consider the consequences of extreme (largest and smallest) values of the ratio θ_0/θ_1, equalling v and $1/v$, respectively given $v \geq 1$, and determined by the largest value of v in both cases.

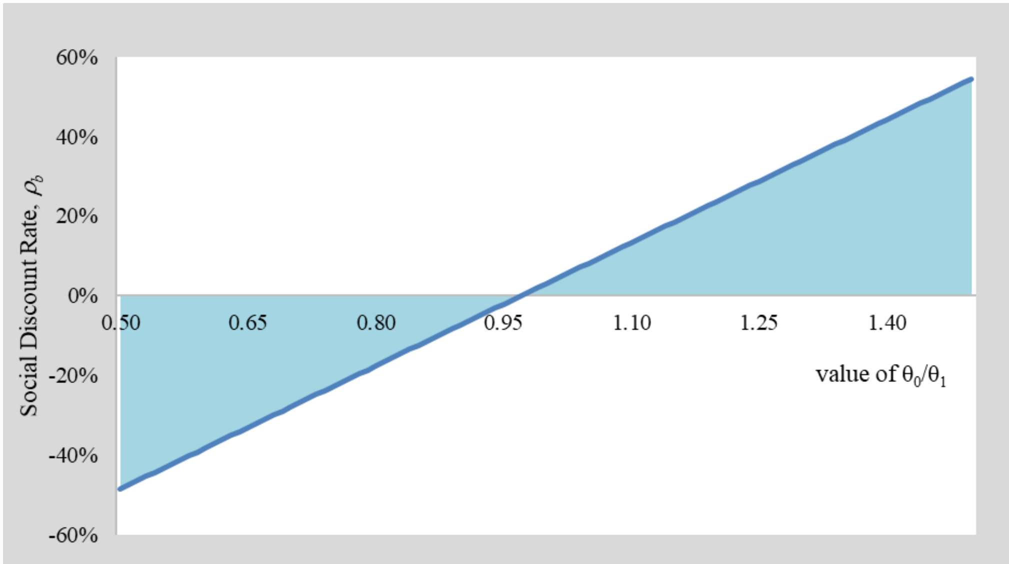

Figure 2 Values of social discount rate in Bradford's two-period model versus the ratio cost and benefit prices, θ_0/θ_1, when r_c = 3%.

where ρ_b is Bradford's discount rate,

$$\rho_b = (1 + r_c)\frac{\theta_0}{\theta_1} - 1 \tag{13}$$

This social discount rate, as illustrated in Figure 2 when r_c = 3% and θ_0/θ_1 ranges from between 0.5 to 1.5, can vary in excess of -/+ 50 percent. This range of potential shadow prices reflects our discussion in section 4, that a reasonable value based on a long-run growth equilibrium is close to 1.5. More generally, Bradford concludes that quite a wide range of social discount rates are possible for benefits over a one-year horizon.

The possibility of discount rates ranging from –50 to +50 percent might appear somewhat hopeless as a practical matter. However, this is the relevant range for benefits one year in the future embedded in a much longer (perhaps infinite) horizon model. We now turn to deriving the social discount rate for benefits arising further in the future.

Table 1 Generalized benefit stream from a dollar of project cost

Period	**t = 0**	**t = 1**	**t = 2**	**…**	**t = T**
Project Costs vs. (net) Benefits in terms of undiscounted project dollars	-1	B_1	B_2	…	B_T

3. A multi-period model for the social discount rate

Consider a public investment project or regulatory program where one dollar of cost is expended at the beginning of the initial period, but now a time series of benefits follows in future periods, $\{B_t, t = 1,2,\dots,T\}$. The multi-period cost-benefit flow is summarized in Table 1.

Note that this generalized problem nests the previous examples where $B_t = B$ and either $T = 1$ or $T \rightarrow \infty$. Following the previous examples, we are looking for the social discount rate that converts future benefits into equivalent dollars of current costs. As before, we begin by considering the NPV in terms of household consumption of a single project dollar spent, a modified version of (2), (6) and (11).

$$NPV = -\theta_0 + \sum_{t=1}^{T} \frac{\theta_1 B_t}{(1 + r_c)^t} \tag{14}$$

As before $\theta_0 = \alpha v + (1 - \alpha)$ is the consumption-equivalent cost of an initial project dollar spent based on a weighted average of the shadow price of capital and the (numeraire) price of consumption, and $\theta_1 = \beta v + (1 - \beta)$ is the same conversion for future benefits. Here, we assume without loss of generality that all benefits accrue to household consumption and capital in the same way over time.[9]

We can then construct the alternative NPV' in terms of dollars of initial project cost:

$$NPV' = \frac{NPV}{\theta_0} = -1 + \sum_{t=1}^{T} \frac{\left(\frac{\theta_1}{\theta_0}\right) B_t}{(1+r_c)^t}$$
$$= -1 + \sum_{t=1}^{T} \frac{B_t}{\left(1 + \left((1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}} - 1\right)\right)^t} \tag{15}$$

So, the implied social discount rate ρ_t at any point in time is given by

$$\rho_t = (1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}} - 1 \tag{16}$$

This definition (16) for social discount rate as a function of horizon t is one of our two main results. We can make three immediate observations about (16). First, using the shadow-price approach, the appropriate social discount rate depends on how costs and benefits accrue with respect to capital and consumption, captured by θ_0/θ_1, as well as the underlying shadow price v used to define θ_0 and θ_1. Second, the social discount rate varies over time. Third, as $t \to \infty$, the social discount rate converges to the consumption interest rate.

