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# ECONOMIC INCENTIVES AND POLITICAL INSTITUTIONS: SPENDING AND VOTING IN SCHOOL BUDGET REFERENDA

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## ABSTRACT

Allocation of resources in the local public sector involves economic and political forces. Spending for elementary and secondary education is a major area of public expenditure. In many states, the bulk of this spending is subject to referendum. In addition, grants-in-aid from state governments to local school districts form an important component of the district revenues. This paper has two main features. One is the characterization of local spending when the state aid structure is of the closed-end matching grant type. Under this structure, local tax price is endogenous, since the amount of state subsidy depends on the district's spending choice. The other main feature is the linking of spending proposals to referendum outcomes. In this way, our model makes use of voting data to shed light on the extent to which referenda constrain spending. The empirical setting is public school budget referenda in 544 New York school districts for the 1975-76 school year. Our econometric results and simulations based on them reveal considerable sensitivity of spending to the form of the grant structure, as well as to the referendum requirement. In addition, large school districts appear to behave more like "budget-maximizers" than do small districts, where proposals appear to be more in line with "median voter" demands.

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# ECONOMIC INCENTIVES AND POLITICAL INSTITUTIONS: SPENDING AND VOTING IN SCHOOL BUDGET REFERENDA

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## 1. Introduction

While public spending faces many of the same resource allocation tradeoffs that affect the market equilibrium for private goods, the decision as to how much to spend is not decentralized among a multitude of invisible hands but is made through a political process that directly aggregates individual preferences. In many American localities, referenda are a key aspect of the budgetary process. The major type of expenditure decision made this way is spending for elementary and secondary education by local public school districts. Public school spending is therefore a useful vehicle for the study of local public goods provision processes -- and is a quantitatively important area of resource allocation in its own right.

A large empirical literature has examined various aspects of the determinants of school spending. For the most part, this literature has focused on specifying an expenditure equation, without linking observed spending proposals to referendum outcomes. One might well think, however, that the vote outcomes contain information, for they may reveal the extent to which public school officials view the referendum process as a constraint on spending.

Resources to school districts flow not only from the locality but also from higher level governments, especially the state. As state aid accounts for a substantial component of public school spending throughout the U.S., it clearly plays an important role in the determination of local expenditures. A frequently used form of state aid for education is the closed-end matching grant. Such grants provide aid that subsidizes local spending up to some predetermined level, beyond which the subsidy stops. The local "tax price" of public spending is therefore less at low levels of spending than at high ones. A choice of spending level by the school district is therefore also a choice of tax price. This endogeneity needs to be captured in an econometric specification of spending in the presence of closed-end matching grants.

The structure of constraints and incentives thus combines political and economic elements. In this paper, we estimate a model that allows for simultaneous determination of two key outcomes of budget referenda: an *economic outcome*, as captured by per-student spending levels; and a *political outcome*, the vote results of the budget referenda. The empirical setting is 544 non-city school districts in New York State in the 1975-76 school year. All noncity districts were required to have at least one referendum on school spending and received closed-end matching aid from the state for basic operating expenditures.[1]

Our model combines a spending equation and a vote equation. This specification allows us to ask questions about the responsiveness of public spending to local characteristics (income, property wealth), as in conventional studies. It also provides evidence on the link between local spending and referendum voting. In addition, it allows us to study the effects of possible changes in state aid structure (e.g., replacing closed-end matching grants by lump-sum block grants). We can also look at the expected impact of changing referendum rules (e.g., requiring supramajorities for passage).

To develop the model, we begin in Section 2 by specifying a spending equation that captures underlying demand effects. While whose demand is effective is determined politically, an explicit discussion of the political process is deferred to later in the paper.

Our spending equation keys--within a conventional log-linear structure--on the effects of tax price and income. The estimation of these effects has to pay particular attention to the endogeneity of tax price. Other investigators have noted that this endogeneity makes OLS inappropriate. Their approach to dealing with the problem typically has been to use instrumental variables in a fairly *ad hoc* way. We use a full-information maximum likelihood method presented by Moffitt (1984, 1986) to obtain consistent estimates of income and price elasticities. Our estimates, particularly of price elasticity, are quite different from those obtained by OLS, and lead to quite different conclusions about the responsiveness of spending to the incentives posed by the matching program.

Our spending equation allows for an error structure in which one of the error components can be interpreted as a "shift" in expenditures relative to some average demand at given tax price and income. This shift may be thought of as coming from discretionary actions of agents (formally, the school board) that we characterize as the agenda "setter." A positive value of the "setter shift" corresponds to high spending, relative to a "typical" district with similar economic characteristics.

In Section 3, we construct a voting equation and link it to the expenditure equation. What should enter the voting equation? Unlike spending, where, at a minimum, demand is characterized in terms of income and price effects, there is no clear indication that any exogenous variables are relevant to voting. Consider a stylized example. Assume that all districts are characterized by setters aiming to maximize the budget (Romer and Rosenthal, 1979). Then, for reasons on which we elaborate in Section 4, the vote outcome should be close to 50 per cent Yes. No exogenous variables should enter the vote equation because the setter will have taken these variables into account in setting spending. To clarify presentation through a simple model, we follow the spirit of this setter model in section 3 and estimate a voting equation without exogenous variables.

The spending and voting equations are linked by an error components model. *Ceteris paribus*, high operating budget requests should get fewer votes than low ones. More precisely, errors in the spending equation should covary negatively with those in the voting equation. A "setter shift" common to both equations permits testing for the expected negative covariance: a high spending shift should tend to be accompanied by a low vote.

In section 4, we seek to move toward a better specification of the political process. In studying the political process, one would like to have knowledge of the preferences of the setter. Is the setter seeking an extremely high budget, or are ambitions more modest? We hypothesize that bureaucrats looking for large budgets (per student) are more likely to be found in large school districts. Therefore, we use size, measured as enrollment, as a proxy for the ambitions of setters. On the other hand, size may also proxy for demand effects and for the heterogeneity of voter preferences. We also consider further elaborations of the basic model. In addition to capturing aspects of the political process, we have a narrower concern with sociodemographic variables that might proxy for effects on demand that are not captured by income and price variables. If such variables are capturing purely demand effects they, like income and price, should have no effect on vote outcome, though they may have spending effects.

In Section 5, we present a variety of "what if" analyses based on our empirical results. These explore how school spending might be affected by changes in the grant-in-aid formula or by instituting supramajorities in referenda. The conclusion is in Section 6.

## 2. Expenditure Estimation with Closed-End Matching Grants

With the local tax structure given by a proportional property tax, a school district's budget constraint is:

$$NS = tK + NA,$$

where N is number of students, S is per-student spending, K is total market value of taxable property, t is the effective tax rate, and A is the amount of grants-in-aid per student.

Under a closed-end matching grant, the state government subsidizes local spending at rate m, but above a combined state-local spending limit of $S_k$ per student the subsidy stops, so that districts spending above $S_k$ (including state aid) receive, in effect, a lump-sum grant totaling $NmS_k$. The district budget constraint under this kind of grant is:

$$NS = tK + N \min(mS, mS_k), \tag{1}$$

and the local tax rate can be written as:

$$t = \begin{cases} \dfrac{(1 - m)S}{K/N} & \text{if } S \le S_k \\[2ex] \dfrac{S - mS_k}{K/N} & \text{if } S > S_k \end{cases} \tag{2}$$

Consider an individual voter with utility function U(C,S), where C is consumption of a numeraire good. Let Y be this voter's income and H the value of his *taxable* property. Using (2), this voter's budget constraint can be written as:

$$C = Y - tH$$

i.e.,

$$C = \begin{cases} Y - P_1 S & \text{if } S \le S_k \\ Y + P_2 mS_k - P_2 S & \text{if } S > S_k \end{cases} \tag{3}$$

where

$$P_1 = \frac{(1 - m)H}{K/N} \tag{4}$$

and

$$P_2 = H/(K/N) \tag{5}$$

The voter is faced with a budget constraint that has a kink at $S = S_k$. (See Figure 1.) To the left of $S_k$, the effective tax price is $P_1$, and to the right it is $P_2$. Spending above $S_k$ is not matched, but the second segment of the budget constraint incorporates the "income equivalent" of the lump-sum grant of $mS_k$ per student, so that "effective income" along this segment is equal to $Y + P_2 mS_k$. Because $1 > m > 0$, the budget constraint is convex.

Maximizing U(C,S) subject to (3) yields a "most-preferred" level of S for the voter. The political process may be viewed as aggregating the basic individual preferences into an expenditure decision. In this section, we largely put the political process aside and simply postulate *desired spending* $S^d$ in a school district is given by

$$S^d = g(P,M;\ \epsilon_d) \tag{6}$$

The function $g(\cdot)$ is an underlying demand function, with P being tax price and M denoting "effective income". In a standard median-voter model it would be interpreted as the median of desired spending levels. In previous work (Romer and Rosenthal, 1979, 1982a), two of us have argued that (a) the median voter is not necessarily the "representative" voter in referenda, and (b) estimates of price and income elasticities probably should not be interpreted as corresponding to "median" demand even when "median" variables (tax-price, income) are used as regressors.[2]

For our empirical work, we specialize (6) to the log-linear form, so that

$$\mathcal{S}^d = f(P,M) + \epsilon_d \tag{7}$$

where $\mathcal{S}^d \equiv \ln S^d$

and $$f(P,M) = \beta_0 + \beta_1 \ln P + \beta_2 \ln M \ .$$

In this specification, we choose to interpret f(P,M) as the demand, given income and price, of the "representative" voter in an "average" district. In this context, the random error term $\epsilon_d$ captures variations across districts in the representative voter's most-preferred spending level. This variation may be due to cross-district heterogeneity of two types. First, it may indicate unobserved differences in underlying demand. Second, it may be due to differences in the extent to which setters in different districts reflect the preferences of relatively high-demand voters; i.e., variation across districts in the political process that establishes the characteristics of the "representative" voter.

In the presence of matching grants, a choice of $\mathrm{S}^d$ is also simultaneously a choice of tax price, since choosing a level of spending also selects a particular segment of the budget constraint (or a location at the kink). Let

$$M_1 = Y$$

and $$M_2 = Y + P_2 m \mathrm{S}_k \ .$$

Let $S_k = \ln \mathrm{S}_k$. Then the desired segment location can be written as:

$$S^d = \begin{cases} f(P_1,M_1) + \epsilon_d & \text{if } f(P_1,M_1) + \epsilon_d \leq S_k \\ S_k & \text{if } f(P_1,M_1) + \epsilon_d > S_k \geq f(P_2,M_2) + \epsilon_d \\ f(P_2,M_2) + \epsilon_d & \text{if } f(P_2,M_2) + \epsilon_d > S_k \end{cases} \qquad (8)$$

Observed spending need not correspond to desired spending. The two may differ due to random measurement or specification errors, so that actual expenditures S are related to desired spending as follows:

$$S = S^d + \epsilon_r \qquad (9)$$

where $S \equiv \ln \mathrm{S}$. We want to estimate (9), with $S^d$ as specified in (8).

