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Carsten Eckel
Stephen R. Yeaple

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ABSTRACT

International trade is primarily conducted by large, multiproduct firms (MPFs) that pay above average wages and exhibit high productivity. In this paper we show that if firms can invest in management technologies for identifying worker skill then they will enjoy a form of market power in the labor market that artificially lowers their labor costs. This market failure results in excessive consumption of resources by large, productive exporting firms relative to the social optimum. Trade liberalization then has an ambiguous effect on aggregate welfare: lower trade costs increase access to foreign goods but also exacerbates the labor market distortion as resources are transferred to large firms. The model highlights the need to know why firms "excel" before drawing welfare conclusions regarding cross firm reallocations of resources.

Carsten Eckel
University of Munich
Department of Economics
Ludwigstr. 28 VG
80539 Munich
Germany
carsten.eckel@lrz.uni-muenchen.de

Stephen R. Yeaple
Department of Economics
The Pennsylvania State University
520 Kern Building
University Park, PA 16802-3306
and NBER
sry3@psu.edu

1 Introduction

Exporting firms are rare, and they display characteristics that suggest that they are superior to non-exporting firms. They tend to be larger, more productive, sell a wider array of goods, and pay higher wages than do smaller firms (Bernard et al., 2007). Given these facts, it is tempting to believe that reallocation of resources from small, non-exporting firms to exporting firms likely increases real output. Indeed, large, exporting firms may be inefficiently small for a number of reasons: they charge larger mark-ups (e.g. Nocco et al., 2017), they may be suppressed by anti-dumping policies that target the most efficient firms (Ruhl, 2014), or they may have higher labor costs because of labor institutions that force them to share rents with their workers (e.g. Egger and Kreickemeier, 2009).

The literature has been less focused on the possibility that market imperfections might induce exporters to be too large. Recent empirical studies suggest that some component of the productivity advantages of exporting firms is determined by the structure and the quality of their workers (e.g. Irarrazabal et al, 2013). This is potentially problematic because hiring and recruiting decisions are subject to numerous imperfections, as a larger labor literature has pointed out (e.g. Greenwald, 1986). In this paper we will show that these imperfections can lead to distortions in the allocation of labor across firms and affect the welfare implications of (trade-induced) reallocations of labor. Our results highlight the need to know why firms "excel" before drawing welfare conclusions regarding cross firm reallocations of resources.

Our findings also have important policy implications. We show that market failures in the labor market may actually favor large, exporting firms and that this may explain in part their apparent superior performance relative to smaller, non-exporters. As a result, subsidies to smaller firms may actually be welfare improving. This finding is in line with policies in many countries that favor small, non-exporting businesses while imposing regulations that restrict the largest firms.

In this paper we analyze the welfare effects of trade liberalization in a version of the Yeaple (2005) model of endogenous firm heterogeneity in which information asymmetries give rise to labor market inefficiencies. In the model, workers' ability is private information on the labor market. Firms can adopt high-tech technologies which are better implemented by high ability workers or they can use an old technology in which worker ability is less crucial. To implement the high-technology, firms must invest in a human resources screening technology to identify the quality of job applicants.

In this setting, firms that invest in the high technology have two advantages relative to those that do not. First, their choice of technology lowers their marginal cost as it does in the original Yeaple (2005) paper. Second, their information advantage in the labor market confers onto them a form of market power. Because they select the best workers, adopting firms pay high wages, yet in equilibrium these wages don't fully compensate workers for their ability so that adopting firms have lower effective labor costs than non-adopting firms.[1] These lower effective labor costs allow these firms to expand into non-core activities, such as more product lines (multiproduct firms) or exporting.

In equilibrium, the labor market recruiting advantage of large multiproduct exporters induces them to consume too many resources relative to the social optimum. This is because the marginal (most skilled) worker in small firms is paid a wage that reflects the average productivity of the labor pool and not her (high) productivity relative to the pool. We show that an employment subsidy that lowers the cost of labor to small firms can achieve the socially optimal level of output.[2]

Our model has important implications for trade liberalization. In the absence of a corrective subsidy to smaller firms, the impact of trade liberalization on aggregate welfare is ambiguous. On the one hand, a trade liberalization directly raises welfare through cheaper access to foreign produced goods as in standard models. On the other hand, a trade liberalization leads to a reallocation of resources from small to large firms and so worsens the market imperfection. Whatever the aggregate welfare impact, our model predicts that trade liberalization worsens income inequality as the real income of high ability workers rises and the real income of low ability workers falls.

The key assumptions of our paper that skilled workers have a comparative advantage using low marginal cost technologies and that firms make managerial investments in human resource management have received growing support in the empirical literature that uses matched employer-employee data. For instance, Bender et al (2016) consider detailed employer-employee data from Germany. They show that average employee ability is higher for firms using advanced management practices and that a substantial portion of the productivity advantages of these firms can be attributed to their use of better workers.[3] Further, the authors directly document on-going selection by higher productivity firms of better-than-

[1] In many models that generate the result that productive firms pay higher wages, labor institutions require firms to share rents with workers (e.g. Egger and Kreickemeier, 2009). Here, exactly the opposite obtains. Information asymmetries give employers an edge in the labor market and so workers share rents on their skill with firms.

[2] In our baseline model, we consider a scenario in which firms can enter freely as either small or large firms. We show in the appendix that if firms are intrinsically heterogeneous that the result continues to hold. Rather than there being too many large firms and too few small firms, large firms are too large and small firms are too small relative to the social optimum.

[3] Friedrich (2017) uses matched employer-employee data for Belgium to show that high productivity firms invest in identifying more talented managers and then subsequently invest more heavily in their human capital accumulation. He models this empirical phenomenon as stemming from internal labor markets that arise from asymmetric learning and firm-specific human capital.

average employees,[4] writing

> ....better managed firms are able to build up a superior stock of employees through selective hiring and attrition. In particular, examining job inflows and outflows at the plants in our sample, we find that those with higher management scores are more likely to recruit higher ability workers (measured by the permanent component in their earnings) and are less likely to lay off or fire the highest skilled workers in the period between 2004 and 2009. (p. 3)

As in Yeaple (2005), our model predicts wage stratification across firms within industries as workers select into firms that have adopted different technologies and that wage dispersion across firms should rise in response to trade liberalization. Unlike Yeaple (2005), our model also predicts greater wage dispersion within large, exporting firms than within small, non-exporters as it is these larger firms that have the informational advantage that allows them to compete for talent in the work force. Recent research by Becker et al (2017) confirms that this is indeed the case among German firms: as firms become larger residual wage dispersion within firms grows.

Our paper is related to a small but growing literature that highlights the implications of factor market imperfections for the welfare implications of trade liberalization. For instance, Helpman, Itskhoki, and Redding (2010) present a model in which human capital spillovers among employees in the presence of imperfect information regarding worker skills leads more productive firms to invest more intensively in a worker screening technology. As in our paper, they analyze the wage inequality implications of trade liberalization. Unlike our paper, they do not stress the implications of the factor market frictions for corrective government policies concerning that friction or how trade liberalization affects the aggregate welfare effects in the presence of imperfections in the labor market.[5]

Our paper contributes to the literature that explores how market imperfections in the presence of heterogeneous firms may affect the welfare impact of trade. Much of the recent literature has focused primarily on the product market by investigating the role of international trade on the reallocation of resources across firms that charge different mark-ups over their marginal cost (for example, see Arkolakis et al., 2017, Nocco et al., 2017, and Edmonds et al., 2015). In these settings, the key resource problem is that the most efficient firms

[4] Using similar German data, Card et al. (2013) establish that a significant portion of rising inequality among German workers can be attributed to increasing plant-level productivity heterogeneity and rising assortativeness in the assignment of workers to establishments.

[5] While imperfect observability of skill and endogenous and costly screening efforts of parent firms are the focus of our paper and of Helpman et al. (2010), there are substantial differences in market structure that lead to very different outcomes. For instance, unemployment arises in their setting but is not an implication of our model.

are too small from a social point of view because they charge the highest mark-ups. In our setting, there is too much entry of large firms that stems from their artificially low cost of labor.[6]

Our paper also makes contact with the literature on multiproduct firms and international trade. We adopt the flexible manufacturing apparatus present in Eckel and Neary (2010) and demonstrate that multiproduct and single product firms arise endogenously in equilibrium. As in Eckel and Neary (2010) and Bernard, Redding, and Schott (2011), trade liberalization induces firms to pare their high cost product lines that are sold only domestically but the reallocation of labor from small to large firms has the implication that the share of multiproduct firms in total output expands.

The remainder of this paper is organized into four sections. Section 1 introduces the model assumptions and characterizes the equilibrium. Section 2 provides an analysis of the welfare implications of labor market imperfections. The resource allocation and welfare implications of international trade liberalization are explored in section 3. Section 4 provides concluding comments.

