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MONEY MARKETS, COLLATERAL AND MONETARY POLICY

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ABSTRACT

Interbank money markets have been subject to substantial impairments in the recent decade, such as a decline in unsecured lending and substantial increases in haircuts on posted collateral. This paper seeks to understand the implications of these developments for the broader economy and monetary policy. To that end, we develop a novel general equilibrium model featuring heterogeneous banks, interbank markets for both secured and unsecured credit, and a central bank. The model features a number of occasionally binding constraints. The interactions between these constraints - in particular leverage and liquidity constraints - are key in determining macroeconomic outcomes. We find that both secured and unsecured money market frictions force banks to either divert resources into unproductive but liquid assets or to de-lever, which leads to less lending and output. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output. We show how central bank policies which increase the size of the central bank balance sheet can attenuate this decline.

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1 Introduction

Interbank money markets are essential to the liquidity management of banks. They are also important for monetary policy implementation as interbank rates are often central banks' target rates. Money market trade is subject to a number of frictions, which displayed themselves forcefully during the Global Financial Crisis, with the unsecured segment “freezing” (see, e.g., Heider, Hoerova, and Holthausen (2015)) and the secured segment facing “runs” due to haircut increases on riskier collateral (see, e.g., Gorton and Metrick (2012)). Yet, the question of what impact the frictions in bank liquidity management have on the broader economy is largely unaddressed.

In this paper, we take a step towards understanding the impact of frictions in money markets on bank lending, real activity and monetary policy. We develop a novel model featuring heterogeneous banks, interbank money markets for both secured and unsecured credit, and a central bank that can conduct open market operations as well as lend to banks against collateral. As a particular advance compared to the existing literature, banks may both face leverage constraints and liquidity constraints: the interaction of these constraints is at the heart of our analysis. To highlight the key features of our framework, we first present a partial equilibrium banking model which we then embed in a general equilibrium setup.

Each period in the model is sub-divided into a morning and an afternoon. In the morning, banks choose their assets (loans, bonds and money) and liabilities (central bank funding and deposits), subject to a leverage constraint as proposed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2011). On the liability side, central bank funding must be backed by bond collateral. Deposit funding is uncollateralized but it exposes banks to idiosyncratic liquidity shocks in the afternoon, as formulated by Bianchi and Bigio (2013). These liquidity shocks can be managed by borrowing or lending in interbank money markets. Banks face an exogenous probability of being “connected,” defined as the ability to borrow in the unsecured market in the afternoon. Those banks that are unable to borrow in the unsecured market, the “unconnected” banks, can satisfy withdrawals either by acquiring bonds in the morning to obtain collateralized funding in the private market in the afternoon and/or by bringing money into the afternoon (self-insurance). All collateralized borrowing is subject to a haircut, with haircuts in the private market being potentially different from haircuts set by the central bank.

Five inequality constraints on banks emerge as crucial: the “morning” leverage constraint, a collateral constraint vis-a-vis the central bank and three short-sale constraints. We show that

one cannot a priori impose any of these constraints to bind or to be slack: on the contrary, each of these may turn on or off and each can be crucial for the macroeconomic outcomes, as we traverse the parameter space and for different monetary policies. We view this as a novel and central contribution of our paper. Usually, a single inequality is studied and equality is often imposed. By contrast, our five-dimensional inequality space offers a rich set of interactions. Different parameter values then generate different types of bottlenecks, which an astute central bank all needs to take into account and which we argue to be key to understanding the financial system. We investigate the role of these constraints per conducting a steady-state comparative static analysis, when varying the severity of a particular money market friction and imposing a particular monetary policy. Given the high dimensionality of the constraints space, we deliberately chose the steady-state comparative statics as the more illuminating mode of analysis compared to a fully dynamic and stochastic, but likely opaque approach. Additionally, with persistent money market frictions, a steady-state comparative statics appears to be more appropriate in any case.

Indeed, our modelling framework is motivated by two major and persistent money market developments that occurred over the past fifteen years: a decline in the unsecured interbank market and a corresponding increased reliance on the secured market, which consequently exposes banks more substantially to the concurrent increase in collateral haircuts. We document these developments using data for the euro area, but similar changes have been observed in the US (see footnote 2 on the next page).

The first development is documented in Figure 1. While the total turnover was split about equally between unsecured and secured market segments in 2003, the turnover in the unsecured market declined five-fold and was just five percent of total by 2019.¹ The decline in the relative importance of the unsecured market started several years before the global financial crisis of 2008, and further steepened with the onset of the financial and sovereign debt crisis in the euro area.

The second development is the declining value of assets used as collateral in the secured market, which had two sources in recent years. First, there were large and abrupt increases in haircuts on some asset classes. In the euro area, haircuts on government bonds of some euro area countries increased to 80 percent or higher during the sovereign debt crisis (Table 1). Even outside the period of the sovereign crisis, in a relatively calm year such as 2017, private market haircuts remained heterogeneous across countries and did not return to the

¹By contrast, turnover levels in the secured market actually increased between 2003 and 2019.

pre-crisis levels. At the same time, haircuts applied by the European Central Bank (ECB) on the same collateral were much lower than private market haircuts and remained largely stable throughout this period. Second, the stock of safe (AAA-rated) assets fell due to rating downgrades, which reduced the availability of high-quality collateral that could be pledged in the secured market. In the euro area, the downgrades also affected sovereign bonds, with the ratio of AAA-rated government debt to GDP falling from a pre-crisis average of 29 percent to just 14 percent (average over 2015-2019; see Figure 4).²

Different observers may attribute these two developments to different underlying causes. For example, perhaps the private sector haircuts and high yields on certain sovereign bonds reflect a dysfunctional system or a bad equilibrium, which the ECB appropriately seeks to correct, see e.g. Roch and Uhlig (2018). Conversely, perhaps these haircuts are due to the appropriate rational assessment of default risks of the underlying bonds, while the ECB haircuts are too small. These varying points of view are parts of a heated and contentious debate in Europe, to which we do not wish to contribute in this paper. Instead, our focus is on the response of the system, if these private haircuts increase compared to those charged by the central bank, focusing on the benign branch of events, where no defaults occur. For that reason, we do not explicitly model how these haircuts arise, but instead treat them as an exogenous parameter. We view our results as a positive rather than normative analysis, providing an important piece of an all-encompassing view. We likewise treat the fraction of “unconnected” banks, which can only use the secured interbank market, as an exogenous parameter in our analysis.

We therefore use the general equilibrium model to provide two sets of steady state comparative statics scenarios, varying either the fraction of unconnected banks or varying the private sector haircut on government bonds. Both types of money market frictions force banks to either divert resources into unproductive but liquid assets (bonds or money rather than productive capital) or to de-lever (raise fewer deposits as it is deposit funding that exposes banks to liquidity shocks). This leads to less lending and output in the economy. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output, in the absence of central bank intervention. Policies that increase the size of the central bank balance sheet (outright purchases or collateralized lending) alleviate the

²In the US, the size of the interbank money market declined from the estimated \$100 billion before the financial crisis to less than \$5 billion in 2018 (see Kim, Martin, and Nosal (2020)). For the US secured market, Gorton and Metrick (2012) provide evidence of increases in average haircuts on risky collateral from around zero in early 2007 to 50% in late 2008, contributing to the emergence of “repo runs” during the financial crisis.

bank liquidity constraint by expanding the money supply and attenuate the decline in lending and output. They may, of course, unduly increase the risk exposure by the central bank, but this is outside of our analysis for the reasons stated.

A key contribution of our paper is to allow for three different avenues for central bank liquidity provision. We consider three instruments in particular: central bank holdings of government bonds, the interest rate on central bank loans, and haircuts on accepted collateral. We relate these instruments to the stylized versions of monetary policies pursued by central banks around the world in recent years: i) a pre-crisis policy characterized by a constant balance sheet; ii) a policy where the balance sheet is expanded via *collateralized credit operations* (“CO” henceforth), whereby the central bank stands ready to provide the liquidity demanded by banks at a given interest rate and haircut level; and iii) a policy of *outright asset purchases* (“OP” henceforth), whereby the central bank changes the stock of bonds on its balance sheet to achieve a certain inflation goal.³

We calibrate the model to the euro area data and use it to analyze the macroeconomic impact and central bank policies under the two alternative scenarios.

In the first scenario, i.e. when the share of banks with access to unsecured lending is varied, a constant-balance sheet policy or collateralized credit operations make no difference, as there is no advantage to borrow from the central bank compared to borrowing on the private secured market in the afternoon. By contrast, open market asset purchases inject much needed liquidity generally, and can substantially alleviate the negative output effects that would otherwise materialize. In our benchmark calibration, the difference in output between a steady-state with 0.58 share of unconnected banks and that with 0.89 share (average pre-2008 vs 2019 share of secured turnover in total) is around 2 percent in the CO case, and 1 percent in the OP case.

In the second scenario of varying private-sector haircuts and under a constant central bank balance sheet policy, the difference in output between a steady-state with 3 percent haircuts and one with 40 percent haircuts is 3 percent. The key to mitigating the reduction in capital and output is to provide liquidity to the unconnected banks and to prevent their leverage constraint from turning slack. This can now be achieved both with the CO policy by lending to banks against collateral at favorable haircuts or per the OP policy of open market purchases of government bonds.

³The CO and OP policies are reminiscent of the ECB’s fixed-rate full allotment policy implemented since 2008 and the Public Sector Purchase Programme implemented since 2015, respectively.

The paper proceeds as follows. In section 2, we relate our paper to the existing literature. In section 3, we present a partial equilibrium banking model that highlights the key ingredients for the analysis. In section 4, we cast this latter into a general equilibrium model. In section 5, we present the analysis and characterize the equilibrium. In section 6, we illustrate the model predictions through a numerical analysis. Section 7 concludes.

2 Related literature

Our paper is related to the broad literature that investigates the implications of financial frictions for the macroeconomy and for monetary policy as well as to the literature which focuses on frictions in secured and unsecured interbank trade. We now discuss how various elements in our analysis relate to these literatures.

Bank balance sheet constraints and monetary policy

A number of recent papers emphasize the role of banks' balance sheet and leverage constraints for the provision of credit to the real economy and for the transmission of standard and non-standard monetary policies (see e.g. Gertler and Karadi (2011) and Gertler and Kiyotaki (2011)). As in those papers, banks in our model face an enforcement problem and endogenous balance sheet constraints. Additionally, they solve a liquidity management problem that further constrains their actions. Another novel feature of our framework is that we do not impose the various constraints to be binding at all times (as in Brunnermeier and Sannikov (2014); He and Krishnamurthy (2016); Mendoza (2010); Bocola (2016); Justiniano, Primiceri, and Tambalotti (2017)). Typically, however, only one or few occasionally binding constraints are considered. In our calibrated model, five key constraints can switch from binding to slack and vice versa, interacting in complex ways and determining the effectiveness of monetary policy.

Interbank markets and bank liquidity management

The role of interbank markets in banks' liquidity management is explored by a large literature in banking, starting with Bhattacharya and Gale (1987). Several papers analyse frictions in interbank markets that prevent an efficient distribution of liquidity within the banking system (Flannery (1996); Freixas and Jorge (2008); Freixas and Holthausen (2005); Repullo (2005); Freixas, Martin, and Skeie (2011); Afonso and Lagos (2015); Atkeson, Eisfeldt, and Weill (2015)). Some of these frictions have played a particularly important role during the

Global Financial Crisis. Heider, Hoerova, and Holthausen (2015) build a model where asymmetric information about banks’ assets and counterparty risk induce banks to hoard liquidity and contribute to generate a “freeze” of the unsecured money market segment. Martin, Skeie, and von Thadden (2014) characterize when expectations-driven runs in the secured market are possible.

Macroeconomic impact of money market frictions

Some recent papers explore the macroeconomic consequences of the money market frictions that featured prominently during the Global Financial Crisis. Altavilla, Carboni, Lenza, and Uhlig (2018) provide evidence that increases in interbank rate uncertainty, as observed during 2007-2009 and again during the European sovereign crisis, generate a significant deterioration in economic activity. Using a general equilibrium model, Bruche and Suarez (2010) show that freezes in the unsecured money market segment can cause large reallocation of capital across regions, with significant impact on output and welfare. Gertler, Kiyotaki, and Prestipino (2016) point to runs on wholesale banks as a major source of the breakdown of the financial system in 2007-2009, and show in a general equilibrium framework that this can have devastating effects on the real economy. Our paper contributes to this literature by considering both unsecured and secured funding. In our setup, frictions in the unsecured money market segment may in principle be offset by an increased recourse to private secured markets or to central bank funding.

Bank liquidity management and monetary policy

Frictions in the unsecured or secured money markets interact with the effectiveness of monetary policy. Bianchi and Bigio (2013) build a model where banks are exposed to liquidity risk and manage it by borrowing unsecured or by holding a precautionary buffer of reserves. Monetary policy affects lending and the real economy by supplying reserves and thus by changing banks’ trade-off between profiting from lending and incurring greater liquidity risk. In a general equilibrium model that features the same search frictions in the interbank market as in Bianchi and Bigio (2013), Arce, Nuno, Thaler, and Thomas (2019) show that a policy of large central bank balance sheet that uses interest rate policy to react to shocks achieves similar stabilization properties to a policy of lean balance sheet, where QE is occasionally used when the interest rate hits the zero-lower bound. Piazzesi and Schneider (2017) build a model in which the use of inside money by agents for transaction purposes requires banks to handle payments instructions. Banks thus lend or borrow secured in the interbank market, or use

central bank reserves. In this framework, key to the efficiency of the payment system is the provision and allocation of collateral. Policies that exchange reserves for lower quality collateral can be beneficial when high quality collateral is scarce. In our model with both secured and unsecured money markets and a central bank providing collateralized loans or purchasing assets outright, it is the interplay between the bank liquidity and leverage constraints that is key in determining the macroeconomic impact of money market frictions and the effectiveness of central bank policies.

Scarcity of safe assets and the size of central bank balance sheet

The emergence of a shortage of safe assets has been documented and analyzed in a number of recent works (see e.g. Caballero, Farhi, and Gourinchas (2017), Andolfatto and Williamson (2015) and Gorton and Laarits (2018)). Some papers discuss the implications of scarcity for monetary policy. Caballero and Farhi (2017) analyze a situation of a deflationary safety trap and point to policies of “helicopter drops” of money, safe public debt issuances, or swaps of private risky assets for safe public debt as possible ways to mitigate the negative impact of safe asset scarcity. Carlson, Duygan-Bump, Natalucci, Nelson, Ochoa, Stein, and den Heuvel (2016) argue that the central bank could react to safe asset scarcity by maintaining a large balance sheet and a floor system, as large holdings of long-term assets are financed by large amounts of reserves that are safe and liquid assets. Our model enables to compare alternative policies - outright purchases and collateralized credit operations - that can accommodate the increased demand for reserves through an expansion of the balance sheet.

3 The partial equilibrium model

At the heart of our model are the decisions taken by banks. In this section, we provide a simple exposition of the bank problem in a static, partial-equilibrium setting. We keep the specification and notation close to the description of the full model, for ease of comparison.

Consider an economy populated by a continuum of banks (“Lenders”), indexed by $l \in (0, 1)$. Each bank starts the period with net worth n . There is one period, composed of two sub-periods, “morning” and “afternoon”.

At the beginning of the morning, banks are hit by a shock that discloses their type, which can be either “connected” or “unconnected”. We use the subscript c to denote a generic connected bank and u for a generic unconnected bank. Banks then choose their portfolio

of assets and liabilities, taking prices and returns as given, and make a payment to their shareholders.

In the afternoon, banks are hit by idiosyncratic liquidity shocks. We think of liquidity shocks as an act of writing a check on a deposit account or making an electronic payment. Payments between banks are settled using reserves acquired in the morning (outside money) or using interbank loans (IOUs, inside money) which are either unsecured or secured by government bond collateral (e.g., a repo). Unsecured IOUs can only be issued by connected banks. Unconnected banks have to collateralize their IOUs. At the end of the afternoon, the liquidity shocks are reversed and the interbank loans are repaid. Therefore, the end-of-the-afternoon assets and liabilities of a bank are unchanged compared to its assets and liabilities at the beginning of the afternoon.

At the end of the afternoon, assets and liabilities earn their market returns. The banks then take stock of the resulting balance sheet and the remaining net worth \tilde{n}_l . The objective of a bank is to maximize its end-of-period value \tilde{v}_l .

3.1 The morning

At the beginning of the morning, banks learn their type, i.e., whether they are connected or unconnected. After that, they make their asset-liability choice. On the asset side, a bank l can invest in loans to firms k_l (“capital”), in government discount bonds b_l and/or in reserves m_l (“money”). On the liability side, a bank can finance itself by taking in household deposits d_l and/or by taking discount central bank loans with face value f_l (central bank “funding”), which need to be collateralized by government bonds. We denote by $b_l^F \leq b_l$ bonds that a bank pledges with the central bank in the morning. We assume that the central bank imposes a haircut $1 - \eta$, $0 \leq 1 - \eta \leq 1$, on the posted bonds so that the total amount of funding a bank can obtain from the central bank is given by⁴

$$f_l \leq \eta Q b_l^F \tag{1}$$

where Q denotes the market discount price for the bonds and where $Q b_l^F$ represents the collateral value of bonds pledged at the central bank.

