NBER WORKING PAPER SERIES

GLOBAL EFFECTIVE LOWER BOUND AND UNCONVENTIONAL MONETARY POLICY

Jing Cynthia Wu
Ji Zhang

Working Paper 24714
http://www.nber.org/papers/w24714

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2018

Global Effective Lower Bound and Unconventional Monetary Policy
Jing Cynthia Wu and Ji Zhang
NBER Working Paper No. 24714
June 2018
JEL No. E52,F00

ABSTRACT

In a standard open-economy New Keynesian model, the effective lower bound causes anomalies: output and terms of trade respond to a supply shock in the opposite direction compared to normal times. We introduce a tractable two-country model to accommodate for unconventional monetary policy. In our model, these anomalies disappear. We allow unconventional policy to be partially active and asymmetric between the countries. Empirically, we find the US, Euro area, and UK have implemented a considerable amount of unconventional monetary policy: the US follows the historical Taylor rule, whereas the others have done less compared to normal times.

Jing Cynthia Wu
University of Chicago
Booth School of Business
5807 South Woodlawn Avenue
Chicago, IL 60637
and NBER
jwu11@nd.edu

Ji Zhang
Tsinghua University
43 Chengfu Road
PBC School of Finance
Haidian District, Beijing, 100083
China
zhangji@pbcsf.tsinghua.edu.cn

1 Introduction

Since the Great Recession, many major central banks of developed economies have faced the effective lower bound (ELB) for their policy interest rates and resorted to unconventional monetary policy to provide further stimulus. In this extraordinary environment, how do we evaluate the role of unconventional monetary policy theoretically and empirically?

In a standard New Keynesian model (e.g., Eggertsson and Woodford (2003) for a closed economy and Cook and Devereux (2013a) for an open economy), the ELB yields to the classic liquidity trap; that is, the central bank cannot further reduce the policy rate, and monetary policy is completely absent. However, emerging empirical studies provide overwhelming evidence to demonstrate the effectiveness of unconventional monetary policy; see, for example, Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Bauer and Rudebusch (2014), and Wu and Xia (2016) for its domestic impact, and Neely (2015), Bauer and Neely (2014), Bowman et al. (2015), and Chen et al. (2016) for its global effects.

We propose a tractable two-country New Keynesian model that incorporates unconventional monetary policy into an otherwise standard model. We propose a Taylor (1993)-type policy rule to conveniently summarize both conventional and unconventional monetary policy. In earlier work by Wu and Zhang (2017), unconventional monetary policy follows the historical Taylor rule by construction. In this paper, we relax this assumption and allow unconventional policy to be potentially less effective, and two countries can implement them asymmetrically. Our new model nests the traditional model when monetary policy is absent at the ELB and the model in Wu and Zhang (2017) with fully active unconventional monetary policy.

During normal times, a negative supply shock from the home country leads to lower home output and terms of trade. In our model, if a sufficient amount of unconventional monetary policy is implemented, the same results apply for the ELB. On the contrary, the standard model implies a higher output and an appreciation of the terms of trade during a liquidity trap, and we will refer to these as anomalies.

The basic mechanism that leads to these anomalies consists of two channels. First, it transmits through inflation and the real interest rate, which works the same way as in a closed-economy macro model. A negative supply shock leads to higher inflation for home goods. At the ELB, the nominal rate does not move, which lowers the real rate. The lower real rate stimulates demand and hence the equilibrium output of the home country. In the open economy, international trade further amplifies this effect through improved terms of trade.

When we allow the two countries to implement their respective unconventional monetary policy asymmetrically, we find different results for the home and foreign economies. For the home country, its own policy matters most, whereas the foreign economy relies on both central banks. More active home or foreign policy is associated with higher welfare, and the most efficient case is obtained when both countries' unconventional policies follow their historical policy rules.

We explore alternative model and parameter specifications for robustness. The anomalies that the ELB makes home inflation and output larger than normal are robust when we exclude international trade, replace the negative supply shock with a positive demand shock, or implement a fixed exchange rate or CPI-based Taylor rule. We also assess across alternative parameter values the robustness of the anomalies that home output and terms of trade move in the opposite direction at the ELB compared to normal times. We find they are not sensitive to structural parameters, including the Frisch elasticity of labor supply, elasticity of intertemporal substitution, and home bias. Results vary more over parameters governing the preference shock, which creates the ELB environment. We find as long as the ELB lasts for several quarters, the anomalies hold.

Finally, we seek empirical evidence for unconventional monetary policy in the United States, Euro area and United Kingdom. First, we test model implications by comparing how output responds to a supply shock in a structural vector autoregression (VAR) between normal times and the ELB. We find that for all three countries and regions, output decreases with a negative shock to total-factor productivity (TFP) regardless of normal times or the ELB. This result is in contrast to the anomaly presented in the standard New Keynesian model, and suggests that the three central banks have implemented a considerable amount of unconventional monetary policy.

Next, we quantify unconventional monetary policy using the Taylor rule by comparing what has been done with what should have been done according to the historical rule. Both methods point to the same conclusion, whether it's qualitatively from the VAR or quantitatively from the Taylor rule: the United States operates its unconventional monetary policy similarly to the historical Taylor rule, whereas the Euro area and United Kingdom have done less unconventionally compared to what they would normally have done.

The rest of the paper after a brief literature review proceeds as follows. Section 2 describes the theoretical model, and we discuss model implications with and without unconventional monetary policy in Section 3. Section 4 assesses empirical evidence for unconventional monetary policy, and Section 5 concludes.

Literature Our paper is related to several recent papers that investigate policy responses in the global low interest rate environment. Cook and Devereux (2013a) analyze the interaction between monetary and fiscal policy in a global liquidity trap with a two-country New Keynesian model. Cook and Devereux (2013b) compare a currency union to a system with a flexible exchange rate. Fujiwara et al. (2013) focus on the optimal monetary policy. Eggertsson et al. (2016) consider the effectiveness of monetary and fiscal policy during the global secular stagnation, using an open-economy overlapping generations model.

Our empirical analysis of the Taylor rule is related to Hakkio and Kahn (2014). The main difference is we propose alternative ways to compute the quantity for what should have been done, and our methods are less subject to accumulating and compounding errors and uncertainty over time. Our structural VAR results are consistent with Garín et al. (2016)

and Debortoli et al. (2016). The difference is the literature has focused on the United States, and our analysis encompasses the United States, Euro area, and United Kingdom.

2 Model

This section describes a two-country open-economy New Keynesian model. Many model ingredients are standard and similar to Clarida et al. (2002) and Cook and Devereux (2013a). The main difference in our model is we do not restrict our attention to the standard setup for the effective lower bound, that is, the nominal interest rate is zero, and the monetary authority provides no additional stimulus. Instead, we allow a potential role for unconventional monetary policy, which could be completely inactive, fully active, and anywhere in between. See the setup in Subsection 2.5, and economic implications are discussed in Section 3.

The two countries, home and foreign, are the same size and symmetric. Households consume both home and foreign goods with some preference for the domestically produced products. Firms hire labor to produce differentiated goods, and face Calvo (1983)-type price rigidity. The wage paid to workers is determined in a perfectly competitive labor market without any frictions. Complete markets allow perfect international risk sharing. Monetary policy follows a Taylor (1993) rule.

For the most part, we describe the home economy, and the foreign optimization problems are symmetric, which are denoted by an asterisk superscript *. H stands for home-produced goods, and F is foreign goods.

2.1 Households

2.1.1 Optimization problem

Households maximize their utility over consumption and hours worked:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Xi_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{2.1}$$

where $\mathbb{E}$ is the expectation operator and β is the discount factor. Ξ_t is the preference shifter, and its log $\xi_t = \log(\Xi_t)$ follows $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}$, $\varepsilon_{\xi t} \sim N(0, \sigma_\xi^2)$. C_t is consumption and N_t is labor supply. σ is the elasticity of intertemporal substitution and ϕ is the Frisch labor supply elasticity.

Their budget constraint is

$$P_t C_t + \mathbb{E}_t[Q_{t,t+1} B_{t+1}] = B_t + W_t N_t, \tag{2.2}$$

where P_t is the consumer price index (CPI) and W_t is wage. B_{t+1} is the period $t+1$ random payoff of the asset bought at t for B_t, and $Q_{t,t+1}$ is the stochastic discount factor between t and $t+1$.

Households' Euler equation is

$$\beta \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Xi_t} \left(\frac{C_t}{C_{t+1}} \right)^\sigma \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t[Q_{t,t+1}] = \frac{1}{R_t}, \tag{2.3}$$

where $R_t = 1/\mathbb{E}_t[Q_{t,t+1}]$ is the short-term nominal interest rate. Their first-order condition for labor supply satisfies

$$\frac{W_t}{P_t C_t^\sigma} = N_t^\phi. \tag{2.4}$$

2.1.2 Consumption allocation

Households consume both home (H) and foreign (F) goods:

$$C_t = \Phi C_{Ht}^{\nu/2} C_{Ft}^{1-\nu/2}, \tag{2.5}$$

where $\Phi = \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \left(1 - \frac{\nu}{2}\right)^{1-\frac{\nu}{2}}$. Households have a preference bias for domestic goods with $1 < \nu \leq 2$. They allocate $\nu/2$ share of their expenditure to domestic goods, and $1 - \nu/2$ to imported goods, and the demand curves are

$$C_{Ht} = \frac{\nu}{2}\frac{P_t}{P_{Ht}}C_t \tag{2.6}$$

$$C_{Ft} = \left(1-\frac{\nu}{2}\right)\frac{P_t}{P_{Ft}}C_t, \tag{2.7}$$

and the CPI aggregates over prices for homes goods and foreign goods:

$$P_t = P_{Ht}^{\nu/2}P_{Ft}^{1-\nu/2}. \tag{2.8}$$

C_{Ht} is a constant elasticity of substitution (CES) aggregator over differentiated home goods:

$$C_{Ht} = \left(\int_0^1 C_{Ht}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}, \tag{2.9}$$

where the elasticity of substitution $\theta > 1$. The demand curve for each differentiated good i is

$$\frac{C_{Ht}(i)}{C_{Ht}} = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta},$$

where the producer price index (PPI) is

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}}.$$

2.2 Inflation, terms of trade, and exchange rate

Inflation The CPI and PPI inflations are

$$\Pi_t = \frac{P_t}{P_{t-1}} \tag{2.10}$$

$$\Pi_{Ht} = \frac{P_{Ht}}{P_{H,t-1}}. \tag{2.11}$$

Terms of trade The terms of trade are defined as the price of foreign goods relative to domestic goods:

$$\mathcal{T}_t = \frac{P_{Ft}}{P_{Ht}}. \tag{2.12}$$

Exchange rate The law of one price holds

$$\mathcal{E}_t = \frac{P_{Ht}}{P^*_{Ht}} = \frac{P_{Ft}}{P^*_{Ft}}, \tag{2.13}$$

where $\mathcal{E}_t$ is the nominal exchange rate. Our baseline model has a flexible exchange rate. The exchange rate and the terms of trade are related by

$$\mathcal{T}_t = \mathcal{E}_t \frac{P^*_{Ft}}{P_{Ht}}. \tag{2.14}$$

International risk sharing implies

$$\frac{\Xi_t}{C_t^\sigma} = \frac{\Xi_t^*}{(C_t^*)^\sigma} \frac{P_t}{\mathcal{E}_t P_t^*} = \frac{\Xi_t^*}{(C_t^*)^\sigma} \mathcal{T}_t^{1-\nu}. \tag{2.15}$$

Relation to interest rates In the log-linear form, the exchange rate can be written as the domestic and foreign nominal interest rate differential:

$$\mathbb{E}_t \left[\Delta e_{t+1} \right] = r_t - r_t^*, \tag{2.16}$$

where lowercase letters denote logs $r_t = \log(R_t)$ and $e_t = \log(\mathcal{E}_t)$, and * indicates the foreign economy.

The terms of trade can be expressed in terms of the real interest rate differential:

$$\tau_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} (rr^*_{t+k} - rr_{t+k}) \right], \tag{2.17}$$

where $\tau_t = \log(\mathcal{T}_t), \pi_{Ht} = \log(\Pi_{Ht})$, and $rr_t = r_t - \mathbb{E}_t[\pi_{H,t+1}]$ is the home real interest rate. Derivation details can be found in Appendix A.

2.3 Firms

There is a continuum of firms $i \in [0, 1]$ that hire labor and produce differentiated goods with the production function:

$$Y_t(i) = A_t N_t(i), \tag{2.18}$$

where A_t is the exogenous technology process and it follows $\log(A_t) - \log(A) = \rho_A[\log(A_{t-1}) - \log(A)] + \varepsilon_{at}$, where $\log(A)$ is the steady-state value and $\varepsilon_{at} \sim N(0, \sigma_a^2)$. The real marginal cost is

$$MC_t = \frac{(1-g)W_t}{A_t P_{Ht}}, \tag{2.19}$$

where g is the wage subsidy for firms to ensure efficient output level at the steady state.

