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GENERAL EQUILIBRIUM REBOUND FROM ENERGY EFFICIENCY INNOVATION

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ABSTRACT

Energy efficiency improvements "rebound" when economic responses undercut their direct energy savings. I show that general equilibrium channels typically amplify rebound by making consumption goods cheaper but typically dampen rebound by increasing the cost of non-energy inputs to production. Improvements in energy efficiency are especially likely to increase total energy use when they arise in the energy supply sector because they make energy inputs cheaper in all other sectors. When energy and non-energy inputs are substitutes (complements), innovators often direct research efforts towards those consumption good sectors where improvements in efficiency are especially likely to increase (decrease) total energy use.

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1 Introduction

Industrial economies have become much better at converting energy into useful work. For instance, the energy intensity of U.S. output fell by 35% from 1985 to 2011,[1] and the energy intensity of British output has fallen by 80% since 1850 (Fouquet and Pearson, 2003). However, economists since Jevons (1865) have wondered whether improvements in energy efficiency might actually increase aggregate use of energy resources. Further, environmental economists have been especially concerned with the possibility of such large "rebound" effects because of the prominent externalities associated with energy use. Most formal analyses of rebound effects have focused on partial equilibrium settings that hold some prices fixed. Yet Jevons was especially worried about general equilibrium channels,[2] and his concern has been reinforced by computable general equilibrium models that suggest the potential for strong rebound effects through "economy-wide" or "indirect" channels (Allan et al., 2009; Turner, 2013). As a result, many have called for theoretical research to illuminate the channels through which economy-wide rebound arises (e.g., Dimitropoulos, 2007; Turner, 2013; Borenstein, 2015).

I fill the gap in the theoretical literature by developing an analytically tractable general equilibrium framework for studying the implications of improved energy efficiency. I disentangle the channels through which improvements in energy efficiency affect total energy use and I sign the effect in a range of cases. The modeled economy contains an arbitrary number of sectors that produce distinct consumption goods. In the baseline model, each consumption good is produced competitively by combining a labor-capital aggregate with energy, using a constant elasticity of substitution (CES) technology. Each household supplies a single unit of the labor-capital aggregate to the production sector that offers the highest price. Energy is converted to useful work via energy conversion technologies that are specific to each consumption good sector. Energy is itself produced by an additional sector that also employs a CES technology in energy and labor-capital inputs. I study how an improvement in the quality of some sector's energy conversion technology affects prices and energy use throughout the economy.

An engineering estimate of the effects of an efficiency improvement would hold the production of energy services (e.g., useful work or lighting) fixed and calculate the energy resources displaced by the improvement in efficiency. "Rebound" is the percentage of these engineering savings lost through economic responses. A partial equilibrium analysis of rebound holds the prices of consumption goods, energy resources, and the labor-capital aggregate fixed. In this case with fixed prices, I show the result familiar from previous literature (e.g., Saunders,

[1] `https://energy.gov/eere/analysis/energy-intensity-indicators-highlights`

[2] Jevons (1865, VII p. 141) wrote, "Now, if the quantity of coal used in a blast-furnace, for instance, be diminished in comparison with the yield, the profits of the trade will increase, new capital will be attracted, the price of pig-iron will fall, but the demand for it increase; and eventually the greater number of furnaces will more than make up for the diminished consumption of each." This story hinges on changes in prices. I will formally identify this story as an output price channel.

1992; Sorrell and Dimitropoulos, 2008): rebound is proportional to the elasticity of substitution between energy and non-energy inputs. This elasticity captures how firms substitute towards the energy input when improved technology reduces its effective cost. When energy and non-energy inputs are gross substitutes in production (i.e., when this elasticity is greater than 1), rebound is greater than 100%. In this case, an efficiency improvement is said to "backfire," actually increasing consumption of energy resources.

In general equilibrium, all prices adjust to the improved energy conversion technology. Improving the technology in some sector k reduces the cost of producing that sector's consumption good and thus reduces the price of the consumption good. As a result, households substitute towards that consumption good. This substitution increases demand for both energy and non-energy inputs to production in sector k. However, the lower output price also reduces demand for inputs in sector k. The net effect on sector k's demand for energy depends on (i) households' elasticity of substitution across consumption goods and (ii) sector k firms' elasticity of substitution across inputs. This general equilibrium channel increases total energy use if and only if (i) is larger than (ii), which is consistent with standard assumptions in numerical models.

The reduced output price also changes sector k's demand for the labor-capital input, with the direction of this effect matching the direction of the effect on demand for energy inputs. If demand for the labor-capital input increases, then the price of that input must increase in order to clear the market. As a result, households' income increases and the cost of producing each consumption good increases. The former effect works to increase energy use and the latter effect works to reduce energy use. I show that the latter effect dominates. Therefore an outward (inward) shift in demand for the labor-capital input ends up reducing (increasing) energy use. Because demand for the labor-capital input shifts in the same direction as demand for the energy input, the general equilibrium effects arising through the market for the labor-capital input oppose the previously described general equilibrium effects arising through the reduced price of the consumption good.

I connect general equilibrium rebound to parameters that can be estimated in future empirical work and used to understand future numerical modeling. In particular, I show that the general equilibrium component of rebound grows with the value share of energy in the sector with improved technology and with the difference between the elasticity of substitution between the various consumption goods and the elasticity of substitution between energy and non-energy inputs to production. The value share determines the degree to which improved technology reduces the price of that sector's consumption good, and the two elasticities determine how factor demand scales with the price of the consumption good.[3] General

[3]Historical evidence supports the importance of the value share of energy for the possibility of backfire. Rosenberg (1994)[Chapter 9, p. 165] observes, "Historically, new technologies that improved energy efficiency have often led to a significant increase, and not to a reduction, in fuel consumption. This has been especially true in energy-intensive sectors where fuel costs have constituted a large proportion of total costs." Note also that policymakers often try to reduce total energy use by targeting energy efficiency policies towards

equilibrium channels are likely to to be especially important when efficiency improvements arise in sectors with strongly complementary energy and non-energy inputs.

The special case where all consumption good sectors have the same production technology is especially tractable. I show that the two opposing general equilibrium effects exactly cancel in this special case. As a result, backfire occurs if and only if energy and non-energy inputs are substitutes in production, as in the partial equilibrium analysis. However, negative rebound or "super-conservation" (Saunders, 2008) is now also possible because of an additional general equilibrium effect. If energy and non-energy inputs are complements, then energy use falls in sector k. As a result, the energy-producing sector contracts and reduces its demand for energy inputs, which dampens rebound. Consistent with computable general equilibrium models' results (e.g., Turner, 2009), this multiplier effect can even increase energy savings beyond what an engineering analysis would predict. In general, the multiplier effect amplifies any given change in energy use, so that it also makes backfire more severe when the energy and non-energy inputs are substitutes.

I show that rebound tends to be especially severe when improved technology arises in the energy supply sector. Improving the efficiency of energy production reduces the price of the energy inputs to consumption good production. As a result, consumption good producers substitute towards the newly cheap energy input. Further, the reduced cost of producing each consumption good works to reduce the price of each consumption good, with the implications for total energy demand described above. Because an improvement in the efficiency of energy production leads all other producers to substitute towards energy use, backfire occurs for a much broader set of conditions than when new technology arises in some consumption good sector. This result formalizes and confirms the results of numerical models that have emphasized rebound from the energy supply sector (see Allan et al., 2007; Sorrell, 2007; Hanley et al., 2009; Turner, 2009).

Finally, I extend the setting to incorporate a modern model of directed technical change (Acemoglu, 2002, 2007) in order to assess whether innovation is likely to occur in sectors that are especially vulnerable to rebound. Consumption good firms now combine their energy inputs with a continuum of machine varieties, which are supplied by monopolistically competitive firms. Each variety of machine has its own energy conversion technology. Research firms target a consumption good sector, and if they succeed in innovating, they receive a patent to produce their improved machine. I show that the presence of the imperfectly competitive machine inputs only slightly modifies the expression for general equilibrium rebound. When energy and the labor-capital input are complements (weak substitutes), improvements in efficiency generally decrease (increase) total energy use when they arise in a consumption good sector with above-average efficiency. I show that the marginal research firm often targets the most efficient sector when energy and the labor-capital input are complements, in which case profit-driven innovation decreases total energy use. However, I also show that

sectors with a high value share of energy (such as the utility and transportation sectors).

the marginal research firm always targets the most efficient sector when energy and the labor-capital input are weak substitutes, in which case profit-driven innovation increases total energy use. The elasticity of substitution between energy and non-energy inputs remains critical even after allowing for endogenously directed innovation.

There are only a few previous analytic general equilibrium studies of rebound effects.[4] Wei (2007) restricts attention to a single energy good and a single non-energy good, assumes Cobb-Douglas functional forms for all production functions, and analyzes a linear demand system. Wei (2010) considers a setting with only a single consumption good and does not model the production of energy. Böhringer and Rivers (2018) and Fullerton and Ta (2018) linearize models to study improvements in the efficiency of energy service inputs to firms and households, respectively, in settings with one or two consumption goods.[5,6] All of this prior work abstracts from the use of energy resources in the production of energy. However, computable general equilibrium models have emphasized the importance of accounting for energy demand by the firms that produce the energy needed for consumption good production (e.g., Allan et al., 2007; Sorrell, 2007; Hanley et al., 2009; Turner, 2009).[7] I show that the energy supply sector is critical to the possibility of super-conservation and that improvements to the

[4]There are quite a few numerical studies with computable general equilibrium models (see Sorrell, 2007; Allan et al., 2009; Turner, 2013; Broberg et al., 2015), which often report quite large effects from general equilibrium channels. There is also a large partial equilibrium literature. See Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), and van den Bergh (2011). Neoclassical growth settings have emphasized how analogues of partial equilibrium income and substitution effects arise after improving the productivity of energy in the broader economy's production function (Saunders, 1992, 2000). Finally, Hart (2018) and Rausch and Schwerin (2018) study the role of rebound in explaining the long-run dynamics of aggregate energy use, emphasizing expanding varieties of energy-using goods and putty-clay production of energy-using capital, respectively.

[5]Fullerton and Ta (2018) contrast the implications of "costless technology shocks" with policy-induced increases in energy efficiency that impose costs on energy users. I here mostly follow other analytic literature and numerical simulations in focusing on the case of costless improvements, which may be more relevant to the study of technical change and which may be more relevant to the study of firms that pay one-time fixed costs to adopt new technologies. However, I do also analyze a setting with directed technical change, in which firms must purchase machines that embody the more efficient technology. See Allan et al. (2007), Allan et al. (2009), Turner (2013), and Broberg et al. (2015) for discussions of costly improvements in the context of computable general equilibrium modeling, and see Borenstein (2015) and Gillingham et al. (2016) for informal discussions.

[6]Some prior analytic models have studied improvements in the efficiency with which households use energy (Chan and Gillingham, 2015; Fullerton and Ta, 2018), but computable general equilibrium models have typically studied improvements in firms' efficiency. I follow the numerical general equilibrium literature in studying improvements in firms' efficiency. The appendix connects the analysis to households who directly purchase energy, as with use of vehicles or appliances.

[7]In their reviews, Greening et al. (2000) emphasize the potential for large adjustments in energy supply and Turner (2013) laments the lack of attention given to energy supply in analyses of rebound effects. Based on numerical experiments, Saunders (2014) conjectures that greater efficiency in energy production will inevitably backfire. I formally demonstrate that backfire is indeed especially likely in this case, but I also show that it is not inevitable.

efficiency of energy supply are much more likely to backfire than are improvements to the efficiency of consumption good production. Further, whereas recent analytic work has used linearization techniques, I explicitly solve for energy use in a dual setting that treats prices as independent variables. My analysis demonstrates precisely which price changes generate each general equilibrium channel and thus further develops intuition for general equilibrium consequences.[8]

The next section describes the setting. Section 3 derives the equilibrium prices and allocation. Section 4 recounts the familiar partial equilibrium analysis. Section 5 analyzes general equilibrium rebound from improvements in the energy efficiency of consumption good producers and energy producers. Section 6 extends the setting to allow for directed technical change and analyzes whether profit-driven research firms target sectors in which improved efficiency will backfire. The final section concludes. The first appendix connects the analysis to a case in which energy services are a direct input to households' utility. The second appendix extends the main analysis to consider the implications for total energy use of improvements in the energy efficiency of every sector (as with general purpose energy technologies), in the productivity of the labor-capital aggregate in some sector, and in total factor productivity in some sector.

