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INTERNATIONAL YIELD CURVES AND CURRENCY PUZZLES

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ABSTRACT

The depreciation rate is often computed as the ratio of foreign and domestic pricing kernels. Using bond prices alone to estimate these kernels leads to currency puzzles: the inability of models to match violations of uncovered interest parity and the volatility of exchange rates. One cannot use information in bonds alone because exchange rates are not spanned by bonds. This view of the puzzles is distinct from market incompleteness. Incorporating exchange rates into estimation of yield curve models helps with resolving the puzzles. It also allows us to connect the differences between international yield curves to characteristics of exchange rates.

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1 Introduction

The asset market view of exchange rates, whereby the depreciation rate is computed as the ratio of foreign and domestic pricing kernels, has become a dominant paradigm in financial economics following the influential work of Backus, Foresi, and Telmer (2001) (B/F/T hereafter). Thus, empirical applications require estimates of the pricing kernels, and looking to bond prices as the relevant source of information appears to be a natural step. Indeed, it was pursued by B/F/T, among others.

A typical finding is that the variation in depreciation rates, inferred via the asset market view, has little to do with that of the observed ones, referred to as the *FX volatility anomaly* (Brandt and Santa-Clara, 2002), and that the inferred depreciation rates cannot replicate the *FX forward premium anomaly,* that is, the well-documented violations of the uncovered interest parity (UIP) hypothesis (B/F/T).

In this paper we argue that one must use information about depreciation rates jointly with bond prices in order to understand the cross-country differences between bonds and their connection to exchange rates. That is because, as we show, depreciation rates are not spanned by bonds. As a result, one cannot use information in bonds alone to infer the dynamics of depreciation rates. This focus is distinct from exploration of market incompleteness as a potential source of FX anomalies (e.g., Lustig and Verdelhan, 2015). Market incompleteness does not imply lack of FX spanning. Lack of FX spanning does imply market incompleteness with respect to bonds, but not necessarily with respect to a larger set of assets.[1]

Further, using bonds to infer pricing kernels naturally connects to the literature on no-arbitrage models of international yield curves. Typically, this research focuses on similarities between the different countries by modeling global/US and local/foreign factors and quantifying their contribution to the overall variation of the curves. The conclusion is that common variation is the major driver of interest rates.

We show that the differences in domestic and foreign bonds must be related to depreciation rates. Indeed, cross-country differences between yields reflect expected depreciation rates and the associated currency risk premiums. Cross-country differences between bond risk premiums reflect currency risk premiums. We demonstrate that in our sample of three countries (USA, Germany, and UK) the second and third principal components of the joint set of three yield curves are approximated by these differences. This evidence suggests an important role for depreciation rates and the associated currency risk premiums in understanding international yield curves.

[1]Markets are complete if there is a price for any asset in any state of the world, in other words one can trade a given set of assets to achieve any possible payoff and, therefore, know its value. If an exchange rate is unspanned by bonds, then the market cannot be complete with respect to bonds alone. The market could still be complete with respect to bonds and exchange rates.

We implement the joint modeling of bonds and currencies using the no-arbitrage affine framework. This approach allows us to estimate the USD-denominated pricing kernels using data on USD-denominated domestic and foreign bond prices, and on exchange rates (referred to as the WFX approach, estimation With FX rates). This is in contrast to the predominant approach that estimates the pricing kernel denominated in currency of a given country using data on bonds of the same country that are denominated in the currency of that country (referred to as the NFX approach, No FX rates used in estimation). This helps us with matching the observed FX rates and, therefore, the differences between the different yield curves.

We show that there is no difference in how the two approaches match yields, consistent with currencies that are unspanned by bonds. However, the exchange rate implied by the NFX approach is grossly misspecified. Its behavior is in line with findings reported by previous studies. In contrast, the WFX approach implies realistic exchange rate behavior. In particular, we can match all the FX moments discussed by B/F/T.

The NFX approach does not allow one to address how currency risk premiums connect bond yields and risk premiums of different countries. We use the WFX model to interpret the differences between the international yield curves. We derive currency and bond risk premiums from our model and we find that they (i) have exposures to factors that were not previously considered in the literature, e.g., differences in long-term interest rate spreads in addition to the usual interest rate differential considered in the UIP literature, and (ii) the factors that are affecting the two types of premiums are not necessarily the same. Thus, studying currency risk premiums or bond risk premiums in isolation does not offer a full picture.

The main lesson from our empirical study is that augmenting the set of assets by exchange rates makes a big difference in the implications for the analysis of international asset markets. In particular, this suggests that a rich collection of international bonds does not complete the markets. Market incompleteness with respect to bonds should be a starting point for an equilibrium model of exchange rates.

Our paper connects with several research themes. We do not provide a grand literature review. Instead, in an attempt to offer clarity, we describe the related work when we cover an appropriate subtopic.

2 Preliminary analysis

The purpose of this section is to connect two observations. First, loosely speaking, the differences between bonds of different countries reflect FX-related quantities. Differences in yields reflect expectations of depreciation rates and currency risk premiums. Differences in bond risk premiums reflect currency risk premiums. Second, despite these relationships, one cannot use bond data alone to say something useful about currencies. This is because depreciation rates are not spanned by yields. The implication is that one has to use FX-data in conjunction with bond data to produce accurate currency-related measurements, or to use properties of currencies to say something about bond behavior.

2.1 Bonds and currencies

Suppose $M_{t,t+i}$ is a USD-denominated i-period pricing kernel. Then the USD-denominated value of any zero-coupon bond of maturity n is

$$P_t^n = E_t(M_{t,t+n} \cdot C_{t,t+n}),$$

where $C_{t,t+i}$ is the cash flow growth between time t and $t+i$. If the bond is issued in USD, then $C_{t,t+i} = 1$; we denote its price by Q_t^n and its yield is $y_t^n = -n^{-1} \log Q_t^n$. If the bond is issued in foreign currency, then $C_{t,t+i} = S_{t+i}/S_t$ with S_t representing the value of one unit of foreign currency in USD; we denote the foreign bond price by $\widehat{Q}_t^n$ and its yield is $\widehat{y}_t^n$ in this case.

The USD bond one-period excess returns, in logs, are:

$$rx_{t+1}^n \equiv p_{t+1}^{n-1} - p_t^n + p_t^1 = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1 \tag{1}$$

with a similar expression for the foreign currency, $\widehat{rx}_{t+1}^n$. Note that the reference rate for foreign excess returns is the short rate of the respective country, $\widehat{y}_t^1$. Therefore, $\widehat{rx}_{t+1}^n$ does not depend on the currency of that country. In logs, this is equivalent to using the US short rate y_t^1 as a reference irrespective of the country and then constructing currency-hedged bond returns.

2.2 Differences between international yields

The yields y_t^n and $\widehat{y}_t^n$ share a simple relationship. We use Δ to denote the one-period time-series difference operator, Δ_c to denote the cross-country difference operator, and lowercase letters to denote logs of variables. We have, under conditional log-normality,

$$\begin{aligned}
\widehat{q}_t^n &= \log E_t \left(e^{\sum_{i=1}^n m_{t+i-1,t+i} + \Delta s_{t+i}} \right) \\
&= q_t^n + E_t \left(\sum_{i=1}^n \Delta s_{t+i} \right) + \frac{1}{2} var_t \left(\sum_{i=1}^n \Delta s_{t+i} \right) + cov_t \left(\sum_{i=1}^n m_{t+i-1,t+i}, \sum_{i=1}^n \Delta s_{t+i} \right).
\end{aligned}$$

After multiplying both sides by $-n^{-1}$, we get the interest rate differential (IRD):

$$\Delta_c y_t^n \equiv y_t^n - \widehat{y}_t^n \quad = \quad es_t^n - srp_t^n + vs_t^n. \tag{2}$$

Here $es_t^n \equiv n^{-1}E_t\left(\sum_{i=1}^n \Delta s_{t+i}\right)$ is the average expected depreciation rate, and $srp_t^n \equiv -n^{-1}cov_t\left(\sum_{i=1}^n m_{t+i-1,t+i}, \sum_{i=1}^n \Delta s_{t+i}\right)$ is the (ex-ante) currency "risk premium." We use the quotation marks because srp_t^n does not reflect the convexity term $vs_t^n \equiv (2n)^{-1}var_t\left(\sum_{i=1}^n \Delta s_{t+i}\right)$. The currency risk premium measures the additional compensation that an investor in foreign bonds requires in order to be exposed to future shocks to the exchange rate.

Likewise, there is a simple currency-related connection between the excess returns on bonds from different countries. Combining equations (1) and (2) we get:

$$\begin{aligned}
\Delta_c rx_{t+1}^n \equiv rx_{t+1}^n - \widehat{rx}_{t+1}^n &= (n-1)\cdot srp_{t+1}^{n-1} - n\cdot srp_t^n + srp_t^1 \\
&- (n-1)\cdot vs_{t+1}^{n-1} + n\cdot vs_t^n - vs_t^1 \\
&- u_{t+1}^n + u_{t+1}^1,
\end{aligned} \tag{3}$$

where, for a given horizon j, $u_{t+1}^j = E_{t+1}\left(\sum_{i=1}^j \Delta s_{t+i}\right) - E_t\left(\sum_{i=1}^j \Delta s_{t+i}\right)$ – is the surprise in expectations of the depreciation rate. Therefore, ignoring convexity, differences in expected log excess returns are driven by the differences in currency risk premiums across different horizons.

These expressions highlight that differences in international yield curves are driven by either investors' expectations of future depreciation rates or by currency risk premiums. Thus the observed differences in yield curves or excess returns, tell us about some combination of them.

2.3 Data

We work with monthly data from the US, UK, and Germany/Eurozone from January 1983 to December 2015 making for $T = 396$ observations per country. All data is aligned to the end of the month. US government yields are downloaded from the Federal Reserve and are constructed by Gurkaynak, Sack, and Wright (2007). All foreign government zero-coupon yields with maturities $12, 24, 36, 48$, and 60 months are downloaded from their respective central banks (Federal Reserve, Bank of England, and Bundesbank). The corresponding nominal exchange rates are from the Federal Reserve Bank of St. Louis. Prior to the introduction of the Euro, we use the German Deutschemark and splice these series together beginning in 1999.

Additionally, we use data on one-month yields to connect our approach to the evidence on UIP regressions. US one month yields are downloaded from CRSP. UK and German yields are harder to obtain. We obtain two data sources for each (investing.com for both; Bank of England for the UK; Federal Reserve Bank of St. Louis for Germany) and use only data that match across the two sources. This approach produces a nearly full sample for the UK, and some missing observations for Germany from September 2007 through October 2010.

2.4 Principal components

We consider three variations of the PC-construction. First, we extract six PCs from US bonds and the bonds of the country corresponding to the depreciation rates. Second, we extract six PCs from bonds of all three countries. Lastly, we extract nine PCs from bonds of all three countries. Table 1 reports the results. Six PCs explain 99.98% of variation in the yields of all three countries. Nine PCs across the three countries explain as much variation as six PCs do for yields of two countries (99.9995%).

Next, we approximate PCs with more simple linear combinations of yields to facilitate interpretation. Specifically, we introduce a vector

$$f_t = \begin{pmatrix} y_t^1 \\ \Delta_c^{€} y_t^1 \\ \Delta_c^{£} y_t^1 \\ y_t^{60,1} \\ \Delta_c^{€} y_t^{60,1} \\ \Delta_c^{£} y_t^{60,1} \end{pmatrix} = \begin{pmatrix} \text{US 1 month yield} \\ \text{US 1 month yield - Euro 1 month yield} \\ \text{US 1 month yield - UK 1 month yield} \\ \text{US slope = US 60 month yield - US 1 month yield} \\ \text{US slope - Euro slope} \\ \text{US slope - UK slope} \end{pmatrix}.$$

Figure 1 shows that the PCs and these factors are similar.

The combination of the last column of Table 1 and Figure 1 tells us that the short-term IRDs, $\Delta_c y_t^1$, serve as PC2 and PC3, approximately. Because of the connection of this spread to expected depreciation rates via equation (2), it is immediately clear that exchange rates play an important role in the behavior of international yield curves. While the US level obviously plays the major role by explaining 93.5% of variation, PC2 and PC3 contribute 4.4% and 1.4%, respectively. These are non-trivial amounts as is well known from the literature on US-only yield curve modeling.

2.5 Using bond data to infer properties of currencies

We argue that, in order to use the differences between international yield curves to characterize exchange rates, one needs to incorporate exchange rate data into a model of yield curves. That is because exchange rates are hidden in the yield curve. An exchange rate is hidden if the exchange rate cannot be replicated by a linear combination of yields (Duffee, 2011). That implies that an exchange rate cannot be spanned by bonds.

This concept is different from market completeness. If the market is complete with respect to bonds and currencies, and currencies are spanned by bonds, then the market is complete with respect to the bonds only. However, if exchange rates cannot be spanned by bonds then the market could still be complete, just with respect to a larger set of assets (bonds and exchange rates). Further, markets may be incomplete regardless of the bonds' ability to span exchange rates.

We proceed in three steps. First, we motivate our unspanning assertion in three ways. We argue that the existing evidence is already pointing that way. Then, we implement motivating regressions along the lines of regressions that are used to motivate unspanned macro variables in the term structure literature. Finally, we offer a fully-worked analytical example of a two-factor model with unspanned exchange rates.

Second, we implement an affine term structure model. We specify the dynamic properties of $M_{t,t+i}$ and S_{t+i}, and estimate the model using data on bonds $Q = \{Q_t^n\}$, $\widehat{Q} = \{\widehat{Q}_t^n\}$, and the corresponding exchange rate $S = \{S_t\}$. To emphasize the data used in estimation, we denote the estimated pricing kernel by $M_{t,t+i}(Q, \widehat{Q}, S)$ (or subset of these assets) in contrast to the true (unknown) pricing kernel $M_{t,t+i}$. We test whether exchange rates are hidden in this setting.

Third, we show what happens if the pricing kernel is estimated using bond data alone, $M_{t,t+i}(Q, \widehat{Q})$. We discuss the benefits of adding the exchange rate data for both models of the term structure and for capturing the exchange rate puzzles.

3 Motivating unspanned exchange rates

3.1 Relation to the literature

A long-standing tradition in the reduced-form no-arbitrage literature is to specify dynamics of $M_{t,t+i}$ and a foreign-currency-denominated i-period pricing kernel, $\widehat{M}_{t,t+i}$. The latter implies a value of a foreign-currency-denominated foreign-issued bond

$$\widehat{Q}_t^n = E_t(\widehat{M}_{t,t+n}).$$

Estimating a model of $M_{t,t+1}$ and $\widehat{M}_{t,t+1}$ using data on Q and $\widehat{Q}$, one obtains estimates of the pricing kernels $M_{t,t+1}(Q)$, and $\widehat{M}_{t,t+1}(\widehat{Q})$.

Next, researchers infer the depreciation rate via

$$S_{t+1}/S_t = \widehat{M}_{t,t+1}(\widehat{Q})/M_{t,t+1}(Q). \tag{4}$$

This relationship is valid under two assumptions. First, the markets are complete. Second, the estimated pricing kernel matches the true pricing kernel up to estimation noise, that is, Q and $\widehat{Q}$ span the space of all assets. Put differently, the markets are complete with respect to bonds.

There are variations in this approach where $M_{t,t+1}$ and $\widehat{M}_{t,t+1}$ are estimated simultaneously resulting in $M_{t,t+1}(Q, \widehat{Q})$ and $\widehat{M}_{t,t+1}(Q, \widehat{Q})$. Examples include, but are not limited to, Ahn (2004); Backus, Foresi, and Telmer (2001); Brennan and Xia (2006); Jotikasthira, Le, and Lundblad (2015); Kaminska, Meldrum, and Smith (2013). Sarno, Schneider, and Wagner

(2012) are similar, but they also incorporate information about conditional expectations of the depreciation rates into their estimation procedure.

Most papers report that the depreciation rate from (4) does not resemble the observed depreciation rates. Most prominently, researchers document the forward premium and volatility anomalies. These results might simply manifest a model misspecification. However, the inherent empirical flexibility of affine models and sophistication of the authors involved suggest to us that bonds, on their own do not posses the information needed to capture the behavior of exchange rates.