⁴It might be better to state the collateral constraint either as $Q^F f_l \leq \eta Q b_l^F$ in terms of money borrowed now or as $f_l \leq \eta b_l^F$ in terms of repayment later. Our equation is a hybrid, which one may justify by a concern, that the bank sells its bonds in t and holds cash to repay its central bank loan in $t + 1$. Ultimately, only the product ηQ matters here, in any case: so all three versions are equivalent in this static model, when adjusting η is allowed.

For consistency with the general model, we impose that banks pay a fixed fraction ϕ of the net worth n to their shareholders as a dividend at the end of the morning. Let Q^F denote the discount price on central bank loans. The balance sheet of a bank in the morning, before dividends are distributed, looks as follows:

Assets	Liabilities
k_l (capital)	d_l (deposits)
Qb_l (bond holdings)	$Q^F f_l$ (central bank loans)
ϕn (dividends)	n (net worth)
m_l (reserves)	

The balance sheet identity therefore writes as:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n \quad (2)$$

At the end of the afternoon, when the idiosyncratic liquidity shocks are reversed, the balance sheet of a bank is unchanged compared to the morning and asset returns accrue. In order to also account for these returns, let R_k denote the market return on capital and R_d the market return on deposits. The net worth \tilde{n} at the end of the afternoon will therefore be:

$$\tilde{n}_l = R_k k_l + b_l + m_l - R_d d_l - f_l \quad (3)$$

We assume that the end-of-the-afternoon value \tilde{v}_l of a bank is a fixed multiple of this residual net worth,

$$\tilde{v}_l = \tilde{\psi} \tilde{n}_l \quad (4)$$

for some parameter $\tilde{\psi}$.⁵

As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), we assume that there is a moral hazard constraint in that bank managers may run away with a fraction λ of their assets at the end of the morning. Define the end-of-the-morning value v_l of a bank, before dividends ϕn are paid to the household, as

$$v_l = \phi n + \tilde{v}_l \quad (5)$$

⁵In the general framework described in section 4, a subtle difference arises due to the presence of inflation, π : the residual net worth will then be $\tilde{n}_l = (R_k k_l + b_l + m_l - R_d d_l - f_l)/\pi$. This matters only numerically in terms of the constraint (4), and could be incorporated by using a numerically different value for $\tilde{\psi}$ here compared to the general framework.

We assume that the leverage constraint is given by

$$\lambda (k_l + Qb_l + m_l) \leq v_l \quad (6)$$

where $0 \leq \lambda \leq 1$ is a leverage parameter. Implicitly, we assume that the same leverage parameter holds for all assets, and that bankers can run away with all assets, including government bonds that may have been pledged as collateral with the central bank.⁶ We impose that asset positions and liability positions cannot be negative, $k_l \geq 0$, $b_l \geq 0$, $m_l \geq 0$, $d_l \geq 0$ and $f_l \geq 0$.

3.2 The afternoon

In the afternoon, banks face a liquidity management problem. We model this problem using the device introduced by Bianchi and Bigio (2017). At the beginning of the afternoon, banks experience idiosyncratic liquidity shocks. A bank l with end-of-morning deposits d_l experiences a shock $\omega_l d_l$. Negative (positive) ω_l denote incoming (outgoing) payments. Here, $\omega_l \in (-\infty, \omega^{\max}]$ is a random variable, which is iid across banks l and is distributed according to $F(\omega)$, and ω^{\max} is a parameter, $0 \leq \omega^{\max} \leq 1$. Payment shocks average out across all banks, $E[\omega_l] = 1$, so that total deposits remain unchanged. Payments between banks are settled using reserves m_l obtained in the morning (outside money) or using interbank loans (IOUs) which are either unsecured or secured by government bond collateral. At the end of the afternoon, the liquidity shocks are reversed and the interbank loans are repaid. Thus, an initial afternoon liquidity shock creates only a temporary liquidity need that banks must satisfy, in line with the idea of payments circulating in the system.⁷

We add to the Bianchi and Bigio (2017) structure the distinction between “connected” and “unconnected” bank types. Connected banks can issue unsecured IOUs in the afternoon interbank market. Unconnected banks must secure their IOUs with bonds. In the secured (repo) market, we assume that a lending bank imposes a haircut $0 \leq 1 - \tilde{\eta} \leq 1$. The borrowing bank then pledges an amount $\tilde{b}_l \leq (b_l - b_l^F)$ of bonds - reflecting that the bank can only pledge

⁶Alternatively, one may wish to impose that banks cannot run away with assets pledged to the central bank as collateral. Denoting by b_l^F bonds pledged with the central bank, the collateral constraint would then read as

$$\lambda \left[k_l + Q \left(b_l - b_l^F \right) + m_l \right] \leq v_l$$

or a version in-between this and the in-text equation. Since collateral pledged to the central bank typically remains in the control of banks, we feel that the assumption used in the text is more appropriate.

⁷We follow a long tradition in the banking literature of focusing on the role of interbank money markets in smoothing out idiosyncratic liquidity shocks, as in Bhattacharya and Gale (1987) and Allen and Gale (2000). While analytically convenient, in reality interbank relationships may exhibit more persistent patterns, with some banks being structural borrowers and others structural lenders in interbank markets (Craig and Ma (2018)).

the portion that has not yet been pledged to the central bank - and receives in return the cash amount $\tilde{\eta}b_l$ in the first leg of the repo, repaying the same amount at the end of the afternoon. The end bond position is therefore the one held in the morning, b_l . Taken literally, there is no risk here that this haircut could reasonably insure against, but this is just to keep the model simple. Every bank can lend unsecured, if they so choose.⁸ The interest rate on interbank IOUs is imposed to be zero.⁹

With ω^{\max} as the maximal liquidity shock, unconnected banks thus have to make sure they have enough reserves brought from morning and/or enough unpledged collateral to be able to cover afternoon payment flows as follows:

$$\omega^{\max}d_l \leq m_l + \tilde{\eta}Q(b_l - b_l^F) \quad (7)$$

We denote the above constraint as the unconnected bank's "afternoon constraint".¹⁰

As all the afternoon transactions are reversed at the end of the afternoon and since all within-afternoon interest rates are zero, banks will be entirely indifferent between using any of the available sources of liquidity: what happens in the afternoon stays in the afternoon. The balance sheet at the end of the afternoon, and before asset returns accrue, is the same as the balance sheet at the end of the morning. The only impact of these choices and restrictions is that unconnected banks need to plan ahead of time in the morning to make sure they have enough reserves or collateral in the afternoon, whatever level of withdrawals they may face.

Time line of bank decisions An overview of the timing of bank decisions is in figure 1.

3.3 The formal problem and its solution

The maximization problem of a bank $l \in \{c, u\}$ with net worth n is

$$\max \tilde{v}_l$$

⁸Implicitly, we are assuming that the discount window of the central bank is not open in the afternoon, i.e., that banks need to obtain central bank reserves, if any, in the morning in precaution to liquidity shocks in the afternoon. This captures the fact that the discount window is rarely used for funding liquidity needs and that these liquidity transactions happen "fast", compared to central bank liquidity provision.

⁹This can be justified, if some banks hold positive reserves $m_l > 0$: in that case, there is an excess supply of reserves (payment inflows plus morning reserves) compared to the demand for reserves (payment outflows). The market clearing interbank rate then must fall to the price of the alternative storage technology for keeping reserves, i.e., to zero. If no banks wishes to hold positive reserves in the morning, supply and demand for interbank loans are equal across a range of interest rates. We pick the lowest one, compatible with the storage alternative, implicitly assuming that the borrowing banks have all the bargaining power. Alternatively, one could introduce a minimum reserve requirement, so that always $m_l > 0$.

¹⁰We assume that banks will always find defaulting on the payments worse than any precautionary measure they can take against it, and thus rule out payment caps and bank runs by assumption.

subject to

$$\begin{aligned}
k_l + Qb_l + m_l + \phi n &= d_l + Q^F f_l + n \\
\tilde{n}_l &= R_k k_l + b_l + m_l - R_d d_l - f_l \\
\tilde{v}_l &= \tilde{\psi} \tilde{n}_l \\
v_l &= \phi n + \tilde{v}_l \\
f_l &\leq \eta Q b_l^F \\
b_l^F &\leq b_l \\
\lambda(k_l + Qb_l + m_l) &\leq v_l \\
\text{if } l = u: \quad \omega^{\max} d_u &\leq m_u + \tilde{\eta} Q (b_u - b_u^F)
\end{aligned}$$

together with the non-negativity constraints

$$k_l \geq 0, b_l \geq 0, m_l \geq 0, d_l \geq 0, f_l \geq 0$$

where ψ, λ, ϕ are given parameters and Q, Q^F, R_k, R_d are given market prices and returns.

From here on forward, we assume that $R_k \geq R_d$. This restriction on returns implies that banks are always willing to raise deposits and invest them in capital, ensuring that capital is always non-negative. We will therefore drop k_l from the list of variables subject to the non-negativity constraint.

Define the returns on government bonds and central bank loans as $R_b = 1/Q$ and $R_f = 1/Q^F$. We further assume that $R_d \leq R_f$. Therefore - absent any further considerations - banks will choose deposits rather than central bank loans, as they are a cheaper source of funding. This ensures that also deposits are always non-negative.

In order to solve the bank problem, some observations and simplifications shall prove useful. For the collateral constraint (1), banks will pledge just enough collateral to the central bank to make the constraint binding, nothing more (and we shall impose this as an assumption for the case that the banks are indifferent between that and pledging more). Then, we can write

$$f_l = \eta Q b_l^F. \quad (8)$$

We can also write the afternoon constraint for unconnected banks, equation (7), with equality,

$$\omega^{\max} d_u + \frac{\tilde{\eta}}{\eta} f_u = m_u + \tilde{\eta} Q b_u. \quad (9)$$

Banks will never choose to have this equation hold with strict inequality, given our assumptions regarding the returns.

Substituting out k_l in (3) with the balance sheet constraint (2), b_l^F with (8), v_l with (5) allows us to restate the problem as

$$\begin{aligned}
& \max_{(b_l, m_l, d_l, f_l) \in \mathbf{R}_+^4} \tilde{v}_l \\
\tilde{n}_l &= (R_k - R_d) d_l + (R_k - R_f) Q^F f_l - (R_k - R_b) Q b_l - (R_k - 1) m_l + R_k (1 - \phi) n \\
\tilde{v}_l &= \tilde{\psi} \tilde{n}_l \\
f_l &\leq \eta Q b_l \\
d_l + Q^F f_l &\leq \frac{\tilde{v}_l + \phi n}{\lambda} - (1 - \phi) n \\
\text{if } l = u: & \quad \omega^{\max} d_u + \frac{\tilde{\eta}}{\eta} f_u = m_u + \tilde{\eta} Q b_u
\end{aligned}$$

This is a linear programming problem, with solutions at extrema or zones of indifference, and it can be solved with the help of a few case distinctions. On the asset side, the bank will need to choose in the morning a combination of money, and bonds, for a given choice of liabilities.

Connected banks: For connected banks $l = c$, it is straightforward to see that $b_c = 0, m_c = 0, f_c = 0$. These banks can always satisfy their liquidity needs in the afternoon by issuing unsecured IOUs. Therefore they do not borrow at the central bank in the morning nor use bonds to secure their IOUs in the afternoon. It follows that

$$d_c = \frac{\tilde{v}_c + \phi n}{\lambda} - (1 - \phi) n$$

and

$$k_c = \frac{\tilde{v}_c + \phi n}{\lambda}$$

Unconnected banks: For unconnected banks, compare first relaxing the right hand side of the afternoon constraint (7) by one unit with either money or bonds, given d_u and f_u . Using money results in a loss of $R_k - 1$ units of net worth $\partial \tilde{n}_u$ next period, which arises from the lost revenues per giving up investment in capital for holding money. Using bonds requires an additional bond investment of $1/\tilde{\eta}$, which costs $(R_k - R_b)/\tilde{\eta}$ units of net worth $\partial \tilde{n}_u$ next period, where $R_k - R_b$ arises from the lost revenues per investing in bonds instead of capital..

1. If

$$\frac{R_k - R_b}{\tilde{\eta}} > R_k - 1 \quad (10)$$

then money is cheaper, and $b_u = f_u/(Q\eta)$ as well as $m_u = \omega^{\max}d_u$.

2. If the inequality in (10) is the opposite, with the left-hand-side strictly smaller than the right-hand-side, then bond financing is cheaper for the afternoon. Then $m_u = 0$ and

$$b_u = \frac{\omega^{\max}}{Q\tilde{\eta}}d_u + \frac{1}{Q\eta}f_u.$$

3. With equality in (10), the bank is indifferent between any non-negative value of b_u and m_u that satisfy the afternoon constraint (7) holding with equality.

On the liability side, the bank will need to choose some combination of $d_u \geq 0$ and $f_u \geq 0$. With the previous case distinction regarding the afternoon constraint, an additional marginal unit of deposits ∂d_u earns

$$X_d = R_k - R_d - \omega^{\max} \min \left\{ R_k - 1, \frac{R_k - R_b}{\tilde{\eta}} \right\}$$

of additional marginal net worth $\partial \tilde{n}_u$ next period, where $R_k - R_d$ arises from the spread earned per investing in capital and paying depositors, and where the second term arises from the lost revenues per switching from capital to either money or bonds, in order to satisfy the afternoon constraint. An additional marginal unit of central bank funding ∂f_u earns

$$X_f = (R_k - R_f)Q^F - \frac{R_k - R_b}{\eta}$$

of additional marginal net worth $\partial \tilde{n}_u$. The first term is the spread $R_k - R_f$ earned from investing in capital and repaying the loan from the central bank. The second term is the lost revenues arising from the need to substitute out capital in order to hold more bonds as collateral with the central bank, where $\frac{1}{\eta}$ is the additional loan that pledging one unit of bond at the central bank entitles to receive. Note that bonds pledge always has the same opportunity cost, regardless of the cheapest afternoon source of funding: as (7) states, only the bonds not already pledged to the central bank can be used to relax the afternoon constraint.

With this, we obtain the following case distinction.

1. If $\max\{X_d, X_f\} > 0$, then the leverage constraint will be satisfied with equality, i.e. the bank will fully leverage. Additionally,

- (a) If $X_d > X_f$, then $f_u = 0$ and

$$d_u = \frac{\tilde{v}_u + \phi n}{\lambda} - (1 - \phi) n$$

- (b) If $X_d < X_f$, then $d_u = 0$ and

$$Q^F f_u = \frac{\tilde{v}_u + \phi n}{\lambda} - (1 - \phi) n$$

- (c) If $X_d = X_f$, then the bank is indifferent among any value d_u and f_u between the two bounds arising from the previous two cases, as long as the leverage constraint (6) is satisfied with equality.
2. If $\max\{X_d, X_f\} = 0$, then the leverage constraint (6) will be satisfied with inequality. Additionally,
 - (a) If $0 = X_d > X_f$, then $f_u = 0$ and d_u can be anywhere between 0 and

$$d_u^{\max} = \frac{\tilde{v}_u + \phi n}{\lambda} - (1 - \phi) n$$

The bank is indifferent between all these choices.

- (b) If $X_d < X_f = 0$, then $d_u = 0$ and f_u can be anywhere between 0 and

$$Q^F f_u^{\max} = \frac{\tilde{v}_u + \phi n}{\lambda} - (1 - \phi) n$$

The bank is indifferent between all these choices.

- (c) If $X_d = X_f = 0$, then the bank is indifferent among any value $d_u \geq 0$ and $f_u \geq 0$, as long as (6) is satisfied.
3. If $\max\{X_d, X_f\} < 0$, then $d_u = f_u = 0$.

With this, the portfolio choices of connected and unconnected bank are fully described.

4 The general equilibrium model

We cast the partial equilibrium model from the previous section into a dynamic, nominal, deterministic general equilibrium framework.¹¹ The economy is inhabited by a continuum of households, firms and banks, a government and a central bank. Time is discrete and infinite. Each period is composed of a “morning” and an “afternoon.”

An overview of the timing is in figure 2.

In the morning, households receive nominal payments from their holdings of financial assets and allocate their wealth among money and deposits at banks. They also supply labor to firms, receiving wages in return. The government taxes the labor income of the households, makes payments on its debt and may change the stock of outstanding debt. Banks are hit by a shock that discloses their afternoon “type.” The time line of bank decisions is as outlined in the partial equilibrium model presented in section 3.

In the afternoon, firms use labor and capital to produce a homogeneous output good which is consumed by households. Banks settle the idiosyncratic liquidity shocks with reserves acquired in the morning or with interbank loans which are either unsecured (if issued by the connected banks) or secured with government bond collateral (if issued by the unconnected banks).

Firms and banks are owned by households. Similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2011), banks are operated by bank managers who run a bank on behalf of their owning households. We deviate from those papers in that banks pay a fixed fraction of their net worth to households as a dividend at the end of the morning of every period.

