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THE ECONOMIC CONSEQUENCES OF NOISE TRADERS

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The Economic Consequences of Noise Traders

ABSTRACT

The claim that financial markets are efficient is backed by an implicit argument that misinformed "noise traders" can have little influence on asset prices in equilibrium. If noise traders' beliefs are sufficiently different from those of rational agents to significantly affect prices, then noise traders will buy high and sell low. They will then lose money relative to rational investors and eventually be eliminated from the market.

We present a simple overlapping-generations model of the stock market in which noise traders with erroneous and stochastic beliefs (a) significantly affect prices and (b) earn higher returns than do rational investors. Noise traders earn high returns because they bear a large amount of the market risk which the presence of noise traders creates in the assets that they hold: their presence raises expected returns because sophisticated investors dislike bearing the risk that noise traders may be irrationally pessimistic and push asset prices down in the future.

The model we present has many properties that correspond to the "Keynesian" view of financial markets. (i) Stock prices are more volatile than can be justified on the basis of news about underlying fundamentals. (ii) A rational investor concerned about the short run may be better off guessing the guesses of others than choosing an appropriate β portfolio. (iii) Asset prices diverge frequently but not permanently from average values, giving rise to patterns of mean reversion in stock and bond prices similar to those found directly by Fama and French (1987) for the stock market and to the failures of the expectations hypothesis of the term structure. (iv) Since investors in assets bear not only fundamental but also noise trader risk, the average prices of assets will be below fundamental values; one striking example of substantial divergence between market and fundamental values is the persistent discount on closed-end mutual funds, and a second example is Mehra and Prescott's (1986) finding that American equities sell for much less than the consumption capital asset pricing model would predict. (v) The more the market is dominated by short-term traders as opposed to long-term investors, the poorer is its performance as a social capital allocation mechanism. (vi) Dividend policy and capital structure can matter for the value of the firm even abstracting from tax considerations. And (vii) making assets illiquid and thus no longer subject to the whims of the market -- as is done when a firm goes private -- may enhance their value.

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"People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on the average sell when the [asset]... is low in price and buy when it is high. It does not, of course, follow that speculation is not destabilizing; professional speculators might... make money while a changing body of amateurs regularly lost larger sums. But, while this may happen... the presumption is rather the opposite."

--Milton Friedman (1953), p. 175.

"If the reader interjects that there must surely be large profits to be gained... in the long run by a skilled individual who... purchase[s] investments on the best genuine long-term expectation he can frame, he must be answered... that there are such serious-minded individuals and that it makes a vast difference to an investment market whether or not they predominate... But we must also add that there are several factors which jeopardise the predominance of such individuals in modern investment markets. Investment based on genuine long-term expectation is so difficult... as to be scarcely practicable. He who attempts it must surely... run greater risks than he who tries to guess better than the crowd how the crowd will behave."

-- John Maynard Keynes (1936), p. 157.

"If you're so rich, why aren't you smart?"

-- Anonymous

There is considerable evidence that many investors do not follow economists' advice that the market portfolio should be bought and held. Individual investors typically fail to diversify, holding instead a single stock or a small number of stocks (Lewellen, Lease, and Schlarbaum (1974)). They often pick stocks on advice of the likes of Joe Granville, or of Louis Rukeyser on Wall Street Week. When investors do diversify, they entrust their money to stock-picking mutual funds which charge them high fees while failing to beat the market (Jensen (1968)), and turn their portfolios over as often as twice a year. Institutional investors are more prone to churn portfolios than individual investors, and are notoriously reluctant to pursue a passive investment strategy.

Many prominent market participants see asset markets as little more than casinos. Wojnilower (1980) finds the fact "that so many major financial institutions... try to outperform the market on a monthly or even weekly basis... particularly indicative of a gambling mentality...." Keynes (1936) saw the stock market as a beauty contest in which the judges selected the winners by trying to match as closely as possible the judgments of others. And Graham and Dodd (1934) dwelled on the persistence of deviations of market prices from their fundamental values and argued that the prudent investor should purchase assets that possessed a substantial "margin of safety," that is, were so undervalued that one could achieve more than satisfactory returns either through dividends or through liquidation even if the market valuation were to decline further.

Despite the concern of many participants that irrational noise trading makes financial markets function poorly in spreading risk and allocating capital, financial economists, with the notable exceptions of Shiller (1984), Kyle (1985), Campbell and Kyle (1987), and especially Black (1986), have been reluctant to assign any role to noise traders in studying the behavior of asset prices.[1] Their skepticism stems from the idea that even if many investors do trade irrationally, sophisticated arbitrageurs would trade against them and drive prices close to fundamental values (Fama (1965)). And in the course of such trading, those whose judgments of asset values were sufficiently mistaken to affect prices would lose money to rational, sophisticated investors and so would be driven out of the market (Friedman (1953)).[2]

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[1]See Merton (1985), Miller (1986), and Kleidon (1986).

[2]Hart and Kreps (1986) have challenged Friedman's analysis in a fully rational model. Several other studies have explored the effects of irrational behavior. Haltiwanger and Waldman (1985) study the effects of irrational behavior on prices in the presence of externalities, and Thaler and Russell (1985) examine the same question in a market where

This paper demonstrates that even if noise traders have substantial effects on asset prices economic selection may still work in their favor. Optimistic noise traders might well invest a large share of their wealth in risky assets, and as long as risk taking is rewarded they will earn a higher expected return than sophisticated investors. The wedge between utility and wealth maximization is large enough to allow irrational investors to earn high expected returns even while substantially distorting prices. Moreover, noise traders make the assets they trade more risky by subjecting them to changes in their whims. Risk-averse sophisticated investors then avoid these assets unless compensated for bearing not only fundamental risk but also noise trader created risk.[1] As a consequence, noise traders may depress the prices of and raise the returns on the assets they buy and so provide a further reason for economic selection to operate in their favor. The demise of noise traders is not as certain as has been supposed even by their advocates.

There is a second set of objections to the introduction of irrational noise traders into models of asset prices. It is suggested that they are a kind of deus ex machina who serve to explain only the questionable proposition that asset prices are excessively volatile, and that economists should not sacrifice their traditional presumption in favor of rational behavior in order to account for one single fact. We demonstrate to the contrary that the introduction of noise traders sheds light on several anomalies in the behavior of asset prices. Examining optimal responses to noise traders also helps to illuminate a number of aspects of the behavior of sophisticated investors and firms.

A financial market in which noise trader risk is significant invites a qualitative description often heard from managers, investment advisors, and other observers -- many of whom depend for their livelihood on a competitive market's placing a high monetary value on their insights into the future behavior of asset prices. If noise trading accounts for a large part of the variation in asset prices, it is rational for traders to focus attention on possible predictors of noise traders' future moves. Optimal trading strategies are likely to take the form of market timing, and will not necessarily bear close resemblance to buy and hold. Sophisticated investors trying to take advantage of noise traders will also pick stocks.

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arbitrage is restricted. Neither of the latter studies a competitive market without restrictions on trade.

[1]Noise trader-created risk is present in Campbell and Kyle (1987), although they do not emphasize this particular effect. Very similar effects exist in Stein's (1987) model of heterogeneously informed investors; he observes that noise traders reduce the informational content of prices and in this way drive out sophisticated investors.

Noise trading can also give rise to a number of observed properties of asset prices. If noise trading were prevalent and frequently pushed prices away from fundamental values, firms with market values high relative to their earnings, dividends, book value, or any other size measure would tend to perform poorly, while firms with market values low relative to these benchmarks would do well. In addition, one would expect to find discrepancies between asset prices and fundamental values such as can be seen in the persistent underpricing of closed end mutual fund shares and are suggested by the calculations of Mehra and Prescott (1986) on the relationship between the variability of consumption and the equity risk premium.

The presence of noise traders also makes coherent some of the fears of corporate managers that the short time horizon of the typical American investor harms the economy. Investors with short horizons increase asset price volatility and investors with long horizons stabilize the market and push asset prices closer to fundamental values. Managers are right to complain that the market is short-sighted and undervalues their firms (Donaldson (1984)) and that the short time horizon of investors forces investment projects to pass excessively high rate of return hurdles.

A firm operating in a market full of noise traders will take their presence into account. Its managers will try to reduce the noise trader risk to which their firm's securities are subject by paying dividends, altering the debt equity ratio, and otherwise "packaging" claims to the firm's cash flows to reduce their vulnerability to noise trader risk. If the discount of equity caused by noise traders gets to be so large that it outweighs the benefits of public ownership, managers will find it profitable to take their companies private. As pointed out by Black (1986), leveraged buyouts of undervalued firms make sense in a world where noise traders matter.

We develop our two central arguments -- that market selection may well work for, not against, noise traders and that models with noise traders yield predictions that seem to fit well with many standard financial anomalies -- in five sections and two appendices. Section I below presents a model with two assets which have identical riskless fundamentals, and one of the assets, but not the other, is subject to noise traders' misperceptions. While the only risk in this model comes from changes in noise traders' opinions, prices nevertheless diverge significantly from fundamentals. Section II deals with the survival of noise traders in the basic model and in an extended model in which successful investors are imitated (as in Denton (1985)). Section III presents qualitative implications of the model for the behavior of asset prices and market participants. Section IV presents qualitative implications of the presence of noise traders for real economic activity. Section V concludes. A first appendix discusses the effect of fundamental risk on the survival of noise traders. A second appendix shows that our results, while mathematically more complex, hold as well in a model with a bounded distribution of prices and with fundamental as well as noise trader created price risk.

I. NOISE TRADING AS A SOURCE OF RISK

Noise Trading and Sophisticated Investing

The central feature of the model presented below is the presence of both noise traders and sophisticated investors. Noise traders falsely believe that they have information about the price that the risky asset will sell for in the future. They may get their pseudo-signals from technical analysts, stock brokers, or economic consultants and irrationally believe that they carry information. Or they may, in formulating their investment strategies, exhibit the fallacy of excessive subjective certainty that has been repeatedly demonstrated in experimental contexts since Alpert and Raiffa (1960). Alternatively, noise traders may be motivated by the following chain of reasoning: "The tip I have just received may reflect real knowledge -- in which case I will profit by following it -- or the market may be fully efficient and the tip may be noise. If the market is efficient I will be accepting extra risk, but not an abnormally low expected return, by acting on the tip. Therefore I should invest at least a small amount as long as I give the tip any positive probability of being valid."[1]

The optimal behavior of sophisticated investors in asset markets without noise traders is to buy and sell assets on the basis of fundamental risk characteristics. In the presence of noise traders the optimal behavior of sophisticated investors would involve paying attention to pseudo-signals and acting to exploit noise traders' irrational misperceptions. Sophisticated traders would then optimally exploit noise traders, buying when noise traders depress prices and selling when noise traders push prices up. Sophisticated investors would trade actively on the basis of public information. When

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[1]Many economists' speculations on the "small firm in January" effect were based on this line of reasoning.

viewed from the outside they would resemble noise traders in actively managing their portfolios. These are the sophisticated investors our model examines.

