FINANCIAL HETEROGENEITY AND THE INVESTMENT CHANNEL OF MONETARY POLICY

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ABSTRACT

We study the role of heterogeneity in firms' financial positions in determining the investment channel of monetary policy. Empirically, we show that firms with low leverage or high credit ratings are the most responsive to monetary policy shocks. We develop a heterogeneous firm New Keynesian model with default risk to interpret these facts and study their aggregate implications. In the model, firms with high default risk are less responsive to monetary shocks because their marginal cost of external finance is high. The aggregate effect of monetary policy therefore depends on the distribution of default risk across firms.

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1 Introduction

Aggregate investment is one of the most responsive components of GDP to changes in monetary policy. Our goal in this paper is to understand the role of financial frictions in determining the investment channel of monetary policy. Given the rich heterogeneity in financial positions across firms, a key question is: which firms are the most responsive to changes in monetary policy, and why? The answer to this question is theoretically ambiguous. On the one hand, because financial frictions increase the marginal cost of investment, they may dampen the response of investment to monetary policy for firms more severely affected by financial frictions. On the other hand, to the extent that monetary policy alleviates financial frictions, they may amplify the response of affected firms. This latter view is the conventional wisdom of the literature, often informed by applying the financial accelerator logic across firms.

We address the question of which firms respond the most to monetary policy and why using new cross-sectional evidence and a heterogeneous firm New Keynesian model. Our main empirical result is that firms with low leverage are significantly more responsive to monetary policy shocks than firms with high leverage. We further show that low-leverage firms have high average credit ratings and that highly rated firms are also more responsive to monetary policy, suggesting that these heterogeneous responses are partly driven by default risk. To speak to this evidence, our model embeds heterogeneous firms subject to default risk into the benchmark New Keynesian general equilibrium environment. In the model, monetary policy stimulates investment through a combination of direct effects, due to changes in the real interest rate, and indirect effects, due to changes in firms' cash flows and borrowing costs. Firms with high default risk are less responsive to these changes because their marginal cost of external finance is high. Quantitatively, we replicate our empirical regressions on model-simulated data and recover a similar degree of heterogeneous responses. These heterogeneous responses imply that the aggregate effect of monetary policy depends on the initial distribution of default risk; when default risk in the economy is high, monetary policy is significantly less powerful.

Our empirical work combines monetary policy shocks, measured using high-frequency changes in Fed Funds futures, with firm-level outcomes from quarterly Compustat data. Our main empirical specification estimates how the semi-elasticity of a firm's investment with respect to a monetary policy shock depends on the firm's leverage, conditioning on both firm fixed effects – to capture permanent differences across firms – and sector-by-quarter fixed effects – to capture differences in how sectors respond to aggregate shocks. Quantitatively, our estimates imply that a firm with one standard deviation more leverage than the average firm in our sample is about half as responsive to monetary policy as the average firm. Furthermore, the 50% least-leveraged firms account for nearly all of the total response to monetary policy in our sample.

Although we do not exploit exogenous variation in leverage, we provide suggestive evidence that our results reflect, at least in part, heterogeneity in default risk: low-leverage firms are more likely to have high credit ratings, and conditional on leverage highly rated firms are more responsive to monetary shocks as well. We also provide three key pieces of evidence that these heterogeneous responses are not driven by other firm-level characteristics. First, the results are not driven by permanent heterogeneity in financial positions because they are robust to using within-firm variation in leverage. Second, our results are not driven by differences in past sales growth, realized future sales growth, or size. Third, unobservable factors are unlikely to drive our results because we find similar results if we instrument leverage with past leverage (which is likely more weakly correlated with unobservables).[1]

In order to interpret these empirical results, we embed a model of heterogeneous firms facing default risk into the benchmark New Keynesian framework. There is a group of heterogeneous production firms who invest in capital using either internal funds or external borrowing; these firms can default on their debt, leading to an external finance premium. There is also a group of retailer firms with sticky prices, generating a New Keynesian Phillips curve linking nominal variables to real outcomes. We calibrate the model to match key features of firms' investment, borrowing, and lifecycle dynamics in the micro data. Our model also generates realistic behavior along non-targeted dimensions of the data, such as measured

[1]Another concern is that our monetary policy shocks are correlated with other economic conditions that are in fact driving the differences across firms. Although our shock identification was designed to address this concern, we also show that there are not significant differences in how firms respond to changes in other cyclical variables like GDP growth, the unemployment rate, the inflation rate, or the VIX index.

investment-cash flow sensitivities.

We first use the model to decompose three broad channels through which monetary policy affects firms' investment decisions. First, *direct effects* are driven by changes in the real interest rate, which affect firms' discount factors and the level of borrowing costs. Second, *indirect effects* are driven by changes in aggregate demand conditions, which affect firms' cash flows and credit spreads. Third, *adjustment cost* effects are driven by changes in the relative price of new capital, which dampen the response of investment to the other two channels. Quantitatively, all three of these channels play an important role in driving aggregate transmission in our model. The peak responses of aggregate investment, output, and consumption in the model are in line the peak responses estimated in the data by Christiano, Eichenbaum and Evans (2005).

We then use the model to study the heterogeneous responses to monetary policy across firms. On average, firms increase their investment by engaging in additional borrowing. However, firms with high risk of default face a higher marginal cost of borrowing because an additional unit of borrowing increases their default risk. Since borrowing is the marginal source of investment finance for these firms, this force dampens their response to monetary policy. The implied heterogeneity in responses is qualitatively in line with our empirical results because, on average, firms with low default risk have low leverage. We show that the model is quantitatively consistent with the data by estimating our empirical regression specification on panel data simulated from our model; the coefficient capturing heterogeneous responses in our model is within one standard error of its estimate in the data.

Finally, we show that the aggregate effect of a given monetary shock depends on the distribution of default risk across firms. We perform a simple calculation which exogenously varies the initial distribution of firms in the period of the shock. A monetary shock will generate an approximately 25% smaller change in the aggregate capital stock starting from a distribution with 50% less net worth than the steady state distribution. Under the distribution with low average net worth, more firms have a high risk of default and are therefore less responsive to monetary policy. More generally, this calculation suggests a potentially important source of time-variation in monetary transmission: monetary policy is less powerful when more firms have risk of default.

Related Literature Our paper contributes to four key strands of literature. The first studies the transmission of monetary policy to the aggregate economy. Bernanke, Gertler and Gilchrist (1999) embed the financial accelerator in a representative firm New Keynesian model and find that it amplifies the aggregate response to monetary policy. We build on Bernanke, Gertler and Gilchrist (1999)'s framework to include firm heterogeneity. Consistent with their results, we find that the response of aggregate investment to monetary policy is larger in our model than in a model without financial frictions at all. However, among the 96% of firms affected by financial frictions in our model, those with low risk of default are more responsive to monetary policy than those with high risk of default, generating an additional source of state dependence.

Second, we contribute to the literature that studies how the effect of monetary policy varies across firms. A number of papers, including Kashyap, Lamont and Stein (1994), Gertler and Gilchrist (1994), and Kashyap and Stein (1995) argue that smaller and presumably more credit constrained firms are more responsive to monetary policy along a number of dimensions. We contribute to this literature by showing that low leverage and highly rated firms are also more responsive to monetary policy. These characteristics are essentially uncorrelated with firm size in our sample. In addition, we use a different empirical specification, identification of monetary policy shocks, sample of firms, and time period.[2,3]

Third, we contribute to the literature which studies how incorporating micro-level heterogeneity into the New Keynesian model affects our understanding of monetary transmission. To date, this literature has focused on how household-level heterogeneity affects the consumption channel of monetary policy; see, for example, Auclert (2017); McKay, Nakamura and Steinsson (2015); Wong (2016); or Kaplan, Moll and Violante (2017). We instead explore the role of firm-level heterogeneity in determining the investment channel of monetary policy.

[2]In a recent paper, Crouzet and Mehrotra (2017) find some evidence of differences in cyclical sensitivity by firm size during extreme business cycle events. Our work is complementary to their's by focusing on the conditional response to a monetary policy shock and using our economic model to draw aggregate implications.

[3]Ippolito, Ozdagli and Perez-Orive (2017) study how the effect of high-frequency shocks on firm-level outcomes depends on firms' bank debt. In order to merge in data on bank debt, Ippolito, Ozdagli and Perez-Orive (2017) must focus on the 2004-2008 time period. Given this small sample, Ippolito, Ozdagli and Perez-Orive (2017) do not consistently find significant differences in investment responses across firms. In addition, Ippolito, Ozdagli and Perez-Orive (2017) use a different empirical specification and focus on stock prices as the main outcome of interest.

In contrast to the heterogeneous-household literature, we find that both direct and indirect effects of monetary policy play a quantitatively important role in driving the investment channel. The direct effect of changes in the real interest rates are smaller for consumption than for investment because households attempt to smooth consumption over time.

Finally, we contribute to the literature studying the role of financial heterogeneity in determining the business cycle dynamics of aggregate investment. Our model of firm-level investment builds heavily on Khan, Senga and Thomas (2016), who study the effect of financial shocks in a flexible price model. We contribute to this literature by introducing sticky prices and studying the effect of monetary policy shocks. In addition, we extend Khan, Senga and Thomas (2016)'s model to include capital quality shocks and a time-varying price of capital in order to generate variation in the implicit collateral value of capital, as in the financial accelerator literature (Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999)). Khan and Thomas (2013) and Gilchrist, Sim and Zakrajsek (2014) study related flexible-price models of investment with financial frictions. Our model is also related to Arellano, Bai and Kehoe (2016), who study the role of financial heterogeneity in determining employment decisions.

Road Map Our paper is organized as follows. Section 2 provides the descriptive empirical evidence that the firm-level response to monetary policy varies with leverage and credit rating. Section 3 develops our heterogeneous firm New Keynesian model to interpret this evidence. Section 4 calibrates the model and verifies that it is consistent with key features of the joint distribution of investment and leverage in the micro data. Section 5 uses the model to study the monetary transmission mechanism. Section 6 concludes.

2 Heterogeneous Responses to Monetary Policy: Empirical Evidence

This section provides descriptive empirical evidence on how the response of investment to monetary policy varies across firms. Section 2.1 describes our data sources. Section 2.2 shows that low-leverage firms are more responsive to monetary policy than high-leverage firms.

Section 2.3 provides suggestive evidence that these heterogeneous responses are driven by differences in default risk.

2.1 Data Description

We combine monetary policy shocks with firm-level outcomes from quarterly Compustat.

Monetary Policy Shocks A key challenge in measuring changes in monetary policy is that most of the variation in the Fed Funds Rate is driven by the Fed's endogenous response to aggregate economic conditions. We identify shocks to monetary policy, which are arguably not driven by aggregate conditions, using the high-frequency event-study approach pioneered by Cook and Hahn (1989). This high-frequency identification imposes fewer assumptions to identify shocks than the VAR approach in Christiano, Eichenbaum and Evans (2005) or the narrative approach in Romer and Romer (2004).

Following Gurkaynak, Sack and Swanson (2005) and Gorodnichenko and Weber (2016), we construct our monetary policy shocks $\varepsilon_t^{\mathrm{m}}$ as

$$\varepsilon_t^{\mathrm{m}} = \tau(t) \times (\mathtt{ffr}_{t+\Delta_+} - \mathtt{ffr}_{t-\Delta_-}), \tag{1}$$

where t is the time of the monetary announcement, $\mathtt{ffr}_t$ is the implied Fed Funds Rate from a current-month Federal Funds future contract at time t, Δ_+ and Δ_- control the size of the time window around the announcement, and $\tau(t)$ is an adjustment for the timing of the announcement within the month.[4] We focus on a window of Δ_- = fifteen minutes before the announcement and Δ_+ = forty five minutes after the announcement. Our shock series begins in January 1990, when the Fed Funds futures market opened, and ends in December 2007, before the financial crisis.[5] During this time there were 183 shocks with a mean of approximately zero and a standard deviation of 9 basis points.[6]

[4]This adjustment accounts for the fact that Fed Funds Futures pay out based on the average effective rate over the month. It is defined as $\tau(t) \equiv \frac{\tau_m^n(t)}{\tau_m^n(t) - \tau_m^d(t)}$, where $\tau_m^d(t)$ denotes the day of the meeting in the month and $\tau_m^n(t)$ the number of days in the month.

[5]We stop in December 2007 to study a period of conventional monetary policy, which is the focus of our economic model.

[6]In our economic model, we interpret our measured monetary policy shock as an innovation to a Taylor Rule. An alternative interpretation of the shock, however, is that it is driven by the Fed providing information

Table 1
Monetary Policy Shocks: Summary Statistics

	high frequency	smoothed	sum
mean	-0.0209	-0.0481	-0.0477
median	0	-0.0134	-0.00536
std	0.0906	0.111	0.132
min	-0.463	-0.480	-0.479
max	0.152	0.233	0.261
num	183	79	80

Notes: Summary statistics of monetary policy shocks. "High frequency" shocks are estimated using event study strategy in (1). "Smoothed" shocks are time aggregated to the quarterly frequency using the weighted average (2). "Sum" refers to time aggregating by simply summing all shocks within a quarter.

We time aggregate the high-frequency shocks to the quarterly frequency in order to merge them with our firm-level data. We construct a moving average of the raw shocks weighted by the number of days in the quarter after the shock occurs.[7] Our time aggregation strategy ensures that we weight shocks by the amount of time firms have had to react to them. Table 1 indicates that these "smoothed" shocks have similar features to the original high-frequency shocks. For robustness we will also use the alternative time aggregation of simply summing all the shocks that occur within the quarter, as in Wong (2016). Table 1 shows that the moments of these alternative shocks do not significantly differ from the moments of the smoothed shocks.

Firm-Level Variables We draw firm-level variables from quarterly Compustat, a panel of publicly listed U.S. firms. We use Compustat because it satisfies three key requirements for our study: it is quarterly, a high enough frequency to study monetary policy; it is a long panel, allowing us to use within-firm variation; and it contains rich balance-sheet information,

to the private sector. In Section 2.3 we argue that the information component of Fed announcements does not drive our results.

[7]Formally, the monetary-policy shock in quarter q is defined as

$$\varepsilon_q^{\mathrm{m}} = \sum_{t \in J(q)} \omega^a(t) \varepsilon_t^{\mathrm{m}} + \sum_{t \in J(q-1)} \omega^b(t) \varepsilon_t^{\mathrm{m}} \tag{2}$$

where $\omega^a(t) \equiv \frac{\tau_q^n(t) - \tau_q^d(t)}{\tau_q^n(t)}$, $\omega^b(t) \equiv \frac{\tau_q^d(t)}{\tau_q^n(t)}$, $\tau_q^d(t)$ denotes the day of the monetary-policy announcement in the quarter, $\tau_q^n(t)$ denotes the number of days in the monetary-policy announcement's quarter, and $J(q)$ denote the set periods t contained in quarter q.

allowing us to construct our key variables of interest. To our knowledge, Compustat is the only dataset that satisfies these three requirements. The main disadvantage of Compustat is that it excludes privately held firms which are likely subject to more severe financial frictions.[8] In Section 4, we calibrate our economic model to match a broad sample of firms, not just those in Compustat.

We focus on two measures of investment in our empirical analysis. Our main measure is $\Delta \log k_{jt+1}$, where k_{jt+1} denotes the capital stock of firm j at the end of period t. We use the log-difference specification because investment is highly skewed, suggesting a log-linear rather than level-linear model. We use the net change in log capital rather than the log of gross investment because gross investment often takes negative values. The second measure of investment that we consider is an indicator for whether the firm j has a gross investment rate greater than 1%, $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$. This measure is motivated by the fact that many changes in micro-level investment occur along the extensive margin (see, for example, Cooper and Haltiwanger (2006)). Additionally, by focusing on large investment episodes, this measure is less prone to small measurement error in the capital stock.

Our main measure of leverage is the firms' debt-to-asset ratio ℓ_{jt}. We measure debt as the sum of short term and long term debt and measure assets as the book value of assets. We focus on leverage as our main measure of financial position for two reasons. First, leverage is tightly linked, both empirically and theoretically, to the costs of external finance (see, for example, Kaplan and Zingales, 1997; Whited and Wu, 2006; Tirole, 2010). Second, leverage exhibits considerable within-firm variation, which we use to control for permanent heterogeneity in financial position. In Section 2.3, we also measure financial position using the firm's credit rating, size, and indicator for whether the firm pays dividends.

Appendix A.1 provides details of our data construction, which follows standard practice in the investment literature. Table 2 presents simple summary statistics of the final sample used in our analysis. The mean capital growth rate is roughly 0.4% quarterly with a standard deviation of 9.3%. The mean leverage ratio is approximately 27% with a cross-sectional

[8] Crouzet and Mehrotra (2017) construct a non-public, high-quality quarterly panel using micro data from the Quarterly Financial Reports. A key advantage of this dataset is that covers a much broader set of firm sizes than Compustat. However, it only covers the manufacturing sector and only records a rotating panel for small firms, which limits the ability to use within-firm variation.

standard deviation of 36%.

Table 2
Firm-Level Variables: Summary Statistics

Statistic	$\Delta \log k_{jt}$	$\frac{i_{jt}}{k_{jt}}$	$\mathbb{1}\{\frac{i_{jt}}{k_{jt}} > 1\%\}$	ℓ_{jt}
Average	0.004	0.040	0.732	0.267
Median	-0.004	0.027	1.000	0.204
Std	0.093	0.102	0.443	0.364
Bottom 5%	-0.089	-0.053	0.000	0.000
Top 5%	0.130	0.171	1.000	0.726

Notes: Summary statistics of firm-level outcome variables. $\Delta \log k_{jt+1}$ is the net change in the capital stock. $\frac{i_{jt}}{k_{jt}}$ is the firm's investment rate. $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ is an indicator variable for whether a firm's investment rate is greater than 1%. ℓ_{jt} is the ratio of total debt to total assets.

2.2 Heterogeneous Responses By Leverage

Our baseline empirical specification is

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}, \tag{3}$$

where α_j is a firm j fixed effect, α_{st} is a sector s by quarter t fixed effect, $\varepsilon_t^{\mathrm{m}}$ is the monetary policy shock, ℓ_{jt} is the firm's leverage ratio, Z_{jt} is a vector of firm-level controls, and ε_{jt} is a residual.[9] We lag both leverage ℓ_{jt-1} and the controls Z_{jt-1} to ensure they are predetermined at the time of the monetary shock.[10]

Our main coefficient of interest is β, which measures how the semi-elasticity of investment $\Delta \log k_{jt+1}$ with respect to monetary shocks $\varepsilon_t^{\mathrm{m}}$ depends on the firm's leverage ℓ_{jt-1}. This coefficient estimate is conditional on a number of factors that may simultaneously affect investment and leverage. First, firm fixed effects α_j capture permanent differences in investment behavior across firms. Second, sector-by-quarter fixed effects α_{st} capture differences

[9]The sectors s we consider are: agriculture, forestry, and fishing; mining; construction; manufacturing; transportation communications, electric, gas, and sanitary services; wholesale trade; retail trade; and services. We do not include finance, insurance, and real estate or public administration.

[10]Note that both k_{jt+1} and ℓ_{jt} measure end-of-period stocks. We denote the end-of-period capital stock with k_{jt+1} rather than k_{jt} to be consistent with the standard notation in our economic model in Section 3.