One approach to estimation would use the observed values of P and M (as determined by whether observed spending is above or below $S_k$) to estimate $S = f(P,M) + \epsilon$ (where $\epsilon$ is a random error) by ordinary least squares. Because of the endogeneity of tax-price (and of "effective income" M), this approach will yield inconsistent coefficients. For example, large values of $\epsilon$ may lead to high $S$. But an observation with high $S$ will be assigned the tax-price $P_2$. Similarly, low values of $\epsilon$ will be correlated with assigning the value $P_1$. In other words, the error term and the variables P and M will be correlated.

To obtain consistent estimates, we use a maximum-likelihood approach described by Moffitt (1984, 1986).[3] This approach does not ascribe a particular segment to a given observation. Rather, given an observation, we compute the probability associated with being on each segment (or at the kink). Thus, the probability of observing $S$ is given by:

$$\begin{aligned} \Pr\{S\} = & \Pr\{\epsilon_d + \epsilon_r = S - f(P_1,M_1),\ \epsilon_d \le S_k - f(P_1,M_1)\} \\ & + \Pr\{\epsilon_r = S - S_k,\ S_k - f(P_1,M_1) < \epsilon_d \le S_k - f(P_2,M_2)\} \\ & + \Pr\{\epsilon_d + \epsilon_r = S - f(P_2,M_2),\ \epsilon_d > S_k - f(P_2,M_2)\} \end{aligned} \qquad (10)$$

With assumptions about the error distributions, the above probabilities can be used to derive the likelihood function. We assume that $\epsilon_d$ and $\epsilon_r$ are independent, with $\epsilon_d \sim N(0,\sigma_d)$ and $\epsilon_r \sim N(0,\sigma_r)$. The likelihood function is given in Appendix 1.

We estimated this specification using data for the 544 school districts. The spending variable is per-student approved operating expenditure for the school year 1975-76. In the definition of effective income (M), Y is measured as median household income. The "unmatched" tax-price $P_2$ is median housing value divided by the district's full property valuation per student. The matching rate m varies across districts, from a high of about 0.9 for a district with very low tax base (measured in the 1973-74 school year) to 0.3 for high-wealth districts. Matching stops when approved operating expenditures reach \$1200 per student (i.e., $S_k = 1200$), so wealthy districts with high spending do not receive more than \$360 in operating aid. (Appendix 2 has details of variable definition and data sources.)

Column 1 of Table 1 presents the MLE results. Our estimates of price and income elasticities are within the ranges of those for a variety of states in a number of studies reported in Black *et al.* (1979, Table 2) and Welch (1981) and for Oregon in Filimon *et al.* (1982). The estimated income elasticity is somewhat higher than those reported in most analyses of school spending.[4]

The estimate of $\sigma_d^2$, the setter shift variance, is five times that of $\sigma_r^2$, the variance of the random observation or measurement error. This suggests that differences in "desired spending" across districts (given tax price and income) are more important than random errors in translating desired spending into actual expenditures. The magnitude of $\sigma_d^2$ relative to $\sigma_r^2$ reflects the clustering of observations about the kink point $S_k$ due to the incentive effects of the closed-end matching grant program.[5]

It is instructive to compare the maximum likelihood results with OLS estimates that ignore the endogeneity of tax-price and "effective income". Column 2 of Table 1 shows these estimates when all observations are included. The results are strikingly different. The price elasticity is estimated to be nearly zero.[6] Because OLS does not take into account the process whereby districts select the segment of the budget constraint on which they would prefer to locate, the responsiveness of spending to tax-price is underestimated. Indeed, a naive interpretation of the OLS results might lead one to believe that matching grants were fairly ineffective, since spending appears quite insensitive to price at the margin! The OLS results also provide lower income elasticity estimates than does MLE.

For observations near the kink ($S \approx S_k$), it is not clear what is the "appropriate" tax-price to use in the OLS estimation. We reran the OLS estimates, omitting observations with per-pupil spending within 50 dollars of $S_k$ ($1150 \leq S \leq 1250$). Because of the clustering of expenditures toward the kink, this reduces the number of observations from 544 to 449. For this new set of observations, the OLS price elasticity estimate is greater than the previous estimate -- but still quite close to zero, and much lower than the maximum likelihood estimate. (See Column 3 of Table 1.)

These differences in the results suggest that estimation as well as modeling considerations point to using the full error components specification of the spending equation when we link spending and voting. We turn to this in the next section.

## 3. Spending and Voting in Budget Referenda

Every non-city school district in New York is required to have at least one referendum on the proposed budget. Of the 544 districts in our sample, 142 failed to pass their budget on the first try. Even among those that passed, there was considerable variation in the percentage voting in favor of the proposal.

The variable we use to analyze voting in more detail is the *logit* of the vote outcome on the first election. This is defined as

$$V \equiv \ln \text{(Yes votes/ No votes)} \tag{11}$$

and allows V to range over the interval $(-\infty,+\infty)$. (In our sample we have no unanimous votes.) A district with 50% Yes vote would have $V = 0$.

If one were simply interested in analyzing the effects of a variety of (putatively) exogenous variables on the vote outcome, one might specify an equation of the form

$$V = Z + \epsilon_v \; . \tag{12}$$

In such a specification, Z would be a function of variables that are thought to affect the vote and $\epsilon_v$ is a random error term. Z might include demographic variables or even proposed spending. Equation (12) would then be estimated by itself.

We take a different tack here. Instead of estimating a voting equation separately, we recognize the link between spending and voting by considering the relationship between desired spending and vote outcome.

Suppose that, in a given district, the setter is "aiming" at the voter whose desired spending level is given by f(P,M), as developed in the previous section. With no uncertainty about turnout, this level of spending, if put to the voters, would receive a Yes vote from some fraction of the electorate; let the vote logit corresponding to this be $\alpha$. In this section, we assume that setters "aim" in such a way that $\alpha$ is invariant across districts. If the vote were uncertain due to random effects (e.g., such factors as turnout affected by weather or purely probabilistic voting), then V would be given by

$$V = \alpha + \epsilon_u ,$$

where $\epsilon_u$ is a random variable with mean zero.

Now suppose that the setter considers the pivotal voter to be someone with $S^d = f(P,M) + \epsilon_d$ and makes his budget proposal accordingly. If $\epsilon_d > 0$, this will be someone with ideal point greater than f(P,M). So one would expect a smaller fraction of voters to favor this proposal than a proposal with $\epsilon_d = 0$. That is, in the absence of uncertainty about turnout, we would have $V < \alpha$. Similarly, if $\epsilon_d < 0$, the expected Yes vote would be higher (at least as long as the proposal is for an amount greater than what could be spent without holding a referendum). In this case, we would have $V > \alpha$ in the absence of turnout effects.

These arguments suggest that there should be a negative relationship between the vote logit and the error $\epsilon_d$ from the spending equation. The simplest way to capture this is to suppose that the shift in votes is proportional to $\epsilon_d$, so that given $\epsilon_d$ leads to a shift of $\gamma\epsilon_d$ in V. We make two additional strong assumptions: that the negative relationship is constant across districts; and that in every district, the expected Yes vote is the same when $\epsilon_d = 0$. We can then write the vote equation as:

$$V = \alpha + \gamma\epsilon_d + \epsilon_u . \tag{13}$$

If the negative relationship between spending and voting holds, then we should find $\gamma < 0$.

## Estimating the Two-Equation System

The combined spending-voting system now consists of two equations (9) and (13). Maximum likelihood estimation of the combined system is similar to that of the spending equation alone. To allow the model to include other variables $X_k$ in the expenditure equation, we define:

$$h(\cdot) = f(\cdot) + \sum_{k>2} \beta_k X_k \tag{14}$$

For economy of notation, we will refer to h(P,M), but note that other variables may be included. Similarly, the expression Z in the vote equation may include more than just the constant $\alpha$.

Now, for a given district, we are concerned with the *joint* probability of observing expenditure S and vote logit V:

$$\begin{aligned} \Pr\{S,V\} = \\ &\Pr\{\epsilon_d + \epsilon_r = S - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V - Z,\ \epsilon_d \le S_k - h(P_1,M_1)\} \\ &+ \Pr\{\epsilon_r = S - S_k,\ \gamma\epsilon_d + \epsilon_u = V - Z,\ S_k - h(P_1,M_1) < \epsilon_d \le S_k - h(P_2,M_2)\} \\ &+ \Pr\{\epsilon_d + \epsilon_r = S - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V - Z,\ \epsilon_d > S_k - h(P_2,M_2)\} \end{aligned} \tag{15}$$

In (15), we have again taken $S \equiv \ln S$ and $S_k \equiv \ln S_k$.

To complete the model, we now specify the error structure. We assume that the errors $\epsilon_d$, $\epsilon_r$, and $\epsilon_u$ are independent, and that $\epsilon_d \sim N(0,\sigma_d)$, $\epsilon_r \sim N(0,\sigma_r)$, and $\epsilon_u \sim N(0,\sigma_u)$. As the disturbances $\epsilon_d$ and $\epsilon_r$ relate to a decision made by a single agenda setter (or a small committee) in each district, they are taken to be homoskedastic. In contrast, $\epsilon_u$ is the disturbance associated with the logit of an aggregate proportion. Its variance would therefore depend on the number of individuals making voting decisions. An appropriate heteroskedasticity correction would need to consider turnout factors. In the absence of data on registration, we have taken the variance of $\epsilon_u$ to be proportional to the number of *actual* voters; i.e., $\sigma_u{}^2 = (\sigma_u^*)^2/W$, where W is the number of actual voters and $\sigma_u^*$ is constant across observations. The variance-covariance structure is thus determined by the estimated parameters $\gamma$, $\sigma_d$, $\sigma_r$, and $\sigma_u^*$, and the observed values of W. The likelihood function is given in Appendix 1.

## Results for the Basic Two-Equation Model

The estimated coeffiecients of the "bare-bones" model developed in this section appear in the first column of Table 2. The chief results include:

1. The hypothesized negative relationship between setter errors in spending levels and voting is supported by our estimates. (This finding parallels Oregon results reported for a different specification in Romer and Rosenthal, 1982b.) The estimated $\gamma$ coefficient is negative. Its magnitude is eleven times the estimated standard error.

2. Price and income effects are similar to those in the single-equation model.

3. The estimated effect of a setter shift is substantial. If the sample

mean spending level were the target ($\epsilon_d = 0$), an 11 percent upward "shift" in spending leads to a vote loss such that an expected victory ($\alpha = 0.138$ corresponds to a vote of 53.4%) is turned into an expected defeat ($\alpha - \gamma\epsilon_d = -0.006$ corresponds to a vote of 49.8%). Obviously, given that 142 of 544 first referenda failed, shifts of 11 percent and above were not uncommon. Indeed if $\epsilon_d$ takes on a value equal to $\sigma_d$, the standard deviation of its distribution, the upward shift at the mean is approximately 15 percent.

4. The setter shift appears to be a major source of variation in spending. The size of $\sigma_d$, the disturbance term parameter corresponding to the setter shift, is estimated to be over twice the size of the "random" error parameter, $\sigma_r$.

## 4. The Political Process and Size Effects

### The Political Process: Preliminary Theory and Observations

The negative covariance between spending and voting in the "bare bones" model suggests that a link does indeed exist between a school district's finances and its politics. We therefore turn to a more detailed analysis of how politics can affect expenditure in the context of referendum institutions.