2 Model

In this section, we present the closed economy version of our model. We begin in the next subsection with the model assumptions and then characterize the equilibrium in the next subsection.

2.1 Key Assumptions

2.1.1 Demand

On the demand side, we are not making any new or specific assumption but follow Krugman (1980). Consumers derive utility from the consumption of horizontally differentiated varieties. The utility function of a consumer is CES:

$$U = \left(\int_{i \in \tilde{\Omega}} q\left(i\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $q\left(i\right)$ is the quantity consumed, σ is the elasticity of substitution between any two varieties, and $\tilde{\Omega}$ is the set of potentially consumable varieties.

[6]Costinot et al. (2016) show that in a firm heterogeneity model a social planner can improve a country's welfare by raising tariffs on the most efficient exporters while leaving marginal exporters untaxed. This result is fundamentally different than ours as there is no rationale in their setting for subsidizing little firms.

Direct demand for variety $i \in \Omega$ (the set of actually produced varieties) is then given by

$$x(i) = EP^{\sigma-1} p(i)^{-\sigma}, \qquad (2)$$

where $x(i)$ is economy-wide output of variety i and E is aggregate income in the economy. P stands for the price index, defined by

$$P \equiv \left(\int_{i \in \Omega} p(i)^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}. \qquad (3)$$

2.1.2 Production

There are two types of factors of production: Management H and labor L. Management is a homogeneous factor that is used as our numéraire. As in Yeaple (2005), labor consists of a continuum of heterogeneous workers with skills (or productivity) z. The distribution of skills in the economy is described by the probability density function $g(z)$ with positive support over $[\underline{z}, \infty)$ ($\underline{z} > 0$) and its cumulative distribution function $G(\tilde{z}) = \int_{\underline{z}}^{\tilde{z}} g(z) \, dz$.

Production of a variety $x(i)$ requires a fixed costs f in units of management plus marginal costs in units of (effective) labor. These marginal costs are constant for a specific variety but may vary across varieties. They consist of a unit labor requirement α (in units of effective labor) plus a factor cost component c_j. The factor cost component is firm specific, as denoted by the subscript j.

Unit labor requirements α are given by technology, and all firms have access to the same technology. We follow Eckel and Neary (2010) and assume that all firms possess a certain core competency for a specific variety where their unit labor costs is lowest for all products in their product range. All other products in their product range can then be identified by their (unidimensional) distance to the firm's core competency, denoted by $\omega > 0$. Production of multiple products is subject to flexible manufacturing, which implies that firms can add and drop products to and from their product range freely, but as they add products to their product range and move away from their core competency, unit labor requirements of these products increases. Thus, unit labor requirements α depend on the position ω of a product in a firm's product range, and are increasing in ω:

$$\alpha = \alpha(\omega) \qquad \text{and} \qquad \alpha'(\omega) \equiv \partial\alpha / \partial\omega > 0. \qquad (4)$$

To simplify notation we normalize unit labor requirements at the core to one: $\alpha(0) = 1$.

The productivity of individual workers depends on the skills of these workers and on the technology used by the firm. There are two technologies available. In one technology, call it low-tech, skills of workers are proportionate to their effective supply of labor $a(z)$. In this case a worker with skill z has an effective supply of labor of $a_L(z) = z$. In the other technology, call it high-tech, a worker with skill z has an effective supply of labor of $a_H(z) > z$, where $a_H(\underline{z}) = a_L(\underline{z}) = \underline{z}$, $a'_H(\underline{z}) \geq 1$ and $a''_H(z|\, z > \underline{z}) > 0$. Thus, a worker with a higher skill has an absolute advantage in both technologies, and a comparative advantage in the high-tech technology. This is essentially the same assumption as in Yeaple (2005).

Since the hi-tech technology is superior to the low-tech technology, firms would always prefer to use the hi-tech technology. However, we assume that the hi-tech technology requires knowledge of the true productivity of workers, and this information is not available to all firms. In the absence of a screening technology, firms do not observe the productivity of any given worker. It is this information asymmetry that gives rise to the market imperfection. A screening technology exists but is only available to a firm if it incurs a fixed cost f_m (in units of management). One can think of this screening technology as an investment in a human resource staff that can accurately assess productivity. Firms that have incurred f_m can immediately evaluate the productivity of a worker while firms that have not incurred f_m can never observe the productivity of any worker.

Thus, a firm that has invested in the screening technology knows the productivity of its workers and can use the more advanced technology. A firm that has not invested in this screening technology must use the less advanced technology.

2.1.3 Market Structure and Timing

The market for the homogeneous factor management H is perfectly competitive, and the wage of a unit of management is normalized to one. Workers L are fully informed about their own productivity z but firms know only the distribution of productivity in the population, $G(z)$, which is common knowledge.

This is a one shot game that occurs in five stages. All agents have rational expectations and perfect foresight.

In stage 1, firms enter and decide whether they want to pay f_m and acquire the screening technology or not. This determines their type: Type-m firms pay f_m, type-s firms do not.[7] There is a continuum of firms of both types and their masses will be denoted by n_j ($j \in \{m, s\}$).

Once firms have made their entry and screening technology investments, two labor markets open. Firms that have made the screening investment, $j = m$, operate in one labor market while firms that have not made the screening investment, $j = s$, operate in the other.

[7] We have chosen this notation because type m-firms will turn out to be multi-product firms and type s-firms will be single-product firms. This will be proven below in proposition 2.

Let the set of workers that ultimately choose to be in labor market j be denoted as Z_j.

We refer to the labor market associated with firms $j = m$ as the "frictionless" labor market because all information regarding workers in that labor market is known by all firms. Perfect competition implies that the wage of worker 1 relative to worker 2 with skills $z_1,\ z_2 \in Z_m$ and productivities $a(z_1)$ and $a(z_2)$ satisfy the no arbitrage condition $w_1/w_2 = a(z_1)/a(z_2)$.

We refer to the labor market associated with firms $j = s$ as the "frictional" labor market because individual worker productivies, $z \in Z_s$, are known only to the workers. The inability of firms $j = s$ to verify workers' productivities requires that there must be a single wage $w = w_S$ for all $z \in Z_s$.

In Stage 2, workers choose whether to enter the frictionless or the frictional labor market. They make this choice with perfect foresight regarding the wage they would receive in each labor market.

In stage 3, the set of products produced (ω) is chosen, and the fixed costs f per product is paid. Firms that produce only their core competency product are called single-product firms (SPF), firms that produce multiple products are called multi-product firms (MPF).

In stage 4, both frictionless and frictional labor markets clear.

In stage 5, production occurs and product markets are cleared. Firms compete via monopolistic competition. Individual products are atomistic and there is no strategic interaction.

2.2 Closed Economy Equilibrium

This section characterizes the equilibrium to our closed economy model. Each stage is analyzed in sequence starting from stage 5 and progressing backward to stage 1.

2.2.1 Product Market Clearing

Given demand (2) and a market structure of monopolistic competition, the profit-maximizing price charged by division ω of firm j is a constant mark-up over its marginal costs:

$$p(\omega, c_j) = \frac{\sigma}{\sigma - 1} \alpha(\omega) c_j, \tag{5}$$

where j denotes firm type $j \in \{m, s\}$. Since all firms have access to the same technology, and demands are symmetric across all products, all firms within one type will be symmetric. Since firms of different types are drawing their workers from different labor markets, their factor costs c_j may be different, hence the subscipt j.

In order to simplify notation we define

$$A \equiv (\sigma - 1)^{\sigma-1} \sigma^{-\sigma} EP^{(\sigma-1)}. \tag{6}$$

This parameter A depends only on aggregate income E, the price index P, and the elasticity of substitution σ. Since firms are atomistic, A is exogenous to the firm.

Given (2), (5) and (6), output of variety ω can be written as

$$x(\omega, c_j) = (\sigma - 1) A c_j^{-\sigma} \alpha(\omega)^{-\sigma}, \tag{7}$$

and revenues are

$$p(\omega, c_j) x(\omega, c_j) = \sigma A c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} \tag{8}$$

Finally, profits per product are variable profits $p(\omega, c_j) x(\omega, c_j)/\sigma$ minus fixed costs f:

$$\pi(\omega, c_j) = A c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} - f. \tag{9}$$

2.2.2 Labor Market Clearing

Worker sorting in stage two leads to segmentation of labor markets by firm type. The labor market equilibrium for type $j \in \{m, s\}$ is

$$n_j \int_0^{\omega_j^d} x(\omega, c_j) \alpha(\omega) d\omega = \tilde{L}_j, \tag{10}$$

where ω_j^d is the mass of varieties produced by firms of type j,[8] and $\tilde{L}_j$ is the effective supply of labor available to firms of type j. Since workers sort in stage two, and firms decide on their product range in stage 3, both of these variables are given at this stage, and the labor market equilibrium determines the effective labor cost, c_j, facing firms of type j. In both labor markets $j \in \{m, s\}$ firms are atomistic and take wages as given.