As the general equilibrium model is nominal, we will use capital letters to denote the relevant nominal variables henceforth.

4.1 The household

There is a continuum of identical households. At the beginning of time t , the representative household holds an amount of cash, \widetilde{M}_{t-1}^h , brought from period $t-1$, and receives repayment from banks of deposits opened in the previous period, $R_{d,t-1}D_{t-1}$, where $R_{d,t-1}$ is the gross return on one unit of deposits. The household allocates the nominal funds at hand among

¹¹Our focus is on the analysis of different monetary policies under alternative steady state comparative statics scenarios. We therefore consider a deterministic model and abstract from aggregate risk.

existing nominal assets, namely money, M_t^h , and deposits, D_t ,

$$M_t^h + D_t = R_{d,t-1} D_{t-1} + \widetilde{M}_{t-1}^h. \quad (11)$$

During the day, beginning-of-period money balances are increased by the value of households' revenues and decreased by the value of their expenses. The amount of nominal balances brought by the household into period $t + 1$, \widetilde{M}_t^h , is thus

$$\widetilde{M}_t^h = M_t^h + (1 - \tau_t) W_t h_t + \phi N_t - P_t c_t, \quad (12)$$

where P_t is the price of the consumption good, c_t is the amount of that good consumed, h_t is hours worked, τ_t is the labor tax rate, W_t is the nominal wage level and ϕN_t is the share ϕ of banks' aggregate net worth N_t that is distributed to households.

The household then chooses $c_t > 0, h_t > 0, D_t \geq 0, M_t^h \geq 0$ to maximize the objective function

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t, h_t) + v\left(\frac{M_t^h}{P_t}\right) \right] \quad (13)$$

subject to (11) and (12).

4.2 Firms

A representative final-good firm uses capital k_{t-1} and labor h_t to produce a homogeneous final output good y_t according to the production function

$$y_t = k_{t-1}^\theta h_t^{1-\theta}.$$

It receives revenues $P_t y_t$ and pays wages $W_t h_t$. Capital is owned by the firms, which are in turn owned by banks: effectively then, the banks own the capital, renting it out to firms and extracting a nominal rental rate $P_t r_t$ per unit of capital.

Capital-producing firms buy old capital k_{t-1} from the banks and combine it with final goods I_t (where capital I_t is, exceptionally, used to denote *real* investments) to produce new capital k_t , according to

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

New capital is then sold back to banks. Alternatively and equivalently, one may directly assume that the banks undertake the investments.

4.3 The government

At the beginning of period t , the government has some outstanding debt with face value \bar{B}_{t-1} of which a fraction κ will be repaid. The government furthermore needs to purchase goods $P_t g_t$. It pays for its expenses by taxing labor income, collecting seigniorage from the central bank, as well as issuing discount bonds with a face value $\Delta \bar{B}_t$ to be added to the outstanding debt next period, obtaining nominal resources $Q_t \Delta \bar{B}_t$ for it in period t . We assume that some suitable no-Ponzi condition holds.

The outstanding debt at the beginning of period $t + 1$ will be $\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta \bar{B}_t$. The government budget balance at time t is

$$P_t g_t + \kappa \bar{B}_{t-1} = \tau_t W_t h_t + Q_t \Delta \bar{B}_t + S_t, \quad (14)$$

where S_t are seigniorage payments from the central bank and g_t is an exogenously given process for government expenditures.

The government conducts fiscal policy by adopting a rule for the income tax that stabilizes the real stock of debt, $\bar{b} = \frac{\bar{B}}{P_t}$, at a targeted level \bar{b}^* .¹²

4.4 The central bank

The central bank chooses the total money supply \bar{M}_t and interacts with banks in the “morning”, providing them with funds. These funds take the form of one period loans. In period t , banks obtain loans with face value \bar{F}_t , getting funding in the amount $Q_t^F \bar{F}_t$, where Q_t^F is the common price or discount factor, which is a policy parameter set by the central bank. Banks also repay previous period liabilities, \bar{F}_{t-1} .

The central bank furthermore buys and sells government bonds outright. Let B_{t-1}^C be the stock of government bonds held by the central bank (“C”) at the beginning of period t . The government makes payments on a fraction of these bonds, i.e., the central bank receives cash

¹²A possible specification of the fiscal rule is $(\tau_t - \tau^*) \frac{W_t}{P_t} h_t = \alpha \left(\frac{\bar{B}_t}{P_t} - \bar{b}^* \right)$, see equation (72) in the appendix. Thus, τ_t increases above its target level τ^* , if the real stock of debt \bar{B}_t/P_t is above its target \bar{b}^* . The reaction coefficient α needs to be such that the equilibrium is saddle-path stable and that the fiscal rule ensures a gradual convergence to the desired stock of debt, following aggregate disturbances. Notice, however, that in our quantitative section, we provide a comparison of steady state equilibria: in that analysis, the parameter α plays no role. The target value τ^* is the level of the income tax necessary to stabilize the debt at \bar{b}^* . τ^* can be obtained by combining $\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta \bar{B}_t$ and equation (14) in steady state, together with the rule $b = \bar{b}^*$, to get

$$\tau^* (1 - \theta) y = g + \kappa (1 - Q) \frac{\bar{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi} \right) \bar{b}^* - s,$$

where $s = \frac{S}{P}$ and π is the steady state inflation rate.

payments κB_{t-1}^C . The remaining government bonds in the hands of the central banks are $(1 - \kappa) B_{t-1}^C$. The central bank then changes its stock to B_t^C , at current market prices Q_t , using cash. Thus, $B_t^C = (1 - \kappa) B_{t-1}^C + \Delta B_t^C$, where ΔB_t^C denote gross bond purchases by the central bank.

The central bank balance sheet looks as follows at time t :

Assets	Liabilities
$Q_t^F \bar{F}_t$ (loans to banks)	M_t^h (currency held by HH)
$Q_t B_t^C$ (bond holdings)	M_t (bank reserves)
	S_t (seigniorage)

Let

$$\bar{M}_t = M_t^h + M_t$$

be the total money stock before seigniorage is paid. Note that the seigniorage is paid to the government at the end of the period and therefore becomes part of the currency in circulation next period. The flow budget constraint of the central bank is given by:

$$\bar{M}_t - \bar{M}_{t-1} = S_{t-1} + Q_t^F \bar{F}_t + Q_t B_t^C - R_{b,t} Q_{t-1} B_{t-1}^C - R_{f,t} Q_{t-1}^F \bar{F}_{t-1} \quad (15)$$

where $R_{b,t} = \frac{(1-\kappa)Q_t + \kappa}{Q_{t-1}}$ and $R_{f,t} = \frac{1}{Q_t^F}$. Seigniorage can then be calculated as the residual balance sheet profit,

$$S_t = Q_t^F \bar{F}_t + Q_t B_t^C - \bar{M}_t. \quad (16)$$

4.5 Banks

There is a continuum of banks (“Lenders”), indexed by l , owned by the households. The subscript l for a generic bank encapsulates the entire history of a particular bank (being connected/unconnected over time).

Each period t is divided into morning and afternoon.

Each bank enters the morning with its portfolio from the previous period and end-of-period net worth N_t, l .

We assume that households are perfectly diversified across banks (they hold the entire banking sector). As we will show below, the problem of a bank is linear in net worth. Therefore, the distribution of net worth across banks does not matter for aggregate allocations. We note that, depending on the prior histories of being connected or unconnected, some banks would

be growing over time while some would be shrinking over time. We assume that, regardless of their size, all banks behave competitively and take prices as given.

In the morning, banks learn their afternoon type. With probability ξ_t , they are of the connected type, and able to issue unsecured IOUs in the afternoon. With probability $1 - \xi_t$, they are of the unconnected type and can only issue afternoon IOUs secured by government bonds collateral. We assume this probability to be iid across banks and time. Knowing their type, banks choose their portfolio of assets and liabilities. On the asset side, they can lend to firms (more precisely, finance their capital), hold government bonds and reserves (“money”). On the liability side, they can fund themselves with deposits from households and/or with collateralized loans from the central bank. They distribute dividends to households.

In the afternoon, banks face idiosyncratic liquidity shocks, which reverse at the end of the period, as described in detail in Section 3.2. Then, IOUs are repaid and the portfolio of banks returns to what it was at noon.

4.5.1 Assets and liabilities

In the morning, before paying dividends to shareholders, a generic bank l holds four types of assets, as in Section 3.1.

1. Capital $k_{t,l}$ of firms, or, equivalently, firms, who in turn own the capital. Capital can only be acquired and traded in the morning and evolves according to $k_{t,l} = (1 - \delta) k_{t-1,l} + \Delta k_{t,l}$, where $\Delta k_{t,l}$ is the gross investment of bank l in capital.
2. Bonds with a nominal face value $B_{t,l}$. A fraction κ of the government debt will be repaid. The bank changes its government bond position per market purchases or sales $\Delta B_{t,l}$ in the morning, so that $B_{t,l} = (1 - \kappa) B_{t-1,l} + \Delta B_{t,l}$. If the bank purchases (sells) bonds on the open market, it pays (receives) $Q_t \Delta B_{t,l}$.
3. Cash as a fraction of the bank’s net worth, $\phi N_{t,l}$, earmarked to be distributed to shareholders as dividends at the end of the morning.
4. Reserves (M =“money”) $M_{t,l} \geq 0$. Banks may add to cash (not earmarked for paying shareholders), $M_{t,l} = M_{t-1,l} + \Delta M_{t,l} \geq 0$. (In the afternoon, banks may see temporary inflows or outflows of reserves, until payment shocks reverse at the end of the afternoon.)

Bank l has three types of liabilities at the end of the morning:

1. Deposits $D_{t,l}$. This is owed to households and subject to aggregate withdrawals and additions $\Delta D_{t,l}$, so that $D_{t,l} = R_{d,t-1}D_{t-1,l} + \Delta D_{t,l}$. (Additionally, there are idiosyncratic withdrawals and additions in the afternoon, which subsequently reverse.)
2. Secured loans (F ="funding") from the central bank at face value $F_{t,l}$. A bank l with liabilities $F_{t,l}$ to the central bank needs to pledge an amount of government bonds $B_{t,l}^F$, satisfying the collateral constraint

$$F_{t,l} \leq \eta_t Q_t B_{t,l}^F \quad (17)$$

corresponding to (1). Secured loans from the central bank can only be obtained in the morning.

3. Net worth $N_{t,l}$.

The sum of assets equals the sum of liabilities.

The bank's balance sheet constraint at the end of the morning is given by

$$P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_{t,l} = D_{t,l} + Q_t^F F_{t,l} + N_{t,l}, \quad (18)$$

corresponding to equation (2) in the partial equilibrium model. Note that the afternoon interbank loans do not appear in the morning balance sheet as liquidity shocks only materialize in the afternoon and are in any case reversed by the end of the afternoon.

The collateral constraint at the central bank (17) requires loans not to exceed the value of the bonds pledged, adjusted by the central bank haircut. In equilibrium, banks will pledge just enough collateral to the central bank to make the collateral constraint bind, nothing more (even if indifferent between that and pledging more: then, "bind" is an assumption). Thus, for both types of banks,

$$F_{t,l} = \eta_t Q_t B_{t,l}^F \quad (19)$$

corresponding to equation (8).

Bonds pledged at the central bank are constrained to be non-negative but also not to exceed the amount of bonds acquired in the morning,

$$0 \leq B_{t,l}^F \leq B_{t,l}. \quad (20)$$

Let $V_{t,l}$ be the value of a bank l at the end of the morning, after the type of the bank is known but before dividends are paid, see subsection 4.5.2 below. The bank leverage constraint requires the value of the bank $V_{t,l}$ not to fall below the share λ of the value of the assets that the bank can run-away with:

$$\lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \leq V_{t,l} \quad (21)$$

corresponding to equation (6).

Unconnected banks - who do not have access to the unsecured market in the afternoon - must plan ahead how to cover their afternoon liquidity needs. They must hold enough money and/or have enough government bonds to pledge in the private secured market. Banks can only pledge the portion of bonds not yet pledged to the central bank. The afternoon constraint is given by

$$\omega^{\max} D_{t,l} \leq M_{t,l} + \tilde{\eta}_t Q_t (B_{t,l} - B_{t,l}^F). \quad (22)$$

corresponding to (7).

There are also non-negativity constraints for investing in capital, deposits, cash, in bonds, and for financing from the central bank, for any bank l :

$$0 \leq k_{t,l} \quad (23)$$

$$0 \leq D_{t,l} \quad (24)$$

$$0 \leq M_{t,l} \quad (25)$$

$$0 \leq B_{t,l} \quad (26)$$

$$0 \leq F_{t,l}. \quad (27)$$

4.5.2 The bank problem

Define $V_{t,l}$ as the value of a bank l in the morning of period t after the type of the bank is known, but before dividends are paid. It is the nominal price a household would be willing to pay for a bank l before dividend payments and taking into account the future randomness of net worth due to the future type draws, and is given by

$$V_{t,l} = E \left[\sum_{s=0}^{\infty} \beta^s \frac{u_c(c_{t+s}, h_{t+s})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+s}} \phi N_{t+s,l} \right] \quad (28)$$

This can be rewritten in a recursive fashion. Define $\bar{V}_{t,l}$ as the value of a bank l in the morning, before the type draw for t is known. It is given by

$$\bar{V}_{t,l} = E[V_{t,l}] \quad (29)$$

where the expectation reflects the type draw.

Define $\tilde{V}_{t,l}$ as the value of bank l at the end of the morning, after the distribution of dividends and after the type is known. We have

$$V_{t,l} = \phi N_{t,l} + \tilde{V}_{t,l} \quad (30)$$

corresponding to equation (5). In turn

$$\tilde{V}_{t,l} = \beta \frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \bar{V}_{t+1,l}. \quad (31)$$

Equations (29) (30) and (31) deliver a recursive formulation of (28).

Let $R_{k,t} = \frac{P_t}{P_{t-1}} (r_t + 1 - \delta)$ be the return on holding one unit of capital from $t-1$ to t , and r_t be the rental rate of capital acquired in $t-1$. Net worth $N_{t,l}$ results from the investment of the previous period and is thus given by

$$N_{t,l} = R_{k,t} P_{t-1} k_{t-1,l} + M_{t-1,l} + R_{b,t} Q_{t-1} B_{t-1,l} - R_{d,t-1} D_{t-1,l} - R_{f,t-1} Q_{t-1}^F F_{t-1,l} \quad (32)$$

In the morning, after the type is known, bank l chooses $k_{t,l}$, $B_{t,l}$, $B_{t,l}^F$, $F_{t,l}$, $D_{t,l}$, $M_{t,l}$ to maximize $V_{t,l}$, subject to equation (32) and

$$\begin{aligned} V_{t,l} &\geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \\ 0 &\leq B_{t,l} - B_{t,l}^F \\ P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_{t,l} &= D_{t,l} + Q_t^F F_{t,l} + N_{t,l} \\ F_{t,l} &\leq \eta_t Q_t B_{t,l}^F \end{aligned}$$

as well as

$$\omega^{\max} D_{t,l} \leq M_{t,l} + \tilde{\eta}_t Q_t (B_{t,l} - B_{t,l}^F)$$

if bank l is unconnected in period t .

We shall consider only scenarios in which bank net worth remains positive and banks repay the portion ϕ of their net worth to households at noon of each period.

4.6 The rest of the world

We assume that a share of the stock of government bonds is held by the rest of the world and that foreigners have an elastic demand for those bonds.¹³ Because unconnected banks can buy or sell bonds to foreigners, they can change their bond holdings independently from the government's outstanding stock of debt.

We do not wish to model the foreign sector explicitly. We simply assume that international investors have a demand for domestic bonds, B_t^w , that reacts to movements in the real return,

$$\frac{B_t^w}{P_t} = f\left(\varkappa - \frac{1}{\varrho} \log \tilde{Q}_t \pi_t\right) \quad (33)$$

where $\varrho > 0$, $\varkappa \geq 0$ and where $\tilde{Q}_t^{-1} = \frac{R_{b,t}}{Q_{t-1}}$ denotes the nominal return from investing one unit of money in bonds (correspondingly, $\frac{1}{\tilde{Q}_t \pi_t}$ is the real return). The function $f(\cdot)$ is differentiable and increasing; it provides a non-linear transformation ensuring that the foreign demand does not become negative when the net return becomes zero.¹⁴ Notice that, if $\varrho = 0$, the bond demand is infinitely elastic. In that case, the real return is fixed and foreign holdings take whatever value is needed to clear the bond market.

The flow budget constraint of the foreign sector is

$$Q_t B_t^w + P_t c_t^w = R_{b,t} Q_{t-1} B_{t-1}^w. \quad (34)$$

4.7 Analysis

Here, we describe the decision of households, firms and banks in turn.