[9] As noted in footnote 8, our ultimate focus is on the range of social discount rates; this will be determined by extreme assumptions about how costs versus benefits accrue. The most extreme assumptions will be that costs accrue to capital and benefits to household consumption, and vice versa.

Table 2: Discount rate defined in four special cases

Assumptions	**Case I: Two-period model** (Harberger; Sandmo and Drèze; Sjaastad and Wisecarver; Bradford)	**Case II: Benefits paid as perpetuity** (Drèze; Sjaastad and Wisecarver)	**Case III: Benefits paid at the beginning** (Bradford)	**Case IV: Benefits paid in the distant future**
Benefits Stream	B	$B_t = B$, $\forall t = 1, \dots, \infty$	$B_1 = B$ $B_t = 0, \forall t = 2, \dots, T$	$B_t = 0$, $\forall t = 1, \dots, T-1$ $B_T = B$
Constraint on ratio of cost-benefit shadow price	Implied by two-period model; $v = \dfrac{1 + r_i}{1 + r_c}$ and $\theta_1 = 1$	Yes, $v = r_i / r_c$ and $\theta_1 = 1$	No, $\dfrac{1}{v} < \dfrac{\theta_0}{\theta_1} < v$	No, $\dfrac{1}{v} < \dfrac{\theta_0}{\theta_1} < v$
Social Discount Rate	$\rho^* = \rho_h$ $= (1 + r_c)\theta_0 - 1$ $= \alpha r_i + (1 - \alpha) r_c$	$\rho^* = \rho_h$ $= r_c \theta_0$ $= \alpha r_i + (1 - \alpha) r_c$	$\rho^* = \rho_b$ $= (1 + r_c)\left(\dfrac{\theta_0}{\theta_1}\right) - 1$	$\rho^* = \rho_T$ $= (1 + r_c)\left(\dfrac{\theta_0}{\theta_1}\right)^{\frac{1}{T}} - 1$ $\rho^* \xrightarrow{T \to \infty} r_c$

If we want to define a single social discount rate that generates the same NPV' for this indicated pattern of benefits and a particular ratio θ_0/θ_1, we can find ρ^* such that

$$\sum_{t=1}^{T} \frac{B_t}{(1+\rho^*)^t} = \sum_{t=1}^{T} \frac{B_t}{\left(1+\left((1+r_c)\left(\frac{\theta_0}{\theta_1}\right)^{\frac{1}{t}}-1\right)\right)^t}. \tag{17}$$

Equation (17) is our second key result and provides two more observations: The appropriate constant social discount rate generally depends on the pattern of benefits, $\{B_t, t = 1, \ldots, T\}$. Finally, the highest appropriate rate, based on uncertainty about θ_0/θ_1, will arise when we fix θ_0/θ_1 at its maximum (e.g., most emphasis on current period costs) and the lowest appropriate rate will arise when we fix θ_0/θ_1 at its minimum. These values are given by v and $1/v$, respectively.[10]

As noted, the previous literature focused on more restrictive assumptions. Results showing the social discount rate to be a weighted average of the investment and consumer rates hinged on a two-period model or a perpetuity. Later, Bradford (1975) assumed a one-time benefit payment in the second period of an otherwise multi-period model. We can now see these as three special cases in our framework. Moreover, we consider our model applied to the case of a distant horizon as a fourth special case. These are summarized in Table 2.

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[10] $\theta_0/\theta_1 = v$ implies the case where a hundred percent of the social costs today displaces private capital, and the entire future benefits are allocated to household consumption; the exact opposite is true for $\theta_0/\theta_1 = 1/v$.

We view Case IV in Table 2 as a powerful result; namely, the social discount rate converges over long horizons to the consumption rate of interest.[11] However, the speed of convergence will be determined by the magnitude of the shadow price and the range of possible values for θ_0/θ_1. For a shadow price of capital being close to one, the range of social discount rate quickly narrows down to the rate of return on consumption, while the process is slower with a larger

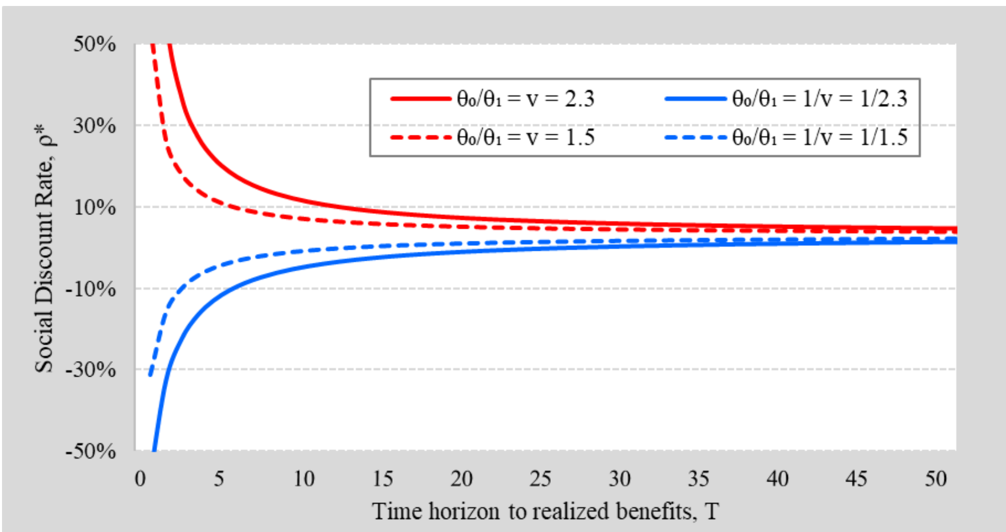

Figure 3 Range of possible social discount rates, versus time horizon, based on a consumer rate of 3 percent, an investment rate of 7 percent, and alternate values of the shadow price of capital (v = 2.3, 1.5).