Market clearing of the numéraire factor (management H) is implied in general equilibrium.

2.2.3 Product Range

The product range is determined at the firm (or conglomerate) level. Firm level profits Π_j consist of all the profits of its divisions minus the fixed cost for the screening technology (for

[8] The superscript d stands for domestic. It is superfluous in the case of the closed economy, but will be useful in the open economy to distinguish domestic sales from exports.

type-m firms):

$$\Pi_j = \int_0^{\omega_j^d} \pi(\omega, c_j)\, d\omega - \mathbb{I}_j f_m, \tag{11}$$

where $\mathbb{I}_j$ is an indicator variable that takes on the value 1 if $j = m$ and 0 otherwise.

The first order condition with respect to the product range requires that $d\Pi_j/d\omega_j^d = 0$. Since firms are atomistic, they are price takers in both labor markets. Using the Leibniz integral rule, the first order condition can be expressed as

$$\frac{d\Pi_j}{d\omega_j^d} = \pi\left(\omega_j^d, c_j\right) = 0. \tag{12}$$

Thus, using (9), the optimal scope ω_j^d is determined by

$$Ac_j^{1-\sigma}\alpha\left(\omega_j^d\right)^{1-\sigma} = f. \tag{13}$$

2.2.4 Worker Sorting

Workers can observe whether a firm has invested in the screening technology or not. Thus, they can decide whether they want to apply for a job in a type-m firm or in a type-s firm by choosing the respective labor pool. There are no differences in non-pecuniary job returns, so this decision is entirely based on differences in wages.

The labor market of type-m firms is perfectly competitive. After screening, the true productivity of workers is known by all firms in this labor market segment, and they can pay a wage to individual workers based on this worker's true productivity. In addition, all firms in this segment use the hi-tech technology. Anticipating correctly the effective wage c_m determined in stage 4, firms of type-m pay

$$w_m(z) = c_m a(z). \tag{14}$$

We drop the index H in $a_H(z)$ because it is not necessary to distinguish the two technologies.

The labor market of type-s firms is only imperfectly competitive. Firms in this labor market segment have not acquired the screening technology and hence never know the true productivity of their workers and cannot use the hi-tech technology. But they do know the distribution of productivities in their labor market pool. Consequently, the wage rate cannot be conditioned on the true productivity of any particular worker, but rather depends on the expected productivity of a representative bundle of workers in this labor market segment:

$$w_s = c_s \mathbb{E}_s(z). \tag{15}$$

Given that wages differ between these two types of firms, each worker can decide whether he or she wants to apply for a job in the frictionless labor market of type-m firms or in the frictional labor market of type-s firm. The wage of a worker with productivity z is thus

$$w = \max\left\{c_s \mathbb{E}_s(z); c_m a(z)\right\}. \tag{16}$$

The following proposition describes the sorting outcome:

Proposition 1 (Sorting) *There exists at least one stable equilibrium that is characterized by a $\tilde{z}$ so that workers with $z > \tilde{z}$ will choose to work for type-m firms, and workers with $z < \tilde{z}$ will choose to work for type-s firms. The critical $\tilde{z}$ is determined by*

$$c_s \bar{z}_s(\tilde{z}) = c_m a(\tilde{z}), \tag{17}$$

where $\bar{z}_s(\tilde{z}) \equiv \int_{\underline{z}}^{\tilde{z}} z dG(z)/G(\tilde{z})$. This equilibrium is stable if $\bar{z}_s(\tilde{z})/a(\tilde{z})$ is decreasing in $\tilde{z}$.

Proof. Assume a $\tilde{z}$ exists, so that $\mathbb{E}_s(z) = \int_{\underline{z}}^{\tilde{z}} z dG(z)/G(\tilde{z}) = \bar{z}_s(\tilde{z})$. Then rewrite condition (17) as $\bar{z}_s(\tilde{z})/a(\tilde{z}) = c_m/c_s$. Using L'Hôpital's rule, we can determine the limits of $\bar{z}_s(\tilde{z})/a(\tilde{z})$ as $\tilde{z}$ approaches the boundaries of the support: $\lim_{\tilde{z}\to\underline{z}}[\bar{z}_s(\tilde{z})/a(\tilde{z})] = 1$ and $\lim_{\tilde{z}\to\infty}[\bar{z}_s(\tilde{z})/a(\tilde{z})] = 0$. Since $\bar{z}_s(\tilde{z})/a(\tilde{z})$ is differentiable, this proves existence of (at least) one equilibrium with $\underline{z} < \tilde{z} < \infty$ for $c_m < c_s$. Furthermore, this equilibrium implies sorting where the most productive workers work for type-m firms and the least productive work for type-s firms: $c_m a(z) > c_s \bar{z}_s(\tilde{z})$ for $z > \tilde{z}$ and $c_m a(z) < c_s \bar{z}_s(\tilde{z})$ for $z < \tilde{z}$. This equilibrium is stable if for $\zeta < \tilde{z}$, $c_s \bar{z}_s(\zeta) > c_m a(\zeta)$, and for $\zeta > \tilde{z}$, $c_s \bar{z}_s(\zeta) < c_m a(\zeta)$. Thus, stability implies that $\bar{z}_s(\zeta)/a(\zeta)$ is decreasing in ζ at $\zeta = \tilde{z}$ and requires that

$$\frac{\tilde{z} g(\tilde{z})}{G(\tilde{z})} \frac{[\tilde{z} - \bar{z}_s(\tilde{z})]}{\bar{z}_s(\tilde{z})} < \frac{a'(\tilde{z})\tilde{z}}{a(\tilde{z})}. \tag{18}$$

Since $\bar{z}_s(\tilde{z})/a(\tilde{z})$ is decreasing globally (from 1 to 0), at least one stable equilibrium must exist. This equilibrium is unique if $\bar{z}_s(\tilde{z})/a(\tilde{z})$ is monotonically decreasing. ■

[FIGURE 1 here]

In Figure 1 we illustrate the equilibrium and its stability graphically. For illustrative purposes, the function $\bar{z}_s(\zeta)/a(\zeta)$ is not monotonic. Clearly, if $c_s \bar{z}_s(\zeta) > c_m a(\zeta)$, a worker with skill ζ earns higher wages in type-s firms then in type-m firms. Thus, if ζ was a sorting cutoff, this would not be an equilibrium because the marginal worker would want to work for type-s firms, leading to an increase in this cutoff. Therefore, a stable equilibrium requires that the $\bar{z}_s(\zeta)/a(\zeta)$-function intersects c_m/c_s from above. In our Figure 1, equilibria $E1$ and $E3$ are stable, $E2$ is unstable. In what follows we only consider stable equilibria, so we assume that (18) holds.

One important implication of the sorting equilibrium is that

$$c_m = \frac{\bar{z}_s(\tilde{z})}{a(\tilde{z})} c_s < c_s. \tag{19}$$

Thus, type-m firms that have invested in the screening technology pay a lower effective wage rate (in efficiency units) than type-s firms with no access to the screening technology. This has to hold in equilibrium because the productivity of the marginal worker is discretely higher than the average productivity of all workers with a lower productivity: $a(\tilde{z}) > \bar{z}_s(\tilde{z})$. Therefore, type-$s$ firms have to pay a premium on the effective wage rate of type-m firms in order to compensate their above-average workers for pooling them with below-average workers and for using the low-tech technology.

Note that the difference in technologies enlarges the wage differences in the two labor market segments, but is not a necessary condition for the labor market segmentation.

Corollary 1 *The difference in technologies between type-m and type-s firms is neither necessary nor sufficient for the sorting equilibrium.*

Proof. If $a_H(z) = a_L(z) = z$ equation (19) reduces to $c_m = [\bar{z}_s(\tilde{z})/\tilde{z}]\, c_s$, where the term $\bar{z}_s(\tilde{z})/\tilde{z}$ is larger than the term $\bar{z}_s(\tilde{z})/a(\tilde{z})$ in proposition 1 but behaves identical at the limits. ■

Yeaple (2005) has shown that differences in technologies combined with comparative advantages of skilled workers in certain types of technologies can lead to positive assortative matching of workers to firms. Here we show that this sorting is reinforced by information asymmetries in the labor market. In fact, we even show that these information asymmetries alone can lead to a sorting equilibrium where skilled workers choose a different working environment than unskilled workers.

In a sorting equilibrium, we can now also determine the effective supplies of labor $\tilde{L}_j$ for the two types of firms from (10):

$$\tilde{L}_s = LG(\tilde{z})\,\bar{z}_s(\tilde{z}) \qquad \text{and} \qquad \tilde{L}_m = L[1 - G(\tilde{z})]\,\bar{z}_m(\tilde{z}), \tag{20}$$

where $\bar{z}_m(\tilde{z}) \equiv \int_{\tilde{z}}^{\infty} a(z)\, dG(z) / [1 - G(\tilde{z})]$.

