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ABSTRACT

Climate change is considered as one of the major global challenges. Although countries past and future contributions to the accumulation of greenhouse gases in the atmosphere are different, all countries are affected, but not necessarily in the same way (e.g. rising sea levels). This is the reason why it is so hard to reach global agreements on this matter. We study this issue in a dynamic game-theoretical model (stochastic differential game) with multiple countries that are open economies, i.e. we allow for international trade between the countries. Our framework involves stochastic dynamics for CO2-emissions and economic output of the countries. Each country is represented by a recursive-preference functional. Despite its complexity, the model is tractable and we can quantify each country's decision on consumption, investment, carbon abatement and the social cost of carbon, explicitly. One key finding is that both the country-specific and global social cost of carbon are increasing in the trade volume. This result is robust to adding capital transfers between countries. Our numerical examples suggest that disregarding trade might lead to a significant underestimation of the SCC.

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A data appendix is available at http://www.nber.org/data-appendix/w24604

1 Introduction

> IPCC (2014): "Effective mitigation [of greenhouse gases] will not be achieved if individual agents advance their own interests independently."

Climate change is considered as one of the major global challenges. Although countries past and future contributions to the accumulation of greenhouse gases (GHGs) in the atmosphere are different, all countries are affected, but not necessarily in the same way. For instance, rising sea levels affect costal areas much more than inland areas. On the other hand, countries that are potentially affected the most are not necessarily the ones emitting most of the GHGs and vice versa. Furthermore, mitigating the effects of climate change is bedeviled by free-rider problems and external effects that make it hard to achieve global agreements to mitigate the potential consequences of anthropological emissions. For instance, it took several years to reach the Paris Agreement, but the result is still considered as imperfect by some (e.g. Rogelj et al. (2016) and UNEP (2016)), binding commitment devices are missing, and some countries are threatening to defect.

This paper proposes a non-cooperative game-theoretical framework that allows us to study crucial issues prevalent in the international efforts to address climate change. Our model is formulated as a repeated game reflecting the fact that dealing with climate change demands continuous actions by all countries. Problems of this type are in general hard to solve, but our formulation is analytically tractable. Assuming that each country's decision making can be characterized by a recursive utility function, we can explicitly calculate the optimal consumption and abatement decisions of all countries as well as the corresponding social cost of carbon (SCC). One important feature of our model is that countries are open economies, i.e. there is potentially international trade between all countries. We can thus study the effect of international trade on the SCC, which is a key contribution of our paper. We find that the SCC are increasing in the trade volume. This effect can be significant and disregarding trade might lead to a severe underestimation of the SCC, both at the country and global level. We show that this result is robust to allowing for capital transfers. Notice that the majority of the existing optimization-based integrated assessment models (IAMs) involve only one representative agent, i.e. by construction there is no trade. The few IAMs with multiple countries typically assume that these countries are autarkies, i.e. they also abstract from international trade.[1]

[1] See the discussion of the literature below.

Furthermore, we show that the number of countries is a crucial determinant of the optimal amount of abatement. In fact, we find that as the number of countries becomes larger, the optimal efforts that each country implements become smaller, leading to less global abatement. In the limit,it may be optimal to do no abatement. This is in line with the nature of the carbon abatement game that due to all its externalities can lead to a Prisoner's dilemma. The framework we develop is general enough to allow us the flexibility to analyze many different situations depending on the structure of the damage functions and the number of countries.

From a formal point of view, this paper offers a closed-form solution to an involved stochastic differential game with recursive preferences (stochastic differential utility) that are typically very challenging to solve. Our model involves several stochastic state variables such as the global temperature and the capital stocks that generate the outputs of the different countries. All countries can decide about how to use their output: Each country can implement carbon abatement strategies to reduce carbon emissions and thus mitigate the increase in global temperature. This decision is plagued by external effects, since the benefits of carbon abatement are shared by all countries, but the expenditures are paid by each country individually. Alternatively, each country can consume or reinvest its output to increase its capital stock. To compare our findings with a cooperative setting, we also provide the solution for a particular problem where a social planner makes all decisions. We can explicitly determine the welfare gains that arise from having a social planner who forces all countries to implement strategies that are optimal from a global perspective and that internalizes all external effects. Of course, this solution is not attainable in a realistic setting since defecting from this global optimal strategy is difficult to punish.

Our work is related to several other papers. Firstly, there are integrated assessment models studying the impact of climate change. The DICE model (Dynamic Integrated Model of Climate and the Economy) is the most common framework to study optimal carbon abatement. It is formulated in a deterministic setting, see for example Nordhaus (1992, 2008), Nordhaus and Sztorc (2013). This framework has been extended by several authors: Crost and Traeger (2014), Jensen and Traeger (2014), and Ackermann et al. (2013) analyze versions where one component is assumed to be stochastic and the decision maker has recursive preferences. Ackermann et al. (2013) introduce transitory uncertainty of the climate sensitivity parameter into the DICE model. A stochastic version is analyzed by Cai et al. (2015). All these papers study frameworks with one representative agent.

Closed-form solutions are only available in few special cases. The most prominent example is the combination of log-utility, Cobb-Douglas production and full depreciation as in Golosov et al. (2014). Traeger (2015) generalizes this setting to recursive preferences and provides a sound description of the carbon cycle and the climate system. An alternative approach is proposed by van den Bremer and van der Ploeg (2018) who combine AK-growth and recursive preferences to solve for the optimal fossil fuel use. These papers are all single-agent models.

There are few papers taking a game-theoretical approach. van der Ploeg and de Zeeuw (1992) analyze a deterministic setting and distinguish between open-loop and feedback Nash equilibrium outcomes. Nordhaus and Yang (1996) is a deterministic game-theoretical version of the DICE model which is called the RICE model. Ackermann et al. (2011) extend the RICE model and focus on a social-planner solution. Tol (2002a,b) considers a static game with deterministic actions called the FUND model to estimate the damages of climate change. Nordhaus (2015) emphasizes the non-cooperative feature of international efforts to mitigate climate change and proposes so-called climate clubs involving external penalties in the form of trade tariffs. All these papers are formulated in a deterministic setting or restrict the optimal abatement strategies to be deterministic. Furthermore, they do not allow for trade, except for Nordhaus (2015). His model, however, is static and does not analyze the effect of trade on the SCC, which in his analysis are exogenously given. By contrast, we determine the SCC endogenously. Hassler and Krusell (2012) analyze a stochastic general-equilibrium version of RICE which is a multi-region version of Golosov et al. (2014). In their model, there is no trade except for trade in oil. They show that in this setup only taxes on oil producers can improve the climate, whereas taxes on oil consumers have no effect. van der Ploeg and de Zeeuw (2016) study the effect of productivity shocks resulting from climate change (tipping point) in cooperative and non-cooperative settings. They show that the cooperative response to these stochastic tipping points requires converging carbon taxes for developing and developed regions. However, they also abstract from international trade.

The remainder of the paper is structured as follows: Section 2 introduces the model setup. Section 3 formalizes the non-cooperative game that all countries face. Section 4 provides the solution to the non-cooperative game. Section 5 shows that the world abatement effort becomes negligible if the number of countries is large. In this case, inattentiveness is optimal. Section 6 contains a detailed analysis of how international trade contributes to the SCC. Section 7 shows that our results regarding international trade are robust to adding capital transfers between

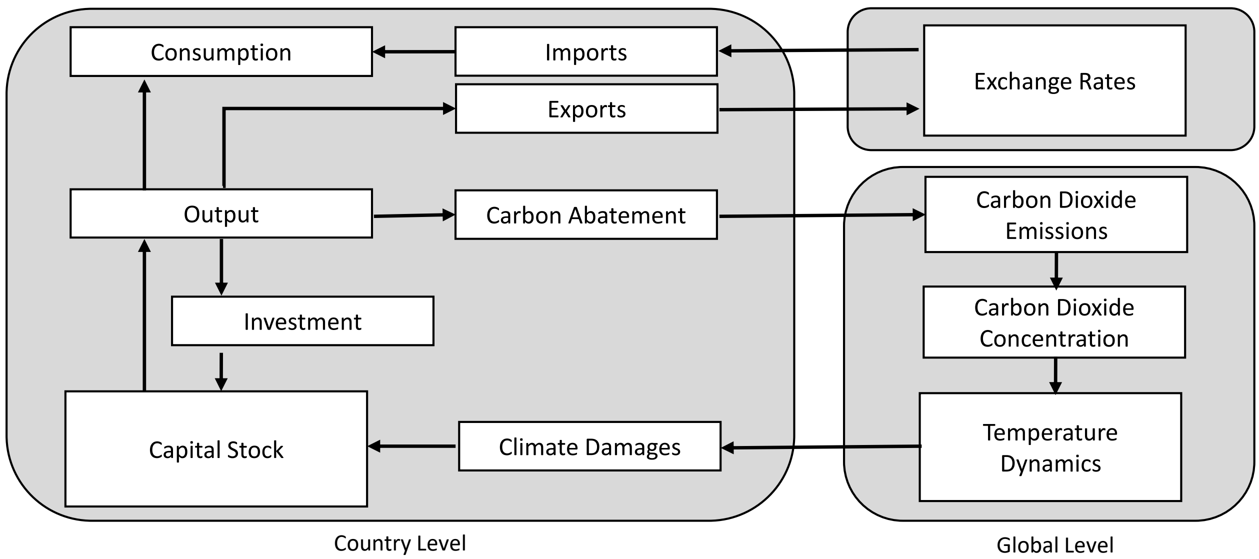

Figure 1: Model Structure. This figure depicts the structure of the model presented in Section 2. The arrows depict the flow of goods and the direction of operation.

countries. Section 8 studies a cooperative version of the game and quantifies welfare effects. Section 9 reports numerical results for a calibration with five regions given by the AR5 Scenario Database of IPCC (2014). It is shown that for this calibration 22.5% of the global SCC are generated by international trade. Section 10 concludes.

2 Model Setup

The world is divided into N heterogeneous regions (syn. countries), which are indexed by $n \in \{1, \dots, N\}$. On a global level, we model carbon dioxide emissions, concentrations and changes in global warming and incorporate these building blocks into an economic analysis. We solve for economic key variables such as the optimal abatement-consumption strategies and the social cost of carbon. The general model structure is depicted in Figure 1.

2.1 Economic Model

Production Following Barro (2006, 2009) and Pindyck and Wang (2013), every country produces output (syn. GDP) using a production technology that is linear in capital (AK-

technology). Formally, output of country n is

$$Y_{nt} = A_n K_{nt}, \tag{1}$$

where A_n is a country-specific constant that models productivity and K_n models capital which is the only factor of production. K_n is the total stock of capital, i.e. it includes physical capital, but also human capital and firm-based intangible capital such as patents. We assume that K_n is measured in the domestic currency.

Budget Constraint Climate change has a negative impact on economic growth. In order to mitigate this impact, each country controls carbon dioxide emissions by choosing an abatement strategy α_n which reduces current CO_2-emissions. This strategy is costly and thus leads to abatement expenditures $\mathcal{A}_n^\alpha$. The budget constraint of country n reads

$$Y_{nt} = \mathcal{I}_{nt} + \mathcal{A}_{nt}^\alpha + \mathcal{C}_{nt}, \tag{2}$$

i.e. output can be used to invest, to abate carbon, or to consume. Notice that $\mathcal{C}_{nt}$ is the part of output that is consumed in country n or exported to another country and consumed there. We allow for international trade and assume that the trade balance is balanced, i.e. exports $\mathcal{EX}_{nt}$ equal imports $\mathcal{IM}_{nt}$. In our framework, the exports and imports of country n are given by

$$\mathcal{EX}_{nt} = \mathcal{C}_{nt} - \mathcal{C}_{nt}^n \quad \text{and} \quad \mathcal{IM}_{nt} = \sum_{k \neq n} \mathcal{P}_{nt}^k \mathcal{C}_{kt}^n,$$

where $\mathcal{C}_k^n$ denotes the amount of consumption units produced in country k and consumed by country n. Furthermore, $\mathcal{P}_n^k$ denotes the exchange rate between country k and n, i.e. the price of the k-currency expressed in terms of the n-currency. Therefore, an even trade balance implies

$$\mathcal{C}_{nt} = \mathcal{C}_{nt}^n + \sum_{k \neq n} \mathcal{P}_{nt}^k \mathcal{C}_{kt}^n. \tag{3}$$

Capital Accumulation Following Pindyck and Wang (2013), capital accumulation in country n is given by

$$\mathrm{d}K_{nt} = \Phi_n(\mathcal{I}_{nt}, \mathcal{A}_{nt}^\alpha, K_{nt})\mathrm{d}t - \xi_n T_t K_{nt}\mathrm{d}t + \sigma_n K_{nt}\mathrm{d}W_{nt}. \tag{4}$$

We model economic damages from climate change as in Dell et al. (2009, 2012). The parameter ξ_n is a country-specific damage parameter that relates global average temperatures T_t to loss of economic growth in country n. The adjustment cost function $\Phi_n(\mathcal{I}_n, \mathcal{A}_n^\alpha, K_n)$ captures effects of depreciation and costs of installing capital and implementing an abatement policy. As in Hayashi (1982), we assume $\Phi_n(\mathcal{I}_n, \mathcal{A}_n, K_n)$ is homogenous of degree one in K_n, i.e. $\Phi_n(\mathcal{I}_n, \mathcal{A}_n, K_n) = \phi_n\left(\frac{\mathcal{I}_n}{K_n}, \frac{\mathcal{A}_n^\alpha}{K_n}\right) K_n$. We choose the following quadratic adjustment cost function

$$\phi_n\left(\frac{\mathcal{I}_n}{K_n}, \frac{\mathcal{A}_n^\alpha}{K_n}\right) = -\delta_n^K + \frac{\mathcal{I}_n}{K_n} - \frac{1}{2}\theta_n\left(\frac{\mathcal{I}_n}{K_n} + \frac{\mathcal{A}_n^\alpha}{K_n}\right)^2, \tag{5}$$

where θ_n is a positive constant that scales the adjustment costs and δ_n^K denotes the depreciation rate of capital.[2] The process $W = (W_{1t}, \ldots, W_{Nt})_{t\geq 0}$ is an N-dimensional Brownian motion, where its components are correlated via $\mathrm{d}\langle W_k, W_n\rangle = \rho_{kn}\mathrm{d}t$, for $k, n = 1, \ldots, N$. The volatility σ_n is assumed to be constant.