The Model

Our basic model is a stripped down overlapping generations model with two-period lived agents (Samuelson (1958), Diamond (1965)). For simplicity, there is no first period consumption, no labor supply decision, and no bequest. As a result, the resources agents have to invest are exogenous. The only decision considered is the portfolio choice of the young.

The model contains two assets that pay identical dividends. One of the assets, the safe asset (s), pays a fixed real dividend r. Asset (s) is in perfectly elastic supply: a unit of it can be created out of and a unit of it turned back into a unit of the consumption good in any period. Its price is therefore always fixed at one. The dividend r paid on asset (s) is thus the riskless rate. The other asset, the unsafe asset (u), always pays the same fixed real dividend r as asset (s). But (u) is not in elastic supply: it is in fixed and unchangeable quantity, normalized at one unit. We will usually interpret (s) as a riskless short-term bond and (u) as the aggregate of equities. The price of (u) in period t is denoted p_t. If all agents accurately perceive that the two assets always pay the same dividends, then assets (u) and (s) will be perfect substitutes and will sell for the same price of one in all periods. But this is not an equilibrium in the presence of noise traders.

The basic model possesses two types of agents: sophisticated investors (denoted "i") who have rational expectations and noise traders (denoted "n"). We assume that noise traders are present in the model in measure μ, that sophisticated investors are present in measure $1-\mu$, and that all agents of a given type are identical.[1] Both types of agents maximize perceived expected utility given their perception of the ex-ante mean of the distribution of the price of (u) at t+1. The representative sophisticated investor young in period t accurately perceives the distribution of returns to holding the risky asset. The representative noise trader young in period t misperceives the expected price of the risky asset by an independent normal random variable ρ_t:[2]

[1] A more general model would consider the interaction of noise traders with different sets of misperceptions.
[2] In this case asset returns have a normal distribution, and so the linear mean-variance approximation to expected utility is exact. The validity of the mean-variance approximation when misperceptions are not normally distributed is considered in the appendix.

$$(1) \qquad \rho_t \sim N(\rho^*, \sigma_\rho^2)$$

The mean misperception ρ^* is a measure of the average "bullishness" of the noise traders, and σ_ρ^2 is the variance of noise traders' misperceptions of the expected return per unit of the risky asset.

Each agent's utility is a constant absolute risk aversion function of wealth when old:

$$(2) \qquad U = -e^{-(2\gamma)w}$$

where γ is the coefficient of absolute risk aversion. Agents choose their portfolio when young to maximize expected utility. Sophisticated investors use the correct probability distribution of next period's prices. Noise traders maximize their own expectation of utility given the dividend that will be paid next period, the one-period variance of p_{t+1}, and their false belief that the distribution of the price of (u) next period has mean ρ_t above its true value. With normally-distributed returns, maximizing the expected value of (2) is equivalent to maximizing (Samuelson (1970)):

$$(3) \qquad \bar{w} - \gamma\sigma_w^2$$

where $\bar{w}$ is the expected final wealth, and σ_w^2 is the one-period ahead variance of wealth.

The sophisticated investor chooses the amount λ_t^i of the risky asset (u) he buys to maximize:

$$(4) \qquad E(U) = \bar{w} - \gamma\sigma_w^2 = c_0 + \lambda_t^i(r + {}_tp_{t+1} - p_t(1+r)) - \gamma(\lambda_t^i)^2\{{}_t\sigma^2_{p_{t+1}}\}$$

where c_0 is a function of first-period labor income, an anterior subscript denotes the time at which an expectation is taken, and we define:

$$(5) \qquad {}_t\sigma^2_{p_{t+1}} = E_t\left\{ (p_{t+1} - E_t(p_{t+1}))^2 \right\}$$

to be the variance of p_{t+1} about its one-period forecast. The representative noise trader maximizes:

$$(6) \qquad E(U) = \bar{w} - \gamma\sigma_w^2 = c_0 + \lambda_t^n(r + {}_tp_{t+1} - p_t(1+r)) - \gamma(\lambda_t^n)^2\{{}_t\sigma^2_{p_{t+1}}\} + \lambda_t^n\{\rho_t\}$$

The only difference between (4) and (6) is the final term in (6) added to capture the noise trader's misperception of the expected return from holding a unit of the risky asset.

Given noise traders' misperception of the one-period return on (u), young noise traders and sophisticated investors maximize (perceived expected) utility by dividing wealth between (u) and (s). The quantities λ_t^n and λ_t^i of the risky asset purchased are functions of the price p_t of the risky asset, of the one-period ahead distribution of the price of (u), and (in the case of noise traders) of their misperception ρ_t of the expected return. When old, agents convert their holdings of (s) to the consumption good, sell their holdings of (u) for price p_{t+1} to the new young, and consume all their wealth.

Any agent wishing to hold asset (u) from period t to period t+1 must consider the possibility that the noise traders will be either bullish or bearish on asset (u) in period t+1. Noise traders with faulty and stochastic expectations create the possibility of capital gains and losses on rational agents' holdings of (u). Asset (u) -- which carries no fundamental risk -- thus becomes risky. The presence of noise traders eliminates the riskless arbitrage demand for asset (u) and breaks the identity between the prices of (u) and (s).

One can think of alternative specifications of noise traders. There are well-defined mappings between misperceptions of returns ρ_t and (a) noise traders' fixing a price p_t at which they will buy and sell, (b) noise traders' purchasing a fixed quantity λ_t^n of the risky asset, or (c) noise traders' mistaking the variance of returns (taking them to be σ^{2*} instead of σ^2).[1] The equilibrium in which noise traders matter found in our basic model exists regardless of which primitive specification of noise traders' behavior is assumed.

The Pricing Function

Solving (4) and (6) yields expressions for agents' holdings of (u):

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[1]Let noise traders set:

$$p_t = 1 - \frac{2\gamma}{r}\sigma^2 + \frac{\mu\rho^*}{r} + \frac{\mu(\rho_t - \rho^*)}{1+r}$$

where σ^2 is the total variance -- the sum of "fundamental" dividend variance, noise trader-generated price variance, and any covariance terms -- associated with holding the risky asset (u) for one period. Alternatively, let noise traders set the quantity of the risky asset that they buy -- whatever its price --

$$\lambda_t^n = 1 + \frac{\rho_t}{(2\gamma)\sigma^2}$$

or let the noise traders misperceive the variance of returns on the risky asset, taking as the variance:

$$\sigma^{2}* = \sigma^2 \left(\frac{\gamma\sigma^2 - \rho_t}{\gamma\sigma^2 + \rho_t} \right)$$

$$(7) \qquad \lambda_t^i = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma\{{}_t\sigma^2_{p_{t+1}}\}}$$

$$(8) \qquad \lambda_t^n = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma\{{}_t\sigma^2_{p_{t+1}}\}} + \frac{\rho_t}{2\gamma\{{}_t\sigma^2_{p_{t+1}}\}}$$

Since the old sell their holdings, the demands of the young must sum to one in equilibrium. Equations (7) and (8) imply that:

$$(9) \qquad p_t = \frac{1}{1+r}\left\{ r + {}_tp_{t+1} - 2\gamma({}_t\sigma^2_{p_{t+1}}) + \mu\rho_t \right\}$$

Equation (9) expresses the risky asset's price in period t as a function of period t's misperception by noise traders (ρ_t), of the technological (r) and behavioral (γ) parameters of the model, and of the characteristics of the one-period ahead distribution of p_{t+1}. If we consider only steady-state equilibria by imposing the requirement that the unconditional distribution of p_{t+1} be identical to the distribution of p_t, then the endogenous one-period ahead distribution of the price of asset (u) can be eliminated from the equilibrium pricing function (9) by solving recursively.[1]

$$(10) \qquad p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}\left({}_t\sigma^2_{p_{t+1}}\right)$$

Inspection of (10) reveals that only the second term is variable, for γ, ρ^*, and r are all constants, and the one-step ahead variance of p_t is a simple unchanging function of the constant variance of a generation of noise traders' misperception ρ_t.

$$(11) \qquad {}_t\sigma^2_{p_{t+1}} = \sigma^2_{p_{t+1}} = \frac{\mu^2\sigma^2_\rho}{(1+r)^2}$$

The final form of the pricing rule for (u), in which the price depends only on exogenous parameters of the model and on public information about present and future misperception by noise traders, is:

[1]The model cannot have well-behaved bubble equilibria, for the safe asset is equivalent to a storage technology that pays a rate of return r greater than the rate of growth of the economy. The number of stationary equilibria does depend on the primitive specification of noise traders' behavior. For example, if noise traders randomly pick each period the price p_t at which they will buy and sell unlimited quantities of the risky asset, then (trivially) there is only one equilibrium. If the noise traders randomly pick the quantity λ_t^i which they purchase, then the fundamental solution in which p_t is always equal to one is an equilibrium in addition to the equilibrium in which noise traders matter.

$$(12) \qquad p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\mu^2\sigma_\rho^2}{r(1+r)^2}$$

Interpretation

The last three terms that appear in (12) and (10) show the impact of noise traders on the price of asset (u). As the distribution of ρ_t converges to a point mass at zero the equilibrium pricing function (12) converges to its fundamental value of one.

The second term in (12) captures the fluctuations in the price of the risky asset (u) due to the variation of noise traders' misperceptions. Even though asset (u) is not subject to any fundamental uncertainty and is so known by a large class of investors, its price varies substantially as noise traders' opinions shift. When a generation of noise traders is more "bullish" than the average generation, they bid up the price of (u). When they are more "bearish" than average, they bid down the price. When they hold their average misperception -- when $\rho_t = \rho^*$ -- the term is zero. As one would expect, the more numerous are noise traders relative to sophisticated investors, the more volatile are asset prices.