2.2.5 Firm Entry

All types of firms can enter and exit freely. Within types, firms are symmetric. This implies that their respective profits are driven down to zero. Given (11), this zero profit condition requires for type-s firms

$$\Pi_s = \int_0^{\omega_s^d} \pi(\omega, c_s)\, d\omega = 0 \tag{21}$$

and for type-m firms

$$\Pi_m = \int_0^{\omega_m^d} \pi(\omega, c_m)\, d\omega - f_m = 0 \tag{22}$$

In this stage, upon entry, type-m firms invest in the screening technology and pay $f_m > 0$.

We can now establish the following proposition regarding firm types:

Proposition 2 (Firm types) *Type-m firms are multi-product firms and type-s firms are single-product firms.*

Proof. For type-s firms, the first order condition for scope (13) and the free entry condition (21) require that $\alpha\left(\omega_s^d\right)^{\sigma-1}\int_0^{\omega_s^d}\alpha(\omega)^{1-\sigma}\,d\omega = \omega_s^d$. Since $\alpha'(\omega) > 0$ and $\alpha(0) = 1$ [from flexible manufacturing (4)], this can only hold for $\omega_s^d = 0$. In addition, we have $d\Pi_j/d\omega_s^d\big|_{\omega_s^d=0} = \pi(0, c_s) = 0$. Therefore, type-$s$ firms produce only their core competency product and have no incentives to add any additional products to their product range (the marginal profits of doing so are zero). They become single-product firms. For type-m firms, (13) and (22) imply that $\alpha\left(\omega_m^d\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}\,d\omega - \omega_m^d = f_m/f$. Since $f_m > 0$ and the left hand side of this equation is clearly increasing in ω_m^d, this condition requires that $\omega_m^d > 0$. Finally, combining (13) for $j \in \{m, s\}$ and $\omega_s^d = 0$ leads to $c_s/c_m = \alpha\left(\omega_m^d\right) > 1$, confirming that our equation (19) holds, and implying that $d\Pi_j/d\omega_m^d\big|_{\omega_m^d=0} = \pi(0, c_m) > \pi(0, c_s) = 0$. Hence, the marginal profits of adding additional varieties evaluated at the core competency variety is positive for type-m firms. Thus, they will become multi-product firms.[9] ■

This proof shows that the sorting equilibrium is essential for multi-product firms to arise. As stated in our proposition 1, sorting implies that multi-product firms pay a lower effective wage rate than single-product firms ($c_m < c_s$). This allows them to expand into less efficient activities and produce varieties further away from their core competency with higher unit labor requirements. They have an incentive to do so because the screening technology is applicable in all divisions within the firm, so that by adding products to their product range they can lower the fixed costs per product.

[9] Note that the set of products produced by single-product firms and the set of products produced by multi-product firms both have positive Lebesgue measure because the cardinality of both sets is $\mathfrak{c} \equiv |\mathbb{R}|$, the cardinality of the continuum (see Briggs and Schaffter, 1979). Individual products have a finite cardinality and, thus, measure zero. A detailed proof is available upon request.

Using our normalization of $\alpha(0) = 1$, the free-entry/zero-profit conditions can now be rewritten as

$$Ac_s^{1-\sigma} = f, \tag{23}$$

for single-product firms and

$$Ac_m^{1-\sigma}\bar{\alpha}_m\left(\omega_m^d\right)^{1-\sigma} = \omega_m^d f + f_m \tag{24}$$

for multi-product firms, where $\bar{\alpha}_m\left(\omega_m^d\right) = \left[\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}\,d\omega\right]^{\frac{1}{1-\sigma}}$ is the mean of unit labor requirements in multi-product firms.

Proposition 3 (Co-existence) *In a free entry equilibrium, both types of firms (single-product firms and multi-product firms) will exist.*

Proof. First note from (23) and (24) that $f_m/\omega_m^d > 0$ and $\bar{\alpha}_m\left(\omega_m^d\right) > 1$, so multi-product firms have higher fixed costs per variety and on average higher unit labor requirements. Therefore, a necessary condition for co-existence is that $c_m < c_s$, which is met [see (19)]. Second, we can show that an equilibrium with only one type of firm is inconsistent with free entry: If $\tilde{z} \to \infty$ (no multi-product firms), $\lim_{\tilde{z}\to\infty} c_m = c_s \lim_{\tilde{z}\to\infty}\frac{\bar{z}_s(\tilde{z})}{a(\tilde{z})} = 0$ and $\lim_{\tilde{z}\to\infty}\Pi_m = +\infty$. Hence, multi-product firms must exist. If $\tilde{z} \to \underline{z}$ (no single-product firms), $\lim_{\tilde{z}\to\underline{z}} c_m = c_s \lim_{\tilde{z}\to\underline{z}}\frac{\bar{z}_s(\tilde{z})}{a(\tilde{z})} = c_s$ and $\Pi_s > \Pi_m$. Hence, single-product firms must exist. ■

Co-existence of single-product firms and multi-product firms is only possible because the screening technology leads to sorting and allows firms to segment labor markets. A free entry equilibrium with access to identical technologies requires that marginal production costs are equalized across firm types, at least at the margin. This is true here, too [see equations (13) for $j \in \{m, s\}$]:

$$c_m\alpha\left(\omega_m^d\right) = c_s\alpha\left(\omega_s^d\right). \tag{25}$$

But marginal production costs consist of two components: A factor cost component c_j and a unit labor requirement component $\alpha(\omega_j)$. Differences in one component require differences in the other: $\alpha\left(\omega_m^d\right) > \alpha\left(\omega_s^d\right) \Longrightarrow c_m < c_s$. Thus, multi-product firms expand into less efficient varieties because they pay a lower effective wage rate.

Propositions 2 and 3 are at the core of our theory. They show how multi-product firms and single-product firms can arise endogenously from ex ante identical firms due to labor market imperfections and different strategies to deal with them. In the frictional labor market, firms pay a wage based on the average productivity of workers in this labor market segment. Such a wage scheme implies an implicit transfer of rents from the more productive workers in this segment to the less productive workers. In the frictionless labor market, firms pay a wage based on the true productivity of workers so that no transfer between workers takes place. Such a wage scheme is particularly beneficial for the more productive workers in the economy who prefer not to be pooled with less productive workers. As a consequence, firms in the frictionless labor market can pay a lower effective wage rate and expand into less efficient products while firms in the frictional labor market have to pay a higher wage rate for pooling and so focus on their core competency to stay competitive.

The differences in the wage schemes between the two labor markets have important implications for the allocative efficiency of resources. In the frictional labor market, rents go from relatively high productive workers to less productive workers. But since all workers are paid the same wage, this does not affect the allocation of resources. In the frictionless labor market, the rents are transferred from (high productivity) workers to firms because firms are paying a lower effective wage. This induces them to expand and implies a misallocation of resources. This will be important in the welfare analysis.

Our framework has a number of interesting implications that are important for empirical work or for welfare analysis. We present them here as corollaries of propositions 1 to 3:

Corollary 2 (Size) *Multi-product firms have higher sales and sell more of their core competency variety than single-product firms.*

Proof. It follows directly from the free entry conditions that multi-product firms have higher fixed costs and thus must have higher revenues in a free entry equilibrium. The output of the core product is determined by (7) when evaluated at $\omega = 0$. Then, it follows directly from $c_m < c_s$ that $x(0, c_m) > x(0, c_s)$. ■

Multi-product firms need higher sales to cover their larger fixed costs, and have higher output of their core variety than a single-product firm because they have lower marginal production costs.

Corollary 3 (Productivity) *Multi-product firms are more productive than single-product firms as measured by revenue per worker.*

Proof. Using (5), (10), and (20), revenues per worker in multi-product firms φ_m can be expressed as

$$\varphi_m \equiv \frac{\int_0^{\omega_m^d} p(\omega)\, x(\omega)\, d\omega}{[1 - G(\tilde{z})]\, L/n_m} = \frac{\sigma}{\sigma - 1} c_m \bar{z}_m \tag{26}$$

Similarly, revenues per worker in single-product firms φ_s can be expressed as

$$\varphi_s \equiv \frac{p(0)\, x(0)}{G(\tilde{z})\, L/n_s} = \frac{\sigma}{\sigma - 1} c_s \bar{z}_s \tag{27}$$

Then, using the sorting condition (17), the ratio of the two productivity measures can be expressed as

$$\frac{\varphi_m}{\varphi_s} = \frac{\bar{z}_m}{a(\tilde{z})} > 1 \tag{28}$$

■

Multi-product firms generate higher revenues per worker because they employ more productive workers and use a more advanced technology.