Firms set prices for differentiated goods in the Calvo fashion. A firm can reset its price with probability $1 - \kappa$ in each period. The reset price satisfies

$$\tilde{P}_{Ht} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k} \kappa^k (1-g) W_{t+k} P_{t+k}^{\theta} Y_{t+k} / A_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k} \kappa^k P_{t+k}^{\theta} Y_{t+k}}, \tag{2.20}$$

where the stochastic discount factor is $Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+2} \ldots Q_{t+k-1,t+k}$. $Y_t = \frac{\int_0^1 Y_t(i) di}{\int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di}$ is the aggregate output and and $Y_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} Y_t$. Firms keep prices constant when they cannot reoptimize. Finally, the PPI evolves according to

$$P_{Ht} = \left[(1-\kappa)\tilde{P}_{Ht}^{1-\theta} + \kappa P_{H,t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.21}$$

2.4 Market clearing and welfare

The goods market-clearing condition is

$$Y_t = C_{Ht} + C^*_{Ht}. \tag{2.22}$$

The labor market clears when

$$N_t = \int_0^1 N_t(i)di. \tag{2.23}$$

Welfare W is defined as the second-order approximation of households' lifetime utility. Adding two countries together, the world welfare is

$$W^W = W + W^*. \tag{2.24}$$

2.5 Monetary policy and the effective lower bound

The conventional monetary policy follows a Taylor interest-rate rule:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1 - \rho_s) \left[\phi_\pi \hat{\pi}_{Ht} + \phi_y x_t\right], \tag{2.25}$$

where hatted variables are log deviations from the steady states $\hat{s}_t = s_t - s$ and $\hat{\pi}_{Ht} = \pi_{Ht} - \pi$, and $x_t = y_t - y^n_t$ is the output gap. $y_t = \log(Y_t)$, $s_t = \log(S_t)$, $s = \log(S)$, $\pi = \log(\Pi)$, and S and Π are the steady-state nominal interest rate and inflation. $y^n_t = \log(Y^n_t)$ is the natural level of output, or the equilibrium output under flexible prices when $\kappa = 0$; see more details in Appendix A.1. ρ_s captures the persistence of the interest-rate rule, and ϕ_π and ϕ_y are the sensitivities of the nominal interest rate to inflation and output, respectively. During normal times, when $s_t \geq 0$, $r_t = s_t$.

Effective lower bound and unconventional monetary policy When the effective lower bound binds $s_t < 0$,[1] the nominal interest rate relevant for the economy is

$$r_t = \lambda s_t. \tag{2.26}$$

The case $\lambda = 0$, $r_t = 0$ corresponds to the ELB in the standard New Keynesian model, where the nominal interest rate is stuck at zero, and the central bank's unconventional monetary policy fails to intervene with the economy.

However, a growing literature argues that unconventional monetary policy has a stimulative effect on the economy that is similar to the effect of conventional policy, which implies $\lambda = 1$; for example, see Wu and Xia (2016), Wu and Zhang (2017), Mouabbi and Sahuc (2017), and Debortoli et al. (2016).

Note (2.26) is a tractable framework to summarize all unconventional monetary policy. Wu and Zhang (2017) theorize how unconventional monetary policy such as QE can be conveniently mapped into a negative shadow interest rate that follows the same Taylor rule, making conventional and unconventional monetary policy comparable.

In this paper, we do not impose $\lambda = 0$ or $\lambda = 1$. Rather, we explore all the possibilities along $\lambda \in [0, 1]$. Quantitative analyses of the theoretical model are in Section 3, and empirical results follow in Section 4.

3 Anomalies at the ELB and unconventional monetary policy

This section first discusses analytically and quantitatively the anomalies at the ELB artificially created by the standard New Keynesian model, which does not capture any effort by

[1] For simplicity, we take 0 as the lower bound, and hence the ELB becomes the zero lower bound. In practice, the lower bound does not necessarily have to be zero (see Wu and Xia (2016)) or a constant (see Wu and Xia (2017)).

the central bank's unconventional monetary policy; that is, $\lambda = 0$ in (2.26). By contrast, we demonstrate these anomalies disappear once unconventional monetary policy is introduced in our model. Next, we relax the model assumptions in two steps. First, we allow different degrees of activeness for unconventional monetary policy $\lambda \in [0, 1]$. Second, we further relax $\lambda = \lambda^*$ and allow the two countries to implement unconventional monetary policy differently, and study their interactions. Finally, we consider several alternative model and parameter specifications.

3.1 Anomalies at the ELB

This section presents the anomalies at the ELB: when a negative supply shock hits the economy, output and terms of trade increase, which is the opposite direction from normal times. These anomalies disappear when unconventional monetary policy is implemented. We first derive some analytical results in a simplified setting to gain some intuition, and then relax the simplifying assumptions and present quantitative results.

3.1.1 Analytical results

In this section, we impose some simplifying restrictions to derive analytical properties to provide some intuition: $\rho_s = 0$ and $\xi_t = 0 \forall t$, so that for any variable z_t, we can write $\mathbb{E}_t[z_{t+1}] = \rho_a z_t$. The analytical analysis also imposes $\phi_y = 0$ for simplicity. We create the ELB environment with an interest-rate peg at the steady state by modifying (2.26) to $\hat{r}_t = \lambda \hat{s}_t$ and $\lambda = 0$. We find the solution that solves for any generic λ first and then impose $\lambda = 0$ for ELB, and ignore other potential equilibria that only arise at the ELB. We will relax all these assumptions in the quantitative Section 3.1.2.

When a supply shock occurs, the inflation differential, output differential, and terms of trade move as functions of technology as follows:

$$\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} = -2\Theta(1+\phi)(1-\rho_a)\sigma_0\Lambda_a\hat{a}_t \quad (3.1)$$

$$\hat{y}_t - \hat{y}^*_t = \Theta(1+\phi)(\lambda\phi_\pi - \rho_a)(D+1)\Lambda_a\hat{a}_t \quad (3.2)$$

$$\hat{\tau}_t = \Theta(1+\phi)(\lambda\phi_\pi - \rho_a)\frac{\sigma(D+1)}{D}\Lambda_a\hat{a}_t, \quad (3.3)$$

where $\hat{a}_t = \log(A_t) - \log(A)$, $\hat{y}_t = y_t - y$, $\hat{\tau}_t = \tau_t - \tau$, and $y = \log(Y)$ and $\tau = \log(\mathcal{T}_t)$ are the steady-state values. $\Theta = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa} > 0$, $\Lambda_a = \frac{1}{\Theta(\sigma/D+\phi)(\lambda\phi_\pi-\rho_a)(D+1)+2\sigma_0(1-\rho_a)(1-\beta\rho_a)}$, $D = [(\nu-1)^2 + \sigma\nu(2-\nu)] > 0$, $\sigma_0 = \sigma - (1-\nu/2)(\sigma-1)\nu\sigma/D > 0$. These equations lead to the following proposition:

Proposition 1 *If $\phi_\pi > 1$ and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} > 0$, and*

- *$\hat{y}_t - \hat{y}^*_t < 0$, $\hat{\tau}_t < 0$ when $\lambda = 1$.*
- *$\hat{y}_t - \hat{y}^*_t > 0$, $\hat{\tau}_t > 0$ when $\lambda = 0$.*

Proof: See Appendix B.1.

The contrast between the two cases in Proposition 1 illustrates the anomalies. To demonstrate intuition, we ignore the home supply shock's foreign effect for now, which we will see is small in the quantitative section. During normal times $\lambda = 1$, a negative home TPF shock lowers the domestic output and terms of trade when the monetary policy obeys the Taylor principal $\phi_\pi > 1$. By contrast, when the ELB binds and the central bank is completely out of the picture, the same shock stimulates its own economy by raising equilibrium output, and increases the terms of trade. In our setting, unconventional monetary policy in (2.25) and (2.26) works the same as the conventional Taylor rule when it is fully active with $\lambda = 1$. Hence, results with unconventional monetary policy are identical to normal times. $\Lambda_a > 0$ is imposed to guarantee inflation moves in the same direction whether $\lambda = 0$ or 1. It is always satisfied for $\lambda = 1$, and when $\lambda = 0$, it is guaranteed by $0 < \rho_a < \bar{\rho}_a$, where the bound is defined in Appendix B.1.

Proposition 1 makes statements about inflation and output differentials, which we then interpret as home quantities by approximating changes to the foreign economy to zero. To see how the home economy moves without approximation, we'll study the special case of $\sigma = 1$ or $\nu = 2$, in which the home shock does not move across the border. The case $\nu = 2$ corresponds to complete home bias and hence no trade, whereas when $\sigma = 1$, wealth and substitution effects in the foreign economy are completely canceled out.

Corollary 1 *If* $(\sigma - 1)(\nu - 2) = 0$, $\phi_\pi > 1$, *and* $\Lambda_a > 0$, $\hat{a}_t < 0$ *implies* $\hat{\pi}^*_{Ft} = 0$, $\hat{y}^*_t = 0$, $\hat{\pi}_{Ht} > 0$, *and*

- $\hat{y}_t < 0$, $\hat{\tau}_t < 0$ *when* $\lambda = 1$.
- $\hat{y}_t > 0$, $\hat{\tau}_t > 0$ *when* $\lambda = 0$.

Proof: See Appendix B.2.

Corollary 1 illustrates similar anomalies to Proposition 1. The difference is when $\sigma = 1$ or $\nu = 2$, the foreign economy does not move in response to the home TFP shock. Hence, we can make precise statements about the home economy.

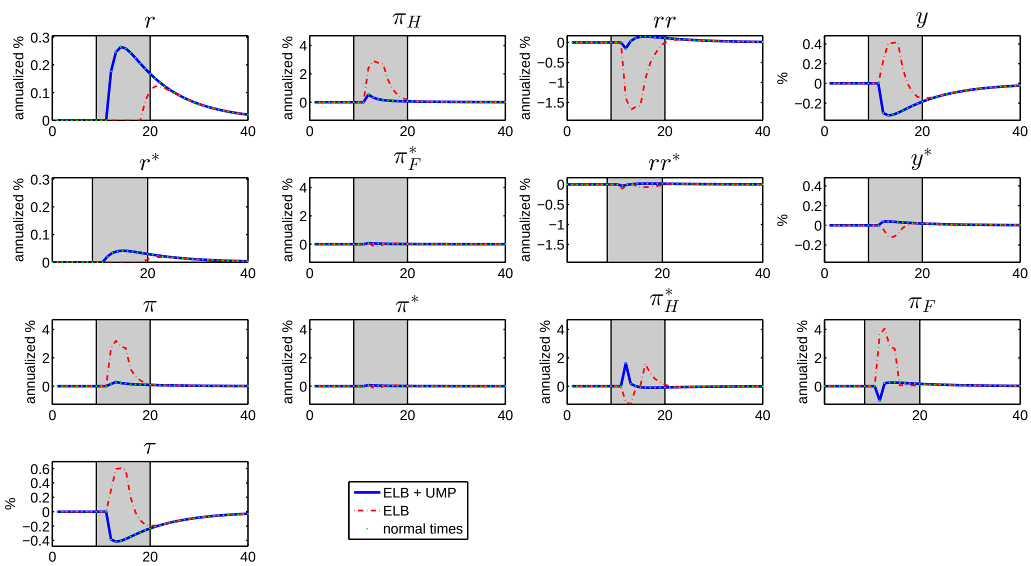
Next, we study how international trade contributes to the anomalies at the ELB in Proposition 1. We compare the case with international trade $\nu < 2$ with the no-trade case $\nu = 2$.

Proposition 2 *If* $\lambda = 0$, $1 < \sigma < \phi$ *and* $\Lambda_a > 0$, $\hat{a}_t < 0$ *implies* $\hat{y}_t - \hat{y}^*_t \geq (\hat{y}_t - \hat{y}^*_t)\,|_{\nu=2} > 0$, *and* $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$.

Proof: See Appendix B.3.

With some mild condition between σ and ϕ, international trade amplifies the impact of the TFP shock on output and terms of trade, which makes the anomalies even more prominent.

Figure 1: **TFP shock in home country**



Notes: A negative technological shock of -0.5% (2 standard deviations) happens in the home country in period 12. To create the ELB environment in the blue and red dashed lines, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23% (about 2/3 of a standard deviation in each period). We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The blue solid lines represent the case in which fully active unconventional monetary policy is implemented. The red dashed lines represent the case in which, when the policy rate is bounded by zero, no unconventional monetary policy is implemented. The green dotted lines represent normal times with the standard Taylor rule, and the only shock that hits the economy is the TFP shock. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflation, percentage changes relative to the steady states for output and terms of trade.

3.1.2 Quantitative illustration

Setup for quantitative analysis We set up a quantitative environment here that we will use hereafter, where we relax all the assumptions imposed in Section 3.1.1. We study economies' responses to a negative TFP shock, which serves as a supply shock. We create the ELB with a sequence of preference shocks. Unlike in the analytical section, we do not assume the ELB will bind forever. Instead, we use the occasionally binding method of Guerrieri and Iacoviello (2015) to solve the model. Model details are in Appendix A, and details for calibration and solution method are in Appendix C.

