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FINANCING VENTURES

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ABSTRACT

The relationship between venture capital and growth is examined using an endogenous growth model incorporating dynamic contracts between entrepreneurs and venture capitalists. At each stage of financing, venture capitalists evaluate the viability of startups. If viable, venture capitalists provide funding for the next stage. The success of a project depends on the amount of funding. The model is confronted with stylized facts about venture capital; viz., statistics by funding round concerning the success rates, failure rates, investment rates, equity shares, and IPO values. Raising capital gains taxation reduces growth and welfare.

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1   Introduction

> "I think the development of the venture capital system has been an example of something which is a successful improvement in risk-bearing. It doesn't exactly remove the risks at the beginning, but at least creates greater rewards at a slightly later stage and therefore encourages, say, small companies to engage in technologically risky enterprises. If you like innovation, you expect 50 percent to 60 percent failure. In a sense if you don't get that, you're not trying hard enough. Venture capital has done much more, I think, to improve efficiency than anything." Kenneth J. Arrow, 1995

The importance of venture capital in the U.S. economy has skyrocketed over the past 50 years. Investment by venture capitalists was roughly $303 million in 1970. This soared to $54 billion by 2015 (both numbers are in $2009). The rise in venture capital (VC) financing is shown in the right-hand-side panel of Figure 1. While the share of VC funding in total investment is still relatively small, around 2 percent in 2015, its punch far exceeds its weight. The fraction of public firms that have been backed at some time by venture capitalists is now around 20 percent, compared with just 4 percent in 1970–see the left-hand-side panel of Figure 1. (See the Data Appendix for the sources of all data used in the paper.) Such firms presently account for about 20 percent of market capitalization. The capitalization line lies below the fraction-of-firms line because VC-backed companies tend to be more recent entrants that are younger and smaller in size, whereas their non-VC-backed counterparts tend to be established incumbents. Today venture capitalists are significant players in job creation and technological innovation. Public firms that were once backed by venture capitalists currently make up a significant fraction of employment and an even larger share of R&D spending, as opposed to virtually nothing in 1970, as the left-hand-side panel of Figure 2 makes clear. The right-hand side of the figure displays their enormous contribution to the generation of patents, both in raw and quality-adjusted terms. The employment share of VC-backed firms is far less than the R&D (and patents) share. This is because VC-backed companies are more R&D intensive than their non-VC-backed
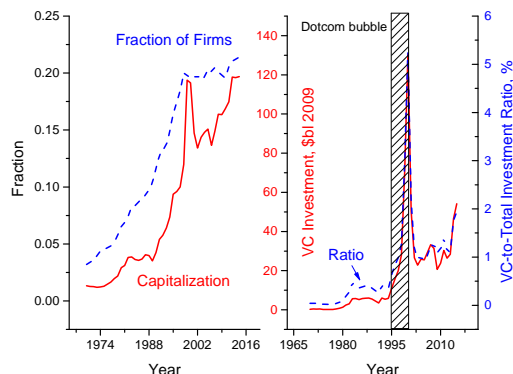
Figure 1: The rise of venture capital, 1970 to 2015. The right-hand-side panel shows investment by venture capitalists. The left-hand-side panel plots both the fraction of public firms financed by venture capitalists and the share of VC-backed public firms in market capitalization.

counterparts. For instance, Google (a VC-backed company) has far fewer employees than General Motors (a non-VC-backed company), but Google invests a lot more in R&D than General Motors.

The VC industry has been an incubator of numerous technological giants in the information and communication technology sector as well as the biotechnology sector, plus an array of star innovators in the service industry. Former VC-backed firms are household names. Table 1 lists the top 30 VC-backed public companies by market capitalization. Figure 3 plots the relative significance of the words "banks" and "venture capital," as reflected by their usage in English language books. As shown, the term venture capital was virtually unused in 1930. The relative significance of venture capital vis-à-vis banks has increased considerably since then.

To address the importance of VC in the U.S. economy, an endogenous growth model is developed. At the heart of the growth model is a dynamic contract between an entrepreneur and a venture capitalist. The venture capitalist invests in the entrepreneur's startup as an active participant. The venture capitalist provides seed money for initial research. The project then enters a funding-round cycle. At the beginning of each funding round the venture capitalist evaluates the worthiness of the project. Those projects that pass the
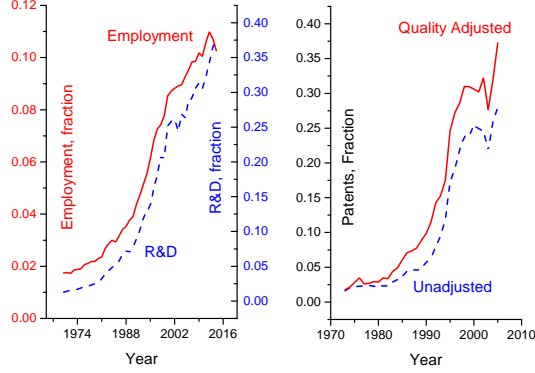
Figure 2: The share of VC-backed firms in employment, R&D spending, and patents. The data in the left-hand-side panel are from 1970 to 2014, while that in the right-hand-side panel spans 1973 to 2005.

TOP 30 VC-BACKED COMPANIES

1	Apple Inc	11	Amgen Inc.	21	Fedex Corp.
2	Cisco Systems Inc.	12	Yahoo Inc.	22	Juniper Networks Inc.
3	Microsoft Corp.	13	Genentech Inc.	23	Nextel Communications Inc.
4	Alphabet Inc.	14	Celgene Corp.	24	Gap Inc.
5	Facebook Inc.	15	Ebay Inc.	25	Viacom Inc.
6	Oracle Corp.	16	Compaq Computer Corp.	26	Veritas Software Corp.
7	Amazon.Com Inc.	17	Starbucks Corp.	27	Salesforce.Com Inc.
8	Sun Microsystems Inc.	18	Micron Technology Inc.	28	Alexion Pharmaceuticals Inc.
9	Gilead Sciences Inc.	19	Applied Materials Inc.	29	Adobe Systems Inc.
10	Dell Inc.	20	Regeneron Pharmaceuticals	30	Twitter Inc.

Table 1: The table shows the top 30 VC-backed companies by market capitalizaton. These companies are identified by matching firm names in VentureXpert and CompuStat.
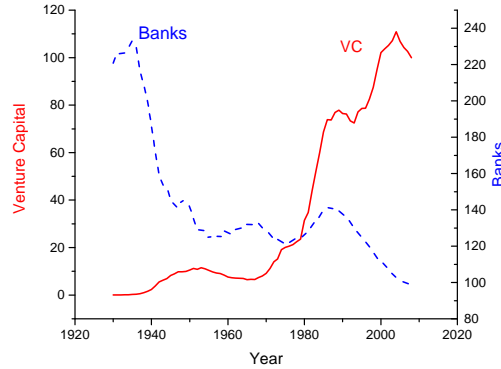
Figure 3: Banks and venture capital, 1930-2008. The figure plots the use of the words "banks" and "venture capital," relative to all words in English language books, using the Google Ngram Viewer. For each series, the value in 2008 is normalized to 100.

evaluation are given funds for development. The contract is designed so that it is not in the entrepreneur's interest to divert funds away from their intended purpose. The venture capitalist can imperfectly monitor at a cost the entrepreneur's use of funds, which helps to ensure incentive compatibility. Those ventures that are successful during a fund round are floated on the stock market. The contract specifies for each funding round the evaluation strategy to gauge the project's worthiness, the amount of VC invested in development, the level of monitoring to avoid malfeasance, and the shares of each party in the proceeds from a potential IPO. The predicted features of the contract are compared with some stylized facts about venture capital: (i) the success and failure rates by funding round, (ii) investment by funding round, (iii) the value of an IPO by duration of the incubation period, and (iv) the venture capitalist's share of equity by funding round. Despite the importance of VC, the majority of U.S. firms are not financed through this channel. So, the analysis includes a traditional sector that produces the majority of output using capital that can be thought of as being financed through regular banks. The key participants in a VC partnership receive the majority of their compensation in the form of stock options and convertible equity. As such, they are subject primarily to capital gains taxation. The analysis examines how innovative activity is affected by the capital gains tax rate.

4

Dynamic contract models have now been used for some time to study consumption/savings cum effort decisions with moral hazard. An early example is Phelan and Townsend (1991), with Karaivanov and Townsend (2014) being representative of more recent work. Dynamic contract frameworks that focus on firms, and VC in particular, are rarer. Bergemann and Hege (1998), Clementi and Hopenhayn (2006), Cole, Greenwood, and Sanchez (2016) and Smith and Wang (2006) develop contracting structures that share some similarities with the one presented here. In Bergemann and Hege (1998) a venture capitalist also learns about a project's type, good or bad, over time. The odds of a good project's success are a linear function of investment. The entrepreneur can secrete some of the funds intended for investment, so there is a moral hazard problem. Given the linear structure of their model, which generates corner solutions, analytical results obtain. In an extension, the venture capitalist can monitor investment or not. If he monitors, then any irregularities are uncovered with certainty. The analysis is done in partial equilibrium. While illuminating some economics about VC, it would be hard to take their streamlined structure to the data. Clementi and Hopenhayn (2006) and Smith and Wang (2016) model long-term credit relationships between entrepreneurs and lenders. Lenders cannot monitor the borrower. These analyses stress the efficiency of long-term contracts. Since they do not focus on VC, they do not formulate the incubation period where a lender supplies funding for research and development while evaluating the worthiness of the startup and monitoring the use of funds.

The current paper borrows Cole, Greenwood, and Sanchez's (2016) flexible-monitoring technology. The more the venture capitalist invests in auditing, the higher the odds that he will detect any irregularities. The venture capitalist can also invest in evaluating a project in each funding round to learn about its type, good or bad, something not allowed in Bergemann and Hege (1998). This feature is important because it allows the odds that a project is good to rise over funding rounds. This works to generate an upward-sloping investment profile by funding round. The odds of a good project's success are an increasing, concave function of investment in development. Additionally, VC is taken to be a competitive industry; this is similar to Cole, Greenwood, and Sanchez's (2016) and Smith and Wang's (2006) assumption

that financial intermediation, more generally, is competitive.

The current analysis is done within the context of an endogenous growth model. Cole, Greenwood, and Sanchez (2016) focus on the impact that financial intermediation, more broadly defined, has on cross-country technological adoption and income levels. As in Akcigit, Celik, and Greenwood (2016), the current work has a distribution of competitive firms operating in general equilibrium. This distribution is continually shifting rightward with technological progress in the economy. A new entrepreneur decides how far to push his productivity relative to the frontier; this is somewhat reminiscent of Parente (1994). The position of the frontier is determined by a classic Romer (1986) type externality. The last three papers noted have no startups. None of the above papers compares the predictions of their models with the VC process in the United States. And none of them examines how innovative activity is affected by the rate of capital gains taxation.

There is, of course, work on VC that does not take a dynamic contract perspective. Silveira and Wright (2016) build a canonical search model of the process where entrepreneurs are matched with venture capitalists, something abstracted from here. Upon meeting, the parties bargain in Nash fashion over each one's investment and how to split the proceeds. Jovanovic and Szentes (2013) focus on a setting where the incubation period for a project is unknown. Unlike entrepreneurs, venture capitalists have deep pockets and can weather supporting a project over a prolonged period of time, if they so choose. A contract specifies the initial investment by the venture capitalist and some fixed split of the profits. The analysis focuses on characterizing and measuring the excess return earned by venture capitalists, due to the latters' scarcity. A tractable stylized Schumpeterian model of VC that has analytical solutions is developed by Opp (2018). He estimates that the welfare benefits of VC are worth 1 to 2 percent of aggregate consumption, despite the fact that VC investment is highly procyclical, which operates to trim the estimates. In his analysis, entrepreneurs do not choose how far to launch their endeavour ahead of the pack. Also, the likelihood of success does not depend on the level of development funding. Since the innovation process is essentially static, there is no investment over time in learning about the project's quality.

Given the static nature of R&D investment, he does not model the stage-financing process; i.e., the success rates, failure rates, investment rates, equity shares, and values of an IPO by funding round.

2   The Rise of Venture Capital as Limited Partnerships

Financing cutting-edge technologies has always been problematic.[1] It is difficult to know whether new ideas are viable, if they will be saleable, and how best to bring them to market. Also, it is important to ensure that entrepreneurs' and investors' incentives are aligned. Traditional financial institutions, such as banks and equity/securities markets, are not well suited to engage in this sort of underwriting. Historically, the introduction of new technologies was privately financed by wealthy individuals. Investors were plugged into networks of inventive activity in which they learned about new ideas, vetted them, and drew on the expertise needed to operationalize them. These financiers are similar to today's "angel investors."

The Brush Electric Company provided such a network for inventors and investors in Cleveland around the turn of the 20th century. Electricity was one of the inventions born during the Second Industrial Revolution. Individuals linked with the Brush Electric Company network spawned ideas for arc lighting, liquefying air, smelting ores electrically, and electric cars and trolleys, among other things. The shops at Brush were a meeting place for inventors; they could develop and debug new ideas with help from others. Investors connected with the Brush network learned about promising new ideas from the scuttlebutt at the shops. They became partners/owners in the firms that they financed. Interestingly, in the Midwest at the time, prolific inventors (those with more than 15 patents) who were principals in companies were much more likely to keep their patents or assign them to the companies where they were principals as opposed to other inventors, who typically sold them to businesses where they had no concern. This aligned the incentives of innovators

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[1] This section draws *heavily* on Lamoreaux, Levenstein, and Sokoloff (2007) for the period prior to World War II and on Kenney (2011) for the period after.

and investors.

World War II and the start of the Cold War ushered in new technologies, such as jets, nuclear weapons, radars, and rockets. There was a splurge of spending by the Defense Department. A handful of VC firms were formed to exploit the commercialization of scientific advances. American Research and Development (ARD), founded by General Georges Doriot and others, was one of these. ARD pulled in money from mutual funds, insurance companies, and an initial public stock offering. The founders knew that it was important for venture capitalists to provide advice to the fledging enterprises in which they were investing. In 1956 ARD invested $70,000 in Digital Equipment Corporation (DEC) in exchange for a 70 percent equity stake. ARD's share was worth $38.5 million when DEC went public in 1966, which represented an annual return of 100 percent. While this investment was incredibly successful, the organizational form of ARD did not come to dominate the industry. The compensation structure of ARD made it difficult for the company to retain the VC professionals needed to evaluate startups and provide the guidance necessary for success.

An alternative organizational form came to emblematize the industry; viz., the limited partnership. This form is exemplified by the formation of Davis and Rock in 1961. These partnerships allowed VC professionals to share in the gains from startups along with the entrepreneurs and investors. Limited partnerships served to align venture capitalists' interests with those of entrepreneurs, investors, and key employees. Money was put in only at the beginning of the partnership. The general partners received management fees as a salary plus a share of the capital gains from the investments, say 40 percent, with the limited partners earning 60 percent. The limited partners had no say in the decisions of the general partners. The partnerships were structured for a limited length of time, say 7 to 10 years. The returns from the partnership were paid out to the investors only when the partnership was dissolved–there were no dividends, interest payments, etc. Therefore, the returns upon dissolution were subject only to capital gains taxation at the investor level. The VC industry also rewarded founders, CEOs, and key employees using stock options. Thus, they too were subject to capital gains taxation and not taxation on labor income. The short time horizon

created pressure to ensure a venture's rapid success.