Corollary 4 (Wages) *Multi-product firms pay higher wages per worker*

Proof. Single-product firms pay a flat wage of $w_s = c_s \bar{z}_s$. Multi-product firms pay wages based on individual productivities. The average wage in multi-product firms is $\bar{w}_m \equiv \int_{\tilde{z}}^{\infty} w(z)\, dG(z) / [1 - G(\tilde{z})] = c_m \bar{z}_m(\tilde{z})$. Again using (17), the relative average wage in multi-product firms is

$$\frac{\bar{w}_m}{w_s} = \frac{\bar{z}_m}{a(\tilde{z})} \left(= \frac{\varphi_m}{\varphi_s} \right) > 1 \tag{29}$$

■

Multi-product firms appear more productive despite paying higher wages because they have a more productive labor pool and pass on the gains from the higher labor productivity only incompletely. In a world where labor productivity could be perfectly observed, multi-product firms as modelled here would not exist.

The following figure shows the profile of wages as a function of worker productivity.

[FIGURE 2 here]

In Figure 2, the thick green line depicts the hockey stick profile of wages as a function of workers' productivities. Workers in the range $z \in [\underline{z}, \tilde{z})$ self-select into the frictional labor market and work for single-product firms. They receive a flat wage given by $w_s = c_s \bar{z}_s$. Above $\tilde{z}$, workers decide to go on the frictionless labor market, work for multi-product firms and receive a wage $w_m(z) = c_m a(z)$. This figure also illustrates nicely why a sorting equilibrium implies that the effective wage c_s in the frictional labor market has to be larger than the effective wage c_m in the frictionless labor market. If single-product firms paid the same effective wage as multi-product firms, $w_s = c_m \bar{z}_s$, then the wage for workers with above-average productivity $z > \bar{z}_s$ would be discretely lower in single-product firms than in multi-product firms $[c_m \bar{z}_s < c_m a(z)$ for all $z \in (\bar{z}_s, \tilde{z})]$. Consequently, this could not be a sorting equilibrium. Instead, single-product firms have to pay a premium on the effective wage rate, $c_s > c_m$, in order to compensate their above-average workers for pooling them with below-average worker, so that $c_s \bar{z}_s = c_m a(\tilde{z})$. Put differently, multi-product firms are able to obtain a rent from their workers in the form of a lower effective wage rate. This rent comes from allowing more productive workers to avoid being pooled with less productive workers. In this sense, workers share rents on their ability with firms rather than the other way around as is common in much of the literature on "fair wages."

2.2.6 General Equilibrium

For completeness we derive aggregate statistics that will be important in the welfare calculations below. With profits driven down to zero, aggregate income consists of labor income and compensation for managers. Since management is used as our numéraire, their compensation is normalized to one:

$$E \equiv L \int_{\underline{z}}^{\infty} w(z)\, dG(z) + H = L \left[c_s \int_{\underline{z}}^{\tilde{z}} z dG(z) + c_m \int_{\tilde{z}}^{\infty} a(z)\, dG(z) \right] + H. \qquad (30)$$

With CES demand, a constant fraction of revenues goes to fixed costs, and variable factors receive the remaining (constant) fraction. In our framework, this implies that $E = \sigma H$, and thus

$$E = \frac{\sigma}{\sigma - 1} L \left\{ c_s G(\tilde{z})\, \bar{z}_s(\tilde{z}) + c_m \left[1 - G(\tilde{z})\right] \bar{z}_m(\tilde{z}) \right\} = \sigma H. \qquad (31)$$

With E determined, and A pinned down by (23), the price index P can be derived easily from (6).

3 Welfare Implications of Labor Market Imperfections

This section analyzes the welfare implications of the information advantage of multiproduct firms. We begin by solving for the optimal allocation of labor to firms as chosen by a social planner that wishes to maximize aggregate real income. We then show that this is less labor than is allocated in a market equilibrium because multiproduct firms expand into marginal products in which they face relatively high labor requirements. We conclude the section by showing that the market imperfection can be improved by a subsidy to employment at small firms.

3.1 The Socially Optimal Allocation of Labor

In this section we want to analyze the determinants of welfare in the closed economy and the social efficiency of the market equilibrium.

Given (1), aggregate welfare W can be expressed as

$$W = \frac{E}{P} = \frac{\sigma}{\sigma - 1} \frac{\bar{w}}{P} L,$$

where

$$\frac{\bar{w}}{P} = \int_{\underline{z}}^{\tilde{z}} \frac{c_s \bar{z}_s (\tilde{z})}{P} dG(z) + \int_{\tilde{z}}^{\infty} \frac{c_m a(z)}{P} dG(z) \tag{32}$$

is the average real wage that consists of the average real wage in SPF (the first term) and the average real wage in MPF (the second term), weighed with the respective employment shares.

Using the definitions of aggregate demand A (6) and income E (31), as well as the free entry condition for SPF (23) and the optimal scope for MPF (13), welfare can be expressed as

$$W = \Phi \left\{ G(\tilde{z}) \bar{z}_s (\tilde{z}) + \frac{1}{\alpha(\omega_m^d)} [1 - G(\tilde{z})] \bar{z}_m (\tilde{z}) \right\}, \tag{33}$$

where $\Phi \equiv (H/f)^{\frac{1}{\sigma-1}} L$ is a constant.

Note that for this expression of welfare we have not used the sorting condition (17). Note also that the optimal scope of MPF ω_m^d does not depend on the sorting condition, either. In fact, ω_m^d is entirely independent of the allocation of labor $\tilde{z}$ and fully determined (implicitly) by the optimal scope of MPF (13) in combination with their free entry condition (24):

$$\frac{\bar{\alpha}_m \left(\omega_m^d\right)^{1-\sigma}}{\alpha \left(\omega_m^d\right)^{1-\sigma}} = \omega_m^d + \frac{f_m}{f} \qquad \longrightarrow \qquad \omega_m^d = \omega_m^d \left[\underset{+}{f_m}, \underset{-}{f}, \alpha(\cdot) \right], \tag{34}$$

with $\omega_m^d [0, f, \alpha(\cdot)] = 0$.

Because of these independences from the sorting condition, we can express welfare as a function of two key variables in our model, the allocation of labor across firm types and the product range of MPF: $W = W\left(\tilde{z}, \omega_m^d\right)$. In equilibrium, both variables are connected through the sorting condition: $a(\tilde{z}) / \bar{z}_s(\tilde{z}) = \alpha\left(\omega_m^d\right)$. However, it is instructive to leave the sorting condition out for a moment in order to analyze how these two variables affect welfare ceteris paribus and to determine the socially optimal allocation of labor $\tilde{z}^*$.

First, for a given allocation of labor $\tilde{z}$, an increase in the product range of MPF ω_m^d (f.ex. due to an increase in f_m) clearly lowers welfare:

$$\frac{\partial W}{\partial \omega_m^d} = -\Phi [1 - G(\tilde{z})] \bar{z}_m (\tilde{z}) \frac{\alpha'\left(\omega_m^d\right)}{\alpha\left(\omega_m^d\right)^2} < 0.$$

The reason for this negative effect is that a larger product range implies an increase in the effective unit labor requirements (due to the flexible manufacturing technology), and this reduces the productivity of workers in MPF and lowers welfare.

Second, for a given product range of MPF ω_m^d, the effect of a change in the allocation of labor on welfare cannot be signed unambiguously:

$$\frac{\partial W}{\partial \tilde{z}} = \Phi \left[\tilde{z} - \frac{a(\tilde{z})}{\alpha(\omega_m^d)} \right] g(\tilde{z}) \gtreqless 0.$$

Intuitively, this effect depends on the sign of $\tilde{z} - a(\tilde{z}) / \alpha\left(\omega_m^d\right)$: The welfare effect of a change in the allocation of labor depends ultimately on how productive the marginal worker is in the two types of firms. Here, $\tilde{z}$ is the productivity of the marginal worker in the low-tech technology of SPF, and $a(\tilde{z}) / \alpha\left(\omega_m^d\right)$ is the productivity of this worker in high-tech MPF (with effective labor supply of $a(\tilde{z})$ and unit labor requirement of $\alpha\left(\omega_m^d\right)$. Since $a(\tilde{z}) / \tilde{z}$ is equal to one at $\tilde{z} = \underline{z}$ and increasing in $\tilde{z}$, welfare as a function of $\tilde{z}$ is increasing for low values of $\tilde{z}$, and decreasing for high values of $\tilde{z}$. This leads us to proposition 4:

Proposition 4 (Social Optimum) *For a given product range $\omega_m^d > 1$, a socially optimal allocation of labor $\tilde{z}^* \in (\underline{z}, \infty)$ exists that satisfies $\tilde{z}^* = a(\tilde{z}^*) / \alpha\left(\omega_m^d\right)$.*

Proof. The first and second derivative of (33) evaluated at $\tilde{z} = \tilde{z}^*$ yield $\partial W / \partial \tilde{z} (\tilde{z}^*) = 0$ and $\partial^2 W / \partial \tilde{z}^2 (\tilde{z}^*) = \Phi \left[a(\tilde{z}) / \tilde{z} - a'(\tilde{z}) \right] \tilde{z} g(\tilde{z}) / a(\tilde{z}) < 0$. Since $\alpha\left(\omega_m^d\right) \in (1, \infty)$, $\lim_{\tilde{z} \to \underline{z}} a(\tilde{z}) / \tilde{z} = 1$ and $\lim_{\tilde{z} \to \infty} a(\tilde{z}) / \tilde{z} = \lim_{\tilde{z} \to \infty} a'(\tilde{z}) = +\infty$, we have $\tilde{z}^* \in (\underline{z}, \infty)$. ■

The intuition for the socially optimal allocation of labor is very intuitive. The productivity of a worker in a particular firm type depends on two factors: The technology employed by the firm (low-tech z in SPF, or high-tech $a(z)$ in MPF), and the unit labor requirements in that firm ($\alpha\left(\omega_s^d\right) = 1$ in SPF, $\alpha\left(\omega_m^d\right) > 1$ in MPF). The second effect is constant for all workers, but the first effect is not. Because workers with higher skills z have a comparative advantage in the high technology, the first effect is small for workers with low skills (low values of z) and large for workers with high skills (high values of z). Hence, it is socially optimal to allocate low-skill workers to SPF that operate with a low-tech technology and a low unit labor requirement, and high-skill workers to MPF with a high-tech technology and high unit labor requirements.

