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ABSTRACT

We revisit the causes, welfare consequences, and policy implications of the dispersion in households' labor market outcomes using a model with uninsurable risk, incomplete asset markets, and home production. Accounting for home production amplifies welfare-based differences across households meaning that inequality is larger than we thought. Home production does not offset differences that originate in the market sector because productivity differences in the home sector are significant and the time input in home production does not covary with consumption expenditures and wages in the cross section of households. The optimal tax system should feature more progressivity taking into account home production.

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1 Introduction

A substantial body of research examines the causes, welfare consequences, and policy implications of the pervasive dispersion across households in their labor market outcomes.[1] The literature trying to understand the dispersion in wages, hours worked, and consumption expenditures across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. It is well known, however, that households spend roughly half as much time in home production activities such as child care, shopping, and cooking as in the market.

While it is understood that home production of goods and services introduces, on average, a gap between households' consumption as recorded in official statistics and their standards of living, little is known about how differences in home production across households affect inequality in standards of living. A priori there are good reasons why home production can change the inferences economists draw from observing dispersion in labor market outcomes. To the extent that households are willing to substitute between market expenditures and time in the production of goods and services, home production will tend to compress welfare differences that originate in the market sector. However, to the extent that household differences in the home sector remain uninsurable and are large relative to the market sector, the home sector itself may emerge as an additional source of welfare differences across households.

We show that incorporating home production in a model with uninsurable risk and incomplete asset markets changes the inferred sources of heterogeneity across households, alters meaningfully the welfare consequences of dispersion, and leads to different policy conclusions. Surprisingly, we infer that inequality across households is larger than what one would infer without incorporating home production.[2] We reach this conclusion because, for households of all ages, productivity

[1]See Heathcote, Perri, and Violante (2010) and Attanasio and Pistaferri (2016) for empirical regularities on household heterogeneity in labor market outcomes.

[2]We use the term dispersion to refer to the variation in observed outcomes (such as time allocation, consumption expenditures, and wages) or inferred sources of heterogeneity (such as permanent or transitory productivity and taste shifters). We use the term inequality to refer to the mapping from dispersion to measures that capture welfare differences across households.

differences in the home sector are large and the time input in home production does not covary negatively with consumption expenditures and wages in the cross section of households. Thus, home production does not offset differences that originate in the market sector. Rather, home production amplifies these differences.

We develop our findings using a general equilibrium model with home production, heterogeneous households that face idiosyncratic risk, and incomplete asset markets. In the spirit of Ghez and Becker (1975), households produce goods with a technology that uses as inputs both expenditures and time. In the home sector, households are heterogeneous with respect to their disutility of work in some activities and with respect to their productivity in other activities. Home production is not tradeable and there are no assets that households can purchase to explicitly insure against differences that originate in the home sector. In the market sector, households are also heterogeneous with respect to their disutility of work and their productivity. The structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences. We retain tractability and prove identification by extending the no-trade result with respect to certain assets for the one-sector model of Heathcote, Storesletten, and Violante (2014) to our model embedding multiple sectors. Therefore, we can characterize the allocations of time and consumption goods in closed form without simultaneously solving for the wealth distribution.

At the core of our approach lies an observational equivalence theorem that allows us to compare our model with home production to a nested model without home production. The observational equivalence theorem states that both models account perfectly for any given cross-sectional data on three observables: consumption expenditures, time spent working in the market sector, and market productivity (wages). However, the inferred sources of heterogeneity that generate the data and inequality will in general differ between the two models. It is essential for our purposes that the two models are observationally equivalent because any differences between the two models is exclusively driven by structural factors and not by their ability to account for cross-sectional data on labor market outcomes. We first infer heterogeneity in market productivity and disutility of work such that the allocations generated by the model without home production match the cross-sectional data on the three observables. Then, we infer the sources of heterogeneity such that the allocations generated by the model with home production match the same cross-sectional data and, additionally, time spent on home activities that are subject to either productivity or preference differences.

To investigate how incorporating home production changes our inferences, we apply our observational equivalence theorem to U.S. data between 1995 and 2016. We use data on consumption expenditures, time spent on the market sector, and market productivity from the Consumption Expenditure Survey (CEX). The CEX does not contain information on time spent on home production. To overcome this problem, we use data from the American Time Use Survey (ATUS) to impute individuals' time spent on home production based on observables that are common between the two surveys. We allow households to have different preferences over time uses such as cooking and cleaning because we find that these activities map closest to occupations that are intensive in manual skills. By contrast, other time uses such as child care and nursing are less intensive in manual skills and we allow households to have different productivities in them.

The key result of our analysis is that the world is more unequal than we thought when we take into account home production. We arrive at our conclusion using four ways to map dispersion in labor market outcomes into welfare-based measures of inequality. First, the standard deviation of equivalent variation across households is roughly 15 percent larger when we incorporate home production. Second, equalizing marginal utilities across households requires transfers with a standard deviation that is roughly 30 percent higher in the model with home production than in the model without home production. Third, an unborn household is willing to sacrifice up to 13 percent of lifetime consumption in order to eliminate heterogeneity in an environment with home production, compared to 6 percent in an environment without home production. Finally, taking into account home production, a utilitarian government would choose a more progressive tax system. For example, a household earning 200,000 dollars would face an average tax rate of 21 percent with home production, compared to 13 percent without home production. One way to understand our inequality result is in terms of the distinction between consumption and expenditures emphasized by Aguiar and Hurst (2005). We find that market expenditures are less dispersed than the market value of total consumption which, in addition to market expenditures, includes the market value of time spent on home production.

Heterogeneity in home productivity rather than disutility of work is essential in amplifying inequality across households. If there was only preference heterogeneity in the home sector, there would be no significant difference in inequality between the model with and the model without home production. Our inference of home productivity is based on an intra-period optimality condition which requires households to consume more in their more productive sector and implies a log-linear relationship between home productivity and three observables (market expenditures, time spent on home production, and market productivity). Home productivity cumulates the variances of these three observables because the covariation between them is relatively small. As a result, we find that significant productivity differences at home across households.

Our results are robust to a battery of sensitivity checks. First, our conclusions are robust to the estimated values of the elasticity of substitution across sectors, the parameter that governs the Frisch elasticity of labor supply, and the progressivity of the tax system. Second, our results apply separately within subgroups of households defined by their age, marital status, number of children, the presence of a working spouse, and education levels. Third, our conclusions are robust to measures of expenditures that range from narrow (food) to broad (total spending including durables). Fourth, our results are robust to even large amounts of measurement error in market expenditures, market hours, and home hours. Fifth, we examine four alternative datasets in which we do not need to impute home production time because they contain information on both expenditures and time use. We confirm our results in the Panel Study of Income Dynamics (PSID) with food expenditures, in a version of the PSID with expanded consumption categories, in a dataset from Japan, and in a dataset from the Netherlands.

There is an extensive literature that examines how non-separabilities and home production affect consumption and labor supply either over the business cycle (Benhabib, Rogerson, and

Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013) or over the life-cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007; Dotsey, Li, and Yang, 2014). In these papers, home production provides a smoothing mechanism against differences that originate in the market sector if households are sufficiently willing to substitute expenditures with time. Our conclusions for the role of home production in understanding cross-sectional patterns differ from this literature because in the data we find that time in home production is not negatively correlated with wages and consumption expenditures in the cross section of households. By contrast, an assumption underlying the business cycle and life-cycle literatures is that decreases in the opportunity cost of time and in consumption expenditures are associated with substantial increases in time spent on home production.

Even though the home production literature has emphasized shocks in the home sector in order to generate higher volatility in labor markets and labor wedges (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Karabarbounis, 2014), little is known about cross-sectional differences in home productivity and tastes. We develop a methodology to infer productivity and preference heterogeneity in the home sector. Our conclusion is that these sources of heterogeneity are important in terms of generating cross-sectional and life-cycle patterns of expenditures and time allocation.

The literature on incomplete markets has started to incorporate home production and non-separabilities into models. Kaplan (2012) argues that involuntary unemployment and non-separable preferences allow an otherwise standard model with self-insurance to account for the variation of market hours over the life-cycle. Blundell, Pistaferri, and Saporta-Eksten (2016) examine consumption inequality in a model in which shocks can also be insured within the family and preferences for hours are non-separable across spouses. Blundell, Pistaferri, and Saporta-Eksten (2018) incorporate child care into a life-cycle partial equilibrium model of consumption and family labor supply. Their paper aims to understand the responsiveness of consumption and time use to transitory and permanent wage shocks and, unlike our paper, it does not quantify the extent to which home production affects inequality.

Another related literature addresses consumption inequality. Earlier work (Deaton and Paxson, 1994; Gourinchas and Parker, 2002; Storesletten, Telmer, and Yaron, 2004; Aguiar and Hurst, 2013) has examined the drivers of life-cycle consumption inequality and their welfare consequences. More recent work focuses on the increase of consumption inequality (Krueger and Perri, 2006; Blundell, Pistaferri, and Preston, 2008; Aguiar and Bils, 2015) and the decline in leisure inequality (Attanasio, Hurst, and Pistaferri, 2014) over time. Our contribution is to introduce home production data into the inequality literature and show that they change the inferences we draw about welfare. Closest to the spirit of our exercise, Jones and Klenow (2016) map differences in consumption levels and dispersion, market hours, and mortality into welfare differences across countries and find that in some cases GDP per capita does not track welfare closely.

Finally, our paper relates to a strand of literature that uses no-trade theorems to derive analytical solutions for a certain class of models with incomplete markets and heterogeneous agents. Constantinides and Duffie (1996) first derived a no-trade theorem in an endowment economy. Krebs (2003) extends the theorem to an environment with capital, in which households invest a constant share of their wealth in physical and human capital and their total income follows a random walk in logs. Most relevant for us, Heathcote, Storesletten, and Violante (2014) extend the no-trade theorem by allowing for partial insurance of wage shocks and flexible labor supply. Our contribution is to extend the theorem in a multi-sector model under log preferences with respect to the consumption function that aggregates market and non-market inputs. Importantly, with log preferences, we do not need to place restrictions on either the stochastic process governing home productivity and preferences or the elasticity of substitution across sectors.

2 Model

We first present the model and characterize its equilibrium in closed form. We then show how to infer the sources of heterogeneity across households such that the model accounts perfectly for cross-sectional data on consumption expenditures, allocation of time, and wages.

2.1 Environment

Demographics. The economy features perpetual youth demographics. We denote by t the calendar year and by j the birth year of a household. Households face a constant probability of survival δ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

Goods and Time. Goods are produced in three sectors with labor. The vector $\mathbf{h} = (h_M, h_N, h_P)$ contains hours worked in each sector. Households derive utility from consuming a bundle of three goods $\mathbf{c} = (c_M, c_N, c_P)$. We denote by c_M the consumption of goods purchased in the market sector. In the home sector, households consume two imperfectly substitutable goods c_N and c_P. The key difference between the two home production sectors is that there is heterogeneity across households in how productively they transform h_N into c_N whereas the production function of c_P is identical across households. To give some concrete examples from our quantitative results, we think of time spent on activities such as child care and nursing as belonging to h_N because these activities are relatively less intensive in manual skills and productivity differences across households are likely to be a significant source of dispersion. We think of time spent on activities such as cooking and cleaning as belonging to h_P because these activities are more intensive in manual skills and differences in preferences are more likely to be important than productivity differences.

Technologies. Households have access to a technology in the market sector and two technologies in the home sector. A household's technology in the market sector is characterized by its (pre-tax) earnings $y = z_M h_M$, where z_M denotes market productivity (wage) and h_M denotes hours worked in the market sector. Aggregate production is given by $\int_\iota z_M(\iota) h_M(\iota) \mathrm{d}\Phi(\iota)$, where ι identifies households and Φ denotes the cumulative distribution function of households. Goods and labor markets are perfectly competitive and the wage per efficiency unit of labor is one.

The government taxes labor income to finance (wasteful) public expenditures G of the market good. If $y = z_M h_M$ is pre-tax earnings, then $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$ is after-tax earnings, where τ_0

determines the level of taxes and τ_1 governs the progressivity of the tax system. When $\tau_1 = 0$ there is a flat tax rate. A higher τ_1 introduces a larger degree of progressivity into the tax system because it compresses after-tax earnings relative to pre-tax earnings.[3]

Production of home goods is given by $c_N = z_N h_N$ and $c_P = z_P h_P$, where z_N and z_P denote home productivities. We will allow z_N to vary across households, whereas we fix z_P to be the same across households. Home goods are consumed in every period and cannot be stored or traded in the market.

Preferences. Households order sequences of goods and time by $\mathbb{E}_j \sum_{t=j}^{\infty} (\beta\delta)^{t-j} U_t(\mathbf{c}_t, \mathbf{h}_t)$, where β is the discount factor and the period utility function is given by:

$$U = \frac{\left[\left(\omega_M c_M^{\frac{\phi-1}{\phi}} + \omega_N c_N^{\frac{\phi-1}{\phi}} + \omega_P c_P^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}\right]^{1-\gamma} - 1}{1-\gamma} - \frac{\Big(\exp(B)(h_M + h_N) + \exp(D) h_P\Big)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \quad (1)$$

The curvature of the utility function with respect to consumption is given by the parameter γ. Consumption is given by a CES aggregator of goods, with an elasticity of substitution between any goods equal to ϕ. The weights ω_M, ω_N, and ω_P govern the preference for each good. The curvature of the utility function with respect to the total effective hours is given by parameter η. Hours are perfect substitutes across sectors. The preference shifter B captures a household's disutility of work in either the market sector or the home sector N. The preference shifter D captures a household's disutility of work in the home sector P. We allow both B and D to vary across households. All parameters γ, η, ϕ, ω_M, ω_N, and ω_P are non-negative and constant across households.[4]

[3] Our tax schedule modifies the tax schedule considered, among others, by Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014) in that τ_1 is applied to market productivity z_M instead of earnings $z_M h_M$. We adopt the specification of after-tax earnings $\tilde{y} = (1-\tau_0) z_M^{1-\tau_1} h_M$ instead of $\tilde{y} = (1-\tau_0)(z_M h_M)^{1-\tau_1}$ because we can only prove the no-trade result in the home production model under the former specification. We argue that this modification does not matter for our results because market productivity z_M and hours h_M are relatively uncorrelated in the cross section of households and most of the cross-sectional variation in earnings $z_M h_M$ is accounted for by z_M. For this reason, our estimate of τ_1 in Section 3.2 is close to the estimates found in Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014).

[4] Our model features a single decision maker within each household. We model hours worked across spouses as perfect substitutes and in our quantitative results we define h_M, h_N, and h_P as the sum of the respective hours worked across spouses. The perfect substitutability of hours (across sectors and spouses) is essential for the no-trade result. We can extend the model for separate disutility of work shifters by spouse.

Our specification of preferences and technologies nests several special cases of interest. The parameterization $\omega_M = 1$ and $\omega_N = \omega_P = 0$ yields the standard model without home production. In this case $(1-\tau_1)\eta$ becomes the Frisch elasticity of hours. If we set $\omega_P = 0$, then we obtain a two-sector model in which the disutility of work B is equalized across sectors and the sectoral allocation of time depends on productivities in the market z_M and at home z_N. If we set $\omega_N = 0$, then we obtain a two-sector model in which market productivity z_M and differences in the disutility of work across sectors, B and D, determine the allocation of time. More broadly, our three-sector model is a special case of the Beckerian model of home production in which expenditures and time combine to produce final utility (Becker, 1965; Ghez and Becker, 1975).

Sources of Heterogeneity. Households are heterogeneous with respect to the disutilities of work B and D and productivities z_M and z_N. For B and z_M we impose a random walk structure that is important for obtaining the no-trade result. Under certain parametric restrictions that we discuss below, we are able to obtain the no-trade result with minimal structure on the process that governs home productivity z_N and preferences D.

Households' disutility of work is described by a random walk process:

$$B_t^j = B_{t-1}^j + \upsilon_t^B. \tag{2}$$

Households' log market productivity $\log z_M$ is the sum of a permanent component α and a more transitory component ε:

$$\log z_{M,t}^j = \alpha_t^j + \varepsilon_t^j \ . \tag{3}$$

The permanent component follows a random walk, $\alpha_t^j = \alpha_{t-1}^j + \upsilon_t^\alpha$. The more transitory component, $\varepsilon_t^j = \kappa_t^j + \upsilon_t^\varepsilon$, equals the sum of a random walk component, $\kappa_t^j = \kappa_{t-1}^j + \upsilon_t^\kappa$, and an innovation υ_t^ε. Finally, households are heterogeneous with respect to their home productivity $z_{N,t}^j$ and preferences D_t^j. Our identification theorem below is based on cross-sectional data and does not require the restriction of z_N and D to a particular class of stochastic processes. We identify a household ι by a sequence $\{z_N^j, D^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$.

For any random walk, we use υ to denote innovations and Φ_{υ_t} to denote distributions of innovations. We allow distributions of innovations to vary over time t. We assume that $z^j_{N,t}$ and D^j_t are orthogonal to the innovations $\{\upsilon^B_t, \upsilon^\alpha_t, \upsilon^\kappa_t, \upsilon^\varepsilon_t\}$ and that all innovations are drawn independently from each other. The distribution of initial conditions of $(z^j_{N,j}, D^j_j, B^j_j, \alpha^j_j, \kappa^j_j)$ can be non-degenerate across households born at j and can vary by birth year j.

Asset Markets. It is convenient to describe the restrictions on asset markets using the definition of an island in the spirit of Heathcote, Storesletten, and Violante (2014). Islands are capturing insurance mechanisms available to households for smoothing more transitory shocks in the market sector. Households are partitioned into islands, with each island consisting of a continuum of households that are identical in terms of their productivity at home z_N, disutilities of work D and B, permanent component of market productivity α, and the initial condition of κ. More formally, household $\iota = \{z^j_N, D^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$ lives on island ℓ consisting of ι's with common initial state $(z^j_{N,j}, D^j_j, B^j_j, \alpha^j_j, \kappa^j_j)$ and sequences $\{z^j_{N,t}, D^j_t, B^j_t, \alpha^j_t\}^\infty_{t=j+1}$.

We now summarize the structure of asset markets. First, households cannot trade assets contingent on $z^j_{N,t}$ and D^j_t. Second, households can trade one-period bonds $b^\ell(s^j_{t+1})$ that pay one unit of market consumption contingent on $s^j_t \equiv (B^j_t, \alpha^j_t, \kappa^j_t, \upsilon^\varepsilon_t)$ with households that live on their island ℓ. Third, households can trade economy-wide one-period bonds $x(\zeta^j_{t+1})$ that pay one unit of market consumption contingent on $\zeta^j_t \equiv (\kappa^j_t, \upsilon^\varepsilon_t)$ with households that live either on their island or on other islands.

To preview the implications of these assumptions, differences in (z_N, D, B, α) across households remain uninsurable by the no-trade result we will discuss below that yields $x(\zeta^j_{t+1}) = 0$ in equilibrium.[5] The more transitory component of productivity ε^j_t becomes fully insurable because households on an island are only heterogeneous with respect to ζ^j_t and can trade bonds $b^\ell(\zeta^j_{t+1})$. As a result, the island structure generates partial insurance with respect to market productivity differences. Anticipating these results, henceforth we call α the uninsurable permanent component

[5]There is still implicit insurance that households obtain by substituting time across sectors. A realization of z_N that leads to low home-produced c_N can be offset by higher purchases in the market c_M if a household desires so. And the converse for a realization of α that households can offset by substituting away from the market toward the home sector.

of market productivity and $\varepsilon = \kappa + \upsilon^{\varepsilon}$ the insurable transitory component of market productivity. We offer some examples of the type of wage shocks accommodated by the framework. Aggregate changes in wages that load differently across households, such as the skill premium, may be more difficult to insure and are captured by α. By contrast, κ may be capturing persistent shocks such as disability and υ^{ε} may be capturing transitory shocks such as unemployment that are more easy to insure using asset markets, family transfers, or government transfers.[6]

Household Optimization. We now describe the optimization problem of a particular household ι born in period j. The household chooses $\{\mathbf{c}_t, \mathbf{h}_t, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}_{t=j}^{\infty}$ to maximize the expected value of discounted flows of utilities in equation (1), subject to the home production technologies, $c_{N,t} = z_{N,t} h_{N,t}$ and $c_{P,t} = z_P h_{P,t}$, and the sequential budget constraints:

$$c_{M,t} + \int_{s^j_{t+1}} q^{\ell}_b(s^j_{t+1}) b^{\ell}(s^j_{t+1}) \mathrm{d}s^j_{t+1} + \int_{\zeta^j_{t+1}} q_x(\zeta^j_{t+1}) x(\zeta^j_{t+1}) \mathrm{d}\zeta^j_{t+1} = \tilde{y}^j_t + b^{\ell}(s^j_t) + x(\zeta^j_t) \ . \tag{4}$$

The left-hand side of the budget constraint denotes expenditures on market consumption $c_{M,t}$, island-level bonds $b^{\ell}(s^j_{t+1})$ at prices $q^{\ell}_b(s^j_{t+1})$, and economy-wide bonds $x(\zeta^j_{t+1})$ at prices $q_x(\zeta^j_{t+1})$. The right-hand side of the budget constraint consists of after-tax labor income $\tilde{y}^j_t$ and bond payouts.

