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THE DEADWEIGHT LOSS FROM "NONNEUTRAL" CAPITAL INCOME TAXATION

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ABSTRACT

This paper develops an overlapping generations general equilibrium growth model with an explicit characterization of the role of capital goods in the production process. The model is rich enough in structure to evaluate and measure simultaneously the different distortions associated with capital income taxation (across sectors, across assets and across time) yet simple enough to yield intuitive analytical results as well.

The main result is that uniform capital income taxation is almost certainly suboptimal, theoretically, but that empirically, optimal deviations from uniform taxation are inconsequential. We also find that though the gains from a move to uniform taxation are not large in absolute magnitude these gains would be offset only by an overall rise in capital income tax rates of several percentage points.

A separate contribution of the paper is the development of a technique for distinguishing intergenerational transfers from efficiency gains in analyzing the effects of policy changes on long-run welfare.

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1. Introduction

Capital income taxation distorts economic behavior in many ways. Since the work of Harberger (1966), economists have been concerned with the inefficient allocation of capital caused by taxing capital income from different sectors of the economy at different rates (e.g. Shoven 1976). Others (e.g. Feldstein 1978, Boskin 1978) have considered the savings disincentives that even uniform capital income taxes impose. In recent years, one type of distortion has received substantial attention: that caused by the nonuniform taxation of different assets, attributable to such factors as investment incentives. An international comparison of capital income taxation in four countries (King and Fullerton 1984) stressed the great variation in effective tax rates in each country, and many authors (e.g. Auerbach 1983, Gravelle 1981, Fullerton and Henderson 1986) have estimated the excess burden caused by such distortions. Indeed, the Tax Reform Act of 1986 had as one of its explicit objectives a "levelling of the playing field" among various capital investments.

Many have criticized the 1986 Act on the grounds that the efficiency gains from more uniform business taxation are small and likely to be more than offset by the distortions induced by the increase in business taxes overall, associated with the widening gap between the taxation of business capital and owner-occupied housing and the increase in the tax burden on saving (see, e.g., Summers 1987). Such criticism suggests that it is important to know the relative magnitudes of the three different types of distortions. Unfortunately, previous growth models used to measure capital income tax distortions have lacked the interasset and intersectoral detail required to measure the first two of these distortions, while those models possessing such detail have lacked an adequate treatment of intertemporal decisions or general equilibrium effects. This paper develops a general equilibrium growth model which is rich enough in structure to evaluate and measure simultaneously these different distortions yet simple enough to yield intuitive analytical results as well.

Few critics of the recent tax changes have disputed the argument that, by itself, a move toward uniform business taxation would increase economic efficiency, if not by a substantial amount. Yet this presumption could be unwarranted. It is a well-known principle of "second-best" welfare economics that, in the presence of some distortions, the introduction of others need not worsen the allocation of social resources. What distinguishes differential capital income taxation from other potential distortions is that it is a production distortion, a type of distortion that should be eschewed even in cases where other distortions exist, if the government taxes away pure profits and has the ability to tax households on all transactions with the production sector and does so in an optimal manner (Diamond and Mirrlees 1971, Stiglitz and Dasgupta 1971). Even without such optimal consumption taxes, results from a model with a single production sector suggest that uniform capital income taxation is still optimal if different types of capital are equally complementary to labor in production, as would be true if the production function were separable into capital and labor and exhibited constant returns to scale (Auerbach 1979).

At the theoretical level, this paper demonstrates that in a more realistic model of the production process the conditions under which uniform capital income taxation is optimal will almost certainly be violated. Even if every production function in the economy is separable into capital and other factors and exhibits constant returns to scale, the location of capital goods in the chain of production and the composition of the capital goods themselves play a role in determining which capital goods should be taxed more heavily than others in order to minimize deadweight loss. At the empirical level, however, we find that the optimal deviation from uniform taxation generates welfare gains that are quite small relative to those generated by a move to uniform taxation from a tax system like that prevailing in the U.S. before 1986.

All of the paper's welfare analysis is based on changes in steady state utility. A problem that has plagued such analysis in the past is that tax policy changes induce not only behavioral distortions but also intergenerational transfers. Without resorting to numerical simulation models (as in Auerbach and Kotlikoff 1987) it has not been possible to distinguish the steady state welfare changes arising from each source. Below, we develop a simple technique for doing so, which at the same time provides insight into the intergenerational incidence of taxation. This technique should be of general use for measuring efficiency gains in steady state models.

2. The Model

There have been many previous approaches to the measurement of deadweight loss from capital income taxation in models with a complex production structure, but each lacks at least one attribute needed for our current objectives. In several papers, Diewert (1981, 1983, 1985) carefully analyzed the deadweight loss caused by capital income taxes in a price-taking production sector. While quite useful in many contexts, particularly for individual sectors of production or small open economies, such results were not intended and cannot be used to evaluate the consumption and savings distortions induced by such taxes in general equilibrium. Using a large-scale numerical general equilibrium model, Fullerton and Henderson (1986, 1987) have considered capital income tax distortions in a model with changing consumer prices but have focused primarily on static deadweight loss measures, with saving not really considered and capital viewed as a primary factor of production. The model considered here is a general equilibrium model in which there is a complex structure of production and dynamic issues are incorporated in a consistent manner.

The notation used in describing the model is summarized in Table 1. We consider the steady state of an overlapping generations closed economy in which individuals live for two periods and there are two primary factors of production, labor and land, each of which in homogeneous. There are N production sectors, with outputs produced using land and labor as well as intermediate goods and capital goods which are, themselves, produced by the N sectors. Each generation has a single representative household that supplies labor in the first period of life and purchases the N commodities in each period, subject to preferences defined over these 2N + 1 goods (including labor).

The population is assumed to grow exogenously at rate n, and the supply of land is also taken to be exogenous. To make these two assumptions consistent with the existence of a steady state, we must allow land to grow at rate n as well. This is accomplished in the model by assuming that each person entering old age receives n units of land per existing unit, with this receipt viewed as a lump-sum transfer. Although not entirely satisfactory, this seems preferable to the more standard practices of omitting land entirely or treating it as a type of reproducible capital.

The N production sectors each behave competitively subject to constant returns to scale. We assume that intermediate inputs enter the production process according to a fixed relationship to gross output but that capital, labor and land may be varied. There are M capital goods, each of which is a linear combination of the N outputs.[1] These include fixed capital goods as well as inventories. Each capital good i is assumed to depreciate exponentially at rate $\delta_i \geq 0$.

Let A be the N*N input output matrix, each column i giving the input requirements of goods $j = 1,...N$ per unit of output i. Let B be the N*M capital goods definition matrix, each column i giving the composition (with column entries summing to one) of capital good i. We let K be the M*N capital requirements matrix, with column i giving the amount of each type of capital j (= 1,...,M) needed to produce a unit of good i. We let h and ℓ be the N-vectors, the i^{th} entry of which are the labor and land requirements per unit of output. By assumption, K, h and ℓ are variable but A and B are fixed.

The N-vector of gross outputs (per capita young) is y. The corresponding consumption goods vector is z. Let $\bar{c}_i = n + \delta_i$ be the ratio of gross investment to capital of type i needed to maintain the steady state level of capital per capita, and define a corresponding diagonal matrix $\bar{C}$ with element ii equal to $\bar{c}_i$. Then the relationship between gross output y and final consumer demand z is:

(1) $$y = z + Ay + B\bar{C}Ky \rightarrow y = (I-A-B\bar{C}K)^{-1}z = \bar{\Omega}^{-1}z$$

that is, gross output equals consumption plus intermediate purchase plus investment. The second term would be omitted in a GNP calculation, so y exceeds GNP in value.

Total labor and land requirements corresponding to y are:

(2) $$E = h'y$$

(3) $$L = \ell'y$$

The gross wage is chosen as numeraire, so that prices are all in units of labor productivity. The net (of tax) wage is denoted w. Let r be the net rate of return to capital, and s_0 the net land rent per unit of land.

The government in this model is assumed to rebate all tax revenues to consumers, with proportional income taxes being assessed on labor, land, and each type of capital. Because of our focus on the long-run consequences of the tax system, we assume capital income taxes are on true economic income, and that there is therefore no distinction in the treatment of new and old capital.[2]

Let τ_L be the unit tax on land, τ the unit tax on labor and t_i the tax per dollar of value on capital type i. By construction, $\tau = 1 - w$. Let the gross land rent $s_0 + \tau_L$ be denoted s. The gross return on capital of type i per dollar of capital is $c_i = r + \delta_i + t_i$. Let $c_{0i} = c_i - t_i = r + \delta_i$, and let C and C_0 be the diagonal matrices with $\{c_i\}$ and $\{c_{0i}\}$ on the diagonals.

