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THE FRAGILITY OF MARKET RISK INSURANCE

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ABSTRACT

Insurers sell retail financial products called variable annuities that package mutual funds with minimum return guarantees over long horizons. Variable annuities accounted for $1.5 trillion or 34 percent of U.S. life insurer liabilities in 2015. Sales fell and fees increased after the 2008 financial crisis as the higher valuation of existing liabilities stressed risk-based capital. Insurers also made guarantees less generous or stopped offering guarantees entirely to reduce risk exposure. We develop an equilibrium model of insurance markets in which financial frictions and market power are important determinants of pricing, contract characteristics, and the degree of market incompleteness.

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The traditional role of life insurers is to insure idiosyncratic risk through products like life annuities, life insurance, and health insurance. With the secular decline of defined benefit pension plans and Social Security around the world, life insurers are increasingly taking on the role of insuring market risk through minimum return guarantees. In the U.S., life insurers sell retail financial products called variable annuities that package mutual funds with minimum return guarantees over long horizons. Variable annuities have grown to be the largest category of life insurer liabilities, larger than traditional annuities and life insurance, and accounted for $1.5 trillion or 34 percent of U.S. life insurer liabilities in 2015. Variable annuities also represent an important share of the mutual fund sector because the underlying assets are invested in mutual funds.

The large size of the variable annuity market reflects its importance for household welfare. In theory, minimum return guarantees could facilitate efficient risk sharing across heterogeneous agents (Dumas 1989, Chan and Kogan 2002) or overlapping generations (Allen and Gale 1997, Ball and Mankiw 2007). From the insurers' perspective, however, minimum return guarantees are long-dated put options on market risk that are difficult to price and hedge. Imperfect hedging leads to risk mismatch that stresses risk-based capital when the valuation of existing liabilities increases with a falling stock market, falling interest rates, or rising volatility. During the 2008 financial crisis, many insurers including Aegon, Allianz, AXA, Hartford, Jackson National, Sun Life, and Voya suffered large increases in variable annuity liabilities ranging from 12 to 106 percent of total equity. Hartford was subsequently bailed out by the Troubled Asset Relief Program in June 2009 because of significant losses on their variable annuity business.[1] Given their size and potential risk, variable annuities are an essential piece of the puzzle for understanding the insurance sector more broadly.

To this end, we construct a new and comprehensive panel data set on the variable annuity market at the contract level. Our data contain quarterly sales, fees, and contract characteristics from 1999:1 (first quarter) to 2015:4 (fourth quarter). We combine these data with the annual financial statements of insurers from 2005 to 2015. The financial statements contain information about the value of variable annuity liabilities and the share of these liabilities that are reinsured. Our data provide a detailed account of how the variable annuity market has evolved over time as the changing valuation of existing liabilities affected balance sheet health.

Quarterly sales of variable annuities grew robustly from $25 billion in 2005:1 to $41 billion in 2007:4 and subsequently fell to $27 billion in 2009:2. At the same time, the average fee on minimum return guarantees increased from 0.59 percent in 2007:4 to 0.96 percent in

[1] Other examples of risk mismatch due to minimum return guarantees include the financial distress of Japanese life insurers in the 1990s and the failure of Equitable Life in 2000 (Kashyap 2002, Roberts 2012).

2009:2, suggesting an important role for a supply shock. After the financial crisis, insurers made the minimum return guarantees less generous or stopped offering guarantees entirely to reduce risk exposure. In the cross section of insurers, sales fell more for insurers that suffered larger increases in the valuation of existing liabilities. These insurers moved their variable annuity liabilities off balance sheet through reinsurance, consistent with the importance of a risk-based capital constraint (Koijen and Yogo 2016).

To interpret this evidence, we develop an equilibrium model of insurance markets in which financial frictions and market power are important determinants of pricing, contract characteristics, and the degree of market incompleteness. Insurers compete in an oligopolistic market by setting the fee and the rollup rate, which is a key contract characteristic that is equivalent to the strike price of a put option. Required capital increases in the rollup rate because of a risk-based capital or an economic risk constraint. An adverse shock to the valuation of existing liabilities increases the shadow cost of capital and drives up the marginal cost of issuing contracts. The insurer not only raises the fee but lowers the rollup rate to reduce risk exposure. When the shadow cost of capital is sufficiently high, the insurer exits the market for minimum return guarantees to avoid additional risk exposure from the sale of contracts.

The demand for variable annuities could be driven by various factors other than the fee and the rollup rate. They include the attractiveness (e.g., tax advantage) of variable annuities relative to other savings products, the diversity of options within contracts, and insurer characteristics that capture reputation in the retail market. To capture these effects, we estimate a differentiated product demand system for the variable annuity market and use the optimal pricing equation to decompose fees into markups versus marginal cost. Marginal cost increased by 16 percent for the average contract during the financial crisis, explaining most of the increase in fees. By exploiting the large cross section of contracts, we further decompose the change in marginal cost into within-insurer variation along contract characteristics versus between-insurer variation. According to the optimal pricing equation, the between-insurer variation identifies the cross-sectional variation in the shadow cost of capital across insurers. The between-insurer variation explains as much as 40 percent of the cross-sectional variation in marginal cost, confirming the importance of financial frictions for variable annuity supply.

Previous research on the supply side of insurance markets has shown the importance of financial frictions and market power in the pricing of catastrophe reinsurance (Froot 2001) and traditional annuities and life insurance (Koijen and Yogo 2015, Koijen and Yogo 2016). Risk-based capital regulation also matters for portfolio choice on asset side (Becker and Ivashina 2015, Ellul, Jotikasthira, Lundblad and Wang 2015). We build on this literature by showing that financial frictions and market power not only affect pricing but also contract characteristics and the degree of market incompleteness. Thus, we develop a more complete theory of insurance markets that is analogous to Rothschild and Stiglitz (1976), which shows how informational frictions could affect pricing, contract characteristics, and the degree of market incompleteness. Our theory could apply to other insurance markets in which insurers bear significant aggregate risk over long horizons, such as the long-term care insurance market (Cutler 1996).

Our work also relates to the mutual fund literature. Previous research has shown that past performance (Chevalier and Ellison 1997, Sirri and Tufano 1998, Wermers 2003) and tax efficiency (Bergstresser and Poterba 2002, Sialm and Starks 2012) are important determinants of mutual fund flows. At the same time, demand is surprisingly inelastic to fees, which suggests an important role for product differentiation and market power (Hortaçsu and Syverson 2004). We study the determinants of supply and demand for variable annuities, which are an important part of the mutual fund sector that has received relatively little attention.

The remainder of this paper proceeds as follows. Section I describes variable annuities and details about their regulation that are relevant for this paper. Section II describes the data construction and summarizes key facts about the variable annuity market. Section III presents a model of variable annuity supply that explains the evidence on pricing and contract characteristics. Section IV estimates a model of variable annuity demand to quantify the importance of financial frictions. Section V concludes.

I. Institutional Background

We start with an example of an actual product to explain how variable annuities work. We then summarize risk-based capital regulation, which is important for understanding how an adverse shock to the valuation of existing liabilities could affect variable annuity supply. We also explain how an economic risk constraint could work in conjunction with a risk-based capital constraint. Finally, we summarize economic and institutional reasons why insurers do not fully hedge variable annuity risk.

A. An Example of a Variable Annuity Product

Insurers sell long-term savings products called variable annuities, which are investments in mutual funds. For an additional fee, insurers offer an optional minimum return guarantee on the mutual fund. Thus, a variable annuity is a retail financial product that packages a mutual fund with a long-dated put option on the mutual fund.[2] To explain how variable annuities work, we start with an example of an actual product.

MetLife Investors USA Insurance Company (2008) offers a variable annuity contract called MetLife Series VA, which comes with various investment options and guaranteed living benefits. In 2008:3, one of the investment options was the American Funds Growth Allocation Portfolio, which is a mutual fund with a target equity allocation of 70 to 85 percent and an annual portfolio expense of 1.01 percent. One of the guaranteed living benefits was a Guaranteed Lifetime Withdrawal Benefit (GLWB). MetLife Series VA has an annual base contract expense of 1.3 percent of account value, and the GLWB has an annual fee of 0.5 percent of account value. Thus, the total annual fee for the variable annuity with the GLWB is 1.8 percent, which is on top of the annual portfolio expense on the mutual fund.

Suppose that an investor were to invest in the American Funds Growth Allocation Portfolio in 2008:3. After 2013:3, the investor withdraws a constant dollar amount each year that is 5 percent of the highest account value ever reached. For example, this behavior describes an investor who invests in a mutual fund five years prior to retirement and subsequently spends down her assets by consuming a constant dollar amount each year. Figure 1 shows the account value of the investor per $1 of initial investment, with the shaded region covering the withdrawal period after 2013:3. The account value fluctuates over time because of uncertainty in investment returns.

The same investor could purchase the GLWB from MetLife and guarantee her investment returns. The GLWB has an annual rollup rate of 5 percent prior to first withdrawal, which means that at each contract anniversary, the guaranteed amount steps up to the greater of the account value and the previous guaranteed amount accumulated at 5 percent. Thus, the GLWB is a put option on the mutual fund that locks in every year to a strike price that accumulates at an annual rate of 5 percent. Figure 1 shows that the guaranteed amount can only increase during five-year accumulation period, protecting the investor from uncertainty in investment returns.

Once the investor enters the withdrawal period, she can annually withdraw up to 5 percent of the highest guaranteed amount ever reached. In our example, the guaranteed amount in 2013:3 is \$1.44, which means that the investor can withdraw up to $\$1.44 \times 0.05 = \$0.072$ per year. Each withdrawal gets deducted from both the account value and the guaranteed amount. The GLWB is a lifetime guarantee in that the investor receives income (i.e., \$0.072 per year) as long as she lives, even after the account is depleted to zero.

[2]Variable annuities have a tax advantage over mutual funds because taxes on earnings can be deferred until withdrawal. We refer to Brown and Poterba (2006) for a complete discussion of how the tax benefit depends complexly on the timing of withdrawal.

During the withdrawal period, the guaranteed amount steps up to the account value at each contract anniversary. In Figure 1, these step-ups occur in 2014:3 and 2016:3 because of high investment returns.

Because the annual rollup rate is 5 percent and the annual fee is 0.5 percent, one may be tempted to conclude that the guaranteed return on the variable annuity is 4.5 percent during the accumulation period. This logic turns out to be incorrect because the guaranteed amount of $1.44 in 2013:3 is only payable as annual income of $0.072 over 20 years (or until the investor's death). Because of the time value of money, the present value of $0.072 per year over 20 years is worth less than $1.44. Appendix A shows the empirical relevance of this issue using the historical term structure of interest rates.

GLWB is the most common type of guaranteed living benefit. The other three types of guaranteed living benefits are Guaranteed Minimum Withdrawal Benefit (GMWB), Guaranteed Minimum Income Benefit (GMIB), and Guaranteed Minimum Accumulation Benefit (GMAB). GMWB is similar to GLWB, except that the investor does not receive income after the account is depleted to zero. GMIB is similar to GLWB, except that guaranteed amount at the beginning of the withdrawal period converts to a life annuity (i.e., fixed income for life). GMAB provides a minimum return guarantee much like the accumulation period of GLWB, but it does not have a withdrawal period with guaranteed income.

If an investor were to die while the variable annuity contract is in effect, her estate receives a standard death benefit that is equal to the remaining account value. For an additional fee, insurers offer four types of guaranteed death benefits (highest anniversary value, rising floor, earnings enhancement benefit, and return of premium) that enhance the death benefit during the accumulation period. Our main focus is on the guaranteed living benefits, so we will not go into the details of the guaranteed death benefits in this paper.