Banks and other financial institutions are not well suited to invest in cutting-edge new ventures. While banks are good at evaluating systematic lending risk, they have limited ability to judge the skill of entrepreneurs, the worth of new technologies, and the expertise to help commercialize them. The Glass-Steagall Banking Act of 1933 prohibited banks from taking equity positions in industrial firms–the act was repealed in 1999. Allstate Insurance Company created a private placements program in the 1960s to undertake VC-type investments. It abandoned the program because it could not compensate the VC professionals enough to retain them. The Employee Retirement Income Security Act of 1974 prevented pension funds (and dissuaded other traditional fiduciaries) from investing in high-risk ventures. The act was reinterpreted in the 1980s to allow pension funds to invest in VC-operating companies, which provided a fillip for the VC industry.

3   Empirical Evidence on Venture Capital and Firm Performance

How does VC affect firm growth and technological innovation? The VC industry is a successful incubator of high-tech and high-growth companies. VC-backed public companies have higher R&D-to-sales ratios than their non-VC-backed counterparts. Following an IPO, they also grow faster in terms of employment and sales. VC-backed companies are embraced as "golden geese" by the investors. They are valued higher than their non-VC-backed counterparts around the time of an IPO. In addition, VC is a potent apparatus for financing technological innovation. VC funding is positively associated with patenting activity by firms. Moreover, patenting depends more on VC funding in those industries where the dependence on external financing is high.

3.1 Venture Capital and Firm Growth

Regression analysis is now conducted to evaluate the performance of VC-backed and non-VC-backed firms along four dimensions for years following an IPO: the R&D-to-sales ratio, the growth rate of employment, the growth of sales revenue, and the (natural logarithm of the) market value of firms. The results are presented in Table 2. The regressions are based on an unbalanced panel of U.S. public companies between 1970 and 2014. To compare VC-backed companies with their non-VC-backed counterparts, a VC dummy is entered as an independent variable that takes the value of 1 if the company is funded by VC before its IPO. In all regressions, industry dummies, year dummies, and a year dummy for the IPO are included. In addition, a cross term is added between the VC dummy and the number of years since the firm's IPO.

As shown by the first row of regression coefficients, VC-backed companies are more R&D intensive and grow faster than their non-VC-backed counterparts. On average the R&D-to-sales ratio of a public VC-backed company is higher than its non-VC-backed counterpart by 5.2 percentage points, and it grows faster–by 4.9 percentage points in terms of employment and 7.0 percentage points in terms of sales revenue. These superior performances translate into higher market values: VC-backed companies are valued 37.3 percent higher than their non-VC-backed counterparts. The difference in performance, however, gradually dwindles over the years, as shown by the negative signs of the regression coefficients in the second row. As a consequence, the performances of VC- and non-VC-backed public companies tend to converge in the long run, though the speed of convergence is fairly low, as revealed by the magnitude of the regression coefficients in the second row.

3.2 Venture Capital and Innovation

Regression analysis now assesses the role of VC in encouraging technological innovation; specifically, the impact of VC funding on patent performance at an annual periodicity is evaluated, both at the firm and industry levels. The regression analysis is based on all

VC- versus Non-VC-Backed Public Companies

Dependent variable	R&D / Sales	Employment growth	Sales growth	ln(Firm value)
VC (= 1, if backed by VC)	0.0521***	0.0490***	0.0696***	0.373***
	(0.00169)	(0.00206)	(0.00270)	(0.0141)
VC × years since IPO	-0.000780***	-0.00304***	-0.00406***	-0.0110***
	(0.000132)	(0.000165)	(0.000215)	(0.00110)
ln(employment)	-0.0133***	-0.00567***	-0.00641***	0.851***
	(0.000248)	(0.000254)	(0.000335)	(0.00170)
Observations	84,116	148,834	149,672	168,549
R-squared	0.383	0.084	0.108	0.737

Table 2: All specifications include year dummies, industry dummies (at the 4-digit SIC), and a year dummy for the IPO. Standard errors are in parentheses and significance at the 1 percent level is denoted by ***.

companies funded by venture capitalists between 1970 and 2015. These VC-funded patentees are identified by matching firm names in VentureXpert and PatentsView.

Firm-Level Regressions. In the firm-level regression analysis, the primary independent variable is (the natural logarithm of) annual VC funding, while the dependent variable is a measure of patenting performance, both in the year and the year after the firm receives the funding. The primary independent variable may suffer from both measurement error and selection issues. So, in some of the regressions, two instrumental variables are used. The first instrumental variable (IV) is the (maximum) rate of capital gains taxation in the state where the VC-funded company is located. The second IV is a Rajan and Zingales (1998) type measure of the dependence on external finance of the industry in which the firm operates. This measure reflects the extent to which outside funds are used in the industry for expenditures on property, plant and equipment, R&D, advertising, and employee training. Both of these datums are exogenous at the level of a startup. In all of the regressions, controls are added for the number of patents held by the firm at the beginning of the year, the age of the firm, and the total amount of private- and federally-funded R&D of the industry in which the firm operates. Additionally, both a year and industry dummy are entered. Last, since both innovation and VC activities are remarkably clustered in California or Massachusetts,

11

a "cluster dummy" for a firm headquartered in California and Massachusetts is included.

The results of the regression analysis are reported in Table 3. Panel A of Table 3 conducts the analysis along the extensive margin analysis; i.e., it examines whether the firm obtains any patents after receiving VC funding. In regressions (1) and (2), the dependent variable is a dummy that takes the value of 1 if the firm files any successful patent applications at the U.S. Patents and Trademark Office (USPTO) within one year after it receives funding. Regressions (3) and (4) focus on "breakthrough" patents, a measure pioneered by Kerr (2010). Breakthrough patents refers to those in the right tail of the citation distribution. Here the dependent variable in regressions (3) and (4) is a dummy variable that takes the value of 1 if the firm files any patents in the top 10 percent of the citation distribution in its cohort (i.e., those patents with the same technological class and same application year). Panel B of Table 3 turns to the intensive margin. In regressions (5) and (6) the dependent variable is the natural logarithm of the number of patents. The natural logarithm of the number of patents is weighted by citations in regressions (7) and (8).

As shown by the positive regression coefficients of VC funding in Panel A, a firm is more likely to file a patent and come up with a breakthrough patent the larger is the funding from a venture capitalist, although the impact of VC funding is somewhat smaller in spurring breakthrough patents than ordinary patents. According to the IV estimates in regressions (6) and (8), a 10 percent increase in VC funding will induce a 3.6 percent boost in patenting one year after funding, and this number goes up to 6.7 percent when the number of patents is adjusted for quality. In addition, across all the regressions in Table 3, the estimates are consistently higher in the IV regressions.

Industry-Level Regressions. The above firm-level regressions are now recast at the 4-digit SIC industry level. The main explanatory variable is the (natural logarithm of the) aggregate amount of VC investment across all industries between 1970 and 2015. The dependent variable is the (natural logarithm of the) number of patents filed by all VC-backed companies in the industry one year after they receive VC funding. To capture the heterogeneous dependence on external finance across industries, a cross term is added between

12

VC Funding and Patenting: Firm-Level Regressions

	Panel A: Extensive Margin Analysis			
Dependent variable	1{Patent > 0}		1{"Breakthrough patent" > 0}	
	Probit	IV	Probit	IV
	(1)	(2)	(3)	(4)
ln(VC funding)	0.141***	0.682***	0.133***	0.635***
	(0.0108)	(0.0590)	(0.0112)	(0.0979)
Observations	9,166	8,132	9,149	8,122
	Panel B: Intensive Margin Analysis			
Dependent variable	ln(Patent)		ln(Patent, quality adj)	
	OLS	IV	OLS	IV
	(5)	(6)	(7)	(8)
ln(VC funding)	0.115***	0.363*	0.155***	0.674*
	(0.00907)	(0.187)	(0.0164)	(0.356)
Observations	5,828	5,207	5,032	4,519
R-squared	0.244		0.123	

Table 3: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

aggregate VC funding and the industry's dependence on external finance. This specification emulates Rajan and Zingales (1998) in the sense that they exploit the variation of financial development across countries, whereas the current analysis taps into fluctuations of aggregate VC investment across time. As in the firm-level regressions, the main independent variable may suffer from both measurement error and selection issues. An instrumental variable is used to address this. The IV follows Kortum and Lerner (2000) and is based on the deregulation of pension funds in 1979, as highlighted in Section 2. To be specific, a "deregulation dummy," which takes the value of 1 after 1979, is used as an instrumental variable. In all of the industry-level regressions, controls are added for the total amounts of private R&D and federally-funded R&D in the industry. A 2-digit industry dummy variable is also included. Since the deregulation dummy is used as an IV, year dummies cannot be used, so common shocks to all industries are controlled for by adding NBER recession dummies as a proxy for the business cycle and the federal funds rate as a proxy for the tightness of monetary policy.

The industry-level regressions are presented in Table 4. As can be seen from the first row

Dependent variable	ln(Patent)		ln(Patent, quality adj)	
	OLS	IV	OLS	IV
ln(agg VC funding)	0.200***	0.151***	0.129***	0.115*
	(0.0381)	(0.0569)	(0.0454)	(0.0681)
ln(agg VC funding) × ind financial dependence	0.1854***	0.1852***	0.192***	0.191***
	(0.00965)	(0.00976)	(0.0117)	(0.0118)
Observations	1,971	1,971	1,890	1,890
R-squared	0.378		0.362	

Table 4: See the main text for a description of the dependent and independent variables. Standard errors are in parentheses. *** denotes significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

of the regression coefficients, the positive signs on aggregate VC funding complement the findings at the firm level. VC investment contributes positively to patenting performance at the industry level. According to the IV estimate in column 2, at the median level of financial dependence across industries, a 10 percent increase in aggregate VC funding will induce a 1.51 percent boost in industry-level patenting within a year. This elasticity is 0.194 in the prepackaged software industry, which accounted for 23 percent of VC investment. In addition, the impact of VC is heterogeneous across industries, as revealed by the cross term between VC funding and the dependence on external finance (see the second row). Since the regression coefficients on the cross terms turn out to be positive, the impact of the fluctuations in aggregate VC investment is more pronounced the higher is the industry's dependence on external finance. For industries in the top quartile of financial dependence, the elasticity is 0.339, versus 0.111 in the bottom quartile.[2] As complementary evidence on the cyclicality of VC activities, Khan and Petratos (2016) document that VC-backed firm entry (the number of startups) and exit (the number of IPOs and M&As) are nearly three and five times, respectively, as volatile as business fixed investment.

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[2] To be conservative, the number for the upper quartile excludes an unrealistic high elasticity for the insurance carrier industry, where there are only two VC-funded firms.

4   The Setting

At center of the analysis is the interplay between an entrepreneur and a venture capitalist, which is governed by an incentive-compatible financial contract. Entrepreneurs have ideas, but no money, while venture capitalists have expertise and money, but no ideas. Each period new entrepreneurs bring ideas of their choosing to a venture capitalist to obtain funding. The parties sign a partnership agreement that has finite duration. Most VC enterprises are operated as partnerships. The share of corporate venture programs in total U.S. VC investment is low, averaging just 9 percent between 1995 and 2015. Also, corporate VC faces many of same challenges as VC partnerships; viz., the uncertainty about a project's quality, the decision about how much to invest at each stage of the development process based on limited information, and the moral hazard problem connected with lending.

At the time the contract is signed, the venture capitalist provides seed money to research initially the idea. After this initial research is finished, the project enters a funding-round cycle that may last for many periods. Some ideas brought by entrepreneurs to the venture capitalist are good, others are bad. Only a good idea has a payoff, and even then this might not happen. Neither party knows whether an idea is good or bad. So, at the beginning of each funding round the venture capitalist evaluates the project at a cost in an attempt to detect whether the venture is bad. Bad projects are terminated. Projects that aren't known to be bad are given development money. The probability of success within a funding round is an increasing function of the level of investment in development undertaken by the entrepreneur. How much of the money the entrepreneur actually uses for development is private information. The venture capitalist can imperfectly monitor development investment at a cost in an attempt to detect any malfeasance. When malfeasance is detected, the venture capitalist drops the venture. If successful, the project will be floated on the stock market or sold to another firm, which yields a reward that will be a function of the idea's type. The reward is split between the entrepreneur and venture capitalist as specified by the partnership agreement. Any profits from floating a VC-funded enterprise are subject to capital gains taxation. All revenue from capital gains taxation is rebated back to the populace in lump-
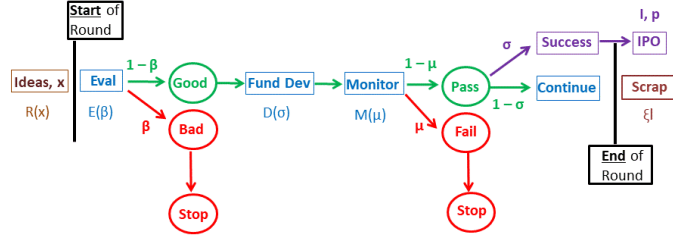
Figure 4: The timing of events within a typical funding round. The research underlying the idea occurs at the very beginning of the funding cycle, or round 0, and is shown to the left of generic funding round. A surviving project can be sold for scrap at the end of the contract, or at the end of round T, as shown to the right of the typical funding round.

sum transfer payments. If the project is not successful, then it enters another funding round, provided the contract has not expired, and the funding cycle goes on. At the time a contract expires, an unsuccessful surviving project can be sold by the venture capitalist for scrap. The timing of events within a generic funding round is shown in Figure 4.

The analysis focuses on a balanced-growth path. The aggregate level of productivity in the VC sector is denoted by $\mathbf{x}$, which represents the aggregate state of the economy. Along a balanced-growth path, $\mathbf{x}$ will grow at the gross rate $\mathbf{g_x} > 1$ so that

$$\mathbf{x}' = \mathbf{g_x}\mathbf{x}.$$

The gross growth rate of aggregate productivity, $\mathbf{g_x}$, is an endogenous variable in equilibrium. It will be a function of the efficiency of the VC system. The gross growth rate in wages, $\mathbf{g}_w$, will be a function of the growth rate of aggregate productivity, $\mathbf{g_x}$. The discussion now proceeds by detailing the stages portrayed in Figure 4.

16

4.1 The Research Stage–Starting a New Venture

Each period a flood of new entrepreneurs in the amount $\mathfrak{e}$ approach venture capitalists to obtain funding for their ideas. An entrepreneur incurs an opportunity cost in the amount $w\mathfrak{o}$ to run a project, where w is the wage rate for labor. The component $\mathfrak{o}$ of this cost is distributed across potential entrepreneurs according to the non-normalized distribution function, $O(\mathfrak{o})$. This distribution function $O(\mathfrak{o})$ is assumed to be Pareto so that

$$O(\mathfrak{o}) = 1 - (v/\mathfrak{o})^{\nu}, \text{ with } \nu, v > 0. \tag{1}$$

Only those potential entrepreneurs who expect the payoff from a startup to exceed their opportunity cost, $w\mathfrak{o}$, will approach a venture capitalist for funding. This criterion determines the number of funded entrepreneurs, $\mathfrak{e}$.

A new entrepreneur is free to choose the type of startup, x, that he wants to develop. In particular, when deciding on the project, the entrepreneur picks x subject to a research cost function of the form

$$i = R(\frac{x}{\mathbf{x}}) = w(\frac{x}{\mathbf{x}})^{\iota}/\chi_R,$$

where $i \geq 0$ is the initial investment in researching the project. The entrepreneur can choose how far ahead the productivity of his firm, x, is from the average level of productivity in the VC sector, $\mathbf{x}$. The more ambitious he is, or the higher x is relative to $\mathbf{x}$, the greater will be the research cost, which rises in convex fashion. The cost of research, $R(x/\mathbf{x})$, rises with the current level of wages, w, which will be a function of the aggregate state of the economy, $\mathbf{x}$. (Think about $R(x/\mathbf{x})/w$ as representing the cost in terms of labor.) This structure provides a mechanism for endogenous growth in the model.