Equilibrium. Given a tax function (τ_0, τ_1), an equilibrium consists of a sequence of allocations $\{\mathbf{c}_t, \mathbf{h}_t, b^{\ell}(s^j_{t+1}), x(\zeta^j_{t+1})\}_{\iota,t}$ and a sequence of prices $\{q^{\ell}_b(s^j_{t+1})\}_{\ell,t}$, $\{q_x(\zeta^j_{t+1})\}_t$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$\int_{\iota \in \ell} b^{\ell}(s^j_{t+1}; \iota) \mathrm{d}\Phi(\iota) = 0 \quad \forall \ell, s^j_{t+1}, \quad \text{and} \quad \int_{\iota} x(\zeta^j_{t+1}; \iota) \mathrm{d}\Phi(\iota) = 0 \quad \forall \zeta^j_{t+1}; \tag{5}$$

and (iii) the goods market clears:

$$\int_{\iota} c_{M,t}(\iota) \mathrm{d}\Phi(\iota) + G = \int_{\iota} z_{M,t}(\iota) h_{M,t}(\iota) \mathrm{d}\Phi(\iota), \tag{6}$$

where government expenditures are given by $G = \int_{\iota} \left[z_{M,t}(\iota) - (1-\tau_0) z_{M,t}(\iota)^{1-\tau_1} \right] h_{M,t}(\iota) \mathrm{d}\Phi(\iota)$.

[6] We refer the reader to Heathcote, Storesletten, and Violante (2008) for a more detailed discussion of how the partial insurance framework relates to frameworks with exogenously imposed incomplete markets or to frameworks in which incompleteness arises endogenously from informational frictions or limited commitment.

2.2 Equilibrium Allocations

The model retains tractability because, under certain parametric restrictions, it features a no-trade result. This section explains the logic underlying this result and its usefulness. Appendix A presents the proof. Our proof follows very closely the proof presented in Heathcote, Storesletten, and Violante (2014). We extend their analysis along two dimensions. First, we allow the disutility of work B to be a random walk instead of a fixed effect. Second, we extend the no-trade result in an environment with multiple sectors.

We begin by guessing that the equilibrium features no trade across islands, that is $x(\zeta^j_{t+1};\iota) = 0, \forall \iota, \zeta^j_{t+1}$. Further, we postulate that equilibrium allocations $\{\mathbf{c}_t(\iota), \mathbf{h}_t(\iota)\}$ solve a sequence of static planning problems. The planner problems consist of maximizing average utility within each island, $\int_{\zeta^j_t} U\left(\mathbf{c}_t(\iota), \mathbf{h}_t(\iota); \iota\right) \mathrm{d}\Phi_t(\zeta^j_t)$, subject to households' home production technologies $c_{N,t}(\iota) = z_{N,t}(\iota) h_{N,t}(\iota)$ and $c_{P,t}(\iota) = z_P h_{P,t}(\iota)$ and an island-level constraint that equates aggregate market consumption to aggregate after-tax earnings $\int_{\zeta^j_t} c_{M,t}(\iota) \mathrm{d}\Phi_t(\zeta^j_t) = \int_{\zeta^j_t} \tilde{y}_t(\iota) \mathrm{d}\Phi_t(\zeta^j_t)$. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and all asset and goods markets clear.

We obtain the no-trade result in two nested versions of the model. The first model sets the preference weight on market consumption to $\omega_M = 1$ and the preference weights on home consumption to $\omega_N = \omega_P = 0$. This is the environment without home production considered by Heathcote, Storesletten, and Violante (2014). The second model sets the curvature of utility with respect to consumption to $\gamma = 1$ for any value of $(\omega_M, \omega_N, \omega_P)$. The home production model nests the model without home production when $\gamma = 1$, which is the case we consider below in our quantitative results.

To understand the no-trade result, we begin with the observation that households on each island ℓ have the same marginal utility of market consumption because they are identical in terms of (z_N, D, B, α) and trade in state-contingent bonds allows them to perfectly insure against $(\kappa, \upsilon^{\varepsilon})$. Considering first the model without home production ($\omega_M = 1$), the common marginal utility of market consumption $\mu(\ell)$ at the no-trade equilibrium is:

$$\mu(\ell)=\frac{1}{c_M^{\gamma}}=\left(\frac{\exp\left((1+\eta)(B-\log(1-\tau_0)-(1-\tau_1)\alpha)\right)}{\int_{\zeta}\exp\left((1+\eta)(1-\tau_1)(\kappa+\upsilon^{\varepsilon})\right)\mathrm{d}\Phi(\zeta)}\right)^{\frac{\gamma}{1+\eta\gamma}}, \tag{7}$$

where for simplicity we have dropped the time subscript from all variables. The no-trade result states that households do not trade bonds across islands, $x(\zeta_{t+1}^{j})=0$. Owing to the random walk assumptions on B and α, equation (7) shows that the growth in marginal utility, μ_{t+1}/μ_t, does not depend on the state vector (B_t^j,α_t^j) that differentiates islands ℓ. As a result, all households value bonds traded across islands identically in equilibrium and hence there are no mutual benefits from trading $x(\zeta_{t+1}^{j})$.

For the economy with home production and $\gamma=1$, we obtain a marginal utility of market consumption:

$$\mu(\ell)=\frac{1}{c_M+\tilde{z}_M\left(h_N+\frac{\exp(D)}{\exp(B)}h_P\right)}=\left(\frac{\exp\left((1+\eta)(B-\log(1-\tau_0)-(1-\tau_1)\alpha)\right)}{\int_{\zeta}\exp\left((1+\eta)(1-\tau_1)(\kappa+\upsilon^{\varepsilon})\right)\mathrm{d}\Phi(\zeta)}\right)^{\frac{1}{1+\eta}}. \tag{8}$$

The marginal utility in equation (8) has the same form as the marginal utility in equation (7) for $\gamma=1$. Therefore, marginal utility growth does not depend on the state vector $(z_{N,t}^{j},D_t^j,B_t^j,\alpha_t^j)$ that differentiates islands and the same logic explains why we obtain the no-trade result in the home production model. For this result we note the importance of log preferences with respect to the consumption aggregator. Log preferences generate a separability between the marginal utility of market consumption and z_N and D and, thus, the no-trade result holds irrespective of the value of the elasticity of substitution across sectors ϕ and further stochastic properties of z_N and D.

The no-trade result is useful because it allows us to derive equilibrium allocations for consumption and time using the sequence of planning problems described previously without solving simultaneously for the wealth distribution.[7] We summarize the equilibrium allocations for both models in Table 1. The rows in the table present the equilibrium values for market consumption

[7] The no-trade result applies to the bonds traded across islands $x(\zeta_{t+1}^{j})=0$ and not to the within-islands bonds $b^{\ell}(s_{t+1}^{j})$ which are traded in equilibrium. However, the bonds $b^{\ell}(s_{t+1}^{j})$ are state-contingent within each island and, therefore, solving for the equilibrium allocations amounts to solving a sequence of static planning problems.

Table 1: Equilibrium Allocations

Variable		No Home Production: $\omega_M = 1$	Home Production: $\gamma = 1$
1.	c_M	$\frac{\exp\left(\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathbb{C}_a^{\frac{1}{1+\eta\gamma}}$	$\frac{1}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)}\mathbb{C}_a^{\frac{1}{1+\eta}}$
2.	h_N		$\frac{\left(\frac{\omega_N}{\omega_M}\right)^\phi\left(\frac{z_N}{\tilde{z}_M}\right)^\phi}{R}\frac{\exp((1-\tau_1)\alpha)}{z_N\exp(B)}\mathbb{C}_a^{\frac{1}{1+\eta}}$
3.	h_P		$\frac{\left(\frac{\omega_P}{\omega_M}\right)^\phi\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D)/z_P}\right)^\phi}{R}\frac{\exp((1-\tau_1)\alpha)}{z_P\exp(B)}\mathbb{C}_a^{\frac{1}{1+\eta}}$
4.	h_M	$\tilde{z}_M^\eta\frac{\exp\left(-\eta\gamma\frac{1+\eta}{1+\eta\gamma}(1-\tau_1)\alpha\right)}{\exp\left(\frac{1+\eta}{1+\eta\gamma}B\right)}\mathbb{C}_a^{-\frac{\eta\gamma}{1+\eta\gamma}}$	$\tilde{z}_M^\eta\frac{\exp(-\eta(1-\tau_1)\alpha)}{\exp(B)}\mathbb{C}_a^{-\frac{\eta}{1+\eta}} - h_N - \frac{\exp(D)}{\exp(B)}h_P$

Table 1 presents the equilibrium allocation in the two models. Parameters γ, η, ϕ, ω_M, ω_N, and ω_P are constant across households. We define the constant $\mathbb{C}_a \equiv \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon)\mathrm{d}\Phi_\zeta(\zeta)$, the after-tax market productivity $\tilde{z}_M \equiv (1-\tau_0)z_M^{1-\tau_1}$, and the rate of transformation $R \equiv 1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi\left(\frac{z_N}{\tilde{z}_M}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D)/z_P}\right)^{\phi-1}$.

and hours in the three sectors (we then obtain $c_N = z_N h_N$ and $c_P = z_P h_P$). For convenience, we have dropped the household index ι from the table. The constant $\mathbb{C}_a$ is common across households and is proportional to a moment of the transitory component of productivity $\exp(\varepsilon)$. All sources of heterogeneity $(z_N, D, B, \alpha, \varepsilon)$ and allocations are ι-specific.

Starting with the model without home production, market consumption c_M depends positively on the tax-adjusted uninsurable permanent productivity component $(1-\tau_1)\alpha$ and negatively on the disutility of work B. By contrast, c_M does not depend on the transitory component of market productivity ε because state-contingent assets insure against variation in ε. The final row shows that market hours h_M increase in the after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$ with an elasticity η. This reflects the substitution effect on labor supply from variations in after-tax market productivity. Conditional on $\tilde{z}_M$, h_M decreases in $(1-\tau_1)\alpha$ which reflects the income effect from changes in the permanent component of market productivity. When $\gamma = 1$, substitution and income effects from variations in α cancel out and h_M depends positively only on the insurable component ε. Finally, h_M decreases in the disutility of work B.

To understand the solutions in the home production model, we note that the relative marginal rate of substitution between consumption and time equals the relative after-tax productivity across sectors:

$$\frac{c_M}{c_P} = \left(\frac{\omega_M}{\omega_P}\right)^{\phi} \left(\frac{\exp(D)}{\exp(B)}\right)^{\phi} \left(\frac{\tilde{z}_M}{z_P}\right)^{\phi}, \quad \text{and} \quad \frac{c_M}{c_N} = \left(\frac{\omega_M}{\omega_N}\right)^{\phi} \left(\frac{\tilde{z}_M}{z_N}\right)^{\phi}. \tag{9}$$

The solution for c_M in the second column of Table 1 is obtained after substituting these optimality conditions in the marginal utility given in equation (8). The solution for c_M has the same form as the solution in the model without home production under $\gamma = 1$ up to the rate of transformation $R \equiv 1 + \left(\frac{\omega_N}{\omega_M}\right)^{\phi} \left(\frac{z_N}{\tilde{z}_M}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^{\phi} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D)/z_P}\right)^{\phi-1}$. This rate describes the incentives of households to shift hours across sectors as a function of relative productivities and relative disutilities of work, given an elasticity of substitution ϕ across sectors.

The second and third rows present solutions for home production time h_N and h_P. Hours h_N increase in productivity z_N when $\phi > 1$, in which case substitution effects from changes in z_N dominate income effects. Hours h_P decrease in disutility D for any value of ϕ. In the final row, we present the solution for hours in the market sector. To understand this expression, we define effective total hours as $h_T = h_M + h_N + \frac{\exp(D)}{\exp(B)} h_P$ and note that the solutions for h_T coincide in the two models under $\gamma = 1$. The expression for h_T shows that it does not depend on α because under $\gamma = 1$ substitution and income effects from permanent changes in wages cancel out. It also shows that h_T increases in ε with a Frisch elasticity of $(1 - \tau_1)\eta$.

2.3 Identification of Sources of Heterogeneity

We demonstrate how to infer the sources of heterogeneity across households, $\{\alpha, \varepsilon, B, D, z_N\}_\iota$, such that the models with and without home production both account perfectly for given cross-sectional data on market expenditures, hours, and productivity.

Observational Equivalence Theorem. Let $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ be some cross-sectional data. Then, for any given parameters $(\eta, \phi, \tau_0, \tau_1)$:

1. There exists unique $\{\alpha, \varepsilon, B\}_\iota$ such that $\{c_M, h_M, z_M\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ under $\omega_M = 1$ for any γ.
2. There exists unique $\{\alpha, \varepsilon, B, D, z_N\}_\iota$ such that $\{c_M, h_M, z_M, h_N, h_P\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$ under $\gamma = 1$ for any $(\omega_M, \omega_N, \omega_P)$.

Table 2: Identified Sources of Heterogeneity

		No Home Production: $\omega_M = 1$
1.	α	$\frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathbb{C}_s\right]$
2.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1+\eta)}\left[\log\left(\frac{c_M}{h_M}\right) + \eta(1-\tau_1)\log z_M - \log \mathbb{C}_s\right]$
3.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta\gamma}{1+\eta}\log c_M - \frac{1}{1+\eta}\log h_M$
		Home Production: $\gamma = 1$
4.	α	$\frac{1}{(1-\tau_1)(1-\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathbb{C}_s\right]$
5.	ε	$\log z_M - \frac{1}{(1-\tau_1)(1-\eta)}\left[\log\left(\frac{c_T}{h_T}\right) + \eta\,(1-\tau_1)\log z_M - \log \mathbb{C}_s\right]$
6.	B	$\frac{\eta}{1+\eta}\log(1-\tau_0) + \frac{\eta(1-\tau_1)}{1+\eta}\log z_M - \frac{\eta}{1+\eta}\log c_T - \frac{1}{1+\eta}\log h_T$
7.	D	$B + \log\left(\frac{\omega_P}{\omega_M}\right) + \frac{1}{\phi}\log\left(\frac{c_M}{h_P}\right) + \frac{\phi-1}{\phi}\log z_P - \log(1-\tau_0) - (1-\tau_1)\log z_M$
8.	z_N	$\left(\frac{\omega_M}{\omega_N}\right)^{\frac{\phi}{\phi-1}}(1-\tau_0)^{\frac{\phi}{\phi-1}} z_M^{\frac{(1-\tau_1)\phi}{\phi-1}}\left(\frac{c_M}{h_N}\right)^{\frac{1}{1-\phi}}$

Table 2 presents the inferred sources of heterogeneity for the environment without home production (first panel) and for the economy with home production (second panel). We define the constant $\mathbb{C}_s \equiv \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_\zeta(\zeta)$, effective total hours as $h_T \equiv h_M + h_N + \frac{\omega_P}{\omega_M}\left(\frac{c_M}{z_P h_P}\right)^{\frac{1}{\phi}}\frac{z_P}{\tilde{z}_M}h_P$, and the market value of total consumption as $c_T \equiv c_M + \tilde{z}_M h_N + \frac{\omega_P}{\omega_M}\left(\frac{c_M}{z_P h_P}\right)^{\frac{1}{\phi}} z_P h_P$.

The theorem uses the fact that, in each model, the equilibrium allocations presented in Table 1 can be uniquely inverted to obtain, up to a constant, the sources of heterogeneity that result in these allocations. The formal proof is presented in Appendix A.5.

Table 2 presents the inferred sources of heterogeneity that allow the model without home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ and the model with home production to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N, \bar{h}_P\}_\iota$. Henceforth, we drop the bar to indicate variables observed in the data since, by appropriate choices of the sources of heterogeneity, both models generate perfectly these data.

To understand how observables inform the sources of heterogeneity, in Table 2 we define effective total hours as:

$$h_T \equiv h_M + h_N + \frac{\omega_P}{\omega_M}\left(\frac{c_M}{z_P h_P}\right)^{\frac{1}{\phi}} \frac{z_P}{\tilde{z}_M} h_P = h_M + h_N + \frac{\exp(D)}{\exp(B)} h_P, \tag{10}$$

and the market value of total consumption as:

$$c_T \equiv c_M + \tilde{z}_M h_N + \frac{\omega_P}{\omega_M}\left(\frac{c_M}{z_P h_P}\right)^{\frac{1}{\phi}} z_P h_P = c_M + \tilde{z}_M\left(h_N + \frac{\exp(D)}{\exp(B)} h_P\right). \tag{11}$$

These expressions first define total hours and consumption only in terms of observables and parameters. The equality uses the inferred sources of heterogeneity to express total hours and consumption in a more intuitive way. Specifically, total hours h_T are the sum of hours in the three sectors, adjusted for disutility differences across sectors. The market value of total consumption c_T is the sum of market consumption, consumption in sector N adjusted with the exchange rate $\frac{\tilde{z}_M}{z_N}$, and consumption in sector P adjusted with the exchange rate $\frac{\tilde{z}_M}{z_P}\frac{\exp(D)}{\exp(B)}$.

Rows 1 to 6 show that, for $\gamma = 1$, the inferred α, ε, and B have the same functional forms between the two models. The difference is that the hours and consumption informative for the sources of heterogeneity in the home production model are h_T in equation (10) and c_T in equation (11), whereas in the model without home production $h_T = h_M$ and $c_T = c_M$. The inferred α depends positively on the consumption-hours ratio $\log(c_T/h_T)$ and market productivity $\log z_M$ and the inferred ε is the difference between $\log z_M$ and α. The inferred B depends on the gap between market productivity $\log z_M$ and consumption $\log c_T$ and hours $\log h_T$.

The new sources of heterogeneity in the home production model are presented in rows 7 and 8. These are inferred by substituting the production functions $c_P = z_P h_P$ and $c_N = z_N h_N$ into the first-order conditions (9) and solving for D and z_N:

$$\frac{\exp(D)}{\exp(B)} = \frac{\omega_P}{\omega_M}\left(\frac{z_P}{\tilde{z}_M}\right)^{\frac{\phi-1}{\phi}}\left(\frac{c_M}{\tilde{z}_M h_P}\right)^{\frac{1}{\phi}}, \quad \text{and} \quad \frac{z_N}{\tilde{z}_M} = \left(\frac{\omega_M}{\omega_N}\right)^{\frac{\phi}{\phi-1}}\left(\frac{\tilde{z}_M h_N}{c_M}\right)^{\frac{1}{\phi-1}}. \tag{12}$$

Holding constant preference weights and relative productivities, we infer a higher relative disutility at home $\exp(D)/\exp(B)$ the higher is a household's market expenditures c_M relative to the market value of producing at home $\tilde{z}_M h_P$. If sectors are substitutes ($\phi > 1$), we infer a higher relative productivity at home $z_N/\tilde{z}_M$ the higher is a household's market value of producing at home $\tilde{z}_M h_N$ relative to market expenditures c_M.

Table 3: Numerical Example

Household	z_M	c_M	h_M	h_N	h_P	α	ε	B	D	z_N	T
1	20	1,000	60			2.90	0.09	-4.00			0
2	20	600	40			2.85	0.14	-3.54			399
1	20	1,000	60	10	50	2.95	0.04	-4.74	-4.74	5.96	0
2	20	600	40	50	30	2.95	0.04	-4.74	-4.74	29.37	-776

Table 3 presents an example with parameters $\tau_0 = \tau_1 = 0$ and $\gamma = \eta = 1$. The upper panel shows inference based on the model without home production and the lower panel shows inference based on the model with home production. For the home production model we use $\omega_M = \omega_N = \omega_P = 1/3$, $z_P = 20$, and $\phi = 2.33$. The last column, labeled T, shows the equivalent variation for household 2 to achieve the utility level of household 1.

A numerical example in Table 3 provides some insights for the mechanisms of the model and draws lessons from the observational equivalence theorem. The economy is populated by two households, there are no taxes, and preference parameters satisfy $\gamma = \eta = 1$. In the upper panel, the economist uses the model without home production to infer the sources of heterogeneity. Household 1 earns a wage $z_M = 20$, spends $c_M = 1,000$, and works $h_M = 60$. Household 2 also earns $z_M = 20$, but spends $c_M = 600$ and works $h_M = 40$. The analytical solutions in Table 2 show that households with a higher expenditures to hours ratio, c_M/h_M, or higher market productivity, z_M, have a higher uninsurable productivity component α. In Table 3 we thus infer that α is higher for household 1 than for household 2 (2.90 versus 2.85). Since both households have the same market productivity and $\alpha + \varepsilon$ add up to (log) market productivity, we infer that household 2 has a higher insurable productivity component ε than household 1. Finally, we infer that household 2 has a higher B because it spends less and works less than household 1 despite having the same market productivity.