Beginning with Barr and Davis(1966), politics was introduced into economic analysis by modelling the spending equation as capturing the median of voters' demands. The median in this model corresponds to a special type of competitive political equilibrium maintained primarily through competitive elections of those responsible for budgeting. A sharp alternative to the median voter hypothesis is the bureaucratic budget-maximization model of Niskanen (1971). This model posits that the agenda for the decision process in a given area of public spending is controlled by those who have high demand for the expenditure. With reference to school spending, one would argue that the agenda is controlled by a coalition including the providers of education services and some high-demand consumers such as relatively affluent families with several children attending public schools. Corollary to the Niskanen-type hypothesis is the proposition that popular election of school board members is an inadequate mechanism for curbing agenda control powers.

In budget referenda, school boards have agenda control powers necessary for a Niskanen "monopoly" outcome to prevail over a "competitive" one. Only school boards can put a budget on the ballot. The board's power depends critically on the implicit or explicit alternative to the proposal they formulate. In a school district where failure to approve the proposal forces closing of the schools, the latitude afforded the agenda setter is considerably greater than in a district where the schools can continue to operate normally even if the proposed budget fails.

Earlier research using data from Oregon referenda (Filimon et al. 1982, Romer and Rosenthal, 1982a, 1982b) was largely motivated by the widespread cross-sectional variation in these alternative or *reversion* levels in that state. In many Oregon localities, reversions are so low that voter failure to approve the proposal can and does result in the closing of the school system. In others, particularly the city of Portland, the reversion is so high that the school board typically elects not to hold a referendum and to operate with the reversion. Several results from this research on Oregon supported the budget-maximization model. These included:

(1) As a group, districts that failed to hold a referendum spent over 99 per cent of their reversion levels.

(2) Districts whose reversions were so low that a school closing was threatened were estimated to spend about 15 per cent more per student than districts with reversions just large enough to avoid the threat of a closing.

It is quite possible that modification of the reversion rules might so change the agenda that there would be little difference between a median voter model and a budget-maximization model. Indeed, New York's institutional structure for referenda differs from Oregon's in the way the reversion is specified. In New York, a non-city school district must hold one referendum each year. If the referendum fails, the district may impose a "contingency" budget. The definition of a "contingency" budget is rather flexible, and allows for the operation of schools, though certain "nonessential" activities such as interscholastic sports may have to be curtailed.[7]

These relatively high reversions imply that districts that go on contingency will not always have, ceteris paribus on the demand side, lower spending than those that do not. Consider the simple case where all voters have school spending preferences that are single-peaked about the ideal point represented by the "most-preferred" level. Suppose that voters are fully informed and are certain to vote. Now consider two districts, one in which the reversion is somewhat above the median voter's ideal point, another in which the reversion is somewhat below. Both districts have budget-maximizing agenda setters. In the first district, the setter will not be able to pass a budget higher than the reversion. Consequently, he will adopt a contingency budget -- the reversion. In the second district, the setter will be able to pass a budget above the median. But this budget will be close to the first district's contingency budget. Thus, when reversions vary more or less symmetrically about medians, one would not expect strong differences in the spending patterns of contingency and non-contingency districts.

In exploring the New York data, we indeed did not observe important differences in the spending patterns of districts that did and did not go on contingency.[8] While this preliminary exploration of the spending data did not disclose any impact of the political process, the relevance of the referendum institution is strongly evident in the referendum outcomes. Several key facts are worth noting.

*First*, the political process is not pro forma. As we noted earlier, 142 districts failed to pass the budget on the first try. Of 120 districts that tried twice, only 65 passed. Even on the third try, eight of twenty-four proposals were rejected. One district took four tries before the budget was approved. Sixty districts chose after one or more tries to go on contingency. Approval is far from automatic.

*Second*, districts do try to avoid going on contingency. There is obviously no general tendency to make a first proposal so high that the voters automatically reject the budget. Even when the first try fails, only 22 of the failing districts opt for contingency. This suggests that the agenda setters regard contingency as costly, possibly resulting in a lower expenditure level than a passed budget.

*Third*, there is widespread variation in the percentage voting Yes on the first try. Though the average share of the Yes vote in all 544 districts is 63 percent, some districts pass with near-unanimity, while many others lose.

*Fourth*, there appears to be considerable variation in referendum results when districts are grouped by size (as measured by enrollment), as in Table 3. On the first try, small districts tend to pass by overwhelming margins. In the larger districts, however, the average percent Yes is close to 50.

What are the sources of these variations? Three factors are relevant. First, districts may exhibit variation in their reversion levels. Second, districts may differ in the degree of heterogeneity of voters' spending preferences. Third, there may be variation across districts in the extent to which agenda setters pursue budget-maximizing strategies. We briefly take up each of these possibilities.

To see the possible impact of differences in reversions, suppose that each voter has spending preferences that are symmetric as well as single-peaked. Consider a district whose cumulative distribution of voter ideal points is given by $F(S)$, with median ideal point $S^0$. (See Figure 2.) Suppose the reversion (contingency spending) is $R_1 < S^0$, and $S^0$ is proposed. Then, abstracting from turnout considerations, voters with ideal points greater than $S_1 = (R_1 + S^0)/2$ will vote in favor of the proposal, while those with lower ideal points will prefer the reversion. The median ideal point would pass with a fraction $1 - F(S_1)$ voting Yes. As the reversion moved closer to $S^0$ (but remained below $S^0$), the fraction voting Yes would decline. If, however, the reversion is $R_2 > S^0$, then those with ideal point less than $S_2 = (R_2 + S^0)/2$ will prefer $S^0$ to $R_2$, and the median ideal point will pass with a fraction $F(S_2)$ voting Yes. Again, moving the reversion closer to $S^0$ (this time from above) reduces the Yes vote.

Now consider a second district with the same median ideal point $S^0$ and the same reversion, but with ideal points distributed according to $G(S)$. If this second district is more heterogeneous, in the sense that $G(S) \left\{\begin{matrix} > \\ < \end{matrix}\right\} F(S)$ for $S \left\{\begin{matrix} < \\ > \end{matrix}\right\} S_0$, then when the median ideal point is proposed against other $R_1$, or $R_2$, the more homogeneous district will have a higher Yes vote.

Because we cannot measure reversions for districts that do not go on contingency, we cannot disentangle the effects of voter heterogeneity and variations in reversions. We will use "within-district heterogeneity" as a short-hand to summarize both of these two factors.

In our example, the agenda setter strove to enact the median ideal point. This may not be the setter's goal in every district. Referring again to Figure 2, suppose two districts had ideal point distributions given by F(S) and reversions $R_1$. Suppose that in one district, the proposal is the median; in the other, the largest proposal that will defeat the contingency. In the second, budget-maximizing district, the proposal ($2S^0$ - $R_1$) would be chosen so as to just get the approval of a majority of voters. The "representative" voter whose ideal point corresponds to the enacted budget is no longer the median voter but is someone with preference for spending in excess of the median. Obviously, the Yes percentage is greater in the district where the median was proposed.

To summarize the implications illustrated by our example, we contrast budget-maximizing setters with median-voter-seeking setters. Districts with budget-maximizers should show relatively little variation in referendum results. Their vote outcomes should be independent of within-district heterogeneity and be close to 50 percent favorable. Their spending will be strongly influenced by the reversion. In contrast, median-voter-seekers should have spending independent of the reversion but vote outcomes highly related to within-district heterogeneity.

One might well expect to see both more heterogeneity of voter preferences and, due to information and free-riding problems, more budget-maximizing behavior in large districts. The closeness of election results in large districts suggests that these districts are more likely to have budget-maximizing agenda setters aiming at relatively higher-than-median expenditures. The actual Yes percentage in such districts may vary about 50 as a result of misinformation about voter preferences and random variation in turnout.[9] To explain that the average is around 50% by a median voter model, one would have to posit both much more voter heterogeneity in large districts than in small districts and reversion levels very close to the median ideal point.

Another suggestive piece of evidence comes from what happens when there is a second referendum in a district whose voters rejected the initial budget proposal. There, regardless of district size, the average percent Yes hovers around 50, exactly as a budget-maximizing model would suggest. By this model, a district failing a first election would be likely to be one in which the agenda setter had decided to play "hardball" with the voters. Budget-maximizing districts would get an outcome close to 50-50 on all tries.[10]

When a district holds more than one referendum, there is almost no correlation in the vote results on successive tries. The correlation between first- and second-election Yes percentage is only 0.21. The second-to-third election correlation is actually negative at -0.05. These results indicate that in "hardball" districts the variation about 50% Yes largely reflects elements, such as turnout variations, not fully subject to the setter's control. The observed variation about the 50-50 point is puzzling if viewed from the median voter perspective. Setters aiming to enact median voter preferences who had misjudged voter preferences on a first try might be expected to make a second proposal that would be widely supported.

The Effect of Community Size

These preliminary observations on voting suggested that we make a systematic effort to ascertain whether the variations in the Yes percentage reflect differences in within-district heterogeneity or indeed differences in expenditure levels sought by agenda setters.

The specification of Section 3 assumed a great deal of homogeneity across school districts. Except for tax price and income, only stochastic effects differentiate school districts in both their approved spending and their voting. Our districts, however, vary greatly in size, ranging from an enrollment of just under 500 students to one of over 21,000. This variation in size may have at least three important influences on voting and spending. First, as just noted, large districts may tend to have budget setters more keyed to budget-maximization, producing both larger budgets and smaller approval margins. Second, large districts may be more heterogeneous than smaller ones. Increasing within-district heterogeneity, *ceteris paribus*, should lower the approval margin. Third, district size may be associated with economies or diseconomies of scale in the production of educational output. Thus, district size may need to be incorporated in the demand function, $h(\cdot)$. We now indicate how each of these three effects can be included in our econometric model.

A. Community Size as an Indicator of Budget-Maximizers

In developing the spending model, we have focused on how political agenda setters may shift spending relative to the demand expressed by some "average" pivotal voter. So far, this shift was captured entirely by a disturbance term. We now allow the shift to depend upon district size. We therefore redefine the "setter shift" as consisting of both this term and a random disturbance term. Our only available measure of size for 1975-76 is enrollment N. We take:

$$\text{setter shift} = \zeta \ln N + \epsilon_d ,$$

where $\zeta$ is a coefficient to be estimated. We would expect $\zeta > 0$. This reformulation of the spending equation then leads to a modified voting equation:

$$Z = \alpha + \gamma\zeta \ln N$$

For $\gamma<0$, the "target" vote logit moves down with the observable portion of the setter shift. The restriction on the coefficient of ln N in the vote equation captures the notion that as spending goes up approval goes down.

B. Community Size as an Indicator of Homogeneity of Voter Preferences.

In addition to being related to the effects of agenda setting strategies on vote outcomes, district size may also proxy for voter heterogeneity. To capture this effect, together with the budget-maximizing effect, we modify Z to:[11]

$$Z = \alpha + \gamma\zeta \ln N + \delta \ln N$$

Because small districts are presumably more homogenous than large ones, we expect $\delta < 0$.