4.2 The Evaluation Stage

Out of the pool of *new* entrepreneurs, the fraction ρ will have good ideas, implying that the fraction $1 - \rho$ have bad ones. The venture capitalist can potentially discover a bad project

by evaluating it. Assume that the venture capitalist can detect within each funding round a bad project with probability β, according to the cost function, $E(\beta; \mathbf{x})$, where E is an increasing, convex function in β. Specifically,

$$E(\beta; \mathbf{x}) = w(\frac{1}{1-\beta} - 1)\beta/\chi_E.$$

The productivity of the evaluation process is governed by χ_E. Note that the marginal cost of evaluating starts at zero when $\beta = 0$ and goes to infinity as β approaches 1. The cost of evaluating rises with the level of wages, w. Think about χ_E as capturing the efficiency of investment in evaluation. Projects that are detected to be bad are thrown out.

4.3 The Development Stage

Ventures that pass the evaluation stage are given development funding. The level of funding depends upon the common prior (held by the entrepreneur and venture capitalist) that the project is good, which evolves across funding rounds. The odds of success during a funding round depend on the entrepreneur's investment in development. In particular, a probability of success, σ, can be secured by undertaking development investment in the amount $D(\sigma; \mathbf{x})$, where D is an increasing, convex function in σ. The development cost function $D(\sigma; \mathbf{x})$ is given the form

$$D(\sigma; \mathbf{x}) = w(\frac{1}{1-\sigma} - 1)\sigma/\chi_D.$$

The development cost function $D(\sigma; \mathbf{x})$ has a similar form to that for $E(\beta; \mathbf{x})$.

There is also a fixed cost, ϕ_t, connected with developing a startup project in round t. This fixed cost rises with the level of wages in the economy. In particular,

$$\phi_t = w_1 \mathbf{g}_w^{t-1} \phi(t),$$

where w_1 represents the round-1 wage rate and $\mathbf{g}_w > 1$ is the gross growth rate in wages (which will be a function of $\mathbf{g_x}$). Additionally, the fixed cost changes by the round of the

project, as reflected by the function $\phi(t)$. The shape of the function $\phi(t)$ will be parameterized using a polynomial that is pinned down from the U.S. VC funding-round data.

4.4 The Monitoring Stage

The venture capitalist provides in a funding round the amount $D(\sigma; \mathbf{x})$ for development. The entrepreneur may decide to spend some smaller amount $D(\widetilde{\sigma}; \mathbf{x}) \leq D(\sigma; \mathbf{x})$ and siphon off the difference, $D(\sigma; \mathbf{x}) - D(\widetilde{\sigma}; \mathbf{x})$. The entrepreneur uses the difference in funds for his own consumption. By diverting funds the entrepreneur reduces the odds of success in the current funding round; i.e., $\widetilde{\sigma} \leq \sigma$. The venture capitalist can dissuade this fraud by engaging in monitoring. Assume that the venture capitalist can pick the odds μ of detecting fraud in a venture during round t according to the strictly increasing, convex cost function, $M_t(\mu; \mathbf{x})$, where

$$M_t(\mu; \mathbf{x}) = w_1 \mathbf{g}_w^{t-1} (\frac{1}{1 - \mu} - 1)\mu / \chi_{M,t}.$$

The cost of monitoring rises with wages in the economy. Additionally, monitoring costs change by the round of the project, as reflected by the term $\chi_{M,t}$; again, $\chi_{M,t}$ represents the productivity of this auditing process in round t. Presumably, as the venture capitalist becomes more familiar with the project, $\chi_{M,t}$ will rise with t. This feature implies that the incentive problem will become less severe over time and helps to generate an upward-sloping funding profile. A polynomial for $\chi_{M,t}$ will be fit to the U.S. VC funding-round data. While motivated by the prototypical costly-state-verification paradigms of Townsend (1979) and Williamson (1986), the monitoring technology employed here is different. In those frameworks, getting monitored is a random variable–in Williamson (1986) only those entrepreneurs declaring a bad outcome are monitored, while in Townsend (1979) some fraction of such entrepreneurs are. The audit will detect any fraud with certainty. By contrast, here everybody gets monitored, but the detection of any fraud is a probabilistic event.

4.5 The Success Stage–Floated Firms

A startup of type x turns into a going concern with productivity x, if successful. A successful VC-backed firm produces output, o, according to the production process

$$o = x^\zeta k^\kappa l^\lambda, \text{ with } \zeta + \kappa + \lambda = 1, \qquad (2)$$

where k and l are the amounts of capital and labor used in production. This structure is borrowed from Akcigit, Celik, and Greenwood (2016). It results in the firm earning pure profits that are linear in its productivity, x. The lure of capturing these profits is what motivates entrepreneurs and venture capitalists. Labor is hired at the wage rate, w, and capital at the rental rate, r. The firm's per period net takings are

$$
\begin{aligned}
T(x; \mathbf{x}) &= \max_{k,l}\{x^\zeta k^\kappa l^\lambda - rk - wl\} \\
&= x(1 - \kappa - \lambda)[(\frac{\kappa}{r})^\kappa(\frac{\lambda}{w})^\lambda]^{1/\zeta}.
\end{aligned}
\qquad (\text{P1})
$$

Clearly, as wages rise, which will be a function of the aggregate state of the economy, $\mathbf{x}$, net takings will shrink for a given level of the firm's productivity, x. Operating firms last stochastically in accordance with the time-invariant survival rate, $\mathfrak{s}$.

A successful VC-backed project is sold for $I(x; \mathbf{x})$, either through an IPO or an M&A, just before production starts. The (gross) reward for a successful IPO is

$$I(x; \mathbf{x}) = \sum_{t=1}^{\infty}(\mathfrak{s}\delta)^{t-1}T(x; \mathbf{g}_{\mathbf{x}}^{t-1}\mathbf{x}), \qquad (3)$$

where δ is the market discount factor. If the startup is successful, the entrepreneur must pay the venture capitalist the amount p. So the entrepreneur will reap the amount $I(x; \mathbf{x}) - p$, which is taxed at the capital gains rate, τ. If a project is not successful, it moves back to the evaluation stage, assuming that the contract has not expired. An ongoing project that has not been successful by the time the contract expires at end of round T can be sold by

the venture capitalist for scrap value. The scrap value for a project in the current period is $\xi I(x; \mathbf{x})$, where $0 < \xi < 1$.

5 The Financial Contract

The financial contract between the entrepreneur and the venture capitalist is cast now. VC is a competitive industry so the entrepreneur shops around to secure the financial contract with the best terms. Venture capitalists cover the cost of research, evaluation, development, and monitoring. They raise the money to do this from savers, to whom they promise a gross rate of return of $1/\delta$. There are no profits on VC activity in equilibrium. The profits that accrue to the entrepreneur are subject to the rate of capital gains taxation, τ. The analysis presumes that there is a maximum of T rounds of potential funding. The timing of events for the contract is shown in Figure 4. The research for the idea is done at the start of the funding-round cycle or in round zero. At the beginning of a generic funding round, the venture capitalist evaluates projects and purges the ones that are found to be bad. Good projects are then given an injection of cash for development. The venture capitalist monitors the use of these funds. If malfeasance is detected, the project is terminated. Some projects will be successful. These are floated in the next period on the stock market. The unsuccessful projects then start another funding round, assuming the number of funding rounds doesn't exceed T. At the end of round T, any unsuccessful surviving projects can be sold by the venture capitalist for scrap.

Let β_t represent the odds of detecting a bad project in round t and σ_t denote the probability of success for a good project. Now suppose that a unit measure of new entrepreneurs approach a venture capitalist for funding. As the funding rounds progress, the numbers of good and bad projects will evolve as shown in Table 5. For example, of the entrepreneurs initially applying for funding, the number ρ will have good projects and $1-\rho$ will have bad ones. In round 1 the venture capitalist will evaluate the applicants and eliminate $(1 - \rho)\beta_1$ bad projects, so that $(1-\rho)(1-\beta_1)$ bad ones will still remain. Of the good projects, the number

EVOLUTION OF PROJECT TYPES ACROSS FUNDING ROUNDS

Round	Number good	Number bad
1	ρ	$(1-\rho)(1-\beta_1)$
2	$\rho(1-\sigma_1)$	$(1-\rho)(1-\beta_1)(1-\beta_2)$
3	$\rho(1-\sigma_1)(1-\sigma_2)$	$(1-\rho)(1-\beta_1)(1-\beta_2)(1-\beta_3)$
$\vdots$	$\vdots$	$\vdots$
t	$\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$	$(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)$

Table 5: The table shows how the number of good and bad projects change across funding rounds assuming that the venture capitalist starts with a unit mass of ventures.

$\rho\sigma_1$ will be successful. So, at the beginning of the second round there will be $\rho(1-\sigma_1)$ good projects in the pool. After the second-round evaluation, $(1-\rho)(1-\beta_1)(1-\beta_2)$ bad projects will still be around. Table 5 specifies how the number of good and bad projects evolves over funding rounds. As can be seen, the number of good and bad projects in funding-round t are given by $\rho\Pi_{j=1}^{t-1}(1-\sigma_j)$ and $(1-\rho)\Pi_{j=1}^{t}(1-\beta_j)$, respectively.

The odds of a project being good in round t are

$$\Pr(\text{Good}|\text{Round}=t) = \frac{\rho\Pi_{j=1}^{t-1}(1-\sigma_j)}{\rho\Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j)}. \tag{4}$$

As time goes by, more and more bad projects are purged from the pool. The number of goods projects will also fall due to the successes. Thus, the odds of being good can rise or fall with the funding round, depending on which type of projects are exiting the pool the fastest, at least theoretically. Without the evaluation technology the odds of a project being good must decline by funding round, since then $\beta_j = 0$ for all j. By this account, the venture capitalist should invest less in a startup as funding rounds progress, something at odds with the data as discussed by Lerner (1979). The introduction of the evaluation technology admits the possibility that "lemons ripen faster than plums."

The contract between the entrepreneur and the venture capitalist will specify for the length of the relationship: (i) the precision of evaluation, as given by the β_t's; (ii) the investments in development as reflected by the σ_t's; (iii) the exactness of monitoring as measured by the μ_t's; and (iv) the payments that an entrepreneur who finds success in

round t must make to the intermediary, or the p_t's. The contract is summarized by the outcome of the following maximization problem in sequence space:

$$C(x; \mathbf{x}) = \max_{\{\beta_t, \sigma_t, \mu_t, p_t\}} (1 - \tau) \sum_{t=1}^{T} \rho \Pi_{j=1}^{t-1}(1 - \sigma_j) \delta^t \sigma_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t], \qquad \text{(P2)}$$

subject to:

1. The round-t incentive constraints

$$\Pr(\text{Good}|\text{Round} = t) \times (1 - \tau) \times \{\delta \sigma_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t]$$

$$+ (1 - \sigma_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j) \delta^{i+1-t} \sigma_i [I(x; \mathbf{g}_{\mathbf{x}}^i \mathbf{x}) - p_i]\}$$

$$\geq (1 - \mu_t) \max_{\widetilde{\sigma}_t} \left(D(\sigma_t) - D(\widetilde{\sigma}_t) \right.$$

$$+ \Pr(\text{Good}|\text{Round} = t) \times (1 - \tau) \times \{\delta \widetilde{\sigma}_t [I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t]$$

$$\left. + (1 - \widetilde{\sigma}_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j) \delta^{i+1-t} \sigma_i [I(x; \mathbf{g}_{\mathbf{x}}^i \mathbf{x}) - p_i]\} \right),$$

$$\tag{5}$$

for $t = 1, \cdots, T$, where $\Pr(\text{Good}|\text{Round} = t)$ is given by (4);

2. The round-0 zero-profit condition

$$\rho \sum_{t=1}^{T} \Pi_{j=1}^{t-1}(1 - \sigma_j) \delta^t \sigma_t p_t + \rho \Pi_{j=1}^{T}(1 - \sigma_j) \delta^T \xi I(x; \mathbf{g}_x^T \mathbf{x})$$

$$- \sum_{t=1}^{T} [\rho \Pi_{j=1}^{t-1}(1 - \sigma_j) + (1 - \rho) \Pi_{j=1}^{t}(1 - \beta_j)] \delta^{t-1} [D(\sigma_t) + \phi_t + M_t(\mu_t)]$$

$$- \sum_{t=1}^{T} [\rho \Pi_{j=1}^{t-1}(1 - \sigma_j) + (1 - \rho) \Pi_{j=1}^{t-1}(1 - \beta_j)] \delta^{t-1} E(\beta_t) - R(\frac{x}{\mathbf{x}}) = 0.$$

$$\tag{6}$$

23

The objective function in (P2) reflects the fact that VC is a competitive industry. A contract must maximize the expected return for the entrepreneur, subject to the two constraints (5) and (6). The term $I(x; \mathbf{g}_{\mathbf{x}}^t \mathbf{x}) - p_t$ gives the payoff to the entrepreneur should the enterprise be floated in round t. The payoff could come from executing stock options or convertible shares. It is taxed at the capital gains rate, τ. The maximized value of objective function, $C(x; \mathbf{x})$, specifies the worth of the financial contract for the entrepreneur. This expected discounted payoff is a function of the entrepreneur's idea, x.

Equation (5) is the incentive compatibility constraint for a round-t project. The left-hand side gives the expected return to the entrepreneur when he undertakes the level of development investment linked with σ_t. The first term in brackets are the Bayesian odds of having a good project at the beginning of round t, conditional on the entrepreneur still dealing with the venture capitalist. The right-hand side gives the return when the entrepreneur deviates and picks the level of development linked with $\widetilde{\sigma}_t$. The level of development represented by $\widetilde{\sigma}_t$ maximizes the value of the deviation. The return from deviating will only materialize if the entrepreneur is not caught cheating, which has the odds $1 - \mu_t$; if caught cheating, which occurs with probability μ_t, then the contract is terminated and the entrepreneur receives nothing. The incentive constraint has a dynamic element to it. If the entrepreneur invests less in development today, he lowers the odds that a good project will be successful in the current period. He increases the probability that a success, if it happens, will occur in the future; thus, an intertemporal tradeoff is involved.

The last equation, or (6), is the zero-profit constraint. The first two terms are the expected present value of the cash that the venture capitalist expects to receive. This includes any scrap value. The remaining terms are the venture capitalist's expected costs. Observe that there is a fixed cost, ϕ_t, connected with operating a startup project in round t. Last, the venture capitalist must cover the initial research cost, $R(x/\mathbf{x})$. Since VC is a competitive industry, the expected present value of the cash inflow exactly offsets the expected present value of the cash outflow.

Now, it is easy to see that the ability of the venture capitalist to monitor the entrepreneur

is important. Focus on the incentive constraint (5). If $\mu_t = 1$, say because the cost of monitoring is zero, then the left-hand side of the constraint will always exceed the right-hand side. This transpires no matter what the solution for $\widetilde{\sigma}_t$ is, as dictated by the right-hand side of (5). In this situation, the first-best solution to problem (P2) can be obtained. Alternatively, suppose $\mu_t = 0$, because the cost of monitoring is infinite. Then, the incentive-compatible contract specifies that $\sigma_t = \widetilde{\sigma}_t$. To see this, pull the $D(\sigma_t)$ term over onto the left-hand side of (5). Note that the terms on the left- and right-hand sides are then the same, except that they involve σ_t on the left and $\widetilde{\sigma}_t$ on the right. But $\widetilde{\sigma}_t$ maximizes the right-hand side, implying that the right-hand side must then equal the left-hand side. This can only be the case if $\sigma_t = \widetilde{\sigma}_t$, which greatly limits the contract and may result in an allocation far from first-best. So if no monitoring is done, then the incentive constraint holds tightly. Why can't the incentive constraint be slack? Suppose it is slack, implying that the associated Lagrange multiplier is zero. Then, no monitoring will be done because it would have no benefit and is costly. But, as just discussed, when $\mu_t = 0$, the constraint must hold tightly–a contradiction. Therefore, the incentive constraint (5) always binds.