Results Figure 1 plots impulse response functions of economic quantities to a negative TFP shock in the home country. Green dots are normal times. Red dashed lines represent that the ELB prevails in both countries. The blue lines plot what happens when both countries implement unconventional monetary policy following the historical Taylor rule, that is, $\lambda = \lambda^* = 1$.

Figure 1 illustrates the same anomalies as we discussed in Section 3.1.1. In response to the negative supply shock, output and terms of trade decrease during normal times or with unconventional monetary policy, whereas they increase when the ELB is binding and central banks are absent. Additionally, we also see a contrast for foreign output, albeit in a smaller scale given the origin of the shock.

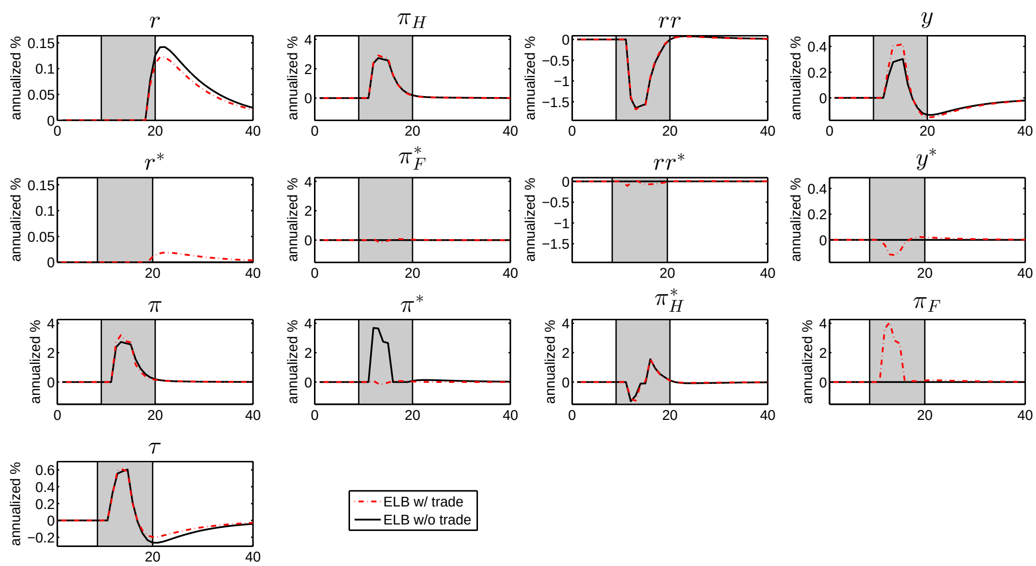
Next, we calculate welfare, which, unlike variables in Figure 1, are non-linear objects. Hence, we compute the total welfare in the presence of all the shocks. We find the case with ELB and no policy intervention suffers from the largest welfare losses for both the home and foreign economies. The losses become much smaller when unconventional monetary policy is fully active. Note that normal times have slightly higher welfare compared to the case with unconventional monetary policy, because the effective lower bound is created by preference shocks, which introduce some inefficiency.

3.1.3 Mechanism

The basic mechanism that leads to these results consists of two channels. First, it transmits through inflation and the real interest rate, which works the same as in a closed-economy macro model; see Wu and Zhang (2017), for example. A negative supply shock leads to a higher inflation for home goods. During normal times or with unconventional monetary policy, the nominal interest rate increases as a response, which leads to a higher real interest rate. However, at the ELB, the nominal rate does not move, which lowers the real rate. The lower real rate stimulates the demand and hence equilibrium output in the home country.

The open-economy model contains an additional international channel. To illustrate this

Figure 2: **Trade vs. no trade at the ELB**

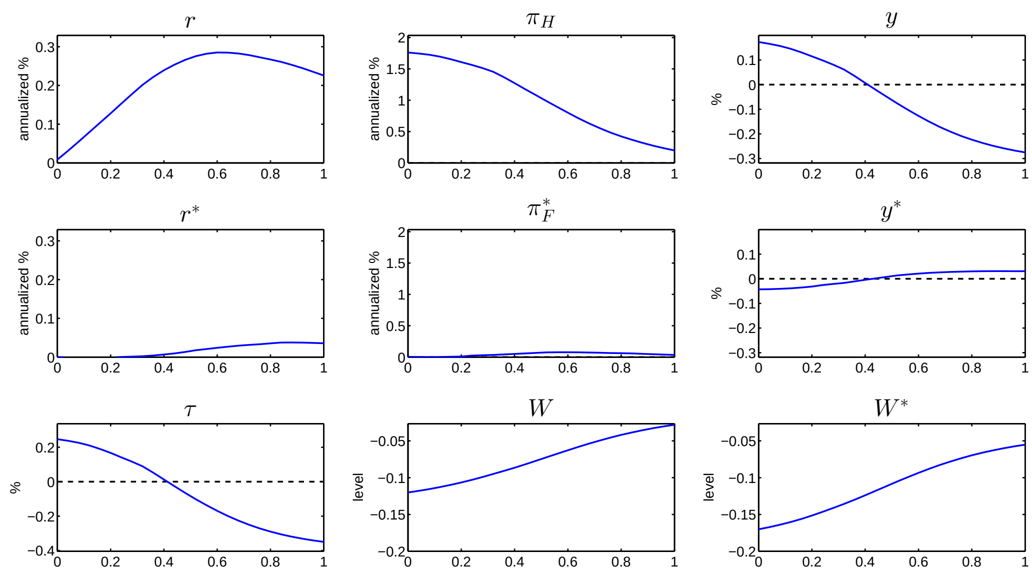


Notes: A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The red dashed lines represent the case with trade, and the black solid lines represent the case without trade $\nu = 2$. In both cases, the policy rate is bounded by zero and no unconventional monetary policy is implemented. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade.

channel, we plot in Figure 2 the ELB cases with and without international trade. The red lines are identical to the red lines in Figure 1. The case without trade is in black, where $\nu = 2$.

Without trade, the foreign economy does not react to the home shock, which is consistent with Corollary 1. International trade brings this shock across the border into the foreign economy, which in turn further amplifies its effect on the home output: The home country is expected to have a lower real interest rate compared to the foreign country, which improves the terms of trade through (2.17). Higher terms of trade imply that home goods are cheap compared to foreign goods, and therefore households shift their demand toward home goods. Hence, international trade reduces foreign output and further increases home production. This result is consistent with Proposition 2.

Figure 3: **Varying degrees of activeness of unconventional monetary policy**



Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted life-time welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

3.2 Partially active unconventional monetary policy

We have studied the limiting cases in which unconventional monetary policy is either completely absent $\lambda = 0$ or fully active $\lambda = 1$. In this section, we explore all possibilities along $\lambda, \lambda^* \in [0, 1]$ and allow unconventional monetary policy to be partially active. Section 3.2.1 imposes $\lambda = \lambda^*$ that both central banks' interventions are equally active, and we turn to the asymmetric case in Section 3.2.2.

3.2.1 Symmetric case

Figure 3 summarizes the impulse response of each economic variable to the TFP shock to a one-dimensional object and plots it as a function of λ, which is the same as λ^*. For the first seven variables, we plot the average response during the ELB. For $\lambda = 0$ ($\lambda = 1$), they are equal to the average values of the red (blue) lines from period 12 to 19 (20) in Figure 1. Both the home output and terms of trade decrease from positive to negative as unconventional monetary policy becomes more active, whereas the foreign output increases from negative to positive. Life-time welfare, W and W^*, increases when unconventional monetary policy becomes more active, and fully active unconventional monetary policy is the closest to being efficient.

Interestingly, the home country's nominal interest rate does not increase monotonically with λ. Combining (2.25) and (2.26), the nominal rate depends on the products $\lambda\pi_{Ht}$ and λy_t. While λ increases, both inflation and output decrease. For small λ, when λ increases, r_t increases. At some point, the rate of decrease in π_{Ht} and y_t overweights the increase of λ, and hence r_t decreases.

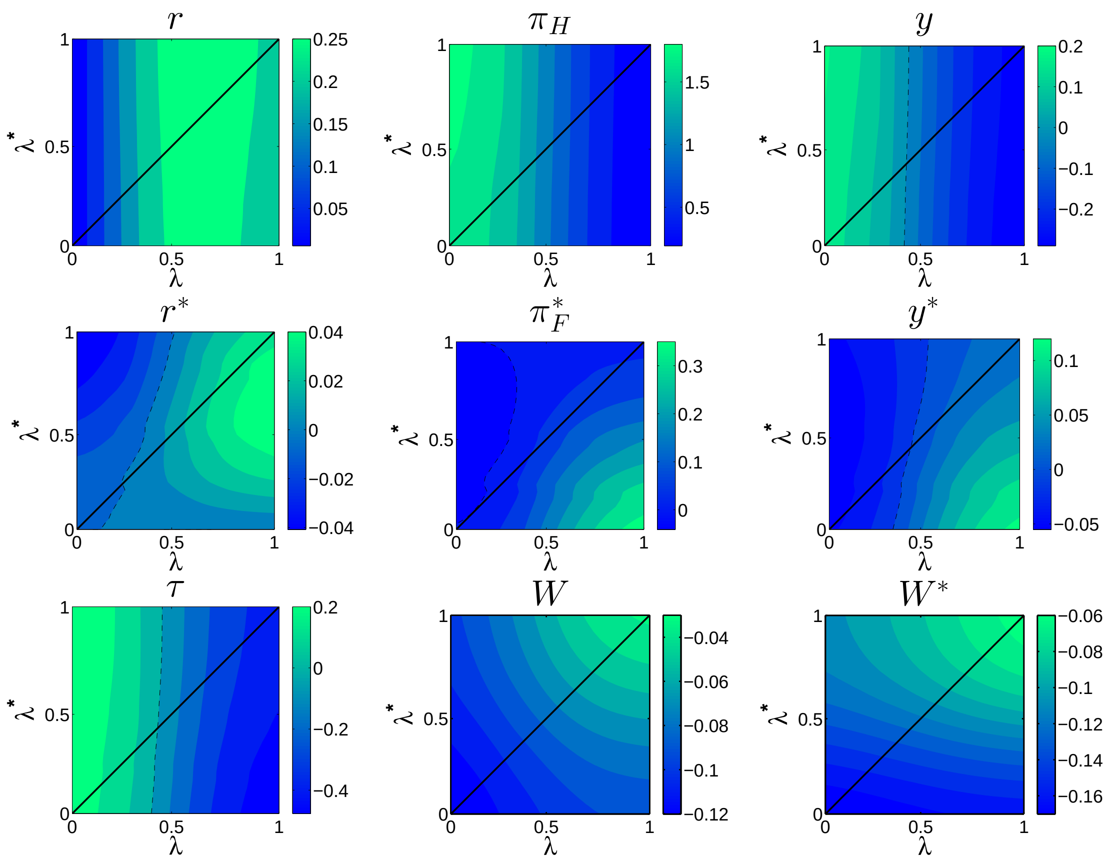
3.2.2 Asymmetric case

Next, we further relax the assumption $\lambda = \lambda^*$ and allow two countries to operate their unconventional monetary policy differently, and study their interactions. Figure 4 plots summary responses, defined in Section 3.2.1, to the TFP shock as functions of λ and λ^*. Different colors represent different values for economic quantities, where light green (dark blue) represents higher (lower) values. The 45-degree lines correspond to the symmetric case $\lambda = \lambda^*$ in Figure 3.

For the home country, mainly its own policy matters: the more active its central bank is in implementing unconventional monetary policy, the lower its output and inflation are. It is similar for the terms of trade: a higher λ is associated with smaller terms of trade.

The foreign economy, as well as welfare, relies on both central banks. A more active home unconventional policy or a less active foreign policy is associated with higher foreign inflation and output. For welfare, more active home or foreign policy is associated with higher welfare. The most efficient case happens when both countries' policies are fully active.