In the lower panel the economist uses the home production model to infer the sources of heterogeneity. In addition to the same data on (c_M, h_M, z_M), now the economist uses that the first household works $h_N = 10$ and $h_P = 50$ hours and the second household works $h_N = 50$ and $h_P = 30$ hours in the two sectors. The inferred α now depends on the ratio of the market value of total consumption to total hours, c_T/h_T, rather than on the ratio of market expenditures to market hours c_M/h_M. Since both households have the same market value of total consumption, $c_T = 2,200$, and the same total hours, $h_T = 120$, the α's are equal. Given the same market productivity, the ε's are also equalized. Given that the two households consume and work the same, the B's are also equalized. Equation (12) shows that D is also the same between the two households because they have the same ratio of home production to market expenditures $z_M h_P/c_M$. As Table 3 shows, all differences in observables between the two households are loaded into home productivity z_N. We infer that z_N is higher for household 2 because it has a higher value of home production to market expenditures $z_M h_N/c_M$ and the sectors are substitutes ($\phi > 1$).

There are two lessons we draw from this example. First, home productivity z_N is dispersed across households and absorbs dispersion one would attribute to (α, ε, B) in the absence of home production. This result generalizes in our quantitative application using U.S. data below where we find that z_N is significantly more dispersed than z_M and that the dispersion in (α, ε, B) is smaller in the home production model.

The second lesson we draw concerns the welfare implications of labor market dispersion. A household's welfare ranking depends on whether the data has been generated by a model with or without home production. The last column of Table 3 shows equivalent variations T, equal to the transfers required for households to achieve a given level of utility if they re-optimize their consumption and hours choices. The reference utility level in Table 3 is the utility of household 1 and, thus, T for household 1 is always equal to zero. In the model without home production, T for household 2 equals 399. In the home production model, the two households are identical in terms of their $(\alpha, \varepsilon, B, D)$, but household 2 has a higher home productivity z_N. Therefore, the welfare ranking changes and T becomes -776.

2.4 Discussion

Before proceeding to the quantitative results, we pause to make three comments. First, we emphasize the importance of developing an equilibrium model that expresses the arguments (c_M, h_M, h_N, h_P) of the utility function in terms of productivity and preference shifters and policy parameters. An alternative approach, followed by Krueger and Perri (2003) in their study of the welfare effects of increasing inequality in the United States and Jones and Klenow (2016) in their study of welfare and GDP differences across countries, is to plug what are endogenous variables in our framework into the utility function and conduct welfare experiments by essentially varying these variables. While our approach comes with additional complexity, it has the conceptual advantage of taking into account equilibrium responses when conducting welfare analyses with respect to changes in more primitive sources of heterogeneity and policies.

Second, we wish to highlight the merits of the Heathcote, Storesletten, and Violante (2014) framework used in our analysis compared to alternative frameworks. First, standard general equilibrium models with uninsurable risk, such as Huggett (1993) and Aiyagari (1994) and extensions with endogenous labor supply such as Pijoan-Mas (2006), Chang and Kim (2007), and Marcet, Obiols-Homs, and Weil (2007), feature self-insurance via a risk-free bond. Solutions to these models are obtained computationally. While the present model also allows households to trade a risk-free bond (by setting $x(\zeta_t^j) = 1$ for all states ζ_t^j), the assumptions on asset markets, stochastic processes, and preferences allow us to derive a no-trade result and characterize equilibrium allocations in closed form. Owing to the analytical results, a major advantage of the Heathcote, Storesletten, and Violante (2014) framework is the transparency and generality of the identification.[8]

Third, our non-parametric approach to identifying the sources of heterogeneity is such that the model accounts perfectly for any given cross-sectional data on market consumption, hours, and wages. Conceptually, our approach is similar to Hsieh and Klenow (2009) who infer wedges in first-order conditions such that firm-level outcomes generated by their model match data analogs. Heathcote, Storesletten, and Violante (2014) also do not impose distributional assumptions on the sources of heterogeneity in order to estimate model parameters and parameters of these

[8]Despite the wealth distribution not being an object of interest within this framework, a dynamic structure with non-labor income is still essential. In a framework without non-labor income, households would maximize utility subject to home production technologies $c_N = z_N h_N$ and $c_P = z_P h_P$ and the budget constraint $c_M = z_M h_M$. Here, observed market productivity z_M is constrained to equal the market consumption to hours ratio c_M/h_M and any choice of (z_M, B, D, z_N) is not sufficient to match data on $(z_M, c_M, h_N, h_P, h_M)$.

distributions. A difference with Heathcote, Storesletten, and Violante (2014) is that these authors select moments in order to estimate parameters using the method of moments. Our approach, instead, does not require restrictions on which moments are more informative for the identification of the sources of heterogeneity.

3 Quantitative Results

We begin by describing the data sources and the parameterization of the model. We then present the inferred sources of heterogeneity across households.

3.1 Data Sources

For the baseline analyses we use data from the Consumer Expenditure Survey (CEX) and the American Time Use Survey (ATUS). We consider married and cohabiting households with heads between 25 and 65 years old who are not students. We drop observations for households with a market productivity below 3 dollars or above 300 dollars per hour in 2010 dollars, with expenditures at the top and bottom one percent, and with respondents who indicated working more than 92 hours in the market or at home. In the ATUS we drop respondents during weekends and in the CEX we keep only households that completed all four interviews. The final sample from CEX/ATUS contains 32,993 households between 1995 and 2016. In all our results, we weight households with the sample weights provided by the surveys.

Data for market expenditures c_M, market productivity z_M, and market hours h_M come from CEX interview surveys collected between 1996 and 2017. Closest to the definition of Aguiar and Hurst (2013), for our baseline analyses c_M is annual non-durable consumption expenditures which include food and beverages, tobacco, personal care, apparel, utilities, household operations (including child care), public transportation, gasoline, reading material, and personal care. Non-durable consumption expenditures exclude health and education. We adjust consumption for household composition and size.

Our measure of income is the amount of wage and salary income before deductions earned over the past 12 months. Individual wages are defined as income divided by hours usually worked in a year, which is the product of weeks worked with usual hours worked per week. We define household market hours h_M as the sum of hours worked by spouses and market productivity z_M as the average of wages of individual members weighted by their market hours.

Data for home hours h_N and h_P come from the ATUS waves between 2003 and 2017. Randomly selected individuals from a group of households that completed their eight and final month interview for the Current Population Survey report their activities on a 24-hour time diary of the previous day. Similar to Aguiar, Hurst, and Karabarbounis (2013), total time spent on home production, $h_N + h_P$, includes housework, cooking, shopping, home and car maintenance, gardening, child care, and care for other household members.

Before using the ATUS data in our analyses, we need to separate total home production time between h_N and h_P. Our approach is to map disaggregated time uses into occupations and then classify in h_N all the time uses mapped into occupations that perform tasks with low manual content and in h_P all time uses mapped into occupations that perform tasks with high manual content. The logic underlying our approach is that time activities that use the same skills as occupations with high manual content are less likely to display significant heterogeneity in terms of productivity. We use the mapping from time uses to occupations together with Occupational Information Network (O*NET) task measures for various activities as described in the appendix of Acemoglu and Autor (2011) to create an index of manual content for each disaggregated time use.[9] We classify activities in h_N if they have a manual index below the median and classify activities in h_P if they have an index above the median.

The CEX does not contain information on time spent on home production. To overcome this difficulty, we impute time use data from the ATUS into the CEX.[10] Our imputation is based

[9]Because there are many such indices, we standardize the task measures to have mean zero and standard deviation of one and take the average across all manual tasks to create a single manual index. We list the mapping for the seven largest time use categories. Child care time is mapped to preschool teachers and child care workers; shopping time is mapped to cashiers; nursing time is mapped to registered nurses and nursing assistants; cooking is mapped to food preparation and serving workers; cleaning is mapped to maids and housekeeping cleaners; gardening is mapped to landscaping and groundskeeping workers; laundry is mapped to laundry and dry-cleaning workers.

[10]In an approach similar to ours, Blundell, Pistaferri, and Saporta-Eksten (2018) use the CEX to impute

on an iterative procedure where individuals in the CEX are allocated the mean home hours (for each of h_N and h_P) of matched individuals from the ATUS based on group characteristics. We begin the procedure by matching individuals based on work status, race, gender, and age. We then proceed to improve these estimates by adding a host of additional characteristics, such as family status, education, disability status, geography, hours worked, and wages, and matching individuals based on these characteristics whenever possible. We first impute home hours to individuals and, similarly to market hours, then sum up these hours at the household level.

Our imputation accounts for approximately two-thirds of the variation in home hours h_N and h_P. In Appendix Table A.1 we confirm that our imputation does not introduce spurious correlations in the merged CEX/ATUS data by showing that the correlation of home hours with market hours and wages conditional on age is similar in magnitude between the ATUS sample of individuals and the merged CEX/ATUS sample of households. In Appendix Tables A.2 and A.3 we show that, conditional on age, married men, women, less educated, and more educated exhibit similar correlations between wages, market hours, and home hours in the ATUS. Further, in Appendix Tables A.4 and A.5 we show that the correlation of total home hours with market expenditures, market hours, and wages conditional on age is similar in magnitude between the CEX/ATUS sample of households and two PSID samples of households that do not require an imputation since they contain information on home hours, market expenditures, market hours, and wages.

Table 4 presents summary statistics of the time allocation of married households in the CEX/ATUS sample along with the value of the manual index and the average wage corresponding to the occupations the time uses are mapped to. Beginning with market hours h_M, we note in the last columns a small decline over the life-cycle. The three largest time uses classified in h_N are child care, shopping, and nursing. These are activities with lower manual content (and typically higher cognitive content) than activities such as cooking, cleaning, gardening, and laundry that we classify in h_P. The allocation of time between the two types of home production is relatively

expenditures to the ATUS.

Table 4: Summary Statistics of Time Allocation of Married Households

	Skills		Hours per week		
	Manual Index	Wage (2010)	All	25-44	45-65
Market hours h_M		26.6	66.1	66.8	65.5
Home hours h_N		15.3	21.4	25.4	17.3
Child care	-0.73	14.6	10.8	14.9	6.7
Shopping	0.08	10.8	6.4	6.5	6.3
Nursing	-0.12	22.1	1.9	1.8	2.0
Home hours h_P		12.5	16.7	16.4	17.0
Cooking	0.41	10.3	7.5	7.4	7.5
Cleaning	0.43	12.0	3.7	3.7	3.6
Gardening	1.27	14.5	2.1	1.7	2.5
Laundry	0.89	11.6	2.0	2.2	1.9

Table 4 presents summary statistics of the time allocation of married households in the merged CEX/ATUS sample. The first column shows the index of manual skills of individual time uses and the second column shows the average wage of the corresponding occupations these time uses are mapped to.

balanced, but there are noticeable differences over the life-cycle. As expected, child care time declines significantly in the second half of working life which generates a decline in h_N over the life-cycle. By contrast, h_P increases moderately over the life-cycle.[11]

3.2 Parameterization

Table 5 presents parameter values for our baseline analyses in the models without home production ($\omega_M = 1$) and home production ($\omega_M < 1$). We estimate the progressivity parameter τ_1 using data from the Annual Social and Economic Supplement of the Current Population Survey between 2005 and 2015. We use information on pre-tax personal income, tax liabilities at the federal and state level, Social Security payroll deductions, as well as usual hours and weeks worked. Our estimate of τ_1 comes from a regression of log after-tax market productivity on log market productivity before taxes. We estimate $\tau_1 = 0.12$ with a standard error below 0.01.[12] We choose $\tau_0 = -0.36$

[11] Our life-cycle profiles are consistent with those reported in Cardia and Gomme (2018), who also embrace the view that child care has a different technology from other home production.

[12] Our definitions of income and wage include the child care and earned income tax credits but exclude government transfers such as unemployment benefits, welfare, and food stamps because we think of fully insurable shocks ε as

Table 5: Parameter Values

Parameter	$\omega_M = 1$	$\omega_M < 1$	Rationale
τ_1	0.12	0.12	$\log\left(\frac{\tilde{y}}{h_M}\right) = \mathbb{C}_\tau + (1-\tau_1)\log z_M$.
τ_0	-0.36	-0.36	Match $G/Y = 0.10$.
γ	1	1	Nesting of models.
η	0.90	0.50	Match $\beta = 0.54$ in $\log h_M = \mathbb{C}_\eta + \beta(\eta)\varepsilon$.
ω_M	1.00	0.40	$\omega_M + \omega_N + \omega_P = 1$.
ω_N	0.00	0.38	$\mathbb{E}z_N = 0.58\mathbb{E}z_M$.
ω_P	0.00	0.22	$z_P = 0.47\mathbb{E}z_M$.
ϕ	—	2.33	$\frac{\Delta_{65-25}\log(c_M/h_N)}{\Delta_{65-25}\log z_M} = \phi(1-\tau_1) = 2.05$.

Table 5 presents parameter values for the models without home production ($\omega_M = 1$) and with home production ($\omega_M < 1$).

to match an average tax rate on labor income equal to 0.10, which equals the average ratio of personal current taxes to income from the national income and product accounts.

For the home production model, we obtained the equilibrium allocations in closed form only under a curvature of the utility function with respect to consumption equal to $\gamma = 1$. We choose $\gamma = 1$ also for model without home production. Is is essential to nest the model without home production, so that welfare differences across the two models do not arise from different curvatures of the utility function with respect to consumption.[13]

Next, we estimate the parameter η for the curvature of the utility function with respect to hours. Our strategy is to choose η in each model such that a regression of log market hours $\log h_M$ on the transitory component of market productivity ε yields a coefficient of 0.54. The target value of 0.54 comes from the meta analysis of estimates of the intensive margin Frisch elasticity from micro variation found in Chetty, Guren, Manoli, and Weber (2012). Consistent with the logic

subsuming these transfers. Our estimated tax parameter is close to the estimate of 0.19 in Heathcote, Storesletten, and Violante (2014). Using their tax function $\log \tilde{y} = \text{constant} + (1-\tau_1)\log y$, we would estimate $\tau_1 = 0.15$. We, therefore, argue that it is relatively inconsequential whether we apply the progressivity parameter $(1-\tau_1)$ to after-tax wages or after-tax labor income.

[13] While welfare effects are sensitive to the value of γ in the model without home production, our inference of α and ε does not depend on γ as seen in Table 2.

of Rupert, Rogerson, and Wright (2000) who argue that estimates of the Frisch elasticities are downward biased in the presence of home production, we estimate $\eta = 0.90$ in the model without home production and $\eta = 0.50$ in the model with home production.[14]

We now describe the estimation of the preference weights $(\omega_M, \omega_N, \omega_P)$ and the elasticity of substitution ϕ which are parameters specific to the home production model. We invert the first-order conditions (9) and take means over the population to solve for the ratios:

$$\frac{\omega_M}{\omega_N} = \frac{\mathbb{E}\left(\frac{c_M}{\tilde{z}_M^{\phi} h_N}\right)^{\frac{1}{\phi}}}{\mathbb{E} z_N^{\frac{1-\phi}{\phi}}}, \quad \text{and} \quad \frac{\omega_M}{\omega_P} = \frac{\mathbb{E}\left(\frac{c_M}{\tilde{z}_M^{\phi} h_P}\right)^{\frac{1}{\phi}}}{z_P^{\frac{1-\phi}{\phi}}\, \mathbb{E}\left(\frac{\exp(D)}{\exp(B)}\right)}, \tag{13}$$

We use these two equations and the normalization $\omega_M + \omega_N + \omega_P = 1$ to solve for the weights for any given value of ϕ. To do so, we require some normalization of the levels of productivity and disutility of work. From the estimates of wages in occupations matched with time uses in Table 4 we set $\mathbb{E} z_N = 0.58 \mathbb{E} z_M$ and $z_P = 0.47 \mathbb{E} z_M$. We normalize $\mathbb{E}\left(\frac{\exp(B)}{\exp(D)}\right) = 1$.[15]

For the elasticity of substitution ϕ, we again use the first-order condition (9) to derive the regression:

$$\log\left(\frac{c_M}{h_N}\right) = \phi \log\left(\frac{\omega_M}{\omega_N}\right) + \phi \log(1-\tau_0) + \phi(1-\tau_1)\log z_M - (\phi - 1)\log z_N. \tag{14}$$

We note that estimation of ϕ using data on c_M/h_N and z_M would lead to biased estimates if z_M and z_N are correlated. For this reason, we take changes over time in equation (14) and use a synthetic panel approach to estimate ϕ based on changes in c_M/h_N and changes in z_M between the beginning and the end of the life-cycle. The identifying assumption is that changes in z_N are uncorrelated with changes in z_M between the beginning and the end of the life-cycle.

[14] Our strategy is conservative in the sense that the inequality difference between the two models becomes larger when we set η to be equal between the two models. The Frisch elasticity for effective total hours h_T is $(1-\tau_1)\eta$ in both models. There are three reasons why η deviates from the targeted elasticity of 0.54. First, the progressivity of the tax system introduces the wedge $1-\tau_1$ between η and the Frisch elasticity for total hours h_T. Second, tastes and home productivity are correlated with market wages. Third, even without such a correlation, the elasticities of market hours h_M differ between the two models because $h_M = h_T$ without home production whereas with home production h_M is negatively correlated to h_N and h_P. In Appendix Table A.6 we present the various labor supply elasticities implied by the two models.

[15] We have confirmed that these normalizations are inconsequential for all our results except for the levels of productivity in the home sector. This is because the products $\omega_N z_N^{(\phi-1)/\phi}$ and $\omega_P z_P^{(\phi-1)/\phi}$ enter the utility function and, therefore, for any given targeted value of $\mathbb{E} z_N$, z_P, and $\mathbb{E}\left(\frac{\exp(B)}{\exp(D)}\right)$, the parameters $(\omega_M, \omega_N, \omega_P)$ will adjust so that the model matches exactly the same data.

This assumption is consistent with the assumptions underlying the no-trade result which requires $z_{N,t+1}$ to be independent of innovations to $z_{M,t+1}$. Both our estimation strategy and the no-trade theorem are consistent with a correlation of productivity across sectors in levels.

We estimate that market and home goods are substitutes with an elasticity of $\phi = 2.33$. Our estimate of the elasticity of substitution is consistent with those found in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4 and Aguiar and Hurst (2007) obtain estimates of around 2.

3.3 Inferred Sources of Heterogeneity

We extract the sources of heterogeneity by plugging the estimated parameters and CEX/ATUS data on $(c_M, h_M, z_M, h_N, h_P)$ into the solutions in Table 2 for each household. In Figure 1 we present the age profiles for the means of the productivity and taste shifters $(\alpha, \varepsilon, B, D, \log z_N)$. To obtain these age profiles, we regress each inferred shifter on age dummies, cohort dummies, and normalized year dummies as in Deaton (1997).[16] We plot the coefficients on age dummies which give the mean of each shifter by age relative to age 25. To reduce noise in the figures, we present the fitted values from locally weighted regressions of the coefficients on age dummies on age.

Recall from the analytical solutions in Table 2, that the permanent component of productivity α grows over the life-cycle when either the ratio of consumption to hours c_T/h_T grows or when wages z_M grow. The transitory component ε falls when the increase in c_T/h_T is large relative to the increase in z_M. In the upper panels of Figure 1 we see that the means of α and ε are similar until roughly age 45 between the two models, but diverge after that. The slower growth of α and the smaller decline in ε in the model with home production reflect the significant decline in home hours h_N in the second part of the life-cycle which implies that c_T/h_T grows by less than c_M/h_M. In the lower panels we see that both models generate a relatively similar increase in the disutility of work B which reflects the faster growth of z_M relative to c_T and h_T over the life-cycle.

[16] Results are similar when we extract the age effect in regressions that either control only for cohort dummies or only for year dummies.