Abatement Costs and Economic Growth For tractability, we assume that the abatement costs $\mathcal{A}_n^\alpha$ are proportional to capital and convex in the abatement policy. More precisely, we assume

$$\mathcal{A}_{nt}^\alpha = a_n(t)\alpha_{nt}^{b_n} K_{nt}, \tag{6}$$

with $b_n > 1$ implying that the costs for the implementation of more stringent abatement policies increase disproportionately. The time-dependent coefficient $a_n(t) > 0$ captures exogenous technological progress and is assumed to decline over time.[3] We refer to a_n as the cost function trend. Combining (1), (2), (4), (5), and (6), we obtain

$$\mathrm{d}K_{nt} = K_{nt}\left[(g_n(\chi_{nt}) - \kappa_n(t, \alpha_{nt}) - \xi_n T_t)\mathrm{d}t + \sigma_n \mathrm{d}W_t^n\right], \tag{7}$$

where $\chi_n = \mathcal{C}_n/Y_n$ is the fraction of output that country n designates for consumption. Furthermore, $g_n(x) = A_n(1-x) - \frac{1}{2}\vartheta_n(1-x)^2 - \delta_n^K$ with $\vartheta_n = \theta_n A_n^2$ denotes the expected economic gross growth rate. The function $\kappa_n(t, \alpha_{nt}) = a_n(t)\alpha_{nt}^{b_n}$ models the costs of abatement relative to

[2]Homogeneous adjustment costs have been widely used in the literature, see, e.g., Hayashi (1982), Jermann (1998), Pindyck and Wang (2013).

[3]The assumptions regarding the abatement cost functions are standard in the IAM literature (e.g., DICE model).

output or capital. Therefore, the expected economic growth rate $\mu_{nt}^K = g_n(\chi_{nt}) - \kappa_n(t, \alpha_{nt}) - \xi_n T_t$ consists of three parts that can be interpreted as follows: (i) the expected gross growth rate $g_n(\chi_n)$ models the growth rate of capital in the absence of climate change, (ii) implementing an abatement strategy α reduces economic growth by $\kappa_n(t, \alpha_{nt})$, (iii) the growth rate is negatively affected by current temperatures via $\xi_n T_t$.

2.2 Climate Model

Atmospheric Carbon Dioxide Concentration The average pre-industrial concentration of carbon dioxide in the atmosphere is denoted by M^{PI}. The dynamics of the current concentration of carbon dioxide in the atmosphere are given by

$$\mathrm{d}M_t = M_t \Big[\Big(\mu_m(t) - \sum_{n=1}^{N} \alpha_{nt} \Big) \mathrm{d}t + \sigma_m \mathrm{d}W_t^m \Big]. \tag{8}$$

The carbon dioxide concentration is measured in parts-per-million (ppm). We use the notations $m_t = \log(M_t)$ and $m^{\mathrm{PI}} = \log(M^{\mathrm{PI}})$. The control variables α_n are the above mentioned abatement policies. The process $W^m = (W_t^m)_{t \geq 0}$ is a Brownian motion that is correlated with $(W_n)_{n=1}^N$ via $\mathrm{d}\langle W^m, W^n \rangle = \rho_{mn} \mathrm{d}t$, for $n = 1, \ldots, N$ and models environmental shocks on the carbon dioxide concentration. The correlation structure makes it possible that carbon dioxide emissions change if there is a shock to economic growth.[4] The volatility of these shocks σ_m is assumed to be constant. Atmospheric carbon dioxide evolves with an expected business-as-usual growth rate μ_m. In other words, μ_m is the growth rate if no country takes additional actions to reduce carbon dioxide emissions. The abatement policies α_n model how additional actions (beyond BAU) reduce carbon dioxide emissions. By definition, these abatement policies were zero in the past. If no abatement policy is chosen and the countries stick to BAU, we also use the notation M^{BAU} instead of M.

[4]By contrast to the DICE model, we do not assume that carbon dioxide emissions are directly proportional to global output, but positively correlated. Our assumption is more in line with historical data. According to IPCC (2014), less than 45% of greenhouse gas emissions in 2010 came from industrial or agricultural production. Maddison and Rehdanz (2008) examine a panel of data for evidence of a causal relationship between GDP and CO_2-emissions. They find in particular that the non-causality hypothesis that GDP does not Granger-cause CO_2-emissions cannot be rejected.

Carbon Dioxide Emissions Our dynamics of the carbon concentration M are formulated in terms of the abatement policies α_n. However, we are also interested in the resulting CO_2-emissions. To back out the implied CO_2-emissions that are consistent with (8), we now consider alternative representation of the CO_2-dynamics where – up to environmental shocks – the change in M is expressed as the difference between CO_2-emissions and the amount of carbon absorbed by natural sinks. Formally, if $\mathcal{E}_{nt}$ denotes the time-t anthropological CO_2-emissions of country n, it is reasonable to postulate that $\mathcal{E}_{nt}$ can be determined from

$$\mathrm{d}M_t = \zeta_e \sum_{n=1}^{N} \mathcal{E}_{nt}\mathrm{d}t - \delta_m(M_t - M^{\mathrm{PI}})\mathrm{d}t + M_t\sigma_m \mathrm{d}W_t^m. \tag{9}$$

Here ζ_e is a factor converting emissions into concentrations[5] and δ_m denotes the decay rate of atmospheric carbon dioxide, i.e. the speed at which CO_2 is absorbed from the atmosphere. By equating (8) and (9), we can solve for the world CO_2-emissions that are consistent with both dynamics. Assuming that the regional BAU-emissions are given by $\mathcal{E}_{nt} = \nu_n(t)\mathcal{E}_t$ where ν_n is a deterministic function,[6] the regional carbon dioxide emissions $\mathcal{E}_n$ are given by

$$\mathcal{E}_{nt} = \frac{M_t}{\zeta_e}\big[\nu_n(t)(\mu_m(t) + \delta_m) - \alpha_{nt}\big] - \nu_n(t)\frac{\delta_m}{\zeta_e}M^{\mathrm{PI}}. \tag{10}$$

Temperature Dynamics Following Nordhaus and Sztorc (2013) and Cai et al. (2015) we assume a two-layer atmosphere-ocean temperature system where temperatures are measured in degrees Celsius (°C). The temperature dynamics are given by

$$\mathrm{d}T_t = \kappa_\tau\left[\eta_\tau \log\frac{M_t}{M_t^{\mathrm{PI}}} + F^{\mathrm{ex}}(t)\right]\mathrm{d}t - \phi T_t \mathrm{d}t + \phi_{21}(T_t^o - T_t)\mathrm{d}t, \tag{11}$$

$$\mathrm{d}T_t^o = \phi_{12}(T_t - T_t^o)\,\mathrm{d}t, \tag{12}$$

where T denotes the atmospheric global average temperature increase relative to pre-industrial levels and T^o the average change in oceanic temperatures. The parameter ϕ_{ij} is the heat diffusion rate from layer i to layer j and ϕ is the rate of atmospheric temperature change by infrared radiation to space. The parameter η_τ is the radiative forcing parameter and κ_τ measures the speed at which temperatures react to changes in radiative forcing. Atmospheric temperature

[5]While carbon dioxide concentration is measured in parts-per-million (ppm), we measure emissions in gigatons of CO_2 ($GtCO_2$). ζ_e thus takes the different units into account.

[6]This can be well calibrated by the RCP 8.5 scenario.

is affected by carbon dioxide concentrations, but also by other greenhouse gases which are treated exogenous as in Nordhaus and Sztorc (2013). This is captured by the deterministic function F^{ex}.

3 Non-cooperative Game

Since every country is affected by the decisions of all other countries, the problem can be formalized as a stochastic differential game. Most popular models (such as RICE, e.g., Nordhaus and Yang (1996), Nordhaus (2010)) consider a social planner who makes the decisions for all regions in order to maximize a global welfare functional. Such a framework can be interpreted as a cooperative game and leads to a globally optimal solution. The more realistic situation is that countries make their decisions themselves (and not the social planner), which leads to a non-cooperative game in the sense of Nash (1950). In such a framework, the countries anticipate the decisions of all other countries and act individually to maximize their national welfare.

3.1 Preferences

At every point in time $t \in [0, \infty)$, each region optimally chooses a consumption strategy and an abatement policy. Every region is affected by its own decision, but also by the decisions of all other regions. We use the notation $\pi \equiv (\mathcal{C}_1^n, \ldots, \mathcal{C}_N^n, \alpha_n)_{n=1}^N$ for a given $(N+1)$-tuple of consumption-abatement strategies. Following Colacito and Croce (2013), among others, each region gains utility from consumption bundles $\mathscr{C}_n$ that are given by

$$\mathscr{C}_{nt} = \prod_{k=1}^{N} (\mathcal{C}_{kt}^n)^{\beta_k^n}, \tag{13}$$

where β_k^n denotes the weight that region n puts on the consumption good produced by region k. The weights satisfy $\sum_{k=1}^N \beta_k^n = 1$ for all $n = 1, \ldots, N$. The time-t utility index J_n^π of region n associated with a given $(N+1)$-tuple of consumption-abatement strategies π is then defined by

$$J_n^\pi(t, x) = \mathbb{E}_t \left[\int_t^\infty f_n(\mathscr{C}_{ns}, J_n^\pi(s, \mathscr{X}_s)) \mathrm{d}s \;\middle|\; \mathscr{X}_t = x \right], \tag{14}$$

where $\mathscr{X}_t = (m_t, T_t, T_t^o, K_{1t}, \dots, K_{Nt})$ is the current state of the world. Furthermore, f_n is the continuous-time Epstein-Zin aggregator for unit EIS given by

$$f_n(\mathscr{C}, J) = \begin{cases} \delta_n(1-\gamma_n)J\log\Big(\frac{\mathscr{C}}{[(1-\gamma_n)J]^{\frac{1}{1-\gamma_n}}}\Big), & \gamma_n \neq 1, \\ \delta_n \log(\mathscr{C}) - \delta_n J, & \gamma_n = 1, \end{cases}$$

For society n, the parameter $\delta_n > 0$ denotes the time-preference parameter and $\gamma_n > 1$ measures the degree of relative risk aversion.[7]

3.2 Nash Equilibrium

Each region maximizes its own utility from per-capita consumption by implementing a consumption-abatement strategy. The regions anticipate the activities of the other regions and choose admissible consumption-abatement strategies $\pi_n = (\mathcal{C}_1^n, \dots, \mathcal{C}_N^n, \alpha_n)$ in order to maximize their utility indexes J_n^π at any point in time $t \in [0, \infty)$. A Nash equilibrium is a situation where no region has a reason to deviate unilaterally from its strategy. For a precise formulation, we use the notation $(\pi_n \mid \pi_{-n}^*) \equiv (\pi_1^*, \dots, \pi_{n-1}^*, \pi_n, \pi_{n+1}^*, \dots, \pi_N^*)$:

Definition 3.1 (Nash Equilibrium). *An $(N+1)$-tuple of consumption-abatement strategies $(\pi_n^*)_{n=1}^N = (\mathcal{C}_1^{n*}, \dots, \mathcal{C}_N^{n*}, \alpha_n^*)_{n=1}^N$ is called a Nash equilibrium if for every strategy $\pi_n = (\mathcal{C}_1^n, \dots, \mathcal{C}_N^n, \alpha_n)$ and all (t, x)*

$$J_n^{(\pi_n|\pi_{-n}^*)}(t,x) \leq J_n^{\pi^*}(t,x)$$

for all regions $n = 1, \dots, N$. The indirect utility functions (syn. value functions) are given by

$$J^n(t,x) = \sup_{\pi_n} \left\{ J_n^{(\pi_n|\pi_{-n}^*)}(t,x) \right\}.$$

We use the terms *optimal strategies* and Nash equilibrium interchangeable. The optimal strategies can be determined by solving a coupled system of Hamilton-Jacobi-Bellman (HJB) equations (see, e.g., Dockner et al. (2000)). The HJB equation of region $n = 1, \dots, N$ is given

[7] Although empirical evidence suggests that $\gamma_n > 1$ is the reasonable specification for the index of relative risk aversion, it is also possible to define aggregator functions for $\gamma_n \in [0, 1]$. For $\gamma_n > 1$ society prefers early resolution of uncertainty and is eager to learn outcomes of random events before they occur. On the other hand, if $\gamma_n < 1$ society prefers late resolution of uncertainty.

by

$$0 = \sup_{\mathcal{C}_1^n,\ldots,\mathcal{C}_N^n,\alpha_n} \Big\{ J_t^n + f_n(\mathscr{C}_n, J^n) + \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \alpha_k\Big] J_m^n + \frac{1}{2}\sigma_m^2 J_{mm}^n + \phi_{12}(\tau - \tau_t^o) J_{\tau^o}^n \tag{15}$$
$$+ \left(\kappa_\tau\left[\eta_\tau(m - m^{\mathrm{PI}}) + F^{\mathrm{ex}}\right] - (\phi + \phi_{21})\tau + \phi_{21}\tau^o\right) J_\tau^n + \sum_{k=1}^N K_k \rho_{m,k}\sigma_m\sigma_k J_{K_k m}^n$$
$$+ \sum_{k=1}^N J_{K_k}^n K_k\left[g_k(\cdot,\chi_k) - \kappa_k(\cdot,\alpha_k) - \xi_k\tau\right] + \frac{1}{2}\sum_{k=1}^N\sum_{l=1}^N K_k K_l \rho_{l,k}\sigma_l\sigma_k J_{K_k K_l}^n \Big\},$$

where subscripts of J^n denote partial derivatives (e.g. $J_t^n = \partial J^n/\partial t$).

3.3 Social Cost of Carbon

Following Nordhaus and Sztorc (2013), among others, we define the social cost of carbon (SCC) as the marginal rate of substitution between atmospheric carbon dioxide and capital. Formally, the country-specific social cost of carbon is given by

$$\mathrm{SCC}_{nt} = -\frac{\partial J_t^n}{\partial M_t}\Big/\frac{\partial J_t^n}{\partial K_{nt}} \tag{16}$$

It thus measures the climate damage of capital caused by an marginal increase of time-t emissions that country n suffers. The global social cost of carbon expressed in terms of the currency of the first country is[8]

$$\mathrm{SCC} = \sum_{n=1}^N \mathcal{P}_1^n\,\mathrm{SCC}_n \tag{17}$$

and quantifies the total damage of all countries. Consequently, global SCC can be interpreted as an optimal global *carbon tax* that internalizes all negative external effects from burning carbon. By contrast, SCC_n only takes country-specific climate damages into account, i.e. implementing this country-specific carbon tax only internalizes external effects within a country.

[8] Notice that SCC_n is measured in the domestic currency of country n since the SCC_n is the derivative of K_n with respect to M and K_n is measured in this currency as well. Therefore, we must multiply all SCC except for one country (here the first one) by the exchange rate. If the first country is the US, then the global SCC is expressed in dollars.