B. Risk-Based Capital Regulation

Insurance regulators and rating agencies use risk-based capital as an important metric of an insurer's financial strength. Risk-based capital is the ratio of accounting equity to required capital:

$$\text{RBC} = \frac{\text{Assets} - \text{Reserves}}{\text{Required capital}}. \tag{1}$$

Reserves in the numerator is an accounting measure of liabilities that may not coincide with market value. Required capital in the denominator is a measure of how much equity could be lost in an adverse scenario. For a sufficiently high risk-based capital ratio, the regulators view the insurer as having enough capital to meet its existing liabilities even in an adverse scenario.

Variable annuity liabilities enter both reserves and required capital in risk-based capital. As summarized in Junus and Motiwalla (2009), Actuarial Guideline 43 since December 2009 determines the reserve value of variable annuities, and the C-3 Phase II regulatory standard since December 2005 determines the contribution of variable annuities to required capital. Actuarial Guideline 43 is a higher reserve requirement than its precursor Actuarial Guideline 39, so insurers were given a phase-in period until December 2012 to fully comply with the new requirement.

To compute reserves and required capital, insurance regulators provide various scenarios for the joint path of Treasury, corporate bond, and equity prices. Insurers simulate the path of equity deficiency for their variable annuity business (net of the hedging programs and reinsurance) under each scenario and keep the highest present value of equity deficiency along each path. Reserves are then computed as a conditional mean over the upper 30 percent of equity deficiencies. This conditional tail expectation builds in a degree of conservatism that is conceptually similar to a correction for risk premia, but reserves need not coincide with the market value of liabilities. Insurers use the same methodology to compute required capital, except that they take a conditional mean over 10 percent of equity deficiencies.

More generous guarantees with higher rollup rates or better coverage of downside market risk relative to fees require higher reserves and more capital. Moreover, minimum return guarantees are long-dated put options on mutual funds whose value increases when the stock market falls, interest rates fall, or volatility rises. Therefore, an adverse scenario like the financial crisis increases both reserves and required capital and put downward pressure on risk-based capital. Insofar as insurers want to avoid a rating downgrade or regulatory action, an adverse shock to the valuation of existing liabilities could affect their ability to issue new liabilities. In Section III, we present a model that formalizes this mechanism through which financial frictions affect variable annuity supply.

In addition to the risk-based capital constraint, the insurer could have an economic risk constraint as part of risk management. An economic risk constraint works similarly to a risk-based capital constraint, except that the relevant measure of assets and liabilities is market value. For example, let ϵ be a multiplicative shock to the leverage ratio due to risk mismatch from variable annuities, whose cumulative distribution function is F. Consider a value-at-risk constraint under which the probability that assets exceed liabilities must exceed a threshold:

$$\Pr\left(\frac{\text{Liabilities}}{\text{Assets}}\epsilon \leq 1\right) = F\left(\frac{\text{Assets}}{\text{Liabilities}}\right) \geq \kappa. \tag{2}$$

We can rewrite this constraint as

$$\frac{\text{Assets} - \text{Liabilities}}{(F^{-1}(\kappa) - 1)\text{Liabilities}} \geq 1, \tag{3}$$

which is similar to risk-based capital (1). An insurer with more conservative risk management has higher $F^{-1}(\kappa)$, either through higher κ or lower risk reflected in the distribution of ϵ.

As a consequence of the financial crisis, the insurer could learn that model uncertainty is higher than previously recognized. In response, the insurer could make risk management more conservative, tightening the economic risk constraint. Thus, an economic risk constraint could work in conjunction with a risk-based capital constraint and affect variable annuity supply.

C. *Reasons for Risk Mismatch*

In theory, insurers could hedge uncertainty in the valuation of variable annuity liabilities through offsetting derivatives positions. In practice, there are important economic and institutional reasons why insurers do not fully hedge variable annuity risk.

An economic reason why insurers do not fully hedge is risk shifting motives that arise from limited liability and the presence of state guaranty funds, especially for stock rather than mutual companies (Lee, Mayers and Smith 1997). A second reason is that someone must bear aggregate risk in general equilibrium, and insurers may have comparative advantage over other types of institutions because their liabilities have a longer maturity and are less vulnerable to runs (Paulson, Rosen, Mohey-Deen and McMenamin 2012). A third reason is that any hedging program would be subject to basis risk and counterparty risk. Basis risk arises from the fact that minimum return guarantees have longer maturity than standard derivative contracts, so any hedging program would be based on an option pricing model, which introduces model uncertainty (Kling, Ruez and Russ 2011).

An institutional reason why insurers do not fully hedge is that existing regulation does not properly reward hedging of market equity. Insurers report accounting equity under statutory accounting principles at the operating company level and under generally accepted accounting principles (GAAP) at the holding company level. Therefore, hedge positions differ depending on whether the insurer targets economic, statutory, or GAAP capital. A hedging program that smoothes market equity could actually increase the volatility of accounting equity under statutory accounting principles or GAAP (Credit Suisse 2012).

Whether insurers target market or accounting equity depends on whether the relevant friction is economic (e.g., value-at-risk constraint) or regulatory. If regulatory frictions are an important consideration, reinsurance could be a more efficient way to relax a risk-based capital constraint than hedging (Koijen and Yogo 2016). Consistent with this view, Section IV shows that insurers used reinsurance to move variable annuity liabilities off balance sheet during the financial crisis.

II. Data on the Variable Annuity Market

A. Data Construction

We use three sources to construct a comprehensive panel data set on the variable annuity market at the contract level. The first data source is Morningstar (2016a), which has quarterly sales of variable annuities at the contract level since 1999. Morningstar provides a textual summary of the prospectus for each contract, from which we extract the history of fees and contract characteristics. The key contract characteristics are the base contract expense, the number of investment options, and the types of guaranteed living and death benefits that are offered.[3] For each guaranteed living benefit, the key characteristics are the type (i.e., GLWB, GMWB, GMIB, or GMAB), the fee, the rollup rate, and the withdrawal rate. Morningstar provides the open and close dates for each contract and guaranteed living benefit, from which we construct the history of when different benefits were offered.

Sales are available at the contract level but not at the benefit level. Therefore, we must aggregate fees and rollup rates over all guaranteed living benefits that a contract offers to construct a panel data set on sales, fees, and characteristics at the contract level. For each date and contract, we first average the fees and rollup rates by the type of guaranteed living benefit. We then use the average fee and rollup rate in the order of GLWB, GMWB, GMIB, and GMAB, based on availability. For example, if a contract does not offer GLWB but offers GMWB, we use the average fee and rollup rate on GMWB. Because GLWB is the most common type of guaranteed living benefit and GMWB is the closest substitute to GLWB, our procedure yields a representative set of fees and rollup rates that are comparable across contracts.

The second data source is the annual financial statements of insurers, which are filed with the NAIC (National Association of Insurance Commissioners 2005–2015). General Interrogatories Part 2 Table 9.2 of the financial statements reports total related account value, the gross amount of reserves, and the reinsurance reserve credit on variable annuities. As we described in Section I, total related account value is the market value of the mutual funds.

[3]We use assets under management by subaccount from Morningstar (2016b) to compute a measure of investment options that adjusts for the non-uniform distribution of assets across subaccounts within a variable annuity contract. Our measure is the inverse of the Herfindahl index over the subaccount shares within each variable annuity contract, which is the number of investment options when the subaccounts are uniformly distributed.

The gross amount of reserves is the accounting value of the minimum return guarantees net of the hedging programs. We define variable annuity liabilities as total related account value plus the gross amount of reserves minus reinsurance reserve credit. For each insurer, we construct *reserve valuation* as the ratio of gross amount of reserves to total related account value. Reserve valuation is an important measure of the option value of variable annuity liabilities (net of the hedging programs). In the cross section, reserve valuation is higher for insurers that have sold more generous guarantees. In the time series, reserve valuation increases when the stock market falls, interest rates fall, or volatility rises.

The third data source is A.M. Best Company (2006–2016), which provides a cleaned and organized version of the main parts of the annual financial statements. Following A.M. Best's definition of financial groups, we aggregate insurance companies' balance sheets up to the group level. Total liabilities are aggregate reserve for life contracts plus liabilities from separate accounts statement. Total equity is capital and surplus. We convert the A.M. Best financial strength rating (coded from A++ to D) to a cardinal measure (coded from 175 to 0 percent) based on risk-based capital guidelines (A.M. Best Company 2011, p. 24).

We merge the A.M. Best data and the NAIC data by the NAIC company code. We then merge the Morningstar data and the NAIC data by company name. The final data set is a quarterly panel on the variable annuity market from 2005:1 to 2015:4, where the start date is dictated by the availability of the NAIC data. For some of the summary statistics that only involve the Morningstar data, we use a longer sample from 1999:1.

B. Summary of the Variable Annuity Market

Table 1 reports summary statistics for the variable annuity market. In 2005, variable annuity liabilities across all insurers was \$1.091 trillion or 36 percent of total liabilities. Variable annuity liabilities have ranged from 34 to 42 percent of total liabilities as its value fluctuates with the market value of the mutual funds. Most recently in 2015, variable annuity liabilities were \$1.486 trillion or 34 percent of total liabilities. The variable annuity market is fairly concentrated as measured by the number of insurers. The total number of insurers fell from 43 in 2008 to 38 in 2015.

As we explained above, reserve valuation (i.e., the ratio of gross amount of reserves to total related account value) measures the option value of variable annuity liabilities. Table 1 shows that reserve valuation aggregated across all insurers increased sharply from 0.9 percent in 2007 to 4.1 percent in 2008. Since 2008, reserve valuation is volatile and remains high relative to the level prior to the financial crisis.

Table 2 reports the top insurers offering variable annuities in 2007, ranked by their variable annuity liabilities. Seven of these insurers (Aegon, Allianz, AXA, Hartford, Jackson

National, Sun Life, and Voya) suffered large increases in reserve valuation ranging from 2.9 to 8.2 percentage points. These increases in reserve valuation are significant shocks because these insurers have high leverage ratios that range from 93 to 97 percent. Across the seven insurers, the increases in gross amount of reserves range from 12 to 106 percent of total equity.

Figure 2 reports quarterly sales of variable annuities across all contracts from 1999:1 to 2015:4. Sales grew robustly from $25 billion in 2005:1 to its peak at $41 billion in 2007:4. Sales subsequently fell during the financial crisis to $27 billion in 2009:2, picked up again to $34 billion in 2011:2, and are $20 billion most recently in 2015:4. For comparison, the same figure shows the aggregate sales of U.S. open-end stock and bond mutual funds (excluding money market funds and funds of funds), which is a larger market and shown on a different scale. Interestingly, sales of variable annuities and mutual funds moved closely together through 2008, but the two time series diverge thereafter as mutual fund sales grew.

The decline in variable annuity sales after 2008 is partly explained by insurers that have exited the market for guaranteed living benefits. Figure 3 reports the number of insurers and contracts offering guaranteed living benefits from 1999:1 to 2015:4. Eleven insurers stopped offering guaranteed living benefits from 2008 to 2015, during which five insurers stopped selling variable annuities altogether as reported in Table 1. This means that some insurers have opted to remain in the variable annuity market but to stop offering minimum return guarantees. Without minimum return guarantees, variable annuities are essentially mutual funds with a potential tax advantage.

The upper panel of Figure 4 reports the average annual fee on open (i.e., currently offered) guaranteed living benefits from 1999:1 to 2015:4. The increase in fees during the financial crisis coincides with the decline in sales, suggesting an important role for a supply shock. The average fee increased from 0.59 percent of account value in 2007:4 to 0.96 percent in 2009:2. Since then, the average fee has increased at a slower pace and was 1.08 percent in 2015:4.