By the zero profits assumption, we may solve for the prices of output and capital goods. The price of output i equals the cost of its intermediate inputs plus the required before-tax returns to labor, capital and land, or:

(4) $$p' = p'A + h' + q'CK + s\ell'$$

where the capital goods price vector q must satisfy:

(5) $$q' = p'B$$

Combining (4) and (5), we obtain:

(6) $$p' = (h' + s\ell')(I-A-BCK)^{-1} = (h' + s\ell')\Omega^{-1}$$

where Ω^{-1} has the same form as $\bar{\Omega}^{-1}$ but is based on C rather than $\bar{C}$. It is also useful to define Ω_0^{-1} based in analogous fashion upon C_0.

The revenue raised and rebated is, per young individual:

(7) $$\hat{R} = q'(C-C_0)Ky + (1-w)E + (s-s_0)L$$

With two representative households alive at any date, we must specify how the tax proceeds are distributed. We let β be the fraction of revenue distributed to the old, and defer until later a discussion of how β is determined. In the household's budget constraint at birth, the value of revenue received is therefore:

$$R = R^1 + \frac{1}{1+r}R^2 = (1-\beta)\hat{R} + \beta\hat{R}(\frac{1+n}{1+r}) = \hat{R}\ [1 + \beta(\frac{1+n}{1+r} - 1)] \quad (8)$$

The (1+n) term in second-period transfers R^2 is due to the fact that $\hat{R}$ is measured per capita young.

The other element of the household's lump sum income is receipts of new land, nL. In capital market equilibrium, the price per unit of land must be:

$$p_L = \frac{s_0}{r} \quad (9)$$

so we may express the household's indirect utility function as:

$$U = V[p, \frac{p}{1+r}, w, R + np_L L] \quad (10)$$

where the first three elements are the 2N + 1 prices corresponding to first-period consumption, second-period consumption, and labor, respectively.

3. Deadweight Loss

We consider the change in utility caused by the introduction of a small set of taxes $d\tau$, $d\tau_L$ and $dt(=dt_1,\ldots,dt_M)$ beginning from a zero-tax steady state. This focus on small taxes is restrictive but permits the use of comparative statics. Total differentiation of (10) yields:

$$\begin{aligned} dU &= -\lambda dp'[z^1 + \frac{z^2}{1+r}] - \lambda p'z^2 d(\frac{1}{1+r}) + \lambda dwE + \lambda dR + \lambda ndp_L L \quad (11) \\ &= -\lambda dp'z - \lambda dp'z^2[\frac{1}{1+r} - \frac{1}{1+n}] - p'z^2 d(\frac{1}{1+r}) + \lambda dwE + \lambda dR + \lambda ndp_L L \end{aligned}$$

where $z = z^1 + z^2/(1+n)$ is total consumption per capital young (as previously defined), z^1 and z^2 are the consumption bundles in each period per person and λ is the marginal utility of income.

From (7) and (8), we obtain:

$$(12)\qquad dR = \{dp'B(C-C_0)Ky + q'(dC-dC_0)Ky - q'(C-C_0)K\bar{\Omega}^{-1}dz + q'(C-C_0)dK\bar{\Omega}^{-1}z$$

$$+ q'(C-C_0)K\bar{\Omega}^{-1}B\bar{C}dK\bar{\Omega}^{-1}z + (1-w)dE - dwE - (ds-ds_0)L\}\cdot\{1+\beta[\frac{1+n}{1+r} - 1]\}$$

$$- \{q'(C-C_0)Ky + (1-w)E + (s-s_0)L\}\beta(\frac{1+n}{1+r})\,\frac{dr}{1+r}$$

Using the definition of p_L in (9) and the fact that $dC_0 = drI$, one may break up the second and last terms in the first set of curly brackets in (12) and then rearrange the terms to obtain,[3] in combination with (11):

$$(13)\qquad \frac{1}{\lambda}dU = -dp'z - dp'z^2[\frac{1}{1+r} - \frac{1}{1+n}] + \{dp'B(C-C_0)Ky + q'dCKy + dsL\}\cdot\beta[\frac{1+n}{1+r} - 1]\}$$

$$+ \{q'dC_0Ky + ds_0L\}(1-\beta)[\frac{1+n}{1+r} - 1] + dp_LL[n - r(\frac{1+n}{1+r})] - dwE\beta[\frac{1+n}{1+r} - 1]$$

$$+ \{(1-w)dE + q'(C-C_0)d(Ky)\}\ \{1 + \beta[\frac{1+n}{1+r} - 1]\}$$

$$+ \{p'z^2 - [q'Ky(1+n) + p_LL(1+n)](1+r) - R^2\}\frac{dr}{(1+r)^2}$$

where

$$(14)\qquad d(Ky) = K\bar{\Omega}^{-1}dz + dK\bar{\Omega}^{-1}z + K\bar{\Omega}^{-1}B\bar{C}dK\bar{\Omega}^{-1}z$$

is the total change in the capital stock vector Ky due to all changes in the demand for capital.

In the closed economy modelled here, the last term in brackets in (13) equals the household's second-period budget constraint: consumption less transfers and principal and interest from assets. Hence it must equal zero.[4]

To simplify expression (13), we need an expression for dp'. By the envelope theorem,

$$(15)\qquad dp' = (h'+s\ell')\Omega^{-1}BdCK\Omega^{-1} + ds\ell'\Omega^{-1} = (q'dCK+ds\ell')\Omega^{-1}$$

Substituting this into (13), we obtain:

$$\frac{1}{\lambda}dU = (q'dCK+ds\ell')(\overline{\Omega}^{-1}-\Omega^{-1})z - dp'z^2[\frac{1}{1+r} - \frac{1}{1+n}]$$

$$+ (q'dCK+ds\ell')y\beta[\frac{1+n}{1+r} - 1] + dp'B(C-C_0)Ky\{1 + \beta[\frac{1+n}{1+r} -1]\}$$

$$+ (q'dC_0K+ds_0\ell')y(1-\beta)[\frac{1+n}{1+r} - 1] + dp_L L[\frac{1+n}{1+r} - 1] - dwE\beta[\frac{1+n}{1+r} - 1]$$

$$+ \{(1-w)dE + q'(C-C_0)d(Ky)\}\cdot\{1 + \beta[\frac{1+n}{1+r} - 1]\}$$

Again by the envelope theorem, if one starts at a Pareto-optimum the first-order change in utility resulting from the introduction of taxes is zero; there is no first-order deadweight loss. Examining (16), however, we see that $dU = 0$ only if, in addition, $r = n$ in the initial steady state. This extra condition is present because dU is not the change in utility in a single consumer static model but rather the change in steady-state utility in an overlapping generations model. Unless this "Golden Rule" condition is satisfied, the introduction of small taxes has first-order effects on steady state utility. In a dynamically efficient economy ($r > n$), these changes represent movements alone the Pareto frontier; if utility increases in the steady state, it must be reduced for some transitional generations. In general, each tax policy change induces intergenerational transfers. To measure the magnitude of a tax policy's distortion, one must account for such transfers, which may be large relative to changes due to efficiency gains or losses (Auerbach and Kotlikoff 1987).

Indeed, after a few lines of algebra, expression (16) may be rewritten in a way that indicates the contribution of intergenerational transfers to changes in steady state utility:

(17) $$\frac{1}{\lambda}dU = (1-w)dE + q'(C-C_0)d(Ky)$$

$$+ (\frac{1}{1+r} - \frac{1}{1+n})[-dp'z^2 + dr(q'Ky+p_L L)(1+n) + (dq'Ky+dp_L L)(1+n)(1+r) + dR^2]$$

The term in brackets in (17) has a straightforward interpretation. It equals the real income loss the household experiences in the second period due to changes in the prices of consumption goods and assets, the rate of return to savings, and government transfers. If this term is negative, then the tax burden is being shifted toward the older generation, a shift that will increase steady state utility if $r > n$. To neutralize this term, it is necessary to rebate enough of the tax revenue to the old, dR^2, to offset the income effect experienced in the second period. Because $dR^2 = \beta d\hat{R}(1+n)$, this amounts to choosing β so that:

$$(18) \quad \beta = \left[dp'\frac{z^2}{1+n} - dr(q'Ky+p_L L)-(dq'Ky+dp_L L)(1+r)\right]/\left[q'd(C-C_0)Ky+d(1-w)E+d(s-s_0)L\right]$$

The term β is interesting in its own right, for it indicates what the incidence of the tax change is across generations. For example, in a simple model with no land and only uniform capital income taxes, if these taxes were fully borne by capital via a decline in r, then dp, dw and dq would all equal zero and $d(C-C_0)$ would equal $-drI$; hence β would equal 1: the tax would be borne fully by the elderly. In other studies of capital income tax incidence in the two-period overlapping generations model (e.g. Diamond 1970, Kotlikoff and Summers 1979), it has been customary to set $\beta = 1$, and indeed the value one chooses for β has been discussed in terms of what the "right "experiment is to derive a "compensated" elasticity of saving with respect to the interest rate (Sandmo 1981). Our analysis suggests that the appropriate generational distribution of tax compensation in turn depends on the incidence of the tax.