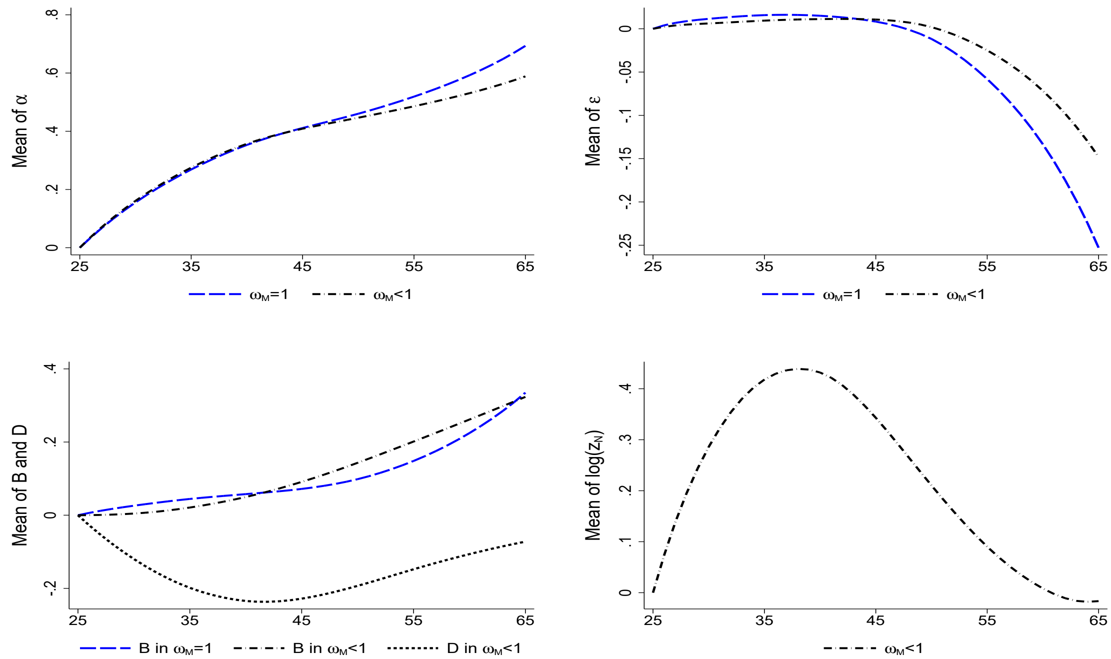


Figure 1: Means of Productivity and Preference Shifters

Figure 1 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D, and home productivity $\log z_N$ for the economy with ($\omega_M < 1$, black dotted lines) and without home production ($\omega_M = 1$, blue dashed lines).

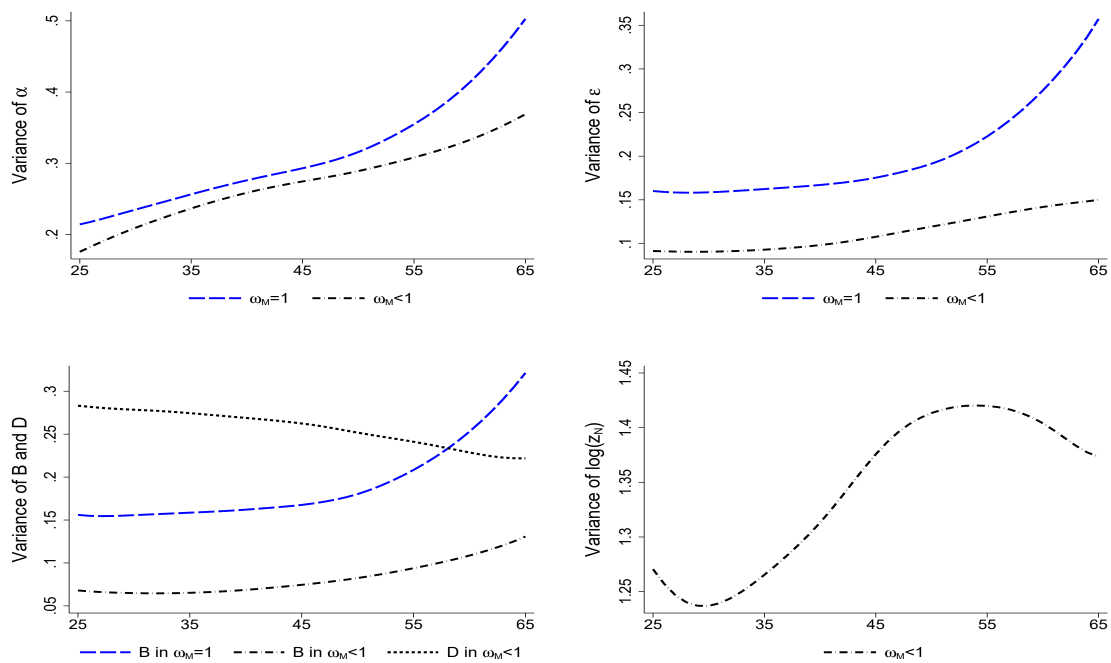


Figure 2: Variances of Productivity and Preference Shifters

Figure 2 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D, and home productivity $\log z_N$ for the economy with ($\omega_M < 1$, black dotted lines) and without home production ($\omega_M = 1$, blue dashed lines).

The model with home production generates a U-shaped profile for home work disutility D which contrasts with the increasing profile for B. To understand this difference, recall from equation (12) that $\frac{\exp(D)}{\exp(B)} \propto \left(\frac{c_M}{h_P}\right)^{1/\phi} \frac{1}{\tilde{z}_M}$. To rationalize the faster growth of z_M relative to c_M during the earlier stages of the life-cycle, the model requires a decline in the disutility D relative to B. By contrast, in later stages of the life-cycle z_M and c_M comove more closely, yielding an upwards slopping profile for D.

The model with home production generates a hump-shaped profile for home productivity z_N. To understand this pattern, recall from equation (12) that $z_N \propto z_M^{\frac{\phi}{\phi-1}} \left(\frac{h_N}{c_M}\right)^{\phi-1}$. Until roughly age 40, z_N tracks market productivity z_M since $\phi > 1$. After age 40, z_N starts to decline despite z_M still rising and, by age 65 z_N has returned to its initial value at age 25. This pattern is generated by the strong decline of hours h_N after age 40. As shown in Table 4, child care is the subcategory of h_N responsible for this decline.

In Figure 2 we present the age profiles for the cross-sectional variances of $(\alpha, \varepsilon, B, D, \log z_N)$, which equal the variances of the residuals for each age from a regression of each shifter on age dummies, cohort dummies, and normalized year dummies. The main result is that the home production model requires significantly smaller variances of α, ε, and B relative to the model without home production. From the solutions in Table 2, we observe that the increasing variance of α over the life-cycle is driven by the increase in the variance of the consumption-hours ratio $\log(c_T/h_T)$ and the increase in the variance of wages $\log z_M$.[17] Because the variance of $\log(c_T/h_T)$ is lower than the variance of $\log(c_M/h_M)$, the home production model generates a lower variance of α. Given that both models match the same variance of $\log z_M$ but the home production model displays a larger covariance between α and ε than the model without home production (see Appendix Table A.7), ε turns out to be less dispersed in the home production model. The variance of B is also smaller in the home production model which reflects the smaller variance of a combination of $\log c_T$ and $\log h_T$ than a combination of $\log c_M$ and $\log h_M$.

[17] To derive analytical solutions, we have not allowed for borrowing constraints that are important when thinking about the comovement of income with consumption at the bottom of the asset distribution. While the transmission mechanism is different than in our model, the presence of borrowing constraints generates comovement between income and consumption in a similar way as α.

As is seen in the lower panels, a fraction of the dispersion in observables is now accounted partly by the dispersion in the disutility of home work D and especially by the large dispersion in home productivity $\log z_N$. To set a benchmark for home productivity, the variance of $\log z_M$ is 0.33 in the data. What explains the almost four times as large dispersion in $\log z_N$? From equation (12) inferred home productivity is given by:

$$\log z_N = \text{constant} + \left(\frac{1}{\phi - 1}\right)(\phi \log \tilde{z}_M + \log h_N - \log c_M)\,. \tag{15}$$

Our result that home productivity is more dispersed than market productivity reflects the fact that $\log z_N$ cumulates the dispersions of three observables, $\log \tilde{z}_M$, $\log h_N$, and $\log c_M$, that are relatively uncorrelated with each other. From equation (15), we see that when ϕ tends to zero and the goods tend to become perfect complements, we obtain $\log z_N = \text{constant} + \log c_M - \log h_N$. In this case the variance of $\log z_N$ is roughly 1.3 because the variance of $\log c_M$ is roughly 0.3, the variance of $\log h_N$ is roughly 1, and the two variables are relatively uncorrelated in the cross-section of households. When ϕ tends to infinity and the goods tend to become perfect substitutes, we obtain $\log z_N = \text{constant} + \log \tilde{z}_M$. In that case, the variance of $\log z_N$ converges to the variance of $\log \tilde{z}_M$. As equation (15) shows, around $\phi = 1$, the variance of $\log z_N$ tends to infinity. To summarize, for any value of ϕ, the variance of $\log z_N$ exceeds the variance of $\log \tilde{z}_M$.

Figure 3 summarizes the properties of home and market productivity.[18] The left panel shows the variance of $\log z_N$ relative to the variance of $\log z_M$ and the middle panel shows the correlation of the two productivities as function of the elasticity of substitution across sectors ϕ. For the variance, we obtain that the variance of $\log z_N$ is larger than the variance of $\log z_M$ for any value of $\phi < 5$ in the figure.[19] For the correlation of the two variables, however, the parameter ϕ becomes crucial. When goods are substitutes as suggested by our estimation, $\phi > 1$, productivity

[18] We focus on home productivity because as we will argue this is the crucial source of heterogeneity driving the inequality gap between the home production model and the model without home production. Appendix Table A.7 presents the correlation matrix of observables with all sources of heterogeneity. Appendix Figure A.1 shows estimates of the distributions of all other sources of heterogeneity.

[19] We note that the argument in the preceding paragraph referred to after-tax market productivity $\log \tilde{z}_M$ whereas in Figure 3 we use the more primitive pre-tax market productivity $\log z_M$. The former measure of productivity is roughly 70 percent as dispersed as the latter because our estimated tax progressivity parameter $\tau_1 = 0.12$ implies a compression of its dispersion relative to the dispersion in pre-tax productivity.

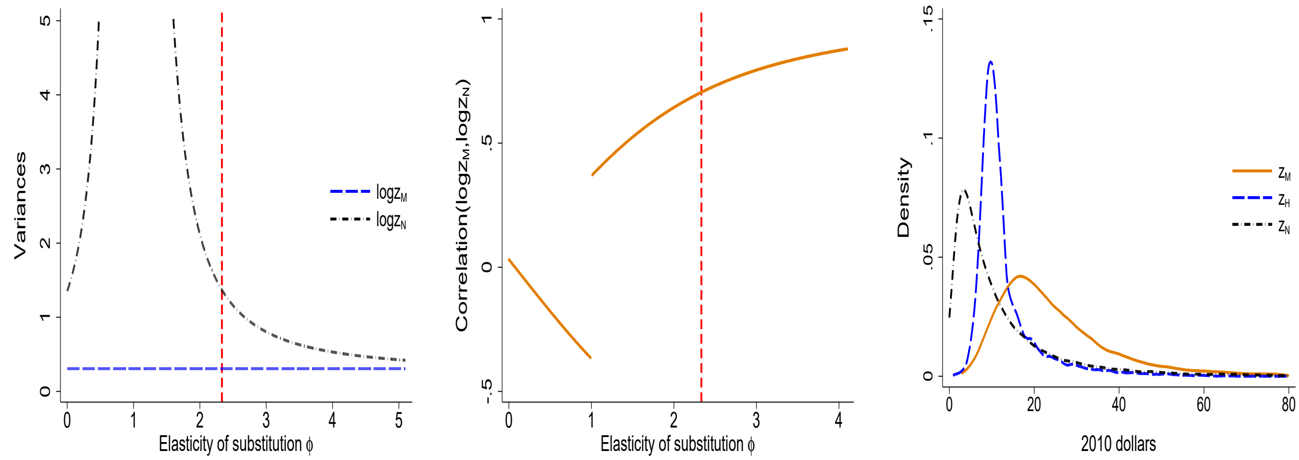


Figure 3: Productivity Moments

The left panel of Figure 3 shows the variance of home productivity $\log z_N$ and market productivity $\log z_M$ and the middle panel shows the correlation between the two variables as a function of the elasticity of substitution across sectors ϕ. The dashed vertical line shows the variances and correlation at our estimated value of $\phi = 2.33$. The right panel plots estimates of the distributions of z_M, $z_H = \frac{h_N}{h_N+h_P} z_N + \frac{h_P}{h_N+h_P} z_P$, and z_N at $\phi = 2.33$.

in the home sector is positively correlated with productivity in the market sector. If goods were complements, $\phi < 1$, the correlation would have typically been negative.

The right panel of Figure 3 plots the distributions of productivities under our estimated $\phi = 2.33$. We define effective home productivity $z_H = \frac{h_N}{h_N+h_P} z_N + \frac{h_P}{h_N+h_P} z_P$. Because z_P is a constant, we find that effective productivity at home z_H is less dispersed than z_N. The means of z_M, z_H, and z_N are 26.6, 15.3, and 15.2 dollars respectively. The fraction of households with productivity exceeding 100$ per hour equals roughly 1 percent, 0.6, and 1.4 percent respectively.

4 Inequality and Home Production

We begin by demonstrating that home production amplifies inequality across households. Next, we argue that heterogeneity in home productivity rather than preferences is crucial in amplifying inequality.

4.1 Home Production Amplifies Inequality

We demonstrate that inequality across households is larger in the home production model than in the model without home production, despite both models generating the same data on market observables.[20] By inequality, we mean a mapping from the dispersion in observed allocations and inferred sources of heterogeneity to measures that capture welfare differences across households. We acknowledge there are various such mappings and, therefore, present four inequality metrics.

4.1.1 Equivalent Variation

The equivalent variation, a broadly used metric in welfare economics, is the change in income required for a household to achieve a reference level of utility. Let $\hat{\iota}$ be a reference household with allocations of consumption and time given by the vector $(\hat{\mathbf{c}}_t, \hat{\mathbf{h}}_t)$, a flow utility $U(\hat{\mathbf{c}}_t, \hat{\mathbf{h}}_t; \hat{\iota})$, and a value function $\hat{V}_t(\hat{\iota})$. For every household ι, we compute the income transfer $T_t(\iota)$ that makes it indifferent between being ι and being $\hat{\iota}$ in the current period, holding constant ι's expectation over all future allocations. Equivalently, the equivalent variation $T_t(\iota)$ solves:

$$\hat{V}_t(\hat{\iota}; \iota) = \max_{\{\mathbf{c}_t, \mathbf{h}_t\}} \left\{ U(\mathbf{c}_t, \mathbf{h}_t; \iota) + \beta\delta\mathbb{E}_t \left[V_{t+1}(\iota') | \iota \right] \right\}, \tag{16}$$

subject to the home production technologies $c_{N,t} = z_{N,t} h_{N,t}$ and $c_{P,t} = z_P h_{P,t}$ and the budget constraint:

$$c_{M,t} = \tilde{y}_t + T_t(\iota) + \overline{\mathrm{NA}}_t(\iota). \tag{17}$$

In equation (16) we define $\hat{V}_t(\hat{\iota}; \iota) \equiv U(\hat{\mathbf{c}}_t, \hat{\mathbf{h}}_t; \hat{\iota}) + \beta\delta\mathbb{E}_t \left[V_{t+1}(\iota') | \iota \right]$ and in equation (17) we keep the net asset position $\overline{\mathrm{NA}}_t(\iota)$ constant at its initial value before the transfer $T_t(\iota)$ is given.

Figure 4 presents the cross-sectional dispersion in equivalent variation for every age.[21] The left panel shows the standard deviation of equivalent variation, standardized by the mean value of market consumption $\int c_M(\iota) \mathrm{d}\Phi(\iota)$ which is constant across models and ages. At age 25, the standard deviation is around 0.6 in both economies. By age 45, however, the standard deviation has increased to more than 0.9 in the home production model, as opposed to below 0.8 in the model

[20] Home production amplifies the level of inequality significantly. Appendix Figure A.2 reports time trends in two of our measures of inequality, the standard deviation of the equivalent variation T and the standard deviation of the transfers t required to equalize marginal utilities. We find that inequality has increased between the beginning of the sample and the mid 2000s, with some leveling off or decline after that. The figure shows that this happens for both the model with and without home production.

[21] In this figure we assume that $\hat{\iota}$ is the household with the median utility. Our results are similar when we define $\hat{\iota}$ as the household with the mean utility, the household with the median utility by age, or the household with the mean utility by age.

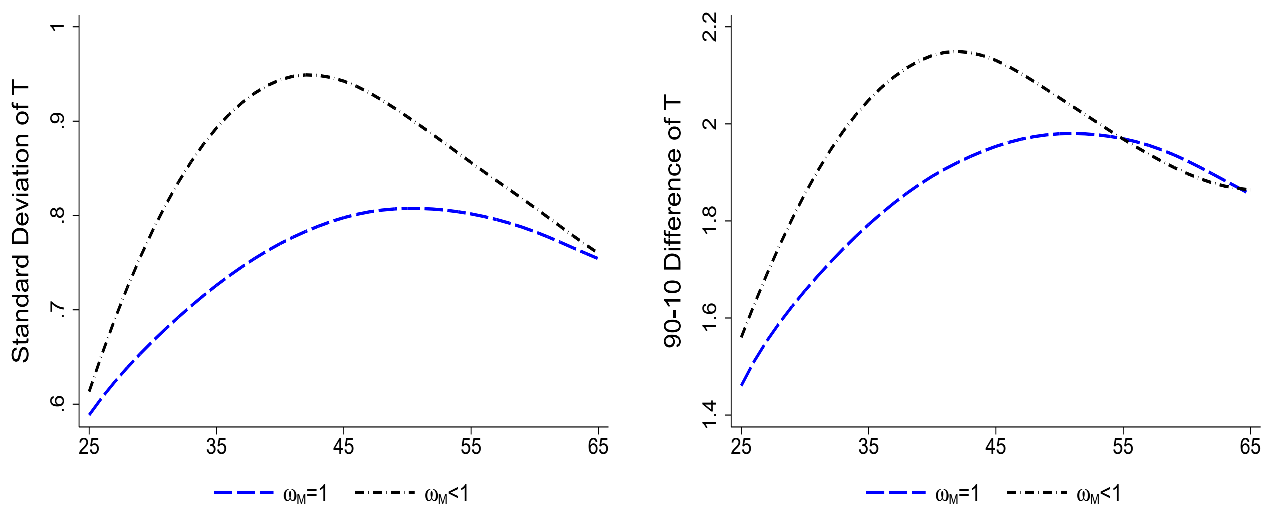


Figure 4: Dispersion in Equivalent Variation

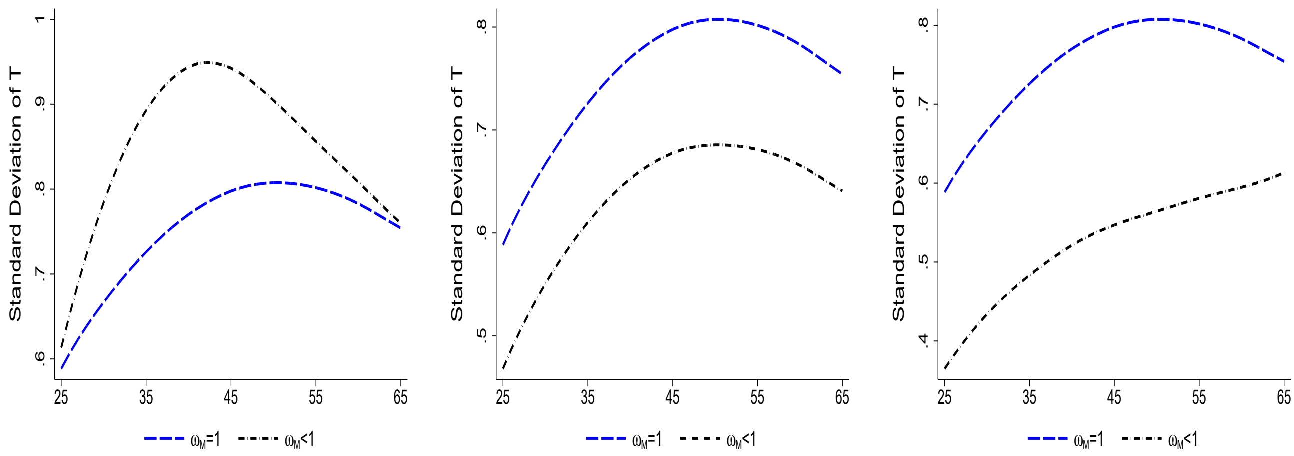
Figure 4 shows the dispersion in equivalent variation T for the model without ($\omega_M = 1$, blue dashed line) and with home production ($\omega_M < 1$, black dotted line) by age. The standard deviation of T is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

without home production. The difference between the two models tends to vanish for households above 60 years old. The right panel shows that we obtain a similarly divergent pattern until age 55 between the two models using the difference between the 90th and 10th percentile in equivalent variation.