3.2 Regressions

One way to establish whether an asset (exchange rate) is spanned by other assets (bonds) is to regress returns of the former on the returns of the latter. To realize a return on an exchange rate, one must convert domestic currency into foreign currency, purchase a foreign (riskless) bond, sell it at a later date and then convert the proceeds back to the domestic currency. In order to avoid exposure to interest rate risk, this has to be a buy-and-hold strategy: $R^{FX}_{t+1} = S_{t+1}/S_t/\widehat{Q}^1_t$. A return on a domestic $n-$period bond is $R^n_{t+1} = Q^{n-1}_{t+1}/Q^n_t$. Thus, one can regress R^{FX}_{t+1} on a set of R^n_{t+1} for a variety of horizons n.

One practical problem with these regressions is that we do not have data on bonds with maturities that are one month apart to compute monthly returns. Thus, the regression could be implemented at an annual frequency only. We regress R^{FX}_{t+12} on R^n_{t+12}, $n = 24, 36, 48, 60$. The R^2, regular and adjusted, from these regressions are reported in Table 2A, in the column labeled "\$ returns." The amount of variation in currency returns that can be hedged with bonds is quite modest, 16% at most.

To check if the exchange rate is spanned by foreign bonds, one must take the perspective of a foreign-currency investor: $\widehat{R}^{FX}_{t+1} = S_t/S_{t+1}/Q^1_t$, and $\widehat{R}^n_{t+1} = \widehat{Q}^{n-1}_{t+1}/\widehat{Q}^n_t$; and implement a regression on these returns. Table 2A, column labeled "€ or £ returns" reports the R^2, which are of the same magnitude as the USD-denominated returns. This evidence establishes that bonds are unable to span the space of currency payoffs.

In addition, we would like to use our regressions results to understand how to model bonds and currencies in an affine context. Consequently, we will focus on establishing whether the depreciation rate is a hidden factor in domestic and foreign yield curves, i.e. that it does not appear as a factor in the cross-section (Duffee, 2011). As we demonstrate in the next subsection, a variable that is hidden in the yield curve is not spanned dynamically by a portfolio of bonds.

There is a well-established literature that primarily focuses on US bonds and seeks to identify hidden variables. Joslin, Priebsch, and Singleton (2014) refer to them as unspanned and use regressions of inflation or output growth on yields to motivate their use in a term structure model. We follow these authors and implement a regression to motivate our conjecture that depreciation rates are unspanned by yields. We regress the individual log depreciation rate on the three versions of principal components (PC) constructed from yields discussed in section 2.4. The timing is $\Delta s_t \sim f_t$ at a monthly frequency.

Table 2B displays the regression results. The largest R^2 is 6.38%.[2] Depreciation rates have very little to do with bond yields. To be clear, we are not imposing this conclusion on our main term structure model that we introduce in the next section. This conclusion serves as a motivation for considering different versions of that model. We will be formally testing, in the context of our model, whether depreciation rates are unspanned.

3.3 Illustration of unspanned exchange rates

We consider a simple example in order to demonstrate the effects of unspanned exchange rates and to preview the Gaussian term structure framework. We emphasize four main points. First, even if markets are complete and one knows the true model, the estimated depreciation rate cannot be identified if it is not observed. As a result, the estimated expected depreciation rate and its volatility are biased. Second, because of that, one cannot realistically decompose yield or bond premium spreads into the expected depreciation rate and currency risk premium. Third, the modeling of bond yields is unaffected by unobserved exchange rates. Fourth, once the exchange rate is introduced into the set of observations, one does not need market completeness to estimate its dynamics.

Setup

Consider a vector $x_t = (y_t^1, \Delta s_t)^\top$ that follows a VAR(1):

$$x_t \quad = \quad \mu_x + \Phi_x x_{t-1} + \Sigma_x \varepsilon_t \qquad\qquad \varepsilon_t \sim \mathrm{N}\left(0, I\right), \tag{5}$$

where the vector μ_x has elements $\mu_{x,i}$, matrix Φ_x has elements $\phi_{x,ij}$, and matrix Σ_x has elements $\sigma_{x,ij}$.

We model the dynamics of the true USD-denominated log pricing kernel

$$m_{t,t+1} \quad = \quad -y_t^1 - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}, \tag{6}$$

with market prices of risk

$$\lambda_t \quad = \quad \Sigma_x^{-1}\left(\lambda_0 + \lambda_x x_t\right).$$

[2] As a reference, Joslin, Priebsch, and Singleton (2014), who argue that inflation and output growth are unspanned by bonds, report R^2 of 86% and 32%, respectively.

For the purposes of this section only, we assume that the matrix λ_x has a special form:

$$\lambda_x = \begin{pmatrix} \lambda_{x,11} & \phi_{x,12} \\ \lambda_{x,21} & \phi_{x,22} \end{pmatrix}.$$

In words, changes in risk premiums due to variations in the depreciation rate exactly offset changes in expectations of future short interest rates and future depreciation rates.

Bond yields

The prices of zero-coupon USD-denominated bonds with maturity n are given by the standard pricing condition

$$Q_t^n = E_t\left(M_{t,t+1}Q_{t+1}^{n-1}\right).$$

As a result, because $\lambda_{x,12} = \phi_{x,12}$, US yields are linear functions of y_t^1 only

$$y_t^n = a_n + b_n y_t^1. \tag{7}$$

See Duffee (2011).

Let e_j denote a unit vector with a one in location j and zeros in all other entries. The currency risk premium is

$$srp_t^1 = -cov_t(m_{t,t+1}, \Delta s_{t+1}) = \mathrm{e}_2^\top \Sigma_x \lambda_t = \lambda_{0,2} + \lambda_{x,21} y_t^1 + \phi_{x,22}\Delta s_t. \tag{8}$$

We can express the USD-denominated pricing kernel in foreign currency:

$$\begin{aligned} \widehat{m}_{t,t+1} &= m_{t,t+1} + \Delta s_{t+1} = m_{t,t+1} + \mathrm{e}_2^\top x_{t+1} \\ &= -\widehat{y}_t^1 - \frac{1}{2}\widehat{\lambda}_t^\top \widehat{\lambda}_t - \widehat{\lambda}_t^\top \varepsilon_{t+1}, \end{aligned} \tag{9}$$

where

$$\widehat{y}_t^1 = \alpha + \beta y_t^1, \tag{10}$$

$$\alpha = -\mathrm{e}_2^\top(\mu_x - \lambda_0) - \mathrm{e}_2^\top \Sigma_x \Sigma_x^\top \mathrm{e}_2/2 = -\mu_{x,2} + \lambda_{0,2} - (\sigma_{x,21}^2 + \sigma_{x,22}^2)/2, \tag{11}$$

$$\beta = 1 - \phi_{x,21} + \lambda_{x,21}, \tag{12}$$

$$\widehat{\lambda}_t = \Sigma_x^{-1}\left(\widehat{\lambda}_0 + \lambda_x x_t\right), \tag{13}$$

$$\widehat{\lambda}_0 = \lambda_0 - \Sigma_x[\mathrm{e}_2^\top \Sigma_x]^\top. \tag{14}$$

The foreign interest rate $\widehat{y}_t^1$ in (10) does not depend on the depreciation rate because $\lambda_{x,22} = \phi_{x,22}$.

The prices of zero-coupon foreign currency bonds with maturity n are given by

$$\widehat{Q}_t^n = E_t\left[\widehat{M}_{t,t+1}\widehat{Q}_{t+1}^{n-1}\right].$$

As a result, because $\lambda_{x,22} = \phi_{x,22}$, foreign yields are linear functions of $\widehat{y}_t^1$ only

$$\widehat{y}_t^n = \widehat{a}_n + \widehat{b}_n \widehat{y}_t^1. \tag{15}$$

Spanning

Equations (7) and (15) imply that the depreciation rate is hidden in the yield curves. We can characterize the implication of these relationships for spanning of currency returns with a portfolio of bonds that was described in section 3.2:

$$\begin{aligned} R^{FX}_{t+1} &= e^{\Delta s_{t+1}+\widehat{y}^1_t} \text{ spanned with } R^n_{t+1} = e^{a_n - a_{n-1} + b_n y^1_t - b_{n-1} y^1_{t+1}}; \\ \widehat{R}^{FX}_{t+1} &= e^{-\Delta s_{t+1}+y^1_t} \text{ spanned with } \widehat{R}^n_{t+1} = e^{\widehat{a}_n - \widehat{a}_{n-1} + \widehat{b}_n \widehat{y}^1_t - \widehat{b}_{n-1} \widehat{y}^1_{t+1}}. \end{aligned}$$

The analytics simplify if we express the return on a portfolio of bonds with a vector of weights w_t in logs. The switch to logs can be justified via a log-linearization: $\log(\tilde{w}_t^\top e^{r_{t+1}}) \approx w_{t0} + w_t^\top r_{t+1}$. We use additional notation: b ($\widehat{b}$) for a vector of loadings b_{n-1} ($\widehat{b}_{n-1}$) for a range of maturities n, and r_{t+1} ($\widehat{r}_{t+1}$) for a vector of log returns $\log R^n_{t+1}$ ($\log \widehat{R}^n_{t+1}$). Then,

$$\begin{aligned} |corr_t(r^{FX}_{t+1}, w_t^\top r_{t+1})| &= \frac{cov_t(\Delta s_{t+1}, w_t^\top b y^1_{t+1})}{var_t^{1/2}(\Delta s_{t+1}) var_t^{1/2}(w_t^\top b y^1_{t+1})} = \frac{\sigma_{x,21}}{(\sigma^2_{x,21} + \sigma^2_{x,22})^{1/2}} < 1, \\ |corr_t(\widehat{r}^{FX}_{t+1}, w_t^\top \widehat{r}_{t+1})| &= \frac{cov_t(\Delta s_{t+1}, w_t^\top \widehat{b} \widehat{y}^1_{t+1})}{var_t^{1/2}(\Delta s_{t+1}) var_t^{1/2}(w_t^\top \widehat{b} \widehat{y}^1_{t+1})} = \frac{\sigma_{x,21}}{(\sigma^2_{x,21} + \sigma^2_{x,22})^{1/2}} < 1. \end{aligned}$$

These correlations imply that $R^2 < 1$ for the spanning regressions.[3]

Thus, a conditional portfolio of bonds cannot span an exchange rate if it is hidden in the yield curve. Nevertheless, if we assume existence of the currency market and the ability to trade an infinite number of bonds, the markets would be complete. In that case $\widehat{m}_{t,t+1}$ would be the true foreign pricing kernel.

Estimation, NFX approach

Suppose we ignore information in the depreciation rates to estimate the term structure model. The issue is whether we can identify the true dynamics of Δs_t:

$$\Delta s_t = \mu_{x,2} + \phi_{x,21} y^1_t + \phi_{x,22} \Delta s_{t-1} + \sigma_{x,21} \varepsilon_{1t} + \sigma_{x,22} \varepsilon_{2t}. \tag{16}$$

Suppose that, in addition, depreciation rates do not predict interest rates (consistent with the evidence to be discussed in a later section), that is, $\phi_{x,12} = 0$, and Σ_x is lower triangular (recursive identification). Then equation (5) implies that the interest rate y^1_t follows a process

$$y^1_t = \mu_{x,1} + \phi_{x,11} y^1_{t-1} + \sigma_{x,11} \varepsilon_{1t},$$

[3] If the depreciation rate were not hidden, then the conditional correlations above would depend on the portfolio weights. These weights then could be chosen to maximize the correlation with a possibility of reaching the value of 1 (if markets are complete with respect to bonds and currencies).

which can be easily estimated. In addition to the parameters in this equation, the coefficients a_n and b_n in equation (7) depend on $\lambda_{0,1}$ and $\lambda_{x,11}$. These parameters can be estimated using US bond data.

The estimated USD pricing kernel is

$$m_{t,t+1}(Q,\widehat{Q}) = m_{t,t+1}(Q) = -y_t^1 - [\sigma_{x,11}^{-1}(\lambda_{0,1} + \lambda_{x,11}y_t^1)]^2/2 - \sigma_{x,11}^{-1}(\lambda_{0,1} + \lambda_{x,11}y_t^1)\varepsilon_{1t+1}.$$

The foreign rate $\widehat{y}_t^1$ is a linear transformation of y_t^1, equation (10). If we do not use Δs_t in estimation, we can only identify α and β. Coefficients $\widehat{a}_n$ and $\widehat{b}_n$ in equation (15) depend on $\widehat{\lambda}_{0,1}$ and $\lambda_{x,11}$. These parameters can be estimated from foreign bonds. Equation (14) implies that

$$\widehat{\lambda}_{0,1} = \lambda_{0,1} - \sigma_{x,11}\sigma_{x,22},$$

Given that we can identify $\lambda_{0,1}$ and $\sigma_{x,11}$ from US bonds, we can infer $\sigma_{x,22}$.

The estimated foreign currency pricing kernel is

$$\widehat{m}_{t,t+1}(Q,\widehat{Q}) = -\widehat{y}_t^1 - [\sigma_{x,11}^{-1}(\widehat{\lambda}_{0,1} + \lambda_{x,11}y_t^1)]^2/2 - \sigma_{x,11}^{-1}(\widehat{\lambda}_{0,1} + \lambda_{x,11}y_t^1)\varepsilon_{1t+1}.$$

Thus, if one were to assume complete markets, the estimated depreciation rate would be

$$\begin{aligned}
\Delta s_{t+1}(Q,\widehat{Q}) &= \widehat{m}_{t,t+1}(Q,\widehat{Q}) - m_{t,t+1}(Q,\widehat{Q}) \\
&= \mu_{x,2} - \lambda_{0,2} + \sigma_{x,11}^{-1}\sigma_{x,22}\lambda_{0,1} + \sigma_{x,21}^2/2 \\
&+ (\phi_{x,21} - \lambda_{x,21} + \sigma_{x,11}^{-1}\sigma_{x,22}\lambda_{x,11})y_t^1 + \sigma_{x,22}\varepsilon_{1t+1}.
\end{aligned}$$

The implied currency risk premium is

$$srp_t^1(Q,\widehat{Q}) = -cov_t(m_{t,t+1}(Q,\widehat{Q}), \Delta s_{t+1}(Q,\widehat{Q})) = \sigma_{x,11}^{-1}\sigma_{x,22}\lambda_{0,1} + \sigma_{x,11}^{-1}\sigma_{x,22}\lambda_{x,11}y_t^1.$$

Comparing that with the true depreciation rate in (16) and true currency premium in (8) we see that we get a bias in both expected depreciation rate, risk premium (forward premium anomaly), and volatility (volatility anomaly).

Ahn (2004) makes depreciation rate a hidden factor by allowing Δs_t to be affected by a shock that does not affect the USD pricing kernel $m_{t,t+1}$. If we set risk premium parameters $\lambda_{0,2}$, $\lambda_{x,21}$, $\lambda_{x,12}$, $\lambda_{x,22}$ and persistence parameters $\phi_{x,12}$, $\phi_{x,22}$ to zero, we obtain a simple version of that model (it has three factors). In this case the NFX method recovers the USD pricing kernel, but not the depreciation rate.

These biases take place even if the markets are complete, but some of the assets needed for market completion (exchange rates) are not used in estimation. The problem is exacerbated if the markets are incomplete. If Δs_t is spanned (matrix λ_x is unrestricted), then Δs_t still cannot be recovered if one uses bonds only for estimation and the markets are incomplete.

Estimation, WFX approach

The WFX approach uses observations on Δs_t for estimation. As a result one can estimate the full dynamics of x_t in (5). The transition from $m_{t,t+1}(Q,\widehat{Q},S)$ to $\widehat{m}_{t,t+1}(Q,\widehat{Q},S)$ by changing denomination via $\widehat{m}_{t,t+1}(Q,\widehat{Q},S) = m_{t,t+1}(Q,\widehat{Q},S) + \Delta s_{t+1}$ does not require any assumptions because it is a simple change in the denomination of the pricing kernel. Once the parameters of Δs_t are estimated, one can use α in (11) to back out the risk premium parameter $\lambda_{0,2}$ and β in (12) to back out $\lambda_{x,21}$.

Implications for international yield curves and currencies

If the depreciation rate is unspanned by bonds, then both estimation approaches would produce identical domestic and foreign yields curves. The WFX approach would allow one to use equations (2) and (3) to decompose the cross-country differences in yields into the currency premium and expected currency components.

From the currency-modeling perspective, the WFX approach allows for identification of realistic dynamics of the depreciation rate. It also allows researchers to connect currency risk premiums to factors driving bond premiums. That is not feasible if one uses observations on currencies alone.

4 The full model

We model the dynamics of the state vector x_t as a Gaussian VAR given by (5). We use $\bar{\mu}_x$ to denote the unconditional mean of the state.

