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ABSTRACT

Expectations of bailouts by central governments incentivize overborrowing by local governments. In this paper, we ask if fiscal rules can correct these incentives to overborrow when central governments cannot commit and if these rules will arise in equilibrium. We address these questions in a reputation model in which the central government can either be a commitment or a no-commitment type and the local governments learn about this type over time. We find that if the central government's reputation is low enough, then fiscal rules can lead to even more debt accumulation relative to the case with no rules. This is because the punishment for violating the fiscal rule worsens the payoffs of preserving reputation. Despite being welfare reducing, binding fiscal rules will arise in the equilibrium of a signaling game due to the incentives of the commitment type to reveal its type.

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1 Introduction

There are numerous examples throughout history in which excessive spending and debt accumulation by subnational governments led to bailouts by central governments. Examples include provinces in Argentina, states in Brazil, länders in Germany, and most recently countries (Greece, Ireland, and Portugal) in the European Union.[1] One view of such events is that the lack of commitment of central governments to not bail out leads to profligating fiscal policies ex-ante, which in turn justifies the bailouts ex-post. This idea has been formally studied by Chari and Kehoe (2007), Chari and Kehoe (2008), and Cooper et al. (2008) in the economics literature and Rodden (2002) in political science. See also Sargent (2012).

A commonly held view is that *fiscal rules* can correct these incentives to overborrow when central governments lack commitment. In practice, fiscal rules take the form of limits to debt-to-GDP or deficit-to-GDP ratios along with some penalty if these are violated. For example, the Stability and Growth Pact (SGP) calls for all EU member countries to keep budget deficits below 3% of GDP and public debt to below 60% of GDP. EU member countries are liable to financial penalties of up to 0.5% of GDP if they repeatedly fail to respect these limits.

When thinking about the design of fiscal rules, a natural question that arises is why central governments can commit to enforcing these rules if they cannot commit to not bail out. In this paper, we ask if fiscal rules can be beneficial if central governments cannot commit and if these rules will arise in equilibrium. We address these questions in a reputation model in which the type of the central government is uncertain: it can be either a commitment type or a no-commitment type. The reputation of a central government is the probability that local governments assign to it being a commitment type. In the tradition of Kreps and Wilson (1982) and Milgrom and Roberts (1982), we focus on the case in which there is a small initial probability that the central government is the commitment type.

Our first main result is that if the reputation of the central government is low enough, then fiscal rules are welfare reducing and lead to even more debt accumulation relative to the case with no rules. This is because the punishment associated with the fiscal rule enforcement makes it more attractive for the no-commitment type to reveal its type earlier relative to an environment without rules. This early resolution of uncertainty makes overborrowing more attractive for the local governments. Our second main result is that despite being welfare reducing, binding fiscal rules can arise in an equilibrium of a signaling game because the commitment type wants to signal its type and it is optimal for the no-commitment type to initially mimic and then not enforce the rule once violated.

[1]See Rodden et al. (2003), Rodden (2006), and Bordo et al. (2013) for further documentation.

We show these results in a stylized three-period model populated by local governments and a benevolent central government. Local governments choose the provision of a local public good and have access to local tax revenues. They can also borrow from the rest of the world at a given interest rate. We first consider the case in which local governments are homogenous and then assume that they can be one of two types: the North and the South. In the latter case, we assume that the North has access to a larger period 0 tax revenue, which leads to a non-degenerate distribution of debt holdings in period 1 along the equilibrium path. The central government does not have tax revenues, but it can impose transfers from one state to another. We consider an institutional setup in which the constitution requires the central government to not impose such transfers (*no-bailout clause*) and local governments to keep their debt below some level or face an output cost if they violate this rule (*fiscal rule*).

The central government can either be a commitment type that enforces the fiscal constitution or a no-commitment type that chooses its policy sequentially. This type is initially unknown to the local governments, which learn about it through the actions of the central government. In period 1 (the intermediate period), the benevolent no-commitment central government faces a trade-off between not enforcing the constitution and preserving its reputation, which incentivizes the local governments to keep future debt accumulation in check.

We first consider the case in which the constitution contains only a no-bailout clause and no fiscal rules. We show that when the central government's initial reputation level is low enough, there is a unique equilibrium in which the no-commitment type central government does not make transfers to the local governments in the intermediate period and so there is no revelation of uncertainty until the terminal period. The central government prefers to delay the revelation of its type because for low enough reputation levels, the costs of early information revelation are first order, while the benefits of equalizing the provision of the local public good in the interim period via a bailout are second order. When local governments are homogenous, these costs are exactly zero on path. When local governments are heterogeneous, the distribution of debt inherited in the interim period is non-degenerate and so these costs are positive. However, if the probability of facing the commitment type is close to zero, the provision of the local public good in the North and the South is almost identical even without a bailout in the interim period, because the South borrows against the bailout transfer it anticipates in the final period.

We next consider a constitution with both a no-bailout clause and a fiscal rule. If the central government's reputation and discount factor are low enough, there exists a unique equilibrium in which fiscal rules are violated in period 0 by the local governments and are not enforced ex-post by the no-commitment type central government. Therefore, in this equilibrium there is *early resolution of uncertainty* (i.e., the central government reveals its type in period 1). The intuition behind this result is that with fiscal rules, the value of preserving reputation is lower, since the enforcement of the constitution now requires the no-commitment type central government to impose costly penalties on the local governments that violate the rule. In particular, unlike in the case without fiscal rules, the costs of enforcing the constitution are no longer second order.

We then compare the debt levels in the equilibrium outcomes with and without rules. Having fiscal rules in the constitution leads to even more debt accumulation relative to the case without rules. The key driver for this result is that the type of the central government is revealed in the interim period with rules (early resolution of uncertainty), and only in the terminal period without rules (late resolution of uncertainty). Knowing the type of the central government in period 1 allows the local governments to condition their new debt issuances on the government type. This in turn lowers the cost of servicing the debt inherited in period 1; hence, the local governments will issue more debt in period 0.

We next consider a planner tasked with designing the optimal fiscal rule taking into account this lack of commitment. We show that if the prior of the central government being the commitment type is low enough, it is strictly optimal to not have fiscal rules.

The previous result raises the question of why we would ever see fiscal rules being instituted in practice if they were welfare reducing. We study a signaling game in which rules are chosen at the beginning of time by the central government. We show that for intermediate values of the central government's discount factor, in the equilibrium of this game, the commitment type chooses to announce a fiscal rule, which is mimicked by the no-commitment type. However, in this equilibrium the rule is not enforced in period 1 by the no-commitment type, leading to early resolution of uncertainty and even more debt accumulation.

This result sheds light on historical and contemporary episodes when fiscal rules were instituted but were not enforced ex-post. A leading example is the SGP in the Eurozone. The SGP was instituted for the newly formed monetary union, under the pressure of Germany, with the intent of constraining fiscal policy in member countries to insulate the European Central Bank (ECB) from the pressure to inflate or monetize the debt of member countries. However, the enforcement of the SGP has been very lax. For example, in 2003 both Germany and France violated it and sanctions were not imposed. Moreover, the sanctionary powers of the European Commission were subsequently weakened. Through the lens of our theory, this corresponds to the case in which the central government reveals its type in the intermediate period. Consistent with our theory, after 2003, the power of the SGP in disciplining fiscal policy was arguably weakened. According to several commentators, this was a major factor in the current European debt crisis in which Greece, Ireland, and Portugal received bailout packages from the European Union and the ECB (the central government), as our theory predicts.

Arguably, after the bailouts to peripheral member countries, the reputation and credibility of the central European institutions were very low. EU member countries and European institutions agreed to impose tough fiscal rules by strengthening the SGP by introducing the so-called "Six-Pack" and "Fiscal Compact" consistent with the prediction of our signaling game. The provisions of the "Six-Pack" were soon violated by Spain and Portugal without any sanction being levied.[2] In 2016 the governor of the Bundesbank, Jens Weidmann, accused the Commission of not enforcing the fiscal rules: "My perception is that the European Commission has basically given up on enforcing the rules of the Stability and Growth Pact."[3]

Another leading example of federal governments with poor fiscal discipline among subnational governments is Brazil, the most decentralized state in the developing world. The fiscal behavior of the states and large municipal governments in Brazil were a major source of macroeconomic instability and resulted in subnational debt crises in 1989, 1993, and 1997. "The federal government took a variety of measures to control state borrowing in the 1990s, and at a first glance it would appear to have had access to an impressive array of hierarchical control mechanisms through the constitution, additional federal legislation, and the central bank. Most of these mechanisms have been undermined however, by loopholes or bad incentives that discourage adequate enforcement" (Rodden et al. (2003) page 222).

In 1997, the federal government assumed the debts of 25 of the 27 states that were unable to service their debt—an amount equivalent to about 13% of GDP. By September 2001, 84% of state debt was held by the national treasury (see Rodden et al. (2003), page 234). After the bailouts in 1997, the Cardoso administration approved the Fiscal Responsibility Law, which instituted "a rule-based system of decentralized federalism that leaves little room for discretionary policymaking at the subnational level. It has been motivated by the recognition that market control over subnational finances should be replaced, or strengthened, by fiscal rules as well as appropriate legal constraints and sanctions for noncompliance, Afonso and De Mello (2000)." So, in a manner similar to Europe, the central government in Brazil imposed stringent fiscal rules when its reputation was arguably low.

Related literature Our paper is related to several strands of literature. First, it is related to the literature that studies the free-rider problem in federal governments when the central government cannot commit (e.g., Chari and Kehoe (2007), Chari and Kehoe (2008), Cooper et al. (2008), Aguiar et al. (2015), Chari et al. (2016), and Rodden (2002)). The main result in this literature is that the inability of the central government (or monetary

[2]See https://www.ft.com/content/f66a5c1d-b023-3d0f-ad02-767a9656d4f9

[3]See https://www.ft.com/content/95e7ee7e-ad8e-11e6-ba7d-76378e4fef24.

authority) to commit not to bail out ex-post leads to overborrowing ex-ante. In such settings, it is often argued that fiscal rules can improve outcomes by lowering the amount of debt issued (e.g., Beetsma and Uhlig (1999)). Our paper contributes to this literature by analyzing the effects of fiscal rules when the government cannot commit to enforcing them.

Fiscal rules have been studied in several environments as the solution to time inconsistency problems. See for instance Athey et al. (2005), Amador et al. (2006), Halac and Yared (2014), and Halac and Yared (2017) in the context of delegation, and Hatchondo et al. (2015) and Alfaro and Kanczuk (2016) in the context of sovereign default. All these papers assume that the agents can commit to rules and do not analyze the enforcement problem, which is the main focus of our paper.

The baseline model uses a reputational setup similar to Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982) with uncertainty about the type of the central government. It also relates to papers that try to account for several features of policy outcomes by studying models in which a government with a hidden type interacts with a continuum of private agents (e.g., Cole et al. (1995), Phelan (2006), and D'Erasmo (2008)). In contrast, in our paper the local governments are strategic and can incentivize the central government to reveal its type via its actions. In addition, we also study the optimal policy in this environment and how varying the costs of maintaining good reputation affects outcomes.

Uncertainty about the type of the central government plays a key role in the provision of incentives to local governments. Nosal and Ordoñez (2013) also consider an environment in which uncertainty can mitigate the time inconsistency problem when a central government cannot commit not to bail out banks. The mechanism is very different: here uncertainty about the type of the central government curbs debt issuances by the local governments, while in their paper it is the uncertainty about banks (local governments) that restraints the central government to not intervene ex-post.

The rest of the paper is organized as follows. In Section 2 we present the model, and in Section 3 we show that fiscal rules promote fiscal indiscipline when local governments are homogeneous. Section 4 demonstrates that our results extend to the case with heterogenous local governments. Section 5 discusses the role of having large (strategic) local governments. In Section 6, we show that imposing no rules is optimal under the veil of ignorance, and in Section 7 we show that rules can arise in the equilibrium of a signaling game. Section 8 concludes the paper.

2 Model

Environment

Let $t = 0, 1, 2$.[4] Consider a small open economy consisting of N states or regions indexed by $i \in \{1, 2, ..., N\}$. The representative citizen in region i has preferences over the local public good provision $\{G_{it}\}$

$$U_i = \sum_{t=0}^{2} \beta^t u(G_{it}).$$

Throughout we make the following assumptions:

Assumption 1. *The period utility function u is strictly increasing, strictly concave, $u \in C^1$, $\lim_{c \to 0} u'(c) = \infty$, and $u(0)$ finite.*

The local public good provision is decided by a benevolent *local government* with local tax revenues $\{Y_{it}\}$. We start by characterizing the simplest possible case, $Y_{it} = Y$ for all i and t. We will later show that our results generalize to cases when tax revenues are heterogeneous across regions and vary over time. The local government can borrow from the rest of the world at a rate $1 + r^*$. Let $q = 1/(1 + r^*)$ be the price of a bond that promises to pay one unit of the consumption good next period. There is also a *central government*. The central government does not have tax revenues, but it can impose transfers from one region to another subject to a budget constraint

$$\sum_{i=1}^{N} T_{it} \leqslant 0,$$

where T_{it} is the transfer to region i in period t.

Efficient allocation

As a benchmark, we consider the efficient allocation in this environment. An allocation is efficient if for some set of Pareto weights $\{\lambda_i\}$ it solves

$$\max_{\{G_{it}\}} \sum_{i=1}^{N} \lambda_i \sum_{t=0}^{2} \beta^t u(G_{it})$$

subject to

$$\sum_{t=0}^{2} \sum_{i=1}^{N} q^t [G_{it} - Y_{it}] \leqslant 0. \tag{1}$$

[4] Our main results extend to any finite horizon economy.

Any efficient allocation must satisfy

$$qu'(G_{it}) = \beta u'(G_{it+1}) \tag{2}$$

and the consolidated budget constraint (1) with equality.

Institutional setup and equilibrium

Consider an institutional setup in which the central government is subject to a fiscal constitution. The fiscal constitution contains two clauses. The first clause states that the central government should not bail the regions out (i.e., $T_{it} = 0$ for all i, t). We call such a provision the *no-bailout clause*. The second clause requires the local governments to keep their debt issued in period 0 below a cap $\bar{b}$. In case $b_{i1} > \bar{b}$, the central government must impose a penalty ψY on the region that violated the rule. We assume that the resources collected from penalties are thrown away.[5] We call this constitutional provision a *fiscal rule*. A fiscal rule is then fully described by $(\bar{b}, \psi)$. To simplify notation, we abstract from a cap on debt issued in period 1 and its associated penalty. All our propositions will extend to the case with a cap on debt issued in period 2.

The central government can be one of two types: a *commitment type*, which follows the prescriptions in the constitution, and a *no-commitment type*, which is not bound to follow the prescriptions of the constitution, as it chooses policies sequentially to maximize an equally weighted average of the utility of citizens in both regions:[6]

$$W_r = \sum_{t \geqslant r}^{2} \frac{1}{N} \sum_{i=1}^{N} \beta^t u(G_{it}).$$

The type of the central government is drawn at the beginning of period 0 and is not known to the local governments. They have a common prior π that the central government is the commitment type. Throughout the paper we consider the probability of facing the commitment type as being close to zero. Local governments cannot rule out the possibility that the central government might always enforce the constitution, no matter what the state of the world is. This is consistent with Kreps and Wilson (1982) and Milgrom and Roberts (1982), who assume that players are "behavioral" with an arbitrarily small probability.

The timing is as follows:

[5] This assumption ensures that the cost of imposing the fiscal rule is nonzero for the central government even if $\pi = 0$.