The third term in (12) captures the deviations of p_t from its fundamental value due to the fact that the average misperception by noise traders is not zero. If noise traders are bullish on average, this "price pressure" effect makes the price of the risky asset higher than it would otherwise be. Optimistic noise traders bear a greater than average share of price risk. Since sophisticated investors bear a smaller share of price risk the higher is ρ^*, they require a lower expected excess return and so are willing to pay a higher price for asset (u).

The final term in (12) is the heart of the model. Sophisticated investors will not hold the risky asset unless compensated for bearing the risk that noise traders will become bearish and the price of the risky asset will fall. Both noise traders and sophisticated investors present in period t believe that asset (u) is mispriced, but because p_{t+1} is uncertain each class is not willing to go too far in betting on this mispricing. At the margin, the returns from enlarging one's position in an asset that everyone agrees is mispriced (but different classes think is mispriced in different directions) are offset by the additional price risk that would be run. Noise traders thus "create their own space": the uncertainty over what next period's noise traders will believe makes the otherwise riskless asset (u) risky, and drives its price down and its return up. This is so despite the fact that both sophisticated investors and noise traders always hold portfolios which possess the same amount of fundamental risk: zero. Any intuition to the effect that investors in the risky asset "ought" to receive higher expected returns because they perform the valuable social function of risk bearing neglects to consider that noise traders' speculation is the only source of risk. For the economy as a whole, there is no risk to be borne.

The reader might suspect that our results are critically dependent on the overlapping generations structure of the model, but this is not accurate. Equilibrium exists as long as the returns to holding the risky asset are always uncertain. In the overlapping generations structure this is assured by the absence of a last period. For if there is a last period in which the risky asset pays a non-stochastic dividend and is liquidated, then both noise traders and sophisticated investors will seek to exploit what they see as riskless arbitrage opportunities. If, say, the total liquidation value of the risky asset is 1+r, the previous period sophisticated investors will try to buy and sell arbitrarily large quantities of asset (u) at a price of one, and noise traders will try to buy and sell arbitrarily large quantities at a price of:

$$(13) \qquad p_t = 1 + \frac{\rho_t}{1+r}$$

The excess demand function for the risky asset will be undefined. But in a model with fundamental dividend risk the assumption that there is no last period, and hence the overlapping generations structure, are not necessary. With fundamental dividend risk no agent will ever be subjectively certain what the return to holding the risky asset will be, and so the qualitative properties of equilibrium in our model hold even with a known terminal date.[1] The overlapping generations structure is therefore not needed when fundamental dividend risk is present.

Our discussion has maintained the assumption that all agents who are not noise traders are sophisticated investors who optimally exploit the presence of noise. A more reasonable assumption is that many traders pursue passive strategies -- neither responding to noise nor trading against noise traders -- as is advised by many finance textbooks. If a large fraction of non-noise trading is of this

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[1]The infinitely extended overlapping generations structure of the basic model does play another function. It assures that each agent's horizon is short. No agent has any opportunity to wait until the price of the risky asset "recovers" before selling. Such an overlapping generations structure may be a fruitful way of modelling the effects on prices of a number of institutional features, like frequent evaluations of money managers' performance, that may lead rational, long-lived market participants to care about short term rather than long term performance. In our model, the horizon of the typical investor is of some importance: as we show below, arbitrage becomes easier as the horizon of agents becomes longer, and prices approach fundamental values.

passive type, then even a very small measure of noise traders can have a large impact on prices. If noise traders wish to sell, they will find in aggregate that they have no one else to sell to. Prices will move until noise traders' no longer wish to sell. The impact of noise trading depends not on the number of noise traders but on the relative numbers of noise traders and of those willing to actively bet against them.[1]

II. THE SURVIVAL OF NOISE TRADERS

The Returns to Noise Trading

We have demonstrated that noise traders can affect prices even though there is no uncertainty about fundamentals. It is often argued that noise traders who affect prices will earn lower returns than the rational, sophisticated speculators they trade with. Hence economic selection will work to weed them out (Friedman (1953)). This argument is flawed. Noise traders' collective shifts of opinion increase the riskiness of and average returns to assets. If noise traders' portfolios are concentrated in assets subject to noise trader risk, noise traders can earn higher rates of return on their investments even though they hold portfolios with no greater degree of fundamental risk than do sophisticated investors.

The conditions under which noise traders earn higher expected returns than sophisticated investors are easily laid out. All agents earn a certain net return of r on their investments in asset (s). The difference between noise traders' and sophisticated investors' total returns given equal initial wealth is the product of the difference in their holdings of the risky asset (u) and of the excess return paid by a unit of the risky asset (u). Call this difference in net returns to the two types of agents ΔR_{n-i}:

$$(14) \qquad \Delta R_{n-i} = (\lambda_t^n - \lambda_t^i)(r + p_{t+1} - p_t(1 + r))$$

The difference between noise traders' and sophisticated investors' demands for asset (u) is simply:

$$(15) \qquad (\lambda_t^n - \lambda_t^i) = \frac{\rho_t}{(2\gamma)_t \sigma^2_{p_{t+1}}} = \frac{(1+r)^2 \rho_t}{(2\gamma)\mu^2 \sigma^2_\rho}$$

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[1]A simple example may help to make our point. Suppose that all investors are convinced that the market is efficient. They will hold the market portfolio. Now suppose that one investor decides to commit his wealth disproportionately to a single security. Its price will be driven to infinity.

The expected value of the excess return on the risky asset (u) as of time t is:

$$(16) \qquad {}_t(r + p_{t+1} - p_t(1 + r)) \; = \; (2\gamma)_t\sigma^2_{p_{t+1}} - \mu\rho_t \; = \; \frac{(2\gamma)\mu^2\sigma^2_\rho}{(1+r)^2} - \mu\rho_t$$

And so:

$$(17) \qquad {}_t(\Delta R_{n-i}) = \rho_t - \frac{(1+r)^2(\rho_t)^2}{(2\gamma)\mu\sigma^2_\rho}$$

The expected excess total return of noise traders will be positive only if both noise traders are optimistic (ρ_t positive, which makes (15) positive) and the risky asset is priced below its fundamental value (which makes (16) positive. Since (17) is the product of (15) and (16), it is positive only if both (15) and (16) have the same sign. Since (16) is guaranteed to be positive if (15) is negative, the expected excess total return can be positive only if both of its factors are positive.

Taking the global unconditional expectations of (17) yields:

$$(18) \qquad E(\Delta R_{n-i}) = \rho* - \frac{(1+r)^2(\rho*)^2 + (1+r)^2\sigma^2_\rho}{(2\gamma)\mu\sigma^2_\rho}$$

Equation (18) makes obvious the requirement that for noise traders to earn higher expected returns, the mean misperception $\rho*$ of returns to holding the risky asset must be positive. The first $\rho*$ on the right hand side of (18) increases noise traders' expected returns through what might be called the "hold more" effect. Noise traders' expected returns relative to those of sophisticated investors are increased when noise traders on average hold more of the risky asset and earn a larger share of the rewards to risk bearing. If $\rho*$ is less than zero, noise traders' changing misperceptions still make the fundamentally riskless asset (u) risky and still push the expected returns to holding asset (u) up, but the rewards paid to risk bearing accrue disproportionately to sophisticated investors, for noise traders on average hold less than their share of the risky asset.

The first term in the numerator incorporates the "price pressure" effect. As noise traders become more bullish, they demand more of the risky asset on average and drive up its price. They thus reduce the return to risk bearing, and hence the differential between their returns and those of sophisticated investors.

The second term in the numerator incorporates the "buy high-sell low" effect. Because noise traders' misperceptions are stochastic, that they have the worst possible market timing. They buy the most of the risky asset (u) just when other noise traders are buying it, which is when they are most likely to suffer a capital loss. The more variable are noise traders' beliefs, the more damage their poor market timing does to their returns.

The denominator incorporates the "create space" effect central to this model. As the variability of noise traders' beliefs increases, the price risk in the system increases. Sophisticated investors are less willing to assume the capital risk they must bear to take advantage of noise traders' misperceptions. If the "create space" effect is large, then the "price pressure" and "buy high-sell low" effects inflict less damage on noise traders' wealth relative to sophisticated investors' wealth. The "create space" effect raises noise traders' relative returns by reducing the extent to which sophisticated traders are willing to exploit noise traders' misperceptions.

Two effects -- "hold more" and "create space" -- tend to raise noise traders' relative expected returns. Two effects -- "buy high-sell low" and "price pressure" -- tend to lower noise traders' relative expected returns. Neither pair clearly dominates. It is clear that noise traders cannot have higher returns if noise traders are on average bearish, for if $\rho*$ does not exceed zero there is no "hold more" effect. It is also clear that noise traders do not have higher returns if they are too bullish, for as $\rho*$ grows large the "price pressure" effect, which increases with $(\rho*)^2$, dominates. For intermediate degrees of average bullishness noise traders earn higher expected returns. And it is clear from (18) that the larger is γ, that is the more risk averse are agents, the larger is the range of $\rho*$ over which noise traders have higher returns.

The higher expected returns of the noise traders come at the cost of holding portfolios with sufficiently higher variance to give noise traders lower expected utility (computed using the true distribution of wealth when old). Since sophisticated investors maximize true expected utility, any trading strategy alternative to theirs that earns a higher mean return must have a variance sufficiently higher to make it unattractive. The average amount of asset (s) that must be given to old noise traders to give them the <u>ex ante</u> expected utility of sophisticated investors can be shown to be:

$$(19) \qquad \frac{(1+r)^2}{(4\gamma)\mu^2}\left\{1+\frac{\rho*^2}{\sigma_\rho^2}\right\}$$

This amount falls with an increase in the stochastic irrationality of noise traders and rises with an increase in the mean misperception of noise traders. The magnitude of noise traders' mistakes grows with $\rho*$, but the extra risk penalty for attempting to exploit noise traders' mistakes grows with $\sigma_\rho{}^2$.