4.7.1 Households and firms

The household maximizes his preferences, equation (13), subject to the budget constraints

$$D_t + M_t^h \leq R_{d,t-1} D_{t-1} + M_{t-1}^h + (1 - \tau_{t-1}) W_{t-1} h_{t-1} + \phi N_{t-1} - P_{t-1} c_{t-1} \quad (35)$$

¹³We introduce the elastic foreign sector demand for two reasons. First, a large fraction of euro area sovereign debt is held by non-euro area residents, and these bondholders actively re-balance their bond positions. Kojien, Koulischer, Nguyen, and Yogo (2016) document that during the Public Sector Purchase Programme implemented by the ECB since March 2015, for each unit of sovereign bonds purchased by the ECB, the foreign sector sold 0.64 of it. Second, when solving the model we will focus on the parameter space in which connected banks choose not to hold bonds. In a closed economy, unconnected banks would have to absorb whatever amount of bonds is issued by the government (after deducting the fixed amount held by the central bank). The price of the bond would have to adjust to clear the market. Such direct link between the bond market and the unconnected banks' decisions would be quantitatively implausible.

¹⁴More specifically, in our numerical analysis, we use the functional form $f(\cdot) = \left(\varkappa - \frac{1}{\varrho} \log \tilde{Q}_t \pi_t\right) \frac{[\arctan(200(1-Q_t)+3.14)]}{3.14}$.

Note that there are further restrictions on the choice variables, i .e. $c_t > 0, h_t > 0, M_t^h > 0$ and $D_t \geq 0$. We do not list these constraints separately for the following reasons. For $c_t > 0, h_t > 0$, and $M_t^h > 0$, we can assure non-negativity with appropriate choice for preferences and per the imposition of Inada conditions. We constrain the analysis a priori to $D_t > 0$, (see section 5 below) despite the possibility that it could be negative when allowing for more generality.¹⁵

Firms choose labor $h_t > 0$, and capital $k_t > 0$ to maximize their profits. The optimality conditions for households and firms are reported in appendix A.

4.7.2 Banks

The following proposition allows us to calculate the value of a bank l in the morning, $V_{t,l}$.

Proposition 1 (linearity) *The problem of bank l is linear in net worth and*

$$\bar{V}_{t,l} = \psi_t N_{t,l} \quad (36)$$

for any bank l and some factor ψ_t which gives the value of a marginal unit of net worth of a bank in the morning, before a bank's type (connected/unconnected) is known. In particular, $\bar{V}_{t,l} = 0$ if $N_{t,l} = 0$.

Proof: The bank problem is linear. Since there are no fixed costs, a bank with twice as much net worth can invest twice as much in the assets. Furthermore, if a portfolio is optimal at some scale for net worth, then doubling every portion of that portfolio is optimal at twice that net worth. Thus the value of the bank is twice as large, giving the linearity above. QED

The proposition above implies that

$$\int_0^1 \bar{V}_{t,l} dl = \psi_t \int_0^1 N_{t,l} dl \quad (37)$$

which gives the value of a marginal unit of net worth at the beginning of period t , for the aggregate banking sector.

¹⁵We have not analyzed this matter for the dynamic evolution of the economy. It may well be that net worth of banks temporarily exceeds the funding needed for financing the capital stock, and that therefore deposits ought to be negative, rather than positive. For now, the attention is on the steady state analysis, however, and on returns to capital exceeding the returns on deposits.

It follows from (30), (31) and (36), that¹⁶

$$\tilde{V}_{t,l} = \tilde{\psi}_t N_{t+1,l} \quad (38)$$

corresponding to (4), and that

$$V_{t,l} = \phi N_{t,l} + \tilde{\psi}_t N_{t+1,l} \quad (39)$$

where

$$\tilde{\psi}_t = \Omega_{t+1} \psi_{t+1} \quad (40)$$

and

$$\Omega_{t+1} = \beta \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \right] \quad (41)$$

We focus on cases where banks choose to raise deposits and to extend loans. When $D_{t,l} > 0$, banks have liquidity shocks in the afternoon, providing a meaningful role for interbank markets to smooth out those shocks. When $k_{t,l} > 0$, there is an active link between financial intermediation and real activity in the economy. Throughout, we check that the conditions ensuring that $D_{t,l} > 0$ and $k_{t,l} > 0$ are satisfied, similar to the conditions laid out in section 3.3. As in Section 3, we explicitly allow for corner solutions for $M_{t,l}$, $B_{t,l}$ and $F_{t,l}$.

These are linear programming problem, maximizing a linear objective subject to linear constraints. So, the solution is either a corner solution or there will be indifference between certain asset classes, resulting in no-arbitrage conditions. The optimality conditions of the problem of the banks are reported in appendix B.

The equilibrium of the model is defined in appendix C. All the equilibrium conditions are also listed there.

In the numerical analysis, it will be useful to refer to what we label “collateral premium,” Λ_{t+1} . This is defined as the difference for banks between the return at time $t+1$ from investing one unit of net worth at time t in capital and the return from investing the same unit of net worth in bonds:

$$\Lambda_{t+1} = \tilde{\psi}_{t+1} (R_{k,t+1} - R_{b,t+1}). \quad (42)$$

If the premium is positive, connected banks always prefer to invest in capital rather than to hold bonds, as capital yields a higher return and these banks do not need to use bonds as collateral. Unconnected banks, instead, may decide to hold bonds despite the lower return because of their

¹⁶This would get a bit more tricky in the stochastic version of this model, as one would then have to take into account the correlation of the random returns and thus the randomness of the net worth in period $t+1$ with the stochastic discount factor.

collateral value. In particular, the collateral premium is positive if the afternoon constraint is binding or if banks are collateral-constrained in their borrowing from the central bank. Formal proof of this argument is given in appendix D.

Given the linearity of the bank problem per proposition 1, we analyze the problem of an average connected or average unconnected bank, denoting them with $l = c$ or $l = u$ in slight abuse of the notation. In the morning, banks do not yet know their type. The net worth of the average connected or unconnected bank is thus the same and given by

$$N_t = \int N_{t,l} dl \quad (43)$$

Likewise, their value at the beginning of the morning is given by $\bar{V}_t = \psi_t N_t$. Once their type is realized, connected and unconnected banks make different choices, resulting in different net worths $N_{t+1,c}$ and $N_{t+1,u}$ per equation (32), where the index c and u here refers to the connection status in t . Average net worth in $t + 1$ is then

$$N_{t+1} = \xi_t N_{t+1,c} + (1 - \xi_t) N_{t+1,u} \quad (44)$$

This is then also the net worth of the average and newly connected or unconnected bank in $t + 1$. Equations (29) (30) and (31) turn into

$$\bar{V}_t = \xi_t V_{t,c} + (1 - \xi_t) V_{t,u} \quad (45)$$

$$V_{t,c} = \phi N_t + \tilde{V}_{t,c} \quad (46)$$

$$V_{t,u} = \phi N_t + \tilde{V}_{t,u} \quad (47)$$

$$\tilde{V}_{t,c} = \beta \frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \frac{N_{t+1,c}}{N_{t+1}} \bar{V}_{t+1} \quad (48)$$

$$= \tilde{\psi}_t N_{t+1,c} \quad (49)$$

$$\tilde{V}_{t,u} = \beta \frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \frac{N_{t+1,u}}{N_{t+1}} \bar{V}_{t+1} \quad (50)$$

$$= \tilde{\psi}_t N_{t+1,u} \quad (51)$$

where, again, we need to keep in mind that c and u in the last two lines refer to the type draw at t , not at $t + 1$.

4.8 Steady state analysis

We characterize a steady state where prices grow at the rate π and all shocks are zero except for the idiosyncratic liquidity shock ω faced by banks. We denote with small letters all real

variables, i.e. the corresponding variables in capital letter divided by the price of the consumption good, P_t . The steady state is characterized by the set of conditions reported in Appendix E.

5 Numerical results

In this section, we calibrate the model to euro area data. We then evaluate the macroeconomic impact of the observed money market developments and the effectiveness of alternative central bank policies.

Our results highlight the complex interactions between various occasionally binding constraints, which is a novel feature of our model. To simplify the numerical analysis, we restrict our attention to regions of the parameter space where connected banks hold neither bonds nor money, nor do they borrow at the central bank, i.e. $b_c = b_c^F = m_c = f_c = 0$. This effectively limits the number of interacting occasionally binding constraints to seven: for both types of banks, $l = u, c$, the leverage constraint (equations (10)); for $l = u$, the afternoon constraint (equation 22), the non-negativity constraints for bonds pledged at the central bank and the constraint that those bonds cannot exceed the stock of bonds held in the morning (summarized in condition (20)), and the non-negativity constraint for money, bonds and loans from the central bank (conditions (25)-(27)).

When a single parameter changes, constraints can turn from binding to slack, and then to binding again, due to the interaction with other constraints. The particular constraint that binds is typically crucial for determining the effectiveness of policy interventions.

5.1 Calibration

In the model, each period is a quarter. In the numerical analysis, we assume the following functional form of the utility function:

$$u(c_t, h_t) + v\left(\frac{M_t^h}{P_t}\right) = \log(c_t) + \frac{1}{\chi} \log\left(\frac{M_t^h}{P_t}\right) - \frac{h_t^{1+\epsilon}}{1+\epsilon}.$$

Table 2 summarize the value of all the parameter under the chosen calibration, which we discuss in turn below.

We set the discount factor at $\beta = 0.994$,¹⁷ and the inverse Frisch elasticity at $\epsilon = 0.4$. The depreciation rate is fixed at $\delta = 0.02$, and the capital income share θ at 0.33. The fraction of government bonds repaid each period, κ , is 0.042, corresponding to an average maturity of the outstanding stock of euro area sovereign bonds of around 6 years.¹⁸ The parameters determining the value of collateral in the private market and at the central bank reflect the data shown in Table 1. The haircuts on government bonds in private markets and at the central bank are set equal to each other, at $1 - \tilde{\eta} = 1 - \eta = 0.03$ (corresponding to a 3% haircut). The private haircut value is taken from LCH.Clearnet, a large European-based multi-asset clearing house, and refers to an average haircut on French, German and Dutch bonds across all maturities in 2010. The value for the central bank haircut matches the haircut imposed by the ECB on sovereign bonds with credit quality 1 and 2 (corresponding to a rating AAA to A-) in 2010.

Two novel parameters of our model, which capture frictions in the funding markets and are key to determining banks' choices, are the share of "unconnected" banks, $1 - \xi$, and the maximum fraction of deposits that households can withdraw in the afternoon, ω^{\max} .

We compute the average pre-crisis value of $1 - \xi$ using data from the Euro Money Market Survey, which underlie Figure 3. We set $1 - \xi = 0.58$, corresponding to the 2003-2007 average share of cumulative quarterly turnover in the secured market in the total turnover, which sums up the turnover in the secured and in the unsecured segments (where 2003 is the first available observation in the survey while 2007 is the last year before the Global Financial Crisis). To assess the impact of the observed decline in unsecured market access, we compute the same average over 2008-2019 (where 2019 is the last available observation). The average value for that period is 0.89.

We determine ω^{\max} using the information embedded in the liquidity coverage ratio (LCR) - a prudential instrument that requires banks to hold high-quality liquid assets (HQLA) in an amount that allows them to meet 30-days liquidity outflows under stress. We implicitly assume here that regulators can estimate with high precision the 30-days outflows in a period of stress. We therefore calibrate ω^{\max} so that the maximum amount of liquidity demand in the model,

¹⁷The inverse of the discount factor $1/\beta$ determines the real rate on household deposits. This rate has been very low in the euro area (in fact, it was negative for overnight deposits both before and after the onset of the financial crisis). To match this stylized fact, we choose a relatively high discount rate β .

¹⁸Average maturity is computed as a weighted average of all maturities of euro area government bonds, with weights given by outstanding amounts in year 2011. Source: Bloomberg, ECB and authors' calculations. Bond level data used in Andrade, Breckenfelder, Fiore, Karadi, and Tristani (2016) give a similar average maturity in 2015, pointing to a stable maturity structure of euro area debt over time.

$\omega^{\max} D_{t,u}$, equates the observed holdings of HQLA.¹⁹ More specifically, we use the European Banking Authority report from December 2013, which provides LCR data for 2012Q4 and covers 357 EU banks from 21 EU countries. Their total assets sum to EUR 33000 billion, and the aggregate HQLA to EUR 3739 billion. We take ω^{\max} to be the ratio of aggregate HQLA over total assets so that $\omega^{\max} = 0.1$.

We choose the parameter of the foreign demand for bonds, \varkappa , to ensure that, if foreign bond holdings take a value consistent with their observed share in total debt, then \tilde{Q} and π also take their average value at that steady state (.955 and 1.005, respectively). The steady state calibration cannot inform us about the elasticity of foreign bond demand ϱ , so we pick a value that produces an elasticity which is in line with available empirical evidence. We take data reported by Koijen, Koulischer, Nguyen, and Yogo (2016) on average foreign holdings of euro area government bonds over the periods 2013Q4-2014Q4 and 2015Q2 to 2015Q4. We compute the percentage change in foreign holdings between the two periods to be -3.3%. We then calculate the percentage change between the same periods in the average real return on euro area government bonds to be -38%.²⁰ We then set ϱ to replicate the observed elasticity of foreign bond holdings with respect to changes in the real return on bonds, i.e. $\varrho = 1.76$. We check robustness to alternative values (not reported) and find little impact on our quantitative analysis.

We are left with six parameters that we calibrate to match the model-based predictions on some key variables from their empirical counterparts: the share of net worth distributed by banks as dividends, ϕ , the share of assets bankers can run away with, λ , the coefficient determining the utility from money holdings for households, χ , the expenditure on public goods, g , the real stock of government bonds purchased by the central bank, b^C , and the targeted stock of real debt in the economy, \bar{b}^* . The targeted variables are: i) average government expenditure to GDP; ii) bank leverage; iii) government bond spread (annual); iv) share of banks' bond holdings in total debt; v) share of foreign sector's bond holdings in total debt; and vi) average inflation (annual). Table 3 reports the value taken by the six variables in

¹⁹In our model, whenever the afternoon constraint binds, banks hold liquid assets in the amount of $M_u + \tilde{\eta}Q (B_u - B_u^F)$ to cover afternoon withdrawals $\omega^{\max} D$. Since $F = 0$ in our calibrated steady state, and net worth is a small fraction of total liabilities, we approximate D with total assets. Alternatively, we can approximate ω^{\max} using the run-off rates on deposits, as specified in the LCR regulation (e.g., run-off rate of 10% means that 10% of the deposits are assumed to possibly leave the bank in 30 days). Run-off rates for deposits range between 5% for the most stable, fully insured deposits to 15% for less stable deposit funding. Our calibration of ω^{\max} at 0.1 is consistent with these rates.

²⁰Notice that the period 2015Q2-2015Q4 coincides with the introduction of the Public Sector Purchase Programme, which was implemented by the ECB in March 2015.

the data (computed over the pre-crisis period, 1999-2006, unless otherwise indicated) and the model prediction under the chosen parameterization.²¹ The model perfectly replicates the six targeted moments. Table 3 also documents a good fit of the model for two non-targeted moments: the ratios of central bank bond holdings to GDP and government debt to GDP.

5.2 Macroeconomic impact and central bank policies

We assess the implications of the observed changes in the money market landscape for the macroeconomy and for central bank policies by means of a comparative statics analysis.

We consider the following monetary policy instruments: the interest rate on central bank loans, $\frac{1}{Q^F}$, the haircut on collateral charged by the central bank, $1 - \eta$, and the stock of government bonds on its balance sheet, b^C . We map these central bank instruments into three types of monetary policies implemented by the ECB in recent years: i) a pre-financial crisis policy characterized by a constant balance sheet (b^C is held constant, banks do not borrow from the central bank as we set $\eta = 0$; inflation is determined endogenously); ii) a CO policy whereby the size of the balance sheet is determined by the demand for funding of the banking sector at a given policy rate (b^C is held constant, $\eta = 0.97$ so that banks may borrow from the central bank; inflation is determined endogenously); and iii) a OP policy whereby the central bank changes the stock of bonds on its balance sheet to achieve an inflation goal of 2% (inflation is fixed in this exercise while b^C is endogenous).

Our benchmark central bank policy is the constant balance sheet policy. We compare outcomes under the benchmark policy to outcomes under a CO policy and to a OP policy of maintaining constant inflation.

²¹The average government expenditure to GDP is computed using data for euro area (EU12) governments from Eurostat. The value of bank leverage is taken from Andrade et al. (2016). The share of banks' bond holdings in total debt is set at the value reported in Kojien, Koulischer, Nguyen, and Yogo (2016) for 2015, 23%. To compute the share of the foreign sector's bond holdings, we first use data from SDW (the ECB database) to calculate the share of central bank's holdings in total government debt. We impute to this item not only outright purchases of government bonds but also collateralized loans extended in refinancing operations (the main instrument through which the ECB injects liquidity in normal times). The ratio to total sovereign debt is 10%. Kojien, Koulischer, Nguyen, and Yogo (2016) report that households hold 3% of government bonds. We then impute to the foreign sector the remaining share, which amounts to 64%. The government bond spread is computed using data from SDW. We build average government bond yields by weighting yields of all euro area government bonds, for all maturities, with the respective amounts in 2011. We then build the spread relative to the overnight rate, the Eonia. Average inflation is computed using quarterly changes of the HICP index taken from SDW.

