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ABSTRACT

We revisit the causes, welfare consequences, and policy implications of the dispersion in households' labor market outcomes using a model with uninsurable risk, incomplete asset markets, and a home production technology. Accounting for home production amplifies welfare-based differences across households meaning that inequality is larger than we thought. Using the optimality condition that households allocate more consumption to their more productive sector, we infer that the dispersion in home productivity across households is roughly three times as large as the dispersion in their wages. There is little scope for home production to offset differences that originate in the market sector because productivity differences in the home sector are large and the time input in home production does not covary with consumption expenditures and wages in the cross section of households. We conclude that the optimal tax system should feature more progressivity taking into account home production.

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1 Introduction

A substantial body of research examines the causes, welfare consequences, and policy implications of the pervasive dispersion across households in their labor market outcomes.[1] The literature trying to understand the dispersion in wages, hours worked, and consumption expenditures across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. It is well known, however, that households spend roughly half as much time in home production activities such as cooking, shopping, and child care as in the market sector.

In this paper we revisit the causes, welfare consequences, and policy implications of labor market dispersion taking into account that households spend a significant amount of their time in home production. A priori there are good reasons why home production can change the inferences economists draw from observing dispersion in labor market outcomes. To the extent that households are willing to substitute between market expenditures and time in the production of goods and services, home production will tend to compress welfare differences that originate in the market sector. However, to the extent that household productivity differences in the home sector remain uninsurable and are large relative to the market sector, the home sector itself may emerge as an additional source of welfare differences across households.

We show that incorporating home production in a model with uninsurable risk and incomplete asset markets changes the inferred sources of heterogeneity across households, alters meaningfully the welfare consequences of dispersion, and leads to different policy conclusions. We find that the dispersion in home productivity across households is roughly three times as large as the dispersion in wages (henceforth referred to as market productivity). Surprisingly, we infer that inequality across households is larger than what one would infer without incorporating home production.[2] We reach this conclusion because, for households of all ages, productivity differences in the home

[1] See Heathcote, Perri, and Violante (2010) and Attanasio and Pistaferri (2016) for empirical regularities on household heterogeneity in labor market outcomes. We review the literature in more detail below.

[2] We use the term dispersion to refer to the variation in observed outcomes (such as time allocation, consumption expenditures, and wages) or inferred sources of heterogeneity (such as permanent or transitory productivity and taste shifters). We use the term inequality to refer to the mapping from dispersion to measures that capture welfare differences across households.

sector are larger than in the market sector and the time input in home production does not covary with consumption expenditures and productivity in the cross section of households. Thus, there is little scope for home production to offset differences that originate in the market sector. Rather, home production amplifies these differences.

We develop our findings using a general equilibrium model with home production, heterogeneous households that face idiosyncratic risk, and incomplete asset markets. In the spirit of Ghez and Becker (1975), households produce goods with a technology that uses as inputs both expenditures and time. Home production is not tradeable and there are no securities that households can purchase to insure against productivity differences that originate in the home sector. Households are also heterogeneous with respect to their disutility of work and their market productivity. The structure of asset markets allows households to insure against transitory shocks in their market productivity but not against permanent productivity differences. We retain tractability and prove identification by extending the no-trade result with respect to certain assets for the one-sector model of Heathcote, Storesletten, and Violante (2014) to our model embedding home production. Therefore, we can characterize the allocations of time and consumption goods in closed form without simultaneously solving for the wealth distribution.

At the core of our approach lies an observational equivalence theorem that allows us to compare our model with home production to a nested model without home production. The observational equivalence theorem states that both models account perfectly for any given cross-sectional data on three observables: consumption expenditures, time spent working in the market sector, and market productivity. We begin by inferring productivity and taste heterogeneity across households such that the allocations generated by the model without home production match the cross-sectional data on the three observables. Then, we infer these sources of heterogeneity such that the allocations generated by the model with home production match the same cross-sectional data. Under our approach, we additionally infer productivity heterogeneity in the home sector so that the model accounts for cross-sectional data on time spent on the home sector.

Our approach builds on the observation that the inferred sources of heterogeneity and inequality will in general differ between the two models, despite both models accounting perfectly for the same cross-sectional data on the three labor market outcomes. It is essential for our purposes that the two models are observationally equivalent in terms of accounting for the three outcomes. The observational equivalence theorem guarantees that any differences between models is exclusively driven by structural factors and not by their ability to account for cross-sectional data on labor market outcomes. Thus, comparing the two models reveals the role of home production for the inferred sources of heterogeneity, for their welfare consequences, as well as for the policy implications that stem from observing dispersion in labor market outcomes.[3]

To investigate how incorporating home production changes our inferences, we apply our observational equivalence theorem to U.S. data between 1996 and 2015. We use data on consumption expenditures, time spent on the market sector, and market productivity from the Consumption Expenditure Survey (CEX). The CEX does not contain information on time spent on home production. To overcome this problem, we use data from the American Time Use Survey to impute individuals' time spent on home production based on observables that are common between the two surveys.

Our inference of home productivity is based on an intra-period first-order condition dictating that households allocate more consumption to their more productive sector. This first-order condition emerges in a large class of models and implies a log-linear relationship between home productivity and three observables (market expenditures, time spent on home production, and market productivity) that have relatively equal variances when expressed in logs. Home productivity cumulates the variances of these three observables because the covariation between them is relatively small. This logic underlies our finding that home productivity is roughly three times as dispersed as market productivity. While the exact magnitude of the dispersion in home productivity depends on the elasticity of substitution between time and expenditures in home production, we show that home productivity is robustly more dispersed than (after-tax) market productivity

[3]Our inference of the sources of heterogeneity resembles the approach undertaken by Hsieh and Klenow (2009) who infer wedges in first-order conditions such that firm-level outcomes generated by their model match data analogs. Despite different underlying models leading to the same inferred wedges, these models may have different macroeconomic and welfare implications.

for any value encompassing perfect complementarity and perfect substitutability between the two sectors. We conclude that home productivity is an important source of within-age and life-cycle differences in consumption expenditures and time allocation across households.

The key result of our analysis is that the world is more unequal when we take into account home production. One way to understand this result is that home productivity differences across households, which remain uninsurable, are significantly larger than differences in market productivity. Another way to understand this result is in terms of the distinction between consumption and expenditures emphasized by Aguiar and Hurst (2005). We find that the market value of consumption is more dispersed than market expenditures. The former measure of consumption is the sum of market expenditures and the market value of time spent on home production. This sum is more dispersed than market expenditures because households with higher market expenditures do not have lower market values of time spent on home production.

We arrive at our conclusion using four ways to map dispersion in labor market outcomes into welfare-based measures of inequality. First, we show that the standard deviation of equivalent variation across households is roughly 45 percent larger when we incorporate home production. Second, we demonstrate that equalizing marginal utilities across households requires transfers with a standard deviation that is roughly 40 percent higher in the model with home production than in the model without home production. Third, an unborn household is willing to sacrifice up to 25 percent of lifetime consumption in order to eliminate heterogeneity in an environment with home production, compared to 7 percent in an environment without home production. Finally, taking into account home production, a utilitarian government would choose a more progressive tax system. For example, a household earning 200,000 dollars would face an average tax rate of 29 percent with home production, compared to 17 percent without home production.

Our results are robust to a battery of sensitivity checks. First, we establish that all our conclusions, with one exception that we discuss below, remain unchanged for any value of the elasticity of substitution between sectors. We also show the robustness of our results to parameters that govern the Frisch elasticity of labor supply and the progressivity of the tax system. Second, we demonstrate that our results apply separately within groups of households defined by their age, marital status, number of children, and education levels. Third, we confirm the robustness of our conclusions to measures of expenditures that range from narrow (food) to broad (total spending including durables). Fourth, we allow for measurement error in market expenditures, market hours, and home hours and show that our results are robust to even large amounts of measurement error. Fifth, we document that the lack of covariation between time spent on home production and market expenditures and productivity, which is the key empirical regularity driving our inferences, is also observed in datasets from the Netherlands and Japan which contain direct information on both expenditures and time use.

Our paper contributes to three strands of literature. There is an extensive literature that examines how non-separabilities and home production affect consumption and labor supply either over the business cycle (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson, and Wright, 1997; Baxter and Jermann, 1999; Aguiar, Hurst, and Karabarbounis, 2013) or over the life-cycle (Rios-Rull, 1993; Aguiar and Hurst, 2005, 2007; Dotsey, Li, and Yang, 2014). In these papers, home production provides a smoothing mechanism against differences that originate in the market sector if households are sufficiently willing to substitute expenditures with time. Our conclusions for the role of home production in understanding cross-sectional patterns differ from this literature because in the data we find that time in home production does not correlate cross-sectionally with wages and consumption expenditures. By contrast, an assumption underlying the business cycle and life-cycle literatures is that decreases in the opportunity cost of time and in consumption expenditures are associated with substantial increases in time spent on home production.

The home production literature has emphasized that shocks in the home production technology allow models to generate higher volatility in labor markets and labor wedges (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991; Karabarbounis, 2014). However, little is known about cross-sectional differences in home productivity. Our analysis suggests that home productivity is three times as dispersed as market productivity across households. We stress that our inference of home productivity is based on an intra-period first-order condition that emerges in a large class of models and, therefore, can be applied in various other settings.

The literature on incomplete markets has started to incorporate home production and non-separabilities into models. Kaplan (2012) argues that involuntary unemployment and non-separable preferences allow an otherwise standard model with self insurance to account for the variation of market hours over the life-cycle. Blundell, Pistaferri, and Saporta-Eksten (2016) examine consumption inequality in a model in which shocks can also be insured within the family and preferences for hours are non-separable across spouses. Blundell, Pistaferri, and Saporta-Eksten (2017) incorporate child care into a life-cycle partial equilibrium model of consumption and family labor supply. Their paper aims to understand the responsiveness of consumption and time use to transitory and permanent wage shocks and, unlike our paper, it does not quantify the extent to which home production affects inequality.

The second related literature addresses consumption inequality. Earlier work (Deaton and Paxson, 1994; Gourinchas and Parker, 2002; Storesletten, Telmer, and Yaron, 2004; Aguiar and Hurst, 2013) has examined the drivers of life-cycle consumption inequality and their welfare consequences. More recent work focuses on the evolution of consumption inequality over time (Krueger and Perri, 2006; Blundell, Pistaferri, and Preston, 2008; Aguiar and Bils, 2015). Our contribution is to introduce home production into the consumption inequality literature because, as we show, home production data change the inferences we draw about welfare. Closest to the spirit of our exercise, Jones and Klenow (2016) map differences in consumption, work, mortality, and inequality into welfare differences across countries and find that in some cases GDP per capita does not track welfare closely.

Finally, our paper relates to a strand of literature that uses no-trade theorems to derive analytical solutions for a certain class of models with incomplete markets and heterogeneous agents. Constantinides and Duffie (1996) first derived a no-trade theorem in an endowment economy. Krebs (2003) extends the theorem to an environment with capital, in which households invest a constant share of their wealth in physical and human capital and their total income follows a random walk in logs. Most relevant for us, Heathcote, Storesletten, and Violante (2014) extend the no-trade theorem by allowing for partial insurance of productivity shocks and flexible labor supply. Our contribution to this literature is to extend the theorem in a two-sector model under log preferences with respect to the consumption function that aggregates market and non-market inputs. Importantly, with log preferences, we do not need to place restrictions on either the stochastic process governing home productivity or the elasticity of substitution between the two sectors.

2 Model

We develop a general equilibrium model with home production, heterogeneous households that face idiosyncratic risk, and incomplete asset markets. We first present the environment and characterize the equilibrium in closed form. We then show how to infer the sources of heterogeneity across households such that the model accounts perfectly for cross-sectional data on consumption expenditures, allocation of time, and wages.

2.1 Environment

Preferences. The economy features perpetual youth demographics. We denote by t the calendar year and by j the birth year of a household. Households face a constant probability of survival δ in each period. Each period a cohort of mass $1-\delta$ is born, keeping the population size constant with a mass of one.

Denoting the discount factor by β, households order sequences of market consumption expenditures c_M, non-market (home) consumption c_N, market hours h_M, and non-market (home) hours h_N by:

$$\mathbb{E}_j \sum_{t=j}^{\infty} (\beta\delta)^{t-j}\, U_t\left(c_{M,t}, c_{N,t}, h_{M,t}, h_{N,t}\right), \tag{1}$$

where the period utility function is given by:

$$U = \frac{\left[\left(\omega {c_M}^{\frac{\phi-1}{\phi}} + (1-\omega){c_N}^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}\right]^{1-\gamma} - 1}{1-\gamma} - \exp\left(B\right)\frac{\left(h_M + h_N\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\,. \tag{2}$$

Utility from consumption is given by a CES aggregator of market and home consumption. The elasticity of substitution between the two sectors is given by ϕ, with $\phi > 1$ denoting substitutability between the two sectors. Parameter ω governs the preference for market relative to home consumption, with $\omega = 1$ denoting the nested model without home production. The curvature of the utility function with respect to consumption is given by γ, while η governs the Frisch elasticity for total hours. Finally, B captures a household's disutility of work.

Technology. Households have access to a technology in the market sector and a technology in the home sector. A household's technology in the market sector is characterized by its (pre-tax) earnings $y = z_M h_M$, where z_M denotes market productivity (wage) and h_M denotes hours worked in the market sector. Aggregate production is given by $\int_\iota z_M(\iota)h_M(\iota)\mathrm{d}\Phi(\iota)$, where ι identifies households and Φ denotes the cumulative distribution function of households. Goods and labor markets are perfectly competitive and the wage per efficiency unit of labor equals unity.[4]

The government taxes labor income to finance a given level of (wasteful) public expenditures G of the market good. If $y = z_M h_M$ is pre-tax earnings, then $\tilde{y} = (1-\tau_0)z_M^{1-\tau_1}h_M$ is after-tax earnings, where τ_0 determines the level of taxes and τ_1 governs the progressivity of the tax system. When $\tau_1 = 0$ there is a flat tax rate. A higher τ_1 introduces a larger degree of progressivity into the tax system because it compresses after-tax earnings relative to pre-tax earnings.[5]

A household's technology in the home sector is analogous to the earnings technology in the market sector before taxes. Production of home goods is given by $c_N = z_N h_N$, where z_N denotes home productivity and h_N denotes home hours. The home good is consumed in every period and it cannot be stored or traded.

Our specification of preferences and technologies is a special case of the Beckerian model of

[4]We study a stationary economy for convenience. Our results are easily extended to environments with aggregate shocks.

[5]Our tax schedule modifies the tax schedule considered, among others, by Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014) in that the τ_1 parameter is applied to market productivity z_M instead of earnings $z_M h_M$. We adopt the specification of after-tax earnings $\tilde{y} = (1-\tau_0)z_M^{1-\tau_1}h_M$ instead of $\tilde{y} = (1-\tau_0)\left(z_M h_M\right)^{1-\tau_1}$ because we can only prove the no-trade result in the home production model under the former specification. We argue that this modification does not matter for our results because market productivity z_M and hours h_M are relatively uncorrelated in the cross section of households and most of the cross-sectional variation in earnings $z_M h_M$ is accounted for by z_M. For this reason, our estimate of τ_1 in Section 3.2 is close to the estimates found in Guner, Kaygusuz, and Ventura (2014) and Heathcote, Storesletten, and Violante (2014).

home production (Becker, 1965; Ghez and Becker, 1975). Households have preferences over two goods. The first good, which we label consumption $c(c_M, h_N)$ and is given by the CES aggregator in the utility function (2), uses market expenditures and time to produce utility. The second good, which we label leisure, $1 - h_M - h_N$, uses only time as an input in the production of utility.

Sources of Heterogeneity. Households are heterogeneous with respect to the disutility of work B, market productivity z_M, and home productivity z_N. For B and z_M we impose a random walk structure that is important for obtaining the no-trade result. Under certain parametric restrictions that we discuss below, we are able to obtain the no-trade result with minimal structure on the process that governs z_N.[6]

Households' disutility of work is described by a random walk process:

$$B_t^j = B_{t-1}^j + \upsilon_t^B. \tag{3}$$

Households' log market productivity $\log z_M$ is the sum of a permanent component α and a more transitory component ε:

$$\log z_{M,t}^j = \alpha_t^j + \varepsilon_t^j \, . \tag{4}$$

The permanent component follows a random walk, $\alpha_t^j = \alpha_{t-1}^j + \upsilon_t^\alpha$. The more transitory component, $\varepsilon_t^j = \kappa_t^j + \upsilon_t^\varepsilon$, equals the sum of a random walk component, $\kappa_t^j = \kappa_{t-1}^j + \upsilon_t^\kappa$, and an innovation υ_t^ε. Finally, households are heterogeneous with respect to their home productivity levels $z_{N,t}^j$. Our identification theorems below are based on cross-sectional data and, therefore, we do not restrict z_N to a particular class of stochastic processes. Given the stochastic processes that govern heterogeneity across households, we identify a household ι by a sequence $\{z_N^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$.