[6] The redistribution motive generates an incentive for the central government to bail out the local government with higher debt. We would obtain similar results if bailouts were motivated by spillovers, as in Tirole (2015).

- At $t = 0$, the local governments choose the local public good provision G_{i0} and debt b_{i1} subject to the budget constraint

$$G_{i0} \leqslant Y_{i0} + qb_{i1}.$$

- At $t = 1$, if the central government is the no-commitment type, it decides whether to make transfers $\{T_{i1}\}$ or not and whether to enforce the penalty if the fiscal rule is violated by a local government. After observing the central government's actions, the local governments update their prior about the central government type and decide the provision of the local public good G_{i1} and new debt issuance b_{i2} subject to

$$G_{i1} + b_{i1} \leqslant Y + T_{i1} + qb_{i2} - \psi_1 Y \mathbb{I}_{\{b_{i1} > \bar{b}_1 \text{ and central government enforces fiscal rule}\}}.$$

- At $t = 2$, if the central government is the no-commitment type, it decides whether to make a transfer $\{T_{i2}\}$ or not. Next, the local governments choose G_{i2} subject to budget constraints

$$G_{i2} + b_{i2} \leqslant Y + T_{i2}.$$

We assume that the local government can commit to repaying its debt. This can be motivated by the existence of high default costs, which makes repayment always optimal for the local government.

We now define the states, payoffs, and beliefs at each node of the game tree.

Period 2 The state in the last period is the distribution of debt among local governments, $b_2 = (b_{i2})_{i \in \{1,2,\ldots,N\}}$. If the central government is the no-commitment type, it will choose transfers $T_{i2}(b_2)$ such that the consumption of the local public good is equalized between regions[7]: $T_{i2}(b_2) = b_{i2} - \frac{\sum_{j=1}^{N} b_{j2}}{N}$ so that

$$G_{i2} = Y - \frac{\sum_{j=1}^{N} b_{j2}}{N},$$

and it will not impose the penalty if the fiscal rule is violated. We refer to this situation as *debt mutualization*. The value for the central government is

$$W_2(b_2) = \sum_{i=1}^{N} \frac{1}{N} u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right),$$

[7]Note that there is no benefit to preserving reputation, since the world ends after period 2.

and the value for a local government is

$$V_{i2}\left(b_2\right) = u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right).$$

If instead the central government is the commitment type, each region will consume $G_{i2} = Y - b_{i2}$. The value for the local government is then

$$V_{i2}^{c}\left(b_2\right) = u\left(Y - b_{i2}\right).$$

Period 1 The state in period 1 is the distribution of debt among the local governments, $b_1 = \left(b_{i1}\right)_{i\in\{1,2,\ldots,N\}}$ and the prior on the type of the central government, π. Let σ be the equilibrium strategy of the central government in period 1. The central government can either enforce the fiscal constitution or not.[8] We consider equilibria where the law of motion for beliefs follows Bayes' rule and is given by

$$\pi'\left(b_1, \zeta, \pi; \sigma\right) = \begin{cases} \frac{\pi}{\pi + (1-\pi)(1-\sigma(b_1,\pi))} & \text{if } \zeta = 0 \\ 0 & \text{if } \zeta = 1 \end{cases}, \tag{3}$$

where $\zeta = 1$ if the central government does not enforce the fiscal constitution in period 1, and σ denotes the enforcement strategy for the central government and is defined by

$$\sigma\left(b_1, \pi, \psi; \pi'\right) = \begin{cases} 0 & W_1^{e}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) > W_1^{ne}\left(b_1\right) \\ 1 & W_1^{e}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) < W_1^{ne}\left(b_1\right) \\ 0 < \tilde{\sigma} < 1 & W_1^{e}\left(b_1, \pi'\left(b_1, 0, \pi; \sigma\right), \psi\right) = W_1^{ne}\left(b_1\right) \end{cases}, \tag{4}$$

where $\sigma = 1$ means that the constitution is not enforced, while $\sigma = 0$ denotes enforcement; W_1^{e} is the value for the no-commitment type central government if it enforces the fiscal constitution in period 1, and W_1^{ne} is the value for the no-commitment type central government if it does not enforce the fiscal constitution in period 1. We will describe these value functions in detail in what follows.

We now analyze the decision of the local governments. Suppose first that there is enforcement so that the posterior of the central government's type remains constant at π, $\pi'\left(b_1, 0, \pi; \sigma\right) = \pi$. In this case, the local governments choose G_{i1}, b_{i2} to solve

$$V_{i1}^{e}\left(b_1, \pi\right) = \max_{G_{i1}, b_{i2}} u\left(G_{i1}\right) + \beta\pi V_{i2}^{c}\left(b_{i2}\right) + \beta\left(1-\pi\right) V_{i2}\left(b_{i2}, \left(b_{j2}\left(b_1, \pi\right)\right)_{j\neq i}\right) \tag{5}$$

[8] To ease notation, we exclude the case in which the central government enforces only one of the provisions of the fiscal constitution. This is without loss of generality, since it will never be optimal for the central government to do so.

subject to

$$G_{i1} + b_{i1} \leqslant Y_{i1} + qb_{i2} - \psi Y \mathbb{I}_{\{b_{i1} > \bar{b}\}}$$

taking as given the strategy $b_{j2}(b_1, \pi)$ followed by the other local governments.

For later reference, the equilibrium outcome at this node will be given by $\{\mathbf{b}_{i2}(b_1, \pi)\}_{i=1}^{N}$, which solves for all i

$$qu'\left(Y - \left(b_{i1} + \psi Y \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + qb_{i2}\right) = \beta \pi u'(Y - b_{i2}) + \beta(1 - \pi)\frac{u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right)}{N}. \quad (6)$$

If local government i exceeds the debt limit and the punishment is implemented, the continuation outcome is equivalent to one in which local government i enters the period with debt $b_{i1} + \psi Y$, so the debt it issues is $\mathbf{b}_{i2}((b_{-i1}, b_{i1} + \psi Y), \pi)$. Moreover, unless the probability of facing the commitment type is one, the optimality condition (6) differs from the Euler equation (2) that characterizes the efficient allocation. In particular, if $\pi < 1$, there is overborrowing because each local government internalizes only $\frac{1}{N}$ of the marginal cost of repaying its debt if it anticipates a bailout when the central government is the no-commitment type. Note, however, that in any symmetric equilibrium there is never a bailout on path. If a local government were to deviate and borrow a larger amount, it would trigger a transfer in period 2 if the central government is the no-commitment type.[9]

Next, suppose that the fiscal constitution is not enforced, which implies that $\pi'(b_1, 1, \pi; \sigma) = 0$ so that the central government reveals its type. In this case, the value for the local government given a set of transfers T_1 is

$$V_{i1}^{ne}(b_1, 0, T_1) = \max_{G_{i1}, b_{i2}} u(G_{i1}) + \beta V_{i2}(b_{i2}, b_{-i2}(b_1, 0, T_1)) \quad (7)$$

subject to

$$G_{i1} + b_{i1} \leqslant Y_{i1} + T_{i1} + qb_{i2}$$

taking as given the strategy $b_{-i2}(b_1, 0, T)$ followed by the other local governments.

To simplify the exposition, note that if $\pi'(b_1, 1, \pi; \sigma) = 0$, whether there is debt mutualization in period 1 and 2 or only in period 2 is irrelevant in that the equilibrium consumption outcomes are identical.

Lemma 1. *If $\pi'(b_1, 1, \pi; \sigma) = 0$, the continuation values and public good provisions for the local governments are independent of transfers in period 1. In particular, for all budget feasible*

[9] This is similar to the "split-the-bill" problem.

$T_1 = (T_{i1})_{i=1}^{N}$, *the value of a violation of the no-bailout clause is independent of* T_1 *in that*

$$V_{i1}^{ne}\left(b_1, 0, T_1\right) = V_{i1}^{e}\left(b_1, 0\right) = V_{i1}^{ne}\left(B_1\right), \tag{8}$$

where $B_1 = \sum_{i=1}^{N} b_{i1}$.

The proof of this lemma is provided in the Appendix. The intuition is that when $\pi = 0$, a form of Ricardian equivalence holds: when the local governments are certain that they are facing the no-commitment type central government, the timing of transfers is irrelevant. Absent transfers, the local governments with inherited debt above average will simply borrow more to keep current consumption constant, expecting a bailout in the second period. On the other hand, the local governments with inherited debt below average, absent transfers, will reduce new debt issuances because they anticipate a negative transfer in period 2.

As a result of Lemma 1, the value for a local government in the event of non-enforcement depends only on the aggregate level of debt rather than the distribution of debt. Moreover, we can then drop T_1 as a decision variable and assume without loss of generality that transfers equal zero in period 1. The value for the no-commitment type central government is then

$$W_1\left(b_1, \pi, \psi\right) = \left[1 - \sigma\left(b_1, \pi, \psi\right)\right] W_1^{e}\left(b_1, \pi'\left(b_1, 0, \pi\right), \psi\right) + \sigma\left(b_1, \pi, \psi\right) W_1^{ne}\left(B_1\right), \tag{9}$$

where the value of not enforcing is

$$W^{ne}\left(B_1\right) = \sum_{i=1}^{N} \frac{1}{N} V_{i1}^{ne}\left(B_1\right), \tag{10}$$

and the value of enforcing is[10]

$$W_1^{e}\left(b_1, \pi, \psi\right) = \sum_{i} \frac{1}{N}\left[u\left(Y - b_i + q\mathbf{b}_{i2}\left(b, \pi, \psi\right) - \psi Y \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + \beta u\left(Y - \frac{\mathbf{B}_2\left(b, \pi, \psi\right)}{N}\right)\right], \tag{11}$$

where $\mathbf{B}_2 = \sum_i \mathbf{b}_{i2}$, and the equilibrium enforcement strategy is given by (4). Similarly, the value for the commitment type in period 1 is

$$W_1^{c}\left(b_1, \pi, \psi\right) = \sum_{i} \frac{1}{N}\left[u\left(Y - b_i + q\mathbf{b}_{i2}\left(b, \pi, \psi\right) - \psi Y \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b, \pi, \psi\right)\right)\right].$$

[10]Note that $W_1^e \neq \sum_i V_i^e$ since the no-commitment type central government knows that it will mutualize debt in period 2, while the local governments have uncertainty about the central government's type.

Period 0 The state in period 0 is the prior on the type of the central government, π (the realization of Y_{i0} is incorporated by indexing the value functions by t and i). In period $t = 0$, each local government chooses the local public good provision and debt to solve

$$V_{i0}(\pi, \psi) = \max_{G_{i0}, b_{i1}} u(G_{i0}) + \beta [1 - \sigma(b_1, \pi, \psi)] V_{i1}^e(b_{i1}, b_{-i1}, \pi, \psi) \\ + \beta \sigma(b_1, \pi, \psi) [\pi V_{i1}^c(b_{i1}) + \beta(1 - \pi) V_{i1}^e(b_{i1}, b_{-i1}, 0)] \tag{12}$$

subject to the budget constraint

$$G_{i0} \leqslant Y_{i0} + q b_{i1},$$

taking as given the strategies $b_{-i1}(\pi, \psi)$ followed by other local governments, and $\sigma(b_1, \pi, \psi)$ followed by the central government.

For later reference, we also define the value for the no-commitment type central government in period 0,

$$W_0(\pi, \psi) = \sum_{i=1}^{N} \frac{1}{N} u(G_{i0}(\pi, \psi)) + \beta [1 - \sigma(b_1(\pi, \psi), \pi, \psi)] W_1^e(b_1(\pi, \psi), \pi, \psi) \\ + \beta \sigma(b_1(\pi, \psi), \pi, \psi) W_1^e(b_1(\pi, \psi), 0, 0), \tag{13}$$

where $G_{i0}(\pi, \psi)$ and $b_1(\pi, \psi)$ are the decision rules in (12). The value for the commitment type is

$$W_0^c(\pi, \psi) = \sum_{i=1}^{N} \frac{1}{N} u(G_{i0}(\pi, \psi)) + \beta [1 - \sigma(b_1(\pi, \psi), \pi, \psi)] W_1^c(b_1(\pi, \psi), \pi, \psi) \\ + \beta \sigma(b_1(\pi, \psi), \pi, \psi) W_1^c(b_1(\pi, \psi), 1, \psi). \tag{14}$$

Equilibrium definition We can now define a Perfect Bayesian Equilibrium for this institutional setup.

Definition. A Perfect Bayesian Equilibrium is a set of strategies and beliefs for the local governments, $b_{i1}(\pi, \psi)$, $\pi'(b_1, \zeta, \pi)$, $b_{i2}(b_1, \pi, \psi)$, a strategy for the no-commitment type central government, $\sigma(b_1, \pi, \psi)$, and associated value functions, such that (i) given $b_{-i1}(\pi, \psi)$ and $\sigma(b_1, \pi, \psi)$, $b_{i1}(\pi, \psi)$ solves (12); (ii) given $b_{-i2}(b_1, \pi, \psi)$, $b_{i2}(b_1, \pi, \psi)$ solves (5); (iii) $\pi'(b_1, \zeta, \pi)$ satisfies (3); and (iii) $\sigma(b_1, \pi, \psi)$ satisfies (4).

3 Fiscal rules promote fiscal indiscipline

In this section we present the first main result of the paper: if the reputation of the central government is low enough, then fiscal rules lead to even more debt accumulation relative to the case with no rules. This is because the punishment associated with the fiscal rule enforcement makes it more attractive for the no-commitment type to reveal its type earlier relative to an environment without rules. This early resolution of uncertainty makes overborrowing more attractive for the local governments.

Equilibrium outcomes without fiscal rules: Enforcement in period 1

We start by characterizing the equilibrium when the fiscal constitution contains only a no-bailout clause and no fiscal rules. The main result in this section is that without fiscal rules, if the central government's initial reputation is low enough, there exists a unique equilibrium outcome in which the central government type is not revealed in period 1.

Proposition 1 (No revelation of central government type)**.** *Suppose the constitution has no fiscal rules. Then, for π sufficiently small but positive and N sufficiently large, there exists a unique symmetric equilibrium in pure strategies in which the type of the central government is not revealed in period 1. Moreover, the debt issuances $\left\{b_1^{\textit{no-rules}}, b_2^{\textit{no-rules}}\right\}$ satisfy*

$$\begin{aligned} qu'\left(Y + qb_1^{\textit{no-rules}}\right) = {} & \beta u'\left(Y - b_1^{\textit{no-rules}} + q\mathbf{b}_{i2}\left(b_1^{\textit{no-rules}}, \pi\right)\right) \\ & + \beta^2(1-\pi)\, u'\left(Y - \mathbf{b}_{j2}\left(b_1^{\textit{no-rules}}, \pi\right)\right)\frac{N-1}{N}\frac{\partial \mathbf{b}_{-i2}\left(b_1^{\textit{no-rules}}, \pi\right)}{\partial b_{i1}} \end{aligned} \tag{15}$$

and $b_2^{\textit{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\textit{no-rules}}, \pi\right)$.

The proof of this and other propositions is provided in the Appendix.

Since all regions are homogenous, if a pure strategy symmetric equilibrium exists, it must have no bailout on path, as each local government enters period 1 with the same amount of debt.[11] However, off equilibrium, a local government could potentially increase the debt issued in period 0 to induce the central government to bail it out in period 1. In the proof, we show that such a deviation is not profitable provided the reputation of the local government is low enough (i.e., π sufficiently close to zero).