C. Community Size as a Determinant of Demand.

Community size may be related to economies or diseconomies of scale that affect the "representative" voter's demand for spending directly. Such effects can best be thought of as belonging in the demand function, rather than in the setter shift. As such, they will not affect the "target" level of voting. To model these effects, we redefine $h(\cdot)$ in the spending equation as follows:

$$h(\cdot) = \beta_0 + \beta_1 \ln P + \beta_2 \ln M + \beta_3 \ln N.$$

The vote equation is unaffected by this change.

## Summary of the Voting-Spending Models

Incorporating all three size-related effects yields a spending equation of the form

$$S = \beta_0 + \beta_1 \ln P + \beta_2 \ln M + \beta_3 \ln N + \zeta \ln N + \epsilon_d + \epsilon_r \qquad (16)$$

and a vote equation

$$V = \alpha + \delta \ln N + \gamma\zeta \ln N + \gamma\epsilon_d + \epsilon_u. \qquad (17)$$

The parameters $\beta_3$, $\delta$, and $\zeta$ cannot be simultaneously estimated. For example, a model that assumes no within-district heterogeneity effects but the presence of size effects on setter shifts and demands ($\delta = 0$, $\zeta \neq 0$, $\beta_3 \neq 0$) will fit the data exactly as well as a model that assumes no "average" voter demand effects but the presence of setter and within-district heterogeneity effects ($\beta_3 = 0$, $\delta \neq 0$, $\zeta \neq 0$). The parameter $\gamma$ is identified through the error structure.

These considerations, together with our discussion of size effects, suggest an interest in estimating four models:

1. Enrollment in spending but not in voting ($\zeta = \delta = 0$). In this model, size is unrelated to setter shift or within-district heterogeneity but does relate to scale economies.
2. Enrollment in voting but not in spending ($\zeta = \beta_3 = 0$). In this model, the effect of size relates solely to within-district heterogeneity.
3. Size effects limited to setter shift ($\beta_3 = \delta = 0$).
4. Enrollment in both equations, separate coefficients in both equations. This model allows for all three effects, heterogeneity, scale economies, and

budget-maximization. The estimates of the unconstrained coefficients on ln N in the two equations correspond to an interpretation that size has no effect on setter shift ($\zeta = 0$). The estimated coefficients can also be used to solve for estimates corresponding to the other two "extreme" interpretations ($\delta = 0$ or $\beta_3 = 0$).

<u>Estimation Results for the Size Models</u>

Estimation of the various size equations gave results for parameters other than size that were all quite similar to those reported in Section 3. Consequently, we focus the discussion on the size effects.

1. When added to the basic model, demand effects of size appear to be inconsequential. With $\zeta = \delta = 0$, the log-likelihood improves only marginally over the model with no size effects. (Compare rows 1 and 2 in Table 4.) Using the standard likelihood ratio (asymptotic chi-square) test, size, as a pure demand effect is not significant at the .05 level.

2. In contrast, both the models of pure heterogeneity effects (compare Table 4, rows 1 and 3) or of solely setter effects (compare rows 1 and 4) represent highly significant additions to the model with no size effects. These two models are not nested; thus, no direct tests are made between them.

4. In Table 5, we interpret the unconstrained estimates from the second column of Table 2 to show the implied values for $\zeta$, $\delta$, and $\beta_3$ when one of the three parameters is assumed to be zero. As can be seen in the table, the estimated magnitude of $\delta$, the heterogeneity parameter, is relatively insensitive as to which of the other two parameters is dropped from the model. On the other hand, the magnitudes of the demand and setter coefficients vary widely under alternative assumptions. This result, coupled with the slightly lower log-likelihood for the pure within-district

heterogeneity model as against the pure setter model, leads us to conclude that the major effect of size is that of controlling for variations in the heterogeneity of voter preferences.[12]

5. Notwithstanding the importance of heterogeneity, a model that allows for all three size effects (setter, heterogeneity, and demand) is a highly significant improvement over any of the models that allow for only one effect. Size has a statistically significant if modest role in the spending equation. A district with enrollment of 10,000 would be expected to have per student expenditures 9.6% higher than a district with enrollment of 1000 students, *ceteris paribus*.

6. The total effect of size on the proportion approving the budget is very pronounced. If $\epsilon_u$ and $\epsilon_d$ were zero, for example, the percentage approving the budget would be only 43.7% in a district with 20,000 students but reach 50% in a district with 6873 students, 54.6% at the sample mean of 3179 students, and 65% in a district with 500 students.

## Results for Extensions of the Basic Specification

The parameter estimates we have presented so far might exaggerate the effects of policy variables, such as the matching rate whose effect depends on price and income elasticities, because the model was based on a simplified description of the district, characterized solely by price, income, and size. In particular, we omitted common socio-demographic descriptions of demand that other studies often include on an ad hoc basis.

The socio-demographic variables we examined included %BLACK, %Enrolled in PRIVATE Schools, %OLD (65 and over),[13] a dummy for whether the district was in an SMSA, and ln(KIDS), with KIDS defined as total enrollment 1975-76/total households 1970.

When added to the voting equation, these variables (and $M_2$ and $P_2$) made only negligible improvements in the log-likelihood. This is what we would expect from the stylized model of a budget-maximizing setter whose proposed budget "takes out" the effects of these variables.

In the spending equation, ln(KIDS) or being located in an SMSA had no impact. The other three variables had only marginal (albeit statistically significant) effects in some specifications. Column (3) of Table 2 shows a representative run with demographic variables in the expenditure equation.

Our qualitative results, including the key cross-equation linkage represented by the setter shift, were unaffected by adding these auxiliary variables. With the estimates of column (3) of Table 2, the size effect on spending (e.g., as noted in result 5 of the previous subsection) is slightly attenuated. The calculations in result 6 are virtually unchanged. As the bottom half of Table 5 shows, the alternative interpertations of size models are barely affected. Note, however, that the estimated magnitudes and the t-statistics of all non-disturbance coefficients fall moderately, mosy likely because of collinearity among our earlier variables and these auxiliary variables.

## 5. Impact of Aid Formulas and Referendum Rules

To get a sense of the importance of state aid and referendum rules for school spending, we examined a variety of possible policy options. For each district the probability of being on a particular segment of the budget constraint (or at the kink) is endogenous, as is desired location along a given segment. Consequently, to compute the effect of changes in exogenous variables, we must calculate these endogenous probabilities and desired locations for each observation, to get expected spending under the new regime.

State Aid Effects

With the closed end matching grant structure, expected spending is given by:[14]

$$E(S) = \Pr\{\epsilon_d \le S_k - h(P_1,M_1)\} \cdot [h(P_1,M_1) + E(\epsilon_d | \epsilon_d \le S_k - h(P_1,M_1)]$$
$$+ \Pr\{\epsilon_d > S_k - h(P_2,M_2)\} \cdot [h(P_2,M_2) + E(\epsilon_d | \epsilon_d > S_k - h(P_2,M_2)]$$
$$+ \Pr\{S_k - h(P_1,M_1) < \epsilon_d \le S_k - h(P_2,M_2)\} \cdot S_k \quad (18)$$

This expectation can be computed for each district by using the estimated error variances and parameters of $h(\cdot)$ together with the values of exogenous variables.

The model is useful for a wide variety of policy simulations. An obvious one is to examine the effect of changing $S_k$, the cutoff point for matching grants, while leaving the matching rates unchanged. Figure 3 shows variations in expected statewide spending per pupil as well as expected spending in the lowest-spending district, as the cutoff is varied. In this, as in all simulations, we used the estimates reported in column (3) of Table 2. For low values of $S_k$, the simulated spending changes are negligible, as all districts spend significantly above the cutoff, so that changes in the cutoff work only through their relatively weak effects on $M_2$ ($= Y + P_2mS_k$). As the cutoff is increased, some districts are likely to be spending close to the cutoff point (near the kink in the budget constraint). For such districts, increases in $S_k$ cause increases in expected spending far in excess of what would be predicted from simple income effect computations alone -- so the impact of changing the cutoff is magnified. This is the case for the minimum district as the cutoff is varied from about \$800 to about \$1450 per student. Past this point, the minimum district is almost surely spending less than the cutoff, so increases in $S_k$ will have little or no further effect on its spending. Of course, at higher $S_k$ higher-spending districts begin to exhibit the amplified response associated with spending near the cutoff, so the statewide average continues to rise.

As an alternative to the matching grant system, we simulated the effects of a block grant structure. We assumed that each district would receive a lump-sum grant equal to $mS_k$ per student. (This is what a district would receive under the matching grant system if it spent at or above the cutoff.) Such grants are usually nonfungible, in the sense that a district cannot spend less than the amount of the grant on education. This nonfungibility makes even the block grant budget constraint kinked, with the kink occurring at a level of spending equal to $mS_k$ per student. The expression for expected spending becomes:

$$E(S) = \Pr\{\epsilon_d \leq B_k - h(P_2,M_2)\} \cdot B_k$$
$$+ \Pr\{\epsilon_d > B_k - h(P_2,M_2)\} \cdot [h(P_2,M_2) + E(\epsilon_d | \epsilon_d > B_k - h(P_2,M_2)]$$

where $B_k \equiv \ln(mS_k)$.

Not surprisingly, for low values of $S_k$ the spending effect of this aid policy is identical to that of the matching grant system. At higher values of $S_k$, the block grant is less stimulative than the matching grant, since the subsidized tax-price effects are absent. (Note that the identity of the minimum district changes as $S_k$ increases above $1100.)

Finally, Figure 3 also displays what might be called the "revenue sharing" system. In this system, each district gets $mS_k$ per student -- exactly what it would get under the block grant formula. The difference is that now districts are allowed to rebate some (or all) of the grant dollars to individuals, so that the funds are fully fungible. The entire effect of the grant is an income effect. Except at low values of $S_k$, the effect on education spending is much smaller than under the other structures.[15,16]

Supramajorities in Referenda

For an expenditure proposal to pass in a referendum, a simple majority in favor is required. What if this requirement were raised to something in excess of a simple majority (as, for example, is the case for capital levy referenda in some states)? A definitive answer to this question is beyond the scope of this paper, since such a change may involve fundamental readjustments in behavior by both voters and setters. Our model does allow at least a preliminary look at how spending may be affected as the required majorities are changed. In the discussion that follows, we assume that the actual closed-end matching grant state aid structure remains in place.

The presumption behind our vote equation is that, given that a simple majority is required, setters in each district currently aim for a vote (logit) outcome given by $Z + \gamma\epsilon_d$. This corresponds to an environment in which passage requires that $V > 0$. Suppose instead that a supramajority were required, so that for passage a setter needed $V > V_0$, where $V_0$ is some positive constant. For example, a 60% majority would imply $V_0 = 0.405$, and a 2/3 majority corresponds to $V_0 = 0.693$. As a first approximation, suppose that this led setters in each district to aim for a vote greater by $V_0$ than they were currently getting. For districts currently obtaining large majorities or those failing with large majorities voting No, this implies a smaller change in the setter's vote goal than for setters aiming at close to 50%. [If the current aim is 80% Yes (logit = 1.386), moving to a 60% requirement raises the goal to 86% Yes. A district with a 25% Yes goal (logit = -1.099) would move to 33% Yes. A district currently aiming at 51% would move to 61%.]