What drives our inference that inequality is higher with home production? We argue that an important feature of the data driving our inference is that home hours h_N is not negatively correlated with market consumption c_M and market productivity z_M in the cross section of households. We calculate that h_N has a correlation of roughly 0.10 with $\log z_M$ and roughly 0 with $\log c_M$. Thus, home production does not offset heterogeneity that originates in the market sector. Instead, home production exacerbates inequality given the large dispersion in home productivity.[22]

To illustrate this point, Figure 5 shows illustrative analyses in which we change the correlation

[22] We focus on h_N because its low correlation with c_M and z_M is more informative than the low correlations of h_P and further discuss the role productivity and preference heterogeneity in Section 4.2. Given that child care is the largest subcategory of h_N, our estimate of a weakly positive correlation between h_N and z_M is broadly consistent with the findings of Guryan, Hurst, and Kearney (2008) who document that higher educated and higher income parents tend to spend more time with their children. Appendix Tables A.1, A.2, and A.3 demonstrate that the lack of a negative correlation with wages is present both for individuals and households and is present within age, sex, and education groups. Appendix Tables A.4 and A.5 demonstrate that the correlation of home hours with both consumption and wages is similar in magnitude between the CEX/ATUS sample and PSID samples in which home production time is not imputed.



(a) Baseline (b) $\text{corr}(h_N, \log z_M) = -0.8$ (c) $\text{corr}(h_N, \log c_M) = -0.8$

Figure 5: Counterfactuals of Dispersion in Equivalent Variation

Figure 5 shows the dispersion in equivalent variation T for the model without ($\omega_M = 1$, blue dashed line) and with home production ($\omega_M < 1$, black dotted line) by age in the baseline and in counterfactual datasets.

of home hours h_N with other observables in the data. The left panel repeats the age profile of the standard deviation in equivalent variation $T(\iota)$ shown in the left panel of Figure 4. In the other two panels we repeat our inference of $(\alpha, \varepsilon, B, D, z_N)$ and then calculate the equivalent variation $T(\iota)$ in counterfactual data in which the correlation of home hours h_N with market productivity $\log z_M$ and market expenditures $\log c_M$ is -0.8. The figure shows that if the data featured a significantly more negative correlation between h_N and either $\log z_M$ or $\log c_M$, then we would have concluded that inequality in the model with home production is actually lower.

4.1.2 Redistributive Transfers

Our second measure of inequality is the cross-sectional dispersion in redistributive transfers that would equalize marginal utilities. After households choose their allocations of consumption and hours, we allow a utilitarian planner to redistribute aggregate market consumption across households in order to maximize the average of households' utilities. The dispersion in these transfers captures the extent of redistribution required in order to maximize social welfare or, equivalently, to equalize marginal utilities of market consumption. Formally, the problem we consider is to

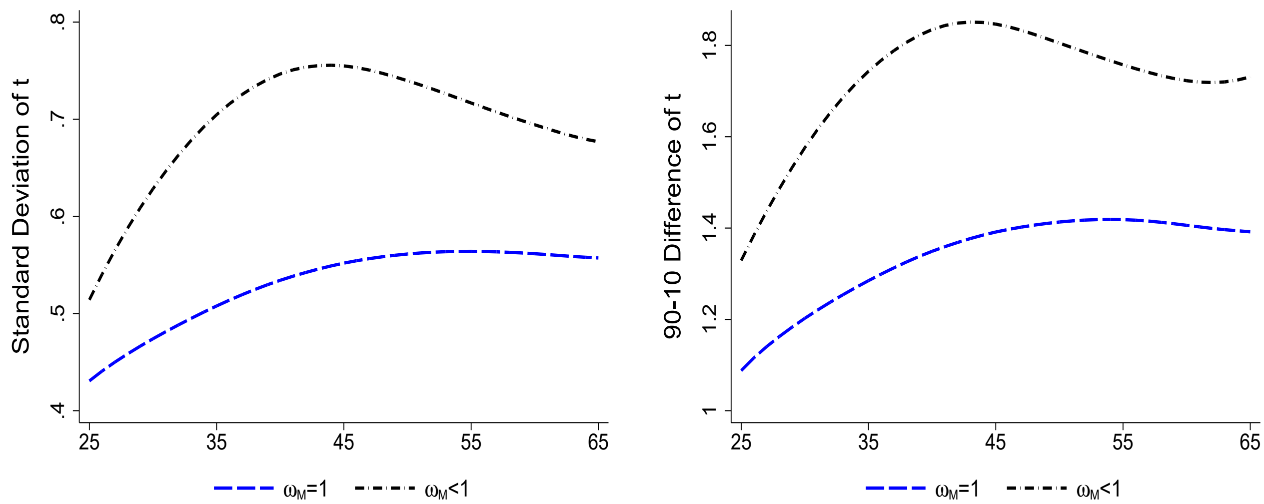


Figure 6: Dispersion in Redistributive Transfers

Figure 6 shows the dispersion in redistributive transfers t for the environment without ($\omega_M = 1$, blue dashed line) and with home production ($\omega_M < 1$, black dotted line) by age. The standard deviation of t is normalized by mean market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$ which is constant across models and ages.

choose transfers $\{t(\iota)\}$ to:

$$\max \int_\iota U(c_M(\iota) + t(\iota), h_M(\iota), h_N(\iota), h_P(\iota))\mathrm{d}\Phi(\iota), \tag{18}$$

subject to aggregate transfers being equal to zero $\int_\iota t(\iota)\mathrm{d}\Phi(\iota) = 0$.

The optimal transfers equal the gap between the average and individual market value of total consumption $c_T(\iota)$:[23]

$$t(\iota) = \int_\iota c_T(\iota)\mathrm{d}\Phi\,(\iota) - c_T(\iota). \tag{19}$$

The dispersion in redistributive transfers $t(\iota)$ differs from the dispersion in equivalent variation $T(\iota)$ in Section 4.1.1 because it leads to an equalization of marginal utilities instead of utility levels. A benefit of dispersion in $t(\iota)$ as a measure of inequality is that it leads to a measure of inequality that transparently depends only on observables and estimated parameters.

The left panel of Figure 6 shows the age profiles for the cross-sectional standard deviation in redistributive transfers $t(\iota)$ for the two models, standardized again by the mean value of market consumption $\int c_M(\iota)\mathrm{d}\Phi(\iota)$. The standard deviation of $t(\iota)$ is larger and increases by more over

[23]We remind the reader that the marginal utility of market consumption under an equilibrium allocation ($c_M + t, c_N, c_P, h_M, h_N, h_P$) equals the inverse of the market value of total consumption c_T given in equation (11).

the life-cycle in the model with home production. We obtain a similar result in the right panel which shows the difference between the 90th and 10th percentile in redistributive transfers $t(\iota)$.

It is instructive to compare our findings using the market value of total consumption $c_T(\iota)$ to other findings in the literature. Some authors such as Frazis and Stewart (2011) and Bridgman, Dugan, Lal, Osborne, and Villones (2012) have embraced the view that home production decreases inequality. Their argument is that, since home hours do not correlate with income in the cross section of households, adding a constant value of home production across households results in a smaller dispersion of total income. Inspection of equation (11) for c_T reveals a fundamental difference in our logic. Home hours in our model are valued at their opportunity cost, which depends on market productivity and sectoral disutilities of work, and the opportunity cost varies across households. Using a constant cost across households to value their home hours does not take into account differences in the productivity of home hours or preferences for home production.[24]

4.1.3 Lifetime Welfare Cost of Heterogeneity

In this section we present the lifetime welfare effect arising from heterogeneity across households. These calculations contrast with our inequality metrics thus far which have ignored dynamic considerations. Specifically, we calculate the fraction of lifetime consumption that a household is willing to sacrifice ex-ante to be indifferent between being born in the baseline environment with heterogeneity and allocations $\{\mathbf{c}_t, \mathbf{h}_t\}$ and a counterfactual environment in which dimensions of heterogeneity are shut down. The allocations in the counterfactual economy are denoted by $\{\hat{\mathbf{c}}_t, \hat{\mathbf{h}}_t\}$ and are generated using the equations in Table 1 after shutting down particular dimensions of heterogeneity.[25]

[24]A reasonable concern with using wages to value home hours is that some households or members of the household may be at a corner solution. In practice, we are not concerned that this biases our results for three reasons. First, in our baseline CEX/ATUS sample of married households the fraction of households with either zero market hours or zero home hours per year is less than one percent. Further, sensitivity analyses presented in Section 5 confirm our inequality results for a sample of singles and for a subsample of married households with a working spouse for which valuation at market wages is less concerning. Finally, our notion of inequality in consumption allows for a wedge between the market wages and marginal value of home hours h_P arising from preference differences across sectors.

[25]For the counterfactuals in this section, and consistently with our definition of equilibrium in which G is an endogenous variable, we choose to keep constant the tax parameters (τ_0, τ_1) because we prefer to evaluate more direct welfare effects from heterogeneity rather than more nuanced effects arising from changes in the tax

Table 6: Lifetime Welfare Cost of Heterogeneity

	No Home Production: $\omega_M = 1$		Home Production: $\omega_M < 1$	
No dispersion in ...	λ_p	λ	λ_p	λ
z_M, z_N, B, D	0.055	0.065	0.065	0.129
z_M, z_N	0.055	0.078	0.065	0.166
z_M	0.055	0.078	0.065	0.118
z_N	—	—	0.000	0.140

Table 6 shows changes in aggregate labor productivity λ_p and welfare λ for the environment without ($\omega_M = 1$) and with ($\omega_M < 1$) home production. In each row we shut down combinations of sources of heterogeneity.

The share of lifetime consumption that households are willing to sacrifice ex-ante to be indifferent between the actual and counterfactual economy is given by the λ that solves:

$$\mathbb{E}_{j-1} V\left(\{c_t, h_{M,t}, h_{N,t}\}\right) = \mathbb{E}_{j-1} V\left(\{(1-\lambda)\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}\}\right) , \tag{20}$$

where $c_t = \left(\omega_M c_{M,t}^{\frac{\phi-1}{\phi}} + \omega_N c_{N,t}^{\frac{\phi-1}{\phi}} + \omega_P c_{P,t}^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$ denotes the CES aggregator of goods in the utility function (1). When $\lambda > 0$, households prefer the counterfactual to the actual allocation. Benabou (2002) and Floden (2001) have emphasized that total welfare effects from eliminating heterogeneity, λ, arise both from level effects when aggregate allocations change and effects capturing changes in the dispersion of allocations across households. Therefore, alongside λ, we discuss how heterogeneity influences aggregate labor productivity $\int_\iota z_M(\iota) h_T(\iota) \mathrm{d}\Phi(\iota) / \int_\iota h_T(\iota) \mathrm{d}\Phi(\iota)$. We denote by λ_p the percent change in aggregate labor productivity between the counterfactual and the baseline allocation. Dispersion in market productivity z_M decreases aggregate labor productivity because h_T is negatively correlated with z_M in both models.

In the first row of Table 6, we shut down all sources of heterogeneity and both models collapse to a representative household economy. The welfare cost of heterogeneity λ is almost 13 percent in the model with home production as opposed to 6.5 percent in the model without home production. The difference between the two models reflects predominately the differential cost of dispersion

parameters in order to satisfy the government budget constraint. By contrast, when we calculate optimal taxes (τ_0, τ_1) in Section 4.1.4, we keep constant G to its initial equilibrium value.

in allocations rather than aggregate productivity changes λ_p which are relatively similar across models.[26]

The larger dispersion costs of heterogeneity in the home production model reflect the costs of dispersion in productivity rather than preferences. To see this, in the second row we shut down heterogeneity in productivities z_M and z_N while we maintain heterogeneity in the disutilities of work B and D. We find even larger welfare effects than row 1 and, thus, conclude that heterogeneity in B and D is not important for the welfare effects of eliminating all heterogeneity. In the third row, we shut down only heterogeneity in market productivity z_M and find that eliminating this source of dispersion carries larger welfare gains in the model with home production than in the model without. In the fourth row, we shut down heterogeneity in home productivity z_N only. We find large welfare effects, which illustrates the key role of z_N heterogeneity for the welfare losses.

4.1.4 Optimal Tax Progressivity

This section contrasts the optimal progressivity of the tax system between the model with and without home production. Relative to our previous inequality metrics, the optimal taxation exercise mixes redistribution with efficiency concerns because the optimal progressivity of the tax system increases with redistributive motives and decreases with the efficiency losses from distorting labor allocations. However, this exercise allows us to more directly link our inequality result to policy.

Given government expenditures G fixed at its initial equilibrium level, the government chooses tax function parameters $\tau \equiv (\tau_0, \tau_1)$ to maximize utilitarian welfare:

$$\max_{\tau} \int_{\iota} U(\mathbf{c}(\tau), \mathbf{h}(\tau); \iota) \mathrm{d}\Phi(\iota) \ , \tag{21}$$

[26] The welfare effects in Table 6 reflect heterogeneity both within age and over the life-cycle because each counterfactual imposes a constant value of the source of heterogeneity for households of all ages. We have repeated these exercises by shutting down only the within-age heterogeneity and allowing each source of heterogeneity to take its mean value over the life-cycle as shown previously in Figure 1. Appendix Table A.8 shows similar welfare effects to those shown in Table 6 and, therefore, we conclude that the welfare effects predominately reflect the within-age component of heterogeneity.

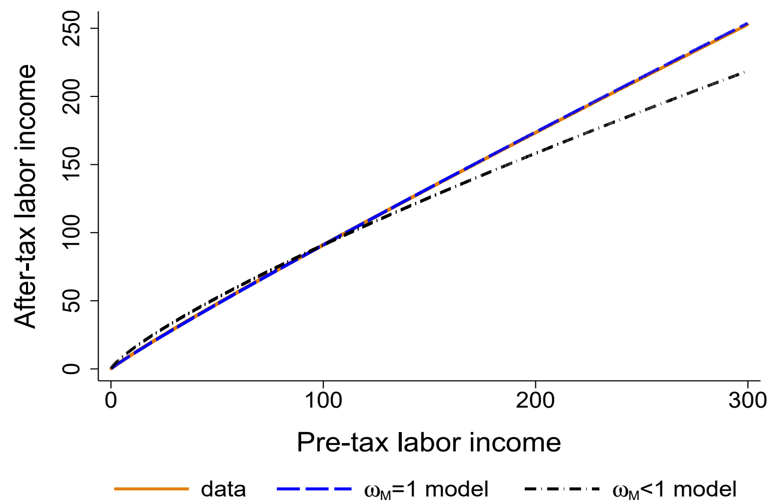


Figure 7: Optimal Tax Function

Figure 7 displays the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ under the parameters estimates for the United States (orange solid line), under the optimal tax function for the model without home production ($\omega_M = 1$, blue dashed line), and under the optimal tax function with home production ($\omega_M < 1$, black dotted line).

subject to the government budget constraint:

$$\int_\iota \left[z_M - (1-\tau_0){z_M}^{1-\tau_1}\right] h_M(\tau) \mathrm{d}\Phi(\iota) = G. \tag{22}$$

In formulating this Ramsey problem, we have assumed a stationary environment in which the government faces an identical cross section of households in each year.

In Figure 7 we plot the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ (in thousands of 2010 dollars). The orange solid curve shows the relationship between y and $\tilde{y}$ under the parameter $\tau_1 = 0.12$ that we estimated in the data for the United States. The blue dashed and black dotted curves show this relationship under the optimal $\tau_1 = 0.11$ for the model without home production and the optimal $\tau_1 = 0.28$ for the model with home production. The relationship between y and $\tilde{y}$ is significantly more concave in the model with home production. To give an example, consider a household earning 200 thousand dollars. Under the optimal tax schedule in the model without home production the household faces an average tax rate of 13 percent, while in the model with home production the average tax rate increases to 21 percent.

Table 7: The Role of Home Productivity and Preferences in Amplifying Inequality

Statistics	No Home Production	Home Production		
		Productivity	Baseline	Preferences
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.21	0.13	0.04
τ_1	0.11	0.35	0.28	0.15

Table 7 shows the four inequality metrics for the model without home production, the home production model with only productivity heterogeneity, the baseline home production model with both productivity and preference heterogeneity, and the home production model with only preference heterogeneity. The preference weights $(\omega_M, \omega_N, \omega_P)$ in the four models are given by $(1, 0, 0)$, $(0.45, 0.55, 0)$, $(0.40, 0.38, 0.22)$, $(0.54, 0, 0.46)$. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. The estimated values for η are 0.90, 0.53, 0.50, and 0.57.

4.2 The Role of Productivity and Preference Heterogeneity

Using four different metrics of inequality, we have demonstrated that home production amplifies inequality across households. In this section we explore the roles of productivity heterogeneity and preference heterogeneity at home in amplifying inequality across households. We compare our baseline model with both productivity and preference heterogeneity to two nested versions of the home production model. In the first version, we set the preference weight $\omega_P = 0$ and, therefore, differences in the allocation of time across households originate from productivity. In the second version, we set the preference weight $\omega_N = 0$ and, therefore, differences in the allocation of time across households originate from preferences.

Table 7 summarizes our results. The first column presents the four inequality metrics (averaged across all ages) in the model without home production and the last three columns present the metrics in the three versions of the home production model. In the home production model with only productivity heterogeneity, all inequality metrics are magnified relative to the baseline with both productivity and preference heterogeneity. If there was only preference heterogeneity, there would be no significant difference in inequality between the model with and the model without home production. We conclude that productivity heterogeneity in the home sector rather than preference heterogeneity is important in amplifying inequality across households.

5 Sensitivity Analyses

In this section we present various sensitivity analyses in the CEX/ATUS sample with respect to the parameterization, the measures of consumption, the subsamples of the population, and measurement error in observables. Each row in Table 8 corresponds to a different sensitivity analysis. For both models, the columns show the standard deviation in equivalent variation T, the standard deviations in transfers t required to equalize marginal utilities, the ex-ante lifetime welfare loss λ from shutting down all heterogeneity, and the degree of progressivity τ_1 in an optimal tax system. In each exercise, we repeat our analysis of identifying the sources of heterogeneity $(\alpha, \varepsilon, B, D, z_N)$ and then calculate the inequality metrics. The first row of the table repeats these statistics for our baseline case.

Rows 2 to 9 vary parameters of the model. Relative to our estimated value $\tau_1 = 0.12$, changing the progressivity of the tax system to $\tau_1 = 0.06$ as in Guner, Kaygusuz, and Ventura (2014) or to $\tau_1 = 0.19$ as in Heathcote, Storesletten, and Violante (2014) does not alter significantly any result. We also obtain a similar result when we change the target for the average labor income tax G/Y to 0.05 or 0.15. In rows 6 and 7 we change the target coefficient from the regression of $\log h_M$ on ε used to identify the parameter η which governs the curvature of the utility function with respect to effective total hours. Raising η to target a coefficient of 0.70 as suggested by Pistaferri (2003) results in larger inequality in both models, but in all cases inequality is higher in the model with home production.

In rows 8 and 9, we vary the elasticity of substitution ϕ. The value of the $\text{Std}(t)$ measure of inequality is relatively insensitive to ϕ. When $\phi = 0.5$ and goods are complements, the $\text{Std}(T)$, λ, and τ_1 metrics of inequality increase substantially. Intuitively, the complementarity between goods implies that home production amplifies differences in the market sector even more. When $\phi = 20$ and goods are almost perfect substitutes, we still find that inequality is higher with home production according to the $\text{Std}(T)$, $\text{Std}(t)$, and λ metrics but to a lesser extent than with lower

Table 8: Sensitivity Analyses of Inequality Metrics

		No Home Production: $\omega_M = 1$				Home Production: $\omega_M < 1$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.11	0.90	0.73	0.13	0.28
	Parameter Values								
2.	$\tau_1 = 0.06$	0.78	0.55	0.07	0.17	0.93	0.74	0.15	0.30
3.	$\tau_1 = 0.19$	0.78	0.55	0.06	0.02	0.88	0.72	0.11	0.24
4.	$G/Y = 0.05$	0.78	0.55	0.06	0.09	0.91	0.73	0.13	0.28
5.	$G/Y = 0.15$	0.78	0.55	0.06	0.14	0.90	0.73	0.13	0.28
6.	Target Frisch = 0.4	0.68	0.55	0.03	-0.67	0.80	0.73	0.11	0.11
7.	Target Frisch = 0.7	0.85	0.55	0.09	0.30	0.98	0.73	0.14	0.34
8.	$\phi = 0.5$	0.78	0.55	0.06	0.11	1.93	0.70	0.53	0.52
9.	$\phi = 20$	0.78	0.55	0.06	0.11	0.85	0.71	0.10	-0.80
	Definition of Consumption Expenditures								
10.	Food expenditures	0.82	0.56	0.05	0.02	0.92	0.75	0.13	0.25
11.	All expenditures	0.88	0.63	0.08	0.29	0.99	0.83	0.13	0.33
	Marital, Employment, Family, and Education Groups								
12.	Singles	0.89	0.61	0.01	0.06	0.90	0.71	0.09	0.15
13.	Non-working spouse	0.79	0.55	0.11	0.23	1.32	1.07	0.22	0.34
14.	Working spouse	0.78	0.54	0.06	0.14	0.85	0.70	0.11	0.27
15.	No children	0.79	0.55	0.11	-0.01	0.81	0.67	0.19	0.18
16.	One child	0.78	0.55	0.07	0.14	0.85	0.72	0.12	0.30
17.	Two or more children	0.77	0.53	0.04	0.20	0.96	0.77	0.19	0.33
18.	Less than college	0.78	0.55	0.03	-0.16	0.86	0.71	0.07	0.18
19.	College or more	0.76	0.58	0.06	-0.04	0.86	0.68	0.16	0.24

Table 8 presents each sensitivity analysis in a row. Columns show the four inequality metrics for the model without home production ($\omega_M = 1$) and the model with home production ($\omega_M < 1$).