The central government does not reveal its type in period 1. So, when the local governments choose their debt issuance in period 1, they are still uncertain about the type of the central government and about the probability of receiving a transfer in the terminal period 2. Given these expectations, debt issuances along the equilibrium path

[11] While this trivially holds with identical local governments, we show in a later section that the argument extends to the case with heterogeneity.

are characterized by equation (15) and $b_2^{\text{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\text{no-rules}}, \pi\right)$. The first two terms of condition (15) resemble those in a standard intertemporal Euler equation, while the last term on the right hand side captures strategic effects in the debt issuance decision. Each local government understands that its choice of debt issuance in period 0 will affect the debt issuance decisions of the other $N-1$ local governments in period 1, which in turn affects the utility of the local government in period 2 in case of debt mutualization (which happens with probability $1-\pi$). Notice that this term vanishes as $N \to \infty$ since $(N-1)\,\partial\mathbf{b}_{-i2}\left(b_1^{\text{no-rules}}, \pi\right)/\partial b_{i1} \to -1/q$, as shown in Lemma 3 in the Appendix.

Equilibrium outcomes with fiscal rules: No enforcement in period 1

We now consider the case in which the fiscal constitution has a no-bailout clause and a fiscal rule. We say that fiscal rules are *binding* if the debt limits are lower than the equilibrium outcome without fiscal rules, $\bar{b}_1 < b_1^{\text{no-rules}}$. When the central government's reputation is low and the fiscal rule is binding, there is a unique equilibrium in which the local government violates the fiscal rule in period 0 and the no-commitment central government does not enforce the punishment in period 1. This leads to the revelation of the central government's type in period 1.

Proposition 2 (Early revelation of central government type)**.** *Suppose the constitution has binding fiscal rules. Then, for π and β sufficiently small but positive and N sufficiently large, there exists a unique symmetric equilibrium in pure strategies in which the fiscal rule is violated in period 0 and not enforced by the no-commitment type in period 1 so that the type of the central government is revealed in period 1. Moreover, the debt issuances $\left\{b_1^{rules}, b_2^{rules,c}, b_2^{rules,nc}\right\}$ satisfy*

$$
\begin{aligned}
qu'\left(Y + qb_1^{rules}\right) = {} & \beta\pi u'\left(Y - \left(b_1^{rules} + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules} + \psi, 1\right)\right) \\
& + \beta\left(1-\pi\right)u'\left(Y - b_1^{rules} + q\mathbf{b}_{i2}\left(b_1^{rules}, 0\right)\right) \\
& + \beta^2\left(1-\pi\right)u'\left(Y - \mathbf{b}_{j2}\left(b_1^{rules}, 0\right)\right)\frac{N-1}{N}\frac{\partial\mathbf{b}_{-i2}\left(b_1^{rules}, 0\right)}{\partial b_{i1}},
\end{aligned}
\tag{16}
$$

$b_2^{rules,c} = \mathbf{b}_{i2}\left(b_1^{rules} + \psi Y, 1\right)$, *and* $b_2^{rules,nc} = \mathbf{b}_{i2}\left(b_1^{rules}, 0\right)$.

The key step to establish the proposition is to show that in period 1, the no-commitment type central government when faced with debt $b_{i1} = b_1^{\text{rules}} > \bar{b}_1$ for all i prefers to not enforce the punishment ψ and reveal its type ($\pi' = 0$ thereafter) than to enforce the punishment and enjoy the reputation gain,[12] and therefore lower distortions in the local govern-

[12] The posterior jumps to one as the local governments expect only the commitment type to enforce the fiscal rule.

ment's Euler equations. In the Appendix, we show that this is true when β is sufficiently low.

Hence, when local governments choose the new debt levels in period 1, they know with certainty the type of the central government they are facing. This shows up in equation (16), where the right side of the Euler equation is contingent on the type of the central government.

Note for future reference that the strategic term in equation (16), the last term on the right side, does not vanish as $N \to \infty$. This is because while $\frac{N-1}{N}\frac{\partial \mathbf{b}_{-i2}\left(b_1^{\text{rules}},0\right)}{\partial b_{i1}} \to 0$ as in the case without rules, $u'\left(Y - \mathbf{b}_{j2}\left(b_1^{\text{rules}},0\right)\right) \to \infty$ because local governments exhaust their debt capacity in period 1 if they know for sure they face the no-commitment type. We can show that the product converges to a finite negative number. However, we will show that the sum of the second and third terms on the right hand side converges to zero as $N \to \infty$.

Comparing debt levels

We next show that when the central government's reputation is low, binding fiscal rules promote *more* fiscal indiscipline than a constitution without fiscal rules—that is, the debt levels in this equilibrium are higher than in the equilibrium without fiscal rules. The key driver for this result is that the type of the central government is revealed in period 1 with rules and so the local governments can condition their new debt issuances on the government type.

Proposition 3 (Fiscal rules promote fiscal indiscipline when reputation is low.)**.** *Under the assumptions of Proposition 1 and 2, the level of debt issued in period 0 is higher with binding fiscal rules than without. Moreover, contingent on facing the no-commitment type, the debt issued in period 1 is higher with binding fiscal rules than without.*

Consider first the debt issued in period 0. For the case without fiscal rules, we can combine (15) with (6) to obtain a condition that characterizes the debt issuance in period 0:

$$u'\left(Y + qb_1\right)q = \frac{\beta^2\pi}{q}u'\left(Y - \mathbf{b}_{i2}\left(b_1,\pi\right)\right) + \frac{\beta^2\left(1-\pi\right)}{qN}u'\left(Y - \mathbf{b}_{i2}\left(b_1,\pi\right)\right) + \frac{\beta^2\left(1-\pi\right)}{N}u'\left(Y - \mathbf{b}_{i2}\left(b_1,\pi\right)\right)\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1,\pi\right)}{\partial b_{i1}}. \tag{17}$$

For the case with fiscal rules, we can combine (16) with (6) to obtain

$$u'(Y+qb_1)\,q = \frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi,1\right)\right)+\frac{\beta^2(1-\pi)}{qN}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right) \tag{18}$$
$$\frac{\beta^2(1-\pi)}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}}.$$

These two optimality conditions are identical with the exception that with no fiscal rules (condition (17)), debt issued in period 1 is not conditional on the type of the central government, $b_{i2}=\mathbf{b}_{i2}(b_1,\pi)$. With binding fiscal rules (condition (17)), debt issued in period 1 is conditional on the type of the central government, and is either $\mathbf{b}_{i2}(b_1,1)$ if the central government is the commitment type (with probability π) or $\mathbf{b}_{i2}(b_1,0)$ if the central government is the no-commitment type. We next show that the early revelation of the central government's type in the equilibrium with fiscal rules induces the local governments to issue more debt.

Taking the limit as N goes to infinity, since $\lim_{N\to\infty}u'(Y-\mathbf{b}_{i2}(b_1,\pi))<\infty$, as shown in Lemma 2 in the Appendix, condition (17) reduces to

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right) \tag{19}$$

as the second and third terms on the right side converge to zero. Condition (18) instead reduces to

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right), \tag{20}$$

because

$$\lim_{N\to\infty}\frac{\beta u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{N}\frac{1}{q}=-\lim_{N\to\infty}\frac{u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{N}\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}}=u'\left(Y(1+q)-b_{i1}\right),$$

as shown in Lemma 2 and 3 in the Appendix. We can then compare the right hand side of (19) and (20). We know that for a small enough π, $\mathbf{b}_{i2}(b_1,\pi)>\mathbf{b}_{i2}(b_1+\psi Y,1)$, because as $\pi\to 0$, $\mathbf{b}_{i2}(b_1,\pi)\to Y$ but $\mathbf{b}_{i2}(b_1+\psi Y,1)$ is bounded away from Y (see Lemma 2 for details). This observation along with the concavity of u implies that

$$\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right)<\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right).$$

Therefore, from (19) and (20) we see that the expected marginal cost of issuing debt in period 0 is lower when there is early revelation of the central government's type. Hence, local governments will issue more debt in period 0 because of the lower expected marginal cost.

Intuitively, if the central government reveals its type only in period 2, even if a local government is confident it will receive a bailout in period 2, it does not borrow a lot in period 0 because it knows that if the central government is the commitment type, consumption in period 2 will be very low. If instead the central government reveals its type in period 1, the local government will borrow more because in the unlikely event that the central government is the commitment type, the local government can spread the losses associated with not receiving a bailout over period 1 and period 2. The latter is preferable and so the government has a higher incentive to borrow more in period 0 because it can better insure the risk of facing the commitment type.

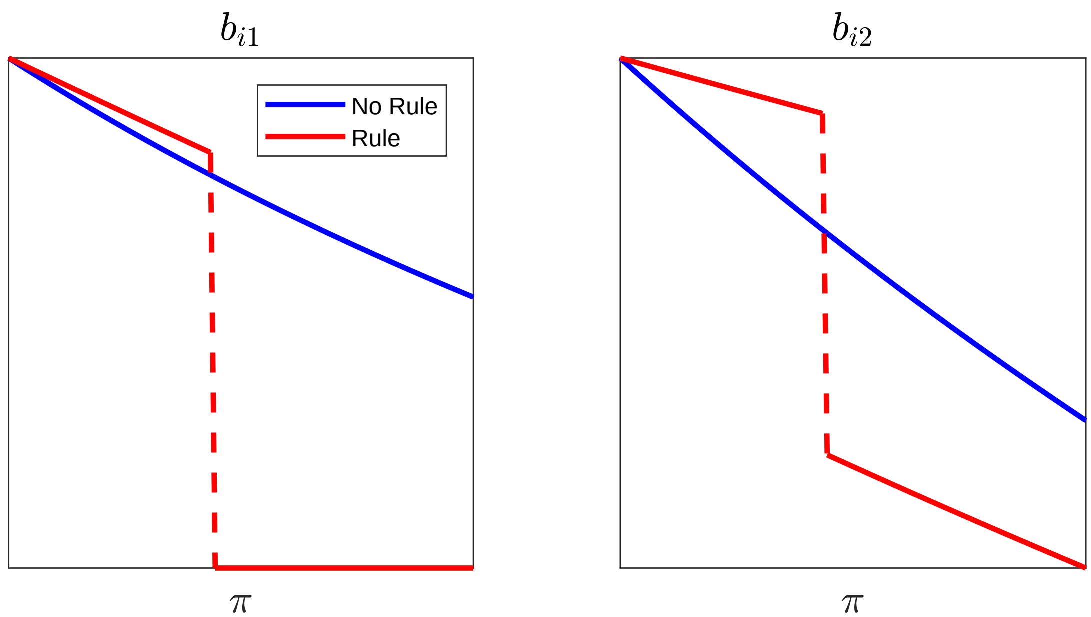
Consider now debt issuances in period 1 if the central government is the no-commitment type. In this case, debt issued in period 1 is higher with rules than without for two reasons: first, the inherited debt is larger; second, the local governments face no uncertainty about the type of central government and therefore internalize only $1/N$ of the cost of issuing debt, while with rules they internalize the full cost with probability π and $1/N$ the cost with probability $1-\pi$. This argument concludes the proof of Proposition 3.

Strategic effects By considering the limit as the number of regions goes to infinity, we abstract from strategic effects in debt issuances. Strategic effects will further incentivize local governments to borrow more when there is early revelation of the central government's type. The strategic interaction in debt choices between the two local governments is captured by the third term on the right side of (18) and (17). The elasticity of the debt issuance by the other local governments in period 1 to the debt issued by local government i in period 0 is decreasing in π. Formally,

$$\frac{\partial \mathbf{b}_{i2}\left(b_1, \pi'\right)}{\partial b_{-i1}} < \frac{\partial \mathbf{b}_{i2}\left(b_1, \pi\right)}{\partial b_{-i1}} < 0 \tag{21}$$

for $\pi' > \pi$. The intuition is straightforward. If $\pi = 0$, as is the case when the central government's type is revealed at $t = 1$, then a given local government has a high incentive to adjust its period 1 debt issuance in response to the inherited debt of the other local governments. This is because at $\pi = 0$, the local governments know there will be debt mutualization with probability one next period. If instead there is no early revelation and $\pi > 0$, then there is debt mutualization in period 2 only with probability $1-\pi$ and so a local government's debt issuance will be less sensitive to debt issued in the previous period by the other local governments. The increased sensitivity implies that if a local government borrows an additional unit in period 0, it will receive a larger transfer in period 2 conditional on facing the no-commitment type when $\pi = 0$ than when $\pi > 0$. This strategic channel contributes to the lower cost of servicing debt internalized by local

Figure 1: Equilibrium outcomes: Debt issued in period 1 and 2



governments when there are fiscal rules.

Numerical example We illustrate the proposition in Figure 1.[13] The two panels of Figure 2 display the debt issued by a representative local government along the equilibrium path without rules (blue line) and with rules (red line) as a function of the prior in period 0 that the central government is the commitment type.

When π is low enough, debt issuances in period 0 are higher with rules. The same is true in period 1 conditional on facing the no-commitment type. When the initial prior π is not close to zero, we cannot characterize the equilibrium analytically. Numerically, we show that when instead π is above a threshold, there exists an equilibrium in which rules are followed, the central government does not reveal its type in period 1, and total indebtedness is lower than in the case without rules. Hence fiscal rules may be effective in reducing debt only when the central government's reputation is sufficiently high. But when the central government's reputation is high, the gains from reducing indebtedness are smaller: debt is decreasing in π because the local governments expect that they will not receive a bailout with a high probability. Therefore, fiscal rules are detrimental exactly when the problem of overborrowing is most severe, while they are effective only when the gains from enforcement are relatively low.

Given the assumptions in our model, the non-enforcement of any one of the clauses in the constitution implies full revelation of the central government's type. As a result, the

[13]To construct the figure, we compute the equilibrium outcome with $u(c) = c - \frac{\alpha}{2}c^2$ and $N = 2$.

no-commitment type will never choose to enforce one of the clauses but not the other. However, the arguments that establish the results above will extend to a situation in which the central government can somehow commit to the no-bailout clause but not to the fiscal rule, so long as the non-enforcement of the fiscal rule lowers reputation and increases the likelihood of a bailout.

Our characterization is consistent with the experience of several federal states in which fiscal rules were instituted and often violated by subnational governments. It also provides a rationale for why subnational governments kept on borrowing excessively after the central governments deviated from the fiscal constitution. Arguably, this is what happened in the European Monetary Union (EMU) after the violation of Maastricht treaty in 2005 and the subsequent relaxation of the rules and penalties. This is also consistent with the experience in Brazil where "[d]ebt burden continued to grow in the 1990s. Despite the previous crises and bailouts - or perhaps because of them - the states continued to increase spending." (Rodden et al. (2003)).

4 Heterogeneous local governments

So far we have assumed that the local governments are identical. As a result, in any symmetric equilibrium outcome, the central government is never tempted to impose transfers between regions, because all governments have the same debt position. In this section, we show that our arguments extend to the case in which the local governments are heterogeneous and so the central government might find it optimal to bail out the poorer local governments along the equilibrium path.

Suppose that $N = 2M$ with $M \geqslant 1$. We partition the local governments in two groups: the North, $n \in \mathcal{N} = \{1, ..., M\}$, and the South, $s \in \mathcal{S} = \{M+1, M+2, ..., 2M\}$, and for all $n \in \mathcal{N}$ and $s \in \mathcal{S}$ we let[14]

$$Y_{n0} > Y_{s0}, \quad Y_{nt} = Y_{st} = Y \quad \text{for } t = 1, 2$$

and so the North is "richer" at time 0 relative to the South. This creates incentives for the no-commitment type central government to bail out the South and reveal its type.