Figure 4: **Asymmetric unconventional monetary policy**

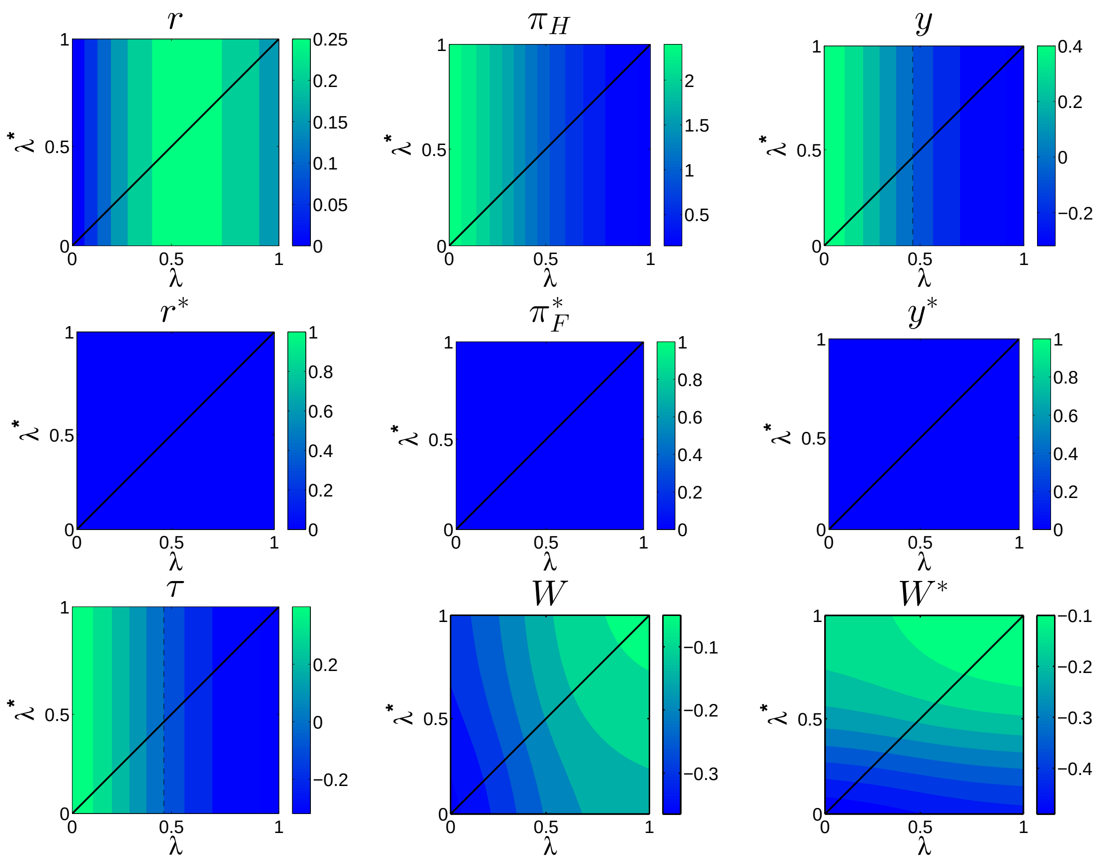


Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted life-time welfare. X-axis: λ; Y-axis: λ^*. The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output and terms of trade, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours.

3.3 Alternative specifications

This section explores alternative specifications and serves as a robustness check. Section 3.3.1 excludes international trade. Section 3.3.2 inspects alternative parameter spaces. Section 3.3.3 studies a demand shock. Sections 3.3.4 and 3.3.5 assess alternative monetary policy rules.

Figure 5: **No-trade case**

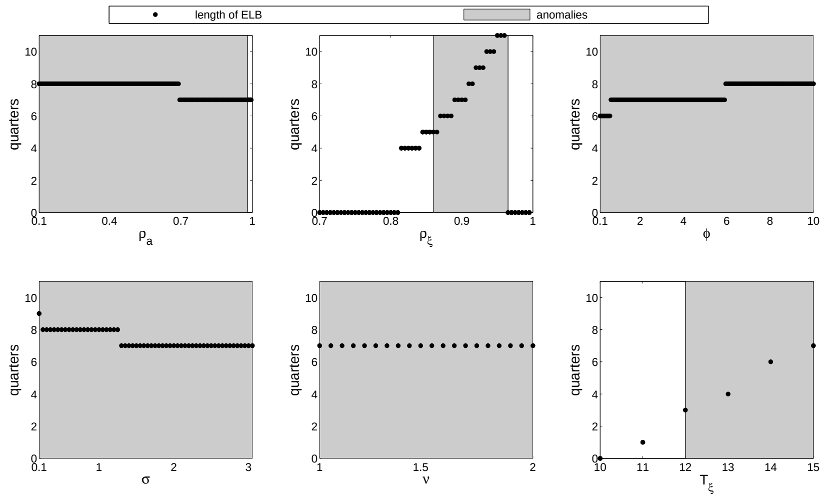


Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted life-time welfare. X-axis: λ; Y-axis: λ^*. The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output and terms of trade, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours. The no-trade case is created by $\nu = 2$.

3.3.1 No-trade case

Figure 5 plots the summary responses to the TFP shock as functions of both λ and λ^* for the case with no international trade, which is instrumented by $\nu = 2$. Unlike in Figure 4, the home economic indicators and terms of trade only move with the home policy indicator λ. The foreign economy in the second row does not move regardless of monetary policy. Welfare, on the other hand, still depends on monetary policies of both countries.

Figure 6: **Anomalies with alternative parameter values**



Notes: X-axis: ρ_a in the top left panel, ρ_ξ in the top middle panel, ϕ in the top right panel, σ in the bottom left panel, ν in the bottom middle panel, and T_ξ in the bottom right panel. Y-axis: time in quarters. Black dots: the number of ELB periods after the TFP shock. Gray shades: anomalies exist; white areas: anomalies do not exist.

3.3.2 Alternative parameter space

This section assesses the robustness of anomalies discussed in Subsection 3.1 across alternative parameter values, where we define anomalies when the maximum response of y and τ are positive.[2]

Figure 6 illustrates the existence of anomalies when we vary the persistence of the TFP dynamics ρ_a, the persistence of the preference shifter ρ_ξ, Frisch elasticity of labor supply ϕ, elasticity of intertemporal substitution σ, home bias ν, and the length of preference shocks T_ξ, one at a time, and set other parameters as in the baseline calibration. Gray areas mark that anomalies exist, whereas white areas correspond to the parameter space where anomalies do not appear.

The anomalies are not sensitive to structural parameters $\rho_a, \phi, \sigma, \nu$: they exist as long as $\rho_a < 0.98$. This finding is consistent with the condition $0 < \rho_a < \bar{\rho}_a$ in Proposition 1 that guarantees $\Lambda_a > 0$. They always exist for all $\phi \in [0.1, 5]$, $\sigma \in [0.1, 3]$, and $\nu \in [1, 2]$.

Results vary more over parameters related to the preference shock. The gray shades

[2]Results for an alternative definition, the average response of y or τ being positive, are very similar.

correspond to $0.86 \leq \rho_\xi \leq 0.9650$ or $T_\xi \geq 12$. Fundamentally, whether anomalies exist depends on how long the ELB lasts, which varies substantially over ρ_ξ and T_ξ. When ρ_ξ is too small or too large or when T_ξ is too small, the number of ELB periods is not large enough to generate anomalies. In the case of ρ_ξ (T_ξ), anomalies are supported if ELB lasts six (three) quarters or longer.

3.3.3 Demand shock

We have presented our main results with a supply shock. In this section, we illustrate what happens when a demand shock occurs. To derive analytical results, we impose the same simplifying restrictions as in Section 3.1.1. To obtain implications for the home economy, we further impose $\nu = 2$ and express the economies as follows:

$$\hat{y}_t = (1 - \rho_\xi)(1 - \beta\rho_\xi)\Lambda_\xi\hat{\xi}_t \tag{3.4}$$

$$\hat{\pi}_{Ht} = \Theta(\sigma + \phi)(1 - \rho_\xi)\Lambda_\xi\hat{\xi}_t \tag{3.5}$$

$$\hat{\tau}_t = -\Theta(\sigma + \phi)(\lambda\phi_\pi - \rho_\xi)\Lambda_\xi\hat{\xi}_t \tag{3.6}$$

$$\hat{y}^*_t = \hat{\pi}^*_{Ft} = 0, \tag{3.7}$$

where $\hat{\xi}_t = \xi_t - \xi$ and $\Lambda_\xi = \frac{1}{\Theta(\sigma+\phi)(\lambda\phi_\pi-\rho_\xi)+\sigma(1-\rho_\xi)(1-\beta\rho_\xi)}$. Hence,

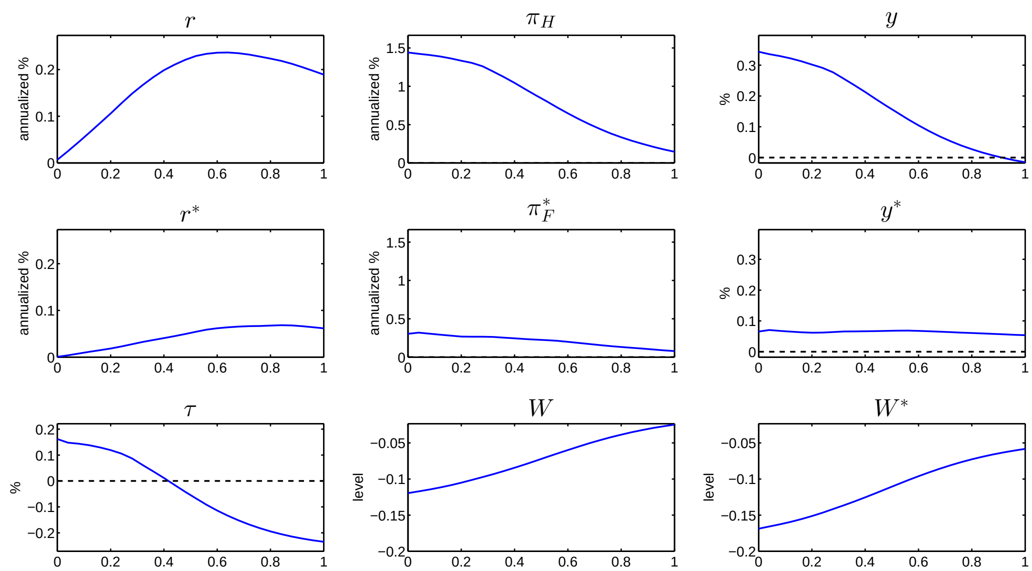
Proposition 3 *If $\nu = 2$, $\phi_\pi > 1$, and $\Lambda_\xi > 0$, $\hat{\xi}_t > 0$ implies $\hat{\pi}^*_{Ft} = 0$, $\hat{y}^*_t = 0$, and*

- $\hat{\pi}_{Ht}|_{\lambda=0} > \hat{\pi}_{Ht}|_{\lambda=1} > 0$
- $\hat{y}_t|_{\lambda=0} > \hat{y}_t|_{\lambda=1} > 0$
- $\hat{\tau}_t|_{\lambda=1} < 0$, $\hat{\tau}_t|_{\lambda=0} > 0$.

Proof: See Appendix B.4.

When there is a positive demand shock and no international trade, inflation and output always increase regardless of whether monetary policy is implemented. The anomalies exist

Figure 7: **Demand shock**



Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's preference shock of 0.77% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the negative preference shock. W and W^* are the discounted life-time welfare. X-axis: $\lambda = \lambda^*$.Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

in the sense that the ELB amplifies this effect. Moreover, the terms of trade still move in opposite directions for $\lambda = 0$ and $\lambda = 1$. Similar to Λ_a in the case of a supply shock, we impose $\Lambda_\xi > 0$ to guarantee inflation moves in the same direction whether $\lambda = 0$ or 1. $\Lambda_\xi > 0$ is guaranteed by $0 < \rho_\xi < \bar{\rho}_\xi$, where the bound is defined in Appendix B.4.

For numerical analysis, we relax parameter restrictions and introduce a preference shock in period 12 to serve as a positive demand shock. Figure 7 illustrates how impulse responses vary over the unconventional monetary policy indicator λ. Many of the patterns look very similar to those in Figure 3. Inflation and output in the home country and terms of trade decrease when λ increases. While home output stays mostly positive, terms of trade move from positive to negative. On the other hand, welfare increases when unconventional monetary policy becomes more active. The home country's nominal interest rate displays a humped shape.

Whether it is a negative supply shock or a positive demand shock, the basic mechanism works similarly through the inflation and real interest-rate channel. A positive home demand shock increases its CPI inflation, which involves both home and foreign goods. At the ELB, the nominal rates do not move, which lowers the real rates. Lower real rates further stimulate demand. When unconventional monetary policy is active, the nominal interest rates in both countries increase in response, which increases the real interest rates and lowers demand. Due to the origin of the shock, the movement of the domestic real rate is larger than its foreign counterpart. Therefore, at the ELB, real rates decrease and terms of trade increase, whereas with fully active unconventional monetary policy, terms of trade decrease.

The difference between the demand and supply shocks is their direct impact on output. While a positive demand shock increases consumption, a negative supply shock decreases output. Therefore, the equilibrium output is higher in the case of a demand shock.