[11] This result appears similar to a result in Little and Mirrlees (p. 283, 1974) and Squire and van der Tak (p. 142, 1975). However, these authors explicitly ignore (our) developed country context where capital income is taxes (footnote 1, page 285, Little and Mirrlees 1974). They have in mind a developing country where capital markets fail to equate investment demand and savings supply through the interest rate. Instead, the return to investment exceeds the consumer rate of interest. By construction, the shadow price of investment (in terms of consumption) must fall over time. This does not necessarily lead the interest rates to converge. Rather, it will prioritize projects that past a cost benefit analysis even when the shadow price of capital is high; other projects will wait. If the rates converge in their model, it is because capital markets begin to function (and absent taxes on capital income).

shadow price value, as illustrated by Figure 3. In the following section, we consider two approaches to specifying the shadow price of capital and, in turn, estimating the range of possible social discount rates at each horizon.

4. The shadow price of capital

We consider two approaches to estimate the shadow price of capital in our calculations. Both approaches start with consideration of a small change in capital today, tracing the impacts over time, and then computing the net present value of the resulting changes in consumption at the consumer interest rate. The key parameter in the resulting expression is the saving rate. In one approach we consider a reasonable way to bound the saving rate. In the other, we calculate the savings rate assuming an underlying Ramsey growth model.

We have already noted the general approach in Marglin (1963a, 1963b) and Sjaastad and Wisecarver (1977). They assumed any change in capital generated a perpetuity of flows at rate r_i all accruing to consumption, and, in turn, showed $v = r_i/r_c$. However, a more careful consideration requires thinking about whether flows accrue to consumption or investment, what to assume about tax revenue, and the role of depreciation (Bradford 1975; Mendelsohn 1981, 1983; Lind 1982; Lyon 1990).

Following Lyon (1990), we specify a constant saving rate s from incremental gross capital returns, equal to net returns r_i plus depreciation μ. We then think about stepping through the sequence of events following an incremental change in capital. A change in private investment ΔK_t in period t produces a return equal to the gross rate of return from capital prior to depreciation $\Delta Y_{t+1} = (r_i + \mu)\Delta K_t$ in period $t + 1$. This return will be divided between reinvestment, $\Delta Z_{t+1} = s(r_i + \mu)\Delta K_t$, taxes on the net capital return, $\tau r_i \Delta K_t$, and direct consumption $\Delta Y_{t+1} - \Delta Z_{t+1} - \tau r_i \Delta K_t$, in period $t + 1$. Capital stock in the next period will increase by the amount reinvested net of depreciation.

As a rule of thumb in welfare analysis, the change in taxes is assumed to generate current-period government spending that increases current-period consumption dollar-for-dollar.[12] Thus the total flow to household consumption is $\Delta Y_{t+1} - \Delta Z_{t+1}$.

The dynamics of ΔK_t-induced flows are illustrated in Figure 4 for the first period after an incremental change in the capital stock. The flow to consumption (including the benefit from government spending) is given by $\Delta C_{t+1} = (1 - s)(r_i + \mu)\Delta K_t$. Tracing out the next period based on the new $\Delta K_{t+1} = \big(s(r_i + \mu) + (1 - \mu)\big)\Delta K_t$ yields

$$\begin{aligned} \Delta C_{t+2} &= (1 - s)(r_i + \mu)\Delta K_{t+1} \\ &= (1 - s)(r_i + \mu)\big(s(r_i + \mu) + (1 - \mu)\big)\Delta K_t \end{aligned} \tag{18}$$

Repeating this every period and constructing the discounted sum of consumption changes at the consumption rate of interest yields (see Appendix B for additional details):

$$v = \frac{(1 - s)(r_i + \mu)}{r_c + \mu - s(r_i + \mu)}. \tag{19}$$

[12] One could assume tax revenues flow at least partly into government projects / investment that in turn provide consumer benefits in future periods, and/or that government funds exceed the value of consumption. This is easily accomodated in the current framework by considering that each dollar of such government investment will have an equivalent current-period consumption value greater than one—what is referred to as the marginal value of public funds (MVPF). Allowing this shadow price to be v_G, the numerator in (19) would be replaced by $(1 - s)(r_i + \mu) + (v_G - 1)\tau r_i$. That is, there is a "bonus" each period from tax revenue being diverted into more valuable public projects. Based on recent MVPF estimates (Hendren 2014), this does not significantly alter our results, raising our preferred estimate of *v* from 1.5 to 1.7.