4.1 Bonds denominated in US dollars

We model the dynamics of the USD-denominated log pricing kernel

$$m_{t,t+1}(\cdot) \quad = \quad -\delta_{i,0} - \delta_{i,x}^{\top} x_t - \frac{1}{2}\lambda_t^{\top}\lambda_t - \lambda_t^{\top}\varepsilon_{t+1},$$

with market prices of risk in equation (7). The notation $(\cdot)$ highlights that we are not specifying the true pricing kernel $M_{t,t+1}$, but only its component that correctly prices a set of assets to be specified later, either $(Q,\widehat{Q})$, or $(Q,\widehat{Q},S)$. Regardless of the choice of assets, our model of the component of the USD pricing kernel that values them correctly is the same.

The prices of zero-coupon USD-denominated bonds with maturity n are given by the standard pricing condition

$$Q_t^n \quad = \quad E_t\left(M_{t,t+1}(\cdot)Q_{t+1}^{n-1}\right).$$

As a result, US yields are linear functions of the factors

$$y_t^n \quad = \quad a_n + b_{n,x}^{\top} x_t.$$

Expressions for the bond loadings can be found in Appendix A.1.

4.2 Bonds denominated in foreign currency

4.2.1 The NFX approach

In this case $M_{t,t+1}(\cdot) = M_{t,t+1}(Q, \widehat{Q})$. Here the state vector x_t is presumed to span all bonds. We model the dynamics of the log pricing kernel denominated in foreign currency as

$$\widehat{m}_{t,t+1}(Q, \widehat{Q}) \quad = \quad -\widehat{\delta}_{i,0} - \widehat{\delta}_{i,x}^{\top} x_t - \frac{1}{2}\widehat{\lambda}_t^{\top}\widehat{\lambda}_t - \widehat{\lambda}_t^{\top}\varepsilon_{t+1} \tag{17}$$

with market prices of risk

$$\widehat{\lambda}_t \quad = \quad \Sigma_x^{-1}\left(\widehat{\lambda}_0 + \widehat{\lambda}_x x_t\right).$$

We suppress country-specific notation for simplicity.

The prices of zero-coupon foreign currency bonds with maturity n are given by

$$\widehat{Q}_t^n \quad = \quad E_t\left[\widehat{M}_{t,t+1}(Q, \widehat{Q})\widehat{Q}_{t+1}^{n-1}\right].$$

As a result, foreign yields are linear functions of the factors

$$\widehat{y}_t^n \quad = \quad \widehat{a}_n^N + \widehat{b}_{n,x}^{N\top} x_t.$$

where the bond loadings can be found in Appendix A.2.

This strategy is similar to the ones undertaken in the literature on international yield curves. There is some variation: some authors distinguish between global and country-specific factors; some authors estimate the USD-denominated kernel using USD bond data only, $M_{t,t+1}(Q)$, as a first step, and then proceed with estimating $\widehat{M}_{t,t+1}(Q, \widehat{Q})$. We use the joint data and allow yields from each country to load on all the factors allowing the data to speak to which bonds load on which factors. Assuming market completeness with respect to bonds, one can infer the log depreciation rate as:

$$\Delta s_{t+1} = \widehat{m}_{t,t+1} - m_{t,t+1} = \widehat{m}_{t,t+1}(Q, \widehat{Q}) - m_{t,t+1}(Q, \widehat{Q}). \tag{18}$$

4.2.2 The WFX approach

In this case $M_{t,t+1}(\cdot) = M_{t,t+1}(Q, \widehat{Q}, S)$. Here the state vector x_t is presumed to span all bonds and exchange rates. The log depreciation rate Δs_t is assumed to be a linear function of the state vector:

$$\Delta s_t = \delta_{s,0} + \delta_{s,x}^\top x_t. \tag{19}$$

The prices of zero-coupon foreign currency bonds with maturity n are given by

$$\widehat{Q}_t^n \;\; = \;\; E_t \left(M_{t,t+1}(Q, \widehat{Q}, S) \frac{S_{t+1}}{S_t} \widehat{Q}_{t+1}^{n-1} \right). \tag{20}$$

As a result, foreign yields are linear functions of the factors

$$\widehat{y}_t^n \;\; = \;\; \widehat{a}_n^W + \widehat{b}_{n,x}^{W\top} x_t.$$

where the bond loadings can be found in Appendix A.3. Bauer and de los Rios (2014); Graveline and Joslin (2011) model the FX rate directly as well, but do not explore the implications of such an approach for the FX anomalies, or differences between the yield curves.

Given the estimated model, we can express the USD-denominated pricing kernel in foreign currency:

$$\widehat{m}_{t,t+1}(Q, \widehat{Q}, S) = m_{t,t+1}(Q, \widehat{Q}, S) + \Delta s_{t+1}. \tag{21}$$

This equation is an accounting identity that does not require any assumptions in contrast to (18). Combining equations (19) and (21), we can write

$$\widehat{m}_{t,t+1}(Q, \widehat{Q}, S) \;\; = \;\; -\widehat{y}_t^1 - \frac{1}{2} \widehat{\lambda}_t^\top \widehat{\lambda}_t - \widehat{\lambda}_t^\top \varepsilon_{t+1},$$

where $\widehat{\lambda}_t$ has the same functional form as (13), and

$$\widehat{\lambda}_0 \;\; = \;\; \lambda_0 - \Sigma_x [\delta_{s,x}^\top \Sigma_x]^\top. \tag{22}$$

Thus, in contrast to many theoretical models of exchange rates, $M(Q, \widehat{Q}, S)$ and $\widehat{M}(Q, \widehat{Q}, S)$ are asymmetric. The asymmetry arises via the constant component of the risk premium – an implication of the constant volatility model of depreciation rates. Changing that feature would introduce further asymmetry via the time-variation in risk premium.

4.3 Discussion

Our approach appears similar to the exploration of incomplete markets in affine settings studied by B/F/T and Lustig and Verdelhan (2015). However, there is an important conceptual difference. These authors specify a model of the true domestic and foreign pricing kernels, $M_{t,t+1}$ and $\widehat{M}_{t,t+1}$, respectively. Then they discuss conditions under which the markets could be incomplete and how many assets are required to span the markets.

Despite similar-looking equations, we do not assert dynamics of the full pricing kernel. Instead, we specify the dynamics of that part of the kernel that matches the properties of a given set of assets: domestic and foreign bonds in the NFX case; domestic and foreign bonds, and exchange rates in the WFX case. Intuitively, we are constructing something similar to the pricing kernel projection on a given set of assets. It is not a literal projection because we are using a model and the full sample of data to estimate it.

Each approach has its virtues. Which one is the most relevant depends on the research question being asked. Our approach is attractive because we do not have to take a stance on market completeness to estimate $M_{t,t+1}(Q,\widehat{Q},S)$, or $M_{t,t+1}(Q,\widehat{Q})$ and $\widehat{M}_{t,t+1}(Q,\widehat{Q})$. As a result, we can characterize the joint behavior of bonds and exchange rates. A drawback of our approach is that we cannot determine which assets are needed to complete the markets. We cannot give a full characterization of market incompleteness and how it impacts one's ability to understand some of the currency puzzles.

It has been argued in the literature that one needs to incorporate time-varying volatility to resolve the FX volatility anomaly in the context of the NFX approach (Anderson, Hammond, and Ramezani, 2010; Brandt and Santa-Clara, 2002). Of course, FX volatility is time-varying and adding that feature is an obvious extension if one is interested in FX option valuation or other aspects of FX dynamics. We do not model stochastic volatility to emphasize the point that there is no volatility anomaly even in a Gaussian model if the WFX approach is used.

Finally, B/F/T and many authors following them explore so-called square-root, or CIR, state variables instead of the Gaussian ones used here. This distinction plays no role here. We are simply looking for models that are capable of realistic fit to the yield curve data. Starting from Dai and Singleton (2000) and many papers following them, the literature has concluded that Gaussian models are more flexible in capturing yield co-movement and risk premiums. These models have been a de-facto standard for the last 15 years. A square-root factor could be helpful in capturing time-varying volatility of interest rates, but, absent data on interest rate derivatives, it is very hard to identify empirically (Bikbov and Chernov, 2011).

5 Results

5.1 Empirical approach

We extend the estimation procedure of Joslin, Singleton, and Zhu (2011) to international yield curves and use Bayesian MCMC to implement it. The approach has two ingredients.

First, risk premiums λ_t and $\widehat{\lambda}_t$ are estimated by specifying risk-adjusted dynamics of the state that is implied by the specification of the pricing kernels. The mapping into risk-adjusted parameters and identifying restrictions are discussed in Appendix B.1. Second, the state x_t is observable and is a linear transformation of yields and, in our case, exchange rates.

The choice of state vector is motivated by the PC analysis in section 2.4. Specifically, in the case of the NFX model, the state is $x_t^N = f_t$. In the WFX model, we complement the state vector by adding the two depreciation rates

$$x_t^W = \left(\Delta s_t^{€}, \Delta s_t^{£}, x_t^{N\top}\right)^\top.$$

Because all the state variables in x_t are observable, the free parameters that govern the dynamics of the state, $\mu_x, \Phi_x, \Sigma_x\Sigma_x^\top$, are identifiable directly from the VAR in equation (5). These parameters therefore require no identifying restrictions. Restrictions are required on the factor loadings and the risk-premium parameters λ_0 and λ_x. These restrictions are necessary to exactly identify the model and are not over-identifying restrictions. Here we highlight those restrictions that are easy to explain and will be helpful in interpreting some of the evidence.

The choice of depreciation rates as state variables implies that factor loadings and intercepts for the exchange rates are restricted as follows:

$$\delta_{s,0}^{€} = 0, \quad \delta_{s,x}^{€} = \mathrm{e}_1, \tag{23}$$

$$\delta_{s,0}^{£} = 0, \quad \delta_{s,x}^{£} = \mathrm{e}_2. \tag{24}$$

The IRDs $\Delta_c^{€} y_t^1$ and $\Delta_c^{£} y_t^1$ are the 4th and 5th elements of the state vector in (5). That imposes restrictions on the risk premium parameters. Equation (20) implies, for $n = 1$,

$$\begin{aligned}
\widehat{y}_t^1 &= -\log E_t(M_{t,t+1}(Q, \widehat{Q}, S)e^{\Delta s_{t+1}}) \\
&= y_t^1 - \delta_{s,0} - \delta_{s,x}^\top \mu_x - \delta_{s,x}^\top \Phi_x x_t + \delta_{s,x}^\top \lambda_0 + \delta_{s,x}^\top \lambda_x x_t - \delta_{s,x}^\top \Sigma_x \Sigma_x^\top \delta_{s,x}/2 \\
&= y_t^1 - \mathrm{e}_j^\top (\mu_x + \Sigma_x \Sigma_x^\top \mathrm{e}_j/2 - \lambda_0) - \mathrm{e}_j^\top (\Phi_x - \lambda_x) x_t,
\end{aligned}$$

where we imposed restrictions (23), (24) in the last row and $j = 1$, or 2, depending on the exchange rate. Therefore, the elements of the first two rows of λ_x are equal to the elements of the first two rows of Φ_x with the exception of the (1,4) and (2,5) elements that have to be equal to the corresponding element of Φ_x minus 1. Likewise, we can derive restrictions on λ_0.

Similar restrictions are obtained by recalling that the differences in slopes are part of the state vector as well. Following similar derivation steps, one can show that

$$\Delta_c y_t^{60} - \Delta_c y_t^1 = \text{const} + 60^{-1}\mathrm{e}_j^\top \left([\Phi_x - \lambda_x] + \ldots + [\Phi_x - \lambda_x]^{60} - 60[\Phi_x - \lambda_x]\right) x_t.$$

So, all the loadings on x_t have to be equal to zero, except for the 7th and 8th entries (depending on whether $j = 1$ or 2) where they have to be equal to 1. That requirement places non-linear restrictions on λ_x.

5.2 Basic properties of the estimated models

5.2.1 Estimates and fit

We report the estimated parameters in Tables 3 - 5. Table 6 displays pricing errors. The overall message from this set of Tables is that both models fit the given collection of domestic and foreign yields similarly in terms of small errors and similarity of estimated parameters that correspond to yield-based states.

The differences in the approaches are manifested by two extra sets of risk premium parameters in Table 4 for the NFX model and by extra parameters corresponding to depreciation rates in Table 5 for the WFX model. The WFX model exhibits a slight deterioration in fitting yields at shorter maturities suggesting some tension in fitting yields and depreciation rate with the same set of states.

5.2.2 Exchange rates are unspanned by bonds

From the perspective of modeling the yield curve, the two depreciation rates are the new factors in the WFX model as compared to the NFX model. Figure 2 demonstrates that they are unspanned by showing how bonds of different countries load on the two depreciation rates in the WFX model. While there are some departures from zero, none of them are statistically significant. Given that the monthly standard deviation of each depreciation rate is about 0.029 (10% per year), the largest monthly movement in a yield (UK at 5 months) is 0.2 basis points ($0.7 \times 10^{-3} \times 0.029 \times 100^2$) for one standard deviation move in a depreciation rate. This is not an economically significant amount either.

In contrast to the literature on hidden/unspanned variables that focusses on their ability to forecast bond excess returns, we make no such claims here. Inspection of the estimated persistence matrix Φ_x in Table 5 suggests that the depreciation rates have little to do with forecasting of future yields. Indeed, the first two columns of Φ_x are statistically indistinguishable from zero.[4] There is one exception: the lagged GBP depreciation rate predicts the EUR one. Depreciation rates do not help forecasting bond risk premiums either as the relevant elements of the risk premium matrix λ_x are zero as well.

5.2.3 The role of other factors

We describe how the other factors impact yields by reporting the term structure of factor loadings in Figure 3. We show these loadings for the WFX model, while the NFX model (not reported) displays similar patterns. At some level, the figure should not be surprising.

[4] That is consistent with our assumption $\phi_{x,12} = 0$ in section 3.3.

It shows that the US short interest rate and slope act as level and slope factors for all countries. The departures from the US factors, differences in short rates or differences in slopes, primarily affect yields of the corresponding foreign country.

These bond loadings could tempt a researcher to label US variables as global and the other variables as local, a language that is commonly used in the literature on international yield curve models. One has to be careful with such an interpretation because the choice of US factors as a de-facto reference is arbitrary. We could have considered an equivalent factor rotation where the German short rate and slope are the reference factors and the rest are defined relative to that. Nevertheless, these loadings suggest a modular approach towards modeling multiple yield curves: start with a benchmark and its yield-based factors, then add as many countries as needed via factors that increment over the benchmark. One has to deal with parameter proliferation when more than three countries are considered at once. There is no choice but to impose overidentifying parameter restrictions; a problem we do not address in this paper. See Graveline and Joslin (2011) for a recent effort along these lines.

5.2.4 Implications for exchange rates

From the perspective of modeling depreciation rates, yields are an important ingredient in their dynamics. While risk-adjusted expected depreciation rates depend on their respective interest rate differential (IRD) only, an implication of covered interest parity (CIP), the true expectations depend on other yield factors as well. At first glance, the model cannot replicate the violations of uncovered interest parity (UIP) documented by Bilson (1981); Fama (1984); Tryon (1979). For instance, the loading of the expected Euro depreciation rate on the respective IRD is positive. However, the estimated model is consistent with a multivariate regression with potentially correlated regressors, while the original UIP regressions are simple linear regressions with one regressor. In the next section, we explore what the WFX model implies for the specific univariate regressions studied in the literature.

While the WFX model matches the depreciation rate by construction, we need to infer them in the case of the NFX model. As pointed out in equation (18), we can do so only under the assumption of complete markets. Figure 4 displays the inferred and observed depreciation rate. They are clearly different in terms of their scale and dynamic patterns. In the next section we provide more details on these differences.

As noted in the context of equation (22), the modeled pricing kernels are asymmetric. The estimated model verifies that indeed estimated parameter values are such that asymmetry between $M(Q, \widehat{Q}, S)$ and $\widehat{M}(Q, \widehat{Q}, S)$ holds. Thus, depending on the setup of the general equilibrium model, the marginal rates of substitution of domestic and foreign economic agents might have to be asymmetric to capture the behavior of exchange rates.

5.3 B/F/T

Results of B/F/T represent a challenge to affine no-arbitrage models. It appears to be difficult to replicate a certain set of stylized facts about interest rates and exchange rates simultaneously. They propose a model that succeeds in matching some of the properties of FX and yet generates unrealistic yield curves. Indeed, B/F/T state: "The implied yield curve, for instance, is hump shaped with long yields reaching as high as 80 percent per annum." We revisit their analysis in the context of our models.