For any random walk, we use υ to denote the innovation to the process and Φ_{υ_t} to denote the distribution of the innovation. We allow the distributions of innovations to vary over time t and the initial distributions to vary over cohorts j. We assume that $z_{N,t}^j$ is orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

[6]The stochastic processes for market productivity z_M that we use is also used by Heathcote, Storesletten, and Violante (2014) in a model without home production. While these authors assume that B is a fixed effect, we generalize their results in the case where B follows a random walk over the life-cycle.

Asset Markets. It is convenient to describe the restrictions on asset markets using the definition of an island as in Heathcote, Storesletten, and Violante (2014). Households are partitioned into islands, with each island consisting of a continuum of households that are identical in terms of their productivity at home z_N, disutility of work B, permanent component of market productivity α, and the initial condition of κ. More formally, household $\iota = \{z_N^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}$ lives on island ℓ consisting of households with common initial state $(z_{N,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$ and sequences $\{z_{N,t}^j, B_t^j, \alpha_t^j\}_{t=j+1}^\infty$.

We now summarize the structure of asset markets. First, households cannot write contracts contingent on $z_{N,t}^j$. Second, households can write any contract contingent on $s_t^j \equiv (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon)$ with households that live on their island. Third, households can trade contracts contingent on $\zeta_t^j \equiv \{\kappa_t^j, \upsilon_t^\varepsilon\}$ with households that live either on their island or on other islands.

The structure of asset markets implies that z_N, B, and α differences across households remain uninsurable since either trade is not allowed or allowed only with households that have identical sources of heterogeneity. The more transitory component of productivity $\varepsilon_t^j = \kappa_t^j + \upsilon_t^\varepsilon$ becomes fully insurable because households on an island are heterogeneous with respect to ζ_t^j and can trade securities contingent on ζ_t^j. Anticipating these results, henceforth we call α the uninsurable permanent component of market productivity and ε the insurable transitory component of market productivity.

Household Optimization. We now describe the optimization problem of a household ι born in period j. We denote the purchases of state-contingent bonds within an island by $b^\ell(s_{t+1}^j)$ and the purchases of economy-wide state-contingent bonds by $x(\zeta_{t+1}^j)$. The household chooses $\{c_{M,t}, c_{N,t}, h_{M,t}, h_{N,t}, b^\ell(s_{t+1}^j), x(\zeta_{t+1}^j)\}_{t=j}^\infty$ to maximize the expected value of discounted flows of utilities in equation (1), subject to the home production technology, $c_{N,t} = z_{N,t} h_{N,t}$, and the sequence of budget constraints:

$$c_{M,t} + \int_{s_{t+1}^j} q_b^\ell(s_{t+1}^j) b^\ell(s_{t+1}^j) \mathrm{d}s_{t+1}^j + \int_{\zeta_{t+1}^j} q_x(\zeta_{t+1}^j) x(\zeta_{t+1}^j) \mathrm{d}\zeta_{t+1}^j = \tilde{y}_t^j + b^\ell(s_t^j) + x(\zeta_t^j) \ . \qquad (5)$$

The left-hand side of the budget constraint denotes the expenditures of households on market consumption $c_{M,t}$, island-level bonds $b^\ell(s_{t+1}^j)$ at prices $q_b^\ell(s_{t+1}^j)$, and economy-wide bonds $x(\zeta_{t+1}^j)$

at prices $q_x(\zeta^j_{t+1})$. The right-hand side of the budget constraint consists of after-tax labor income $\tilde{y}^j_t$ and bond payouts.

Equilibrium. Given a public policy (τ_0, τ_1, G), an equilibrium is a sequence of allocations $\{c_{M,t}, c_{N,t}, h_{M,t}, h_{N,t}, b^\ell(s^j_{t+1}), x(\zeta^j_{t+1})\}_{\iota,t}$, and a sequence of prices $\{q^\ell_b(s^j_{t+1})\}_{\ell,t}$, $\{q_x(\zeta^j_{t+1})\}_t$ such that: (i) the allocations solve households' problems; (ii) asset markets clear:

$$\int_{\iota\in\ell} b^\ell(s^j_{t+1};\iota)\mathrm{d}\Phi(\iota) = 0 \ \ \forall \ell, s^j_{t+1}, \qquad \text{and} \qquad \int_\iota x(\zeta^j_{t+1};\iota)\mathrm{d}\Phi(\iota) = 0 \ \ \forall \zeta^j_{t+1}; \tag{6}$$

and (iii) the goods market clears:

$$\int_\iota c_{M,t}(\iota)\mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t}(\iota)h_{M,t}(\iota)\mathrm{d}\Phi(\iota), \tag{7}$$

where government expenditures are given by $G = \int_\iota \left[z_{M,t}(\iota) - (1-\tau_0)z_{M,t}(\iota)^{1-\tau_1}\right] h_{M,t}(\iota)\mathrm{d}\Phi(\iota)$.

2.2 Equilibrium Allocations

We next characterize equilibrium allocations in closed form. We retain tractability by showing that, under certain parametric restrictions, our model features a no-trade result. This section explains the logic underlying this result and its usefulness. Appendix A presents the proof.

We begin by guessing that the equilibrium features no trade between islands, that is $x(\zeta^j_{t+1};\iota) = 0, \forall \iota, \zeta^j_{t+1}$. Further, we postulate that equilibrium allocations for $\{c_{M,t}, c_{N,t}, h_{M,t}, h_{N,t}\}_{\iota,t}$ can be characterized by solving a sequence of static planning problems. The planner problems maximize average utility within each island subject to household's home production technology and an island-level constraint that equates aggregate market consumption to after-tax earnings. We verify our guess by demonstrating that, at the postulated allocations, households solve their optimization problems and all asset and good markets clear. Finally, we obtain asset prices that sustain the no-trade equilibrium using the intertemporal marginal rates of substitution evaluated at the solutions of the planning problems.

We obtain the no-trade result in two nested versions of our model. The first model sets the weight on market consumption to $\omega = 1$ and corresponds to the environment without home production considered by Heathcote, Storesletten, and Violante (2014). The second model sets the curvature of utility with respect to consumption to $\gamma = 1$ for any value of ω. We label these as the "no home production" and "home production" model respectively. The home production model nests the no home production model when $\gamma = 1$, which is the case we will consider below in our quantitative results.

The no-trade result is useful because it allows us to derive equilibrium allocations for consumption and time without solving simultaneously for the wealth distribution. Crucially, we note that the no-trade result applies to the bonds traded across islands $x(\zeta^j_{t+1}) = 0$ and not to the within-islands bonds $b^\ell(s^j_{t+1})$ which are traded in equilibrium. The bonds $b^\ell(s^j_{t+1})$ are state-contingent within each island and solving for the equilibrium allocations amounts to solving a sequence of static planning problems.

To understand how our model with non-separable preferences retains such a tractability, we begin with the observation that households on each island ℓ have the same marginal utility of market consumption because they are identical in terms of (z_N, B, α) and trade in state-contingent securities allows them to perfectly insure against $(\kappa, \upsilon^\varepsilon)$. Considering first the no home production model ($\omega = 1$), the common marginal utility of market consumption $\mu(\ell)$ at the no-trade equilibrium equals:

$$\mu(\ell; \omega = 1) = c_M^{-\gamma} = \left(\frac{(1-\tau_0)^{-1} \exp\left(\eta B - (1+\eta)(1-\tau_1)\alpha\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa + \upsilon^\varepsilon)\right) \mathrm{d}\Phi(\zeta)} \right)^{\frac{\gamma}{1+\gamma\eta}}, \tag{8}$$

where for simplicity we have dropped the time subscript from all variables. The no-trade result states that households do not trade securities across islands, $x(\zeta^j_{t+1}) = 0$. Owing to the random walk assumptions on B and α, equation (8) shows that the growth in marginal utility, μ_{t+1}/μ_t, does not depend on the state vector (B^j_t, α^j_t) that differentiates islands ℓ. As a result, all households value securities traded across islands identically in equilibrium and there are no mutual benefits from trading these securities.

For the economy with home production and $\gamma = 1$, we obtain a marginal utility of market consumption:

$$\mu(\ell; \gamma = 1) = \left(c_M + (1-\tau_0) z_M^{1-\tau_1} h_N\right)^{-1} = \left(\frac{(1-\tau_0)^{-1} \exp\left(\eta B - (1+\eta)(1-\tau_1)\alpha\right)}{\int_\zeta \exp\left((1+\eta)(1-\tau_1)(\kappa + \upsilon^\varepsilon)\right) \mathrm{d}\Phi(\zeta)} \right)^{\frac{1}{1+\eta}}. \tag{9}$$

Table 1: Equilibrium Allocations

Variable		No Home Production: $\omega = 1$	Home Production: $\gamma = 1$
1.	c_M	$\frac{\exp\left((1-\tau_1)\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\alpha\right)}{\exp(B)^{\frac{1}{\frac{1}{\eta}+\gamma}}}\mathbb{C}^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}}$	$\frac{1}{R}\frac{\exp((1-\tau_1)\alpha)}{\exp(B)^{\frac{1}{\frac{1}{\eta}+1}}}\mathbb{C}^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+1}}$
2.	$h_M + h_N$	$\tilde{z}_M^{\eta}\frac{\exp\left(-\gamma\eta\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}(1-\tau_1)\alpha\right)}{\exp(B)^{\frac{1}{\frac{1}{\eta}+\gamma}}}\mathbb{C}^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}$	$\tilde{z}_M^{\eta}\frac{\exp(-\eta(1-\tau_1)\alpha)}{\exp(B)^{\frac{1}{\frac{1}{\eta}+1}}}\mathbb{C}^{-\frac{1}{\frac{1}{\eta}+1}}$
3.	h_N		$\frac{1}{\tilde{z}_M^j}\frac{\exp((1+\eta)(1-\tau_1)\alpha)}{\exp(B)^{\frac{1}{\frac{1}{\eta}+1}}}\left(1-\frac{1}{R}\right)\mathbb{C}^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+1}}$

Table 1 presents the equilibrium allocation in the two models. We define the constant $\mathbb{C} \equiv \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_\zeta(\zeta)$, the rate of transformation $R \equiv 1+\left(\frac{1-\omega}{\omega}\right)^{\phi}(z_N/\tilde{z}_M)^{\phi-1}$, and the after-tax market productivity $\tilde{z}_M \equiv (1-\tau_0)z_M^{1-\tau_1}$.

The marginal utility in equation (9) has the same form as the marginal utility in equation (8) for $\gamma = 1$. Therefore, marginal utility growth does not depend on the state vector $(z^j_{N,t}, B^j_t, \alpha^j_t)$ that differentiates islands and the same logic explains why we obtain the no-trade result in the home production model. For this result we note the importance of log preferences with respect to consumption ($\gamma = 1$). We also note that, given $\gamma = 1$, the result does not depend on the value of the elasticity of substitution between sectors ϕ or the stochastic properties of z_N.

Given the no-trade result, we solve for the equilibrium allocations using the sequence of planning problems. We summarize the equilibrium allocations for both models in Table 1. The rows in the table present the equilibrium values for market consumption c_M, total hours $h_M + h_N$, and hours in the home sector h_N (so market hours equal the difference between row 2 and row 3). The first column presents equilibrium values for the no home production model ($\omega = 1$) and the second column presents equilibrium values for the home production model ($\gamma = 1$). For convenience, we have dropped the household index ι from the table. The constant $\mathbb{C}$ is the only common variable across households ι, while all sources of heterogeneity and allocations are ι-specific.

Starting with the no home production model in the first column, c_M depends positively on the tax-adjusted uninsurable permanent productivity component $(1-\tau_1)\alpha$ and negatively on the disutility of work B. Market consumption c_M does not depend on the transitory component of market productivity ε because state-contingent contracts insure against variation in ε. Hours are supplied only in the market sector, so $h_M + h_N = h_M$. The second row shows that hours increase in the after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$ with an elasticity η. This reflects the substitution effect on labor supply from variations in after-tax market productivity. Conditional on $\tilde{z}_M$, hours decrease in $(1-\tau_1)\alpha$ which reflects the income effect from uninsurable variations in market productivity. When $\gamma = 1$, substitution and income effects from variations in α cancel out and hours depend positively only on the insurable component ε. Finally, hours decrease in the disutility of work B.

In the home production model in the second column, the solution for c_M is the same as in the no home production model under $\gamma = 1$ up to a rate of transformation $R \equiv 1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} (z_N/\tilde{z}_M)^{\phi-1}$. This rate reflects the fact that households tend to shift consumption in the sector in which productivity is higher. The expression for total hours $h_M + h_N$ is the same between the two models under $\gamma = 1$, with the difference being that η now governs the response of total hours instead of market hours. Finally, in the last row we present the expression for home production hours h_N. Home hours h_N increase in home productivity z_N when the two sectors are substitutes ($\phi > 1$).

2.3 Identification of Sources of Heterogeneity

In this section we demonstrate how to infer the sources of heterogeneity across households, $\{\alpha, \varepsilon, B, z_N\}_\iota$, such that the models without and with home production both account perfectly for a given cross-sectional data on market expenditures, hours, and productivity. The observational equivalence theorem states that the two models are equivalent in terms of accounting for the same data on cross-sectional market outcomes. This allows us to compare the two models and isolate the role of home production for the causes, welfare consequences, and policy implications of labor market dispersion.

Observational Equivalence Theorem. Let $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N\}_\iota$ be some cross-sectional data.

Then, for any given parameters $(\eta, \phi, \tau_0, \tau_1)$:

1. There exists $\{\alpha, \varepsilon, B\}_\iota$ such that under $\omega = 1$: $\{c_M, h_M, z_M\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$.
2. There exists $\{\alpha, \varepsilon, B, z_N\}_\iota$ such that under $\gamma = 1$: $\{c_M, h_M, z_M, h_N\}_\iota = \{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N\}_\iota$.

The theorem uses the fact that, in each model, the equilibrium allocations presented in Table 1 can be uniquely inverted to obtain the sources of heterogeneity that lead to these allocations. The formal proof is presented in Appendix B.

Table 2 presents the inferred sources of heterogeneity that allow the no home production model to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M\}_\iota$ and the home production model to generate the cross-sectional data $\{\bar{c}_M, \bar{h}_M, \bar{z}_M, \bar{h}_N\}_\iota$. Henceforth, we drop the bar to indicate variables observed in the data since, by appropriate choices of the sources of heterogeneity, both models generate perfectly these data.

A simple numerical example in Table 3 provides intuition for the observational equivalence theorem and draws lessons from it. Consider an economy with two types of households, no taxes $\tau_0 = \tau_1 = 0$, and preference parameters $\gamma = \eta = 1$. In the first panel, the economist does not observe home production data and uses the no home production model to infer the sources of heterogeneity. The economist observes household 1 earning a wage of $z_M = 20$, spending $c_M = 1,000$, and working $h_M = 60$. Household 2 also earns $z_M = 20$, but spends $c_M = 600$ and works $h_M = 40$. Row 1 of Table 2 shows that households with a higher expenditures to hours ratio, c_M/h_M, or higher after-tax market productivity, $\tilde{z}_M$ have a higher uninsurable productivity component α. In Table 3 we infer that α is higher for household 1 than for household 2 (2.88 vs. 2.83) because household 1 has a higher c_M/h_M than household 2. Since both households have the same market productivity and $\alpha + \varepsilon$ add up to (log) market productivity, we infer that household 2 has a higher insurable productivity component ε than household 1 (0.16 vs. 0.11). Finally, we infer that household 2 has a higher disutility of work B because it spends less and works less than household 1.