In addition to the fee, the rollup rate is an important contract characteristic for guaranteed living benefits. The lower panel of Figure 4 reports the average rollup rate on open guaranteed living benefits available from 1999:1 to 2015:4. The average rollup rate increased from 2.4 percent in 2005:1 to 4.0 percent in 2007:4, coinciding with a period of robust sales growth. The average rollup rate remained high through the financial crisis. Coinciding with the decline in sales since 2011, the average rollup rate has decreased from 4.9 percent in 2011:2 to 3.4 percent in 2015:4.

The fact that the average rollup rate did not immediately respond during the financial crisis may seem surprising. However, the average rollup rate in Figure 4 represents the intensive margin conditional on offering a contract with a minimum return guarantee. Insurers can also respond through the extensive margin by offering contracts without minimum return guarantees. Indeed, Figure 4 shows that the share of contracts with guaranteed living benefits decreased immediately during the financial crisis.

III. A Model of Variable Annuity Supply

As we discussed in Section I, risk-based capital and economic risk constraints are important determinants of variable annuity supply and provide a narrative for the aggregate facts in Section II. Insurers suffered an adverse shock to risk-based capital from the increased valuation of existing liabilities during the financial crisis. In addition, insurers could have made risk management more conservative in response to higher model uncertainty. As the shadow cost of capital increased, insurers raised fees to pass through a higher marginal cost. Insurers also lowered rollup rates or stopped offering minimum return guarantees entirely to reduce risk exposure. Higher fees and lower rollup rates make variable annuities less attractive to investors, explaining the decline in sales.

We formalize this narrative through a simple model of how an insurer chooses the fee and the rollup rate in the presence of financial frictions and market power. To simplify the notation and the presentation, we model the insurer's optimization problem as a one-time choice. We refer to our previous work for a dynamic version in which the insurer chooses the optimal price in every period (Koijen and Yogo 2015, Koijen and Yogo 2016). Relative to our previous work, the novel modeling ingredient is the optimal choice of contract characteristics, and the novel insight is that the insurer changes contract characteristics to reduce risk exposure. Thus, we develop a more complete theory of the supply side of insurance markets that explains pricing, contract characteristics, and the degree of market incompleteness.

A. Variable Annuity Market

We start with high-level assumptions about financial markets that are standard in an option pricing model. There is a mutual fund whose price evolves exogenously over time. Let S_t be the mutual fund price per share in period t. By the absence of arbitrage, there exists a strictly positive stochastic discount factor $M_{t,t+s}$ that discounts a payoff in period $t+s$ to its price in period t. For example, the mutual fund price satisfies $S_t = \mathbb{E}_t[M_{t,t+s}S_{t+s}]$.

In period t, an insurer sells a variable annuity, which is a combination of the mutual fund and a minimum return guarantee. The variable annuity price is P_t per dollar of account value, so that the fee is $P_t - 1$. The minimum return guarantee is over two periods, and the gross rollup rate $r_t \geq 0$ is the guaranteed return per period. Thus, the investor's payoff upon withdrawal in period $t+2$ is

$$X_{t,t+2} = \max\left\{r_t^2, \frac{S_{t+2}}{S_t}\right\} = \frac{S_{t+2}}{S_t} + \underbrace{\max\left\{r_t^2 - \frac{S_{t+2}}{S_t}, 0\right\}}_{\text{put option}}. \tag{4}$$

The minimum return guarantee is a put option whose strike price is the cumulative rollup rate. When $r_t = 0$, the variable annuity is a mutual fund because the put option is always worthless. We assume that the investor cannot insure downside market risk outside of variable annuities, so the insurance market is incomplete when $r_t = 0$.

The frictionless value of the variable annuity at issuance in period t is

$$V_{t,t} = \mathbb{E}_t[M_{t,t+2}X_{t,t+2}] \tag{5}$$

per dollar of account value. More generally, $V_{t-s,t}$ denotes the frictionless value in period t of a contract that was issued in period $t-s$. Although this notation is slightly cumbersome, it will be important to distinguish the option value of existing liabilities $V_{t-1,t}$ from the option value of new contracts $V_{t,t}$. The frictionless value $V_{t,t}$ is the sum of 1 for the account value of the mutual fund and $V_{t,t}-1$ for the option value of the minimum return guarantee.

For the purposes of our theory, we do not need parametric assumptions about the option pricing model (e.g., Black and Scholes 1973). We just need to assume that the partial derivatives of option value have the usual signs. Namely, the put option value decreases in the mutual fund price, decreases in the riskless interest rate, increases in volatility, and increases in the rollup rate. In the language of Greeks in the option pricing literature, we assume that delta is negative, rho is negative, vega is positive, and dual delta is positive.

We also make minimal assumptions about variable annuity demand. Demand is continuous, continuously differentiable, strictly decreasing in price, and strictly increasing in the rollup rate. An institutional feature of the variable annuity market is that the rollup rate is always positive (i.e., $r_t \geq 1$) or $r_t = 0$ in the case of mutual funds with no minimum return guarantees. That is, insurers never offer a variable annuity with a negative rollup rate in the range $r_t \in (0,1)$, presumably because investors have a psychological aversion to "negative interest rates". To model this institutional feature, we simply assume that the insurer's choice of the rollup rate is constrained to be in the set $\mathcal{R} = \{0\} \bigcup [1,\infty)$.

B. Balance Sheet Dynamics

We now describe how variable annuity sales affect the insurer's balance sheet. Let Q_t be the account value of new contracts, excluding the option value of minimum return guarantees, that the insurer sells in period t. Let B_t be the total account value of mutual funds (or "separate accounts" in actuarial terms) at the end of period t. The account value evolves according to

$$B_t = \frac{S_t}{S_{t-1}} B_{t-1} + Q_t. \tag{6}$$

Current account value is the previous account value revalued at the current mutual fund price plus the account value of new contracts.

Let A_t be the insurer's assets at the end of period t, excluding the account value of the mutual funds. In actuarial terms, A_t represents the general account assets, and $A_t + B_t$ are total assets. The assets evolve according to

$$A_t = R_{A,t} A_{t-1} + (P_t - 1) Q_t, \tag{7}$$

where $R_{A,t}$ is an exogenous gross return on assets in period t. Current assets are the gross return on previous assets plus the fees on new contracts. Section I discussed economic and institutional reasons why insurers do not fully hedge variable annuity risk. Following that discussion, we assume that $R_{A,t}$ could be imperfectly correlated with the option value of existing liabilities, leading to risk mismatch.

Let L_t be the insurer's liabilities at the end of period t, excluding the account value of the mutual funds. In actuarial terms, L_t represents the general account liabilities, and $L_t + B_t$ are total liabilities. The liabilities evolve according to

$$L_t = \frac{V_{t-1,t} - S_t / S_{t-1}}{V_{t-1,t-1} - 1} L_{t-1} + (V_{t,t} - 1) Q_t. \tag{8}$$

Current liabilities are previous liabilities revalued at current cost plus the cost of new contracts. The principle of reserving requires that the cost $V_{t,t} - 1$ be recorded on the liability side to back the fees $P_t - 1$ on the asset side.

The following T account summarizes the insurer's balance sheet, which emphasizes that there is no risk mismatch for mutual funds in the separate account.

Assets	Liabilities	
B_t	B_t	(separate account)
A_t	L_t	(general account)
	$A_t - L_t$	(equity)

C. Financial Frictions

We define the insurer's statutory capital at the end of period t as

$$K_t = \underbrace{A_t - L_t}_{\text{equity}} - \underbrace{\phi_t L_t}_{\text{required capital}}. \tag{9}$$

Statutory capital is equity minus required capital that is proportional to liabilities.[4] Following the discussion in Section I, $\phi_t > 0$ could represent the risk weight on minimum return guarantees under the C-3 Phase II regulatory standard. As equation (8) shows, required capital increases in the option value of existing liabilities $V_{t-1,t}$. Therefore, required capital increases when the stock market falls, interest rates fall, or volatility rises. Required capital also increases in the option value of new contracts $V_{t,t}$. Therefore, required capital for new contracts increases in the rollup rate, decreases in interest rates, and increases in volatility.

Following the discussion in Section I, low statutory capital could lead to a rating downgrade or regulatory action, which have adverse consequences in both retail and capital markets. We model the cost of financial frictions through a cost function

$$C_t = C(K_t), \tag{10}$$

which is continuous, twice continuously differentiable, strictly decreasing, and strictly convex. The cost function is decreasing because higher statutory capital reduces the likelihood of a rating downgrade or regulatory action. The cost function is convex because these benefits of higher statutory capital have diminishing returns. Statutory capital would not matter if equity issuance were costless. Therefore, implicit in our specification of the cost function are financial frictions that make equity issuance costly.

An alternative interpretation of equation (9) is that the insurer has an economic risk constraint, such as the value-at-risk constraint described in Section I. As a consequence of the financial crisis, the insurer learned that model uncertainty is higher than previously recognized and made risk management more conservative. An increase in ϕ_t could capture such tightening of an economic risk constraint. A permanent increase in ϕ_t could lead to very persistent effects on variable annuity supply that is consistent with the evidence in Section II.

[4]The formulation of statutory capital as a difference rather than as a ratio is for mathematical convenience in the derivations that follow. However, Koijen and Yogo (2015) show that the two formulations are similar because a constraint on statutory capital such as $K_t \geq 0$ can be rewritten as a risk-based capital constraint $\frac{A_t - L_t}{\phi_t L_t} \geq 1$.

D. *Optimal Pricing and Contract Characteristics*

The insurer chooses the price P_t and the rollup rate $r_t \in \mathcal{R}$ on the variable annuity to maximize firm value in an oligopolistic market, where we assume the existence of a Nash equilibrium. Firm value is the profit from variable annuity sales minus the cost of financial frictions:

$$J_t = (P_t - V_{t,t})Q_t - C_t. \tag{11}$$

To simplify notation, we define the price elasticity of demand as $\epsilon_{P,t} = -\frac{\partial \log(Q_t)}{\partial \log(P_t)}$ and the elasticity of demand to the rollup rate as $\epsilon_{r,t} = \frac{\partial \log(Q_t)}{\partial \log(r_t)}$. We also define the shadow cost of capital as

$$c_t = -\frac{\partial C_t}{\partial K_t} > 0. \tag{12}$$

The shadow cost of capital represents the importance of financial frictions, which decreases in statutory capital by the convexity of the cost function. The following proposition, which we prove in Appendix B, characterizes the optimal price and rollup rate.

PROPOSITION 1: *The optimal price is*

$$P_t = \left(1 - \frac{1}{\epsilon_{P,t}}\right)^{-1} \underbrace{\left(V_{t,t} + \frac{c_t \phi_t (V_{t,t} - 1)}{1 + c_t}\right)}_{\textit{marginal cost}}. \tag{13}$$

At an interior optimum, the optimal rollup rate is

$$r_t = \left(\frac{\partial V_{t,t}}{\partial r_t}\right)^{-1} \frac{\epsilon_{r,t}}{\epsilon_{P,t} - 1} \left(V_{t,t} - \frac{c_t \phi_t}{1 + c_t(1 + \phi_t)}\right) > 1. \tag{14}$$

Otherwise, $r_t \in \{0, 1\}$ *is optimal.*

The optimal price (13) is a product of two terms. The first term is the Bertrand pricing formula, under which the optimal price decreases in the price elasticity of demand because of market power. The second term is the marginal cost of issuing contracts, which is greater than the frictionless value $V_{t,t}$ because of financial frictions. Marginal cost increases in the shadow cost of capital c_t and the capital requirement ϕ_t.[5]

[5] Equation (13) implies that marginal cost *decreases* in the shadow cost of capital if $\phi_t < 0$. In Koijen and Yogo (2015), the prices of traditional annuities decreased during the financial crisis because the effective capital requirement was negative for those products.