4 Solution to the Non-cooperative Game

This section presents the main results for the non-cooperative game. In particular, we solve for a Nash-equilibrium and determine the social cost of carbon.

Theorem 4.1 (Unit EIS). *If $\gamma_n \neq 1$ for country $n \in \{1, \ldots, N\}$, then its indirect utility function is given by*

$$J^n(t, m, \tau, \tau^o, K_1, \ldots, K_N) = \frac{1}{1-\gamma_n} \left(\prod_{k=1}^{N} K_k^{\beta_k^n} \right)^{1-\gamma_n} \exp\left\{ (\gamma_n - 1)\left(\widehat{p}_m^n m + \widehat{p}_\tau^n \tau + \widehat{p}_{\tau^o}^n \tau^o\right) + p^n(t) \right\} \tag{18}$$

with

$$\widehat{p}_\tau^n = \frac{\sum_{k=1}^N \beta_k^n \xi_k}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n + \phi_{12}}}, \qquad \widehat{p}_{\tau^o}^n = \frac{\widehat{p}_\tau^n \phi_{21}}{\delta_n + \phi_{12}}, \qquad \widehat{p}_m^n = \widehat{p}_\tau^n \frac{\kappa_\tau \eta_\tau}{\delta_n}, \tag{19}$$

and p^n is given in (48)*. The optimal controls are given by*

$$\alpha_{nt}^* = \left(\frac{\widehat{p}_m^n}{\beta_n^n} \frac{1}{a_n(t) b_n} \right)^{\frac{1}{b_n - 1}} = \left(\frac{1}{\beta_n^n \delta_n} \left(\sum_{k=1}^N \beta_k^n \xi_k \right) \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n + \phi_{12}}} \frac{1}{a_n(t) b_n} \right)^{\frac{1}{b_n - 1}}, \tag{20}$$

$$\mathcal{C}_{nt}^* = \frac{\vartheta_n - A_n + \sqrt{(\vartheta_n - A_n)^2 + 4\vartheta_n \frac{\delta_n}{\beta_n^n}}}{2\vartheta_n} Y_{nt}, \tag{21}$$

$$\mathcal{C}_{kt}^{n*} = \beta_n^k \mathcal{C}_{kt}^*. \tag{22}$$

The country-specific social cost of carbon is given by

$$\mathrm{SCC}_{nt} = \frac{\widehat{p}_m^n}{\beta_n^n} \frac{K_{nt}}{M_t} = \frac{Y_{nt}}{\beta_n^n \delta_n A_n M_t} \left(\sum_{k=1}^N \beta_k^n \xi_k \right) \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n + \phi_{12}}}, \tag{23}$$

and the global SCC follows from (17). The equilibrium exchange rates are

$$\mathcal{P}_{nt}^k = \frac{\beta_k^n \mathcal{C}_{nt}^*}{\beta_n^k \mathcal{C}_{kt}^*}. \tag{24}$$

This solution constitutes a Nash equilibrium and the goods markets clear under the equilibrium exchange rates (24), i.e. a general equilibrium obtains. Condition (46) ensures that the

	γ_n	δ_n	β_n^n	ξ_k	η_τ	κ_τ	ϕ	ϕ_{12}	ϕ_{21}	σ_τ	A_n	ϑ_n	$a_n(t)$	b_n
SCC_n	0	−	−	+	+	+	−	+	−	+	0	0	0	0
α_n	0	−	−	+	+	+	−	+	−	+	0	0	−	+
$\mathcal{C}_n$	0	+	−	0	0	0	0	0	0	0	−	+	0	0
$\mathcal{I}_n$	0	NU	+	−	−	−	+	−	+	−	+	−	+	−

Table 1: Influence of the model input parameters. The table summarizes the influence of several parameters on the SCC, and optimal decisions. A positive influence is labeled by +, a negative by −, and independence by 0. NU indicates that the influence is not unique.

investments of all countries are positive.[9]

Proof. See Appendix A.1. □

Notice that the exchange rates $\mathcal{P}_{nt}^k$ take the usual form as for instance in Colacito and Croce (2013). The optimal abatement policies $(\alpha_n)_{n=1}^N$, consumption decisions, and the SCC depend on several parameters. Table 1 summarizes the qualitative effects of these input parameters on the SCC and the optimal decisions.

Optimal Abatement and the SCC The SCC does not depend on cost parameters since it measures marginal damage that is not affected by abatement costs. For all other parameters, the effects on (20) and (23) go in the same direction since higher values of the SCC induce more abatement efforts.

Effect of damage-related terms: The term $\sum_{k=1}^N \beta_k^n \xi_k$ is a weighted average of country-specific damage parameters weighted by the country's Cobb-Douglas weights. It relates to the economic impact of climate change on economic growth, i.e. it measures the severity of climate change. Intuitively, country n only cares about the damage in country k if it consumes a significant amount of the good produced in country k. This is measured by the size of the weight β_k^n scaling the damage parameter ξ_k of country k. In other words, if the weight β_k^n is small, then the abatement policy is not significantly affected.[10]

Effect of the climate system: The effect of the climate system parameters is reasonable: First, the higher the climate sensitivity parameter η_τ or the speed κ_τ at which temperatures reacts to changes in atmospheric CO_2, the higher is the SCC and, in turn, the more are the incentives

[9] See Appendix A.1 for a discussion of this condition. It is satisfied in all our calibrations.

[10] The effect of β_n^n is extensively discussed in Section 6.

to abate carbon dioxide emissions. Second, intuitively ϕ captures the speed at which the space absorbs heat from the atmosphere and thus higher values make climate change more transitory. Therefore, the abatement policy is decreasing in ϕ. Finally, the term $\frac{\phi_{21}\delta_n}{\delta_n+\phi_{12}}$ captures the impact of the temperature exchange between oceans and atmosphere. Since ϕ_{21} models heat diffusion from atmosphere to oceans, its effect is negative on abatement policies, whereas the opposite is true for ϕ_{12}.

Effect of abatement costs: The optimal abatement policies depend on the costs of abatement. Optimal abatement is more stringent if the cost function trend $a_n(t)$ is smaller. We also find that countries implement more abatement if the abatement cost function is more convex. This is because incremental improvements are relatively cheaper than drastic actions. As mentioned above, the parameters of the abatement cost function do not influence the SCC.

Effect of preference parameters: It is well-known that a higher time preference rate δ_n reduces the social cost of carbon and the demand for abatement. On the other hand, we find that risk aversion does not affect the results at all. This confirms the earlier findings of Crost and Traeger (2014) and Jensen and Traeger (2014). This is however only true in the case of unit EIS.

Optimal Consumption The optimal consumption strategies $(\mathcal{C}_1^n, \ldots, \mathcal{C}_N^n)_{n=1}^N$ are proportional to output, i.e. the countries consume and export constant fractions of their output. The total amount of optimal consumption units $\mathcal{C}_n^*$ produced in region n is increasing in the time-preference rate δ_n. A higher time-preference rate puts implicitly more weight on the present such that society cares less for the future. This reduces the demand for abatement and investment and thus increases aggregate consumption. Optimal consumption is also increasing in the capital adjustment cost parameter ϑ_n. This is because higher capital adjustment costs reduce the efficiency of investments such that consumption becomes more attractive. Similarly, a higher productivity A_n makes investments more efficient and thus reduces aggregate consumption.

Corollary 4.2 (Log-Utility)**.** *Assume $\gamma_n = 1$ for some $n = 1, \ldots, N$. The indirect utility function of region n, J^n is given by*

$$J^n(t, m, \tau, \tau^o, K_1, \ldots, K_N) = \sum_{k=1}^{N} \beta_k^n \log(K_k) - \widehat{p}_m^n m - \widehat{p}_\tau^n \tau - \widehat{p}_{\tau^o}^n \tau^o + p_{\log}^n(t), \tag{25}$$

where $\widehat{p}^n_m$, $\widehat{p}^n_\tau$, and $\widehat{p}^n_{\tau^o}$ are given as in (19) and $p^n_{\log}$ is stated in Appendix A.2. The optimal controls, the SCC, and the exchange rate are given by (20)-(24).

Proof. See Appendix A.2. □

5 Optimal Inattentiveness

This section addresses the question of how the Nash-equilibrium is affected by a changing number of countries involved. We show that under some mild regularity conditions the world-wide abatement effort decreases with the number of countries. In particular, it can in fact be optimal to have almost zero carbon abatement if the number of countries becomes large. Therefore, our model is consistent with little abatement efforts observed in reality.

Notation To compare abatement policies of models with different numbers of countries, we introduce the following notation: $\alpha_n^{(N)}$ denotes the abatement policy of region $n \in \{1, \dots, N\}$ in a model with N regions. We use similar notations for cost function trends $a_n^{(N)}(t)$. For the polar case of a global model with only one aggregated country, we drop the superscript index and write $a(t)$ instead. The structure of the CO_2-dynamics (8) suggests that we define the world abatement policy as

$$\overline{\alpha}_t^{(N)} = \sum_{n=1}^{N} \alpha_{nt}^{(N)}.$$

We consider the homogenous case where the world is successively split up in more and more homogenous countries. Homogeneity means that the countries face the same impact of climate change, release the same emissions, and have the same costs for abatement. The only parameters that are allowed to differ are the consumption weights so that every country can have a home bias which is captured by the size of $(\beta_n^n)^{(N)}$, i.e. the weight of country n on the domestic good in a setting with N countries.

We now assume that implementing an abatement strategy in the case with one country generates the same or fewer costs than in the disaggregated settings with more than one country, i.e.

$$\mathcal{A} \leq \sum_{k=1}^{N} \mathcal{P}_1^k \mathcal{A}_k^{(N)} \quad \Longleftrightarrow \quad a\overline{\alpha}^b \sum_{k=1}^{N} \mathcal{P}_1^k K_k^{(N)} \leq \sum_{k=1}^{N} a_k^{(N)} \left(\alpha_k^{(N)}\right)^b \mathcal{P}_1^k K_k^{(N)}.$$

In the homogenous case, we have $\alpha_k^{(N)} = \overline{\alpha}/N$ and $a_k^{(N)}\big(\alpha_k^{(N)}\big)^b$ is the same for all k. Therefore, we obtain

$$a_k^{(N)} \geq a \cdot N^b.$$

Then we can show the following result.

Proposition 5.1 (Abatement Limit for Homogenous Countries)**.** *Assume that the weights $(\beta_n^n)^{(N)}$ on the domestic goods are uniformly bounded away from zero. Then the optimal world abatement policies vanish as the number of countries goes to infinity, i.e.*

$$\lim_{N\to\infty} \sum_{n=1}^{N} \alpha_{nt}^{(N)} = 0 \tag{26}$$

for all $t \geq 0$.

Proof. See Appendix B. □

Proposition 5.1 shows that in a world with many countries, global abatement activities are very small if the countries do not cooperate. However the global social cost of carbon is not decreasing in the number of countries. This shows in particular that it can be optimal to have almost zero carbon abatement even if the global SCC is large. This result relaxes the well-known relation between high abatement and high SCC. It also raises the question how abatement would look like if the countries cooperated. We analyze this situation in Section 8. To illustrate Proposition 5.1 we consider an example. Figure 2 depicts the optimal emission control rate in 2015 as a function of the number of countries. It turns out that optimal abatement decreases rapidly in the number of regions. The results of the homogenous case also hold in heterogeneous settings if the countries are not becoming too diverse. In particular, we cannot allow one country to dominate the limit.[11]

6 SCC and International Trade

The countries in our model are open economies that are allowed to trade their goods with each other. Every country consumes a consumption bundle (13) potentially containing all goods which are available world-wide. Empirically, countries typically have a home bias for their own

[11] The corresponding results are available upon request.

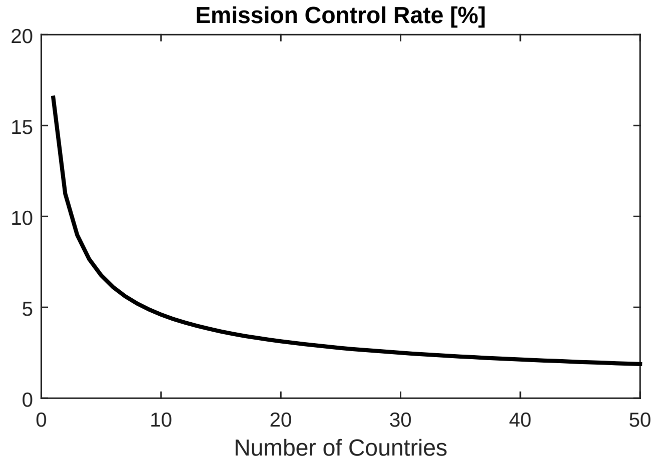

Figure 2: Optimal Emission Control Rate. The graph depicts the optimal emission control rate in 2015 as a function of the number of countries. The figure is based on our calibration of aggregated parameters that we discuss in Appendix E.

good that is captured by the weight β_n^n. In turn, this means that countries do import significant amounts and the share of trade can be calibrated to the data. We now address the question of how international trade influences the size of the SCC, both for a country and globally. By (23), the country-specific SCC can be rewritten as

$$\mathrm{SCC}_n = \frac{Y_n}{\delta_n A_n M}\left(\xi_n + \sum_{k\neq n}\frac{\beta_k^n}{\beta_n^n}\xi_k\right)\frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n+\phi_{12}}}$$

In a closed-economy without trade, the country is only consuming its own good, i.e. $\beta_n^n = 1$ and $\beta_k^n = 0$ for all $k \neq n$, and thus its SCC become

$$\mathrm{SCC}_n^{closed} = \frac{Y_n}{\delta_n A_n M}\xi_n \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n+\phi_{12}}},$$

which are the country-specific SCC of a closed economy. Therefore, the adjustment for international trade is driven by the weighted damage term in brackets

$$\sum_{k\neq n}\frac{\beta_k^n}{\beta_n^n}\xi_k,$$

which captures the relative importance of the damage in country k that delivers goods to country n. This importance increases the fraction of weights of the foreign and domestic good

β^n_k/β^n_n. Consequently, we get the following result.

Proposition 6.1 (SCC in Open and Closed Economies)**.** *The SCC in an open economy is higher than in a closed economy. More precisely,*

$$\mathrm{SCC}_n^{open} = \mathrm{SCC}_n^{closed} + \mathrm{SCC}_n^{trade}$$

where the additional SCC from trade are given by

$$\mathrm{SCC}_n^{trade} = \frac{\sum_{k\neq n}\beta^n_k\xi_k}{\beta^n_n\xi_n}\mathrm{SCC}_n^{closed} \tag{27}$$

i.e. the SCC of a closed economy is additionally scaled by the ratio of the global damages in other countries from whom country n is importing goods over the domestic damage in country n.