Table 7 replicates Table I of B/F/T in our sample and complements it by displaying model implications for the same set of facts. Both models do well in replicating facts about interest rates. This is not surprising given that B/F/T focused on short rates and they serve as state variables in our model. There is some deterioration for Euro/Germany because of the aforementioned missing observations on one-month yields.

The differences in FX implications are drastic. This is consistent with Figure 4, but offers other angles. The inferred depreciation rate is 25 times more volatile in the NFX model than in the data, and the mean is greater by two orders of magnitude. The NFX model is nowhere close to the results of UIP regressions.

The WFX model replicates all of these moments perfectly, by construction. Does it come at the cost of a poor fit of the yield curve as in B/F/T? Table 6 foreshadows the answer. Table 8 reports the yield-curve summary statistics in the B/F/T style. There is not much of a trade-off in fitting both yields and FX rates for the WFX model.

One way to interpret the NFX results is that markets are in fact incomplete with respect to bonds, so it is incorrect to use equation (18) to infer FX rates.

To clarify, while the B/F/T methodology is close to NFX, it is not identical. B/F/T construct their model to match the UIP violations and volatility of the depreciation rate. Thus, they use information about depreciation rates, but not about their joint dynamics with yields. That is why they can match some basic facts about currencies, but the resulting model cannot match yields. That is a manifestation unspanned currencies. The only way for B/F/T to succeed empirically is to incorporate this lack of spanning into their model of exchange rates.

More generally, one might wonder if it is possible to use the presented evidence to construct M and $\widehat{M}$ such that they produce a realistic depreciation rate via (18). It is possible to do so, in theory, by taking the estimated $M(Q, \widehat{Q}, S)$ and $\widehat{M}(Q, \widehat{Q}, S)$. Section 3.3 offers a more explicit version of that in equations (6) and (9). The issue is, as pointed out in section 3.3, that one cannot estimate these models without the joint data on yields and currencies.

5.4 International yields and risk premiums

This section studies how the differences between foreign and US bonds relate to FX rates. The connection is highlighted in equations (2) and (3). The discussion in section 2.4 demonstrates that, at the most basic level, expectations about future depreciation rates and the associated currency risk premiums that drive the second and third principal components of yields is an important building block for international yield curves. We use the estimated WFX model to illustrate how the relationships between yields and currencies may work.

5.4.1 Differences in yields

Figure 5 displays the time-series properties of the IRDs for three maturities ($n = 1$, 12, and 60 months) and their respective ingredients. Both the Euro and UK IRDs could be negative and positive. Their time-variation has a strong cyclical component. Their local minimums coincide with the peak of foreign recessions.

We compute the decomposition of the yield differentials via (2). Given that the volatility is constant in our model, we focus on srp_t^n and es_t^n. Alvarez, Atkeson, and Kehoe (2007) point out that if exchange rates are random walks (in logs), then es_t^n is constant and the time-variation in the IRDs is driven by srp_t^n. The Figure shows that there is strong time-variation in both components, and the two have a strong positive correlation.

The cyclical properties of IRDs are driven by the fact that, despite the strong correlation, currency risk premiums become much larger than expected depreciation rates during the recessions. To understand this pattern, we have to establish which variables drive the two ingredients. Thus, we establish elements of x_t that affects es_t^n and srp_t^n significantly.

Empirically, factors that have the same economic meaning, relative to the respective country, load similarly across the UK and EUR. For example, each of these countries have similar loadings on the US level and slope factors. Likewise, the interest rate differential, $\Delta_c y_t^1$, slope differential, $\Delta_c y_t^{60,1}$, and depreciation rate Δs_t each load on their own country in a way that is similar in magnitude across countries. To make inference more precise, we follow the spirit of pooled or fixed-effects regressions used in the context of currency risk premiums (Ang and Chen, 2010; Bansal and Dahlquist, 2000) and take an average of the loadings across the UK and EUR at each maturity for those factors that have the same economic interpretation.[5]

Table 9 shows the averaged loadings of expected future depreciation rates es_t^n and srp_t^n on x_t across horizons. We display only those elements of x_t that have a significant loading for

[5]It is straightforward to conduct inference from a Bayesian perspective that we take in this paper. The factor loadings are a function of the posterior distribution of the model parameters. Our estimates of the country-specific loadings are averages across the draws of the Markov-chain Monte Carlo (MCMC) algorithm. Thus an average loading is the average of the averages, which allows to take into account parameter uncertainty in a natural fashion.

at least one horizon. We see that departures of the depreciation rates from random walk, or, equivalently, departures of es_t^n from a constant, are affected by two foreign variables. Those are the depreciation rate of the other country and the spread in slopes of the other country and the US – variables that one would not be able to identify in a bilateral model of exchange rates.[6]

These two foreign variables appear in srp_t^n as well. That is the driving force behind the strong correlation between es and srp evident in Figure 5. The loadings on these two variables are approximately the same for both ingredients of the IRD (they must be the same for $n = 1$ and $n = 60$ because of the assumptions required to identify the model, see section 5.1).

As a result of the similarities, the difference $es_t^n - srp_t^n$ from equation (2) is a linear combination of $\Delta_c y_t^1$ and $\Delta_c y_t^{60,1}$ of the respective country. That is intuitive because the difference is equal to $\Delta_c y_t^n$, up to a constant, and exchange rates are not spanned by yields. So Δs_t should not be affecting this variable. The US factors y_t^1 and $y_t^{60,1}$ are common to all bonds and should roughly cancel out in the difference $\Delta_c y_t^n$. Finally, the Euro variables should have a small impact on the UK variables and vice versa.

The aforementioned cyclical differences between the curves is driven not only by $\Delta_c y_t^1$ – a traditional variable used to gauge currency risk premiums. The long term $\Delta_c y_t^{60,1}$ plays a role as well. This conclusion highlights the advantage of bringing in information from bonds to speak to the properties of currencies.

Using currencies is useful to speak to bond properties as well. Because there are no common variables correlated with es_t^n and $\Delta_c y_t^n$ in a significant fashion, it would have been impossible to decompose $\Delta_c y_t^n$ into the FX-related components had we studied bonds only. More generally, this is a manifestation of bonds' inability to span currencies.

5.4.2 Differences in risk premiums

Currency risk premiums, srp_t^n, are at the center of bond risk premium differentials as well, equation (3). Most of the research in international bond risk premiums does not exploit that connection to currencies. The focus in this literature is largely on the extent to which the risk premiums are influenced by global vs local factors. Usually the US variables stand in for global variables. The research settings vary from global only (Ilmanen, 1995) to a mix of global and local (Dahlquist and Hasseltoft, 2013) to local only (Brooks and Moskowitz, 2017). Driessen, Melenberg, and Nijman (2003) look for a common structure in bond returns thereby using a mix of the US and non-US variables. Ang and Chen (2010) use bond-inspired factors that go beyond the IRDs of UIP regressions to predict currency premiums.

[6] While this evidence is based on a modest cross-section of only two countries, it is sufficiently promising to be pursued in future research.

Our estimated no-arbitrage model implies how srp_t^n and $E_t \Delta_c rx_{t+1}^n$ relate to the state variables. Thus, we can discuss these interrelationships within the model. Thus, our approach is similar to that of Driessen, Melenberg, and Nijman (2003) as our model allows a UK variable to affect Euro objects and vice-versa. Our approach extends that of Ang and Chen (2010) as we rely on an explicitly estimated model to select the candidate forecasting variables. Further, we study the cross-country differences between the bond risk premiums. That allows us to amplify the variables that are relevant for predicting foreign bond returns beyond what is needed for the US ones.

Table 9 shows how srp_t^n and $E_t \Delta_c rx_{t+1}^n$ load on the state variables in the model. We display only those elements of x_t that have at least one significant loading. We also show loadings for $E_t srp_{t+1}^n$ as it is one of the building blocks for $E_t \Delta_c rx_{t+1}^n$.

The key lesson from this table is that, despite using the same model to derive these quantities, a different set of factors could be important even for closely related objects. First, moving from srp_t^n to $E_t srp_{t+1}^n$ we retain significance only in the variables of the respective country: one-period IRD and difference in slopes. The importance of the IRD is, of course, in line with the UIP literature. The difference in slopes is significant at longer maturities.

Second, we combine currency risk premiums of different maturities to obtain the bond premiums' differential via equation (3). As a result, we gain a new significant variable – the level of interest rates. Methodologically, this is possible because the original set of considered variables, x_t, is the same. It is just that different linear combinations of these variables appearing in srp_t^n vs $E_t \Delta_c rx_{t+1}^n$ lead to varying significance levels.

Economically, the currency and bond risk premiums are related to different variables. The difference between the US and foreign bond premiums is, in particular, related to the "global" factor, that is, the level of US interest rates. None of these conclusions would have emerged if one studied currency or bond premiums separately.

6 Conclusion

We connect differences in international yield curves to exchange rates. In order to account for these risks, we combine estimation of yield curves with estimation of exchange rate dynamics. This exercise is unnecessary if bonds span exchange rates. We find drastic differences in results relative to a benchmark model estimated without exchange rates. Both models fit yields accurately, but the benchmark implies exchange rates that are grossly incompatible with observed behavior. Besides capturing realistic behavior of exchange rates, our main model speaks to the sources of the differences between the US and foreign yield curves, and their respective bond risk premiums.

Both differences are driven by currency risk premiums, and our model suggests factors that are correlated with these premiums. Specifically, the currency risk premium is not only affected by the interest rate differential that appears in UIP regressions. Among other variables, it is affected by the difference in the US and foreign 5-year term spreads. That exposure to long-term interest rates drives cyclical patterns in the yield-curve differences. The differences in bond risk premiums are not only affected by "local" variables, that is, departures of foreign variables from their US counterparts. The differences are also correlated with the US short interest rate suggesting a global importance of the US markets.

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Appendix A Bond prices

Appendix A.1 U.S. bonds

The price of a one month bond is

$$Q_t^1 \;=\; E_t\left[\exp\left(m_{t,t+1}\right)\right] \;=\; \exp\left(\bar{a}_1 + \bar{b}_{1,x}^\top x_t\right)$$

where $\bar{a}_1 \;=\; -\delta_{i,0}$ and $\bar{b}_{1,x} \;=\; -\delta_{i,x}$. The U.S. short rate is

$$i_t \;=\; \delta_{i,0} + \delta_{i,x}^\top x_t$$

The price of an n-period U.S. bond is

$$\begin{aligned}
Q_t^n \;&=\; E_t\left[\exp\left(m_{t,t+1}\right) Q_{t+1}^{n-1}\right] \;=\; E_t\left[\exp\left(-\delta_{i,0} - \delta_{i,x}^\top x_t - \frac{1}{2}\lambda_t^\top\lambda_t - \lambda_t^\top\varepsilon_{t+1} + \bar{a}_{n-1} + \bar{b}_{n-1,x}^\top x_{t+1}\right)\right] \\
&=\; \exp\left(\bar{a}_{n-1} - \delta_{i,0} - \delta_{i,x}^\top x_t - \frac{1}{2}\lambda_t^\top\lambda_t\right) E_t\left[\exp\left(-\lambda_t^\top\varepsilon_{t+1} + \bar{b}_{n-1,x}^\top\left[\mu_x + \Phi_x x_t + \Sigma_x\varepsilon_{t+1}\right]\right)\right] \\
&=\; \exp\left(\bar{a}_{n-1} - \delta_{i,0} - \delta_{i,x}^\top x_t + \bar{b}_{n-1,x}^\top\left[\mu_x + \Phi_x x_t\right] - \lambda_t^\top\Sigma_x^\top\bar{b}_{n-1,x} + \frac{1}{2}\bar{b}_{n-1,x}^\top\Sigma_x\Sigma_x^\top\bar{b}_{n-1,x}\right) \\
&=\; \exp\left(\bar{a}_n + \bar{b}_{n,x}^\top x_t\right)
\end{aligned}$$

where the loadings are

$$\begin{aligned}
\bar{a}_n \;&=\; \bar{a}_{n-1} - \delta_{i,0} + \bar{b}_{n-1,x}^\top\left(\mu_x - \lambda_\mu\right) + \frac{1}{2}\bar{b}_{n-1,x}^\top\Sigma_x\Sigma_x^\top\bar{b}_{n-1,x} \\
\bar{b}_{n,x} \;&=\; \left(\Phi_x - \lambda_\phi\right)^\top\bar{b}_{n-1,x} - \delta_{i,x}
\end{aligned}$$

We can write this in terms of the U.S. risk neutral parameters

$$\begin{aligned}
\bar{a}_n \;&=\; \bar{a}_{n-1} - \delta_{i,0} + \bar{b}_{n-1,x}^\top\mu_x^* + \frac{1}{2}\bar{b}_{n-1,x}^\top\Sigma_x\Sigma_x^\top\bar{b}_{n-1,x} \\
\bar{b}_{n,x} \;&=\; \Phi_x^{*,\top}\bar{b}_{n-1,x} - \delta_{i,x}
\end{aligned}$$

U.S. yields are $y_t = a_n + b_{n,x}^\top x_t$ with $a_n = -n^{-1}\bar{a}_n$ and $b_{n,x} = -n^{-1}\bar{b}_{n,x}$.

Appendix A.2 Foreign bonds in the NFX model

The price of a one month bond is

$$\widehat{Q}_t^1 \;=\; E_t\left[\exp\left(\widehat{m}_{t,t+1}\right)\right] \;=\; \exp\left(\widehat{\bar{a}}_1^N + \widehat{\bar{b}}_{1,x}^{N,\top} x_t\right)$$

where $\widehat{\bar{a}}_1^N \;=\; -\widehat{\delta}_{i,0}$ and $\widehat{\bar{b}}_{1,x}^N \;=\; -\widehat{\delta}_{i,x}$. The foreign short rate is

$$\widehat{i}_t \;=\; \widehat{\delta}_{i,0} + \widehat{\delta}_{i,x}^\top x_t$$

Using the same calculations as above, the price of an n-period foreign bond in the NFX model is

$$\begin{aligned}
\widehat{Q}_t^n \;&=\; E_t\left[\exp\left(\widehat{m}_{t,t+1}\right)\widehat{Q}_{t+1}^{n-1}\right] \;=\; E_t\left[\exp\left(-\widehat{\delta}_{i,0} - \widehat{\delta}_{i,x}^\top x_t - \frac{1}{2}\widehat{\lambda}_t^\top\widehat{\lambda}_t - \widehat{\lambda}_t^\top\varepsilon_{t+1} + \widehat{\bar{a}}_{n-1}^N + \widehat{\bar{b}}_{n-1,x}^{N,\top} x_{t+1}\right)\right] \\
&=\; \exp\left(\widehat{\bar{a}}_{n-1}^N - \widehat{\delta}_{i,0} - \widehat{\delta}_{i,x}^\top x_t + \widehat{\bar{b}}_{n-1,x}^{N,\top}\left[\mu_x + \Phi_x x_t\right] - \widehat{\lambda}_t^\top\Sigma_x^\top\widehat{\bar{b}}_{n-1,x}^N + \frac{1}{2}\widehat{\bar{b}}_{n-1,x}^{N,\top}\Sigma_x\Sigma_x^\top\widehat{\bar{b}}_{n-1,x}^N\right) \\
&=\; \exp\left(\widehat{\bar{a}}_n^N + \widehat{\bar{b}}_{n,x}^{N,\top} x_t\right)
\end{aligned}$$

where the loadings are

$$
\begin{aligned}
\widehat{a}_n^N &= \widehat{a}_{n-1}^N - \widehat{\delta}_{i,0} + \widehat{b}_{n-1,x}^{N,\top}\left(\mu_x - \widehat{\lambda}_\mu\right) + \frac{1}{2}\widehat{b}_{n-1,x}^{N,\top}\Sigma_x\Sigma_x^\top \widehat{b}_{n-1,x}^N \\
\widehat{b}_{n,x}^N &= \left(\Phi_x - \widehat{\lambda}_\phi\right)^\top \widehat{b}_{n-1,x}^N - \widehat{\delta}_{i,x}
\end{aligned}
$$

We can write this in terms of the foreign risk neutral parameters

$$
\begin{aligned}
\widehat{a}_n^N &= \widehat{a}_{n-1}^N - \widehat{\delta}_{i,0} + \widehat{b}_{n-1,x}^{N,\top}\widehat{\mu}_x^* + \frac{1}{2}\widehat{b}_{n-1,x}^{N,\top}\Sigma_x\Sigma_x^\top \widehat{b}_{n-1,x}^N \\
\widehat{b}_{n,x}^N &= \widehat{\Phi}_x^{*,\top}\widehat{b}_{n-1,x}^N - \widehat{\delta}_{i,x}
\end{aligned}
$$

Foreign yields are $\widehat{y}_t = \widehat{a}_n^N + \widehat{b}_{n,x}^{N,\top} x_t$ with $\widehat{a}_n^N = -n^{-1}\widehat{a}_n^N$ and $\widehat{b}_{n,x}^N = -n^{-1}\widehat{b}_{n,x}^N$.