5.2.1 Reduced access to the unsecured market

The first exercise we conduct aims at analyzing the macroeconomic effects of a shrinking unsecured money market segment. In this comparative statics exercise, the share of unconnected banks, $1 - \xi$, varies between 0.58 and 0.97. Figures 5 and 6 show the results for the constant balance sheet policy and for the OP policy, respectively.

In both figures, the solid red line denotes the share of unconnected banks under our benchmark calibration ($1 - \xi = 0.58$). In Figure 5, the green dashed lines indicate the level of $1 - \xi$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. In addition, in Figure 6, the orange dashed lines indicate the level of $1 - \xi$ at which unconnected banks stop holding bonds. As we shall see, these two constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. The amount of deposits raised by connected and unconnected banks is of a broadly comparable magnitude. Unconnected banks, however, invest less in capital than connected banks, as they need to invest part of the funds in bonds to be pledged in the secured market in the afternoon. At this point, the return on bonds is higher than the return on money (not shown), and unconnected banks choose not to hold money to satisfy their afternoon liquidity needs.

If more banks in the economy are unconnected (moving rightward in both figures), a larger number of banks faces an afternoon liquidity constraint, which raises the aggregate demand for bonds and the bond price. In the region where $1 - \xi < 0.8$ (< 0.81 in OP case), the real return on bonds falls for foreign investors, inducing them to sell part of their bond holdings to domestic banks. The amount of bonds held by each unconnected bank, b_u , nonetheless mildly declines, as more banks need to hold bonds as collateral, and the supply of bonds is fixed. When the share of unconnected banks increases further, i.e., when $1 - \xi$ exceeds 0.8 (< 0.81 in OP case), the high price of bonds lowers the return on bonds to the point when it is equalized with the return on money. From this point onward (indicated by the green dashed lines), unconnected banks also use money to self-insure against afternoon withdrawals. That is, their demand for money increases.

Under the constant balance sheet policy (Figure 5), the supply of money is fixed. Higher demand for money by unconnected banks is accommodated by an increase in inflation and the deposit rate, which induces households to reduce their money holdings. Scarce money

balances are therefore reallocated from households to unconnected banks. A higher nominal rate requires an increase in inflation,²² which raises the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts downward pressure on aggregate capital and, correspondingly, upward pressure on the return on capital. As the net worth of unconnected banks decline, and there is an increasing share of those, the aggregate net worth which is equally distributed to all banks in the morning declines. This results in a tightening of the run-away constraint of connected banks which induces them to also reduce their investment in capital and their deposit intake. Therefore, aggregate deposits and capital fall and so does output. Quantitatively, the decline in output between a steady-state with 0.58 share of unconnected banks and that with 0.89 share (pre- to post-2008 average share of secured turnover in total) is around 1.1 percent.

Under the CO policy, the outcome is the same as under the constant balance sheet policy. This is because central bank funding is not used in this case (and therefore the central bank balance sheet remains constant) as deposit funding is less expensive than - and therefore preferred to - central bank funding.

Under the OP policy (Figure 6), the central bank can expand its balance sheet by purchasing bonds and thus increase the supply of money to help relax the afternoon constraint of unconnected banks. When $1 - \xi$ exceeds 0.83 (indicated by the orange dashed lines) and the price of bonds is high, unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. As inflation is kept constant, the opportunity cost of holding money is constant (and low) as well. Aggregate capital and output still fall simply because the share of unconnected banks - who invest less in capital - increases in the economy. As this effect is driven by the change in the relative share of banks in the economy, it is not something that the central bank can affect. Quantitatively, the decline in output between a steady-state with 0.58 share of unconnected banks and that with 0.89 share (pre- to post-2008 average share of secured turnover in total) is 0.9 percent.

In sum, reduced access to the unsecured market can reduce investment and output via two channels. First, since unconnected banks need to satisfy liquidity shocks by holding bonds and/or by holding money, they can invest less in capital. Therefore, as the share of unconnected

²²This is an artefact of our steady-state analysis in which the Fisher equation holds. An alternative way to think about the adjustment in response to a higher demand for real money balances when the nominal money supply is fixed is that the price level must decrease so that the real money supply increases. That is, increased demand for scarce money balances necessitates deflation in the short-run.

banks in the economy increases, capital and output decrease. Central bank policy cannot do anything about this channel. Second, as more banks become unconnected, bonds and money become more scarce, tightening the afternoon constraint, reducing aggregate deposits, investment in capital and, consequently, output. Central bank policy can mitigate the second channel if it provides money to banks at a low opportunity cost by maintaining a constant low inflation (OP policy). In the comparison between steady-states with 0.58 versus 0.89 shares of unconnected banks, the first channel dominates and therefore there is little difference between policies. However, if we compared a steady-state with 0.58 share of unconnected banks to that with 0.97 (share of secured turnover in total in 2019, then the contraction in output would be 3.1 percent in the constant balance sheet or CO case, and only 1.1 percent in the OP case.

5.2.2 Reductions in collateral value

In this subsection, we analyze the macroeconomic effects of changing collateral value by comparing different private haircuts in the secured market. In this comparative statics exercise, the private haircut moves from the benchmark pre-crisis value of 3 percent to 80 percent. Figures 7, 8 and 9 show the results under the policies of constant balance sheet, CO and OP, respectively.

In these figures, the solid red line denotes the secured market haircut under our benchmark calibration ($1 - \tilde{\eta} = 0.03$). The green dashed lines indicate the level of $1 - \tilde{\eta}$ at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of $1 - \tilde{\eta}$ at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The cyan dashed lines indicate the level of $1 - \tilde{\eta}$ at which unconnected banks start borrowing from the central bank so that the multiplier μ_u^F becomes zero. The magenta dashed lines indicate the level of $1 - \tilde{\eta}$ at which unconnected banks pledge their entire bond holdings at the central bank and no longer use secured market ($b_u = b_u^F$ and the collateral constraint binds). The orange dashed lines indicate the level of $1 - \tilde{\eta}$ at which unconnected banks no longer hold bonds. As we shall see, these five constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. At higher haircut levels (moving rightward in all figures), it becomes more difficult for unconnected banks to satisfy their liquidity needs in the secured market.

Under the constant balance sheet policy (Figure 7), at higher haircut levels, bond collateral value in the private market initially increase up to the point when unconnected banks start demanding money to self-insure against afternoon liquidity shocks (as of $1 - \tilde{\eta} = 0.10$, indicated by the green dashed lines). As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in inflation and the deposit rate, which increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. The reduction in net worth of unconnected banks induces a reduction in the net worth allocated also to connected banks. This tightens the run away constraint of connected banks, which therefore also reduce their investment in capital and deposit intake. When the haircut reaches 0.23, unconnected banks are very constrained in the secured market but they cannot increase their money holdings any further as households' money holdings are at a minimum and the central bank is not ready to increase the supply of money. At this point, unconnected banks become so constrained in the afternoon that they dramatically reduce their deposit intake. Their leverage constraint turns slack. Bond prices collapse. From here onwards unconnected banks' deposit intake and therefore investment in capital continues to fall. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they face a tight leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is 3%.

Both the CO and OP policies are able to substantially mitigate output contractions by preventing the leverage constraint of unconnected banks from turning slack.

Under the CO policy (Figure 8), this is achieved by unconnected banks accessing central bank funding as haircut in the secured market reaches 0.19 (indicated by the cyan dashed lines). Unconnected banks reduce their deposit funding (as their afternoon constraint is tight due to the high secured market haircut) and substitute it with the central bank funding (which is subject to a much more favorable haircut of 0.03). As the central bank provides funding to banks, its balance sheet expands and so does the money supply. Therefore, unconnected bank can further increase their money holdings, without the need for a reallocation of money holdings from households (indeed, households increase their money holdings again as inflation and the nominal interest rate declines). As the private haircut increases above 0.30 (indicated by the

magenta dashed lines), unconnected banks pledge all their bond collateral at the central bank and stop using the secured market to manage their afternoon liquidity needs, relying solely on money holdings instead. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is just 0.4%.

Under the OP policy (Figure 9), the central bank prevents the leverage constraint of unconnected banks from turning slack by purchasing bonds and thus increasing the supply of money which helps relax the afternoon constraint of unconnected banks. When private haircut reaches 0.10, unconnected banks start selling bonds to the central banks and - to a much smaller extent - to foreigners. The bond price decreases. When the private haircut reaches 0.16 (indicated by the orange dashed lines), unconnected banks sell off their entire bond holdings to the central bank and choose to hold money instead to satisfy the afternoon constraint. From this point onwards, the economy is insulated from further increases in the secured market haircut. Deposits, capital, and output stabilize. Quantitatively, the decline in output between a steady-state with 3% private haircut and one with 40% haircut is just 0.1%.

In sum, the key to stabilizing output when haircuts in the private market increase is to expand the central bank balance sheet either through a provision of collateralized loans to banks (using more favorable haircuts and the CO policy) or through bond purchases which replace bonds that become less valuable as collateral in the private market with money so that banks can self-insure against liquidity shocks (OP policy). The differential effectiveness of the OP and CO policies under our calibration in terms of mitigating output reductions is driven by the fact that these policies operate on different bank constraints. OP work directly towards relaxing the afternoon liquidity constraint: OP make deposit funding more attractive at the margin, by providing money to banks at a low opportunity cost. This prevents large declines in deposits and capital of the unconnected banks which could otherwise occur for high haircut levels. By contrast, CO offer banks additional (central bank) funding, once the deposits are too expensive at the margin. In that case, unconnected banks top up deposit funding with central bank funding, which prevents large declines in capital of the unconnected banks. As central bank funding is more costly than deposit funding since it requires holding costly collateral, lending and output decline more under CO policy than under the OP policy.

6 Conclusions

We developed a general equilibrium model featuring heterogeneous banks, interbank markets for both secured and unsecured credit, and a central bank that can conduct open market operations as well as lend to banks against collateral. The model features a number of occasionally binding constraints. The interactions between these constraints - in particular leverage and liquidity constraints - are key in determining macroeconomic outcomes.

We use the model to answer three questions: How do money market frictions affect the macroeconomy? How do bank leverage and liquidity constraints interact? What does this imply for central bank policies?

We find that both secured and unsecured money market frictions force banks to either divert resources into unproductive but liquid assets (bonds or money) or to de-lever (raise fewer deposits as it is deposit funding that exposes banks to liquidity shocks). This leads to less lending and output in the economy. If the liquidity constraint is very tight, the leverage constraint may turn slack. In this case, there are large declines in lending and output (up to 5% in our calibrated example), in the absence of central bank intervention. Central bank policies that increase the size of the central bank balance sheet (via outright purchases or credit operations) can prevent the leverage constraint from turning slack and significantly attenuate the decline in lending and output. The outright purchases are more effective than credit operations in terms of mitigating output reductions - with the output decline under the outright purchase policy as small as 0.1% - as they work directly towards relaxing the afternoon liquidity constraint.

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Tables

Table 1: ECB vs private haircuts on government bonds

	ECB haircuts		Private haircuts	
	CQS1-2	CQS3	Germany	Portugal
2010	2.8	7.8	3.2	8.1
2011	2.8	7.8	3.4	39.4
2012	2.8	7.8	3.4	77.5
2013	2.8	7.8	3.4	80.0
2014	2.2	9.4	3.4	80.0
2019	2.2	9.4	2.8	27.1

Notes: The table presents ECB and private market (here: LCH.Clearnet) haircuts on government bonds. For each year, the haircuts are calculated as simple averages of haircuts on fixed coupon government bonds with maturities between 0 and 30 years. For ECB haircuts, CQS1-2 refers to government bonds with credit quality 1 and 2, corresponding to a rating AAA to A-; CQS3 refers to government bonds with credit quality 3, corresponding to a rating BBB+ to BBB-. Source: ECB and LCH.Clearnet.

Table 2: Parameter values

Parameter	Description	Value
θ	Capital share in income	0.330
δ	Capital depreciation rate	0.020
β	Discount rate households	0.994
ϵ	Inverse Frisch elasticity	0.400
χ^{-1}	Coefficient in households' utility	0.006
g	Government spending	0.566
κ^{-1}	Average maturity bonds (years)	5.952
ϕ	Fraction net worth paid as dividends	0.025
ξ	Fraction banks with access to unsecured market	0.420
$\tilde{\eta}$	Haircut on bonds set by banks	0.970
η	Haircut on bonds set by central bank	0.970
λ	Share of assets bankers can run away with	0.701
ω^{\max}	Max possible liquidity demand as share of deposits	0.100
\varkappa	Intercept foreign demand function	10.120
B_C	Bonds held by central bank	0.968
B^*	Stock of debt	7.443
ϱ	Parameter foreign bond demand	1.757
Q^F	Price central bank loans	0.997

Table 3: Calibration

Targeted variables	Data	Model
Govt expenditure/GDP	0.20	0.20
Bank leverage	6.00	6.00
Govt bond spread (annual)	0.002	0.002
Share bonds held by banks	0.23	0.23
Share bonds foreign sector	0.64	0.64
Inflation (annual)	0.02	0.02
Non-targeted variables	Data	Model
CB bond holdings/GDP	0.06	0.08
Govt debt/GDP	0.69	0.66

Figures

Figure 1: Timeline of bank decisions

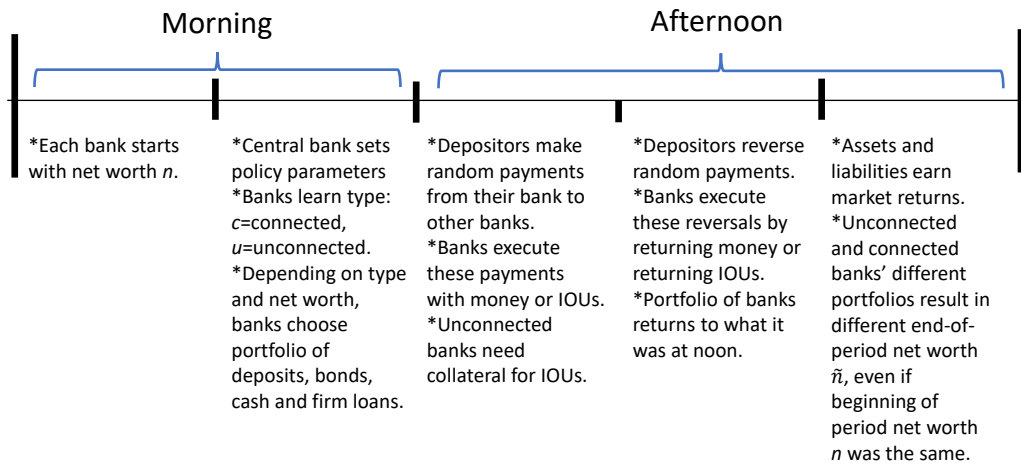


Figure 2: Timeline general equilibrium model

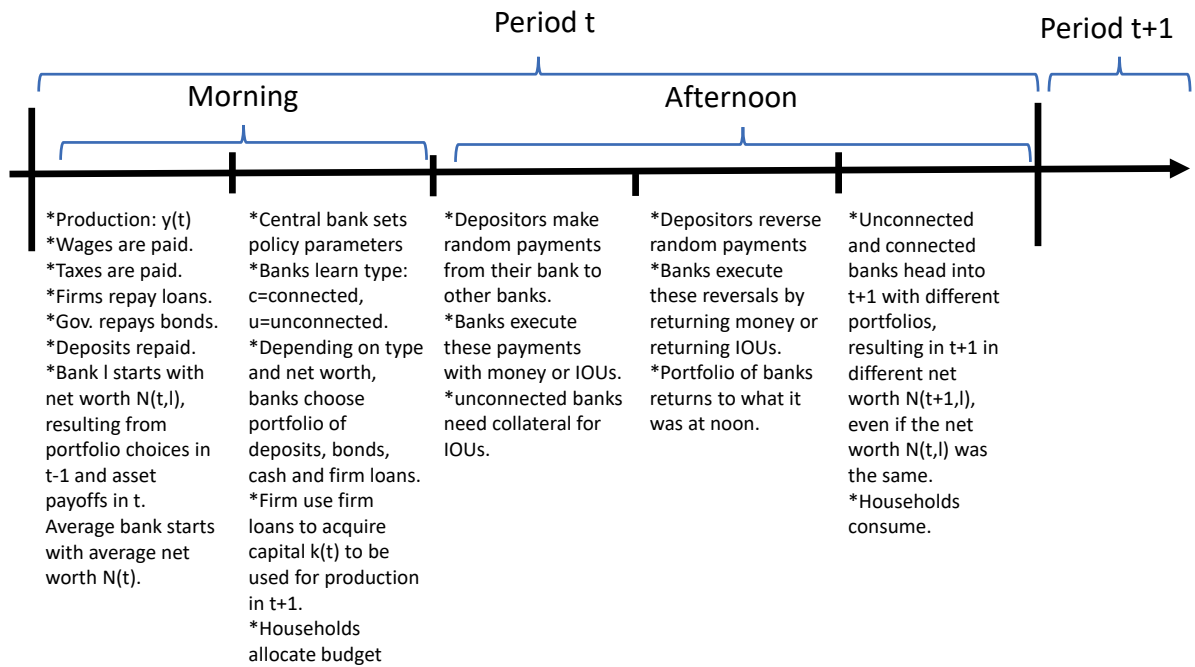
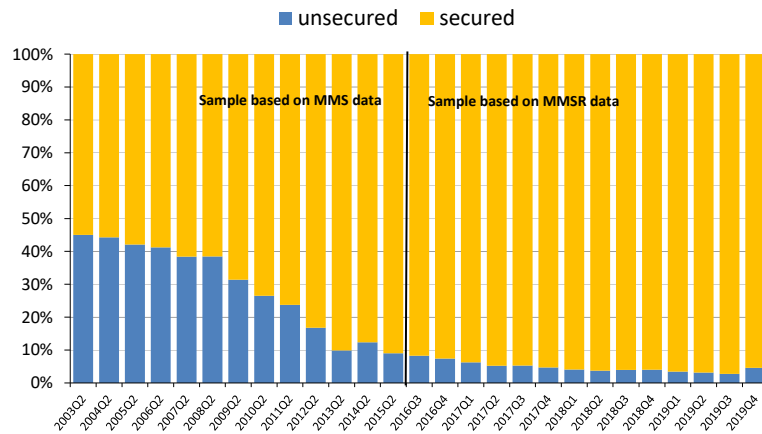
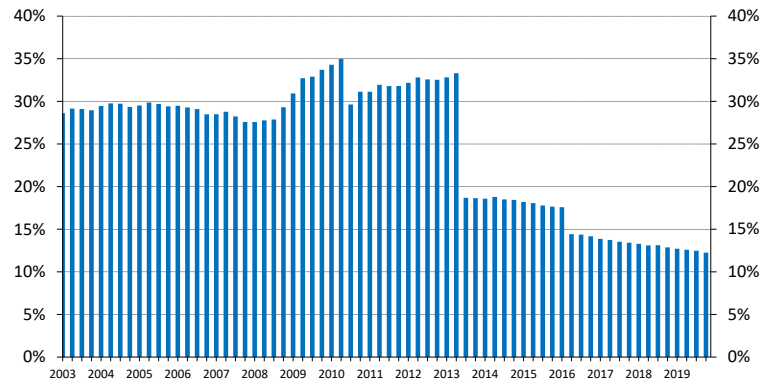