Of course, we do not observe vote goals; we observe V and can compute Z using parameter estimates and exogenous variables. For a given value of $V_0$, we can calculate the expected spending in each district, conditional on the error in the vote equation being equal to $V + V_0 - Z$ instead of $V - Z$:

$$
\begin{aligned}
E(S) = {} & \\
& \Pr\{\epsilon_d \le S_k - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z\} \\
& \quad \cdot [h(P_1,M_1) + E(\epsilon_d | \epsilon_d \le S_k - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z)] \\
& + \Pr\{\epsilon_d > S_k - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z\} \\
& \quad \cdot [h(P_2,M_2) + E(\epsilon_d | \epsilon_d > S_k - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z)] \\
& + \Pr\{S_k - h(P_1,M_1) < \epsilon_d \le S_k - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z\} \cdot S_k
\end{aligned} \tag{19}
$$

Appendix 1 provides further details of the computation of (19).

Roughly speaking, changing the vote goal affects expected spending in two ways, both of them tending to decrease spending as $V_0$ is increased. First, since $\gamma < 0$, V is decreasing in the setter shift $\epsilon_d$. Thus the value of $\epsilon_d$ consistent with a supramajority is smaller, *ceteris paribus*, than when simple majority is the rule. This in turn implies that, along each segment of the budget constraint, desired spending $S^d$ is lower for a given district with supramajority than simple majority. Second, the probability of being on the segment with spending below the closed-end cutoff is increased.

Table 6 presents computations of expected spending as $V_0$ varies. (The first row of the table corresponds to the simple majority case.) The second column of the table gives the implied value of %Yes for a district whose current goal is 50%. [These values are given by $100/(1+e^{-V_0})$.] The third column shows the expected statewide spending per pupil. The last two columns give, respectively, the smallest and largest expected changes in per-pupil spending in the sample of 544 districts. Statewide per-pupil expenditures drop by about 5% as the passage requirement moves from 50% Yes to 60% Yes, and by about 9% if two-thirds majority is required. This apparently small overall effect is due, in part, to the fact that there are many relatively small districts where referenda currently pass with comfortable majorities. In such districts, the effect of instituting suparamajorities is expected to be small. (The smallest change -- less than 50 cents per pupil! -- corresponds to a district that was actually passing its budget with a whopping 94% majority.) Nonetheless, there are districts for which changing the vote requirement does have considerable spending impact. These are, as anticipated, districts that currently obtain close to 50% majorities; the district for which the simulated effect is greatest (shown in the last column of Table 6) is a large district whose actual vote was 45%. This district's proposed spending is reduced by 17% as the passage requirement moves from 50% to 60% Yes, and by 28% if 2/3 majority is required.

## 6. Conclusion

Our simulations and the preceding analysis recognized that legislated policies of state governments have two potential avenues for changing the incentives that influence spending by local governments. First, the state can modify the incentives posed by grants-in-aid. Until recently, empirical analysis has largely ignored the endogeneity problem posed by closed-end matching grants. Our estimates, based on a model that addresses endogeneity directly, lead to price and income elasticity estimates similar to those found in studies of states where lump-sum grants have been used. Thus, in contrast to some of the previous literature, we find that the closed-end grants are effective vehicles for both stimulating local spending and reducing the variance in spending across school districts. Second, the state can alter the political process that local districts use to make spending decisions. Although the political process is ignored (or encapsulated in median voters) in empirical analyses that contain only a spending equation, we have pointed to an important linkage between economic outcomes and political ones--higher than "normal" spending receives lower than normal support from the voters.

We want to stress that our results should be taken as suggestive, and should be interpreted in light of the necessarily rudimentary structure of the model. For tractability, we have relied on what is pretty much a "bare bones" specification. Except for the particular care taken with the role of state aid, our spending equation has at its heart a formulation that follows the by now almost canonical representation of Bergstrom and Goodman (1973).[17] In the voting equation, we have abstracted from a variety of considerations that may be important, especially those relating to endogenous voter turnout or strategic setter (and, perhaps, voter) behavior in light of the possibility for more than one referendum.[18] More refined modelling, together with more detailed data, will be needed for further progress on the linkage between spending and voting. Our results point to the importance of the linkage, and to the potentially broad range of responses to changes in aid or referendum policies.

## FOOTNOTES

1. Our sample consists of all non-city New York school districts that had 1975-76 enrollment (measured as weighted average daily attendance) over 484, a total of 549 districts. Of these we eliminated one district with data errors and four districts that passed referenda unanimously. The unanimous districts were deleted in order to keep the dependent variable in our voting equation (see Section 3) finite. Aid under the matching formula represented 42% of total basic (so-called "approved") operating expenses of the schools in our sample.

2. In settings of incomplete information, there is no compelling reason to focus on the median even if one argues for the competitive model in which school budgets would represent the convergent equilibrium platforms of two candidates in a majority rule election. Hinich (1977) showed that there would rarely be convergence to the median in a model where voting occurred probabilistically but the probabilities were specified in an *ad hoc* way. Ledyard (1984) obtains the same result with fully rational, non-probabilistic voting in the case of a game of incomplete information. Using preferences that are quadratic about the ideal point, he provides an example of convergence to the mean. As the informational assumptions in Ledyard's work are clearly more compelling than those in the original Hotelling (1929) - Bowen (1943) model, it is fair to say that the competitive theory of elections places few, if any, restrictions on the observations economists can make on public spending outcomes.

3. Welch (1981) has an extended discussion of the consistency problem in the context of closed-end matching grants. He uses a two-step estimation technique based on Heckman's (1976) approach to the censored sample problem. As Moffitt (1984) points out, the maximum likelihood estimation

we use is preferable, particularly with the error structure as we specify it. Our specification of the spending equation is formally equivalent to the model discussed by Moffitt.

4. The Oregon results are based on a sample from an institutional setting in which state aid was all in the form of block grants, so the elasticity estimates are not subject to the econometric problems raised with closed-end matching grants. For the other studies cited, except possibly Welch (1981), comparisons with our New York results are suspect, since matching grants may not have been adequately accounted for. A number of studies include a district wealth as well as an income variable, further confounding direct comparisons.

5. In our data, 95 of the 544 observations are within \$50 of $S_k$ (=\$1200). For discussion of the relationship between the relative magnitudes of $\sigma_d$ and $\sigma_r$ and clustering of data around budget constraint kink points, see Moffitt (1984).

6. Using a Pennsylvania sample, Welch (1981) reports similar results when comparing his two-stage estimates with OLS estimates. Megdal (1984) reports OLS price elasticities of -0.08 to -0.14 for New Jersey school districts operating in a setting of closed-end matching grants.

7. A district operating under a contingency budget is permitted to levy taxes to cover administrative and teachers' salaries, as well as other "ordinary" expenses. Indeed, a document reviewing the system of education finance in New York noted: "One of the reasons for allowing districts to operate under 'contingency' or austerity budgets has been to prevent the type of school closings which have occurred in other states following defeats on local tax and referenda votes. ... Administrative and legal interpretations of a 'contingent' expense made over the years permit a

district to increase its expenditures and/or total property tax levies while operating on a contingency basis after its budget has been disapproved by the voters of the district." New York State (1979, p. 127). Munley (1984) found that districts that adopted contingency budgets in 1975-76 but had passed referenda the prior year had, on average, slightly lower spending for 1975-76 than in the previous year, in real terms.

8. We looked at the relationship between amounts proposed to voters and the district's *ex post* budget. This relationship was no different for districts on contingency than for those not on contingency. Nor was there any significant systematic difference in actual expenditures between contingency and non-contingency districts when a variety of demand-related variables were controlled for. These preliminary data analyses simply treated being on a contingency budget as a dichotomous variable. Because we cannot measure the reversion for those districts that did not go on contingency, we cannot directly investigate whether variations in reversions were related to variations in spending.

9. Given the availability of multiple referenda to the agenda setter, failure might well be expected on the first try (Romer and Rosenthal, 1979).

10. The slight increase above 50% Yes on the third try is consistent with some theoretical results. Budget-maximizing setters confronted with uncertainty would aim for a higher expected Yes percentage on successive tries, with the largest increase coming on the last try (Romer and Rosenthal, 1979). In turn, it appears that most districts in New York regard three as the practical limit on attempts.

11. We do not have data (e.g., on income distributions) that capture within-district heterogeneity more directly. This makes it necessary to rely on size as a proxy.

12. Munley (1982) analyzes factors that influence the degree of dispersion in the distribution of voter ideal points for spending by New York State school districts. That analysis is based on a model with somewhat restrictive data requirements and assumptions about referendum voting patterns, conditions satisfied by only 54 of the 544 districts considered here. The results of that analysis suggest that the degree of dispersion in voter ideal points does increase with community size.

13. Denzau and Grier (1984) performed a vast number of specification searches using 1970-71 New York school expenditure data. Their results suggest that of the additional conditioning variables most frequently used in spending equations, % of population nonwhite, % enrolled in private school, and % of population below the poverty line have non-negligible effects on estimated spending (in a simple linear model).

14. With this formulation, we have not explicitly conditioned spending on a particular vote outcome. We have taken the *expected* vote outcome in each district to be equal to the vote outcome predicted by the vote equation; i.e., $E(V) = Z$. By contrast, in (19) below, expected spending is conditioned on $\gamma\epsilon_d + \epsilon_u$.

15. The effect of changes in state aid on the highest-spending district is negligible throughout the range of all our simulations. Its expected (and actual) spending is around $3350 per student, and changes only marginally, through the effects of aid on $M_2$.

16. Grant impacts would be greater if there were "flypaper effects", so that, even under revenue sharing grants, the response of spending from grants

exceeded the response due to income effects alone. We tested for the presence of flypaper by allowing for separate coefficients on aid in the spending equation. Our results generally rejected the flypaper hypothesis, though in some of these specifications, our highly nonlinear model ran into convergence difficulties.

17. Oates (1986) has an illuminating discussion of some controversies related to empirical work on local public goods spending.

18. Romer *et al.* (1984) estimate a model in which spending and voting linkages are explored in the context of a sequence of referenda. Their spending model does not deal with the complexities introduced by the closed-end matching grant structure. Inman (1978) presents a specification of a spending equation that attempts to capture voter turnout effects.

## REFERENCES


Barr, James, and Otto Davis (1966) "An Elementary Political and Economic Theory of Expenditures of Local Governments," *Southern Economic Journal* 33: 149-165.

Bergstrom, Theodore C., and Robert P. Goodman (1973) "Private Demands for Public Goods," *American Economic Review* 63: 280-296.

Black, David E., Kenneth A. Lewis, and Charles R. Link (1979) "Wealth Neutrality and the Demand for Education," *National Tax Journal* 32: 157 - 164.

Bowen, Howard R. (1943) "The Interpretation of Voting in the Allocation of Economic Resources," *Quarterly Journal of Economics* 58: 27-48.

Denzau, Arthur, and Kevin Grier (1984) "Determinants of Local School Spending: Some Consistent Estimates," *Public Choice* 44: 375-383.