ϕ. The main difference with our baseline arises in terms of the optimal progressivity which is significantly affected by the value of ϕ. Because a higher value of ϕ increases the efficiency losses from a progressive tax system, we obtain a lower τ_1 in the model with home production and $\phi = 20$ than in the model without home production.

In rows 10 and 11 we show that our results are robust under two alternative measures of market expenditures c_M. In row 10 we use food only whereas in row 11 we use all expenditures including health, education, and durables. The inequality metrics and the gap between the two models are generally similar to the baseline which used nondurable consumption excluding health and education. From the four metrics, the optimal progressivity τ_1 is the most sensitive to the measure of consumption.

In rows 12 to 19 of Table 8 we repeat our analyses in subsamples of households defined along their marital status, employment status of the spouse of the head, number of children, and education. Repeating our analyses for different samples allows us to explore whether our inequality results reflect within group inequality or inequality across groups. Additionally, verifying our results at the subgroup level is reassuring because one would expect dimensions of heterogeneity that we did not model, such as spousal employment at the extensive margin or number of children, to be less important within more narrowly defined groups.

Our results are remarkably stable at the subgroup level, with the home production model always generating more inequality than the model without home production according to all four metrics. Row 12 shows the sample of singles, for which the inequality gap between models is generally smaller. Rows 13 and 14 show subsamples of married households according to whether the spouse is working or not. Reassuringly for the mechanisms we have stressed, we obtain a larger inequality gap for the group of non-working spouses for which we would expect home productivity differences to be more important. In rows 15 to 17 we differentiate according to the number of children present in the household. We obtain larger inequality gaps among households with more children, which highlights the importance of time spent on child care for our results. Finally, rows 18 and 19 show results for married households with a head who has not completed college and with a head who has completed college or more. Our results are similar to the baseline with the exception of the optimal progressivity τ_1 which declines substantially in the model without home production.

Table 9 examines the sensitivity of our results to measurement error. The first row repeats the

Table 9: Inequality Metrics and Measurement Error

		No Home Production: $\omega_M = 1$				Home Production: $\omega_M < 1$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
1.	Baseline	0.78	0.55	0.06	0.11	0.90	0.73	0.13	0.28
	Consumption $x = c_M$								
2.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.74	0.51	0.06	0.15	0.88	0.69	0.13	0.29
3.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.61	0.41	0.05	0.17	0.79	0.60	0.13	0.29
4.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.45	0.26	0.02	0.19	0.69	0.47	0.12	0.29
	Market Hours $x = h_M$								
5.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.79	0.55	0.07	0.13	0.90	0.73	0.13	0.28
6.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.79	0.55	0.07	0.18	0.90	0.74	0.13	0.29
7.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.80	0.55	0.07	0.25	0.88	0.73	0.13	0.33
	Home Hours $x = \{h_N, h_P\}$								
8.	$\sigma_m^2/\text{var}(\log x) = 0.20$	0.78	0.55	0.06	0.11	0.93	0.74	0.13	0.27
9.	$\sigma_m^2/\text{var}(\log x) = 0.50$	0.78	0.55	0.06	0.11	0.89	0.73	0.14	0.27
10.	$\sigma_m^2/\text{var}(\log x) = 0.80$	0.78	0.55	0.06	0.11	0.78	0.70	0.15	0.28

Table 9 presents each sensitivity analysis in a row. Columns show the four inequality metrics for the model without home production ($\omega_M = 1$) and the model with home production ($\omega_M < 1$).

baseline case without measurement error. For the other rows, we consider a classical measurement error model in which the reported value of a variable x for household ι is:

$$\log x(\iota) = \log x^*(\iota) + m(\iota), \tag{23}$$

where x^* is the true and unobserved value of variable x and m denotes a classical measurement error with variance σ_m^2. Rows 2 to 4 show results with measurement error in market consumption ($x = c_M$), rows 5 to 7 show results with measurement error in market hours ($x = h_M$), and rows 8 to 10 show results with measurement error in home hours ($x = \{h_N, h_P\}$). For each variable we show measurement errors that absorb 20, 50, and 80 percent of the variance of the observed variable.

Our process is to draw measurement error with variance σ_m^2 across households for each variable and then use the true values x^* from equation (23) as the data for the extraction of the sources of heterogeneity $(\alpha, \varepsilon, B, D, z_N)$. The table shows the calculation of the four inequality metrics for the same parameters as in our baseline case without measurement error. We find small differences relative to our baseline results. Inequality tends to decline with measurement error in consumption, but not differentially across the two models. For market hours, measurement error affects only the optimal progressivity τ_1, but we always find that progressivity is higher in the home production model. Finally, most of our results are robust to measurement error of up to 80 percent of the variance of home hours. At that level, the dispersion in equivalent variation in the model with home production is equalized to the dispersion in the model without home production. We still obtain higher inequality with home production using the other three metrics of inequality.

6 Other Datasets and Countries

We show the similarity of the inequality results between the CEX/ATUS and three alternative datasets, the Panel Study of Income Dynamics (PSID), the Japanese Panel Survey of Consumers (JPSC), and the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS).

6.1 Comparison between CEX/ATUS and PSID

The PSID has two advantages relative to the CEX/ATUS. It has a panel dimension and contains information on both expenditures and time spent on home production. However, we prefer using the CEX/ATUS sample for our baseline analyses for three reasons. First, the PSID survey question covers aggregated time spent on home production, which does not allow us to separate credibly home hours h_N in the sector with productivity heterogeneity from home hours h_P in the sector with preference heterogeneity. Second, the PSID has lower quality of time use data as compared to the time diaries from the ATUS. In particular, it is not clear if respondents include activities such as child care and shopping in their reported home hours.[27] Third, food is the

[27] The survey question is "About how much time do you spend on housework in an average week? I mean time spent cooking, cleaning, and doing other work around the house."

only measure of consumption which is consistently covered across surveys. Later surveys cover expanded categories but the sample size is significantly smaller than the CEX/ATUS sample.

We use two versions of the PSID. In the version in which c_M includes only expenditures on food, we have 69,951 observations between 1975 and 2014 for 10,992 households. In the version in which c_M includes food, utilities, child care expenses, clothing, home insurance, telecommunication, transportation, and home repairs, we have 13,626 observations between 2004 and 2014. PSID does not have information that allows us to disaggregate time spent on home production between h_N and h_P. To make the analyses as comparable as possible to CEX/ATUS, we consider three cases. The first is when all home hours belong to h_N in the sector with productivity heterogeneity. The second case, which is more similar to our benchmark in the CEX/ATUS, is that home hours are split equally between the two sectors.[28] The third case is when all home hours belong to h_P in the sector with preference heterogeneity.

Table 10 reassesses our conclusions regarding inequality.[29] The first panel repeats the findings of Table 7 in the CEX/ATUS for the four inequality metrics in the model without home production, the home production model with only productivity heterogeneity, the baseline home production model with both productivity and preference heterogeneity, and the home production model with only preference heterogeneity. The second panel reports these statistics for the version of the PSID that includes an expanded set of consumption categories. The third and fourth panels report these statistics for the CEX/ATUS and PSID datasets when we restrict our measure of consumption to include only food.

Our conclusions regarding inequality and the role of productivity heterogeneity are stable across the four datasets. First, the baseline model with home production generates higher inequality than the model without home production. Second, in the model with only productivity heterogeneity, all inequality metrics are magnified relative to the baseline with both productivity

[28] Differences still exist, however, because h_N and h_P in the CEX/ATUS are far from perfectly correlated.

[29] To isolate differences in the samples rather than differences stemming from parameters, we keep parameters fixed at their values shown in Table 5. We follow a similar strategy with the JPSC and the LISS datasets later. The exception is the preference weights $(\omega_M, \omega_N, \omega_P)$ that we recalibrate to hit the same targets as in the CEX/ATUS using equations (13).

Table 10: Inequality and Home Production: CEX/ATUS and PSID

CEX All	No Home Production	Home Production		
Statistics		Productivity	Baseline	Preferences
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.21	0.13	0.04
τ_1	0.11	0.35	0.28	0.15
PSID All	**No Home Production**	**Home Production**		
Statistics		Productivity	Baseline	Preferences
std(T)	0.59	0.88	0.64	0.56
std(t)	0.40	0.62	0.51	0.45
λ	0.10	0.17	0.13	0.08
τ_1	0.22	0.27	0.25	0.21
CEX Food	**No Home Production**	**Home Production**		
Statistics		Productivity	Baseline	Preferences
std(T)	0.82	1.15	0.92	0.79
std(t)	0.56	0.84	0.75	0.67
λ	0.05	0.21	0.13	0.03
τ_1	0.02	0.33	0.25	0.11
PSID Food	**No Home Production**	**Home Production**		
Statistics		Productivity	Baseline	Preferences
std(T)	0.61	0.98	0.69	0.59
std(t)	0.40	0.67	0.54	0.46
λ	0.10	0.22	0.17	0.11
τ_1	0.44	0.45	0.40	0.34

Table 10 shows the four inequality metrics for the model without home production, the home production model with only productivity heterogeneity, the baseline home production model with both productivity and preference heterogeneity, and the home production model with only preference heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The preference weights $(\omega_M, \omega_N, \omega_P)$ for the home production models are given by $(0.45, 0.55, 0)$, $(0.40, 0.38, 0.22)$, $(0.54, 0, 0.46)$ in the first panel; $(0.51, 0.49, 0)$, $(0.46, 0.32, 0.22)$, $(0.60, 0, 0.40)$ in the second panel; $(0.44, 0.56, 0)$, $(0.39, 0.38, 0.22)$, $(0.54, 0, 0.46)$ in the third panel; $(0.49, 0.51, 0)$, $(0.43, 0.33, 0.24)$, $(0.57, 0, 0.43)$ in the fourth panel.

and preference heterogeneity. Third, if there was only preference heterogeneity, there would be no significant difference in inequality between the model with and the model without home production. The only significant change in the PSID relative to the CEX/ATUS is in the optimal progressivity τ_1 which displays a smaller difference between the two models.[30]

Our results using the PSID are particularly reassuring because we do not take a stance about the classification of time uses between h_N and h_P. Therefore, the result that inequality is higher with home production does not hinge on which activities are subject to productivity heterogeneity and which activities are subject to preference heterogeneity. What is important for this result is that some portion of home production time is subject to productivity heterogeneity.

6.2 Comparison between US, Japan, and the Netherlands

In this section, we repeat our analyses to datasets from other countries. As in the PSID, these datasets have in general limited information that allows us to disaggregate time spent on home production between h_N and h_P. To make the analyses as comparable as possible to CEX/ATUS, we consider the three cases of all home hours belonging to h_N in the sector with productivity heterogeneity, of splitting home hours equally between the two sectors, and of all home hours belonging to h_P in the sector with preference heterogeneity. We apply the same sampling restrictions as in the CEX/ATUS and focus our analyses on married households.

The first dataset is the Japanese Panel Survey of Consumers (JPSC; see, for example, Lise and Yamada, 2018). The JPSC records information for time spent on commuting, working, studying, home production and child care, leisure, and sleeping, personal care and eating. For home hours we use the variable for home production and child care and for market hours we use the hours worked. To calculate the home and market hours for a given week, we weight the time use on workdays and days off by the number of days worked. Our measure of consumption

[30] Appendix Figures A.3 and A.4 display the age profiles for the means and variances of the sources of heterogeneity $(\alpha, \varepsilon, B, D, \log z_N)$ from the version of the PSID with food in the baseline case which splits home hours equally between h_N and h_P. The difference relative to the means and variances we extracted using the CEX/ATUS is that we obtain these age profiles by regressing each inferred shifter on age and year dummies and an individual fixed effect. Therefore, these profiles reflect the within-household evolution of the sources of heterogeneity. Despite this difference, most of age profiles in the PSID are quantitatively similar to the age profiles in the CEX/ATUS.

expenditures includes food, utilities, apparel, transport, culture and leisure, communication, trips and activities, house and land rent. Our findings are robust to broader and narrower definitions of consumption expenditures. Our final dataset has 12,423 observations between 1998 and 2014. The second dataset is the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS; see, for example, Cherchye, Demuynck, De Rock, and Vermeulen, 2017), administered by CentERdata in the Netherlands. The dataset is based on a representative sample of Dutch households who participate in monthly surveys. We use the three waves (2009, 2010, and 2012) that contain information on time use. Home production time includes household chores, child care, and administrative chores. Market hours are measured by time spent on paid work, which includes commuting time. Consumption expenditures include food, utilities, home maintenance, transportation, daycare, and child support. The final dataset has 978 observations.

Table 11 summarizes our results. The first panel repeats our findings in the CEX/ATUS and the other panels show inequality statistics in the JPSC and the LISS. Our conclusions regarding inequality and the role of productivity heterogeneity are stable in other countries as well. Namely, the baseline model with home production always generates higher inequality than the model without home production. All inequality statistics are magnified in the home production model with only productivity heterogeneity, whereas with only preference heterogeneity there would be no significant difference in inequality between the models with and without home production.

7 Conclusion

The literature examining the causes, welfare consequences, and policy implications of the substantial labor market dispersion we observe across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. We revisit these issues taking into account that households spend a significant amount of their time in home production. Our model incorporates non-separable preferences between expenditures and time and home productivity and preference differences across households into a standard incomplete markets model with uninsurable risk.

Table 11: Inequality and Home Production: US, Japan, and the Netherlands

CEX/ATUS Statistics	No Home Production	Home Production Productivity	Home Production Baseline	Home Production Preferences
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.21	0.13	0.04
τ_1	0.11	0.35	0.28	0.15
JPSC Statistics	**No Home Production**	**Productivity**	**Baseline**	**Preferences**
std(T)	0.66	0.99	0.76	0.67
std(t)	0.46	0.68	0.60	0.56
λ	0.04	0.11	0.07	0.02
τ_1	-0.15	0.19	0.11	0.03
LISS Statistics	**No Home Production**	**Productivity**	**Baseline**	**Preferences**
std(T)	0.64	1.12	0.80	0.64
std(t)	0.45	0.77	0.63	0.54
λ	0.03	0.21	0.12	0.02
τ_1	-0.80	-0.12	-0.24	-0.80

Table 11 shows the four inequality metrics for the model without home production, the home production model with only productivity heterogeneity, the baseline home production model with both productivity and preference heterogeneity, and the home production model with only preference heterogeneity. Parameters τ_0, τ_1, and ϕ are held constant to their values shown in Table 5. For each column, the values for η are given by 0.90, 0.53, 0.50, and 0.57 (constant across panels). The preference weights $(\omega_M, \omega_N, \omega_P)$ for the home production models are given by $(0.45, 0.55, 0)$, $(0.40, 0.38, 0.22)$, $(0.54, 0, 0.46)$ in the first panel; $(0.46, 0.54, 0)$, $(0.38, 0.33, 0.28)$, $(0.50, 0, 0.40)$ in the second panel; $(0.46, 0.54, 0)$, $(0.40, 0.36, 0.24)$, $(0.56, 0, 0.44)$ in the third panel.

We reach several substantial conclusions. We find that home production amplifies welfare-based differences across households and inequality is larger than we thought. Our result is surprising given that a priori one could expect that home production tends to compress welfare differences that originate in the market sector when households are sufficiently willing to substitute between market expenditures and time in the production of home goods. We show that home productivity is an important source of within-age and life-cycle differences in consumption expenditures and time allocation across households. Through the lens of the model, we infer that home production does not offset differences that originate in the market sector because productivity differences in the home sector are large and the time input in home production does not covary with consumption expenditures and wages in the cross section of households. These results support the view that the optimal tax system should feature more progressivity when incorporating the fact that households can produce goods and services at home.

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Inferring Inequality with Home Production

Online Appendix

Job Boerma and Loukas Karabarbounis

A Proofs

In this appendix, we derive the equilibrium allocations presented in Table 1 in the main text and prove the observational equivalence theorem. We proceed in four steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 2 in the main text. Finally, we show how to invert the equilibrium allocations and solve for the sources of heterogeneity that lead to these allocations.

A.1 Preliminaries

In what follows, we define the following state vectors. The idiosyncratic shifters that differentiate households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can trade bonds within each island contingent on the vector s^j:

$$s_t^j = (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{z_N^j, D^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, we denote the history of all sources of heterogeneity up to period t with the vector:

$$\sigma_t^j = (z_{N,t}^j, D_t^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, ..., z_{N,j}^j, D_j^j, B_j^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(.|.)$. For example, the probability that we observe σ_t^j conditional on σ_{t-1}^j is $f^{t,j}(\sigma_t^j|\sigma_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to the processes and Φ_υ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^B}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi_{z_{N,t}}^j, \Phi_{D_t}^j\}$, and the initial distributions to vary over cohorts j, $\{\Phi_{\alpha,j}^j, \Phi_{B,j}^j, \Phi_{\kappa,j}^j\}$. We assume that both $z_{N,t}^j$ and D_t^j are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

A.2 Planner Problems

In every period t and in every island ℓ, the planner solves a static problem that consists of finding the allocations that maximize average utility for households on the island subject to an aggregate resource constraint and household-specific home production technologies. We omit t and ℓ from the notation for convenience.