In our model, international trade of country n can be quantified by

$$\mathcal{T}_n = \mathcal{C}_n - \mathcal{C}^n_n = \mathcal{C}_n - \beta^n_n\mathcal{C}_n = \mathcal{C}_n(1-\beta^n_n) = \chi_n Y_n(1-\beta^n_n) \tag{28}$$

which is the amount of goods produced minus the amount of the domestic good consumed (in monetary units). In other words, this is the amount of goods that is exported to and imported from other countries. Aggregating over all countries, we obtain

$$\mathcal{T} = \sum_{n=1}^{N}\mathcal{P}^n_1\mathcal{T}_n = \sum_{n=1}^{N}\chi_n\mathcal{P}^n_1 Y_n(1-\beta^n_n),$$

which is the global trade volume expressed in the currency of the first country. Solving (28) for Y_n and substituting into (27) yields

$$\mathrm{SCC}_n^{trade} = \frac{\kappa_\tau\eta_\tau}{M}\varpi_n\mathcal{T}_n$$

where

$$\varpi_n = \frac{1}{\chi_n(1-\beta^n_n)\delta_n A_n}\frac{\sum_{k\neq n}\beta^n_k\xi_k}{\beta^n_n}\frac{1}{\delta_n+\phi+\frac{\phi_{21}\delta_n}{\delta_n+\phi_{12}}}.$$

We define the weights $w_n = \mathcal{P}^n_1\varpi_n/\sum_k\mathcal{P}^k_1\varpi_k$. Then the additional social cost of carbon from trade can be written as a weighted average of the country-specific amount of international trade

$$\mathrm{SCC}^{trade} = \sum_{n=1}^{N} \mathcal{P}_1^n \, \mathrm{SCC}_n^{trade} = \frac{\kappa_\mathcal{T} \eta_\mathcal{T}}{M} \left(\sum_{n=1}^{N} \mathcal{P}_1^n \varpi_n \right) \sum_{n=1}^{N} w_n \mathcal{T}_n,$$

which is expressed in the currency of the first country. Finally, we consider the special case where all countries are homogenous except for their weights on foreign goods β_k^n, $k \neq n$. For all other parameters we can thus drop the indices n. Furthermore, we set $\beta^{home} = \beta_n^n$, which is the homogenous weight on the domestic good. Then ϖ_n is also independent of n and we thus get

$$\varpi = \frac{1}{\chi \delta A} \frac{\xi}{\beta^{home}} \frac{1}{\delta + \phi + \frac{\phi_{21}\delta}{\delta + \phi_{12}}},$$

since $\sum_{k \neq n} \beta_k^n \xi_k = \xi \sum_{k \neq n} \beta_k^n = \xi(1 - \beta^{home})$. Hence, we arrive at the following result.

Corollary 6.2 (SCC from Trade for Homogenous Countries)**.** *If all countries are homogeneous except for their weights for foreign goods, the additional global SCC are linear in global trade* $\mathcal{T} = \sum_n \mathcal{P}_1^n \mathcal{T}_n$*:*

$$\mathrm{SCC}^{trade} = \frac{1}{\chi \delta A M} \frac{\xi}{\beta^{home}} \frac{\kappa_\mathcal{T} \eta_\mathcal{T}}{\delta + \phi + \frac{\phi_{21}\delta}{\delta + \phi_{12}}} \mathcal{T}.$$

7 Capital Transfers

There are two interesting questions that arise: Would a country be willing to do abatement in another country if this can be achieved via transfers? If the answer is positive, what is the effect on the SCC? A way to address these points in our framework is that we allow country n to donate some of their imports of country k so that country k can implement additional abatement.[12] There are two possible scenarios.

1st scenario without a commitment device. In this case, all countries are allowed to optimize consumption, abatement, and transfers simultaneously, i.e. we add transfers as an additional decision variable in (15). One can show that in such a setting optimal abatement stays the same in all countries.[13] Only the financing of the abatement policies changes since some of it might be financed by transfers. This might not be satisfying for the giving countries.

[12]This is in line with our earlier assumption that the source of abatement in a country is its output.
[13]The proof is available upon request.

2nd scenario with a commitment device. Here it is assumed that there is a commitment device that binds a receiving country to maintain its optimal abatement expenditures before transfers and to use the transfers to implement additional abatement. To determine the equilibrium in this scenario, we suggest that decisions are made in *three steps*: First, all countries optimize over consumption and abatement without transfers. This leads to the solution of the non-cooperative game presented in Section 4. Second, all countries determine whether it is optimal for them to make transfers to other countries. To do so, we assume that countries optimize over transfers and again over consumption, but keep abatement from the first step fixed. Third, countries that do not receive any transfers are allowed to reoptimize both abatement and consumption.

As explained above, we model a transfer from country n to country k in such a way that country n leaves some of its imports of good k in country k, i.e. $\mathcal{C}_k^n$ is reduced by the size of the transfer $\mathcal{T}_k^n$. Formally, the consumption bundles (13) and the abatement expenditures can be rewritten in the presence of capital transfers as

$$\widehat{\mathscr{C}}_n = \prod_{k=1}^{N} (\mathcal{C}_k^n - \mathcal{T}_k^n)^{\beta_k^n}, \qquad \widehat{\mathcal{A}}_n = \mathcal{A}_n + \sum_{k=1}^{N} \mathcal{T}_n^k. \tag{29}$$

Let $\mathcal{A}_n^*$ denote the optimal abatement expenditures of the non-cooperative game that are determined in Section 4. In the first scenario, $\mathcal{A}_n$ can be different from the optimal abatement $\mathcal{A}_n^*$ without transfers. In the second scenario, $\mathcal{A}_n$ is identical to $\mathcal{A}_n^*$ for receiving countries due to the commitment device, but it can differ for countries that do not receive any transfers. In any case, we impose the constrained that transfers must be positive, i.e. $\mathcal{T}_n^k \geq 0$ for all $n, k \in \{1, \dots, N\}$. The following proposition summarizes the effects of optimal capital transfers on the SCC in both scenarios.

Proposition 7.1 (SCC and Capital Transfers)**.** *The country-specific social cost of carbon is given by*

$$\mathrm{SCC}_{nt} = \frac{Y_{nt}}{\beta_n^n \delta_n A_n M_t} \Big(\sum_{k=1}^{N} \beta_k^n \xi_k \Big) \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \frac{\phi_{21}\delta_n}{\delta_n + \phi_{12}}},$$

i.e. initially the social cost of carbon is the same for the non-cooperative game with and without capital transfers.

Proof. See Appendix C. ∎

Recall that in the first scenario the optimal abatement policies with and without transfers are identical, i.e. transfers have no effect on global abatement activities. In the second scenario, it is obvious that receiving countries implement higher abatement policies than in the case without capital transfers. In the proof of Proposition 7.1, we also show the following for countries which do not receive transfers: These countries do not see any need to change their abatement even after transfers, i.e. the optimal abatement policies before and after transfers are identical. Therefore, no country reduces its abatement efforts, but receiving countries implement more stringent policies. Consequently, transfers with a commitment device have a positive effect on global abatement activities.

8 Cooperative Game

This section compares the solution of the non-cooperative game with a situation where a social planner (e.g. the United Nations) chooses an consumption-abatement strategy to maximize a social welfare functional.

8.1 Social Planner Problem

The social planner's utility index associated with a given $(N+1)$-tuple of consumption-abatement strategies $\pi = (\mathcal{C}_1^n, \ldots, \mathcal{C}_N^n, \alpha_n)_{n=1}^N$ is defined as the weighted sum of utility indices

$$V^\pi(t,x) = \sum_{n=1}^{N} \varphi_{nt} J_n^\pi(t,x), \tag{30}$$

where the regional utility indices J_n^π is defined in (14) and the utility weights $(\varphi_{nt})_{n=1}^N$ satisfy $\varphi_{nt} > 0$ and $\sum_{n=1}^N \varphi_{nt} = 1$ for all $t \geq 0$.

Definition 8.1 (Social Planner Solution)**.** *For a given set of utility weights $(\varphi_n)_{n=1}^N$, an $(N+1)$-tuple of consumption-abatement strategies $(\widehat{\pi}_n)_{n=1}^N = (\widehat{\mathcal{C}}_1^n, \ldots, \widehat{\mathcal{C}}_N^n, \widehat{\alpha}_n)_{n=1}^N$ is called a social planner solution if for every π and all (t,x)*

$$V^\pi(t,x) \leq V^{\widehat{\pi}}(t,x).$$

The social planner's indirect utility function is defined by

$$V(t,x)=\sup_{\pi}\left\{V^{\pi}(t,x)\right\}.$$

8.2 Optimal Strategy

We provide a closed-form solution to a social planner problem where the regions have log-utility. Due to their tractability, logarithmic preferences are commonly used in the literature (see, e.g., Golosov et al. (2014)). We assume that the time-preference rates δ_n are the same across countries, i.e. $\delta_n=\delta$ for some δ. To simplify our analysis, we consider the special case of constant utility weights.[14]

Theorem 8.2 (Social Planner Solution)**.** *The regional utility indices associated with the optimal strategy are given by*

$$V^n(t,m,\tau,\tau^o,K_1,\dots,K_N)=\sum_{k=1}^{N}\beta_k^n\log(K_k)-\widehat{p}_m^n m-\widehat{p}_\tau^n\tau-\widehat{p}_{\tau^o}^n\tau^o+p^{n,\mathrm{SP}}(t) \tag{31}$$

where $\widehat{p}_m^n$, $\widehat{p}_\tau^n$, and $\widehat{p}_{\tau^o}^n$ are given by (19) and $p^{n,\mathrm{SP}}$ is stated in (52). The social planner's indirect utility function is given by

$$V(t,m,\tau,\tau^o,K_1,\dots,K_N)=\sum_{k=1}^{N}\varphi_n V^n(t,m,\tau,\tau^o,K_1,\dots,K_N). \tag{32}$$

The optimal strategy is given by

$$\alpha_{nt}^{\mathrm{SP}}=\left(\frac{1}{\delta\sum_{\ell=1}^{N}\varphi_\ell\beta_n^\ell}\Big(\sum_{\ell=1}^{N}\varphi_\ell\sum_{k=1}^{N}\beta_k^\ell\xi_k\Big)\frac{\kappa_\tau\eta_\tau}{\delta+\phi+\phi_{21}-\frac{\phi_{21}\phi_{12}}{\delta+\phi_{12}}}\frac{1}{a_n(t)b_n}\right)^{\frac{1}{b_n-1}}, \tag{33}$$

$$\mathcal{C}_{nt}^{\mathrm{SP}}=\frac{\vartheta_n-A_n+\sqrt{(\vartheta_n-A_n)^2+4\vartheta_n\frac{\delta\varphi_n}{\sum_{\ell=1}^{N}\varphi_\ell\beta_n^\ell}}}{2\vartheta_n}Y_{nt}, \tag{34}$$

$$\mathcal{C}_{kt}^{n,\mathrm{SP}}=\beta_n^k\mathcal{C}_{kt}^{\mathrm{SP}}. \tag{35}$$

[14] A generalization for time-varying Negishi weights is available from the authors upon request.

The social cost of carbon is given by

$$\mathrm{SCC}_{nt} = \frac{Y_{nt}}{\beta_n^n \delta_n A_n M_t} \left(\sum_{k=1}^{N} \beta_k^n \xi_k \right) \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \phi_{21} - \frac{\phi_{21}\phi_{12}}{\delta_n + \phi_{12}}} \tag{36}$$

and the equilibrium exchange rates are

$$\mathcal{P}_{nt}^k = \frac{\beta_k^n \mathcal{C}_{nt}^*}{\beta_n^k \mathcal{C}_{kt}^*}. \tag{37}$$

Proof. See Appendix A.1. □

Notice that the formula for the social cost of carbon is the same as in the non-cooperative game, but due to different abatement policies, it is evaluated at a different carbon dioxide concentration and outputs. Furthermore, one can perform a similar decomposition of the SCC with and without trade as in Section 6.

8.3 Comparison with the Non-cooperative Game

Strategy Analysis Although the qualitative form of the social cost of carbon is the same for both games, there are substantial differences in the optimal consumption-abatement strategies between the cooperative and the non-cooperative game.

Corollary 8.3. *Assume $N > 1$, $\xi_k \geq 0$ for all $k = 1, \dots, N$.*

(a) The optimal abatement policies are more stringent in the social planner problem than in the non-cooperative game, i.e. $\alpha_{nt}^{\mathrm{SP}} > \alpha_{nt}^*$, for all $n = 1, \dots, N$ and all $t \geq 0$.

(b) The optimal consumption strategies are more modest in the social planner problem than in the non-cooperative game, i.e. $\chi_{nt}^{\mathrm{SP}} < \chi_{nt}^*$, for all $n = 1, \dots, N$ and all $t \geq 0$.

(c) Initially, the social cost of carbon is the same for the social planner problem and for the non-cooperative game.

Proof. The results immediately follow from Corollary 4.2 and Theorem 8.2. □

Notice that in our setting the effect of coordination on optimal investments is negative, i.e. investments are higher when coordinated actions is not possible. Coordinate action leads to rather stringent abatement policies such that the damaging effect of climate change is less pronounced. Therefore, regions shift their financial efforts from investments to abatement.

Welfare Analysis We now analyze the welfare effect that occurs when coordinated action is not possible and all countries maximize their individual utility only. For this purpose, we determine the associated wealth equivalent utility gains or losses by comparing the regional indirect utility functions in the situation of the non-cooperative game and the social planner solution. For region n the welfare effect $w_n = w_n(t, m, \tau, \tau^o, K_1, \dots, K_N)$ of coordinated abatement relative to uncoordinated abatement is defined as the solution of

$$V^n(t, m, \tau, \tau^o, K_1, \dots, K_N) = J^n(t, m, \tau, \tau^o, K_1, \dots, K_{n-1}, K_n(1 + w_n), K_{n+1}, \dots, K_N),$$

i.e. w_n is the percentage of additional capital that would make region n indifferent between the two situations. The welfare effects of coordinated or uncoordinated abatement relative to BAU is defined analogously. Notice that the regional utility indices associated with the optimal strategies only differ in the corresponding time-dependent terms.[15] The following corollary summarizes the results.