We next allow the economist to observe home hours h_N and use the home production model with an elasticity $\phi = 1.5$ to infer the sources of heterogeneity. In addition to the same data on

Table 2: Summary of Identified Sources of Heterogeneity

		No Home Production: $\omega = 1$
1.	α	$\frac{(1+\eta)^{-1}}{(1-\tau_1)}\left[\log\left(\frac{c_M}{h_M}\right) - \log(1-\tau_0) + \eta(1-\tau_1)\log z_M - \log \mathbb{C}\right]$
2.	ε	$\log z_M - \alpha$
3.	B	$\left(1+\frac{1}{\eta}\right)[\log(1-\tau_0) + (1-\tau_1)\,\alpha] - \left(\frac{1}{\eta}+\gamma\right)\log c_M + \frac{1}{\eta}\log \mathbb{C}$
		Home Production: $\gamma = 1$
4.	α	$\frac{(1+\eta)^{-1}}{(1-\tau_1)}\left[\log\left(\frac{c_M+\tilde{z}_M h_N}{h_M+h_N}\right) - \log(1-\tau_0) + \eta(1-\tau_1)\log z_M - \log \mathbb{C}\right]$
5.	ε	$\log z_M - \alpha$
6.	B	$\left(1+\frac{1}{\eta}\right)[\log(1-\tau_0) + (1-\tau_1)\,\alpha - \log(c_M + \tilde{z}_M h_N)] + \frac{1}{\eta}\log \mathbb{C}$
7.	z_N	$\tilde{z}_M^{-\frac{\phi}{1-\phi}}\left(\frac{h_N}{c_M}\right)^{\frac{1}{\phi-1}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\phi}{1-\phi}}$

Table 2 summarizes the inferred sources of heterogeneity for the environment without home production (first panel) and for the economy with home production (second panel). We define the constant $\mathbb{C} \equiv \int(1-\tau_0)\exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_\zeta(\zeta)$ and the after-tax market productivity $\tilde{z}_M = (1-\tau_0)z_M^{1-\tau_1}$.

Table 3: Numerical Example

Household	z_M	c_M	h_M	h_N	α	ε	B	z_N	EV
1	20	1,000	60		2.88	0.11	-8.00		0
2	20	600	40		2.83	0.16	-7.09		399
1	20	1,000	60	20	2.91	0.09	-8.63	8.00	0
2	20	600	40	40	2.91	0.09	-8.63	26.07	-642

Table 3 presents an example with parameters $\tau_0 = 0$, $\tau_1 = 0$, $\gamma = 1$, $\eta = 1$, $\omega = 0.5$, and $\phi = 1.5$. The first panel shows inference based on the no home production model and the second panel shows inference based on the home production model. The column labeled EV shows the equivalent variation for household 2 to achieve the utility level of household 1.

(c_M, h_M, z_M), now the economist observes the first household working 20 hours in the home sector and the second household working 40 hours in the home sector. How does this additional data change the inferences we draw? As shown in Table 2, the inferred α now depends on the ratio of the market value of consumption to total hours, $(c_M + \tilde{z}_M h_N)/(h_M + h_N)$, rather than on the ratio of market expenditures to market hours c_M/h_M. Since both households have the same market value of consumption, $c_M + \tilde{z}_M h_N = 1,400$, and the same total hours, $h_M + h_N = 80$, the α's are equalized across households. Given the same market productivity, the ε's are also equalized. Given that the two households consume and work the same, the B's are also equalized. As Table 3 shows, all differences in observables between the two households are loaded into home productivity z_N. We infer that z_N is higher for household 2 because it works more than household 1 in the home sector and $\phi > 1$.

There are two lessons we draw from this illustrative example. First, home productivity z_N is significantly dispersed across households and absorbs part of the dispersion one would attribute to (α, ε, B) in the absence of home production. This result generalizes in our quantitative application using U.S. data below where we find that z_N is three times as dispersed as z_M and that the dispersion in (α, ε, B) is smaller in the home production model.

The second lesson we draw concerns the welfare implications of labor market dispersion. Household's welfare ranking is different depending on whether the data has been generated by a model with or without home production. The last column of Table 3 shows equivalent variations (EV). The equivalent variation equals the transfer we have to give to households in order to achieve a given level of utility, allowing households to re-optimize their consumption and hours choices. The reference utility level in Table 3 is assumed to be the utility of household 1 and, thus, the EV for household 1 is always equal to zero. In the no home production model, the EV for household 2 equals 399. In the home production model, the two households are identical in terms of their (α, ε, B), but household 2 has a higher home productivity z_N. Therefore, the welfare ranking changes and the EV becomes -642.

2.4 Discussion of Our Approach

Before proceeding to the quantitative results, we pause to make two comments regarding the merits of our approach compared to alternative approaches. First, standard general equilibrium models with uninsurable risk, such as Huggett (1993) and Aiyagari (1994) and extensions with endogenous labor supply such as Aiyagari and McGrattan (1998), Pijoan-Mas (2006), Chang and Kim (2007), and Marcet, Obiols-Homs, and Weil (2007), feature self insurance via a risk-free bond. Solutions to these models are obtained computationally. While our model also allows households to trade a risk-free bond (by setting $x(\zeta_{t+1}^{j}) = 1$ for all states ζ_{t+1}^{j}), the assumptions on asset markets, stochastic processes, and preferences allow us to derive a no-trade result and characterize equilibrium allocations in closed form. Owing to the analytical results, a major advantage of our framework is the transparency and generality of the identification.

Despite the wealth distribution not being an object of interest within our framework, a dynamic structure with non-labor income is still essential. In a framework without non-labor income households maximize utility subject to home production technology $c_N = z_N h_N$ and the flow budget constraint $c_M = z_M h_M$. Here, observed market productivity z_M is constrained to equal the market consumption to hours ratio c_M/h_M. The model is underidentified because any choice of (z_M, B, z_N) does not suffice to match data on (z_M, c_M, h_N, h_M). The structure we have adopted allows us to express four observables, (z_M, c_M, h_N, h_M), as a function of four productivity and taste shifters, $(\alpha, \varepsilon, B, z_N)$, and invert the solution to the household problem to identify the shifters. The key in the partial insurance economy is that the two components of market productivity, α and ε, affect differentially market consumption and market hours.

Second, our approach to identifying the sources of heterogeneity is such that the model accounts perfectly for any given cross-sectional data on market consumption, hours, and wages. Heathcote, Storesletten, and Violante (2014) impose distributional assumptions on the sources of heterogeneity and estimate parameters of these distributions using method of moments. Our approach, instead, does not impose parametric assumptions on the distributions. Our non-parametric approach is appealing in light of the work of Guvenen, Karahan, Ozkan, and Song

(2016) who use administrative data to document significant deviations of the distribution of individual earnings growth from lognormality. Conceptually, our approach is more similar to Hsieh and Klenow (2009) who infer wedges in first-order conditions such that firm-level outcomes generated by their model match data analogs.

3 Quantitative Results

This section presents the quantitative results. We begin by describing the data sources and the parameterization of the model. We then present the sources of heterogeneity across households.

3.1 Data Sources

For our quantitative results, we use cross-sectional data on market consumption, market productivity (wages), market hours, and home hours. The data are drawn from the Consumer Expenditure Survey (CEX), the American Time Use Survey (ATUS), and the Current Population Survey (CPS).

Consumer Expenditure Survey. Data for market expenditures c_M, market productivity z_M, and market hours h_M come from the CEX. The CEX is the most comprehensive data on consumption expenditures by households in the United States. This survey is conducted by the U.S. Bureau of Labor Statistics and covers over 5,000 households per wave. We combine the information from the interview surveys collected between 1996 and 2016.

The CEX contains spending for detailed categories. For our baseline analyses, c_M is annual non-durable consumption expenditures which include food and beverages, tobacco, personal care, apparel, utilities, household operations (including child care), public transportation, gasoline, reading material, and personal care. Non-durable consumption expenditures exclude health and education. This definition is closest to the one used by Aguiar and Hurst (2013), but Section 5 shows the robustness of our results to alternative measures that range from a narrower definition (food) to a broader definition (total spending including durables) of expenditures.

Our measure of income is the amount of salary before deductions earned over the past 12 months. Individual wages are defined as income divided by hours usually worked in a year, which is the product of weeks worked and the usual hours worked per week. Our model features a single decision maker within each household. For our baseline analyses that focus on married and cohabiting households, household market hours h_M are defined as the sum of hours worked by spouses. For the households' market productivity z_M, we use the average of wages of individual members weighted by their hours of work.

American Time Use Survey. Data for home hours h_N come from the ATUS waves between 2003 and 2016. The ATUS is conducted by the U.S. Bureau of Labor Statistics, with individuals that are randomly selected from a group of households that completed their eight and final month interview for the CPS. Each individual's response is based on a 24-hour time diary of the previous day and reported activities are assigned to specific categories by survey personnel. We define time spent on home production similar to Aguiar, Hurst, and Karabarbounis (2013). Home production time includes time spent on activities such as housework, preparing meals, shopping, home and car maintenance, garden activities, child care, and care for other household members.

The ATUS does not contain information on market expenditures. We, therefore, impute time use data from the ATUS into the CEX. Our imputation is based on an iterative procedure where individuals in the CEX are allocated the mean home hours of matched individuals from the ATUS based on group characteristics. We begin the procedure by matching individuals based on work status, race, gender, and age. We then proceed to refine these estimates by adding a host of additional characteristics, such as family status, education, disability status, geography, hours worked, and wages, and matching individuals based on these characteristics whenever possible. Similar to market hours, we first impute home hours to individuals and then sum up these hours at the household level.[7]

[7] The Panel Study of Income Dynamics (PSID) contains information on time spent on home production. We prefer using the CEX/ATUS sample because the PSID contains restricted measures of consumption in the earlier years, has lower quality of time use data, and has a significantly smaller sample size. Estimates of the average home hours in the PSID typically are higher than in ATUS (Achen and Stafford, 2005). In an approach similar to ours, Blundell, Pistaferri, and Saporta-Eksten (2017) use the CEX to impute expenditures to the ATUS. In sensitivity analyses presented below, we confirm that the key moments that underlie our inference are similar between the CEX/ATUS sample and two alternative datasets from the Netherlands and Japan that contain information on both consumption expenditures and time spent on home production.

Table 4: Parameter Values

Parameter	$\omega = 1$	$\omega < 1$	Rationale
τ_1	0.12	0.12	$\log(\tilde{y}/h_M) = \text{constant} + (1-\tau_1)\log z_M$.
τ_0	-0.36	-0.36	Match $G/Y = 0.10$.
γ	1	1	Nesting of models.
η	1.14	1.14	Frisch elasticity $\eta(1-\tau_1) = 1$.
ω	—	0.53	Normalize $\mathbb{E}z_N = \mathbb{E}z_M$.
ϕ	—	2.20	Equalize $\text{corr}(\log p, B)$ across models.

Table 4 describes the parameter values of the models without home production ($\omega = 1$) and with home production ($\omega < 1$). We define productivity $p \equiv \frac{h_M}{h_M+h_N} z_M + \frac{h_N}{h_M+h_N} z_N$.

Current Population Survey. We estimate the parameters of the tax function (τ_0, τ_1) using data from the CPS Annual Social and Economic Supplement (ASEC) covering all years between 2005 and 2015. The ASEC applies to households surveyed for the CPS March sample and contains labor force and income information for household members above the age of 15. In particular, we use information on pre-tax personal income, tax liabilities at the federal and state level, Social Security payroll deductions, as well as usual hours and weeks worked.

Sample Adjustments. For our baseline analyses, we consider married and cohabiting households with heads between 25 and 65 years of age. We keep in the sample only households that completed all four interviews in the CEX. Consistent with our definition of consumption expenditures that excludes educational spending, we exclude households in which the head is a student. We drop observations for households with a market productivity below 3 dollars or above 300 dollars per hour in 2010 dollars. Furthermore, we drop outliers in market expenditures, market hours, and home hours. The final sample contains 26,851 households. We adjust all variables for household composition and size. In all our results, we weight households with the sample weights provided by the surveys. Appendix C contains further details of our sample construction.

3.2 Parameterization

Table 4 presents the parameter values for our baseline analyses in the no home production model ($\omega = 1$) and the home production model ($\omega < 1$). We estimate the parameters of the tax function using data from the CPS. To estimate the progressivity parameter τ_1, we regress log after-tax market productivity on log market productivity before taxes. We estimate $\tau_1 = 0.12$ with a standard error below 0.01.[8] We choose $\tau_0 = -0.36$ to match an average tax rate on labor income equal to 0.10.

We set the curvature of the utility function with respect to consumption to $\gamma = 1$. For the home production model, we obtained the equilibrium allocations in closed form under $\gamma = 1$. We choose $\gamma = 1$ also for the no home production model. For our purposes it is essential to nest the no home production model in the home production model, so that welfare differences across the two models do not arise from a different curvature of the utility function with respect to consumption.

We choose the parameter $\eta = 1.14$ so that the Frisch elasticity $\eta(1-\tau_1)$ of total hours $h_M + h_N$ equals one in both models. Following the logic in Rupert, Rogerson, and Wright (2000) who argue that estimates of the Frisch elasticities are biased in the presence of home production, we note that the elasticities of market hours h_M differ between the two models. Under $\omega = 1$ we obtain a Frisch elasticity of market hours equal to 1, whereas under $\omega < 1$ we obtain a Frisch elasticity of market hours equal to 2.5.[9]

We next describe the parameters that are specific to the home production model, ω and ϕ. Without loss, the weight parameter $\omega = 0.53$ is chosen to equalize average productivity across sectors, $\mathbb{E}z_N = \mathbb{E}z_M$.[10] We choose the elasticity of substitution between the two sectors, such

[8] Our estimated tax parameter lies in between the estimates of 0.06 in Guner, Kaygusuz, and Ventura (2014) and the estimate of 0.19 in Heathcote, Storesletten, and Violante (2014). Using their tax function $\log \tilde{y} = \text{constant} + (1-\tau_1)\log y$, we would estimate $\tau_1 = 0.15$. We, therefore, argue that it is relatively inconsequential whether we apply the progressivity parameter $(1-\tau_1)$ to after-tax wages or after-tax labor income.

[9] The Frisch elasticity is defined as the change in log hours in response to a unit change in log wages, holding constant the marginal utility of wealth. We also estimate Marshallian elasticities by holding constant non-labor income. For total hours, the two models generate similar elasticities that are roughly zero. For market hours, the $\omega = 1$ model generates a roughly zero elasticity and the $\omega < 1$ model generates an elasticity of roughly 0.3.

[10] This normalization is inconsequential for all our results, conditional on matching the average time spent on home production. This is because for any given targeted value of $\mathbb{E}z_N/\mathbb{E}z_M$, the parameter ω adjusts to so that the model matches the average time spent on home production.

that the correlation between households' productivity, $p = \frac{h_M}{h_M+h_N} z_M + \frac{h_N}{h_M+h_N} z_N$, and disutility of work, B, is as close as possible between the two models. We find that $\phi = 2.2$. Our targeted moment ensures that the welfare comparisons are not contaminated by productive households having different disutility in one of the economies. Our estimate of the elasticity of substitution is consistent with those found in the literature. For example, most estimates of Rupert, Rogerson, and Wright (1995) for couples fall between roughly 2 and 4 and Aguiar and Hurst (2007) obtain estimates of around 2.[11]

3.3 Inferred Sources of Heterogeneity

In Figure 1 we present the age profiles for the means of $(\alpha, \varepsilon, B, z_N)$. To obtain these age profiles, we first infer the productivity and preference shifters from the equations in Table 2 using data on (c_M, h_M, z_M, h_N). For each shifter, we regress the shifter on age dummies, cohort dummies, and normalized year dummies as in Deaton (1997).[12] The coefficients on the age dummies give the means of each shifter by age group relative to age 25. To reduce noise in the figures, we present the fitted values from locally weighted regressions of the coefficients on the age dummies on age.

In the upper panels of Figure 1 we see that the means of the market productivity components α and ε are similar until roughly age 45 between the no home production model ($\omega = 1$) and the home production model ($\omega < 1$). After age 45, the no home production model implies a larger increase in the uninsurable component α and a larger decline in the insurable component ε than the home production model. In the lower panels we see that, whereas both models generate an increase in the disutility of work B over the life-cycle, this increase is significantly smaller in the home production model (around 0.3 log points by age 65). Instead, the home production model generates a large increase in home productivity z_N over the life-cycle. To understand

[11] Our inference is based on a moment condition imposed on unobserved variables. We note that we cannot follow the typical approach in the literature of estimating ϕ from the covariation between observed relative hours and observed wages. The observational equivalence theorem implies that for any ϕ we can identify productivity and preference shifters so that the model matches cross-sectional data for each household. So, for any given ϕ, our choices of the sources of heterogeneity guarantees that the model matches the covariation between relative hours and wages. Equivalently, we argue that a regression of relative hours on wages suffers from an omitted variable bias because productivity in the home sector and the disutility of work are correlated with wages.