2 Setting

There are N consumption goods, produced in quantity c_i for $i \in \{1, ..., N\}$. The representative household obtains utility from consuming these goods:

$$u(C), \text{ where } C \triangleq \left(\sum_{i=1}^{N} c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} .$$

Utility $u(\cdot)$ is monotonically increasing in the consumption index C. $\epsilon > 1$ denotes the elasticity of substitution between the different varieties of consumption good. The price of each good is p_i.

Each consumption good is produced competitively using quantity X_i of labor-capital aggregate and quantity R_i of energy resources:[9]

$$c_i = \left(\kappa_i [\chi_i X_i]^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \kappa_i)[A_i R_i]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}} .$$

The production function has a constant elasticity of substitution $\sigma_i > 0$, with $\sigma_i \neq 1$. I will be especially interested in the case where $\sigma_i < \epsilon$ because it is consistent with empirical

[8] I also analyze improvements in the productivity of non-energy inputs, following up on speculation (e.g., Saunders, 1992; Sorrell, 2007; Saunders, 2013) that such improvements may be especially likely to increase energy use.

[9] I will use "energy" and "resources" interchangeably.

evidence that energy and non-energy inputs are either complements or weak substitutes and because computable general equilibrium models tend to use small values for σ_i (see Broberg et al., 2015). I drop the subscript when considering special cases with identical σ_i for all consumption good firms. $\kappa_i \in (0,1)$ is the distribution parameter. The productivity of energy resources and of the labor-capital aggregate are determined by $A_i > 0$ and $\chi_i > 0$, respectively. We can interpret $A_i R_i$ as energy services such as heating, lighting, or mechanical motion, with A_i controlling the conversion from R_i into energy services.

The same energy resources are used in each sector. In equilibrium, each sector pays price p_R for each unit of energy resource. Energy resources are produced competitively via a CES function of the labor-capital aggregate and energy:

$$R = \left(\kappa_{N+1}[\chi_{N+1}X_{N+1}]^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}} + (1-\kappa_{N+1})[A_{N+1}R_{N+1}]^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}}\right)^{\frac{\sigma_{N+1}}{\sigma_{N+1}-1}}.$$

R is the total quantity of energy produced for the economy, with $\sigma_{N+1} > 0, \neq 1$, $\kappa_{N+1} \in (0,1)$, and $A_{N+1}, \chi_{N+1} > 0$. Assume that $(1-\kappa)A_{N+1}^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}} < 1$ so that energy producers' profit function is concave in R_{N+1}. Energy production can be interpreted as the extraction of oil or as the generation of electricity.

There is a continuum of households, of measure X. Each household is endowed with one unit of the labor-capital aggregate, which it sells to some sector i. In equilibrium, each sector pays price p_X for each unit of labor-capital aggregate. The representative household's budget constraint is then $\sum_{i=1}^{N} p_i c_i \leq p_X X$. For simplicity, I will often refer to X as labor and to p_X as the wage.

3 Equilibrium Prices and Allocations

I study market equilibria.

Definition 1. *An* equilibrium *is given by consumption good prices ($\{p_i\}_{i=1}^{N}$), a price for the labor-capital aggregate (p_X), a price for energy resources (p_R), demands for inputs ($\{X_i, R_i\}_{i=1}^{N+1}$), and demands for consumption goods ($\{c_i\}_{i=1}^{N}$) such that: (i) (X_i, R_i) maximizes profits of producers of consumption good i, (ii) (X_{N+1}, R_{N+1}) maximizes profits of energy resource producers, (iii) $\{c_i\}_{i=1}^{N}$ maximizes household utility, (iv) firms make zero profits, and (v) the prices p_X, p_R, and $\{p_i\}_{i=1}^{N}$ clear the markets for the labor-capital aggregate, for energy resources, and for consumption goods.*

The equilibrium prices clear all factor markets, all firms maximize profits within competitive markets, and households maximize utility subject to their budget constraint.

The representative household solves the following maximization problem:

$$\max_{\{c_i\}_{i=1}^N} u\left(\left(\sum_{i=1}^N c_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}\right), \text{ subject to } \sum_{i=1}^N p_i c_i \leq p_X X.$$

Households will choose to sell all of their endowment.[10] Letting λ be the shadow value of the budget constraint, the first-order condition for c_i is

$$\frac{\lambda p_i}{u'(C)} = \left(\frac{c_i}{C}\right)^{-\frac{1}{\epsilon}}.$$

Let P be the ideal price index, so that $\sum_{i=1}^N p_i c_i = P\,C$. Households' first-order condition for C implies that $P = u'(C)/\lambda$. I choose the price index as the numeraire: $P = 1$. The household budget constraint then implies that $C = p_X X$ in equilibrium. Aggregate household demand for good i becomes

$$c_i = \left(\frac{p_i}{P}\right)^{-\epsilon} C = X\, p_i^{-\epsilon} p_X. \tag{1}$$

Now consider the input mix chosen by firms in sector $i \in \{1, .., N+1\}$. Firms solve:

$$\max_{X_i, R_i}\left\{p_i\ \left(\kappa_i[\chi_i X_i]^{\frac{\sigma_i-1}{\sigma_i}} + (1-\kappa_i)[A_i R_i]^{\frac{\sigma_i-1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i-1}} - p_X X_i - p_R R_i\right\},$$

where $p_{N+1} \triangleq p_R$. The first-order conditions are:

$$p_X = p_i \kappa_i \chi_i^{\frac{\sigma_i-1}{\sigma_i}}\left(\frac{X_i}{c_i}\right)^{-\frac{1}{\sigma_i}}, \tag{2}$$

$$p_R = p_i(1-\kappa_i) A_i^{\frac{\sigma_i-1}{\sigma_i}}\left(\frac{R_i}{c_i}\right)^{-\frac{1}{\sigma_i}}, \tag{3}$$

where $c_{N+1} \triangleq R$. Rearranging the first-order conditions to solve for X_i and R_i and substituting into the zero-profit condition required by competitive markets, we obtain:

$$p_i = \left(p_X^{1-\sigma_i}\chi_i^{\sigma_i-1}\kappa_i^{\sigma_i} + p_R^{1-\sigma_i} A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\right)^{\frac{1}{1-\sigma_i}}. \tag{4}$$

In the energy-producing sector, substituting p_R for p_i in equation (4) and rearranging yields:

$$p_R = \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1}{1-\sigma_{N+1}}} p_X, \tag{5}$$

[10] Allan et al. (2007), Broberg et al. (2015), and Böhringer and Rivers (2018) consider the role of alternate assumptions about labor supply.

where our assumption that $(1-\kappa_{N+1})A_{N+1}^{\frac{\sigma_{N+1}-1}{\sigma_{N+1}}} < 1$ ensures that $p_R > 0$. Rearranging equations (2) and (3) and then substituting for p_R from equation (5), we have factor demand in the energy-producing sector:

$$X_{N+1} = \kappa_{N+1}^{\sigma_{N+1}} \chi_{N+1}^{\sigma_{N+1}-1} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R, \tag{6}$$

$$R_{N+1} = (1-\kappa_{N+1})^{\sigma_{N+1}} A_{N+1}^{\sigma_{N+1}-1} R. \tag{7}$$

Now consider factor demand in the consumption good sectors. Rearrange equations (2) and (3) and substitute for c_i from equation (1) to obtain, for $i \in \{1, ..., N\}$:

$$X_i = X \chi_i^{\sigma_i - 1} \left(\frac{\kappa_i}{p_X} \right)^{\sigma_i} p_i^{\sigma_i - \epsilon} p_X, \tag{8}$$

$$R_i = X A_i^{\sigma_i - 1} \left(\frac{1-\kappa_i}{p_R} \right)^{\sigma_i} p_i^{\sigma_i - \epsilon} p_X. \tag{9}$$

Substituting for p_R from equation (5), the latter equation becomes:

$$R_i = X\, A_i^{\sigma_i-1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{-\sigma_i}{1-\sigma_{N+1}}} p_i^{\sigma_i - \epsilon} p_X^{1-\sigma_i}. \tag{10}$$

Using equation (5) to substitute for p_R in equation (4), we have:

$$p_i = \left(\chi_i^{\sigma_i-1} \kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{1}{1-\sigma_i}} p_X. \tag{11}$$

Substituting from equation (11), equations (8) and (10) become:

$$X_i = X\, \chi_i^{\sigma_i-1} \kappa_i^{\sigma_i} \left(\chi_i^{\sigma_i-1} \kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i-\epsilon}{1-\sigma_i}} p_X^{1-\epsilon}, \tag{12}$$

$$R_i = X\, A_i^{\sigma_i-1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{-\sigma_i}{1-\sigma_{N+1}}}$$
$$\left(\chi_i^{\sigma_i-1} \kappa_i^{\sigma_i} + A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i-\epsilon}{1-\sigma_i}} p_X^{1-\epsilon}. \tag{13}$$

Market-clearing for the labor-capital aggregate implies that

$$X = \sum_{i=1}^{N+1} X_i$$
$$= \sum_{i=1}^{N} X\, \chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} \left(\chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i - \epsilon}{1-\sigma_i}} p_X^{1-\epsilon}$$
$$+ \chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R.$$

Rearranging, we have:

$$p_X^{1-\epsilon} = \frac{X - \chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R}{X \sum_{i=1}^{N} \chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} \left(\chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} \right)^{\frac{\sigma_i - \epsilon}{1-\sigma_i}}}. \tag{14}$$

Market-clearing for energy implies that

$$R = \sum_{i=1}^{N+1} R_i.$$

We could obtain a closed-form solution for R by substituting for R_i from equation (13) and then for p_X from equation (14). Doing so shows that the equilibrium is unique and that energy use increases linearly in X.

Finally, define the value share (or cost share) of energy and the labor-capital aggregate in sector i as α_{Ri} and α_{Xi}, respectively. These are:

$$\alpha_{Ri} \triangleq \frac{p_R R_i}{p_i c_i} = \frac{A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}}{\chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}},$$
$$\alpha_{Xi} \triangleq \frac{p_X X_i}{p_i c_i} = \frac{\chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i}}{\chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} + A_i^{\sigma_i - 1} (1-\kappa_i)^{\sigma_i} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}}.$$

4 Partial Equilibrium Rebound

I begin by reviewing partial equilibrium rebound from a 1% improvement in the efficiency of energy production in some sector $k \in \{1, ..., N+1\}$.

First, the simplest "engineering" calculation does not consider changes in prices and does not allow for factor substitution by firms: it fixes c_k, $\chi_k X_k$, and $A_k R_k$. Let E_k be the energy services used prior to the improvement in A_k: $E_k \triangleq A_k R_k$. Totally differentiate and set $\mathrm{d}E_k = 0$ to hold the production of energy services fixed: $0 = R_k \mathrm{d}A_k + A_k \mathrm{d}R_k$. The energy resource savings from a 1% improvement in A_k become:

$$Savings^{eng} \triangleq -A_k \frac{\mathrm{d}E_k}{\mathrm{d}A_k} = R_k.$$

The engineering calculation predicts that a 1% improvement in the efficiency of energy conversion leads to a 1% reduction in energy use.

Economists have long noted that improving the efficiency of energy conversion lowers the relative price of energy inputs, which leads profit- or utility-maximizing agents to increase their use of energy inputs. This substitution towards energy inputs is called rebound and is often analyzed in a partial equilibrium setting in which the prices of energy inputs to energy production, of non-energy inputs to consumption-good production, and of consumption good outputs are held fixed.[11] Use equation (10) to write $R_k(A_k, p_k, p_R, p_X)$. The partial equilibrium calculation does not allow p_k, p_R, or p_X to change with A_k, so that

$$Savings^{PE} \triangleq -A_k \frac{\partial R_k(A_k, p_k, p_R, p_X)}{\partial A_k} = (1 - \sigma_k) R_k.$$

Partial equilibrium rebound, as a fraction of the no-rebound or "engineering" savings from an improvement in energy efficiency, is then

$$Rebound^{PE} \triangleq \frac{Savings^{eng} - Savings^{PE}}{Savings^{eng}} = \sigma_k.$$

Partial equilibrium rebound is equal to σ_k, a result familiar from many studies (e.g., Saunders, 1992; Sorrell and Dimitropoulos, 2008). This analysis suggests that a 1% improvement in energy efficiency most strongly reduces energy use when it occurs in a sector with high energy use (large R_k) and a small elasticity of substitution between energy and non-energy inputs (small σ_k). Partial equilibrium rebound goes to zero as $\sigma_k \to 0$, in which case the firm has a Leontief production function and so has no scope to adjust its input mix. Following Saunders (2008), I say that "super-conservation" occurs when rebound is negative and that "backfire" occurs when rebound is greater than 1 (so that improving efficiency actually increases energy use).