Using the definition of β in (18), we obtain a very simple expression for first-order deadweight loss, which equals zero under the assumption that taxes are initially zero. Calculation of the deadweight loss from taxation thus requires a second-order Taylor approximation:

(19) $\Delta U = \frac{1}{\lambda}[dU + 1/2d^2U]$

From (17), (18) and (19), and the assumption that β is varied to keep the second term in (17) equal to zero, we obtain:

(20) $\Delta U = 1/2d\tau dE + 1/2q'dTd(Ky) = 1/2d\tau dE + 1/2\sum_{i=1}^{M} dt_i q_i d(Ky)_i$

where $T = C - C_0$. Note that $(Ky)_i$ is the economy's total capital stock of type i. This expression is very much in the spirit of Harberger's (1966) original formulation of the problem of measuring the deadweight loss from capital income taxation, although the sources of change in the capital stock are more complicated here. As shown in (14), d(Ky) incorporates changes in capital resulting from altered production levels of consumption and capital goods and altered capital-output ratios. It is worth noting that, because land is in fixed supply, it is absent from expression (20).

It is possible to express this deadweight loss in a form more useful for determining the additional efficiency cost of nonuniform capital income taxation. Define $d\hat{t}$ to be the uniform capital income tax rate that would yield the same revenue (and, since there is no first-order deadweight loss, the same costs of production) as the actual taxes imposed on capital. Let $\hat{c} = r + \hat{t}$ and $\hat{C} = \hat{c}I$. By the definition of $d\hat{t}$, we have:

(21) $q'(dC-dC_0)Ky = q'dTKy = d\hat{t}q'Ky$

and, defining $d\tilde{T} = dT - d\hat{t}I$ as the differential capital income tax matrix with diagonal terms $d\tilde{t}_i = dt_i - d\hat{t}$,

(22) $q'd\tilde{T}Ky = 0$

Finally, define

(23) $$d\tilde{p}' = p'Bd\tilde{T}K\bar{\Omega}^{-1}$$

Note that, when $r = n$ (and $\Omega = \bar{\Omega}$), $d\tilde{p}$ is the change in p, holding s and $\hat{c}$ constant, due to the tax perturbation $d\tilde{T}$. Also note that, by (22),

(24) $$d\tilde{p}'z = p'Bd\tilde{T}Ky = q'd\tilde{T}Ky = 0$$

Using these definitions, we rewrite (20) as:

(25) $$\Delta U = 1/2q'd\tilde{T}d(Ky|z) + 1/2d\tilde{p}'dz + 1/2d\hat{t}q'd(Ky) + 1/2d\tau dE$$

where $d(Ky|z)$ is the change in Ky, holding z fixed. To simplify (25), note that total differentiation of the second-period budget constraint (given in (13)), combined with the assumption that the second term in (17) equals to zero, yields:

(26) $$\frac{p'}{1+r}dz^2 = q'd(Ky)(1+n)$$

which says that the change in expenditures on second-priced consumption equals the change in saving, consistent with the method of choosing β above.

Substitution of (26) into (25) yields:

(27) $$\Delta U = 1/2q'd\tilde{T}d(Ky|z) + 1/2d\tilde{p}'z + \frac{1/2d\hat{t}p'}{(1+r)(1+n)}dz^2 + 1/2d\tau dE$$

The last two terms in (27) represent the deadweight loss from uniform capital income taxation and labor income taxation, based on the tax wedges and behavioral changes associated with each. The first two terms each vanish when capital taxes are uniform and $d\tilde{T} = 0$. We may therefore interpret their sum as (minus) the deadweight loss from differential capital income taxation. If these terms were negative for any $d\tilde{T} \neq 0$ and did not affect the last two terms regardless of the values chosen for τ, τ_L and $\hat{t}$, it would be optimal to have uniform capital income taxation. However, it is easy to see that this may not be so. Each of the first two terms has another condition under which it vanishes; for the first, that $d(Ky|z)$ is proportional to Ky (by (22)) and for the second, that dz is proportional to z (by (24)). Unless these equiproportional reductions in capital Ky and output z coincide with uniform taxation, there will be two separate ways to set these deadweight loss terms to zero. Moreover, starting from uniform taxation, it seems likely that one could cause the terms to become positive by introducing negative differential taxes $\tilde{t}_i$ or price perturbations $\tilde{p}_i$ for capital goods or final consumption goods whose proportional reductions are particularly large. The intuition is that optimal tax rules typically call for equiproportional reductions in taxed activities. If this does not occur under uniform capital taxation, differential taxation may improve efficiency even while introducing a new set of distortions.

4. The Desirability of Uniform Taxation

To simplify the discussion of when such "second-best" gains will be available, we limit our consideration in this section to the special case in which r=n. Relaxing this restriction would simply reinforce the negative theoretical results that obtain.

Define:

$$(28) \qquad \pi = \begin{pmatrix} p \\ \frac{p}{1+r} \end{pmatrix}; \quad \pi_0 = \begin{pmatrix} p \\ \frac{p}{1+r+\hat{t}} \end{pmatrix}$$

to be the price vectors for first and second period consumption based on the net and gross interest rates, respectively. Also, let

$$(29) \qquad d\tilde{\pi} = \begin{pmatrix} d\tilde{p} \\ \frac{d\tilde{p}}{1+r} \end{pmatrix}$$

be the distortion to π associated with nonuniform capital income taxation. Also, note that

$$(30) \qquad d(Ky|z) = dKy + K\bar{\Omega}^{-1}B\bar{C}dKy = (I+K\bar{\Omega}^{-1}B\bar{C})dKy = \Lambda' dKy$$

Using (28), (29) and (30), we rewrite (27) as:

$$(31) \qquad \Delta U = 1/2q'd\tilde{T}\Lambda'dKy + 1/2d\tilde{\pi}'dx + 1/2d(\pi-\pi_0)'dx$$

where

$$(32) \qquad x = \binom{z^1}{z^2}$$

The changes dK and dx are determined by the underlying technology and preferences. Note that

$$(33) \qquad dKy = \sum_{i=1}^{n} dK^i y_i = (\sum_i y_i \tilde{H}^i)d\rho = \tilde{H}d\rho$$

where K^i is the capital stock vector for industry i, $\tilde{H}^i$ is the M*(M+1) submatrix of industry i's unit cost function Hessian omitting the rows for labor and land and the column for labor, and

$$(34) \qquad \rho = \binom{Cq}{s}$$

is the vector of capital and land user costs. It will be useful to decompose dρ into its components (and define the terms θ and $\hat{\rho}$):

$$(35) \qquad d\rho = d\binom{Cq}{s} = \binom{dCq + Cdq}{ds} = \binom{dCq + C(B'\Omega'^{-1}K'dCq) + CB'\Omega'^{-1}\ell ds}{ds}$$

$$= \binom{\Lambda dCq + \Theta ds}{ds} = \binom{\Lambda d\tilde{T}q}{0} + \binom{\Lambda d\hat{C}q + \Theta ds}{ds}$$

$$= \binom{\Lambda d\tilde{T}q}{0} + d\hat{\rho}$$

In expression (35), $d\hat{\rho}$ is the change in gross capital and land factor prices attributable to uniform taxes. Note that $d\hat{\rho}$ is not necessarily independent of $d\tilde{T}$, in general equilibrium.

Now, consider preferences. The change in the consumption vector dx is:

$$(36) \qquad dx = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} d\pi = S d\pi$$

where S is the 2N*2N matrix derived by striking the first row and column (corresponding to leisure) from the Slutsky matrix.

Decomposing $d\pi$ yields:

$$(37) \qquad d\pi = d\begin{pmatrix} p/w \\ p/w(1+r) \end{pmatrix} = \begin{pmatrix} \Omega'^{-1}K'dCB'p + \Omega'^{-1}\ell ds - dwp \\ (\Omega'^{-1}K'dCB'p + \Omega'^{-1}\ell ds - dwp)/(1+r) - \frac{p}{(1+r)^2}dr \end{pmatrix}$$

$$= \begin{pmatrix} d\tilde{p} \\ \frac{d\tilde{p}}{1+r} \end{pmatrix} + \begin{pmatrix} \Omega'^{-1}K'd\hat{C}B'p + \Omega'^{-1}\ell ds - dwp \\ (\Omega'^{-1} dCB'p + \Omega'^{-1}\ell ds - dwp)/(1+r) - \frac{p}{(1+r)^2}dr \end{pmatrix}$$

$$= d\tilde{\pi} + d\hat{\pi}$$

where $d\hat{\pi}$ is the change in consumption price attributable to uniform taxes.