Figure 3: Quarterly turnover in unsecured and secured interbank money markets



Notes: The figure presents cumulative quarterly turnover in the euro area unsecured and secured interbank money markets for 2003-2019 (quarterly data; percentages of total). Source: Euro Area Money Market Survey (MMS) until 2015; Money Market Statistical Reporting (MMSR) transactions-based data thereafter. The MMS was conducted once a year, with each data point corresponding to the second quarter of the respective year; it was discontinued in 2015. The MMS panel comprised 98 euro area banks. The MMSR data start in the third quarter of 2016. The sample presented here refers to 38 banks, all of which also participated in the MMS.

Figure 4: Ratio of safe (AAA-rated) euro area government debt to GDP



Notes: The figure presents breakdown of euro area government debt outstanding (for 19 euro area countries, EA 19) according to the credit rating for 2003-2019 (quarterly data; ratio to GDP, in %). Country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P. The kinks in the chart correspond to dates when specific euro area countries moved from "at least one AAA" to "non AAA"-rated. This happened in 2009 Q3 for Ireland, in 2010 Q3 for Spain, in 2013 Q3 for France, and in 2016 Q2 for Austria. Source: ECB.

Figure 5: Comparative statics: changing the share of banks with no access to unsecured funding, $1 - \xi$, constant balance sheet policy
Notes: Red solid lines denote the calibrated steady state. Green dashed lines denote the share of unconnected banks at which the non-negativity conditions on their cash holdings become slack. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation rate, and percent deviation of output from steady-state.

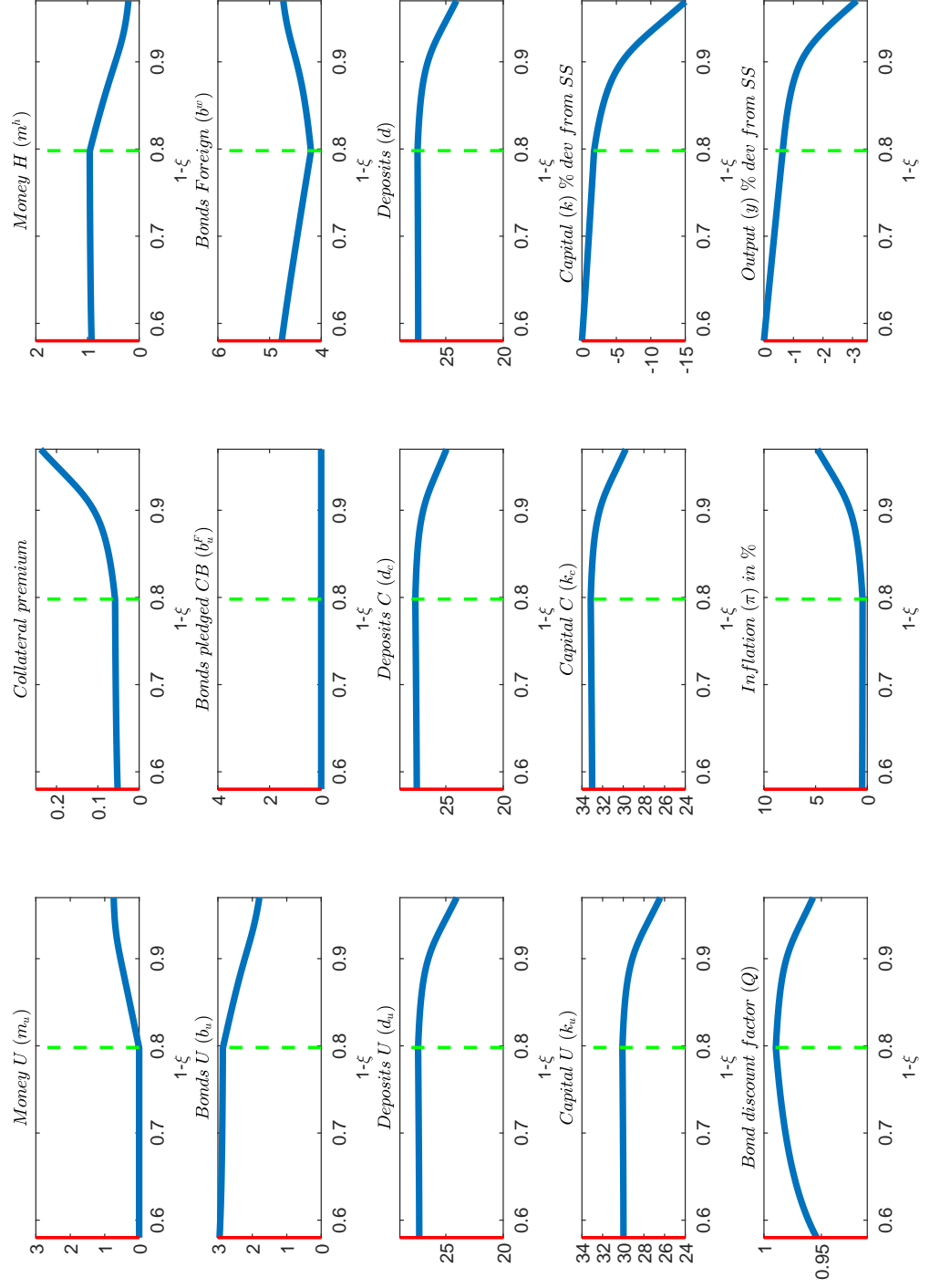


Figure 6: Comparative statics: changing the share of banks with no access to unsecured funding, $1 - \xi$, OP policy

Notes: Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.

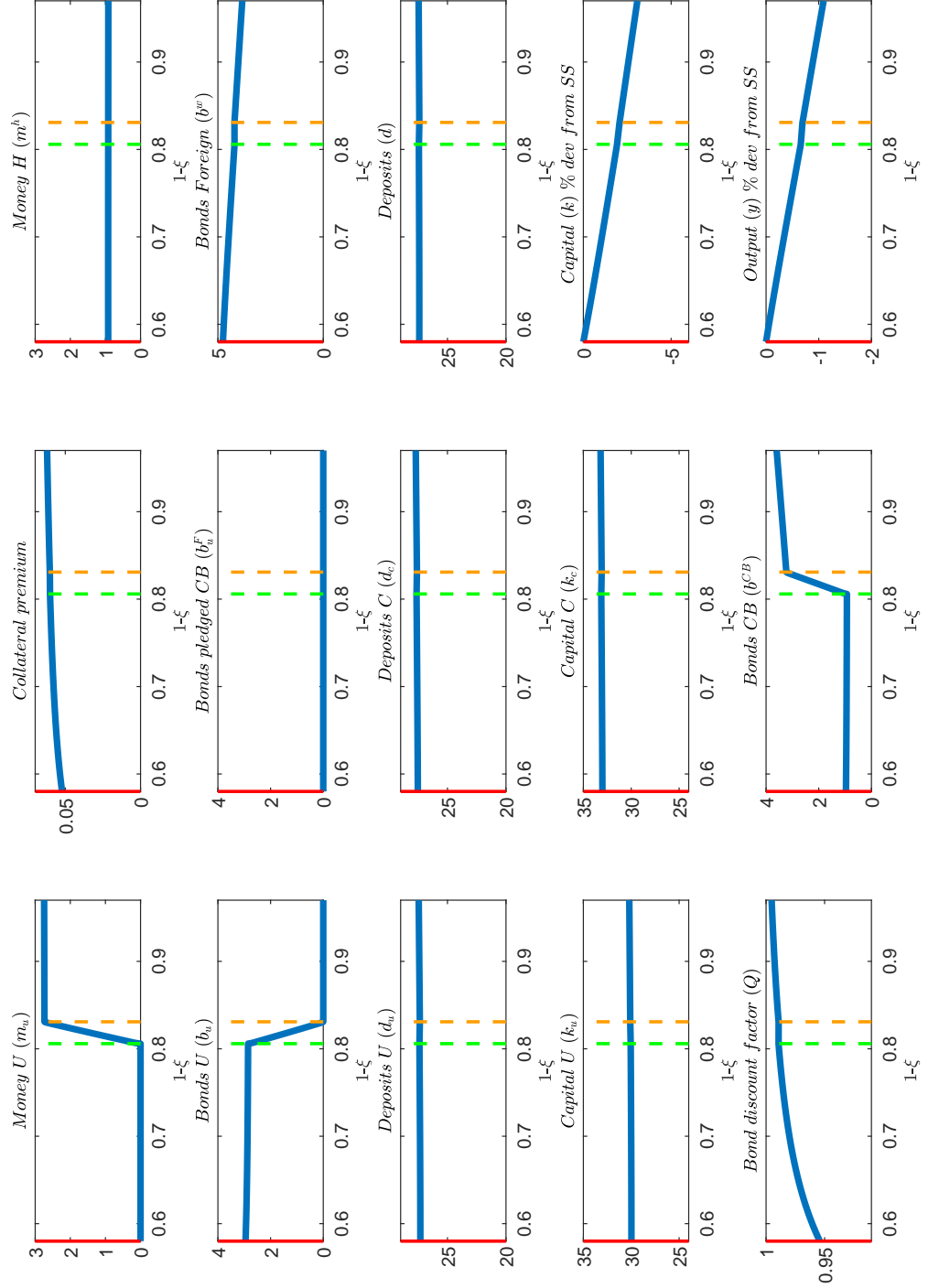


Figure 7: Comparative statics: changing the value of the private haircut, $1 - \tilde{\eta}$, constant balance sheet policy.

Notes: Red solid lines denote the calibrated steady state. Green and blue dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and the run-away constraint of unconnected banks become slack, respectively. First row: money holdings of unconnected banks, money holdings of households, and collateral premium. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation rate, and percent deviation of output from steady-state.

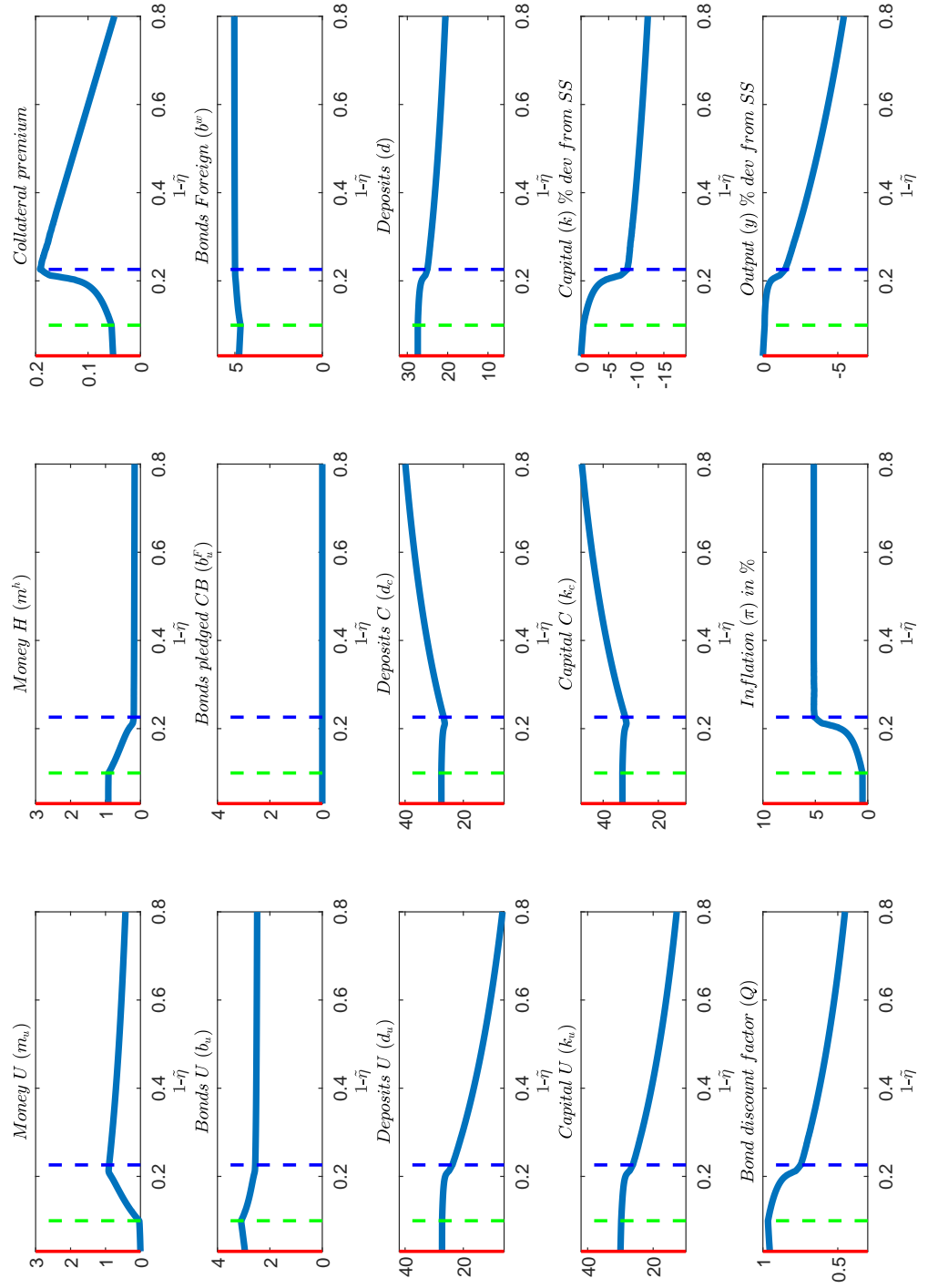


Figure 8: Comparative statics: changing the value of the private haircut, $1 - \tilde{\eta}$, CO policy

Notes: Red solid lines denote the calibrated steady state. Green, cyan and magenta dashed lines denote the level of private haircuts at which the non-negativity conditions on cash holdings and on bonds pledged at the central bank become slack, and the constraint on bonds pledged at the central bank not exceeding bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, money holdings of households, and collateral premium. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, inflation rate, and percent deviation of output from steady-state.