[12] All results reported in the paper are similar when we extract the age effect in regressions that either control only for cohort dummies or only for year dummies.

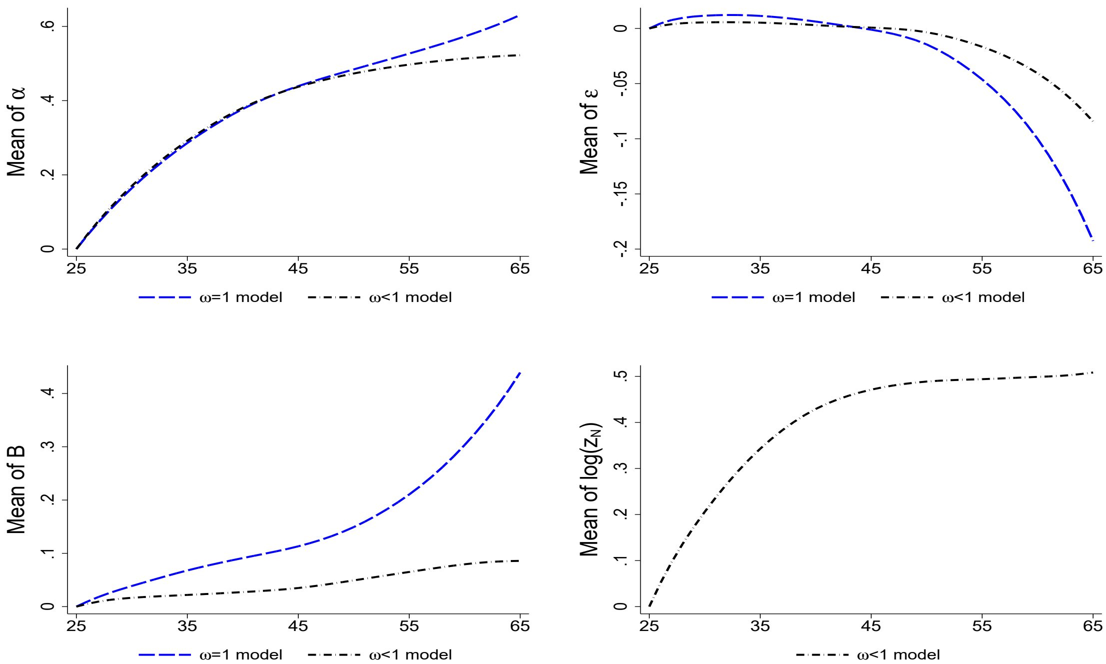


Figure 1: Means of Preference and Productivity Shifters

Figure 1 plots the age means of uninsurable component of market productivity α, insurable component of market productivity ε, disutility of work B, and home productivity $\log z_N$ for the economy with ($\omega < 1$, black) and without home production ($\omega = 1$, blue).

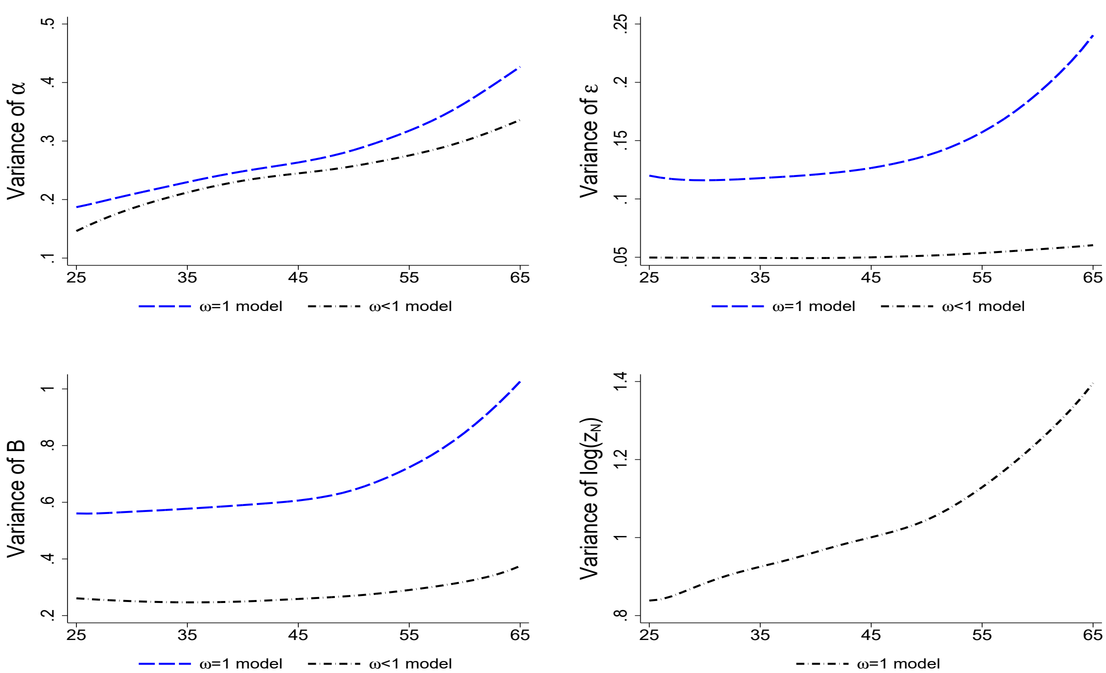


Figure 2: Variances of Preference and Productivity Shifters

Figure 2 plots the age variances of uninsurable component of market productivity α, insurable component of market productivity ε, disutility of work B, and home productivity $\log z_N$ for the economy with ($\omega < 1$, black) and without home production ($\omega = 1$, blue).

the difference between the two models, recall that market expenditures c_M are humped shaped and market hours h_M tend to decline over the life-cycle. The life-cycle increase in z_N allows the home production model to generate these life-cycle patterns for c_M and h_M. The home production model, thus, requires a significantly smaller increase in B over the life-cycle to match these patterns.

In Figure 2 we present the age profiles for the cross-sectional variances of $(\alpha, \varepsilon, B, z_N)$, which equal the variances of the residuals for each age from a regression of each shifter on age dummies, cohort dummies, and normalized year dummies. The main result is that the home production model generates significantly smaller variances of α, ε, and B. As is seen in the last panel, z_N absorbs a substantial fraction of the dispersion in observables. As a benchmark, the variance of $\log z_M$ is roughly 0.3, so home productivity z_N is roughly three times as dispersed as z_M in the cross-section of households.

What explains the large dispersion in home productivity across households? Our inference of home productivity comes from a standard first-order condition that emerges in a large class of models and states that households shift consumption to the sector with the higher (after-tax) productivity:

$$\frac{c_M}{c_N} = \left(\frac{\omega}{1-\omega}\right)^{\phi} \left(\frac{\tilde{z}_M}{z_N}\right)^{\phi}, \tag{10}$$

where $\tilde{z}_M = (1-\tau_0) z_M^{1-\tau_1}$. After substituting the home production technology $c_N = z_N h_N$ in equation (10), we obtain:

$$\log z_N = \text{constant} + \left(\frac{1}{\phi - 1}\right) (\phi \log \tilde{z}_M + \log h_N - \log c_M). \tag{11}$$

Equation (11) expresses $\log z_N$ in the left-hand side as a function of the observables $\log \tilde{z}_M$, $\log h_N$, and $\log c_M$, for a given value of the elasticity of substitution ϕ between sectors.

The intuition for our result that home productivity is more dispersed than market productivity in the cross-section of households is that $\log z_N$ cumulates the dispersions of three observables, $\log \tilde{z}_M$, $\log h_N$, and $\log c_M$, that are relatively uncorrelated with each other. From equation (11), we see that when ϕ tends to zero and the two goods tend to become perfect complements, we

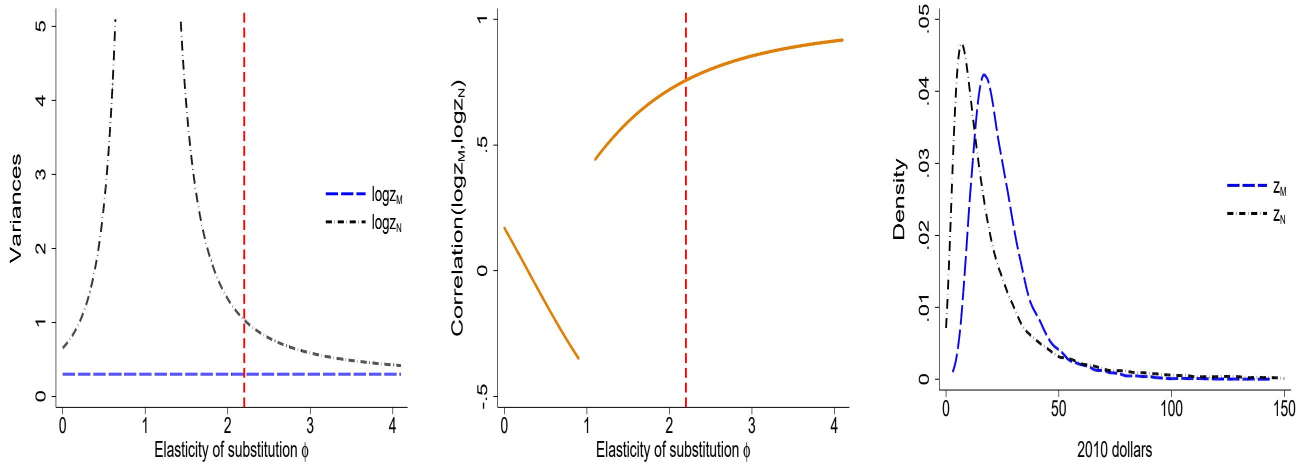


Figure 3: Patterns of Sectoral Productivity

The left panel of Figure 3 shows the variance of home productivity $\log z_N$ and market productivity $\log z_M$ and the middle panel shows the correlation between the two variables as a function of the elasticity of substitution between the two sectors ϕ. The dashed line shows the variances and correlation at our baseline value of $\phi = 2.2$. The right panel plots the distributions of z_M and z_N at $\phi = 2.2$.

obtain $\log z_N = \text{constant} + \log c_M - \log h_N$. In this case the variance of $\log z_N$ is roughly twice as high as the variance of $\log \tilde{z}_M$. The reason is that $\log c_M$ and $\log h_N$ are relatively uncorrelated in the cross-section of households for each age and that the variances of $\log c_M$ and $\log h_N$ are both roughly equal to 0.3. When ϕ tends to infinity and the two goods tend to become perfect substitutes, we obtain $\log z_N = \text{constant} + \log \tilde{z}_M$. In that case, the variance of $\log z_N$ tends to the variance of $\log \tilde{z}_M$. As equation (11) shows, around $\phi = 1$, the variance of $\log z_N$ tends to infinity. To summarize, for any value of ϕ, the variance of $\log z_N$ is larger than the variance of $\log \tilde{z}_M$.

Figure 3 summarizes the properties of home and market productivity. The left panel shows the variance of $\log z_N$ relative to the variance of $\log z_M$ and the middle panel shows the correlation between the two productivities as function of the elasticity of substitution between sectors ϕ. For the variance, we obtain that the variance of $\log z_N$ is larger than the variance of $\log z_M$ for any value of $\phi < 4$ in the figure.[13] For the correlation between the two variables, however,

[13] We note that the argument in the preceding paragraph referred to after-tax market productivity $\log \tilde{z}_M$ whereas in Figure 3 we use the more primitive pre-tax market productivity $\log z_M$. The former measure of productivity is roughly 70 percent as dispersed as the latter because our estimated tax progressivity parameter $\tau_1 = 0.12$ implies a compression of its dispersion relative to the dispersion in pre-tax productivity.

the parameter ϕ becomes crucial. We find that when the two sectors are substitutes, $\phi > 1$, productivity in the home sector is positively correlated with productivity in the market sector. When the two sectors are complements, $\phi < 1$, the correlation is lower and can be either negative or positive. While we find the $\phi > 1$ case more intuitive, in our sensitivity results in Section 5, we show that our conclusions regarding inequality hold irrespective of whether ϕ is above or below one.

The right panel of Figure 3 plots the distributions of home productivity z_N and market productivity z_M in the cross-section of households of all ages for an elasticity $\phi = 2.2$. As it is seen in the figure, z_N is significantly more dispersed than z_M. While both productivities have a mean of roughly 27 dollars, the median of z_M is roughly 22 dollars and the median of z_N is roughly 14 dollars. The z_N distribution is also significantly more skewed and thicker-tailed, with a mode of roughly 10 dollars. At the top of the distribution, we find that roughly 3 percent of households have a z_N that exceeds 100 dollars, as compared to roughly 1 percent for z_M.

Finally, in Table 5 we present the correlation matrix of observables and sources of heterogeneity in the no home production model in the upper panel and the home production model in the lower panel. We note that the correlations between observables and each source of heterogeneity are unconditional and, therefore, do not hold constant other sources of heterogeneity that may be correlated. The sources of heterogeneity are allowed to be correlated because the no-trade result does not require the initial draws of heterogeneity to be uncorrelated with each other.

The unconditional correlations among observables (z_M, c_M, h_M) are identical between the two models since both models account perfectly for the same data. Turning to the sources of heterogeneity, we observe that in both models the uninsurable component of market productivity α is positively correlated with market consumption c_M and negatively related with market hours h_M. In the no home production model α is negatively correlated with the insurable component ε, whereas in the home production model the two components of market productivity are essentially uncorrelated. The disutility of work B is negatively correlated with consumption and hours in both models, but the correlations are stronger in the no home production model. Finally, home

Table 5: Unconditional Correlations

Model $\omega = 1$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	α	ε	B	$\log z_N$
$\log z_M$	1.00	0.28	-0.11	–	0.77	0.39	0.45	–
$\log c_M$		1.00	0.13	–	0.68	-0.55	-0.60	–
$\log h_M$			1.00	–	-0.42	0.44	-0.63	–
$\log h_N$				–	–	–	–	–
α					1.00	-0.29	0.18	–
ε						1.00	0.42	–
B							1.00	–

Model $\omega < 1$	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	α	ε	B	$\log z_N$
$\log z_M$	1.00	0.28	-0.11	0.04	0.91	0.41	0.39	0.76
$\log c_M$		1.00	0.13	-0.04	0.57	-0.57	-0.57	-0.24
$\log h_M$			1.00	-0.30	-0.26	0.31	-0.26	-0.29
$\log h_N$				1.00	0.10	-0.12	-0.36	0.51
α					1.00	-0.01	0.14	0.57
ε						1.00	0.64	0.57
B							1.00	0.44

Table 5 presents unconditional correlations in the no home production model ($\omega = 1$) and the home production model ($\omega < 1$).

productivity z_N is negatively correlated with market consumption c_M and market hours h_M and positively correlated with home hours h_N. We emphasize that the correlations for z_N depend on the value of the elasticity of the substitution. We find these correlations reasonable, corroborating our choice of an elasticity of substitution $\phi > 1$.

4 Inequality

In this section we present our inequality results. By inequality, we mean a mapping from the dispersion in observed allocations and inferred sources of heterogeneity to measures that capture welfare differences across households. We acknowledge that there are various such mappings and, therefore, present four inequality measures. These are the cross-sectional dispersion in equivalent variation, the cross-sectional dispersion in transfers required to equalize marginal utilities, the ex-ante lifetime welfare loss from heterogeneity, and the degree of progressivity in an optimal tax system.

Our conclusion is that inequality is larger when the data generating process is the model with home production. We reach this conclusion using all four different measures of inequality. The appealing feature of the observational equivalence theorem is that we obtain this inequality result despite both models generating the same data on market outcomes.

4.1 Equivalent Variation

The first inequality metric we study is the dispersion in equivalent variation across households. The equivalent variation, a broadly used metric in welfare economics, equals the change in income required for an optimizing household to achieve a reference level of utility. Let $\hat{\iota}$ be a reference household with allocations of consumption and time given by the vector $\hat{x}_t$, a flow utility $U(\hat{x}_t; \hat{\iota})$, and a value function $\hat{V}_t(\hat{\iota})$. For every household ι, we compute the income change $T_t(\iota)$ that makes it indifferent between being ι and being $\hat{\iota}$ in the current period, holding constant ι's expectation over all future allocations. Equivalently, the equivalent variation $T_t(\iota)$ solves:

$$\hat{V}_t(\hat{\iota}) = \max_{\{c_{M,t}, h_{M,t}, h_{N,t}\}} \left\{ U(c_{M,t}, h_{M,t}, h_{N,t}; \iota) + \beta\delta\mathbb{E}_t \left[V_{t+1}(\iota')|\iota \right] \right\}, \tag{12}$$

subject to the home production technology $c_{N,t} = z_{N,t} h_{N,t}$ and the budget constraint:

$$c_{M,t} = \tilde{y}_t + T_t(\iota) + \overline{\mathrm{NA}}_t(\iota). \tag{13}$$

In equation (12) we define $\hat{V}_t(\hat{\iota}) \equiv U(\hat{x}_t; \hat{\iota}) + \beta\delta\mathbb{E}_t [V_{t+1}(\iota')|\iota]$ and in equation (13) we keep the net asset position $\overline{\mathrm{NA}}_t(\iota)$ constant at its initial value before the transfer $T_t(\iota)$ is given.