Table 3

Heterogeneity in the Response to Monetary Policy Shocks

A) Dependent variable: $\Delta \log k$

	(1)	(2)	(3)
leverage × ffr shock	-0.93***	-0.73**	-0.74**
	(0.34)	(0.29)	(0.31)
ffr shock			1.38
			(0.99)
Observations	233182	233182	233182
R^2	0.107	0.119	0.104
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)	(3)
leverage × ffr shock	-5.22***	-4.80***	-4.59***
	(1.42)	(1.29)	(1.35)
ffr shock			4.01
			(4.39)
Observations	233182	233182	233182
R^2	0.212	0.217	0.204
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Notes: Results from estimating variants of the baseline specification

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt+1}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

in how broad sectors are exposed to aggregate shocks. Finally, the firm-level controls Z_{jt} include the level of leverage ℓ_{jt}, total assets, sales growth, current assets as a share of total assets, and a fiscal quarter dummy.

Table 3 reports the results from estimating the baseline specification (3). We perform two normalizations to make the estimated coefficient β easily interpretable. First, we standardize leverage ℓ_{jt} over the entire sample, so that the units of leverage are standard deviations relative to its mean value in our sample. Second, we normalize the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive value corresponds to a cut in interest rates. Standard errors are clustered two-ways to account for correlation within firms and within quarters. This clustering strategy is conservative, leaving less than 80 time-series observations.

Panel (A) of Table 3 shows that firms with higher leverage are less responsive to monetary policy shocks. Column (1) reports the interaction coefficient β without the firm-level controls

Z_{jt-1}, and implies that a firm with one standard deviation more leverage than the average firm has a nearly one unit lower semi-elasticity of investment. Adding firm-level controls Z_{jt-1} in column (2) does not substantially change this point estimate.

A natural way to assess the economic significance of our estimated interaction coefficient β is to compare it to the average effect of a monetary policy shock. However, in our baseline specification (3), the average effect is absorbed by the sector-by-quarter fixed effect α_{st}. Column (3) relaxes this restriction by estimating

$$\Delta \log k_{jt+1} = \alpha_j + \gamma \varepsilon_t^{\mathrm{m}} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}_1' Z_{jt-1} + \mathbf{\Gamma}_2' Y_t + \varepsilon_{jt}, \tag{4}$$

where Y_t is a vector of aggregate controls for GDP growth, the inflation rate, the unemployment rate, and the VIX index. The average investment semi-elasticity is roughly 1.4. Hence, our point estimate in column (2) indicates that a firm with leverage one standard deviation higher than the average firm has an investment semi-elasticity roughly half as large as the average firm. However, this magnitude should be interpreted with care because the estimated average effect γ is not statistically significant due to the fact that the time-series variation in the monetary shocks $\varepsilon_t^{\mathrm{m}}$ is small and we cluster our standard errors at the quarterly level.

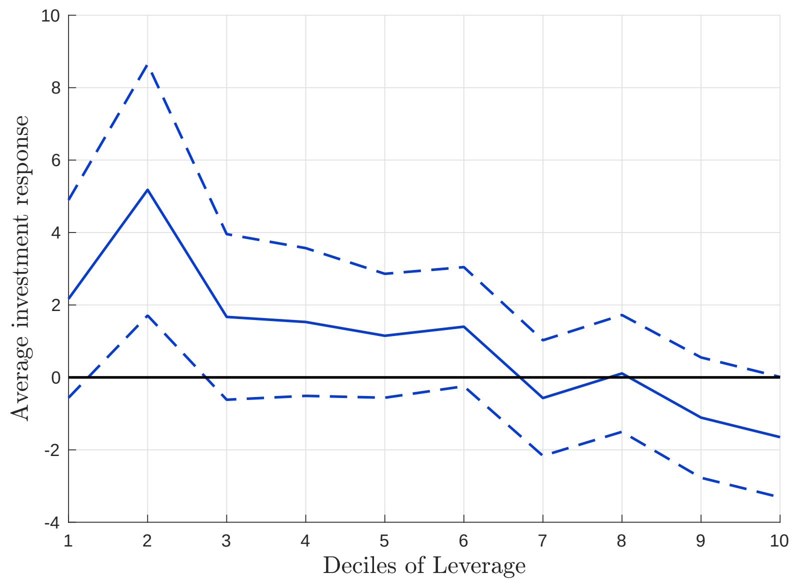
Panel (B) shows that all of these results hold for the extensive margin measure of investment $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as well. Quantitatively, firms with one cross-sectional standard deviation higher leverage are nearly 5% less likely to invest following a one percentage point expansionary monetary policy shock.

Aggregate Implications In order to further assess the economic significance of these heterogeneous responses and whether the heterogeneity survives aggregation, we estimate the regression

$$\Delta \log K_{jt+1} = \mathbf{\Gamma}' Y_t + \beta_j \varepsilon_t^{\mathrm{m}} + \varepsilon_{jt}, \tag{5}$$

where the outcome $\Delta \log K_{jt+1}$ is the total investment done by firms in the j^{th} decile of the leverage distribution in quarter t, and again Y_t contains controls for aggregate GDP growth, the inflation rate, the unemployment rate, and the VIX index. Figure 1 plots the aggregated semi-elasticities β_j against decile j. The aggregated semi-elasticity declines fairly steadily

FIGURE 1: Aggregated Semi-Elasticity With Respect To Monetary Policy Shocks



Notes: Semi-elasticity of aggregated investment with respect to monetary policy shocks for deciles of leverage distribution. Reports estimated semi-elasticities β_j from specification

$$\Delta \log K_{jt+1} = \mathbf{\Gamma}' Y_t + \beta_j \varepsilon_t^{\mathrm{m}} + \varepsilon_{jt}$$

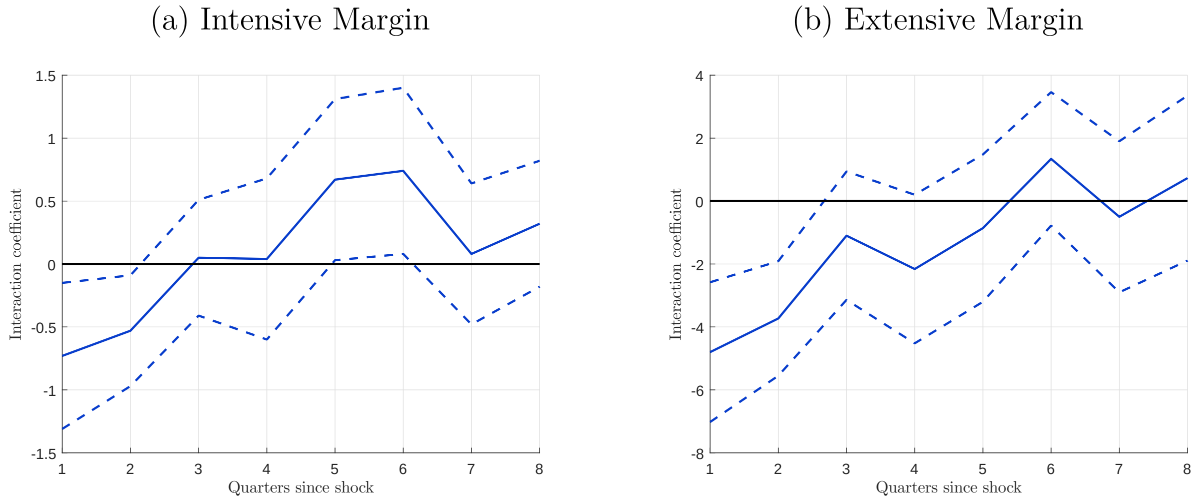
where $\Delta \log K_{jt+1}$ is the aggregated investment of firms with leverage in the jth decile of the leverage distribution in quarter t and Y_t is a vector containing GDP growth, the inflation rate, the unemployment rate, and the VIX index. Dotted lines provide 90% standard error bands. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates).

with leverage, even though this specification is far less structured and more aggregated than our benchmark (3). Furthermore, the aggregated semi-elasticity is essentially zero past the 6^{th} decile of the leverage distribution, indicating that the total effect of monetary policy is driven almost entirely by low-leverage firms.

Dynamics To study the dynamics of these differential responses across firms, we estimate the Jorda (2005)-style projection

$$\Delta \log k_{jt+h} = \alpha_{jh} + \alpha_{sth} + \beta_h \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}'_h Z_{jt-1} + \varepsilon_{jth}, \tag{6}$$

Figure 2: Dynamics of Differential Response to Monetary Shocks



Notes: dynamics of the interaction coefficient between leverage and monetary shocks over time. Reports the coefficient β_h over quarters h from

$$\Delta \log k_{jt+h} = \alpha_{jh} + \alpha_{sth} + \beta_h \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}'_h Z_{jt-1} + \varepsilon_{jt},$$

where α_{jh} is a firm fixed effect, α_{sth} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Standard errors are two-way clustered by firms and time. Dashed lines report 90% error bands. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

where $h \geq 1$ indexes quarters in the future. The coefficient β_h measures how the response of investment in quarter $t+h$ to a monetary policy shock in quarter t depends on the firm's leverage in quarter $t-1$. Panel (a) of Figure 2 plots the dynamics of the coefficient β_h estimated in (6). The interaction coefficient returns to zero three quarters after the initial shock, although the dynamics are somewhat hump-shaped after that. Panel (b) estimates (6) using the extensive margin measure of investment and finds that differences across firms disappear after six quarters. Taking these two results together, we conclude that the differential response to monetary shocks across firms is fairly short-lived, and therefore focus on the impact period for the rest of the paper.

It is important to note that the short-lived dynamics of the cross-sectional differences that we find here are not necessarily in conflict with the long-lived and hump-shaped dynamics of aggregate variables typically estimated in VARs. The cross-sectional differences are

TABLE 4
STOCK PRICES

	(1)	(2)	(3)
leverage × ffr shock	-8.22***	-8.22**	-6.12*
	(3.82)	(3.82)	(3.33)
ffr shock			6.22**
			(1.88)
Observations	32274	32274	32274
R^2	0.128	0.128	0.073
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Notes: Results from estimating the regression $r^{\mathrm{e}}_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon^{\mathrm{m}}_t + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$ where $r^{\mathrm{e}}_{jt+1} = \frac{p_{jt+1} - p_{jt}}{p_{jt}}$ is the change in the firm's stock price on the announcement day, α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon^{\mathrm{m}}_t$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Standard errors are two-way clustered by firms and quarter. We have normalized the sign of the monetary shock $\varepsilon^{\mathrm{m}}_t$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

simply a different object than aggregate investment.[11] One explanation for the hump-shaped response of aggregate investment is that hump-shaped responses of other variables, such as consumption demand, spill over to investment through general equilibrium linkages. In this case, it is unclear that these spillovers would apply differentially across firms by leverage. Another explanation is that the hump-shaped aggregate dynamics reflect frictions to capital demand itself; again, it is unclear that such frictions should affect firms differentially by leverage.

Supporting Evidence From Stock Prices Stock prices provide a natural reality check on our findings because they encode the extent to which monetary policy shocks are good news for firms. Additionally, stock prices are available at a high frequency, so they are not subject to time-aggregation bias. We therefore estimate the equation

$$r^{\mathrm{e}}_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon^{\mathrm{m}}_t + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}, \tag{7}$$

[11] Gertler and Karadi (2015) show that the high-frequency monetary shocks generate aggregate impulse responses that are similar to the VAR literature using an instrumental variable VAR strategy.

where $r^{\mathrm{e}}_{jt+1} = \frac{p_{jt+1} - p_{jt}}{p_{jt}}$ is the percentage change in the firm's stock price between the beginning and end of the trading *day* in which a monetary policy announcement occurs. Accordingly, the time period in t is a day and the monetary policy shock $\varepsilon^{\mathrm{m}}_t$ is the original high-frequency shock. The firm-level covariates are the quarterly observations on day t.

Table 4 shows that stock prices of low-leverage firms are significantly more responsive to monetary policy shocks. Quantitatively, increasing leverage by one standard deviation decreases the exposure of stock returns to the monetary policy shock by more than eight percentage points. Hence, these results suggest that the stock market understands monetary policy expansions are better news for low-leverage firms. The average response of stock returns to the monetary policy shock is about six percentage points.

Robustness and Additional Results A possible concern with our empirical evidence so far is that our monetary policy shocks may be correlated with other business cycle conditions which themselves drive differences across firms. Although our high-frequency shock identification is designed to address this concern, as a further check we interact leverage with various business cycle indicators in

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta_1 \ell_{jt-1} \varepsilon^{\mathrm{m}}_t + \beta_2 \ell_{jt-1} Y_t + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where Y_t is GDP growth, the inflation rate, the unemployment rate, or the VIX index. Table 5 shows that the estimated coefficients β in this regression are not economically meaningful or statistically different from zero for any of the business cycle indicators.

Appendix A.1 reports a number of additional robustness checks on our findings. The first set of robustness checks concerns the variation in the monetary shock. First, following Gurkaynak, Sack and Swanson (2005) we decompose monetary policy announcements into a "target" component that affects the level of the yield curve and a "path" component that affects the slope of the yield curve. We find that all of the differences across firms are driven by the target component. This result indicates that our results are primarily driven by the effect of Fed policy announcements on short-term interest rates rather than on expectations of growth in the future, which would affect long-term rates more than short-

TABLE 5
MONETARY SHOCKS VS. BUSINESS CYCLE CONDITIONS

	(1)	(2)	(3)	(4)	(5)
leverage × ffr shock	-0.85***	-0.73***	-0.74***	-0.83***	-0.96***
	(0.29)	(0.27)	(0.28)	(0.28)	(0.31)
leverage × dlog gdp	-0.08				-0.08
	(0.08)				(0.07)
leverage × dlog cpi		-0.05			-0.06
		(0.09)			(0.09)
leverage × ur			0.00		0.00
			(0.00)		(0.00)
leverage × vix				0.00	0.00
				(0.00)	(0.00)
Observations	239579	239579	239579	239579	239579
R^2	0.118	0.118	0.118	0.118	0.118
Firm controls	yes	yes	yes	yes	yes

Notes: Results from estimating variants of the baseline specification $\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} Y_t + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter, and Y_t is GDP growth (dlog gdp), the inflation rate (dlog cp), the unemployment rate (ur), or the VIX index (vix). Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

term. Second, we restrict our sample to post-1994 observations, after which time monetary policy announcements became more transparent. We find similar results, though with less statistical power due to the smaller sample. Third, we instrument the BAA spread with the monetary shock instead of using the shock directly and find similar results. Fourth, we decompose the shocks into expansionary and contractionary episodes and find that almost all the differential responses across firms are driven by expansionary monetary policy episodes.

The second set of robustness checks concerns our measure of leverage. First, we run our benchmark specification with leverage defined using debt net of liquid assets and find similar results. Second, we separately split out short-term debt, long-term debt, and other liabilities, and find consistent differential responses for all three subcomponents of leverage.[12]

[12]This decomposition sheds light on the role of the "debt overhang" hypothesis in driving our results. Under this hypothesis, equity holders of highly leveraged firms capture less of the return on investment; since equity holders make the investment decision, they will choose to invest less following the monetary policy shock. However, because investment is long lived, this hypothesis would predict much stronger differences by long

Table 6
Within-Firm Variation in Leverage

	(1)	(2)	(3)
leverage × shock	-0.95**	-0.77**	-0.75**
	(0.42)	(0.36)	(0.35)
ffr shock			1.38
			(1.00)
Observations	239523	239523	239523
R^2	0.106	0.118	0.103
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Results from estimating

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\mathbb{E}_j[\ell_{jt}]$ is the average leverage of firm j in the sample, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Standard errors are two-way clustered by firms and quarter. We have normalized the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

2.3 Suggestive Evidence that Default Risk Drives Heterogeneous Responses

We now provide suggestive evidence that the heterogeneous responses by leverage documented in Section 2.2 are driven, at least in part, by heterogeneity in default risk. Before doing so, we provide evidence against the hypotheses that our results are driven by permanent heterogeneity in financial positions across firms or by heterogeneity in other time-varying firm characteristics.

Permanent Heterogeneity in Financial Positions In the economic model we develop in Section 3, low-leverage firms are less affected by financial frictions because they have low risk of default. The existence of permanent heterogeneity in firms' financial positions may break this tight positive relationship between leverage and default risk. For example, if low-

term debt. We find that this is not the case; if anything, the differences across firms are stronger for debt due in less than one year.

leverage firms have poor collateral which limits their ability to borrow, then low-leverage firms may actually be the most affected by financial frictions. Another example is that low-leverage firms hold low debt because they are permanently riskier, which leads to higher costs of investment finance.

We argue that permanent heterogeneity in financial positions does not drive our results by estimating the specification

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where $\mathbb{E}_j[\ell_{jt}]$ is the average leverage of firm j in our sample.[13] Permanent heterogeneity in leverage is differenced out of the interaction $(\ell_{jt-1} - \mathbb{E}_j[\ell_{jt}])\varepsilon_t$. Table 6 shows that our benchmark results are stable if we only use within-firm variation in leverage.

Heterogeneity in Other Firm Characteristics Table 7 shows that our main results are not driven by firms' sales growth, realized future sales growth, or size. It expands the baseline specification as:

$$\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta_\ell \ell_{jt-1}\varepsilon_t^{\mathrm{m}} + \beta_y y_{jt}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where y_{jt} is lagged sales growth, realized future sales growth in one year, or lagged size. In each case, the coefficient on leverage ℓ_{jt-1} remains stable. Hence, firm-level shocks or characteristics that are correlated with these additional variables do not drive the heterogeneous responses by leverage.[14]

Table 8 provides evidence that unobservable factors do not drive the heterogeneous responses by leverage either. We instrument leverage ℓ_{jt-1} in our baseline specification (3) with past leverage (ℓ_{jt-5} or ℓ_{jt-9}). If unobserved factors drive both leverage and the response to monetary policy, and these factors are more weakly correlated with lagged leverage, we would expect these instrumental variables coefficients to be smaller than our baseline results.

[13] Our sample selection focuses on firms with at least forty quarters of data to precisely estimate the average leverage $\mathbb{E}_j[\ell_{jt}]$.

[14] Our result that large firms are more sensitive to monetary policy shocks is broadly consistent with Kudlyak and Sanchez (2017), who find that, in Compustat, large firms are also more responsive to the 2007 financial crisis.