Filimon, Radu, Thomas Romer, and Howard Rosenthal (1982) "Asymmetric Information and Agenda Control: The Bases of Monopoly Power in Public Spending," *Journal of Public Economics* 17: 51-70.

Heckman, James J. (1976) "The Common Structure of Statistical Models of Truncation and Limited Dependent Variables and a Simple Estimator for Such Models," *Annals of Economic and Social Measurement* 5: 475-492.

Hinich, Melvin (1977) "Equilibrium in Spatial Voting: The Median Voter Result is an Artifact," *Journal of Economic Theory* 16: 208-219.

Hotelling, Harold (1929) "Stability in Competition," *Economic Journal* 39: 41-57.

Inman, Robert P. (1978) "Testing Political Economy's 'As If' Proposition: Is the Median Income Voter Really Decisive?" *Public Choice* 33: 45-65.

Ledyard, John (1984) "The Pure Theory of Large Two-Candidate Elections," *Public Choice* 44: 7-41.

Maddala, G. S. (1977) *Econometrics*, New York: McGraw-Hill

Megdal, Sharon B. (1984) "A Model of Local Demand for Education," *Journal of Urban Economics* 16: 13-30.

Moffitt, Robert A. (1984) "The Effects of Grants-in-Aid on State and Local Expenditure: The Case of AFDC," *Journal of Public Economics* 23: 279-305.

Moffitt, Robert A. (1986), "The Econometrics of Piecewise-Linear Budget Constraints," *Journal of Business and Economic Statistics* 4: 317-327.

Munley, Vincent (1982) "An Alternate Test of the Tiebout Model," *Public Choice* 38: 211-218.

Munley, Vincent (1984). "Has the Median Voter Found a Ballot Box that He Can Control?" *Economic Inquiry,* 22: 323-336.

New York State (1979). Division of the Budget, Education Study Unit, "Current Components of New York State's Educational Finance System: A Review" Albany, N.Y.

Niskanen, William (1971) *Bureaucracy and Representative Government*, Chicago: Aldine.

Oates, Wallace E. (1986) "The Estimation of Demand Functions for Local Public Goods: Issues in Specification and Interpretation," University of Maryland working paper.

Romer, Thomas, and Howard Rosenthal (1979) "Bureaucrats vs. Voters: On the Political Economy of Resource Allocation by Direct Democracy," *Quarterly Journal of Economics* 93: 563-587.

Romer, Thomas, and Howard Rosenthal (1982a) "Median Voters or Budget Maximizers: Evidence from School Expenditure Referenda," *Economic Inquiry* 20:556-78.

Romer, Thomas, and Howard Rosenthal (1982b) "An Exploration in the Politics and Economics of Local Public Services," *Zeitschrift fur Nationalokonomie/Journal of Economics*, Supplement 2.

Romer, Thomas, Howard Rosenthal, and Krishna Ladha (1984), "If at First you Don't Succeed: Budgeting by a Sequence of Referenda," in H. Hanusch (ed.), *Public Finance and the Quest for Efficiency*, Detroit: Wayne State University Press.

Welch, W. P. (1981) "Estimating School District Expenditure Functions under Conditions of Closed-End Matching Aid," *Journal of Urban Economics* 10: 61-75.

Table 1.
**Expenditure Estimates**

(Dependent Variable = ln S)

| Parameter | (1) Maximum Likelihood | (2) OLS, All Obs. | (3) OLS, omit 1150<S<1250 |
|---|---|---|---|
| Constant $\beta_0$ | -2.864 (0.370) | -0.061 (0.323) | -0.230 (0.348) |
| Ln price $\beta_1$ | -0.271 (0.012) | -0.034 (0.012) | -0.074 (0.015) |
| Ln income $\beta_2$ | 1.035 (0.037) | 0.761 (0.033) | 0.778 (0.035) |
| "Setter shift" $\sigma_d$ | 0.143 (0.0067) | | |
| "Random" error $\sigma_r$ | 0.064 (0.0057) | | |
| SEE | | 0.161 | 0.159 |
| -2 ln Likelihood | -732.08 | | |
| $\overline{R}^2$ | | 0.537 | 0.546 |
| No. of obs. | 544 | 544 | 449 |

Estimated standard errors in parentheses

S = Approved operating expenditure per student, 1975/76

"Income" is adjusted for formula aid for districts with S > 1200

Table 2.
**Estimated Coefficients of the Two-Equation Model**

| | (1) | (2) | (3) |
|---|---|---|---|
| <u>Spending</u> | | | |
| Constant $\beta_0$ | -2.334 (0.346) | -2.314 (0.351) | -1.291 (0.403) |
| Ln P $\beta_1$ | -0.266 (0.012) | -0.274 (0.012) | -0.240 (0.012) |
| Ln M $\beta_2$ | 0.980 (0.035) | 0.946 (0.038) | 0.849 (0.043) |
| Ln N in spending $\beta_3 + \zeta$ | | 0.040 (0.011) | 0.033 (0.011) |
| % Black $\beta_4$ | | | 0.0066 (0.0011) |
| % Private $\beta_5$ | | | 0.0025 (0.0010) |
| % Old $\beta_6$ | | | -0.0029 (0.0017) |
| <u>Vote</u> | | | |
| Vote Intercept $\alpha$ | 0.138 (0.026) | 2.094 (0.195) | 2.019 (0.197) |
| Setter shift $\gamma$ | -1.385 (0.126) | -1.169 (0.131) | -1.143 (0.144) |
| Ln N in vote $\delta + \gamma\zeta$ | | -0.237 (0.023) | -0.228 (0.023) |
| <u>Disturbances</u> | | | |
| Setter shift $\sigma_d$ | 0.140 (0.0070) | 0.141 (0.0067) | 0.131 (0.0072) |
| Expend. Error $\sigma_r$ | 0.069 (0.0067) | 0.067 (0.0062) | 0.067 (0.0064) |
| Vote Error $\sigma_u^*$ | 16.949 (0.540) | 16.189 (0.472) | 16.340 (0.483) |
| -2 ln Likelihood | 411.84 | 339.77 | 304.97 |

<u>Note:</u> These estimates employ a heteroskedasticity correction for the Vote Error. For a given observation, $\sigma_u = \sigma_u^*/\sqrt{W}$ , where W is the total number of votes cast in the referendum.

Estimated standard errors in parentheses.

Table 3.
Summary of Referendum Outcomes

| | District Size | | |
|---|---|---|---|
| | Under 3000 | 3000-6000 | Over 6000 |
| **First Referenda** | | | |
| Number | 362 | 111 | 71 |
| Mean % Yes | 63.9 | 55.0 | 49.9 |
| Std. Dev. of % Yes | 15.8 | 11.1 | 10.3 |
| Number Failing | 75 | 34 | 33 |
| To Contingency | 14 | 5 | 3 |
| **Second Referenda** | | | |
| Number | 61 | 29 | 30 |
| Mean % Yes | 51.4 | 50.8 | 48.1 |
| Std. Dev. of % Yes | 9.6 | 8.2 | 8.0 |
| Number Failing | 25 | 14 | 16 |
| To Contingency | 12 | 9 | 10 |
| **Third Referenda** | | | |
| Number | 13 | 5 | 6 |
| Mean % Yes | 54.8 | 52.7 | 55.3 |
| Std. Dev. of % Yes | 9.6 | 8.5 | 11.0 |
| Number Failing | 5 | 1 | 2 |
| To Contingency | 5 | 1 | 1 |

Table 4.
**Likelihoods of Estimated Size Models**

| | -2 ln Likelihood |
|---|---|
| ln N omitted<br>$\beta_3 = \zeta = \delta = 0$ | 411.84 |
| ln N in Exp. only<br>$\zeta = \delta = 0$ | 410.29 |
| ln N in Vote only<br>$\beta_3 = \zeta = 0$ | 351.39 |
| ln N as setter effect<br>$\beta_3 = \delta = 0$ | 361.80 |
| ln N in Exp. and Vote unconstrained | 339.77 |

Table 5.
**Alternative Interpretations of the Models**

| | Estimated Coefficient | | |
|---|---|---|---|
| | $\zeta$ | $\delta$ | $\beta_3$ |
| Column (2), Table 2 | | | |
| No Setter Size Effects | 0 | -0.237 | 0.040 |
| No Heterogeneity Effects | 0.203 | 0 | -0.163 |
| No Demand Effects | 0.040 | -0.190 | 0 |
| Column (3), Table 2 | | | |
| No Setter Size Effects | 0 | -0.228 | 0.033 |
| No Heterogeneity Effects | 0.199 | 0 | -0.166 |
| No Demand Effects | 0.033 | -0.190 | 0 |

Table 6
Spending Effects of Changing Vote Requirements

| $V_0$ | $100/(1+e^{-V_0})$ | Statewide Expected Spending per Pupil | Max Δ | Min Δ |
|---|---|---|---|---|
| 0.0 | 50.0 | 1659.68 | —— | —— |
| 0.1 | 52.5 | 1638.36 | -0.04 | -137.34 |
| 0.2 | 55.0 | 1617.56 | -0.08 | -268.41 |
| 0.3 | 57.4 | 1597.27 | -0.11 | -393.48 |
| 0.4 | 59.9 | 1577.50 | -0.15 | -512.84 |
| 0.5 | 62.2 | 1558.25 | -0.19 | -626.74 |
| 0.6 | 64.6 | 1539.54 | -0.22 | -735.43 |
| 0.7 | 66.8 | 1521.37 | -0.26 | -839.16 |
| 0.8 | 69.0 | 1503.75 | -0.30 | -938.15 |
| 0.9 | 71.1 | 1486.69 | -0.33 | -1032.61 |
| 1.0 | 73.1 | 1470.18 | -0.37 | -1122.76 |

Note: Δ is the change in expected spending per pupil, relative to $V_0 = 0.0$

Simulations are based on estimates reported in column (3) of Table 2.

## APPENDIX 1

### Derivation of Likelihood Functions

A. Expenditure Equation

For a given observation, (10) provides the probability density of a particular value of $S$. We assume that $\epsilon_d$ and $\epsilon_r$ are distributed independently, with $\epsilon_d \sim N(0,\sigma_d)$ and $\epsilon_r \sim N(0,\sigma_r)$.