While the bulk of our analyses focuses on the effects of the choice of fiscal constitution on the equilibrium debt holdings of the local governments, it is worth noting that from a utilitarian perspective, commitment to the fiscal constitution need not always be optimal when local governments are heterogeneous. The reason for this is that a utilitarian planner values redistribution and as a result bailouts can be valuable as a means of equaliz-

[14] Adding heterogeneity in tax revenues Y_{it} for $t > 0$ leaves the results unchanged.

ing consumption. Nevertheless, this comes at a cost of distorting the local government's Euler equation, which leads to overborrowing as compared to the efficient benchmark. However, for Y_{s0} and Y_{n0} sufficiently close to each other, the second effect will always dominate and the ex-ante welfare associated with the commitment type is strictly larger than that of the no-commitment type. This is the region of the parameter space we will restrict our attention to.

Next, we show that without fiscal rules, provided that π is small enough and $Y_{n0} - Y_{s0}$ is not too large, it is still optimal for the central government to delay revealing its type and not provide a bailout in period 1:

Proposition 4 (No bailout in period 1 when credibility is low.)**.** *Suppose the constitution has no fiscal rules. Then, for π and $Y_{n0} - Y_{s0}$ sufficiently small but positive and N sufficiently large, there exists a unique symmetric equilibrium in pure strategies in which the type of the government is not revealed in period 1. Moreover, the debt issuances $\left\{b_{i1}^{\textit{no-rules}}, b_{i2}^{\textit{no-rules}}\right\}_{i=n,s}$ satisfy*

$$q u'\left(Y_{i0} + q b_{i1}^{\textit{no-rules}}\right) = \beta u'\left(Y - b_{i1}^{\textit{no-rules}} + q \mathbf{b}_{i2}\left(b_1^{\textit{no-rules}}, \pi\right)\right) + \beta^2 (1-\pi) u'\left(Y - \mathbf{b}_{i2}\left(b_1^{\textit{no-rules}}, \pi\right)\right) \frac{N-1}{N} \frac{\partial \mathbf{b}_{-i2}\left(b_1^{\textit{no-rules}}, \pi\right)}{\partial b_{i1}} \tag{22}$$

and $b_{i2}^{\textit{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\textit{no-rules}}, \pi\right)$.

The proof of this proposition mainly follows the same steps as the proof of Proposition 1 with one exception: since the local governments in the North and in the South will enter period 1 with different levels of debt, we now have to show that the central government wants to enforce the constitution in period 1, when its reputation is sufficiently low.

To understand this step, let us consider the costs and benefits of enforcing the fiscal constitution in period 1. By enforcing the no-bailout clause, the central government preserves its reputation. A higher π in turn promotes fiscal responsibility, because the local governments expect to repay their debt without a bailout from the central government with higher probability. Hence the benefits of enforcing are associated with a reduction of the distortions in the local government's Euler equations (6) relative to the efficient one (2).

The costs of enforcing the no-bailout clause are associated with the high inequality in the provision of the local public good. A higher π will induce the South government to borrow less in period 1 and cut the consumption of the local public good relative to the North. This dispersion in the consumption of the local public good across regions (North and South) is costly from the perspective of the benevolent central government.

For π close to zero, if the central government enforces the constitution and does not bail out, there is essentially no inequality of the local public good consumption, since the local governments expect a bailout with high probability in period 2 and so the costs of not redistributing are second order. However, the benefits from inducing more fiscal discipline are first order, since the Euler equation is distorted relative to the efficient allocation. Hence, it is optimal for the central government to not bail the regions out (or to enforce the constitution) when its reputation is very low.

In our model, we do not allow the central government to make transfers in period 0. If we did allow for transfers, notice that the no-commitment type might not want to, since it would reveal its type in period 0, thus leading to large distortions in the Euler equations for both periods. Moreover, for π close to zero, we can use a similar argument to 1 to show that in period 0, local governments borrow so that there is very little inequality in consumption. As a result, the central government will not choose to reveal its type until the last period.

Next, we state the analogue of Proposition 2 and 3 for the economy with heterogeneous local governments:

Proposition 5 (Early revelation of central government's type.)**.** *Suppose the constitution has binding fiscal rules. Then, for π, $Y_{n0} - Y_{s0}$, and β sufficiently small but positive and N sufficiently large, there exists a unique symmetric equilibrium in pure strategies in which the fiscal rule is violated in period 0 and not enforced by the no-commitment type in period 1 so that the type of the central government is revealed in period 1. Moreover, the debt issuances $\left\{b_1^{rules}, b_2^{rules,c}, b_2^{rules,nc}\right\}$ satisfy*

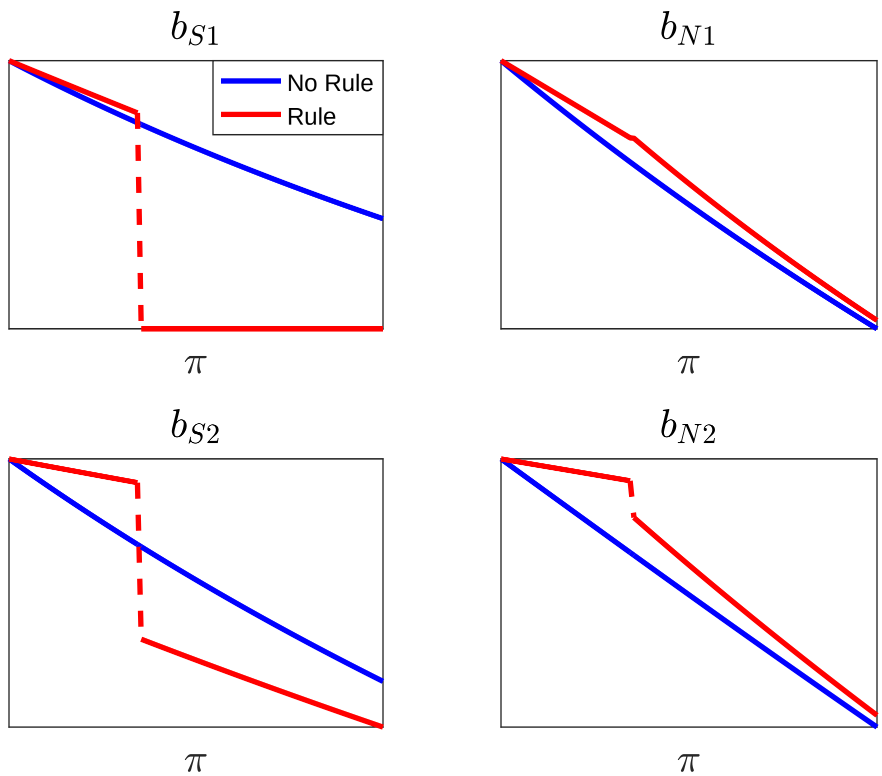
$$\begin{aligned} qu'\left(Y_{i0} + qb_1^{rules}\right) &= \beta\pi u'\left(Y - \left(b_1^{rules} + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules} + \psi, 1\right)\right) \\ &+ \beta\left(1-\pi\right)u'\left(Y - b_1^{rules} + q\mathbf{b}_{i2}\left(b_1^{rules}, 0\right)\right) \\ &+ \beta^2\left(1-\pi\right)u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(b_1^{rules}, 0\right)}{N}\right)\frac{1}{N}\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1^{rules}, 0\right)}{\partial b_{i1}}, \end{aligned} \tag{23}$$

$b_2^{rules,c} = \mathbf{b}_{i2}\left(b_1^{rules} + \psi, 1\right)$, *and* $b_2^{rules,nc} = \mathbf{b}_{i2}\left(b_1^{rules}, 0\right)$.

Proposition 6 (Fiscal rules promote fiscal indiscipline when reputation is low.)**.** *Under the assumptions of Proposition 4 and 5, the level of debt issued in period 0 is higher with binding rules than without. Moreover, contingent on facing the no-commitment type, the debt issued in period 1 is higher with binding fiscal rules than without.*

As in the case without heterogeneity, we show that along the equilibrium outcome with fiscal rules the rule is violated and not enforced ex-post by the no-commitment type central government that reveals its type in period 1. The early revelation in turn promotes fiscal indiscipline. The proofs of the two propositions essentially follow from continuity of the equilibrium outcome in $Y_{n0} - Y_{s0}$ given the results in Proposition 2 and 3.

Figure 2: Equilibrium outcomes: Debt issued in period 1 and 2 with heterogeneous governments



We illustrate Proposition 6 with a numerical example. The four panels of Figure 2 display the debt issued by the South and North along the equilibrium path without rules (blue line) and with rules (red line) as a function of the prior in period 0 that the central government is the commitment type. To make the two graphs comparable, we assume that in the equilibrium with fiscal rules no transfers are made in period 1, so the different debt levels issued in period 1 do not reflect the different pattern of transfers. As shown in Lemma 1, this does not affect public good provision or the aggregate amount of debt. Next, as shown in Proposition 6, for π close to zero the debt issued in period 0 by all local governments is higher with rules than without. The same is true in period 1.

As in the case with homogenous local governments, when the central government's reputation is above a threshold, the South respects the fiscal rule and the central government does not reveal its type in period 1. In this case, total indebtedness is lower than in the case without rules. Note that the North still borrows more with rules because it now anticipates that the South will borrow less, which implies that it will have to transfer less in the event of a bailout and so its expected marginal utility of consumption is lower. So, again, fiscal rules may be effective in reducing debt only when the central government's reputation is sufficiently high.

5 Large vs. small local governments

In this section we briefly discuss the role of having "large" local governments for our results. We show that when $N = \infty$, there exists another equilibrium in addition to the one that is the limit of N finite economies in which fiscal rules can be beneficial.

We now show that if $N = \infty$ and all local governments are small, in addition to the equilibrium we characterized above (the limit of N finite), there exists another equilibrium with fiscal rules where all local governments obey the rule and therefore fiscal rules achieve their intended role of curbing debt issuance by local governments.

Proposition 7. *Under the same assumptions of Proposition 2, if $N = \infty$, there exists another equilibrium in which fiscal rules are followed in period 0.*

The key intuition for Proposition 7 is that with non-atomistic local governments, there are no costs for the central government to enforce the penalty for a violation of the fiscal rule by an *individual* local government that has measure zero. Hence, if one local government expects that other local governments will respect the fiscal rule, it is optimal for it to respect the rule as well and so there is an equilibrium in which fiscal rules can curb indebtedness and where the local governments internalize the free-rider problem.

This result is fragile: there is always an equilibrium where the rule is ignored by all the local governments and not enforced. In particular, if a government expects the other governments to violate the rule, it will find it optimal to violate the rule as well since it anticipates that the rule will not be enforced ex-post. This type of multiplicity is similar to the one in Farhi and Tirole (2012) and Chari and Kehoe (2015).

Proposition 7 suggests that the forces we emphasize in this paper are more likely to be relevant when the local governments are relatively large. This result may help to rationalize why when two large countries such as Germany and France violated the SGP in 2003, no sanctions were imposed by the European institutions. More generally, Eyraud et al. (2017) provides suggestive evidence that compliance with the SGP rules has been lower among the largest countries. However, it may be possible for institutions such as the IMF to enforce penalties on a small country to preserve their reputation.

6 Optimal fiscal rules

We now turn to analyzing whether the optimal fiscal constitution should have fiscal rules. By *optimal* we mean the fiscal constitution that induces the maximal average welfare for the citizens who believe that the central government is the commitment type with probability π and the no-commitment type with probability $1 - \pi$. We show that if the central government's reputation is low, it is optimal to have no fiscal rules.

Proposition 8. *Under the assumptions of Proposition 1 and 2, then the optimal fiscal rule without commitment has* $\psi = 0$*.*

This proposition follows as a corollary of Proposition 3 and 6. Fiscal rules can be welfare improving only if they restrain the local governments from overborrowing. But we showed that the rules actually induce more borrowing for low π. Hence, rules only have costs relative to the outcome without rules. In particular, (i) rules promote more borrowing in period 0 when the local governments are already overborrowing relative to the efficient benchmark; (ii) if the central government is the commitment type, there are output costs associated with the enforcement of rules; and (iii) if the central government is the no-commitment type, there is also more borrowing from period 1 to period 2, which is also detrimental for welfare. Hence, it is not optimal to have fiscal rules in the constitution when reputation is low. This is the outcome that would be chosen in period 0 by a benevolent central government under the veil of ignorance.

Next we analyze if fiscal rules can be instituted when the benevolent central government chooses a fiscal constitution after its type is realized.

7 Equilibrium fiscal constitution

In this section, we study the *equilibrium fiscal constitution*, that is, the fiscal constitution that arises as the outcome of a signaling game between the two types of government in period 0. We show that if the commitment type is sufficiently patient, it is optimal for it to announce fiscal rules that will promote early resolution of uncertainty in period 1, and the no-commitment type will choose to mimic the strategy of the commitment type in period 0 and also announce such rules (and violate them in period 1). This outcome can arise even though it is ex-ante efficient to impose no rules, as shown in the previous section.

More formally, we add an additional stage to the policy game described in Section 2. In the initial stage, given the prior π about the type of central government, the central government chooses to write a fiscal constitution. A fiscal constitution has a no-bailout clause and a fiscal rule $(\psi, \bar{b})$ with $\psi \leqslant \bar{\psi}$. After observing the chosen fiscal constitution, the local governments update their prior about the type of the central government, and the subsequent equilibrium outcome is an equilibrium outcome of the policy game described in the previous sections.

Definition (Equilibrium fiscal constitution.)**.** An equilibrium fiscal constitution is an equilibrium outcome of the signaling game between the two types of the central governments. Given a prior π, an equilibrium of the signaling game is a strategy for the commitment type central government ψ^c, a strategy by the non-commitment type ψ^{nc}, and beliefs π_0' such that (i) beliefs evolve according to

$$\pi_0'(\psi, \pi; \psi^c, \psi^{nc}) = \begin{cases} \pi & \text{if } \psi = \psi^{nc} = \psi^c \\ 0 & \text{if } \psi = \psi^{nc} \neq \psi^c \\ 1 & \text{if } \psi = \psi^c \neq \psi^{nc} \\ 0 & \text{if } \psi \notin \{\psi^c, \psi^{nc}\} \end{cases}; \tag{24}$$

(ii) given ψ^{nc}, the strategy for the commitment type ψ^c is optimal, in that for all ψ

$$W_0^c\left(\pi_0'(\psi^c, \pi; \psi^c, \psi^{nc}), \psi^c\right) \geqslant W_0^c\left(\pi_0'(\psi, \pi; \psi^c, \psi^{nc}), \psi\right),$$

where W_0^c is defined in (14); (iii) given ψ^c, the strategy ψ^{nc} for the no-commitment type is optimal, in that for all ψ

$$W_0\left(\pi_0'(\psi^{nc}, \pi; \psi^c, \psi^{nc}), \psi^{nc}\right) \geqslant W_0\left(\pi_0'(\psi, \pi; \psi^c, \psi^{nc}), \psi\right),$$

where W_0 is defined in (13).