3.3.4 Fixed exchange rate

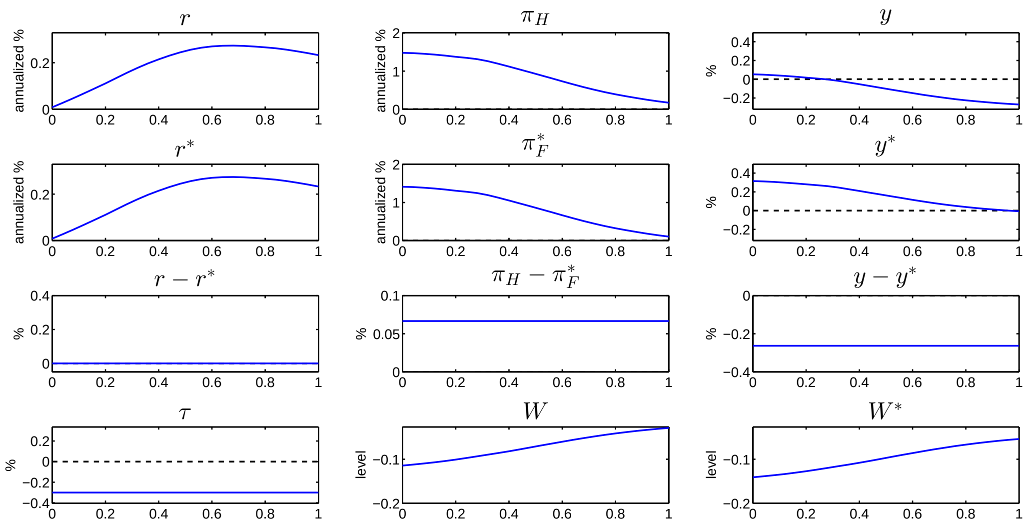
Our baseline model implements a flexible exchange rate. An alternative is a fixed exchange rate regime, and we implement it by letting the foreign country follow the home country's interest rate and react to the exchange rate:

$$\hat{r}_t^* = \hat{r}_t - \phi_e \hat{e}_t, \tag{3.8}$$

where $\hat{e}_t = \log(\mathcal{E}_t) - \log(\mathcal{E})$, and $\mathcal{E} = 1$ is the steady-state exchange rate. ϕ_e is the sensitivity of the foreign nominal interest rate to the exchange rate, and $\phi_e > 0$ ensures the equilibrium $r_t^* = r_t$, and $\hat{e}_t = 0$. See Appendix B.5 for details.

Imposing the same restrictions as in Section 3.1.1, we can solve the economy as functions

Figure 8: **Fixed exchange rate**



Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted life-time welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

of the TFP shock:

$$\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} = 2\sigma_0\Theta(1+\phi)(1-\rho_a)\Lambda^F_a\hat{a}_t \tag{3.9}$$

$$\hat{y}_t - \hat{y}^*_t = 2\rho_a(1+\phi)(D+1)\Lambda^F_a\hat{a}_t \tag{3.10}$$

$$\hat{\tau}_t = \frac{2\rho_a\sigma}{D}(1+\phi)(D+1)\Lambda^F_a\hat{a}_t, \tag{3.11}$$

where $\Lambda^F_a = \frac{1}{2\sigma_0(1-\rho_a)(1-\beta\rho_a)-\Theta\rho_a(\sigma/D+\phi)(D+1)}$. Equations (3.9) - (3.11) show that with the fixed exchange-rate rule, the terms of trade, inflation differential, and output differential behave the same regardless of how active unconventional monetary policy is. See Appendix B.5 for derivation. Figure 8 illustrates the same pattern numerically when we relax the assumptions on the parameters; see the third row and τ.

Compared to Figure 3, Figure 8 shows that for smaller λ, while the home economy seems to be less volatile under the fixed exchange rate rule, the foreign economy is much more susceptible to the home shock. Here is the basic mechanism. As a result of the negative TFP shock, the inflation rate for home goods increases. With a fixed exchange rate, the complementarity between the home and foreign goods puts upward pressure on the foreign goods and downward pressure on the home goods. In equilibrium, the two inflation rates increase by similar amounts. When monetary policy intervention is insufficient, real interest rates decrease both domestically and internationally. Consequently, this shock stimulates the home and foreign economies. The foreign output increases more than the domestic counterpart because the TFP shock has a direct negative impact on the home output. Similar to Figure 3, welfare in Figure 8 also increases when unconventional monetary policy becomes more active.

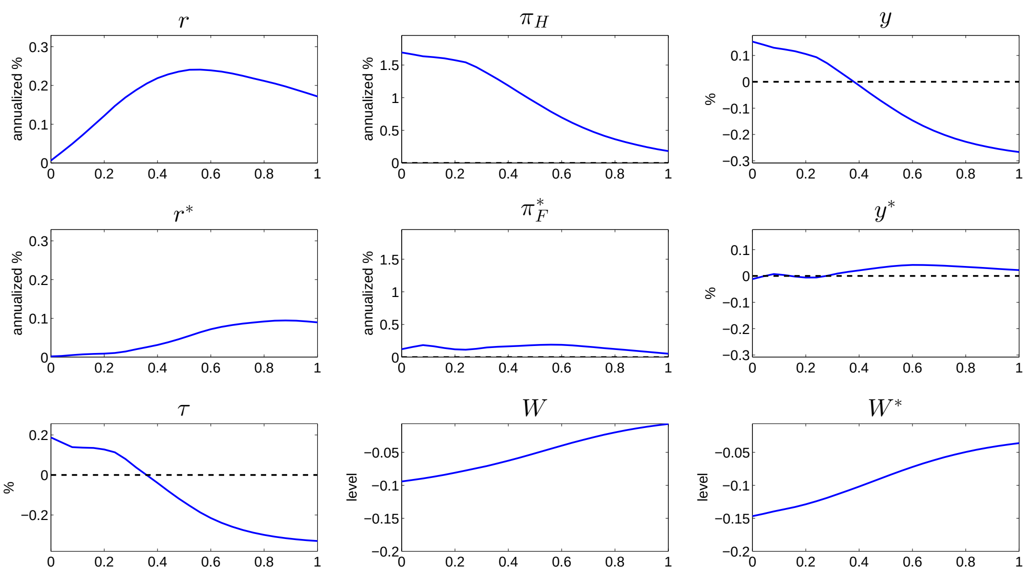
3.3.5 CPI - based Taylor rule

Our baseline specification of the Taylor rule in (2.25) relies on the PPI inflation. A viable alternative is to have the central bank respond to the CPI inflation instead:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1 - \rho_s) \left[\phi_\pi \hat{\pi}_t + \phi_y x_t \right]. \tag{3.12}$$

Figure 9 shows how economic quantities vary with $\lambda = \lambda^*$ when the central bank adopts the alternative Taylor rule. The economies behave similarly to those with the PPI-based rule in Figure 3. The impulse responses for the domestic economy and the terms of trade are lower if the monetary policy is implemented based on the CPI inflation for most λ, whereas the foreign quantities are higher in this case.

Figure 9: **CPI vs. PPI - based Taylor rule**

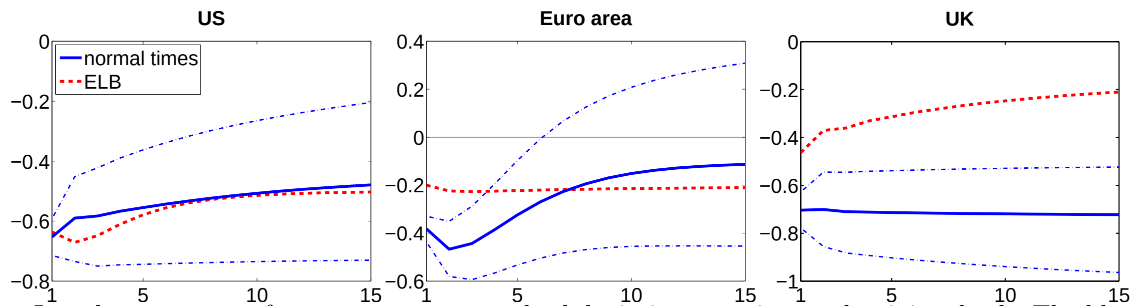


Notes: For all the variables but W and W^*, we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occur in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted life-time welfare. X-axis: $\lambda = \lambda^*$.Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

4 Empirical evidence on unconventional monetary policy

This section empirically investigates unconventional monetary policies at the ELB in the United States, Euro area, and United Kingdom, and compares them with their corresponding conventional policies. First, we test model implications by comparing impulse responses in a vector autoregression between normal times and the ELB. This exercise allows us to draw some qualitative conclusions for the three countries and regions. Next, to further draw some quantitative conclusions, we rely on the Taylor rule to compare what has been done with what should have been done.

Figure 10: **Impulse response of output to a productivity shock**



Notes: Impulse responses of output to a one standard deviation negative productivity shock. The blue lines are normal times, with solid being medians and dashed lines representing 68% confidence intervals. The red dashed lines are the median impulse responses at the ELB. X-axis: time in quarters; Y-axis: changes in output in standard deviations.

4.1 Vector autoregression

This section analyzes unconventional monetary policy in a VAR framework. We quantify empirically how output responds to a TFP shock in the United States, Euro area, and United Kingdom. Then we compare our empirical results with implications from our theoretical model in Section 3 to draw conclusions.

Following Galí and Gambetti (2009), we measure TFP with labor productivity. Our VAR has two variables: the growth rate of labor productivity, $\Delta(y_t - n_t)$, and the log of per-capita hours, n_t. We use a first-order VAR for the sake of short sample in the quarterly frequency. We identify the TFP shock through the Cholesky decomposition by ordering labor productivity first, which assumes that a shock to hours has no contemporaneous impact on labor productivity growth.

We estimate the VAR for the pre-ELB and ELB samples separately. The two samples span from 1985Q2 - 2007Q4[3] and 2009Q - 2015Q4 for the United States, 1999Q1 - 2009Q1 and 2009Q2 - 2017Q4 for the Euro area, and 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4 for the UK. The detailed data sources for the three countries and regions are in Appendix D.

Figure 10 plots the impulse response of output to a one standard deviation negative productivity shock for the three countries and regions. Blue represents normal times with

[3]We end the pre-ELB sample prior to the Great Recession.

medians in the solid lines, and 68% confidence intervals in the dashed lines. Red represents the central tenancies at the ELB. We find that for all three countries and regions, output decreases with a negative TFP shock regardless of normal times or the ELB. This similarity result is in contrast to the anomaly presented by the standard New Keynesian model in Subsection 3.1, and is evidence for unconventional monetary policy.

The left panel is for the United States. We find the impulse response at the ELB is initially slightly lower than normal times, and the red and blue lines track each other closely after 5 quarters. Moreover, the red line is well within the confidence interval in blue. This result suggests $\lambda \approx 1$ and potentially slightly larger than 1, which is consistent with Garín et al. (2016) and Debortoli et al. (2016).

The middle panel is for the Euro area, and the right panel is for the UK. Both of them show that output decreases less at the ELB than in normal times. The differences between the red and blue lines are statistically significant in both cases, with the UK being more pronounced. These findings suggest a less active unconventional monetary policy for both the Euro area and UK, and overall, we find $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$.

4.2 Taylor rule

In Subsection 4.1, the VAR qualitatively sorts the activeness of unconventional monetary policy among the three regions and countries. However, the key parameter λ enters the VAR non-linearly, and consequently, the VAR does not provide a direct numerical measure for it. For this purpose, we direct attention to the Taylor rule in this section.

We calculate how active unconventional monetary policy is by comparing what has been done at the ELB, measured by the shadow rates of Wu and Xia (2016) and Wu and Xia (2017),[4] with the interest rates implied by the historical Taylor rule.

[4]Shadow rates are downloaded from Cynthia Wu's website: https://sites.google.com/view/jingcynthiawu/shadow-rates.

We estimate the historical Taylor rule,

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 \pi_{Ht} + \beta_3 x_t + \varepsilon_t, \tag{4.1}$$

which is the empirical version of (2.25), via ordinary least squares, using the pre-ELB sample, and label the estimates $\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3$. When the ELB is binding, the desired interest rate implied by the historical Taylor rule can be calculated as follows:

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 s_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.2}$$

Next, we calculate the activeness of unconventional monetary policy by comparing the implemented monetary policy at ELB and the desired interest rate. Specifically, we regress observed shadow rate r_t on the imputed $\tilde{s}_t$ per (2.26).

Now we turn our attention to s_{t-1} in (4.2). We propose two methods below to proxy it.

Simple method The simple method uses the observed shadow rate r_{t-1} as a proxy for s_{t-1}. Hence, (4.2) becomes

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 r_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.3}$$

The benefit of the simple method is that the shadow rate is observable to us. Hence, the calculation is simple and robust.