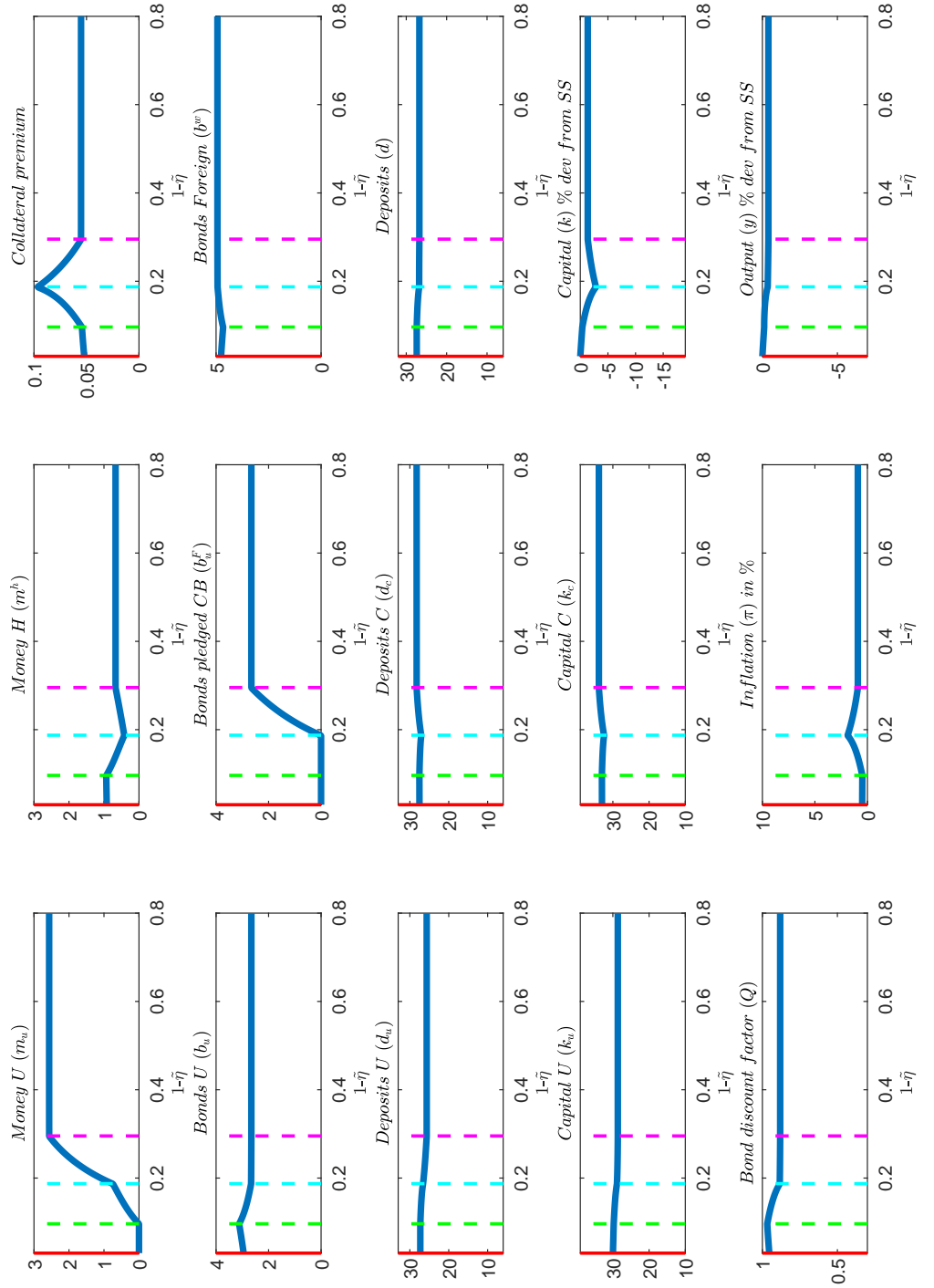
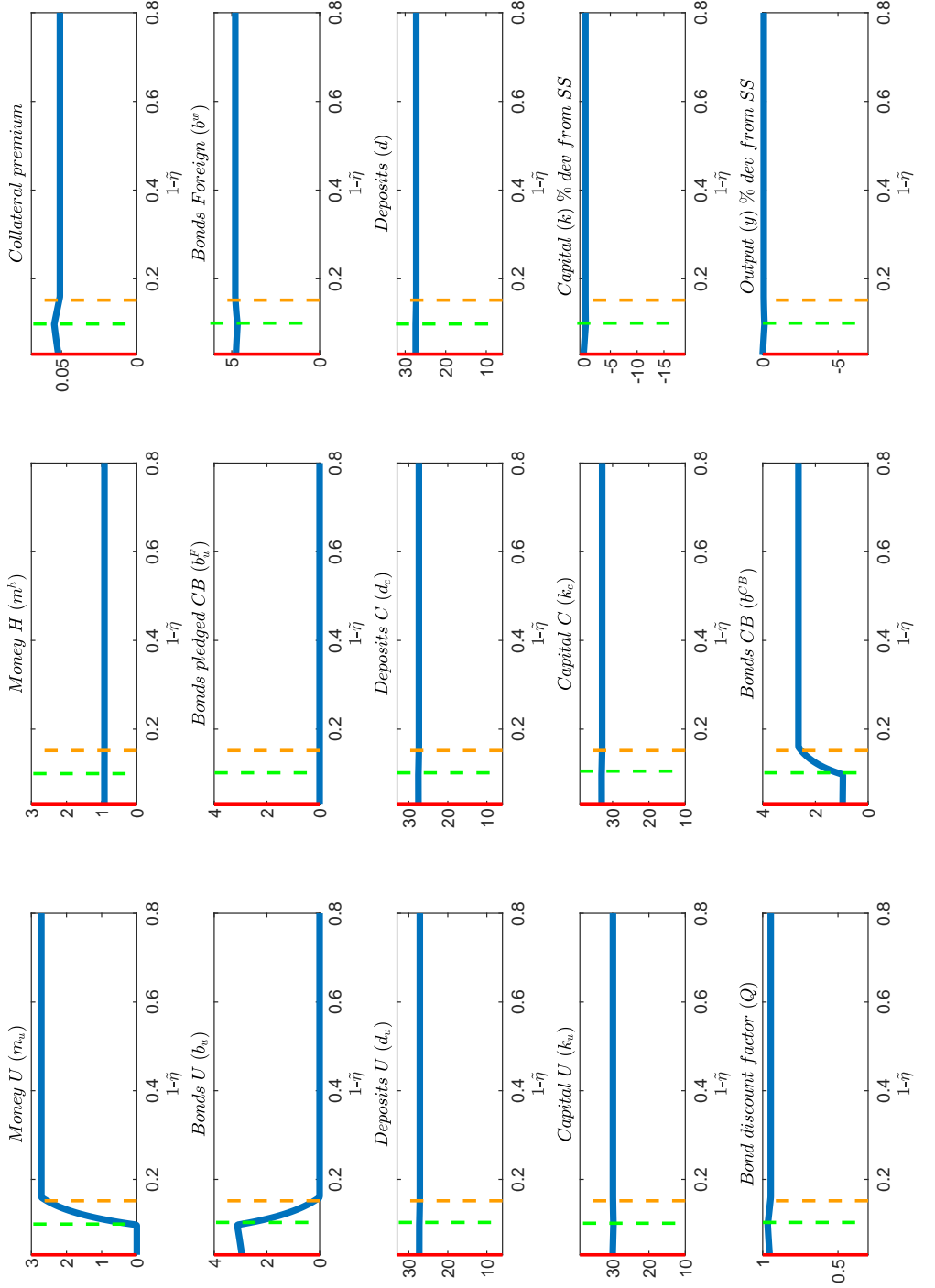


Figure 9: Comparative statics: changing the value of the private haircut, $1 - \tilde{\eta}$, OP policy

Notes: Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the level of private haircuts at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, money holdings of households, and collateral premium. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.



Online Appendix

A Optimality conditions of households and firms

The optimality conditions of the households are given by:

$$\begin{aligned} -\frac{u_l(c_t, h_t)}{u_c(c_t, h_t)} &= (1 - \tau_t) \frac{W_t}{P_t} \\ v_M(m_t^h) &= u_c(c_t, h_t) (R_{d,t} - 1) \\ \frac{u_c(c_{t-1}, h_{t-1})}{P_{t-1}} &= \beta R_{d,t} \left[\frac{u_c(c_t, h_t)}{P_t} \right]. \end{aligned}$$

First-order conditions arising from the problem of the firms are

$$\begin{aligned} y_t &= \gamma_t k_{t-1}^\theta h_t^{1-\theta}, \\ W_t h_t &= (1 - \theta) P_t y_t, \\ r_t k_{t-1} &= \theta y_t, \\ k_t &= (1 - \delta) k_{t-1} + I_t. \end{aligned}$$

B The problem of the banks

To characterize numerically the choices of banks, it is useful to define $\mu_{t,l}^{BC}$ as the Lagrange multiplier on the budget constraint (18), $\mu_{t,l}^{RA}$ as the Lagrange multiplier on the leverage constraint (21), $\mu_{t,l}^{CC}$ as the Lagrange multiplier on the collateral constraint (19), $\mu_{t,u}$ as the Lagrange multiplier on the afternoon funding constraint of the unconnected banks (22), $\mu_{t,l}^M \geq 0$, $\mu_{t,l}^F \geq 0$, and $\mu_{t,l}^B \geq 0$ as the Lagrange multipliers on the constraints $M_{t,l} \geq 0$, $F_{t,l} \geq 0$ and $B_{t,l} \geq 0$, respectively, and $\mu_{t,l}^C \geq 0$ as the Lagrange multiplier on the collateral constraint at the central bank, $B_{t,l}^F \leq B_{t,l}$.

We use the convention stated at the end of subsection 4.7.2 and analyze the problem of an average connected or average unconnected bank, starting the period with average net worth N_t and denoting their choices and results with subindex $l = c$ or $l = u$ respectively. The first-order conditions characterizing the choice of banks $l = u$ and $l = c$ for capital, bonds,

money, are given by

$$\begin{cases}
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t R_{k,t+1} = \mu_{t,l}^{BC} + \lambda \mu_{t,l}^{RA} & \text{for } l = c, u \\
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t R_{b,t+1} = \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C & \text{for } l = c \\
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t R_{b,t+1} = \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,l}^C - \mu_{t,u} \tilde{\eta}_t & \text{for } l = u \\
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t + \mu_{t,c}^M = \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda & \text{for } l = c \\
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t + \mu_{t,c}^M = \mu_{t,l}^{BC} + \mu_{t,l}^{RA} \lambda - \mu_{t,u} & \text{for } l = u
\end{cases} \quad (52)$$

Those characterizing banks' choices for deposits, central bank funding, and bonds to be pledged at the central bank, are

$$\begin{cases}
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t R_{d,t} = \mu_{t,l}^{BC} & \text{for } l = c \\
(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t R_{d,t} = \mu_{t,l}^{BC} - \omega^{\max} \mu_{t,u} & \text{for } l = u
\end{cases} \quad (53)$$

$$(1 + \mu_{t,l}^{RA}) \tilde{\psi}_t = \mu_{t,l}^{BC} Q_t^F - \mu_{t,l}^{CC} + \mu_{t,l}^F \text{ for } l = c, u \quad (54)$$

$$\begin{aligned}
\mu_{t,l}^{CC} \eta_t &= \mu_{t,l}^C & \text{for } l = c \\
\mu_{t,l}^{CC} \eta_t &= \mu_{t,u} \tilde{\eta}_t + \mu_{t,l}^C & \text{for } l = u
\end{aligned}$$

The complementary slackness conditions are

$$\mu_{t,l}^F F_{t,l} = 0 \quad (55)$$

$$\mu_{t,l}^M M_{t,l} = 0 \quad (56)$$

$$\mu_{t,l}^C (B_{t,l} - B_{t,l}^F) = 0 \quad (57)$$

$$\mu_{t,l}^{RA} [\phi N_{t,A} + \tilde{V}_{t,l} - \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l})] = 0 \quad (58)$$

$$\mu_{t,l}^B B_{t,l} = 0 \quad (59)$$

for $l = u, c$, and

$$\mu_{t,u} [\omega^{\max} D_{t,u} - M_{t,u} - \tilde{\eta}_t Q_t (B_{t,u} - B_{t,u}^F)] = 0$$

for unconnected banks only.

C Equilibrium

Consider the average connected or unconnected bank, starting the period with net worth N_t .

Denote its investment and deposit choices with $k_{t,c}$, $B_{t,c}$, $B_{t,c}^F$, $F_{t,c}$, $D_{t,c}$, $M_{t,c}$ as well as $k_{t,u}$,

$B_{t,u}$, $B_{t,u}^F$, $F_{t,u}$, $D_{t,u}$, $M_{t,u}$. Put differently, given that $N_t = \int N_{t,l} dl$, we have

$$\begin{aligned} k_{t,c} &= \int_{\{l : l \text{ is connected at } t\}} k_{t,l} dl \\ k_{t,u} &= \int_{\{l : l \text{ is unconnected at } t\}} k_{t,l} dl \end{aligned}$$

etc.. Since the decision problems are linear in net worth, we can proceed by focusing on the decision problem of the average bank.

An equilibrium is a vector of sequences such that:

1. Given $P_t, \tau_t, W_t, R_{d,t-1}, Z_t$, the representative household chooses $c_t > 0, h_t > 0, D_t \geq 0, M_t^h \geq 0$ to maximize their objective function

$$\max E_t \left[\sum_{t=0}^{\infty} \beta^t \left[u(c_t, h_t) + v\left(\frac{M_t^h}{P_t}\right) \right] \right]$$

subject to

$$D_t + M_t^h \leq R_{d,t-1} D_{t-1} + M_{t-1}^h + (1 - \tau_{t-1}) W_{t-1} h_{t-1} + \phi N_{t-1} - P_{t-1} c_{t-1}.$$

2. Final good firms choose capital and labor to maximize their expected profits from production, which makes use of the technology

$$y_t = \gamma_t k_{t-1}^\theta h_t^{1-\theta}.$$

3. Capital-producing firms choose how much old capital k_{t-1} to buy from banks and to combine with final goods I_t to produce new capital k_t , according to the technology

$$k_t = (1 - \delta) k_{t-1} + I_t.$$

4. Given the paths for the endogenous variables $c_t, h_t, r_t, P_t, Q_t, Q_t^F, \eta_t$, and exogenous sequence for $\tilde{\eta}_t$, the average connected or unconnected bank $l \in \{c, u\}$ chooses $k_{t,l}, B_{t,l}$,

$B_{t,l}^F$, $F_{t,l}$, $D_{t,l}$, $M_{t,l}$ to maximize its noon-post-type-revelation-before-dividend-payment-value $V_{t,l}$ subject to

$$\begin{aligned}
V_{t,l} &= \phi N_{t,l} + \tilde{V}_{t,l} \\
\tilde{V}_{t,l} &= \beta \frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} \frac{P_t}{P_{t+1}} \bar{V}_{t+1,l} \\
\bar{V}_{t,l} &= E[V_{t,l}] \\
V_{t,l} &\geq \lambda (P_t k_{t,l} + Q_t B_{t,l} + M_{t,l}) \\
0 &\leq B_{t,l} - B_{t,l}^F \\
P_t k_{t,l} + Q_t B_{t,l} + M_{t,l} + \phi N_t &= D_{t,l} + Q_t^F F_{t,l} + N_t \\
F_{t,l} &= \eta_t Q_t B_{t,l}^F \\
N_{t+1,l} &= R_{k,t+1} P_t k_{t,l} + M_{t,l} + R_{b,t+1} Q_t B_{t,l} - R_{d,t} D_{t,l} - R_{f,t} Q_t^F F_{t,l}
\end{aligned}$$

as well as

$$\omega^{\max} D_{t,l} \leq M_{t,l} + \tilde{\eta}_t Q_t (B_{t,l} - B_{t,l}^F)$$

if $l = u$, i.e. if the bank is unconnected in period t .

5. Aggregation

$$k_t = \xi_t k_{t,c} + (1 - \xi_t) k_{t,u} \quad (60)$$

$$B_t = \xi_t B_{t,c} + (1 - \xi_t) B_{t,u} \quad (61)$$

$$B_t^F = \xi_t B_{t,c}^F + (1 - \xi_t) B_{t,u}^F \quad (62)$$

$$F_t = \xi_t F_{t,c} + (1 - \xi_t) F_{t,u} \quad (63)$$

$$D_t = \xi_t D_{t,c} + (1 - \xi_t) D_{t,u} \quad (64)$$

$$M_t = \xi_t M_{t,c} + (1 - \xi_t) M_{t,u} \quad (65)$$

$$\bar{V}_t = \xi_t \bar{V}_{t,c} + (1 - \xi_t) \bar{V}_{t,u} \quad (66)$$

$$N_{t+1} = \xi_t N_{t+1,c} + (1 - \xi_t) N_{t+1,u} \quad (67)$$

(where we recall that “ c ” and “ u ” in the last equation refer to the type status in t , not $t + 1$).

6. The central bank chooses the total amount of money supply \bar{M}_t , the haircut parameter η_t , the discount factor on central bank funds Q_t^F , the bond purchases B_t^C as well as the

seigniorage payment S_t . It satisfies the balance sheet constraint

$$S_t = Q_t^F \bar{F}_t + Q_t B_t^C - \bar{M}_t \quad (68)$$

and the budget constraint

$$\begin{aligned} \bar{M}_t = & Q_{t-1}^F \bar{F}_{t-1} + Q_{t-1} B_{t-1}^C + Q_t^F \bar{F}_t \\ & - R_{f,t-1} Q_{t-1}^F \bar{F}_{t-1} + Q_t B_t^C - R_{b,t} Q_{t-1} B_{t-1}^C \end{aligned} \quad (69)$$

7. The government satisfies the debt evolution constraint, the budget constraint and the tax rule

$$\bar{B}_t = (1 - \kappa) \bar{B}_{t-1} + \Delta \bar{B}_t \quad (70)$$

$$P_t g_t + \kappa \bar{B}_{t-1} = \tau_t W_t h_t + Q_t \Delta \bar{B}_t + S_t \quad (71)$$

$$(\tau_t - \tau^*) \frac{W_t}{P_t} h_t = \alpha \left(\frac{\bar{B}_t}{P_t} - b^* \right) \quad (72)$$

where b^* is a target for the real debt level and where τ^* is the steady state tax rate. Equation (72) thus implies that b^* is the steady state real debt level.