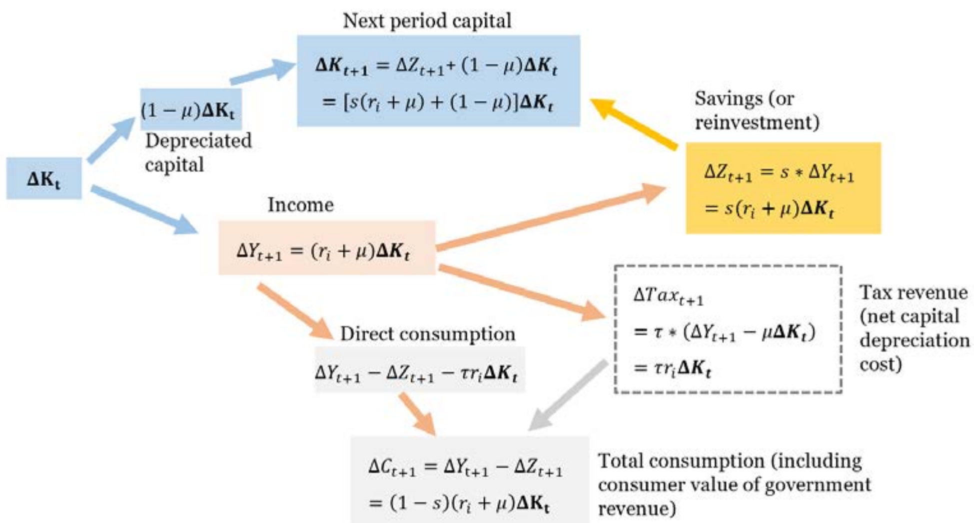

Figure 4 Dynamic flows of an incremental change in capital stock in period t

One way to understand (19) is to consider the numerator and denominator. For a given capital stock deviation in any period, the numerator gives the portion flowing to household welfare. That is, it equals the gross return $r_i + \mu$, times the non-reinvested fraction $1 - s$. The denominator then reflects the adjustments necessary to value an initial capital stock deviation over time. The consumption discount rate governs the household valuation, lowering the value over time. The term $s(r_i + \mu) - \mu$ governs the change in the actual capital stock deviation each period. The deviation increases by the reinvestment each period $s(r_i + \mu)$ and decreases by the amount of depreciation μ.

Note that if $r_i = r_c$ and there is no distortion between the value of capital and consumption, the expression (19) for v simplifies to $v = 1$.

Expression (19) replaces the problem of identifying the shadow price of capital with the problem of identifying values for the savings rate and capital depreciation (in addition to the consumption and investment interest rates). One approach is to think about possible bounding values. In both Mendelsohn (1983) and Lyon (1990), they consider the condition necessary for the shadow price of capital to be finite. This amounts to a condition that consumption grows more slowly than the consumption interest rate[13].

However, we would argue that a more reasonable condition for a steady-state economy is to have a non-explosive capital stock (i.e. $\Delta K_{t+1} \leq \Delta K_t$). That is, if one adds or subtracts a little capital from the economy, it goes back to its original equilibrium or path. This implies reinvestment $s(r_i + \mu)\Delta K_t$ should be less than depreciation, $\mu\Delta K_t$, or

$$s \leq \frac{\mu}{r_i + \mu}. \tag{20}$$

Using this expression to replace the savings rate in (19), the numerator simplifies to $(1 - s)(r_i + \mu) \leq r_i$ and the denominator to $r_c + \mu - s(r_i + \mu) \geq r_c$. So, the shadow price of capital at steady state is less than the ratio of the two interest rates. We also know this ratio equals one if no distortion from taxes are imposed to the market ($r_i = r_c$). Putting these together

$$1 \leq v \leq \frac{r_i}{r_c}. \tag{21}$$

That is, we have replicated Marglin's (and others) shadow price of capital as a maximum shadow price under what we believe are more general / appropriate conditions.

[13] This condition is specified in (B8) of Appendix B.

If we focus on the government's values of 7 and 3 percent for r_i and r_c, respectively, we have the upper bound for v being 7/3. It is this value of the shadow price that motivated the solid blue and red lines in Figure 3. Based on those values, the range of social discount rate becomes 1.2 to 4.8 percent after 50 years. It narrows to 2.5 to 3.5 percent after 175 years.

Instead of looking for bounding values, our second approach turns to a structural model to see how the shadow price relates the saving rate to underlying technology or preference parameters, and to then specify those parameters. In Appendix C, we use the Ramsey growth model with a Cobb-Douglas production function to derive the steady-state rate of savings (s^*) with a tax distortion τ on the labor and capital income (such that $r_c = (1-\tau)r_i$). That yields

$$s^* = \frac{(\mu + g + n)a}{\mu + r_i}. \tag{22}$$

Here, g is the growth rate of labor-augmenting productivity, n is the population growth rate, and a is the capital-output elasticity (capital share) in the production function. As before, μ is the depreciation rate of capital and $r_i = r_c/(1-\tau)$ is the investment rate of interest, equal to the grossed up consumption interest rate. In the Ramsey model, this consumption rate of interest is, in turn, related to pure time preference, utility curvature, and productivity growth, which we have subsumed into r_c. Choosing parameters from the literature (see Table C - 1), along with r_i = 7 percent and r_c = 3 percent, yields $s \approx 23\%$ from (22) and $v \approx 1.5$ from (19). The range of social discount rates at different future horizons based on this value of v is indicated by the dashed line in Figure 1. In particular, the range of social discount rates narrows to 1.6 to 4.4 percent after 30 years and 2.1 to 3.8 percent after 50. After 100 years, the range is 2.6 to 3.4 percent.