Let

$$U_A = \begin{bmatrix} \epsilon_d + \epsilon_r \\ \epsilon_d \end{bmatrix} \quad \text{and} \quad \Omega_A = \begin{bmatrix} \sigma_d^2 + \sigma_r^2 & \sigma_d^2 \\ \sigma_d^2 & \sigma_d^2 \end{bmatrix}$$

Then $U_A \sim N_2(0,\Omega_A)$. Using results on the relationships between marginals and conditionals of multivariate normal variables (see, for example, Maddala (1977), pp. 454-455) gives the following for the first of the three terms in (10):

$$\Pr\{\epsilon_d + \epsilon_r = S - f(P_1,M_1);\ \epsilon_d \leq S_k - f(P_1,M_1)\} = \frac{\phi(s_1)\Phi(t_1)}{\sigma_w} \qquad \text{(A1)}$$

where $\phi(\cdot)$ is the standard normal pdf, $\Phi(\cdot)$ is the standard normal cdf, and

$$\sigma_w^2 = \sigma_d^2 + \sigma_r^2$$

$$s_1 = [S - f(P_1,M_1)]/\sigma_w$$

$$t_1 = (\sigma_w u_1 - \sigma_d s_1)/\sigma_r$$

$$u_1 = [S_k - f(P_1,M_1)]/\sigma_d$$

The third term in (10) has a form similar to (A1). The middle term in (10) is simply the product of the independent probabilities associated with $\epsilon_d$ and $\epsilon_r$. The full expression for the likelihood of an observation is:

$$\ell = \frac{\phi(s_1)\ \Phi(t_1)}{\sigma_w} + \frac{\phi[(S - S_k)/\sigma_r]\ [\Phi(u_1) - \Phi(u_2)]}{\sigma_r} + \frac{\phi(s_2)\ [1 - \Phi(t_2)]}{\sigma_w}$$

where, for i = 1,2,

$$s_i = [S - f(P_i, M_i)]/\sigma_w$$

$$t_i = (\sigma_w u_i - \sigma_d s_i)/\sigma_r$$

$$u_i = [S_k - f(P_i, M_i)]/\sigma_d$$

We assume that errors across observations are independent. The log likelihood for the entire sample is then the sum of ln $\ell$ over all the observations.

B. The Two-Equation System

We now have the additional disturbance term $\epsilon_u$ from the vote equation. We assume that the distributions of $\epsilon_d$ and $\epsilon_r$ are as before, that $\epsilon_u \sim N(0,\sigma_u)$, and that $\epsilon_d$, $\epsilon_r$, and $\epsilon_u$ are independent. Let

$$U_B = \begin{bmatrix} \epsilon_d + \epsilon_r \\ \gamma\epsilon_d + \epsilon_u \\ \epsilon_d \end{bmatrix} \quad \text{and} \quad \Omega_B = \begin{bmatrix} \sigma_w^2 & \gamma\sigma_d^2 & \sigma_d^2 \\ \gamma\sigma_d^2 & \sigma_v^2 & \gamma\sigma_d^2 \\ \sigma_d^2 & \gamma\sigma_d^2 & \sigma_d^2 \end{bmatrix}$$

where $\sigma_w^2 = \sigma_d^2 + \sigma_r^2$ and $\sigma_v^2 = \gamma^2\sigma_d^2 + \sigma_u^2$. Then $U_B \sim N_3(0,\Omega_B)$. The derivation of the likelihood corresponding to (15) is similar to the derivation in the single-equation case -- complicated, of course, by the presence of the additional disturbance term.

The likelihood corresponding to (15) is:

$$\ell = \frac{e^{-q_1/2}\Phi(z_1)}{2\pi\sigma_s} + \frac{\phi\left(\frac{S - S_k}{\sigma_r}\right)\phi\left(\frac{V - Z}{\sigma_v}\right)[\Phi(y_2) - \Phi(y_1)]}{\sigma_r\sigma_v}$$

$$+ \frac{e^{-q_2/2}[1 - \Phi(z_2)]}{2\pi\sigma_s} \qquad \text{(A2)}$$

where, for i = 1,2

$$q_i = (1/\sigma_s)^2 \left[ \sigma_v^2[S - h(P_i, M_i)]^2 - 2\gamma\sigma_d^2[S - h(P_i, M_i)](V - Z) + \sigma_w^2(V - Z)^2 \right]$$

$$z_i = \frac{\sigma_s^2[S_k - h(P_i, M_i)] - \sigma_d^2\sigma_u^2[S - h(P_i, M_i)] - \gamma\sigma_d^2\sigma_r^2(V - Z)}{\sigma_r\sigma_d\sigma_u\sigma_s}$$

$$y_i = \frac{\sigma_v^2[S_k - h(P_i, M_i)] - \gamma\sigma_d^2(V - Z)}{\sigma_d\sigma_u\sigma_v}$$

and

$$\sigma_s^2 = \sigma_r^2\sigma_v^2 + \sigma_d^2\sigma_u^2 .$$

The first term in (A2) is derived as follows. Let

$$U_1 = \begin{bmatrix} \epsilon_d + \epsilon_r \\ \gamma\epsilon_d + \epsilon_u \end{bmatrix}, \qquad \Omega_1 = \begin{bmatrix} \sigma_w^2 & \gamma\sigma_d^2 \\ \gamma\sigma_d^2 & \sigma_v^2 \end{bmatrix},$$

and $$\Omega_2 = \begin{bmatrix} \sigma_d^2 \\ \gamma\sigma_d^2 \end{bmatrix} .$$

Note that $U_1 \sim N_2(0,\Omega_1)$ and the distribution of $\epsilon_d | U_1$ is $N(\Omega_2'\Omega_1^{-1}U_1, \sigma_d^2 - \Omega_2'\Omega_1^{-1}\Omega_2)$. [Cf. Maddala (1977, p. 455).] The first term of (A2) follows from this and the relationship $Pr\{U_1, \epsilon_d\} = Pr\{U_1\} \cdot Pr\{\epsilon_d | U_1\}$. Other terms in (A2) can be derived similarly.

Again the log likelihood for the sample is the sum of ln $\ell$ over all the observations.

A technical matter should be noted. For $S^d$ to have the properties of a demand function for arbitrary values of $\epsilon_d$, the paramters of h(·) must be such that $h(P_1,M_1) \geq h(P_2,M_2)$. In estimation, however, it may be the case that for some parameters, one or more of the probabilities given in (15) is computed to be negative. We dealt with this possibility by recognizing that if $h(P_1,M_1) \leq h(P_2,M_2)$, then an observation has zero probability of being at the kink, and the likelihood for such an observation is computed according to the following expression:

$$\Pr\{S,V\} =$$

$$\Pr\{\epsilon_d + \epsilon_r = S - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V\text{-}Z,\ \epsilon_d \leq S_k - h(P_2,M_2)\}$$

$$+ \Pr\{\epsilon_d + \epsilon_r = S - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V\text{-}Z,\ \epsilon_d \geq S_k - h(P_2,M_2)\}$$

(A corresponding expression applies to the one-equation model.) At convergence for the results we report in the Tables, the condition $h(P_1,M_1) \geq h(P_2,M_2)$ is satisfied for all observations.

C. Vote Simulations

To evaluate (19), note that the conditional distribution of $\epsilon_d$ given $\gamma\epsilon_d + \epsilon_u$ is

$$N\left[\frac{\gamma\sigma_d^2}{\sigma_v^2}(V+V_0-Z),\ \frac{\sigma_d^2\sigma_u^2}{\sigma_v^2}\right]$$

and

$$\Pr\{\epsilon_d \leq S_k - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V+V_0-Z\} = \Phi(r_1),$$

where $\Phi$ is the N(0,1) cdf

and $$r_1 = \frac{\sigma_v^2[S_k - h(P_1,M_1)] - \gamma\sigma_d^2(V+V_0-Z)}{\sigma_d\sigma_u\sigma_v}$$

Similarly, $\Pr\{\epsilon_d > S_k - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V+V_0-Z\} = 1 - \Phi(r_2)$

where $r_2 = [\sigma_v^2[S_k - h(P_2,M_2)] - \gamma\sigma_d^2(V+V_0-Z)]/\sigma_d\sigma_u\sigma_v$ .

$$E\{\epsilon_d | \epsilon_d < S_k - h(P_1,M_1),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z\}$$

$= \mu - \sigma\phi(r_1)/\Phi(r_1)$, where $\phi$ is the N(0,1) pdf,

$\mu = \gamma\sigma_d^2(V+V_0-Z)/\sigma_v^2$ and $\sigma = \sigma_d\sigma_u/\sigma_v$.

Similarly, $E\{\epsilon_d | \epsilon_d \geq S_k - h(P_2,M_2),\ \gamma\epsilon_d + \epsilon_u = V + V_0 - Z\}$

$= \mu + \sigma\phi(r_2)/[1 - \Phi(r_2)]$ .

So $E(S) = \Phi(r_1)[h(P_1,M_1) + \mu - \sigma\phi(r_1)/\Phi(r_1)]$

$+ S_k[\Phi(r_2) - \Phi(r_1)]$

$+ [1 - \Phi(r_2)][h(P_2,M_2) + \mu + \sigma\phi(r_2)/[1 - \Phi(r_2)]]$

## APPENDIX 2

### Data

S : Approved operating expenditures 1975-76 school year, divided by weighted average daily attendance, 1975-76.

N : Weighted average daily attendance, 1975-76.

K : Full valuation of real property, 1975-76.

The above data, as well as vote results, were obtained from the New York State Department of Education.

To derive "effective income", M, and tax price P, we proceeded as follows. For district i (i = 1,... ,544), located in county k (k = 1,...,57),

$$Y_i = Y_i^m \ (I_k^{75}/I_k^{70})$$

$$H_i = H_i^m \ (I_k^{75}/I_k^{70})$$

where $Y_i^m$ is district median income from the 1970 Census, and $H_i^m$ is district median house value from the 1970 Census. $I_k^{75}$ and $I_k^{70}$ are, respectively, New York State estimates of 1975 and 1970 per capita income in county k, as published in *New York State Statistical Yearbook, 1979-80*.

District i's matching rate $m_i$ is determined by a formula that depends on the district's full valuation of real property, as measured in the year preceding the school year in which referenda are held. So for the 1975-76 school year, the 1973-74 full valuations are used (since the first referenda for 1975-76 budgets are held during the 1974-75 school year. In essence, the matching rate formula has the following form:

$$m_i = \begin{cases} (1200 - .015\bar{K}_i)/1200 & \text{if } \bar{K}_i \leq 52785 \\ (461 - .001\bar{K}_i)/1200 & \text{if } 52785 < \bar{K}_i \leq 101000 \\ 0.3 & \text{if } \bar{K}_i > 101000 \end{cases}$$

where $\bar{K}_i$ is 1973-74 full valuation of real property divided by 1974-75 enrollment.*

Tax prices $P_1$ and $P_2$ were calculated for each district using the above definitions of $H_i$ and $m_i$. Note that in the definition of tax price the current (i.e., 1975-76) value of $K_i$ is used.

**Sample Descriptive Statistics**

| Variable | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|
| S | 1541.70 | 412.55 | 1023.94 | 3347.27 |
| $P_1$ | 0.223 | 0.116 | 0.0057 | 0.712 |
| $P_2$ | 0.634 | 0.238 | 0.020 | 1.441 |
| $M_1$ | 15667.0 | 4329.7 | 9321.9 | 45064.7 |
| $M_2$ | 16159.4 | 4312.2 | 9779.6 | 45440.6 |
| N | 3179.3 | 3178.2 | 484 | 21030 |
| %BLACK | 2.01 | 4.98 | 0.0 | 72.31 |
| %PRIVATE | 8.96 | 8.02 | 0.0 | 39.00 |
| %OLD | 18.92 | 4.10 | 6.82 | 36.43 |
| V | 0.489 | 0.781 | -1.297 | 4.205 |
| YES/(YES+NO)* | 0.603 | 0.153 | 0.215 | 0.985 |

*This equals $1/(1+e^{-V})$ for each observation.