The optimal rollup rate (14) is a product of three terms. First, the optimal rollup rate decreases in the sensitivity of option value to the rollup rate. This is because a higher rollup rate increases the option value of the minimum return guarantee and decreases statutory capital through higher required capital. Second, the optimal rollup rate increases in the elasticity of demand to the rollup rate and decreases in the price elasticity of demand. This is the traditional demand channel through which the insurer optimally chooses the rollup rate to exploit market power. Third, the optimal rollup rate decreases in the shadow cost of capital and the capital requirement. The insurer lowers the rollup rate to reduce risk exposure when statutory capital is low.

When the shadow cost of capital is sufficiently high, the insurer offers mutual funds with no minimum return guarantees (i.e., $r_t = 0$). That is, the insurer exits the market for minimum return guarantees to avoid additional risk exposure from the sale of contracts. The general insight is that financial frictions affect contract characteristics and could even lead to market incompleteness in the extreme case.

The shadow cost of capital is not directly observed. However, reserve valuation $V_{t-1,t}$ (i.e., the option value of existing liabilities) can be measured empirically and is positively related to the shadow cost of capital. Therefore, we derive comparative statics for the optimal price and rollup rate with respect to reserve valuation. For a general demand function, equations (13) and (14) do not yield clean comparative statics because the demand elasticities could depend on the price and the rollup rate. For the purposes of obtaining analytical insights, we assume constant demand elasticities in the following corollary to Proposition 1. We refer to Appendix B for an example of a constant elasticity demand function.

COROLLARY 1: *If demand elasticities ϵ_P and ϵ_r are constant, the optimal price increases in reserve valuation (i.e., $\frac{\partial P_t}{\partial V_{t-1,t}} > 0$), and the optimal rollup rate decreases in reserve valuation (i.e., $\frac{\partial r_t}{\partial V_{t-1,t}} < 0$). Therefore, sales decrease in reserve valuation (i.e., $\frac{\partial Q_t}{\partial V_{t-1,t}} < 0$).*

Corollary 1 provides a narrative for the aggregate facts in Section II. Insurers suffered an adverse shock to risk-based capital as reserve valuation increased during the financial crisis. In addition, insurers could have made risk management more conservative in response to higher model uncertainty. As the shadow cost of capital increased, insurers raised fees to pass through a higher marginal cost. Insurers also lowered rollup rates or stopped offering minimum return guarantees entirely to reduce risk exposure. Higher fees and lower rollup rates make variable annuities less attractive to investors, explaining the decline in sales.

E. *Evidence from the Cross Section of Insurers*

We now provide some evidence from the cross section of insurers that is consistent with Corollary 1. We look for broad patterns at the insurer level that could be summarized by a simple scatter plot and leave more formal analysis at the contract level for Section IV. Depending on the contract characteristics of existing liabilities, different insurers could experience different shocks to reserve valuation during the financial crisis. Insurers that sold more generous guarantees prior to the financial crisis would have suffered larger increases in reserve valuation than those that sold less generous guarantees. In addition, insurers that sold more generous guarantees could have made risk management more conservative after the financial crisis as they learned that model uncertainty is higher than previously recognized. Thus, changes in reserve valuation should be negatively related to sales growth in the cross section of insurers.

The upper panel of Figure 5 is a scatter plot of sales growth versus the change in reserve valuation from 2007 to 2010. The linear regression line shows that sales growth is negatively related to the change in reserve valuation. On the bottom right are insurers like AXA and Genworth that essentially closed their variable annuity business as they suffered large increases in reserve valuation. On the left side of the figure are a cluster of six insurers (Fidelity Investments, MassMutual, New York Life, Northwestern, Ohio National, and Thrivent Financial) that did not offer a GLWB in 2007, which tends to be more generous than other types of guaranteed living benefits. Reserve valuation did not change much for these insurers because they sold less generous guarantees.

Insurers could relax a risk-based capital constraint by moving liabilities off balance sheet through reinsurance (Koijen and Yogo 2016). If insurers that suffered large increases in reserve valuation were in fact constrained, they should move variable annuity liabilities off balance sheet through reinsurance. The bottom panel of Figure 5 is a scatter plot of the change in percent of variable annuity reserves reinsured versus the change in reserve valuation from 2007 to 2010. The linear regression line shows that the change in percent of variable annuity reserves reinsured is positively related to the change in reserve valuation. On the one hand, AXA increased the share of variable annuity reserves reinsured by 64 percentage points as its reserve valuation increased by 12 percentage points from 2007 to 2010. On the other hand, the six insurers that did not offer a GLWB in 2007 did not experience any change in reserve valuation or reinsurance activity. This particular evidence is difficult to explain with an economic risk constraint alone and suggest that an important role for a risk-based capital constraint.

IV. Importance of Financial Frictions

Variation in fees across insurers and over time could come from supply- or demand-side effects. We need a model of variable annuity demand to disentangle these effects and to quantify the importance of financial frictions in explaining variable annuity supply. Therefore, we estimate a differentiated product demand system for the variable annuity market at the contract level, which provides an internally consistent framework to model market equilibrium and to decompose fees into markups versus marginal cost.

A. A Model of Variable Annuity Demand

A life-cycle model of consumption and portfolio choice is a fully structural approach to modeling variable annuity demand (Horneff, Maurer, Mitchell and Stamos 2009, Horneff, Maurer, Mitchell and Stamos 2010, Koijen, Nijman and Werker 2011). These models could explain the demand for variable annuities relative to other savings products, but they are not designed to explain demand across contracts differentiated by fees, rollup rates, and other characteristics. Therefore, we take a different approach and model variable annuity demand based on the random coefficients logit model (Berry, Levinsohn and Pakes 1995), which is a tractable and micro-founded model of product differentiation and market power.

Let $P_{i,t}$ be the annual fee on contract i in period t. Let $\mathbf{x}_{i,t}$ be a vector of observable characteristics of contract i in period t including the rollup rate, which are determinants of demand. Let $\xi_{i,t}$ be an unobserved (to the econometrician) characteristic of contract i in period t. The probability that an investor with realized preference parameters (α, β) buys contract i in period t is

$$q_{i,t}(\alpha, \beta) = \frac{\exp\{\alpha P_{i,t} + \beta' \mathbf{x}_{i,t} + \xi_{i,t}\}}{1 + \sum_{j=1}^{I} \exp\{\alpha P_{j,t} + \beta' \mathbf{x}_{j,t} + \xi_{j,t}\}}, \tag{15}$$

where I is the total number of contracts. If the investor does not buy a variable annuity, she buys an "outside asset" instead, which happens with probability $1 - \sum_{i=1}^{I} q_{i,t}(\alpha, \beta)$.

Let $F(\alpha, \beta)$ be the cumulative distribution function of the preference parameters. The coefficient on fees α is lognormally distributed, and the vector of coefficients β is normally and independently distributed. Integrating equation (15) over the distribution of investors, the market share for contract i in period t is

$$Q_{i,t} = \int q_{i,t}(\alpha, \beta)\, dF(\alpha, \beta). \tag{16}$$

The price elasticity of demand for contract i in period t is

$$-\frac{\partial \log(Q_{i,t})}{\partial \log(P_{i,t})} = \frac{P_{i,t}}{Q_{i,t}} \int -\alpha q_{i,t}(\alpha,\beta)(1 - q_{i,t}(\alpha,\beta))\, dF(\alpha,\beta). \tag{17}$$

B. Empirical Specification

Our estimation sample is all variable annuity contracts with guaranteed living benefits from 2005:1 to 2015:4. Because sales are at the contract level, we measure total annual fee as the sum of the annual base contract expense and the annual fee on the guaranteed living benefit. We measure the demand for outside assets as sales of open-end stock and bond mutual funds as well as variable annuity contracts without guaranteed living benefits, which are close substitutes to mutual funds.

The contract characteristics in our specification are the rollup rate, the number of investment options, and a dummy for whether the contract offers a guaranteed death benefit. The latter two characteristics capture the diversity or the complexity of options within contracts (Célérier and Vallée 2017). We also include the A.M. Best rating and insurer fixed effects to capture reputation or perceived quality of the insurer in the retail market. The unobserved characteristic $\xi_{i,t}$ in equation (15) captures other demand factors that are difficult to measure such as relative tax advantages. Finally, the intercept captures the attractiveness of variable annuities relative to the outside asset.

According to the model of variable annuity supply in Section III, the insurer optimally chooses the fee and the rollup rate, so they are jointly endogenous with the demand shock. We start with the usual identifying assumption that characteristics other than the fee and the rollup rate that enter demand are exogenous. Furthermore, we assume that reserve valuation and the share of variable annuity reserves reinsured are valid instruments that affect marginal cost, but they do not enter demand directly. Reserve valuation is a relevant instrument that is correlated with the fee and the rollup rate according to Corollary 1. Because our specification includes insurer fixed effects, the demand elasticities are identified from the time-series variation in the instruments within each insurer.

Our identifying assumption is valid under two scenarios. The first scenario is that risk-based capital regulation is overly conservative and that the insurer is more than adequately capitalized. Under this scenario, the shadow cost of capital represents pure regulatory frictions, which can only affect demand by driving up marginal cost and prices. The second scenario is that the insurer does not have adequate capital, and reserve valuation contains information about default probability that is not fully reflected in ratings (e.g., if ratings are slow to adjust). Under this scenario, the identifying assumption requires that investors face information acquisition costs so that demand depends only on readily available information such as ratings. Some investors may not bother to acquire information altogether because their claims are insured by the state guaranty funds (Deng, Leverty and Zanjani 2017).

In addition to reserve valuation and the share of variable annuity reserves reinsured, we use the square of these instruments as well as A.M. Best rating identify the variance of the random coefficients. Following the usual methodology, we estimate the random coefficients logit model by two-step generalized method of moments. We approximate the integral over the distribution of preference parameters through a simulation with 500 draws.

C. Estimated Model of Variable Annuity Demand

Table 3 reports the estimated mean and standard deviation of the random coefficients for the model of variable annuity demand. The mean coefficient on the fee is -3.35 with a standard error of 0.25. The standard deviation of the random coefficient on the fee is 0.75 and statistically significant. These estimates imply an average price elasticity of 11.2 with a standard deviation of 0.7 in 2007:4. The average price elasticity varies between 9 and 13 throughout the sample period. The coefficient on the rollup rate is 0.84 with a standard error of 0.54. The signs of these coefficients confirm that demand decreases in the fee and increases the rollup rate.

Demand also increases in the number of investment options, the availability of a guaranteed death benefit, and the A.M. Best rating. The coefficient on the number of investment options is 0.06 with a standard error of 0.02, and the coefficient on the dummy for guaranteed death benefit is 1.12 with a standard error of 0.31. The coefficient on the A.M. Best rating, which is standardized, is 1.07 with a standard error of 0.45. This means that a standard deviation increase in the rating increases demand by 107 percent.

Our preferred specification limits the random coefficients to the fee. For robustness, we have estimated a richer model in which the coefficients on the rollup rate or the A.M. Best rating are also random. However, the estimate of the standard deviation converged to zero or had large standard errors that indicated that the richer model is poorly identified. The identification problem arises from the fact that the variation in market shares can only identify a limited covariance structure for the random coefficients.

D. Marginal Cost

The estimated model of variable annuity demand implies an estimate of price elasticity for each contract, from which we can infer marginal cost through equation (13). A slight complication arises in taking equation (13) to the data. Equation (13) was derived assuming that the insurer offers only one contract, whereas actual insurers offer multiple contracts and presumably choose fees accounting for cross-price elasticities across contracts. Therefore, in Appendix C, we derive a more general version of equation (13) for a multi-product insurer and describe how to estimate marginal cost based on the estimated model of variable annuity demand.