Appendix A.3 Foreign bonds in the WFX model

The price of a one month bond is

$$
\widehat{Q}_t^1 = E_t\left[\exp\left(m_{t,t+1} + \Delta s_{t+1}\right)\right] = \exp\left(\widehat{a}_1^W + \widehat{b}_{1,x}^{W,\top} x_t\right)
$$

where $\widehat{a}_1^W = \delta_{s,0} - \delta_{i,0} + \delta_{s,x}^\top\left(\mu_x - \lambda_\mu\right) + \frac{1}{2}\delta_{s,x}^\top\Sigma_x\Sigma_x^\top\delta_{s,x}$ and $\widehat{b}_{1,x}^W = \left(\Phi_x - \lambda_\phi\right)^\top\delta_{s,x} - \delta_{i,x}$. Using the same calculations as above, the price of an n-period foreign bond in the WFX model is

$$
\begin{aligned}
\widehat{Q}_t^n &= E_t\left[\exp\left(m_{t,t+1} + \Delta s_{t+1}\right)\widehat{Q}_{t+1}^{n-1}\right] \\
&= \exp\left(\widehat{a}_{n-1}^W + \delta_{s,0} - \delta_{i,0} - \delta_{i,x}^\top x_t + \left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\left[\mu_x + \Phi_x x_t\right]\right) \\
&\quad \exp\left(-\lambda_t^\top\Sigma_x^\top\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right) + \frac{1}{2}\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\Sigma_x\Sigma_x^\top\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)\right) \\
&= \exp\left(\widehat{a}_n^W + \widehat{b}_{n,x}^{W,\top} x_t\right)
\end{aligned}
$$

where the loadings are

$$
\begin{aligned}
\widehat{a}_n^W &= \widehat{a}_{n-1}^W - \delta_{i,0} + \left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\left(\mu_x - \lambda_\mu\right) + \frac{1}{2}\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\Sigma_x\Sigma_x^\top\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right) \\
\widehat{b}_{n,x}^W &= \left(\Phi_x - \lambda_\phi\right)^\top\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right) - \delta_{i,x}
\end{aligned}
$$

We can write this in terms of the U.S. risk neutral parameters

$$
\begin{aligned}
\widehat{a}_n^W &= \widehat{a}_{n-1}^W - \delta_{i,0} + \left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\mu_x^* + \frac{1}{2}\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right)^\top\Sigma_x\Sigma_x^\top\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right) \\
\widehat{b}_{n,x}^W &= \Phi_x^{*,\top}\left(\widehat{b}_{n-1,x}^W + \delta_{s,x}\right) - \delta_{i,x}
\end{aligned}
$$

Foreign yields are $\widehat{y}_t = \widehat{a}_n^W + \widehat{b}_{n,x}^{W,\top} x_t$ with $\widehat{a}_n^W = -n^{-1}\widehat{a}_n^W$ and $\widehat{b}_{n,x}^W = -n^{-1}\widehat{b}_{n,x}^W$.

Appendix B Estimation

Appendix B.1 Parameterization and identification

In the main text, we report estimates of the market price of risk parameters λ_0 and λ_x that enter the stochastic discount factor. These parameters are defined in terms of the observable state vector x_t as defined in the text. In practice, these parameters λ_0 and λ_x require identifying restrictions that are not easy to impose directly. The term structure literature solves the problem of imposing the necessary identifying restrictions by parameterizing the model in terms of the identifiable risk neutral parameters under a latent factor rotation. Under the latent factor rotation, the restrictions are easy to impose. We follow this literature and explain how to impose these restrictions in this appendix.

First, we note that the risk neutral parameters μ_x^* and Φ_x^* of the observable state vector x_t are related to the market prices of risk as

$$\begin{aligned} \mu_x^* &= \mu_x - \lambda_0 \\ \Phi_x^* &= \Phi_x - \lambda_x \end{aligned}$$

where μ_x and Φ_x are the drift and autocovariance of x_t under the "real-world" probabilities.

In this appendix, we use a 'tilde' to denote any parameters $\tilde{\theta}$ or state variables $\tilde{x}_t$ under the latent factor rotation. In our implementation, we make one minor change relative to the term structure literature. We define $\tilde{x}_t$ such that the first two elements are the observed depreciation rates while the remaining yield factors are latent, i.e. we have

$$\tilde{x}_t = \begin{pmatrix} \Delta s_t \\ \tilde{g}_t \end{pmatrix}$$

where $\tilde{g}_t$ are latent yield factors. Therefore, when we rotate from $\tilde{x}_t$ to x_t, the first two elements of the state remain the same.

With this definition of $\tilde{x}_t$ in hand, the observed factors x_t are related to the latent factors $\tilde{x}_t$ through the linear transformation

$$x_t = \Gamma_0 + \Gamma_1 \tilde{x}_t$$

where the vector Γ_0 and matrix Γ_1 are described below in Appendix B.3.

The risk neutral dynamics under the latent factor rotation are

$$\begin{aligned} \Delta s_t &= \tilde{\delta}_{s,0} + \tilde{\delta}_{s,x}^{\top} \tilde{x}_t \\ i_t &= \tilde{\delta}_{i,0} + \tilde{\delta}_{i,x}^{\top} \tilde{x}_t \\ \tilde{x}_t &= \tilde{\mu}_x^* + \tilde{\Phi}_x^* \tilde{x}_{t-1} + \tilde{\Sigma}_x \varepsilon_t^* \end{aligned}$$

Under this rotation, the identifying restrictions require imposing the following restrictions

$$\begin{aligned} \tilde{\delta}_{s,0} &= 0 \\ \tilde{\delta}_{s,x} &= \mathrm{e}_i \qquad i = 1,2 \\ \tilde{\delta}_{i,x} &= \begin{pmatrix} \tilde{\delta}_{i,s} \\ \iota \end{pmatrix} \\ \tilde{\mu}_x^* &= \begin{pmatrix} \tilde{\mu}_s^* \\ \tilde{\mu}_g^* \end{pmatrix} = \begin{pmatrix} \tilde{\mu}_s^* \\ 0 \end{pmatrix} \\ \tilde{\Phi}_x^* &= \begin{pmatrix} \tilde{\Phi}_s^* & \tilde{\Phi}_{sg}^* \\ \tilde{\Phi}_{gs}^* & \tilde{\Phi}_g^* \end{pmatrix} = \begin{pmatrix} \tilde{\Phi}_s^* & \tilde{\Phi}_{sg}^* \\ 0 & \tilde{\Phi}_g^* \end{pmatrix} \end{aligned}$$

In addition, the matrix $\tilde{\Phi}_g^*$ is restricted to be a matrix of eigenvalues. In general, the eigenvalues may be distinct and real, complex, or repeated. We follow the standard approach in the term structure literature and assume that this matrix is diagonal with distinct, real eigenvalues ordered from largest to smallest.

For the loadings on the U.S. short rate $\tilde{\delta}_{i,x}$, the first 2 elements associated with depreciation rate are estimable while the remaining loadings are restricted to be a vector of ones ι.

Appendix B.2 Observables

We stack the U.S. and foreign nominal yields of different maturities into vectors $y_t = \left(y_t^1, \ldots, y_t^{60}\right)$ and $\widehat{y}_t = \left(\widehat{y}_t^1, \ldots, \widehat{y}_t^{60}\right)$ as well as their bond loadings, e.g. $A = (a_1, \ldots, a_{60})^\top, B = (b_{1,x}, \ldots, b_{60,x})^\top$ and $\widehat{A} = (\widehat{a}_1, \ldots, \widehat{a}_{60})^\top, \widehat{B} = \left(\widehat{b}_{1,x}, \ldots, \widehat{b}_{60,x}\right)^\top$.

The system of observation equations used in the model are

$$\begin{aligned} \Delta s_t &= \delta_{s,0} + \delta_{s,x} x_t \\ y_t &= A + B x_t \\ \widehat{y}_t &= \widehat{A} + \widehat{B} x_t \end{aligned}$$

where, in our application, the vector $\widehat{y}_t$ includes both Euro and U.K. yields. We define the overall vector of observables as

$$Y_t = \begin{pmatrix} \Delta s_t \\ y_t \\ \widehat{y}_t \end{pmatrix}$$

Next, we define two selection matrices W_1 and W_2 that select out of Y_t linear combinations of depreciation rates and yields. Together, the matrices $\left(W_1^\top; W_2^\top\right)^\top$ must be full rank. These linear combinations are

$$\begin{aligned} Y_t^{(1)} &= W_1 Y_t \\ Y_t^{(2)} &= W_2 Y_t \end{aligned}$$

The vector $Y_t^{(1)}$ is a linear combination of observables that we assume to be measured without error. The vector $Y_t^{(2)}$ is observed with error.

Appendix B.3 Rotating the state vector to observables

In our implementation, we choose W_1 so that the state vector is $x_t = Y_t^1 = W_1 Y_t$ with x_t defined as in the text

$$x_t = \begin{pmatrix} \Delta s_t^{€} \\ \Delta s_t^{£} \\ y_t^1 \\ y_t^1 - \widehat{y}_t^{€,1} \\ y_t^1 - \widehat{y}_t^{£,1} \\ y_t^{60,1} \\ y_t^{60,1} - \widehat{y}_t^{€,60,1} \\ y_t^{60,1} - \widehat{y}_t^{£,60,1} \end{pmatrix}$$

In order for x_t to have exactly this definition, we follow the term structure literature and assume that this linear combination of the observables Y_t is observed without measurement error.

The matrix W_2 is a selection matrix full of zeros and ones that selects unique linear combinations out of Y_t that are not used in x_t. Specifically, W_2 is defined such that the second set of linear combinations $Y_t^{(2)} = W_2 Y_t$ includes the 12, 24, 36, and 48 month U.S. yields as well as the 12, 24, 36, and 48 month foreign yields (both Euro and U.K.).

To implement this rotation in practice, we note that the observables Y_t are related to the latent state vector $\tilde{x}_t$ as

$$Y_t = \begin{pmatrix} \Delta s_t \\ y_t \\ \widehat{y}_t \end{pmatrix} = \begin{pmatrix} \tilde{\delta}_{s,0} \\ \tilde{A} \\ \tilde{\widehat{A}} \end{pmatrix} + \begin{pmatrix} \tilde{\delta}_{s,x}^{\top} \\ \tilde{B} \\ \tilde{\widehat{B}} \end{pmatrix} \tilde{x}_t = \tilde{C} + \tilde{D}\tilde{x}_t$$

where the vector $\tilde{C}$ and matrix $\tilde{D}$ are appropriately defined. In this case, the bond loadings $\tilde{A}, \tilde{\widehat{A}}, \tilde{B}, \tilde{\widehat{B}}$ are calculated under the latent factor rotation. Next, we pre-multiply Y_t above by W_1 and substitute in for the observed state variables

$$\begin{aligned} W_1 Y_t &= W_1\tilde{C} + W_1\tilde{D}\tilde{x}_t \\ &= W_1\tilde{C} + W_1\tilde{D}\Gamma_1^{-1}(x_t - \Gamma_0) \\ &= W_1\left(\tilde{C} - \tilde{D}\Gamma_1^{-1}\Gamma_0\right) + W_1\tilde{D}\Gamma_1^{-1}x_t. \end{aligned}$$

In order for $x_t = W_1 Y_t$, it implies that the rotation matrices Γ_0 and Γ_1 must satisfy the restrictions

$$\begin{aligned} \Gamma_0 &= W_1\tilde{C} \\ \Gamma_1 &= W_1\tilde{D} \end{aligned}$$

Given these matrices, we can map between the parameters of the observable rotation of the state vector x_t and the latent factor rotation $\tilde{x}_t$ through the transformations

$$\begin{aligned} \Phi_x^* &= \Gamma_1\tilde{\Phi}_x^*\Gamma_1^{-1} \\ \mu_x^* &= (I - \Phi_x^*)\,\Gamma_0 + \Gamma_1\tilde{\mu}_x^* \end{aligned}$$

Appendix B.4 Prior distributions

- Let $S_y = \Sigma_y\Sigma_y^{\top}$ with dimension $d_{y_2} \times d_{y_2}$. Note that $Y_t^{(2)}$ has dimension $d_{y_2} \times 1$. We assume S_y has a diffuse inverse Wishart distribution $S_y \sim \text{Inv-W}\left(\underline{\Omega}_y, \underline{\nu}_y\right)$ with degrees of freedom $\underline{\nu}_y = 0$ and scale matrix $\underline{\Omega}_y = 0$.
- The matrix Σ_x is lower triangular. We place inverse Gamma $\sigma_i^2 \sim \text{IG}(\alpha_i, \beta_i)$ on each of the diagonal elements where $i = s, g$. The subscript s stands for depreciation rate while the subscript g stands for yield factor. We set $\alpha_s = 3.3$ and $\beta_s = 0.0015$. We set $\alpha_g = 3.05$ and $\beta_g = 8e^{-8}$.
- We place a prior on the free parameters of the unconditional means $\left(\bar{\mu}_x, \bar{\tilde{\mu}}_x^*\right)$ directly instead of the drifts $(\mu_x, \tilde{\mu}_x^*)$. First, we calculate the unconditional sample mean of the factors $\hat{\bar{\mu}}_x$. Our prior for each element of $\bar{\mu}_x$ is a normal distribution centered at the sample mean. Then, we choose the variance of this distribution to be large enough to cover the support of the data. Our priors are
  - depreciation rates: $\bar{\mu}_s \sim \text{N}\left(\hat{\bar{\mu}}_s, 0.0003\right)$
  - U.S. level: $\bar{\mu}_i \sim \text{N}\left(\hat{\bar{\mu}}_i, 0.000003\right)$
  - Foreign Level differentials: $\bar{\mu}_{\Delta_c i} \sim \text{N}\left(\hat{\bar{\mu}}_{\Delta_c i}, 0.000003\right)$
  - U.S. slope: $\bar{\mu}_{sl} \sim \text{N}\left(\hat{\bar{\mu}}_{sl}, 0.000003\right)$
  - Foreign slope differential: $\bar{\mu}_{\Delta_c sl} \sim \text{N}\left(\hat{\bar{\mu}}_{\Delta_c sl}, 0.000003\right)$

Identifying restrictions on $\bar{\tilde{\mu}}_x^*$ only allow for only 2 free parameters, which are the parameters associated with the depreciation rates (i.e. the first two factors in $\tilde{x}_t$). Our prior for this variable is the same as the unconditional depreciation rate under the real world probability but multiplied by a factor of 100.

  - depreciation rates: $\bar{\tilde{\mu}}_s \sim \mathrm{N}\left(\hat{\tilde{\mu}}_s, 0.03\right)$

- We parameterize the matrix $\tilde{\Phi}_x^*$ as

$$\tilde{\Phi}_x^* = \begin{pmatrix} \tilde{\Phi}_s^* & \tilde{\Phi}_{sg}^* \\ 0 & \tilde{\Phi}_g^* \end{pmatrix}$$

  Our priors on the sub-matrices are as follows:

  - $\tilde{\Phi}_g^*$ is a diagonal matrix of real, ordered eigenvalues. Let $a_1 = -1$ and $b = 1$. We parameterize them as $\tilde{\Phi}_{g,11}^* = a_1 + (b - a_1)U_1$ and $\tilde{\Phi}_{g,jj}^* = a_{j-1} + (b - a_{j-1})U_j$ for $j = 2, \ldots, d_g$. This transformation ensures that they are increasing and contained in the interval $[-1, 1]$. We then place priors on $\tilde{\Phi}_{g,jj}^*$ via $U_j \sim \mathrm{Beta}\,(12, 12)$.
  - We place the same prior on $\tilde{\Phi}_s^*$ and $\tilde{\Phi}_{sg}^*$ as under the risk neutral dynamics but where the covariance matrix is multiplied by a factor of 100.