Lemma 1 *(Always monitor) The incentive constraint (5) holds tightly for all funding rounds with $0 < \mu_t < 1$.*

Remark 1 *(One-shot versus multi-shot deviations) The incentive constraints in (5) prevent one-shot deviations from occuring in any funding round. Lemma 4 in the Theory Appendix establishes that this is equivalent to using a single consolidated round-0 incentive constraint with multi-shot deviations.*

Remark 2 *(Self financing) If an entrepreneur has any funds, he should invest them all. This does not change the generic form of the contract problem. The entrepreneur's funds can merely be subtracted from the expected present value of the fixed costs, or the ϕ_t's, in (6). (See Cole, Greenwood, and Sanchez (2016, Lemmas 1 and 6)). What matters is how much the entrepreneur borrows, net of his own investment. The entrepreneur's funds can be incorporated in problem (P2) by simply transforming the fixed costs.*

6   The Choice of Idea

The entrepreneur is free to pick the type of venture, x, that he pitches to the venture capitalist. He selects the one that maximizes his expected discounted profits. Therefore, x will solve

$$V(\mathbf{x}) = \max_x C(x; \mathbf{x}), \tag{P3}$$

where the value of the entrepreneur's contract, or $C(x; \mathbf{x})$, is specified by problem (P2). The shape of the $C(x; \mathbf{x})$ function determines the value of x picked by the entrepreneur. So if better intermediation increases the marginal return from x, then VC will increase growth. Note that the cost of researching x, or $R(x/\mathbf{x})$, is embedded in the zero-profit condition (6) connected with problem (P2). This problem will give a decision rule of the form

$$x = X(\mathbf{x})\mathbf{x}.$$

The function $V(\mathbf{x})$ gives an entrepreneur's expected discounted payoff from a startup.

7   The Flow of New Startups

Recall that an entrepreneur incurs an opportunity cost in the amount $w\mathfrak{o}$ to run a project. Therefore, only those new entrepreneurs with $w\mathfrak{o} \leq V(\mathbf{x})$ will choose to engage in a startup. Now, $\mathfrak{o}$ is distributed according the cumulative distribution function $O(\mathfrak{o})$. Therefore, $O(V(\mathbf{x})/w)$ entrepreneurs will approach the venture capitalist for funding. Consequently, the number of new entrants, $\mathfrak{e}$, is given by

$$\mathfrak{e} = O(V(\mathbf{x})/w). \tag{7}$$

8 The Non-VC-Funded Sector

Most firms are not funded by venture capitalists. To capture this, suppose there are always $\mathfrak{m}$ firms operating that were not funded by VC. All firms in the non-VC-funded sector are same. These non-VC-funded firms produce using a production function that is identical to a VC-funded firm with one exception: their productivity differs. Specifically, they produce in line with

$$o = z^\zeta k^\kappa l^\lambda, \text{with } \zeta + \kappa + \lambda = 1,$$

where z represents their productivity. Suppose that

$$z = \omega \mathbf{x}, \text{with } \omega < 1.$$

Thus, firms in the non-VC-funded segment of the economy are on average less productive than the ones in the VC segment, but will be dragged along by the latter. Average productivity in the VC sector is defined in Section 9. For more micro-founded theories about how ideas diffuse through in a economy (either by buying or imitating them) see Akcigit, Celik, and Greenwood (2016), Jovanovic and MacDonald (1994), Lucas and Moll (2014), and Perla and Tonetti (2014).

The non-VC-funded firm's profit maximization problem is

$$\max_{k,l}\{z^\zeta k^\kappa l^\lambda - rk - wl\}. \tag{8}$$

One can think about these firms as raising the funds for capital through traditional intermediation at the gross interest rate $1/\delta$. VC-funded firms also raise capital this way after they are floated. On this, Midrigan and Xu (2014) argue that producing establishments can quickly accumulate funds internally and thus rapidly grow out of any borrowing constraints. Therefore, modeling producing firms as having frictionless access to capital markets may not be grossly at variance with reality–Moll (2014) discusses how the ability to self-finance is tied with the degree of persistence in technology shocks.

9 Balanced-Growth Equilibrium

The analysis focuses on analyzing a balanced-growth path for the model. Along a balanced-growth path, the rental rate on capital, r, is some fixed number. In particular, the rental rate on capital will be

$$r = 1/\delta - \mathfrak{d}, \tag{9}$$

where δ is the market discount factor and $\mathfrak{d}$ is the depreciation factor on capital. In balanced growth, the market discount factor, δ, in turn is given by

$$\delta = \widehat{\delta} \mathbf{g}_w^{-\varepsilon}, \tag{10}$$

where $\widehat{\delta}$ is the representative agent's discount factor and ε denotes his coefficient of relative risk aversion.[3]

The idea distribution for VC-backed firms will now be characterized. To this end, let $\mathfrak{n}_t$ represent the number of VC-backed firms that are operating with an idea, x_{-t}, that was generated t periods ago. Attention will now be turned to specifying the number $\mathfrak{n}_t$. Now, no firms will operate in the VC-backed sector with productivity level x, since this type is not operational yet. Each period, $\mathfrak{e}$ new entrepreneurs will be funded by the venture capitalist. Hence, $\mathfrak{n}_1 = \mathfrak{e}\rho\sigma_1$ firms will operate with an idea generated one period ago, x_{-1}. Likewise, there will be $\mathfrak{n}_2 = \mathfrak{e}\rho\sigma_1\mathfrak{s} + \mathfrak{e}\rho(1-\sigma_1)\sigma_2$ firms operating with a two-period-old idea, x_{-2}. So, the number of firms operating with an idea, x_{-t}, from $t \leq T$ periods ago is

$$\mathfrak{n}_t = \mathfrak{e} \sum_{i=1}^{t} \rho\Pi_{j=1}^{i-1}(1-\sigma_j)\sigma_i\mathfrak{s}^{t-i}, \text{ for } t = 1, \cdots, T. \tag{11}$$

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[3] That is, in the background there is a representative consumer/worker who inelastically supplies one unit of labor and has a utility function (in period 1) of the form

$$\sum_{t=1}^{\infty} \widehat{\delta}^{t-1} c_t^{1-\epsilon}/(1-\epsilon),$$

where c_t is his period-t consumption.

The venture capitalist only funds entrepreneurs for T periods. Consequently, the number of operational firms with an idea from more than T periods ago is

$$\mathfrak{n}_{T+j} = \mathfrak{s}^j \mathfrak{n}_T, \text{ for } j \geq 1. \tag{12}$$

The total number of operational VC-backed firms, $\mathfrak{n}$, is given by

$$\mathfrak{n} = \sum_{t=1}^{T} \mathfrak{n}_t + \sum_{t=T+1}^{\infty} \mathfrak{n}_t = \sum_{t=1}^{T} \mathfrak{n}_t + \frac{\mathfrak{n}_T \mathfrak{s}}{1 - \mathfrak{s}}.$$

In a stationary equilibrium the distribution function over VC-funded firms using an age-t idea will remain constant; that is, $\mathfrak{n}_t' = \mathfrak{n}_t$. It is easy to from (11) that this will be true provided that $\mathfrak{e}$ and the σ_i's are constant.

In balanced growth the wage rate, w, will grow at some constant gross rate, $\mathbf{g}_w$. To determine this growth rate, note that a VC-funded firm with productivity level x will hire labor in the amount

$$l(x; w) = \left(\frac{\kappa}{r}\right)^{\kappa/\zeta} \left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} x, \tag{13}$$

where again w and r are the current wage and rental rates, respectively. For a non-VC-funded firm, just replace the x with a z in the above formula. In general equilibrium, the labor market must clear each period. Suppose that there is one unit of labor available in aggregate. To calculate the aggregate demand for labor, sum over all operating firms' demands for labor, both in the VC- and non-VC-backed sectors. Equilibrium in the labor market requires that

$$\sum_{t=1}^{T} \mathfrak{n}_t l(x_{-t}; w) + \sum_{t=T+1}^{\infty} \mathfrak{n}_t l(x_{-t}; w) + \mathfrak{m} l(z; w) = 1,$$

where $\mathfrak{m}$ is the measure of firms in the non-VC-funded sector. Along a balanced-growth path, the productivity of the latest idea will grow at rate $\mathbf{g}_x$. Therefore, the above condition

can be recast as

$$\sum_{t=1}^{T} \mathfrak{n}_t l(x_{-1}\mathbf{g_x}^{1-t}; w) + \sum_{t=T+1}^{\infty} \mathfrak{n}_t l(x_{-1}\mathbf{g_x}^{1-t}; w) + \mathfrak{m}l(\omega\mathbf{x}; w) = 1.$$

Using equations (12) and (13), this can be expressed as

$$\left(\frac{\kappa}{r}\right)^{\kappa/\zeta} \left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta} [x_{-1}(\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}) + \mathfrak{m}\omega\mathbf{x}] = 1.$$

The solution for wages, w, obtained from the above labor-market clearing condition, is

$$w = \lambda \left(\frac{\kappa}{r}\right)^{\kappa/(\zeta+\lambda)} \underbrace{[x_{-1}(\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}) + \mathfrak{m}\omega\mathbf{x}]}_{=\mathfrak{n}\mathbf{x}}{}^{\zeta/(\zeta+\lambda)}, \tag{14}$$

where aggregate productivity in the VC sector, $\mathbf{x}$, is

$$\mathbf{x} \equiv \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}/(1 - (\mathfrak{s}/\mathbf{g_x}))]}{\sum_{t=1}^{T} \mathfrak{n}_t + \mathfrak{n}_T \mathfrak{s}/(1 - \mathfrak{s})} = \frac{x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}/(1 - (\mathfrak{s}/\mathbf{g_x}))]}{\mathfrak{n}}.$$

As can be seen, wages rise with the aggregate state of the economy, $\mathbf{x}$, which grows at rate $\mathbf{g_x}$. Therefore, wages will grow at the gross growth rate $\mathbf{g_x}^{\zeta/(\zeta+\lambda)}$, so that

$$\frac{w'}{w} \equiv \mathbf{g}_w = \mathbf{g_x}^{\zeta/(\zeta+\lambda)}.$$

Attention is now turned to determining the growth rate in aggregate productivity, $\mathbf{g_x}$. All new entrepreneurs will pick the same type of project, x. Now

$$\mathbf{g_x} = \mathbf{x}'/\mathbf{x} = x'/x.$$

Recall that

$$x = X(\mathbf{x})\mathbf{x},$$

and

$$\mathbf{x} = x_{-1}[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}]/\mathfrak{n}.$$

Therefore,

$$\mathbf{g_x} = \frac{x}{x_{-1}} = \frac{X(\mathbf{x})\mathbf{x}}{x_{-1}} = X(\mathbf{x})[\sum_{t=1}^{T} \mathfrak{n}_t \mathbf{g_x}^{1-t} + \frac{\mathfrak{n}_T \mathfrak{s} \mathbf{g_x}^{-T}}{1 - (\mathfrak{s}/\mathbf{g_x})}]/\mathfrak{n}. \tag{15}$$

This is a nonlinear equation in $\mathbf{g_x}$.

It is easy to see that the aggregate capital stock and output grow at the same rate as wages. The demand for capital by a type-x VC-backed firm is

$$k(x; w) = (\frac{\kappa}{r})^{(1-\lambda)/\zeta}(\frac{\lambda}{w})^{\lambda/\zeta}x.$$

From this it is easy to deduce that $k(\boldsymbol{g}_x x; \mathbf{g}_w w) = \mathbf{g}_w k(x; w)$. The same is true for a non-VC-backed firms; just replace x with z to get $k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w k(z; w)$. Let the aggregate capital stock in the current period be represented by $\mathbf{k}$ and that for next period by $\mathbf{k}'$. Then $\mathbf{k}' = \sum_{t=1}^{\infty} \mathfrak{n}_t k(\mathbf{g_x} x_{-t}; \mathbf{g}_w w) + \mathfrak{m} k(\boldsymbol{g}_x z; \mathbf{g}_w w) = \mathbf{g}_w[\sum_{t=1}^{\infty} \mathfrak{n}_t k(x_{-t}; w) + \mathfrak{m} k(z; w)] = \mathbf{g}_w \mathbf{k}$, so that the aggregate capital stock grows at gross rate $\mathbf{g}_w$. A similar argument can be used to show that aggregate output grows at the same rate.

Definition *(Balanced-Growth Path) For a given subjective discount factor and coefficient of relative risk aversion, $\widehat{\delta}$ and ε, a balanced-growth path consists of (i) a financial contract, $\{\beta_t, \sigma_t, \mu_t, p_t\}$, between entrepreneurs and the venture capitalist; (ii) a set of labor inputs for VC- and non-VC-funded firms, $l(x; w)$ and $l(z; w)$; (iii) values for the contract, an IPO, and a startup, $C(x; \mathbf{x})$, $I(x; \mathbf{x})$, and $V(\mathbf{x})$; (iv) a project type, x, for new entrepreneurs; (v) an inflow of new entrepreneurs, $\mathfrak{e}$; (vi) a rental rate for capital, r, and a market discount factor, δ; (vii) an idea distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$; (viii) a wage rate, w; and (ix) a gross growth rate of aggregate productivity, $\mathbf{g_x}$, such that:*

1. *The financial contract, $\{p_t, \sigma_t, \mu_t, \beta_t\}$, solves problem (P2), given the function $I(x; \mathbf{x})$ and x, $\mathbf{g_x}$, and $\mathbf{x}$. The solution to this problem gives the expected return to a new entrepreneur from the contract, $C(x; \mathbf{x})$.*

2. *The VC-funded firm maximizes its profits, given x, r, and w, as specified by problem (P1). This determines the value of its IPO, $I(x; \mathbf{x})$, as presented in (3). The solution to the firm's maximization problem gives the rule for hiring labor (13). Analogously,*

a non-VC-funded firm maximizes its profits, given z, r and w, as specified by problem (8).

3. *A new entrepreneur picks the project type, x, to solve problem (P3), given the value of the contract, $C(x; \mathbf{x})$, as a function of x and $\mathbf{x}$. This determines the expected value of a startup, $V(\mathbf{x})$.*

4. *The inflow of new entrepreneurs, $\mathfrak{e}$, is regulated by (1) and (7), taking as given the value of the startup, $V(\mathbf{x})$.*

5. *The rental rate on capital, r, and the market discount factor, δ, are governed by (9) and (10), given $\mathbf{g}_w$.*

6. *The idea distribution for VC-funded firms, $\{\mathfrak{n}_t\}_{t=1}^{\infty}$, is specified by (11) and (12).*

7. *The market-clearing wage rate, w, is given by (14) and grows at the gross rate $\mathbf{g}_w = \mathbf{g_x}^{\zeta/(\zeta+\lambda)}$.*

8. *Aggregate productivity in the VC sector, $\mathbf{x}$, grows at the gross rate $\mathbf{g_x}$ specified by (15).*

The lemma below establishes that the setup will have a balanced-growth path.