We can characterize the equilibrium of this game by considering the fiscal rule chosen by the commitment type given the prior π. The problem for the commitment type in period 0 is

$$W_0^c = \max\left\{W_0^{c,sep}, W_0^{c,pool}\right\},$$

where $W_0^{c,sep}$ is the value for the commitment type if it chooses a fiscal rule that ensures separation in period 1:

$$\begin{aligned} W_0^{c,sep} = \max_{\psi,\bar{b}} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{er}(\pi, \psi)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1}^{er}(\pi, \psi) + q \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi, \psi), 1, \psi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi, \psi), 1, \psi\right)\right) \end{array} \right] \end{aligned}$$

subject to

$$W_1^{ne}\left(b_{i1}^{er}(\pi, \psi)\right) = W_1\left(b_{i1}^{er}(\pi, \psi), 0, 0\right) \geqslant W_1\left(b_{i1}^{er}(\pi, \psi), 1, \psi\right).$$

Conversely, $W_0^{c,pool}$ is the value for the commitment type if the fiscal constitution it chooses is such that the no-commitment type will have an incentive to enforce the rule in period

1:

$$W_0^{c,pool} = \max_{\psi,\bar{b}} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi,\psi)\right) +$$

$$+ \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1}^{lr}(\pi,\psi) + q \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\psi),\pi,\psi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\psi),\pi,\psi\right)\right) \end{array} \right]$$

subject to

$$W_1\left(b_{i1}^{lr}(\pi,\psi),\pi,\psi\right) \geqslant W_1^{ne}\left(b_{i1}^{lr}(\pi,\psi)\right) = W_1\left(b_{i1}^{lr}(\pi,\psi),0,0\right).$$

In setting up the problem we assumed that it was optimal for the no-commitment type to mimic the strategy of the commitment type in period 0. In the next proposition we provide sufficient conditions for this to be the case.

Suppose first that the commitment type can only choose between two levels of penalties, $\psi \in \{0, \bar{\psi}\}$. The next proposition shows that if the commitment type central government is sufficiently patient, then there exists a unique equilibrium fiscal constitution that has fiscal rules. Moreover, the no-commitment type central government prefers to mimic the strategy of the commitment type in period 0 and chooses a constitution with fiscal rules despite knowing that it will not enforce the constitution in period 1.

Proposition 9. *Under the assumptions of Proposition 4–6 with $Y_{n0} > Y_{s0}$, there exists $\underline{\beta}$ and $\bar{\beta} > \underline{\beta}$ such that:*

1. *For $\beta < \underline{\beta}$, there exists a unique fiscal constitution with no fiscal rules and $\psi = 0$.*

2. *For $\beta \in [\underline{\beta}, \bar{\beta}]$, there exists a unique constitution with fiscal rules that are violated by local governments, and there is early resolution of uncertainty in period 1.*

When the central government's reputation is sufficiently close to zero, for intermediate values of the discount factor β, fiscal rules arise in equilibrium even if they are going to be violated by the local governments. The commitment type chooses to do so to reveal its type in period 1. From its perspective, this has benefits, because in period 1 the reputation of the central government will jump from almost zero to one, promoting fiscal discipline going forward. In particular, the local government's decision will satisfy the Euler equation and so is efficient from period 1 onward.[15] But this also has costs. As we have shown in Proposition 3 and 6, instituting fiscal rules promotes overborrowing and fiscal indiscipline in period 0. When β is above the cutoff $\underline{\beta}$ defined in the Appendix, the benefits outweigh the costs. Conditional on the commitment type announcing a fiscal

[15] Of course, the commitment type central government would like to redistribute resources from the North to the South, but in our setup it has no instruments to do so.

rule, for π close to zero, the no-commitment type always prefers to mimic the strategy of the commitment type. Intuitively, the reputation cost of not mimicking the strategy of the commitment type is of first order, while the benefit of equalizing consumption is of second order when π is close to zero.

If instead β is below such cutoff, the commitment type prefers not to institute the rule, and clearly the no-commitment type chooses to do the same. Finally, for the no-commitment type not to enforce the rule in period 1, we need to impose an upper bound $\bar{\beta}$ (defined in the Appendix) on the discount factor. In the Appendix we show that $\underline{\beta} < \bar{\beta}$ when countries are heterogeneous in period 0, $Y_{n0} > Y_{s0}$. In our baseline case with $Y_{n0} = Y_{s0}$, $\underline{\beta} = \bar{\beta}$ and so rules are never adopted in equilibrium.

How do we reconcile Proposition 9 for intermediate values of the discount factor, $\beta \in \left[\underline{\beta}, \bar{\beta}\right]$, with Proposition 8, which establishes the optimality of no rules when the central government's reputation is low enough regardless of the discount factor? The idea is that the commitment type gains from imposing a rule, $W_0^{c,sep} > W_0^{c,pool}$, but the no-commitment type loses from the imposition of the fiscal rule that induces early revelation of uncertainty, $W_0^{nc,sep} < W_0^{nc,pool}$. In expectation, not knowing the type of the central government, imposing rules and inducing early separation lowers utility, $\pi W_0^{c,sep} + (1-\pi) W_0^{nc,sep} < \pi W_0^{c,pool} + (1-\pi) W_0^{nc,pool}$ and so the optimal policy under the veil of uncertainty calls for no rules, as shown in Proposition 8.

Next, suppose that the commitment type can choose ψ in an interval $\left[0, \bar{\psi}\right]$. The proof for the above proposition is identical except that the objects $\underline{\beta}, \bar{\beta}$ now depend on the optimal choice ψ and thus are no longer defined in terms of fundamentals. However, if the equations defining these bounds are well defined, then the proposition holds in this case as well.

Proposition 9 rationalizes why we often observe central governments with low reputation setting up tough fiscal rules. Examples include the case of the Eurozone after the European debt crisis and the bailouts in Greece, Portugal, Ireland, and Spain with the institution of the "Six-Pack", and the case of Brazil after the bailouts in 1997 and the Fiscal Responsibility Law approved by the Cardoso administration. In both cases, the reputation of the central government was low because of the recent bailouts to local governments.

8 Conclusion

Fiscal rules are often thought to be useful in federal states when the central government cannot commit to no-bailout clauses. In this paper, we ask if this is indeed the case when the central government also cannot commit to imposing these rules. We show that in a reputation model in which the local governments are uncertain whether the central government can commit or not, outcomes with rules can attain lower welfare than outcomes without rules. Moreover, the outcomes associated with fiscal rules are worse exactly when there is a high probability that the central government cannot commit. Our results shed light on the multitude of examples throughout history when fiscal rules were instituted but not enforced. Our analysis of the equilibrium constitution suggests that stringent fiscal rules arise when the central government's reputation is low even though they are not optimal under the veil of ignorance.

One interesting extension would be to study the infinite horizon dynamic game. This would be particularly interesting in the context of an environment where the local governments cannot commit to repay debt, to study the joint dynamics of debt, central government's reputation, and interest rate spreads on local government debt. This may help to understand the dynamics of interest rates during the European debt crisis, where, according to several commentators, much of the dynamics of spreads was attributable to political risk or the willingness of the European institutions to bail out members in crisis.

This paper does not provide a meaningful theory of the instances in which fiscal rules were effective in reducing debt. One such example is the United States.[16] A simplistic answer, which would be consistent with our theory, would be to say that the US central government has a high reputation. However, we believe that differences in institutional features might help account for the differences in the efficacy of fiscal rules, and should be an important avenue for future research.

On a related note, it is worth considering what kinds of policies can prevent overborrowing even when the central government's reputation is very low. Our results suggest that policies which constrain the actions of the central government are more likely to work than those which constrain the actions of the local governments (and are sustained by punishments). For example, if there was a cap on the amount of tax revenues the central government could access, this would reduce the underlying free-rider problem. See Rodden (2006) for a similar argument. However, this would also reduce the amount of consumption insurance possible and as a result the optimal cap would trade off the costs of consumption smoothing with the benefits of lowering debt. We leave this and similar extensions to future work.

In this paper, we assumed that the central government is benevolent and maximizes the utility of the local governments. Another possibility is to study institutional settings where local governments' representatives vote to impose sanctions on the local governments that violate the rule. This is left for future research.

Finally, in our analysis we take as given the policy instruments available to the central

[16] See Arellano et al. (2016) for a discussion of dynamics of debt for states in the US.

government, such as the form of the fiscal rules. It would be useful to study a mechanism design problem, similar to Golosov and Iovino (2016), to understand the optimal mechanism when the central government lacks commitment. One can then ask if the optimal mechanism can be implemented using arrangements that resemble the types of fiscal rules used in practice.

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A Omitted Proofs

Proof of Lemma 1

Let $\pi = 0$ and consider the continuation equilibrium in period 1 after the central government made transfers $T_1 = \{T_{i1}\}$ such that $\sum_{i=1}^{N} T_{i1} = 0$. Debt issuances $\{b_{i2}(T_1)\}$ must satisfy

$$qu'\left(Y - b_{i1} + T_{i1} + qb_{i2}(T_1)\right) = \frac{\beta}{N}u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}(T_1)}{N}\right) \quad \text{for all } i$$

We can then see that if $\{b_{i2}(0)\}$ solves the system above for $T_1 = 0$ then

$$b_{i2}(T_1) = b_{i2}(0) - \frac{1}{q}T_{i1} \quad \text{for all } i$$

solve the system given for T_1 such that $\sum_{i=1}^{N} T_{i1} = 0$ and it leaves public good provisions in period 1 and 2 unchanged. Hence the value is unaffected by the transfers in period 1 when $\pi = 0$. Q.E.D.

Proof of Proposition 1

Assume first that local governments expect that the central government would not make any transfer in period 1 and it will mutualize debt in period 2 with probability $1 - \pi$. The optimality condition of problem (12) and the envelope condition from problem (5) gives that debt issuance in period 0 satisfies (15) and debt issuance in period 1 is $b_2^{\text{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\text{no-rules}}, \pi\right)$. Given symmetry, b_{i1} is constant for all i and so in period 1 the no-commitment type central government has no incentive to implement a transfer in period 1.

We are left to show that an individual government has no incentives to increase its debt and force the central government to make a transfer. Suppose local government i chooses $b_{i1} > b_1^{\text{no-rules}}$ to induce the central government to bailout region i in period 1. The value for the best deviation is

$$V_i^{dev} = \max_{b_{i1}} u\left(Y + qb_{i1}\right) +$$
$$+ \beta\pi\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right)\right]$$
$$+ \beta\left(1-\pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)}{N}\right)\right]$$

subject to $W_1^{ne} \geqslant W_1^e$

$$W_1^e = \frac{(N-1)}{N}\left[u\left(Y - \bar{b}_1 + q\mathbf{b}_{-i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)}{N}\right)\right]$$
$$+ \frac{1}{N}\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)}{N}\right)\right],$$
$$W_1^{ne} = \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)}{N}\right)\right]$$

Let V_i be the value along the conjectured equilibrium and $\Delta V_i = V_i - V_i^{dev}$. Note that by construction, $b_1^{\text{no-rules}}$ solves

$$V_i = \max_{b_{i1}} u\left(Y + qb_{i1}\right) +$$
$$+ \beta\pi\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right)\right]$$
$$+ \beta\left(1-\pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)}{N}\right)\right]$$

Note that for $\pi = 0$ the constraint $W_1^{ne} \geqslant W_1^e$ is binding because the unconstrained solution is $b_{i1} = b_1^{\text{no-rules}}$ which induces a degenerate debt distribution so there are no costs to enforce the constitution but benefits because the reputation jumps to one and so there are no intertemporal distortions going forward. Hence for $\pi = 0$ we have that $V_i\left(\pi = 0\right) > V_i^{dev}\left(\pi = 0\right)$ and by continuity in π we know that the deviation is not profitable for $\pi > 0$ but sufficiently close to zero. This shows that the constructed outcome is an equilibrium outcome.

We are left to show that such equilibrium is unique (among symmetric pure strategy equilibrium). Since local governments are homogeneous the distribution of debts among local governments is degenerate along the equilibrium path hence we cannot have non-enforcement of the fiscal constitution in period one. Therefore an equilibrium with non-enforcement cannot exist. With enforcement in period 1, the optimal debt issuances are the ones characterized above and so the equilibrium is unique. Q.E.D.

Proof of Proposition 2

Consider first the problem a local government i that expects that i) other local governments are going to violate the fiscal rule, ii) the no-commitment type central government is not going to enforce the fiscal rule punishment in period 1. Consequently, local government i expects to learn the type of the central government in period 1. The problem for the local government in time 0 is then:

$$\Omega(\pi) = \max_{b_{i1}} u(Y + qb_{i1}) + \beta\pi V_{i1}\left(\left(b_1^{\text{rules}} - \psi, b_{i1} - \psi\right), 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)$$

The optimality condition is:

$$qu'(Y - qb_{i1}) = \beta\pi \frac{\partial V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 1\right)}{\partial b_{i1}} - \beta(1-\pi)\frac{\partial V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)}{\partial b_{i1}}$$

and using the envelope conditions for $V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 1\right)$ and $V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)$ we obtain

$$\begin{aligned} qu'(Y + qb_{i1}) = {} & \beta\pi u'\left(Y - (b_{i1} + \psi) + q\mathbf{b}_{i2}(b_1 + \psi, 1)\right) \\ & + \beta(1-\pi) u'\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)\right) \\ & + \beta^2(1-\pi) u'\left(Y - \frac{\sum_{j=1}^{N} \mathbf{b}_{j2}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)}{N}\right) \sum_{j=1, j\neq 1}^{N} \frac{1}{N}\frac{\partial \mathbf{b}_{j2}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)}{\partial b_{i1}}, \end{aligned} \tag{25}$$

imposing symmetry we obtain condition (16) in the text. Notice that it is optimal to choose b_1 above the rule if everybody else choose $b_1 > \bar{b}_1$ instead of choosing $b_{i1} = \bar{b}_1$ and attaining value

$$\bar{\Omega}(\pi) = u\left(Y + q\bar{b}_1\right) + \beta\pi V_{i1}\left(\bar{b}_1, 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_1^{\text{rules}}, \bar{b}_1\right), 0\right)$$

In fact, for N large enough

$$\bar{\Omega}(\pi) \approx u\left(Y + q\bar{b}_1\right) + \beta\pi V_{i1}\left(\bar{b}_1, 1\right) + \beta(1-\pi) V_{i1}\left(b_1^{\text{rules}}, 0\right)$$

so for N large enough

$$\Omega(\pi) - \bar{\Omega}(\pi) = \left[u(Y + qb_1) - u\left(Y + q\bar{b}_1\right)\right] + \beta\pi\left[V_{i1}(b_1 + \psi, 1) - V_{i1}\left(\bar{b}_1, 1\right)\right]$$

which is positive if π is sufficiently small as

$$\left[u\left(Y+qb_1\right)-u\left(Y+q\bar{b}_1\right)\right]>0,\quad \left[V_{i1}\left(b_1+\psi,1,N\right)-V_{i1}\left(\bar{b}_1,1,N\right)\right]<0$$

The last step to establish that the conjectured equilibrium exists is to show that the no-commitment type central government when faced with debt $b_{i1}=b_1^{\text{rules}}$ for all i prefers to not enforcing the punishment ψ and revealing its type ($\pi'=0$ thereafter) than enforcing the punishment and having the posterior jumps to one (as the local governments expect only the commitment type to enforce the fiscal rule). That is, it must be that

$$W_1\left(b_1^{\text{rules}}+\psi,1\right)\leqslant W_1\left(b_1^{\text{rules}},0\right)$$

which is true if β is sufficiently small. In fact, for N large enough (see Lemma 2)

$$\begin{aligned} W_1\left(b_1^{\text{rules}},0\right)&=u\left((1+q)\,Y-b_1^{\text{rules}}\right)+\beta u\,(0)\\ W_1\left(b_1^{\text{rules}}+\psi,1\right)&=u\left(Y-\left(b_1^{\text{rules}}+\psi\right)+q\mathbf{b}_{i1}\left(b_1^{\text{rules}}+\psi,1\right)\right)+\beta u\left(Y-\mathbf{b}_{i1}\left(b_1^{\text{rules}}+\psi,1\right)\right)\end{aligned}$$

so in order for $W_1\left(b_1^{\text{rules}}+\psi,1\right)\leqslant W_1\left(b_1^{\text{rules}},0\right)$ it must be that

$$\beta\leqslant\bar{\beta}\left(b_1^{\text{rules}}\left(\bar{\beta}\right)\right)=\frac{u\left((1+q)\,Y-b_1^{\text{rules}}\right)-u\left(Y-\left(b_1^{\text{rules}}+\psi\right)+q\mathbf{b}_{i1}\left(b_1^{\text{rules}}+\psi,1\right)\right)}{u\left(Y-\mathbf{b}_{i1}\left(b_1^{\text{rules}}+\psi,1\right)\right)-u\,(0)}$$

where the right side of the expression above implicitly defines the maximal discount factor.