TABLE 7
INTERACTION WITH OTHER FIRM-LEVEL COVARIATES

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
leverage × ffr shock	-0.74**		-0.74***		-0.81***		-0.73**
	(0.28)		(0.28)		(0.30)		(0.28)
sales growth × ffr shock		-0.09	-0.09*				
		(0.27)	(0.05)				
future sales growth × ffr shock				-0.55	-0.53		
				(0.40)	(1.39)		
size × ffr shock						0.35	0.37***
						(0.31)	(0.07)
Observations	239523	239523	239523	227513	227513	239523	239523
R^2	0.118	0.115	0.118	0.119	0.122	0.115	0.118
Firm controls	yes	yes	yes	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes	yes	yes

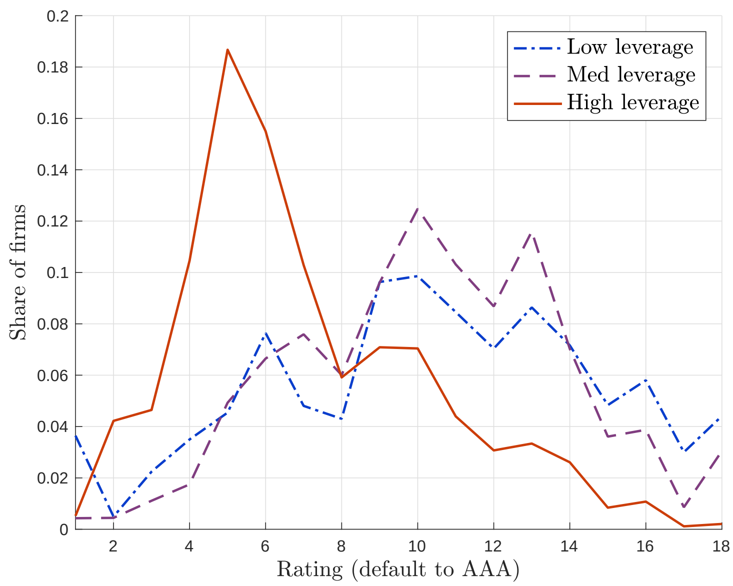
Notes: Results from estimating variants of the baseline specification $\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta y_{jt}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, y_{jt} is the firm's lagged sales growth, future sales growth, or lagged size, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Columns (2) and (4) additionally include an interaction between leverage ℓ_{jt-1} and the monetary policy shock $\varepsilon_t^{\mathrm{m}}$. Standard errors are two-way clustered by firms and quarter. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

TABLE 8
INSTRUMENTING LEVERAGE WITH PAST LEVERAGE

	(1)	(2)
leverage × ffr shock	-0.80	-2.64***
	(1.27)	(0.95)
Observations	230621	221468
R^2		
Firm controls, Time-Sector FE	yes	yes
Instrument	4q lag	8q lag

Notes: Results from estimating and IV strategy for the baseline specification $\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Leverage in $t-4$ and $t-8$ are used as instruments for leverage in $t-1$. Standard errors are two-way clustered by firms and time. We have normalized the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage x_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

Figure 3: Distribution of Credit Ratings, Conditional on Leverage



Notes: Conditional distribution of credit ratings by leverage. "Low leverage" refers to observations in the bottom tercile of leverage. "Medium leverage" refers to observations in the middle tercile of leverage. "High leverage" refers to observations in the top tercile of leverage.

Instead, Table 8 shows that the estimated coefficients increase in this instrumental variables specification. This result is consistent with measurement error creating attenuation bias in our baseline specification (3).

Heterogeneity in Credit Ratings and Other Measures of Financial Positions We now argue that the heterogeneous responses by leverage are driven, at least in part, by heterogeneity in default risk. Our argument has two main components. First, firms with low leverage on average have high credit ratings. Figure 3 plots the distribution of firm-level credit ratings for conditional on having low, medium, and high leverage. Most of the mass of the high-leverage distributions in concentrated in the left tail, below credit rating category 8 (BB). In contrast, most of the mass of medium- and particularly high-leverage distributions are in the right tail of the credit rating categories. Table 21 in Appendix A.1 shows that this negative relationship between leverage and credit rating is also true conditional on the set of controls that enter our baseline regression (3).

Table 9

Heterogeneous Responses by Credit Rating

	(1)	(2)	(3)
leverage × ffr shock	-0.73**		-0.71**
	(0.29)		(0.29)
$\mathbb{1}\{\texttt{rating}_{it} \geq AA\}\times$ ffr shock		2.50**	2.37**
		(1.14)	(1.16)
Observations	233232	233182	233182
R^2	0.119	0.119	0.119
Firm controls	yes	yes	yes
Time sector FE	yes	yes	yes
Time clustering	yes	yes	yes

Notes: Results from estimating variants of the baseline specification $\Delta \log k_{jt+1} = \alpha_j + \alpha_{st} + \beta y_{jt-1}\varepsilon_t^{\mathrm{m}} + \boldsymbol{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, y_{jt-1} is the firm's leverage or an indicator for having a credit rating above AA, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Standard errors are two-way clustered by firms and time. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

The second component of our argument is that highly rated firms are more responsive to monetary policy. Table 9 estimates our baseline specification (3) with an additional interaction for credit rating; the coefficient estimate in column (2) implies that firms with a rating above AA have a 2.5 unit higher semi-elasticity with respect to monetary policy. This increase nearly triples the response relative to the average firm. Column (3) shows that this relationship continues to hold even conditional on leverage, consistent with the idea that both leverage and credit rating are imperfect proxies for firms' default risk.

Appendix A.1 Table 22 explores heterogeneity by size, cash flows, and dividend payments. Overall, the interaction between these variables is weaker than the interactions with leverage and credit ratings. Nonetheless, larger firms, firms with higher cash flows, and dividend-paying firms – characteristics typically associated with less severe financial frictions – are more responsive to monetary policy shocks.

3 Model

This section develops a heterogeneous firm New Keynesian model in order to interpret the cross-sectional evidence in Section 2 and draw out aggregate implications. Our model embeds a corporate finance-style model of investment subject to default risk into the dynamic New Keynesian framework.

3.1 Environment

Time is discrete and infinite. We describe in the model in three blocks: an investment block, which captures heterogeneous investment responses to monetary policy; a New Keynesian block, which generates a Phillips Curve; and a representative household.

3.1.1 Investment Block

Our investment block contains a fixed mass of heterogeneous production firms that invest in capital subject to financial frictions. It builds heavily on the flexible-price model developed in Khan, Senga and Thomas (2016). Besides incorporating sticky prices, we extend Khan, Senga and Thomas (2016)'s framework in three additional ways. First, we add idiosyncratic capital quality shocks, which help us match observed default rates. Second, we incorporate aggregate adjustment costs in order to generate time-variation in the relative price of capital. Third, we assume that new entrants have lower initial productivity than average firms, which helps us match lifecycle dynamics.

Production Firms Each period there is a fixed mass M of production firms.[15] Each heterogeneous production firm $j \in [0, M]$ produces an undifferentiated good y_{jt} using the production function

$$y_{jt} = z_{jt}(\omega_{jt}k_{jt})^{\theta}n_{jt}^{\nu}, \tag{8}$$

where z_{jt} is an idiosyncratic total factor productivity shock, ω_{jt} is an idiosyncratic capital quality shock, k_{jt} is the firm's capital stock, n_{jt} is the firm's labor input, and $\theta + \nu < 1$. The

[15] We describe the entry and exit process below, which keeps the total mass of firms fixed.

idiosyncratic TFP shock follows an log-AR(1) process

$$\log z_{jt+1} = \rho z_{jt} + \varepsilon_{jt+1}, \text{ where } \varepsilon_{jt+1} \sim N(0, \sigma^2). \tag{9}$$

The capital quality shock is i.i.d. across firms and time and follows the log-normal process[16]

$$\log \omega_{jt} \sim N(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2).$$

Each period, each firm j makes a series of decisions in order to maximize its value. First, with probability π_d the firm receives an i.i.d. exit shock and must exit the economy at the end of the period. Firms that do not receive the exit shock will be allowed to continue into the next period.

Conditional on the realization of the exit shock, the firm decides whether or not to default. If the firm defaults it immediately and permanently exits the economy. In order to continue, the firm must pay back the face value of its outstanding debt, B_{jt}, and pay a fixed operating cost ξ in units of the final good. We assume that in the event of default the equity holders do not directly recover any resources from the firm, so the firm's value upon default is zero. Lenders recover a fraction of the capital stock, and the remaining capital is rebated lump-sum to the representative household. We describe the lender's problem in more detail below.

Firms that do not default produce using the production function (8). The firms sell their undifferentiated output in a competitive market at price P_t. In order to produce, firms use their pre-existing stock of capital k_{jt} and hire labor n_{jt} from a competitive labor market at wage W_t. After production, firms that received the idiosyncratic exit shock sell their undepreciated capital and exit the economy.

Continuing firms make investment and financing decisions. The price of capital in period t is Q_t. Firms have two sources of investment finance, each of which is subject to a friction. First, firms can use external finance by issuing new nominal debt with face value

[16] We additionally assume that the idiosyncratic shock processes are bounded, which is important in our definition of unconstrained firms below. The idiosyncratic TFP shock is constrained to be in the interval $\left[-\frac{2.5\sigma}{\sqrt{1-\rho^2}}, \frac{2.5\sigma}{\sqrt{1-\rho^2}}\right]$ and the capital quality shock is in the interval $[-2.5\sigma_\omega, 2.5\sigma_\omega]$.

B_{jt+1}. Lenders offer a price schedule $\mathcal{Q}_t(z_{jt}, k_{jt+1}, B_{jt+1})$ which is decreasing in the amount of borrowing B_{jt+1} because firms may default on this borrowing (we derive this price schedule below). Second, firms can use internal finance by lowering dividend payments D_{jt}. Firms cannot issue new equity, which bounds dividend payments $D_{jt} \geq 0$.[17] Dividend payments in period t are given by[18]

$$D_{jt} = \max_n P_t z_{jt}(\omega_{jt} k_{jt})^\theta n_{jt}^\nu - W_t n_{jt} - B_{jt} - \xi + Q_t(1-\delta)\omega_{jt} k_{jt} - Q_t k_{jt+1} + \mathcal{Q}_t(z, k_{jt+1}, B_{jt+1}) B_{jt+1}.$$

Note that the capital quality shock ω_{jt} affects the value of undepreciated capital in addition to the value of capital in production.

Lenders There is a representative financial intermediary that lends resources from the household to the production firms at the firm-specific price schedule $\mathcal{Q}_t(z_{jt}, k_{jt+1}, B_{jt+1})$. These lenders are competitive, so the schedule $\mathcal{Q}_t(z_{jt}, k_{jt+1}, B_{jt+1})$ simply prices the firm's default risk in period $t+1$. In the event of default the lender recovers a fraction α of the market value of the firm's undepreciated capital stock $Q_{t+1}(1-\delta)\omega_{jt+1} k_{jt+1}$.

Entry Each period, a mass $\overline{\mu}_t$ of new firms enter the economy. We assume that the mass of new entrants is equal to the mass of firms that exit the economy so that the total mass of production firms is fixed in each period t. Each of these new entrants $j \in [0, \overline{\mu}_t]$ draws an idiosyncratic productivity shock z_{jt} from the time-invariant distribution

$$\mu^{\text{ent}}(z) \sim \log N\left(-m\frac{\sigma}{\sqrt{(1-\rho^2)}}, s\frac{\sigma}{\sqrt{(1-\rho^2)}}\right),$$

[17]The non-negative dividend constraint captures two key facts about external equity documented in the corporate finance literature. First, firms face significant costs of issue new equity, both direct flotation costs (see, for example, Smith (1977)) and indirect costs (for example, Asquith and Mullins (1986)). Second, firms issue external equity very infrequently (DeAngelo, DeAngelo and Stulz (2010)). The specific form of the non-negativity constraint is widely used in the macro literature because it allows for efficient computation of the model in general equilibrium. Other potential assumptions include proportional costs of equity issues (e.g., Gomes, 2001; Cooley and Quadrini, 2001; Hennessy and Whited, 2005; Gilchrist, Sim and Zakrajsek, 2014) and quadratic costs (e.g., Hennessy and Whited, 2007).

[18]We assume that firms sell their undepreciated capital stock back to the capital good producer (described below). This assumption implies that firms value their undepreciated capital stock at its market value.

where $m \geq 0$ and $s \geq 0$ are parameters. We calibrate these parameters to match the average size and growth rates of new entrants, motivated by the evidence in Foster, Haltiwanger and Syverson (2016) that young firms have persistently low levels of measured productivity.[19] New entrants also draw capital quality from its ergodic distribution, are endowed with k_0 units of capital from the household, and have zero units of debt. They then proceed as incumbent firms.

3.1.2 New Keynesian Block

The New Keynesian block of the model is designed to parsimoniously generate a New Keynesian Phillips curve relating nominal variables to the real economy. Following Bernanke, Gertler and Gilchrist (1999), we keep the nominal rigidities separate from the investment block of the model.[20]

Retailers There is a fixed mass of retailers $i \in [0, 1]$. Each retailer producers a differentiated variety $\widetilde{y}_{it}$ using the heterogeneous production firms' good as its only input. The production function is simply

$$\widetilde{y}_{it} = y_{it},$$

where y_{it} is the amount of the undifferentiated good demanded by retailer i. Retailers are monopolistic competitors who set their prices $\widetilde{P}_{it}$ subject to the demand curve generated by the final good producer (described below). Retailers pay a quadratic adjustment cost $\frac{\varphi}{2}\left(\frac{\widetilde{P}_{it}}{\widetilde{P}_{it-1}} - 1\right)^2 Y_t$ to adjust their price, where Y_t is the final good.

[19] Foster, Haltiwanger and Syverson (2016) argue that these low levels of measured productivity among young firms demand across firms rather than physical productivity. We remain agnostic about the interpretation of TFP in our model. Without the assumption that entrants have lower average productivity than existing firms, default risk would be disproportionately concentrated in a small group of young firms.

[20] Our formulation separates investment and price-setting decisions of firms and can be literally interpreted as representing perfectly competitive wholesalers whose product is then marked up by monopolistically competitive retailers. Studying the joint dynamic decision of investment and price setting under financial frictions and nominal rigidities is outside the scope of our paper.

Final Good Producer There is a representative final good producer who produces the final good Y_t using the production function

$$Y_t = \left(\int \widetilde{y}_{it}^{\frac{\gamma-1}{\gamma}} \mathrm{d}i \right)^{\frac{\gamma}{\gamma-1}},$$

where γ is the elasticity of substitution over intermediate goods.

Capital Good Producer There is a representative capital good producer who produces aggregate capital K_{t+1} using the technology

$$K_{t+1} = \Phi(\frac{I_t}{K_t})K_t + (1-\delta)K_t, \tag{10}$$

$\Phi(\frac{I_t}{K_t}) = \frac{\delta^{1/\phi}}{1-1/\phi}\left(\frac{I_t}{K_t}\right)^{1-1/\phi} - \frac{\delta}{\phi-1}$ and I_t are units of the final good used to produce capital.[21]

Monetary Authority The monetary authority sets the nominal risk-free interest rate R_t^{nom} according to the Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \log \Pi_t + \varepsilon_t^{\text{m}}, \text{ where } \varepsilon_t^m \sim N(0, \sigma_m^2),$$

where Π_t is gross inflation of the final good price, φ_π is the weight on inflation in the reaction function, and ε_t^{m} is the monetary policy shock. ε_t^{m} is the only aggregate disturbance in the model. In our quantitative work, we will characterize the effect of an unexpected change in ε_t^{m} followed by a perfect foresight transition path back to steady state.

[21] We use external adjustment costs rather than internal adjustment costs for two reasons. First, external adjustment costs generate time-variation in the price of capital, which allows us to study changes in the collateral value of capital. Second, because capital is liquid at the firm level, we can reduce the number of individual state variables, which is useful in the computation of the model.

3.1.3 Household

There is a representative household with preferences over consumption C_t and labor supply N_t represented by the expected utility function

$$\mathbb{E}_0 \sum_t^{\infty} \beta^t \left(\log C_t - \Psi N_t \right),$$

where β is the discount factor and Ψ controls the disutility of labor supply. The household owns all firms in the economy.

3.2 Equilibrium

We now characterize and define the model's equilibrium.

3.2.1 New Keynesian Block

We begin with the New Keynesian block of the model. As usual, the final good producer's profit maximization problem gives the demand curve $\left(\frac{\widetilde{P}_{it}}{\widetilde{P}_t}\right)^{-\gamma} Y_t$ where $\widetilde{P}_t = \left(\int \widetilde{P}_{it}^{1-\gamma} \mathrm{d}i\right)^{\frac{1}{1-\gamma}}$ is the price index. We take the final good as the numeraire.

Retailers are symmetric and face real marginal cost $p_t = \frac{P_t}{\widetilde{P}_t}$ in their price-setting decision. After aggregation, this yields the familiar New Keynesian Phillips Curve:

$$\log \Pi_t = \frac{\gamma - 1}{\varphi} \log \frac{p_t}{p^*} + \beta \mathbb{E}_t \log \Pi_{t+1}, \tag{11}$$

where $\Pi_t = \frac{\widetilde{P}_t}{\widetilde{P}_{t-1}}$ is gross inflation and $p^* = \frac{\gamma-1}{\gamma}$ is the steady state relative price of the heterogeneous production firm output.[22] The Phillips Curve links the New Keynesian block to the investment block through the relative price p_t. When aggregate demand for the final good Y_t increases, retailers must increase production of their differentiated goods because of the nominal rigidities; this force increases demand for the production firms good y_{it}, which increases its relative price p_t and generates inflation through the Phillips Curve (11).

The final good, which is aggregate GDP in our model, is simply the total output of

[22] We focus directly on the linearized formulation for computational simplicity.

production firms:

$$Y_t = \int_0^M z_{jt}(\omega_{jt}k_{jt})^\theta n_{jt}^\nu \mathrm{d}j. \tag{12}$$

From the capital good producer's profit maximization problem, the relative price of capital $q_t = \frac{Q_t}{\widetilde{P}_t}$ is given by

$$q_t = \frac{1}{\Phi'(\frac{I_t}{K_t})} = \left(\frac{I_t/K_t}{\delta}\right)^{1/\phi}. \tag{13}$$

3.2.2 Investment Block

We characterize the decisions of the heterogeneous production firms recursively. The individual state variable of a production firm is z, its draw of the idiosyncratic productivity shock; ω, its draw of the capital quality shock; k, its stock of capital; and B, the face value of outstanding debt. The aggregate state contains the monetary policy shock $\varepsilon_t^{\mathrm{m}}$ and the distribution of firms over their individual states.

Default Firms in our model only default when they have no feasible choice. To see this, first note that the non-negativity constraint on dividends implies that firms' profits are weakly positive, which in turn implies that the value of not defaulting is weakly positive. Since we assume the value of default is zero, firms with a feasible choice will always weakly prefer to continue. Let $\chi_t^1(z,\omega,k,B)$ and $\chi_t^2(z,\omega,k,B)$ be indicators for firms not defaulting conditional on receiving or not receiving the exit shock. We show in Proposition 1 below that default decisions are characterized by a simple threshold rules.

Exiting Firms' Decision Rules If the firm receives the idiosyncratic exit shock and does not default, its value is

$$V_t^{\mathrm{exit}}(z,\omega,k,B) = \max_n P_t z(\omega k)^\theta n^\nu - W_t n + Q_t(1-\delta)\omega k - B - \widetilde{P}_t \xi,$$

where we have absorbed the aggregate state into the time subscript t for notational simplicity. Because the firm is not continuing into the next period, it simply chooses its labor input n to maximize its current revenue net of labor costs, sells its undepreciated capital, pays back the face value of its debt, pays its fixed operating cost, and exits the economy.