- We separate $\tilde{\delta}_{i,x}$ into two sub-vectors $\tilde{\delta}_{i,s}$ and $\tilde{\delta}_{i,g}$. We place a prior on the free parameters of the factor loadings $\tilde{\delta}_{i,s}$. Our identifying restriction is that $\tilde{\delta}_{i,g} = \iota$. The parameters of $\tilde{\delta}_{i,s}$ are estimable and we assume that each entry is independent and distributed as $\tilde{\delta}_{i,g} \sim \mathrm{N}\,(0, 0.01)$.

Appendix B.5 Log-likelihood function

The log-likelihood function is

$$\mathcal{L} = \log p\,(Y_1, \ldots, Y_T|\theta) = \sum_{t=1}^{T} \log p\,(x_t|x_{t-1}, \theta) + \sum_{t=1}^{T} \log p\left(Y_t^{(2)}|x_t; \theta\right)$$

where x_0 are assumed to be known. The density $p\,(x_t|x_{t-1}; \theta)$ is determined by the VAR dynamics of the factors x_t while the second term comes from the linear combination of yields observed with error

$$Y_t^{(2)} = C^{(2)} + D^{(2)}x_t + \Sigma_y \eta_t, \qquad \eta_t \sim \mathrm{N}\,(0, \mathrm{I}),$$

where $C^{(2)} = W_2 C$ and $D^{(2)} = W_2 D$ and

$$\begin{aligned} C &= \tilde{C} - \tilde{C}\Gamma_1^{-1}\Gamma_0, \\ D &= \tilde{D}\Gamma_1^{-1}. \end{aligned}$$

This likelihood function assumes that there are no missing values in either $Y_t^{(1)}$ or $Y_t^{(2)}$. In practice, this is not the case. We impute these missing values during the MCMC algorithm using the Kalman filter.

Appendix B.6 Estimation

Let θ denote all the parameters of the model and define $f_{1:T} = (f_1, \ldots, f_T)$ and $Y_{1:T} = (Y_1, \ldots, Y_T)$. In practice, some data points are missing which implies that some of the factors f_t are missing. We use $Y_{1:T}^o$ and $Y_{1:T}^m$ to denote the observed and missing data, respectively. The joint posterior distribution over the parameters and missing data is given by

$$p\,(\theta, Y_{1:T}^m|Y_{1:T}^o) \quad \propto \quad p\,(Y_{1:T}^o|\theta)\, p\,(\theta)\,,$$

where $p\,(Y_{1:T}^o|\theta)$ is the likelihood and $p\,(\theta)$ is the prior distribution. We use Markov-chain Monte Carlo to draw from the posterior.

Appendix B.6.1 MCMC algorithm

We provide a brief description of the MCMC algorithm. Let $S_y = \Sigma_y \Sigma_y'$ and $S_x = \Sigma_x \Sigma_x'$ denote the covariance matrices. We use a Gibbs sampler that iterates between drawing from each of the full conditional distributions.

- Place the model in linear, Gaussian state space form as described in Appendix B.6.2. Draw the missing data and unconditional means $(Y_{1:T}^m, \bar{\mu}_x, \bar{\mu}_x^*)$ from their full conditional distribution using the Kalman filter and simulation smoothing algorithm. Given the full data $Y_t^{o,m} = (Y_t^o, Y_t^m)$, we can recalculate the factors $x_t = W_1 Y_t^{o,m}$.
- Let $\bar{x}_t = x_t - \bar{\mu}_x$ denote the demeaned factors. We draw the free elements of Φ_x from their full conditional distribution using standard results for Bayesian multiple regression. We write the VAR as a regression model
$$\bar{x}_t = X_t \phi_x + \Sigma_x \varepsilon_t$$
where $\phi_x = \text{vec}(\Phi_x)$ and the regressors X_t contain lagged values of $\bar{x}_{t-1}$. Draws from this model are standard.
- Draw the free elements of S_x from their full conditional using a random-walk Metropolis algorithm. In this step, we avoid conditioning on the parameters S_y, Φ_x by analytically integrating these parameters out of the likelihood.
- Draw the eigenvalues Λ_x^* from their full conditional using random-walk Metropolis. To avoid conditioning on S_y, Φ_x, we draw from the marginal distribution that analytically integrates these values out of the likelihood.
- Draw the elements of $\tilde{\delta}_{s,x}$ from their full conditional using random-walk Metropolis. To avoid conditioning on S_y, Φ_x, we draw from the marginal distribution that analytically integrates these values out of the likelihood.
- The full conditional posterior of S_y is an inverse Wishart distribution $S_y \sim \text{Inv-Wish}\left(\bar{\nu}, \bar{\Omega}\right)$ where $\bar{\nu} = \underline{\nu} + T$ and $\bar{\Omega} = \underline{\Omega} + \sum_{t=1}^{T} \eta_t \eta_t^{\top}$.

Appendix B.6.2 State space form

In our data set, some of the yields contain missing values. We impute them using the Kalman filter. Given that $x_t = Y_t^{(1)}$, we can write the model in VAR form as

$$\begin{pmatrix} Y_t^{(1)} \\ Y_t^{(2)} \end{pmatrix} = \begin{pmatrix} \mu_x \\ A^{(2)} + B^{(2)}\mu_x \end{pmatrix} + \begin{pmatrix} \Phi_x & 0 \\ B^{(2)}\Phi_x & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1}^{(1)} \\ Y_{t-1}^{(2)} \end{pmatrix} + \begin{pmatrix} \Sigma_x & 0 \\ B^{(2)}\Sigma_x & \Sigma_y \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}$$

Next we translate this system back into Y_t using the fact that

$$Y_t = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} Y_t^{(1)} \\ Y_t^{(2)} \end{pmatrix}$$

to get

$$\begin{aligned} Y_t = & \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \mu_x \\ A^{(2)} + B^{(2)}\mu_x \end{pmatrix} + \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \Phi_x & 0 \\ B^{(2)}\Phi_x & 0 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} Y_{t-1} \\ & + \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_x & 0 \\ B^{(2)}\Sigma_x & \Sigma_y \end{pmatrix} \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \end{aligned}$$

This structure implies that Y_t is a reduced-rank VAR of the form

$$Y_t = \mu_Y + \Phi_Y Y_{t-1} + \Sigma_Y \varepsilon_{Y,t} \qquad \varepsilon_{Y,t} \sim \text{N}(0, I)$$

where

$$\mu_Y = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \mu_x \\ A^{(2)} + B^{(2)}\mu_x \end{pmatrix} \qquad \Phi_Y = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \Phi_x & 0 \\ B^{(2)}\Phi_x & 0 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

$$\Sigma_Y = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_x & 0 \\ B^{(2)}\Sigma_x & \Sigma_y \end{pmatrix} \qquad \varepsilon_{Y,t} = \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}$$

We place this model in the following linear, Gaussian state space form

$$Y_t = Z\alpha_t + d + u_t \qquad u_t \sim \mathrm{N}(0, H), \tag{B.1}$$

$$\alpha_{t+1} = T\alpha_t + c + Rv_t \qquad v_t \sim \mathrm{N}(0, Q). \tag{B.2}$$

where the initial condition is $\alpha_1 \sim \mathrm{N}\left(a_{1|0}, P_{1|0}\right)$.

Let $\bar{\mu} = \left(\bar{\mu}_{x,u}^{\top} \; \bar{\mu}_{x,u}^{*,\top} \; \delta_{\widehat{c},0}\right)^{\top}$ denote the vector of unrestricted unconditional means that enter $\bar{\mu}_x$ and $\bar{\mu}_x^*$ plus the intercept $\delta_{\widehat{c},0}$. The vector of intercepts μ_Y can be written as a linear function of the unconditional means

$$\mu_Y = S_{\mu,0} + S_{\mu,1}\bar{\mu}$$

We draw unconditional means jointly with the missing data by including them in the state vector. We define the system matrices from (B.1)-(B.2) as

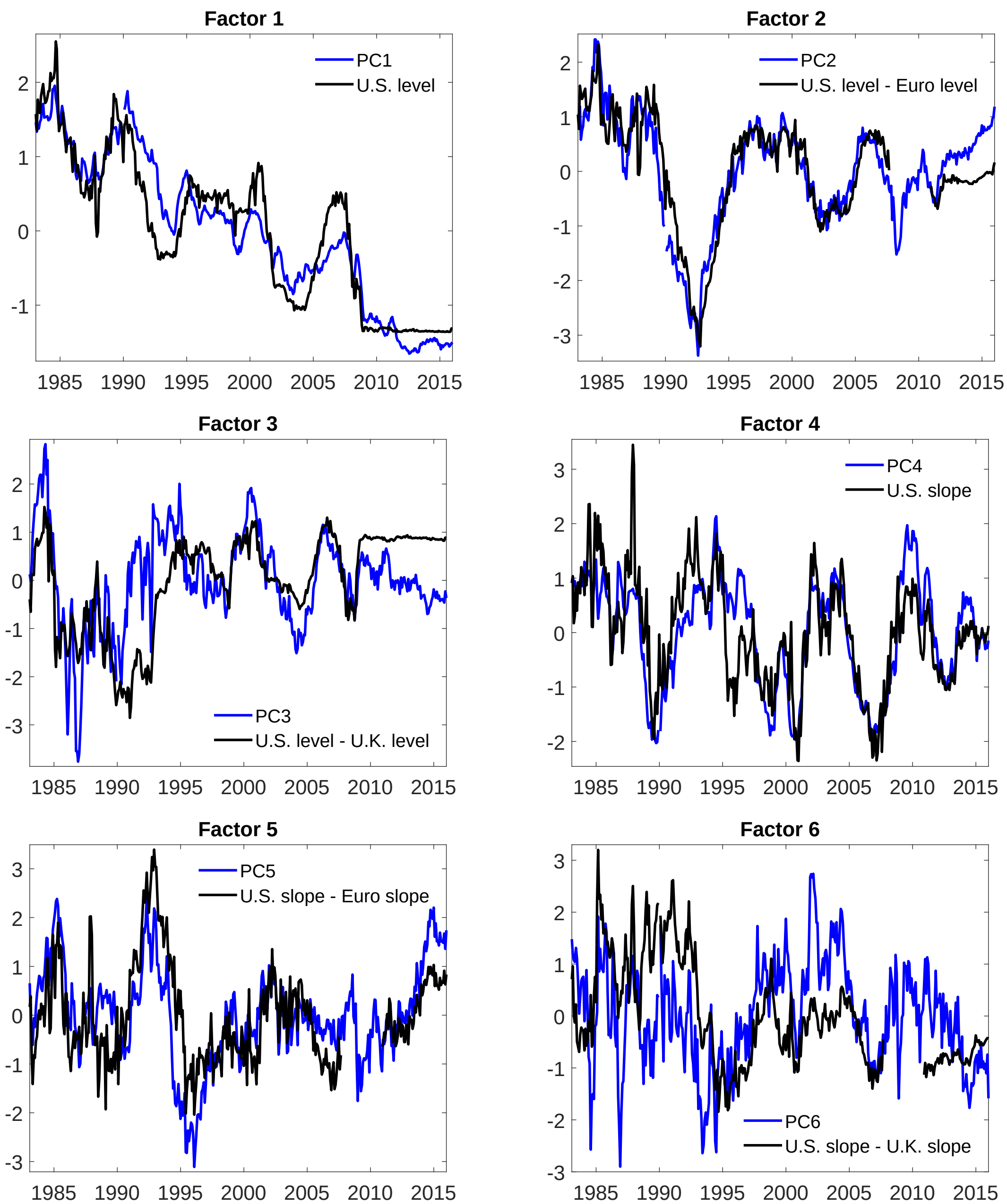
$$d = 0 \qquad Z = \begin{pmatrix} \mathrm{I} & 0 \end{pmatrix} \qquad H = 0 \qquad Q = \Sigma_Y \Sigma_Y^{\top}$$

$$\alpha_t = \begin{pmatrix} Y_t \\ \bar{\mu} \end{pmatrix} \qquad T = \begin{pmatrix} \Phi_Y & S_{\mu,1} \\ 0 & \mathrm{I} \end{pmatrix} \qquad c = \begin{pmatrix} S_{\mu,0} \\ 0 \end{pmatrix} \qquad R = \begin{pmatrix} \mathrm{I} \\ 0 \end{pmatrix}$$

$$a_{1|0} = \begin{pmatrix} S_{\mu,1}\bar{m}_{\mu} \\ \bar{m}_{\mu} \end{pmatrix} \qquad P_{1|0} = \begin{pmatrix} \Sigma_Y\Sigma_Y^{\top} + S_{\mu,1}V_{\mu}S_{\mu,1}^{\top} & S_{\mu,1}V_{\mu} \\ V_{\mu}S_{\mu,1}^{\top} & V_{\mu} \end{pmatrix}$$

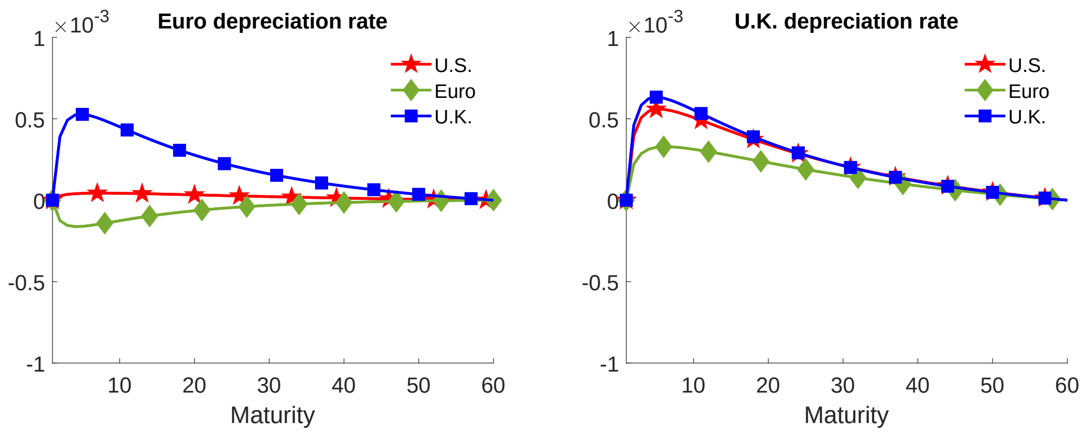
where the prior on the unconditional means is $\bar{\mu} \sim \mathrm{N}(\bar{m}_{\mu}, V_{\mu})$. We use the Kalman filter and simulation smoothing algorithm to draw the missing values and the unconditional means jointly.

Figure 1
Principal components and yield factors



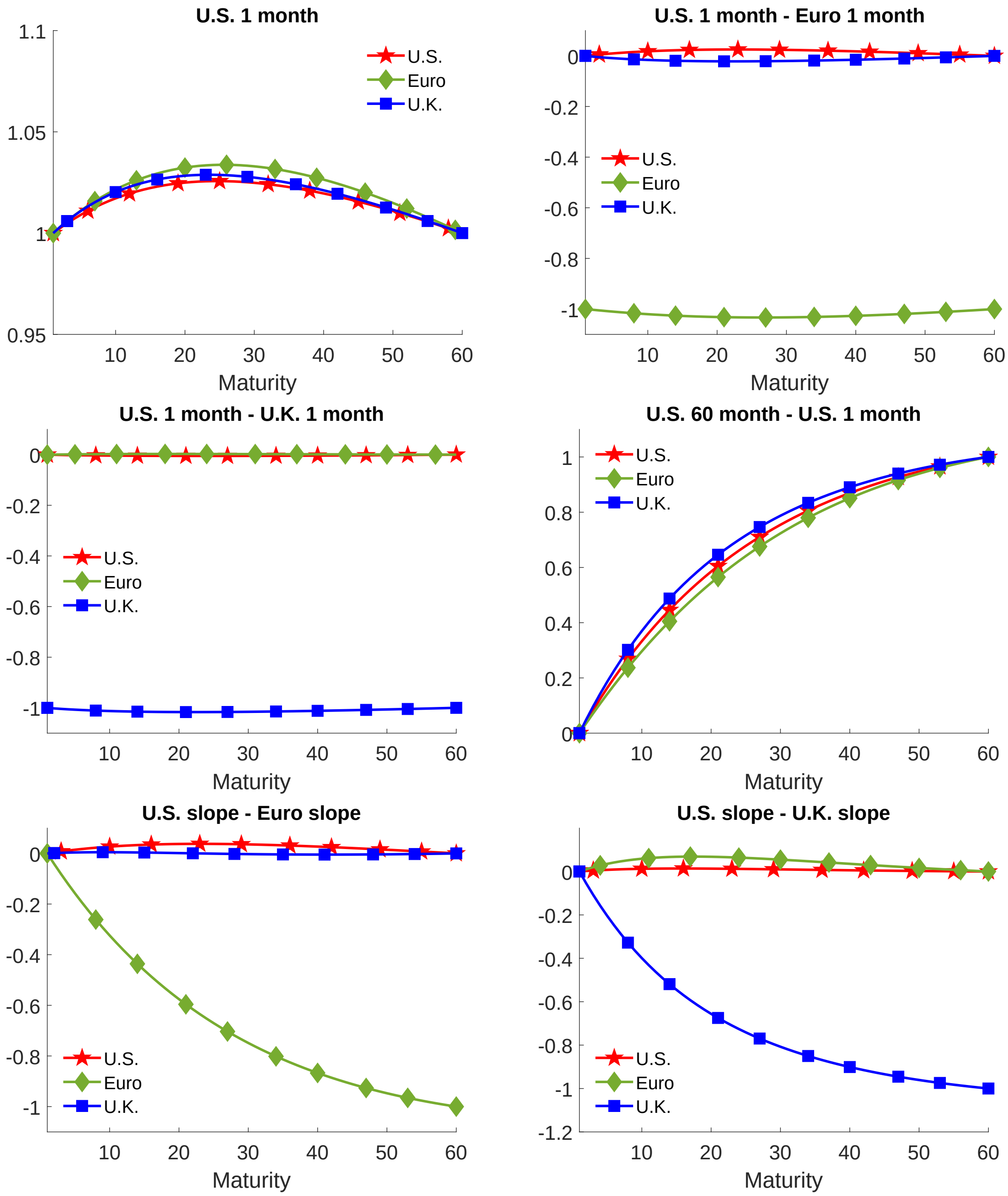
Notes: Plots of the first six principal components (blue) and the yield factors in the state vector x_t. All variables have been standardized to have mean zero and variance one in the plot to make them comparable.