Figure 4 presents the cross-sectional dispersion in equivalent variation for every age. Units are in thousands of 2010 dollars.[14] The left panel shows the standard deviation of equivalent variation. At age 25, the standard deviation equals roughly 9 thousand dollars in both economies. By age 65, however, the standard deviation has increased to 24 thousand in the home production model

[14] In Figure 4 we assume that $\hat{\iota}$ is the household with the median utility. Our results are similar when we define $\hat{\iota}$ as the household with the mean utility, the household with the median utility in every age, or the household with the mean utility in every age.

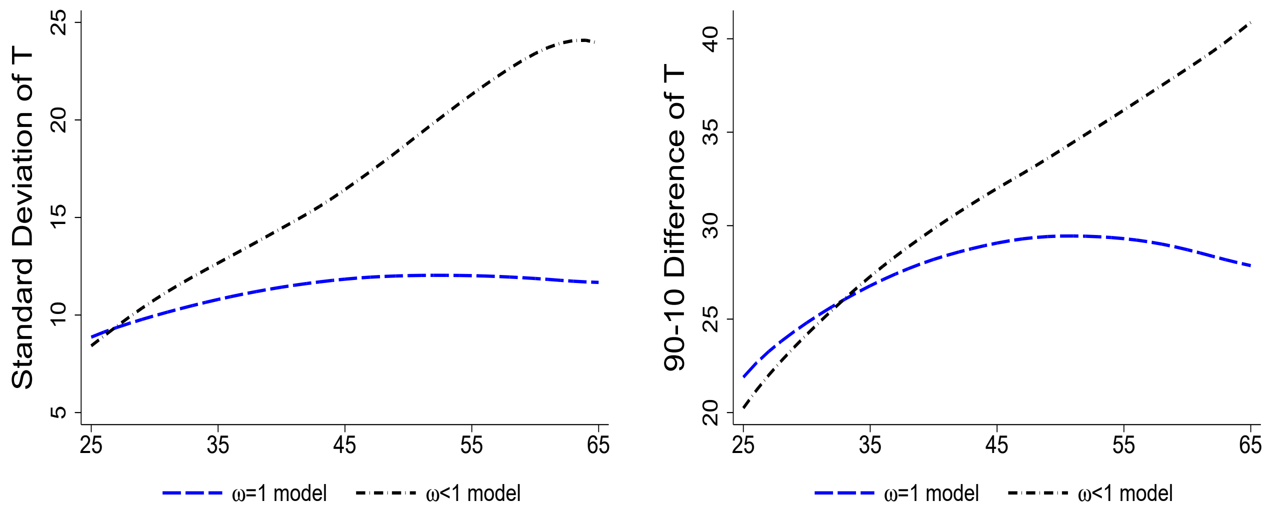


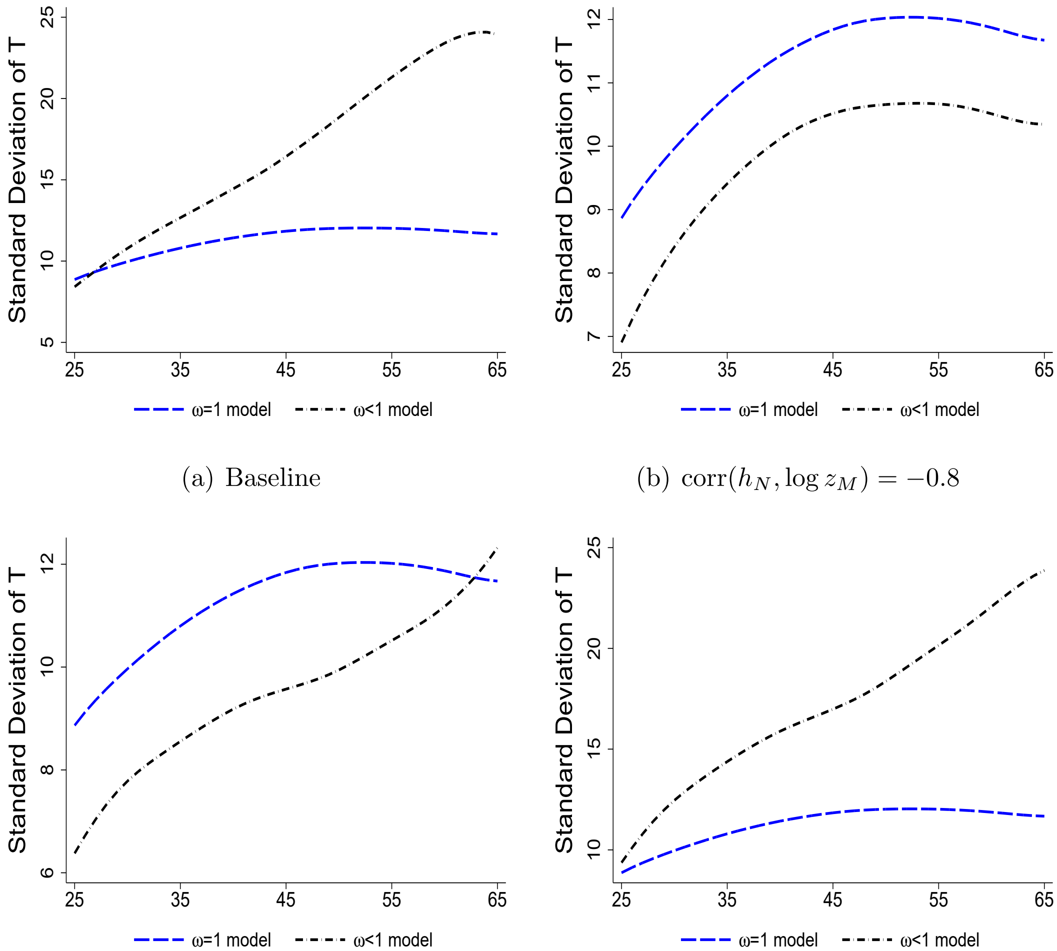
Figure 4: Inequality: Dispersion in Equivalent Variation

Figure 4 shows the dispersion in equivalent variation T for the environment with ($\omega = 1$, blue) and without home production ($\omega < 1$, black) by age. Units are in thousands of 2010 dollars.

as opposed to 11 thousand in the no home production model. The right panel shows that we obtain the same divergent pattern between the two models using the difference between the 90th and 10th percentile in equivalent variation.

What drives our inference that inequality is higher with home production? We argue that an important feature of the data that drives our inference is that home hours are not significantly correlated with market consumption and market productivity in the cross section of households (see Table 5). Households with lower consumption expenditures or lower market productivity do not spend more time on home production. Thus, home production does not offset heterogeneity that originates in the market sector. Instead, home production exacerbates inequality given the large dispersion in home productivity.

To illustrate this point, Figure 5 shows counterfactual analyses in which we change the correlation of home hours with other observables in the data. The first panel repeats the age profile of standard deviation in equivalent variation $T(\iota)$ shown in the left panel of Figure 4. In the other three panels we repeat our inference of $(\alpha, \varepsilon, B, z_N)$ and then calculate the equivalent variation $T(\iota)$ in counterfactual data in which the correlation of home hours h_N with market productivity $\log z_M$, market expenditures $\log c_M$, and market hours h_M is -0.8 (as opposed to roughly 0 for



(a) Baseline

(b) $\text{corr}(h_N, \log z_M) = -0.8$

(c) $\text{corr}(h_N, \log c_M) = -0.8$

(d) $\text{corr}(h_N, h_M) = -0.8$

Figure 5: Counterfactuals of Dispersion in Equivalent Variation

Figure 5 shows the dispersion in equivalent variation T for the model with ($\omega = 1$, blue) and without home production ($\omega < 1$, black) by age in the baseline economy and in counterfactual datasets.

the first two and -0.40 for the latter in the true data). The figure shows that if the data featured a significantly more negative correlation between home hours and market productivity or market consumption, then we would have concluded that inequality in the model with home production is actually lower. A lower correlation between hours in the two sectors does not change the inequality difference between the two economies.[15]

[15] Rios-Rull (1993) uses PSID data between 1969 and 1982 to argue that in the cross section of individuals, higher wage earners spent less time in home production. In addition to the period covered and quality differences in the measure of home hours between the PSID and the ATUS, there are several additional factors that may explain our different conclusions. Rios-Rull (1993) does not condition on age when examining the covariance between wages and home hours, focuses on individuals rather households and excludes individuals who stay at home, and presents the covariation between wages and home hours in the between-wage group dimension of the data. In additional analyses presented in Section 5, we show the low correlation between wages and home hours is also observed in

4.2 Redistributive Transfers

Our second measure of inequality is the cross-sectional dispersion in transfers required to equalize marginal utilities. After households choose their allocations of market consumption, market hours, and home hours, we allow a utilitarian planner to redistribute aggregate market consumption across households in order to maximize the average of households' utilities. The dispersion in these transfers captures the extent of redistribution required in order to maximize social welfare or, equivalently, to equalize marginal utilities. The utilitarian planner chooses transfers in terms of market consumption $\{t(\iota)\}$ to solve:

$$\max \int_{\iota} U(c_M(\iota) + t(\iota), h_M(\iota), h_N(\iota)) \mathrm{d}\Phi(\iota), \tag{14}$$

subject to aggregate transfers being equal to zero:

$$\int_{\iota} t(\iota) \mathrm{d}\Phi(\iota) = 0. \tag{15}$$

The optimal allocation involves redistributing aggregate market consumption to equalize marginal utilities of market consumption.[16] This requires transfers equal to:

$$\omega = 1: \quad t(\iota) = \int_{\iota} c_M(\iota) \mathrm{d}\Phi\,(\iota) - c_M(\iota), \tag{16}$$

$$\omega < 1: \quad t(\iota) = \int_{\iota} \left(c_M(\iota) + \tilde{z}_M(\iota) h_N(\iota)\right) \mathrm{d}\Phi\,(\iota) - \left(c_M(\iota) + \tilde{z}_M(\iota) h_N(\iota)\right). \tag{17}$$

The dispersion in redistributive transfers $t(\iota)$ differs from the dispersion in equivalent variation $T(\iota)$ in Section 4.1 because it leads to an equalization of marginal utilities instead of utility levels. As equations (16) and (17) show, the benefit of the dispersion in $t(\iota)$ is that it leads to a transparent measure of inequality that depends only on observables and the tax parameters (τ_0, τ_1). In particular, in the no home production model, the transfer equals the difference between average market expenditures and a household's market expenditures $c_M(\iota)$. In the home production model, the transfer equals the difference between the average market value of consumption and a household's market value of consumption $c_M(\iota) + \tilde{z}_M(\iota) h_N(\iota)$.

two alternative datasets from the Netherlands and Japan.

[16] We remind the reader that the marginal utility of market consumption under an equilibrium allocation $(c_M + t, c_N, h_M, h_N)$ is equal to $(c_M + t + \tilde{z}_M h_N)^{-1}$.

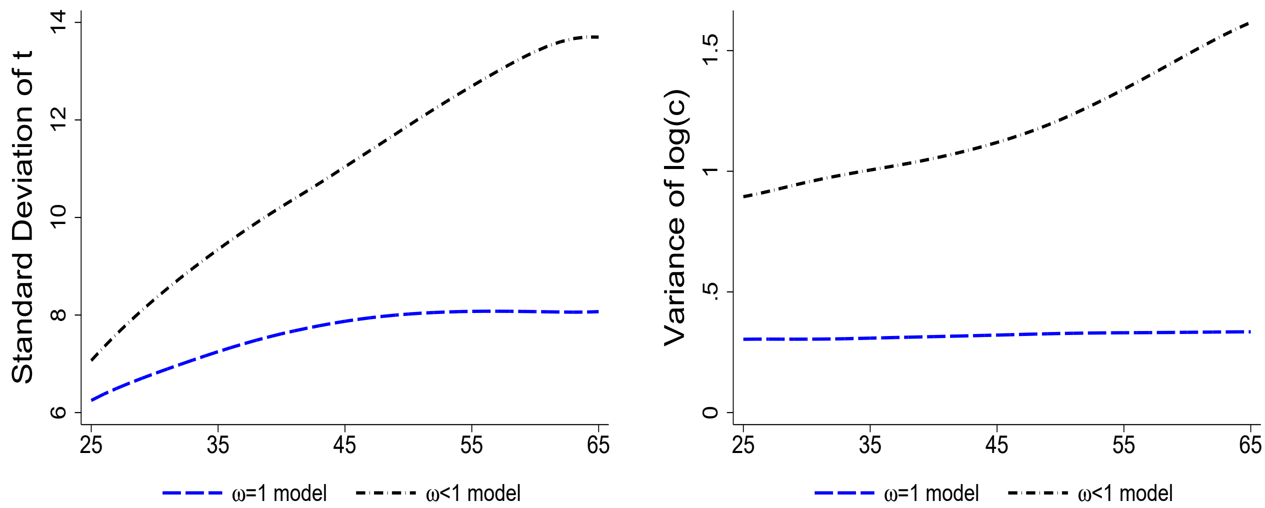


Figure 6: Inequality: Dispersion in Redistributive Transfers and Consumption

The left panel of Figure 6 shows the cross-sectional dispersion in redistributive transfers t for the model with ($\omega = 1$, blue) and without home production ($\omega < 1$, black). The right panel shows the variance of $\log c$ in the two models, where $c = \left(\omega c_M^{\frac{\phi-1}{\phi}} + (1-\omega)c_N^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$.

The left panel of Figure 6 shows the age profiles for the cross-sectional standard deviation in redistributive transfers $t(\iota)$ for the two models. Units are in thousands of 2010 dollars. The standard deviation of $t(\iota)$ is larger and increases by more over the life-cycle in the model with home production. Similar to our conclusions using the measure of inequality based on the equivalent variation $T(\iota)$, home production generates more inequality because in the cross-section of households higher market expenditures c_M are not offset by lower market value of home production $\tilde{z}_M h_N$. In fact, the correlation between these variables is weakly positive (0.15).

The right panel of Figure 6 shows the age profiles for the cross-sectional variance in log consumption, where consumption is $c = \left(\omega c_M^{\frac{\phi-1}{\phi}} + (1-\omega)c_N^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}$. In the no home production model, $\log c = \log c_M$ and the variance of log consumption starts at roughly 0.3 and is weakly increasing over the life-cycle. As the figure shows, the variance of $\log c$ in the home production model is roughly three times as large as in the no home production model and increases significantly over the life-cycle.

4.3 Lifetime Welfare Cost of Heterogeneity

In this section we present the lifetime welfare effect arising from heterogeneity across households. These calculations contrast with our inequality measures thus far which have ignored dynamic considerations. Specifically, we calculate the fraction of lifetime consumption that a household is willing to sacrifice ex-ante to be indifferent between being born in the baseline environment with heterogeneity and allocations $\{c_t, h_{M,t}, h_{N,t}\}$ and a counterfactual environment in which dimensions of heterogeneity are shut off. The allocations in the counterfactual environment are denoted by $\{\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}\}$ and are generated using the equations in Table 1 after shutting off particular dimensions of heterogeneity.

The share of lifetime consumption that households are willing to sacrifice ex-ante to be indifferent between the actual and counterfactual economy is the λ that solves:

$$\mathbb{E}_{j-1}V\left(\{c_t, h_{M,t}, h_{N,t}\}\right) = \mathbb{E}_{j-1}V\left(\{(1-\lambda)\hat{c}_t, \hat{h}_{M,t}, \hat{h}_{N,t}\}\right) . \tag{18}$$

When $\lambda > 0$, households prefer the counterfactual to the actual allocation. Benabou (2002) and Floden (2001) have emphasized that total welfare effects from heterogeneity, λ, arise both from level effects when aggregate allocations change and effects capturing changes in the dispersion of allocations across households. Therefore, alongside λ, we discuss how heterogeneity influences the aggregate market value of consumption $\int_\iota \left(c_M(\iota) + \tilde{z}_M(\iota) h_N(\iota)\right) \mathrm{d}\Phi(\iota)$ and aggregate labor productivity $\int_\iota \left(h_M(\iota) + h_N(\iota)\right) z_M(\iota) \mathrm{d}\Phi(\iota) \big/ \int_\iota \left(h_M(\iota) + h_N(\iota)\right) \mathrm{d}\Phi(\iota)$. We denote by λ_c and λ_p the percent change in aggregate consumption and aggregate labor productivity between the counterfactual and the baseline allocation.