Using (33), (35), (36) and (37), we rewrite (31) as:

$$(38) \qquad \Delta U = 1/2q'd\tilde{T}\Lambda'\tilde{H}d\rho + 1/2d\pi'Sd\pi + 1/2d(\pi-\pi_0)Sd\pi$$

$$= 1/2q'd\tilde{T}\Lambda'\hat{H}\Lambda d\tilde{T}q + 1/2d\tilde{\pi}'Sd\tilde{\pi} + 1/2d(\pi-\pi_0)'Sd\tilde{\pi}$$

$$+ 1/2[q'd\tilde{T}\Lambda'\tilde{H}d\hat{\rho} + d\tilde{\pi}'Sd\hat{\pi} + d(\pi-\pi_0)'Sd\tilde{\pi}]$$

where $\hat{H}$ is M*M and derived from $\tilde{H}$ by striking both the row <u>and</u> column corresponding to land.

In Appendix A we prove the following result, which indicates that the conditions that guarantee the optimality of uniform taxation are quite restrictive. Note that under uniform capital taxation we may characterize all behavioral changes in terms of response to changes in $\hat{c}$, s, r, and w, since changes in p and q are themselves functions of changes in $\hat{c}$ and s.

Proposition 1: For an initial steady state in which r=n, and arbitrary taxes $d\hat{t}$, $d\tau$ and $d\tau_L$, it is optimal to set $d\tilde{T} = 0$ if the following proportionality conditions are met:

(1) $$\frac{\partial(Ky)}{\partial\hat{c}} \sim Ky$$

(2) $$\frac{\partial(Ky)}{ds} \sim Ky$$

(3) $$\frac{\partial(Ky)}{\partial r} \sim Ky$$

(4) $$\frac{\partial(Ky)}{\partial w} \sim Ky$$

The first of these conditions represents restrictions on technology and (since $\hat{c}$ and s affect p) preferences together, while the last two represent restrictions only on preferences, since r and w do not affect producers.

If the conditions set out by Proposition 1 are satisfied, then we have an easily calculable measure of the additional deadweight loss caused by differential capital income taxation, with a piece due to distorted production and one due to distorted consumption:

(39) $$DWL = -1/2(\Lambda d\tilde{T}q)'\hat{H}(\Lambda d\tilde{T}q) - 1/2 d\tilde{\pi}' S d\tilde{\pi}$$

Previous measures that were based only on production distortions (e.g., Auerbach 1983, Gravelle 1981) had nothing analogous to the second term in (39). The customary treatment of capital as a primary factor of production is equivalent to assuming that $\Lambda = I$, that capital goods prices are given and that capital is not used in the production of other capital.

The validity of this deadweight loss measure is subject to question, however, because some of the proportionality conditions of Proposition 1 are quite stringent. Conditions (3) and (4) do not seem particularly unreasonable. They would be satisfied if $dz/dw \sim z$ and $dz/dr \sim z$, i.e., if a change in the wage rate or the interest rate had no effect on the composition of goods purchased by consumers. Under such conditions, the government would not wish to tax goods differentially in order to alleviate distortions introduced by changes in r and w. An example of a utility function that would satisfy these conditions is:

$$(40) \qquad U = U[\phi(z^1), \phi(z^2), E]$$

where ϕ is a homogeneous function. Here, only changes in p will affect the composition of demand among commodities.

The other two conditions, however, are more restrictive. These are the conditions under which the government would not wish to use differential capital taxes to improve the distortions caused by changing the relative (to labor) producer prices of capital and land, $\hat{c}$ and s, respectively. Consider condition (1). Expanding $\frac{d(Ky)}{d\hat{c}}$, we have,

$$(41) \qquad \frac{d(Ky)}{\partial \hat{c}} = \frac{\partial(Ky|z)}{\partial \hat{c}} + K\bar{\Omega}^{-1}\frac{\partial z}{\partial \hat{c}} = \Lambda'\hat{H}\Lambda q + K\Omega^{-1}\tilde{S}\Omega'^{-1}K'q$$

where $\tilde{S} = [S_{11} + \frac{S_{12}}{1+n} + \frac{S_{21}}{1+n} + \frac{S_{22}}{(1+n)^2}]$. Each of the terms on the right-hand side of (41) has an intuitive interpretation. Consider first the production term, $\Lambda'\hat{H}\Lambda q$. If there were no land, and we ignored Λ, then the remaining term $\hat{H}q$ would be proportional to Ky if, in every production function, capital goods were of equal complementarity to labor <u>and</u> did not depreciate (or depreciated at the same rate). This would follow from the fact that $d\hat{c}q$ would be proportional to $\hat{c}q$, so that relative prices among capital goods would not change. This is the condition needed in a one-sector model (Auerbach 1979) to ensure the optimality of uniform taxation. Even this condition will fail to be met with such equal complementarity to labor once we consider the terms Λ' and Λ.[5] These vary according to a capital good's place in the production process. One may measure these differences in production characteristics in the following manner. Suppose that $\hat{H}q \sim Ky$. Then $\Lambda'\hat{H}\Lambda q \sim Ky$ if $\Lambda q \sim q$ and $\Lambda' Ky \sim Ky$. Element i of Λq equals $d(c_i q_i)/d\hat{c}$, and exceeds q_i to the extent that q_i itself depends on the cost of capital. hence, the ratio of $(\Lambda q)_i$ to q_i measures that capital intensity of capital good i. Similarly, element i of $\Lambda' Ky$ equals the change in the aggregate capital stock of type i with respect to an equiproportional change in the capital intensity of production, as measured by the matrix K. Since the derivative holding output fixed would be Ky, the ratio of $(\Lambda' Ky)_i$ to $(Ky)_i$ measures the importance of capital good i in the production capital. The use of a capital good may go down proportionally more than other capital goods if either of these ratios is large, suggesting that if capital taxation is "too high," we might wish to tax such capital goods less heavily.[6]

A similar difficulty is posed by the second term on the right-hand side of (41), $K\Omega^{-1}\tilde{S}\Omega'^{-1}K'q$. One would like this term to be proportional to Ky. The matrix $\tilde{S}$ multiplied by p equals $-\frac{\partial z}{\partial w}$ (see (37)). Thus, if one ignores $\Omega'^{-1}K'B$, then $K\Omega^{-1}\tilde{S}\Omega'^{-1}K'q = K\Omega^{-1}\tilde{S}p = -K\Omega^{-1}\frac{\partial z}{\partial w}$, which is proportional to Ky if $\frac{\partial z}{\partial w} \sim z$. We have argued that this latter proportionality condition is not unreasonable. However, unless $\Omega'^{-1}K'Bp$ is proportional to p, the whole term will still not be proportional to Ky. Since $\Omega'^{-1}K'Bp$ equals the derivative of p with respect to $\hat{c}$, the term $\hat{c}(\Omega'^{-1}K'Bp)_i/p_i$ measures the elasticity of good i's price with respect to a change in the interest rate. Goods whose production is capital intensive may have their demand especially discouraged. We may then, following the previous argument, wish to tax less heavily the capital used especially in the production of such consumption goods.

Thus, even with "standard" restrictions on preferences and technology (which themselves are undoubtedly violated but may be seen as reasonable benchmarks) there are three reasons why $\frac{\partial(Ky)}{\partial\hat{c}}$ is not likely to be proportional to Ky and hence some form of nonuniform capital taxation may be desirable[7]:

(1) capital goods vary in capital intensity (direct and indirect ($\Lambda q \not\propto q$);

(2) capital goods enter in different ways into the production of other capital goods ($\Lambda' Ky \not\propto Ky$); and

(3) consumption goods vary in their capital intensity (direct and indirect) ($\hat{c}\Omega'^{-1}K'Bp \not\propto p$).

These complications all disappear in a one-sector model.

Adding land to the analysis does not make uniform taxation any more likely, of course. One can go through the same type of analysis and derive the same sort of reasons why $\frac{\partial(Ky)}{\partial s}$ is not proportional to Ky, even if capital goods are equal complements to land in production. Similarly, making some capital income taxes equal but different to others (for example, owner-occupied housing capital) need not be desirable.

We now return briefly to the discussion of production efficiency in our introduction. We have not shown the "production efficiency" theorem to be false, only that if its conditions are not satisfied, production efficiency is unlikely to remain optimal. In this model, however, these conditions are quite restrictive. They require a 100 percent tax on land rent which is equivalent to pure profits, plus optimal commodity taxes on each of the 2N consumption goods (if labor is chosen as numeraire) (see Auerbach 1985). These 2N taxes can only be achieved using the two taxes $d\hat{c}$ and $d\tau$ if it is optimal to tax goods in each period uniformly. Then, the optimal price distortions would consist of a uniform tax on second period consumption and a uniform tax, perhaps at a different rate, on first period consumption, and these could be simulated exactly by varying the wage rate and the interest rate in the household's budget constraint.