[11] In some settings, energy services are modeled as a direct input to utility. The appendix derives a partial equilibrium direct income effect that is equal to the budget share of energy.

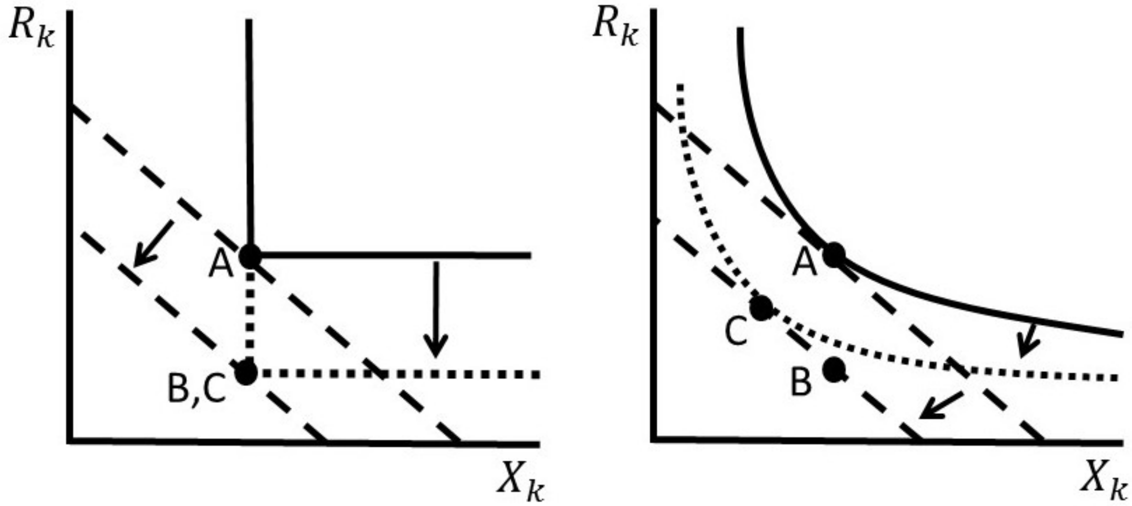
Figure 1 graphically describes the partial equilibrium effect. It plots the combinations of R_k and X_k that generate a given quantity of output c_k, and it also plots the isocost line (dashed). Prior to the improvement in energy efficiency, the firm's profit-maximizing point is at point A, where the solid isoquant is tangent to the isocost line. Improving the efficiency of energy conversion technology changes the isoquant to the dotted line. The improvement in efficiency shifts the frontier by more in regions of heavy energy use. The engineering calculation of the change in energy use holds X_k fixed, so it finds the point B on the altered isoquant that is directly below point A. Point B is on a lower isocost line. The vertical distance between points A and B defines the energy resource savings. The partial equilibrium calculation recognizes that the firm will reoptimize its input mix to return to a point of tangency with the isocost line. As $\sigma_k \to 0$ (left), point B is also the point of tangency with the altered isoquant. For larger σ_k (right), the new point of tangency (labeled C) is to the left and above point B. The vertical distance between points B and C determines partial equilibrium rebound. As σ_k becomes larger, the isoquant becomes flatter and the vertical distance between points B and C grows. For $\sigma_k > 1$ (not pictured), point C is above point A, in which case rebound is greater than 100% (a case of "backfire"). The general equilibrium analysis will account for how the firm moves to a different isocost line in order to restore the zero-profit condition and will account for how changes in factor prices rotate the isocost line.

5 General Equilibrium Rebound

The previous, partial equilibrium analysis held factor and output prices fixed and asked how energy use changed in the sector with improved energy efficiency. However, pollution and other externalities are often related to the total change in energy use, including changes in other sectors induced by changes in factor and output prices.[12] I now consider this total, general equilibrium change in energy use from an improvement in some A_k. The appendix considers the general equilibrium consequences of an improvement in some χ_k and of an improvement in sector k's total factor productivity.

Market-clearing in the energy supply sector required that $R = \sum_{i=1}^{N+1} R_i$. Equation (9) gives R_i as a function of A_i, p_i, p_R, and p_X for $i \in \{1, ..., N\}$, and equation (7) gives R_{N+1} as a function of R and A_{N+1}. Equation (4) gives p_i as a function of χ_i, A_i, p_R, and p_X $i \in \{1, ..., N\}$. Equation (5) gives p_R as a function of χ_{N+1}, A_{N+1}, and p_X. And equation (14) gives p_X as a function of each A_i, of each χ_i, and of R. Now let variable y indicate some A_k

[12]Many authors have observed that changes in energy use do not map into changes in welfare: rebound effects are not necessarily bad from a welfare perspective. However, those same authors nonetheless often focus on changes in energy use because that question has been of interest to historians, to policymakers, and to environmental economists concerned with externalities.



(a) $\sigma_k = 0$ (b) $\sigma_k \in (0, 1)$

Figure 1: Improving the quality of energy conversion technology A_k changes the isoquants of sector k's production technology from the solid line to the dotted line, with the dashed lines indicating the isocost lines. Point A indicates the initial equilibrium. The gap between point A and point B along the y-axis is the no-rebound calculation of energy resource savings, and the gap between point B and point C along the y-axis defines partial equilibrium rebound.

or some χ_k, for $k \in \{1, ..., N+1\}$. We have:

$$\frac{\mathrm{d}R}{\mathrm{d}y} = \sum_{i=1}^{N} \left[\frac{\partial R_i}{\partial y} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial y} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R} \right) \frac{\partial p_R}{\partial y} \right.$$
$$\left. + \left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_X} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R} \right) \frac{\partial p_R}{\partial p_X} + \frac{\partial R_i}{\partial p_X} \right) \left(\frac{\partial p_X}{\partial y} + \frac{\partial p_X}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y} \right) \right]$$
$$+ \frac{\partial R_{N+1}}{\partial y} + \frac{\partial R_{N+1}}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y}.$$

Solving for $\mathrm{d}R/\mathrm{d}y$, we have:

$$\frac{\mathrm{d}R}{\mathrm{d}y} = \frac{\sum_{i=1}^{N} \left[\frac{\partial R_i}{\partial y} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial y} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R} \right) \frac{\partial p_R}{\partial y} + \left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_X} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R} \right) \frac{\partial p_R}{\partial p_X} + \frac{\partial R_i}{\partial p_X} \right) \frac{\partial p_X}{\partial y} \right] + \frac{\partial R_{N+1}}{\partial y}}{1 - \sum_{i=1}^{N} \left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_X} + \left(\frac{\partial R_i}{\partial p_R} + \frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R} \right) \frac{\partial p_R}{\partial p_X} + \frac{\partial R_i}{\partial p_X} \right) \frac{\partial p_X}{\partial R} - \frac{\partial R_{N+1}}{\partial R}}.$$

Let $\theta_{a,b}$ represent the elasticity of a with respect to b: $\theta_{a,b} \triangleq [b/a][\partial a/\partial b]$. We then have:

$$\theta_{R,y} = \frac{\overbrace{\sum_{i=1}^{N+1} \theta_{R_i,y} R_i}^{\text{PE effect}} + \overbrace{\sum_{i=1}^{N} \theta_{R_i,p_i}\theta_{p_i,y} R_i}^{\text{Output price effect}} + \overbrace{\sum_{i=1}^{N} \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R} \right) \theta_{p_R,y} R_i}^{\text{Resource supply effect}}}{R - \sum_{i=1}^{N} \left(\theta_{R_i,p_i}\theta_{p_i,p_X} + \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R} \right) \theta_{p_R,p_X} + \theta_{R_i,p_X} \right) \theta_{p_X,R} R_i - \theta_{R_{N+1},R} R_{N+1}}$$
$$+ \frac{\overbrace{\sum_{i=1}^{N} \theta_{R_i,p_X}\theta_{p_X,y} R_i}^{\text{Income effect}} + \overbrace{\sum_{i=1}^{N} \left(\theta_{R_i,p_i}\theta_{p_i,p_X} + \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R} \right) \theta_{p_R,p_X} \right) \theta_{p_X,y} R_i}^{\text{Input cost effect}}}{R - \sum_{i=1}^{N} \left(\theta_{R_i,p_i}\theta_{p_i,p_X} + \left(\theta_{R_i,p_R} + \theta_{R_i,p_i}\theta_{p_i,p_R} \right) \theta_{p_R,p_X} + \theta_{R_i,p_X} \right) \theta_{p_X,R} R_i - \theta_{R_{N+1},R} R_{N+1}}.$$

The denominator is equal to $R\frac{\sum_{i=1}^{N} R_i}{R}\frac{X}{\sum_{i=1}^{N} X_i}$. It accounts for how changes in total energy production affect the energy-producing sector's demand for energy resources and for the labor-capital aggregate (which manifests itself as a change in p_X). If energy resources were somehow produced without using either energy or the labor-capital aggregate ($R_{N+1} = X_{N+1} = 0$), then the denominator would reduce to R, which derives from the definition of the elasticity. Through the denominator, increasing R_{N+1}/R (i.e., reducing $\sum_{i=1}^{N} R_i/R$) increases the magnitude of $\theta_{R,y}$ because expansions and contractions of the energy supply sector amplify changes in energy demand from other sectors, and increasing X_{N+1}/X (i.e., reducing $\sum_{i=1}^{N} X_i/X$) reduces the magnitude of $\theta_{R,y}$ because it acts like making resource supply less elastic.[13] Substituting, we find:

$$\theta_{R,y} = \frac{1}{\sum_{i=1}^{N} R_i} \frac{\sum_{i=1}^{N} X_i}{X} \Bigg\{ \overbrace{\sum_{i=1}^{N+1} \theta_{R_i,y} R_i}^{\text{PE effect}} - \overbrace{\sum_{i=1}^{N} (\epsilon - \sigma_i) \theta_{p_i,y} R_i}^{\text{Output price effect}} - \overbrace{\sum_{i=1}^{N} \left(\sigma_i + (\epsilon - \sigma_i)\alpha_{Ri}\right) \theta_{p_R,y} R_i}^{\text{Resource supply effect}}$$
$$+ \overbrace{\underbrace{\theta_{p_X,y} \sum_{i=1}^{N} R_i}_{\text{Income effect}} \quad \underbrace{-\epsilon\, \theta_{p_X,y} \sum_{i=1}^{N} R_i}_{\text{Input cost effect}}}^{\text{Wage effect}} \Bigg\}.$$

The five terms in braces determine total rebound. The first term captures the partial equilibrium channels: it reflects how an improvement in some technology y affects factor use at constant prices. The second term captures how changes in technology affect factor demand via output prices. Improving any sector's technology decreases its output price ($\theta_{p_i,y} \leq 0$), which increases energy use if and only if $\epsilon > \sigma_i$ (from equation (9), $-\theta_{R_i,p_i} = \epsilon - \sigma_i$ for $i \in \{1, ..., N\}$). The lower output price reduces producers' demand for inputs but also leads households to substitute towards good i. The first effect is controlled by σ_i and the second is controlled by ϵ. When $\sigma_i > \epsilon$, factor demand in sector i falls as the output price falls, and when $\epsilon > \sigma_i$, factor demand in sector i increases as the output price falls.

The third term is relevant when we consider improvements in the technology used to produce energy. These improvements reduce the cost of energy inputs to consumption good sectors ($\theta_{p_R,y} \leq 0$). This reduction in the cost of energy directly works to increase energy use ($-\theta_{R_i,p_R} = \sigma_i$ for $i \in \{1, ..., N\}$), and by reducing the output price ($-\theta_{p_i,p_R} < 0$), the reduction in the cost of energy indirectly increases energy use if and only if $\epsilon > \sigma_i$.