Continuing Firms' Decision Rules If the firm does not receive the idiosyncratic exit shock and does not default, its value is

$$V_t^{\text{cont}}(z,\omega,k,B) = \max_{n,k',B'} P_t z(\omega k)^\theta n^\nu - W_t n + Q_t(1-\delta)\omega k - B - \widetilde{P}_t\xi - Q_t k' + \mathcal{Q}_t(z,k',B')B'$$
$$+ \mathbb{E}_t\left[\hat{\Lambda}_{t,t+1}\left(\pi_d \chi^1_{t+1}(z',\omega',k',B')V^{\text{exit}}_{t+1}(z',\omega',k',B') + (1-\pi_d)\chi^2_{t+1}(z',\omega',k',B')V^{\text{cont}}_{t+1}(z',\omega',k',B')\right)\right]$$
$$\text{s.t. } P_t z(\omega k)^\theta n^\nu - W_t n + Q_t(1-\delta)\omega k - B - \widetilde{P}_t\xi - Q_t k' + \mathcal{Q}_t(z,k',B')B' \geq 0, \tag{14}$$

where $\hat{\Lambda}_{t,t+1} = \beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{\widetilde{P}_t}{\widetilde{P}_{t+1}}$ is the nominal stochastic discount factor. The firm chooses its labor input, investment, and borrowing to maximize the value of its current dividends plus its continuation value. In making these choices, the firm takes the debt price schedule $\mathcal{Q}_t(z,k',B')$ as given and cannot pay negative dividends.

It is convenient to make two simplifications to the firm's decision problem. First, we write the problem relative to the price level $\widetilde{P}_t$; to that end, let $b=\frac{B}{\widetilde{P}_t}$, $b'=\frac{B'}{\widetilde{P}_t}$, $\Lambda_{t,t+1}=\beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$, and $v_t(z,\omega,k,b) = \frac{V_t^{\text{cont}}(z,\omega,k,B)}{\widetilde{P}_t}$. Second, we combine capital k, debt b, and the capital quality shock ω into a composite state variable $x = \max_n p_t z(\omega k)^\theta n^\nu - w_t n + q_t(1-\delta)\omega k - b - \xi$. The composite state variable x is sufficient because capital is a liquid asset and capital quality is i.i.d. over time. We often refer to x as "cash on hand" because it is the total amount of resources available to the firm other than additional borrowing. The normalized value function over this composite state variable solves the Bellman equation

$$v_t(z,x) = \max_{k',b'} x - q_t k' + \mathcal{Q}_t(z,k',b')b' + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\pi_d\chi^1(x')x' + (1-\pi_d)\chi^2_{t+1}(z',x')v_{t+1}(z',x')\}\right)\right]$$
$$\text{such that } x - q_t k' + \mathcal{Q}_t(z,k',b')b' \geq 0 \tag{15}$$
$$x' = \max_{n'} p_{t+1}z'(\omega' k')^\theta (n')^\nu - w_{t+1}n' + q_{t+1}(1-\delta)\omega' k' - \frac{b'}{\Pi_{t+1}} - \xi,$$

where $\chi^1(x)$ and $\chi^2_t(z,x)$ are the normalized default indicators. Proposition 1 characterizes the decision rules which solve this Bellman equation.

***Proposition* 1.** *Consider a firm at time t that is eligible to continue into the next period, has idiosyncratic productivity z, and has cash on hand x. The firm's optimal decision is characterized by one of the following three cases.*

(i) ***Default**: there exists a threshold $\underline{x}_t(z)$ such that the firm defaults if $x < \underline{x}_t(z)$.*

(ii) ***Unconstrained**: there exists a threshold $\overline{x}_t(z)$ such that the firm is **financially unconstrained** if $x > \overline{x}_t(z)$. Unconstrained firms follow the capital accumulation policy*

$$k_t'(z,x) = k_t^*(z) = \left(\frac{1}{q_t} \frac{\mathbb{E}_t \left[\Lambda_{t+1} A \hat{\theta} p_{t+1}^{\frac{1}{1-\nu}} w_{t+1}^{-\frac{\nu}{1-\nu}} z'^{\frac{1}{1-\nu}} \right]}{1 - (1-\delta)\mathbb{E}_t \left[\Lambda_{t+1} \frac{q_{t+1}}{q_t} \right]} \right)^{\frac{1}{1-\hat{\theta}}}, \tag{16}$$

where $A = \left(\nu^{\frac{\nu}{1-\nu}} - \nu^{\frac{1}{1-\nu}} \right) e^{-\frac{\sigma_\omega^2}{2}\hat{\theta}(1-\hat{\theta})}$ and $\hat{\theta} = \frac{\theta}{1-\nu}$, for each period t. Unconstrained firms are indifferent over any combination of b' and d such that they remain unconstrained for every period with probability one. We assume that unconstrained firms choose borrowing $b' = b_t^*(z)$ defined by

$$\begin{aligned} b_t^*(z) = \Pi_{t+1} \min_{z'} \{ \max_{n'} \{ p_{t+1} z' (\underline{\omega} k_t^*(z))^\theta (n')^\nu - w_{t+1} n' \} + q_{t+1}(1-\delta)\underline{\omega} k_t^*(z) - \xi \\ + \min\{ \mathbb{E}_t \left[\Lambda_{t+1} \right] b_{t+1}^*(z')/\Pi_{t+1} - q_{t+1} k_{t+1}^*(z'), 0 \} \}, \end{aligned} \tag{17}$$

where $\underline{\omega} = -2.5\sigma_\omega$ is the lower bound on capital quality ω. The policy $b_t^*(z)$ is the maximum amount of borrowing that an unconstrained firm can do while still guaranteeing it will not default with probability one.

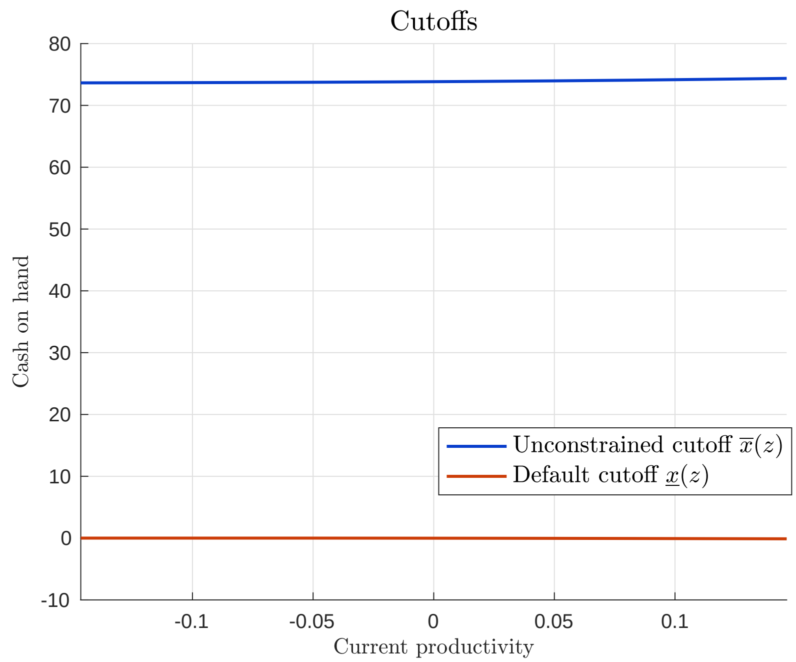
(iii) ***Constrained**: firms with $x \in [\underline{x}_t(z), \overline{x}_t(z)]$ are **financially constrained**. Constrained firms' optimal investment $k_t'(z,x)$ and borrowing $b_t'(z,x)$ decisions solve the Bellman equation (15) and pay zero dividends.*

Proof. See Appendix A.2. ■

Proposition 1 partitions the individual state space (z, x) into three distinct regions, which Figure 4 plots in steady state for the calibrated parameter values from Section 4. Firms with low cash on hand $x < \underline{x}_t(z)$ default. As discussed above, firms only default when they have no feasible choice of capital investment k' and financial investment b' that satisfies the non-negativity constraint on dividends,

$$x - q_t k' + \mathcal{Q}_t(z, k', b')b' \geq 0.$$

FIGURE 4: Partition of Individual State Space



Notes: Partition of individual state space for the calibrated parameters from Section 4 in steady state. Firms below the red line have $x < \underline{x}_t(z)$ and default. Firms above the blue line area have $x > \overline{x}_t(z)$ and are unconstrained. Firms in between the two lines have $x \in [\underline{x}_t(z), \overline{x}_t(z)]$ and are constrained according to the definition in Proposition 1.

The minimum amount of "cash-on-hand" x that a firm can have and still satisfy this constraint is

$$\underline{x}_t(z) = \xi - \max_{k',b'} \left(\mathcal{Q}_t(z, k', b')b' - q_t k' \right).$$

The threshold $\underline{x}_t(z)$ is decreasing in productivity z because firms with high productivity face more favorable borrowing rates.

Firms with high cash on hand $x > \overline{x}_t(z)$ are *financially unconstrained* in the sense that they can follow the frictionless capital accumulation policy (16) for their entire lifetime and have zero probability of default. Any combination of external financing b' and internal financing d that leaves these firms unconstrained is an optimal decision; in this sense, the Modigliani-Miller theorem holds for unconstrained firms. Following Khan, Senga and Thomas (2016), we resolve this indeterminacy by imposing the *maximum borrowing policy* $b_t^*(z)$ defined in (17). $b_t^*(z)$ is the highest level of debt which firms can incur and be guaranteed to, with probability one, not default.[23]

Firms with intermediate cash on hand $x \in [\underline{x}_t(z), \overline{x}_t(z)]$ do not default but are not financially unconstrained; hence, we refer to these firms as *financially constrained*. Constrained firms set $d = 0$ because the value of resources inside the firm, used to lower default risk, is higher than the value of resources outside the firm. Setting $d = 0$ implies

$$q_t k' = x + \mathcal{Q}_t(z, k', b')b'. \tag{18}$$

Constrained firms' investment expenditures are therefore financed by either their own cash on hand x or new borrowing $\mathcal{Q}_t(z, k', b')b'$. Financially constrained firms can be either *risky constrained* firms, who pay a credit spread over the risk-free rate, from *risk-free constrained* firms, who do not currently pay a credit spread but have a positive probability of default in some future state.

Lenders In real terms, a loan to a firm is an asset that pays $\frac{1}{\Pi_{t+1}}$ units of the final good if the firm does not default and pays $\min\{\frac{\alpha q_{t+1}\omega' k'}{b'/\Pi_{t+1}}, 1\}$ if the firm does default. Therefore, its price is

$$\mathcal{Q}_t(z, k', b') = \mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{1}{\Pi_{t+1}} - \left(\pi_d(1 - \chi^1(x')) + (1 - \pi_d)(1 - \chi^2_{t+1}(z', x'))\right)\left(\frac{1}{\Pi_{t+1}} - \min\{\frac{\alpha q_{t+1}(1-\delta)\omega' k'}{b'/\Pi_{t+1}}, 1\}\right)\right)\right], \tag{19}$$

where $x' = \max_{n'} p_{t+1} z(\omega' k')^\theta (n')^\nu - w_t n' + q_{t+1}(1 - \delta)\omega' k' - b' - \xi$ is the implied cash-on-hand and $\chi^1(x) = \mathbb{1}\{x \geq 0\}$ and $\chi^2_t(z, x) = \mathbb{1}\{x \geq \underline{x}_t(z)\}$ are indicator variables for not defaulting.

Distribution of Firms The distribution of firms in production is composed of incumbents who do not default and new entrants who do not default. Mathematically, this distribution

[23]The "with probability one" statement does not take into account the monetary policy shock, which is completely unexpected by firms. However, since we only analyze expansionary shocks, monetary policy in our model does not induce any firm to default.

$\hat{\mu}_t(z,x)$ is given by

$$\hat{\mu}_t(z,x) = \int \big(\pi_d \chi^1(x_t(z,\omega,k,b)) + (1-\pi_d)\chi_t^2(z, x_t(z,\omega,k,b))\big)\,\mathrm{d}\mu_t(z,\omega,k,b) \\ + \overline{\mu}_t \int \big(\pi_d \chi^1(x_t(z,\omega,k_0,0)) + (1-\pi_d)\chi_t^2(z, x_t(z,\omega,k_0,0))\big)\, g(\omega)\mathrm{d}\omega \mathrm{d}\mu^{\text{ent}}(z), \tag{20}$$

where $x_t(z,\omega,k,b) = \max_n p_t z(\omega k)^\theta n^\nu - w_t n + q_t(1-\delta)\omega k - b - \xi$ is the implied cash-on-hand x of a firm with state (z,ω,k,b) and $g(\omega)$ is the PDF of capital quality shocks.

The evolution of the distribution of firms $\mu_t(z,\omega,k,b)$ is given by

$$\mu_{t+1}(z',\omega',k',b') = \int (1-\pi_d)\chi_t^2(z, x_t(z,\omega,k,b)) 1\{k_t'(z, x_t(z,\omega,k,b)) = k'\} \\ \times \mathbb{1}\{\frac{b_t'(z, x_t(z,\omega,k,b))}{\Pi_{t+1}} = b'\} p(\varepsilon | e^{\rho \log z + \varepsilon} = z') g(\omega')\mathrm{d}\varepsilon \mathrm{d}\mu_t(z,\omega,k,b) \\ + \overline{\mu}_t \int (1-\pi_d)\chi_t^2(z, x_t(z,\omega,k_0,0)) 1\{k_t'(z, x_t(z,\omega,k_0,0)) = k'\} \\ \times \mathbb{1}\{\frac{b_t'(z, x_t(z,\omega,k_0,0))}{\Pi_{t+1}} = b'\} p(\varepsilon | e^{\rho \log z + \varepsilon} = z') g(\omega')\mathrm{d}\varepsilon \mathrm{d}\mu^{\text{ent}}(z), \tag{21}$$

where $p(\varepsilon | e^{\rho \log z + \varepsilon} = z')$ denotes the density of draws ε such that $e^{\rho \log z + \varepsilon} = z'$.

3.2.3 Equilibrium Definition

An **equilibrium** of this model is a set of $v_t(z,x)$, $k_t'(z,x)$, $b_t'(z,x)$, $n_t(z,x)$, $\mathcal{Q}_t(z,k',b')$, Π_t, Δ_t, Y_t, q_t, $\mu_t(z,\omega,k,b)$, $\hat{\mu}_t(z,x)$, $\Lambda_{t,t+1}$, w_t, C_t, and I_t such that

(i) Production firms optimization: $v_t(z,x)$ solves the Bellman equation (15) with associated decision rules $k_t'(z,x)$, $b_t'(z,x)$, and $n_t(z,x)$.

(ii) Financial intermediaries price default risk according to (19).

(iii) New Keynesian block: Π_t, p_t, and q_t satisfy (11) and (13).

(iv) The distribution of firms in production $\hat{\mu}_t(z,x)$ satisfies (20) and the distribution $\mu_t(z,\omega,k,b)$ evolves according to (21).

(v) Household block: the stochastic discount factor is given by $\Lambda_{t,t+1} = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$. The wage must satisfy $w_t = \Psi C_t$. The stochastic discount factor and nominal interest rate are

Table 10
Fixed Parameters

Parameter	Description	Value
Household		
β	Discount factor	0.99
Firms		
ν	Labor coefficient	0.64
θ	Capital coefficient	0.21
δ	Depreciation	0.026
New Keynesian Block		
ϕ	Aggregate capital AC	4
γ	Demand elasticity	10
φ_π	Taylor rule coefficient	1.25
φ	Price adjustment cost	90

Notes: Parameters exogenously fixed in the calibration.

linked through the Euler equation for bonds, $1 = \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{R_t^{\text{nom}}}{\Pi_{t+1}}\right]$.

(vi) Market clearing: aggregate investment is implicitly defined by $K_{t+1} = \Phi(\frac{I_t}{K_t})K_t + (1-\delta)K_t$, where $K_t = \int k\mathrm{d}\mu_t(z,\omega,k,b)$. Aggregate consumption is defined by $C_t = Y_t - I_t - \xi$.[24]

4 Calibration and Steady State Analysis

We now calibrate the model and verify that its steady state behavior is consistent with key features of the micro data. In Section 5, we use the calibrated model to study the effect of a monetary policy shock ε_t^{m}.

4.1 Calibration

We calibrate the model in two steps. First, we exogenously fix a subset of parameters. Second, we choose the remaining parameters in order to match moments in the data.

[24] We normalize the mass of firms in production to 1, so ξ is the total resources lost from the fixed operating costs.

Fixed Parameters Table 10 lists the parameters that we fix. The model period is one quarter, so we set the discount factor $\beta = 0.99$. We set the coefficient on labor $\nu = 0.64$. We choose the coefficient on capital $\theta = 0.21$ to imply a total returns to scale of 85%. Capital depreciates at rate $\delta = 0.026$ quarterly to match the average aggregate investment rate of nonresidential fixed investment reported in Bachmann, Caballero and Engel (2013).

We choose the elasticity of substitution in final goods production $\gamma = 10$, implying a steady state markup of 11%. This choice implies that the steady state labor share is $\frac{\gamma-1}{\gamma}\nu \approx 58\%$, close to the current U.S. labor share reported in Karabarbounis and Neiman (2013). We choose the coefficient on inflation in the Taylor rule $\varphi_\pi = 1.25$, in the middle of the range commonly considered in the literature. Finally, we set the price adjustment cost parameter $\varphi = 90$ to generate the slope of the Phillips Curve equal to 0.1, as in Kaplan, Moll and Violante (2017).

Fitted Parameters We choose the parameters listed in Table 11 to match the empirical moments reported in Table 12. The first set of parameters govern the idiosyncratic shocks: ρ and σ control the AR(1) process for TFP and σ_ω controls the i.i.d. process for capital quality. The second set of parameters govern the frictions to external finance: the fixed operating cost ξ controls how often firms default and the recovery rate α controls the credit spread conditional on default. The final set of parameters govern the firm lifecycle: the parameters m and s control the productivity distribution of new entrants, k_0 controls the initial capital stock of new entrants, and π_d is the probability of receiving an exogenous exit shock.

We target four key sets of statistics in our calibration.[25] First, we target the dispersion of plant-level investment rates in Census microdata reported by Cooper and Haltiwanger (2006).[26] The dispersion of investment rates places discipline on the degree of idiosyncratic risk faced by firms. Cooper and Haltiwanger (2006)'s sample is a balanced panel of plants that have survived at least sixteen years; to mirror this sample selection in the model, we

[25]At each step of this moment-matching process, we choose the disutility of labor supply Ψ to generate a steady state employment rate of 60%.

[26]An issue with this empirical target is that production units in our model correspond more closely to firms than to plants. We prefer to use the plant-level data from Cooper and Haltiwanger (2006) because it carefully constructs measures of retirement and sales of capital to measure negative investment, which is important in our model because capital is liquid.

Table 11

Fitted Parameters

Parameter	Description	Value
Idiosyncratic shock processes		
ρ	Persistence of TFP	0.86
σ	SD of innovations to TFP	0.03
σ_ω	SD of capital quality	0.04
Financial frictions		
ξ	Operating cost	0.02
α	Loan recovery rate	0.91
Firm lifecycle		
m	Mean shift of entrants' prod.	2.92
s	SD of entrants' prod	1.11
k_0	Initial capital	0.46
π_d	Exogeneous exit rate	0.02

Notes: Parameters chosen to match the moments in Table 12.

condition on firms that have survived for twenty years, but our calibration results are robust to different choices of this cutoff.

The second set of moments we target are related to firms' use of external finance. Following Bernanke, Gertler and Gilchrist (1999), we target a mean default rate of 3% as estimated in a survey of businesses by Dun and Bradstreet. We target an average annual credit spread implied by BAA rated corporate bond yields to the ten-year Treasury yield.[27] Finally, we target the average firm-level gross leverage ratio of 34.4% from the microdata underlying the Quarterly Financial Reports, as reported in Crouzet and Mehrotra (2017).