Can there be another equilibrium when local governments obey the rule? Suppose all local governments obey the rule. Since the rule is binding, it must be that $b_{i1}=\bar{b}_1$ for all i. Hence the value in this proposed equilibrium is

$$\bar{V}_i=u\left(Y+q\bar{b}_1\right)+\beta u\left(Y-\bar{b}_1+q\mathbf{b}_2\left(\bar{b}_1,\pi\right)\right)+\beta u\left(Y-\mathbf{b}_2\left(\bar{b}_1,\pi\right)\right)$$

We want to show that this cannot be an equilibrium because if local government i deviates by choosing $b_i>\bar{b}_1$ it gets a higher payoff. At the new state $\left(\bar{b}_1,b_{i1}\right)$, the no-commitment type does *not* enforce iff $W_1^{ne} > W_1^e$ where

$$W_1^e = \frac{(N-1)}{N}\left[u\left(Y-\bar{b}_1+q\mathbf{b}_{-i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)+\beta u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)}{N}\right)\right]$$
$$+\frac{1}{N}\left[u\left(Y-\left(b_{i1}+\psi\right)+q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)+\beta u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)}{N}\right)\right],$$
$$W_1^{ne} = \left[u\left(Y-b_{i1}+q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)\right)+\beta u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right]$$

The value for the deviation with enforcement is

$$V_i^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) = u\left(Y+qb_{i1}\right)+\beta u\left(Y-b_{i1}-\psi+q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)$$
$$+\beta\pi u\left(Y-\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)+\beta(1-\pi) u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)}{N}\right)$$

The value for the deviation without enforcement is

$$V_i^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) = u\left(Y+qb_{i1}\right)+$$
$$+\beta\pi\left[u\left(Y-b_{i1}-\psi+q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)+\beta u\left(Y-\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), \pi\right)\right)\right]$$
$$+\beta(1-\pi)\left[u\left(Y-b_{i1}+q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}+\psi\right), 0\right)\right)+\beta u\left(Y-\frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right]$$

If $\pi \to 0$ then

$$\lim_{\pi\to 0}\left[W_1^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right)-W_1^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right)\right]$$
$$=\frac{1}{N}\left[u\left((1+q)Y-\frac{(N-1)\bar{b}_1+b_{i1}}{N}\right)-u\left((1+q)Y-\frac{(N-1)\bar{b}_1+b_{i1}}{N}-\psi\right)\right]>0$$

and

$$\lim_{\pi\to 0} V_i^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) > \lim_{\pi\to 0} \bar{V}_i\left(\bar{b}, \pi\right)$$

So the deviation is profitable for π sufficiently small so there is only one equilibrium. Q.E.D.

Lemmas used in the proof of Proposition 3

To prove Proposition 3 we use the following two lemmas:

Lemma 2. *Under the assumptions of Propositions 1–3, as $N \to \infty$, the continuation equilibrium in period 1 given inherited debt b_1 and posterior π is such that:*

1. *If* $\pi > 0$, $\lim_{N\to\infty} \mathbf{b}_2(b_1,\pi) \to b_2 < Y$;

2. *If* $\pi = 0$, $\lim_{N\to\infty} \mathbf{b}_2(b_1,\pi) \to Y$ *and* $\lim_{N\to\infty} \frac{1}{N} u'(Y - \mathbf{b}_{i2}(b_1,0)) = \frac{q}{\beta} u'(Y - b_1 + qY) > 0$.

Proof. We know from the text that along a symmetric equilibrium outcome, it must be that

$$qu'(Y - b_1 + qb_2(b_1,\pi,N)) = \beta\pi u'(Y - b_2(b_1,\pi,N)) + \beta(1-\pi)\frac{1}{N}(Y - b_2(b_1,\pi,N))$$

whenever $\sum_i \mathbf{b}_{i2}(b_1,\pi,N)/N < Y$.

Consider part 1. Clearly, for each finite N, $b_2 < Y$ so the Euler equation holds with equality: $\forall N < \infty$,

$$qu'(Y - b_1 + qb_2) = \beta\pi u'(Y - b_2) + \beta(1-\pi)\frac{1}{N}u'(Y - b_2)$$

Suppose by way of contradiction that $b_2(b_1,\pi,N) \to Y$ as $N \to \infty$. Then the right side goes to ∞ while the left side goes to $qu'(Y - b_1 + qY)$ finite. This is a contradiction.

Consider part 2. For all $N < \infty$ we have that $b_2(b_1,0,N) < Y$ because of the Inada condition $\lim_{c\to 0} u'(c) = \infty$ so the Euler equation holds with equality for all N:

$$qu'(Y - b_1 + qb_2) = \beta\frac{1}{N}u'(Y - b_2) \tag{26}$$

Suppose by way of contradiction that $b_2(b_1,\pi,N) \to b_2 < Y$. Then the left side converges to a positive number, $qu'(Y(1+q) - b_1)$, while the right side converges to zero. Obtaining a contradiction.

Finally, since the Euler equation (26) holds for any N and $b_2(b_1,\pi,N) \to b_2 < Y$, given the continuity of u', it must be that

$$\lim_{N\to\infty}\frac{1}{N}u'(Y - b_2) = \frac{q}{\beta}u'(Y - b_1 + qY).$$

□

It follows immediately from the Lemma above that, if the posterior equals zero, the value of a continuation equilibrium is

$$u(Y - b_1 + qY) + \beta u(0) = u(Y(1+q) - b_1) + \beta u(0).$$

Hence we need to assume that $u(0)$ is finite for the enforcement decision not to be trivial.

Lemma 3. *Suppose* $\pi = 0$. *Then for all* i,

$$\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}}=-\frac{1}{q}$$

Proof. Step 1:$\lim_{N\to\infty} G_1\left(\pi=0\right)=0$.
We know from Lemma 1 that the equilibrium allocations are identical whether or not there are transfers by the central government in period 1. In the case in which there are transfers $T_{i1} = b_{i1} - \sum_i \frac{1}{N} b_{i1}$, the foc wrt b_{i1} and b_{i2} respectively are

$$u'\left(G_0\right)q=\beta\left[\frac{1}{N}u'\left(G_1\right)+\frac{\beta}{N}u'\left(G_2\right)\sum_{j\neq i}\frac{\partial b^t_{j2}}{\partial b^t_{i1}}\right] \tag{27}$$

$$u'\left(G^t_1\right)q=\frac{\beta}{N}u'\left(G^t_2\right) \tag{28}$$

where the superscript t denotes outcomes with transfers. Therefore

$$\sum_{j\neq i}\frac{\partial b^t_{j2}}{\partial b^t_{i1}}=\frac{u'\left(G_0\right)\frac{qN}{\beta}-u'\left(G_1\right)}{\beta u'\left(G_2\right)}=\frac{u'\left(G_0\right)\frac{qN}{\beta}-u'\left(G_1\right)}{Nu'\left(G_1\right)q}=\frac{\frac{u'(G_0)}{u'(G_1)}\frac{q}{\beta}-\frac{1}{N}}{q} \tag{29}$$

We know from Lemma 2 in Appendix B that $\lim_{N\to\infty} G_2\left(0\right)=0$. Now suppose by way of contradiction that $\lim_{N\to\infty} G_1\left(0\right)>0$. Then from (29) we see that

$$\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial b^t_{j2}}{\partial b^t_{i1}}=\frac{u'\left(G_0\right)}{\beta u'\left(G_1\right)}>0$$

Next, we can combine (27) and (28) to obtain

$$u'\left(G_0\right)q=\beta\frac{u'\left(G_1\right)}{N}\left[1+q\sum_{j\neq i}\frac{\partial b^t_{j2}}{\partial b^t_{i1}}\right]$$

If $G_1>0$ then the term $\frac{u'(G_1)}{N}$ converges to zero as $N\to\infty$, while the argument above establishes that the limit of $q\sum_{j\neq i}\frac{\partial b^t_{j2}}{\partial b^t_{i1}}$ is finite. Therefore as $N\to\infty$ the RHS of the above equation converges to zero while the LHS is finite. This is a contradiction. Since the equilibrium outcome with transfer in period 1 and the one without are equivalent when $\pi=0$ then $\lim_{N\to\infty} G_1\left(\pi=0\right)=0$.
Step 2: $\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}=-\frac{1}{q}$.
Now consider the case in which there are no transfers in period 1. In this case the first

order conditions imply that

$$\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1,0,N\right)}{\partial b_{i1}}=\frac{u'\left(G_0\right)\frac{qN}{\beta}-u'\left(G_1\right)N}{\beta u'\left(G_2\right)}=N\left(\frac{u'\left(G_0\right)\frac{q}{\beta}-u'\left(G_1\right)}{Nu'\left(G_1\right)q}\right)=\frac{\frac{u'(G_0)}{u'(G_1)}\frac{q}{\beta}-1}{q}$$

Since we just established that $\lim_{N\to\infty} G_1 = 0$ taking limits on both sides of the above equations yields the result. □

Proof of Proposition 8

Let $\pi > 0$ be sufficiently small. The ex-ante value of not- introducing a fiscal rule can be written as

$$\Omega^{\text{no-rule}} = u\left(Y + qb_1^{\text{no-rule}}\right) + \beta V_1\left(b_1^{\text{no-rule}}, \pi\right)$$

where b_1^{rule} is implicitly defined in (15). Similarly, the ex-ante value of introducing a rule can be written as

$$\Omega^{\text{rule}} = u\left(Y + qb_1^{\text{rule}}\right) + \beta\pi V_1\left(b_1^{\text{rule}} + \psi, 1\right) + \beta\left(1-\pi\right)V_1\left(b_1^{\text{rule}}, 0\right)$$

where b_1^{rule} is implicitly defined in (16). Combining the two expressions above we obtain

$$\begin{aligned}\Omega^{\text{no-rule}} - \Omega^{\text{rule}} = &\left[u\left(Y + qb_1^{\text{no-rule}}\right) - u\left(Y + qb_1^{\text{rule}}\right)\right] + \beta\pi\left[V_1\left(b_1^{\text{no-rule}}, \pi\right) - V_1\left(b_1^{\text{rule}} + \psi, 1\right)\right] \\ &+ \beta\left(1-\pi\right)\left[V_1\left(b_1^{\text{no-rule}}, \pi\right) - V_1\left(b_1^{\text{rule}}, 0\right)\right]\end{aligned}$$

Note that evaluating at $\pi = 0$, $\Omega^{\text{no-rule}} - \Omega^{\text{rule}} = 0$. Differentiating the right side with respect to π we obtain

$$\begin{aligned}\frac{\partial\left[\Omega^{\text{no-rule}} - \Omega^{\text{rule}}\right]}{\partial\pi} = &\left[qu'\left(Y + qb_1^{\text{no-rule}}\right) + \beta\frac{\partial V_{i1}\left(b_1^{\text{no-rule}}, \pi\right)}{\partial b_{i1}} + \beta\left(N-1\right)\frac{\partial V_{i1}\left(b_1^{\text{no-rule}}, \pi\right)}{\partial b_{-i1}}\right]\frac{\partial b_1^{\text{no-rule}}}{\partial\pi} \\ &- \left[qu'\left(Y + qb_1^{\text{rule}}\right) + \beta\pi\frac{\partial V_{i1}\left(b_1^{\text{rule}} + \psi, 1\right)}{\partial b_{i1}} + \beta\pi\left(N-1\right)\frac{\partial V_{i1}\left(b_1^{\text{rule}}, 1\right)}{\partial b_{-i1}}\right]\frac{\partial b_1^{\text{rule}}}{\partial\pi} \\ &- \left[\beta\left(1-\pi\right)\frac{\partial V_{i1}\left(b_1^{\text{rule}} + \psi, 0\right)}{\partial b_{i1}} + \beta\left(1-\pi\right)\left(N-1\right)\frac{\partial V_{i1}\left(b_1^{\text{rule}}, 0\right)}{\partial b_{-i1}}\right]\frac{\partial b_1^{\text{rule}}}{\partial\pi} \\ &+ \beta\frac{\partial V_1\left(b_1^{\text{no-rule}}, \pi\right)}{\partial\pi} - \beta\left[V_1\left(b_1^{\text{rule}} + \psi, 1\right) - V_1\left(b_1^{\text{rule}}, 0\right)\right] \\ &= \beta\left(N-1\right)\left[\frac{\partial V_{i1}\left(b_1^{\text{no-rule}}, \pi\right)}{\partial b_{-i1}}\frac{\partial b_1^{\text{no-rule}}}{\partial\pi} - \left(\pi\frac{\partial V_{i1}\left(b_1^{\text{rule}}, 1\right)}{\partial b_{-i1}} + \left(1-\pi\right)\frac{\partial V_{i1}\left(b_1^{\text{rule}}, 0\right)}{\partial b_{-i1}}\right)\right. \\ &+ \beta\frac{\partial V_1\left(b_1^{\text{no-rule}}, \pi\right)}{\partial\pi} - \beta\left[V_1\left(b_1^{\text{rule}} + \psi, 1\right) - V_1\left(b_1^{\text{rule}}, 0\right)\right]\end{aligned}$$

and evaluating at $\pi = 0$, since $b_1^{\text{no-rule}}(\pi = 0) = b_1^{\text{rule}}(\pi = 0)$, we obtain

$$\frac{\partial\left[\Omega^{\text{no-rule}} - \Omega^{\text{rule}}\right]}{\partial\pi}|_{\pi=0} = \beta(N-1)\left[\frac{\partial V_{i1}\left(b_1^{\text{no-rule}}, 0\right)}{\partial b_{-i1}}\left(\frac{\partial b_1^{\text{no-rule}}}{\partial\pi} - \frac{\partial b_1^{\text{rule}}}{\partial\pi}\right)\right]$$
$$+\beta\frac{\partial V_1\left(b_1^{\text{no-rule}}, 0\right)}{\partial\pi} - \beta\left[V_1\left(b_1^{\text{rule}} + \psi, 1\right) - V_1\left(b_1^{\text{rule}}, 0\right)\right]$$

Now

$$\frac{\partial V_{i1}\left(b_1^{\text{no-rule}}, 0\right)}{\partial b_{-i1}} < 0, \quad \frac{\partial b_1^{\text{no-rule}}}{\partial\pi} < \frac{\partial b_1^{\text{rule}}}{\partial\pi}, \quad \frac{\partial V_1\left(b_1^{\text{no-rule}}, 0\right)}{\partial\pi} > 0$$

so the first two terms are positive and by construction

$$V_1\left(b_1^{\text{rule}} + \psi, 1\right) - V_1\left(b_1^{\text{rule}}, 0\right) < 0$$

so the derivative is positive. Hence for $\pi > 0$ but sufficiently small we have that $\Omega^{\text{no-rule}} - \Omega^{\text{rule}} > 0$ and so the ex-ante value of not- introducing a fiscal rule is higher than the ex-ante value of introducing the fiscal rule.. Q.E.D.