Table 6 presents our results for the model without ($\omega = 1$) and with ($\omega < 1$) home production. In the first row, we shut off all sources of heterogeneity and both models collapse to a representative household economy. The welfare costs of heterogeneity are more than three times larger in the model with home production ($\lambda = 0.25$ vs $\lambda = 0.07$). This difference mostly reflects the differential cost of dispersion in allocations across households rather than level effects because aggregate productivity and consumption do not change significantly in the home production model.[17]

[17]Higher dispersion in the insurable component of market productivity ε increases aggregate market consumption

Table 6: Lifetime Consumption Equivalence

	No Home Production: $\omega = 1$			Home Production: $\omega < 1$		
No dispersion in ...	λ_c	λ_p	λ	λ_c	λ_p	λ
1. z_M, z_N, B	-0.10	0.06	0.07	0.00	0.03	0.25
2. z_M, z_N	-0.02	0.06	0.09	0.04	0.03	0.25
3. z_M	-0.02	0.06	0.09	0.04	0.03	0.14
4. z_N	—	—	—	0.00	0.00	0.23

Table 6 shows the percent changes in aggregate consumption λ_c, aggregate labor productivity λ_p, and welfare λ for the environment without ($\omega = 1$) and with ($\omega < 1$) home production. In each row we shut down combinations of sources of heterogeneity.

The larger dispersion costs of heterogeneity in the home production model mostly reflect the large dispersion in home productivity z_N. To see this, in the second row we shut down all sources except for heterogeneity in the disutility of work B. We find similar results to row 1 and, thus, conclude that heterogeneity in B does not generate significant welfare effects. In the third row we shut off heterogeneity in z_M only and in the fourth row we shut off heterogeneity in z_N only. The welfare effects of shutting off heterogeneity in z_N are larger than the welfare costs under all other sources of heterogeneity.[18]

The welfare effects we calculate in Table 1 reflect heterogeneity both within age and over the life-cycle because each counterfactual imposes a constant value of the source of heterogeneity for households of all ages. We have repeated these exercises by shutting off only the within-age heterogeneity and allowing each source of heterogeneity to take its mean value over the life-cycle as shown previously in Figure 1. We find similar results and, therefore, conclude that the welfare effects predominately reflect the within-age component of heterogeneity.

as shown in the constant $\mathbb{C}$ in the c_M equation of Table 1. Higher dispersion in market productivity z_M increases aggregate labor productivity because market hours are weakly negatively correlated with wages as shown in Table 5.

[18]We have also calculated the welfare costs of market incompleteness, defined as the λ between the baseline and a counterfactual allocation in which market productivity varies only because of the insurable component ($\log z_{M,t} = \bar{\alpha} + \varepsilon_t$). We obtain $\lambda = 0.29$ for the no home production model. The welfare cost is not far from those calculated in Pijoan-Mas (2006), who finds $\lambda = 0.16$ in a model with a risk-free bond and capital accumulation, and Heathcote, Storesletten, and Violante (2008), who find $\lambda = 0.22$ in a model characterized by a no-trade result and no capital accumulation. In the home production model we obtain $\lambda = 0.33$ and, thus, conclude that the welfare costs of market incompleteness are not substantially affected by home production.

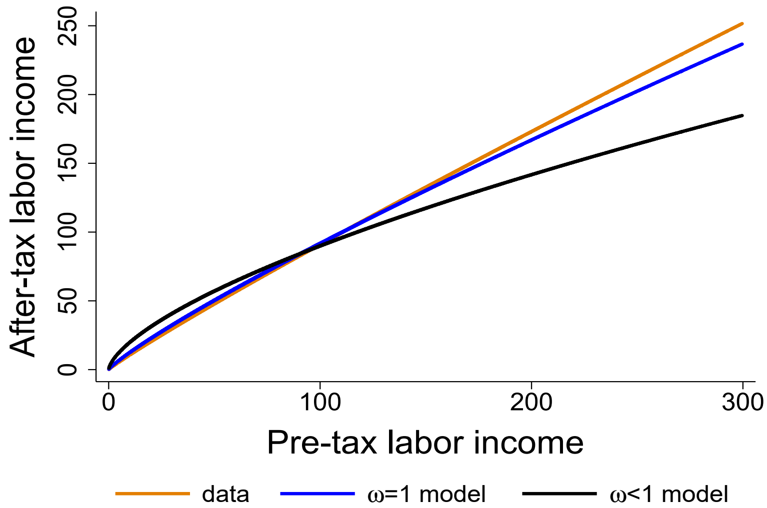


Figure 7: Optimal Tax Function

Figure 7 displays the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ under the parameters estimates for the United States (orange curve), under the optimal tax function for the model without home production ($\omega = 1$, blue), and under the optimal tax function with home production ($\omega < 1$, black).

4.4 Optimal Tax Progressivity

This section contrasts the optimal progressivity of the tax system between the home production model and the no home production model. Relative to our previous inequality measures, the optimal taxation exercise mixes redistribution with efficiency concerns because the optimal progressivity of the tax system increases with redistributive motives and decreases with the efficiency losses from distorting labor allocations. However, this exercise allows us to more directly link our inequality result to public policy.

Given an exogenous level of government expenditures G, the Ramsey government chooses tax function parameters $\tau \equiv (\tau_0, \tau_1)$ to maximize social welfare:

$$\max_{\tau} \int_{\iota} U(c_M(\tau), h_M(\tau), h_N(\tau); \iota) \mathrm{d}\Phi(\iota) \ , \tag{19}$$

subject to the government budget constraint:

$$\int_{\iota} \left[z_M - (1 - \tau_0) {z_M}^{1-\tau_1} \right] h_M(\tau) \mathrm{d}\Phi(\iota) = G. \tag{20}$$

In formulating the Ramsey problem, we have assumed a stationary environment in which the government faces an identical cross section of households in each year.

In Figure 7 we plot the relationship between pre-tax labor income y and after-tax labor income $\tilde{y}$ (in thousands of 2010 dollars). The orange curve shows the relationship between y and $\tilde{y}$ under the parameter $\tau_1 = 0.12$ that we estimated in the data for the United States. The blue and black curves show this relationship under the optimal $\tau_1 = 0.21$ for the model without home production and the optimal $\tau_1 = 0.42$ for the model with home production. As is seen in the figure, the progressivity of the system in the no home production model is relatively similar to the one we estimated in the data. However, the relationship between y and $\tilde{y}$ becomes significantly more concave in the model with home production. To give an example, consider a household earning 200 thousand dollars. Under the optimal tax schedule in the no home production model the household faces an average tax rate of 17 percent, while in the home production model the average tax rate increases to 29 percent.

5 Sensitivity Analyses

Table 7 shows the robustness of our inequality results to a variety of sensitivity analyses. Each row in the table corresponds to a different robustness exercise. For both models, the columns show the standard deviation in equivalent variation T, the standard deviations in transfers t required to equalize marginal utilities, the ex-ante lifetime welfare loss from shutting off all heterogeneity λ, and the degree of progressivity τ_1 in an optimal tax system. In each robustness exercise, we repeat our analysis of identifying the sources of heterogeneity $(\alpha, \varepsilon, B, z_N)$ and then calculate the inequality metrics. The first row of the table repeats these statistics for our baseline case.

In rows 2 to 7 we vary the parameters of the model. Relative to our estimated value $\tau_1 = 0.12$, changing the progressivity of the tax system to $\tau_1 = 0.06$ as in Guner, Kaygusuz, and Ventura (2014) or to $\tau_1 = 0.19$ as in Heathcote, Storesletten, and Violante (2014) does not alter significantly any result. Changing the parameter that governs the Frisch elasticity η also does not affect our results significantly. By contrast, the parameter ϕ that governs the substitutability between the market and the home sector affects substantially the magnitude of the inequality statistics. When $\phi = 0.5$ and the two sectors are complements, three measures of inequality in-

Table 7: Sensitivity Analyses of Inequality Measures

		No Home Production: $\omega = 1$				Home Production: $\omega < 1$			
		Std(T)	Std(t)	λ	τ_1	Std(T)	Std(t)	λ	τ_1
	Benchmark								
1.	Baseline	11.6	7.8	0.07	0.21	16.8	11.1	0.25	0.42
	Parameter Values								
2.	$\tau_1 = 0.06$	11.5	7.8	0.07	0.21	17.7	11.5	0.28	0.42
3.	$\tau_1 = 0.19$	11.8	7.8	0.08	0.20	16.0	10.7	0.23	0.42
4.	$\eta(1-\tau_1) = 0.7$	10.9	7.8	0.06	0.03	15.9	11.1	0.25	0.39
5.	$\eta(1-\tau_1) = 1.5$	12.5	7.8	0.09	0.33	17.9	11.1	0.26	0.44
6.	$\phi = 0.5$	—	—	—	—	40.3	11.1	0.65	0.66
7.	$\phi = 5$	—	—	—	—	15.2	11.1	0.15	-0.65
	Definition of Consumption Expenditures								
8.	Food expenditures	5.2	3.4	0.07	0.15	7.3	4.8	0.25	0.41
9.	All expenditures	37.7	25.9	0.09	0.38	52.4	35.7	0.25	0.49
	Marital Status, Children, and Education								
10.	Singles	9.0	5.9	0.03	0.15	11.2	7.4	0.20	0.38
11.	Married, no children	11.1	7.4	0.12	0.14	15.1	9.8	0.30	0.40
12.	Married, 1 child	11.8	7.9	0.08	0.22	15.3	11.0	0.23	0.42
13.	Married, 2+ children	12.0	8.0	0.06	0.25	18.6	12.4	0.27	0.42
14.	Less than college	10.9	7.2	0.04	0.00	14.4	9.7	0.20	0.39
15.	College or more	12.6	8.3	0.08	0.10	18.5	11.9	0.26	0.41
	Classical Measurement Error								
16.	Consumption	8.6	5.5	0.05	0.23	14.5	8.8	0.24	0.38
17.	Market hours	11.8	7.8	0.07	0.25	16.7	11.1	0.25	0.42
18.	Home hours	11.6	7.8	0.07	0.21	14.4	10.4	0.23	0.41

Table 7 summarizes our sensitivity analyses in each row. In columns we show the four inequality measures for the no home production model ($\omega = 1$) and the home production model ($\omega < 1$).

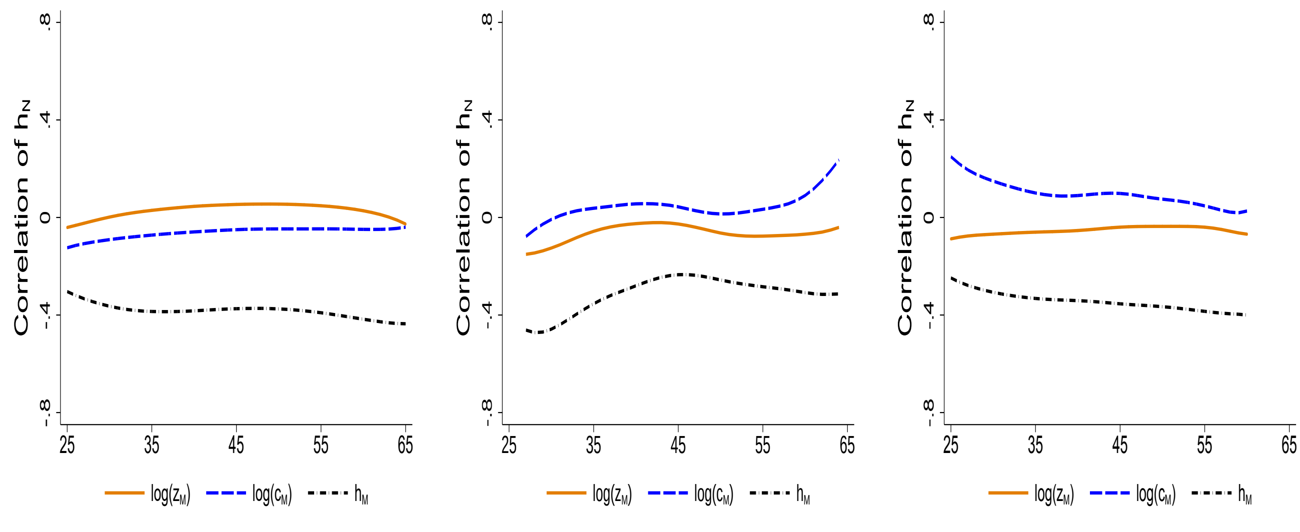
crease substantially. Intuitively, when the two goods are complements home production amplifies differences in the market sector even more. When $\phi = 5$ and the two goods are significantly more substitutable than in the baseline, we still find that inequality is higher with home production according to the three first metrics. By contrast, the progressivity of the tax system is significantly affected by the value of ϕ because a higher value of ϕ increases the sensitivity of hours to changes in the tax system.

In rows 8 and 9 of Table 7 we show that our results remain very similar under two alternative measures of market expenditures c_M. In row 8 we use food only whereas in row 9 we use all expenditures including durables. The standard deviations of T and t change significantly relative to the baseline in which we use non-durables excluding health and education because the average expenditure in food is significantly smaller than non-durables and the average expenditure on non-durables is significantly smaller than all expenditures. We note that the standard deviations in the home production model are always roughly 40 percent larger than in the no home production. Similarly, the welfare loss from heterogeneity λ is stable across different measures of consumption expenditures for both models. Optimal progressivity τ_1 increases when we use more comprehensive measures of consumption, but is always higher in the model with home production.

In our baseline analysis we considered only married households. In rows 10 to 15 we repeat our analyses in different subsamples of households defined along their marital status, number of children, and education. This is an important exercise for two reasons. First, it reveals whether our inequality result applies within groups or across groups. Second, one would expect heterogeneity in preferences or other unobservable characteristics that we did not model to be smaller within more narrowly defined demographic groups.

Row 10 shows the subsample of singles. Rows 11 to 13 show different subsamples of married, differentiated according to the number of children present in the household. Rows 14 and 15 show results for married households with a head who has not completed college and with a head who has completed college or more. The main message of these analyses is that our results are remarkably stable, with the home production model always generating more inequality than the no home production model according to all four different metrics.

Next, we establish that our main findings are robust to measurement error in the survey data. We assume that the reported value of a variable equals $x = x^* \exp(m)$, where x^* is the true value



(a) US (CEX) (b) Netherlands (LISS) (c) Japan (JPSC)

Figure 8: Correlations of Home Hours

Figure 8 shows the cross-sectional correlation of home hours h_N with market productivity $\log z_M$, market consumption $\log c_M$, and market hours h_M in three datasets.

and m denotes classical measurement error. In rows 16 to 18 we assume that the measurement error explains 50 percent of the variance of $\log x$ for each variable $x = (c_M, h_M, h_N)$. We find no significant changes relative to our baseline results.[19]

Finally, in Figure 8 we confirm that the key empirical regularities that underlie our inferences are similar between the CEX/ATUS sample and two alternative datasets that contain direct information on both consumption expenditures and time spent on home production. The first one is the Longitudinal Internet Studies for the Social Sciences from the Netherlands (LISS; see, for example, Cherchye, Demuynck, De Rock, and Vermeulen, 2017) and the second is the Japanese Panel Survey of Consumers (JPSC; see, for example, Lise and Yamada, 2017). While the relatively smaller sample sizes preclude us from repeating all of our analyses in these two datasets, in the figure we show that the within-age correlation between home hours h_N and market productivity $\log z_M$, market consumption $\log c_M$, and market hours h_M is similar between the three datasets. Home hours are weakly correlated with market productivity and consumption and the correlation between hours in the two sectors is roughly -0.4 in all three datasets.

[19] In unreported results, we have confirmed the robustness of our results to measurement errors of up to 80 percent of the variance of the reported variables and to measurement error that is allowed to be positively or negatively correlated with income.