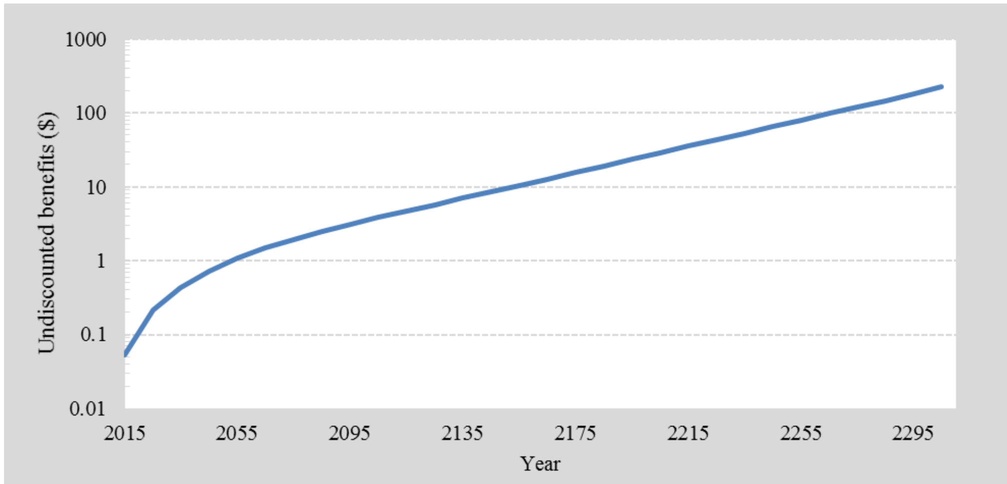

Figure 5 Undiscounted climate damages from one incremental metric ton of CO2 emitted in 2015 under a 2.2 percent economic growth scenario

5. Social discount rate for Climate Change

In section 3, we have shown that for public investment project or regulatory analyses with long-term cost-benefit consequences, the social discount rate calculation contains less uncertainty when future benefits are mostly paid in the distant future. In the extreme (Case IV in Table 2), the effective discount rate converges to the consumption rate of return. This result is quite powerful, but the precise question about the appropriate range hinges on the shadow price of capital as well as the actual pattern of future benefits over time. Having discussed the shadow price of capital in section 4, we now turn to the pattern of future benefits. In particular, we focus on climate change mitigation benefits.

Figure 5 illustrates an increasing sequence of undiscounted future climate damages from an incremental ton of CO_2 emitted in 2015 under a 2.2 percent economic growth scenario (NAS 2017). The benefits of avoiding these damages each year, discounted to the base year, 2015, would then be used to construct a

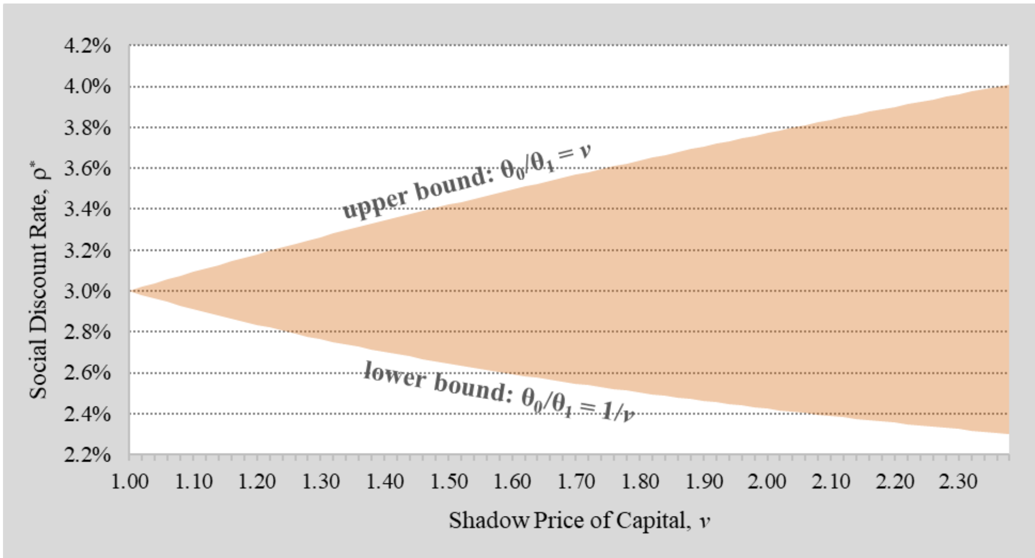

Figure 6 The social discount rate for climate damages is bounded around a narrow range around the 3 percent consumption rate of interest. The value of ρ^* falls approximately between 2.6 and 3.4 percent when $v = 1.5$, and between 2.3 and 4.0 percent when $v = 2.33$.

benefit estimate for tons reduced in 2015.[14] As noted earlier, each year in the future will generally have a different range of discount rates given by (16). However, can we plug this pattern of benefits, $\{B_t\}$, into the expression (17) to define a range of rates ρ^* appropriate for the entire pattern of benefits over time. The only uncertainty defining those rates is the ratio θ_0/θ_1. As discussed in section 3, this ratio ranges from $1/v$ to v, depending on the distribution of costs and benefits over capital and consumption. In section 4, assuming the consumption and investment interest rates are 3 and 7 percent, we argued the

[14] The calculation would need to be repeated for each year of avoided emissions in the cost-benefit analysis. That is, for each year *t* where emission reductions occur, compute a sequence of undiscounted future climate damages from incremental emissions in year *t* and then discount those damages back to year *t* to value emission reductions in that year.

range of values for v is between 1 and 7/3 (≈ 2.33). For each value in this interval, we can solve for the upper and lower bound of ρ^* using equation (17). Figure 6 plots those bounding social discount rates over this range for the shadow price of capital v. At the extreme $v = 2.3$, the range is between 2.3 and 4.0 percent. Based on Ramsey model calculation, we suggested a preferred shadow price close to 1.5. Again reading off Figure 6, we can see that this indicates a narrower ange for the social discount rate of 2.6 to 3.4 percent when applied to climate change.