The final line captures wage effects. Its first term is an income effect: a 1% increase in the wage increases demand for energy by 1% because preferences are homothetic ($\theta_{R_i,p_X} = 1$). The other term reflects how a higher wage reduces energy demand by raising the cost of producing each consumption good i, both directly and by raising the cost of producing its energy inputs. The latter channel works like the resource supply effect, but in reverse. On net, the input cost effect dominates the income effect and a higher wage therefore reduces energy use.[14]

Define

$$Savings^{GE} \triangleq -A_k \frac{\mathrm{d}R}{\mathrm{d}A_k}.$$

[13]As we will see in the analysis of super-conservation, the effect of R_{N+1}/R is the multiplier effect discussed in Turner (2009). The effect of X_{N+1}/X is related to discussions of the "energy price effect" in Borenstein (2015) and the "macroeconomic price effect" in Gillingham et al. (2016).

[14]Intuitively, making the labor-capital aggregate relatively scarcer must constrain energy demand, not amplify energy demand.

Following Saunders (2008), I say that backfire occurs when $\theta_{R,A_k} > 0$ and that super-conservation occurs when $Savings^{GE} > Savings^{eng}$.

5.1 Improved energy efficiency in a consumption good sector

Now consider the consequences of improving A_k in some sector $k \in \{1, ..., N\}$:

$$\theta_{R,A_k} = \frac{\sum_{i=1}^N X_i}{X} \Bigg\{ \overbrace{(\sigma_k - 1)\frac{R_k}{\sum_{i=1}^N R_i}}^{\text{PE effect}} + \overbrace{(\epsilon - \sigma_k)\alpha_{Rk}\frac{R_k}{\sum_{i=1}^N R_i}}^{\text{Output price effect}} \overbrace{- (\epsilon - \sigma_k)\alpha_{Rk}\frac{X_k}{\sum_{i=1}^N X_i}}^{\text{Wage effect}} \Bigg\},$$

where I bring the outer $1/\sum_{i=1}^N R_i$ into the braces. The partial equilibrium effect increases energy use if and only if $\sigma_k > 1$, in accord with the analysis in Section 4. The output price effect and the wage effect work in opposite directions and are both proportional to the value share of energy resources in sector k. When $\epsilon > \sigma_k$, demand for inputs in sector k increases as its output price falls, which works to increase energy use directly but also decreases energy use by raising the wage. These two general equilibrium channels then increase energy use when sector k is especially resource-intensive because the composition of household purchases becomes more resource-intensive and because the change in aggregate labor demand is small. If, instead, $\epsilon < \sigma_k$, then these two general equilibrium channels increase resource use when sector k is especially X-intensive because changes in sector k's marginal product of labor have an especially strong effect on the wage.

The following proposition considers a case in which each consumption good sector has the same production function.

Proposition 1. *Assume that either $N = 1$ or κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *The general equilibrium channels vanish, leaving $\theta_{R,A_k} = \frac{X_k}{X}(\sigma - 1)$.*
2. *Backfire occurs if and only if $\sigma > 1$.*
3. *If $R_{N+1}/X_{N+1} > R_k/X_k$, then there exists $\hat{\sigma} \in (0, 1)$ such that super-conservation occurs if and only if $\sigma < \hat{\sigma}$. If $R_{N+1}/X_{N+1} < R_k/X_k$, then super-conservation does not occur for any $\sigma > 0$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j/\sum_{i=1}^N R_i = X_j/\sum_{i=1}^N X_i = 1/N$ for all $j \in \{1, ..., N\}$. Substituting into θ_{R,A_k} and then using $\sum_{i=1}^N X_i = NX_k$ and $\sum_{i=1}^N R_i = NR_k$ yields the first part of the proposition. The second part of the proposition follows from that result and the definition of backfire as occurring if and only if $\theta_{R,A_k} > 0$. To establish the third part of the proposition, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\theta_{R,A_k} R > R_k \Leftrightarrow 0 > \frac{R_k}{R} + \frac{X_k}{X}(\sigma - 1) \Leftrightarrow \sigma < \frac{\frac{X_k}{X} - \frac{R_k}{R}}{\frac{X_k}{X}}.$$

The result follows straightforwardly from this expression, the assumption that $\sigma > 0$, and the fact that $X_k/X < R_k/R$ if and only if $R_{N+1}/X_{N+1} < R_k/X_k$. □

If $N = 1$, then there is no scope for substitution when input costs change, and if $N > 1$ but consumption good sectors are symmetric, then substitution does not affect energy use. In either case, we are left with the partial equilibrium effect. Backfire then arises if and only if $\sigma > 1$, as in Section 4.

However, even though only the partial equilibrium effect remains, θ_{R,A_k} is not identical to the pure partial equilibrium result, which would be $(\sigma - 1)R_k/R$. The difference arises because the general equilibrium analysis accounts for effects on energy supply. Whereas the partial equilibrium analysis of changes in R_k in Section 4 showed that super-conservation was impossible, we now find that super-conservation is possible if σ is small and energy production is especially energy-intensive. When σ is small, the partial equilibrium effect on sector k reduces sector k's demand for resources. That reduction in resource demand leads the energy-producing sector to contract, which in turn reduces demand for energy inputs in that sector. This multiplier effect amplifies the energy savings from any given reduction in energy demand in the consumption good sectors. When σ is sufficiently small, the total energy savings can be even greater than predicted by an engineering analysis.[15]

Now imagine that consumption good sectors differ only in their energy efficiency A. Let $\bar{A} \triangleq \sum_{i=1}^{N} A_i/N$ indicate the average A across the consumption good sectors. Refer to the combination of the output price and wage effects as the general equilibrium channels. We have:

Proposition 2. *Assume that κ_i, σ_i, and χ_i do not vary with i for $i \in \{1, ..., N\}$ and that $Var(A)$ is small relative to $\bar{A}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma > \epsilon$ and $A_k < \bar{A}$, then the general equilibrium channels are positive and $\theta_{R,A_k} > 0$.*
2. *If $\sigma \in (1, \epsilon)$ and $A_k > \bar{A}$, then the general equilibrium channels are positive and $\theta_{R,A_k} > 0$.*
3. *If $\sigma < 1$ and $A_k > \bar{A}$, then the general equilibrium channels are negative and $\theta_{R,A_k} < 0$.*

[15] This analysis clarifies the conditions under which the changes in energy suppliers' demand for energy inputs considered in Turner (2009, 664) can lead to super-conservation. Of course, this multiplier effect works the other way as well, as it also amplifies backfire. Finally, note that Turner (2009) also discusses disinvestment effects, which can arise only in a dynamic model with imperfectly adjustable capital stocks.

Proof. Analyze the general equilibrium channels. From equations (12) and (13), we have:

$$\begin{aligned}\frac{R_k}{\sum_{i=1}^N R_i}-\frac{X_k}{\sum_{i=1}^N X_i}=&\left\{\frac{A_k^{\sigma-1}Z_k}{\sum_{i=1}^N A_i^{\sigma-1}Z_i}-\frac{Z_k}{\sum_{i=1}^N Z_i}\right\}\\ =&Z_k\left\{\frac{A_k^{\sigma-1}\sum_{i=1}^N Z_i-\sum_{i=1}^N A_i^{\sigma-1}Z_i}{\left[\sum_{i=1}^N A_i^{\sigma-1}Z_i\right]\left[\sum_{i=1}^N Z_i\right]}\right\}\\ =&\frac{NZ_k}{\left[\sum_{i=1}^N A_i^{\sigma-1}Z_i\right]\left[\sum_{i=1}^N Z_i\right]}\left\{\left[A_k^{\sigma-1}-\frac{1}{N}\sum_{i=1}^N A_i^{\sigma-1}\right]\frac{1}{N}\sum_{i=1}^N Z_i-Cov\left(A_i^{\sigma-1},Z_i\right)\right\},\end{aligned}$$

where

$$Z_i\triangleq\left(\chi_i^{\sigma_i-1}\kappa_i^{\sigma_i}+A_i^{\sigma_i-1}(1-\kappa_i)^{\sigma_i}\left[\frac{\chi_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}}\right)^{\frac{\epsilon-\sigma_i}{\sigma_i-1}}.$$

Using a second-order Taylor series expansion around $A_i=\bar{A}$, we have:

$$\frac{1}{N}\sum_{i=1}^N A_i^{\sigma-1}\approx\bar{A}^{\sigma-1}+\frac{1}{2}(\sigma-1)(\sigma-2)\bar{A}^{\sigma-3}Var(A).$$

Using a second-order Taylor series expansion of Z_i around $A_i=\bar{A}$, we have:

$$\frac{1}{N}\sum_{i=1}^N A_i^{\sigma-1}\approx\bar{Z}+\frac{1}{2}\bar{Z}\bar{\alpha}_R\bar{A}^{-2}Var(A)\Big[(\epsilon-2\sigma+1)\bar{Z}\bar{\alpha}_R+(\sigma-2)(\epsilon-\sigma)\Big],$$

where $\bar{Z}$ indicates Z_i evaluated at $A_i=\bar{A}$ and $\bar{\alpha}_R$ indicates the value share of energy in consumption good production evaluated at $\bar{A}$. And using first-order Taylor expansions of $A_i^{\sigma-1}$ and Z_i around $A_i=\bar{A}$, we have:

$$Cov\left(A_i^{\sigma-1},Z_i\right)\approx(\epsilon-\sigma)(\sigma-1)\bar{Z}\bar{A}^{\sigma-3}\bar{\alpha}_R Var(A).$$

Substituting and using the assumption that $Var(A)/\bar{A}$ is small, we find:

$$\frac{R_k}{\sum_{i=1}^N R_i}-\frac{X_k}{\sum_{i=1}^N X_i}\approx\frac{NZ_k\bar{Z}}{\left[\sum_{i=1}^N A_i^{\sigma-1}Z_i\right]\left[\sum_{i=1}^N Z_i\right]}\left\{A_k^{\sigma-1}-\bar{A}^{\sigma-1}\right\}. \qquad (15)$$

If $\sigma>\epsilon$, then (15) is negative if $A_k<\bar{A}$. The first result follows from noting that all terms in θ_{R,A_k} are positive if $\sigma>\epsilon$ and $\frac{R_k}{\sum_{i=1}^N R_i}-\frac{X_k}{\sum_{i=1}^N X_i}<0$.

If $\sigma \in (1, \epsilon)$, then (15) is positive if $A_k > \bar{A}$. The second result follows from noting that all terms in θ_{R,A_k} are positive if $\sigma \in (1, \epsilon)$ and $\frac{R_k}{\sum_{i=1}^N R_i} - \frac{X_k}{\sum_{i=1}^N X_i} > 0$.

Finally, if $\sigma < 1$, then (15) is negative if $A_k > \bar{A}$. The third result follows from noting that all terms in θ_{R,A_k} are negative if $\sigma < 1$ and $\frac{R_k}{\sum_{i=1}^N R_i} - \frac{X_k}{\sum_{i=1}^N X_i} < 0$.

□

The proposition identifies sufficient conditions under which greater efficiency either avoids or generates backfire. Improved efficiency is likely to avoid backfire when σ is small and the improvement occurs in a sector that has above-average efficiency. As households substitute towards this sector with above-average efficiency, the general equilibrium channels reinforce the partial equilibrium channel.[16] If $\sigma > 1$, then the conditions for backfire depend on the relationship between σ and ϵ because these two parameters determine the implications of changes in consumption good prices for energy demand. When $\sigma \in (1, \epsilon)$, sector k's factor demand increases as its output price falls. The output price falls when the effect of improved A_k outweighs the effect of a higher wage, which occurs when sector k is especially energy-intensive (because $A_k > \bar{A}$). In this case, the general equilibrium channels reinforce the partial equilibrium channel and backfire is unambiguous. Similar logic applies when $\sigma > \epsilon$, noting that now sector k's factor demand falls as its output price falls.