Iterative method To measure s_{t-1} more accurately in the case of small λ, we leverage the relationship in (2.26), and replace (4.2) with

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 r_{t-1}/\tilde{\lambda} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \tag{4.4}$$

Now we face a fixed-point problem: (4.4) relies on $\tilde{\lambda}$ to compute $\tilde{s}_t$, whereas to obtain $\tilde{\lambda}$, we regress r_t on $\tilde{s}_t$. We propose an iterative procedure to solve this fixed-point problem. First, we give an initial guess for λ: $\tilde{\lambda}^{(0)}$. Then, we iterate over the following two steps until converged:

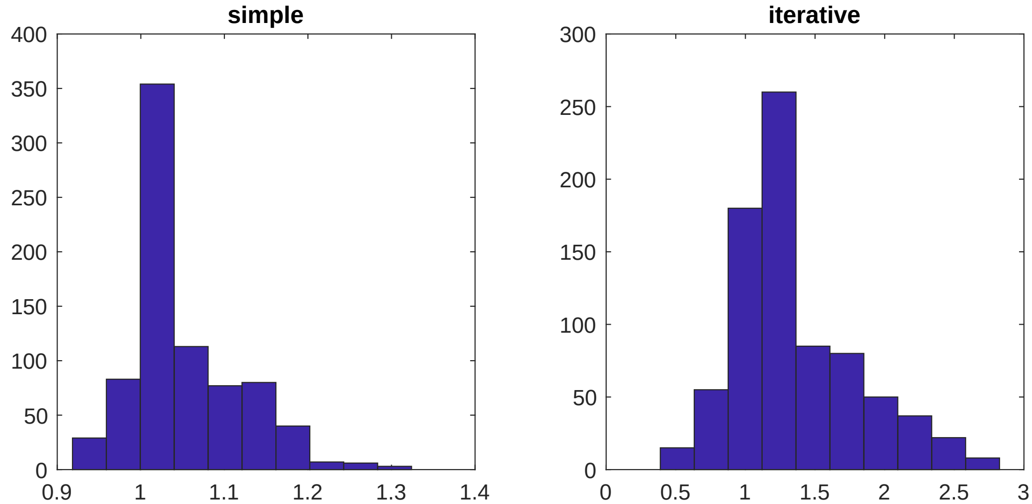
1. Based on $\tilde{\lambda}^{(i)}$, compute $\{\tilde{s}_t^{(i)}\}_{t=1}^T$ using (4.4).

2. Regress r_t on $\tilde{s}_t^{(i)}$ and compute $\tilde{\lambda}^{(i+1)}$.

Empirical results We begin with the United States. We measure π_{Ht} with the inflation based on the GDP price deflator, x_t is the real GDP minus potential GDP, and before the ELB, r_t corresponds to the effective fed funds rate. The pre-ELB and ELB samples are the same as in Subsection 4.1. The details of the data are in Appendix D. The estimate of the simple method is 1.02, and is 1.12 from the iterative method. We conclude that the US unconventional monetary policy is as active as, if not more active than, the historical Taylor rule.

The Taylor rule is known to vary over different sample periods, and researchers' choices of sample periods in the literature are far from unanimous. We quantify the variation of our estimates by varying the pre-ELB estimation sample: the beginning of the sample ranges from $t_0 \in \{1982Q1 : 1990Q1\}$ and the end of the sample varies from $t_1 \in \{2003Q1 : 2008Q4\}$, which covers the majority of popular choices. We compute a λ for each combination of t_0 and t_1 and plot its distribution across all possible combinations in Figure 11. The left panel plots the distribution for the simple method, and the right panel uses the iterative method. They both center around 1: the median for the simple method is 1.03, and 1.19 for the iterative method. The standard error for the simple method is 0.065, and it is 0.45 for the iterative method. The iterative method displays a larger variation across different sample periods than the simple method. On the other hand, the results from the simple method might be biased if λ is far from 1. This is the classic bias-variance tradeoff. The high persistence of the interest rate r_t contributes to both issues.

Figure 11: **Distribution of λ for the United States**



Notes: $t_0 \in \{1982Q1 : 1990Q1\}$, and $t_1 \in \{2003Q1 : 2008Q4\}$. For each combination of t_0 and t_1, estimate a λ from t_0 to t_1. Then plot the distribution across all possible combinations of t_0 and t_1. Left panel: simple method; right panel: iterative method.

For the Euro area and UK, quarterly real potential GDP is not available. Hence, we replace x_t in (4.1) with output growth Δy_t, measured by the growth rate of the real GDP. The pre-ELB r_t for the Euro area is the 3-month Euro Interbank Offered Rate (Euribor), and it is the Bank of England policy rate for the United Kingdom. The details of data are in Appendix D. For the Euro area, $t_0 \in \{1998Q2 : 1999Q1\}$ and $t_1 \in \{2009Q1 : 2011Q3\}$. The ELB period is from $t_1 + 1$ to 2017Q4. The median estimate for the simple (iterative) method is 0.998 (0.63) with a standard error of 0.031 (1.07). For the United Kingdom, $t_0 \in \{1993Q1 : 2003Q1\}$, $t_1 = 2009Q1$, and the ELB period is from 2009Q2 to 2017Q4. The median from the simple (iterative) method is 0.98 (0.39), with a standard error of 0.10 (4.10).

In summary, all three central banks have implemented a considerable amount of unconventional monetary policy: the United States operates following the historical Taylor rule, the Euro area and UK are less active, or $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$, which is consistent with what we find in Subsection 4.1. Note the variations across samples are larger for the Euro area and especially for the UK than for the United States, especially for the UK. That is partly due to a shorter sample.

5 Conclusion

We have introduced a new open-economy New Keynesian model. Our model provides a tractable framework that allows for unconventional monetary policy when the ELB is binding. We find when unconventional monetary policy operates following the historical Taylor rule, the anomalies in a standard model, namely, that output and terms of trade increase in response to a negative supply shock, disappear. These anomalies in the standard model are robust to alternative model and parameter specifications. Our model allows unconventional policy to be partially active and potentially asymmetric between the two countries. Empirically, we assess unconventional monetary policy across the United States, Euro area, and United Kingdom. Both the VAR analysis and the Taylor rule point to the same conclusion: the United States has operated its unconventional monetary policy following the historical Taylor rule. Although both the Euro area and UK have also implemented a considerable amount of unconventional policies, they have done less than what they normally would have.

References

Bauer, Michael D. and Christopher J. Neely, "International channels of the Fed's unconventional monetary policy," *Journal of International Money and Finance*, 2014, *44*, 24 – 46.

_ **and Glenn D. Rudebusch**, "The Signaling Channel for Federal Reserve Bond Purchases," *International Journal of Central Banking*, 2014, *10* (3), 233–289.

Benigno, Gianluca and Pierpaolo Benigno, "Exchange rate determination under interest rate rules," *Journal of International Money and Finance*, 2008, *27* (6), 971–993.

Bowman, David, Juan M. Londono, and Horacio Sapriza, "U.S. unconventional monetary policy and transmission to emerging market economies," *Journal of International Money and Finance*, 2015, *55*, 27 – 59. Macroeconomic and financial challenges facing Latin America and the Caribbean after the crisis.

Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework.," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Chen, Qianying, Andrew Filardo, Dong He, and Feng Zhu, "Financial crisis, US unconventional monetary policy and international spillovers," *Journal of International Money and Finance*, 2016, *67*, 62 – 81.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno, "Risk Shocks," *American Economic Review*, 2014, *104* (1), 27–65.

Clarida, Richard, Jordi Gal, and Mark Gertler, "A simple framework for international monetary policy analysis," *Journal of Monetary Economics*, 2002, *49* (5), 879 – 904.

Cook, David and Michael B. Devereux, "Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap," *American Economic Journal: Macroeconomics*, July 2013, *5* (3), 190–228.

— **and Michael B Devereux**, "Exchange Rate Flexibility under the Zero Lower Bound: the Need for Forward Guidance," November 2013.

Debortoli, Davide, Jordi Galí, and Luca Gambetti, "On the Empirical (Ir)Relevance of the Zero Lower Bound Constraint," 2016. Universitat Pompeu Fabra working paper.

Eggertsson, Gauti and Michael Woodford, "The Zero Interest-Rate Bound and Optimal Monetary Policy.," *Brookings Papers on Economic Activity*, 2003, (1), 139–211.

Eggertsson, Gauti B., Neil R. Mehrotra, Sanjay R. Singh, and Lawrence H. Summers, "A Contagious Malady? Open Economy Dimensions of Secular Stagnation," *IMF Economic Review*, 2016, *64* (4), 581–634.

Fernández-Villaverde, Jesus, Grey Gordon, Guerrón-Quintana, and Juan F. Rubio-Ramírez, "Nonlilnear Adventures at the Zero Lower Bound," *Journal of Economic Dynamics and Control*, 2015, *57*, 182–204.

Fujiwara, Ippei, Tomoyuki Nakajima, ao Sudo, and Yuki Teranishi, "Global liquidity trap," *Journal of Monetary Economics*, 2013, *60*, 936–949.

Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack, "The financial market effects of the Federal Reserve's large-scale asset purchases," *International Journal of Central Banking*, 2011, *7*, 3–43.

Galí, Jordi and Luca Gambetti, "On the Sources of the Great Moderation," *American Economic Journal: Macroeconomics*, 2009, *1* (1), 26–57.

Garín, Julio, Robert Lester, and Eric Sims, "Are supply shocks contractionary at the ZLB? Evidence from utilization-adjusted TFP data.," 2016. NBER working paper No. 22311.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily.," *Journal of Monetary Economics*, 2015, *70*, 22–38.

Hakkio, Craig S. and George A. Kahn, "Evaluating monetary policy at the zero lower bound," *The Macro Bulletin, Federal Reserve Bank of Kansas City*, 7 2014.

Hamilton, James D. and Jing Cynthia Wu, "The effectiveness of alternative monetary policy tools in a zero lower bound environment," *Journal of Money, Credit, and Banking*, 2012, *44 (s1)*, 3–46.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011, *2*, 215–265.

Mouabbi, Sarah and Jean-Guillaume Sahuc, "Evaluating the macroeconomic effects of the ECB's unconventional monetary policies," 2017. Working paper.

Neely, Christopher J., "Unconventional monetary policy had large international effects," *Journal of Banking & Finance*, 2015, *52*, 101 – 111.

Taylor, John B., "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 1993, *39*, 195–214.

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the macroeconomic impact of monetary policy at the zero lower bound.," *Journal of Money, Credit, and Banking*, 2016, *48* (2-3), 253–291.

_ **and** _ , "Negative Interest Rate Policy and Yield Curve," 2017. Chicago Booth Working paper.

_ **and Ji Zhang**, "A Shadow Rate New Keynesian Model," 2017. NBER Working Paper No. 22856.

Appendix A Model

Appendix A.1 Flexible-price equilibrium

For any variable Z_t, Z_t^n represents its flexible-price counterpart. In the flexible-price economy, real marginal cost is a constant

$$\frac{\theta-1}{\theta} \equiv MC_t^n = \frac{(1-g)(W_t^n/P_{Ht}^n)}{A_t} = \frac{(1-g)(W_t^n/P_t^n)(\mathcal{T}_t^n)^{1-\nu/2}}{A_t}, \tag{A.1}$$

and the optimal wage subsidy satisfies $\frac{\theta}{\theta-1} = 1-g$. Combining the labor-supply condition, $(C_t^n)^\sigma (N_t^n)^\phi = W_t^n/P_t^n$, the production function, $Y_t^n = A_t N_t^n$, and the real marginal cost (A.1), we have

$$(C_t^n)^\sigma (N_t^n)^\phi = A_t (\mathcal{T}_t^n)^{\nu/2-1}. \tag{A.2}$$

The risk-sharing condition holds as follows:

$$\frac{\Xi_t}{(C_t^n)^\sigma} = \frac{\Xi_t^*}{(C_t^{n*})^\sigma}\frac{P_t^n}{\mathcal{E}_t P_t^{n*}} = \frac{\Xi_t^*}{(C_t^{n*})^\sigma}(\mathcal{T}_t^n)^{1-\nu}. \tag{A.3}$$

The market-clearing conditions are

$$Y_t^n = \frac{\nu}{2}(\mathcal{T}_t^n)^{1-\nu/2} C_t^n + (1-\frac{\nu}{2})(\mathcal{T}_t^n)^{\nu/2} C_t^{n*} \tag{A.4}$$

$$Y_t^{n*} = \frac{\nu}{2}(\mathcal{T}_t^n)^{-1+\nu/2} C_t^{n*} + (1-\frac{\nu}{2})(\mathcal{T}_t^n)^{-\nu/2} C_t^{n}. \tag{A.5}$$

Appendix A.2 Log-linearized equations

Log-linearizing the consumption-savings decision in (2.3) and its foreign counterpart yields[5]

$$\hat{r}_t - \mathbb{E}_t\left[-(\hat{\xi}_{t+1}-\hat{\xi}_t) + \sigma(\hat{c}_{t+1}-\hat{c}_t) + \hat{\pi}_{t+1}\right] = 0 \tag{A.6}$$

$$\hat{r}_t^* - \mathbb{E}_t\left[-(\hat{\xi}_{t+1}^*-\hat{\xi}_t^*) + \sigma(\hat{c}_{t+1}^*-\hat{c}_t^*) + \hat{\pi}_{t+1}^*\right] = 0. \tag{A.7}$$

The labor-supply decision in (2.4) becomes

$$\hat{w}_t = \hat{c}_t + \phi\hat{n}_t \tag{A.8}$$

$$\hat{w}_t^* = \hat{c}_t^* + \phi\hat{n}_t^*. \tag{A.9}$$

The market-clearing condition in (2.22) becomes

$$\hat{y}_{Ht} = \left[\frac{\nu}{2}\hat{c}_t + \left(1-\frac{\nu}{2}\right)\hat{c}_t^*\right] + \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t \tag{A.10}$$

$$\hat{y}_{Ft}^* = \left[\frac{\nu}{2}\hat{c}_t^* + \left(1-\frac{\nu}{2}\right)\hat{c}_t\right] - \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t. \tag{A.11}$$

The international risk-sharing condition (2.15) is

$$\hat{\xi}_t - \sigma\hat{c}_t = \hat{\xi}_t^* - \sigma\hat{c}_t^* + p_t - e_t - p_t^* = \hat{\xi}_t^* - \sigma\hat{c}_t^* + (1-\nu)\hat{\tau}_t. \tag{A.12}$$

The production function in (2.18) becomes

$$\hat{y}_t = \hat{a}_t + \hat{n}_t \tag{A.13}$$

$$\hat{y}_t = \hat{a}_t^* + \hat{n}_t^*. \tag{A.14}$$

[5] We will omit "log-linearize" and "foreign counterpart" hereafter for brevity.