Of course, finding the social discount rate for climate change mitigation benefits is not an end unto itself. The goal is the estimate per ton benefit (and ultimately a conduct a complete CBA). Using the pattern of benefits in Figure 5 and a particular discount rate, we can compute the net present value of mitigation benefits from 1 ton of reduced emissions. When we do this with the original 3 and 7 percent rates, we find a social cost of CO_2 of \$49 and \$5.9 (in 2015\$ per metric ton CO_2), respectively. The ratio, roughly 8:1, was noted in the introduction. If we instead use the preferred shadow price of capital and implied range of 2.6 to 3.4 percent for the social discount rate, we estimate the social cost of CO_2 as \$77 and \$34, respectively, or a ratio of about 2:1. This is still a rather wide range of potential benefits. However, we see that correct attention to the uncertain impacts on consumption and investment, and consequences for the social discount rate, does eliminate the low end of estimates suggested by recent government estimates (U.S. EPA 2017). Moreover, the estimates are now more centered around the 3% estimate of \$49.

6. Conclusions

The choice of discount rates has a significant consequence for the evaluation of public policy with significant consequences in the future. As shown in the CBAs for Clean Power Plan, the magnitude of foregone climate benefits is reduced by a factor of between six and nine when the discount rate is increased from 3 percent to 7 percent (U.S. EPA 2017). The underlying basis for these discount rates has been divergent views about whether to use a consumption or investment rate of interest. With significant taxes on income, particularly capital income, there will be divergence between the rate of return on investment and the after-tax return on household savings. The economic literature has tended to advocate using a consumption rate in conjunction with shadow prices to convert impacts on investment into consumption equivalents. However, absent clear guidance on how to implement this in practice, government practice has resorted to using alternative consumption and investment rates, generally giving both equal weight. This particular approach follows early results by Marglin (1963a, 1963b), Harberger (1972), and Sjaastad and Wisecarver (1977) but assumes a particular pattern of investment returns and benefit flows—either a simple two-period model or a perpetuity. It also assumes benefits accrue entirely to consumption.

In this paper, we have demonstrated that the correct range of rates based on the shadow-price approach depends on the shadow price of capital *and* the time horizon. For example, if the shadow price of capital is our preferred value of 1.5, and 3 and 7 percent are the correct consumption and investment rates of interest, then the range of appropriate social discount rates is 2.2 to 3.8 percent after 50 years. We demonstrated that the appropriate social discount rate converges to the consumer rate over longer time horizons. More generally, we have demonstrated how to construct the appropriate range of social discount rates for any given temporal pattern of benefits, conditional on the government's assumed consumption and investment rates. That is, we describe the range of possible social discount rates that would be appropriate based on the given pattern of benefits and making alternate assumptions about whether costs (at the beginning of the initial period) or benefits (in all future periods) fall more on investment or consumption. Applied to the NAS pattern of benefits from climate change and our preferred shadow price of capital, the appropriate discount rate is 2.6 to 3.4 percent. This, in turn, leads to estimates of the social cost of CO_2 of \$77 and \$34, respectively. This is much narrower than the range indicated by discount rates of 3 and 7 percent.

Importantly, we believe this paper provides a strong caution against the investment interest rate as a benchmark for discounting in government CBA for projects with long horizons and/or when benefits, like costs, can fall on consumption or investment. Over long horizons, we have shown that the appropriate rate converges to the consumption rate, regardless of the shadow price and the incidence on consumption or investment. And when there is uncertainty about whether benefits, as well as costs, accrue to investment or consumption, the appropriate social rate of discount should be roughly centered on the consumption rate of interest.

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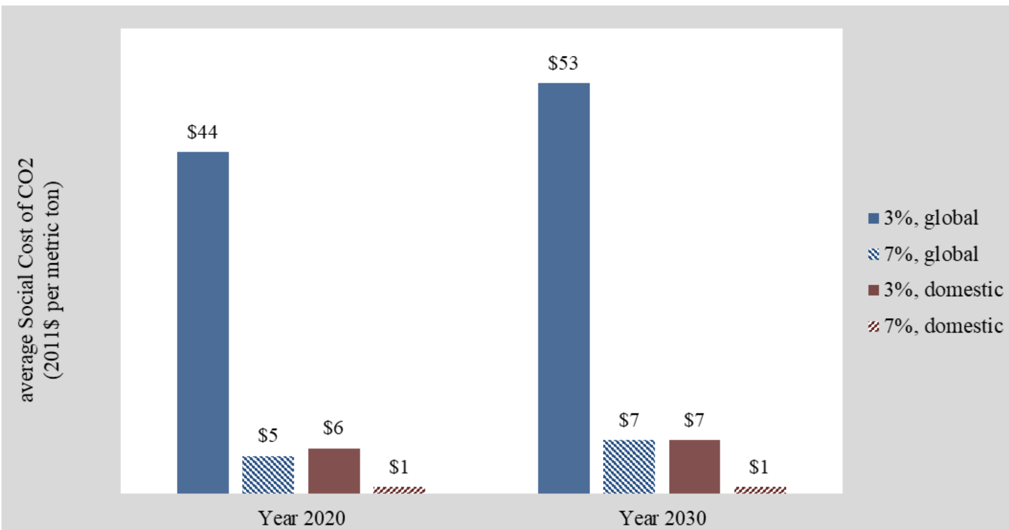
Appendices

A. Impact of discount rate choice on the cost-benefit analysis for repealing Clean Power Plan



Source: Table 3-7, Appendix C.3 (U.S. EPA 2017) and Table 4-2 (U.S. EPA 2015). Numbers are rounded to the nearest integer.

Figure A - 1 Interim SC-CO_2 estimates for 2020 and 2030 (in 2011 dollars per metric ton CO_2). Values are rounded to the nearest integer.