Now let consumption good sectors differ only in their efficiency χ of non-energy inputs, with $\bar{\chi} \triangleq \sum_{i=1}^N \chi_i / N$ indicating the average χ across the consumption good sectors. We have the following corollary:

Corollary 3. *Assume that κ_i, σ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$ and that $Var(\chi)$ is small relative to $\bar{\chi}$. Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma > \epsilon$ and $\chi_k > \bar{\chi}$, then the general equilibrium channels are positive and $\theta_{R,A_k} > 0$.*
2. *If $\sigma \in (1, \epsilon)$ and $\chi_k < \bar{\chi}$, then the general equilibrium channels are positive and $\theta_{R,A_k} > 0$.*
3. *If $\sigma < 1$ and $\chi_k < \bar{\chi}$, then the general equilibrium channels are negative and $\theta_{R,A_k} < 0$.*

Proof. Follows from the proof of Proposition 2, noting that now

$$\frac{R_k}{\sum_{i=1}^N R_i} - \frac{X_k}{\sum_{i=1}^N X_i} = \frac{-N Z_k}{\left[\sum_{i=1}^N \chi_i^{\sigma-1} Z_i\right]\left[\sum_{i=1}^N Z_i\right]} \left\{ \left[\chi_k^{\sigma-1} - \frac{1}{N} \sum_{i=1}^N \chi_i^{\sigma-1} \right] \frac{1}{N} \sum_{i=1}^N Z_i - Cov\left(\chi_i^{\sigma-1}, Z_i\right) \right\}$$

and

$$Cov\left(\chi_i^{\sigma-1}, Z_i\right) \approx (\epsilon - \sigma)(\sigma - 1)\bar{Z}\bar{\chi}^{\sigma-3}\bar{\alpha}_X Var(\chi).$$

□

[16] We can now also obtain super-conservation for a new reason: the general equilibrium channels can be negative when households substitute away from sectors that are more energy-intensive.

Finally, consider two corner cases: one in which sector k uses only energy resources ($\kappa \to 0$) and one in which sector k does not use any energy resources ($\kappa \to 1$).

Proposition 4. *Consider an improvement in A_k for some $k \in \{1, ..., N\}$.*

1. *If $N = 1$, then $\theta_{R,A_k} \to 0$ as $\kappa_k \to 0$. If $N > 1$, then $\theta_{R,A_k} > 0$ as $\kappa_k \to 0$.*
2. *$\theta_{R,A_k} \to 0$ as $\kappa_k \to 1$.*

Proof. From equation (12), $X_k \to 0$ as $\kappa_k \to 0$. We also then have $\alpha_{Rk} \to 1$ as $\kappa_k \to 0$. Therefore $\theta_{R,A_k} \to \frac{\sum_{i=1}^N X_i}{X} \frac{R_k}{\sum_{i=1}^N R_i}[\epsilon - 1]$ as $\kappa_k \to 0$. If $N = 1$, then $\sum_{i=1}^N X_i = 0$, but if $N > 1$, then $\sum_{i=1}^N X_i > 0$. The first result follows from noting that $\epsilon > 1$.

From equation (13), $R_k \to 0$ as $\kappa_k \to 1$. We also then have $\alpha_{Rk} \to 0$ as $\kappa_k \to 1$. The second result follows. □

As sector k becomes highly energy-intensive (κ_k becomes small), the wage effects become small because sector k doesn't use much of the labor-capital aggregate. Further, because the value share of energy goes to 1, the output price p_k declines by an especially large amount. The resulting reduction in demand for R_k exactly offsets the partial equilibrium substitution towards R_k. We are left with the engineering savings and the effect of household substitution towards consumption good k. The latter always dominates when there are multiple consumption goods. If, on the other hand, sector k does not use any energy resources to begin with (κ_k is large), then an improvement in A_k is of limited importance and we have no change in use of energy resources.

5.2 Improved energy efficiency in the energy-producing sector

Next consider the consequences of improving A_{N+1}:

$$\theta_{R,A_{N+1}} = \frac{\sum_{i=1}^N X_i}{X}\Bigg\{ \overbrace{(\sigma_{N+1} - 1)\frac{R_{N+1}}{\sum_{i=1}^N R_i}}^{\text{PE effect}} + \overbrace{\frac{R_{N+1}}{\sum_{i=1}^N R_i} \sum_{i=1}^N (\sigma_i + (\epsilon - \sigma_i)\alpha_{Ri}) \frac{R_i}{\sum_{j=1}^N R_j}}^{\text{Resource supply effect}}$$
$$+ \overbrace{\frac{R_{N+1}}{\sum_{i=1}^N R_i}\left(\sigma_{N+1}\frac{X_{N+1}}{\sum_{j=1}^N X_j} - \sum_{i=1}^N (\epsilon - \sigma_i)\alpha_{Ri}\frac{X_i}{\sum_{j=1}^N X_j}\right)}^{\text{Wage effect}} \Bigg\}.$$

The partial equilibrium effect is the same as in Section 5.1. However, the general equilibrium effects are quite different. First, we now have a resource supply effect arising because the improvement in efficiency reduces the price of energy resources. This price reduction increases use of energy resources because consumption good producers substitute towards the newly cheap energy inputs (controlled by σ_i) and because the price of each consumption good falls (with implications determined by $\epsilon - \sigma_i$, as described earlier). The resource supply effect increases energy resource use if $\epsilon > \sigma_i$ for all $i \in \{1, ..., N\}$. Second, the reduced cost of energy resources tends to increase the marginal product of the labor-capital aggregate in each consumption good sector and thereby raise the wage. This higher wage leads energy producers to substitute towards the energy input (controlled by σ_{N+1}) and also works to make production more expensive in each consumption good sector, which increases demand for energy resources if and only if $\sigma_i > \epsilon$.

The following propositions analyze especially tractable special cases:

Proposition 5. *Assume that either $N = 1$ or κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in A_{N+1}.*

1. *The general equilibrium channels are positive.*
2. *Backfire occurs if $\sigma + \sigma_{N+1} > 1$.*
3. *If $R_{N+1}/X_{N+1} > R_i/X_i$, then super-conservation can occur only if $\sigma + \sigma_{N+1} < 1$. If $R_{N+1}/X_{N+1} < R_i/X_i$, then super-conservation does not occur for any $\sigma, \sigma_{N+1} > 0$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^N R_i = X_j / \sum_{i=1}^N X_i = 1/N$ for all $j \in \{1, ..., N\}$. The first part of the proposition follows. Substituting into $\theta_{R,A_{N+1}}$, we have:

$$\theta_{R,A_{N+1}} = \frac{\sum_{i=1}^N X_i}{X} \frac{R_{N+1}}{\sum_{i=1}^N R_i} \left\{ \sigma_{N+1} + \sigma - 1 + \sigma_{N+1} \frac{X_{N+1}}{\sum_{i=1}^N X_i} \right\}.$$

The second part of the proposition follows. To prove the third part, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\theta_{R,A_{N+1}} R > R_{N+1} \Leftrightarrow 0 > \frac{R_{N+1}}{R} + \theta_{R,A_{N+1}}.$$

As $\sigma, \sigma_{N+1} \to 0$, $\theta_{R,A_{N+1}} \to -\frac{\sum_{i=1}^N X_i}{X} \frac{R_{N+1}}{\sum_{i=1}^N R_i}$ from above and $Savings^{GE} > Savings^{eng}$ if and only if $0 > 1 - \frac{\sum_{i=1}^N X_i}{X} \frac{R}{\sum_{i=1}^N R_i}$. The third part of the proposition follows from noting that $\sum_{i=1}^N X_i = N X_i$ and $\sum_{i=1}^N R_i = N R_i$ and from the fact that $X_i/X < R_i/R$ if and only if $R_{N+1}/X_{N+1} < R_i/X_i$. .

□

Proposition 6. *Assume that, for $i \in \{1, ..., N\}$, each $\sigma_i \approx 1$ and κ_i does not vary with i. Consider an improvement in A_{N+1}.*

1. *The general equilibrium channels are positive.*

2. Backfire occurs.

Proof. The value share of energy in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. From $\theta_{R,A_{N+1}}$, the general equilibrium channels are proportional to $\sum_{j=1}^{N} X_j + \sigma_{N+1} X_{N+1}$. The first part of the proposition follows. We also have

$$\theta_{R,A_{N+1}} \approx \frac{R_{N+1}}{\sum_{i=1}^{N} R_i} \sigma_{N+1}.$$

The second part of the proposition follows. □

In contrast to Proposition 1, the general equilibrium channels in Proposition 5 are positive even when $N = 1$ and even when consumption good sectors are symmetric. The reason is that general equilibrium channels now account for the effect of an outward shift in energy supply on every consumption good producer's input cost. As the cost of energy falls, consumption good firms substitute towards the newly cheap energy input. As a result, the sufficient condition for backfire becomes less demanding than in Proposition 1 and the necessary condition for super-conservation becomes even more demanding than in Proposition 1. As before, backfire occurs when the partial equilibrium effect favors backfire, but now backfire can also occur when the partial equilibrium effect is as negative as can be (i.e., even as $\sigma_{N+1} \to 0$). And Proposition 6 shows that backfire can now occur even when consumption good production is Cobb-Douglas. Resources and the labor-capital aggregate must be strong complements in every sector that uses energy if improvements in the efficiency of energy production are not to backfire.

6 The Direction of Technical Change

Thus far we have considered the consequences of more efficient technology. I now consider which sector would tend to attract research effort and I analyze the conditions under which endogenous innovation will tend to increase or reduce total resource use. I endogenize innovation by extending the setting to allow for directed technical change in the fashion of Acemoglu (2002, 2007): innovations will be driven by the market value of patents to improved technologies. I consider which sector a marginal research firm would target, with the predetermined technology parameters A reflecting both the incoming quality of technology and any pre-existing allocation of research effort.

Modify the production function for consumption goods to include an inner nest in which energy is combined with machines, which are produced according to the Dixit-Stiglitz model of monopolistic competition. The number of machines of variety j produced in sector i is z_{ij}, with the continuum of varieties indexed on the unit interval. Production of sector i's consumption good becomes:

$$c_i = \left(\kappa_i [\chi_i X_i]^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\kappa_i) \left[R_i^{1-\gamma} \int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}$$

for $\gamma \in (0,1)$. We recover the previous setting as $\gamma \to 0$. Machine producers have a monopoly on their variety j, sell their machines at price p_{ij}, and have marginal cost of ζ units of energy resources. Each household own a share in each machine producer. Research firms choose which sector to target and are randomly allocated to a variety within that sector, as in Acemoglu et al. (2012). They succeed in innovating with probability η, in which case they improve the quality of their machine variety to $(1+\rho)A_{ij}$ and receive a patent for its production.

6.1 Equilibrium

Consumption good firms now solve:

$$\max_{X_i, R_i, z_{ij}} \left\{ p_i \left(\kappa_i [\chi_i X_i]^{\frac{\sigma_i - 1}{\sigma_i}} + (1-\kappa_i) \left[R_i^{1-\gamma} \int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}} - p_X X_i - p_R R_i - \int_0^1 p_{ij} z_{ij} \, \mathrm{d}j \right\}.$$

The first-order conditions become:

$$\begin{aligned} p_X =& p_i \kappa_i \chi_i^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{X_i}{c_i} \right)^{-\frac{1}{\sigma_i}}, \\ p_R =& p_i (1-\kappa_i)(1-\gamma) \left[\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j \right]^{\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{R_i}{c_i} \right)^{-\frac{1}{\sigma_i}} R_i^{-\gamma \frac{\sigma_i - 1}{\sigma_i}}, \\ p_{ij} =& p_i (1-\kappa_i) \gamma R_i^{(1-\gamma)\frac{\sigma_i - 1}{\sigma_i}} \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} A_{ij}^{1-\gamma} z_{ij}^{\gamma - 1}. \end{aligned} \tag{16}$$

Rearranging the latter condition, we find

$$z_{ij} = \left[\frac{p_i}{p_{ij}} (1-\kappa_i) \gamma \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i - 1}{\sigma_i}} A_{ij}.$$

Demand is isoelastic, so the monopolist chooses a constant mark-up over marginal cost: $p_{ij} = \zeta p_R/\gamma$.[17] Substituting into z_{ij}, profit-maximizing machine production is:

$$z_{ij} = \left[p_i \frac{\gamma^2}{\zeta p_R}(1-\kappa_i) \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} A_{ij}.$$

Note that:

$$\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j = \int_0^1 A_{ij}^{1-\gamma} \left(\left[p_i \frac{\gamma^2}{\zeta p_R}(1-\kappa_i) \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} A_{ij} \right)^{\gamma} \mathrm{d}j.$$