Corollary 8.4 (Welfare Effects)**.** *If a coordinated abatement policy is implemented by a social planner, then the welfare improvement of region n is*

$$\begin{aligned} w_n = \exp\Big\{ \frac{1}{\beta_n^n} \int_t^\infty \mathrm{e}^{-\delta(s-t)} \Big(\sum_{k=1}^N \beta_k^n \big[\kappa_k(s, \alpha_{ks}^*) - \kappa_k(s, \alpha_{ks}^{\mathrm{SP}}) \big] + p_m^n \sum_{k=1}^N \big[\alpha_{ks}^{\mathrm{SP}} - \alpha_{ks}^* \big) \big] \mathrm{d}s \\ + \sum_{k=1}^N \beta_k^n \Big[\delta \log \Big(\frac{\chi_k^{n,\mathrm{SP}}}{\chi_k^{n*}} \Big) + g_k(\chi_k^{\mathrm{SP}}) - g_k(\chi_k^*) \Big] \Big) \Big\} - 1. \end{aligned}$$

Proof. The statement follows immediately from Corollary 4.2 and Theorem 8.2. □

[15] This result is only true in case of constant utility weights. Taking time-varying utility weights into account yields more pronounced differences between the utility indices. These results are available from the authors upon request.

Region	Description	K_{n0}	g_{n0}	χ_n	σ_n	$\xi_n \cdot 10^3$
OECD90	OECD countries in 1990	517.7	0.024	0.84	0.0164	0.182
ASIA	Asia excl. OECD90, Middle-East and REF	168.3	0.056	0.73	0.0157	0.390
LAM	Latin America and the Caribbean	66.3	0.042	0.77	0.0157	0.208
REF	Eastern Europe and Former Soviet Union	27.4	0.025	0.79	0.0415	0.208
MAF	Middle-East and Africa	80.9	0.054	0.74	0.0235	0.546

Table 2: Definition of Regions and Callibration. The table summarizes the regions used in our numerical examples. It also reports the initial values (in 2015) of capital (trillion 2005-USD) and other relevant parameters.

9 Numerical Example

This section presents some results for the calibration discussed in Appendix E. We consider a model with five heterogeneous regions as in the representative concentration pathways (RCPs) provided by the AR5 Scenario Database of IPCC (2014).[16] Table 2 summarizes the definitions of these regions and the remaining parameters. We determine the optimal abatement policy, the resulting emissions, the evolution of real GDP as well as the evolution of the carbon dioxide concentration and global average temperature changes over the next 100 years for various scenarios.

9.1 BAU Scenario

Figure 3 shows the median business-as-usual (BAU) evolution of the key variables. The five areas in (a) and (c) depict emissions and GDP of the five regions that are the OECD90 region (darkest area), followed by ASIA, LAM, REF, and MAF (lightest area). Details on the parameter choice can be found in Appendix E. Under BAU, our calibrated model predicts annual global CO_2-emissions of about 106 $GtCO_2$ by the end of the century which is close to the predictions of the DICE model (103 $GtCO_2$) and the RCP 8.5 scenario (106 $GtCO_2$). These emissions yield an atmospheric carbon dioxide concentration of approximately 800 ppm in 2100 and an average temperature increase of about 3.9°C compared to 3.8°C in DICE-2013R.

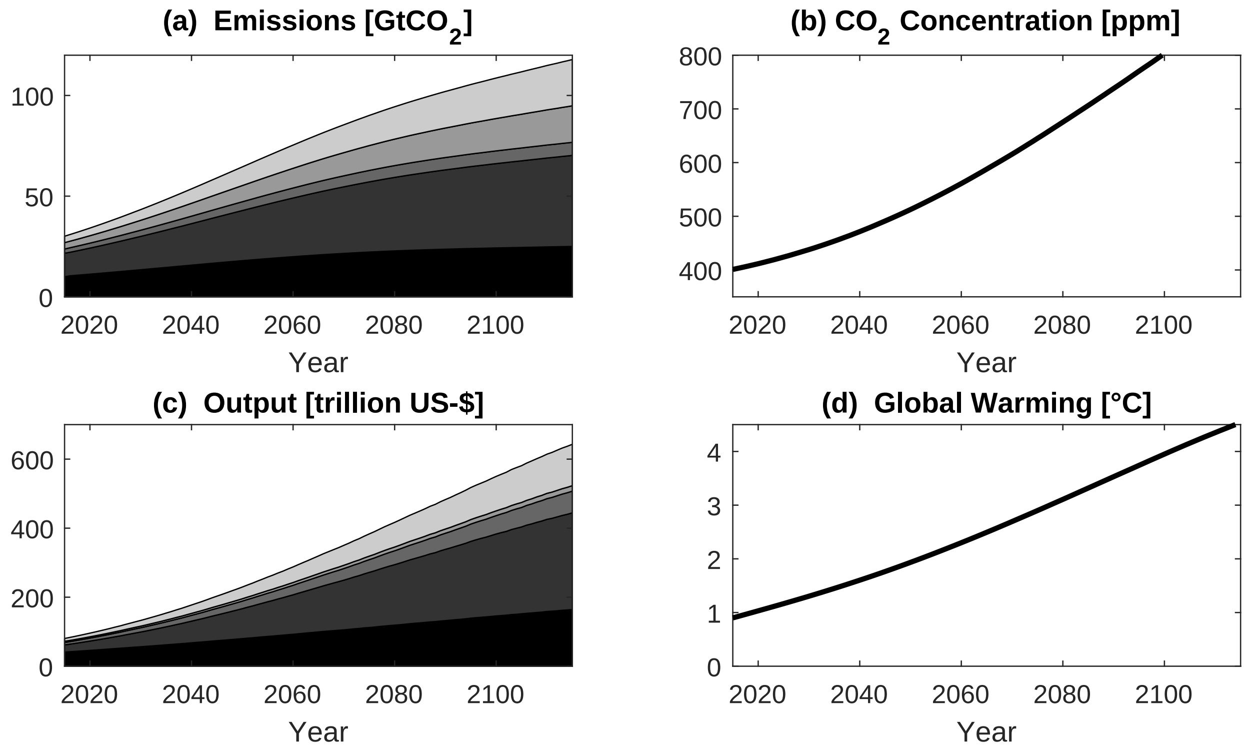

Figure 3: BAU Evolution. Based on the calibration, the graphs depict the BAU evolution of (a) carbon dioxide emissions, (b) atmospheric carbon dioxide, (c) output, (d) global average temperature increase. The five areas in (a) and (c) depict emissions and output of the five regions. These are the OECD90 region (darkest area), followed by ASIA, LAM, REF, and MAF (lightest area).

9.2 Non-cooperative Game

Figure 4 presents the results for the non-cooperative game. It turns out that due to the uncoordinated action the optimal abatement policies are not sufficient to stabilize the climate system. For instance, optimally controlled emissions lead to a global average temperature increase of 3.5°C by the year 2100 compared to a BAU temperature increase of 3.9°C. Therefore, the temperature increase is almost the same in the non-cooperative game as in the BAU scenario. Furthermore, Table 3 reports the social cost of carbon. For 2015, we obtain a global SCC of 18.65 US-dollars, which is well in the range of other models such as the DICE model. Notice that international trade contributes significantly to the SCC. Ignoring the effects of international trade would predict a social cost of carbon of only 14.46, which is about 22.5% smaller

[16]The database is available at https://tntcat.iiasa.ac.at/AR5DB.

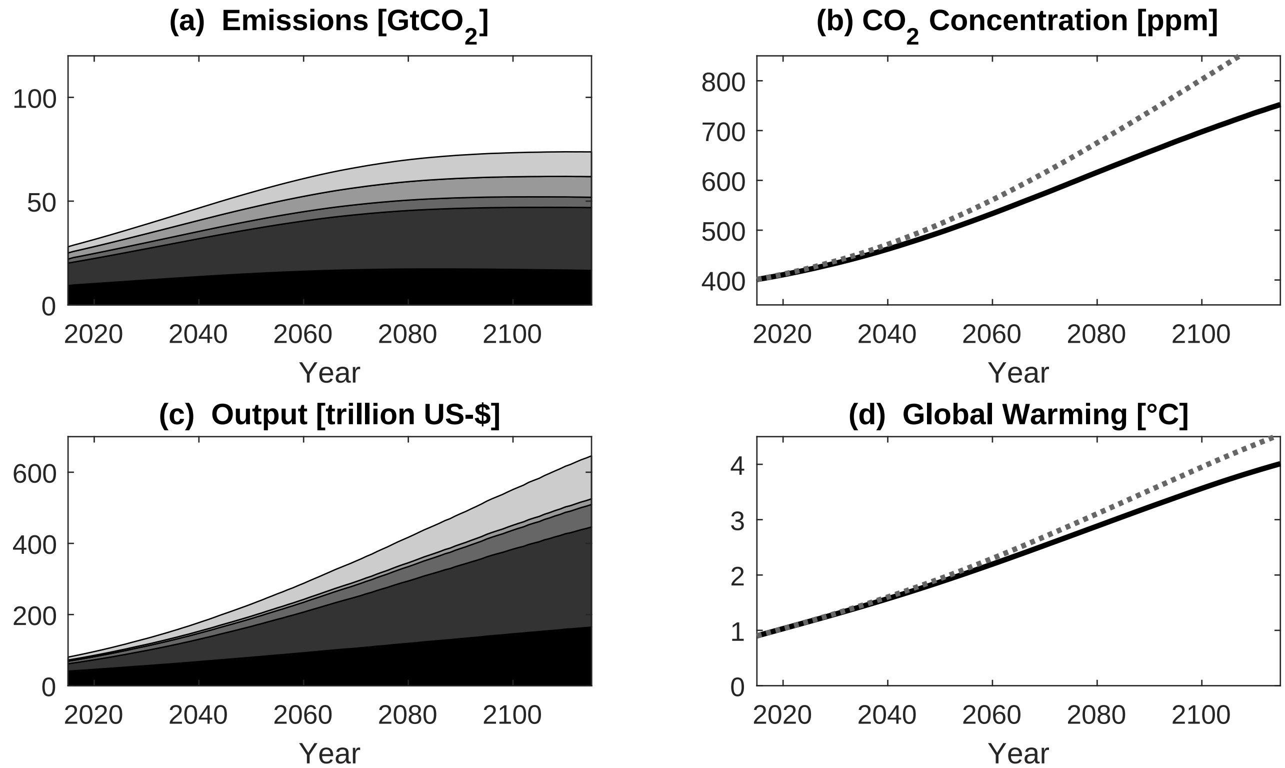

Figure 4: Solution to Non-cooperative Game. Based on the calibration, the graphs depict the optimally controlled evolution of (a) carbon dioxide emissions, (b) atmospheric carbon dioxide, (c) output, (d) global average temperature increase. The five areas in (a) and (c) represent emissions and output from the five regions under consideration. The areas represents the OECD90 region (darkest area), followed by ASIA, REF, LAM, and MAF (lightest area).

than in the benchmark case.

9.3 Social Planner Solution

For illustrative purposes, we solve a social planner problem with constant utility weights that mirror the current distribution of population, i.e. we choose $\varphi_n = P_n / \sum_{k=1}^{N} P_k$ where P_n denotes the current population of region n. Figure 5 depicts the evolution of the key variables for this set of utility weights.[17] Graphs (a) and (b) show that countries act more drastically than in the non-cooperative game. Following the globally optimal abatement policy, the median

[17]Notice that in this example we do not determine the endogenous time-varying Negishi-weights which would lead to a Pareto improvement. These results are available from the authors upon request. In our example the welfare effects for OECD90 and REF are negative as these regions are relatively little affected by climate change and have small populations. Therefore, the social planner puts little weight on these regions.

Region		2015	2035	2055	2075	2095	2115	2150	2200
OECD90	SCC_n [$/tCO$_2$]	8.32	11.35	13.52	15.17	16.66	17.45	19.75	24.80
	SCC_n^{trade}[$/tCO$_2$]	2.05	2.80	3.34	3.75	4.11	4.31	4.88	6.12
ASIA	SCC_n [$/tCO$_2$]	5.17	11.65	19.22	26.47	32.19	36.22	41.72	50.39
	SCC_n^{trade}[$/tCO$_2$]	0.80	1.80	2.97	4.10	4.99	5.61	6.46	7.81
LAM	SCC_n [$/tCO$_2$]	1.33	2.44	3.55	4.55	5.41	6.04	7.20	9.53
	SCC_n^{trade}[$/tCO$_2$]	0.41	0.76	1.10	1.41	1.68	1.87	2.24	2.96
REF	SCC_n [$/tCO$_2$]	0.64	0.90	1.09	1.29	1.43	1.49	1.86	2.61
	SCC_n^{trade}[$/tCO$_2$]	0.19	0.26	0.31	0.37	0.41	0.42	0.53	0.74
MAF	SCC_n [$/tCO$_2$]	3.19	6.97	11.56	16.03	19.84	22.91	27.85	33.90
	SCC_n^{trade}[$/tCO$_2$]	0.74	1.62	2.68	3.72	4.60	5.31	6.46	7.86
Global	SCC [$/tCO$_2$]	18.65	33.31	48.94	63.51	75.53	84.11	98.38	121.23
	SCC_n^{trade}[$/tCO$_2$]	4.19	7.24	10.40	13.35	15.79	17.52	20.57	25.49

Table 3: SCC for the Non-cooperative Game. The table reports the median evolution of the regional and global social cost of carbon for selected years. It also reports the part of the SCC that results from international trade.

global CO_2-emissions peak in the year 2070 and CO_2-concentration in 2120. From this point onwards, the decay capacity of natural carbon dioxide sinks such as oceans and forests exceeds anthropological emissions, and thus the carbon concentration declines. Furthermore, following the abatement strategies leads to a median increase in the world temperature of 2.9°C by the year 2100, which is well in line with the optimal temperature path in DICE (see Graph (d)). Table 4 reports the social cost of carbon which are systematically higher than in the non-cooperative game. As in the non-cooperative game international trade contributes significantly to the social cost of carbon.