Figure 2
Depreciation rate loadings for the WFX model



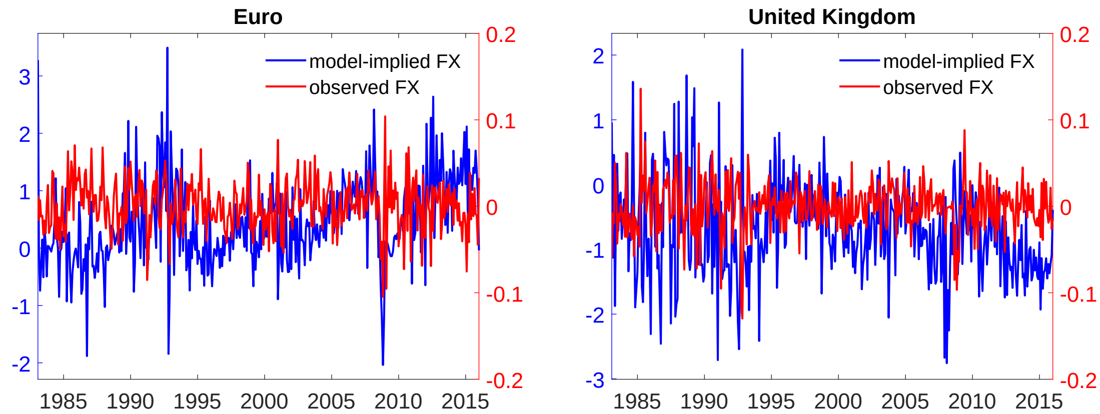
Notes: We plot loadings on depreciation rates that are used by the WFX model to establish U.S. and foreign bond yields for multiple horizons (0-60 month, x-axis). Different lines represent loadings for bonds of different countries. None of the lines are signifciantly different from zero. We do not report confidence intervals to avoid clutter.

Figure 3
Yield factor loadings for the WFX model



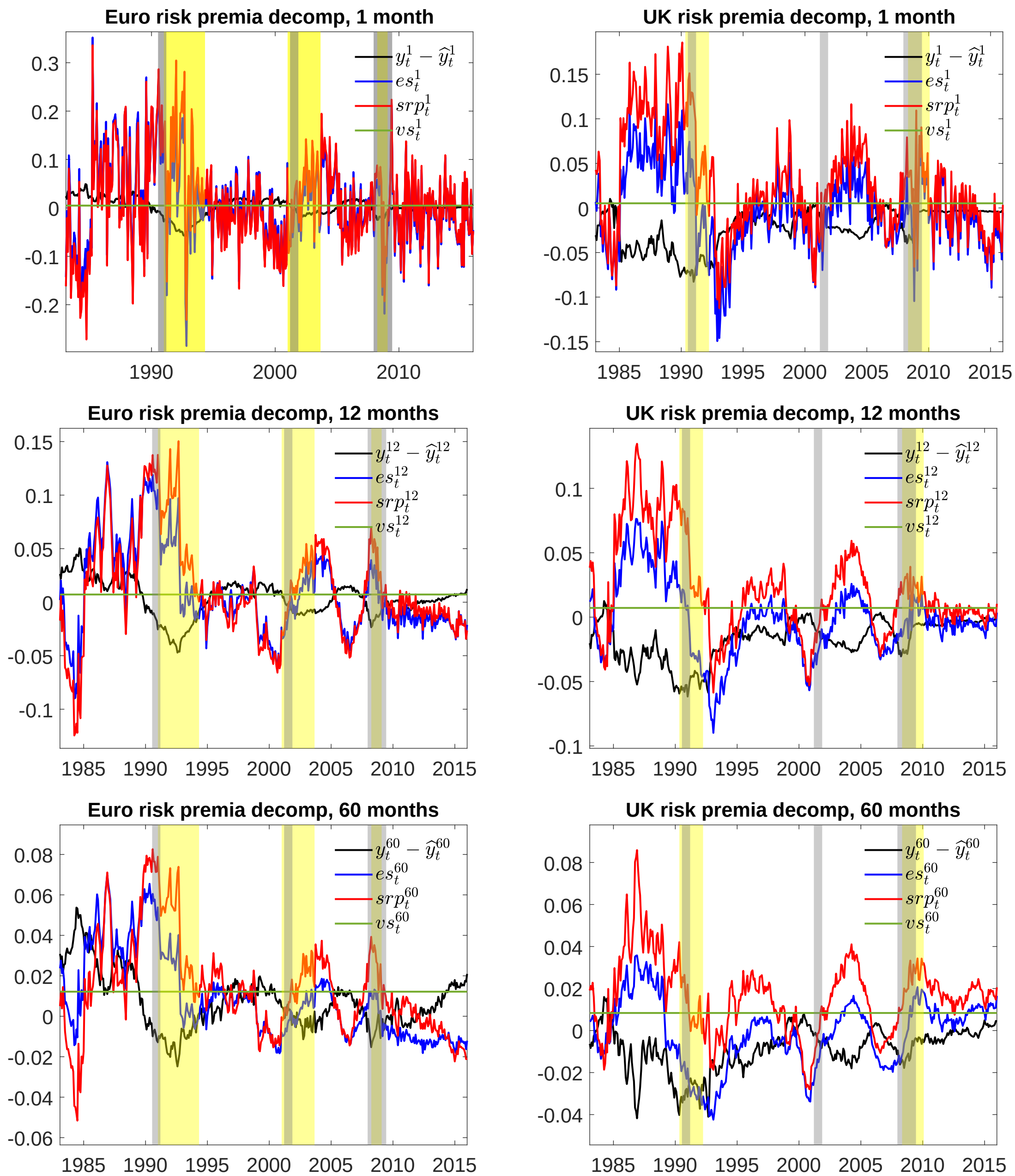
Notes: We plot yield factor loadings that are used by the WFX model to establish U.S. and foreign bond yields for multiple horizons (0-60 month, x-axis). Different lines represent loadings for bonds of different countries.

Figure 4
NFX-implied and observed FX rates



Notes: We plot the depreciation rates $\Delta s_{t+1} = \widehat{m}_{t,t+1}(Q, \widehat{Q}) - m_{t,t+1}(Q, \widehat{Q})$ *implied by the NFX model (blue, vertical axis left) against the observed depreciation rates (red, vertical axis right).*

Figure 5
Decomposition of the time-series of differences in yield curves



Notes: We plot the conditional differences in the yield curves across maturities and its components. All values have been multiplied by 12. Vertical gray bars are U.S. recessions as measured by the NBER. Vertical yellow bars are German and U.K. recessions.

Table 1: Principal components

PC's	6 PCs (US + €)	6 PCs (US + £)	9 PCs (all countries)
1	91.5142	96.1800	93.4792
2	99.3414	99.3000	97.9044
3	99.7994	99.8800	99.3445
4	99.9820	99.9800	99.8226
5	99.9924	99.9960	99.9304
6	99.9995	99.9995	99.9807
7	-	-	99.9923
8	-	-	99.9969
9	-	-	99.9995

We report per cent of variation in international yield curves explained by principal components for various scenarios of data used.

Table 2: R^2 from spanning regressions, %

Panel A. Regression of currency returns on bond returns

FX	Type of R^2	\$ returns	€ or £ returns
€	R^2	15.65	12.80
	R^2_{adj}	14.53	11.65
£	R^2	9.59	14.42
	R^2_{adj}	8.39	13.28

Panel B. Regression of depreciation rate on principal components of yields

FX	Type of R^2	6 PCs (US + country)	6 PCs (all countries)	9 PCs (all countries)
€	R^2	2.90	5.01	6.38
	R^2_{adj}	1.40	3.54	4.20
£	R^2	0.98	2.78	2.92
	R^2_{adj}	-0.55	1.28	0.65

We report R^2, regular and adjusted, expressed in percent for spanning regressions. In panel A we regress annual currency returns of a given country (obtained by investing in a foreign one-period bond) on annual bond returns of maturities $n = 2$, 3, 4, and 5 years expressed in the same units (USD or foreign). In panel B we regress monthly depreciation rate of a given country vis-a-vis the USD on principal components constructed from yields on US bonds, bonds of that country, and, in the last column, bonds of the third country as well.

Table 3: Posterior mean and standard deviation, state dynamics; NFX model.

x_t^N		x_{1t}	x_{2t}	x_{3t}	x_{4t}	x_{5t}	x_{6t}
	$\bar{\mu}_x \times 1200$	Φ_x					
x_{1t}	3.814	0.976	-0.005	0.045	0.075	-0.051	0.064
	(0.791)	(0.015)	(0.034)	(0.041)	(0.043)	(0.062)	(0.057)
x_{2t}	0.389	-0.025	1.052	-0.029	0.024	0.137	-0.062
	(0.584)	(0.012)	(0.026)	(0.031)	(0.033)	(0.047)	(0.043)
x_{3t}	-2.262	-0.021	-0.029	1.037	0.054	-0.022	0.110
	(0.624)	(0.015)	(0.034)	(0.039)	(0.041)	(0.060)	(0.055)
x_{4t}	1.603	0.008	-0.052	0.009	0.955	-0.117	0.015
	(0.248)	(0.012)	(0.027)	(0.032)	(0.034)	(0.048)	(0.045)
x_{5t}	0.477	0.020	-0.118	0.062	0.060	0.694	0.099
	(0.221)	(0.014)	(0.032)	(0.038)	(0.040)	(0.057)	(0.053)
x_{6t}	1.403	0.002	0.081	-0.146	-0.093	0.089	0.752
	(0.358)	(0.013)	(0.030)	(0.036)	(0.037)	(0.054)	(0.049)
	$\delta_{i,x}$	$\Sigma_x \times \sqrt{12} \times 100$					
x_{1t}	1	0.161	0	0	0	0	0
	(—)	(0.006)	(—)	(—)	(—)	(—)	(—)
x_{2t}	0	0.098	0.076	0	0	0	0
	(—)	(0.005)	(0.003)	(—)	(—)	(—)	(—)
x_{3t}	0	0.029	0.083	0.125	0	0	0
	(—)	(0.008)	(0.007)	(0.004)	(—)	(—)	(—)
x_{4t}	0	-0.034	-0.064	-0.015	0.100	0	0
	(—)	(0.006)	(0.006)	(0.005)	(0.004)	(—)	(—)
x_{5t}	0	-0.057	-0.098	0.009	0.066	0.070	0
	(—)	(0.007)	(0.006)	(0.005)	(0.004)	(0.003)	(—)
x_{6t}	0	-0.029	-0.067	-0.084	0.045	0.012	0.068
	(—)	(0.007)	(0.006)	(0.005)	(0.004)	(0.004)	(0.002)

Posterior mean and standard deviations of $\bar{\mu}_x, \Phi_x, \Sigma_x$ from the NFX model. The state variables are: $x_{1t} = y_t^1$, $x_{2t} = y_t^1 - \widehat{y}_t^{€,1}$, $x_{3t} = y_t^1 - \widehat{y}_t^{£,1}$, $x_{4t} = y_t^{60} - y_t^1$, $x_{5t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{€,60} - \widehat{y}_t^{€,1}\right)$, $x_{6t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{£,60} - \widehat{y}_t^{£,1}\right)$.

Table 4: Posterior mean and standard deviation, risk premiums; NFX model.

x_t^N		x_{1t}	x_{2t}	x_{3t}	x_{4t}	x_{5t}	x_{6t}
	$\lambda_0 \times 1200$	λ_x					
x_{1t}	0.154	-0.038	-0.005	0.032	-0.046	-0.012	0.040
	(0.066)	(0.015)	(0.034)	(0.041)	(0.043)	(0.062)	(0.057)
x_{2t}	-0.279	-0.018	0.055	-0.026	0.052	0.053	-0.063
	(0.073)	(0.012)	(0.026)	(0.031)	(0.033)	(0.047)	(0.043)
x_{3t}	0.037	-0.023	-0.026	0.039	0.043	-0.017	0.016
	(0.077)	(0.015)	(0.034)	(0.039)	(0.041)	(0.061)	(0.055)
x_{4t}	-0.057	0.024	-0.053	0.024	0.059	-0.158	0.043
	(0.054)	(0.012)	(0.027)	(0.032)	(0.034)	(0.049)	(0.045)
x_{5t}	0.083	0.017	-0.114	0.063	0.058	-0.250	0.097
	(0.065)	(0.014)	(0.032)	(0.038)	(0.040)	(0.058)	(0.053)
x_{6t}	0.089	0.003	0.081	-0.144	-0.088	0.087	-0.165
	(0.061)	(0.014)	(0.031)	(0.036)	(0.037)	(0.054)	(0.050)
	$\widehat{\lambda}_0^{€} \times 1200$	$\widehat{\lambda}_x^{€}$					
x_{1t}	-0.203	-0.029	0.012	0.041	0.005	0.005	0.049
	(0.084)	(0.015)	(0.034)	(0.041)	(0.043)	(0.062)	(0.057)
x_{2t}	-0.288	-0.022	0.064	-0.027	0.035	0.109	-0.058
	(0.075)	(0.012)	(0.026)	(0.031)	(0.033)	(0.047)	(0.043)
x_{3t}	0.017	-0.023	-0.018	0.042	0.043	-0.002	0.063
	(0.083)	(0.015)	(0.034)	(0.039)	(0.041)	(0.060)	(0.055)
x_{4t}	0.060	0.017	-0.057	0.016	0.016	-0.152	0.036
	(0.054)	(0.012)	(0.027)	(0.032)	(0.034)	(0.049)	(0.045)
x_{5t}	0.054	0.019	-0.117	0.063	0.059	-0.278	0.097
	(0.064)	(0.014)	(0.032)	(0.038)	(0.040)	(0.058)	(0.053)
x_{6t}	0.087	0.003	0.080	-0.145	-0.088	0.085	-0.201
	(0.060)	(0.013)	(0.030)	(0.036)	(0.037)	(0.054)	(0.050)
	$\widehat{\lambda}_0^{£} \times 1200$	$\widehat{\lambda}_x^{£}$					
x_{1t}	0.130	-0.044	0.013	0.033	-0.087	0.022	0.028
	(0.083)	(0.015)	(0.035)	(0.041)	(0.044)	(0.063)	(0.059)
x_{2t}	-0.434	-0.016	0.062	-0.025	0.058	0.038	-0.060
	(0.090)	(0.012)	(0.026)	(0.031)	(0.033)	(0.048)	(0.043)
x_{3t}	-0.091	-0.025	-0.015	0.038	0.033	0.004	-0.010
	(0.083)	(0.015)	(0.034)	(0.039)	(0.041)	(0.061)	(0.056)
x_{4t}	-0.061	0.029	-0.057	0.028	0.089	-0.173	0.060
	(0.058)	(0.012)	(0.027)	(0.032)	(0.034)	(0.049)	(0.046)
x_{5t}	0.112	0.017	-0.113	0.064	0.057	-0.233	0.097
	(0.066)	(0.014)	(0.032)	(0.038)	(0.040)	(0.058)	(0.053)
x_{6t}	0.088	0.004	0.081	-0.143	-0.084	0.082	-0.141
	(0.063)	(0.014)	(0.031)	(0.036)	(0.037)	(0.055)	(0.050)

The state variables are: $x_{1t} = y_t^1$, $x_{2t} = y_t^1 - \widehat{y}_t^{€,1}$, $x_{3t} = y_t^1 - \widehat{y}_t^{£,1}$, $x_{4t} = y_t^{60} - y_t^1$, $x_{5t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{€,60} - \widehat{y}_t^{€,1}\right)$, $x_{6t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{£,60} - \widehat{y}_t^{£,1}\right)$.