Noise traders receive higher average consumption than sophisticated investors, and sophisticated investors receive higher average consumption than in fundamental equilibrium, yet the productive resources available to society -- its per period labor income, its ability to create the productive asset (s), and the unit amount of asset (u) yielding its per period dividend r -- are unchanged by the presence of noise trading. The source of extra returns is made clear by the following thought experiment. Imagine that before some date τ there are no noise traders. Up until time τ both assets sell at a price of one. At τ it is unexpectedly announced that in the next generation noise traders will appear. The price p_τ of the asset (u) drops; those who hold asset (u) in period τ suffer a capital loss. This capital loss is the source of the excess returns and of the higher consumption in the equilibrium with noise. The period τ young have more to invest in (s) because they pay less to the old for the stock of asset (u). If at time ω it became known that noise traders had permanently withdrawn -- perhaps because the government had credibly committed itself to undo the purchases and sales of noise traders -- then those who held (u) at time ω would capture the present value of what would otherwise have been future excess returns as p_ω jumped to one. The fact that the generations that suffer and benefit from the arrival and departure of noise traders are pushed off to $-\infty$ and $+\infty$ in the basic model creates the appearance of a free lunch.[1]

The fact that "bullish" noise traders can earn higher returns in the market than sophisticated traders implies that "market selection" does not necessarily eliminate irrational behavior that affects prices.[2] Since noise traders' wealth can increase faster than sophisticated investors', it is not possible to make any blanket statement to the effect that there can be a stable population of irrational agents only if they are continuously subsidized.

[1]In practice, the cost of future noise trader risk in a security will be paid by whoever sells it to the public. In the case of stock, the cost will be paid by the entrepreneur.
[2]There is a sense in which "market selection" could work against noise traders. The greater variance of noise traders' returns may give long-lived agents a high probability of having low wealth and a low probability of having very high wealth. Since the criterion of "fitness" might be not a high expected wealth but a small probability of a very low realization of wealth, market selection might work against such a trader even though the expected value of wealth is high We pursue these issues in our follow up paper (De Long, Shleifer, Summers, and Waldmann (in preparation)).

Inheritance of Beliefs but Not Wealth

Our two-period model precludes our treating the effects of wealth accumulation by noise traders. We turn instead to a model of "imitation" rather than "accumulation" in which the issues of survival can be directly addressed by allowing the fraction of agents who are noise traders to be determined by the emulation effects developed in Denton (1985).

There are no bequests. Each generation earns exogenous labor income when young and consumes all of its wealth when old. Each individual has one child; and children are predisposed to be of the same type (sophisticated investor or noise trader) as their parent. But there is some switching. If noise traders earn a higher return in any period, a fraction of the young who would otherwise be sophisticated investors become noise traders:

$$(20) \qquad \mu_{t+1} = \mu_t + \zeta(R_n - R_i)$$

where R_n and R_i are the realized returns of period t old, and where μ_t is the fraction of the population that are noise traders. There are $(1-\mu_t)$ sophisticated investors and μ_t noise traders.

Equation (20) is a simple learning rule; investment strategies that were successful in the previous generation win converts. It is similar to simple adaptive learning rules used by Bray (1982) and Lucas (1986).[1] Equation (20) aims to capture the idea that success breeds imitation, which in many cases is a plausible theory of investor behavior. Witness the well-known 1966 "President's Report" of then Ford Foundation President McGeorge Bundy (1967):

> It is far from clear that trustees have reason to be proud of their performance in making money for their colleges. We recognize the risks of unconventional investing, but the true test of performance in the handling of money is the record of achievement, not the opinion of the respectable. We have the preliminary impression that over the long run caution has cost our colleges and universities much more than imprudence or excessive risk taking.

Bundy's intention to change the management of the Ford Foundation's portfolio in the late 1960's exemplifies the switch from acting like a risk-averse sophisticated investor to acting like a noise trader.

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[1]One might argue that the single reasonable learning rule would be to take the past distribution of noise traders' and sophisticated investors' total wealth, evaluate the utility associated with each realized total wealth, and calculate the expected utility from following each strategy. In our view, this amounts to saying that an agent who understands the deep structure of the model and performs the calculations that a sophisticated utility-maximizing investor would perform as part of his decision among investment strategies would not choose to be a noise trader. We do not disagree.

Casting aside "the opinion of the respectable" -- that the large returns earned by go-go fund managers in the 1960's were achieved by riding the crest of the noise trader wave -- Bundy views "the record of [recent past] achievement" as the rational criterion for choosing among portfolio strategies. The Ford Foundation abandoned its prudence and shifted to a high β portfolio that had probably already attracted the attention of large numbers of other noise traders.[1]

The model with imitation is easily solved if $\zeta << 1$. If ζ is significant at the scale of any one generation, then those investing in period t will have to calculate what the effect of the distribution of returns will be on the division of those young in period t+1 between noise traders and sophisticated investors. If ζ is small, then the calculation of returns can be carried out as if the population will be divided between noise traders and sophisticated investors in the proportions μ_t and $1-\mu_t$ forever.

Equation (12), the pricing rule for p_t with μ noise traders, requires only that a subscript be added to the fraction of noise traders μ_t to give the limit of the pricing rule for the model with variable proportions as the parameter ζ converges to zero:

$$(12') \qquad p_t = 1 + \frac{\mu_t(\rho_t - \rho^*)}{1+r} + \frac{\mu_t \rho^*}{r} - \frac{(2\gamma)\mu_t^2\sigma_\rho^2}{r(1+r)^2}$$

The expected return gap between noise traders and sophisticated investors is equation (17) when the proportion of noise traders is fixed at μ. With the proportion μ_t variable, the limit of the expected return gap as ζ converges to zero is given by:

$$(17') \qquad {}_t(\Delta R_{n-i}) = \rho_t - \frac{(1+r)^2(\rho_t)^2}{(2\gamma)\mu_t\sigma_\rho^2}$$

Over time μ_t will tend to grow or shrink as (17') is greater or less than zero. It is thus immediate clear that although there is a steady state value for μ_t, this steady state is unstable. As the noise trader share declines, sophisticated investors' willingness to bet against noise traders increases. Sophisticated investors thus earn more money from their exploitation of noise traders' misperceptions, and the gap

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[1]As a model in which noise traders produce transitory components in aggregate stock prices would predict, the years up to the late 1960's that saw equities earn high rates of return were followed by years in which equities earned low rates of return. As many a university official can ruefully attest, anyone who paid attention to Bundy would have lost a great deal as a result of holding a portfolio with a high β that had been probably bid up by noise traders during the 1960's.

between the expected returns earned by noise traders and sophisticated investors becomes negative. If the noise trader share μ_t is below:

$$(21) \qquad \mu* = \frac{(\rho*^2 + \sigma_\rho^2)(1+r)^2}{2\rho*(\gamma\sigma_\rho^2)}$$

then the noise trader share will tend to shrink. If μ_t is greater than $\mu*$, noise traders will create so much price risk as to make sophisticated investors very reluctant to speculate against them. Noise traders will therefore earn higher expected returns than sophisticated investors and will grow in number. In the long run noise traders dominate the market or disappear, as is shown in figure 1.

This result contradicts our intuition, which suggests that rather than approaching zero or two the number of noise traders should converge to an intermediate steady-state value. Appendix I demonstrates that this is indeed the case if the current model is further extended to allow for fundamental dividend risk. In such a model, the noise trader population may settle down to a steady state value or noise traders may drive sophisticated investors out of existence. The noise trader share, however, is never driven to zero in a model in which asset (u) possesses fundamental risk.

These results do not imply that all types of noise traders will flourish in financial markets. Only "bullish," and not too "bullish," noise traders will flourish. But this is sufficient to make our point. In any population there are likely to be noise traders of different stripes. As long as economic selection works in favor of some of them, there is no basis for assuming that their effects on asset prices can be neglected in the long run.

There is a further selection argument that works in favor of noise traders and that is obscured by our two period formulation. Noise traders who are lucky in their guesses get wealthier, and as a result are likely to increase their faith in their guesses and invest even more aggressively. Those who are unlucky will lose both money and faith. Even if noise traders as a group do not earn higher than average returns, as time passes a larger and larger share of noise trader wealth will come to be concentrated in the hands of those noise traders who are most convinced of the value of their judgments.

III. NOISE TRADING AND ASSET MARKET BEHAVIOR[1]

This section describes a number of respects in which models allowing for the presence of noise traders provide a more realistic description of asset markets than models that postulate that all agents are rational. In the presence of noise trading, investment strategies similar to those pursued by highly paid market professionals may pay off, asset returns exhibit mean reversion documented by a great deal of empirical work, and asset prices diverge on average from fundamental values as suggested indirectly by Mehra and Prescott (1986) and directly by the comparison of the portfolio and market values of closed end mutual funds.

What Do Traders Do?

In a world without noise traders, rational sophisticated investors would trade for one of three reasons: to consume (or save), to rebalance portfolios, or to exploit inside information. Trading would be relatively infrequent, especially in markets where little private information is available like those for treasury securities, foreign exchange, and index futures. Sophisticated market participants might be concerned with information bearing on fundamental values, but they would have little concern with indicators of what other traders who lack significant private information are doing.

This description does not ring true as a characterization of the activities of very highly paid and not naive market participants. Professional money managers eschew passive strategies, instead seeking to discover "What is Mister Johnson [of Fidelity] doing?...<u>What three stocks does Mister Johnson like best?</u> What's going to happen next?" (Smith (1968), emphasis in original). The volume of trading is far greater than can be satisfactorily rationalized on the basis of the standard motives. Wojnilower (1980) concludes that "it defies belief that turnover of thirty-eight billion dollars a day in just one segment of the fixed-income market [U.S. Treasury securities] is required to fix the rational allocation of capital." In the foreign exchange markets, two to three hundred billion dollars' worth of currencies change hands every day, an amount far greater than necessary for trade and capital account transactions. And annual trading in index futures approaches the total value of stock market capitalization. Highly compensated market analysts devote a great deal of effort to examining patterns in

[1]Much of this section follows insights developed in Black (1986).

prices and in the volume of trade that have no clear connection with the determinants of fundamental value. And many of them profess to be more concerned with indicators of the extent of the demand for securities a day ahead than with underlying fundamental values.