10 Conclusion

This paper derives the optimal abatement decisions of multiple countries in a non-cooperative game-theoretical framework that takes the repeated-game feature of this problem into account. All countries are open economies, i.e. we allow for international trade between the countries. We offer a tractable continuous-time setup leading to a stochastic differential game that can be solved explicitly. In fact, we provide closed-form solutions for all key decision variables such as consumption, abatement and investment of each country. Furthermore, we can explicitly

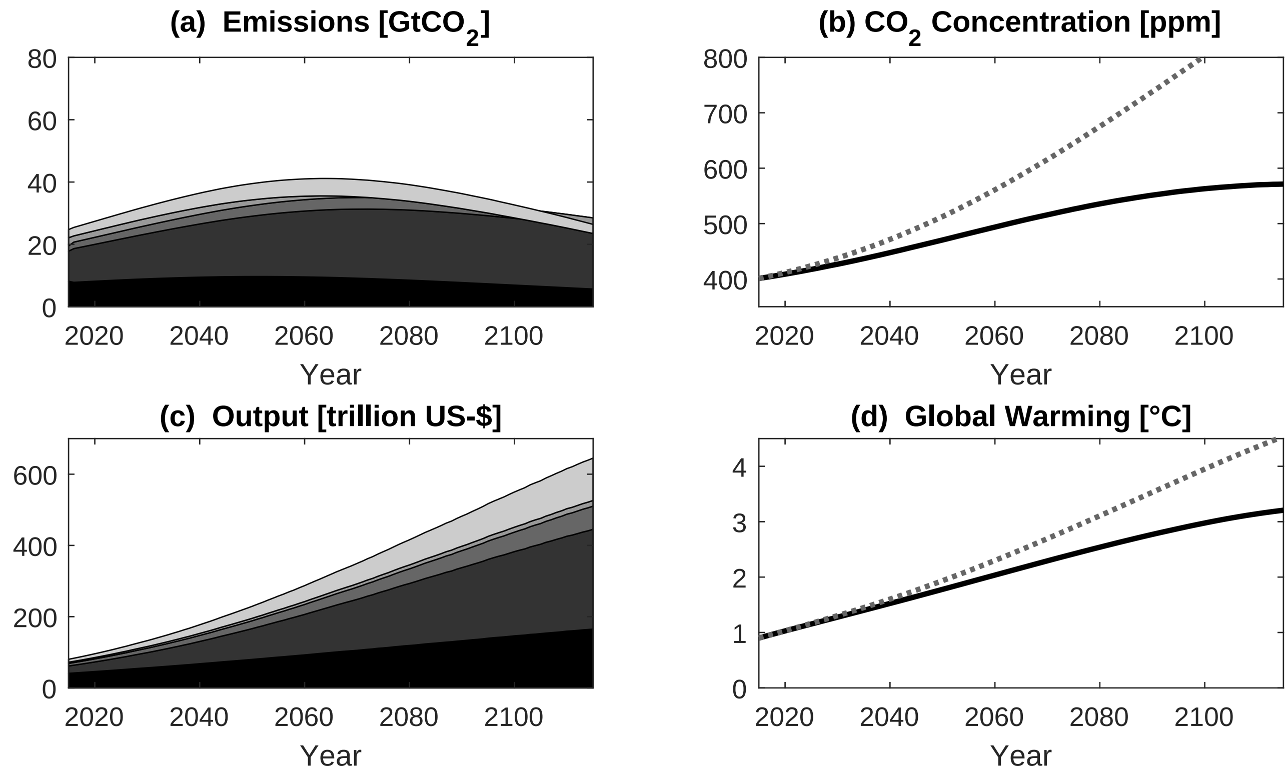

Figure 5: Social Planner Solution. Based on the calibration, the graphs depict the optimally controlled evolution of (a) carbon dioxide emissions, (b) atmospheric carbon dioxide, (c) output, (d) global average temperature increase. The five areas in (a) and (c) represent emissions and output of the five regions. These are the OECD90 region (darkest area), followed by ASIA, LAM, REF, and MAF (lightest area).

quantify the social cost of carbon. One important finding is that the SCC is increasing in the trade volume, both for a specific country and globally. This shows that determining the SCC in models without trade can significantly underestimate the SCC. Our framework can also be applied to quantify country-specific welfare losses of uncoordinated carbon actions. The particular size of a country-specific welfare loss can guide policy makers that are willing to take actions against the effects of climate change.

Region	w_n		2015	2035	2055	2075	2095	2115	2150	2200
OECD90	-0.39%	SCC_n [$/t$CO_2$]	8.32	11.66	14.50	17.29	20.44	23.28	31.20	50.88
		SCC_n^{trade}[$/t$CO_2$]	2.05	2.88	3.58	4.27	5.04	5.74	7.70	12.56
ASIA	1.35%	SCC_n [$/t$CO_2$]	5.17	11.95	20.57	30.13	39.54	48.49	66.64	106.32
		SCC_n^{trade}[$/t$CO_2$]	0.80	1.86	3.18	4.66	6.13	7.51	10.32	16.47
LAM	0.63%	SCC_n [$/t$CO_2$]	1.33	2.44	3.55	4.55	5.41	6.04	7.20	9.53
		SCC_n^{trade}[$/t$CO_2$]	0.41	0.76	1.10	1.41	1.68	1.87	2.24	2.96
REF	-2.40%	SCC_n [$/t$CO_2$]	0.64	0.90	1.09	1.29	1.43	1.49	1.86	2.61
		SCC_n^{trade}[$/t$CO_2$]	0.19	0.26	0.31	0.37	0.41	0.42	0.53	0.74
MAF	2.01%	SCC_n [$/t$CO_2$]	3.19	6.97	11.56	16.03	19.84	22.91	27.85	33.90
		SCC_n^{trade}[$/t$CO_2$]	0.74	1.62	2.68	3.72	4.60	5.31	6.46	7.86
Global	1.01%	SCC [$/t$CO_2$]	18.65	34.27	52.73	72.99	93.87	114.25	160.03	261.48
		SCC_n^{trade}[$/t$CO_2$]	4.19	7.46	11.23	15.36	19.65	23.86	33.53	55.12

Table 4: SCC for the Social Planner Solution. The table reports the median evolution of the regional and global social cost of carbon for selected years. Additionally it reports the welfare effects compared to the non-cooperative game.

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A Proofs for Section 4

A.1 Proof for Theorem 4.1 (Unit EIS)

We consider the HJB equation (15) of country n from the main text. Our first goal is to reformulate this equation in terms of the controls α_n and

$$\chi_k^n = \mathcal{C}_k^n / Y_k, \tag{38}$$

which is the definition of the fraction of the k-th country output that is consumed in country n. We thus rewrite (13) as follows

$$\mathscr{C}_n = \prod_{k=1}^{N} (\chi_k^n Y_k)^{\beta_k^n} = \prod_{k=1}^{N} (\chi_k^n A_k K_k)^{\beta_k^n}.$$

Furthermore, dividing (3) by Y_n yields $\chi_n = \chi_n^n + \sum_{k \neq n} \chi_k^n \mathcal{P}_n^k Y_k / Y_n$. We now conjecture that the equilibrium exchange rate $\mathcal{P}_n^k$ is of the form

$$\mathcal{P}_n^k = \omega_n^k Y_n / Y_k \tag{39}$$

for constants ω_n^k that clear the good markets and will be determined later on. Applying conjecture (39), we get an alternative representation of the budget constraint (13) for consumption expressed in terms of χ_n and χ_ℓ^n:

$$\chi_n = \sum_{\ell=1}^{N} \omega_n^\ell \chi_\ell^n \tag{40}$$

with $\omega_n^n = 1$. Therefore, we can rewrite (15) as follows

$$\begin{aligned} 0 = \sup_{\chi_1^n, \dots, \chi_N^n, \alpha_n} \Big\{ & J_t^n + f_n\Big(\prod_{k=1}^{N} (\chi_k^n A_k K_k)^{\beta_k^n}, J^n \Big) + \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^{N} \alpha_k \Big] J_m^n + \frac{1}{2}\sigma_m^2 J_{mm}^n + \phi_{12}(\tau - \tau_t^o) J_{\tau^o}^n \\ & + \Big(\kappa_\tau \big[\eta_\tau (m - m^{\mathrm{PI}}) + F^{\mathrm{ex}} \big] - (\phi + \phi_{21})\tau + \phi_{21}\tau^o \Big) J_\tau^n + \sum_{k=1}^{N} K_k \rho_{m,k} \sigma_m \sigma_k J_{K_k m}^n \\ & + \sum_{k=1}^{N} J_{K_k}^n K_k \Big[g_k\Big(\cdot, \sum_{\ell=1}^{N} \omega_k^\ell \chi_\ell^k \Big) - \kappa_k(\cdot, \alpha_k) - \xi_k \tau \Big] + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} K_k K_\ell \rho_{\ell,k} \sigma_\ell \sigma_k J_{K_k K_\ell}^n \Big\}. \end{aligned} \tag{41}$$

The first-order conditions for abatement α_n and consumption χ_ν^n read

$$\frac{\partial \kappa_n(\cdot, \alpha_n)}{\partial \alpha_n} = -\frac{J_m^n}{J_{K_n}^n}\frac{1}{K_n} \tag{42}$$

$$J_{K_n}^n K_n \frac{\partial g_n\left(\cdot, \sum_{\ell=1}^N \omega_n^\ell \chi_\ell^n\right)}{\partial x}\omega_n^\nu = \delta_n(\gamma_n - 1)J^n \beta_\nu^n \frac{1}{\chi_\nu^n} \tag{43}$$

Multiplying (43) by χ_ν^n, summing over $\nu = 1, \dots, N$, and using (40) leads to an algebraic equation for χ_n:

$$\frac{\partial g_n(\cdot, \chi_n)}{\partial x}\chi_n = \delta_n(\gamma_n - 1)\frac{J^n}{J_{K_n}^n}\frac{1}{K_n}.$$

The fractions χ_ν^n can then be found by substituting back into (43). To determine the indirect utility function J^n, we now substitute the conjecture

$$J^n(t, m, \tau, \tau^o, K_1, \dots, K_N) = \frac{1}{1-\gamma_n}\prod_{k=1}^N K_k^{(1-\gamma_n)\beta_k^n} \exp\left\{p_m^n m + p_\tau^n \tau + p_{\tau^o}^n \tau^o + p^n(t)\right\}, \tag{44}$$

for some regional-specific constants $p_m^n, p_\tau^n, p_{\tau^o}^n$ into the HJB system:

$$\begin{aligned}
0 = \sup_{\chi_1^n, \dots, \chi_N^n, \alpha_n} \Big\{ & \dot{p}_t^n + \delta_n(1-\gamma_n)\sum_{k=1}^N \beta_k^n \log\left(\chi_k^n A_k\right) - \delta_n\left[p_m^n m + p_\tau^n \tau + p_{\tau^o}^n \tau^o\right] - \delta_n p^n \\
& + p_m^n\Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \alpha_k\Big] + p_\tau^n\left(\kappa_\tau\left[\eta_\tau(m - m^{\mathrm{PI}}) + F^{\mathrm{ex}}\right] - (\phi + \phi_{21})\tau + \phi_{21}\tau^o\right) + p_{\tau^o}^n \phi_{12}(\tau - \tau^o) \\
& + (1-\gamma_n)\sum_{k=1}^N \beta_k^n\Big[g_k\Big(\cdot, \sum_{\ell=1}^N \omega_k^\ell \chi_\ell^k\Big) - \kappa_k(\cdot, \alpha_k) - \xi_k \tau\Big] + \frac{1}{2}\sigma_m^2 (p_m^n)^2 + (1-\gamma_n)p_m^n \sigma_m \sum_{k=1}^N \beta_k^n \rho_{m,k}\sigma_k \\
& - \frac{1}{2}(1-\gamma_n)\sum_{k=1}^N \beta_k^n \sigma_k^2 + \frac{1}{2}(1-\gamma_n)^2 \sum_{k=1}^N \sum_{\ell=1}^N \beta_k^n \beta_\ell^n \rho_{\ell,k}\sigma_\ell \sigma_k \Big\}.
\end{aligned}$$

We choose p_m^n, p_τ^n, $p_{\tau^o}^n$ such that the separation holds true, i.e.

$$p_\tau^n = \frac{(\gamma_n - 1)\sum_{k=1}^N \beta_k^n \xi_k}{\delta_n + \phi + \phi_{21} - \frac{\phi_{21}\phi_{12}}{\delta + \phi_{12}}}, \qquad p_{\tau^o}^n = \frac{p_\tau^n \phi_{21}}{\delta_n + \phi_{12}}, \qquad p_m^n = p_\tau^n \frac{\kappa_\tau \eta_\tau}{\delta_n}.$$

The simplified HJB-system is thus given by:

$$
\begin{aligned}
0 = \sup_{\chi_1^n,\dots,\chi_N^n,\alpha_n} \Big\{ & \frac{\partial p^n}{\partial t} - \delta_n p^n + \delta_n(1-\gamma_n)\sum_{k=1}^N \beta_k^n \log\left(\chi_k^n A_k\right) + p_m^n \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \alpha_k\Big] \\
& + (1-\gamma_n)\sum_{k=1}^N \beta_k^n \Big[g_k\Big(\cdot, \sum_{\ell=1}^N \omega_k^\ell \chi_\ell^k\Big) - \kappa_k(\cdot,\alpha_k)\Big] + p_\tau^n \kappa_\tau \Big[-\eta_\tau m^{\mathrm{PI}} + F^{\mathrm{ex}}\Big] \\
& + \frac{1}{2}\sigma_m^2 (p_m^n)^2 + (1-\gamma_n) p_m^n \sigma_m \sum_{k=1}^N \beta_k^n \rho_{m,k}\sigma_k \\
& - \frac{1}{2}(1-\gamma_n)\sum_{k=1}^N \beta_k^n \sigma_k^2 + \frac{1}{2}(1-\gamma_n)^2 \sum_{k=1}^N \sum_{\ell=1}^N \beta_k^n \beta_\ell^n \rho_{\ell,k}\sigma_\ell\sigma_k \Big\}.
\end{aligned}
$$

Using the conjecture (44) the first-order condition (42) of the optimal abatement policy becomes

$$
\beta_n^n \frac{\partial}{\partial \alpha_n}\kappa_n(t,\alpha_n) = \frac{p_m^n}{\gamma_n - 1}.
$$

Therefore, the optimal abatement policies are given by (20). Similarly, the first-order conditions (43) for the optimal consumption rates imply the following system of equations

$$
0 = \delta_n \beta_k^n - \beta_n^n \Big[A_n - \vartheta_n\Big(1 - \sum_{\ell=1}^N \omega_n^\ell \chi_\ell^n\Big)\Big]\omega_n^k \chi_k^n, \qquad k = 1,\dots,N,
$$

where the solutions are

$$
\chi_n^* = \frac{\vartheta_n - A_n + \sqrt{(\vartheta_n - A_n)^2 + 4\vartheta_n \frac{\delta_n}{\beta_n^n}}}{2\vartheta_n}, \qquad \chi_k^{n*} = \frac{\beta_k^n}{\omega_n^k}\chi_n^*, \qquad k = 1,\dots,N. \tag{45}
$$

To avoid degenerate cases with negative investments, we must impose the restriction on the parameters that

$$
\frac{\mathcal{A}_n^\alpha}{Y} + \frac{\mathcal{C}_n}{Y} = \frac{\kappa_n(\cdot,\alpha_n^*)}{A_n} + \chi_n^* \le 1. \tag{46}
$$