Rearranging, we find:

$$\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j = \left(\left[p_i \frac{\gamma^2}{\zeta p_R}(1-\kappa_i) \left(\frac{1}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\gamma \frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}} A_i^{\frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}}, \tag{17}$$

where A_i is the average technology in sector i. Equation (16) then implies:

$$R_i = p_R^{-\sigma_i}(1-\kappa_i)^{\sigma_i}(1-\gamma)^{\gamma+\sigma_i(1-\gamma)} \left(\frac{\gamma^2}{\zeta} \right)^{(\sigma_i-1)\gamma} A_i^{(\sigma_i-1)(1-\gamma)} p_i^{\sigma_i} c_i. \tag{18}$$

Demand for R_{N+1} is unchanged from equation (7), and equation (5) still defines p_R as a function of p_X. Machine production in sector i is:

$$\begin{aligned}
\int_0^1 z_{ij} \mathrm{d}j &= \left[p_i \frac{\gamma^2}{\zeta p_R}(1-\kappa_i) \left(\frac{\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j}{c_i} \right)^{-\frac{1}{\sigma_i}} \right]^{\frac{1}{1-\gamma}} R_i^{\frac{\sigma_i-1}{\sigma_i}} A_i \\
&= \left[p_i \frac{\gamma^2}{\zeta p_R}(1-\kappa_i) \right]^{\frac{\sigma_i}{\gamma+\sigma_i(1-\gamma)}} c_i^{\frac{1}{\gamma+\sigma_i(1-\gamma)}} R_i^{\frac{\sigma_i-1}{\sigma_i} \frac{\sigma_i(1-\gamma)}{\gamma+\sigma_i(1-\gamma)}} A_i^{\frac{(1-\gamma)(\sigma_i-1)}{\gamma+\sigma_i(1-\gamma)}} \\
&= \frac{\gamma^2}{\zeta} \frac{1}{1-\gamma} R_i,
\end{aligned} \tag{19}$$

where I substitute from equations (17) and (18) and simplify. Finally, producers of machine variety j in sector i obtain profits of

$$\pi_{ij}(A_{ij}) \triangleq \frac{\zeta p_R}{\gamma} z_{ij} - \zeta p_R z_{ij} = \frac{1-\gamma}{\gamma} \zeta p_R z_{ij}.$$

[17] In line with much literature, the monopolist does not account for its effect on $\int_0^1 A_{ij}^{1-\gamma} z_{ij}^{\gamma} \, \mathrm{d}j$.

Substituting for p_R from equation (5), we have:

$$\pi_{ij}(A_{ij}) = \frac{1-\gamma}{\gamma}\zeta z_{ij}\left[\frac{\chi_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1}{1-\sigma_{N+1}}} p_X. \tag{20}$$

Consumption good firms' zero-profit condition becomes

$$p_i c_i = p_X X_i + p_R R_i + \int_0^1 p_{ij} z_{ij}\,\mathrm{d}j,$$

which yields

$$p_i = \left\{ p_X^{1-\sigma_i}\kappa_i^{\sigma_i}\chi_i^{\sigma_i-1} + \frac{1}{1-\gamma}p_R^{1-\sigma_i}(1-\kappa_i)^{\sigma_i}(1-\gamma)^{\gamma+\sigma_i(1-\gamma)}\left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i-1)\gamma} A_i^{(\sigma_i-1)(1-\gamma)}\right\}^{\frac{1}{1-\sigma_i}}.$$

Substituting for p_R from equation (5), we obtain:

$$\begin{aligned} p_i = & p_X\left\{\kappa_i^{\sigma_i}\chi_i^{\sigma_i-1} + \frac{1}{1-\gamma}(1-\kappa_i)^{\sigma_i}(1-\gamma)^{\gamma+\sigma_i(1-\gamma)}\left(\frac{\gamma^2}{\zeta}\right)^{(\sigma_i-1)\gamma}\right. \\ & \left.\left[\frac{\chi_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} A_i^{(\sigma_i-1)(1-\gamma)}\right\}^{\frac{1}{1-\sigma_i}}. \end{aligned} \tag{21}$$

The representative household's budget constraint is now $\sum_{i=1}^N p_i c_i \leq p_X X + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij})\,\mathrm{d}j$. The household budget constraint implies that $C = p_X X + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij})\,\mathrm{d}j$ in equilibrium. Aggregate household demand for good i becomes

$$\begin{aligned} c_i = & p_i^{-\epsilon}\left(p_X X + \sum_{i=1}^N \int_0^1 \pi_{ij}(A_{ij})\,\mathrm{d}j\right) \\ = & p_i^{-\epsilon} p_X\left(X + \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1}\kappa_{N+1}^{\sigma_{N+1}}}{1-A_{N+1}^{\sigma_{N+1}-1}(1-\kappa_{N+1})^{\sigma_{N+1}}}\right]^{\frac{1}{1-\sigma_{N+1}}}\sum_{i=1}^N \gamma R_i\right), \end{aligned} \tag{22}$$

where we substitute from equation (20) and then from equation (19).

Following the analysis in Section 3 but using equations (21) and (22), we obtain equilibrium demand for the labor-capital aggregate, for $i \in \{1, ..., N\}$:

$$X_i = \chi_i^{\sigma_i - 1} \kappa_i^{\sigma_i} p_X^{1-\epsilon} \left(X + \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1}{1-\sigma_{N+1}}} \sum_{i=1}^{N} \gamma R_i \right)$$
$$\left\{ \kappa_i^{\sigma_i} \chi_i^{\sigma_i - 1} + \frac{1}{1-\gamma} (1-\kappa_i)^{\sigma_i} (1-\gamma)^{\gamma + \sigma_i(1-\gamma)} \left(\frac{\gamma^2}{\zeta} \right)^{(\sigma_i - 1)\gamma} \right.$$
$$\left. \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{1-\sigma_i}{1-\sigma_{N+1}}} A_i^{(\sigma_i - 1)(1-\gamma)} \right\}^{\frac{\sigma_i - \epsilon}{1-\sigma_i}}. \tag{23}$$

Demand for X_{N+1} is unchanged from equation (6). Market-clearing for the labor-capital aggregate implies that

$$X = \sum_{i=1}^{N+1} X_i$$
$$= \sum_{i=1}^{N} X_i + \chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}} \left[\frac{\chi_{N+1}^{\sigma_{N+1}-1} \kappa_{N+1}^{\sigma_{N+1}}}{1 - A_{N+1}^{\sigma_{N+1}-1} (1-\kappa_{N+1})^{\sigma_{N+1}}} \right]^{\frac{\sigma_{N+1}}{1-\sigma_{N+1}}} R. \tag{24}$$

Market-clearing for energy resources implies that

$$R = \sum_{i=1}^{N+1} R_i + R_z,$$

where R_z is energy resource demand by machines. Using equation (19), we have:

$$R_z \triangleq \sum_{i=1}^{N} \zeta \int_0^1 z_{ij} \mathrm{d}j = \frac{\gamma^2}{1-\gamma} \sum_{i=1}^{N} R_i.$$

Market-clearing, R_z, and R_{N+1} from equation (7) imply:

$$\sum_{i=1}^{N} R_i = \frac{1-\gamma}{1-\gamma+\gamma^2} (R - R_{N+1}) = \frac{1-\gamma}{1-\gamma+\gamma^2} \left(1 - (1-\kappa_{N+1})^{\sigma_{N+1}} A_{N+1}^{\sigma_{N+1}-1} \right) R. \tag{25}$$

Using this in equations (23) and (24) allows us to explicitly solve for p_X as a function of R. Market-clearing for energy resources then yields a closed-form solution for R. As in the main analysis, the equilibrium is unique and total energy use increases linearly in X.

6.2 The effect of improved efficiency in some consumption good sector

Noting that R_i now depends on R through c_i, we have:

$$\begin{aligned}\frac{\mathrm{d}R}{\mathrm{d}y} =&\frac{\gamma^2-\gamma+1}{1-\gamma}\sum_{i=1}^{N}\left[\frac{\partial R_i}{\partial y}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial y}+\left(\frac{\partial R_i}{\partial p_R}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial y}\right.\\ &\left.+\left(\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_X}+\left(\frac{\partial R_i}{\partial p_R}+\frac{\partial R_i}{\partial p_i}\frac{\partial p_i}{\partial p_R}\right)\frac{\partial p_R}{\partial p_X}+\frac{\partial R_i}{\partial p_X}\right)\left(\frac{\partial p_X}{\partial y}+\frac{\partial p_X}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y}\right)+\frac{\partial R_i}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y}\right]\\ &+\frac{\partial R_{N+1}}{\partial y}+\frac{\partial R_{N+1}}{\partial R}\frac{\mathrm{d}R}{\mathrm{d}y}.\end{aligned}$$

Following the earlier analysis, we obtain:

$$\begin{aligned}\theta_{R,A_k} =&(1-\gamma)\frac{\sum_{i=1}^{N}X_i}{X}\Bigg\{\overbrace{(\sigma_k-1)\frac{R_k}{\sum_{i=1}^{N}R_i}}^{\text{PE effect}}\\ &+\overbrace{\sum_{i=1}^{N}(\epsilon-\sigma_i)(\alpha_{Ri}+\alpha_{zi})\frac{R_i}{\sum_{i=1}^{N}R_i}}^{\text{Output price effect}}\overbrace{-\sum_{i=1}^{N}(\epsilon-\sigma_i)(\alpha_{Ri}+\alpha_{zi})\frac{R_i}{\sum_{i=1}^{N}R_i}}^{\text{Wage effect}}\Bigg\},\end{aligned}$$

for $k \in \{1, ..., N\}$. The only two differences with respect to the expression in Section 5.1 are the leading $1-\gamma$ and the replacement of α_{Ri} with $\alpha_{Ri}+\alpha_{zi}$, which is the combined value share of energy resources and machines. It is easy to see that Propositions 1 and 2 apply here.[18] Therefore, when all consumption good sectors have the same production technology, the next research firm generates backfire if and only if $\sigma_i > 1$. And when consumption good sectors differ only in their average energy efficiency, additional research effort increases energy use if $\sigma_i \in (1, \epsilon)$ and researchers target sectors that are already relatively efficient but additional research effort reduces energy use if $\sigma_i < 1$ and researchers target sectors that are already relatively efficient.

6.3 Researchers' incentives

I now consider which sectors researchers target. A research firm's expected profit from targeting sector i is:

$$\Pi_i \triangleq \eta\int_0^1 \pi_{ij}((1+\rho)A_{ij})\,\mathrm{d}j.$$

[18]The only difference, which is irrelevant to the analysis in this section, is that the condition for super-conservation to be possible must be modified to reflect R_z.

The expected profit in sector i relative to sector j is Π_i/Π_j. Using equations (20) and (19), we find:

$$\frac{\Pi_i}{\Pi_j} = \frac{R_i}{R_j}.$$

Now assume that consumption good sectors differ only in the average quality of their energy technology. Substituting from equations (18) and (22), we have:

$$\frac{\Pi_i}{\Pi_j} = \left(\frac{p_i}{p_j}\right)^{\sigma-\epsilon} \left(\frac{A_i}{A_j}\right)^{(1-\gamma)(\sigma-1)}.$$

Defining p_i from equation (21) and holding p_X constant (because we are interested in the difference in A_i across sectors rather than in the effect of changing some sector's A_i), we have:

$$\begin{aligned}\frac{\mathrm{d}[\Pi_i/\Pi_j]}{\mathrm{d}A_i} =&(\sigma-1)(1-\gamma)A_i^{-1}\frac{\Pi_i}{\Pi_j} + (\sigma-\epsilon)p_i^{-1}\frac{\partial p_i}{\partial A_i}\frac{\Pi_i}{\Pi_j} \\ =&A_i^{-1}\frac{\Pi_i}{\Pi_j}(1-\gamma)\left[\sigma - 1 + (\epsilon-\sigma)(\alpha_{Ri}+\alpha_{zi})\right]. \end{aligned} \tag{26}$$

We now have the following proposition:

Proposition 7. *Assume that κ_i, σ_i, and χ_i do not vary with i for $i \in \{1, ..., N\}$ and that $Var(A)$ is small relative to $\bar{A}$. Without loss of generality, let $A_N \geq A_i$ for all $i \in \{1, .., N-1\}$. Consider a marginal increase in the number of research firms.*

1. *If $\sigma \in (1, \epsilon)$, then the additional innovation occurs in sector N and total energy use increases.*
2. *There exists $\hat{\sigma} < 1$ such that if $\sigma \in (\hat{\sigma}, 1)$ then the additional innovation occurs in sector N and total energy use decreases. $\hat{\sigma} = 0$ if $\epsilon \geq 1/(\alpha_{RN} + \alpha_{zN})$.*

Proof. If equation (26) is positive for all A_i with $i \in \{1, ..., N\}$, then the marginal research firm targets the sector with the largest A_i. Section 6.2 established that Proposition 2 still holds in this setting with monopolistically competitive machine production. Thus, under the given assumptions, innovation in the sector with the largest A_i increases total energy use if $\sigma \in (1, \epsilon)$ and decreases total energy use if $\sigma < 1$. Equation (26) is positive if $\sigma \in (1, \epsilon)$ and if σ is not too much smaller than 1. Noting that $\alpha_{Ri} + \alpha_{zi}$ decreases in A_i when $\sigma < 1$, equation (26) is positive for all $\sigma < 1$ if $\epsilon > 1/(\alpha_{RN} + \alpha_{zN})$. The proposition follows. □

Proposition 2 described how the effect of improved efficiency can depend on whether the sector that improves its efficiency was already more or less efficient than average, and we saw in Section 6.2 that this proposition still applies in our extension to monopolistically competitive machine production. We now see that profit-driven innovation will often direct itself towards the most efficient sector when $\sigma < \epsilon$. As a result, profit-driven innovation directs itself towards sectors that produce backfire when $\sigma \in (1, \epsilon)$, but profit-driven innovation often reduces energy use when $\sigma < 1$.