A.2.1 No Home Production, $\omega_M = 1$

The planner chooses an allocation $\{c_M(\iota), h_M(\iota)\}$ to maximize:

$$\int_Z \left[\frac{c_M(\iota)^{1-\gamma} - 1}{1-\gamma} - \frac{(\exp(B(\iota))h_M(\iota))^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta) \ , \tag{A.5}$$

subject to an island resource constraint for market goods:

$$\int_Z c_M(\iota)\, \mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M(\iota)\, h_M(\iota)\, \mathrm{d}\Phi_\zeta(\zeta) \, . \tag{A.6}$$

Denoting by $\mu(\alpha, B)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M(\iota)]: \ c_M(\iota)^{-\gamma} = \mu(\alpha, B), \tag{A.7}$$

$$[h_M(\iota)]: \ \exp(B(\iota))^{1+\frac{1}{\eta}} h_M(\iota)^{\frac{1}{\eta}} = \tilde{z}_M(\iota)\mu(\alpha, B). \tag{A.8}$$

Equation (A.7) implies that market consumption is equalized for every ι on the island and, thus, there is full consumption insurance. Combining equations (A.6) to (A.8), we solve for market consumption and market hours for every ι:

$$c_M(\iota) = \left[\frac{\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)}{\exp\left(\eta\left(1+\frac{1}{\eta}\right)B(\iota)\right)} \right]^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}}, \tag{A.9}$$

$$h_M(\iota) = \tilde{z}_M(\iota)^\eta \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}}{\exp\left(\left(1+\frac{1}{\eta}\right)B(\iota)\right)^{\frac{1}{\eta}+\gamma}}. \tag{A.10}$$

A.2.2 Home Production, $\omega_M < 1$

The planner chooses $\{c_M(\iota), c_N(\iota), c_P(\iota), h_M(\iota), h_N(\iota), h_P(\iota)\}$ to maximize:

$$\int_Z \left[\log\left(\omega \cdot \mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} - \frac{\left(\exp(B(\iota))\big(h_M(\iota)+h_N(\iota)\big)+\exp(D(\iota))h_P(\iota)\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta), \tag{A.11}$$

where $\omega \equiv (\omega_M, \omega_N, \omega_P)$ and $\mathbf{c} \equiv (c_M, c_N, c_P)$, subject to the island market resource constraint (A.6) and the home production technologies:

$$c_N(\iota) = z_N(\iota)\, h_N(\iota), \tag{A.12}$$

$$c_P(\iota) = z_P(\iota)\, h_P(\iota). \tag{A.13}$$

Denoting by $\mu(\alpha, B, D, z_N)$ the multiplier on the island resource constraint and by $\chi(\iota)$ and $\Lambda(\iota)$ the multipliers on the household's home production constraints, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M(\iota)]: \quad \left(\omega \cdot \mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1} \omega_M c_M(\iota)^{-\frac{1}{\phi}} = \mu(\alpha, B, D, z_N), \tag{A.14}$$

$$[c_N(\iota)]: \quad \left(\omega \cdot \mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1} \omega_N c_N(\iota)^{-\frac{1}{\phi}} = \chi(\iota), \tag{A.15}$$

$$[c_P(\iota)]: \quad \left(\omega \cdot \mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1} \omega_P c_P(\iota)^{-\frac{1}{\phi}} = \Lambda(\iota), \tag{A.16}$$

$$[h_M(\iota)]: \quad \left(\exp(B(\iota))\big(h_M(\iota)+h_N(\iota)\big)+\exp(D(\iota))h_P(\iota)\right)^{\frac{1}{\eta}} = \tilde{z}_M(\iota)\frac{\mu(\alpha, B, D, z_N)}{\exp(B(\iota))}, \tag{A.17}$$

$$[h_N(\iota)]: \quad \left(\exp(B(\iota))\big(h_M(\iota)+h_N(\iota)\big)+\exp(D(\iota))h_P(\iota)\right)^{\frac{1}{\eta}} = z_N(\iota)\chi(\iota)/\exp(B(\iota)), \tag{A.18}$$

$$[h_P(\iota)]: \quad \left(\exp(B(\iota))\big(h_M(\iota)+h_N(\iota)\big)+\exp(D(\iota))h_P(\iota)\right)^{\frac{1}{\eta}} = z_N(\iota)\Lambda(\iota)/\exp(D(\iota)). \tag{A.19}$$

Combining equations (A.14) to (A.19), we solve for the ratio of consumptions:

$$\frac{c_M(\iota)}{c_N(\iota)} = \left(\frac{\omega_M}{\omega_N}\right)^\phi \left(\frac{\tilde{z}_M(\iota)}{z_N(\iota)}\right)^\phi, \tag{A.20}$$

$$\frac{c_M(\iota)}{c_P(\iota)} = \left(\frac{\omega_M}{\omega_P}\right)^{\phi}\left(\frac{\tilde{z}_M(\iota)\,/\exp(B(\iota))}{z_N(\iota)\,/\exp(D(\iota))}\right)^{\phi}. \tag{A.21}$$

Substituting these ratios into equations (A.14) to (A.16), we derive:

$$c_M(\iota) = \frac{1}{\mu(\alpha, B, D, z_N)} \frac{1}{1 + \left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^{\phi}\left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.22}$$

$$c_N(\iota) = \frac{1}{\mu(\alpha, B, D, z_N)} \frac{\left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^{\phi}\left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.23}$$

$$c_P(\iota) = \frac{1}{\mu(\alpha, B, D, z_N)} \frac{\left(\frac{\omega_P}{\omega_M}\right)^{\phi}\left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^{\phi}\left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}. \tag{A.24}$$

These expressions, combined with the home production technologies (A.12) and (A.13), yield solutions for $\{c_M(\iota), c_N(\iota), c_P(\iota), h_M(\iota), h_N(\iota), h_P(\iota)\}$ given a multiplier $\mu(\alpha, B, D, z_N)$. The multiplier is equal to the inverse of the market value of total consumption:

$$\begin{aligned} c_M(\iota) + \tilde{z}_M(\iota)\,h_N(\iota) + \frac{\exp(D(\iota))}{\exp(B(\iota))}\tilde{z}_M(\iota)\,h_P(\iota) &= c_M(\iota) + \frac{\tilde{z}_M(\iota)}{\tilde{z}_N(\iota)}c_N(\iota) + \frac{\exp(D(\iota))\,/z_P(\iota)}{\exp(B(\iota))\,/\tilde{z}_M(\iota)}c_P(\iota) \\ &= \frac{1}{\mu(\alpha, B, D, z_N)}. \end{aligned} \tag{A.25}$$

The first equality follows from the home production technologies (A.12) and (A.13) and the second equality follows from equations (A.22) to (A.24).

Substituting equation (A.17) into equation (A.6), we obtain the solution for $\mu(\alpha, B, D, z_N)$:

$$\mu(\alpha, B, D, z_N) = \frac{\exp(B(\iota))}{\left(\int_Z \tilde{z}_M(\iota)^{1+\eta}\,\mathrm{d}\Phi_\zeta(\zeta)\right)^{\frac{1}{1+\eta}}}. \tag{A.26}$$

The denominator is an expectation independent of ζ. Therefore, μ is independent of ζ. We also note that $\mu(\alpha, B, D, z_N)$ in the model with home production equals $\mu(\alpha, B)$ in the model without home production under $\gamma = 1$. Given this solution for $\mu(\alpha, B, D, z_N)$, we obtain the solutions:

$$c_M(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))} \frac{1}{1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.27}$$

$$c_N(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))} \frac{\left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.28}$$

$$c_P(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))} \frac{\left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.29}$$

$$h_N(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{z_N(\iota)\exp(B(\iota))} \frac{\left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.30}$$

$$h_P(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{z_P(\iota)\exp(B(\iota))} \frac{\left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi}}{1 + \left(\frac{\omega_N}{\omega_M}\right)^\phi \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^\phi \left(\frac{\exp(B(\iota))/\tilde{z}_M(\iota)}{\exp(D(\iota))/z_P(\iota)}\right)^{\phi-1}}, \tag{A.31}$$

$$h_M(\iota) = \tilde{z}_M(\iota)^\eta \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{1}{1+\frac{1}{\eta}}}}{\exp\big(B(\iota)\big)} - h_N(\iota) - \frac{\exp(D(\iota))}{\exp(B(\iota))} h_P(\iota).$$

A.3 Postulating Equilibrium Allocations and Prices

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade across islands, $x(\zeta_{t+1}^j; \iota) = 0, \forall \iota, \zeta_{t+1}^j$.

2. We postulate that the solutions $\{c_{M,t}(\iota), h_{M,t}(\iota)\}$ for the model without home production and $\{c_{M,t}(\iota), c_{N,t}(\iota), c_{P,t}(\iota), h_{M,t}(\iota), h_{N,t}(\iota), h_{P,t}(\iota)\}$ for the model with home production from the planner problems in Section A.2 constitute components of the equilibrium for each

model.

3. We use the sequential budget constraints to postulate equilibrium holdings for the bonds $b^\ell(s_t^j;\iota)$ that are traded within islands. For the models without home production these are given by:

$$b^\ell(s_t^j;\iota) = \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n \frac{\mu_{t+n}(\alpha_{t+n}^j, B_{t+n}^j)}{\mu_t(\alpha_t^j, B_t^j)}\left(c_{M,t+n}(\iota) - \tilde{y}_{t+n}(\iota)\right)\right], \tag{A.32}$$

where $\tilde{y} = \tilde{z}_M h_M = (1-\tau_0) z_M^{1-\tau_1} h_M$ denotes after-tax labor income.

For the model with home production, bonds $b^\ell(s_t^j;\iota)$ are given by the same expression but using the marginal utility $\mu(\alpha, B, D, z_N)$ instead of $\mu(\alpha, B)$. As shown above, the two marginal utilities are characterized by the same equation (A.25) under $\gamma = 1$.

4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s_{t+1}^j;\iota)$ and $x(\zeta_{t+1}^j;\iota)$. For the model without home production, we obtain:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \exp\left(-(1-\tau_1)\,\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \\ &\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j), \end{aligned} \tag{A.33}$$

$$\begin{aligned} q_x(Z_{t+1}) &= \beta\delta \int \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}(\upsilon_{t+1}^B) \int \exp\left(-(1-\tau_1)\,\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}(\upsilon_{t+1}^\alpha) \\ &\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \mathbb{P}\left(\left(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon\right) \in Z_{t+1}\right), \end{aligned} \tag{A.34}$$

where $A \equiv (1+\eta)(1-\tau_1)$. For the model with home production, we obtain the same expressions under $\gamma = 1$.

A.4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section A.3 constitutes an equilibrium by showing that the postulated equilibrium allocations solve the households' problem and that all markets clear.

A.4.1 Household Problem

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

No Home Production, $\omega_M = 1$. The optimality conditions are:

$$(\beta\delta)^{t-j}\, c_{M,t}^{-\gamma} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{\mu}_t, \tag{A.35}$$

$$(\beta\delta)^{t-j} \exp(B_t)^{1+\frac{1}{\eta}} \left(h_{M,t}\right)^{\frac{1}{\eta}} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{z}_{M,t}^j \tilde{\mu}_t, \tag{A.36}$$

$$q_b^\ell(s_{t+1}^j) = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}, \tag{A.37}$$

$$q_x(Z_{t+1}) = \int \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.38}$$

Comparing the planner solutions to the household solutions we verify that they coincide for market consumption and hours when the multipliers are related by:

$$\tilde{\mu}_t = (\beta\delta)^{t-j} f^{t,j}(\sigma_t^j|\sigma_j)\mu(\alpha_t^j, B_t^j). \tag{A.39}$$

Therefore, the Euler equations become:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \tag{A.40}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.41}$$

Home Production, $\omega_M < 1$. We denote total hours, taking into account the respective disutility, by $\tilde{h} = \exp(B)(h_M + h_N) + \exp(D)(h_P)$. Using again the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and $\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)$, we write the optimality conditions directly as:

$$\frac{\tilde{z}_{M,t}}{\exp(B_t)}\left(\omega\cdot\mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1}\omega_M(c_{M,t})^{-\frac{1}{\phi}}=\tilde{h}_t^{\frac{1}{\eta}}, \tag{A.42}$$

$$\frac{z_{N,t}}{\exp(B_t)}\left(\omega\cdot\mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1}\omega_N(c_{N,t})^{-\frac{1}{\phi}}=\tilde{h}_t^{\frac{1}{\eta}}, \tag{A.43}$$

$$\frac{z_{P,t}}{\exp(D_t)}\left(\omega\cdot\mathbf{c}(\iota)^{\frac{\phi-1}{\phi}}\right)^{-1}\omega_P(c_{P,t})^{-\frac{1}{\phi}}=\tilde{h}_t^{\frac{1}{\eta}}, \tag{A.44}$$

$$q_b^\ell(s_{t+1}^j)=\beta\delta\int\frac{\mu(\alpha_{t+1}^j,B_{t+1}^j,D_{t+1}^j,z_{N,t+1}^j)}{\mu(\alpha_t^j,B_t^j,D_t^j,z_{N,t}^j)}f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}z_{N,t+1}^j\mathrm{d}D_{t+1}^j, \tag{A.45}$$

$$q_x(Z_{t+1})=\beta\delta\int\frac{\mu(\alpha_{t+1}^j,B_{t+1}^j,D_{t+1}^j,z_{N,t+1}^j)}{\mu(\alpha_t^j,B_t^j,D_t^j,z_{N,t}^j)}f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}\upsilon_{t+1}^B\mathrm{d}\upsilon_{t+1}^\alpha\mathrm{d}z_{N,t+1}^j\mathrm{d}D_{t+1}^j. \tag{A.46}$$

A.4.2 Euler Equations

We next verify that the Euler equations are satisfied at the postulated equilibrium allocations and prices.

No Home Production, $\omega_M=1$. Using the marginal utility of market consumption of the planner problem $\mu(\alpha_t^j,B_t^j)$, we write the Euler equation for the bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta\frac{\mu(\alpha_{t+1}^j,B_{t+1}^j)}{\mu(\alpha_t^j,B_t^j)}f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \qquad \text{(A.47)}\\ &=\beta\delta\frac{\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}B_{t+1}^j\right)\left[\int\left(\tilde{z}_{M,t+1}^j\right)^{1+\eta}\mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}B_t^j\right)\left[\int\left(\tilde{z}_{M,t}^j\right)^{1+\eta}\mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \end{aligned}$$

where the second line follows from equations (A.7) and (A.9). Using that B_t^j follows a random walk-process with innovation υ_t^B we rewrite $q_b^\ell(s_{t+1}^j)$ as:

$$q_b^\ell(s_{t+1}^j)=\beta\delta\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\frac{\left[\int\left(\tilde{z}_{M,t+1}^j\right)^{1+\eta}\mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\left[\int\left(\tilde{z}_{M,t}^j\right)^{1+\eta}\mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.48}$$

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$ we use that:

$$\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\left((1-\tau_1)\left(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right).$$

Given that $A = (1+\eta)(1-\tau_1)$, the expectation over the random variables in the numerator is given by:

$$\begin{aligned}&\int \exp\left(A\left(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right) \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\\ &= \int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j) \int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)\,,\end{aligned} \tag{A.49}$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa^j,t}(\kappa_t^j) \int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon). \tag{A.50}$$

As a result, the price $q_b^\ell(s_{t+1}^j)$ is:

$$\begin{aligned}q_b^\ell(s_{t+1}^j) = \beta\delta \exp&\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\exp\left(-(1-\tau_1)\,\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\\ &\times\left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int \exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j),\end{aligned} \tag{A.51}$$

where $f^{t+1,j}(s_{t+1}^j|s_t^j) = f(\upsilon_{t+1}^B)f(\upsilon_{t+1}^\alpha)f(\upsilon_{t+1}^\kappa)f(\upsilon_{t+1}^\varepsilon)$. This confirms our guess in equation (A.33). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in square brackets is independent of the state vector that differentiates islands ℓ. As a result, all islands ℓ have the same bond prices, $q_b^\ell(s_{t+1}^j) = Q_b\left(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha\right)$.

We next calculate the bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$:

$$\begin{aligned}q_b^\ell(\mathcal{V}_{t+1}) = \beta\delta \int_{\mathcal{V}^B}&\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^B\right)\mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right)\int_{\mathcal{V}^\alpha}\exp\left(-(1-\tau_1)\,\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)\\ &\times\left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int \exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}.\end{aligned} \tag{A.52}$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim that does not depend on the realization of $(\upsilon_{t+1}^B, \upsilon_{t+1}^\alpha)$:

$$q_b^\ell(\mathbb{V}_{t+1}) = \beta\delta \int_{\mathbb{V}^B} \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right) \int_{\mathbb{V}^\alpha} \exp\left(-\left(1-\tau_1\right)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.53}$$

All islands face the same price $q_b^\ell(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the prices of bonds x and b that are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and given by:

$$q_x(Z_{t+1}) = \mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right) Q_b(\mathbb{V}_{t+1}), \tag{A.54}$$

where $\mathbb{P}\left((\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon) \in Z_{t+1}\right)$ denotes the probability of $(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon)$ being a member of Z_{t+1}. The expression for $q_x(Z_{t+1})$ confirms our guess in equation (A.34)

Home Production, $\omega_M < 1$. For the model with home production, we use the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)$ to write the Euler equation for the bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{t+1}^j, z_{N,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}z_{N,t+1}^j \mathrm{d}D_{t+1}^j \tag{A.55}$$
$$= \beta\delta \int \frac{\exp\left(B_{t+1}^j\right)\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{1}{1+\eta}}}{\exp\left(B_t^j\right)\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{1}{1+\eta}}} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}z_{N,t+1}^j \mathrm{d}D_{t+1}^j.$$

where the second equality follows from equation (A.26). Using equations (A.49) and (A.50), and the fact that $z_{N,t+1}^j$ is orthogonal to the other innovations, the price $q_b^\ell(s_{t+1}^j)$ simplifies to:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\upsilon_{t+1}^B - (1-\tau_1)\,\upsilon_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{1}{1+\eta}} f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.56}$$

The price $q_b^\ell(s_{t+1}^j)$ is identical to equation (A.51) for the model without home production under $\gamma = 1$. The remainder of the argument is identical to the argument for the model without home production.

A.4.3 Household's Budget Constraint

We now verify our guess for the bond positions $b_t^\ell(s_t^j)$ and confirm that the household budget constraint holds at the postulated equilibrium allocations. The proof to this claim is identical for both models. We define the deficit term by $d_t \equiv c_{M,t} - \tilde{y}_t$. Using the expression for the price $q_b^\ell(s_{t+1}^j)$ in equation (A.40), the budget constraint at the no-trade equilibrium is given by:

$$b_t^\ell(s_t^j) = d_t + \beta\delta \int\int\int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, D_{t+1}^j, z_{N,t+1}^j)}{\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)} b_{t+1}^\ell(s_{t+1}^j) f^{t+1}(\sigma_{t+1}^j|\sigma_t^j)\mathrm{d}s_{t+1}^j \mathrm{d}z_{N,t+1}^j \mathrm{d}D_{t+1}^j.$$

By substituting forward using equation (A.40), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.32) and show that the household budget constraint holds at the postulated equilibrium allocations.

A.4.4 Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations that solve the planner problems satisfy the aggregate goods market clearing condition:

$$\int_\iota c_{M,t}(\iota)\mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t}(\iota)h_{M,t}(\iota)\mathrm{d}\Phi(\iota). \tag{A.57}$$

A.4.5 Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota x(\zeta_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $x(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$, $\forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated bond holdings in equation (A.32) into the asset market clearing conditions:

$$\begin{aligned}\int b^\ell(s_t^j)\mathrm{d}\Phi(\iota) &= \int \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{t+n}^j, z_{N,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)} d_{t+n}\right]\mathrm{d}\Phi(\iota)\\
&= \sum_{n=0}^{\infty}(\beta\delta)^n \int\int \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, D_{t+n}^j, z_{N,t+n}^j)}{\mu(\alpha_t^j, B_t^j, D_t^j, z_{N,t}^j)} d_{t+n} f(\sigma_{t+n}^j|\sigma_{t-1}^j)\mathrm{d}\sigma_{t+n}^j \mathrm{d}\Phi(\iota).\end{aligned}$$

For simplicity we omit conditioning on σ^j_{t-1} and write the density function as $f(\sigma^j_{t+n}|\sigma^j_{t-1}) = f(\{\upsilon^B_{t+n}\})f(\{\upsilon^\alpha_{t+n}\})f(\{\upsilon^\kappa_{t+n}\})f(\{\upsilon^\varepsilon_{t+n}\})f(\{z_{N,t+n}\})f(\{D_{t+n}\})$. Further, the expression for the growth in marginal utility is identical between the two models and we denote it by $\mathcal{Q}\left(\upsilon^B_{t+n}, \upsilon^\alpha_{t+n}\right) \equiv \frac{\mu(\alpha^j_{t+n}, B^j_{t+n}, D^j_{t+n}, z^j_{N,t+n})}{\mu(\alpha^j_t, B^j_t, D^j_t, z^j_{N,t})} = \frac{\mu(\alpha^j_{t+n}, B^j_{t+n})}{\mu(\alpha^j_t, B^j_t)}$. Hence, we write aggregate bond holdings $\int b^\ell(s^j_t)\mathrm{d}\Phi(\iota)$ as:

$$\sum_{n=0}^{\infty}(\beta\delta)^n \int\int \mathcal{Q}\left(\upsilon^B_{t+n}, \upsilon^\alpha_{t+n}\right) d_{t+n} f(\{\upsilon^B_{t+n}\})f(\{\upsilon^\alpha_{t+n}\})f(\{\upsilon^\kappa_{t+n}\})f(\{\upsilon^\varepsilon_{t+n}\})f(\{z_{N,t+n}\})\ldots$$

$$\ldots\; f(\{D_{t+n}\})\mathrm{d}\{\upsilon^B_{t+n}\}\mathrm{d}\{\upsilon^\alpha_{t+n}\}\mathrm{d}\{\upsilon^\kappa_{t+n}\}\mathrm{d}\{\upsilon^\varepsilon_{t+n}\}\mathrm{d}\{z^j_{N,t+n}\}\mathrm{d}\{D^j_{t+n}\}\mathrm{d}\Phi(\iota)$$

$$= \sum_{n=0}^{\infty}(\beta\delta)^n \int d_{t+n} f(\{\upsilon^\kappa_{t+n}\})f(\{\upsilon^\varepsilon_{t+n}\})\mathrm{d}\{\upsilon^\kappa_{t+n}\}\mathrm{d}\{\upsilon^\varepsilon_{t+n}\}\mathrm{d}\Phi(\iota)$$

$$\times \int \mathcal{Q}\left(\upsilon^B_{t+n}, \upsilon^\alpha_{t+n}\right) f(\{\upsilon^B_{t+n}\})f(\{\upsilon^\alpha_{t+n}\})f(\{z_{N,t+n}\})f(\{D_{t+n}\})\mathrm{d}\{\upsilon^B_{t+n}\}\mathrm{d}\{\upsilon^\alpha_{t+n}\}\mathrm{d}\{z_{N,t+n}\}\mathrm{d}\{D_{t+n}\}.$$

Recalling that the deficit terms equal $d_t = c_{M,t} - \tilde{y}_t$, the bond market clearing condition holds because the first term is zero by the island-level resource constraint.