This condition is satisfied in all our calibrations. In all our calibrations the consumption rate χ_n^* is much bigger than the relative abatement cost, $\kappa_n(\cdot,\alpha_n^*)/A_n$. Consequently, condition (46) is typically satisfied if χ_n^* is sufficiently below one. Notice that for $\delta_n \in [0, A_n\beta_n^n]$ we obtain

$$
\frac{\vartheta_n - A_n}{\vartheta_n} \le \chi_n^* \le 1.
$$

Hence, a necessary condition for $\chi_n^* < 1$ is that

$$\frac{\delta_n}{\beta_n^n} < A_n. \tag{47}$$

Finally, the function p^n is given by

$$p^n(t) = \int_t^\infty \mathrm{e}^{-\delta_n(s-t)} \eta_n(s) \mathrm{d}s, \tag{48}$$

where the deterministic function η_n is given by

$$\begin{aligned}
\eta_n = {} & \delta_n(1-\gamma_n) \sum_{k=1}^N \beta_k^n \log\left(\chi_k^{n*} A_k\right) + p_m^n \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \alpha_k^*\Big] + p_\tau^n \kappa_\tau \Big[-\eta_\tau m^{\mathrm{PI}} + F^{\mathrm{ex}}\Big] \\
& + (1-\gamma_n) \sum_{k=1}^N \beta_k^n \Big[g_k(\chi_k^*) - \kappa_k(\cdot, \alpha_k^*)\Big] + \frac{1}{2}\sigma_m^2 (p_m^n)^2 + (1-\gamma_n) p_m^n \sigma_m \sum_{k=1}^N \beta_k^n \rho_{m,k} \sigma_k \\
& - \frac{1}{2}(1-\gamma_n) \sum_{k=1}^N \beta_k^n \sigma_k^2 + \frac{1}{2}(1-\gamma_n)^2 \sum_{k=1}^N \sum_{\ell=1}^N \beta_k^n \beta_\ell^n \rho_{\ell,k} \sigma_\ell \sigma_k.
\end{aligned}$$

This solution constitutes a Nash equilibrium, but good markets do not necessarily clear. Market clearing of the good produced in country n obtains if supply $\mathcal{C}_n$ is equal to the demands $\mathcal{C}_n^k$, $k = 1, \ldots, N$, i.e.

$$\mathcal{C}_n = \sum_{k=1}^N \mathcal{C}_n^k \qquad \Longleftrightarrow \qquad \chi_n = \sum_{k=1}^N \chi_n^k \tag{49}$$

where we use definition (38). Choosing the constant in the equilibrium exchange rate to be

$$\omega_n^k = \frac{\chi_n^{k*}}{\chi_k^{n*}}$$

and substituting into (40) we obtain market clearing (49). The first-order condition for consumption can be rewritten as

$$\chi_k^{n*} = \frac{\beta_k^n}{\omega_n^k} \chi_n^* = \frac{\beta_k^n \chi_k^{n*}}{\chi_n^{k*}} \chi_n^* \qquad \Longrightarrow \qquad \chi_n^{k*} = \beta_k^n \chi_n^*$$

Therefore, the equilibrium exchange rate (39) is

$$\mathcal{P}_n^k = \frac{\chi_n^{k*}}{\chi_k^{n*}} \frac{Y_n}{Y_k} = \frac{\beta_k^n \chi_n^*}{\beta_n^k \chi_k^*} \frac{Y_n}{Y_k} = \frac{\beta_k^n \mathcal{C}_n^*}{\beta_n^k \mathcal{C}_k^*}$$

Notice that also $\mathcal{P}_n^k = \mathcal{C}_n^{k*}/\mathcal{C}_k^{n*}$. Furthermore, the SCC (23) directly follows from

$$\mathrm{SCC}_n = \frac{\partial J^n}{\partial M} \Big/ \frac{\partial J^n}{\partial K_n} = \frac{\partial J^n}{\partial m}\frac{\partial m}{\partial M} \Big/ \frac{\partial J^n}{\partial K_n}.$$

A.2 Proof of Corollary 4.2 (Log Utility)

For log-utility, we substitute the conjecture (25)

$$J^n(t, m, \tau, \tau^o, \lambda, K_1, \dots, K_N) = \sum_{k=1}^{N} \beta_k^n \log(K_k) - p_m^n m - p_\tau^n \tau - p_{\tau^o}^n \tau^o + p_{\log}^n(t),$$

into the HJB equation and repeat the same steps as above. The function $p_{\log}^n$ is given by $p_{\log}^n(t) = \int_t^\infty \mathrm{e}^{-\delta_n(s-t)} \eta_n(s) \mathrm{d}s$ where

$$\begin{aligned}
\eta_n = \delta_n \sum_{k=1}^{N} \beta_k^n \log\left(\chi_k^{n*} A_k\right) - p_m^n \Big[\mu_m - \sum_{k=1}^{N} \alpha_k^* - \frac{1}{2}\sigma_m^2\Big] - p_\tau^n \kappa_\tau \Big[-\eta_\tau m^{\mathrm{PI}} + F^{\mathrm{ex}}\Big] \\
+ \sum_{k=1}^{N} \beta_k^n \Big[g_k(\chi_k^*) - \kappa_k(\cdot, \alpha_k^*) - \frac{1}{2}\sigma_k^2\Big].
\end{aligned}$$

B Proof of Proposition 5.1

We consider a model with N identical countries. The optimal world abatement policy is

$$\begin{aligned}
\overline{\alpha}^{(N)} &= \sum_{n=1}^{N} \left(\Big(\frac{1}{\delta_n} \sum_{k=1}^{N} \beta_k^n \xi_k\Big) \frac{1}{\beta_n^n} \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \phi_{21} - \frac{\phi_{21}\phi_{12}}{\delta_n + \phi_{12}}} \frac{1}{a_n^{(N)}(t) b} \right)^{\frac{1}{b-1}}, \\
&\leq \sum_{n=1}^{N} \left(\Big(\frac{1}{\delta_n} \sum_{k=1}^{N} \beta_k^n \xi_k\Big) \frac{1}{\beta_n^n} \frac{\kappa_\tau \eta_\tau}{\delta_n + \phi + \phi_{21} - \frac{\phi_{21}\phi_{12}}{\delta_n + \phi_{12}}} \frac{1}{a^{(1)}(t) b} \right)^{\frac{1}{b-1}} \left(\frac{1}{N}\right)^{\frac{b}{b-1}}, \\
&= \overline{\alpha}^{(1)} \left(\frac{1}{\beta_n^n}\right)^{\frac{1}{b-1}} N^{-\frac{1}{b-1}}
\end{aligned}$$

This implies $\lim_{N\to\infty} \sum_{n=1}^{N} \alpha_n^{(N)} = 0$ as $b > 1$. □

C Proof of Proposition 7.1

In the first scenario, the result is obvious since abatement policies stay the same. To prove the result in the second scenario, we follow the three-step procedure explained in the main text.

1st step. The solution to the optimization problem of the first step is given by Theorem 4.1.

2nd step. To formulate the optimization problem of the second step (over consumption and transfers), we express transfers in relative terms and use the notation $\varepsilon_k^n = \mathcal{T}_k^n/\mathcal{C}_k^n$. By (6) and (29), the abatement policy of country k after transfers is given by

$$\widehat{\alpha}_k = \left(\frac{a_k(t)(\alpha_k^*)^{b_k} + \sum_{\ell \neq k} A_k \chi_k^\ell \varepsilon_k^\ell}{a_k(t)} \right)^{1/b_k}.$$

The HJB equation of country n thus reads

$$\begin{aligned} 0 = \sup_{\chi_1^n,\dots,\chi_N^n,(\varepsilon_k^n)_{k\neq n}} &\Big\{ J_t^n + f_n\Big(\prod_{k=1}^N (\chi_k^n A_k K_k (1-\varepsilon_k^n))^{\beta_k^n}, J^n\Big) + \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \widehat{\alpha}_k\Big] J_m^n + \frac{1}{2}\sigma_m^2 J_{mm}^n \\ &+ \phi_{12}(\tau - \tau_t^o) J_{\tau^o}^n + \big(\kappa_\tau\big[\eta_\tau(m - m^{\mathrm{PI}}) + F^{\mathrm{ex}}\big] - (\phi + \phi_{21})\tau + \phi_{21}\tau^o\big) J_\tau^n + \sum_{k=1}^N K_k \rho_{m,k}\sigma_m\sigma_k J_{K_k m}^n \\ &+ \sum_{k=1}^N J_{K_k}^n K_k \Big[g_k\Big(\cdot, \sum_{\ell=1}^N \omega_k^\ell \chi_\ell^k\Big) - \kappa_k(\cdot, \alpha_k^*) - \xi_k \tau\Big] + \frac{1}{2}\sum_{k=1}^N\sum_{\ell=1}^N K_k K_\ell \rho_{\ell,k}\sigma_\ell\sigma_k J_{K_k K_\ell}^n \Big\}. \end{aligned}$$

To determine the indirect utility function J^n, we now substitute the conjecture (44) into the HJB system and choose p_m^n, p_τ^n, $p_{\tau^o}^n$ such that the separation holds true, i.e.

$$p_\tau^n = \frac{(\gamma_n - 1)\sum_{k=1}^N \beta_k^n \xi_k}{\delta_n + \phi + \phi_{21} - \frac{\phi_{21}\phi_{12}}{\delta + \phi_{12}}}, \qquad p_{\tau^o}^n = \frac{p_\tau^n \phi_{21}}{\delta_n + \phi_{12}}, \qquad p_m^n = p_\tau^n \frac{\kappa_\tau \eta_\tau}{\delta_n},$$

which are exactly the same sensitivities as in the case without money transfers. The simplified HJB-system is thus given by:

$$\begin{aligned} 0 = \sup_{\chi_1^n,\dots,\chi_N^n,(\varepsilon_k^n)_{k\neq n}} &\Big\{ \frac{\partial p^n}{\partial t} - \delta_n p^n + \delta_n(1-\gamma_n)\sum_{k=1}^N \beta_k^n \log\left(\chi_k^n A_k (1-\varepsilon_k^n)\right) \\ &+ p_m^n \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^N \left(\frac{a_k(t)(\alpha_{kt}^*)^{b_k} + \sum_{\ell\neq k} A_k \chi_k^\ell \varepsilon_k^\ell}{a_k(t)} \right)^{1/b_k} \Big] \end{aligned} \tag{50}$$

$$+ (1-\gamma_n)\sum_{k=1}^{N}\beta_k^n\Big[g_k\Big(\cdot,\sum_{\ell=1}^{N}\omega_k^\ell\chi_\ell^k\Big)-\kappa_k(\cdot,\alpha_k^*)\Big]+p_\tau^n\kappa_\tau\Big[-\eta_\tau m^{\mathrm{PI}}+F^{\mathrm{ex}}\Big]+\frac{1}{2}\sigma_m^2(p_m^n)^2$$
$$+ (1-\gamma_n)p_m^n\sigma_m\sum_{k=1}^{N}\beta_k^n\rho_{m,k}\sigma_k-\frac{1}{2}(1-\gamma_n)\sum_{k=1}^{N}\beta_k^n\sigma_k^2+\frac{1}{2}(1-\gamma_n)^2\sum_{k=1}^{N}\sum_{\ell=1}^{N}\beta_k^n\beta_\ell^n\rho_{\ell,k}\sigma_\ell\sigma_k\Big\}.$$

The first-order conditions then imply the following non-linear system for the optimal consumption and transfer decisions

$$\widehat{p}_m^n\left(\frac{a_k(t)(\alpha_{kt}^*)^{b_k}+\sum_{\ell\neq k}A_k\varepsilon_k^\ell\chi_k^\ell}{a_k(t)}\right)^{\frac{1-b_k}{b_k}}\frac{A_k\chi_k^n}{b_ka_k(t)}-\frac{\delta_n\beta_k^n}{1-\varepsilon_k^n}=\lambda_{kt}^n$$
$$\widehat{p}_m^n\left(\frac{a_k(t)(\alpha_{kt}^*)^{b_k}+\sum_{\ell\neq k}A_k\varepsilon_k^\ell\chi_k^\ell}{a_k(t)}\right)^{\frac{1-b_k}{b_k}}\frac{A_k\varepsilon_k^n}{b_ka_k(t)}+\frac{\delta_n\beta_k^n}{\chi_k^n}+\beta_n^n[A_n-\vartheta_n(1-\chi_n)]\omega_n^k=0$$

where $\lambda_k^n \geq 0$ is the Kuhn-Tucker multiplier associated with the non-negativity constraint $\varepsilon_k^n \geq 0$. Due to its non-linearity, the above system cannot be solved in closed-form, but it defines a set of state-independent optimal controls which does not compromise our separation (44).

3rd step. Now, assume that country n does not receive any transfers, i.e. $\varepsilon_n^\ell = 0$ for all $\ell \neq n$. Such a country is allowed to reoptimize abatement after transfers. If this country reoptimizes α_n in (50), then the first-order condition is identical to the case without transfers. Therefore, the optimal abatement policy stays the same. However, if abatement does not change, then consumption also does not change since it has already been optimized in the second step.