Proof of Proposition 9

Given the punishment ψ and $\beta \leqslant \bar{\beta}$ where $\bar{\beta}$ is defined in the proof of Proposition 2, we know that for π small enough that the only two possible equilibria are i) the debt limit is never binding and ii) there is separation in period 1 and early resolution of uncertainty. The commitment type will then impose a binding rule if and only if $W_0^{c,sep} < W_0^{c,pool}$, where: $W_0^{c,sep}$is the value of separation for the commitment type

$$W_0^{c,sep} = \max_{\bar{b}} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi, \psi, \bar{b})\right) +$$
$$+\beta\sum_i \frac{1}{N}\left[\begin{array}{c} u\left(Y - \left(b_{i1}^{er}(\pi, \psi, \bar{b}) + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}\right) + q\mathbf{b}_{i2}\left(b_1^{er}(\pi, \psi, \bar{b}) + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \\ +\beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{er}(\pi, \psi, \bar{b}) + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \end{array}\right]$$

subject to $\bar{b} < b_{s1}^{\text{no-rules}}(\pi)$ with $b_{i1}^{er}(\pi, \psi)$ that solves equation (23) given π, ψ and $\bar{b}$ (note that the rule is going to be violated for sure by the South, $b_{s1}^{er}(\pi, \psi, \bar{b}) > \bar{b}$, but it may be satisfied by the North); $W_0^{c,pool}$ is the value for the commitment type if it imposes no fiscal rules:

$$W_0^{c,pool} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - b_{i1}^{lr}(\pi) + q \mathbf{b}_{i2}\left(b_1^{lr}(\pi), \pi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{lr}(\pi), \pi\right)\right) \end{array} \right]$$

with $b_{i1}^{lr}(\pi)$ that solves equation (22) given π.

Let $\Delta(\pi, \psi) = W_0^{c,sep} - W_0^{c,pool}$. As $\pi \to 0$, $\Delta(\pi, \psi) \to$

$$\beta \sum_i \frac{1}{N} \left[u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right] \\ -\beta \sum_i \frac{1}{N} \left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}(b_1, 0)\right) + \beta u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]$$

since $b_{i1}^{er}\left(0, \psi, \bar{b}\right) = b_{i1}^{lr}\left(0, 0, \bar{b}\right) = b_{i1}^{lr}(0) = b_{i1}$. Notice that if $\Delta(0, \psi) < 0$, then for π small $W_0^{c,sep} < W_0^{c,pool}$. This implies that the commitment type will optimally choose to set $\psi = 0$ and there will be no separation in period 1.

Define

$$\underline{\beta} \equiv \frac{\sum_i \left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right]}{\sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]}$$

where from now on we use $b_{i1} = b_{i1}^{er}\left(0, \psi, \bar{b}\right) = b_{i1}^{lr}(0)$. Then clearly for $\beta < \underline{\beta}$, the unique constitution will feature no fiscal rules because $\Delta(0, \psi) < 0$. Conversely, if $\beta > \underline{\beta}$, then for π close to zero, $W_0^{c,sep} > W_0^{c,pool}$. To show that this is an equilibrium, we need to show that the no-commitment type does indeed not want to enforce the constitution in period 1 (and induce separation) for $\beta > \underline{\beta}$. We know from the proof of Proposition 2 and Proposition 5 that if $\beta < \bar{\beta}$, where

$$\bar{\beta} \equiv \frac{\sum_i \left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right]}{N \left[u\left(Y - \frac{\mathbf{B_2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)}{N}\right) - u\left(Y - \frac{\mathbf{B_2}(b_1, 0)}{N}\right)\right]},$$

with $\mathbf{B_2}(b_1, \pi) = \sum_i \mathbf{b}_{i2}(b_1, \pi)$, for π close to zero, the no-commitment will strictly prefer to not enforce the rule at $t = 1$. To show that this a well defined interval, we need to show that $\bar{\beta} > \underline{\beta}$. This is true if

$$0 > Nu\left(Y - \frac{\mathbf{B_2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)$$
$$- \left[Nu\left(Y - \frac{\mathbf{B_2}(b_1, 0)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right].$$

For this to be true we need $\mathbf{b}_{s2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) < \mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0)$. From the first order conditions for $\mathbf{b}_{i2}(b_1, 0)$ we have

$$u'\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) q = \frac{\beta}{N} u'\left(Y - \frac{\mathbf{B}_{i2}(b_1, 0)}{N}\right)$$

This implies that

$$\mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0) = \frac{b_{s1} - b_{n1}}{q} \tag{30}$$

Next from the first order conditions for $\mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)$ we have

$$u'\left(Y - \psi Y \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) q = \beta u'\left(Y - q\mathbf{b}_{i2}\left(b_1^{er} + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)$$

Then, if the rule is not binding for the North:

$$\begin{aligned} &u'\left(Y - \psi Y - b_{s1} + q b_{s2}\right) - u'\left(Y - b_{n1} + q b_{n2}\right) \\ &= \beta u'\left(Y - q b_{s2}\right) - \beta u'\left(Y - q b_{n2}\right) > 0 \end{aligned}$$

and so

$$\mathbf{b}_{s2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}(b_1, 1) < \frac{\psi Y + b_{s1} - b_{n1}}{q} \tag{31}$$

If instead the rule is binding for the North as well we have

$$\mathbf{b}_{s2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}(b_1, 1) < \frac{b_{s1} - b_{n1}}{q} \tag{32}$$

So from (30) and (31)-(32) it follows that for ψ small enough, $\mathbf{b}_{s2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) < \mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0)$ and so $\bar{\beta} > \underline{\beta}$.

Finally, we need to show that the no-commitment type will mimic the commitment type in period 0 and announce the same rule. The value of mimicking is given by

$$\begin{aligned}
W_0^m(\pi) =& \sum_i u\left(Y_{i0} + q b_{i1}^{er}(\pi)\right) + \beta W_1^{er}\left(b_1^{er}(\pi)\right) \\
=& \sum_i \left[u\left(Y_{i0} + q b_{i1}^{er}(\pi)\right) + \beta u\left(Y - \psi Y - b_{i1}^{er}(\pi) + q\mathbf{b}_{i2}\left(b_{i1}^{er}(\pi), 0\right)\right)\right. \\
& \left. + \beta^2 u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(b_{i1}^{er}(\pi), 0\right)}{N}\right)\right]
\end{aligned}$$

while the value of not mimicking is just $W_0^m(0)$. We will establish that $\frac{\partial}{\partial\pi} W_0^m(0) > 0$, which in turn implies that if π is close to 0, the no-commitment type will always find it optimal to mimic. Differentiating $W_0^m(\pi)$ wrt π yields

$$\begin{aligned}
\frac{\partial}{\partial\pi} W_0^m(0) = \sum_i & \left[u'\left(G_{i0}\right) q \frac{\partial b_{i1}^{er}(\pi)}{\partial\pi} - \beta u\left(G_{i1}\right) \frac{\partial b_{i1}^{er}(\pi)}{\partial\pi} + \right. \\
& \left. + u'\left(G_{i1}\right) q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial b_{j1}^{er}(\pi)}{\partial\pi} - \frac{\beta^2}{N} u'\left(G_{i2}\right) \frac{\partial \mathbf{B}_2}{\partial b_{j1}} \frac{\partial b_{j1}^{er}(\pi)}{\partial\pi}\right]
\end{aligned}$$

Recall the first order conditions for the local government in periods 1 and 2

$$\begin{aligned}
u'\left(G_{i0}\right) q &= \beta u'\left(G_{i1}\right) + \frac{\beta^2}{N} u'\left(G_{i2}\right) \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}}{\partial b_{i1}} \\
u'\left(G_{i1}\right) q &= \frac{\beta}{N} u'\left(G_{i2}\right)
\end{aligned}$$

Substituting these into the previous equation yields

$$\begin{aligned}
\frac{\partial}{\partial\pi} W_0^m(0) =& \sum_i u'\left(G_{i1}\right) q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial b_{-i1}^{er}(0)}{\partial\pi} \\
=& u\left(G_{i1}\right) q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial B_1^{er}(0)}{\partial\pi} > 0
\end{aligned}$$

since at $\pi = 0$, $\frac{\partial}{\partial b_{N1}} \mathbf{b}_{S2}\left(b_1, 0\right) = \frac{\partial}{\partial b_{S1}} \mathbf{b}_{N2}\left(b_1, 0\right) < 0$ and $\partial B_1^{er}(0) / \partial\pi < 0$. Q.E.D.

B Additional Appendix (Not for Publication)

Additional Lemmas

Here we provide the proof for some lemmas we use in the main proofs. We next prove a Lemma that we use in the proof of Lemma 4 used to prove Proposition 3:

Lemma 4. *If* $b_1 = \{b_{i1}\}$ *is degenerate in that* $b_{i1} = b_{j1}$ *for all* i, j *then* $\lim_{N\to\infty} \frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial \pi} < \infty$.

Proof. By applying the implicit function theorem to (6) we obtain

$$\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial \pi} = \frac{\beta \frac{N-1}{N} u'(Y - \mathbf{b}_{i2}(b_1,0))}{\left[q^2 u''(Y - b_1 + q\mathbf{b}_{i2}(b_1,0)) + \frac{\beta}{N} u''(Y - \mathbf{b}_{i2}(b_1,\pi))\right]}$$

so

$$\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial \pi} = \left(1 - \frac{1}{N}\right) \frac{\beta \frac{1}{N} u'(Y - \mathbf{b}_{i2}(b_1,0))}{\left[q^2 u''(Y - b_1 + q\mathbf{b}_{i2}(b_1,0)) + \frac{\beta}{N} u''(Y - \mathbf{b}_{i2}(b_1,\pi))\right]}$$

As $N \to \infty$, the above converges to

$$\frac{\beta \frac{1}{N}\sum_{j\neq i} u'(0)}{\left[q^2 u''(Y - b_1 + qY) + \beta \frac{u''(0)}{N}\right]}$$

We know that $\beta \frac{1}{N}\sum_{j\neq i} u'(0)$ converges to a finite number. If $\beta\frac{u''(0)}{N}$ converges to a finite constant or zero then the above converges to a finite number. If it converges to ∞ then the above converges to zero. In both cases, as $N \to \infty$ $\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial \pi}$ converges to a finite number. □

Lemma 5. *i) For all* π, $W_1^e(\cdot,\pi)$ *is continuous and differentiable.*
ii) For all b*, for* π *small enough,* $W_1^e(b,\cdot)$ *is increasing in* π.

Proof. Part i). For convenience, rewrite (11):

$$W_1^e(b,\pi) = \sum_i \frac{1}{N}\left[u(Y - b_i + q\mathbf{b}_{i2}(b,\pi)) + \beta u\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b,\pi)}{N}\right)\right]$$

The fact that W_1^e is continuous and differentiable in b follows from continuity and differentiability of u and $\mathbf{b_2}$.

Part ii). Consider the derivative with respect to π:

$$\frac{\partial W_1^e(b,\pi)}{\partial \pi} = \sum_i \frac{1}{N}\left[qu'(G_{i1})\frac{\partial \mathbf{b}_{i2}}{\partial \pi} - \beta\frac{u'(G_{i2})}{N}\frac{\partial \mathbf{B}_{2i}}{\partial \pi}\right]$$

While we cannot sign this term in general, at $\pi = 0$ since $qu'(G_{i1}) = \frac{\beta}{N} u'(G_{i2})$, we have

$$\frac{\partial W_1^e(b,\pi)}{\partial \pi} = -\beta \sum_i \frac{u'(G_{i2})}{N^2} \sum_{j \neq i} \frac{\partial \mathbf{b}_{-i2}}{\partial \pi} = -\beta \frac{u'(G_{i2})}{N} \frac{(N-1)}{N} \frac{\partial \mathbf{B}_{-i2}}{\partial \pi} > 0$$

since we have established earlier that $\frac{\partial \mathbf{B}_2}{\partial \pi} < 0$ at $\pi = 0$. So for π close to zero W_1^e is increasing in π. □

Proof of Proposition 4

Assume first that local governments expect that the central government would not make any transfer in period 1 and it will mutualize debt in period 2 with probability $1 - \pi$. The optimality condition of problem (12) and the envelope condition from problem (5) gives that debt issuance in period 0 satisfies (22) and debt issuance in period 1 is $b_{i2}^{\text{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\text{no-rules}}, \pi\right)$ for $i = n, s$.

We next show that facing a distribution of debt $\left\{b_{i1}^{\text{no-rules}}\right\}_{i=n,s}$ in period 1 the central government does not have incentives to implement a transfer if π is small enough. To this end, define

$$\mathcal{W}(\pi) \equiv W^e\left(b_1^{\text{no-rules}}(\pi), \pi\right) - W^e\left(b_1^{\text{no-rules}}(\pi), 0\right)$$

as the difference between the value of enforcing the constitution (no-bailout) and not (bailout) for the government that inherits debts $b_1^{\text{no-rules}} = \left\{b_{i1}^{\text{no-rules}}\right\}_{i=n,s}$. Note that since W^e is continuous and b_1^{lr} is a continuous function of π, then $\mathcal{W}$ is continuous in π. Moreover $\mathcal{W}(0) = 0$ from Lemma 1and differentiating $\mathcal{W}$ we obtain:

$$\mathcal{W}'(\pi) = \sum_i \left[\frac{W^e\left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial b_{1i}} - \frac{\partial W^e\left(b_1^{\text{no-rules}}(\pi), 0\right)}{\partial b_{1i}}\right] \frac{\partial b_1^{\text{no-rules}}(\pi)}{\partial \pi} + \frac{\partial W^e\left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial \pi}$$

Evaluating the expression above at $\pi = 0$ we obtain

$$\mathcal{W}'(0) = \frac{\partial W^e\left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial \pi} > 0$$

as wanted. That W^e is increasing in π for π close to zero is established in Lemma 5 part ii).