The final two sets of moments are informative about firm lifecycle dynamics. We target the average size of firms one year old and two years old relative to the average size of all firms in the economy. The relative size of one year old firms is informative about the size of new entrants, and the difference between the sizes of one and two year old firms is informative about how quickly young firms grow. We also target the average exit rate and the share of

[27] We target credit spreads because the debt price schedule is central to the economic mechanisms in our model. To the extent that observed credit spreads are driven by risk premia rather than risk-neutral pricing of default risk, we may overstate the importance of default risk in our calibration. We do not believe this is a major concern because our calibrated debt recovery rate is broadly in line with estimated loss in default from the corporate finance literature. For robustness, we also directly targeted estimates of the cost of default from this literature, rather than the level of spreads, and found similar steady state behavior.

Table 12
Model Fit

Moment	Description	Data	Model
Investment behavior (annual)			
$\sigma\left(\frac{i}{k}\right)$	SD investment rate	33.7%	31.8%
Financial behavior (annual)			
$\mathbb{E}\left[\text{default rate}\right]$	Mean default rate	3.00%	2.01%
$\mathbb{E}\left[\text{credit spread}\right]$	Mean credit spread	2.35%	2.54%
$\mathbb{E}\left[\frac{b}{k}\right]$	Mean gross leverage ratio	34.4%	33.6%
Firm Growth (annual)			
$\mathbb{E}[n_1]/\mathbb{E}[n]$	Size of age 1 firms (relative to mean)	28%	42%
$\mathbb{E}[n_2]/\mathbb{E}[n]$	Size of age 2 firms (relative to mean)	36%	66%
Firm Exit (annual)			
$\mathbb{E}\left[\text{exit rate}\right]$	Mean exit rate	8.7%	7.88%
$\mathbb{E}\left[M_1\right]/\mathbb{E}\left[M\right]$	Share of firms at age 1	10.5%	7.4%
$\mathbb{E}\left[M_2\right]/\mathbb{E}\left[M\right]$	Share of firms at age 2	8.1%	6.1%

Notes: Empirical moments targeted in the calibration. Investment behavior drawn from the distribution of plant-level investment rates in Census microdata, 1972-1988, reported in Cooper and Haltiwanger (2006). These investment moments are drawn from a balanced panel; we mirror this sample selection in the model by computing investment moments for firms who have survived at least twenty years. The mean default rate is from Dun and Bradstreet survey, as reported by Bernanke, Gertler and Gilchrist (1999). The average firm-level gross leverage ratio is taken from the micro data underlying the Quarterly Financial Reports, and is reported in Crouzet and Mehrotra (2017). The average credit spread is measured as the yield on BAA rated corporate bonds relative to a ten-year Treasury bond. The mean exit rate is computed from the Business Dynamics Statistics (BDS). The average size of firms age one and two is relative to the average size of firms the economy, and also drawn from the BDS. The shares of firms at age one and two are also drawn from the BDS.

firms in the economy at age one and two. The difference in shares of age one and two firms is informative about the exit rate of young firms. All of these statistics are computed from the Business Dynamics Statistics (BDS), the public-release sample of statistics aggregated from the Census' Longitudinal Business Database (LBD).

Table 12 shows that our model matches the targeted moments reasonably well.[28] The model closely matches the dispersion of investment rates, which captures the degree of idiosyncratic risk faced by firms. The model also closely matches the average gross leverage ratio and the average credit spreads, but it underpredicts the mean default rate. Firms in our model grow too quickly relative to the data, which is not surprising because we do not

[28] We do not match the moments exactly because our model is nonlinear. We use simulated annealing to minimize the weighted sum of squared errors implied by these moments.

include other frictions to firm growth such as capital adjustment costs or customer accumulation. Finally, the model underpredicts the total amount of firm exit (due to the fact that it underpredicts the average default rate), but it does provide a good match of the ratio of exit rates of age 1 to age 2 firms.

The calibrated parameters in Table 11 are broadly comparable to existing estimates in the literature. Idiosyncratic TFP shocks are less persistent and more volatile than aggregate productivity shocks, consistent with direct measurements of plant- or firm-level productivity. The calibrated loan recovery rate is 91%, in line with the low estimated costs of default in the literature. New entrants start with significantly lower productivity and capital than the average firm.

4.2 Financial Heterogeneity in the Model and the Data

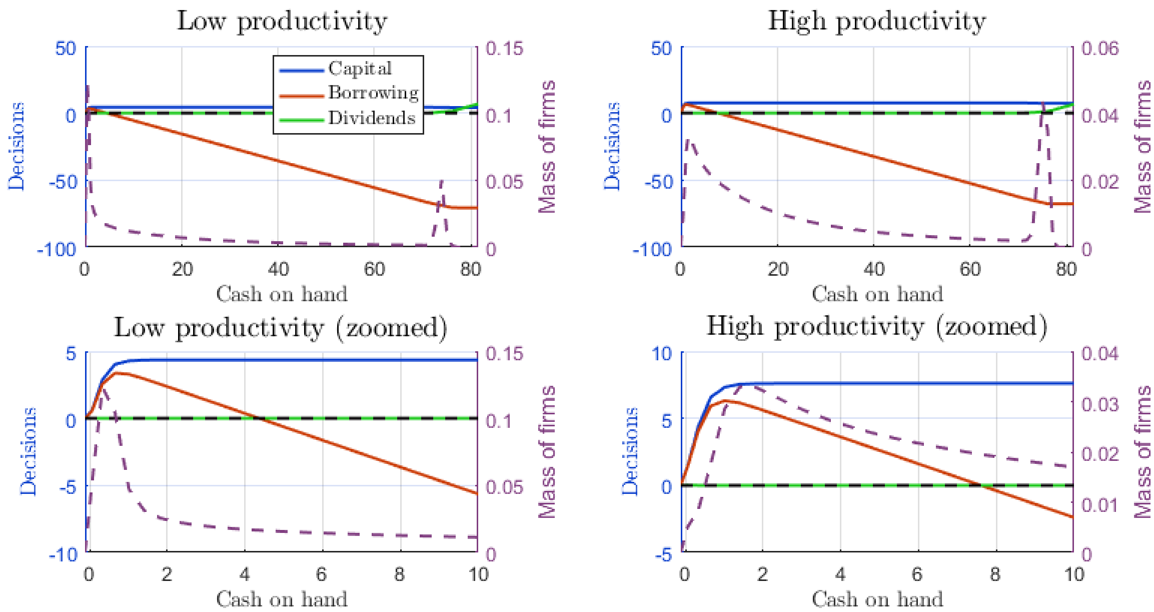
We now analyze firms' decision rules in our calibrated steady state and show that the financial heterogeneity in our model is broadly comparable to that in the data.

Firms' Decision Rules Figure 5 plots the investment, borrowing, and dividend payment decisions of firms. The top row of the figure plots the decision rules over the entire state space. Firms with cash-on-hand x below the default threshold $\underline{x}_t(z)$ do not operate. Once firms clear this default threshold, they lever up to increase their capital to its optimal scale $k_t^*(z)$. Once capital is at its optimal level $k_t^*(z)$, firms use additional cash-on-hand to pay down their debt until they reach the unconstrained threshold $\overline{x}_t(z)$. Unconstrained firms set $k' = k_t^*(z)$ and $b' = b_t^*(z)$, which do not depend on cash on hand x. Only unconstrained firms pay positive dividends.

The curvature in the policy functions over the region with low cash on hand x reflects the role of financial frictions in firms' decisions. Without frictions, all non-defaulting firms would borrow the amount necessary to reach the optimal scale of capital $k_t^*(z)$. However, firms with low cash-on-hand x would need to borrow a substantial amount, increasing their risk of default and therefore borrowing costs. Anticipating these higher borrowing costs, low cash on hand x firms accumulate capital below its optimal scale.

The right axis of Figure 5 plots the stationary distribution of firms. 53.1% of firms pay

Figure 5: Steady State Decision Rules

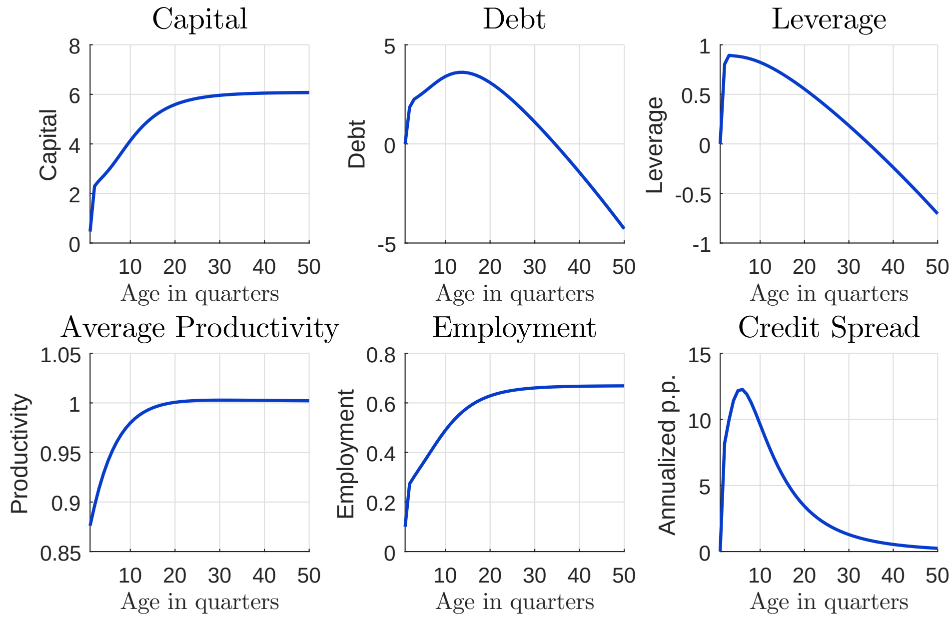


Notes: Left column plots decision rules and stationary distribution of firms conditional on idiosyncratic productivity one standard deviation below the mean. Right column plots the same objects conditional on productivity one standard deviation above the mean. The left y-axis measures the decision rules (capital accumulation, borrowing, and dividend payments) as a function of cash-on-hand x. The right y-axis measures the stationary distribution of firms. Top row plots these functions over the entire space of cash on hand. Bottom row plots these functions for low levels of cash on hand only.

a risk premium, i.e., are "risky constrained." These firms are in the region with curved policy functions described above. 43% of firms are constrained but do not currently pay a risk premium, i.e., are "risk-free constrained." These firms have achieved their optimal scale of capital $k_t^*(z)$ and have linear borrowing policies. The remaining 3.9% of firms are unconstrained. Due to our assumed debt accumulation policy, unconstrained firms pay out any cash on hand $x > \overline{x}_t(z)$ as dividends.

Figure 5 makes clear that there are two key sources of financial heterogeneity in the model. First, reading the graphs from left to right captures heterogeneity due to lifecycle dynamics; young firms accumulate debt in order to reach their optimal level of capital $k_t^*(z)$ and then pay down that debt over time. Second, moving from the left to the right column captures heterogeneity due to idiosyncratic productivity shocks; a positive shock increases the optimal scale of capital $k_t^*(z)$, again leading firms to first accumulate and then decumulate

Figure 6: Lifecycle Dynamics in Model



Notes: Average capital, debt, leverage, productivity, employment, and credit spread conditional on age in steady state.

debt.[29,30]

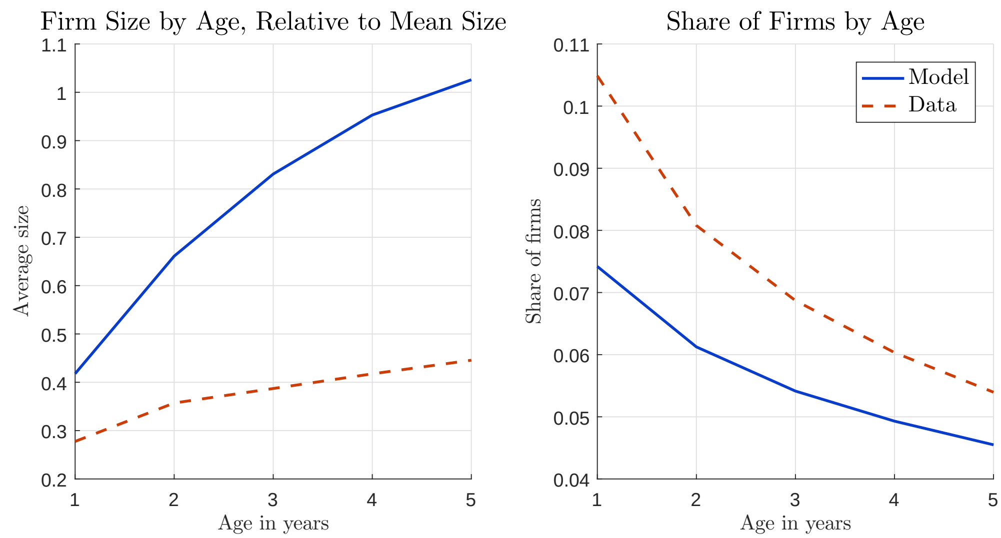
Comparing Lifecycle Dynamics to the Data Figure 6 plots the dynamics of key variables over the firm lifecycle. New entrants begin with a low initial capital stock k_0 and, on average, a low draw of idiosyncratic productivity z. As described above, young firms take on new debt in order to finance investment, which increases their default risk and credit spreads. Over time, as firms accumulate capital and productivity reverts to its mean, they reach their optimal capital stock $k_t^*(z)$ and begin paying down their debt.

Figure 7 shows that these lifecycle dynamics are in line with key features of the data. The left panel plots the average size of firms by age. In the data, young firms are substantially smaller than average and take many years to catch up. Qualitatively, our model captures this prolonged growth process; however, quantitatively, growth in our model is too rapid

[29] A third source of financial heterogeneity are the capital quality shocks, which simply generate variation in firms' cash on hand x.

[30] Buera and Karmakar (2017) study how the aggregate effect of an interest rate shock depends on these two sources of heterogeneity in a simple two-period model.

Figure 7: Comparison of Lifecycle Dynamics to the Data



Notes: Left panel plots the average employment of firms by age, relative to the average employment in the population. Right panel plots the share of firms by age. Model: steady state of the calibrated model; Data: computed from the Business Dynamics Statistics (BDS).

because we do not include other frictions to firm growth such as capital adjustment costs or customer accumulation. The right panel of Figure 7 plots the share of firms in the economy in different age groups. The curve is downward-sloping because firms exit over time. In the model, the only source of curvature is state-dependent exit due to default. Although the model underpredicts the overall level of the curve, it provides a good match of the slope.

Investment and Leverage Heterogeneity in the Data Table 13 shows that our model is broadly consistent with key features of the distributions of investment and leverage not targeted in the calibration. The top panel analyzes the distribution of investment rates in the annual Census data reported by Cooper and Haltiwanger (2006). We present the corresponding statistics in our model for a selected sample – conditioning on firms that survive at least twenty years to mirror the selection into the LRD – and in the full sample. Although we have calibrated the selected sample to match the dispersion of investment rates, the mean and autocorrelation of investment rates in the selected sample are also reasonable. The mean investment rate in the full sample is higher than the selected sample because the full sample includes young, growing firms.

TABLE 13
INVESTMENT AND LEVERAGE HETEROGENEITY

Moment	Description	Data	Model (selected)	Model (full)
Investment heterogeneity (annual LRD)				
$\mathbb{E}\left[\frac{i}{k}\right]$	Mean investment rate	12.2%	8.83%	20.6%
$\sigma\left(\frac{i}{k}\right)$	SD investment rate (calibrated)	33.7%	31.8%	38.5%
$\rho\left(\frac{i}{k}, \frac{i}{k}_{-1}\right)$	Autocorr investment rate	0.058	-0.26	-0.26
Leverage heterogeneity (quarterly Compustat)				
$\sigma\left(\frac{b}{k}\right)$	SD leverage ratio	36.4%	76.4%	77.0%
$\rho\left(\frac{b}{k}, \frac{b}{k}_{-1}\right)$	Autocorr leverage ratio	0.94	0.92	0.95
Joint investment and leverage (quarterly Compustat)				
$\rho\left(\frac{i}{k}, \frac{b}{k}\right)$	Corr. of leverage and investment	-0.08	-0.16	-0.02

Notes: Statistics about the cross-sectional distribution of investment rates and leverage ratios in steady state. Data for investment heterogeneity are drawn from Cooper and Haltiwanger (2006). Model (selected) for investment heterogeneity corresponds to firms alive for longer than twenty years in a panel simulation, time aggregated to the annual frequency. Model (full) corresponds to the full sample of firms in a panel simulation, time aggregated to the annual frequency. Data for leverage heterogeneity drawn from quarterly Compustat data. Model (selected) for leverage heterogeneity corresponds to firms alive for longer than ten years in a panel simulation. Model (full) correpsonds to the full sample of firms in a panel simulation.

The middle and bottom panels of Table 13 compare the model-implied distribution of investment rates and leverage to quarterly Compustat data. We mirror the sample selection into Compustat by conditioning on firms that survive for at least ten years. According to Wilmer et al. (2017), the median time to IPO has ranged from roughly six to eight years over the last decade.[31] Our model provides a close match of the persistence of leverage and its correlation with investment in the selected sample. However, the standard deviation of leverage ratios is about twice as large as in the data.

Table 14 shows that the model generates a positive measured investment-cash flow sensitivity, consistent with the data. Following Gomes (2001), we compute investment-cash flow sensitivity using the regression

$$\frac{i_{jt}}{k_{jt}} = \alpha_j + \alpha_t + a_1 \frac{\mathtt{CF}_{jt-1}}{k_{jt}} + a_2 \mathtt{q}_{jt-1} + \varepsilon_{jt}, \tag{22}$$

where $\mathtt{CF}_{jt}$ is cash flow and $\mathtt{q}_{jt}$ is Tobin's q. The coefficient a_1 captures the statistical co-

[31] Our results are robustness to sensitivity analysis around this cutoff.

Table 14
Measured Investment-Cash Flow Sensitivity

	Without cash flow		**With cash flow**	
	Data	*Model*	*Data*	*Model*
Tobin's q	0.01***	0.06	0.01***	0.02
cash flow			0.02***	0.08
R^2	0.097	0.065	0.106	0.086

Notes: Results from estimating the regression (22). Data refers to quarterly Compustat data. We measure cash flow as earnings before tax, depreciation, and amortization (EBITDA) and Tobin's q as the market to book value of the firm. Model refers to simulating a panel of firms from the calibrated model, conditional on surviving at least ten years. We measure cash flow as the firm's cash-on-hand x and Tobin's q as the ratio of market value to the book value of capital, k.

movement of investment with cash flow, conditional on the fixed effects and Tobin's q. In the model, we identify cash flow as the firm's cash on hand x and Tobin's q as the ratio of the market value of the firm to the book value of its capital stock, k. In quarterly Compustat, we identify cash flow as earnings before tax, depreciation, and amortization (EBITDA) and Tobin's q as the market to book value of the firm.

The model's implications for regression (22) are consistent with two key features of the data. First, the coefficient on cash flow a_1 is positive, indicating that increases in cash flows are associated with increases in investment. Second, the inclusion of cash flow as a regressor in (22) significantly increases the R^2 of the regression, indicating that cash flow has predictive power for investment. However, the quantitative magnitude of the cash flow coefficient is larger in the model than the data.