3.2 Efficiency of Market Equilibrium and Incentives for Subsidies

Now that we know the socially optimal allocation of labor we can compare the sorting equilibrium to the social optimum. This leads to proposition 5:

Proposition 5 (Sorting Efficiency) *The sorting equilibrium leads to a socially inefficient allocation of labor across firm types. Compared to the social optimum, employment in MPF is too high in the sorting equilibrium.*

Proof. The social optimum $\tilde{z}^*$ requires that $\alpha\left(\omega_m^d\right) = a\left(\tilde{z}^*\right)/\tilde{z}^*$. The sorting equilibrium $\tilde{z}$ (17) together with (23) and (13) leads to $\alpha\left(\omega_m^d\right) = a\left(\tilde{z}\right)/\bar{z}_s\left(\tilde{z}\right)$. Since ω_m^d is independent of the allocation of labor, we obtain $a\left(\tilde{z}^*\right)/\tilde{z}^* = a\left(\tilde{z}\right)/\bar{z}_s\left(\tilde{z}\right)$, and, thus, $\tilde{z} < \tilde{z}^*$. ■

Because workers in SPF do not receive a remuneration based on their own true productivity but are pooled instead with all (relatively low-skilled) workers working for SPF, the allocation in the sorting equilibrium is based not on the actual productivity of workers in SPF but on the average productivity. And since the average productivity is lower for the marginal worker $[\bar{z}_s\left(\tilde{z}\right) < \tilde{z}]$, working for SPF is less attractive and fewer workers self-select into the SPF labor pool.

The misallocation of labor manifests itself as too many MPF and not enough entry by SPF. Hence, while MPF have desirable characteristics, they employ too many workers. In this sense, there is "too much of a good thing."

This misallocation of labor creates an incentive to subsidize employment in SPF. Since $\tilde{z} < \tilde{z}^*$, it follows that $\partial W/\partial \tilde{z}\left(\tilde{z} < \tilde{z}^*\right) > 0$, and a reallocation of labor from MPF to SPF (an increase in $\tilde{z}$) could potentially increase welfare.

To see how such a subsidy can increase welfare assume that the government can subsidize employment in SPF and finance this subsidy with a non-distorting per capita tax on income. Then, only two equations would change: Equation (23) and (30):

$$A\left[c_s\left(1-s\right)\right]^{1-\sigma} = f, \tag{35}$$

$$E = \left[c_s \int_{\underline{z}}^{\tilde{z}} z dG\left(z\right) + c_m \int_{\tilde{z}}^{\infty} a\left(z\right) dG\left(z\right)\right] L + H - s c_s \bar{z}_s\left(\tilde{z}\right) G\left(\tilde{z}\right) L, \tag{36}$$

where s is the subsidy rate, $c_s\left(1-s\right)$ are after subsidy effective labor costs in SPF, and $s c_s \bar{z}_s\left(\tilde{z}\right) G\left(\tilde{z}\right) L$ is the total subsidy paid.

Because the subsidy is only paid to SPF, equations (13) and (24) are not affected, and the product range of MPF continues to be determined by (34). However, the subsidy does affect the allocation of labor $\tilde{z}$. Equations (17), (13) and (35) now yield

$$\frac{\left(1-s\right)}{\alpha\left(\omega_m^d\right)} = \frac{\bar{z}_s\left(\tilde{z}\right)}{a\left(\tilde{z}\right)}, \tag{37}$$

implying a positive relation between the subsidy and the share of employment in SPF for a given ω_m^d [the sign is clearly positive because of (18)]:

$$\frac{d\tilde{z}}{ds} = (1-s)^{-1}\left[\frac{a'(\tilde{z})}{a(\tilde{z})} - \frac{\tilde{z}-\bar{z}_s(\tilde{z})}{\bar{z}_s(\tilde{z})}\frac{g(\tilde{z})}{G(\tilde{z})}\right]^{-1} > 0 \tag{38}$$

Using the same approach as in the previous subsection, the subsidy rate cancels out of the welfare expression and we obtain the same expression for welfare as in (33). Thus, the subsidy affects welfare only through the allocation of labor $\tilde{z}$, $dW/ds = (\partial W/\partial \tilde{z})(d\tilde{z}/ds)$, and the optimal subsidy s^* is where $\partial W/\partial \tilde{z} = 0$, or implicitly

$$\tilde{z}(s^*) = \frac{a[\tilde{z}(s^*)]}{\alpha(\omega_m^d)}. \tag{39}$$

Proposition 6 (Subsidy) *There exists an optimal subsidy rate on employment in SPF* $s^* \in (0,1)$ *that corrects the misallocation of labor and reaches the social optimum, so that* $\tilde{z}(s^*) = \tilde{z}^*$.

In this setup, the market imperfections in the labor market create an incentive to subsidize small, single-product firms. These firms are too small to cover the costs of screening workers, and as a consequence need to pool their workers and pay a wage based on the average productivity of their work force. This strategy allows them to survive, but it creates a misallocation of labor due to the fact that the marginal worker has a higher productivity than the average worker. As a consequence, the employment share of small, single-product firms is too small compared to the social optimum, and a subsidy on employment in SPF can be welfare improving.[10]

The corrective subsidy raises welfare because it induces a reallocation of workers from MPF to SPF through the exit of MPF and the entry of SPF. Actual firm product ranges do not change as these are pinned down by free entry and optimal scope conditions.[11] We show in the appendix that under the alternative assumption of a fixed number of MPF and SPF (so that there is no free entry condition) that the misallocation of labor manifests itself as MPF that produce too many product lines relative to SPF. In this case, “Too much of good thing” manifests itself as MPF that are too large and inefficient relative to the social optimum. In that case the corrective subsidy works by inducing SPF to expand their product offerings while inducing MPF to reduce their product offering.

[10] The intuition is similar to that pointed out in Greenwald and Stiglitz (1986) in a different context.

[11] Indeed as in Dixit and Stiglitz (1977) these firms are optimally sized.

4 Open Economy

Let us now consider international trade in an open economy setting with two identical countries. International trade is costly in two dimensions: Entering a foreign market creates fixed costs of exporting f^x, and shipping goods to foreign locations is subject to variable (iceberg) trade costs $\tau > 1$. We follow Melitz (2003) and assume that these trade costs are sufficiently high so that the following condition is met: $f^x \tau^{\sigma-1} > f$.

Profits per product in the domestic market continue to be given by (9). Profits per product in an export market are given by

$$\pi^x(\omega, c_j) = A\tau^{1-\sigma} c_j^{1-\sigma} \alpha(\omega)^{1-\sigma} - f^x. \tag{40}$$

Combining this with the free entry condition in the domestic market leads to our first proposition in the open economy case:

Proposition 7 (Export selection) *Single-product firms do not export.*

Proof. Since $f^x \tau^{\sigma-1} > f$ (by assumption), it follows from (23) that $A\tau^{1-\sigma} c_s^{1-\sigma} < f^x$: The revenues generated by single-product firms in foreign markets are smaller than the fixed costs of entering these markets. Thus, single-product firms do not enter foreign markets and do not export. ■

The intuition behind proposition 7 is analogous to the intuition behind proposition 2. Since $c_s > c_m$, single-product firms pay a higher effective wage rate and have higher marginal production costs. Thus, they can only survive in the market if they focus on the lowest cost activities, like producing only their core competency varieties (proposition 2) and servicing only the domestic market (proposition 7). Multi-product firms, in contrast, have lower marginal production costs, so they can expand into less efficient activities, such as exporting.