7 Conclusions

We have decomposed the general equilibrium consequences of improvements in energy efficiency. We have seen that these consequences are likely to be especially important when improvements occur in sectors with a large value share of energy and when they occur in sectors that produce the energy used as an input to consumption good production. General equilibrium consequences may be more important than partial equilibrium consequences when improvements arise in sectors that have strong complementarities between energy and non-energy inputs. Profit-driven innovation will tend to improve efficiency in sectors that generate backfire when energy and non-energy inputs are substitutes but will often work to reduce energy use when energy and non-energy inputs are complements. Other work has emphasized the roles of trade and labor supply (e.g., Allan et al., 2007; Hanley et al., 2009; Broberg et al., 2015), distortions such as non-marginal cost pricing (Borenstein, 2015), the costs imposed by policy mandates to increase efficiency (Fullerton and Ta, 2018), the introduction of new varieties of energy-using goods (Hart, 2018), the dynamics of capital allocation (Turner, 2009), and the dynamics of capital accumulation (Wei and Liu, 2017). Future work should integrate these and other features into the present setting. Future work should also use the present analysis to understand the results of computable general equilibrium models.

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Appendix

The first section connects the analysis to a case in which households purchase energy directly. The second section analyzes the implications for total energy use of improvements in the energy efficiency of every sector, in the productivity of the labor-capital aggregate, and in total factor productivity.

A Partial equilibrium analysis with energy services as a direct input to utility

I here provide a partial equilibrium analysis of a representative setting in which energy services are a direct input to utility (e.g., Chan and Gillingham, 2015), as with gasoline purchases or household appliances. Define an additional consumption good as $A_0 R_0$, where R_0 is energy purchased by the household and A_0 is the household's efficiency of energy conversion. There are still N standard consumption goods, produced in quantity c_i for $i \in \{1, ..., N\}$. The representative household's utility is now:

$$u(C), \text{ where } C \triangleq \left((A_0 R_0)^{\frac{\epsilon-1}{\epsilon}} + \sum_{i=1}^{N} c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

As before, utility $u(\cdot)$ is monotonically increasing in the consumption index C and $\epsilon > 1$ denotes the elasticity of substitution between the different varieties of consumption good. The price of resources is p_R and the price of each consumption good is p_i.

The representative household solves the following maximization problem:

$$\max_{\{c_i\}_{i=1}^N} u\left(\left((A_0 R_0)^{\frac{\epsilon-1}{\epsilon}} + \sum_{i=1}^{N} c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \right), \text{ subject to } p_R R_0 + \sum_{i=1}^{N} p_i c_i \leq p_X X.$$

Households will choose to sell all of their endowment. Letting λ be the shadow value of the budget constraint, the first-order condition for R_0 is

$$\frac{\lambda p_R}{u'(C)} = \left(\frac{R_0}{C} \right)^{-\frac{1}{\epsilon}} A_0^{\frac{\epsilon-1}{\epsilon}}.$$

As before, let P be the ideal price index, so that $p_R R_0 + \sum_{i=1}^{N} p_i c_i = P\,C$. Households' first-order condition for C implies that $P = u'(C)/\lambda$. I choose the price index as the numeraire: $P = 1$. The household budget constraint then implies that $C = p_X X$ in equilibrium. Aggregate household demand for resources becomes

$$R_0 = p_R^{-\epsilon} A_0^{\epsilon-1} C \quad \text{(A-1)}$$

$$= X\, p_R^{-\epsilon} A_0^{\epsilon-1} p_X. \quad \text{(A-2)}$$

Now consider how household resource use responds to an improvement in A_0, holding p_R, p_X, and each p_i fixed. Differentiating equation (A-2), we have:

$$\theta_{R_0,A_0} = \epsilon - 1.$$

This is the standard partial equilibrium result that resource use increases if and only if resources and other inputs are substitutes. Equation (A-2) holds total consumption fixed because it holds real income fixed. In order to see what has been called the "direct income effect" of improved A_0, instead differentiate equation (A-1) to obtain:

$$\theta_{R_0,A_0} = \epsilon - 1 + \frac{\partial C}{\partial A_0}\frac{A_0}{C} = \epsilon - 1 + \left(\frac{A_0 R_0}{C}\right)^{\frac{\epsilon-1}{\epsilon}}.$$

Using the first-order condition for R_0, it is easy to show that the household's budget share of resources is $\left(\frac{A_0 R_0}{C}\right)^{\frac{\epsilon-1}{\epsilon}}$. Define that budget share as $\alpha_{R_0} \in (0,1)$. We then have:

$$\theta_{R_0,A_0} = \epsilon - 1 + \alpha_{R_0}.$$

The direct income effect is α_{R_0}, which accounts for how improvements in A_0 make the household able to achieve a higher consumption index. Purchases of R_0 increase in proportion to their budget share. Indirect income effects (absent from that derivation) account for how purchases of other consumption goods also increase in proportion to their budget shares and thus increase demand for their resource inputs.

B Additional Results

B.1 Improved energy efficiency in every sector

I here consider the consequences of improving the energy efficiency of a process or engine that is used in all sectors. Formally, consider improving every A_i by 1%, for $i \in \{1, ..., N+1\}$:

$$\begin{aligned}\sum_{i=1}^{N+1} \theta_{R,A_i} =& \frac{\sum_{i=1}^N X_i}{X}\Bigg\{ \sum_{i=1}^{N+1}(\sigma_i - 1)\frac{R_i}{\sum_{j=1}^N R_j} + \frac{R_{N+1}}{\sum_{i=1}^N R_i}\left(\sum_{i=1}^N \sigma_i \frac{R_i}{\sum_{j=1}^N R_j} + \sigma_{N+1}\frac{X_{N+1}}{\sum_{i=1}^N X_i}\right) \\ &+ \frac{R}{\sum_{i=1}^N R_i}\sum_{i=1}^N (\epsilon - \sigma_i)\alpha_{Ri}\left(\frac{R_i}{\sum_{j=1}^N R_j} - \frac{X_i}{\sum_{j=1}^N X_j}\right)\Bigg\}.\end{aligned}$$

We now have the following proposition:

Proposition 8. *Assume that κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in every sector's technology A_i.*

1. *$\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ if $\sigma > 1$.*

2. *$\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ if $\sigma_{N+1} > \frac{\sum_{i=1}^{N} X_i}{X} \frac{R}{R_{N+1}}$.*

3. *As $\sigma_{N+1} \to 0$, $\sum_{i=1}^{N+1} \theta_{R,A_i} > 0$ if and only if $\sigma > 1$ for all $i \in \{1, ..., N\}$.*

4. *If $R_{N+1}/X_{N+1} > \sum_{i=1}^{N} R_i / \sum_{i=1}^{N} X_i$, then super-conservation can occur only if $\sigma < 1$ and σ_{N+1} is sufficiently small. If $R_{N+1}/X_{N+1} < \sum_{i=1}^{N} R_i / \sum_{i=1}^{N} X_i$, then super-conservation does not occur for any $\sigma, \sigma_{N+1} > 0$.*

Proof. Because each consumption good sector has identical parameters, we have $R_j / \sum_{i=1}^{N} R_i = X_j / \sum_{i=1}^{N} X_i = 1/N$ for all $j \in \{1, ..., N\}$. We then have:

$$\begin{aligned}\sum_{i=1}^{N+1} \theta_{R,A_i} =& \frac{\sum_{i=1}^{N} X_i}{X} \left\{ (\sigma - 1) + (\sigma_{N+1} - 1) \frac{R_{N+1}}{\sum_{j=1}^{N} R_j} + \sigma \frac{R_{N+1}}{\sum_{j=1}^{N} R_j} + \sigma_{N+1} \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} \frac{R_{N+1}}{\sum_{j=1}^{N} R_j} \right\} \\ =& \frac{\sum_{i=1}^{N} X_i}{X} \left\{ (\sigma - 1) \frac{R}{\sum_{j=1}^{N} R_j} + \sigma_{N+1} \frac{X}{\sum_{i=1}^{N} X_i} \frac{R_{N+1}}{\sum_{j=1}^{N} R_j} \right\}.\end{aligned}$$

The first three parts of the proposition follow.

To prove the final part of the proposition, note that

$$Savings^{GE} > Savings^{eng} \Leftrightarrow -\sum_{i=1}^{N+1} \theta_{R,A_i} R > \sum_{i=1}^{N+1} R_i \Leftrightarrow 0 > \sum_{i=1}^{N+1} \theta_{R,A_i} + 1.$$

For given R_i and X_i, this must hold at very small σ and σ_{N+1} if it holds anywhere. As $\sigma, \sigma_{N+1} \to 0$, this condition becomes

$$0 > 1 - \frac{\sum_{i=1}^{N} X_i}{X} \frac{R}{\sum_{j=1}^{N} R_j}.$$

The result follows straightforwardly from this expression and the fact that $\sum_{i=1}^{N} X_i / X > \sum_{i=1}^{N} R_i / R$ if and only if $R_{N+1}/X_{N+1} > \sum_{i=1}^{N} R_i / \sum_{i=1}^{N} X_i$.

□

B.2 Improved efficiency of the non-energy input in a consumption good sector

Now consider the consequences of improving χ_k in some sector $k \in \{1, ..., N\}$:

$$\theta_{R,\chi_k} = \frac{\sum_{i=1}^{N} X_i}{X} \left\{ \overbrace{(\epsilon - \sigma_k) \alpha_{Xk} \frac{R_k}{\sum_{i=1}^{N} R_i}}^{\text{Output price effect}} + \overbrace{(1 - \sigma_k - (\epsilon - \sigma_k)\alpha_{Xk}) \frac{X_k}{\sum_{i=1}^{N} X_i}}^{\text{Wage effect}} \right\}.$$

This is nearly identical to θ_{R,A_k}, but for two differences. First, we now have α_{Xk} in place of α_{Rk}, reflecting that the response of output prices to χ_k depends on the value share of X rather than the value share of R. Second, we now have $1-\sigma_k$ in place of $\sigma_k - 1$, reflecting that an increase in X_k through partial equilibrium channels (proportional to $\sigma_k - 1$, as for the response of R_k to A_k) reduces energy use by increasing the wage. Finally, note that $\theta_{R,\chi_k} \to \theta_{R,A_k}$ as $\kappa \to 0.5$ and $\sigma_k \to 1$, reflecting that the two types of factor-augmenting technical change are equivalent in a Cobb-Douglas specification.