5 Monetary Policy Analysis

We now analyze the effect of a monetary policy shock $\varepsilon_t^{\mathrm{m}}$. To fix ideas before the quantitative analysis, Section 5.1 theoretically characterizes the channels through which monetary policy affects firms' investment decisions. Section 5.2 computes aggregate impulse responses to a monetary policy shock in our calibrated model. Section 5.3 studies the heterogeneous effects of monetary policy across firms and shows that our model is consistent with the empirical results from Section 2. Finally, Section 5.4 shows that the aggregate effect of monetary policy depends on the distribution of net worth.

5.1 Channels of Monetary Transmission

We derive analytical expressions decomposing the effect of a one-time, unexpected innovation to the Taylor rule $\varepsilon_t^{\mathrm{m}}$ followed by a perfect foresight transition back to steady state. This MIT shock approach allows for clean analytical results because there is no distinction between ex-ante expected and ex-post realized real interest rates. We will quantitatively evaluate the strength of these channels in response to persistent monetary policy shocks in Section 5.2 through Section 5.4.

Unconstrained Firms Totally differentiating the unconstrained capital decision (16), the monetary shock $\varepsilon_t^{\mathrm{m}}$ perturbs unconstrained firms' investment decisions by

$$\frac{\mathrm{d}\log k'}{\mathrm{d}\varepsilon_t^{\mathrm{m}}} = \frac{1-\nu}{1-\nu-\theta}\left[\underbrace{-\frac{R_t}{R_t-(1-\delta)\frac{q_{t+1}}{q_t}}\frac{\partial \log R_t}{\partial \varepsilon_t^{\mathrm{m}}}}_{\text{discounting}} - \underbrace{\frac{\partial \log q_t}{\partial \varepsilon_t^{\mathrm{m}}}}_{\text{capital price}} + \underbrace{\frac{(1-\delta)\frac{q_{t+1}}{q_t}}{R_t-(1-\delta)\frac{q_{t+1}}{q_t}}\frac{\partial \log \frac{q_{t+1}}{q_t}}{\partial \varepsilon_t^{\mathrm{m}}}}_{\text{capital gains}}\right]$$
$$+\frac{1}{1-\nu-\theta}\left[\underbrace{\frac{\partial \log p_{t+1}}{\partial \varepsilon_t^{\mathrm{m}}} - \nu\frac{\partial \log w_{t+1}}{\partial \varepsilon_t^{\mathrm{m}}}}_{\text{capital revenue}}\right], \tag{23}$$

where $R_t = \frac{R_t^{\mathrm{nom}}}{\Pi_{t+1}}$ is the real interest rate between periods t and $t+1$.

The expression (23) decomposes the effect of monetary policy on unconstrained firms' investment into four distinct channels. The *discounting* channel isolates the direct effect of changing the real interest rate on investment decisions through the discounting future revenues. The *capital price* channel isolates the effect of monetary policy on the relative price of new capital. The *capital gains* channel isolates the effect of monetary policy on the change in the value of the firms' capital between periods t and $t+1$. Finally, the *capital revenue* channel isolates the effect of monetary policy on the marginal revenue product of capital through the relative price of the firms' output, p_{t+1} net of real labor costs νw_{t+1}. The capital revenue channel measures the net effect of both of these terms. Note that there is no heterogeneity in unconstrained firms' responses because monetary policy only impacts their decisions through aggregate prices.

Constrained Firms Since constrained firms set $d = 0$, it in instructive to totally differentiate the flow of funds constraint (18) to get the decomposition

$$\frac{\mathrm{d}\log k'}{\mathrm{d}\varepsilon_t^{\mathrm{m}}} = \underbrace{-\frac{\partial \log q_t}{\partial \varepsilon_t^{\mathrm{m}}}}_{\text{capital price}} + \underbrace{\frac{\partial \log x}{\partial \varepsilon_t^{\mathrm{m}}}\frac{x}{q_t k'}}_{\text{cash flow}} + \underbrace{\frac{\partial \log(\mathcal{Q}_t(z,k',b')b')}{\partial \varepsilon_t^{\mathrm{m}}}\frac{\mathcal{Q}_t(z,k',b')b'}{q_t k'}}_{\text{borrowing cost}}. \tag{24}$$

This expression should be interpreted with caution because it involves derivatives of the endogenous variables on both sides of the equality and does not fully characterize the firms' portfolio choice problem between k' and b'. However, the expression is nonetheless instructive in highlighting three channels through which monetary policy affects constrained firms' investment decisions. As with unconstrained firms, the *capital price* channel isolates how monetary policy affects the relative price of new capital.

The *cash flow* channel in (24) isolates how monetary policy affects firms' internal resources for financing investment. Differentiating the definition of cash-on-hand x allows us to further characterize how monetary policy affects firms' cash flows:

$$\frac{\partial \log x}{\partial \varepsilon_t^{\mathrm{m}}} = \frac{1}{1-\nu-\theta}\left(\frac{\partial \log p_t}{\partial \varepsilon_t^{\mathrm{m}}} - \nu\frac{\partial \log w_t}{\partial \varepsilon_t^{\mathrm{m}}}\right)\frac{\iota_t(z,k)}{x} + \frac{\partial \log q_t}{\partial \varepsilon_t^{\mathrm{m}}}\frac{q_t(1-\delta)\omega k}{x} + \frac{\partial \log \Pi_t}{\partial \varepsilon_t^{\mathrm{m}}}\frac{b/\Pi_t}{x}, \tag{25}$$

where $\iota_t(z,k) = \max_n p_t z k^\theta n^\nu - w_t n$. The expression (25) contains three ways that monetary policy affects cash flows. First, monetary policy affects current revenues by changing the relative price of output p_t net of real labor costs νw_t. Second, monetary policy affects the value of firms' undepreciated capital stock by changing the relative price of capital q_t. Finally, monetary policy changes the real value of outstanding nominal debt through inflation Π_t.

The *borrowing cost* channel in (24) isolates how monetary policy affects firms' external resources from new borrowing. Monetary policy can change either how much debt the firm takes on, b', or the price of that debt $\mathcal{Q}_t(z,k',b')$. It is convenient to characterize the effect of monetary policy on the borrowing rate $\widehat{R}_t(z,k',b') = \frac{1}{\mathcal{Q}_t(z,k',b')}$:

$$\frac{\partial \log \widehat{R}_t(z,k',b')}{\partial \varepsilon_t^{\mathrm{m}}} = \frac{\partial \log R_t}{\partial \varepsilon_t^{\mathrm{m}}} + (\widehat{R}_t(z,k',b') - R_t)\frac{\partial \log \Theta_t(z,k',b')}{\partial \varepsilon_t^{\mathrm{m}}}, \tag{26}$$

where $\Theta_t(z,k',b') = \mathbb{E}_t\left[\left(\pi_d(1-\chi^1(x')) + (1-\pi_d)(1-\chi^2_{t+1}(z',x'))\right)\left(\frac{1}{\Pi_{t+1}} - \min\{\frac{\alpha q_{t+1}(1-\delta)\omega' k'}{b'/\Pi_{t+1}}, 1\}\right)\right]$ is the expected cost of default to the lender. Monetary policy affects borrowing costs $\widehat{R}_t(z,k',b')$ through two channels. First, it affects the real risk-free rate R_t, which shifts the level of the interest rate schedule $\widehat{R}_t(z,k',b')$. Second, if the firm incurs a positive external finance premium $\widehat{R}_t(z,k',b') - R_t$, monetary policy additionally affects the credit spread of the firm by changing either default probabilities or loan recovery rates in $\Theta_t(z,k',b')$.

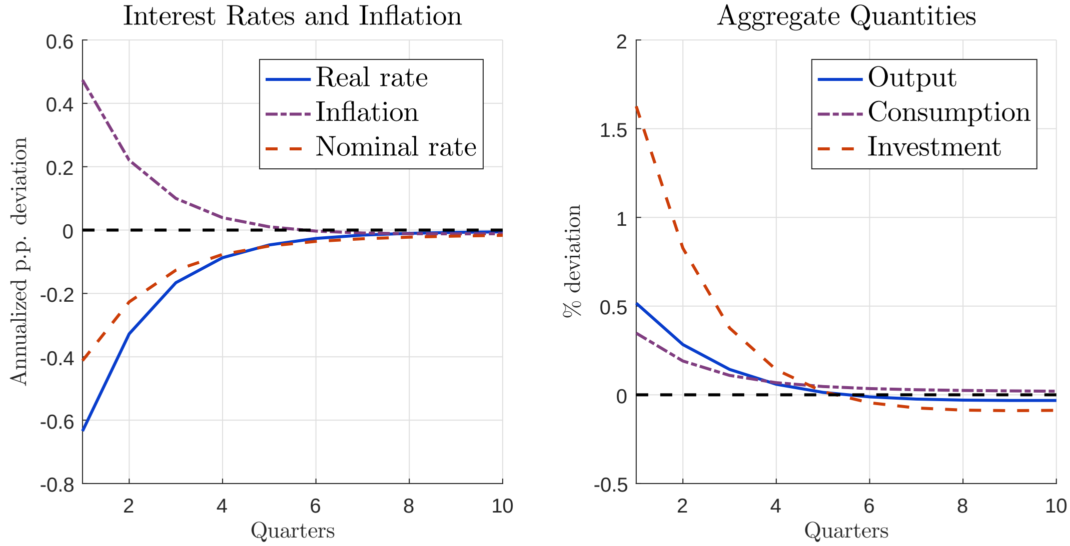
Direct, Indirect, and Adjustment Cost Effects In the quantitative analysis, we will group the channels derived above into three different categories. First, the *direct effect* works through changes in the real interest rate, holding all other prices fixed. The direct effect stimulates investment through the discounting and borrowing cost channels discussed above. Second, the *indirect effect* works through changes in the relative price of output p_t net of the real labor costs νw_t, inflation Π_t, and how the price of capital revalues undepreciated capital. These indirect effects stimulate unconstrained firms through the capital gains and capital revenue channels, and stimulate constrained firms through the cash flow and borrowing cost channels. Finally, the *adjustment cost* effect dampens the response of investment by changing the relative price of capital on new investment.

5.2 Aggregate Response to Monetary Policy

Our quantitative analysis studies the effect of a persistent MIT shock to the Taylor rule. The economy is initially in steady state and unexpectedly receives a $\varepsilon_0^{\mathrm{m}} = -0.0025$ innovation to the Taylor rule which reverts to 0 according to $\varepsilon_{t+1}^{\mathrm{m}} = \rho_m \varepsilon_t^{\mathrm{m}}$ with $\rho_m = 0.5$. We compute the perfect foresight transition path of the economy as it converges back to steady state.[32]

[32]Allowing for persistence in the monetary policy shocks themselves is a simple way to create inertia in response to a monetary shock. In the representative firm version of the model, the response is very similar to a version of the model in which the innovations are transitory but the Taylor rule includes interest rate smoothing.

Figure 8: Aggregate Responses to Expansionary Monetary Shock

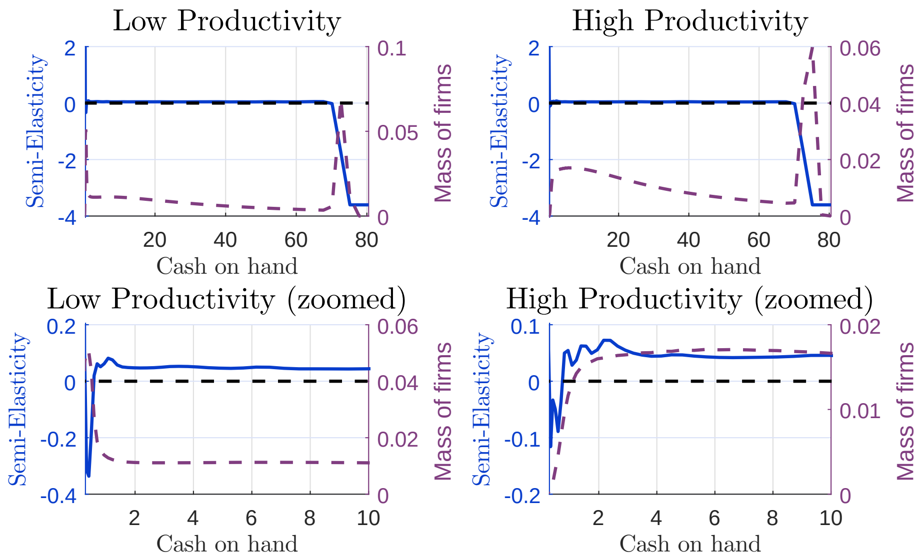


Notes: Aggregate impulse responses to a $\varepsilon_0^{\mathrm{m}} = -0.0025$ innovation to the Taylor rule which decays at rate $\rho_m = 0.5$. Computed as the perfect foresight transition in response to a series of unexpected innovations starting from steady state.

Figure 8 plots the responses of key aggregate variables to this expansionary shock. The shock lowers the nominal interest rate; because prices are sticky, this also lowers the real interest rate. The lower real interest rate stimulates investment demand through the direct effect discussed in Section 5.1. It also stimulates consumption demand from the household through the Euler equation. Higher aggregate demand for goods then raises inflation, cash flows, and the price of capital, activating the indirect and adjustment cost effects. This process increases investment by 1.6%, output by 0.5%, and consumption by 0.35% for a 0.4% change in the annualized nominal interest rate, broadly in line with the peak effect of monetary policy estimated in Christiano, Eichenbaum and Evans (2005).[33]

[33] Our model does not generate the hump-shaped aggregate responses emphasized by Christiano, Eichenbaum and Evans (2005). We could do so by incorporating adjustment costs to investment rather than capital. However, we prefer to focus on capital adjustment costs because they are a parsimonious way to capture movements in the relative price of capital.

Figure 9: Semi-Elasticity of Capital to Monetary Policy Shock



Notes: Left column plots the semi-elasticity of capital with respect to monetary policy and stationary distribution of firms conditional on idiosyncratic productivity one standard deviation below the mean. Right column plots the same objects conditional on idiosyncratic productivity one standard deviation above the mean. The left y-axis measures the semi-elasticity of capital with respect to the monetary policy shock (measured in annual percentage points with a normalized sign). The right y-axis measures the stationary distribution of firms. Top row plots these functions over the entire space of cash on hand. Bottom row plots these functions for low levels of cash on hand only.

5.3 Which Firms Drive Monetary Transmission?

We now show that the heterogeneous responses to monetary policy across firms in our model is consistent with the data. We primarily focus on the response of *financially constrained* firms, who make up more than 96% of firms in our model and drive the majority of the aggregate response. We first show that the response of constrained firms is decreasing in their default risk. We then show that this heterogeneity is quantitatively consistent with the empirical results documented in Section 2.

Heterogeneous Responses by Default Risk Figure 9 plots the semi-elasticity of capital with respect to the monetary policy shock across the individual state space. Completely unconstrained firms have a large negative elasticity; as we explain below, their negative response is due to the fact that the adjustment cost effect outweighs the stimulative effects of monetary policy. This strong negative response dominates the figure, so the bottom row

Table 15
Regression Results

	Model		Data	
	(1)	(2)	(1)	(2)
leverage × ffr shock	−1.193	−0.955	-0.93^{***}	-0.73^{***}
			(0.34)	(0.29)
R^2	0.151	0.216	0.107	0.119
Time FE	yes	yes	yes	yes
Firm controls	no	yes	no	yes

Notes: Results from running the baseline specification $\Delta \log k_{jt} = \alpha_j + \alpha_t + \beta \ell_{jt-1}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$ on model-simulated data, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is the firm's leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage and size. We have normalized the sign of the monetary shock $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean. The sample period is four quarters before the monetary shock through ten quarters after the shock. To mirror the sample selection into Compustat, we condition on firms that have survived at least ten years. "Data" refers to results in Table 3.

of Figure 9 focuses on the region with low cash on hand x. In this region, the semi-elasticity is increasing in cash on hand; since firms in this region with higher cash on hand have lower probability of default, this also implies the semi-elasticity is decreasing in the probability of default. Furthermore, since firms with high leverage have lower cash on hand, the above results imply that our model is qualitatively consistent with the empirical findings in Section 2.

Model-Implied Regression Coefficients In order to quantify the magnitude of this heterogeneity, we simulate a panel of firms in response to the monetary shock and estimate the regression specification (3) on the simulated panel. As in Section 4, we mirror the sample selection into Compustat by conditioning on firms that have survived at least ten years. We identify the innovation to the Taylor rule $\varepsilon_t^{\mathrm{m}}$ with the high-frequency shocks that we measure in the data.[34] We estimate the regression using data from one year before the shock to ten

[34] In our model, the change in the nominal interest rate is smaller than the innovation to the Taylor rule because the monetary authority responds to the increased inflation. This fact may lead to an inconsistency between the monetary shocks in the model and the measured shocks in the data, which are based on changes in expected rates. Our implicit assumption is that the feedback effect through the Taylor rule takes sufficient time that it is not incorporated into the measure of the high-frequency shocks. Because we use the one-month futures, our assumption requires that the monetary authority respond to inflation with at most a one month lag.

quarters after the shock.

Table 15 shows that the estimated interaction coefficient in the model is within one standard error of the empirical estimate. Columns (1) estimates the regression (3) without any firm-level controls Z_{jt}. In both the model and the data, a firm with one standard deviation more leverage than the average firm has an investment semi-elasticity approximately one percentage point lower than the average firm. Columns (2) includes firm-level controls Z_{jt} and shows that this conclusion does not substantially change. The R^2 of the regressions are higher in our model, indicating that the data contain more unexplained sources of variation in investment.[35]

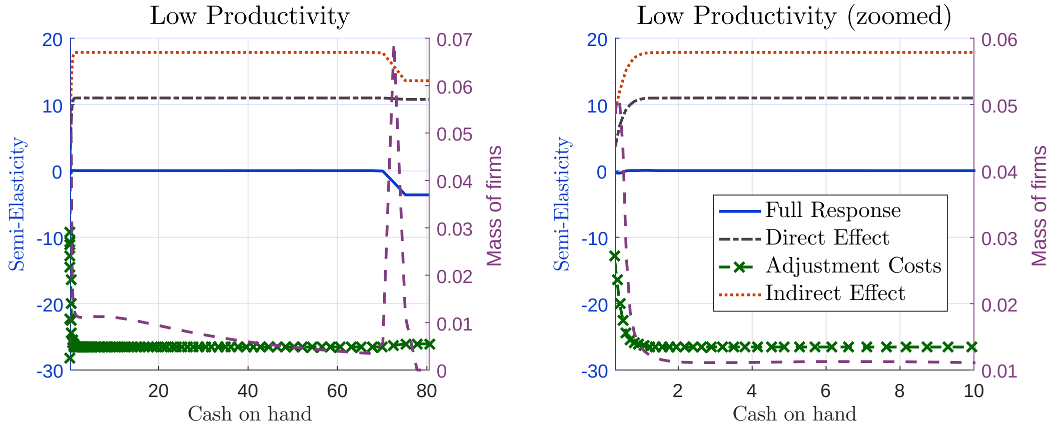
Channels Driving Heterogeneous Responses On average, firms in our model respond to the monetary policy shock by financing additional investment with new borrowing. Risk-free constrained firms face a constant marginal cost of new borrowing because additional borrowing does not impact their risk of default. However, risky constrained firms face an upward-sloping marginal cost because additional borrowing increases their risk of default. The upward-sloping marginal cost of new borrowing then dampens the response of the risky constrained firms.[36]

Figure 10 shows that the direct, indirect, and adjustment cost effects defined in Section 5.1 all play an important role in driving these heterogeneous responses to monetary policy. We compute the contribution of each of these channels by feeding in the relevant series of prices to the firms in our model, holding all other prices fixed at their steady state values. Both the direct and indirect effects have a strong positive effect on investment. However, these stimulative effects are dampened for high-risk firms through the mechanism described above. The adjustment cost effect, due to a higher relative price of new investment, has a strong negative impact on investment demand. Financially unconstrained firms have a lower

[35] Our results are somewhat sensitive to the number of periods we include in the regression. To investigate this sensitivity, we ran our baseline specification (3) using only the period of the shock. Because this specification only includes one quarter of data, we cannot estimate the fixed effects and the coefficient on leverage simply captures cross-sectional heterogeneity in how firms respond to the shock. Even in this much simpler setting, the estimated coefficient is strongly negative without controls. However, the coefficient significantly falls with controls because we exploiting different sources of variation than in our baseline specification.