Figure 6 reports the total annual fee and marginal cost of variable annuities with guaranteed living benefits from 2005:1 to 2015:4, averaged across contracts and weighted by sales. Marginal cost increased by 16 percent from 1.85 percent of account value in 2007:4 to 2.15 percent in 2009:2. Insurers passed through this cost increase to investors as the total annual fee increased from 2.04 to 2.38 percent of account value in the same period.

For contract i sold by insurer n in period t, we can rewrite marginal cost (13) in logarithms as

$$\log(\mathrm{MC}_{i,n,t} - 1) = \underbrace{\log(V_{i,n,t} - 1)}_{\text{option value}} + \underbrace{\log\left(1 + \frac{c_{n,t}\phi_{n,t}}{1 + c_{n,t}}\right)}_{\gamma_{n,t}}. \tag{18}$$

This equation provides a decomposition of marginal cost into the frictionless option value and insurer-time fixed effects $\gamma_{n,t}$. These fixed effects capture cross-sectional variation in the shadow cost of capital across insurers within each period.

Suppose that the frictionless option value depends on a vector $\mathbf{x}_{i,n,t}$ of contract characteristics that includes the rollup rate, the number of investment options, and a dummy for guaranteed death benefit. Then we can rewrite equation (18) as a panel regression model

$$\log(\mathrm{MC}_{i,n,t} - 1) = \beta' \mathbf{x}_{i,n,t} + \nu_{i,n,t} + \gamma_{n,t}, \tag{19}$$

where $\nu_{i,n,t}$ is the residual that represents unobserved contract characteristics. This model allows us to decompose the change in marginal cost into within-insurer variation along contract characteristics versus between-insurer variation due to the shadow cost of capital.

Table 4 reports estimates of the panel regression model (19). A coefficient of 0.048 for the main effect on the rollup rate means that marginal cost increases by 4.8 percent per one percentage point increase in the rollup rate. The interaction of the rollup rate with the year dummies are all negative, which means that the frictionless option value associated the rollup rate peaked in 2010 (i.e., the omitted year in the interactions). The number of investment options and the dummy for guaranteed death benefit, whose coefficients have smaller magnitude, are less important determinants of the frictionless option value.

Table 4 does not report the insurer-date fixed effects because they are too numerous to report, but Figure 7 summarizes their economic importance. The between-insurer variation is especially important during the financial crisis, explaining about 30 percent of the cross-sectional variation in marginal cost. The between-insurer variation is also important after 2012, explaining as much as 40 percent of the cross-sectional variation in marginal cost. This timing coincides with the higher reserve requirements that came into effect under Actuarial Guideline 43 as described in Section I. The persistence after the financial crisis is also consistent with a permanent change in an economic risk constraint as described in Section III. When interpreted through Proposition 1, Figure 7 confirms the importance of cross-sectional variation in the shadow cost of capital across insurers in explaining the cross section of fees.

V. Conclusion

The traditional insurance literature focuses on products such as life annuities, life insurance, and health insurance that insure idiosyncratic risk. This literature shows that informational frictions lead to variation in prices and contract characteristics across different types of individuals (Finkelstein and Poterba 2004). However, the main business of life insurers is now savings products that insure market risk through minimum return guarantees. Although we focus on the U.S. because of data availability, guaranteed return products are important globally and represent a major share of life insurer liabilities in Austria, Denmark, France, Germany, Netherlands, and Sweden (European Systemic Risk Board 2015, Hombert and Lyonnet 2017). The key frictions in this market are financial frictions and market power, which lead to variation in prices and contract characteristics across insurers and over time.

This paper also has important implications for the literature on financial intermediation. Mutual funds are traditionally pure pass-through institutions with no risk mismatch. However, an important and growing part of the mutual fund sector that is sold through life insurers is subject to risk mismatch through minimum return guarantees. In that sense, life insurers are becoming more like pension funds because they have risky assets and guaranteed liabilities. The persistent under-funding of pension funds may foreshadow similar problems for life insurers in the future (Novy-Marx and Rauh 2011). The fact that life insurers are publicly traded and subject to market discipline could lead to additional challenges that are not present for under-funded pension funds.

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Table 1: Summary Statistics for the Variable Annuity Market

	VA liabilities			
Year	Billion $	Percent of total liabilities	Number of insurers	Reserve valuation (percent)
2005	1,091	36	45	0.9
2006	1,296	39	46	0.8
2007	1,461	42	44	0.9
2008	1,068	34	43	4.1
2009	1,170	34	42	3.4
2010	1,325	36	42	2.5
2011	1,342	35	41	4.9
2012	1,416	36	38	3.9
2013	1,590	37	40	1.8
2014	1,584	37	38	2.2
2015	1,486	34	38	2.9

Reserve valuation is the ratio of gross amount of reserves to total related account value.

Table 2: Top Insurers by Variable Annuity Liabilities

Insurer	VA liabilities in 2007 (billion $)	Change from 2007 to 2008: Reserve valuation (percent)	Change from 2007 to 2008: Reserves (percent of equity)
Metropolitan Life	143	3.2	7
AXA	140	8.2	106
Prudential	122	1.4	13
Voya	121	4.2	42
Hartford	120	2.9	13
AIG	105	0.8	1
Lincoln	97	1.3	15
John Hancock	95	1.8	27
Ameriprise	81	1.0	13
Aegon	63	7.3	29
Pacific Life	56	1.5	13
Nationwide	46	1.7	14
Jackson National	33	3.6	12
Sun Life	29	4.0	36
Allianz	23	5.3	35
New York Life	19	2.2	2
Genworth	17	0.5	1
Northwestern	12	0.2	0
Ohio National Life	11	2.1	21
Fidelity Investments	10	1.0	8
Security Benefit	10	1.3	12
MassMutual	6	1.7	0
Financial for Lutherans	3	0.4	5

Reserve valuation is the ratio of gross amount of reserves to total related account value. The change in gross amount of reserves on variable annuities is reported as a percent of total equity in 2007. The sample includes all insurers with at least $1 billion of variable annuity sales in 2007.

Table 3: Estimated Model of Variable Annuity Demand

Variable	Mean	Standard deviation
Fee	-3.35	0.75
	(0.25)	(0.35)
Rollup rate	0.84	
	(0.54)	
Investment options	0.06	
	(0.02)	
Guaranteed death benefit	1.12	
	(0.31)	
A.M. Best rating	1.07	
	(0.45)	
Observations	9,141	

The random coefficients logit model of demand is estimated by two-step generalized method of moments. The specification includes insurer fixed effects whose coefficients are not reported for brevity. The instruments are log reserve valuation, share of variable annuity reserves reinsured, and the squares of these variables and A.M. Best rating. Heteroscedasticity-robust standard errors are reported in parentheses. The sample includes all contracts with guaranteed living benefits from 2005:1 to 2015:4.

Table 4: Cross-Sectional Variance Decomposition of Marginal Cost

Variable	Rollup rate	Investment options	Guaranteed death benefit
Main effect	0.048	0.002	0.028
	(0.003)	(0.001)	(0.032)
Interaction with			
2005	-0.036	-0.001	-0.091
	(0.009)	(0.001)	(0.040)
2006	-0.030	0.001	-0.051
	(0.008)	(0.001)	(0.041)
2007	-0.035	0.001	-0.065
	(0.006)	(0.001)	(0.039)
2008	-0.024	0.002	-0.009
	(0.006)	(0.001)	(0.041)
2009	-0.012	-0.002	0.038
	(0.005)	(0.001)	(0.044)
2011	-0.011	0.002	0.057
	(0.008)	(0.001)	(0.049)
2012	-0.029	-0.001	0.152
	(0.012)	(0.002)	(0.057)
2013	-0.035	-0.003	0.097
	(0.016)	(0.002)	(0.043)
2014	-0.042	-0.002	0.083
	(0.013)	(0.002)	(0.042)
2015	-0.027	0.000	0.075
	(0.012)	(0.001)	(0.061)

Log marginal cost is regressed onto the rollup rate, the number of investment options, a dummy for guaranteed death benefit, and their interaction with year fixed effects. The omitted year is 2010. The specification also includes insurer-date fixed effects whose coefficients are not reported for brevity. Heteroscedasticity-robust standard errors are reported in parentheses. The sample includes all contracts with guaranteed living benefits from 2005:1 to 2015:4.

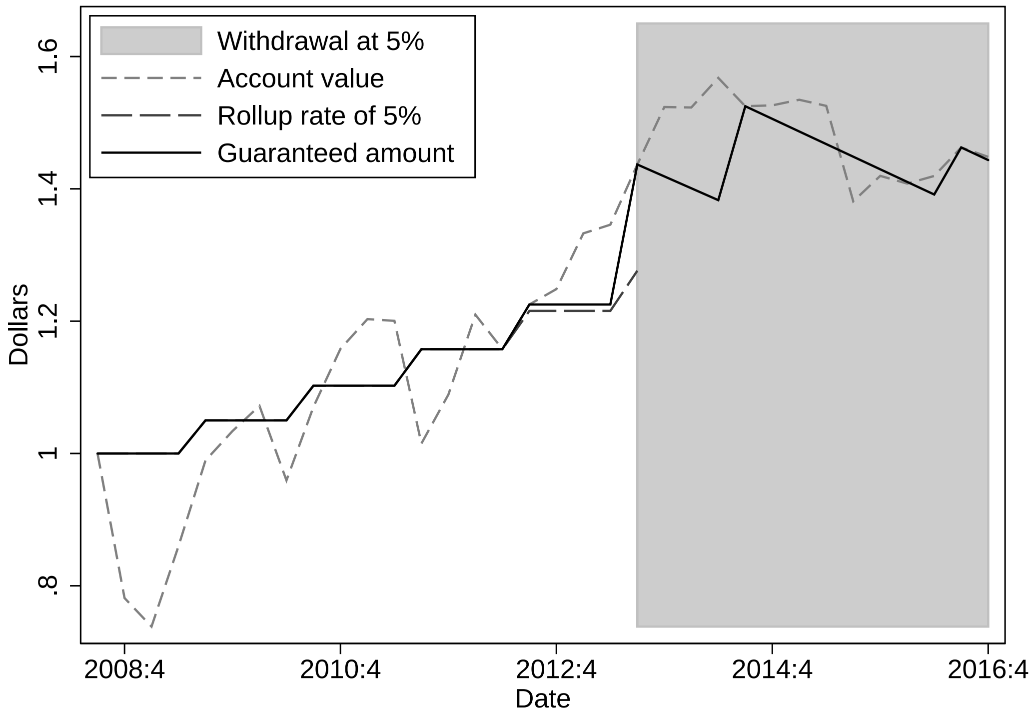


Figure 1: Example of a Guaranteed Living Withdrawal Benefit

This example shows the evolution of account value and the guaranteed amount for MetLife Series VA with GLWB from 2008:3 to 2016:4. The investment option is the American Funds Growth Allocation Portfolio. The investor is assumed to annually withdraw 5 percent of the highest guaranteed amount after 2013:3. For simplicity, this example abstracts from the impact of fees on account value and the guaranteed amount.