We are left to show that an individual government has no incentives to increase its debt and force the central government to make a transfer. The relevant deviation is for the South. Suppose that a single local government $i \in \mathcal{S}$ chooses $b_{i1} > b_1^{\text{no-rules}}$ to induce the central government to bailout region i in period 1. The value for the best deviation is

$$\begin{aligned}
V_i^{dev} &= \max_{b_{i1}} u\left(Y + qb_{i1}\right) + \\
&\quad + \beta\pi\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right)\right] \\
&\quad + \beta\left(1-\pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)}{N}\right)\right]
\end{aligned}$$

subject to $W_1^{ne} \geqslant W_1^e$ where

$$\begin{aligned}
W_1^e &= \frac{M}{N} u\left(Y - b_{n1}^{\text{no-rules}} + q\mathbf{b}_{n2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \frac{(M-1)}{N} u\left(Y - b_{s1}^{\text{no-rules}} + q\mathbf{b}_{s2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right.\right.\right. \\
&\quad + \frac{1}{N} u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 1\right)}{N}\right) \\
W_1^{ne} &= u\left(Y - b_{i1} + q\mathbf{b}_{s2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)}{N}\right)
\end{aligned}$$

Let V_i be the value along the conjectured equilibrium and $\Delta V_i = V_i - V_i^{dev}$. Note that by construction, $b_1^{\text{no-rules}}$ solves

$$\begin{aligned}
V_i &= \max_{b_{i1}} u\left(Y + qb_{i1}\right) + \\
&\quad + \beta\pi\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right)\right] \\
&\quad + \beta\left(1-\pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)}{N}\right)\right]
\end{aligned}$$

Note that for $\pi = 0$ now the constraint $W_1^{ne} \geqslant W_1^e$ can be either slack or binding. (This is because now enforcing the constitution has redistributional costs.) If there is no heterogeneity in period 0 incomes, then the constraint is binding and so the same argument provided in the proof of Proposition 1 establishes that the deviation is not profitable. In particular we know in that case that $V_i > V^{dev}$. But then by continuity for small enough heterogeneity, the constraint $W_1^{ne} \geqslant W_1^e$ will continue to bind and as a result the deviation will not be profitable.

We are left to show that such equilibrium is unique (among symmetric pure strategy equilibrium). Suppose by way of contradiction there exists another equilibrium in which the South receives a bailout in period 1. We now show it is optimal for a local government i in the North to increase its consumption today and receives a bailout as well. In particular, assume that local government i chooses the same debt level chosen by the South along the equilibrium, $b_{i1}^{dev} = b_{s1}$. The no-commitment type central government still has an incentive to mutualize debt in period 1 and the payoff for local government i is

$$V_i^{dev} = u\left(Y_{n0} + qb_{s1}\right) + \beta\left(1-\pi\right) V_1\left(\left(b_1, b_{s1}\right), 0\right) + \beta\pi V_1\left(b_{s1}, 1\right)$$

while the value in equilibrium is

$$V_i = u\left(Y_{n0} + qb_{n1}\right) + \beta\left(1-\pi\right) V_1\left(\left(b_1, b_{n1}\right), 0\right) + \beta\pi V_1\left(b_{n1}, 1\right)$$

Clearly, $u\left(Y_{n0} + qb_{s1}\right) > u\left(Y_{n0} + qb_{n1}\right)$ and for N large enough $V_1\left(\left(b_1, b_{s1}\right), 0\right) = V_1\left(\left(b_1, b_{n1}\right), 0\right)$ since the continuation value for $\pi = 0$ depends only on average debt and government's i decision does not affect average if N is large enough. So if π is small enough the deviation is profitable, a contradiction. Q.E.D.

Proof of Proposition 5

Consider first the problem a local government i that expects that i) other local governments are going to violate the fiscal rule, ii) the no-commitment type central government is not going to enforce the fiscal rule punishment in period 1. Consequently, local government i expects to learn the type of the central government in period 1. The problem for the local government in time 0 is then:

$$\Omega_i\left(\pi\right) = \max_{b_{i1}} u\left(Y_{i0} + qb_{i1}\right) + \beta\pi V_{i1}\left(\left(b_1^{rules} - \psi, b_{i1} - \psi\right), 1\right) + \beta\left(1-\pi\right) V_{i1}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)$$

The optimality condition is:

$$qu'\left(Y_{i0} - qb_{i1}\right) = \beta\pi \frac{\partial V_{i1}\left(\left(b_1^{rules}, b_{i1}\right), 1\right)}{\partial b_{i1}} - \beta\left(1-\pi\right) \frac{\partial V_{i1}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)}{\partial b_{i1}}$$

and using the envelope conditions for $V_{i1}\left(\left(b_1^{rules}, b_{i1}\right), 1\right)$ and $V_{i1}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)$ we obtain

$$\begin{aligned} qu'\left(Y + qb_{i1}\right) = {} & \beta\pi u'\left(Y_{i0} - \left(b_{i1} + \psi\right) + q\mathbf{b}_{i2}\left(b_1 + \psi, 1\right)\right) \\ & + \beta\left(1-\pi\right) u'\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)\right) \\ & + \beta^2\left(1-\pi\right) u'\left(Y - \frac{\sum_{j=1}^{N} \mathbf{b}_{j2}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)}{N}\right) \sum_{j=1, j\neq 1}^{N} \frac{1}{N} \frac{\partial \mathbf{b}_{j2}\left(\left(b_1^{rules}, b_{i1}\right), 0\right)}{\partial b_{i1}}, \end{aligned} \tag{33}$$

Notice that it is optimal to choose b_1 above the rule if everybody else choose $b_1 > \bar{b}_1$ instead of choosing $b_{i1} = \bar{b}_1$ and attaining value

$$\bar{\Omega}_i(\pi) = u\left(Y_{i0} + q\bar{b}_1\right) + \beta\pi V_{i1}\left(\bar{b}_1, 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_1^{\text{rules}}, \bar{b}_1\right), 0\right)$$

In fact, for N large enough

$$\bar{\Omega}_i(\pi) \approx u\left(Y_{i0} + q\bar{b}_1\right) + \beta\pi V_{i1}\left(\bar{b}_1, 1\right) + \beta(1-\pi) V_{i1}\left(b_1^{\text{rules}}, 0\right)$$

so for N large enough

$$\Omega_i(\pi) - \bar{\Omega}_i(\pi) = \left[u\left(Y_{i0} + qb_1\right) - u\left(Y_{i0} + q\bar{b}_1\right)\right] + \beta\pi\left[V_{i1}\left(b_1 + \psi, 1\right) - V_{i1}\left(\bar{b}_1, 1\right)\right]$$

which is positive if π is sufficiently small as

$$\left[u\left(Y_{i0} + qb_1\right) - u\left(Y_{i0} + q\bar{b}_1\right)\right] > 0, \quad \left[V_{i1}\left(b_1 + \psi, 1, N\right) - V_{i1}\left(\bar{b}_1, 1, N\right)\right] < 0$$

The last step to establish that the conjectured equilibrium exists is to show that the no-commitment type central government when faced with debt $b_{i1} = b_1^{\text{rules}}$ for all i prefers to not enforcing the punishment ψ and revealing its type ($\pi' = 0$ thereafter) than enforcing the punishment and having the posterior jumps to one (as the local governments expect only the commitment type to enforce the fiscal rule). That is, it must be that

$$W_1\left(b_1^{\text{rules}} + \psi, 1\right) \leqslant W_1\left(b_1^{\text{rules}}, 0\right)$$

which is true if β is sufficiently small. In fact, for N large enough:

$$W_{i1}\left(b_1^{\text{rules}}, 0\right) \approx \sum_i \frac{1}{N} u\left((1+q)Y - b_{i1}^{\text{rules}}\right) + \beta u(0)$$

$$W_1\left(b_1^{\text{rules}} + \psi, 1\right) \approx \sum_i \frac{1}{N}\left[u\left(Y - \left(b_1^{\text{rules}} + \psi\right) + q\mathbf{b}_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)\right)\right]$$

so in order for $W_1\left(b_1^{\text{rules}} + \psi, 1\right) \leqslant W_1\left(b_1^{\text{rules}}, 0\right)$ it must be that

$$\beta \leqslant \bar{\beta}\left(b_1^{\text{rules}}\left(\bar{\beta}\right)\right) = \frac{\sum_i\left[u\left((1+q)Y - b_1^{\text{rules}}\right) - u\left(Y - \left(b_1^{\text{rules}} + \psi\right) + q\mathbf{b}_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)\right)\right]}{\sum_i\left[u\left(Y - \mathbf{b}_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)\right) - u(0)\right]}$$

where the right side of the expression above implicitly defines the maximal discount factor.

Can there be another equilibrium when local governments obey the rule? Suppose all local governments obey the rule. Since the rule is binding, it must be that $b_{j1} = \bar{b}_1$ for some j. Hence the value in this proposed equilibrium is

$$\bar{V}_i = u\left(Y_{i0} + q\bar{b}_1\right) + \beta u\left(Y - \bar{b}_1 + q\mathbf{b}_{i2}\left(\bar{b}_1, \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\bar{b}_1, \pi\right)}{N}\right)$$

We want to show that this cannot be an equilibrium because if local government i deviates by choosing $b_i > \bar{b}_1$ it gets a higher payoff. At the new state $\left(\bar{b}_1, b_{i1}\right)$, the no-commitment type does *not* enforce iff $W_1^{ne} > W_1^e$ where

$$\begin{aligned}
W_1^e &= \sum_{j \neq i} \frac{1}{N}\left[u\left(Y - b_{i1} + q\mathbf{b}_{-i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right] \\
&\quad + \frac{1}{N}\left[u\left(Y - \left(b_{i1} + \psi\right) + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right], \\
W_1^{ne} &= \sum_j \frac{1}{N}\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right]
\end{aligned}$$

The value for the deviation with enforcement is

$$\begin{aligned}
V_i^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) &= u\left(Y + qb_{i1}\right) + \beta u\left(Y - b_{i1} - \psi + q\mathbf{b}_{i2}\left(\left(b_{-i1}, b_{i1} + \psi\right), \pi\right)\right) \\
&\quad + \beta\pi u\left(Y - \mathbf{b}_{i2}\left(\left(b_{-i1}, b_{i1} + \psi\right), \pi\right)\right) + \beta\left(1 - \pi\right) u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_{-i1}, b_{i1} + \psi\right), \pi\right)}{N}\right)
\end{aligned}$$

The value for the deviation without enforcement is

$$\begin{aligned}
V_i^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) &= u\left(Y + qb_{i1}\right) + \\
&\quad + \beta\pi\left[u\left(Y - b_{i1} - \psi + q\mathbf{b}_{i2}\left(\left(b_{-i1}, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_{-i1}, b_{i1} + \psi\right), \pi\right)\right)\right] \\
&\quad + \beta\left(1 - \pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_{-i1}, b_{i1} + \psi\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_{-i1}, b_{i1}\right), 0\right)}{N}\right)\right]
\end{aligned}$$

Then,

$$\begin{aligned}
&\lim_{\pi \to 0}\left[W_1^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) - W_1^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right)\right] \\
&= \frac{1}{N}\left[u\left(Y - \left(b_{i1} + \psi\right) + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right] \\
&- \frac{1}{N}\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right] < 0
\end{aligned}$$

and

$$\lim_{\pi \to 0} V_i^{ne}\left(\left(b_{-i1}, b_{i1}\right), \pi\right) > \lim_{\pi \to 0} \bar{V}_i\left(b_{-i1}, \bar{b}, \pi\right)$$

So the deviation is profitable for π sufficiently small so there is only one equilibrium.

Proof of Proposition 6

For the case without fiscal rules, we can combine (22) with (6) to obtain a condition that characterizes the debt issuance in period 0: for $i = n, s$

$$u'(Y_{i0} + qb_{i1})\, q = \frac{\beta^2 \pi}{q} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) + \frac{\beta^2 (1-\pi)}{qN} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) + \frac{\beta^2 (1-\pi)}{N} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}(b_1, \pi)}{\partial b_{i1}} \tag{34}$$

For the case with fiscal rules, we can combine (23) with (6) to obtain for $i = n, s$

$$u'(Y_{i0} + qb_{i1})\, q = \frac{\beta^2 \pi}{q} u'\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) + \frac{\beta^2 (1-\pi)}{qN} u'(Y - \mathbf{b}_{i2}(b_1, 0)) \tag{35}$$

$$\frac{\beta^2 (1-\pi)}{N} u'(Y - \mathbf{b}_{i2}(b_1, 0)) \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}(b_1, 0)}{\partial b_{i1}}$$

Taking the limit as N goes to infinity of the two expressions above, using Lemma 2 and Lemma 3, we obtain

$$u'(Y_{i0} + qb_{i1})\, q = \frac{\beta^2 \pi}{q} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) \tag{36}$$

and

$$u'(Y_{i0} + qb_{i1})\, q = \frac{\beta^2 \pi}{q} u'(Y - \mathbf{b}_{i2}(b_1 + \psi Y, 1)) \tag{37}$$

respectively. We know that for π small enough $\mathbf{b}_{i2}(b_1, \pi) > \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)$. This is because as $\pi \to 0$ then $\mathbf{b}_{i2}(b_1, \pi) \to Y$ and $\mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)$ is bounded away from Y. (See Lemma 2 for details.) This observation and concavity of u imply that for all $i = n, s$

$$\frac{\beta^2 \pi}{q} u'\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi Y \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) < \frac{\beta^2 \pi}{q} u'(Y - \mathbf{b}_{i2}(b_1, \pi))$$

so from (36) and (37) the expected marginal cost of issuing debt in period 0 is lower when there is early revelation about the central government's type. Hence, both type of local governments will issue more debt in period 0 because of the lower expected marginal cost. The argument for debt issuances in period 1 being higher with rules is exactly the same as the homogeneous case. Q.E.D.

Proof of Proposition 7

Suppose all local governments obey the rule. Since the rule is binding, it must be that $b_{i1} = \bar{b}_1$ for all i. Hence the value in this proposed equilibrium is

$$\bar{V}_i = u\left(Y + q\bar{b}_1\right) + \beta u\left(Y - \bar{b}_1 + q\mathbf{b}_2\left(\bar{b}_1, \pi\right)\right) + \beta u\left(Y - \mathbf{b}_2\left(\bar{b}_1, \pi\right)\right)$$

We want to show that this cannot be an equilibrium because if local government i deviates by choosing $b_i > \bar{b}_1$ it gets a higher payoff. At the new state $\left(\bar{b}_1, b_{i1}\right)$, the no-commitment type does *not* enforce iff $W_1^{ne} > W_1^e$ where

$$\begin{aligned}
W_1^e = &\frac{(N-1)}{N}\left[u\left(Y - \bar{b}_1 + q\mathbf{b}_{-i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right] \\
&+ \frac{1}{N}\left[u\left(Y - \left(b_{i1} + \psi\right) + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right], \\
W_1^{ne} = &\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right]
\end{aligned}$$

The value for the deviation with enforcement is

$$\begin{aligned}
V_i^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) = &\, u\left(Y + qb_{i1}\right) + \beta u\left(Y - b_{i1} - \psi + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) \\
&+ \beta\pi u\left(Y - \mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta\left(1 - \pi\right) u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)
\end{aligned}$$

The value for the deviation without enforcement is

$$\begin{aligned}
V_i^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) = &\, u\left(Y + qb_{i1}\right) + \\
&+ \beta\pi\left[u\left(Y - b_{i1} - \psi + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right)\right] \\
&+ \beta\left(1 - \pi\right)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right]
\end{aligned}$$

Note that for N large enough and $\pi > 0$,

$$\begin{aligned}
\lim_{N\to\infty} W_1^e = &\lim_{N\to\infty}\left[u\left(Y - \bar{b}_1 + q\mathbf{b}_{-i2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1} + \psi\right), \pi\right)}{N}\right)\right] \\
&> \lim_{N\to\infty}\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(\bar{b}_1, b_{i1}\right), 0\right)}{N}\right)\right] \\
&= \lim_{N\to\infty} W_1^{ne}\left(\left(\bar{b}_1, b_{i1}\right), \pi\right)
\end{aligned}$$

and so

$$\lim_{N\to\infty} V_i^e\left(\left(\bar{b}_1, b_{i1}\right), \pi\right) < \lim_{N\to\infty} \bar{V}_i\left(\bar{b}, \pi\right)$$

So the deviation is not profitable. Hence at $N \to \infty$ there are two equilibria. Q.E.D.