In October 2017, the Trump administration released a revised CBA of the Clean Power Plan (CPP) as part of a regulatory impact analysis associated with the CPPs proposed repeal. This 2017 CBA changed two key assumptions compared to the 2015 CBA. First, the revised CBA calculates climate damages (or foregone climate benefits) based on a U.S. domestic social cost of carbon dioxide (SC-CO_2) estimate, rather than a global SC-CO_2. The domestic SC-CO_2

is approximated as 10 to 20 percent of the global values used in the 2015 analysis. This brings down the average SC-CO_2 value by a factor of seven (U.S. EPA 2017). Second, while the 2015 CBA uses 3 percent as the central discount rate value, the 2017 CBA presents results for discount rates of 3 and 7 percent without a central value. The SC-CO_2 estimates are about 6 to 9 times lower based on 7 rather than 3 percent (U.S. EPA 2017). Equally weighting these two rates, the estimate of the average global SC-CO_2 in year 2020 drops from \$44 (based on 3%) to \$24.5 (mid-point of estimates based on 3 and 7 percent). The average domestic SC-CO_2 drops from \$6 to \$3. That is, the new discount rate approach lowers the effective SC-CO_2 – and estimated climate benefits in the CBA – by 44%.

B. Expression of shadow price of capital

An incremental change in capital at the beginning of period t, ΔK_t, will transfer to a stream of future capital and consumption. The dynamics is illustrated by Figure 4. A change in capital ΔK_t in period t will result in changes in both gross income Y_{t+1} and savings Z_{t+1} in the following period.

$$\Delta Y_{t+1} = (r_i + \mu)\Delta K_t \tag{B1}$$

$$\Delta Z_{t+1} = s \cdot \Delta Y_{t+1} = s(r_i + \mu)\Delta K_t \tag{B2}$$

The incremental change in next-period capital K_{t+1} is the sum of direct changes in post-depreciation capital and indirect changes in savings.

$$\Delta K_{t+1} = \Delta Z_{t+1} + (1 - \mu)\Delta K_t = [s(r_i + \mu) + 1 - \mu]\Delta K_t \tag{B3}$$

The proportion of income remains after savings is the dollars for next-period consumption.

$$\Delta C_{t+1} = \Delta \mathrm{Y}_{\mathrm{t+1}} - \Delta Z_{t+1} = (1 - s)(\mathrm{r_i} + \mathrm{\mu})\Delta K_t \tag{B4}$$

The shadow price of capital, v_t, is the present value of all future consumption changes following period t discounted with the consumption rate of interest r_c. Hence, we can derive the following difference equation for equilibrium state shadow price and savings:

$$v_t = \frac{(1 - s)(r_i + \mu)}{1 + r_c} + \frac{s(r_i + \mu) + (1 - \mu)}{1 + r_c} * v_{t+1} \tag{B5}$$

or equivalently the sum of consumption streams as a geometric sequence:

$$v_t = \frac{(1 - s)(\mu + r_i)}{1 - r_c} * [1 + \gamma + \gamma^2 + \gamma^3 + \cdots], \tag{B6}$$

where $\gamma = \frac{s(\mu+r_i)+1-\mu}{1+r_c}$. Therefore, at a steady state of the economy, the shadow price of capital (v) has the following relationship with savings, consumption rate of return, and pre-tax marginal rate of return to investment.

$$v = \frac{(1-s)(r_i+\mu)}{r_c+\mu-s(r_i+\mu)} \tag{B7}$$

We can place some bounds on this expression by considering practical restrictions. For example, both Mendelsohn (1983) and Lyon (1990) focus on condition for a less-than-infinity v_t; namely the need to have a non-explosive consumption flow. This requires the propensity of savings to be bounded by a depreciation adjusted ratio of the two interest rates:

$$\gamma < 1 \rightarrow s < \frac{r_c+\mu}{r_i+\mu} \tag{B8}$$

However, we think a more reasonable condition for a steady-state economy is to have $\Delta K_{t+1} < \Delta K_t$ in (B3). This avoids a situation where a small perturbation leads to a permanent (or explosive) shift. Intuitively, savings out of gross income must be less than capital depreciation.

$$s \leq \frac{\mu}{r_i+\mu} \tag{B9}$$

This condition (B9) leads the shadow price of capital to be constrained above by the ratio of interest rates.

$$1 \leq v \leq \frac{r_i}{r_c} \tag{B10}$$

C. Steady-state shadow price in the Ramsey growth model

C-1. Ramsey model set-up

The Ramsey model set-up follows the framework in Cass (1965). Aggregate output, Y_t, is produced with two inputs: labor L_t, and capital K_t. Let A_t denote labor-augmenting productivity. The production function, $Y_t = F(K_t, A_t L_t)$, is assumed to have declining marginal product of capital, and is homogeneous of degree 1 in capital and labor[15]. Both capital and labor are essential inputs for production[16].

Productivity A_t and population L_t are assumed to grow exponentially at an exogenous rate g $(g > 0)$, and n $(n > 0)$ respectively.

$$A_t = A_0 e^{gt} \tag{C1}$$

$$L_t = L_0 e^{nt} \tag{C2}$$

Variables are redefined by standardizing them with respect to $A_t L_t$. Standardized production output, y_t, can thus be written as a function of standardized capital, $k_t \stackrel{\text{def}}{=} \frac{K_t}{A_t L_t}$.

$$y_t = \frac{Y_t}{A_t L_t} = F\left(\frac{K_t}{A_t L_t}, 1\right) = F(k_t, 1) \stackrel{\text{def}}{=} f(k_t) \tag{C3}$$

Total production output Y_t is allocated between consumption C_t, savings Z_t, and tax payment (with tax rate τ). We treat depreciation (μ) as tax-deductible expense.