For example, suppose there were no land in the model and preferences were such that all goods (in first <u>and</u> second periods) were equal complements to labor. It would then be optimal to set $d\hat{t} = o$ and have a wage tax. With the resulting equiproportional reduction in first and second period consumption, there would be no change in the fraction of labor income saved by households, and hence no change in the capital-labor ratio or the ratio of gross factor returns. Hence, $d\hat{c} = 0$. With $d\hat{c} = 0$, the first two pieces of the last term in (38) would vanish. The remaining piece would vanish, and $d\hat{\pi}$ would remain unaffected by $d\tilde{T}$, because such preferences would lead to the satisfaction of the fourth condition of Proposition 1, since $\frac{\partial z}{\partial w} \sim z$.[8]

Under such circumstances, however, taxing capital at all would be taxing capital "too much." The <u>presence</u> of a capital income tax would make it optimal to utilize asset specific capital income tax rates unless assumptions (1) through (3) of Proposition 1 were also satisfied. A standard argument against such production distortions is that we are really unsure about the various production and consumption complementarities needed to decide <u>how</u> to vary asset specific taxes (Auerbach 1982). However, our results suggest that such readily observed factors as the capital intensity of different industries could be more directly relevant than the structure of production and utility functions.

5. Empirical Specification

It is important to know how strong the argument against uniform taxation is, whether the foregoing analysis is of more than theoretical interest. To address this and related questions, we consider a simplified model of the private U.S. economy which is sufficiently disaggregate to capture the important aspects of the model. There are three factors of production, labor, land and capital, and nine production sectors. Among the capital goods (which also include inventories) are three fixed capital goods: equipment, nonresidential structures, and residential structures. The industries, based on standard national income accounting definitions, are:

1. Agriculture
2. Mining
3. Construction
4. Durable goods manufacturing
5. Nondurable goods manufacturing
6. Transportation, communication and utilities
7. Wholesale and retail trade
8. Finance, insurance and real estate
9. Other services

Note that housing services, including the imputed rent on owner-occupied housing, are the primary component of industry 8. Our base year for calculations is 1981, the latest year for which input-output data were available. In that year, total production (y) ranged among the industries from approximately 200 billion dollars each in agriculture and mining to approximately one trillion dollars each in durable and nondurable goods manufacturing. The corresponding matrices A, B, K and $\bar{C}$ and the vector ℓ are presented in Appendix B along with an explanation of their derivation. This appendix also sketches the method of solving for the change in equilibrium resulting from taxation.

We assume throughout that $s = r$ initially, which is simply a choice of land units that makes $p_L = 1$. We let $r = .06$ and $n = .03$ for the base case.

The production function for each sector is assumed to be of the nested CES form, with the elasticity of substitution among land and capital goods equal to ω and the elasticity between each of these and labor to be σ. In the base case calculation, each is set equal to 1 (yielding the Cobb-Douglas form) for every industry. The assumption about σ is empirically reasonable. There has been little research about the magnitude of ω.

The household's utility function is also assumed to be of the nested CES form, with leisure in the first-period nest, an intratemporal elasticity of substitution ε in each period (with equal consumption weights) and an intertemporal elasticity of substitution γ. The labor supply elasticity is a function not only of ε and γ but also of the intensity parameter of leisure in the utility function. It is convenient here to choose this parameter indirectly by specifying the fraction of hours worked in the initial equilibrium, λ. Given ε and γ, a higher λ implies a smaller labor supply elasticity, since the leisure demand elasticity is unaffected and the labor-leisure ratio increases.

Our base case parameters are $\gamma = .25$, $\varepsilon = 1$ and $\lambda = .6$. As discussed by Auerbach and Kotlikoff (1987), this value of γ is consistent with the empirical literature, though some outlying estimates approach 1. The values of ε and λ are taken from Ballard et al. (1985), who, for a two-period life cycle model of the household, assumed a similar value for λ and a Cobb-Douglas utility function among first-period commodities and then found an elasticity of substitution between goods and leisure also near 1 to be consistent with observed uncompensated labor supply elasticities.

6. Basic Results and Sensitivity Analysis

We begin with a description of our base case, "pre-tax reform" simulation. The tax parameters used are .45 for inventories, .4 for nonresidential structures, .05 for equipment, .15 for residential structures, .4 for land, and .25 for labor. The business tax rates are representative of those measured for the pre-1986 period (e.g., Auerbach 1987, Fullerton and Henderson 1987). The residential tax rate may be viewed as a weighted average of 0 for owner-occupied housing and .4 for rental housing, although one could argue that the former number is too low (homeowners do pay property taxes, for example) and the latter too high (given the use of rental housing as a tax shelter). The tax rate on labor is consistent with aggregate statistics relating personal taxes to personal income. The nondistortionary tax on land matters only in the calculation of β, the compensation parameter.

The deadweight loss in this case equals 1.003 percent of the present value of each individual's consumption. This is smaller than other estimates of the total deadweight loss of the tax system (e.g., Ballard, Shoven and Whalley 1985), for which there are several potential explanations, including differences in modelling and parameter assumptions and the omission here of several specific taxes (such as social security and excise taxes). However, our main concern here is with the magnitude of deadweight loss resulting from nonuniform capital income taxation.

Setting all nonresidential capital income taxes equal to the average rate for these taxes, .280, causes a reduction in deadweight loss to .925 percent of lifetime consumption, representing a gain of only .078 percent. Such small gains, which translate into a few billion dollars per year, are consistent with earlier results. Moving to fully uniform capital income taxes, including residential capital, at an equal-revenue-yield rate of .222, yields a further efficiency gain of just .030 percent of lifetime consumption, perhaps a billion dollars per year. Thus, moving to uniform taxation yields a total welfare gain of .108 percent of lifetime consumption. Although these efficiency gains are small, so are the efficiency losses associated with small changes in capital income tax rates. For example, it would take a nearly 9 percentage point increase in the average tax rate on nonresidential capital to offset the utility gain from shifting to uniform nonresidential capital taxation. This gain is of the rough order of magnitude of the increase that occurred with the introduction of the Tax Reform Act of 1986.

Next, we consider the robustness of our results. In Table 2, we present the total deadweight loss for each of the three tax configurations just presented, for a variety of parameter variations. The base case results are repeated at the top for convenience.

The parameter variations are intended to be extreme ones. (Note that halving λ is equivalent to doubling the assumed labor supply elasticity). The smallest welfare gains, about 40 percent the size of those in the base case, occur when $r = .03$. It is clear why lowering the rate of return lowers the estimated gain from equalizing taxes based on that rate of return. The "correct" real return to use is unclear given the absence of uncertainty from this model and the divergence between observed marginal products of capital and real interest rates. The largest welfare gains are estimated for the case where $\gamma = 1$, nearly double those estimated for the base case. However, halving or doubling the magnitude of the welfare gains reported for the base case does not change the basic qualitative result: gains from equalizing tax rates on nonresidential capital are small, and gains from going beyond this to equalize tax rates for residential and nonresidential capital are smaller still.

7. Optimal Capital Taxation

The Proposition in Section 4 showed how unlikely it is for uniform capital income taxes to be necessarily optimal. But it is not yet clear how important this result is empirically. Even though the utility function chosen satisfies expression (40), and hence the third and fourth conditions of Proposition 1 are satisfied, the other two conditions are not. This is quite evident from Table 3, which reports the capital intensity of capital and consumption goods. As one would expect, housing is much more capital intensive than other consumption goods. Likewise, equipment is clearly the most capital intensive capital good.

Indeed, it is possible to increase utility by deviating from uniform taxation. Starting from a position with all capital taxes equal to .22 (the equal-yield uniform tax case considered above), a grid search algorithm located a maximum utility level at a tax rate of .16 on equipment, .23 on nonresidential and residential structures, and .27 on inventories. While this deviation is reasonably large, and is not surprising given the very high capital intensity of equipment, the utility gain is small, about .005 percent of consumption, or less than 5 percent of the size of moving from the initial tax system to unifrom taxation. Put another way, virtually all of the increase in utility that can be achieved through the manipulation of capital income taxes is achieved through moving to uniform taxation.

By coincidence, the case for uniform taxation is strengthened if only nonresidential tax rates may be varied. This is probably because the residential tax rate being too low justifies heavier taxation of the capital good used most heavily in building housing, equipment. Starting from the equal yield nonresidential tax rate of .28, we find an optimum tax rate of .26 for equipment, .28 for nonresidential structures and .30 for inventories, with an additional welfare gain of only .0005 percent of consumption.

Given the inevitable uncertainty that must exist about the correct model specification, these results represent a fairly strong argument against attempting to vary capital income tax rates from a uniform structure for "optimal taxation" purposes. They indicate that the indirect effects of capital taxation on capital allocation that work through capital goods prices and consumption goods prices are unimportant relative to the direct effects of taxation considered in the existing literature. Perhaps this should not be a complete surprise, since the indirect price effects of capital costs are attenuated by any specific capital good's small share of value added and value added's fractional contribution to the price of gross output. It is relatively difficult to alter relative output or capital goods prices significantly using differential capital taxes, so the potential benefits of doing so are small relative to the direct deadweight costs of doing so.