Equation (13) continues to determine the optimal product range at home (ω_m^d). The first order condition for the optimal product range of products exported, ω_m^x, is:

$$A c_m^{1-\sigma} \tau^{1-\sigma} \alpha(\omega_m^x)^{1-\sigma} = f^x \tag{41}$$

Proposition 8 (Export range) *The range of products exported by multi-product firms is smaller than the range of products sold domestically. It is positive for sufficiently low values of f_m/f.*

Proof. Equations (13) and (41) yield $\alpha(\omega_m^x)/\alpha(\omega_m^d) = (f/f^x)^{\frac{1}{\sigma-1}}/\tau$. Since $f^x\tau^{\sigma-1} > f$, $\alpha(\omega_m^x)/\alpha(\omega_m^d) < 1$ and, thus, $\omega_m^x < \omega_m^d$. By the same logic, $\omega_m^x > 0$ implies that

$\alpha\left(\omega_m^d\right)^{\sigma-1} > \tau^{\sigma-1} f^x/f > 1$, which in turn requires that f_m/f is sufficiently large (see proof of proposition 2). ■

Since trade is costly, multi-product firms export fewer products than they sell at home. This result is analogous to the selection result in Melitz (2003) and has been pointed out in the context of multi-product firms with CES demand by Bernard, Redding and Schott (2011).

Since single-product firms continue to be active on the domestic market only, their free entry condition has not changed [see equations (21) and (23)]. The free entry condition for multi-product firms changes to $\Pi_m = \int_0^{\omega_m^d} \pi(\omega, c_m)\, d\omega + \int_0^{\omega_m^x} \pi^x(\omega, \tau, c_m)\, d\omega - f_m = 0$, or

$$A c_m^{1-\sigma} \bar{\alpha}_m \left(\omega_m^d, \omega_m^x, \tau\right)^{1-\sigma} = \omega_m^d f + \omega_m^x f^x + f_m, \tag{42}$$

where the mean of unit labor requirements in multi-product firms is now $\bar{\alpha}_m\left(\omega_m^d, \omega_m^x, \tau\right) = \left[\int_0^{\omega_m^d} \alpha(\omega)^{1-\sigma}\, d\omega + \tau^{1-\sigma} \int_0^{\omega_m^x} \alpha(\omega)^{1-\sigma}\, d\omega\right]^{\frac{1}{1-\sigma}}$.

By substituting (13) into (41) and both into (42) we obtain two equations that simultaneously determine ω_m^d and ω_m^x as a function of f_m, f, f^x, and τ,

$$\frac{\alpha\left(\omega_m^d\right)}{\alpha\left(\omega_m^x\right)} = \left(\frac{f^x}{f}\right)^{\frac{1}{\sigma-1}} \tau, \tag{43}$$

$$f \int_0^{\omega_m^d} \left[\frac{\alpha(\omega)}{\alpha\left(\omega_m^d\right)}\right]^{1-\sigma} d\omega + f^x \int_0^{\omega_m^x} \left[\frac{\alpha(\omega)}{\alpha\left(\omega_m^x\right)}\right]^{1-\sigma} d\omega = \omega_m^d f + \omega_m^x f^x + f_m, \tag{44}$$

with solutions

$$\omega_m^d = \omega_m^d\Big(\underset{+}{f_m}, \underset{-}{f}, \underset{+}{f^x}, \underset{+}{\tau}\Big) \qquad \text{and} \qquad \omega_m^x = \omega_m^x\Big(\underset{+}{f_m}, \underset{+}{f}, \underset{-}{f^x}, \underset{-}{\tau}\Big). \tag{45}$$

The signs are derived in the appendix.

The intuition behind these relations is rather straightforward. If the two export-specific cost factors f^x and τ rise, exporting becomes more expensive, and exporting firms reduce the range of products exported (ω_m^x falls). This lowers competition for domestic firms, and they can expand in response (ω_m^d rises). The fixed cost f is a cost factor specific to domestic production, so the intuition is the same with a reverse sign. Finally, the cost of screening f_m is a fixed cost that is the same for all MPF. If it rises, fewer MPF will survive in the market, and the surviving firms will be able to sell a larger product range.

Knowing ω_m^d, the critical skill level $\tilde{z}$ can be determined by combining the optimal product range at home (13) and the zero profit condition for single-product firms (23) with the sorting condition (17):

$$\frac{\bar{z}_s(\tilde{z})}{a(\tilde{z})} = \frac{1}{\alpha(\omega_m^d)}. \tag{46}$$

Note that the right hand side of this equation is between 0 and 1 and decreasing in ω_m^d. Given proposition 1, this determines $\tilde{z}$ as a (positive) function of ω_m^d.

The equations for income (31) and the sorting condition (17) determine simultaneously the two effective wages c_m and c_s for a given $\tilde{z}$:

$$c_m = (\sigma - 1)\left\{a(\tilde{z})G(\tilde{z}) + [1 - G(\tilde{z})]\bar{z}_m(\tilde{z})\right\}^{-1}\frac{H}{L}, \tag{47}$$

$$c_s = (\sigma - 1)\frac{a(\tilde{z})}{\bar{z}_s(\tilde{z})}\left\{a(\tilde{z})G(\tilde{z}) + [1 - G(\tilde{z})]\bar{z}_m(\tilde{z})\right\}^{-1}\frac{H}{L}. \tag{48}$$

Wages are then given by $w(z) = c_m a(z)$ in multi-product firms and $w_s = c_s \bar{z}_s(\tilde{z})$ in single-product firms.

Since only multi-product firms expand into foreign markets, the labor market clearing conditions for the two labor markets in the open economy case are:

$$n_s(\sigma - 1)Ac_s^{-\sigma} = LG(\tilde{z})\bar{z}_s(\tilde{z}), \tag{49}$$

$$n_m(\sigma - 1)Ac_m^{-\sigma}\bar{\alpha}_m\left(\omega_m^d, \omega_m^x, \tau\right)^{1-\sigma} = L[1 - G(\tilde{z})]\bar{z}_m(\tilde{z}), \tag{50}$$

The two labor market clearing conditions pin down the measures of single- and multi-product firms, n_s and n_m.

This concludes our description of the equilibrium in the open economy case. We can now study how this equilibrium changes in response to a trade shock.

5 Comparative Statics

For our comparative statics we focus on changes in variable trade costs τ.[12] However, our results are qualitatively identical to changes in fixed costs (see 45). Hence our comparative statics really cover a wider range of adjustments typically associated with trade liberalization. Now consider a fall in variable trade costs τ:

Proposition 9 (Firm organization) *Trade liberalization leads to an expansion of the range of products exported by multi-product firms, and a reduction in the range of products sold domestically:* $d\ln\omega_m^x/d\ln\tau < 0$ *and* $d\ln\omega_m^d/d\ln\tau > 0$.

[12] We consider only stable equilibria (see discussion following proposition 1).

Proof. See equation (45). ■

In a model without labor market imperfections, the expansion of export sales increases demand for labor and leads to a rise in real wages for all workers. Ultimately, this raises welfare, too. Here, however, the labor market (and welfare) consequences are very different.

First, the sorting equilibrium is affected: Knowing how ω_m^d changes, we can calculate the change in $\tilde{z}$ from (46):

Proposition 10 (Sorting threshold) *Trade liberalization leads to a fall in the threshold value for sorting* $\tilde{z}$.

Proof. From (46) in combination with (18) and proposition 9 we obtain

$$\frac{d\ln\tilde{z}}{d\ln\tau}=\left[\frac{a'(\tilde{z})\,\tilde{z}}{a(\tilde{z})}-\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\left(\frac{\tilde{z}}{\bar{z}_s(\tilde{z})}-1\right)\right]^{-1}\varepsilon_\alpha\left(\omega_m^d\right)\frac{d\ln\omega_m^d}{d\ln\tau}>0, \tag{51}$$

where $\varepsilon_\alpha\left(\omega_m^d\right)\equiv\alpha'\left(\omega_m^d\right)\omega_m^d/\alpha\left(\omega_m^d\right)>0$. ■

Note that fixed and variable trade costs affect only ω_m^d and ω_m^x directly. All other changes below are driven by changes in $\tilde{z}$ through equation (46). One immediate consequence of the change in $\tilde{z}$ is:

Corollary 5 (Employment) *As* $\tilde{z}$ *falls, employment is pulled out of single-product firms* $LG(\tilde{z})$ *and into multi-product firms* $L[1-G(\tilde{z})]$.

Since only multi-product firms export, only they benefit from the reduction in trade costs. This leads to an expansion of economic activity of multi-product firms at the expense of single-product firms.