All of these aspects of actual market behavior are to be expected if noise trading is important. There will be profits to be made by trading against noise traders. Data on volume and price patterns might help to gauge what noise traders have done and are going to do. In the absence of fundamen-tal risk, past volume data are uninformative because past price data alone reveal $\rho*$, σ_ρ^2, and ρ_t. In the presence of fundamental risk, volume may provide worthwhile clues to whether movements in asset prices reflect rational bets made on the basis of inside information or irrational bets made by noise traders. Changes in asset demands and supplies can have significant effects on prices even if they convey no information about fundamentals and involve only a trivial proportion of outstanding asset stocks. Shleifer (1986) and Harris and Gurel (1986) have demonstrated that the inclusion of stocks in the S&P 500 index, and the consequent demand for these stocks by index funds, has a substantial impact on stock prices even though it has no implications at all for fundamental values.[1] Price pressure effects can also be invoked as explanations for the calendar effects recently documented by financial economists (Rozeff (1985)). These effects are not surprising if prices are set not according to some representative agent's valuation of fundamentals but instead to balance the demands and supplies of noise traders and sophisticated investors.[2]

We have written as if noise traders and sophisticated investors can be easily distinguished. This is not the case over the horizons relevant for investors. As Summers (1986) and Poterba and Summers (1987) stress, even if noise traders regularly drove prices thirty percent away from fundamentals, speculators who optimally exploited the deviations would require hundreds of years to statistically demonstrate the superiority of their strategy. For anyone who believes that there are irrational investors out there, but that he is not one of them, trading will be optimal. Noise traders will be those

[1]We regard it as instructive that leading brokerage firms should collect daily data on short interest in individual securities and regard such data as valuable proprietary information.

[2]An analogy makes our point. If no one liked to gamble, there would be no casinos. Since some people like to gamble, other individuals rationally compete in devoting substantial resources to building casinos and betting against those who have a taste for gambling. In the case of casinos, it is clear who is exploiting whom. In other cases, it may not be so clear who is gaining and who is losing welfare as a result of their betting strategy. It is clear that the private returns to the house's efforts to exploit gamblers are greater than the social returns. Hirshleifer (1971) was among the first to observe that trying to learn today what will be public tomorrow is a rent seeking activity.

whose guesses about what average opinion will expect average opinion to be are false relatively often. Sophisticated investors will be those whose guesses are more often on the mark. But those more correct on average -- those with higher actual ex ante expected utility -- will not necessarily be wealthier. And both groups will, like the investors we observe, trade actively and be concerned about information bearing on the supply and demand of securities.

Volatility and Mean Reversion in Asset Prices

In our model with noise traders absent -- with both $\rho*$ and $\sigma_\rho{}^2$ set equal zero -- the price of (u) is always equal to its fundamental value of one. When noise traders are present the price of asset (u) -- identical to asset (s) in all fundamental respects --- is excessively volatile in the sense that its price moves more than can be explained on the basis of changes in fundamental values. The variance of the price of (u) is as given by (11):

$$(11) \qquad {}_t\sigma^2_{p_{t+1}} = \sigma^2_{p_{t+1}} = \frac{\mu^2\sigma^2_\rho}{(1+r)^2}$$

None of this variance can be justified by changes in fundamentals: there are no changes in expected future dividends in our model, or in any variable relevant to the determination of required returns.

A large body of evidence demonstrates that asset prices respond rapidly to news about fundamental values. However, an accumulating body of evidence suggests that it is difficult to account for all of the volatility of asset prices in terms of news. While Shiller's (1981) claim that the stock market wildly violated variance bounds imposed by the requirement that prices be discounted present values relied on controversial statistical procedures (Kleidon (1986)), other evidence supporting the general conclusion that asset price movements do not all reflect changes in fundamental values is more clear cut. For example, Roll (1985) considers the relatively straightforward orange juice futures market, where the principle source of relevant news is weather. He demonstrates that a substantial share of the movement in prices cannot be attributed to news about the weather that bears on fundamental values. French and Roll (1986) demonstrate that over intervals where there is reason to suppose that the amount of news about fundamentals is constant the market moves much more when open; this suggests that volatility is imparted by trading quite apart from the effects of news. Similarly, Campbell and Kyle (1987) conclude on the basis of analyses of the changes in dividends and discount rates that follow stock price movements that a large fraction of market movements cannot be attributed to news about fundamental values.

If asset prices respond to noise and if the errors of noise traders are not permanent, then asset prices should exhibit mean reverting behavior. For example, if noise traders' misperceptions follow an AR (1) process, it is easily demonstrated that serial correlation in returns will decay geometrically as in the "fads" example of Summers (1986). As Shiller (1986) and Summers (1986) and stress, even with long time series it is likely to be difficult to detect slowly decaying transitory components in asset prices. Since the same problems of identification that plague econometricians affect speculators, actual market forces are likely to be less effective in limiting the effects of noise trading than in our model where rational investors fully understand the process describing the behavior of noise traders.

Moreover, even if sophisticated investors accurately diagnose the process describing the behavior of noise traders, if misperceptions are serially correlated they will not be willing to bet nearly as heavily against noise traders: the risks of capital loss remain, and they are balanced by a smaller expected return since the price is not expected to move all the way back to its fundamental value in the next period. Therefore a higher unconditional variance of asset prices about their mean will still be consistent with noise traders' earning higher returns.

An example of how rapidly unconditional price variance grows as misperceptions become persistent can be provided by assuming that misperceptions follow an AR(1) process with innovation η_t and autoregressive parameter ϕ. In this case the unconditional variance of the price of asset (u) about its mean is:[1]

$$\text{(22)} \qquad \sigma_p^2 = \frac{\mu^2\sigma_\rho^2}{(r + (1-\phi))^2} = \frac{\mu^2\sigma_\eta^2}{(r + (1-\phi))^2(1-\phi^2)}$$

Noise traders who earn higher expected returns than sophisticated investors can thus cause larger

[1]Of course demand for assets depends not on the unconditional but on the conditional price risk. The variance of the price of (u) about its one step ahead anticipated value is:

$$ {}_t\sigma^2_{p_{t+1}} = \frac{\mu^2\sigma_\eta^2}{(r + (1-\phi))^2}$$

in the case of serially correlated misperceptions.

deviations of prices from fundamental values of misperceptions are serially correlated. The difference in expected returns is given by:

$$(23) \qquad E(\Delta R_{n-i}) = \rho* - \frac{(r + (1-\phi))^2(\rho*)^2}{(2\gamma)\mu\sigma^2_\eta} - \frac{(r + (1-\phi))^2}{(2\gamma)\mu(1-\phi^2)}$$

Transitory components in asset prices that exhibit persistence can be very large in size and still be consistent with noise traders' earning higher returns than sophisticated investors. And the fact that prices revert to means implies that measures of scale have predictive power for asset returns: when prices are high relative to dividends, prices are going to fall in our model.

There is significant evidence that stock prices indeed exhibit mean reverting behavior. Fama and French (1986) demonstrate using data for the 1926-1985 period that long horizon stock returns exhibit negative serial correlation. Building on their work, Poterba and Summers (1987) use data for the entire 1871-1986 period for the United States and for a number of other countries to demonstrate that transitory components cannot convincingly be ascribed to changes in ex ante returns caused by macroeconomic variables. DeBondt and Thaler (1985, 1987) provide related evidence using data on individual firms. These results would all be predicted by, and are consistent with, our model. In fact, the idea that noise trading distorts prices forms the basis for fundamentalist and contrarian investment strategies. These strategies call for purchasing securities with low price earnings ratios that are seen as irrationally out of favor. Practiced with apparent success by patient investors like Benjamin Graham and Warren Buffett, this strategy laid out by Graham and Dodd (1934) requires both an eye for situations in which firms are selling for below their fundamental values and substantial patience.

Many studies including Mankiw and Summers (1984) and Mankiw (1986) note that anomalies exactly paralleling the price earnings ratio anomaly are present in the bond market. Long rates have predictive power for future short rates, but it is nonetheless the case that when long rates exceed short rates, they tend to fall and not to rise as predicted by the expectations hypothesis. While convincing stories about changing risk factors have yet to be provided, this behavior is exactly what one would expect if noise trading distorted long bond yields.

Asset Prices and Fundamental Values

One of the strongest predictions of the efficient markets hypothesis is that assets ought to sell for their fundamental values. In most cases, fundamental value is difficult to measure, and so this prediction cannot be directly tested. But the fundamental value of a closed-end fund is easily assessed: since the fund pays dividends equal to the sum of the dividends paid by the stocks in its portfolio, a closed end fund should sell for the market price of its portfolio. Yet closed end funds sell and have sold at large and substantially fluctuating discounts (Herzfeld (1980), Malkiel (1977)) which have been relatively small during the bull markets of the late 1960's and the 1980's and were large during the bear markets of the 1970's.

Models in which noise trader risk plays a significant role provide a natural explanation for the gap between the fundamental and market values of closed end funds.[1] Anyone investing in a closed end fund faces not only the risk that the fund's portfolio may decline but also the risk that the spread between the market and fundamental value of the fund may increase. This extra risk would make rational sophisticated investors unwilling to hold the fund at the market value of the fund's portfolio. And this extra risk would make rational sophisticated investors' demand for closed-end fund shares less than perfectly elastic. A rational investor would not buy heavily as the spread widens because doing so would entail accepting the risk that noise traders will be even more averse to the fund when one wants to sell. A rational investor would then take a large capital loss even if the assets in which the fund has invested earn a high return.[2]

[1]Alternative explanations of closed end fund discounts that rely on transactions costs or fears of mismanagement cannot explain the substantial correlated fluctuations of the discounts of different funds. Agency cost models of the closed-end fund discount also make the starting-up of closed end mutual funds unintelligible. In our model, noise traders will sometimes believe that the fund's managers can earn more than the market return on their portfolios. When noise traders are sufficiently optimistic about closed end funds, it will pay entrepreneurs to purchase stock shares, repackage them in a closed end fund, and then sell the shares of the closed end fund to noise traders.

[2]One can see how the fact that closed-end fund shares are subject not only to fundamental risk -- risk affecting the value of the fund's portfolio -- but also noise trader risk -- risk that the closed-end fund discount might change -- affects investment decisions in the investment advice given by one of the leading students of the closed-end fund discount, Malkiel (1973). Malkiel confidently recommended in 1973 that investors purchase then heavily (20 to 30%) discounted closed-end fund shares: such an investor would do better than by picking stocks or investing in an open-end fund unless "the discount widened in the future." The confidence of Malkiel's recommendation stemmed from his belief that "this... risk is minimized... [since] discounts [now]... are about as large as they have ever been historically." And the obverse is Malkiel's belief that the holder of a closed-end fund should be prepared to sell if the discount narrowed -- not only if the discount disappeared, but also if the discount narrowed. The latest version of A Random Walk Down Wall Street (1985) does not recommend the purchase of closed-end fund shares in spite of the fact that many closed-end funds still sell at discounts. The noise trader risk that discounts may widen again in the future is a disadvantage that apparently weighs heavily against the relatively small advantages given by the present small discount.