A.5 Observational Equivalence Theorem

In this appendix we derive the identified sources of heterogeneity presented in Table 2. Our strategy is to invert the equilibrium allocations presented in Table 1 and solve for the unique sources of heterogeneity that lead to these allocations. We note that the identification is defined up to a constant because the constant $\mathbb{C}_s$ that appears in the equations of Table 2 depends on the ε's.

A.5.1 No Home Production, $\omega_M = 1$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}\}_\iota$ and parameters $\gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide the solution for c_M with the solution for h_M to obtain:

$$\frac{c_{M,t}}{h_{M,t}} = (1-\tau_0)\, z_{M,t}^{-\eta(1-\tau_1)} \exp((1-\tau_1)(1+\eta)\alpha_t) \int_{\zeta_t} \exp((1-\tau_1)(1+\eta)\varepsilon_t)\mathrm{d}\Phi_{\zeta^j_t}(\zeta^j_t)\,. \tag{A.58}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Finally, we can use the solution for $c_{M,t}$ or $h_{M,t}$ in Table 1 to solve for a unique value of B_t.

A.5.2 Home Production, $\omega_M < 1$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}, h_{N,t}, h_{P,t}\}_\iota$ and parameters $\omega, \phi, \gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t, z_{N,t}, D_t\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι.

Dividing the solution for h_N with the solution for c_M we obtain z_N from the following equation:

$$\tilde{z}_{M,t}\frac{h_{N,t}}{c_{M,t}} = \left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z^j_{N,t}}{\tilde{z}^j_{M,t}}\right)^{\phi-1}. \tag{A.59}$$

Next, we divide the solutions for h_P with the solution for h_N, we solve for the ratio of disutilities $\exp(D)/\exp(B)$:

$$\frac{h_{P,t}}{h_{N,t}} = \left(\frac{\omega_P}{\omega_N}\right)^{\phi}\left(\frac{\bar{z}^j_{P,t}}{z^j_{N,t}}\right)^{\phi-1}\left(\frac{\exp(B_t)}{\exp(D_t)}\right)^{\phi}. \tag{A.60}$$

Next, we divide the solution for h_T with the solution for c_M and use equation (A.59) to obtain:

$$\begin{aligned}\frac{h_{M,t} + h_{N,t} + \frac{\exp(D_t)}{\exp(B_t)}h_{P,t}}{c_{M,t}} &= \frac{z_{M,t}^{\eta(1-\tau_1)}}{1-\tau_0}\frac{\exp(-(1+\eta)(1-\tau_1)\alpha^j_t)}{\int_{Z_t}\exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_{\zeta^j,t}(\zeta^j_t)} \\ &\times\left[1+\left(\frac{\omega_N}{\omega_M}\right)^{\phi}\left(\frac{z^j_{N,t}}{\tilde{z}^j_{M,t}}\right)^{\phi-1} + \left(\frac{\omega_P}{\omega_M}\right)^{\phi}\left(\frac{\exp\left(B_t\right)/\tilde{z}_{M,t}}{\exp\left(D_t\right)/\bar{z}_{P,t}}\right)^{\phi-1}\right]\end{aligned} \tag{A.61}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Next, we can identify B using the first-order conditions with respect to market consumption and equations (A.25), (A.59) and (A.60) to obtain:

$$\exp\big(\left(1+\eta\right)B_t\big) = \frac{\left(\frac{\bar{c}_{M,t}}{\tilde{z}_{M,t}} + h_{N,t} + \left(\frac{\omega_P}{\omega_M}\right)\left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}}\left(\bar{z}_{P,t}\right)^{\frac{\phi-1}{\phi}}\frac{h_{P,t}}{\tilde{z}_{M,t}}\right)^{-\eta}}{\bar{h}_{M,t} + h_{N,t} + \left(\frac{\omega_P}{\omega_M}\right)\left(\frac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}}\left(\bar{z}_{P,t}\right)^{\frac{\phi-1}{\phi}}\frac{h_{P,t}}{\tilde{z}_{M,t}}}. \tag{A.62}$$

Finally, once we know B, we can solve for D from equation (A.60).

B Additional Results

In this appendix we present summary statistics from various datasets and additional results and sensitivity analyses.

- Table A.1 shows summary statistics of wages and hours for married individuals in the ATUS and for married households in the CEX in which we have imputed home hours. The ATUS sample excludes respondents during weekends and, so, market hours are noticeably higher.
- Tables A.2 and A.3 show summary statistics of wages and hours for married individuals in the ATUS by sex and education.
- Tables A.4 and A.5 present summary statistics of wages, hours, and expenditures in the CEX and PSID samples.
- Table A.6 presents various labor supply elasticities implied by the two models.
- Table A.7 presents the correlation matrix of observables and sources of heterogeneity in the two models.
- Figure A.1 presents distributions of the sources of heterogeneity in the two models.
- Figure A.2 presents time trends in two metrics of inequality for the two models.
- Table A.8 presents the welfare effects of eliminating heterogeneity within age groups.
- Figures A.3 and A.4 present the life-cycle means and variances of the sources of heterogeneity in the version of the PSID with food expenditures. We obtain these age profiles by regressing each inferred source of heterogeneity on age and year dummies and an individual fixed effect. Therefore, these age profiles reflect the within-household evolution of the sources of heterogeneity.

Table A.1: ATUS (Raw) versus CEX (Imputed) Samples

	ATUS Married Individuals			CEX Married Households		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	42.1	41.9	42.2	66.1	66.8	65.5
Mean h_N	12.5	14.6	10.5	21.4	25.4	17.3
Mean h_P	10.6	10.7	10.5	16.7	16.4	17.0
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	-0.15	-0.14	-0.14
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	0.10	0.16	0.11
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	0.02	0.00	0.03
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.25	-0.36	-0.23
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.42	-0.42	-0.41
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.15	0.20	0.17

Table A.2: Correlations in ATUS Married by Sex

	ATUS All			ATUS Men			ATUS Women		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.02	0.00	0.04	0.04	0.02	0.06
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	0.03	0.07	0.01	0.03	0.05	0.01
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	-0.02	0.00	-0.04	-0.08	-0.08	-0.09
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.40	-0.41	-0.39	-0.44	-0.47	-0.43
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.39	-0.38	-0.41	-0.46	-0.44	-0.47
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.06	0.09	0.05	0.07	0.10	0.07

Table A.3: Correlations in ATUS Married by Education

	ATUS All			ATUS Less than College			ATUS College or More		
Age	All	25-44	45-65	All	25-44	45-65	All	25-44	45-65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.05	0.04	0.03	0.05	0.02	0.07
$\text{corr}(z_M, h_N)$	0.01	0.04	-0.01	-0.01	0.01	-0.01	-0.02	0.02	-0.04
$\text{corr}(z_M, h_P)$	-0.08	-0.06	-0.09	-0.05	-0.03	-0.06	-0.07	-0.06	-0.09
$\text{corr}(h_M, h_N)$	-0.44	-0.46	-0.42	-0.42	-0.44	-0.41	-0.47	-0.50	-0.45
$\text{corr}(h_M, h_P)$	-0.45	-0.44	-0.46	-0.45	-0.43	-0.46	-0.45	-0.45	-0.45
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.08	0.12	0.06	0.14	0.17	0.13

Table A.4: CEX/ATUS (1995-2016) versus PSID (1975-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	67.7	65.3	70.3
Mean $h_N + h_P$	38.0	41.8	34.3	25.9	27.1	24.7
corr(z_M, h_M)	-0.15	-0.14	-0.14	-0.15	-0.15	-0.14
corr$(z_M, h_N + h_P)$	0.09	0.12	0.10	0.00	0.02	-0.02
corr(z_M, c_M^{food})	0.22	0.21	0.22	0.28	0.29	0.27
corr$(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.24	-0.28	-0.20
corr(h_M, c_M^{food})	0.10	0.09	0.12	0.06	0.06	0.07
corr$(h_N + h_P, c_M^{\text{food}})$	-0.03	-0.01	-0.02	0.01	0.03	-0.01

Table A.5: CEX/ATUS (1995-2016) versus PSID (2004-2014) Moments

	CEX/ATUS			PSID		
Age	All	25-44	45-65	All	25-44	45-65
Mean h_M	66.1	66.8	65.5	64.8	67.6	62.0
Mean $h_N + h_P$	38.0	41.8	34.3	24.3	24.1	24.6
corr(z_M, h_M)	-0.15	-0.14	-0.14	-0.09	-0.15	-0.06
corr$(z_M, h_N + h_P)$	0.09	0.12	0.10	-0.01	0.03	-0.03
corr(z_M, c_M^{nd})	0.25	0.25	0.25	0.26	0.29	0.25
corr$(h_M, h_N + h_P)$	-0.42	-0.49	-0.42	-0.23	-0.27	-0.20
corr(h_M, c_M^{nd})	0.14	0.16	0.13	0.20	0.21	0.20
corr$(h_N + h_P, c_M^{\text{nd}})$	-0.05	-0.04	-0.03	-0.03	-0.03	-0.03

Table A.6: Labor Supply Elasticities

	Controls	$\omega_M = 1$	$\omega_M < 1$
Marshallian	net assets and taste shifters	0.05	0.33
Hicksian	utility and taste shifters	0.08	0.34
Frisch	marginal utility and taste shifters	0.79	1.55

Table A.7: Unconditional Correlations

$\omega_M = 1$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D	$\log z_N$
$\log z_M$	1.00	0.29	-0.07	—	—	0.70	0.42	0.42	—	—
$\log c_M$		1.00	0.13	—	—	0.69	-0.50	-0.55	—	—
$\log h_M$			1.00	—	—	-0.46	0.50	-0.71	—	—
$\log h_N$				—	—	—	—	—	—	—
$\log h_P$					—	—	—	—	—	—
α						1.00	-0.35	0.23	—	—
ε							1.00	0.26	—	—
B								1.00	—	—
D									—	—
$\log z_N$										—

$\omega_M < 1$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D	$\log z_N$
$\log z_M$	1.00	0.29	-0.07	0.07	-0.02	0.82	0.42	0.45	-0.58	0.69
$\log c_M$		1.00	0.13	0.00	-0.06	0.66	-0.54	-0.43	-0.01	-0.15
$\log h_M$			1.00	-0.17	-0.30	-0.32	0.38	-0.48	0.06	-0.21
$\log h_N$				1.00	0.18	0.13	-0.08	-0.29	-0.36	0.69
$\log h_P$					1.00	0.07	-0.15	-0.04	-0.70	0.12
α						1.00	-0.18	0.23	-0.41	0.46
ε							1.00	0.41	-0.34	0.46
B								1.00	-0.06	0.31
D									1.00	-0.65
$\log z_N$										1.00

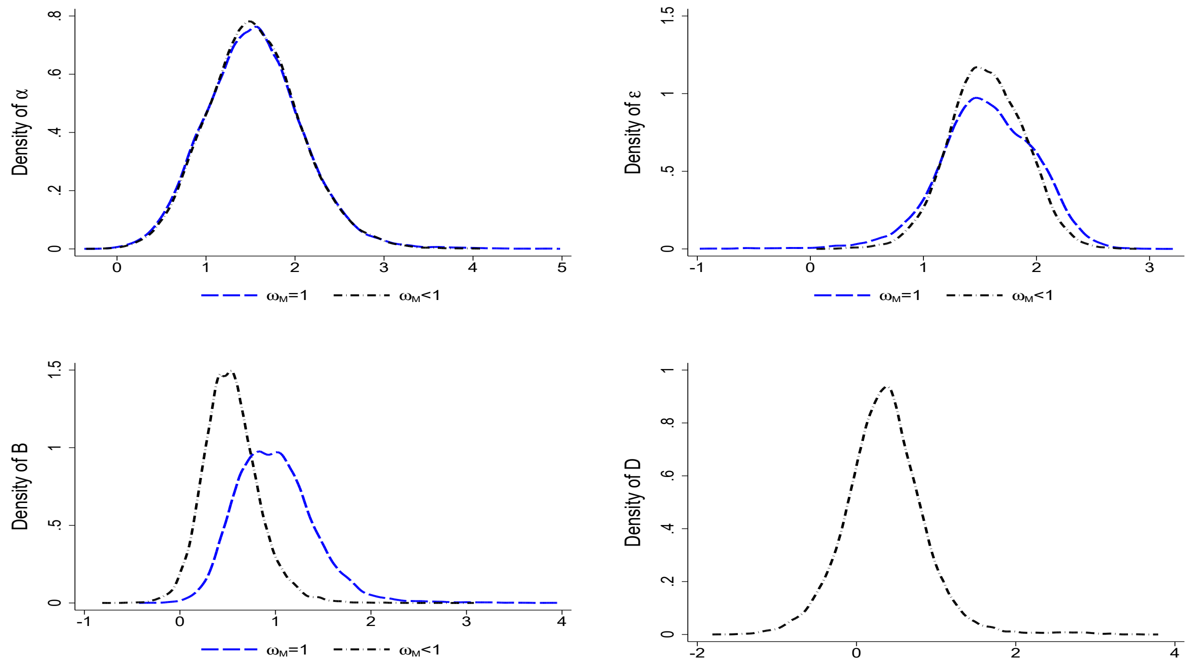

Figure A.1: Distributions of Sources of Heterogeneity

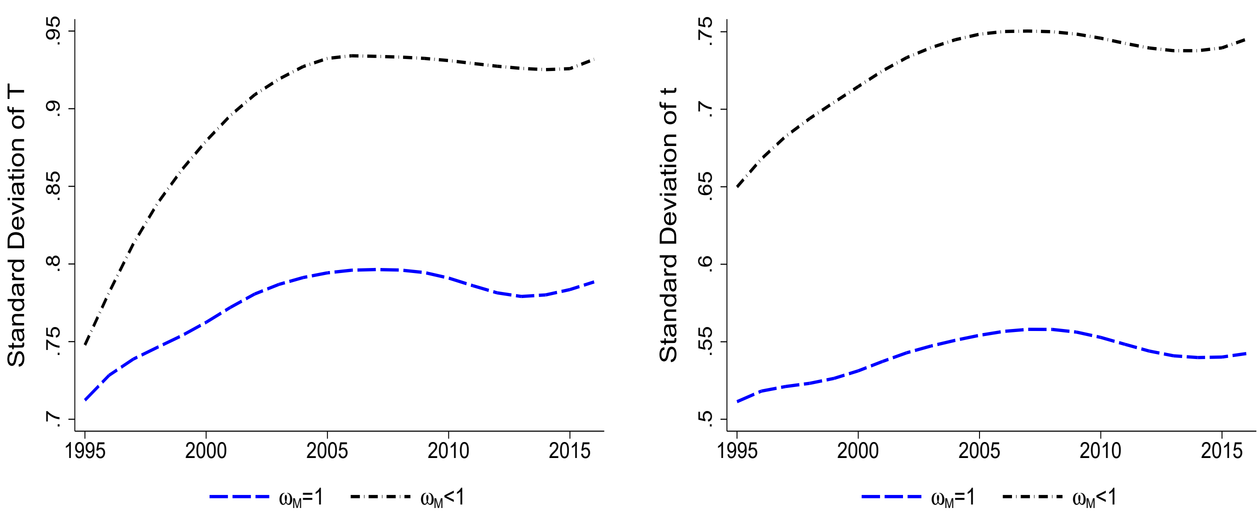

Figure A.2: Trends in Inequality Measures

Table A.8: Within-Age Heterogeneity and Lifetime Consumption Equivalence

No within-age dispersion in ...	$\omega_M = 1$ model	$\omega_M < 1$ model
z_M, z_N, B, D	0.07	0.15
z_M, z_N	0.08	0.17
z_M	0.08	0.12
z_N	—	0.13

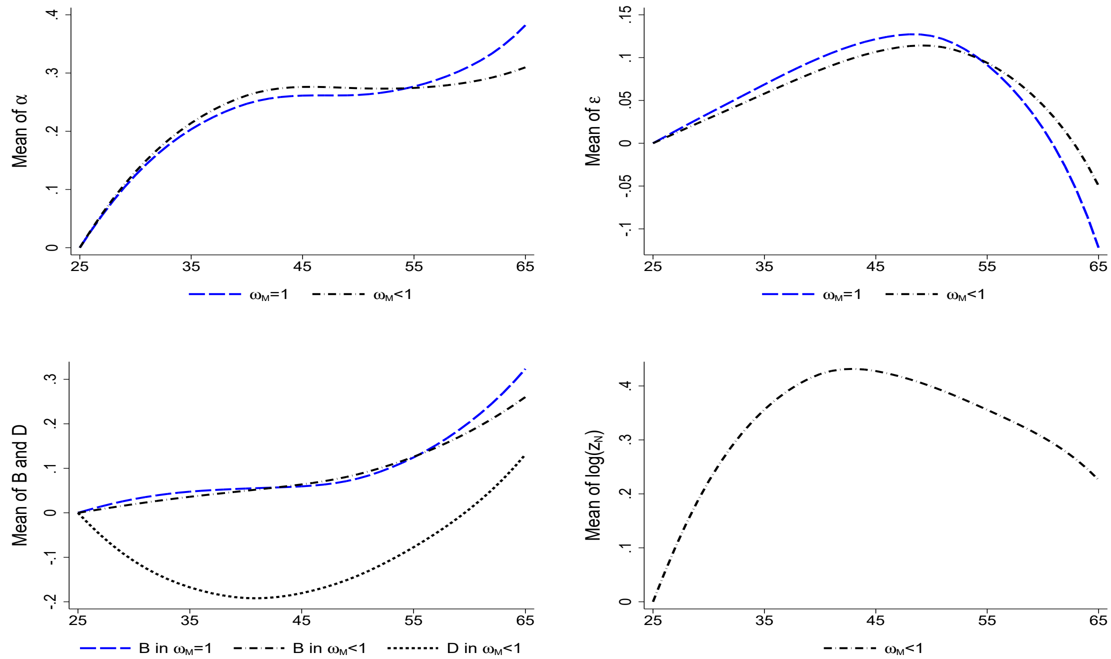

Figure A.3: Means of Productivity and Preference Shifters (PSID Food)

Figure A.3 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D, and home productivity $\log z_N$ for the economy with ($\omega_M < 1$, black dotted lines) and without home production ($\omega_M = 1$, blue dashed lines).

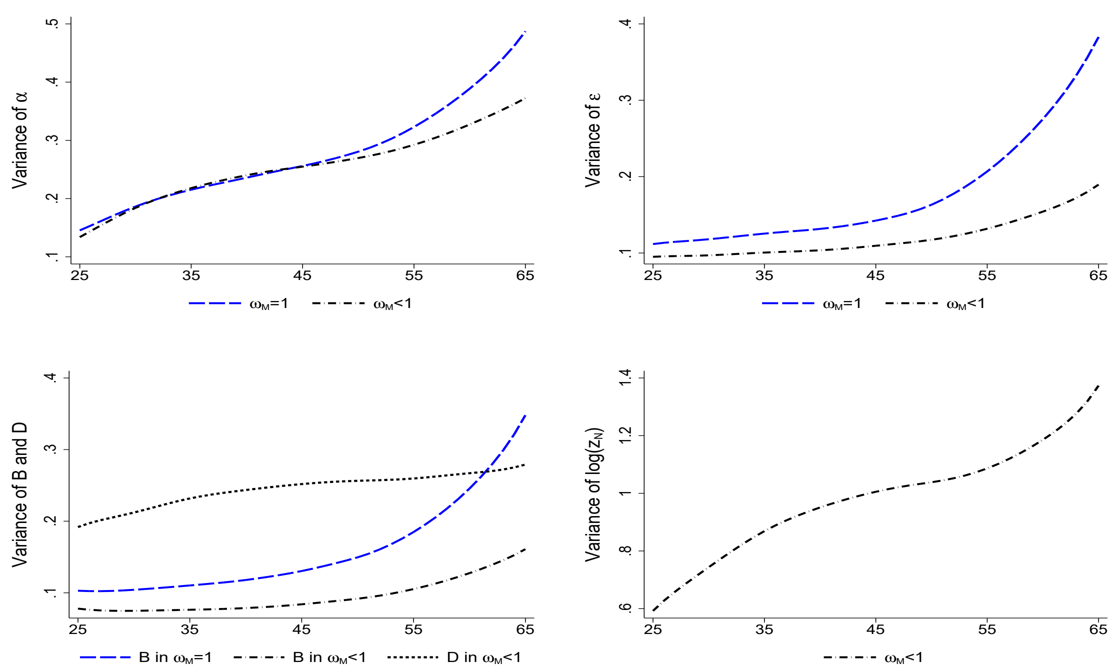

Figure A.4: Variances of Productivity and Preference Shifters (PSID Food)

Figure A.4 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutilities of work B and D, and home productivity $\log z_N$ for the economy with ($\omega_M < 1$, black dotted lines) and without home production ($\omega_M = 1$, blue dashed lines).