6 Conclusion

The literature examining the causes, welfare consequences, and policy implications of the substantial labor market dispersion we observe across households typically abstracts from the possibility that households can produce goods and services outside of the market sector. We revisit these issues taking into account that households spend a significant amount of their time in home production. Our model incorporates non-separable preferences between expenditures and time and home productivity differences across households into a standard incomplete markets model with uninsurable risk.

We reach several substantial conclusions. We find that home production amplifies welfare-based differences across households and inequality is larger than we thought. Our result is surprising given that a priori one could expect that home production tends to compress welfare differences that originate in the market sector when households are sufficiently willing to substitute between market expenditures and time in the production of goods. We show that home productivity is an important source of within-age and life-cycle differences in consumption expenditures and time allocation across households. Though the lens of the model, we infer that the dispersion in home productivity across households is roughly three times as large as the dispersion in their wages. There is little scope for home production to offset differences that originate in the market sector because productivity differences in the home sector are large and the time input in home production does not covary with consumption expenditures and wages in the cross section of households. We conclude that the optimal tax system should feature more progressivity when incorporating home production.

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Inferring Inequality with Home Production

Online Appendix

Job Boerma and Loukas Karabarbounis

A Equilibrium Allocations

In this appendix, we derive the equilibrium allocations presented in Table 1. We proceed in three steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 2.

A.1 Preliminaries

In what follows, we define the following state vectors. The idiosyncratic shifters that differentiate households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can write contracts within each island contingent on the vector s^j:

$$s_t^j = (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{z_N^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, we denote the history of all sources of heterogeneity up to period t with the vector:

$$\sigma_t^j = (z_{N,t}^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, ..., z_{N,j}^j, B_j^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(.|.)$. For example, the probability that we observe σ_t^j conditional on σ_{t-1}^j is $f^{t,j}(\sigma_t^j|\sigma_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to the processes and Φ_{υ_t} to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^B}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi^j_{z_{N,t}}\}$, and the initial distributions to vary over cohorts j, $\{\Phi^j_{\alpha,j}, \Phi^j_{B,j}, \Phi^j_{\kappa,j}\}$. We assume that $z^j_{N,t}$ is orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

A.2 Planner Problems

In every period t and in every island ℓ, the planner solves a static problem that consists of finding the allocations that maximize average utility for households on the island subject to an aggregate resource constraint and household-specific home production technologies. We omit t and ℓ from the notation for convenience.

A.2.1 No Home Production, $\omega = 1$

The planner chooses an allocation $\{c_M(\iota), h_M(\iota)\}$ to maximize:

$$\int_Z \left[\frac{c_M(\iota)^{1-\gamma} - 1}{1-\gamma} - \exp(B(\iota))\frac{h_M(\iota)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right] \mathrm{d}\Phi_\zeta(\zeta)\ , \tag{A.5}$$

subject to an island resource constraint for market goods:

$$\int_Z c_M(\iota)\, \mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M(\iota)\, h_M(\iota)\, \mathrm{d}\Phi_\zeta(\zeta)\, . \tag{A.6}$$

Denoting by $\mu(\alpha, B)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M(\iota)] :\ c_M(\iota)^{-\gamma} = \mu(\alpha, B), \tag{A.7}$$

$$[h_M(\iota)] :\ \exp(B(\iota))h_M(\iota)^{\frac{1}{\eta}} = \tilde{z}_M(\iota)\mu(\alpha, B). \tag{A.8}$$

Equation (A.7) implies that market consumption is equalized for every ι on the island and, thus, there is full consumption insurance. Combining equations (A.6) to (A.8), we solve for market consumption and market hours for every ι:

$$c_M(\iota) = \left[\frac{\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)}{\exp(B(\iota))^\eta}\right]^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}}, \tag{A.9}$$

$$h_M(\iota) = \tilde{z}_M(\iota)^\eta \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}}{\exp\left(B(\iota)\right)^{\frac{1}{\frac{1}{\eta}+\gamma}}}. \tag{A.10}$$

A.2.2 Home Production, $\omega < 1$

The planner chooses $\{c_M(\iota), c_N(\iota), h_M(\iota), h_N(\iota)\}$ to maximize:

$$\int_Z \left[\log\left(\omega c_M(\iota)^{\frac{\phi-1}{\phi}} + (1-\omega) c_N(\iota)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}} - \exp(B(\iota)) \frac{\left(h_M(\iota) + h_N(\iota)\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right] \mathrm{d}\Phi_\zeta(\zeta), \tag{A.11}$$

subject to the island market resource constraint (A.6) and the home production technologies:

$$c_N(\iota) = z_N(\iota) h_N(\iota). \tag{A.12}$$

Denoting by $\mu(\alpha, B, z_N)$ the multiplier on the island resource constraint and by $\Lambda(\iota)$ the multiplier on the household's home production constraint, the solution to this problem is characterized by the following first-order conditions (for every household ι):

$$[c_M(\iota)]: \quad \left[\omega c_M(\iota)^{\frac{\phi-1}{\phi}} + (1-\omega) c_N(\iota)^{\frac{\phi-1}{\phi}}\right]^{-1} \omega c_M(\iota)^{-\frac{1}{\phi}} = \mu(\alpha, B, z_N), \tag{A.13}$$

$$[c_N(\iota)]: \quad \left[\omega c_M(\iota)^{\frac{\phi-1}{\phi}} + (1-\omega) c_N(\iota)^{\frac{\phi-1}{\phi}}\right]^{-1} (1-\omega) c_N(\iota)^{-\frac{1}{\phi}} = \Lambda(\iota), \tag{A.14}$$

$$[h_M(\iota)]: \quad \exp(B(\iota))\left(h_M(\iota) + h_N(\iota)\right)^{\frac{1}{\eta}} = \tilde{z}_M(\iota)\mu(\alpha, B, z_N), \tag{A.15}$$

$$[h_N(\iota)]: \quad \exp(B(\iota))\left(h_M(\iota) + h_N(\iota)\right)^{\frac{1}{\eta}} = z_N(\iota)\Lambda(\iota). \tag{A.16}$$

Combining equations (A.13) to (A.16), we obtain the consumption ratio:

$$\frac{c_M(\iota)}{c_N(\iota)} = \left(\frac{\tilde{z}_M(\iota)}{z_N(\iota)}\right)^\phi \left(\frac{\omega}{1-\omega}\right)^\phi. \tag{A.17}$$

Substituting this expression into equations (A.13) and (A.14), we derive:

$$c_M(\iota) = \frac{1}{\mu(\alpha, B, z_N)} \left(1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}\right)^{-1}, \tag{A.18}$$

$$c_N(\iota) = \frac{1}{\mu(\alpha, B, z_N)} \frac{\left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{\left(1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}\right)} = z_N(\iota) h_N(\iota). \tag{A.19}$$

These expressions yield solutions for $\{c_M(\iota), c_N(\iota), h_M(\iota), h_N(\iota)\}$ as a function of the multiplier $\mu(\alpha, B, z_N)$. The multiplier is equal to the inverse of the market value of consumption:

$$c_M(\iota) + \tilde{z}_M(\iota) h_N(\iota) = c_M(\iota) + \frac{\tilde{z}_M(\iota)}{z_N(\iota)} c_N(\iota) = \frac{1}{\mu(\alpha, B, z_N)}. \tag{A.20}$$

The first equality follows from equation (A.12) and the second equality follows from equations (A.18) and (A.19).

Substituting equation (A.15) into equation (A.6), we obtain a solution for $\mu(\alpha, B, z_N)$:

$$\mu(\alpha, B, z_N) = \left(\frac{\exp(B(\iota))^{\eta}}{\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)}\right)^{\frac{1}{1+\eta}}. \tag{A.21}$$

The denominator is an expectation independent of ζ. Therefore, μ is independent of ζ. We also note that $\mu(\alpha, B, z_N) = \mu(\alpha, B)$. Given the solution for μ, we obtain the solutions:

$$c_M(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))^{\frac{1}{1+\frac{1}{\eta}}}} \frac{1}{1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}}, \tag{A.22}$$

$$c_N(\iota) = \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))^{\frac{1}{1+\frac{1}{\eta}}}} \frac{\left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}}, \tag{A.23}$$

$$\begin{aligned} h_M(\iota) = {} & \tilde{z}_M(\iota)^{\eta} \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)\right]^{-\frac{1}{1+\frac{1}{\eta}}}}{\exp(B(\iota))^{\frac{1}{1+\frac{1}{\eta}}}} \\ & - \frac{1}{z_N(\iota)} \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))^{\frac{1}{1+\frac{1}{\eta}}}} \frac{\left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}}, \end{aligned} \tag{A.24}$$

$$h_N(\iota) = \frac{1}{z_N(\iota)} \frac{\left[\int_Z \tilde{z}_M(\iota)^{1+\eta} \, \mathrm{d}\Phi_{\zeta}(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B(\iota))^{\frac{1}{1+\frac{1}{\eta}}}} \frac{\left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi}}{1 + \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_N(\iota)}{\tilde{z}_M(\iota)}\right)^{\phi-1}}. \tag{A.25}$$

A.3 Postulating Equilibrium Allocations and Prices

We postulate an equilibrium in four steps.

1. We postulate that the equilibrium features no trade between islands, $x(\zeta_t^j;\iota) = 0$.

2. We postulate that the solutions $\left\{c_{M,t}(\iota), c_{N,t}(\iota), h_{M,t}(\iota), h_{N,t}(\iota)\right\}$ to the planner problem in Section A.2 constitute components of the equilibrium.

3. We use the sequential budget constraints to postulate equilibrium holdings for the bonds $b^\ell(s_t^j;\iota)$ that are traded within islands:

$$b^\ell(s_t^j;\iota) = \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu_{t+n}(\alpha_{t+n}^j, B_{t+n}^j)}{\mu_t(\alpha_t^j, B_t^j)} \left(c_{M,t+n}(\iota) - \tilde{y}_{t+n}(\iota)\right)\right], \tag{A.26}$$

   where $\tilde{y} = \tilde{z}_M h_M = (1-\tau_0) z_M^{1-\tau_1} h_M$ denotes after-tax labor income.

4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s_{t+1}^j;\iota)$ and $x(\zeta_{t+1}^j;\iota)$:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) = {} & \beta\delta \exp\left(\frac{\gamma}{\gamma + \frac{1}{\eta}} \upsilon_{t+1}^B\right) \exp\left(-(1-\tau_1)\,\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \\ & \times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j | s_t^j), \end{aligned} \tag{A.27}$$

$$\begin{aligned} q_x(Z_{t+1}) = {} & \beta\delta \int \exp\left(\frac{\gamma}{\gamma + \frac{1}{\eta}} \upsilon_{t+1}^B\right) \mathrm{d}\Phi_{\upsilon_{t+1}^B}(\upsilon_{t+1}^B) \int \exp\left(-(1-\tau_1)\,\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^\alpha\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}(\upsilon_{t+1}^\alpha) \\ & \times \left[\frac{\int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \mathbb{P}\left(\left(\upsilon_{t+1}^\kappa, \upsilon_{t+1}^\varepsilon\right) \in Z_{t+1}\right), \end{aligned} \tag{A.28}$$

   where $A \equiv (1+\eta)(1-\tau_1)$.

A.4 Verifying the Equilibrium Allocations and Prices

We verify that the equilibrium postulated in Section A.3 constitutes an equilibrium by showing that the postulated equilibrium allocations solve the households' problem and that all markets clear.

A.4.1 Household Problem

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

No Home Production, $\omega = 1$. The optimality conditions are:

$$(\beta\delta)^{t-j}\, c_{M,t}^{-\gamma} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{\mu}_t, \tag{A.29}$$

$$(\beta\delta)^{t-j} \exp(B_t) \left(h_{M,t}\right)^{\frac{1}{\eta}} f^{t,j}(\sigma_t^j|\sigma_j) = \tilde{z}_{M,t}^j \tilde{\mu}_t, \tag{A.30}$$

$$q_b^\ell(s_{t+1}^j) = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}, \tag{A.31}$$

$$q_x(Z_{t+1}) = \int \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.32}$$

Comparing the planner solutions to the household solutions we verify that they coincide for market consumption and hours when the multipliers are related by:

$$\tilde{\mu}_t = (\beta\delta)^{t-j} f^{t,j}(\sigma_t^j|\sigma_j)\mu(\alpha_t^j, B_t^j). \tag{A.33}$$

Therefore, the Euler equations become:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \tag{A.34}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha. \tag{A.35}$$

Home Production, $\omega < 1$. We denote total hours by $h = h_M + h_N$. Using again the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and $\mu(\alpha_t^j, B_t^j, z_{N,t}^j)$, we write the optimality conditions directly as:

$$\tilde{z}_{M,t} \left[\omega(c_{M,t})^{\frac{\phi-1}{\phi}} + (1-\omega)(c_{N,t})^{\frac{\phi-1}{\phi}}\right]^{-1} \omega(c_{M,t})^{-\frac{1}{\phi}} = \exp(B_t)(h_t)^{\frac{1}{\eta}}, \tag{A.36}$$

$$z_{N,t} \left[\omega(c_{M,t})^{\frac{\phi-1}{\phi}} + (1-\omega)(c_{N,t})^{\frac{\phi-1}{\phi}}\right]^{-1} (1-\omega)(c_{N,t})^{-\frac{1}{\phi}} = \exp(B_t)(h_t)^{\frac{1}{\eta}}, \tag{A.37}$$

$$q_b^\ell(s_{t+1}^j) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, z_{N,t+1}^j)}{\mu(\alpha_t^j, B_t^j, z_{N,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}z_{N,t+1}^j, \tag{A.38}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j, z_{N,t+1}^j)}{\mu(\alpha_t^j, B_t^j, z_{N,t}^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \mathrm{d}\upsilon_{t+1}^B \mathrm{d}\upsilon_{t+1}^\alpha \mathrm{d}z_{N,t+1}^j. \tag{A.39}$$

A.4.2 Euler Equations

We next verify that the Euler equations are satisfied at the postulated equilibrium allocations and prices.

No Home Production, $\omega = 1$. Using the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha_t^j, B_t^j)$, we write the Euler equation for the bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as:

$$\begin{aligned} q_b^\ell(s_{t+1}^j) &= \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \\ &= \beta\delta \frac{\exp\left(\frac{\gamma}{\frac{1}{\eta}+\gamma} B_{t+1}^j\right)\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\exp\left(\frac{\gamma}{\frac{1}{\eta}+\gamma} B_t^j\right)\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j), \end{aligned} \tag{A.40}$$

where the second line follows from equations (A.7) and (A.9). Using the fact that B_t^j follows a random walk-process with innovation υ_t^B we rewrite $q_b^\ell(s_{t+1}^j)$ as:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\frac{\gamma}{\gamma+\frac{1}{\eta}} \upsilon_{t+1}^B\right) \frac{\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(s_{t+1}^j|s_t^j). \tag{A.41}$$

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$ we use that:

$$\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\left((1-\tau_1)\left(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right).$$

Given that $A = (1+\eta)(1-\tau_1)$, the expectation over the random variables in the numerator is given by:

$$\begin{aligned} &\int \exp\left(A\left(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon\right)\right) \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j) \\ =& \int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j) \int \exp\left(A\upsilon_{t+1}^\kappa\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa) \int \exp\left(A\upsilon_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon), \end{aligned} \tag{A.42}$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals:

$$\int \exp(A\kappa_t^j)\mathrm{d}\Phi_{\kappa^j,t}(\kappa_t^j) \int \exp\left(A\upsilon_t^\varepsilon\right) \mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon). \tag{A.43}$$

As a result, the price $q_b^\ell(s_{t+1}^j)$ is:

$$q_b^\ell(s_{t+1}^j) = \beta\delta \exp\left(\frac{\gamma}{\gamma+\frac{1}{\eta}}\upsilon_{t+1}^B\right)\exp\left(-\left(1-\tau_1\right)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)$$
$$\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} f^{t+1,j}(s_{t+1}^j|s_t^j), \tag{A.44}$$

where $f^{t+1,j}(s_{t+1}^j|s_t^j) = f(\upsilon_{t+1}^B)f(\upsilon_{t+1}^\alpha)f(\upsilon_{t+1}^\kappa)f(\upsilon_{t+1}^\varepsilon)$. This confirms our guess in equation (A.27). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in square brackets is independent of the state vector that differentiates islands ℓ. As a result, all islands ℓ have the same bond prices, $q_b^\ell(s_{t+1}^j) = Q_b\left(\upsilon_{t+1}^B,\upsilon_{t+1}^\alpha\right)$.