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[15] $F(\theta K_t, \theta A_t L_t) = \theta F(K_t, A_t L_t), \forall \theta > 0$

[16] Given $A_t > 0$, then $F(0, A_t L_t) = F(K_t, A_t * 0) = 0$

$$Y_t = \tau Y_t - \tau\mu K_t + C_t + Z_t \tag{C4}$$

Similarly, consumption is also standardized by $A_t L_t$: $c_t \stackrel{\text{def}}{=} C_t/(A_t L_t)$.

$$Z_t = A_t L_t \cdot [(1-\tau)f(k_t) + \tau\mu k_t - c_t] \tag{C5}$$

With capital depreciation ($\mu > 0$), gross investment involves net investment $\dot{K}$ and replacement investment μK. Equation (C6) holds because net increase in gross capital comes from positive savings net out capital depreciation.

$$\dot{K} = Z_t - \mu K_t = A_t L_t \cdot [(1-\tau)(f(k_t) - \mu k_t) - c_t] \tag{C6}$$

Since

$$\dot{K} = \frac{d(A_t L_t k_t)}{dt} = \dot{A} L_t k_t + A_t \dot{L} k_t + A_t L_t \dot{k}\ , \tag{C7}$$

and by assumption in (C1) and (C2), $\dot{A} = A_t g$, and $\dot{L} = L_t n$, we can derive the differential equation to describe how the standardized capital varies over time by combining these two equations. Namely,

$$\dot{k} = (1-\tau)(f(k_t) - \mu k_t) - c_t - (g+n)k_t \tag{C8}$$

Turning to household preferences, we assume a time-invariant instantaneous utility function $U(.)$ that depends on per capita consumption $A_t c_t$. More specifically, assume U is an isoelastic utility function.

$$U(x) = \begin{cases} \dfrac{x^{1-\eta}}{1-\eta}, & \eta > 0 \ and \ \eta \neq 1 \\ \ln(x), & \eta = 1 \end{cases} \tag{C9}$$

So, the marginal utility function is

$$U'(x) = x^{-\eta}. \tag{C10}$$

To find the market equilibrium under perfect competition, we solve the social planner's problem (C11)[17] to find the optimal growth path $\{(c_t, k_t): t \geq 0\}$ that maximizes social welfare. Social welfare is the aggregated utility of all consumers, where the discount rate of pure time preference is δ ($\delta > n$).

$$\max_{c \geq 0} \int_0^\infty e^{-(\delta - n)t} U(A_t c_t)\, dt$$
$$\text{s.t. } \dot{k} = (1-\tau)(f(k_t) - \mu k_t) - c_t - (g+n)k_t \tag{C11}$$

C-2. Estimate steady-state shadow price

We assume a solution (c_t^*, k_t^*) to the necessary conditions for the social planner's problem (C11) that converges to a finite positive steady state (c^*, k^*). The equilibrium conditions can be found by letting $\dot{c} = \dot{k} = 0$.

$$\dot{k} = 0 \rightarrow\ c^* = (1-\tau)(f(k^*) - \mu k^*) - (g+n)k^* \tag{C12}$$

$$\dot{c} = 0 \rightarrow\ \delta + g\eta = (1-\tau)(f'(k^*) - \mu) \tag{C13}$$

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[17] Tax rate fixed at τ. Initial population stock is assumed to be $L_0 = 1$.

Table C - 1 Parameters in the Ramsey model for numeric estimation of shadow price of capital

Parameter	Value in the numeric example	Source
Growth rate of productivity, g	2%	NAS (2017)
Growth rate of labor, n	1%	NAS (2017)
Capital depreciation rate, μ	10%	Nordhaus (2017)
Output elasticity of capital, a	0.3	Nordhaus (2017)

Assuming Cobb-Douglas production, so $y_t = k_t^a$, these two equations can be solved for steady-state capital and consumption.

$$k^* = \left[\frac{\mu}{a} + \frac{\delta + g\eta}{a(1-\tau)}\right]^{\frac{1}{a-1}} = \left[\frac{\mu + r_i}{a}\right]^{\frac{1}{a-1}} \tag{C14}$$

$$c^* = (1-\tau)((k^*)^a - \mu k^*) - (g+n)k^* \tag{C15}$$

Here, we have simplified the expression for k^* by recognizing that $\delta + g\eta$ is what we have been calling the consumer interest rate r_c. It is reflects the consumer's willingness to trade consumptions across periods. And, $(\delta + g\eta)/(1-\tau) = f'(k^*) - \mu$ is what we have been calling the investment interest rate r_i. It reflects the pre-tax (and post-depreciation) return to investment.

From (C5), the equilibrium propensity of savings z^*/y^* can be written as a function of the parameters.

$$\begin{aligned} s^* = 1 - \tau - \frac{\tau\mu k^* - c^*}{f(k^*)} &= [\mu + g + n] \cdot (k^*)^{1-a} \\ &= \frac{(\mu + g + n)a}{\mu + r_i} \end{aligned} \tag{C16}$$

We parameterize the Ramsey model in Table C - 1 to determine the savings rate s^* above and shadow price of capital in (19). In particular, we take r_c = 3% and r_i = 7% (based on OMB guidelines). This leaves μ, g, n, and a. We assume depreciate μ = 10%, economic growth g = 2%, population growth n = 1%, and the capital-output elasticity a = 0.3. This yields a steady-state saving rate s^* = 23%, and the value of shadow price v is about 1.5.