The following proposition is the analogue of Proposition 1:

Proposition 9. *Assume either that $N = 1$ or that κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$. Consider an improvement in χ_k for some $k \in \{1, ..., N\}$. Then $\theta_{R,\chi_k} > 0$ if and only if $\sigma < 1$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^{N} R_i = X_j / \sum_{i=1}^{N} X_i = 1/N$ for all $j \in \{1, ..., N\}$. The proposition follows. □

The condition for backfire is now reversed from the case of an improvement in A_k. The usual partial equilibrium analysis suggests that X_k increases if and only if $\sigma > 1$. In that case, the wage must increase to clear the market for the labor-capital aggregate, which reduces energy use through the wage effect. However, if $\sigma < 1$, then the partial equilibrium reduction in X_k reduces the wage, which increases energy use through the wage effect.

We now have the analogues of Proposition 2, Corollary 3, and Proposition 12:

Proposition 10. *Assume that κ_i, σ_i, and χ_i do not vary with i for $i \in \{1, ..., N\}$ and that $Var(A)$ is small relative to $\bar{A}$. Consider an improvement in χ_k for some $k \in \{1, ..., N\}$.*

1. *If $\sigma > \epsilon$ and $A_k > \bar{A}$, then the general equilibrium channels are negative and $\theta_{R,\chi_k} < 0$.*
2. *If $\sigma \in (1, \epsilon)$ and $A_k < \bar{A}$, then the general equilibrium channels are negative and $\theta_{R,\chi_k} < 0$.*
3. *If $\sigma < 1$ and $A_k < \bar{A}$, then the general equilibrium channels are positive and $\theta_{R,\chi_k} > 0$.*

Proof. If $\sigma > \epsilon$, then (15) is positive if $A_k > \bar{A}$. The first result follows from noting that all terms in θ_{R,χ_k} are negative if $\sigma > \epsilon$ and $\frac{R_k}{\sum_{i=1}^{N} R_i} - \frac{X_k}{\sum_{i=1}^{N} X_i} > 0$.

If $\sigma \in (1, \epsilon)$, then (15) is negative if $A_k < \bar{A}$. The second result follows from noting that all terms in θ_{R,χ_k} are negative if $\sigma \in (1, \epsilon)$ and $\frac{R_k}{\sum_{i=1}^{N} R_i} - \frac{X_k}{\sum_{i=1}^{N} X_i} < 0$.

Finally, if $\sigma < 1$, then (15) is positive if $A_k < \bar{A}$. The third result follows from noting that all terms in θ_{R,χ_k} are positive if $\sigma < 1$ and $\frac{R_k}{\sum_{i=1}^{N} R_i} - \frac{X_k}{\sum_{i=1}^{N} X_i} > 0$.

□

Corollary 11. *Assume that κ_i, σ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$ and that $Var(\chi)$ is small relative to $\bar{\chi}$. Consider an improvement in χ_k for some $k \in \{1, ..., N\}$.*

1. *If* $\sigma > \epsilon$ *and* $\chi_k < \bar{\chi}$*, then the general equilibrium channels are negative and* $\theta_{R,\chi_k} < 0$*.*

2. *If* $\sigma \in (1, \epsilon)$ *and* $\chi_k > \bar{\chi}$*, then the general equilibrium channels are negative and* $\theta_{R,\chi_k} < 0$*.*

3. *If* $\sigma < 1$ *and* $\chi_k > \bar{\chi}$*, then the general equilibrium channels are positive and* $\theta_{R,\chi_k} > 0$*.*

Proof. Follows from the proofs of Corollary 3 and Proposition 10. □

Proposition 12. *Consider an improvement in* χ_k *for some* $k \in \{1, ..., N\}$*.*

1. $\theta_{R,\chi_k} < 0$ *as* $\kappa_k \to 1$*.*

2. $\theta_{R,\chi_k} \to 0$ *as* $\kappa_k \to 0$*.*

Proof. From equation (13), $R_k \to 0$ as $\kappa_k \to 1$. We also then have $\alpha_{Xk} \to 1$ as $\kappa_k \to 1$. The first result follows from noting that $\epsilon > 1$.

From equation (12), $X_k \to 0$ as $\kappa_k \to 0$. We also then have $\alpha_{Xk} \to 0$ as $\kappa_k \to 0$. The second result follows. □

The logic is as in the main text, noting that an increase in X_k corresponds to a decrease in R_k through wage effects.

B.3 Improved efficiency of the non-energy input in the energy-producing sector

Now consider the consequences of improving χ_{N+1}:

$$\theta_{R,\chi_{N+1}} = \frac{\sum_{i=1}^N X_i}{X} \left\{ \overbrace{\sum_{i=1}^N (\sigma_i + (\epsilon - \sigma_i)\alpha_{Ri}) \frac{R_i}{\sum_{j=1}^N R_j}}^{\text{Resource supply effect}} + \overbrace{\left(\frac{X_{N+1}}{\sum_{i=1}^N X_i} - \sum_{i=1}^N (\epsilon - \sigma_i)\alpha_{Ri} \frac{X_i}{\sum_{j=1}^N X_j} \right)}^{\text{Wage effect}} \right\}.$$

We then have:

Proposition 13. *Consider an improvement in* χ_{N+1}*.*

1. *If either* $N = 1$ *or* κ_i*,* σ_i*,* χ_i*, and* A_i *do not vary with* i *for* $i \in \{1, ..., N\}$*, then* $\theta_{R,\chi_{N+1}} > 0$ *and there exists* $\hat{\sigma} \in (0, 1)$ *such that* $\theta_{R,\chi_{N+1}} > 1$ *if and only if* $\sigma > \hat{\sigma}$*.*

2. *If, for* $i \in \{1, ..., N\}$*, each* $\sigma_i \approx 1$ *and* κ_i *does not vary with* i*, then* $\theta_{R,\chi_{N+1}} \approx 1$*.*

3. $\theta_{R,\chi_{N+1}} \to 1$ *as every* $\kappa_i \to 0$ *for* $i \in \{1, ..., N\}$*.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^{N} R_i = X_j / \sum_{i=1}^{N} X_i = 1/N$ for all $j \in \{1, ..., N\}$. We then have:

$$\theta_{R,\chi_{N+1}} = \frac{\sum_{i=1}^{N} X_i}{X} \left\{ \sigma + \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} \right\}.$$

This increases in σ and is equal to 1 if $\sigma = 1$. The first part of the proposition follows.

The value share of energy in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. Substituting into $\theta_{R,\chi_{N+1}}$, we have:

$$\theta_{R,\chi_{N+1}} \approx \frac{\sum_{i=1}^{N} X_i}{X} \left\{ 1 + \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} \right\} = 1.$$

The second part of the proposition follows.

From equation (12), $X_i \to 0$ as $\kappa_i \to 0$. We then have $\theta_{R,\chi_{N+1}} \to X_{N+1}/X$, which goes to 1. The third part of the proposition follows.

□

The primary difference with respect to the analysis of Section 5.2 is that now there are no partial equilibrium effects. Instead, the partial equilibrium change in X affects R through the wage effects. Partial equilibrium substitution towards X in energy production is offset by substitution away from X due to the higher wage, so that we are left with the "engineering" savings in X_{N+1} increasing energy use by lowering the wage. As a result, backfire must arise when the output price channels in the resource supply and wage effects offset each other (whether because $N = 1$ or because sectors are symmetric). That backfire is driven both by the reduction in the wage and also by consumption good producers' substitution towards newly cheap energy resources. In fact, not only does backfire occur, but a 1% improvement in X_{N+1} can increase total energy use by more than 1%. Finally, if consumption good sectors only use energy resources ($\kappa_i \to 0$ for $i \in \{1, ..., N\}$), then the only effect that matters is the increase in R_{N+1} due to the "engineering" savings in X_{N+1} reducing the wage, so energy use increases by the same percentage that χ_{N+1} improved.

B.4 Improved total factor productivity in a consumption good sector

Now consider improving total factor productivity in some consumption good sector. Note that $\theta_{R,TFP_k} = \theta_{R,A_k} + \theta_{R,\chi_k}$.[19] We then have:

$$\theta_{R,TFP_k} = \frac{\sum_{i=1}^{N} X_i}{X} [\epsilon - 1] \left[\frac{R_k}{\sum_{i=1}^{N} R_i} - \frac{X_k}{\sum_{i=1}^{N} X_i} \right].$$

Proposition 14. *Consider an improvement in TFP_k for some $k \in \{1, ..., N\}$.*

1. *If κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, then $\theta_{R,TFP_k} = 0$.*
2. *If κ_i, σ_i, and χ_i do not vary with i for $i \in \{1, ..., N\}$ and $Var(A)$ is small relative to $\bar{A}$, then:*
   - *(a) If $\sigma > 1$, then $\theta_{R,TFP_k} > 0$ if and only if $A_k > \bar{A}$.*
   - *(b) If $\sigma < 1$, then $\theta_{R,TFP_k} > 0$ if and only if $A_k < \bar{A}$.*

Proof. Because each consumption good sector has identical parameters, we have $R_j / \sum_{i=1}^{N} R_i = X_j / \sum_{i=1}^{N} X_i = 1/N$ for all $j \in \{1, ..., N\}$. The first part of the proposition follows from previous results. The second part follows from θ_{R,TFP_k} and the proof of Proposition 2. □

Now consider improving total factor productivity in all consumption good sectors at once.

Proposition 15. $\sum_{i=1}^{N} \theta_{R,TFP_i} = 0$.

Proof. Follows directly from the given expression for θ_{R,TFP_k}. □

B.5 Improved total factor productivity of energy production

Following previous analysis, an improvement in total factor productivity in the energy-producing sector yields the following change in total energy use:

$$\begin{aligned}\theta_{R,TFP_{N+1}} = &\frac{\sum_{i=1}^{N} X_i}{X} \Bigg\{ \left(\sigma_{N+1} \frac{X}{\sum_{i=1}^{N} X_i} - 1 \right) \frac{R_{N+1}}{\sum_{i=1}^{N} R_i} + \frac{R}{\sum_{i=1}^{N} R_i} \sum_{i=1}^{N} \sigma_i \frac{R_i}{\sum_{j=1}^{N} R_j} \\ &+ \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} + \frac{R}{\sum_{i=1}^{N} R_i} \sum_{i=1}^{N} (\epsilon - \sigma_i) \alpha_{Ri} \left(\frac{R_i}{\sum_{j=1}^{N} R_j} - \frac{X_i}{\sum_{j=1}^{N} X_j} \right) \Bigg\}.\end{aligned}$$

We then have:

[19] We can write $\chi_k \triangleq TFP_k \tilde{\chi}_k$ and $A_k \triangleq TFP_k \tilde{A}_k$ in sector k firms' production functions. The claim follows from totally differentiating $R(A_k, \chi_k)$ with respect to TFP_k.

Proposition 16. *If either $N = 1$ or κ_i, σ_i, χ_i, and A_i do not vary with i for $i \in \{1, ..., N\}$, then there exists $\hat{\sigma} < 1$ such that $\theta_{R,TFP_{N+1}} > 0$ if $\sigma_{N+1} + \sigma > \hat{\sigma}$.*

Proof. If either $N = 1$ or each consumption good sector has identical parameters, then $R_j / \sum_{i=1}^{N} R_i = X_j / \sum_{i=1}^{N} X_i = 1/N$ for all $j \in \{1, ..., N\}$. We then have:

$$\theta_{R,TFP_{N+1}} = \frac{\sum_{i=1}^{N} X_i}{X} \left\{ \left(\sigma_{N+1} \frac{X}{\sum_{i=1}^{N} X_i} + \sigma - 1 \right) \frac{R_{N+1}}{\sum_{i=1}^{N} R_i} + \sigma + \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} \right\}.$$

The proposition follows. □

Proposition 17. *Assume that, for $i \in \{1, ..., N\}$, each $\sigma_i \approx 1$ and that κ_i does not vary with i. Then $\theta_{R,TFP_{N+1}} > 1$.*

Proof. The value share of energy in each consumption good sector is approximately κ_i when $\sigma_i \approx 1$ and is then independent of i when, in addition, κ_i is independent of i. Substituting into $\theta_{R,\chi_{N+1}}$, we have:

$$\theta_{R,TFP_{N+1}} = \frac{\sum_{i=1}^{N} X_i}{X} \left\{ \sigma_{N+1} \frac{X}{\sum_{i=1}^{N} X_i} \frac{R_{N+1}}{\sum_{i=1}^{N} R_i} + 1 + \frac{X_{N+1}}{\sum_{i=1}^{N} X_i} \right\}.$$

The proposition follows.

□