Lemma 2 *(Balanced Growth) Let $x' = \mathbf{g_x}x$ and $\mathbf{x}' = \mathbf{g_x}\mathbf{x}$ for all time. If $\beta_t, \sigma_t, \mu_t, p_t$ and $C(x; \mathbf{x})$ solve the contract specified by (P2) for $(x, \mathbf{x})$, then $\sigma'_t = \sigma_t, \mu'_t = \mu_t, \beta'_t = \beta_t, \widetilde{\sigma}'_t = \widetilde{\sigma}_t, p'_t = \mathbf{g}_w p_t$, and $C(x'; \mathbf{x}') = \mathbf{g}_w C(x; \mathbf{x})$ will solve it for $(x', \mathbf{x}')$. Likewise, if it is optimal in (P3) to pick x for $\mathbf{x}$, then it is optimal to choose $x' = \mathbf{g_x}x$ for $\mathbf{x}'$. The gap between the frontier, x, and average productivity in the VC sector, $\mathbf{x}$, as measured by $x/\mathbf{x}$, is time invariant. The inflow of new entrepreneurs, $\mathfrak{e}$, is a constant, so that $\mathfrak{e}' = \mathfrak{e}$.*

Proof. See Theory Appendix. ∎

10   Calibration

As discussed in Section 2, VC partnerships are of a limited duration, usually between 7 to 10 years. So, the analysis assumes that an entrepreneur's contract with a venture capitalist has 7 potential funding rounds each lasting 1.5 years. Thus, partnerships are structured to last

at most 10.5 years. The decreasing returns to scale parameter in the production function (2) is taken from Guner, Ventura, and Xu (2008), which requires setting $\zeta = 0.20$. The exponents for the inputs are picked so that capital earns 1/3 of nonprofit income and labor receives 2/3. The survival rate of a firm is selected so that on average a publicly listed firm lives 25 years, as in the U.S. economy. The depreciation rate on capital, $1 - \eth$, is taken to be 7 percent. Last, Henrekson and Sanandaji (2016) report that the key personnel connected with VC startups are taxed in the United States at a 15 percent capital gains rate. So, set $\tau = 0.15$.[4]

The model is calibrated to match several data targets, listed in Table 7. For the most part, the model's parameter values are *jointly* determined as a function of the data targets. Still, some data targets play a much more central role in identifying a parameter. Over the period 1948 to 2015, U.S. GDP per hours worked grew at 1.8 percent per year. This fact is targeted in the calibration procedure. The parameter governing the efficiency of doing research, χ_R, is important for determining the economy's growth rate. The long-run interest rate is set to 4 percent, a typical value. A standard value of 2 is assigned for the coefficient of relative risk aversion. The market discount factor is the reciprocal of the equilibrium interest rate, and it will change as the growth rate of the economy, $\mathbf{g}_w$, changes. At the calibrated equilibrium, the representative agent's annual discount factor is determined by the formula to $\widehat{\delta} = (1 - 0.04)/(1.018)^{-2}$; cf. (10). This yields a yearly interest rate of 4 percent.

To calibrate the elasticity of the research cost function, ι, the following firm-level regression is run using VentureXpert data:

$$\ln(\text{IPO value}) = \underset{(0.154)}{0.390^{**}} \times \ln(\text{VC funding}) + \text{Controls, obs.} = 1{,}145,$$

$$(16)$$

where the controls are the logarithm of the firm's employment, the firm's age at IPO, a 2-

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[4] The capital gains tax rate has varied across time in the United States. The 15 percent rate was instituted under President Bush in 2003. The maximum rate rose to 20 percent in 2012 under President Obama.

digit SIC industry dummy variable, the logarithm of the aggregate level of VC funding, and a cluster dummy for whether the venture capitalist was located in California or Massachusetts. Three instrumental variables are also used: the capital gains tax rate (which varies across states and time), dependence on external finance (which varies across industries), and the deregulation dummy. The coefficient shows the impact of a firm's VC funding on its IPO value and is used to identify a value for ι, as discussed next.

To identify ι, the impact of a change in firm-level VC funding on its IPO value is calculated for the model. This calculation is broken down into two steps. First, the elasticity of $I(x; \mathbf{x})$ with respect to x is computed. Second, the elasticity of VC funding with respect to x is totted up numerically. This is done in partial equilibrium to match the results of the firm-level regression. The ratio of these two elasticities gives the elasticity of IPO value with respect to VC funding. Thus, the following object is computed for the model:

$$\text{IPO VALUE ELASTICITY} = \frac{d \ln \text{IPO}/d \ln x}{d \ln(\text{VC FUNDING})/d \ln x}.$$

Ideally, this should have a value of 0.390.

Another key elasticity in the model is the shape parameter, ν, for the Pareto distribution governing the opportunity cost of entrepreneurship. This regulates the inflow of entrepreneurs. Henrekson and Sanandaji (2016) report that a one percent increase in a country's effective tax rate on VC activity leads to a one percent decline in the VC investment-to-GDP ratio. This elasticity is targeted to recover the shape parameter, ν. This parameter can be selected after calibrating the remaining parameters because the scale parameter, υ, can adjusted, given the choice for ν, such that the number of entrepreneurs is constant. This normalization for υ implies that all the other moments used in the calibration will not change.

The process for the efficiency of round-t monitoring, $\chi_{M,t}$, is taken to be a cubic:

$$\chi_{M,t} = \log(a_0 + a_1 \times t + a_2 \times t^2 + a_3 \times t^3).$$

This requires specifying four parameters, namely a_0, a_1, a_2 and a_3. Additionally, the monitoring parameters are selected to match the venture capitalist's share of equity by funding round (this pattern is taken up below). The more efficient monitoring is, the higher will be the venture capitalist's share of equity, as will be seen in Section 11.

The time profile for the fixed cost, $\phi(t)$, is governed by the quartic

$$\phi(t) = \exp(b_0 + b_1 \times t + b_2 \times t^2 + b_3 \times t^3 + b_4 \times t^4).$$

Five parameters, b_0, b_1, b_2, b_3, and b_4, govern this specification. The pattern of VC investment by funding round (discussed below) determines these parameters.

Bernstein, Giroud, and Townsend (2016) estimate the impact on investment of a venture capitalist's time cost for monitoring. To do this, they examine the effect of changes in airline routes that reduce the commuting time a venture capitalist spends visiting a startup. They find that the introduction of a new airline route (the treatment) leads to a 4.6 to 5.2 percent increase in VC investment. The average reduction in travel time is significant. The lead investor visits the company site roughly 20 times per year and spends approximately 12 hours traveling and 5 hours at the company per visit, which amounts to 100 contact hours annually.[5] On average, a treatment saves roughly 2 hours per trip, or 40 hours per year of a venture capitalist's time. Accordingly, the treatments correspond to fairly large reductions in monitoring costs: a reduction of 2 hours per trip translates into a 12.4 percent reduction in monitoring costs. Bernstein, Giroud, and Townsend (2016) argue that most of the resources spent by a venture capitalist on monitoring is time. So, assume that monitoring is done using labor in the model.

The size of this micro-level elasticity depends in the model, among other things, on the quality of the projects, captured by, ρ. As the share of good projects rises, the success rate for ventures increases while the failure rate falls. The payoff from investing in research and development hence rises. So, does the return from monitoring because more funds are being

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[5] The time spent visiting the company is quoted in the unpublished version of Bernstein, Giroud, and Townsend (2016).

invested. Therefore, the size of the treatment effect moves up with ρ. Therefore matching, in partial equilibrium, the Bernstein, Giroud, and Townsend's (2016) treatment effect helps to tie down the fraction of good ideas, ρ.

Next, projects that are funded by venture capitalists have an average success rate per funding round of 2.0 percent and a failure rate of 3.2 percent. The calibration procedure attempts to match these two statistics. To construct these statistics for the model, note that the success rate in funding-round t is just the number of IPOs divided by the mass of surviving firms:

$$\text{SUCCESS RATE}_t = \frac{\text{IPOs}_t}{\text{SURVIVING FIRMS}_t} = \frac{\sigma_t \rho \Pi_{j=1}^{t-1}(1-\sigma_j)}{\rho \Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^t(1-\beta_j)}.$$

The analogous definition for the failure rate in round t is

$$\text{FAILURE RATE}_t = \frac{\text{FAILURES}_t}{\text{SURVIVING FIRMS}_t} = \frac{\beta_t(1-\rho)\Pi_{j=1}^{t-1}(1-\beta_j).}{\rho \Pi_{j=1}^{t-1}(1-\sigma_j) + (1-\rho)\Pi_{j=1}^t(1-\beta_j).}.$$

Not surprisingly, the development efficiency parameter, χ_D, is instrumental for determining the average success rate, while the evaluation efficiency parameter, χ_E, impinges heavily on the average failure rate–this is discussed in Section 11.

Puri and Zarutskie (2012, Table I) report that ratio of employment in a VC-backed firm to a non-VC-backed one is 58.14. This is a calibration target. For the model, the employment ratio is

$$\text{EMPLOYMENT RATIO} = \frac{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta}\mathfrak{n}\mathbf{x}/\mathfrak{n}}{\left(\frac{\kappa}{r}\right)^{\kappa/\zeta}\left(\frac{\lambda}{w}\right)^{(\zeta+\lambda)/\zeta}\mathfrak{m}\omega\mathbf{x}/\mathfrak{m}} = \frac{1}{\omega}.$$

This ratio pins down the productivity of a non-VC-backed firm relative to a VC-backed one, or ω.

Data on the scrap value of unsuccessful ventures are, unfortunately, not readily available. So, the parameter ξ governing the scrap value of a firm is identified by attempting to match the observed cash multiple for VC investments. The cash multiple is the ratio of the venture capitalist's cash receipts to disbursements, and is used as a crude measure of the ex post

return on a VC investment. A venture capitalist's receipts will include the scrap value on those unsuccessful projects that are still surviving at the end of the contract.

The upshot of the calibration procedure is now discussed. The parameter values resulting from the calibration procedure are presented in Table 6, which also gives the basis for their identification. First, the model matches the average success and failure rates very well, as shown in Table 7. And, the model replicates perfectly the ratio of VC-backed employment to non-VC backed employment. The IPO elasticity is duplicated. And the model matches exactly the Henrekson and Sanandaji (2016) tax rate elasticity. The monitoring-cost treatment effect lies within the range of estimates reported by Bernstein, Giroud, and Townsend (2016).

Next, note how investment in a project by a venture capitalist increases with the funding round (see the top panel of Figure 5). This time profile is a calibration target. Given the limited life span of a VC partnership, there is considerable pressure to bring a project to fruition as quickly as possible. This is true in the model too, which displays the same increasing profile of funding. Two features help to generate this. The first is that bad projects get purged over time through the evaluation process. The second is that the cost of monitoring drops as the venture capitalist becomes more familiar with the project, which reduces the incentive problem. Without these features, funding would fall over time. Last, since investment increases over time, one would expect that the venture capitalist's share of the enterprise will too. The bottom panel of Figure 5 illustrates this. The model does very well on this account. Again, the calibration procedure focuses on this feature of the data.

The time profiles for the success and failure rates are *not* targeted in the calibration procedure. As shown in the middle panel of Figure 6, in the data the odds of success decline by funding round or with the passage of time. While the model captures the average success across funding rounds very well, it has some difficulty mimicking the declining time profile. Failure rates also decline with time, and the model does very well on this dimension. Now turn to the bottom panel of Figure 6. Observe that the value of an IPO drops with the incubation time for the project. In the model, as time passes, the value of a project declines

Parameter value	Description	Identification
Firms		
$\kappa = 1/3 \times 0.80$	Capital's share	Standard
$\lambda = 2/3 \times 0.80$	Labor's share	Standard
$1 - \mathfrak{d} = 0.07$	Depreciation rate	Standard
$\mathfrak{s} = 0.96$	Firm survival rate	Expected life of Compustat firms
$\chi_R = 4.7$	Research efficiency, x	Growth rate
$\iota = 2.56$	Research cost elasticity, x	Regression (16)
$\nu = 0.025$	Pareto shape parameter	H&S (2016) tax elasticity
$\upsilon = 0.57$	Pareto scale parameter	Normalization
Consumers		
$\varepsilon = 2$	CRRA	Standard
$\widehat{\delta} = 0.994$	Discount factor	4% risk-free rate
VC		
$T = 7$	Number of funding rounds	Partnership length (10.5 years)
$\rho = 0.21$	Fraction of goods ideas	BG&T (2016) treatment effect
$\chi_D = 0.0335$	Development efficiency, σ	Average success rate
$\chi_E = 0.0360$	Evaluation efficiency, β	Average failure rate
$a = \{-1.12, -0.12, 0.321, -0.018\}$	Monitoring efficiency, μ	Equity share by round
$b = \{-0.89, 0.80, 0.25,$ $\qquad -0.12, 0.013\}$	Fixed costs, ϕ	VC funding by round
$\tau = 0.15$	Capital gains tax rate	H&S (2016)
$\xi = 0.375$	Scrap value fraction	Cash multiple
Non-VC-funded		
$m = 40$	Number non-VC firms	Relative empl. non-VC firms
$\omega = 1/58$	Relative prod of non-VC firms	Relative size of non-VC firms

Table 6: The parameter values used in the baseline simulation.

because aggregate productivity in the VC sector catches up with the productivity of the entrepreneur's venture; "the thrill is gone," so to speak. It is a bit surprising that the framework can match almost perfectly this feature of the data, which is not targeted.

11   Thought Experiments

The analysis stresses the ability of a venture capitalist to evaluate, develop, and monitor startup projects. The importance of these three factors is now investigated one by one and

| Target	Source	Data	Model
Economic growth	BEA	1.80%	1.78%
Cash Multiple	Gompers et al. (2016, Table 12)	5.5	5.6
Success Rate	Puri and Zarutskie (2012, Table VI.B)	2.0%	2.0%
Failure Rate	Puri and Zarutskie (2012, Table VI.B)	3.2%	3.3%
VC funding	Crunchbase	Figure 5	
Equity Share	Crunchbase	Figure 5	
IPO Value Elasticity–firm level	Regression (16)	0.39	0.39
Tax Elasticity of VC Inv/GDP	Henrekson and Sanandaji (2016)	-1.0	-1.0
Monitoring-Cost Treatment	Bernstein et al. (2016, Tables IAVI & IAVII)	4.6 to 5.2%	4.9%
VC Employment Share	Puri and Zarutskie (2012, Table I)	5.5%	4.8%
Employment ratio	Puri and Zarutskie (2012, Table I)	58.1	58.1

Table 7
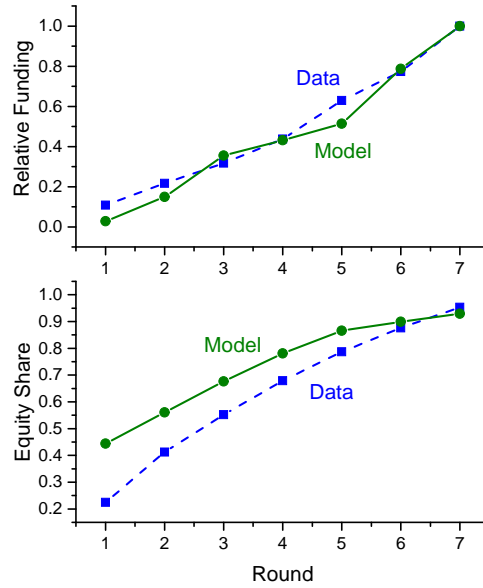
Figure 5: Investment and equity share by funding round–data and model. The upper panel shows the venture capitalist's investment by funding round. Funding in the last round is normalized to 1.0. The lower panel charts the venture capitalist's share of equity by funding round.
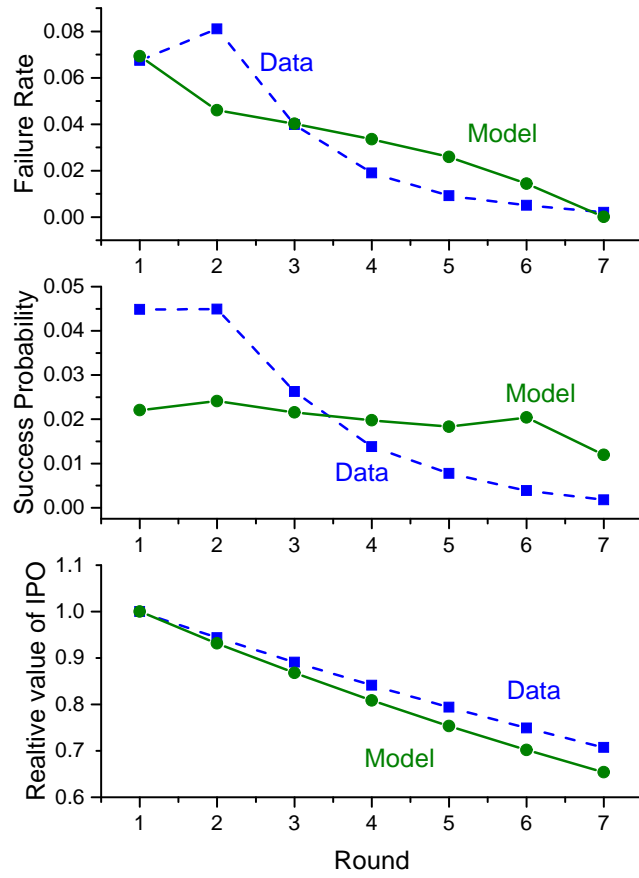
Figure 6: The odds of success and failure by funding round and the value of an IPO by the duration of funding–data and model. The value of an IPO that occurs during first funding round is normalized to 1.0. None of these profiles is targeted in the calibration.

then the efficiencies of each debased in tandem to approximate the success rate of non-VC methods of finance.