Finally, the SCC directly follows from

$$\mathrm{SCC}_n=\frac{\partial J^n}{\partial M}\Big/\frac{\partial J^n}{\partial K_n}=\frac{\partial J^n}{\partial m}\,\frac{\partial m}{\partial M}\Big/\frac{\partial J^n}{\partial K_n}.$$

D Proof of Theorem 8.2

For log-utility, the social planner's HJB equation is given by

$$0=\sup_{(\mathcal{C}_1^n,\dots,\mathcal{C}_N^n,\alpha_n)_{n=1}^N}\Big\{V_t+\delta\sum_{k=1}^{N}\varphi_k\log(\mathscr{C}_k)-\delta V+\Big[\mu_m-\frac{1}{2}\sigma_m^2-\sum_{k=1}^{N}\alpha_k\Big]V_m$$
$$+\Big(\kappa_\tau\big[\eta_\tau(m-m^{\mathrm{PI}})+F^{\mathrm{ex}}\big]-(\phi+\phi_{21})\tau+\phi_{21}\tau^o\Big)V_\tau+\phi_{12}(\tau-\tau^o)V_{\tau^o}$$

$$+ \sum_{k=1}^{N} V_{K_k} K_k \Big[g_k \Big(\cdot, \sum_{\ell=1}^{N} \omega_k^\ell \chi_\ell^k \Big) - \kappa_k(\cdot, \alpha_k) - \xi_k \tau \Big] + \frac{1}{2}\sigma_m^2 V_{mm}$$
$$+ \frac{1}{2} \sum_{k=1}^{N} \sum_{l=1}^{N} K_k K_l \rho_{l,k} \sigma_l \sigma_k V_{K_k K_l} + \sum_{k=1}^{N} K_k \rho_{m,k} \sigma_m \sigma_k V_{K_k m} \Big\},$$

By definition $V(t, m, \tau, \tau^o, K_1, \ldots, K_N) = \sum_{n=1}^{N} \varphi_n V^n(t, m, \tau, \tau^o, K_1, \ldots, K_N)$. We substitute the conjecture

$$V^n(t, m, \tau, \tau^o, K_1, \ldots, K_N) = \sum_{k=1}^{N} \beta_k^n \log(K_k) - p_m^n m - p_\tau^n \tau - p_{\tau^o}^n \tau^o + p^{n,\mathrm{SP}}(t)$$

into the HJB equation and reformulate the HJB equation in terms of the controls α_n and χ_k^n.

$$0 = \sup_{(\chi_1^n, \ldots, \chi_N^n, \alpha_n)_{n=1}^N} \Big\{ \sum_{n=1}^{N} \varphi_n \Big(\dot{p}^{n,\mathrm{SP}} + \delta \sum_{k=1}^{N} \beta_k^n \log(\chi_k^n A_k) + \delta \Big(p_m^n m + p_\tau^n \tau + p_{\tau^o}^n \tau^o - p^{n,\mathrm{SP}} \Big)$$
$$- \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^{N} \alpha_k \Big] p_m^n - \phi_{12}(\tau - \tau^o) p_{\tau^o}^n - \Big(\kappa_\tau \big[\eta_\tau (m - m^{\mathrm{PI}}) + F^{\mathrm{ex}} \big] - (\phi + \phi_{21})\tau + \phi_{21}\tau^o \Big) p_\tau^n$$
$$+ \sum_{k=1}^{N} \beta_k^n \Big[g_k \Big(\cdot, \sum_{\ell=1}^{N} \omega_k^\ell \chi_\ell^k \Big) - \kappa_k(\cdot, \alpha_k) - \xi_k \tau \Big] \Big) \Big\},$$

We choose p_m^n, p_τ^n, and $p_{\tau^o}^n$ as stated in the theorem and obtain simplified HJB equation

$$0 = \sup_{(\chi_1^n, \ldots, \chi_N^n, \alpha_n)_{n=1}^N} \Big\{ \sum_{n=1}^{N} \varphi_n \Big[\dot{p}^{n,\mathrm{SP}} + \delta \sum_{k=1}^{N} \beta_k^n \log(\chi_k^n A_k) - \delta p^{n,\mathrm{SP}} - \Big[\mu_m - \frac{1}{2}\sigma_m^2 - \sum_{k=1}^{N} \alpha_k \Big] p_m^n$$
$$- \kappa_\tau p_\tau^n \big[F^{\mathrm{ex}} - \eta_\tau m^{\mathrm{PI}} \big] + \sum_{k=1}^{N} \beta_k^n \big[g_k \Big(\cdot, \sum_{\ell=1}^{N} \omega_k^\ell \chi_\ell^k \Big) - \kappa_k(\cdot, \alpha_k) - \frac{1}{2}\sigma_k^2 \big] \Big] \Big\},$$

The optimal abatement policies satisfy the first order conditions

$$\sum_{\ell=1}^{N} \varphi_\ell \beta_n^\ell \frac{\partial}{\partial \alpha_n} \kappa_n(\cdot, \alpha_n) = \sum_{\ell=1}^{N} \varphi_\ell p_m^\ell$$

Therefore, the optimal abatement policies are given by (33). Similarly, the optimal consumption strategies satisfy the first-order conditions

$$\frac{\varphi_n \delta_n \beta_k^n}{\chi_k^n} = \beta_n^n \Big[A_n - \vartheta_n \Big(1 - \sum_{\ell=1}^{N} \omega_n^\ell \chi_\ell^n \Big) \Big] \omega_n^k$$

where the solutions are

$$\chi_n^* = \frac{\vartheta_n - A_n + \sqrt{(\vartheta_n - A_n)^2 + 4\vartheta_n \frac{\delta_n \varphi_n}{\sum_{\ell=1}^N \varphi_\ell \beta_n^\ell}}}{2\vartheta_n}, \qquad \chi_k^{n*} = \frac{\beta_k^n}{\omega_n^k} \chi_n^*, \qquad k = 1, \ldots, N. \tag{51}$$

Therefore, the function $p^{n,\mathrm{SP}}$ is given by

$$p^{n,\mathrm{SP}}(t) = \int_t^\infty \mathrm{e}^{-\delta(s-t)} \eta^{n,\mathrm{SP}}(s) \mathrm{d}s, \tag{52}$$

where the deterministic function $\eta^{n,\mathrm{SP}}$ is given by

$$\begin{aligned} \eta^{n,\mathrm{SP}} &= \delta_n \sum_{k=1}^N \beta_k^n \log \Big(\chi_k^{n,\mathrm{SP}} A_k \Big) - p_m^n \Big[\mu_m - \frac{1}{2} \sigma_m^2 - \sum_{k=1}^N \alpha_k^{\mathrm{SP}} \Big] - p_\tau^n \kappa_\tau \Big[F^{\mathrm{ex}} - \eta_\tau m^{\mathrm{PI}} \Big] \\ &\quad + \sum_{k=1}^N \beta_k^n \Big[g_k(\chi_k^{\mathrm{SP}}) - \kappa_k(\cdot, \alpha_k^{\mathrm{SP}}) - \frac{1}{2} \sigma_k^2 \Big]. \end{aligned} \tag{53}$$

Market clearing thus implies that optimal consumption is given by (35) and the exchange rates are given by (37) and

□

E Details on the Parameter Selection

E.1 Economic Model

Economic Growth We use the model predictions from RICE (in particular GDP) and calibrate the growth rate parameters A_n, δ_n^K and ϑ_n such that the economic model closely matches the median evolution of GDP and consumption. For this purpose we aggregate national data from RICE into the five regions used in our model.[18] Additionally we use historical data from the website of the International Monetary Fund to estimate the volatility and correlation parameters of capital growth.[19]

Abatement Costs The first step is to calibrate the global average abatement cost function such that it closely matches the global average abatement costs in the DICE-2013R model, i.e such such $a(t)\alpha^b \approx Aa_{\mathrm{DICE}}(t)\varepsilon^b$ where $\varepsilon_t = 1 - \frac{\mathcal{E}_t}{\mathcal{E}_t^{\mathrm{BAU}}}$ denotes the emission control rate.[20] For this purpose, we iteratively solve a global model. We set $b = 2.8$ as in DICE and start with an initial guess a^0 for the deterministic part of the cost function. Having solved the problem, we update our conjecture for a and set

$$a^{i+1}(t) = A\, a_{\mathrm{DICE}}(t)\mathbb{E}\left[\left(\frac{\varepsilon_t^i}{\alpha_t^i}\right)^b\right]$$

where ε_t^i and α_t^i denote the optimal emission control rate and α^i the optimal abatement policy in iteration i, respectively. We find that this iterative calibration converges quickly such that the differences between DICE abatement costs and our abatement costs become negligible after five iterations.

Impact of Climate Change We combine the global estimates of climate damages from DICE-2013R model with regional estimates from Stanton et al. (2012) to calibrate the damage parameters. Since DICE-2013R uses a level impact of climate change,[21] we need to translate the level impact in an equivalent growth rate impact to estimate the global average damage parameter. On a global level we choose the damage parameter ξ such that the average GDP losses in the year 2100 coincide for both damage specifications. Formally, the following equation implicitly determines the global average damage parameter ξ

$$\mathbb{E}\left[\mathrm{e}^{-\xi\int_0^t T_s \mathrm{d}s}\right] = \mathbb{E}\left[D^N(T_t)\right],$$

where t denotes the year 2100. We obtain an average damage parameter of $\xi = 0.00026$. Finally, we scale the global average damage parameter such that the regional heterogeneity is in line

[18] This data is available at https://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/28461
[19] The data is available at: https://www.imf.org/external/data.htm
[20] The function a_{DICE} can be found in Nordhaus and Sztorc (2013).
[21] DICE-2013R uses an inverse-quadratic damage function of the form $D(T) = \frac{1}{1+0.000266\,T^2}$

with the estimates provided by Stanton et al. (2012) and the worldwide damages coincide with the global model in the year 2015, i.e. $\xi \sum_{n=1}^{N} \mathcal{P}_1^n K_n = \sum_{n=1}^{N} \mathcal{P}_1^n K_n \xi_n$. Here $\mathcal{P}_1^n$ is the exchange rate to the US dollar. The results are summarized in Table 2. The calibration implies that the climate damages for MAF is three times as high as for the industrialized region OECD90 which is consistent with other studies such as Dell et al. (2012).

E.2 Climate Model

We calibrate the climate model such that the median BAU evolution of CO_2-concentration, global CO_2-emissions, and global average temperature increase are close to the corresponding evolutions in the DICE-2013R model. Additionally our calibration also relies on data on the historical carbon dioxide concentration in the atmosphere as well as forward looking data on regional carbon dioxide emissions from the RCP 8.5 scenario.

Atmospheric Carbon Dioxide We fix the pre-industrial athmospheric carbon dioxide concentration at $M^{\mathrm{PI}} = 280$ ppm. Furthermore, in the year 2015 ($t = 0$) the carbon dioxide concentration was $M_0 = 401$ ppm. To calibrate (8) we chose the drift rate μ_m such that the drift of the average BAU evolution is close to the drift rate implied by the baseline scenario in the DICE-2013R model. This can be captured in the following way:

$$\mu_m(t) = a_1 \exp\left\{ -\left(\frac{t-b_1}{c_1}\right)^2 \right\} + a_2 \exp\left\{ -\left(\frac{t-b_2}{c_2}\right)^2 \right\}$$

where $a_1 = 0.0086$, $b_1 = 44.88$, $c_1 = 59.11$, $a_2 = 0.005$, $b_2 = 132.5$, $c_2 = 61.42$. In a second step we calibrate the volatility of carbon dioxide shocks such that the model matches the historical variability. For this purpose we use data on the historical carbon dioxide concentration in the atmosphere.[22] Calculating the standard deviation of the log changes of M we obtain the volatility $\sigma_m = 0.0016$.

Carbon Dioxide Emissions Equation (9) links changes in the atmospheric CO_2-concentration to the CO_2-emissions. Following Nordhaus (1992), among others, we assume a carbon dioxide residence time of 120 years implying $\delta_m = 0.0083$. To determine the conversion factor ζ_e, we discretize (9) and obtain $M_{t+1} - M_t + \delta_m(M_t - M^{\mathrm{PI}}) = \zeta_e \mathcal{E}_t$ where $\mathcal{E}_t$ denotes global carbon

[22]Source: Mauna Loa Observatory, Hawaii. Data available at http://co2now.org/Current-CO2/CO2- Now/

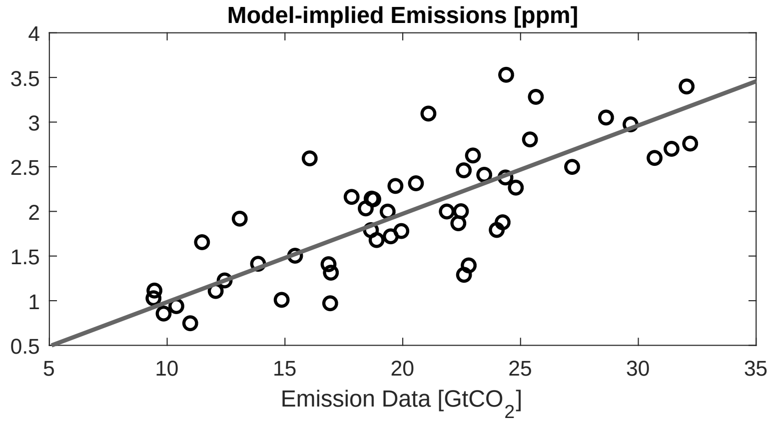

Figure 6: Calibration of the Carbon Dioxide Model. The figure depict pairs of historical carbon dioxide emissions (measured in GtCO2) and emission triggered increases in carbon dioxide concentrations (measured in ppm). The grey line depicts the related regression line.

	2015	2030	2050	2075	2100	2150	2200
ν_{OECD}	0.34	0.31	0.28	0.26	0.22	0.19	0.18
ν_{ASIA}	0.38	0.39	0.39	0.39	0.38	0.38	0.37
ν_{LAM}	0.07	0.07	0.07	0.06	0.06	0.05	0.04
ν_{REF}	0.10	0.11	0.12	0.13	0.15	0.17	0.18
ν_{MAF}	0.11	0.12	0.14	0.16	0.19	0.21	0.23

Table 5: Emission Calibration. The table summarizes the regional fractions describing the share of the five regions in the global BAU-emissions based on RCP 8.5 data which is avalailable at https://tntcat.iiasa.ac.at/AR5DB.

dioxide emissions. Therefore, we estimate ζ_e by a least-squares minimization

$$\zeta_e = \arg\min_{\zeta} \sum_{i=1}^{I} \left[M_{i+1} - M_i + \delta_m(M_i - M^{\text{PI}}) - \zeta \mathcal{E}_i\right]^2$$

yielding a conversion factor of $\zeta_e = 0.0989$. This is the slope of the regression line in Figure 6. This calibration predicts annual global CO_2-emissions of about 106 $GtCO_2$ by the end of the century which is close to the predictions of the DICE model (103 $GtCO_2$) and the RCP 8.5 scenario (106 $GtCO_2$). In a second step we use RCP 8.5 predictions on regional CO_2-emissions to determine the regional emission shares ν_n. Table 5 summarizes the calibration results.

Global Average Temperature For the parameters determining the climate system, we follow Cai et al. (2015) and choose $\phi = 0.047$, $\phi_{12} = 0.0048$, $\phi_{21} = 0.01$, $\kappa_\tau = 0.037$, $\eta_\tau = 5.48$ corresponding to an equilibrium climate sensitivity of 3°C. The model predicts a end-of-century global average temperature increase of about 3.9°C compared to 3.8°C in DICE-2013R.

E.3 Preference Parameters

In our benchmark calibration, we choose logarithmic preferences ($\gamma_n = \psi_n = 1$) and a time preference rate of $\delta_n = 0.015$ which is a standard assumption in the IAM literature. Besides, we calibrate the consumption preferences such that the model is in line with consumption and import data from Worldbank.[23] This implies preferences for the domestic good β_n^n in the range of 60% for MAF to 85% for OECD90.

[23]The data is available at https://data.worldbank.org/