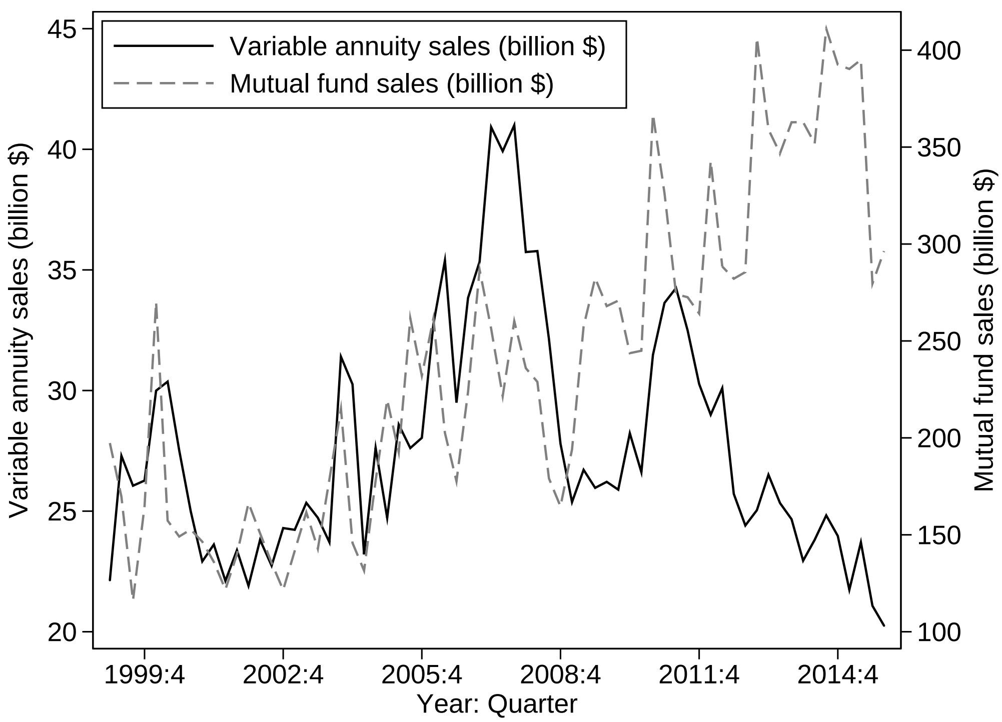


Figure 2: Variable Annuity Sales

The left axis reports quarterly sales of variable annuities across all contracts from 1999:1 to 2015:4. The right axis reports the aggregate sales of U.S. open-end stock and bond mutual funds (excluding money market funds and funds of funds).

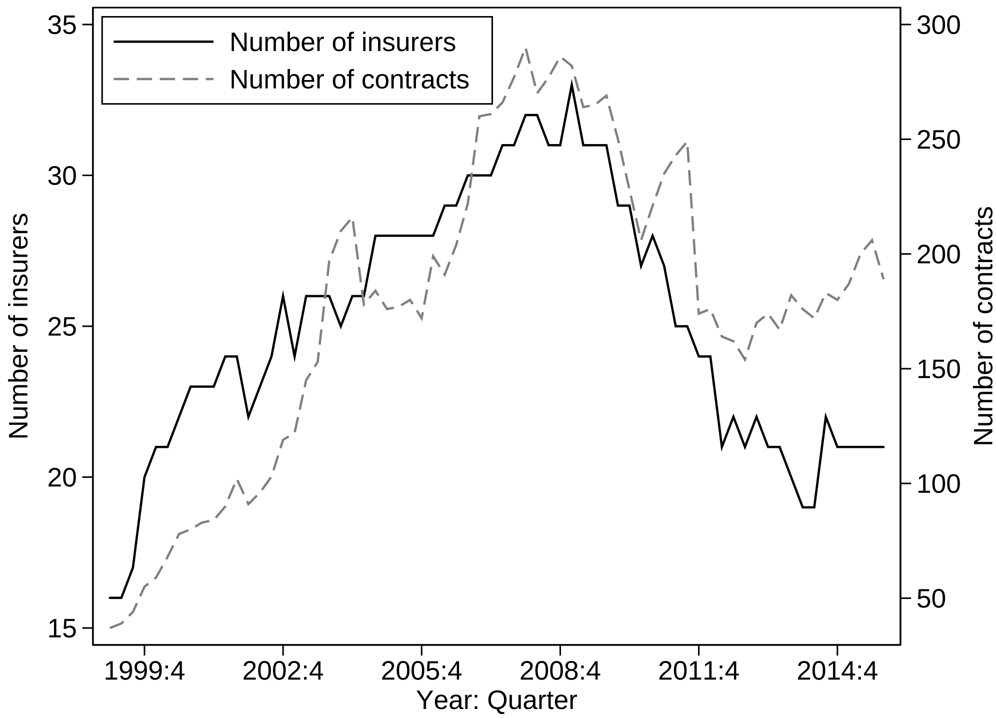
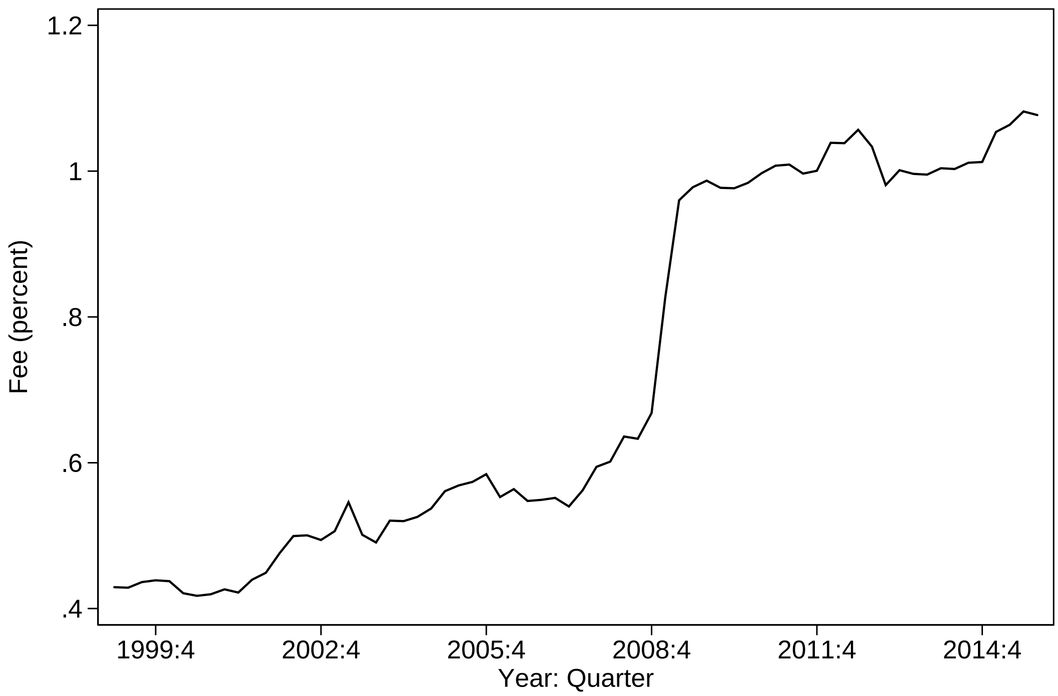


Figure 3: Number of Insurers and Contracts Offering Guaranteed Living Benefits
The sample includes all contracts with guaranteed living benefits from 1999:1 to 2015:4.



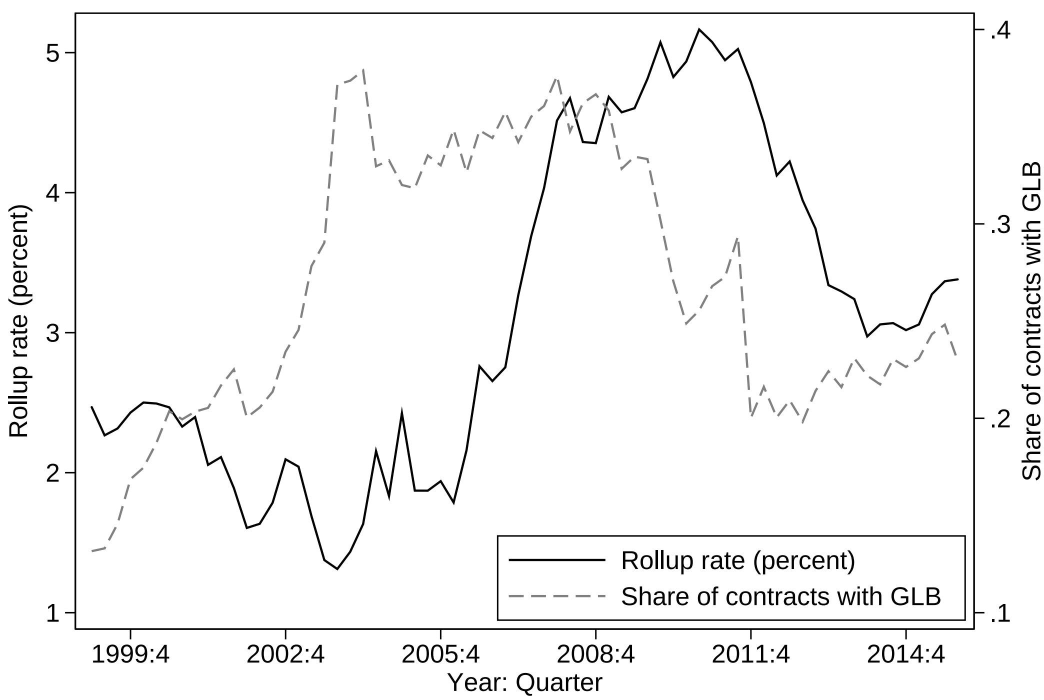


Figure 4: Fees and Rollup Rates on Guaranteed Living Benefits

The upper panel reports the average annual fee (weighted by sales) on open guaranteed living benefits. The lower panel reports the average rollup rate (weighted by sales) on open guaranteed living benefits and the share of contracts with guaranteed living benefits (GLB). The sample includes all contracts with guaranteed living benefits from 1999:1 to 2015:4.