11.1 Changes in Monitoring Efficiency, $\chi_{M,t}$

How important is the venture capitalist's ability to monitor the use of funds by entrepreneurs? Figure 7 shows the general equilibrium impact of improving the efficiency of monitoring in the model. To undertake this thought experiment, the monitoring efficiency profile, $\{\chi_{M,1}, \cdots, \chi_{M,7}\}$, is changed by scalar, which takes the value of 1 for the baseline calibration. Monitoring efficiency is measured in terms of the percentage deviation of this scalar from its baseline value. As monitoring efficiency improves, there is an increase in the average odds of detecting fraud across funding rounds (see the top panel). The venture capitalist's share of equity rises, on average, because it is now easier to ensure that funds are not diverted. Compliance with the contract can still be guaranteed when the entrepreneur is given a lower share of an IPO. The venture capitalist must still earn zero profits, however. Part of the increased return to the venture capitalist is soaked up by letting the new entrepreneur be more ambitious about his choice of technique, which raises the initial cost of research, $R(x/\mathbf{x})$; the rest of the increased return is absorbed by increased investment in development. As a result, VC-backed firms have a higher level of productivity and are more successful. This results in a higher share of employment for VC-backed firms (as shown in the middle panel). Additionally, the economy's growth rate moves up, which results in a welfare gain (measured in terms of consumption; see the bottom panel).[6]

11.2 Changes in Evaluation Efficiency, χ_E

The importance of efficiency in evaluation is examined now. The results are displayed in Figure 8, where χ_E is measured in terms of percentage deviations from the baseline

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[6] See Akcigit, Celik, and Greenwood (2016, Section 5.1) for details on how the welfare gain is computed. In the current work, the initial level of consumption across balanced-growth paths is held fixed, though, as opposed to aggregate productivity.
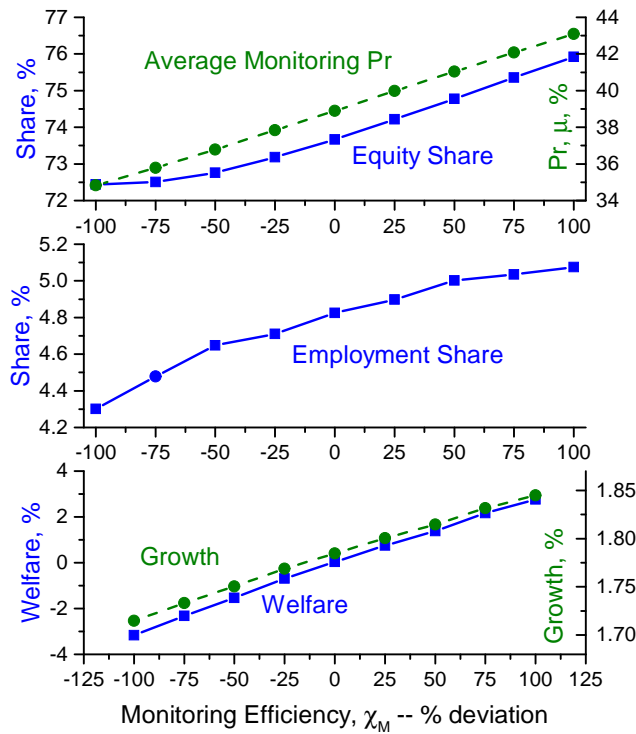
Figure 7: Efficiency in monitoring, $\chi_{M,t}$. The top panel shows how the average probability of detecting fraud and the venture capitalist's share of equity vary with efficiency in monitoring. The middle panel illustrates how the share of VC-backed firms in employment responds. Growth and welfare are displayed in the bottom panel. Monitoring efficiency is measured in terms of the percentage deviation from the baseline equilibrium.

equilibrium. As evaluation becomes more efficient, the odds of detecting a bad project increase. Hence, the average failure rate across funding rounds moves up (see the top panel). The success rate rises, both due to the purging of bad projects and the resulting increased VC investment in development. The purging of bad projects dominates the exit of good ones so that the fraction of good projects in the last round increases with χ_E (as the middle panel illustrates). The fact that it is more profitable to invest in research and development is reflected by an upward movement in the share of VC-backed firms in employment. Economic growth and welfare move up in tandem as evaluation efficiency improves (see the bottom panel).

11.3 Changes in Development Efficiency, χ_D

Finally, impact of changes in development efficiency is studied. Again, χ_D is measured in terms of percentage deviations from the baseline calibration. As it becomes less expensive to develop a project, the odds of success improve. The failure rate also rises because fewer good projects remain in the pool over time. VC-backed firms' share of employment picks up, as it is more profitable to fund a project. Last, economic growth and welfare rise with development efficiency.

11.4 Debasing Venture Capital–An Approximation to Non-VC Forms of Financing

Venture capitalists lend development and evaluation expertise to startups that alternative forms of finance, such as angel investors, banking, and more recently crowdfunding, do not. Arguably, venture capitalists are also better at monitoring projects. Wealthy people have always been willing to lend seed money to startups, as discussed in Section 2. This is what angel investors do today. The sheer size of financing needed as a startup evolves goes well beyond an angel investor's pockets. The average investment per deal of an angel investor was $510,000 in 2014. In contrast, the average venture capitalist invests $4 million and
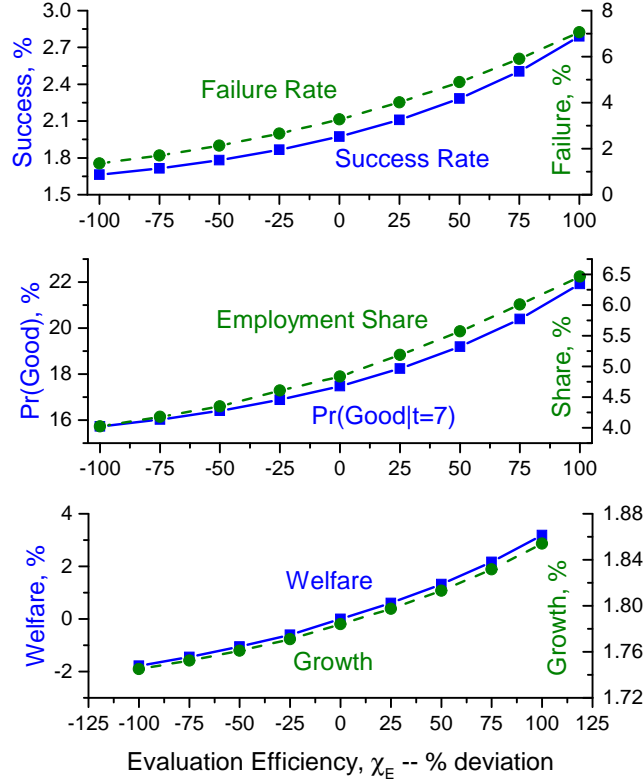
Figure 8: Efficiency in evaluation, χ_E. The top panel shows how the average failure and success rates across funding rounds vary with efficiency in evaluation. The middle panel illustrates how the odds of a project being good in the seventh round and the employment share of VC-backed firms respond. Growth and welfare are illustrated in the bottom panel. Evaluation efficiency is measured in terms of the percentage deviation from the baseline equilibrium.

$14 million in seed-stage and later-stage deals. These investments are 8 times and 28 times larger than those of angel investors. VC organizations feature substantially higher levels of professionalism and specialization than angel investors: all the roles of a VC organization (e.g., evaluation, development, and monitoring) are rolled up into one single angel investor.

To approximate alternative forms of finance, some empirical evidence from Puri and Zarutskie (2012) is used. They track the performance of VC- and non-VC-financed firms using the Longitudinal Business Database (LBD). They identify firms in the LBD as VC-financed if they can be matched to the VentureSource and VentureXpert databases. They
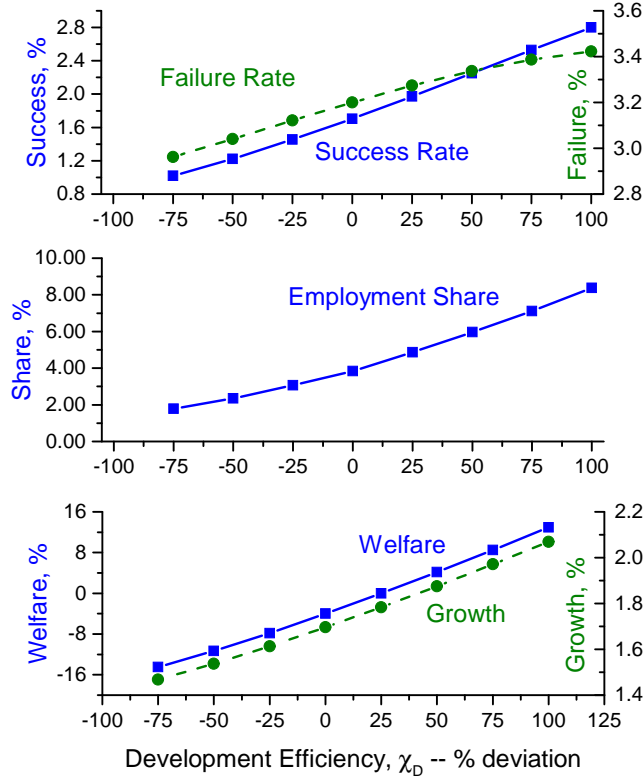
Figure 9: Efficiency in development, χ_D. The top panel shows how the average failure and success rates across funding rounds vary with efficiency in development. The middle panel illustrates how the share of VC-backed firms in employment responds. Growth and welfare are illustrated in the bottom panel. Development efficiency is measured in terms of the percentage deviation from the baseline equilibrium.

match each VC-financed firm to a non-VC-financed firm based on four characteristics: age, 4-digit SIC code, geographical region, and employment size. They find that VC-financed and non-VC-financed firms are observationally identical at the time the former first receive VC financing. Based on this comparison, they report that the average ratio of the success rate of non-VC-financed firms to the success rate of observationally identical VC-financed firms is 0.30.[7] [8]

To approximate more traditional forms of finance in the model, the efficiency of development, evaluation, and monitoring are all debased in an equiproportional manner to render the same average success-odds ratio for a startup. In order for this ratio to be comparable with its empirical counterpart, this recalibration is done in partial equilibrium. The 0.30 ratio is reproduced by reducing in tandem development, evaluation, and monitoring efficiency to 55 percent of their original values.[9] The upshot of this exercise is shown in Table 8. Alternative forms of finance have a much lower success rate (1.1 versus 2.0 percent) than do VC-financed projects. The ratio of 1.1/2.0 is larger than 0.30 because there are general equilibrium effects, inducing a drop in wages, that partially offset the reduction in financing efficiency. The financier's share of the project declines considerably. Since monitoring is less efficient, a larger share of the project must be given to the entrepreneur to ensure that he will invest all of the development funds. The drop off in the success rate and the financier's share of equity lead to less research and development in the debased VC-backed firms. The IPO value of a startup drops a lot, by 43.7 percent. This is in the ballpark of the 31.1 percent drop predicted by the firm-value regression in Table 2 for a non-VC-backed firm (relative to

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[7] Venture capitalists could match with better firms based on unobservable characteristics. Given that startups are very young with little in terms of employment and patents, it might difficult to control empirically for this selection effect. To the extent that such selection effects are important, the results in this section constitute an upper bound for the effect of VC financing.

[8] This number is based on Table VI.B (p. 2271) of Puri and Zarutskie (2012). First, Puri and Zarutskie's cumulative success rates are first differenced to get the yearly rates. Second, the success-odds ratio (of non-VC-financed firms to VC-financed firms) is calculated year by year. Third, an average is taken over the years. Only acquisitions are considered to be successes in this calculation, because Puri and Zarutskie (2012) don't report yearly IPO numbers for non-VC-financed firms. This is presumably because IPOs are virtually non existent for non-VC-financed firms. This implies that the estimated success ratio is conservative in nature.

[9] The results are quite similar when only development efficiency is debased.

An Alternative form of Finance

Variable	Baseline	Debased economy
Success	2.0	1.1
VC Empl. share	4.8	2.2
Equity Share	73.7	69.6
Δ IPO value	0	-43.7
Growth	1.8	1.5
Welfare loss	0	11.6

Table 8: All numbers are in percentages.

a VC-backed one). As a result, there is less employment in VC-backed firms. Growth also drops. This generates a large welfare loss.

12   Capital Gains Taxation

Most VC-funded firms in the United States are setup as partnerships. CEOs, central employees, founders, and investors are paid in terms of convertible equity and stock options. These financial assets payoff only under certain well-specified contingencies and serve to align the incentives of key participants.[10] Interestingly, the returns on convertible equity and stock options are taxed in the United States at the capital gains rate, which is 15 percent. The IRS lets companies assign artificially low values to these instruments when they are issued. So, effectively, participants are only subject to taxation at the time of an acquisition/IPO. In other countries the rate of taxation on VC-funded startups is much higher. Figure 10 illustrates for a cross section of countries how VC investment as a percentage of GDP falls with the tax rate on VC profits. The data are from Henrekson and Sanandaji (2016). To obtain the tax rates on VC profits, they asked the local offices of PricewaterhouseCoopers in 22 countries to calculate the effective tax rate for a representative VC startup. So, for example, PricewaterhouseCoopers calculate that is 30 percent in France, 47.5 percent in Germany, and 72 percent in Italy. Using this data in a regression analysis, Henrekson and

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[10] Celik and Tian (2018) analyze how established firms with better corporate governance (as proxied by the equity share of institutional investors) also tend to remunerate executives more in terms of incentive pay than do other firms, which leads to higher levels of innovation.
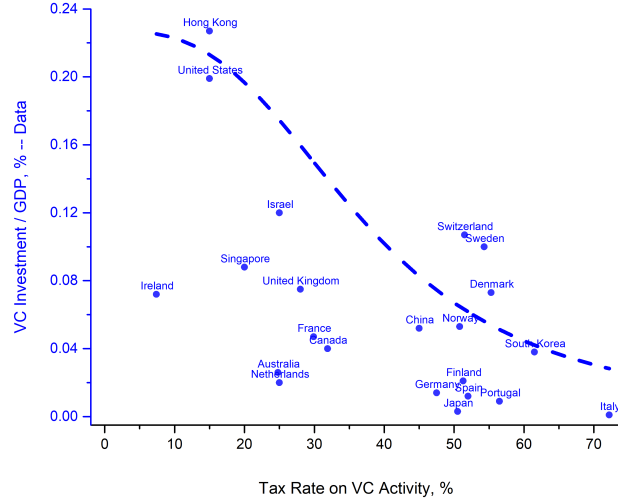
47

Figure 10: The cross-country relationship between the tax rate on VC profits and the VC-investment-to-GDP ratio–data and model. The numbers are expressed as percentages.