8. The Generational Tax Burden

As discussed above, the compensation term β is interesting in its own right, for it tells us what fraction of the tax system is borne by the old and what fraction by the young. This is of particular importance when a shift in the structure of taxation is contemplated.

In the initial tax system considered above, $\beta = .32$; the old must receive just under one-third of the revenue collected to offset the second-period loss in real income induced by taxation. By comparison, the old account for .38 of all consumption. Each component of the overall tax system has a very different age-based burden. Labor income taxes alone would require

$\beta = .09$, indicating that only a small fraction of these taxes are shifted to the elderly. A some what higher fraction of capital income taxes are shifted to the young, with $\beta = .82$ when only capital income taxes are imposed. Land taxes are not shifted at all, of course, and so for them $\beta = 1$.

The shift in capital income taxes to uniform taxation reduces the overall value of β very slightly to .31, as the interest rate increases, but this is smaller than the incidence shift that would attend a change in the tax base, from income to consumption, for example (assuming that β for a consumption tax would be roughly equal to the older cohort's consumption share).

9. Conclusion

The main result of this paper is that uniform capital income taxation is almost certainly suboptimal from a theoretical standpoint because capital goods enter the production process at different levels and through the production of different commodities, but that uniform taxation is empirically close in efficiency terms to the "optimal" system of differential capital income taxes. This empirical result is unlikely to be sensitive to reasonable parameter variations within the model, though one may conceive of other models in which it might not hold. However, the benefits of deviations from uniform taxation remain to be demonstrated.

Though the gains from a move to uniform taxation from a typical tax system are not large, either, they do produce an efficiency gain that would be offset only by a rise in capital income tax rates of several percentage points. This suggests that, in broad terms at least, the recent U.S. tax reform is unlikely to have caused a major net change in long-run efficiency through its changes in capital income taxation.

Appendix A

This appendix presents a proof of Proposition 1. Our strategy will be to show that, under conditions (1) through (4), $d\hat{c}$, ds, dr and dw are all invariant with respect to $d\tilde{T}$. Combined with the conditions themselves, this will imply that the entire last term in (38) always vanishes and that $d\hat{\pi}$ does not depend on $d\tilde{T}$.

Note that, since $d\tau$ is assumed fixed, $dw = -d\tau$ does not depend on $d\tilde{T}$. Also, since $d\hat{c} = dr + d\hat{t}$, $d\hat{c}$ is invariant to changes in $d\tilde{T}$ if and only if dr is.

The proof relies on the assumption that r = n, the saving-investment identity (26) and the fact that land is fixed. Expanding (26), we obtain:

$$\text{(A1)}\quad \frac{p'}{1+r}(S_{21}S_{22})d\pi = (1+n)\{q'\Lambda'\tilde{H}d\rho + p'BK\bar{\tilde{\Omega}}^{-1}[S_{11} + \frac{1}{1+n}S_{21},\ S_{12} + \frac{1}{1+n}S_{22}]d\pi\}$$

Since this must hold regardless of $d\tilde{T}$, <u>if</u> $d\hat{c}$ and ds are to remain fixed as $d\tilde{T}$ changes it must be true that:

$$\text{(A2)}\quad \frac{p'}{1+r}(S_{21}S_{22})d\tilde{\pi} = (1+n)\{q'\Lambda'\hat{H}\Lambda d\tilde{T}q + p'BK\Omega^{-1}[S_{11}+ \frac{1}{1+n}S_{21},\ S_{12} + \frac{1}{1+n}S_{22}]d\tilde{\pi}\}$$

But the right-hand side of (A2) equals[9] $(q'd\tilde{T}\frac{\partial(Ky)}{\partial\hat{c}})'$, which equals zero by (22) and assumption (1) of the proposition.

The left-hand side of (A2) equals[10] $-(1+r)(d\tilde{p}'\frac{\partial z}{\partial\hat{r}})'$. Since $d\tilde{p}'\frac{\partial z}{\partial r} = q'd\tilde{T}\frac{\partial(Ky)}{\partial r}$ (see (24)), this term must equal zero by assumption (3). Thus, holding ds and $d\hat{c}$ constant as $d\tilde{T}$ changes is <u>consistent</u> with the assumptions of the proposition. Using an expression based on the conservation of land, we will show that this is also the case there. This will give us two independent equations in changes in $d\hat{c}$ and ds for which constant $d\hat{c}$ and ds are consistent with the assumptions. Since these equations are both linear, this must be the unique solution. We know that $dL = d\ell'\bar{\tilde{\Omega}}^{-1}z + \ell'\bar{\tilde{\Omega}}^{-1}(B\bar{C}dKy+dz) = 0$.

Expanding this yields:

$$\text{(A3)} \qquad H'_L d\rho + \Theta'\tilde{H}d\rho + \ell'\bar{\bar{\Omega}}^{-1}[S_{11} + \frac{1}{1+n}S_{21}, S_{12} + \frac{1}{1+n}S_{22}]d\pi = 0$$

where Θ' is as defined in (35) and H_L is the last column of $\tilde{H}$. For this to be maintained for any $d\tilde{T}$, and $d\hat{c}$ and ds to remain constant, it must be true that:

$$\text{(A4)} \qquad \hat{H}_L d\tilde{T}q + \Theta'\hat{H}d\tilde{T}q + \ell'\bar{\bar{\Omega}}^{-1}[S_{11} + \frac{1}{1+n}S_{21}, S_{12} + \frac{1}{1+n}S_{22}]d\tilde{\pi} = 0$$

where $\hat{H}_L$ is the vector H_L with its last element omitted.

But this is just[11] $[q'd\tilde{T}\frac{\partial(Ky)}{\partial s}]'$, which equals zero by assumption (2). Thus, assumptions (1), (2) and (3) imply that $d\hat{c}$ and ds are independent of $d\tilde{T}$. This implies that $d\hat{\pi}$ is, as well.

The last condition, condition (4), guarantees that the first two pieces of the last term in (38), which sum to $q'd\tilde{T}d(Ky)$, always vanish, since then $d(Ky)$ must be proportional to Ky regardless of the relative movements of w, r, $\hat{c}$ and s. Finally, the last term in brackets in (38), vanishes by conditions (3) and (4) because it equals[12] $-\{d\tilde{p}'[\frac{\partial z}{\partial w}d\tau + \frac{\partial z}{\partial r}d\hat{c}]\}'$. Hence, the entire final term in (38) always vanishes and the third term in (38), representing the deadweight loss from uniform taxation $d\hat{c}$, $d\tau$ and $d\tau_L$, is not affected by the introduction of asset specific capital taxes.

We will not attempt to prove that conditions (1) - (4) together are also necessary for the general optimality of uniform capital income taxation. However, it seems unlikely that less restrictive sufficient conditions exist. For example, if only condition (4) is violated, one may intuitively see how to vary $d\tilde{T}$ and improve welfare. In that case, the last term in (38) reduces to $d(\pi-\pi_0)'Sd\tilde{\pi}$.[13] Also, $d\hat{\pi}$ and hence $d(\pi-\pi_0)'Sd\hat{\pi}$ is independent of $d\tilde{T}$. Thus, one should choose a perturbation $d\tilde{T}$ in a direction that causes $d(\pi-\pi_0)'Sd\tilde{\pi}$ to be positive. By scaling down the size of this perturbation, one can make the two quadratic forms in (38) arbitrarily small in absolute value compared to this positive term, since they are second-order terms. This will increase ΔU over its value when capital income taxes are uniform.

Appendix B

This appendix presents and describes the calculation of the various matrices and vectors that summarize the U.S. economy's 1981 production structure. It then outlines how these are used to solve for the economy's equilibrium.

A

The 9×9 input-output matrix A was taken directly from a machine readable version of the table given in Planting (1987), with the 79 basic industries aggregated into nine. We ignored the inputs and outputs from the remaining "industries" that are appended to the input-output table for national income accounting purposes. The only one of these that is not negligible is government industry, which is just over 5 percent of total commodity output and does not enter as an intermediate input.

Adding each aggregate industry's value added to the sum of its intermediate inputs gives its total output, which is then divided into the input levels to obtain input coefficients. By construction, therefore, each industry's output price is set to unity. The resulting matrix is:

	Output Industry								
Input Industry	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	.236	.000	.001	.005	.086	.000	.002	.003	.006
(2)	.001	.050	.006	.015	.162	.088	.000	.000	.002
(3)	.011	.037	.002	.008	.007	.032	.008	.033	.019
(4)	.018	.039	.292	.351	.034	.026	.008	.002	.036
(5)	.163	.023	.064	.064	.296	.105	.040	.011	.098
(6)	.029	.024	.030	.049	.058	.177	.063	.022	.055
(7)	.041	.012	.079	.052	.045	.021	.020	.004	.031
(8)	.065	.048	.014	.016	.012	.023	.063	.146	.053
(9)	.022	.018	.095	.042	.044	.050	.134	.057	.098
All Inputs	.586	.251	.583	.602	.744	.522	.338	.278	.398

Examining this matrix, we note that, as one might expect, intermediate inputs are least important in the extractive industry, while they are the most important in manufacturing.