Table 5: Posterior mean and standard deviation; WFX model.

x_t^W		$\Delta s_t^{€}$	$\Delta s_t^{£}$	x_{1t}	x_{2t}	x_{3t}	x_{4t}	x_{5t}	x_{6t}
	$\bar{\mu}_x \times 1200$		Φ_x						
$\Delta s_t^{€}$	0.974	0.019	0.204	-0.696	0.725	-3.710	-0.230	0.274	-2.029
	(2.911)	(0.054)	(0.053)	(0.891)	(1.706)	(1.953)	(2.139)	(2.891)	(2.663)
$\Delta s_t^{£}$	-0.019	0.061	0.010	-0.869	0.510	-0.678	2.211	-4.172	1.589
	(2.568)	(0.057)	(0.056)	(0.933)	(1.774)	(2.010)	(2.217)	(2.993)	(2.766)
x_{1t}	3.802	-4.61e-04	4.57e-04	0.976	1.39e-04	0.038	0.068	-0.036	0.054
	(0.784)	(9.53e-04)	(9.20e-04)	(0.015)	(0.034)	(0.041)	(0.041)	(0.060)	(0.057)
x_{2t}	0.395	-6.73e-05	6.86e-05	-0.026	1.055	-0.033	0.022	0.142	-0.067
	(0.567)	(7.65e-04)	(7.28e-04)	(0.012)	(0.026)	(0.031)	(0.032)	(0.047)	(0.044)
x_{3t}	-2.248	-0.001	5.70e-04	-0.022	-0.026	1.030	0.052	-0.018	0.103
	(0.605)	(9.30e-04)	(8.96e-04)	(0.015)	(0.034)	(0.040)	(0.041)	(0.059)	(0.056)
x_{4t}	1.603	-3.99e-04	2.81e-04	0.009	-0.056	0.010	0.958	-0.123	0.017
	(0.246)	(7.42e-04)	(7.16e-04)	(0.012)	(0.027)	(0.032)	(0.033)	(0.047)	(0.044)
x_{5t}	0.465	-6.38e-04	2.26e-04	0.021	-0.120	0.060	0.059	0.694	0.098
	(0.219)	(9.17e-04)	(8.82e-04)	(0.014)	(0.032)	(0.038)	(0.039)	(0.057)	(0.053)
x_{6t}	1.393	-2.49e-04	3.85e-04	0.002	0.077	-0.145	-0.088	0.081	0.754
	(0.352)	(8.27e-04)	(7.99e-04)	(0.013)	(0.030)	(0.036)	(0.037)	(0.053)	(0.050)
	$\lambda_0 \times 1200$		λ_x						
$\Delta s_t^{€}$	-1.469	0.019	0.204	-0.696	-0.275	-3.710	-0.230	0.274	-2.029
	(3.925)	(0.054)	(0.053)	(0.891)	(1.706)	(1.953)	(2.139)	(2.891)	(2.663)
$\Delta s_t^{£}$	-1.803	0.061	0.010	-0.869	0.510	-1.678	2.211	-4.172	1.589
	(4.136)	(0.057)	(0.056)	(0.933)	(1.774)	(2.010)	(2.217)	(2.993)	(2.766)
x_{1t}	0.093	2.19e-04	-1.84e-04	-0.029	-0.006	0.040	-0.019	-0.044	0.050
	(0.064)	(0.001)	(0.001)	(0.015)	(0.034)	(0.041)	(0.042)	(0.060)	(0.057)
x_{2t}	0.024	-3.83e-04	-2.85e-04	-0.025	0.044	-0.031	0.009	0.050	-0.051
	(0.052)	(0.001)	(0.001)	(0.012)	(0.026)	(0.031)	(0.033)	(0.047)	(0.044)
x_{3t}	-0.041	-3.20e-04	7.00e-04	-0.020	-0.037	0.027	0.065	-0.023	-0.013
	(0.066)	(0.002)	(0.001)	(0.015)	(0.034)	(0.040)	(0.041)	(0.059)	(0.056)
x_{4t}	0.004	-0.001	9.26e-04	0.015	-0.048	0.008	0.024	-0.114	0.022
	(0.053)	(0.001)	(0.001)	(0.012)	(0.027)	(0.032)	(0.033)	(0.047)	(0.044)
x_{5t}	-0.020	-3.21e-04	5.81e-04	0.019	-0.106	0.058	0.072	-0.234	0.080
	(0.064)	(0.001)	(0.001)	(0.014)	(0.032)	(0.038)	(0.040)	(0.057)	(0.053)
x_{6t}	0.063	-9.88e-04	2.56e-04	7.70e-04	0.090	-0.142	-0.103	0.089	-0.150
	(0.060)	(0.002)	(0.001)	(0.014)	(0.030)	(0.036)	(0.037)	(0.053)	(0.051)
	$\delta_{i,x}$		$\Sigma_x \times \sqrt{12} \times 100$						
$\Delta s_t^{€}$	0	9.728	0	0	0	0	0	0	0
	(—)	(0.352)	(—)	(—)	(—)	(—)	(—)	(—)	(—)
$\Delta s_t^{£}$	0	5.311	8.714	0	0	0	0	0	0
	(—)	(0.477)	(0.310)	(—)	(—)	(—)	(—)	(—)	(—)
x_{1t}	1	-0.006	-0.018	0.157	0	0	0	0	0
	(—)	(0.008)	(0.008)	(0.005)	(—)	(—)	(—)	(—)	(—)
x_{2t}	0	-0.005	-0.015	0.096	0.076	0	0	0	0
	(—)	(0.006)	(0.006)	(0.005)	(0.003)	(—)	(—)	(—)	(—)
x_{3t}	0	-0.006	0.002	0.030	0.083	0.125	0	0	0
	(—)	(0.008)	(0.008)	(0.008)	(0.007)	(0.004)	(—)	(—)	(—)
x_{4t}	0	-0.006	2.59e-04	-0.041	-0.060	-0.015	0.096	0	0
	(—)	(0.006)	(0.006)	(0.006)	(0.005)	(0.005)	(0.003)	(—)	(—)
x_{5t}	0	-0.009	0.007	-0.062	-0.095	0.008	0.063	0.070	0
	(—)	(0.008)	(0.008)	(0.007)	(0.006)	(0.005)	(0.004)	(0.003)	(—)
x_{6t}	0	-0.006	-0.006	-0.036	-0.063	-0.084	0.041	0.011	0.068
	(—)	(0.007)	(0.007)	(0.007)	(0.006)	(0.005)	(0.004)	(0.004)	(0.003)

Posterior mean and stand. dev. of the WFX model. The state variables are: $x_{1t} = y_t^1$, $x_{2t} = y_t^1 - \widehat{y}_t^{€,1}$, $x_{3t} = y_t^1 - \widehat{y}_t^{£,1}$, $x_{4t} = y_t^{60} - y_t^1$, $x_{5t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{€,60} - \widehat{y}_t^{€,1}\right)$, $x_{6t} = \left(y_t^{60} - y_t^1\right) - \left(\widehat{y}_t^{£,60} - \widehat{y}_t^{£,1}\right)$

Table 6: Pricing errors across countries

	NFX			WFX		
	US	Euro	UK	US	Euro	UK
y_t^{12}	0.11	0.09	0.10	0.12	0.09	0.10
y_t^{24}	0.08	0.07	0.08	0.09	0.07	0.09
y_t^{36}	0.05	0.05	0.06	0.06	0.05	0.06
y_t^{48}	0.02	0.02	0.03	0.03	0.02	0.03

Posterior mean estimates of the pricing errors in annualized percentage points, $100 \times \sqrt{\text{diag}\left(\Sigma_y \Sigma_y' \times 12\right)}$ *,for the US, Euro, and UK for both the NFX and WFX model. These are reported for yields of different maturity that are not part of the state* x_t*.*

Table 7: Properties of Currency Prices and Interest Rates

Panel A: Summary statistics									
	mean	NFX	WFX	st.dev.	NFX	WFX	autocorr	NFX	WFX
	Δs_t								
Euro	0.909	478.258	0.909	9.99	250.73	9.99	0.150	0.346	0.150
UK	-0.264	-703.672	-0.264	10.18	249.25	10.18	0.055	0.181	0.055
	Short rates								
US	3.725	3.725	3.725	0.79	0.79	0.79	0.985	0.985	0.985
Euro	3.673	3.475	3.459	0.69	0.69	0.70	0.988	0.992	0.992
UK	5.942	5.942	5.942	1.16	1.16	1.16	0.990	0.990	0.990
	$y_t^1 - \widehat{y}_t^1$								
Euro	0.364	0.251	0.266	0.56	0.55	0.55	0.974	0.974	0.974
UK	-2.217	-2.217	-2.217	0.61	0.61	0.61	0.969	0.969	0.969
Panel B: UIP regressions									
	$\Delta s_{t+1} \; = \; a + b(y_t^1 - \widehat{y}_t^1) + \varepsilon_{t+1}$								
	$\hat{a}$	NFX	WFX	$\hat{b}$	NFX	WFX			
Euro	0.0014	0.4123	0.0009	-1.2169	-103.9271	-0.9458			
	(0.0017)	(0.0017)	(0.0501)	(1.1654)	(27.1927)	(1.1422)			
UK	-0.0016	-0.6529	-0.0016	-0.8387	-33.9749	-0.8387			
	(0.0019)	(0.0533)	(0.0019)	(1.1580)	(25.6371)	(1.1580)			

We replicate Table I of B/F/T. We report the sample mean, sample standard deviation, and sample autocorrelation.

Table 8: Sample moments of data versus model-implied yields

	mean			st.dev.			autocorr		
	data	NFX	WFX	data	NFX	WFX	data	NFX	WFX
US									
y_t^1	3.725	3.725	3.725	0.791	0.791	0.791	0.985	0.985	0.985
y_t^{12}	4.354	4.160	4.159	0.877	0.848	0.820	0.991	0.989	0.989
y_t^{24}	4.652	4.542	4.534	0.885	0.867	0.839	0.990	0.989	0.989
y_t^{36}	4.907	4.852	4.842	0.874	0.865	0.846	0.989	0.989	0.989
y_t^{48}	5.130	5.109	5.103	0.856	0.853	0.844	0.989	0.989	0.989
y_t^{60}	5.326	5.326	5.326	0.837	0.837	0.837	0.988	0.988	0.988
Euro									
$\widehat{y}_t^1$	3.673	3.476	3.459	0.690	0.693	0.698	0.992	0.992	0.992
$\widehat{y}_t^{12}$	3.817	3.772	3.787	0.736	0.716	0.720	0.991	0.992	0.992
$\widehat{y}_t^{24}$	4.022	4.030	4.047	0.734	0.727	0.727	0.990	0.992	0.992
$\widehat{y}_t^{36}$	4.228	4.243	4.253	0.727	0.725	0.724	0.990	0.991	0.991
$\widehat{y}_t^{48}$	4.413	4.422	4.425	0.716	0.716	0.715	0.989	0.990	0.990
$\widehat{y}_t^{60}$	4.573	4.573	4.573	0.702	0.702	0.702	0.989	0.989	0.989
UK									
$\widehat{y}_t^1$	5.942	5.942	5.942	1.163	1.163	1.163	0.990	0.990	0.990
$\widehat{y}_t^{12}$	5.785	5.841	5.887	1.070	1.087	1.065	0.990	0.991	0.991
$\widehat{y}_t^{24}$	5.895	5.913	5.934	1.028	1.032	1.014	0.990	0.991	0.991
$\widehat{y}_t^{36}$	6.010	6.022	6.022	0.992	0.994	0.984	0.990	0.990	0.990
$\widehat{y}_t^{48}$	6.116	6.123	6.118	0.964	0.966	0.961	0.990	0.990	0.990
$\widehat{y}_t^{60}$	6.208	6.208	6.208	0.942	0.942	0.942	0.989	0.989	0.989

Reduced-form moments vs model-implied moments from the main model.

Table 9: Significant factors

	es_t^n		srp_t^n				$E_t srp_{t+1}^n$		$E_t \Delta_c rx_{t+1}^n$				
n	$\Delta^{cross} s_t$	$\Delta_c^{cross} y_t^{60,1}$	$\Delta^{cross} s_t$	$\Delta_c^{own} y_t^1$	$\Delta_c^{own} y_t^{60,1}$	$\Delta_c^{cross} y_t^{60,1}$	$\Delta_c^{own} y_t^1$	$\Delta_c^{own} y_t^{60,1}$	y_t^1	$\Delta_c^{own} y_t^1$	$\Delta_c^{cross} y_t^1$	$\Delta_c^{own} y_t^{60,1}$	$\Delta_c^{cross} y_t^{60,1}$
1	0.133	-3.097	0.133	-0.979	0.925	-3.097	-1.356	-0.138	0	0	0	0	0
	(0.035)	(1.877)	(0.035)	(1.180)	(1.862)	(1.877)	(1.026)	(1.554)					
2	0.068	-2.707	0.068	-1.150	0.403	-2.704	-1.424	-0.281	0.022	-0.035	0.034	-0.018	0.037
	(0.019)	(1.695)	(0.019)	(1.091)	(1.687)	(1.695)	(1.000)	(1.454)	(0.12)	(0.020)	(0.019)	(0.032)	(0.032)
11	0.013	-1.418	0.013	-1.552	-0.785	-1.406	-1.593	-0.938	0.196	0.138	0.032	0.567	0.018
	(0.004)	(1.055)	(0.004)	(0.892)	(1.053)	(1.055)	(0.880)	(0.986)	(0.089)	(0.161)	(0.150)	(0.255)	(0.257)
12	0.012	-1.350	0.012	-1.559	-0.840	-1.337	-1.589	-0.972	0.212	0.196	0.008	0.692	-0.010
	(0.004)	(1.022)	(0.004)	(0.883)	(1.021)	(1.022)	(0.874)	(0.961)	(0.096)	(0.175)	(0.163)	(0.276)	(0.279)
23	0.006	-0.911	0.006	-1.512	-1.183	-0.903	-1.473	-1.175	0.369	1.113	-0.409	2.499	-0.482
	(0.002)	(0.800)	(0.002)	(0.817)	(0.810)	(0.801)	(0.815)	(0.788)	(0.168)	(0.322)	(0.311)	(0.511)	(0.519)
24	0.006	-0.887	0.006	-1.503	-1.200	-0.880	-1.461	-1.185	0.381	1.212	-0.455	2.690	-0.534
	(0.002)	(0.788)	(0.002)	(0.812)	(0.797)	(0.788)	(0.811)	(0.777)	(0.175)	(0.336)	(0.326)	(0.534)	(0.542)
59	0.003	-0.490	0.003	-1.218	-1.389	-0.490	-1.128	-1.256	0.732	5.089	-2.268	9.964	-2.681
	(0.001)	(0.539)	(0.001)	(0.670)	(0.550)	(0.539)	(0.669)	(0.547)	(0.455)	(0.885)	(0.910)	(1.435)	(1.460)
60	0.002	-0.484	0.002	-1.212	-1.389	-0.484	-1.122	-1.254	0.741	5.201	-2.321	10.171	-2.748
	(0.001)	(0.535)	(0.001)	(0.666)	(0.546)	(0.535)	(0.666)	(0.543)	(0.463)	(0.902)	(0.927)	(1.462)	(1.487)

We display factors that affect various objects of interest with significant loadings at least at one of the horizons. The loadings are implied by the WFX model, and computed as cross-country averages. Superscripts "cross" refers to our factor from a different country, and "own" to the factor from the same country. The objects affected by these factors are listed in first row of the table.