Combining (2.19) and the labor-supply decision (2.4) results in the real marginal costs:

$$\hat{mc}_t = \phi\hat{n}_t + \sigma\hat{c}_t - \hat{a}_t + (1-\nu/2)\hat{\tau}_t \tag{A.15}$$
$$\hat{mc}_t^* = \phi\hat{n}_t^* + \sigma\hat{c}_t^* - \hat{a}_t^* - (1-\nu/2)\hat{\tau}_t. \tag{A.16}$$

The CPI price in (2.8) yields to

$$p_t = \frac{\nu}{2}p_{Ht} + \left(1-\frac{\nu}{2}\right)p_{Ft} \tag{A.17}$$
$$p_t^* = \frac{\nu}{2}p_{Ft}^* + \left(1-\frac{\nu}{2}\right)p_{Ht}^*. \tag{A.18}$$

The definitions of CPI (2.10) and PPI inflation (2.11) are

$$\hat{\pi}_t = p_t - p_{t-1} \tag{A.19}$$
$$\hat{\pi}_t^* = p_t^* - p_{t-1}^* \tag{A.20}$$
$$\hat{\pi}_{Ht} = p_{Ht} - p_{H,t-1} \tag{A.21}$$
$$\hat{\pi}_{Ft} = p_{Ft} - p_{F,t-1}. \tag{A.22}$$

Combining (2.19), (2.20), and (2.21), the dynamics for the PPI inflation can be written as

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta\hat{mc}_t \tag{A.23}$$
$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta\hat{mc}_t^*, \tag{A.24}$$

where $\Theta = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa} > 0$. The definitions for terms of trade (2.12) and nominal exchange rate (2.13) are

$$\hat{\tau}_t = p_{Ft} - p_{Ht} \tag{A.25}$$
$$p_{Ht} = e_t + p_{Ht}^* \tag{A.26}$$
$$p_{Ft} = e_t + p_{Ft}^*. \tag{A.27}$$

Combining (A.19) - (A.22) and (A.25) - (A.27), the CPI inflation can be expressed as a function of PPI inflation and terms of trade:

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \left(1-\frac{\nu}{2}\right)\Delta\hat{\tau}_t \tag{A.28}$$
$$\hat{\pi}_t^* = \hat{\pi}_{Ft}^* - \left(1-\frac{\nu}{2}\right)\Delta\hat{\tau}_t. \tag{A.29}$$

The labor-supply decision (A.2) in the flexible-price economy becomes

$$\sigma\hat{c}_t^n + \phi\hat{n}_t^n = a_t + (\nu/2-1)\hat{\tau}_t^n \tag{A.30}$$
$$\sigma\hat{c}_t^{n*} + \phi\hat{n}_t^{n*} = a_t^* - (\nu/2-1)\hat{\tau}_t^n. \tag{A.31}$$

The international risk-sharing condition (A.3) in the flexible-price economy is

$$\sigma(\hat{c}_t^n - \hat{c}_t^{n*}) - (\hat{\xi}_t - \hat{\xi}_t^*) - (\nu-1)\hat{\tau}_t^n = 0. \tag{A.32}$$

The market-clearing conditions (A.4) and (A.5) in the flexible-price economy are

$$\hat{y}_{Ht}^n = \left[\frac{\nu}{2}\hat{c}_t^n + \left(1-\frac{\nu}{2}\right)\hat{c}_t^{n*}\right] + \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t^n \tag{A.33}$$
$$\hat{y}_{Ft}^{n*} = \left[\frac{\nu}{2}\hat{c}_t^{n*} + \left(1-\frac{\nu}{2}\right)\hat{c}_t^n\right] - \nu\left(1-\frac{\nu}{2}\right)\hat{\tau}_t^n. \tag{A.34}$$

The output gaps are defined as

$$x_t = y_t - y_t^n \tag{A.35}$$
$$x_t^* = y_t^* - y_t^{n*}. \tag{A.36}$$

Equations (A.6) to (A.36) and the monetary policy rules (2.25) and (2.26) and their foreign counterparts summarize all equilibrium conditions.

Appendix A.3 Exchange rate, terms of trade, and interest rates

Combining the Euler equations (A.6) and (A.7) with the international risk-sharing condition (A.12), we obtain (2.16). Combining (2.16), (A.25), and (A.27), we get

$$\hat{\tau}_t = (\hat{r}_t^* - \mathbb{E}_t[\hat{\pi}_{F,t+1}^*]) - (\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{H,t+1}]) + \mathbb{E}_t[\hat{\tau}_{t+1}] = \hat{rr}_t^* - \hat{rr}_t + \mathbb{E}_t[\hat{\tau}_{t+1}]. \tag{A.37}$$

Solving (A.37) forward under the stationarity condition $\lim_{k\to\infty} \mathbb{E}_t\hat{\tau}_{t+k} = 0$, we obtain (2.17).

Appendix B Analytical results and proofs

Appendix B.1 Proof of Proposition 1

In Appendix B.1 to Appendix B.3, there is only a home country's TFP shock, that is, $\hat{a}_t^* = \hat{\xi}_t = \hat{\xi}_t^* = 0$. Combining the market-clearing conditions (A.10) and (A.11) and the international risk-sharing condition (A.12), terms of trade can be expressed as a function of relative output:

$$\hat{\tau}_t = \sigma/D(\hat{y}_t - \hat{y}_t^*). \tag{B.1}$$

Combining the Euler equations (A.6) and (A.7) with the market-clearing conditions (A.10) and (A.11), international risk sharing (A.12), and the definition of terms of trade (A.25), we get the IS curves for the home and foreign countries:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma_0}(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{H,t+1}) + K_2(\mathbb{E}_t\hat{y}_{t+1}^* - \hat{y}_t^*) \tag{B.2}$$
$$\hat{y}_t^* = \mathbb{E}_t\hat{y}_{t+1}^* - \frac{1}{\sigma_0}(\hat{r}_t^* - \mathbb{E}_t\hat{\pi}_{F,t+1}^*) + K_2(\mathbb{E}_t\hat{y}_{t+1} - \hat{y}_t), \tag{B.3}$$

where $\sigma_0 = \sigma - K_1$, $K_1 = (1-\nu/2)(\sigma-1)\nu\sigma/D = \frac{\sigma}{2}\frac{D-1}{D}$, $D = [(\nu-1)^2 + \sigma\nu(2-\nu)]$, and $K_2 = K_1/\sigma_0$. Take the difference between the home and foreign IS curves,

$$(\hat{r}_t - \hat{r}_t^*) - \mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) = \sigma_0(1-K_2)\mathbb{E}_t[(\hat{y}_{t+1} - \hat{y}_{t+1}^*) - (\hat{y}_t - \hat{y}_t^*)]. \tag{B.4}$$

The monetary policy rules are

$$\hat{r}_t = \lambda\phi_\pi\hat{\pi}_{Ht} \tag{B.5}$$
$$\hat{r}_t^* = \lambda\phi_\pi\hat{\pi}_{Ft}^*. \tag{B.6}$$

Substitute them into (B.2) - (B.4):

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma_0}(\lambda\phi_\pi\hat{\pi}_{Ht} - \mathbb{E}_t\hat{\pi}_{H,t+1}) + K_2(\mathbb{E}_t\hat{y}_{t+1}^* - \hat{y}_t^*) \tag{B.7}$$
$$\hat{y}_t^* = \mathbb{E}_t\hat{y}_{t+1}^* - \frac{1}{\sigma_0}(\lambda\phi_\pi\hat{\pi}_{Ft}^* - \mathbb{E}_t\hat{\pi}_{F,t+1}^*) + K_2(\mathbb{E}_t\hat{y}_{t+1} - \hat{y}_t) \tag{B.8}$$
$$\lambda\phi_\pi(\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^*) - \mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) = \sigma_0(1-K_2)\mathbb{E}_t[(\hat{y}_{t+1} - \hat{y}_{t+1}^*) - (\hat{y}_t - \hat{y}_t^*)]. \tag{B.9}$$

Combining the labor-supply conditions (A.8) and (A.9), production functions (A.13) and (A.14), the risk-sharing condition (A.12), and the market-clearing conditions (A.10) and (A.11), the real marginal costs can be derived as

$$\begin{aligned}\hat{mc}_t &= \phi\hat{n}_t + \sigma\hat{c}_t - \hat{a}_t + (1-\nu/2)\hat{\tau}_t \\ &= \phi\hat{y}_t - (1+\phi)\hat{a}_t + \sigma\hat{c}_t + (1-\nu/2)\hat{\tau}_t \\ &= \phi\hat{y}_t - (1+\phi)\hat{a}_t + \sigma\hat{y}_t - \sigma(1-\nu/2)(\nu - \frac{\nu-1}{\sigma})\hat{\tau}_t + (1-\nu/2)\hat{\tau}_t \\ &= K\hat{y}_t - (1+\phi)\hat{a}_t + K_1\hat{y}_t^*, \end{aligned} \tag{B.10}$$

where $K = \sigma + \phi - K_1$. The foreign country's counterpart is

$$\hat{mc}_t^* = K\hat{y}_t^* + K_1\hat{y}_t. \tag{B.11}$$

Combining (B.10) and (B.11) with (A.23) and (A.24), the New Keynesian Phillips curves (NKPCs) are

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta K\hat{y}_t - \Theta(1+\phi)\hat{a}_t + \Theta K_1\hat{y}_t^* \tag{B.12}$$
$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta K\hat{y}_t^* + \Theta K_1\hat{y}_t. \tag{B.13}$$

The difference is

$$\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = \beta\mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) + \Theta(K-K_1)(\hat{y}_t - \hat{y}_t^*) - \Theta(1+\phi)\hat{a}_t. \tag{B.14}$$

Next, we solve the system of equations in (B.9) and (B.14). When $\lambda\phi_\pi > 1$, the Blanchard-Kahn condition is satisfied, and the system has a unique solution, which is (3.1), (3.2). Next, (B.1) implies (3.3).

In our model, $\Theta > 0$, $1+\phi > 0$, $1-\rho_a > 0$, $D > 0$, $D+1 > 0$, $\sigma > 0$, $\sigma_0 > 0$.

- When $\lambda = 1$ and $\phi_\pi > 1$, $\Lambda_a > 0$ and $\lambda\phi_\pi - \rho_a > 0$.
- When $\lambda = 0$, the denominator of Λ_a is a convex quadratic function of ρ_a with one root between 0 and 1 and another root larger than 1. We solve the root within the unit circle $\bar{\rho}_a = \frac{2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)-\sqrt{[2\sigma_0(1+\beta)+\Theta(\sigma/D+\phi)(D+1)]^2-16\sigma_0^2\beta}}{4\sigma_0\beta}$, and $0 < \rho_a < \bar{\rho}_a$ guarantees $\Lambda_a > 0$. Moreover, $\lambda\phi_\pi - \rho_a < 0$.