8. The foreign sector chooses the amount of domestic bonds to hold

$$\frac{B_t^w}{P_t} = f \left(\varkappa - \frac{1}{\varrho} \log \tilde{Q}_t \pi_t \right), \quad (73)$$

and satisfies the budget constraint

$$Q_t B_t^w + P_t c_t^w = R_{b,t} Q_{t-1} B_{t-1}^w. \quad (74)$$

9. Markets clear:

$$c_t + g_t + I_t + c_t^w = y_t \quad (75)$$

$$\bar{B}_t = B_t + B_t^C + B_t^w \quad (76)$$

$$\bar{F}_t = F_t \quad (77)$$

$$\bar{M}_t = M_t + M_t^h \quad (78)$$

D Collateral premium

Combine the first and third conditions of (52), for $l = u$, to get

$$(1 + \mu_{t,u}^{RA}) \tilde{\psi}_{t+1} (R_{k,t+1} - R_{b,t+1}) = \mu_{t,u}^C + \mu_{t,u} \tilde{\eta}_t \quad (79)$$

Since $\mu_{t,u}^C \geq 0$, $\mu_{t,u}^{RA} \geq 0$ and $\mu_{t,u} \geq 0$, it must be that $\tilde{\psi}_{t+1} (R_{k,t+1} - R_{b,t+1}) \geq 0$.

We show that the collateral premium is strictly positive if the afternoon constraint is binding or unconnected banks are collateral-constrained in their borrowing from the central bank. First, if unconnected banks are constrained in the afternoon, $\mu_{t,u} > 0$. It follows that $\tilde{\psi}_{t+1} (R_{k,t+1} - R_{b,t+1}) > 0$. Second, if unconnected banks are collateral-constrained at the central bank, $\mu_{t,u}^C > 0$ and the same conclusion follows.

E The equations characterizing the steady state

We characterize the steady state of the model.

Define a generic variable as the corresponding capital letter variable, divided by the contemporaneous price level, i.e. $x_t = \frac{X_t}{P_t}$. The steady state is characterized by the following conditions:

1. 4 household equations:

$$\begin{aligned} R_d &= \frac{\pi}{\beta} \\ -\frac{u_h(c, h)}{u_c(c, h)} &= (1 - \tau) w \\ v_M(m^h) &= u_c(c, h) (R_d - 1) \\ c &= (1 - \tau) w h + \left(\frac{1}{\beta} - 1\right) \pi d + (1 - \pi) m^h + \phi n \end{aligned}$$

2. 4 firms' equations:

$$\begin{aligned} y &= \gamma k^\theta h^{1-\theta} \\ wh &= (1 - \theta) y \\ rk &= \theta y \end{aligned}$$

and

$$I = \delta k.$$

3. 5 central bank equations: 2 equations

$$s = Q^F \bar{f} + Q b^C - \bar{m}$$

$$\bar{m} = \left[Q^F - \frac{1}{\pi} (1 - Q^F) \right] \bar{f} + \left[Q - \kappa \frac{1}{\pi} (1 - Q) \right] b^C$$

plus the value of 3 variables (policy instruments): η, Q^F, b^C .

Note that the seigniorage revenue of the central bank is given by the interest rate payments on its assets:

$$s = \frac{1}{\pi} (1 - Q^F) \bar{f} + \kappa \frac{1}{\pi} (1 - Q) b^C.$$

4. 2 government equations:

$$\bar{b} = \bar{b}^*$$

$$\tau^* (1 - \theta) y = g + \kappa (1 - Q) \frac{\bar{b}^*}{\pi} - Q \left(1 - \frac{1}{\pi} \right) \bar{b}^* - s.$$

where g is exogenous.

5. 4 market clearing equations:

$$\bar{f} = f$$

$$\bar{m} = m + m^h$$

$$\bar{b} = b + b^C + b^w$$

$$y = c + c^w + g + I$$

where the market clearing condition for the goods market (last equation above) is redundant due to the Walras law.

6. 44 bank equations:

(a) 2 equations common to c and u banks,

$$\bar{v} = \psi n$$

$$\tilde{\psi} = \beta \frac{1}{\pi} \psi$$

(b) 20 equations for $l = c, u$:

$$k_l + Qb_l + m_l + \phi n = d_l + Q^F f_l + n$$

$$\phi n + \tilde{v}_l = \lambda (k_l + Qb_l + m_l)$$

$$v_l = \phi n + \tilde{v}_l$$

$$\tilde{v}_l = \tilde{\psi} n_l$$

$$n_l = R_k k_l + R_b b_l + m_l - R_d d_l - R_f f_l$$

$$f_l = \eta Q b_l^F$$

$$\mu_l^F f_l = 0$$

$$\mu_l^M m_l = 0$$

$$\mu_l^C (b_l - b_l^F) = 0$$

$$\mu_l^B b_l = 0$$

(c) 7 equations for unconnected banks:

$$(1 + \mu_u^{RA}) \tilde{\psi} R_k = \mu_u^{BC} + \lambda \mu_u^{RA} \quad (80)$$

$$(1 + \mu_u^{RA}) \tilde{\psi} R^B + \mu_u^B = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u^C - \mu_u \tilde{\eta} \quad (81)$$

$$(1 + \mu_u^{RA}) \tilde{\psi} + \mu_u^M = \mu_u^{BC} + \lambda \mu_u^{RA} - \mu_u \quad (82)$$

$$(1 + \mu_u^{RA}) \tilde{\psi} R_d = \mu_u^{BC} - \omega^{\max} \mu_u \quad (83)$$

$$(1 + \mu_u^{RA}) \tilde{\psi} = \mu_u^{BC} - \mu_u^C \frac{1}{Q^F} + \mu_u^F \frac{1}{Q^F} \quad (84)$$

$$\mu_u^C = \mu_u^{CC} \eta - \mu_u \tilde{\eta} \quad (85)$$

$$0 = \mu_u [\omega^{\max} d_u - m_u - \tilde{\eta} Q (b_u - b_u^F)] \quad (86)$$

(d) 6 equations for connected banks:

$$(1 + \mu_c^{RA}) \tilde{\psi} R_k = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (87)$$

$$(1 + \mu_c^{RA}) \tilde{\psi} R_b + \mu_c^B = \mu_c^{BC} + \lambda \mu_c^{RA} - \mu_c^C \quad (88)$$

$$(1 + \mu_c^{RA}) \tilde{\psi} + \mu_c^M = \mu_c^{BC} + \lambda \mu_c^{RA} \quad (89)$$

$$(1 + \mu_c^{RA}) \tilde{\psi} = \mu_c^{BC} \quad (90)$$

$$(1 + \mu_c^{RA}) \frac{\tilde{\psi}}{Q^F} = \mu_c^{BC} - \mu_c^C Q^F + \mu_c^F \frac{1}{Q^F} \quad (91)$$

$$\mu_c^C = \mu_c^{CC} \eta \quad (92)$$

(e) 7 bank aggregation equations:

$$k = \xi k_c + (1 - \xi) k_u$$

$$d = \xi d_c + (1 - \xi) d_u$$

$$b = \xi b_c + (1 - \xi) b_u$$

$$f = \xi f_c + (1 - \xi) f_u$$

$$m = \xi m_c + (1 - \xi) m_u$$

$$\bar{v} = \xi v_c + (1 - \xi) v_u$$

$$n = \xi n_c + (1 - \xi) n_u$$

(where we recall that “ c ” and “ u ” in the last equation refer to the type status in t , not $t + 1$).

7. 2 rest of the world equations

$$b^w = f \left(\varkappa - \frac{1}{\varrho} \log \widetilde{Q}_t \pi_t \right)$$

$$Qb^w + c^w = R_b Qb^w.$$

These are 63 equations (one redundant by the Walras law) in 62 endogenous variables:

$$\left\{ \begin{array}{l} y, k, c, c^w, l, d, n, m^h, b, b^w, f, m, \bar{v}, \bar{b}, \tau^*, \\ \psi, \tilde{\psi}, \mu_u, w, r, Q, R_d, \pi, I, s, \bar{f}, \bar{m}, b^C \end{array} \right\}$$

plus

$$\{k_l, m_l, f_l, b_l, b_l^F, d_l, n_l, v_l, \tilde{v}_l, \mu_l^F, \mu_l^M, \mu_l^{RA}, \mu_l^{BC}, \mu_l^{CC}, \mu_l^C, \mu_l^B\},$$

plus the value of the three policy instruments

$$\eta^A, Q^F, b^C,$$

and of the following exogenous variables: $g, \xi, \tilde{\eta}$.

F Additional comparative statics

In this Appendix, we present results of a comparative statics exercise which aims to capture the effects of safe asset scarcity, a concern which became particularly pronounced in the aftermath of the Global Financial Crisis. In the euro area, the ratio of AAA-rated government debt to GDP declined from 29% pre-crisis (average over 2003-2007) to just 14% post-2015 (average over 2015-2019; a country is taken as AAA-rated if the country is AAA-rated by at least one of the following three rating agencies: Moody's, Fitch, S&P). To analyze the macroeconomic effects of this development, the supply of government bonds \bar{b} in our model is halved from the initial steady-state level of 7.50 units to 3.75 units. Figures 10 and 11 show the results for the constant balance sheet and the OP policy, respectively.

In both figures, the solid red line denotes the supply of government bonds under our benchmark calibration ($\bar{b} = 7.50$). The green dashed lines indicate the level of \bar{b} at which unconnected banks start holding money so that the multiplier μ_u^M becomes zero. The blue dashed lines indicate the level of \bar{b} at which the leverage constraint of unconnected banks turns slack and the multiplier μ_u^{RA} becomes zero. The orange dashed lines indicate the level of \bar{b} at which unconnected banks stop holding bonds. As we shall see, these three constraints will play a major role in this exercise.

In the calibrated steady-state (at the solid red line), the collateral premium on bonds is positive and the afternoon constraint binds for unconnected banks. If the stock of government bonds is lower (moving rightward in both figures), it becomes more difficult for unconnected banks to obtain collateralized funding of any kind.

Under the constant balance sheet policy (Figure 10), the figures resemble what happens as private haircuts increase. In particular, if bonds are more scarce (as at the level indicated by the green dashed lines), unconnected banks demand money to self-insure against afternoon liquidity shocks. As the supply of money is fixed under this policy, higher demand for money by unconnected banks is accommodated by the decrease of money holdings by households. This is facilitated by the increase in inflation and thus in the deposit rate. Higher inflation increases the opportunity cost of holding money for unconnected banks and further tightens their afternoon constraint. Unconnected banks respond by reducing their deposit intake and, therefore, investment in capital. This puts a downward pressure on aggregate capital and, correspondingly, an upward pressure on the return on capital. For the connected banks, this tightens their leverage constraint and, therefore, they reduce their investment in capital and

their deposit intake. When the supply of bonds reaches the level corresponding to the blue dashed line, unconnected banks are very constrained in the secured market but they cannot increase money holdings any further as households reduced their money holdings to a minimum. At this point, unconnected banks become so constrained in the afternoon that they must reduce their deposit intake. Their leverage constraint turns slack. Connected banks are able to pick up some of the deposits from unconnected banks but only up to a limit as they face the leverage constraint. As a result, aggregate deposits, capital and output decline. Quantitatively, if the stock of bonds is halved, output contracts by 3.3 percent.

Under the CO policy, the outcome is the same as under the constant balance sheet policy. This is because providing collateralized central bank funding through CO when bonds are scarce cannot mitigate output contractions.

By contrast, OP policy is very effective in stabilizing output in this case. It achieves this by substituting scarce bonds with another liquid asset - money - while maintaining the opportunity cost of holding money low. Specifically, when the stock of government bonds reaches the level corresponding to the green dashed line, unconnected banks sell their entire bond holdings to the central bank. For steady-states with a lower stock of government bonds, a lower stock of government bonds is reflected only in lower foreign bond holdings. Quantitatively, if the stock of bonds is halved, output contracts only by 0.1 percent under the OP policy.

Figure 10: Comparative statics: changing the stock of government bonds, \bar{b} , constant balance sheet policy

Notes: Red solid lines denote the calibrated steady state. Green, blue and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack, their leverage constraint becomes slack, and the non-negativity condition on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, money holdings of households, and collateral premium. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, net deposit rate, and percent deviation of output from steady-state.

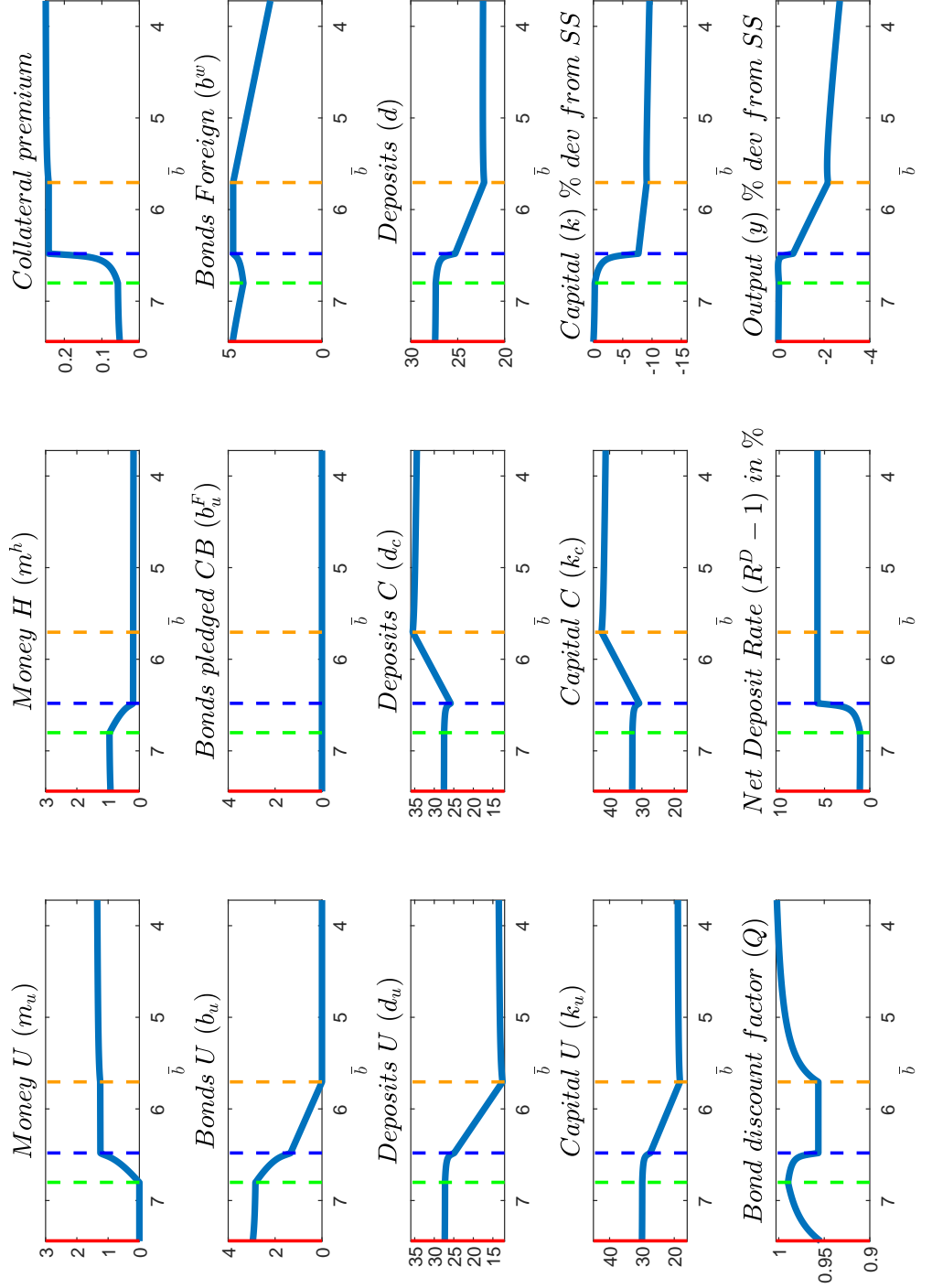


Figure 11: Comparative statics: changing the stock of government bonds, \bar{b} , OP policy

Notes: Red solid lines denote the calibrated steady state. Green and orange dashed lines denote the stock of government bonds at which the non-negativity condition on cash holdings of unconnected banks becomes slack and the one on their bond holdings becomes binding, respectively. First row: money holdings of unconnected banks, collateral premium, and money holdings of households. Second row: bonds held by unconnected banks, bonds they pledge at the central bank, and bonds held by foreigners. Third row: deposits raised by unconnected and connected banks, and aggregate deposits. Fourth row: investment in capital by unconnected and connected banks, and percent deviation of capital from steady-state level. Fifth row: bond discount factor, central bank bond holdings, and percent deviation of output from steady-state.