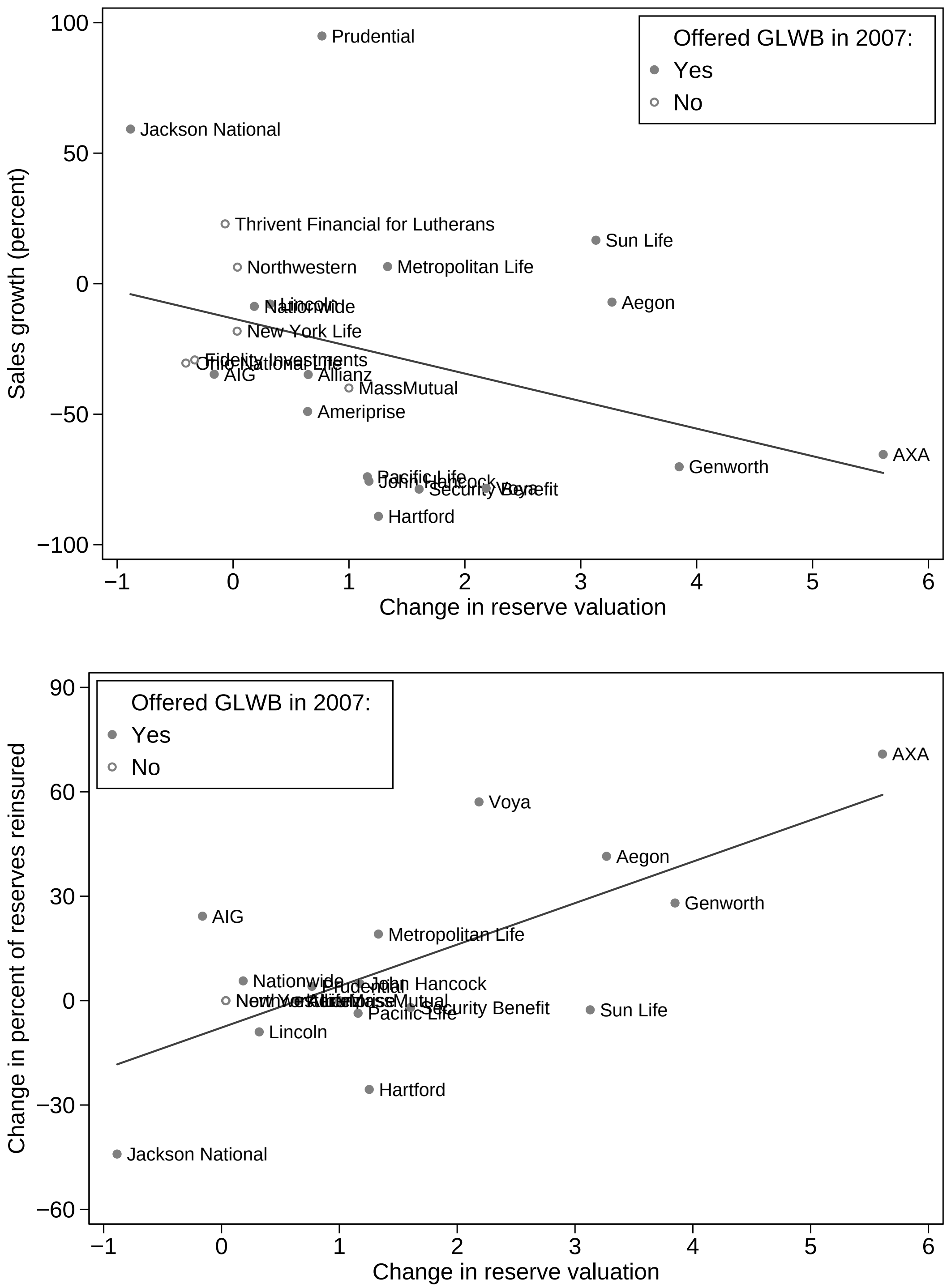


Figure 5: Impact of Change in Reserve Valuation across Insurers

The upper panel is a scatter plot of sales growth versus the change in reserve valuation from 2007 to 2010. The lower panel is a scatter plot of the change in percent of reserves reinsured versus the change in reserve valuation from 2007 to 2010. Both panels report a linear regression line through the scatter points. The sample includes all insurers with at least $1 billion of variable annuity sales in 2007.

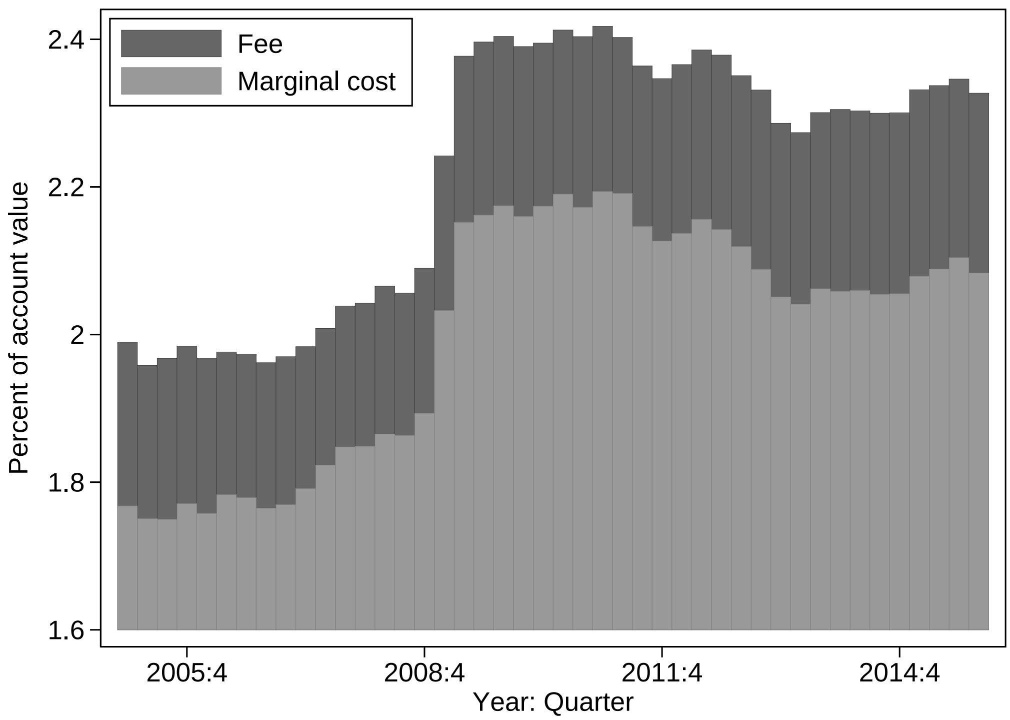


Figure 6: Fees and Marginal Cost

The optimal pricing equation is used to estimate marginal cost by contract and date. Marginal cost is then averaged across contracts (weighted by sales) by date. The sample includes all contracts with guaranteed living benefits from 2005:1 to 2015:4.

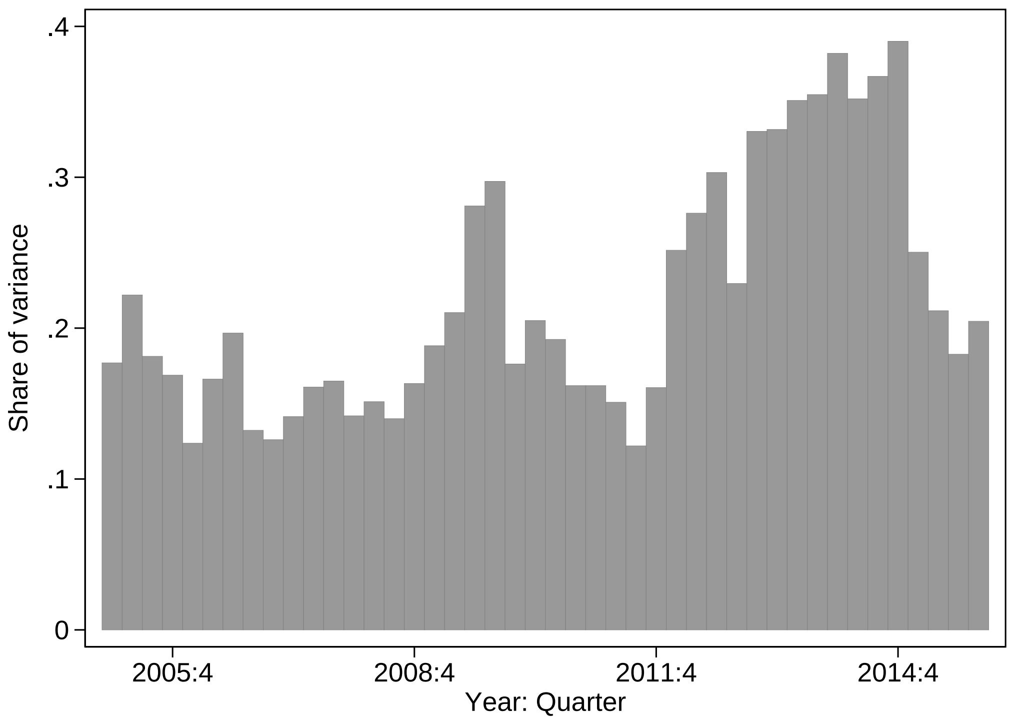


Figure 7: Variation in Marginal Cost across Insurers

This figure reports the share of cross-sectional variance in marginal cost explained by insurer fixed effects, implied by the regression model in Table 4. The sample includes all contracts with guaranteed living benefits from 2005:1 to 2015:4.

Appendix A. A Lower Bound on Fees

The guaranteed amount at the end of the accumulation period can be written as a sum of the cumulative rollup rate and the payoff of a call option. Thus, we derive a lower bound on fees based only on the rollup rate to assess whether an annual fee such as 1.8 percent on MetLife Series VA with GLWB is justified by a rollup rate of 5 percent. We show that the implied fee based on the rollup rate is actually negative because the time value of money during the withdrawal period more than offsets the high rollup rate during the accumulation period. Therefore, the high fees cannot be explained by the high rollup rate and must instead be attributed to the call option value, market power, or financial frictions.

Following the notation in the paper, let S_t be the mutual fund price per share in period t. Let $M_{t,t+s}$ be a strictly positive stochastic discount factor that discounts a payoff in period $t+s$ to its price in period t. Then the term structure of riskless interest rates is given by the usual pricing formula $Y^s_{t,t+s} = \mathbb{E}_t[M_{t,t+s}]^{-1}$. That is, $Y_{t,t+s}$ is the gross yield on a zero-coupon bond of maturity s in period t.

Consider a GLWB with an annual fee v per dollar of account value, a gross annual rollup rate of r, an annual withdrawal rate of w, an accumulation period of T_a years, and a withdrawal period of T_w years. For simplicity, we assume that the withdrawal rate, the accumulation period, and the withdrawal period are all fixed. We also assume that there are no step-ups during the withdrawal period. For a contract issued in period t, the guaranteed amount at the end of the accumulation period in period $t + T_a$ is

$$X_{t,t+T_a} = \max\left\{r^{T_a}, \frac{S_{t+T_a}}{S_t}\right\} = r^{T_a} + \underbrace{\max\left\{0, \frac{S_{t+T_a}}{S_t} - r^{T_a}\right\}}_{\text{call option}}. \tag{A1}$$

For each dollar of account value, the zero-profit condition equates one plus the present value of fees to the present value of guaranteed income:

$$1 + \mathbb{E}_t\left[\sum_{s=1}^{T_a} M_{t,t+s}\frac{vS_{t+s}}{S_t}\right] = 1 + T_a v = \mathbb{E}_t\left[\sum_{s=1}^{T_w} M_{t,t+T_a+s} w X_{t,t+T_a}\right]. \tag{A2}$$

Because $X_{t,t+T_a} \geq r^{T_a}$, a lower bound on fees based only on the rollup rate is

$$v \geq \frac{1}{T_a}\left(\sum_{s=1}^{T_w} \frac{wr^{T_a}}{Y^{T_a+s}_{t,t+T_a+s}} - 1\right). \tag{A3}$$

This equation shows that the rollup rate in the numerator is offset by the time value of money in the denominator because the guaranteed amount is only payable as annual income over T_w years. We show the empirical relevance of this issue by computing the lower bound on fees, using the historical zero-coupon Treasury yield curve (Gürkaynak, Sack and Wright 2007).

Figure A1 reports the lower bound on fees for an annual rollup rate of 5 percent, an annual withdrawal rate of 5 percent, and a withdrawal period of 20 years. To see the sensitivity of the results to the accumulation period, the figure reports the lower bound for an accumulation period of 10 and 20 years. The lower bound on fees is negative for most of the sample period and becomes positive only after 2011:4 for the 20-year accumulation period. This means that the high fees cannot be explained by a rollup rate of 5 percent and must instead be attributed to the call option value, market power, or financial frictions.