■

Appendix B.2 Proof of Corollary 1

When $\sigma = 1$ or $\nu = 2$, $K_1 = K_2 = 0$, so that $\sigma_0 = \sigma$, $K = \sigma + \phi$, and $D = 1$. For the foreign economy, (B.8) and (B.13) yield to

$$\hat{\pi}_{Ft}^* = \hat{y}_t^* = 0. \tag{B.15}$$

The solution to (B.7) and (B.12) for the home economy is

$$\hat{y}_t = \Theta(\lambda\phi_\pi - \rho_a)(1+\phi)\Lambda_a\hat{a}_t \tag{B.16}$$
$$\hat{\pi}_{Ht} = -\Theta(1-\rho_a)(1+\phi)\Lambda_a\hat{a}_t, \tag{B.17}$$

and (B.1) implies $\hat{\tau}_t = \sigma\hat{y}_t$.■

Appendix B.3 Proof of Proposition 2

When $\lambda = 0$, (3.2) and (3.3) become

$$\hat{y}_t - \hat{y}_t^* = -\rho_a\Theta(1+\phi)(D+1)\Lambda_a\hat{a}_t \tag{B.18}$$
$$\hat{\tau}_t = -\rho_a\Theta(1+\phi)\frac{\sigma(D+1)}{D}\Lambda_a\hat{a}_t. \tag{B.19}$$

First,

$$\frac{\partial D}{\partial \nu} = 2(1-\sigma)(\nu - 1) < 0,$$

given $\sigma > 1$. Next, take the derivative of the coefficient in (B.18) with respect to D:

$$\begin{aligned} & \frac{\partial\left[-\rho_a\Theta(1+\phi)(D+1)\Lambda_a\right]}{\partial D} \\ = \quad & -\rho_a\Theta(1+\phi)\Lambda_a\left\{1+(D+1)\Lambda_a\left[\rho_a\Theta(\phi-\sigma/D^2)+\frac{\sigma(1-\beta\rho_a)(1-\rho_a)}{D^2}\right]\right\}. \end{aligned} \tag{B.20}$$

Note $\phi - \sigma/D^2$ is an increasing function of D and hence a decreasing function of ν. Therefore, $\phi - \sigma/D^2 \geq \phi - \sigma/D^2|_{\nu=2} = \phi - \sigma > 0$, and $\frac{\partial[-\rho_a\Theta(1+\phi)(D+1)\Lambda_a]}{\partial D} < 0$. That is, $-\rho_a\Theta(1+\phi)(D+1)\Lambda_a$ is increasing in ν and negative. When $\hat{a}_t < 0$, $\hat{y}_t - \hat{y}_t^* \geq \hat{y}_t - \hat{y}_t^*|_{\nu=2} > 0$.

Next, for the coefficient in (B.19),

$$\begin{aligned} & \frac{\partial\left[-\rho_a\sigma\Theta(1+\phi)\frac{D+1}{D}\Lambda_a\right]}{\partial D} \\ = \quad & -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{-\frac{1}{D^2}+(D+1)\Lambda_a\left[\rho_a\Theta(\phi-\sigma/D^2)+\frac{\sigma(1-\beta\rho_a)(1-\rho_a)}{D^2}\right]\right\} \\ = \quad & -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{\sigma(1-\beta\rho_a)(1-\rho_a)(D+1)\Lambda_a-1}{D^2}+(D+1)\Lambda_a\rho_a\Theta(\phi-\sigma/D^2)\right\} \\ = \quad & -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{1}{D^2}\frac{\sigma(1-\beta\rho_a)(1-\rho_a)(D+1)-1/\Lambda_a}{1/\Lambda_a}+(D+1)\Lambda_a\rho_a\Theta(\phi-\sigma/D^2)\right\} \\ = \quad & -\rho_a\sigma\Theta(1+\phi)\Lambda_a\left\{\frac{1}{D^2}\frac{\sigma/D(1-\beta\rho_a)(1-\rho_a)(D+1)(D-1)+\rho_a\Theta(\sigma/D+\phi)(D+1)}{1/\Lambda_a}\right. \\ & \left.+(D+1)\Lambda_a\rho_a\Theta(\phi-\sigma/D^2)\right\}. \end{aligned} \tag{B.21}$$

D is decreasing in ν: $D \geq D|_{\nu=2} = 1$. Therefore, (B.21) is negative, and $-\rho_a\Theta(1+\phi)\frac{\sigma(D+1)}{D}\Lambda_a$ is negative and increasing in ν, or when $\hat{a}_t < 0$, $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$. ■

Appendix B.4 Proof of Proposition 3

In this section, there is only a home country's demand shock, and $\hat{a}_t = \hat{a}_t^* = \hat{\xi}_t^* = 0$. The terms of trade can be expressed as a function of the relative output:

$$\hat{\tau}_t = \sigma/D(\hat{y}_t - \hat{y}_t^*) - \frac{1}{\nu - 1}\hat{\xi}_t. \tag{B.22}$$

The IS curves are

$$\hat{y}_t - \mathbb{E}_t\hat{y}_{t+1} = -\frac{1}{\sigma_0}(\lambda\phi_\pi\hat{\pi}_{Ht} - \mathbb{E}_t\hat{\pi}_{H,t+1}) + K_2\mathbb{E}_t(\hat{y}_{t+1}^* - \hat{y}_t^*) - \frac{D+\nu-1}{2D\sigma_0}\mathbb{E}_t\Delta\hat{\xi}_{t+1} \tag{B.23}$$

$$\hat{y}_t^* - \mathbb{E}_t\hat{y}_{t+1}^* = -\frac{1}{\sigma_0}(\lambda\phi_\pi\hat{\pi}_{Ft}^* - \mathbb{E}_t\hat{\pi}_{F,t+1}^*) + K_2\mathbb{E}_t(\hat{y}_{t+1} - \hat{y}_t) + \frac{\nu-1-D}{2D\sigma_0}\mathbb{E}_t\Delta\hat{\xi}_{t+1}. \tag{B.24}$$

The NKPCs are

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta K\hat{y}_t + \Theta K_1\hat{y}_t^* + \frac{K_1(\nu-1)}{\sigma}\hat{\xi}_t \tag{B.25}$$

$$\hat{\pi}_{Ft}^* = \beta\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \Theta K\hat{y}_t^* + \Theta K_1\hat{y}_t - \frac{K_1(\nu-1)}{\sigma}\hat{\xi}_t. \tag{B.26}$$

When $\nu = 2$, we have $D = 1$, $K_1 = K_2 = 0$, $\sigma_0 = \sigma$, and $K = \sigma + \phi$, so that (B.23) to (B.26) can be simplified to

$$\hat{y}_t - \mathbb{E}_t\hat{y}_{t+1} = -\frac{1}{\sigma}(\lambda\phi_\pi\hat{\pi}_{Ht} - \mathbb{E}_t\hat{\pi}_{H,t+1}) - \frac{1}{\sigma}\mathbb{E}_t\Delta\hat{\xi}_{t+1}, \tag{B.27}$$

$$\hat{y}^*_t - \mathbb{E}_t\hat{y}^*_{t+1} = -\frac{1}{\sigma}(\lambda\phi_\pi\hat{\pi}^*_{Ft} - \mathbb{E}_t\hat{\pi}^*_{F,t+1}), \tag{B.28}$$

$$\hat{\pi}_{Ht} = \beta\mathbb{E}_t\hat{\pi}_{H,t+1} + \Theta K\hat{y}_t, \tag{B.29}$$

$$\hat{\pi}^*_{Ft} = \beta\mathbb{E}_t\hat{\pi}^*_{F,t+1} + \Theta K\hat{y}^*_t. \tag{B.30}$$

Solving the system of equations (B.22) and (B.27) to (B.30), we obtain (3.4) - (3.7).

When $\lambda = 0$, the denominator of Λ_ξ is a quadratic function of ρ_ξ, with one root between 0 and 1 and the other root larger than 1. $\Lambda_\xi > 0$ requires $0 < \rho_\xi < \bar{\rho}_\xi$, where

$$\bar{\rho}_\xi = \frac{[1+\beta+\Theta(1+\phi))] - \sqrt{[1+\beta+\Theta(1+\phi))]^2 - 4\beta}}{2\beta} < 1.$$

Given $\Lambda_\xi > 0$, we have $\Lambda_\xi|_{\lambda=0} > \Lambda_\xi|_{\lambda=1} > 0$ because

$$\Theta(\sigma+\phi)(\phi_\pi - \rho_\xi) + \sigma(1-\rho_\xi)(1-\beta\rho_\xi) > -\rho_\xi\Theta(\sigma+\phi) + \sigma(1-\rho_\xi)(1-\beta\rho_\xi).$$

Therefore, the coefficients in (3.4) and (3.5) are larger when $\lambda = 0$ than when $\lambda = 1$. For the terms of trade, the term $\lambda\phi_\pi - \rho_\xi$ in (3.6) is positive when $\lambda = 1$ and negative when $\lambda = 0$. ■

Appendix B.5 Fixed exchange rate

The IS curve (B.4), NKPC (B.14), and the home country's monetary policy rule (B.5) are the same as in the flexible exchange rate economy, whereas the foreign country's monetary policy becomes (3.8).

Substituting (3.8) into (2.16), we have $\mathbb{E}_t\hat{e}_{t+1} = (1+\phi_e)\hat{e}_t$. $\phi_e > 0$ ensures a unique and non-explosive equilibrium, which is $e_t = e$, where e is the steady-state value of the exchange rate. Therefore,

$$\hat{e}_t = 0. \tag{B.31}$$

A similar argument can be found in Benigno and Benigno (2008).

Substitute (B.5), (3.8), and (B.31) into (B.4) and (B.14), and we obtain

$$-\mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}^*_{F,t+1}) = \sigma_0(1-K_2)\mathbb{E}_t[(\hat{y}_{t+1} - \hat{y}^*_{t+1}) - (\hat{y}_t - \hat{y}^*_t)]$$
$$\hat{\pi}_{Ht} - \hat{\pi}^*_{Ft} = \beta\mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}^*_{F,t+1}) + \Theta(K-K_1)(\hat{y}_t - \hat{y}^*_t) - \Theta(1+\phi)\hat{a}_t.$$

The solution of this system is given by (3.9) - (3.10). And (3.10) and (B.1) yield to (3.11).

Appendix C Setup for quantitative analysis

Calibration We calibrate structural parameters according to the standard macro and international literature. The discount factor is $\beta = 0.99$, so the steady-state quarterly risk-free nominal interest rate is 1%. The intertemporal elasticity of substitution is $\sigma = 2$, and the Frisch labor-supply elasticity is $\phi = 3$. The elasticity of substitution among differentiated goods is $\theta = 6$, implying the steady-state price markup is 1.2. The price stickiness parameter is $\kappa = 0.75$, meaning the average time between two price adjustments is one year. The persistence of the Taylor rule is $\rho_r = 0.8$, and the sensitivities of the policy rate to inflation and output are $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. $\nu = 1.5$ implies a significant home bias. The persistence and standard deviation of the TFP shock are $\rho_a = 0.9$ and $\sigma_a = 0.0025$, according to Fernández-Villaverde et al. (2015). The persistence and standard deviation of the preference shock are $\rho_\xi = 0.9$ and $\sigma_\xi = 0.023$, according to Christiano et al. (2014).

ELB environment To create an ELB environment, we impose a series of negative preference shocks on both countries. The shocks occur in periods 1-15, and the total shock size in each country is 23%. These shocks push down the nominal interest rate to zero at period 9 and keep it there until period 20 when there is no unconventional monetary policy.

Negative TFP shock In addition to the preference shocks, we hit the home country with a one-time negative TFP shock with a size of -0.5% at period 12.

Solution method When $\lambda = 1$, the model is linear, so we use the standard method for solving the rational expectations models. When $\lambda < 1$, we use the occasional binding method of Guerrieri and Iacoviello (2015).

Appendix D Data

- Shadow rates are downloaded from Cynthia Wu's website:
  https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates.
- The U.S. macroeconomic variables are downloaded from the Database of the Federal Reserve Bank of St. Louis (FRED) at http://research.stlouisfed.org/fred2/.
  - Real GDP (GDPC): billions of chained 2009 dollars, seasonally adjusted.
  - Real potential GDP (GDPPOT): billions of chained 2009 dollars, not seasonally adjusted.
  - GDP deflator (GDPDEF): index 2009=100, seasonally adjusted.
  - Effective federal funds rate (FEDFUNDS): percent.
  - Real output per hour of all persons (nonfarm business sector) (OPHNFB): index 2009=100, seasonally adjusted.
  - Hours of all persons (nonfarm business sector) (HOANBS): index 2009=100, seasonally adjusted.
  - Civilian noninstitutional population (CNP16OV): thousands of persons.
- The Euro area macroeconomic variables are from the ECB Statistical Data Warehouse at http://sdw.ecb.europa.eu/home.do.
  - Real GDP: reference year 1995, calendar and seasonally adjusted.
  - GDP deflator: index 1995=1, calendar and seasonally adjusted.
  - Policy rate: 3-month Euribor.
  - Euro area 19 (fixed composition) total economy labor productivity (per hours worked): index, chain linked volume (rebased), calendar and seasonally adjusted.
  - Euro area 19 (fixed composition) total economy hours worked: hours, calendar and seasonally adjusted.
  - Euro area 19 (fixed composition) employment: thousands of persons, calendar and seasonally adjusted.
- The UK macroeconomic variables are downloaded from the Office for National Statistics at https://www.ons.gov.uk/ and the FRED.
  - Real GDP: seasonally adjusted.
  - GDP deflator: index 1995=1, seasonally adjusted.
  - Bank of England policy rate: percent per annum.
  - Output per hour: index 2015=100, seasonally adjusted.
  - Average actual weekly hours of work for all workers: millions, seasonally adjusted.
  - Population aged 16 and over: thousands of persons.