[36] In fact, for low enough values of cash on hand, these firms actually decrease investment and pay down their debt.

FIGURE 10: Decomposition of Semi-Elasticity of Capital to Monetary Policy Shock



Notes: Semi-elasticity of capital and stationary distribution of firms conditional on idiosyncratic productivity one standard deviation below the mean. Left column plots over the entire state space while right column focuses on low levels of cash on hand x. "Direct effect" refers to only the real interest rate changes, holding all other prices fixed at steady state. "Adjustment cost" refers to changing the price of capital for new investment only. "Indirect effect" refers to changing all other prices.

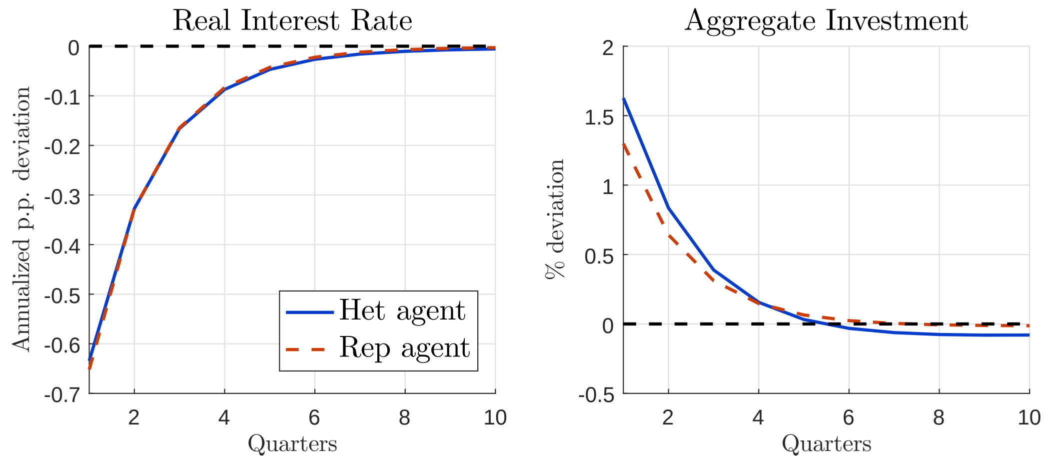
semi-elasticity because they are less exposed to the indirect effects of monetary policy.

The fact that both the direct and indirect effects play a quantitatively important role driving the response of investment to monetary policy is in contrast to Auclert (2017)'s and Kaplan, Moll and Violante (2017)'s decomposition of the consumption channel. They find that the contribution of the direct effect is small relative to indirect effects in incomplete-markets heterogeneous household models. The direct effect is larger in our model because consumption-smoothing motives imply that households' consumption demand is less sensitive to changes in the real interest rate than firms' investment demand.

5.4 Aggregate Implications of Financial Heterogeneity

In this subsection, we study two ways in which financial heterogeneity matters for understanding the aggregate monetary transmission mechanism. First, we show that the aggregate effect of monetary policy is larger in our model than in a comparable version of the model without financial frictions (which collapses to a representative firm). Second, we show that the aggregate effect of a given monetary policy shock in our model significantly depends on the initial distribution of net worth.

Figure 11: Aggregate Impulse Responses in Full Model vs. Rep Firm Model

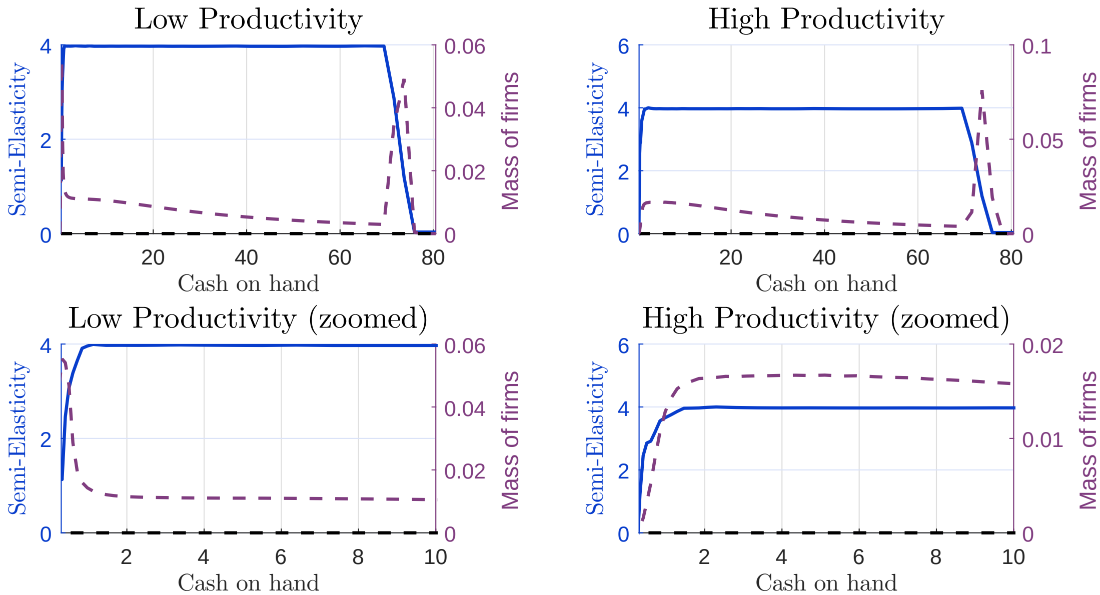


Notes: Aggregate impulse responses to a $\varepsilon_0^{\mathrm{m}} = -0.0025$ innovation to the Taylor rule which decays at rate $\rho_m = 0.5$. Computed as the perfect foresight transition in response to a series of unexpected innovations starting from steady state. "Het agent" refers to calibrated heterogeneous firm model from the main text. "Rep agent" refers to a version of the model in which the heterogeneous production sector is replaced by a representative firm with the same production function and no financial frictions.

Comparison to Frictionless Model We eliminate financial frictions by removing the non-negativity constraint on dividends; in this case, the investment block of the model collapses to a financially unconstrained representative firm (see Khan and Thomas (2008) Appendix B). Figure 11 shows that the impact effect of monetary policy on investment is 25% larger in our full model than in the representative firm benchmark. Hence, despite the fact that risky constrained firms are less responsive than risk-free constrained firms, both types of constrained firms are more responsive than completely unconstrained firms.

To understand this result, Figure 12 plots the semi-elasticity of capital with respect to the monetary policy shock for firms in our model, assuming that they face the equilibrium path of prices from the representative firm model. By construction, the response of unconstrained firms with $x \geq \overline{x}(z)$ is the same as the representative firm in the frictionless benchmark. In contrast, both the risky and risk-free constrained firms are significantly more responsive than the unconstrained firms. Both types of constrained firms are more responsive because their marginal value of additional cash-on-hand is strictly larger than for completely unconstrained firms. Within constrained firms with low cash-on-hand, risky constrained firms are less responsive than risk-free constrained firms, consistent with the results in Section 5.3.

Figure 12: Semi-Elasticity of Capital w.r.t. Monetary Shock, Rep Firm Model Prices



Notes: Left column plots the semi-elasticity of capital and stationary distribution of firms conditional on idiosyncratic productivity one standard deviation below the mean. Right column plots the same objects conditional on idiosyncratic productivity one standard deviation above the mean. The left y-axis measures the semi-elasticity of capital with respect to the monetary policy shock (measured in annual percentage points and absolute value). The right y-axis measures the stationary distribution of firms. Top row plots these functions over the entire space of cash on hand. Bottom row plots these functions for low levels of cash on hand only. Decision rules are computed given the equilibrium path of prices from the representative firm model.

Higher investment demand from constrained firms puts additional upward pressure on the relative price of capital q_t in the general equilibrium of our full model. Unconstrained firms, who have a small positive response facing the representative firm model's prices, now have a large negative response.

State Dependence of Aggregate Transmission We now show that the aggregate effect of monetary policy is smaller when the initial distribution of firms contains more risky firms. In order to illustrate the quantitative magnitude of this mechanism, we perform a simple calculation: we take the semi-elasticity of capital with respect to monetary policy as fixed and vary the initial distribution of firms.[37]

[37] This exercise does not allow for prices to vary with the initial distribution. However, the exercise is a nevertheless an important necessary condition for the general equilibrium model to generate state dependence. We perform the simple exercise of fixing the elasticities and varying the distribution for two reasons.

We vary the initial distribution of firms in production $\hat{\mu}(z,x)$ by taking the weighted average of two reference distributions. The first reference distribution is the steady-state distribution $\hat{\mu}^*(z,x)$. The second reference distribution $\widetilde{\mu}(z,x)$ assumes that the conditional distribution of cash-on-hand for every level of productivity is equal to the distribution of cash-on-hand conditional on the lowest realization of productivity in steady state. We normalize the second reference distribution so that the marginal distribution of productivity is the same as in the steady state distribution. Hence, $\widetilde{\mu}(z,x)$ is an example of a distribution in which firms of all productivity levels have a poor distribution of cash on hand. We then compute the initial distribution as a weighted average of these two reference distributions, $\hat{\mu}(z,x) = \hat{\omega}\widetilde{\mu}(z,x) + (1-\hat{\omega})\hat{\mu}^*(z,x)$. We vary $\hat{\omega} \in [0,1]$ to trace out linear combinations of distributions between the steady state ($\hat{\omega} = 0$) and the low cash on hand ($\hat{\omega} = 1$) distributions. We then compute the change in the aggregate capital stock in response to the monetary policy shock for each of these initial distributions.

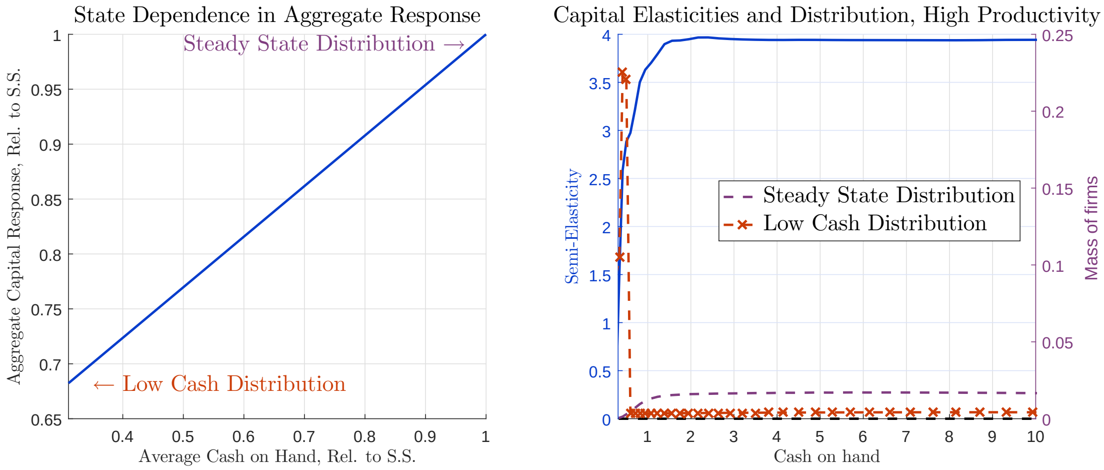
The left panel of Figure 13 shows that the change in the aggregate capital stock is 30% smaller starting from the low-cash distribution $\widetilde{\mu}(z,x)$ than starting from the steady state distribution $\hat{\mu}^*(z,x)$, and the response varies linearly in between these two extremes. Average cash-on-hand is 70% lower and there are twice as many risky constrained firms in the low-cash distribution than in the steady state distribution. The right panel of Figure 13 shows that this effect is due to the fact that the low-cash distribution $\widetilde{\mu}(z,x)$ places more mass in the region of the state space where the elasticity of capital with respect to the monetary policy shock is low.

These results suggests a potentially powerful source of time-variation in the aggregate transmission mechanism: monetary policy is less powerful when net worth is low and default risk is high. A limitation of this analysis is that we have varied the initial distribution exogeneously. The natural next step in this analysis is to incorporate with various business cycle

First, since we do not have to re-compute the equilibrium transition path for each initial distribution, we can investigate state dependence with respect to a large number of initial distributions. Second, this exercise clearly isolates the impact of varying the initial distribution from the additional changes to firms' policy rules arising from changes in prices.

In this exercise, markets do not clear for a given initial distribution of cash-on-hand. We use the elasticities from the representative firm prices plotted in Figure 12 so that markets do not clear for any initial distribution. If we had used the equilibrium elasticities, markets would clear for some initial distributions and not others, potentially biasing our interpretation of the results.

Figure 13: Aggregate Response Depends on Initial Distribution



Notes: Dependence of aggregate response on initial distribution. We compute the change in aggregate capital for different initial distributions using the response to monetary policy computed under the price path from the representative firm model. We vary the initial distribution of firms in production $\hat{\mu}(z,x)$ by taking the weighted average of two reference distributions. The first reference distribution is the steady-state distribution $\hat{\mu}^*(z,x)$. The second reference distribution $\widetilde{\mu}(z,x)$ assumes that the conditional distribution of cash-on-hand for every level of productivity is equal to the distribution of cash-on-hand conditional on the lowest realization of productivity in steady state. We normalize the second reference distribution so that the marginal distribution of productivity is the same as in the steady state distribution. We then compute the initial distribution as a weighted average of these two distributions, $\hat{\mu}(z,x) = \hat{\omega}\widetilde{\mu}(z,x) + (1-\hat{\omega})\hat{\mu}^*(z,x)$. Left panel varies $\hat{\omega} \in [0,1]$ and plots the change in the aggregate capital stock upon impact against the average cash-on-hand of the initial distribution. Right panel plots the semi-elasticity of capital with respect to the shock over cash on hand for high productivity firms. The steady state distribution corresponds to $\hat{\omega} = 0$ and the low-cash distribution corresponds to $\hat{\omega} = 1$.

shocks into our model and study the types of distributions that actually arise in equilibrium.

6 Conclusion

In this paper, we have argued that financial frictions dampen the response of investment for firms with high default risk. Our argument had two main components. First, we showed in the micro data that firms with high leverage or low credit ratings invest significantly less than other firms following a monetary policy shock. Second, we built a heterogeneous firm New Keynesian model with default risk that is quantitatively consistent with these empirical results. In the model, monetary policy stimulates investment through a combination of direct and indirect effects. High-risk firms are less responsive to these changes because their marginal cost of investment finance is higher than for low-risk firms. The aggregate effect of monetary policy is primarily driven by these low-risk firms, which suggests a novel form of state dependence: monetary policy is less powerful when default risk in the economy is greater.

Our results may be of independent interest to policymakers who are concerned about the distributional implications of monetary policy across firms. An often-discussed goal of monetary policy is to provide resources to viable but credit constrained firms; for example, in a 2010 speech then-chairman Ben Bernanke said that "over the past two years, the Federal Reserve and other agencies have made a concerted effort to stabilize our financial system and our economy. These efforts, importantly, have included working to facilitate the flow of credit to viable small businesses (Bernanke (2010))." Many policymakers' conventional wisdom, built on the financial accelerator mechanism, suggests that constrained firms will significantly increase their capital investment in response to expansionary monetary policy. Our results imply that, instead, expansionary policy will stimulate the less risky firms in the economy.

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A Appendix

A.1 Empirical Work

This appendix describes the firm-level variables used in the empirical analysis of the paper, based on quarterly Compustat data. The definition of the variables and sample selection follow standard practices in the literature (see, for example, Whited, 1992; Gomes, 2001; Eisfeldt and Rampini, 2006; Clementi and Palazzo, 2015).

A.1.1 Data Construction

Variables

1. *Investment, intensive margin* (baseline measure): defined as $\Delta \log(k_{jt+1})$, where k_{jt+1} denotes the capital stock of firm j at the end of period t. For each firm, we set the first value of k_{jt+1} to the level of gross plant, property, and equipment (`ppegtq`, item 118) in the first period in which this variable is reported in Compustat. From this period onwards, we compute the evolution of k_{jt+1} using the changes of net plant, property, and equipment (`ppentq`, item 42), which is a measure net investment with significantly more observations than `ppegtq` (net of depreciation). If a firm has a missing observation of `ppentq` located between two periods with nonmissing observations we estimate its value using a linear interpolation with the values of `ppentq` right before and after the missing observation; if two or more consecutive observations are missing we do not do any imputation. We only consider investment spells with 40 quarters or more in order to precisely estimate fixed effects.

2. *Investment, extensive margin*: defined as $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$, where $i_{jt} = k_{jt+1} - (1-\delta_j)k_{jt}$ denotes gross investment. We measure δ_j using depreciation rates of Fixed Asset Tables from NIPA at the sector level.

3. *Leverage*: defined as the ratio of total debt (sum of `dlcq` and `dlttq`, items 45 and 71) to total assets (`atq`, item 44).

4. *Net leverage*: defined as the ratio of total debt minus net current assets (`actq`, item 40, minus `lctq`, item 49) to total assets.

5. *Real Sales Growth*: measured as log-differences in sales (`saleq`, item 2) deflated using CPI.

6. *Size*: measured as the log of total assets.

7. *Cash flow*: measured as EBITDA divided by capital stock.

8. *Dividend payer*: defined as a dummy variable taking a value of one in firm-quarter observations in which the firm paid dividends to preferred stock of the company (constructed using `dvpq`, item 24).

9. *Tobin's q*: defined as the ratio market to book value of assets. The market value of assets is measured as the book value, plus the market value of common stock, minus the book value of common stock `ceq`, plus deferred taxes and investment tax credit (item `txditcq`, item 52). The market value of common stock is computed as the product of price at quarter close (`prccq`) and common shares outstanding (`cshoq` item 61). We winsorize 1% of observations in each tail of the distribution.

10. *Sectoral dummies*. We consider the following sectors: (i) agriculture, forestry, and fishing: `sic` < 10; (ii) mining: `sic`$\in [10, 14]$; (iii) construction: `sic`$\in [15, 17]$; (iv) manufacturing: `sic`$\in [20, 39]$; (v) transportation, communications, electric, gas, and sanitary services: `sic`$\in [40, 49]$; (vi) wholesale trade: `sic`$\in [50, 51]$; (vii) retail trade `sic`$\in [52, 59]$; (viii) services: `sic`$\in [70, 89]$.

Sample Selection Our empirical analysis excludes:

1. Firms in finance, insurance, and real estate sectors (`sic`$\in [60, 67]$) and public administration (`sic`$\in [91, 97]$).