We next calculate the bond price for a set of states $\mathcal{V}_{t+1}\subseteq\mathbb{V}_{t+1}$:

$$q_b^\ell(\mathcal{V}_{t+1}) = \beta\delta\int_{\mathcal{V}^B}\exp\left(\frac{\gamma}{\gamma+\frac{1}{\eta}}\upsilon_{t+1}^B\right)\mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right)\int_{\mathcal{V}^\alpha}\exp\left(-\left(1-\tau_1\right)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.45}$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim that does not depend on the realization of $(\upsilon_{t+1}^B,\upsilon_{t+1}^\alpha)$:

$$q_b^\ell(\mathbb{V}_{t+1}) = \beta\delta\int_{\mathbb{V}^B}\exp\left(\frac{\gamma}{\gamma+\frac{1}{\eta}}\upsilon_{t+1}^B\right)\mathrm{d}\Phi_{\upsilon_{t+1}^B}\left(\upsilon_{t+1}^B\right)\int_{\mathbb{V}^\alpha}\exp\left(-\left(1-\tau_1\right)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon_{t+1}^\alpha\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\alpha}\left(\upsilon_{t+1}^\alpha\right)$$
$$\times\left[\frac{\int\exp\left(A\upsilon_{t+1}^\kappa\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)\int\exp\left(A\upsilon_{t+1}^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_{t+1}^\varepsilon}(\upsilon_{t+1}^\varepsilon)}{\int\exp\left(A\upsilon_t^\varepsilon\right)\mathrm{d}\Phi_{\upsilon_t^\varepsilon}(\upsilon_t^\varepsilon)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.46}$$

All islands face the same price $q_b^\ell(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the prices of bonds x and b that are contingent on the same set of states must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and given by:

$$q_x(Z_{t+1}) = \mathbb{P}\left(\left(\upsilon_{t+1}^\kappa,\upsilon_{t+1}^\varepsilon\right)\in Z_{t+1}\right)q_b(\mathbb{V}_{t+1}), \tag{A.47}$$

where $\mathbb{P}\left((\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1}) \in Z_{t+1}\right)$ denotes the probability of $(\upsilon^{\kappa}_{t+1}, \upsilon^{\varepsilon}_{t+1})$ conditional on Z_{t+1}. The expression for $q_x(Z_{t+1})$ confirms our guess in equation (A.28)

Home Production, $\omega < 1$. For the model with home production, we use the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha^j_t, B^j_t, z^j_{N,t})$ to write the Euler equation for the bonds $b^{\ell}(s^j_{t+1})$ at the postulated equilibrium as:

$$\begin{aligned} q^{\ell}_b(s^j_{t+1}) &= \beta\delta \int \frac{\mu(\alpha^j_{t+1}, B^j_{t+1}, z^j_{N,t+1})}{\mu(\alpha^j_t, B^j_t, z^j_{N,t})} f^{t+1,j}(\sigma^j_{t+1}|\sigma^j_t) \mathrm{d}z^j_{N,t+1} \\ &= \beta\delta \int \frac{\exp\left(\frac{1}{1+\frac{1}{\eta}} B^j_{t+1}\right)\left[\int \left(\tilde{z}^j_{M,t+1}\right)^{1+\eta} \mathrm{d}\Phi_{\zeta^j_{t+1}}(\zeta^j_{t+1})\right]^{-\frac{1}{1+\eta}}}{\exp\left(\frac{1}{1+\frac{1}{\eta}} B^j_t\right)\left[\int \left(\tilde{z}^j_{M,t}\right)^{1+\eta} \mathrm{d}\Phi_{\zeta^j_t}(\zeta^j_t)\right]^{-\frac{1}{1+\eta}}} f^{t+1,j}(\sigma^j_{t+1}|\sigma^j_t) \mathrm{d}z^j_{N,t+1}. \end{aligned} \tag{A.48}$$

where the second equality follows from equation (A.21). Using equations (A.42) and (A.43), and the fact that $z^j_{N,t+1}$ is orthogonal to the other innovations, the price $q^{\ell}_b(s^j_{t+1})$ simplifies to:

$$\begin{aligned} q^{\ell}_b(s^j_{t+1}) &= \beta\delta \exp\left(\frac{\upsilon^B_{t+1}}{1+\frac{1}{\eta}} - (1-\tau_1)\,\upsilon^{\alpha}_{t+1}\right) \\ &\times \left[\frac{\int \exp\left(A\upsilon^{\kappa}_{t+1}\right) \mathrm{d}\Phi_{\upsilon^{\kappa}_{t+1}}(\upsilon^{\kappa}_{t+1}) \int \exp\left(A\upsilon^{\varepsilon}_{t+1}\right) \mathrm{d}\Phi_{\upsilon^{\varepsilon}_{t+1}}(\upsilon^{\varepsilon}_{t+1})}{\int \exp\left(A\upsilon^{\varepsilon}_t\right) \mathrm{d}\Phi_{\upsilon^{\varepsilon}_t}(\upsilon^{\varepsilon}_t)}\right]^{-\frac{1}{1+\eta}} f^{t+1,j}(s^j_{t+1}|s^j_t). \end{aligned} \tag{A.49}$$

The price $q^{\ell}_b(s^j_{t+1})$ is identical to equation (A.44) for the model without home production under $\gamma = 1$. The remainder of the argument is identical to the argument for the model without home production.

A.4.3 Household's Budget Constraint

We now verify our guess for the bond positions $b^{\ell}_t(s^j_t)$ and confirm that the household budget constraint holds at the postulated equilibrium allocations. The proof to this claim is identical for both models. We define the deficit term by $D_t \equiv c_{M,t} - \tilde{y}_t$. Using the expression for the price $q^{\ell}_b(s^j_{t+1})$ in equation (A.34), the budget constraint at the no-trade equilibrium is given by:

$$b^{\ell}_t(s^j_t) = D_t + \beta\delta \int\int \frac{\mu(\alpha^j_{t+1}, B^j_{t+1}, z^j_{N,t+1})}{\mu(\alpha^j_t, B^j_t, z^j_{N,t})} b^{\ell}_{t+1}(s^j_{t+1}) f^{t+1}(\sigma^j_{t+1}|\sigma^j_t) \mathrm{d}s^j_{t+1} \mathrm{d}z^j_{N,t+1}.$$

By substituting forward using equation (A.34), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.26) and show that the household budget constraint holds at the postulated equilibrium allocations.

A.4.4 Goods Market Clearing

Aggregating the resource constraints in every island, we obtain that the allocations that solve the planner problems satisfy the aggregate goods market clearing condition:

$$\int_\iota c_{M,t}(\iota)\mathrm{d}\Phi(\iota) + G = \int_\iota z_{M,t}(\iota)h_{M,t}(\iota)\mathrm{d}\Phi(\iota). \tag{A.50}$$

A.4.5 Asset Market Clearing

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota x(\zeta_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $x(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\mathrm{d}\Phi(\iota) = 0$, $\forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated bond holdings in equation (A.26) into the asset market clearing conditions:

$$\begin{aligned}\int b^\ell(s_t^j)\mathrm{d}\Phi(\iota) &= \int \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, z_{N,t+n}^j)}{\mu(\alpha_t^j, B_t^j, z_{N,t}^j)} D_{t+n}\right]\mathrm{d}\Phi(\iota)\\ &= \sum_{n=0}^{\infty}(\beta\delta)^n \int\int \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, z_{N,t+n}^j)}{\mu(\alpha_t^j, B_t^j, z_{N,t}^j)} D_{t+n} f(\sigma_{t+n}^j|\sigma_{t-1}^j)\mathrm{d}\sigma_{t+n}^j\mathrm{d}\Phi(\iota).\end{aligned}$$

For simplicity we omit conditioning on σ_{t-1}^j and write the density function as $f(\sigma_{t+n}^j|\sigma_{t-1}^j) = f(\{\upsilon_{t+n}^B\})f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{z_{N,t+n}\})$. Further, the growth of marginal utility is identical between the two models and we denote it by $\mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^\alpha\right) \equiv \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j, z_{N,t+n}^j)}{\mu(\alpha_t^j, B_t^j, z_{N,t}^j)} = \frac{\mu(\alpha_{t+n}^j, B_{t+n}^j)}{\mu(\alpha_t^j, B_t^j)}$. As a result, we write aggregate bond holdings $\int b^\ell(s_t^j)\mathrm{d}\Phi(\iota)$ as:

$$\begin{aligned}&\sum_{n=0}^{\infty}(\beta\delta)^n\int\int \mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^\alpha\right) D_{t+n} f(\{\upsilon_{t+n}^B\})f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{z_{N,t+n}\})\mathrm{d}\Phi(\iota)\\ =&\sum_{n=0}^{\infty}(\beta\delta)^n\int\int D_{t+n} f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})\mathrm{d}\{\upsilon_{t+n}^\kappa\}\mathrm{d}\{\upsilon_{t+n}^\varepsilon\}\mathrm{d}\Phi(\iota)\\ &\qquad\times\int \mathcal{Q}\left(\upsilon_{t+n}^B, \upsilon_{t+n}^\alpha\right) f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^B\})f(\{z_{N,t+n}\})\mathrm{d}\{\upsilon_{t+n}^B\}\mathrm{d}\{\upsilon_{t+n}^\alpha\}\mathrm{d}\{z_{N,t+n}^j\}.\end{aligned}$$

Recalling that the deficit terms equal $D_t = c_{M,t} - \tilde{y}_t$, the bond market clearing condition holds because the first term is zero by the island-level resource constraint.

B Observational Equivalence Theorem

In this appendix we derive the identified sources of heterogeneity $\{\alpha_t, \varepsilon_t, B_t\}_\iota$ presented in Table 2. Our strategy is to invert the equilibrium allocations presented in Table 1 and solve for the unique sources of heterogeneity that lead to these allocations. We note that the identification is defined up to a constant because the constant $\mathbb{C}$ that appears in the equations of Table 2 depends on the ε's.

B.1 No Home Production, $\omega = 1$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}\}_\iota$ and parameters $\gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide market consumption with home hours to obtain:

$$\frac{c_{M,t}}{h_{M,t}} = (1-\tau_0)\, z_{M,t}^{-\eta(1-\tau_1)} \exp((1-\tau_1)(1+\eta)\alpha_t) \int_{\zeta_t} \exp((1-\tau_1)(1+\eta)\varepsilon_t) \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\ . \tag{A.51}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Finally, we can use the solution for $c_{M,t}$ or $h_{M,t}$ in Table 1 to solve for a unique value of B_t.

B.2 Home Production, $\omega < 1$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}, h_{N,t}\}_\iota$ and parameters $\omega, \phi, \gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t, z_{N,t}\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide the equilibrium allocation of home hours by market consumption to obtain:

$$\tilde{z}_{M,t}\frac{h_{N,t}}{c_{M,t}} = \left(\frac{1-\omega}{\omega}\right)^{\phi} \left(\frac{z_{N,t}^j}{\tilde{z}_{M,t}^j}\right)^{\phi-1}\ . \tag{A.52}$$

Next, we divide total hours by market consumption, and use (A.52), to obtain:

$$\frac{h_{M,t}+h_{N,t}}{c_{M,t}} = \frac{z_{M,t}^{\eta(1-\tau_1)}}{1-\tau_0} \exp(-(1+\eta)(1-\tau_1)\alpha_t^j) \frac{\left(1+(1-\tau_0){z_{M,t}}^{1-\tau_1}\frac{h_{N,t}}{c_{M,t}}\right)}{\int_{Z_t} \exp((1+\eta)(1-\tau_1)\varepsilon_t)\mathrm{d}\Phi_{\zeta^j,t}(\zeta_t^j)}\ . \tag{A.53}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Using equation (A.52), we next obtain a unique value for $z_{N,t}$. Finally, we can use the solution for $c_{M,t}$, $h_{M,t}$, or $h_{N,t}$ in Table 1 to solve for a unique value of B_t.

C Description of Data Sources

In this appendix we describe the data sources and the sample construction in more detail.

C.1 Consumer Expenditure Survey

We integrate information from surveys conducted between 1996 and 2016. For each quarter, we combine the family data (FMLI), which provide household expenditures, with the member data (MEMI), which contain individual income. Surveys that have been conducted in the first quarter of year t may be recorded in year t as well as year $t-1$. The two records are not always identical. When a survey is recorded twice, we use the last recorded information for each variable. We merge the information collected across the quarterly surveys to obtain annual data. We allocate each household to the calendar year for which most of its recorded surveys apply.

We deflate food and alcoholic beverages with its deflator and all other spending categories using the consumer price index. For our baseline analyses, our definition of non-durable consumption expenditures includes (*i*) food and alcoholic beverages, (*ii*) tobacco and smoking supplies, (*iii*) personal care, (*iv*) utilities, fuels, and public services, (*v*) household operations (which includes child care), (*vi*) public and other transportation, (*vii*) gasoline and motor oil, (*viii*) apparel and services, (*ix*) reading materials, as well as (*x*) miscellaneous items.

C.2 American Time Use Survey

We use the ATUS waves between 2003 and 2016. Our definition of time spent on home production is identical to Aguiar, Hurst, and Karabarbounis (2013) and includes activities such as cooking, child care, gardening, shopping, caring for others, as well as home, garden, and car maintenance.

We merge the information on time use with background characteristics of the respondents using the additional CPS, roster, and respondent files in the ATUS. These files provide data on variables such as age, sex, race, education, labor force status, earnings, hours worked, number of children, and the age of the youngest child. We use this information to obtain a large set of shared individual characteristics between the ATUS and CEX. We use this set of common characteristics to impute information from the ATUS into the CEX.

C.3 Sample Adjustment and Imputation

In our baseline specification, we focus on married and cohabiting households with a household head that is between age 25 and age 65, excluding students. We restrict the sample to households that have completed all four CEX interviews. We exclude outliers in terms of food consumption, non-durable consumption, and total spending (bottom and top 1 percent), in terms of market hours (above 92 hours per week), and in terms of wages (below 3 dollars or above 300 dollars).

To impute home hours from the ATUS into the CEX we use an iterative procedure based on mean hours conditional on group characteristics. To do so, we construct four groups of variables:

1. Work status, race, gender, age.
2. All previous and cohort, family status, and education.
3. All previous and disability status, retirement status, and geographic information.
4. All previous and hours and wage conditional on working.

For every individual in the CEX, we first evaluate at the finest (fourth) level whether there exist individuals in the ATUS that share the same group characteristics. When such individuals exist in the ATUS, we impute the mean of their home hours to the individual in the CEX. If such a group does not exist, we move to a more coarse level (third), and so on. We repeat this procedure for each observation in the CEX. Applying this procedure, we account for approximately three quarters of the variation in home hours. To ensure that our imputation does not introduce spurious correlations in the merged CEX/ATUS data, we have confirmed that the correlation of home hours with market hours and wages conditional on age is similar between the ATUS sample and the merged CEX/ATUS sample.

C.4 Longitudinal Internet Studies for the Social Sciences

The Longitudinal Internet Studies for the Social Sciences (LISS) panel is administered by CentERdata in the Netherlands. The panel is based on a representative sample of Dutch households who participate in monthly surveys. We use the three waves of this panel (2009, 2010, and 2012) that contain information on time use.

We define home production time as close as possible to the ATUS. Specifically, home production time includes household chores, child care, and administrative chores. Market hours are measured by time spent on paid work, which includes commuting time. Consumption expenditures include food, utilities, home maintenance, transportation, daycare, and child support. Our findings are robust to broader and narrower definitions of consumption expenditures.

As in our analysis for the United States, we consider married and cohabiting households with a head between 25 and 65 years of age that completed both the time use and the consumption expenditure survey. We drop outliers in terms of total expenditures, non-durable consumption, and hours worked. Our final sample contains 1,242 observations.

C.5 Japanese Panel Survey of Consumers

The Japanese Panel Survey of Consumers (JPSC) covers the periods between 1998 and 2014. The JPSC records information for time use on workdays as well as on days off. The dataset includes time spent on commuting, working, studying, home production and child care, leisure, and sleeping, personal care and eating. For home hours we use the variable for home production and child care and for market hours we use the hours worked. To calculate the home and market hours for a given week, we weight the time use on workdays and days off by the number of days worked. Our measure of consumption expenditures includes food, utilities, apparel, transport, culture and leisure, communication, trips and activities, house and land rent. Our findings are robust to broader and narrower definitions of consumption expenditures.

We focus on married households with complete information on time use as well as expenditures. We restrict the sample to households with a head between age 25 and age 60 to avoid having less than 40 observations per age group. This leaves us with 14,548 observations for 2,225 households.