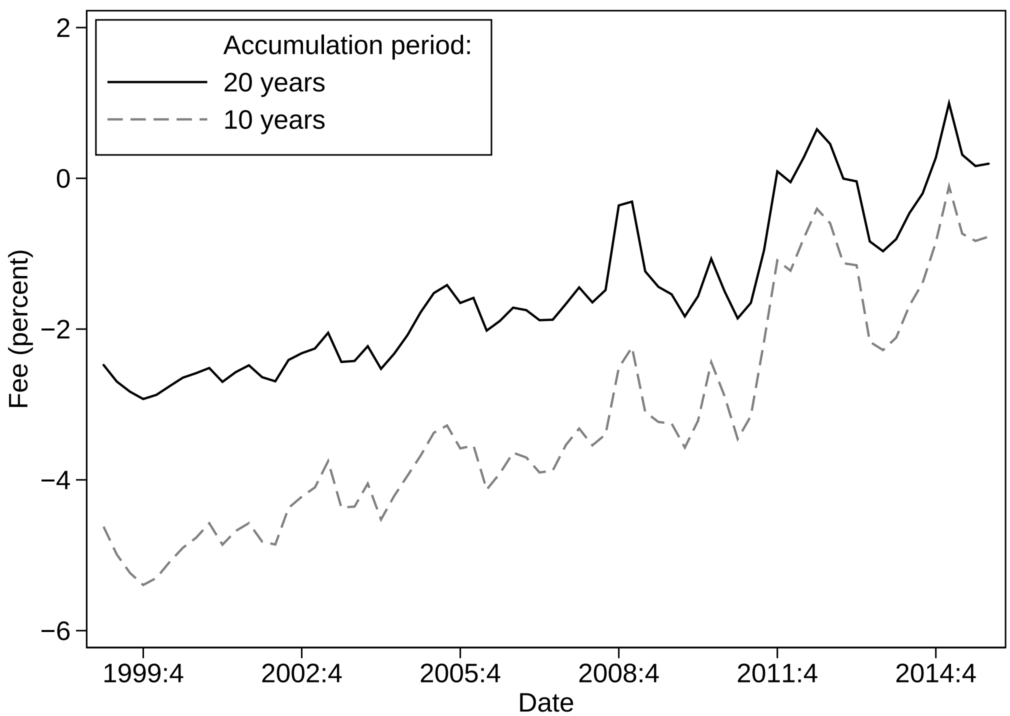


Figure A1: Lower Bound on Fees Based on the Rollup Rate
The lower bound on fees is based on an annual rollup rate of 5 percent, an annual withdrawal rate of 5 percent, and a withdrawal period of 20 years. The calculation uses an average of the zero-coupon Treasury yield curve within each quarter from 1999:1 to 2015:4, assuming that the yield curve is flat beyond 30 years.

Appendix B. Proofs

Proof of Proposition 1: Substituting equations (6), (7), and (8) into equation (9), we have

$$K_t = R_{K,t}K_{t-1} + (P_t - V_{t,t} - \phi_t(V_{t,t} - 1))Q_t, \tag{B1}$$

where

$$R_{K,t} = \frac{A_{t-1}}{K_{t-1}}R_{A,t} - \frac{(1+\phi_t)L_{t-1}}{K_{t-1}}\frac{V_{t-1,t} - S_t/S_{t-1}}{V_{t-1,t-1} - 1} \tag{B2}$$

is the return on statutory capital. The first-order condition for the optimal price is

$$\begin{aligned} \frac{\partial J_t}{\partial P_t} =& \frac{\partial(P_t - V_{t,t})Q_t}{\partial P_t} + c_t\frac{\partial K_t}{\partial P_t} \\ =& Q_t + (P_t - V_{t,t})\frac{\partial Q_t}{\partial P_t} + c_t\left(Q_t + (P_t - V_{t,t} - \phi_t(V_{t,t} - 1))\frac{\partial Q_t}{\partial P_t}\right) \\ =& (1+c_t)Q_t + ((1+c_t)(P_t - V_{t,t}) - c_t\phi_t(V_{t,t} - 1))\frac{\partial Q_t}{\partial P_t} = 0. \end{aligned} \tag{B3}$$

Rearranging, we have

$$P_t = -\left(\frac{\partial Q_t}{\partial P_t}\right)^{-1} Q_t + V_{t,t} + \frac{c_t\phi_t(V_{t,t} - 1)}{1+c_t}. \tag{B4}$$

Equation (13) follows from the definition of price elasticity of demand.

At an interior optimum, the first-order condition for the optimal rollup rate is

$$\begin{aligned} \frac{\partial J_t}{\partial r_t} =& \frac{\partial(P_t - V_{t,t})Q_t}{\partial r_t} + c_t\frac{\partial K_t}{\partial r_t} \\ =& -\frac{\partial V_{t,t}}{\partial r_t}Q_t + (P_t - V_{t,t})\frac{\partial Q_t}{\partial r_t} \\ &+ c_t\left(-\frac{\partial V_{t,t}}{\partial r_t}(1+\phi_t)Q_t + (P_t - V_{t,t} - \phi_t(V_{t,t} - 1))\frac{\partial Q_t}{\partial r_t}\right) \\ =& -\frac{\partial V_{t,t}}{\partial r_t}(1 + c_t(1+\phi_t))Q_t + ((1+c_t)(P_t - V_{t,t}) - c_t\phi_t(V_{t,t} - 1))\frac{\partial Q_t}{\partial r_t} \\ =& -\frac{\partial V_{t,t}}{\partial r_t}(1 + c_t(1+\phi_t))Q_t - (1+c_t)Q_t\left(\frac{\partial Q_t}{\partial P_t}\right)^{-1}\frac{\partial Q_t}{\partial r_t} = 0, \end{aligned} \tag{B5}$$

where the last line follows from substituting equation (B3). Rearranging, we have

$$r_t = \left(\frac{\partial V_{t,t}}{\partial r_t}\right)^{-1} \frac{\epsilon_{r,t}}{\epsilon_{P,t}} \frac{P_t(1+c_t)}{1+c_t(1+\phi_t)}. \tag{B6}$$

Equation (14) follows from this equation and the fact that equation (13) implies

$$\frac{P_t(1+c_t)}{1+c_t(1+\phi_t)} = \left(1 - \frac{1}{\epsilon_{P,t}}\right)^{-1} \left(V_{t,t} - \frac{c_t \phi_t}{1+c_t(1+\phi_t)}\right). \tag{B7}$$

We assume constant demand elasticities in Corollary 1. Before proving the result, we give an example of a demand function with constant demand elasticities to show that our assumption is compatible with the oligopolistic market structure. Let the elasticity of demand for contracts sold by insurer i to the price and the rollup rate of insurer j be $\epsilon_P(i,j) = -\frac{\partial \log(Q_{i,t})}{\partial \log(P_{j,t})}$ and $\epsilon_r(i,j) = \frac{\partial \log(Q_{i,t})}{\partial \log(r_{j,t})}$, respectively. The demand function

$$\log(Q_{i,t}) = \alpha_i - \sum_{j=1}^{I} \epsilon_P(i,j) \log(P_{j,t}) + \sum_{j=1}^{I} \epsilon_r(i,j) \log(r_{j,t}) \tag{B8}$$

has constant demand elasticities. The budget constraint $\sum_{i=1}^{I} P_{i,t} Q_{i,t} = 1$ implies that the cross-price elasticities must satisfy the restrictions

$$P_{i,t} Q_{i,t} - \sum_{j=1}^{I} P_{j,t} Q_{j,t} \epsilon_P(j,i) = 0, \tag{B9}$$

$$\sum_{j=1}^{I} P_{j,t} Q_{j,t} \epsilon_r(j,i) = 0. \tag{B10}$$

Proof of Corollary 1: The partial derivative of price with respect to reserve valuation is

$$\begin{aligned} \frac{\partial P_t}{\partial V_{t-1,t}} &= -\left(1 - \frac{1}{\epsilon_P}\right)^{-1} \frac{\phi_t(V_{t,t}-1)}{(1+c_t)^2} \frac{(1+\phi_t)L_{t-1}}{V_{t-1,t-1}-1} \frac{\partial c_t}{\partial K_t} \\ &= \left(1 - \frac{1}{\epsilon_P}\right)^{-1} \frac{\phi_t(V_{t,t}-1)}{(1+c_t)^2} \frac{(1+\phi_t)L_{t-1}}{V_{t-1,t-1}-1} \frac{\partial^2 C_t}{\partial K_t^2} > 0. \end{aligned} \tag{B11}$$

The partial derivative of the rollup rate with respect to reserve valuation is

$$\text{(B12)} \qquad \frac{\partial r_t}{\partial V_{t-1,t}} = \left(\frac{\partial V_{t,t}}{\partial r_t}\right)^{-1} \frac{\epsilon_r}{\epsilon_P - 1} \frac{\phi_t}{(1 + c_t(1+\phi_t))^2} \frac{(1+\phi_t)L_{t-1}}{V_{t-1,t-1} - 1} \frac{\partial c_t}{\partial K_t}$$
$$= -\left(\frac{\partial V_{t,t}}{\partial r_t}\right)^{-1} \frac{\epsilon_r}{\epsilon_P - 1} \frac{\phi_t}{(1 + c_t(1+\phi_t))^2} \frac{(1+\phi_t)L_{t-1}}{V_{t-1,t-1} - 1} \frac{\partial^2 C_t}{\partial K_t^2} < 0.$$

By the chain rule, the partial derivative of sales with respect to reserve valuation is

$$\text{(B13)} \qquad \frac{\partial Q_t}{\partial V_{t-1,t}} = \frac{\partial Q_t}{\partial P_t} \frac{\partial P_t}{\partial V_{t-1,t}} + \frac{\partial Q_t}{\partial r_t} \frac{\partial r_t}{\partial V_{t-1,t}} < 0.$$

Appendix C. Optimal Pricing for a Multi-Product Insurer

Let $\mathbf{1}$ be a vector of ones, $\mathbf{I}$ be an identity matrix, and $\text{diag}(\cdot)$ be a diagonal matrix (e.g., $\text{diag}(\mathbf{1}) = \mathbf{I}$). A multi-product insurer sets a vector of variable annuity prices $\mathbf{P}_t$ to maximize firm value:

$$\text{(C1)} \qquad J_t = (\mathbf{P}_t - \mathbf{V}_{t,t})' \mathbf{Q}_t - C_t,$$

which generalizes equation (11). The first-order condition for the optimal price is

$$\text{(C2)} \qquad \frac{\partial J_t}{\partial \mathbf{P}_t} = \frac{\partial (\mathbf{P}_t - \mathbf{V}_{t,t})' \mathbf{Q}_t}{\partial \mathbf{P}_t} + c_t \frac{\partial K_t}{\partial \mathbf{P}_t}$$
$$= \mathbf{Q}_t + \frac{\partial \mathbf{Q}_t'}{\partial \mathbf{P}_t} (\mathbf{P}_t - \mathbf{V}_{t,t}) + c_t \left(\mathbf{Q}_t + \frac{\partial \mathbf{Q}_t'}{\partial \mathbf{P}_t} (\mathbf{P}_t - \mathbf{V}_{t,t} - \phi_t (\mathbf{V}_{t,t} - \mathbf{1})) \right)$$
$$= (1 + c_t) \mathbf{Q}_t + \frac{\partial \mathbf{Q}_t'}{\partial \mathbf{P}_t} ((1 + c_t)(\mathbf{P}_t - \mathbf{V}_{t,t}) - c_t \phi_t (\mathbf{V}_{t,t} - \mathbf{1})) = 0.$$

Rearranging this equation, we have

$$\text{(C3)} \qquad \mathbf{P}_t = -\left(\frac{\partial \mathbf{Q}_t'}{\partial \mathbf{P}_t}\right)^{-1} \mathbf{Q}_t + \underbrace{\mathbf{V}_{t,t} + \frac{c_t \phi_t (\mathbf{V}_{t,t} - \mathbf{1})}{1 + c_t}}_{\text{marginal cost}}.$$

That is, the vector of optimal prices are the sum of marginal cost and markups that depend on the matrix of price elasticities across contracts that the insurer offers.

For the random coefficients logit model, the demand vector is

$$\text{(C4)} \qquad \mathbf{Q}_t = \int \mathbf{q}_t(\alpha, \beta) \, dF(\alpha, \beta),$$

and the matrix of price elasticities is

$$\text{(C5)} \qquad \frac{\partial \mathbf{Q}_t'}{\partial \mathbf{P}_t} = \int -\alpha(\text{diag}(\mathbf{q}_t(\alpha,\beta)) - \mathbf{q}_t(\alpha,\beta)\mathbf{q}_t(\alpha,\beta)')\; dF(\alpha,\beta).$$

Thus, given the estimated model of variable annuity demand, we can infer marginal cost through equation (C3).