K

Aside from the three fixed capital goods, we allow nine inventory stocks, one for each industry. This distinction permits each industry's inventories to have a different commodity composition. The nonresidential stocks of equipment and structures were obtained from machine-readable data kindly provided by Matthew Shapiro, obtained by him originally from the Bureau of Economic Analysis and corresponding to the aggregate statistics given in Musgrave (1986). We used the average of the 1980 and 1981 end of year capital stocks, deflated to put them into current 1981 rather than constant 1982 dollars. The residential capital stocks were obtained directly from Musgrave (1986). The 1980 and 1981 end of year current dollar inventory stocks for industries (1), (4), (5) and (7) were obtained from the April 1987 and April 1983 issue of the Survey of Current Business, and averaged. The industry breakdown for the remaining industries (representing about one-tenth of all inventories) was kindly provided us by the BEA. All capital stocks were then divided by the appropriate industry output, to obtain K:

		Industry								
Capital Stock		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Equipment		.54	.28	.11	.26	.20	.83	.19	.20	.14
Structures		.44	.89	.04	.12	.11	.97	.25	.50	.11
Housing		.29	0	0	0	0	0	0	4.30	0
Inventories:	(1)	.46								
	(2)		.04							
	(3)			.06						
	(4)				.24					
	(5)					.13				
	(6)						.06			
	(7)							.50		
	(8)								.00	
	(9)									.01
Total Capital-Output Ratio		1.73	1.21	0.21	0.62	0.44	1.86	0.94	5.00	0.26

Note the enormous differences among industries in capital-output ratios, from .21 in the construction industry to 5.00 in the industry composed mostly of housing services. Also, the major type of capital used varies across industries: structures in mining and housing, equipment in manufacturing and construction, and inventories in trade.

B

The B matrix was obtained from two sources. For fixed capital, we used Table 3 in Silverstein (1985). From this table, one may infer that virtually all capital produced by the construction industry is structures and that virtually all structures are produced by this industry. Given this, one may calculate the source of equipment production from the output of capital goods by the remaining industries.

For inventories, a breakdown of inventories by stage of fabrication is available only for manufacturing in the February 1986 Survey of Current Business. For these industries, we assumed that all finished goods and half of work in process inventories were produced by that industry, while the remaining work in process stocks and all new materials inventories had the same composition as the industry's intermediate inputs. For the other seven industries, we simply assumed that all inventory stocks were produced by the industry itself, equivalent to assuming then to be entirely finished goods.

The resulting B matrix is:

Industries of Origin	Asset			Inventories								
	Equipment	Structures	Housing	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	0	0	0	1	0	0	.004	.058	0	0	0	0
(2)	.003	0	0	0	1	0	.014	.109	0	0	0	0
(3)	0	1	1	0	0	1	.007	.004	0	0	0	0
(4)	.803	0	0	0	0	0	.775	.023	0	0	0	0
(5)	.009	0	0	0	0	0	.057	.699	0	0	0	0
(6)	.035	0	0	0	0	0	.044	.039	1	0	0	0
(7)	.144	0	0	0	0	0	.047	.030	0	1	0	0
(8)	.009	0	0	0	0	0	.014	.008	0	0	1	0
(9)	0	0	0	0	0	0	.038	.030	0	0	0	1

ℓ

The stocks of nonresidential land by industry for 1977 were kindly provided by Don Fullerton and Yolanda Henderson who obtained the unpublished data from Barbara Fraumeni and Dale Jorgenson. Earlier aggregate land estimates and a description of the methodology used to calculate them is provided in Fraumeni and Jorgenson (1980). The 1977 stocks were inflated to 1981 prices using the GNP deflator. To obtain an estimate of residential land, we multiplied the 1980 and 1981 land to structures ratios for owner-occupied housing provided by the Federal Reserve's Balance Sheets for the U.S. Economy by our estimated residential capital stocks for each year. This calculation assumes that rental housing, representing about one-third of all housing, has the same land to structures ratio as owner-occupied housing.

The resulting vector ℓ is:

Industry	Land/Output Ratio
(1)	4.240
(2)	0.021
(3)	0.027
(4)	0.037
(5)	0.041
(6)	0.094
(7)	0.103
(8)	1.870
(9)	0.030

As one might expect, land is an important input only in agriculture and housing, but is quite important in those industries.

$\bar{C}$

To obtain the matrix $\bar{C}$, we need depreciation rates. We assume δ to be zero for inventories. For nonresidential equipment and structures, we use the aggregate values estimated by Auerbach and Hines (1987), .137 and .003, respectively. For residential structures, we use the value of .013 estimated by Jorgenson and Sullivan (1981).

Solution of the Model

A step that must be taken before solution of the model is to express all flow variables in units consistent with the two-period overlapping generations model. Letting T be the number of years per period (assumed to be 30 throughout), we multiply the flow vectors y and z and the rates r, n, s and δ by T, and divide the stock-flow ratios K and ℓ by T. This change in units has no direct effect on the production side of the economy. It is needed to make sense of the assumption that the stock of national wealth is held for second-period consumption.

In addition to the parameters given above, we specify the rate of interest (and growth) and the Hessian and Slutsky matrices. The parameters of the Slutsky matrix are chosen to incorporate the constraint that second-period consumption equal principal plus interest on the stock of national wealth. We also constrain expression (1) in the text to hold exactly by starting with observed consumption z and solving for y. This is necessary because the steady-state assumptions about rates of return and depreciation need not hold exactly in any given year. Industries 1-3 have virtually no final sales to consumers, so for convenience consumption of these goods is set to zero.

Through the use of various substitutions, the change in the economy in response to taxation reduces to two linear equations, based on the savings investment identity (A1) and the conservation of land identity (A3), in two variables, dr and ds (when $r \neq n$, the terms Θ' and Λ' in these equations are based on n, while the terms Θ and Λ appearing in the text in (35) are based on r). Once these are obtained, all other endogenous variables can be calculated.

Footnotes

1. This is not a restrictive assumption, as one could specify a unique and general production process for a capital good by making that good one of the N outputs and assuming that such output is not demanded as a final consumption good or an intermediate input.

2. Were this not so, as it is not in reality, the value of old capital, per efficiency unit, would differ from that of a comparable unit of new capital for tax reasons. Such taxes are clearly important in considering the short-run effects of tax reforms, as the effects of the 1981 and 1986 tax changes clearly demonstrate (Auerbach 1983, 1986, Auerbach and Hines 1987). They may also affect the economy in the long run if changes in the relative value of old to new capital, which one may view as a change in government assets, are not offset by explicit debt policy (Auerbach and Kotlikoff 1987). Such long run effects are attributable to changes in the intergenerational burden of taxation rather than the magnitude of distortions. In the analysis presented below, such tax induced changes in capital goods prices would automatically be offset via government transfers, so that only the marginal tax rates matter for long-run analysis. Hence, it is not a further restriction to focus only on income taxes.

3. Details of this and other points of the derivation are available from the author upon request.

4. In an open economy, this term would not necessarily equal zero, since saving and borrowing abroad would be possible, but then it might be the case that dr = 0, in which case the entire last term would still equal zero.

5. That equal complementarity of capital to labor no longer suffices for the optimality of uniform taxation when there is more than one production sector was pointed out by Mackie (1985) and an extension of Auerbach (1979).

6. The idea that we might wish to tax less heavily capital at the bottom of the production chain was suggested by Willig (1983) in a model where the other factor was absent because capital was not itself produced.

7. Conditions (1) and (3) are slightly different. If $\hat{c}\Omega'^{-1}K'B'p \sim p$, then $\hat{c}B'\Omega'^{-1}K'B'p = \hat{c}B'\Omega'^{-1}K'q \sim B'p = q$. Since $\Lambda = I + CB'\Omega'^{-1}K'$, $\Lambda q \sim q$ only if the elements of the diagonal matrix C are equal, i.e., the rates of depreciation are equal.

8. Indeed, this proportionally condition would be satisfied for z^1 and $\frac{1}{1+n}z^2$ separately as well as their sum.

9. To see this, note that $\{p'BK\Omega^{-1}[S_{11}+\frac{1}{1+n}S_{21}, S_{12}+\frac{1}{1+n}S_{22}]d\tilde{\pi}\}'$ equals $d\tilde{p}[S_{11}\frac{\partial\pi^1}{\partial\hat{c}} + S_{12}\frac{\partial\pi^2}{\partial\hat{c}} + \frac{S_{21}}{1+n}\frac{\partial\pi^2}{\partial\hat{c}} + \frac{S_{22}}{1+n}\frac{\partial\pi^2}{\partial\hat{c}}] = d\tilde{p}\frac{\partial z}{\partial\hat{c}}$, while $[q'\hat{\Lambda}'H\tilde{\Lambda}dTq]'$ equals $q'd\tilde{T}\frac{\partial(Ky|z)}{\partial\hat{c}}$. Adding the two yields $q'd\tilde{T}\frac{\partial(Ky)}{\partial\hat{c}}$ (see (25).