2. Firms not incorporated in the United States.

3. Firm-quarter observations with acquisitions (constructed based on `aqcy`, item 94) larger than 5% percent of assets.

4. Firm-quarter observations that satisfy one of the following conditions, aimed at excluding extreme observations:

   i. Investment rate is in the top and bottom 0.5 percent of the distribution.

   ii. Leverage higher than 10.

   iii. Net current assets as a share of total assets higher than 10 or below -10.

   iv. Quarterly real sales growth above 1 or below -1.

A.1.2 Robustness

This appendix contains various results referenced in Section 2 of the main text. Table 16 estimates our baseline specification (3) using the "target" and "path" components of the monetary shock estimated in Campbell et al. (2016). It shows that the differential responses across firms we find in the main text are driven by the target component of the announcement, capturing the effect of the announcement on the level of the yield curve, rather than the path component of the announcement, capturing the effect on the slope of the yield curve. Because the Fed began making formal policy announcements only after 1994, Table 17 estimates our baseline specification (3) using post-1994 data. Low-leverage firms continue to be more responsive in this specification. Table 18 shows that our baseline results hold when we time-aggregate the high-frequency shocks by taking the simple sum within the quarter, rather than the weighted sum in the main text.

Table 19 runs our baseline specification (3) using leverage net of current assets and shows that our results continue to hold. Table 20 decomposes leverage into various types of debt and shows that our results hold for each of these types of debt. Table 21 shows that the negative relationship between leverage and credit rating documented in Figure 3 holds in a regression context, conditional on firm controls.

Table 22 runs our baseline specification with interactions for size, age (proxied by number of years since the firm's IPO), and whether the firm is a dividend payer, as described in the main text.

Table 16
Target vs. Path Decomposition

A) Dependent variable: $\Delta \log k$

	(1)	(2)
leverage × ffr shock	-0.74**	
	(0.28)	
leverage × target shock		-1.23***
		(0.42)
leverage × path shock		1.50
		(4.35)
Observations	239523	233661
R^2	0.118	0.119

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)
leverage × ffr shock	-5.01***	
	(1.26)	
leverage × target shock		-8.16***
		(1.94)
leverage × path shock		1.62
		(19.76)
Observations	239523	233661
R^2	0.215	0.216

Notes: Results from estimating variants of the baseline specification

$$\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Column (2) of both panels runs separate interactions of leverage with the target and path component of interest rates, as defined in Campbell et al. (2016). Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

Table 17
Post-1994 Estimates

A) Dependent variable: $\Delta \log k$

	(1)	(2)	(3)
leverage × ffr shock	-0.51	-0.55	-0.64
	(0.50)	(0.44)	(0.45)
leverage	-0.01***	-0.01***	-0.01***
	(0.00)	(0.00)	(0.00)
ffr shock			-0.05
			(1.54)
Observations	185752	185752	185752
R^2	0.120	0.131	0.116
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)	(3)
leverage × ffr shock	-3.68	-2.83	-3.05
	(2.12)	(1.82)	(1.78)
leverage	-0.05	-0.04	-0.04***
	(5.41)	(7.46)	(0.01)
ffr shock			-0.53
			(8.71)
Observations	47362	47362	47362
R^2	0.337	0.342	0.339
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Notes: Results from estimating variants of the baseline specification

$$\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Only data after 1994 is used in the estimation. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

Table 18
Alternative Time Aggregation

A) Dependent variable: $\Delta \log k$

	(1)	(2)	(3)
leverage × ffr shock (sum)	-0.89***	-0.79***	-0.79***
	(0.33)	(0.28)	(0.29)
ffr shock (sum)			1.02
			(0.82)
Observations	236296	236296	236296
R^2	0.106	0.118	0.103
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)	(3)
leverage × ffr shock (sum)	-3.75***	-3.56***	-3.43***
	(1.19)	(1.10)	(1.14)
leverage	-0.03***	-0.02***	-0.03***
	(0.00)	(0.00)	(0.00)
ffr shock (sum)			2.09
			(3.55)
Observations	236296	236296	236296
R^2	0.211	0.216	0.203
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Notes: Results from estimating variants of the baseline specification $\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. The high-frequency shocks are aggregated to the quarterly frequency simply by summing all shocks within a quarter. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Standard errors are two-way clustered by firms and time. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates).We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

TABLE 19
NET LEVERAGE

A) Dependent variable: $\Delta \log k$

	(1)	(2)	(3)
net leverage × ffr shock	-1.01**	-0.81**	-0.74*
	(0.43)	(0.36)	(0.37)
net leverage	-0.01***	-0.01***	-0.01***
	(0.00)	(0.00)	(0.00)
ffr shock			1.25
			(0.94)
Observations	233182	233182	233182
R^2	0.110	0.119	0.106
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)	(3)
net leverage × ffr shock	-5.34***	-4.87***	-4.14**
	(1.76)	(1.54)	(1.63)
net leverage	-0.04***	-0.02***	-0.03***
	(0.00)	(0.00)	(0.00)
ffr shock			3.56
			(4.26)
Observations	233182	233182	233182
R^2	0.213	0.217	0.204
Firm controls	no	yes	yes
Time sector FE	yes	yes	no
Time clustering	yes	yes	yes

Notes: Results from estimating variants of the baseline specification

$$\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1} \varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage net of current assets, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized net leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

TABLE 20
DECOMPOSITION OF LEVERAGE

A) Dependent variable: $\Delta \log k$

	(1)	(2)	(3)	(4)	(5)
ST debt × ffr shock	-0.54**		-0.58**		
	(0.23)		(0.24)		
LT debt × ffr shock		-0.41	-0.45***		
		(0.26)	(0.04)		
leverage × ffr shock				-0.72**	
				(0.28)	
other liab × ffr shock				-1.30***	
				(0.17)	
liabilities × ffr shock					-3.61
					(3.08)
Observations	239523	239523	239523	239502	239502
R^2	0.117	0.116	0.118	0.118	0.116
Firm controls	yes	yes	yes	yes	yes

B) Dependent variable: $\mathbb{1}\{\frac{i}{k} > 1\%\}$

	(1)	(2)	(3)	(4)	(5)
ST debt × ffr shock	-5.01***		-5.14***		
	(1.34)		(1.37)		
LT debt × ffr shock		-1.72	-1.95		
		(1.19)	(17.69)		
leverage × ffr shock				-4.95***	
				(1.28)	
other liab × ffr shock				-3.25***	
				(0.46)	
liabilities × ffr shock					-12.20
					(10.83)
Observations	239523	239523	239523	239502	239502
R^2	0.215	0.214	0.215	0.215	0.214
Firm controls	yes	yes	yes	yes	yes

Notes: Results from estimating variants of the baseline specification

$$\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta \ell_{jt-1}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt},$$

where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, ℓ_{jt-1} is leverage, $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Column (1) measures leverage using short term debt, column (2) with long term debt, column (3) with total debt, column (4) with other liabilities (such as trade credit), and column (5) with total liabilities. Panel (A) uses the intensive margin measure of investment $\Delta \log k_{jt}$ as the outcome variable and Panel (B) uses the extensive margin measure $\mathbb{1}\left\{\frac{i_{jt}}{k_{jt}} > 1\%\right\}$ as the outcome variable. Standard errors are two-way clustered by firms and quarters. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized each components of leverage over the entire sample, so its units are in standard deviations relative to the mean.

Table 21
Low-Leverage Firms Have Higher Credit Ratings

	(1)	(2)	(3)	(4)
leverage	-0.09***	-0.04***	-0.09***	-0.04***
	(0.01)	(0.01)	(0.02)	(0.01)
sales_growth	-0.01***	-0.00***	-0.01**	-0.00
	(0.00)	(0.00)	(0.00)	(2.21)
size $(t-1)$	0.28***	0.11***	0.28***	0.11
	(0.02)	(0.03)	(0.02)	(2.48)
share current assets	0.01	0.04***	0.02	0.04
	(0.01)	(0.01)	(0.01)	(5.33)
Observations	49201	49166	49201	49166
R^2	0.261	0.826	0.282	0.828
Firm controls	yes	yes	yes	yes
Firm FE	no	yes	no	yes
Time sector FE	no	no	yes	yes

Notes: Results from estimating variants of the baseline specification $\mathbb{1}\{\texttt{rating}_{it} \geq A\} = \alpha_i + \alpha_{st} + \mathbf{\Gamma}' Z_{it-1} + \varepsilon_{it}$, where $\mathbb{1}\{\texttt{rating}_{it} \geq A\}$ is an indicator variable for whether the firm's credit rating is above AA, α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Standard errors are two-way clustered by firms and quarter. We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

Table 22
Interaction With Other Measures of Financial Positions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
leverage × ffr shock	-0.74**		-0.73**		-0.76**		-0.74**
	(0.28)		(0.28)		(0.29)		(0.28)
size × ffr shock		0.35	0.37***				
		(0.31)	(0.07)				
cash flows × ffr shock				0.24	0.28		
				(0.54)	(1.34)		
$\mathbb{I}\{$dividends $> 0\}\times$ ffr shock						0.13	0.39
						(0.58)	(4.83)
Observations	239523	239523	239523	237890	237890	239232	239232
R^2	0.118	0.115	0.118	0.120	0.122	0.116	0.118
Firm controls	yes	yes	yes	yes	yes	yes	yes
Time sector FE	yes	yes	yes	yes	yes	yes	yes
Time clustering	yes	yes	yes	yes	yes	yes	yes

Notes: Results from estimating variants of the baseline specification $\Delta \log k_{jt} = \alpha_j + \alpha_{st} + \beta y_{jt-1}\varepsilon_t^{\mathrm{m}} + \mathbf{\Gamma}' Z_{jt-1} + \varepsilon_{jt}$, where α_j is a firm fixed effect, α_{st} is a sector-by-quarter fixed effect, y_{jt} is the firm's size (measured by log of current assets), cash flows, or an indicator for whether the firm pays dividends. $\varepsilon_t^{\mathrm{m}}$ is the monetary shock, and Z_{jt-1} is a vector of firm-level controls containing leverage, sales growth, size, current assets as a share of total assets, and an indicator for fiscal quarter. Columns (2) and (4) additionally include an interaction between leverage ℓ_{jt-1} and the monetary policy shock $\varepsilon_t^{\mathrm{m}}$. Standard errors are two-way clustered by firms and time. We have normalized the sign of the monetary shocks $\varepsilon_t^{\mathrm{m}}$ so that a positive shock is expansionary (corresponding to a decrease in interest rates). We have standardized leverage ℓ_{jt} over the entire sample, so its units are in standard deviations relative to the mean.

A.2 Proof of Proposition 1

We prove Proposition 1 in steady state; extending the proof to include transition dynamics is straightforward. To clarify the economic mechanisms, we work with a simple version of the model that abstracts from capital-quality shocks ($\sigma_\omega = 0$), has zero recovery value of debt ($\alpha = 0$), and has no exogenous exit shocks ($\pi_d = 0$). The proof in the full model follows the same steps with more complicated notation.

Default Threshold As discussed in the main text, firms only default when they have no feasible choice which satisfies the non-negativity constraint on dividends, i.e., there is no (k', b') such that $x - k' + \mathcal{Q}(z, k', b')b' \geq 0$. Define the default threshold $\underline{x}(z) = \min_{k',b'} k' - \mathcal{Q}(z, k', b')b'$. Note that the largest feasible dividend payment of a firm is $x - \underline{x}(z)$. If $x \geq \underline{x}(z)$, then $\arg\min_{k',b'} k' - \mathcal{Q}(z, k', b')b'$ is a feasible choice and the firm will not default. On the other hand, if $x < \underline{x}(z)$, then $d \leq 0$ for all (k', b'), violating feasibility.

With this notation in hand, the Bellman equation of a continuing firm in this simple case is

$$v(z, x) = \max_{k',b'} x - k' + \mathcal{Q}(z, k', b')b' + \beta\mathbb{E}\left[v(z', x')\mathbb{1}\{x' > \underline{x}(z')\}|z, k', b'\right] \text{ s.t. } d \geq 0, \quad (27)$$

where $\underline{x}(z')$ is the default threshold.

Although the continuation value is kinked at the default point, it is never optimal for a firm to choose this point (see Clausen and Strub (2017) and the discussion in Arellano et al. (2016)). Hence, the first order conditions are necessary at the optimum.

Unconstrained Firms Define the *unconstrained capital accumulation rule* $k^*(z)$ as

$$k^*(z) = \underset{k'}{\text{argmax}} -k' + \beta\mathbb{E}\left[\iota(z', k') + (1 - \delta)k'|z\right],$$

where $\iota(z, k) = \max_n zk^\theta n^\nu - wn$. After some algebra, one can show that the expression in the main text solves this maximization problem (extending the expression to the full model).

We will now fully characterize the decision rules for firms that can afford the unconstrained capital accumulation rule while have zero probability of default in all future states.

We first claim that such a firm is indifferent over any choice of debt b' which leaves the firm unconstrained. To show this, note that since the firm has no default risk it borrows at the risk-free rate β. In this case, the first order condition for borrowing b' is $\beta = \beta$, which is obviously true for any value of b'.

Following Khan, Senga and Thomas (2016), we resolve this indeterminacy by defining the *maximum borrowing policy* $b^*(z)$ as the maximal borrowing b' the firm can do while having zero probability of default in all future states.[38] To derive the maximum borrowing policy $b^*(z)$, first note that if the firm if the firm invests $k^*(z)$ and borrows $b^*(z)$ in the current period, its dividends in the next period are

$$\iota(z', k^*(z)) + (1-\delta)k^*(z) - b^*(z) - \xi - k^*(z') + \beta b^*(z'),$$

for a given realization of z'. The requirement that the firm has zero probability of default in all future states then implies that

$$b^*(z) = \min_{z'} \iota(z', k^*(z)) + (1-\delta)k^*(z) - k^*(z') + \beta b^*(z').$$

Hence, $b^*(z)$ is the largest amount of borrowing the firm can do and be guaranteed to satisfy the non-negativity constraint on dividends.[39]

By construction, if a firm can follow the unconstrained capital accumulation policy $k^*(z)$ and the maximum borrowing policy $b^*(z)$ while satisfying the non-negativity constraint on dividends in the current period, it will also satisfy the non-negativity constraint in all future periods. Moreover, following $k^*(z)$ is indeed optimal for such firms because it solves the associated first-order condition of these firms. Hence, a firm is *unconstrained* and follows these decision rules if and only if $d = x - k^*(z) + \beta b^*(z)$, i.e.,

$$x > \underline{x}(z) \equiv k^*(z) - \beta b^*(z).$$

[38] Khan, Senga and Thomas (2016) refer to this object as the "minimum savings policy."

[39] To derive this expression, first re-arrange the non-negativity constraint on dividends conditional on a realization of the future shocks as an inequality with b' on the left-hand side. This results in a set of inequalities for each possible realization of the future shocks. The min operator ensures that all of these inequalities are satisfied.

Constrained Firms Consider again the constrained Bellman equation (27). We will show that firms with $x \in [\underline{x}(z), \overline{x}(z)]$ pay zero dividends. Invert the default threshold $\underline{x}(z)$ so that the firm defaults if $z' < \underline{z}(k', b')$. The Bellman equation (27) can then be written as

$$v(z,x) = \max_{k',b'} x - k' + \mathcal{Q}(z,k',b')b' + \beta \int_{\underline{z}(k',b')}^{\overline{z}} v(z',x')g(z'|z)\mathrm{d}z' \text{ s.t. } d \geq 0, \tag{28}$$

where $g(z'|z)$ is the density of z' conditional on z, $\overline{z}$ is the upper bound of the support of z, and $\mathcal{Q}_3(z,k',b')$ is the derivative of the debt price schedule with respect to b'.

Letting $\lambda(z,x)$ be the Lagrange multiplier on the $d \geq 0$ constraint, the first order condition for b' is

$$\begin{aligned}&(1+\lambda(z,x))(\mathcal{Q}(z,k',b') + \mathcal{Q}_3(z,k',b')b') = \\ &\quad \beta\left[\int_{\underline{z}(k',b')}^{\overline{z}} (1+\lambda(z',k',b')g(z'|z)\mathrm{d}z' + g(\underline{z}(k',b')|z)v(\underline{z}(k',b'),\hat{x}'(k',b'))\frac{\partial \underline{z}(k',b')}{\partial b'}\right],\end{aligned}$$

where $\hat{x}'(k',b') = \max_{n'} \underline{z}(k',b')(k')^{\theta}(n')^{\nu} - wn' + (1-\delta)k' - b' - \xi$ and $\lambda(z',k',b') = \lambda(z',x')$ for the x' implied by (z',k',b'). The left hand side of this expression measures the marginal benefit of borrowing. The marginal resources the firm receives on borrowing is the debt price, adjusting for the fact that the marginal cost of borrowing changes on existing debt. The firm values those marginal resources using the Lagrange multiplier. The right hand side of this expression measures the discounted marginal cost of borrowing. In states of the world in which the firm does not default, it must give up one unit of resources, which it values using the next period's Lagrange multiplier. In addition, marginal borrowing implies that the firm defaults in additional future states.

Note that the debt price schedule is $\mathcal{Q}(z,k',b') = \beta \int_{\underline{z}(k',b')}^{\overline{z}} g(z'|z)\mathrm{d}z'$, which implies that $\mathcal{Q}_3(z,k',b') = -\beta g(\underline{z}(k',b')|z)\frac{\partial \underline{z}(k',b')}{\partial b'}$. Plugging this into the first order condition gives

$$\begin{aligned}&\beta(1+\lambda(z,x))(\int_{\underline{z}(k',b')}^{\overline{z}} g(z'|z)\mathrm{d}z' - \beta g(\underline{z}(k',b')|z)\frac{\partial \underline{z}(k',b')}{\partial b'} = \\ &\quad \beta\left[\int_{\underline{z}(k',b')}^{\overline{z}} (1+\lambda(z',k',b')g(z'|z)\mathrm{d}z' + g(\underline{z}(k',b')|z)v(\underline{z}(k',b'),\hat{x}'(k',b'))\frac{\partial \underline{z}(k',b')}{\partial b'}\right].\end{aligned} \tag{29}$$

We will now show that constrained firms set $d = 0$. We do so by contradiction: suppose that a constrained firm sets $d > 0$, implying that $\lambda(z, x) = 0$.

First consider a firm that has zero probability of default in the next period, i.e., $\underline{z}(k', b') = \underline{z}$ and $\frac{\partial \underline{z}(k', b')}{\partial b'} = 0$. In this case, the first order condition (29) can be simplified to

$$0 = \int_{\underline{z}}^{\overline{z}} \lambda(z', k', b') g(z'|z) \mathrm{d}z'.$$

Since the firm is constrained, $\lambda(z', k', b') > 0$ for some positive mass of realizations of z', leading to a contradiction.

Now consider a firm that has some positive probability of default, implying that $\underline{z}(k', b') > \underline{z}$ and $\frac{\partial \underline{z}(k', b')}{\partial b'} > 0$. In this case, the first order condition (29) can be rearranged to

$$0 = \int_{\underline{z}(k', b')}^{\overline{z}} \lambda(z', k', b') g(z'|z) \mathrm{d}z' + \frac{\partial \underline{z}(k', b')}{\partial b'} g(\underline{z}(k', b')|z)(b' + v(z', k', b')),$$

where $v(z', k', b') = v(z', x')$ for the x' implied by (z', k', b'). By construction, risky constrained firms engage in strictly positive borrowing $b' > 0$. This implies that the right hand side is strictly greater than zero, leading to a contradiction.