Sanandaji (2016, Table 4) report a strong negative correlation between the tax rates on VC profits and VC investment as a percentage of GDP. The elasticity of the tax rate on VC activity is about -1.0, as mentioned earlier. This feature of the data is matched in the model by calibrating the shape parameter for Pareto distribution, which governs the inflow of entrepreneurs. So, the response of VC activity to taxes is the same in the data and model.

The model can be used as a laboratory to gauge the effect of taxation on other key variables, such as growth and welfare, which is shown in Figure 11. As the tax rate on VC profits rises, not surprisingly economic growth declines. An increase in the tax rate from -15 percent (a subsidy) to 50 percent, causes economic growth in the model to fall from 1.90 percent to 1.58 percent. The effects on growth might appear small, but lowering the tax rate from 50 percent to 15 percent produces a long-run welfare gain of 9.4 percent, when ignoring transitional dynamics. Going further from 15 percent to -15 percent generates an additional welfare gain of 5.5 percent, all measured in terms of consumption.
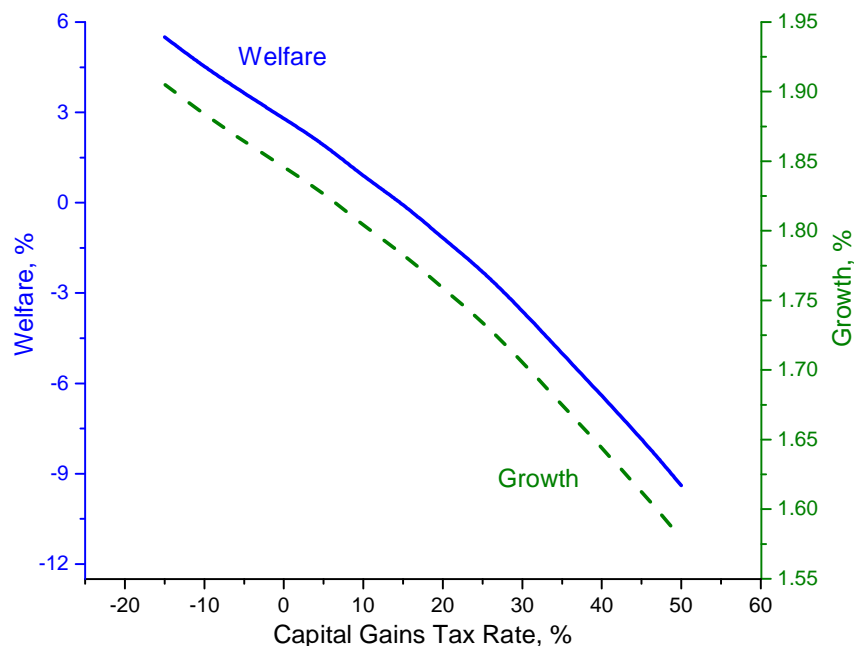
48

Figure 11: Impact of VC profit taxation on economic growth and welfare.

13   What about Growth?

Is the recent rise in VC investment reflected in growth statistics? The answer to this question is nuanced. On the one hand, at the country level VC investment appears to be positively linked with economic growth. A scatter plot between economic growth and VC investment for G7 countries is shown in the upper panel of Figure 12. These are developed nations. As the figure shows, there is a clear positive association between these two variables. The analysis is extended to G20 countries in the bottom panel of the figure. Now the scatter plot includes some poorer countries, where VC investment isn't so prevalent. There is still a positive association, but not surprisingly it is weaker.

To conduct a more formal analysis, some regression analysis is conducted with a sample of 37 economies over the period 1995 to 2014. The sample covers 99 percent of world VC investment and 88 percent of world GDP. In addition, the two-decade sampling period is divided into four sub-periods, each lasting five years. A country is included in the sample if
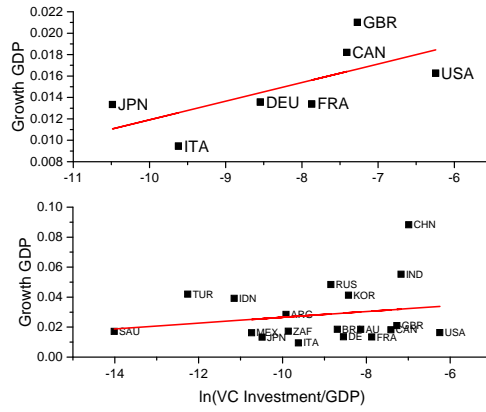
Figure 12: Economic growth and VC investment, 1995-2014. The upper panel shows the relationship between VC investment and growth in G7 countries, while the bottom panel does the same for the G20.

its share of world VC investment between 1995 and 2014 is not less than 0.05 percent.[11] The dependent variable in the regression analysis is the median growth rate of real GDP per capita in each period, while the main explanatory variable is the natural logarithm of the median VC investment-to-GDP ratio. The regressions include the initial levels of real GDP per capita and the Barro and Lee (2013) human capital index. These control variables are the two main factors demonstrated in the empirical literature to be important for economic growth. Moreover, period dummies are included to control for aggregate shocks to all countries. An IV approach is also taken to address the endogeneity issues. Two IVs are used. The first, which follows the strategy pioneered in Barro and Lee (1994), is the median VC investment-to-GDP ratio for each country during the decade preceding the sample period (i.e., 1985 to 1994). The second is a dummy variable for the legal origin of the country, which is equal to 1 for common-law countries. The idea is that common-law legal systems foster better financial development than civil-law legal systems, because of higher judicial independence from the government and the flexibility of the courts to adapt to changing conditions (see Beck, Demirguc-Kunt, and Levine (2005)).

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[11] An exception is Bermuda, which accounted for 0.18 percent of world VC investment. Bermuda is excluded because it is a tax haven. Companies set up offices there, while undertaking virtually no business activity, just to avoid corporate income taxation.

VC Investment and Growth: Cross-Country Regressions

Dependent variable		Growth of GDP, %		
	OLS	IV		
		Pre ln(VC Inv/GDP)	Legal origin	Both
Panel A: G7				
ln(VC Inv/GDP)	0.186**	0.253***	0.227**	0.240***
	(0.0782)	(0.0910)	(0.0899)	(0.0816)
Observations	28	28	28	28
R-squared	0.695			
Panel B: 37-Country Sample				
	0.228**	1.156**	0.421*	0.463*
	(0.112)	(0.501)	(0.254)	(0.260)
Observations	148	120	148	120
R-squared	0.295			

Table 9: See the main text for a description of the dependent and independent variables. Pre ln(VC Inv/GDP) refers to the pre-sample VC investment-to-GDP ratio. Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5 and 10 percent levels.

The main regression results are reported in Table 9. As the table shows, VC and growth are positively correlated. The IV estimate for the G7 countries in the last regression in Panel A shows that a 10 percent increase in the VC investment-to-GDP ratio is connected with a 0.024 percentage point increase in growth. This may seem small, but it implies that increasing the VC investment-to-GDP ratio from the Norwegian level of 0.053 percent, which is the median, to the U.S. level of 0.19 percent would increase growth by 0.31 percentage points.[12]

On the other hand, the impact of VC may not be readily apparent in growth statistics for several reasons. First, technological revolutions, such as the Information Age, may cause disruptions in an economy. Old forms of businesses are displaced by new forms. Online retailing is displacing brick and mortar stores, for example. Greenwood and Yorukoglu (1997) discuss how the dawnings of the First and Second Industrial Revolutions were associated with productivity slowdowns and suggest that the same phenomena characterize the Information

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[12] Relatedly, Sampsa and Sorenson (2011) estimate, using a panel of U.S. metropolitan statistical areas, that VC positively affects startups, employment, and regional income.

Age. Second, measuring investment and output in the information age is difficult. Think about the introduction of cell phones, as discussed in Hulten and Nakumura (2017). Cell phones substitute for traditional land lines, audio players, cameras, computers, navigation systems, and watches, inter alia. Cell phones have free apps. Between 1988 and 2015, land lines fell from 1.7 to 0.3 percent of personal consumption expenditures. Since cell phones constitute 0.15 percent of personal consumption expenditures, this would be measured as a drop or slowdown in GDP. An iPhone 5 would have cost more than $3.56 million to build in 1991.[13] Likewise, global camera production dropped from 120 million units to 40 million over the 2007 to 2014 period. Additionally, investment may be in intangibles, such as software, R&D, retraining workers, reconfiguring products and organizational forms, and branding new products. Corrado, Hulten, and Sichel (2009) estimate that investment in such intangibles is now as large as that in tangibles. Including intangible investment in GDP accounting increases estimates of growth by 10 to 20 percent. McGrattan and Prescott (2005) argue that, after taking intangibles into account, the 1990s was a boom period. Third, technologies flow across national boundaries. So even countries that don't innovate will experience growth from the adoption of new technologies. Out of France, Germany, Japan, the United Kingdom, and the United States, Eaton and Kortum (1999) find that only the United States derived most of its growth from domestic innovation. Comin and Hobijn (2010) document that diffusion lags for new technologies have shrunk over time. Fourth, firms may park offshore the profits from new innovation to avoid taxation. Accounting for this could increase productivity growth in the United States by 0.25 percentage points over the 2004 to 2008 period, according to Guvenen et al. (2017).

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[13] This 2017 guesstimate was done by Bret Swanson, who calculates that the flash memory, processor, and broadband communications of an iPhone 5 would have cost $1.44, $0.62, and $1.5 million in 1991. The cost of these three components adds up to $3.56 million. On top of that, considering the other components (camera, iOS operating system, motion detectors, display, apps, etc.), it would have cost more than $3.56 million to build an iPhone 5 in 1991.

14 Conclusion

Venture capital is important for economic growth. Funding by venture capitalists is positively associated with patenting activity. VC-backed firms have higher IPO values when they are floated. Following flotation they have higher R&D-to-sales ratios. VC-backed firms also grow faster in terms of employment and sales.

An endogenous growth model of the VC process is constructed and taken to the data. In the framework, entrepreneurs bring ideas to venture capitalists for funding. Venture capitalists provide seed money to research the ideas. After this projects enter a funding-round cycle. During each round, projects are: (i) evaluated to assess their ongoing viability; (ii) those that pass are then provided with VC to develop the project; (iii) the use of funds is monitored is done to ensure that there is no malfeasance; and (iv) successful projects are floated on the stock market or sold to other businesses. The evaluation plan, development funding, the monitoring strategy, and the equity share of the venture capitalist are governed by a dynamic contract between the entrepreneur and a venture capitalist. The model is capable of matching several stylized facts of the VC process by funding round. In particular, it mimics the funding-round profiles for the success and failure rates of projects, the injections of VC for development, the venture capitalist's share of equity, and the value of an IPO by the time it takes to go to market. This is done while matching the share of VC-backed firms in total employment, the average size of a VC-backed firm relative to a non-VC-backed one, the elasticity of IPO value with respect to VC funding, the cross-country elasticity of VC investment with respect to profit taxes, and the impact of monitoring costs on VC investment.

The key personnel involved with starting up the enterprises funded by venture capitalists are rewarded in the form of convertible equity and stock options. In the United States, venture capitalists are subject only to capital gains taxation. The rate at which VC-funded startups are taxed in the United States is low relative to other developed countries. Does this promote innovative activity? The analysis suggests that raising the tax on VC-funded startups from the U.S. rate of 15 percent to the Norwegian rate of roughly 50 percent would

shave 0.2 percentage points off growth and lead to a long-run consumption-equivalent welfare loss of 9.4 percent.

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15 Data Appendix

15.1 Figures

- *Figure 1: The rise of venture capital, 1970 to 2015.* Investment by venture capitalists is obtained from the VentureXpert database of Thomson ONE. The fraction of public firms backed by VC companies is created by matching firm names in VentureXpert and CompuStat; the latter are available from Wharton Research Data Services.[14]

- *Figure 2: The share of VC-backed companies in employment, R&D spending, and patents.* The employment and R&D shares of VC-backed public companies are calculated by matching firm names in VentureXpert and CompuStat, as in Figure 1. The share of patents for VC-backed public companies is computed by matching firm names in VentureXpert and the NBER Patent Data Project.[15]

- *Figure 5: Investment and equity share by funding round.* Investment in each funding round is based on the VC-funded deals in Crunchbase between 1981 and 2015. Crunchbase has better funding-round information than VentureXpert. The vertical axis is the mean of funding in a round across all deals, from round 1 (i.e., series A) to round 7

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[14] Source link: https://wrds-web.wharton.upenn.edu/wrds/index.cfm?
[15] Source link: https://sites.google.com/site/patentdataproject/Home

(i.e., series G). Funding is converted into millions of constant \$2009 using the GDP deflator. The mean duration of a funding round in Crunchbase is 1.4 years, which is taken to 1.5 years here. The share of equity transferred to the venture capitalist in each funding round is calculated as the ratio of VC funding in each round to the post-money valuation of the company after the VC investment. For each funding round the mean equity share across all deals is calculated. The vertical axis is the cumulated share of equity transferred to the venture capitalist.

- *Figure 6: The odds of success and failure by funding round and the value of an IPO by the duration of funding.* The underlying data source is Puri and Zarutskie (2012, Table VI.B, p. 2271). The success rate refers to firms that have an IPO or that are acquired by another firm. The acquisitions in Puri and Zarutskie (2012) are converted into successes by multiplying by 0.629. This is based on the fact that the cash multiple for acquisitions is 37.1 percent lower than for IPOs, as reported in Achleitner et al. (2012). In addition, the success and failure rates by funding round are obtained by interpolating the original annual data using a cubic spline to get a periodicity of 1.5 years. Puri and Zarutskie (2012, Table V) classify a firm "as having failed if it disappears from the LBD in its entirety." The value of an IPO, as a function of the duration of VC funding, derives from regression (2) in Table 10 (discussed in Section 15.3).

- *Figure 10: The cross-country relationship between the tax rate on VC activity and the VC investment-to-GDP ratio.* The source for the cross-country data is Henrekson and Sanandaji (2016, Table 1) .

- *Figure 12: Economic growth and VC investment.* VC investment and the growth rate of real GDP per capita are based on VentureXpert of Thomson ONE and the World Development Indicators of the World Bank, respectively.

15.2 Tables

- *Table 1: Top 30 VC-Backed Companies.* As in Figure 1, the list of VC-backed public companies is gathered by matching firm names in VentureXpert and CompuStat.

- *Table 2: VC- versus Non-VC-Backed Public Companies.* The VC-backed public companies are singled out by matching firm names in VentureXpert and CompuStat. Since the R&D-to-sales ratios and growth rates can be very volatile across firms, the top and bottom 5 percent of the outliers are trimmed in this regression. The results are robust to changing the trimming threshold (at the 1 percent versus 5 percent level).