In our model, if noise traders earn higher expected returns than sophisticated investors then the average price for which the asset (u) sells is guaranteed to be below its fundamental value. The expected value of p_t is:

$$(24) \qquad E(p) = p^* = 1 - \frac{2\gamma\mu\sigma_\rho^2}{r(1+r)^2} + \frac{\mu\rho^*}{r}$$

Since noise traders hold more of the risky asset and earn negative capital gains on average, they can earn higher expected returns than sophisticated investors only if the dividend on the unsafe asset amounts to a higher rate of return on average than does the same dividend on the safe asset. For this to hold, the unsafe asset must sell at an average price below its fundamental value of one.

The point that in our model the average prices of assets are different from their fundamental values is of wider application. Consider the analysis of Mehra and Prescott (1986), which rejects the hypothesis that the stochastic processes followed by total consumption and by asset returns satisfy the Euler equation of a representative consumer. An application of Mehra and Prescott's procedure to data generated by our model would produce an equity puzzle similar to that found in U.S. data: the zero covariance of dividends paid on the risky asset with consumption cannot justify the high average rate of return paid on the risky asset. An analyst looking at data generated by our model would also reject the hypothesis that the stochastic processes of aggregate consumption and asset returns satisfy the Euler equation for a representative consumer, since the marginal utility of agents who do satisfy the Euler equation, i.e. young sophisticated investors, is not a function of aggregate consumption (Ingram (1987)).

IV. NOISE TRADING AND REAL ECONOMIC DECISIONS

Tobin's Q and Investment

If the risky asset is interpreted as a claim on equity capital, then for noise traders to earn higher expected returns than sophisticated investors Tobin's q must on average be too low. If physical capital is accumulated up to the point where the cost of a marginal unit is equal to the traded value of equity claims to future rents from capital, on average the capital stock will be below its optimal value. In an average generation, aggregate consumption could be increased if society could purchase additional quantities of asset (u) at a price p*. Yet no investor will find it worth his while to do so. In other words, asset (u) is underpriced on the stock market given the zero covariance of its fundamental return with consumption.

The risk caused by noise trading is a social cost. If it could be reduced either by discouraging noise traders from entering the market or by offsetting their actions, welfare would be enhanced. Suppose, for example, that a wise government was able to credibly promise to stabilize the price of (u) and eliminate the effects of noise trading by engaging in "open market" operations in the securities (s) and (u). The result would be a large capital gain for holders of the unsafe security. While this potential capital gain for the current old that can be realized if it becomes credible that noise trader risk will disappear, private agents themselves cannot bring it about. They cannot affect the economy after their departure, and any individual -- a small actor -- would have to bear an immense amount of risk in any attempt to stabilize the market.

The story told here provides a rationale that might justify government intervention in foreign exchange markets. Government actions to offset noise and reduce noise trader created risk can raise social welfare. Of course, our model biases the case in favor of intervention since the only source of volatility is noise; the standard argument against intervention stresses authorities' proclivity to trade against movements caused by shifting fundamentals.

The fact that Tobin's q is below one on average in our model if noise traders earn higher expected returns does not imply that increasing investment in asset (u) would be socially desirable given the structure and functioning of the asset market. Although asset (u) is underpriced given its fundamental risk, it is overpriced given its market risk. Additional investment in (u) would clearly be desirable if the stock market were not an independent source of risk. However, since more privately liquid capital of type (u) will in equilibrium expose private investors to greater risk, society's acquiring asset (u) at a price of p_t units of asset (s) will unambiguously reduce social welfare in the basic model as long as ρ_t is greater than zero. The utility of sophisticated investors remains unchanged if an infinitesimal amount δ of asset (u) is replaced by $p_t\delta$ of asset (s); the utility of noise traders declines.

Corporate Finance

While our model does not deal with individual securities, it suggests some consequences of noise for corporate financing decisions. In a model with noise traders the Modigliani-Miller theorem does not necessarily apply. To see why it might not, it is instructive to consider the standard "homemade leverage" proof of the theorem. This proof demonstrates that a rational investor can undo any effects of firm leverage and maintain the same real position regardless of a firm's payout policy. It does not suggest that less than rational traders will do so. Given that noise traders in general affect prices, it follows that unless they happen to trade so as to undo the effects of changes in leverage, the Modigliani-Miller theorem will not hold.

An example of failure of investors to trade so as to undo leverage is provided by the pricing of dual purpose investment companies (Litzenberger and Sosin (1977), Malkiel and Firstenberg (1978)). Dual purpose funds are mutual funds which divide their capital into equal numbers of preferred and common shares and invest in a diversified equity portfolio. The preferred stock receives all the dividend income generated by the portfolio and is redeemed at face value at a fixed liquidation date. The common stock receives no dividends while the preferred stock is outstanding, and then receives the balance of the portfolio on liquidation.

The fact that there is a fixed redemption date at which the common stock turns into the investment company's portfolio implies that at earlier dates the common stock is best interpreted as a leveraged claim on the portfolio. An investor purchasing equal fractions of the preferred and the common can undo the increase in leverage arising from the capital structure of the investment company. Yet the sum of the preferred and common of the dual purpose fund often sold much below the value of the fund's portfolio.[1] At the start of 1980 Malkiel recommended that investors purchase a number of dual-purpose funds that sold at discounts from net asset value of over twenty percent (Malkiel (1985)).

It is plausible to think that there is little opportunity for noise traders to become confused about the value and thus disturb the price of assets that have a certain and immediate liquidation value. Noise traders are most likely to become confused about assets that offer fundamentally risky payouts in the

[1] Interpreting the discount solely in terms of transactions or agency costs faces the previously-noted problem of explaining how these funds ever go public and why the discount fluctuates.

distant future. Assets of long duration which promise fundamentally uncertain as opposed to immediate and certain cash payouts may thus be subject to considerably more than their share of noise trader risk.

Consequently, if noise traders fail to pierce the corporate veil, the likely concentration of their misperceptions on assets of long duration makes the packaging of the firm's securities worthwhile. For example, the choice of a firm's debt equity ratio can matter. If noise traders do not misperceive the returns on debt as long as debt is low risk and yet do misperceive the returns on risky and long-duration equity, then keeping some but not too much debt in a firm's capital structure might allow it to receive the highest price for its securities. This might explain why managers appear extremely concerned with maintaining very high bond ratings (Donaldson (1984)).

If noise trader risk associated with a financial asset increases rapidly with the asset's duration, a firm might want to pay dividends rather than reinvest even if there are tax costs to doing so. If dividends can make equity look more like a safe short term bond to noise traders, then paying dividends might reduce the total amount of noise trader risk borne by a firm's securities and might raise the value of equity if the reduction in the discount entailed by noise trader risk exceeds additional shareholder tax liability. Moreover, dividends will not be equivalent to share repurchases unless noise traders perceive the two to be complete substitutes. If investors believe that future stock repurchases are of uncertain value because noise traders disturb the price of equity, then the equity of a firm repurchasing shares can be subject to greater undervaluation than that of a firm paying dividends. A bird in the hand is truly better than one in the bush.

Recent empirical evidence summarized by Jensen (1986) indicates that the more constrained is the allocation of the firm's cash flows, the higher is its valuation by the market. For example, share prices rise when a firm raises dividends, swaps debt for equity, or buys back shares. In contrast share prices fall when a firm cuts dividends or issues new shares. These results are consistent with our model if making the returns to equity more determinate can reduce the noise trader risk that it bears. Increases in dividends that make equity look safer to noise traders may reduce noise trader risk and raise share prices. Swaps of debt for equity will have the same effect, as will share buy backs. As long as a change in capital structure convinces noise traders that a firm's total capital is more like asset

(s) and less like asset (u) than they had previously thought, changes in capital structure will increase value.

Leveraged buyouts can be privately profitable in the presence of noise traders.[1] In these transactions the residual claim to the firm that is subject to noise trader risk is taken out of the equity market and held by management who presumably see only the true fundamental value of the firm and are uninterested in what the firm would bring on the market in a month or a year. If the debt of the now private firm is not traded and is hence free from noise trader risk, this debt plus managers' valuation of their shares are worth more than the debt and equity of the firm when it was public.

The above discussion makes it clear that noise trader risk is a cost that any issuer of a security to be traded in a public market must bear. Both traded equity and traded long term debt will be underpriced relative to fundamentals if their prices are subject to the whims of noise traders' opinions. Why then are securities traded publicly? Put differently, why don't all firms go private to avoid noise trader risk? Presumably firms have publicly traded securities if the benefits, such as a broader base from which to draw capital, a larger pool to use to diversify systematic risk, and liquidity, exceed the costs of the noise trader generated undervaluation. Assets for which these benefits of public ownership are the highest relative to the costs of noise trader risk are probably the assets that will be issued onto and traded on liquid markets. While the issuers of these securities will try to minimize the costs of noise trader risk by "packaging" the securities appropriately, they will not be able to eliminate such risk entirely.

Long Horizons

The presence of noise traders makes coherent and correct a widely-held view of the relative social merits of "speculation" and "investment" that has found little academic sympathy. Many active participants in financial markets have argued that the presence of traders who are looking only for short term profits is socially destructive. The standard economist's refutation of this argument relies on recursion: If one seeks to buy a stock now to sell in an hour, one must calculate its price in an hour. But its price in an hour depends on what those who will purchase it think its price will be a further

[1]Whether they are also socially productive depends on whether the removal of a firm from the market reduces noise trader risk or merely causes the transfer of noise trader risk to other securities.

hour down the road. Anyone who buys an asset -- no matter how short the holding period -- must perform the same present value calculation as someone who intends to hold the asset for fifty years. Since a linked chain of short term "traders" performs the same assessment of values as a single "investor", the claim that "trading" is bad and "investing" good is simply incoherent. Prices will be unaffected by the horizon of the agent as long as the rate of discount and willingness to bear risk are unchanged.