* The actual aid formula is subject to a variety of adjustments, including "hold-harmless" provisions that prevent districts from losing aid if their property value per student changes rapidly, and "maxing out" provisions to prevent large increases in aid. These adjustments were relatively unimportant for 1975-76. By 1978, however, so many districts were in the "hold-harmless" or "maxing out" categories that there were strong pressures to change the formula. See New York State (1979).

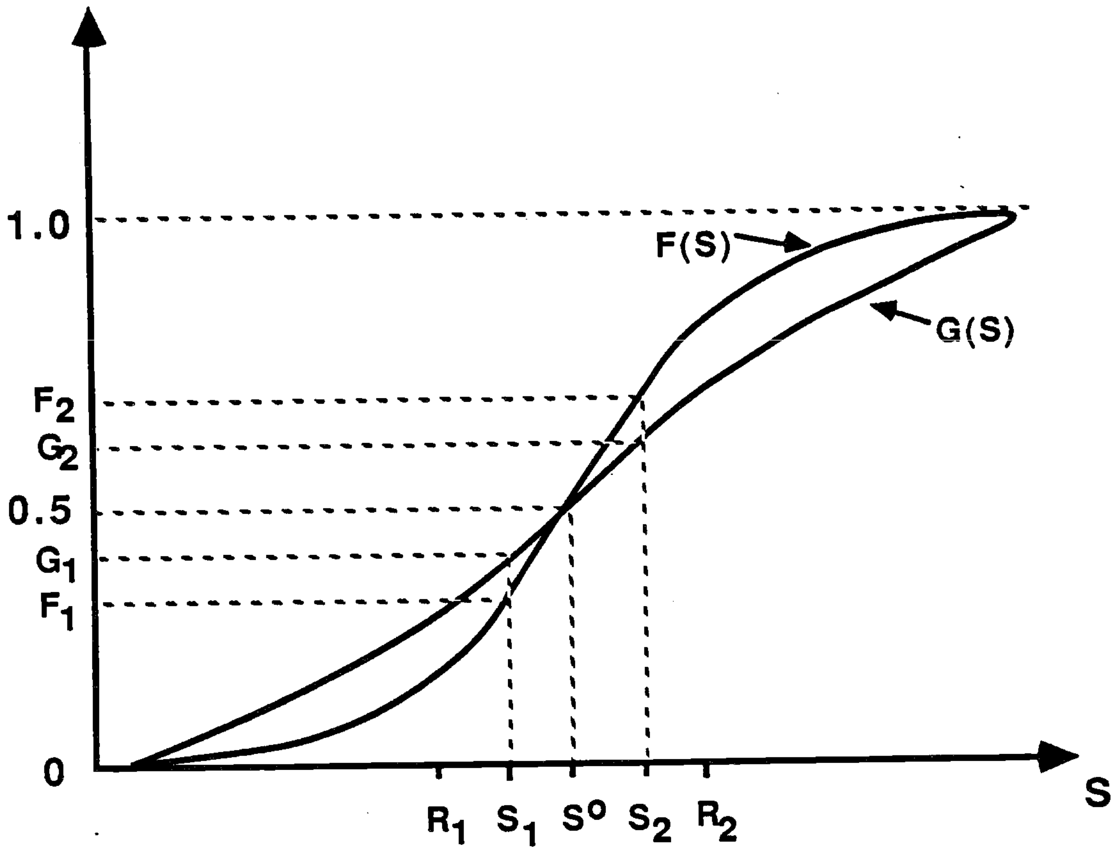



FIGURE 1

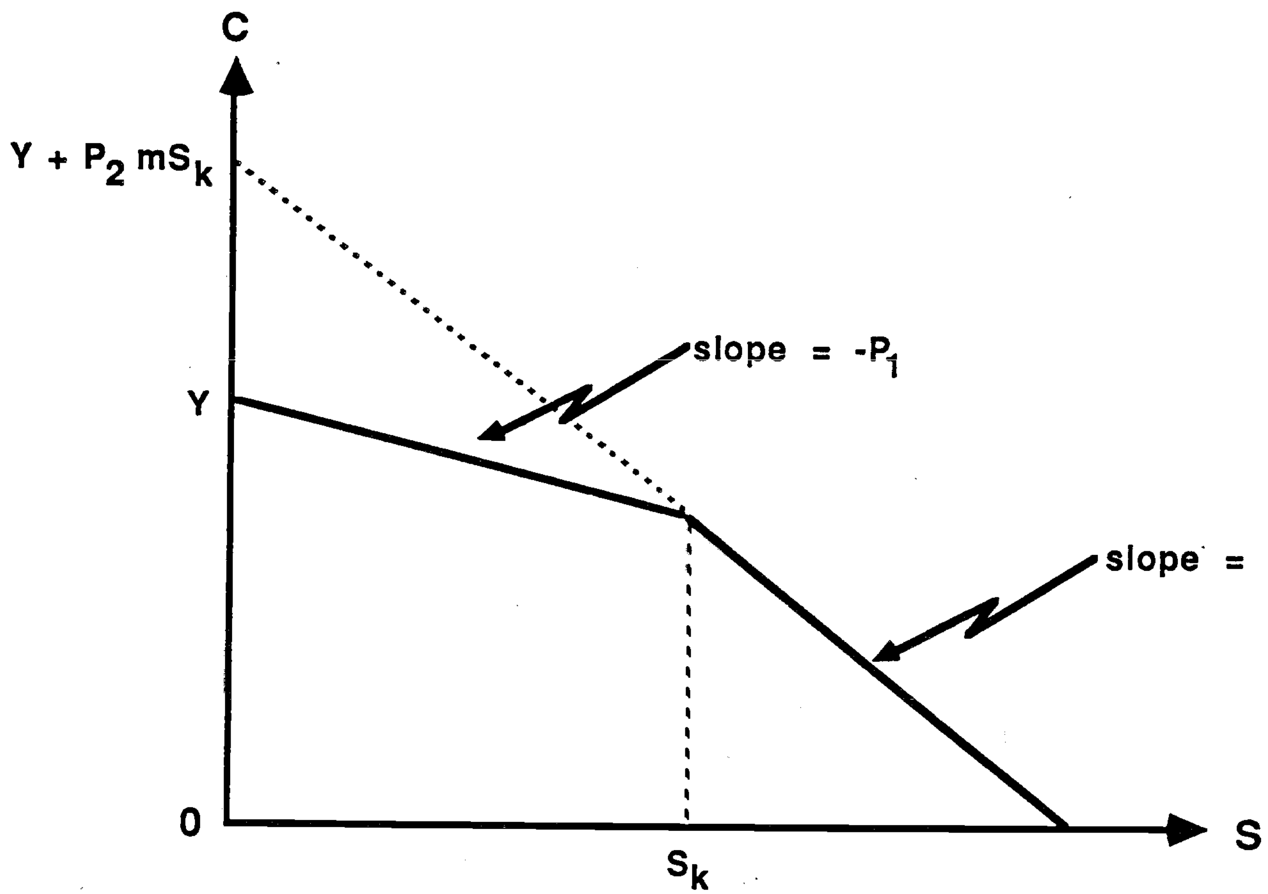
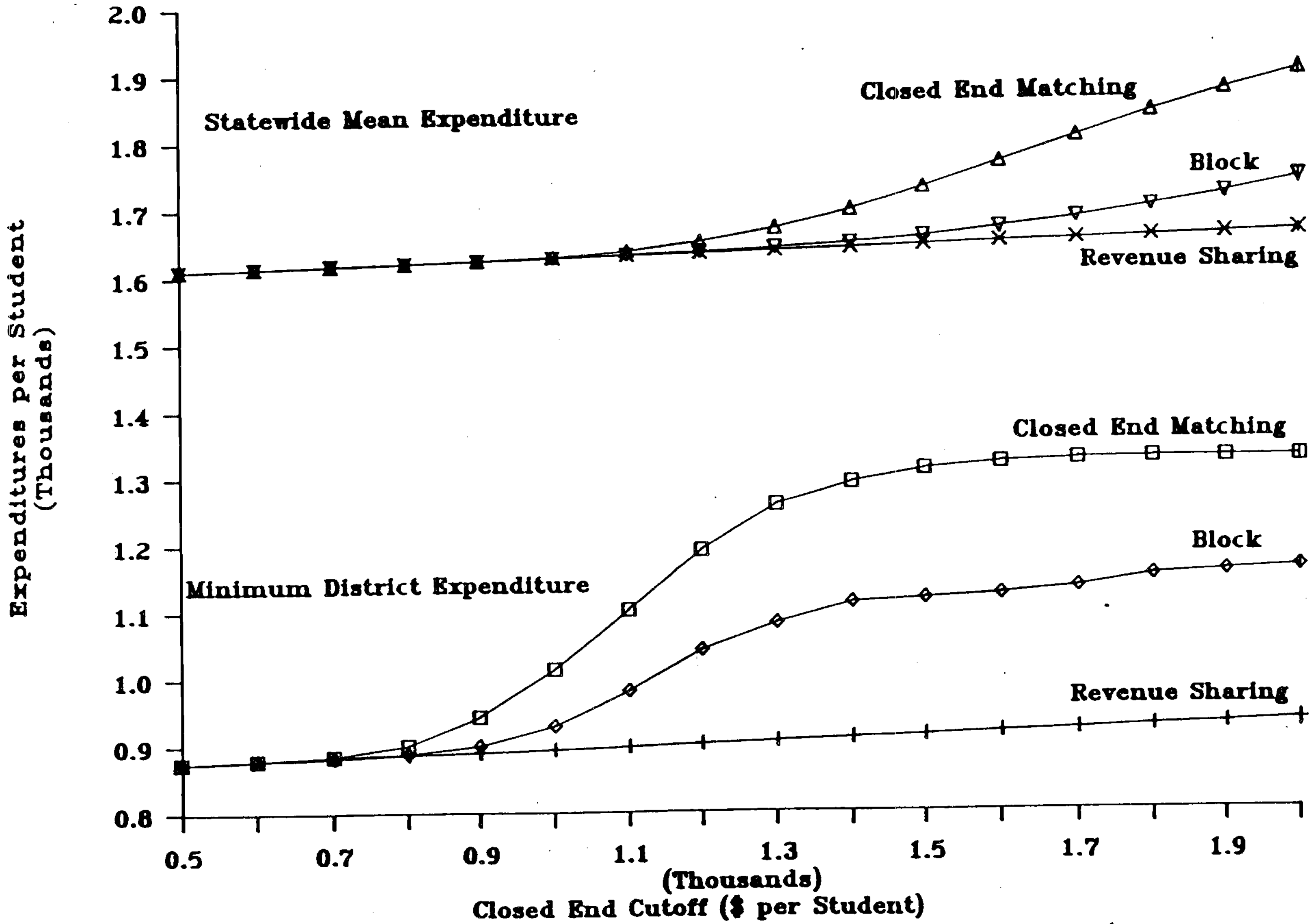



FIGURE 2

FIGURE 3

# Simulated State Aid Programs

**For Alternative Grant-in-Aid Systems**




Note: Simulations are based on the estimates reported in column (3) of Table 2.