- *Table 3: VC and Patenting, Firm-Level Regressions.* The VC-funded patentees are identified by matching firm names in VentureXpert and PatentsView.[16] The capital gain taxes are accessed from TAXSIM, an NBER tax simulation program.[17] In calculating the dependence on external finance, 30 percent of selling, general, and administrative expenses is taken as intangible investment. The industry levels of private- and federally-funded R&D are collected from the Business R&D and Innovation Survey by the National Science Foundation.[18] A truncation adjustment for citations is made following Bernstein (2015). The industry dummies in this regression are at the 2-digit SIC level.

- *Table 4: VC and Patenting, Industry-Level Regressions.* The product of the deregulation dummy and dependence on external finance is used as the IV for the cross term between VC funding and dependence on external finance. The industry panel is based on the 4-digit SIC. The industry dummies in this regression are at 2-digit SIC level.

- *Table 9: VC Investment and Growth, Cross-Country Regressions.* The full sample covers 37 economies between 1995 and 2014. As in Figure 12, VC investment is from VentureXpert and the GDP growth rate is from the World Development Indicators.

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[16] Source link of PatentsView: http://www.patentsview.org/download/.
[17] Source link of TAXSIM: http://users.nber.org/~taxsim/state-rates/.
[18] Source link of BRDIS: https://www.nsf.gov/statistics/srvyindustry/#tabs-2.

The Barro and Lee (2013) human capital index is a measure of educational attainment at the country level. The IVs are the median VC investment-to-GDP ratio (in natural logarithm) for each country between 1985 and 1994, and a dummy variable for legal origin (equal to 1 for common-law countries) à la Beck, Demirguc-Kunt, and Levine (2005).

15.3 Duration of VC Funding and the Value of an IPO

The relationship between the firm's value at an IPO and the number of years it received funding from the venture capitalist is examined using regression analysis. The regressions are based on public companies funded by venture capitalists between 1970 and 2015. These VC-backed companies are identified by matching firm names in CompuStat and VentureXpert. The dependent variable in the regressions is the natural logarithm of the market value of the firms at IPO (in $2009). A three-year average is used for market value because of the notorious volatility of share prices following an IPO. IPOs are excluded when they take more than 11 years for the firms to go public after receiving the first VC funding. This is for two reasons: (i) the sampling period is formulated to be consistent with the model where the maximum duration for each VC investment is 10.5 years, and (ii) only 4.5 percent of the observations occur after 11 years with the data being very noisy. The main explanatory variable is the number of years between the firm's first VC funding and the date of its IPO. The findings are shown in Table 10. The first coefficient in regression (2) is used in Figure 6 to plot the decline in the value of an IPO across successive funding rounds.

16 Theory Appendix

Proofs for Lemmas 2 and 4 are supplied in turn here. Lemma 2 establishes the existence of a balanced-growth path. Lemma 4 shows that solving the contract problem (P2) subject to a sequence of one-shot incentive constraints is equivalent to solving it subject to a single

Dependent variable	ln(Firm value at IPO, real)	
	1	2
years btw first VC funding and IPO	-0.0470***	-0.0385***
	(0.0161)	(0.0146)
firm age at IPO		-0.0246***
		(0.00495)
# of employees at IPO (log)		0.709***
		(0.0375)
year dummy for IPO	N	Y
industry effect	N	Y
Observations	1,042	1,006
R-squared	0.008	0.627

Table 10: Standard errors are in parentheses. ***, **, and * denote significance at the 1, 5 and 10 percent levels.

consolidated round-0 incentive constraint that allows for multi-shot deviations. This is proved using Lemma 3 as an intermediate step.

16.1 Balanced Growth

Lemma 2 *(Balanced Growth) There exists a balanced-growth path of the form outlined in Section 9.*

Proof. Suppose that $\{p_t, \sigma_t, \mu_t, \beta_t\}$ solves the old problem for x and $\mathbf{x}$. It will be shown that $\{\mathbf{g}_w p_t, \sigma_t, \mu_t, \beta_t\}$ solves the new one for $x' = \mathbf{g_x} x$ and $\mathbf{x'} = \mathbf{g_x x}$. First, observe that if $x' = \mathbf{g_x} x$ and $\mathbf{x'} = \mathbf{g_x x}$, then $I(x'; \mathbf{g_x^t x'}) = \mathbf{g}_w I(x; \mathbf{g_x^t x})$. This occurs because $T(x'; \mathbf{x'_t}) = \mathbf{g}_w T(x; \mathbf{x}_t)$. This can be seen from (P1) because x will rise by $\mathbf{g_x}$ and wages by $\mathbf{g}_w$. If $p'_t = \mathbf{g}_w p_t$, then it is immediate from the objective function in (P2) that $C(x'; \mathbf{x'}) = \mathbf{g}_w C(x; \mathbf{x})$. Now, consider the incentive constraint (5). At the conjectured solution, the left-hand side will inflate by the factor $\mathbf{g}_w$. So will the right-hand side because $D(\sigma'_t) - D(\tilde{\sigma}'_t) = \mathbf{g}_w[D(\sigma_t) - D(\tilde{\sigma}_t)]$, since all costs are specified as a function of w. Therefore, the new solution still satisfies the incentive constraint. Move now to the zero-profit constraint (6). Again, the left-hand side will inflate by the factor $\mathbf{g}_w$, since $p'_t = \mathbf{g}_w p_t$, $\phi'_t = \mathbf{g}_w \phi_t$, $D(\sigma'_t) = \mathbf{g}_w D(\sigma_t)$, $M_t(\mu'_t) = \mathbf{g}_w M_t(\mu_t)$,

61

$E(\beta'_t) = \mathbf{g}_w E(\beta_t)$, and $R(x'/\mathbf{x}') = \mathbf{g}_w R(x/\mathbf{x})$. This is trivially true for the right-hand side. Hence, the zero-profit constraint holds at the new allocations. It is easy to deduce from the right-hand side of (5) that the old solution for $\widetilde{\sigma}_t$ will still hold. This can be seen by using the above argument while noting that $D_1(\widetilde{\sigma}'_t) = \mathbf{g}_w D_1(\widetilde{\sigma}_t)$. To sum up, at the conjectured new solution, the objective function and the constraints all scale up by the same factor of proportionality $\mathbf{g}_w$. By cancelling out this factor of proportionality, the new problem reverts back to the old one. Likewise, it is easy to deduce that if x solves problem (P3) for $\mathbf{x}$, then $x' = \mathbf{g}_\mathbf{x} x$ solves it when $\mathbf{x}' = \mathbf{g}_\mathbf{x} \mathbf{x}$. The occurs because problem (P3) also scales up by the factor of proportionality $\mathbf{g}_w$. When $x/\mathbf{x}$ remains constant along a balanced-growth path, then the initial research cost of the project will rise at the same rate as wages, $\mathbf{g}_w$. Additionally, $V(\mathbf{x})$ will grow the same rate as wages, w, so from (7) it is apparent that $\mathfrak{e}$ will remain constant. ■

16.2   One-Shot Deviations versus Multi-Shot Deviations

This is an intermediate step toward solving Lemma 4. To this end, it will be shown that if the incentive constraint (5) holds for round t, when the entrepreneur has not deviated up to and including round $t-1$, then it will also hold when he follows some arbitrary path of deviations up to and including round $t-1$. Let α_t represent that the probability that a project is good at round t as defined by (4). These odds evolve recursively according to

$$\alpha_{t+1} = \frac{(1 - \sigma_t)\alpha_t}{(1 - \sigma_t)\alpha_t + (1 - \beta_{t+1})(1 - \alpha_t)},$$

where $\alpha_1 = \rho/[\rho + (1-\rho)(1-\beta_1)]$. For use in proving Lemma 3, note that α_{t+1} is increasing in α_t and decreasing in σ_t. This implies that if the entrepreneur deviates in round t, so that $\widetilde{\sigma}_t < \sigma_t$, he will be more optimistic about the future, as α_{t+1} will be higher. This increases the value of the α's for future rounds as well. With this notation, the round-t incentive

constraint (5) then reads

$$\alpha_t(1-\tau)\{\delta\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}$$

$$\geq (1-\mu_t)\max_{\widetilde{\sigma}_t}\Big(D(\sigma_t)-D(\widetilde{\sigma}_t)$$

$$+\alpha_t(1-\tau)\{\delta\widetilde{\sigma}_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\widetilde{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}\Big).$$

Lemma 3 *If the incentive constraint (5) holds for round t, when the entrepreneur has not deviated up to and including in round $t-1$, then it will also hold when he follows some arbitrary path of deviations up to and including in round $t-1$.*

Proof. Suppose that the entrepreneur deviates in some manner before round t. Let $\widehat{\alpha}_t$ be the prior associated with this path of deviation. Since the $\widetilde{\sigma}$'s will be less that than the σ's, it follows that $\widehat{\alpha}_t > \alpha_t$. Let $\widehat{\sigma}_t$ be the *optimal* round-t deviation associated with $\widehat{\alpha}_t$. Now,

$$\alpha_t(1-\tau)\{\delta\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}$$

$$\geq (1-\mu_t)\Big(D(\sigma_t)-D(\widehat{\sigma})$$

$$+\alpha_t(1-\tau)\{\delta\widehat{\sigma}_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}\Big),$$

because $\widetilde{\sigma}_t$ is maximal when the prior is α_t, while $\widehat{\sigma}_t$ is not. Next, replace α_t with $\widehat{\alpha}_t$ to get

$$\widehat{\alpha}_t\{\delta\sigma_t[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}$$

$$\geq (1-\mu_t)\Big(D(\sigma_t)-D(\widehat{\sigma}_t)$$

$$+\widehat{\alpha}_t\{\delta\widehat{\sigma}[I(x;\mathbf{g}_\mathbf{x}^t\mathbf{x})-p_t]+(1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x;\mathbf{g}_\mathbf{x}^i\mathbf{x})-p_i]\}\Big),$$

since $\widehat{\alpha}_t > \alpha_t$. Last, if the prior is $\widehat{\alpha}_t$, then $\widehat{\sigma}_t$ is maximal, so the above equation can be rewritten as

$$\widehat{\alpha}_t(1-\tau)\{\delta\sigma_t[I(x; \mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\sigma_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}$$

$$\geq (1-\mu_t)\max_{\widehat{\sigma}_t}\bigg(D(\sigma_t) - D(\widehat{\sigma}_t)$$

$$+ \widehat{\alpha}_t(1-\tau)\{\delta\widehat{\sigma}_t[I(x; \mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t] + (1-\widehat{\sigma}_t)\sum_{i=t+1}^{T}\Pi_{j=t+1}^{i-1}(1-\sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g}_\mathbf{x}^i\mathbf{x}) - p_i]\}\bigg).$$

Hence the round-t incentive constraint hold when for some arbitrary path of deviations up to and including in round $t-1$. ∎

16.3   The Consolidated Round-0 Incentive Constraint

The consolidated round-0 incentive constraint is

$$(1-\tau)\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x; \mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t]$$

$$\geq \max_{\{\widetilde{\sigma}_t\}_{t=1}^{T}}\{\sum_{t=1}^{T}\delta^{t-1}[\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j) + (1-\rho)\Pi_{j=1}^{t}(1-\beta_j)]$$

$$\times (1-\mu_t)[D(\sigma_t) - D(\widetilde{\sigma}_t)]$$

$$+ (1-\tau)\sum_{t=1}^{T}\rho\Pi_{j=1}^{t-1}(1-\widetilde{\sigma}_j)\delta^t\widetilde{\sigma}_t[I(x; \mathbf{g}_\mathbf{x}^t\mathbf{x}) - p_t]\}.$$

$$(17)$$

Lemma 4 *(Equivalence of contracts) A contract $\{\beta_t, \sigma_t, \mu_t, p_t\}$ solves problem (P2) subject to the sequence of one-shot incentive constraints (5) if and only if it solves (P2) subject to the consolidated round-0 incentive constraint (17).*

Proof (by contradiction).   (*Necessity*) Suppose that an allocation satisfies the one-shot incentive compatibility constraints (5) but that it violates the consolidated one (17). This

implies that at some round in the problem with the consolidated constraint it pays to deviate and pick a $\widetilde{\sigma}_t \neq \sigma_t$. Pick the last round of deviation (which may be T). It must be true that $\widetilde{\sigma}_t$ solves the maximization problem

$$
(1 - \mu_t) \max_{\widetilde{\sigma}_t} \Bigg(D(\sigma_t) - D(\widetilde{\sigma}_t)
$$

$$
+ \widehat{\alpha}_t(1 - \tau)\{\delta\widehat{\sigma}[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \widetilde{\sigma}_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\}\Bigg),
$$

where $\widehat{\alpha}_t$ is the prior associated with the path of σ's up to round $t - 1$, which may include previous deviations. But, from Lemma 3, this is less than the value of sticking with the contract or

$$
\widehat{\alpha}_t(1 - \tau)\{\delta\sigma_t[I(x; \mathbf{g_x^t x}) - p_t] + (1 - \sigma_t) \sum_{i=t+1}^{T} \Pi_{j=t+1}^{i-1}(1 - \sigma_j)\delta^{i+1-t}\sigma_i[I(x; \mathbf{g_x^i x}) - p_i]\},
$$

when the round-t one-shot incentive constraint (5) holds, as assumed. Thus, a contradiction emerges.

(*Sufficiency*) Suppose $\{\sigma_t\}_{t=1}^{T}$ satisfies the consolidated incentive constraint, but violates the one-shot incentive constraint at round k. Then, using (4) and (5), it follows that

$$
\rho\Pi_{j=1}^{k-1}(1-\sigma_j)\delta^{k-1}(1-\tau)\{\delta\sigma_k[I(x; \mathbf{g_x^k x})-p_k]+(1-\sigma_k) \sum_{t=k+1}^{T} \Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x; \mathbf{g_x^t x})-p_t]\}
$$

$$
= (1 - \tau) \sum_{t=k}^{T} \rho\Pi_{j=1}^{t-1}(1 - \sigma_j)\delta^t\sigma_t[I(x; \mathbf{g_x^t x}) - p_t]
$$

$$
< \delta^{k-1}(1 - \mu_k)\Bigg([\rho\Pi_{j=1}^{k-1}(1 - \sigma_j) + (1 - \rho)\Pi_{j=1}^{k}(1 - \beta_j)][D(\sigma_k) - D(\widetilde{\sigma}_k)]
$$

$$
+\rho\Pi_{j=1}^{k-1}(1-\sigma_j)(1-\tau)\{\delta\widetilde{\sigma}_k[I(x; \mathbf{g_x^k x})-p_k]+(1-\widetilde{\sigma}_k) \sum_{t=k+1}^{T} \Pi_{j=k+1}^{t-1}(1-\sigma_j)\delta^{t+1-k}\sigma_t[I(x; \mathbf{g_x^t x})-p_t]\}\Bigg).
$$

$$
(18)
$$

The left-hand side gives the payoff in the contract at the optimal solution from round k on, when using the consolidated incentive constraint, while the right-hand side represents the payoff from a one-shot deviation at round k.

Now the objective function for the contract can be written as

$$(1-\tau)\sum_{t=1}^{k-1}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g_x^t x})-p_t]+(1-\tau)\sum_{t=k}^{T}\rho\Pi_{j=1}^{t-1}(1-\sigma_j)\delta^t\sigma_t[I(x;\mathbf{g_x^t x})-p_t].$$

Evaluate this at the optimal solution for the contract when using (17) instead of (5). Next, in this objective function, replace the payoff from round k on, as represented by the left-hand side of (18), with the payoff from the one-shot deviation as given by the right-hand side. This deviation increases the value of the objective function for the entrepreneur under the contract with the time-0 incentive constraint, which contradicts its optimality. ∎