In our model this claim is not true. The horizon of agents matters. If agents live for more than two periods the equilibrium will be closer to the "fundamental" equilibrium then if agents live for two periods. As an example, consider an infinitesimal measure of infinitely lived but risk averse sophisticated traders. Suppose p_t is less than one. An infinitely lived agent can sell short a unit of (s) and buy a unit of (u). He collects a gain of $1-p_t$, and he has incurred no liability in any state of the world. The dividend on (u) will always offset the dividend owed on (s). The fact that an infinitely lived agent can arbitrage assets (s) and (u) without ever facing a settlement date implies that any infinitely-lived sophisticated investor could push the price of (u) to its fundamental value of one.

While long but finite lived agents do not have a riskless arbitrage opportunity, their asset demands are more responsive to price movements than those of two period lived agents. A young three period-lived sophisticated investor forbidden from entering the market in middle age will demand not $\lambda_t{}^i$ but instead:

$$(25) \qquad \lambda_t^i + r(1+r)\left(\frac{1-p_t}{2\gamma({}_t\sigma^2_{p_t})}\right)$$

purchasing more than a two period lived agent if p_t is below its fundamental value and going short more if p_t is higher than its fundamental value. Having a longer horizon allows one to engage in self-insurance by taking advantage of the fact that the two period-ahead price variance is no greater than the one period-ahead price variance. For the longer the holding period, the smaller the excess rate of return necessary to compensate for a given amount of price risk and the greater the chance to earn additional profits from market timing. Changing the maximum "horizon" of the agents in the model has real effects on the behavior of equilibrium prices because returns compound from period to period while price risk does not.

The embedding of the financial market in an overlapping generations model in which agents die after two periods is a device to give rational utility maximizers short horizons. Such a theoretical device may serve as an adequate way to model institutional features of asset markets -- triennial performance evaluations of pension fund money managers, for example -- that may lead even fully rational agents to have short horizons. Realistically, even an agent with a horizon long in terms of time may have a horizon "short" in the context of this model. If there is sufficient dividend risk and if noise trader misperceptions are persistent, then agents might well find it unattractive to buy stocks and hold them for very long periods in the hope that the market will someday recognize their value. For in the meantime, during which the assets might have to be sold, market prices may deviate even further from fundamental values. The claim that short horizons are bad for the economy is both coherent and true in our model.

It is nevertheless unclear that increasing the difficulty of transactions by imposing transaction taxes, and thus removing from the market those with short horizons, is a good idea. Transaction taxes do penalize those with short horizons. But such taxes also reduce the liquidity of each individual's investment. There are two wedges between the market price of capital goods and the fundamental value of their quasi rents: first, capital sells at a discount because it is subject to noise trader-generated price risk; second, capital sells at a discount because it is not as liquid as cash. It is not clear whether transactions taxes push q toward its fundamental value, for they would tend to reduce the first wedge and increase the second, as Keynes (1936) noted.[1]

V. CONCLUSION

The analysis in this paper suggests that traditional objections to introducing noise traders into models of financial assets are ill-founded. Certain types of noise traders are likely to flourish and

[1] General Theory, p. 170: "The spectacle of modern investment markets has sometimes moved me towards the conclusion that to make the purchase of an investment permanent and indissoluble, like marriage... might be a useful remedy for our contemporary evils. For this would force the investor to direct his mind to the long-term prospects and to those only. But a little consideration of this expedient brings us up against a dilemma, and shows us how the liquidity of investment markets often facilitates, though it sometimes impedes, the course of new investment. For the fact that each individual investor flatters himself that his commitment is "liquid" (though this cannot be true for all investors collectively) calms his nerves and makes him much more willing to run a risk."

grow in importance even when rational speculators optimally take advantage of their mistakes. The presence of noise traders and the recognition by sophisticated investors of their presence can together account for a variety of financial market phenomena, including excess price volatility, deviations of average market prices from fundamental values, as well as linkages between speculative prices and investment and financing decisions.

A number of studies, notably Grossman and Stiglitz (1980), Pfleiderer (1984), and Stein (1987) have constructed models of trade among differentially informed agents that are formally similar to the model developed here. In these models there are no irrational traders, but traders have different pieces of information. While the issue is in part semantic, we regard our model with noise traders as more realistic. As the term is conventionally used in economics, someone is "irrational" if two people form different expectations on the basis of the same information. Unless the concept of private information is tautologically equated with one's having one's own distinct opinion, it is hard to see how the assumption of universal rationality can be maintained for all traders in broad markets like those for treasury securities, foreign exchange, and market indices.

Many economists appear to resist the introduction of irrational agents into economic models because of a conviction that the overwhelming comparative advantage of the economics profession lies in the analysis of rational utility-maximizing behavior. We share this belief. Economists have illuminated a wide range of phenomena by placing rational agents in environments that limit in a variety of ways their ability to trade and that afford them different types of technological opportunities. Our analysis has concentrated on describing the interactions that result when maximizing rational investors are placed in an environment where they have the opportunity to trade with persons holding irrational beliefs. Exploring the limits of arbitrage in such settings seems to us to be entirely within the bounds of the conventional economic approach and to take the concept of arbitrage very seriously indeed.

We hope to have demonstrated that a theory in which noise is important is not a theory in which anything can happen; rather, a theory in which noise is important is a theory in which noise creates identifiable consequences. Many theories in the natural sciences -- the ideal gas law and evolution by natural selection come to mind -- derive regular and observable consequences from inescapable random noise. The introduction of noise into models of financial markets has the potential to enlarge the scope of phenomena for which we can give scientific explanations. Taking noise seriously may bear fruit even if economists are never successful at generating predictive theories of its content. We suspect, however, that the experimental evidence on behavior under uncertainty will ultimately enable us to understand the behavior now labelled "noise."

APPENDIX I

FUNDAMENTAL RISK AND INHERITANCE OF BELIEFS

The introduction of fundamental risk changes the conclusions reached by analyzing the "inheritance of beliefs but not wealth" model with imitation of successful types of agents. Instead of the noise trader population tending toward either zero or one depending on the initial state, it now either settles down to a stable equilibrium μ_L or expands until there are no sophisticated investors remaining. The addition of fundamental risk makes it more likely that noise traders earn higher expected returns. Specifically, small populations of noise traders are guaranteed to earn higher expected returns than sophisticated investors.

We first trace how the introduction of normally distributed dividend risk changes the behavior of asset prices, and we then investigate the changed dynamics of the "inheritance of beliefs but not wealth" extension of the basic model. Let asset (u) pay not a certain dividend r but an uncertain dividend:

$$(A1) \qquad r + \varepsilon_t$$

where ε_t is independent and normally distributed with zero mean and constant variance. Asset demands become:

$$(A2) \qquad \lambda_t^i = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma({}_t\sigma^2_{p_{t+1}} + 2\,{}_t\sigma_{\varepsilon_{t+1}p_{t+1}} + \sigma^2_\varepsilon)}$$

$$(A3) \qquad \lambda_t^n = \frac{r + {}_tp_{t+1} - (1+r)p_t}{2\gamma({}_t\sigma^2_{p_{t+1}} + 2\,{}_t\sigma_{\varepsilon_{t+1}p_{t+1}} + \sigma^2_\varepsilon)} + \frac{\rho_t}{2\gamma({}_t\sigma^2_{p_{t+1}} + 2\,{}_t\sigma_{\varepsilon_{t+1}p_{t+1}} + \sigma^2_\varepsilon)}$$

instead of (7) and (8). The only change is the appearance in the denominators of the asset demand functions of the total risk involved in holding asset (u) -- the sum of noise trader price risk and fundamental dividend risk -- instead of simply noise trader-generated price risk.

The pricing function with fundamental risk is transformed from (12) into:

$$(A4) \qquad p_t = 1 + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}\left(\sigma^2_\varepsilon + \frac{2\mu\sigma_{\varepsilon\rho}}{1+r} + \frac{\mu^2\sigma^2_\rho}{(1+r)^2}\right) + \frac{\mu(\rho_t - \rho^*)}{1+r}$$

The price risk term is replaced by the total risk associated with holding (u). The difference between expected returns to noise traders and to sophisticated investors becomes:

$$\text{(A5)} \qquad E(\Delta R_{n-i}) = \rho^* - \frac{\mu\left\{\rho^{*2} + \sigma_\rho^2\right\}}{2\gamma\left(\sigma_\varepsilon^2 + \frac{2\mu\sigma_{\varepsilon\rho}}{1+r} + \frac{\mu^2\sigma_\rho^2}{(1+r)^2}\right)}$$

Adding fundamental risk to the basic model enlarges the range of parameter values over which noise traders earn higher returns than sophisticated investors. While the "hold more," "average price pressure," and "buy high-sell low" effects are not changed by the addition of fundamental risk, the "create space" effect -- the denominator of (A5) -- is increased. Since there is more risk involved in holding asset (u), sophisticated investors are now less willing to trade in order to exploit noise traders' mistakes. The long positions that noise traders take in the risky asset on account of their average "bullishness" are more highly rewarded.

The introduction of fundamental risk alters the dynamics of the model with inheritance of beliefs but not wealth. When the noise trader share μ_t is zero, (A5) is clearly positive. If the proportion of noise traders is small, they have no significant effect on prices. Since on average they hold more of the high-yielding risky asset they will earn higher average returns. When the noise trader share is very large, (A5) is also positive. If there are a larger number of noise traders, they have a significant effect on prices and their demands move prices against them. They buy high and sell low. But their stochastic actions also begin to significantly increase the risk borne by those who hold the asset (u). As risk increases with μ_t^2, the "create space" effect which makes sophisticated investors unwilling to take positions in the risky asset eventually dominates and ensures that noise traders earn higher returns if μ_t is large.

For intermediate values of μ_t, either noise traders or sophisticated investors may have higher expected returns. Setting equation (A5) equal to zero implicitly describes a quadratic equation in μ_t. There are either two or zero values of μ_t that solve $E\{_t(\Delta R_{n-i})\}=0$. If there are no real roots, noise traders will make more money than sophisticated investors and will take over the market no matter how many noise traders there are (see figure 2).[1]

By contrast, when the implicit quadratic in μ_t does have real roots, there will be a stable equilibrium value of μ_t (as shown in figure 3) in the model with inheritance of beliefs but not wealth. This value, the lower of the two roots of the implicit quadratic in μ_t, will govern the long run share of the population who are noise traders if the initial proportion of agents who are noise traders is sufficiently below the upper root μ_H. If μ_t is higher than the upper root, then the proportion of the population who are noise traders will tend to increase until all maximizers are once again driven out in the model with inheritance of beliefs. The large amount of noise trader created risk keeps sophisticated investors from taking positions in the risky asset large enough to give sophisticated investors higher expected returns.