10. To see this, note that since by the symmetry of S, $S'_{21} = S_{12}$, $[p'(S_{21}S_{22})d\tilde{\pi}]$ equals $d\tilde{p}'(S_{12}+\frac{1}{1+n}S_{22})p = -d\tilde{p}'[S_{12}+\frac{1}{1+n}S_{22}]\frac{\partial\pi^2}{dr}(1+r)^2 = -d\tilde{p}'\frac{\partial z}{\partial r}(1+r)^2$.

11. By analogy to the previous case, it is clear that the second part of (A4) equals $(d\tilde{p}'\frac{\partial\pi}{\partial s})'$ (see (37)). Likewise, the first part equals $(q'd\tilde{T}\frac{\partial(Ky|z)}{\partial s})'$ (see (35)).

12. Using (28), we have

$$d(\pi-\pi_0)'Sd\tilde{\pi} = \left(\frac{-d\tau p}{1+r} + \frac{d\hat{t}}{(1+r)^2}p\right)\begin{pmatrix} S_{11}+\frac{1}{1+n}S_{12} \\ S_{21}+\frac{1}{1+n}S_{22}\end{pmatrix}d\tilde{p}$$

$$= -\{d\tilde{p}'[S_{11}+\frac{1}{1+n}S_{21}+S_{12}\frac{1}{1+r}+\frac{1}{1+n}S_{22}\frac{1}{1+r}]d\tau$$

$$+ d\tilde{p}'(S_{12}+\frac{1}{1+n}S_{22})\frac{d\hat{t}}{(1+r)^2}p]\}' = -\{d\tilde{p}'[\frac{\partial z}{\partial w}d\tau + \frac{\partial z}{\partial r}d\hat{t}]\}'$$

13. To see this, note first that only conditions (1) - (3) of the proposition were required for the constancy of $d\hat{\pi}$. The first two pieces of the term in brackets in (38), which together equal $q'd\tilde{T}d(Ky)$, in this case equal

$q'd\tilde{T}\frac{\partial(Ky)}{\partial w}dw = d\tilde{p}'\frac{\partial z}{\partial w}dw$, since the other terms vanish by conditions (1) - (3). From footnote 8, we have that the last piece of the term in brackets in (38) also equals $d\tilde{p}'\frac{\partial z}{\partial w}dw$, since the effect of $d\hat{t}$ vanishes by condition (3) and since $dw = -d\tau$. Thus, the last term in (38) is $1/2[2d\tilde{p}'\frac{\partial z}{\partial w}dw]$ or, from footnote 8, $d(\pi-\pi_0)'Sd\tilde{\pi}$.

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Table 1 (continued)

Scalars (continued)

ω	- elasticity of substitution among capital goods (and land)
σ	- elasticity of substitution between capital (and land) and labor
ϵ	- intratemporal elasticity of substitution in consumption
γ	- intertemporal elasticity of substitution in consumption
λ	- fraction of potential hours worked

Vectors (length in brackets)

h	- [N] labor requirements per unit of output (variable)
ℓ	- [N] land requirements per unit of output (variable)
y	- [N] gross production vector
z	- [N] aggregate consumption vector
z^1	- [N] consumption vector of the younger generation
z^2	- [N] consumption vector of the older generation
x	- [2N] the vector of first- and second-period consumption created by stacking z^1 and z^2
p	- [N] commodity price vector (normalized to one in the initial equilibrium)
q	- [M] capital goods price vector (also normalized to one initially)
$\tilde{p}$	- [M] deviation in p associated with differential taxes $\tilde{T}$
p_0	- [M] hypothetical price vector (also normalized to one initially)
π	- [2N] price vector for first- and second-period consumption, created by stacking p and $p/(1+r)$
π_0	- [2N] net of tax consumer prices
$\tilde{\pi}$	- [2N] relates to π as $\tilde{p}$ does to p
ρ	- [M+1] created by stacking cost of capital, c_i, and land, s
$\hat{\rho}$	- [M] part of ρ due to uniform taxes (relates to ρ as $\hat{\pi}$ does to π)

Table 1 (continued)

Vectors (length in brackets) (continued)

Θ - [M] indicate the importance of land in the production of different capital goods.

H_L - [M+1] last column of $\tilde{H}$ (corresponding to land)

$\hat{H}_L$ - [M] H_L omitting last element

Matrices (size in brackets)

A - [N*N] input output matrix; element ij is the (fixed) intermediate input of good i per unit of output j

B - [N*M] capital goods definition matrix; element ij is the (fixed) amount of good i per unit of capital good j; each column of B sums to one

K - [M*N] capital requirements matrix; element ij is the (variable) amount of type i capital required to produce a unit of output j

S - [2N*2N] Slutsky matrix excluding row and column for leisure

$\tilde{S}$ - [N*N] equal to weighted sum of four blocks of S

$\tilde{H}^i$ - [M*M+1] Hessian of industry i's (=1,,,N] cost function excluding row and column corresponding to labor and row corresponding to land

$\tilde{H}$ - [M*M+1] aggregate Hessian, equal to the sum over i of $\tilde{H}^i y_i$

$\hat{H}$ - [M*M] $\tilde{H}$ with last column omitted

$\bar{\Omega}$ - [N*N] matrix relating gross output to final goods consumed

Ω - [N*N] same as $\bar{\Omega}$ but based on C rather than $\bar{C}$

Ω_0 - [N*N] same as $\bar{\Omega}$ but based on C_0 rather than $\bar{C}$

T - [M*M) diagonal matrix with elements t_i

$\bar{C}$ - [M*M] diagonal matrix with elements $\bar{c}_i$

C - [M*M] diagonal matrix with elements c_i

C_0 - [M*M] diagonal matrix with elements c_{0i}

Table 1 (continued)

<u>Vectors</u> (length in brackets) (continued)

$\hat{C}$ - [M*M] diagonal matrix with elements $\hat{c}_i$

$\tilde{T}$ - [M*M] diagonal matrix of differential capital tax rates equal to $T - \hat{t}I$

Λ - matrix based on where capital goods enter the production process

Table 1

Model Notation

Scalars

N	- number of output industries
M	- number of capital goods
E	- total labor supply, per capita young
L	- total land supply, per capita young
n	- population growth rate
r	- interest rate (after tax)
w	- wage rate (after tax
s	- rate of return to land (before tax)
s_0	- rate of return to land (after tax)
p_L	- $(= s_0/r)$ price of land
δ_i	- rate of geometric decay of type i (=1,...,M) capital
τ	- tax rate on labor income
$\hat{R}$	- tax revenue, per capita young
R^1	- tax revenue rebated to each individual in period 1
R^2	- tax revenue rebated to each individual in period 2
R	- present value of rebated revenue
β	- ratio of R^2 to R, in present value
τ_L	- tax rate on land income
t_i	- tax rate on capital income of type i
$\hat{t}$	- average tax rate on capital goods
$\bar{c}_i$	- $(= n+\delta_i)$ steady-state investment-capital ratio for type i capital
c_i	- $(-r+t_i+\delta_i)$ user cost of type i capital per dollar invested
$\hat{c}_i$	- $(= r+\hat{t}+\delta_i)$ user cost based on uniform capital income tax
c_{0i}	- $(= r+\delta_i)$ type i user cost based on net of tax capital cost

Table 2

Deadweight Loss: Sensitivity Analysis

(Percent of Lifetime Consumption)

Parameter Variation	(1) Initial Tax System	(2) Uniform Business Taxes	(3) Uniform Capital Taxes
Base Case	1.003	.925	.895
$\varepsilon = .5$	.785	.671	.646
$\gamma = 1$	1.951	1.781	1.755
$\lambda = .3$	1.874	1.820	1.789
$\omega = .5$	.964	.924	.903
$\sigma = .5$	.922	.870	.829
$r = .03$	1.254	1.223	1.211

Table 3

The Capital Intensity of Capital and Consumption Goods

	Capital Intensity
Capital Goods	
Equipment	.486
Structures	.159
Residential	.125
Inventories	.158
Consumption Goods:	
Durables	.142
Nondurables	.164
Transportation, Communications, & Utilities	.243
Wholesale & Retail	.135
Finance, Insurance & Real Estate	.422
Other Services	.095

*Capital intensity, discussed in text following Proposition 1, is the derivative of the user cost $d(cq)_i/d\hat{c}$ for capital goods and the price elasticity $\frac{\hat{c}}{p_i}dp_i/d\hat{c}$ for consumption goods, where $\hat{c} = r + \hat{t}$ is the gross return to capital under a uniform capital income tax $\hat{t}$.