[1]As long as noise traders are bullish a homogeneous population of sophisticated investors is not evolutionarily stable. If the set of noise traders is of measure zero, then noise traders always earn higher returns in expected value than sophisticated investors if there is fundamental risk in the model. But if the set of sophisticated investors is of measure zero, then it is not always the case that sophisticated investors earn higher returns in expected value than noise traders.

APPENDIX II

BOUNDED DISTRIBUTIONS OF ASSET PRICES AND THE EXISTENCE OF EQUILIBRIUM

The model presented in the text assumes that asset prices are normally distributed and that dividends are certain. In reality dividends are not certain, and limited liability prevents asset prices from having an unbounded distribution. In this appendix we consider the consequences of the removal of the assumptions that there is no fundamental risk and that asset prices have an unbounded distribution. The first part discusses how the absence of fundamental risk causes equilibrium to fail to exist in the basic model if prices are bounded. The second part presents a model in which prices are bounded, and yet equilibrium exists and noise traders have expected returns as high as do sophisticated investors.

Fundamental Risk and Market Equilibrium

The assumption that returns are normally distributed implies that the linear mean-variance approximation to the constant absolute risk aversion utility function is exact. More importantly, the fact that normally distributed asset prices are unbounded is essential for the existence of equilibrium in the basic model.

Suppose that the distribution of the price of (u) is bounded by p_L and p_H and that p_t is close to p_L. Then a lower bound to the gross rate of return from holding (u) will be equal to: $1 + (r/p_L)$. If p_L is less than one, this lower bound on the distribution of one-period returns will be larger than the sure return on the safe asset (s). Asset (u) will dominate asset (s). Rational sophisticated investors will go infinitely short asset (s) and infinitely long asset (u). Therefore if the distribution of p_t in equilibrium is bounded below, it must be that p_L is greater than or equal to one. Similarly, if the safe asset (s) is not to dominate the risky asset (u) in a neighborhood of p_H, the least upper bound p_H to the distribution of p_t must be less than or equal to one.

Section one of the text showed that setting p_t always equal to one cannot be an equilibrium of the basic model, since the noise traders' demand for the risky asset is then unbounded if their misperception of returns is positive. The existence of a floor below which the price cannot drop implies that when the price approaches the floor, fall beneath $(p_L+r)/(1+r)$ there is a riskless arbitrage opportunity open to sophisticated investors. Since every bounded distribution of prices has a positive chance of reaching states in which a sophisticated investor has a riskless arbitrage opportunity, equilibrium simply does not exist if the distribution of prices is bounded.

A Model with Bounded Prices

The chain of iterative arbitrage arguments that unravels equilibrium if asset prices are bounded can be broken by introducing any mechanism that will keep asset demands bounded when the price of the risky asset approaches its limit. One such mechanism would be the presence of any kind of fundamental risk. A second such mechanism would be the existence of binding margin requirements. And a third would be restrictions on short sales. The latter two mechanisms are themselves easily motivated by the unwillingness of lenders of money or stock to themselves bear any of the risk associated with their debtors' portfolios.

As an example of how these mechanisms can allow the existence of equilibria in which noise traders earn high relative expected returns and prices are never negative, we present a simple model in which fundamental risk plays the role of limiting sophisticated investors' demands for the risky asset. In this model the price of asset (u) takes on only two possible values and noise traders and sophisticated investors earn identical expected returns. Moreover, the presence of noise traders is the source of all variance in the price of the risky asset and of an arbitrarily large share of the variance in the rates of return earned by investments in the risky asset. This model is contrived to serve only the limited purpose of providing a counterexample to the view that our results rely on either the possibility of prices becoming negative or the relative insignificance of noise trader compared to fundamental risk.

Let there be a fixed quantity, one unit, of asset (u) and let asset (s) be in elastic supply at a price of one. Let both assets pay a dividend r. And let there be a chance ε that in any period holdings of the risky asset becomes worthless: the firms that (u) is a claim on are confiscated, shareholders' claims are extinguished, and ownership of asset (u) is transferred to the government and then sold to the young.

Let sophisticated investors and noise traders be endowed when young with wealth w_0, and let both noise traders and sophisticated investors be present in measure 1/2. The utility function of sophisticated investors is taken to be any concave function of wealth when old that satisfies:

$$\text{(A6)} \qquad \lim_{w \to \theta(1+r)} U'(w) = \infty$$

where the level of consumption when old $\theta(1+r)$ is the "subsistence" level of consumption which sophisticated investors are unwilling to risk being unable to attain. The existence of a wealth at which $U'(w)$ becomes infinite and the ε chance of confiscation of holdings of the risky asset together serve to bound sophisticated investors' asset demands. These features perform the same function as wold be performed by margin requirements or restrictions on short sales.

Let noise traders be with probability 1/2 optimistic, in which case they demand a fixed quantity $\lambda^n{}_H$ of the risky asset and the price of the risky asset is at its high value of p_H. With probability 1/2 noise traders are pessimistic, in which case they demand a fixed quantity $\lambda^n{}_L$ of the risky asset and the price of the risky asset is at its low value of p_L. The quantity $\lambda^n{}_H$ is set equal to two so that when optimistic the 1/2 measure of noise traders hold the entire unit of the risky asset. In order for supply to equal demand, sophisticated investors must wish to go neither long nor short in the risky asset, which must therefore pay an expected return when noise traders are optimistic equal to the riskless rate:

$$\text{(A7)} \qquad p_H = \frac{(p_L + 2r)(1 - \varepsilon)}{1 + 2r + \varepsilon}$$

When noise traders are optimistic there are no excess expected returns. All portfolios pay an expected rate of return of r. Noise traders and sophisticated investors will thus have equal unconditional expected returns if they hold equal amounts of the risky asset when noise traders are pessimistic, if:

$$\text{(A8)} \qquad \lambda^i_L = \lambda^n_L = 1$$

Consider a sequence of economies, otherwise alike in structure, for which the probability ε that the holdings of the risky asset will become worthless converges to zero. The values of prices, asset holdings, and expected returns will also converge to limit values.[1] If the amount of wealth that sophisticated investors have available for speculation $w_0 - \theta$ is greater or equal to one, then as the chance of confiscation ε approaches zero $\lambda^i{}_L$ can be equal to one only if the price p_L of the risky asset

[1]Which are not the values taken on in the limit economy. The limit economy, for which ε equals zero, has the price of the risky asset always equal to one in equilibrium.

when noise traders are pessimistic approaches one. Sophisticated investors' demands will be such as to eliminate noise trader risk. As long as ε is small they will buy and sell asset (u) to keep its price always close to one.

If the amount of wealth that sophisticated investors have available for speculation $w_0-\theta$ is less than one, then sophisticated investors' expenditures on the risky asset will be bounded above by $w_0-\theta$. Investing more would force sophisticated investors' consumption below subsistence if holdings of the risky asset did become worthless. As the chance of confiscation ε approaches zero, the price of the risky asset when noise traders are pessimistic approaches:

$$p_L^* = w_0 - \theta \tag{A9}$$

In this case the price of the risky asset when noise traders are optimistic approaches:

$$p_H^* = \frac{(w_0 - \theta) + 2r}{1 + 2r} \tag{A10}$$

Note that the smaller is θ, and hence the larger is the amount of wealth sophisticated investors are willing to commit to speculation, the smaller is the difference between the limits of the prices of (u) in the two states and the closer do the limits of the prices of (u) approach one, which is the limit of the fundamental value of (u) as the chance of confiscation ε approaches zero.

In this formulation as specified, noise traders and sophisticated investors earn identical expected returns. Noise traders would earn higher expected returns if they demanded a little less than all of the risky asset when optimistic. The sophisticated investors would then hold a positive share of the risky asset, which they would be willing to do only if it paid an expected return higher than the riskless rate. Noise traders would also earn higher expected returns if there were additional states in which noise traders held intermediate beliefs about the desirability of the risky asset.

In this model, all variance in the price of the risky asset is due to the presence of noise traders. And an arbitrarily large share of the variance in the rate of return of the risky asset is due to the presence of noise traders. As ε approaches zero, the contribution of fundamental risk to total rate of return variance also approaches zero; fundamental risk is the risk of an event which could be so unlikely as to almost certainly not occur in the sample available to the econometrician. Fundamental risk, however, continues to constrain the behavior of sophisticated investors because it remains significant in utility terms even as it becomes infinitesimal in terms of its contribution to the variability of the rate of return earned by the risky asset.

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FIGURE 1
DYNAMICS OF THE NOISE TRADER SHARE OF THE POPULATION WITH NO FUNDAMENTAL RISK

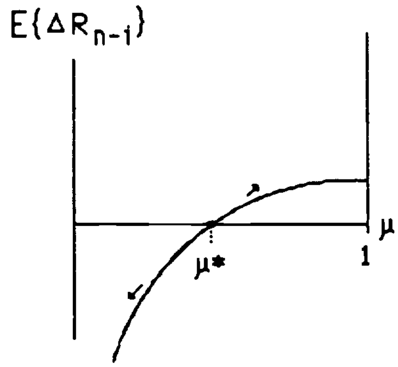

FIGURE 2
DYNAMICS OF THE NOISE TRADER SHARE OF THE POPULATION WITH NO EQUILIBRIA

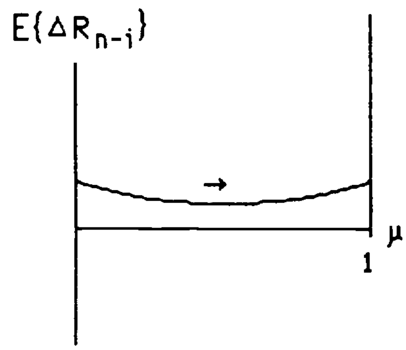

FIGURE 3
DYNAMICS OF THE NOISE TRADER SHARE OF THE POPULATION WITH TWO EQUILIBRIA