As more labor is pulled into multi-product firms and the threshold value for sorting $\tilde{z}$ falls, the difference between the productivity of the marginal worker $a(\tilde{z})$ in MPF and the average productivity of workers in SPF $\bar{z}_s(\tilde{z})$ falls, so that SPF can lower the premium that they pay in terms of effective wages. As a consequence, the relative effective wage rate paid by MPF, c_m/c_s, rises:

$$\frac{d\ln c_m}{d\ln\tau}-\frac{d\ln c_s}{d\ln\tau}=-\left[\frac{a'(\tilde{z})\,\tilde{z}}{a(\tilde{z})}-\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\left(\frac{\tilde{z}}{\bar{z}_s(\tilde{z})}-1\right)\right]\frac{d\ln\tilde{z}}{d\ln\tau}<0. \tag{52}$$

Furthermore, using (47), we can show that wages for individual workers in multi-product firms $[w(z)=c_m a(z)]$ rise and wages in single-product firms $[w_s=w(\tilde{z})=c_m a(\tilde{z})]$ fall:

$$\frac{d\ln w(z)}{d\ln\tau}=-\frac{a'(\tilde{z})\,G(\tilde{z})\,\tilde{z}}{a(\tilde{z})\,G(\tilde{z})+[1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau}<0, \tag{53}$$

$$\frac{d\ln w_s}{d\ln\tau} = \frac{a'(\tilde{z})\,\tilde{z}}{a(\tilde{z})}\frac{[1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}{a(\tilde{z})\,G(\tilde{z})+[1-G(\tilde{z})]\,\bar{z}_m(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau} > 0. \tag{54}$$

These results also imply that relative wages of incumbent workers in multi-product firms, $w(z)/w_s$, rise.

From a welfare prospective it is important to calculate changes in real wages (relative to the price index P). They, too, differ across firm types:

Proposition 11 (Real wages) *Trade liberalization raises real wages in multi-product firms and lowers real wages in single-product firms.*

Proof. Using (6), (23) and (31) we can prove that c_s/P is fixed by exogenous parameters:

$$\frac{c_s}{P} = \frac{\sigma-1}{\sigma}\left(\frac{H}{f}\right)^{\frac{1}{\sigma-1}}. \tag{55}$$

Given (55) the changes in real wages follow directly from changes in $\tilde{z}$: The real wage in single-product firms $w_s/P = \bar{z}_s(\tilde{z})(c_s/P)$ clearly falls because $\bar{z}_s(\tilde{z})$ falls, and $w_m(z)/P = (c_m/P)\,a(z) = (c_m/c_s)(c_s/P)\,a(z)$ clearly rises because c_m/c_s rises:

$$\frac{d\ln w_s - d\ln P}{d\ln\tau} = \frac{g(\tilde{z})\,\tilde{z}}{G(\tilde{z})}\frac{\tilde{z}-\bar{z}_s(\tilde{z})}{\bar{z}_s(\tilde{z})}\frac{d\ln\tilde{z}}{d\ln\tau} > 0 \tag{56}$$

$$\frac{d\ln w_m(z) - d\ln P}{d\ln\tau} = -\left[\frac{\tilde{z}g(\tilde{z})}{G(\tilde{z})}\left(1-\frac{\tilde{z}}{\bar{z}_s(\tilde{z})}\right)+\frac{a'(\tilde{z})\,\tilde{z}}{a(\tilde{z})}\right]\frac{d\ln\tilde{z}}{d\ln\tau} < 0 \tag{57}$$

■

Real wages of workers in single-product firms fall because the most productive workers in their labor pool are pulled away into the frictionless labor markets of multi-product firms. Thus, the remaining workforce is on average less productive, and their real wages fall. Real wages in multi-product firms rise because labor demand for exports increases.

Given our expression of welfare in equation (32) where welfare is expressed as a weighed average of real wages in the two types of firms,

$$W = \frac{\sigma L}{\sigma-1}\left\{\int_{\underline{z}}^{\tilde{z}}\left[\frac{w_s}{P}\right]dG(z)+\int_{\tilde{z}}^{\infty}\left[\frac{w_m(z)}{P}\right]dG(z)\right\} \tag{58}$$

the asymmetric effects on real wages implies that the welfare effects of trade liberalization depend ultimately on the employment shares in the two types of firms. If employment in MPF is high, and real wages of employees in MPF rises, then the welfare effects are more likely to be positive than in a case where employment in these types of firms is actually low.

In addition to these insights from an income-based measure of welfare, we can derive a more thorough understanding of the sources of the welfare effects by studying how the efficiency of production is affected by trade liberalization. For this we express welfare as

$$W = \Phi \left\{ \int_{\underline{z}}^{\tilde{z}} z dG(z) + \int_{\tilde{z}}^{\infty} \left[\frac{a(z)}{\alpha(\omega_m^d)} \right] dG(z) \right\} \tag{59}$$

based on equation (33). Trade liberalization affects the efficiency of production through two effects. First, multi-product firms become "leaner" since ω_m^d falls. This clearly improves the efficiency of production because products with high unit labor requirements are dropped from the product range. Second, employment is shifted from single-product firms to multi-product firms. If the allocation of labor was socially efficient, then this effect would be zero (envelop result). However, as we have established above, because of our sorting condition this allocation is distorted. The productivity of the marginal worker in MPF is smaller than the productivity of that same worker in SPF: $a(\tilde{z}) / \alpha\left(\omega_m^d\right) = \bar{z}_s(\tilde{z}) < \tilde{z}$. Thus, there are already too many workers working in MPF, and trade liberalization actually aggravates this misallocation by moving even more workers into MPF.

The welfare effects are summarized in the following proposition:

Proposition 12 *Trade liberalization has an ambiguous effect on welfare. On the one hand, as usual trade liberalization directly raises welfare by lowering the price of obtaining foreign varieties. On the other hand, trade liberalization induces labor to be reallocated towards multi-product firms, thereby worsening the labor market distortion. The aggregate effect depends on the employment share of multi-product firms.*

Proof. See appendix for a full mathematical proof of the ambiguous welfare effect. ■

Finally, based on our insights from the incentives to subsidize employment in SPF, we can establish the following corollary:

Corollary 6 *A bilateral subsidy can neutralize the negative welfare effect, so that trade is unambiguously welfare increasing.*

Proof. Our proposition 6 proves that a subsidy on employment in SPF can lead to a socially efficient allocation of labor where $\tilde{z}^* = a(\tilde{z}^*) / \alpha\left(\omega_m^d\right)$. In this case, $\partial W / \partial \tilde{z} = 0$, and small changes in the allocation of labor have no effect on welfare. Thus, the only effect that remains is the "leaner" production effect which raises welfare. Since all results are derived for symmetric countries, this result also implies that the subsidy is provided symmetrically. ■

6 Conclusion

In this paper, we present a framework in which large firms' superior human resources management capabilities are a mixed blessing from the point of view of efficient resource allocation. On the one hand, because knowledge of workers' skills is necessary to use a technology adapted for skilled workers, human resource management capabilities allow skilled labor to be used more efficiently. On the other hand, the market power conferred on large, multiproduct firms artificially lowers their labor costs and induces too much entry of large, exporting firms. In such a world, subsidization of employment at small, non-export oriented firms is optimal and gains from trade liberalization can only be ensured given a proper subsidy.

In this paper, we have analyzed only one type of factor market distortion that can give large firms an advantage relative to smaller firms. In an environment in which larger firms are better equipped to influence government policy, it is likely that there are other, perhaps more pernicious forces, that induce large firms to be too large from a social point of view. We hope that this will become a vibrant area of research.

7 Appendix

7.1 Product scopes in the open economy

There are two equations:

$$\alpha\left(\omega_m^d\right) = \left(\frac{f^x}{f}\right)^{\frac{1}{\sigma-1}} \tau\alpha\left(\omega_m^x\right)$$

$$\frac{1}{\alpha\left(\omega_m^d\right)^{1-\sigma}} \int_0^{\omega_m^d} \alpha\left(\omega\right)^{1-\sigma} d\omega - \omega_m^d + \left(\frac{1}{\alpha\left(\omega_m^x\right)^{1-\sigma}} \int_0^{\omega_m^x} \alpha\left(\omega\right)^{1-\sigma} d\omega - \omega_m^x\right) \frac{f^x}{f} = \frac{f_m}{f}$$

The derivatives are

$$(\sigma-1)\frac{\alpha'\left(\omega_m^d\right)\omega_m^d}{\alpha\left(\omega_m^d\right)} d\ln\omega_m^d - (\sigma-1)\frac{\alpha'\left(\omega_m^x\right)\omega_m^x}{\alpha\left(\omega_m^x\right)} d\ln\omega_m^x = d\ln\left(\frac{f^x}{f}\right) + (\sigma-1)\,d\ln\tau$$

and

$$\begin{aligned}
& (\sigma-1)\frac{1}{\alpha\left(\omega_m^d\right)^{1-\sigma}} \int_0^{\omega_m^d} \alpha\left(\omega\right)^{1-\sigma} d\omega \frac{\alpha'\left(\omega_m^d\right)\omega_m^d}{\alpha\left(\omega_m^d\right)} d\ln\omega_m^d \\
& + (\sigma-1)\frac{1}{\alpha\left(\omega_m^x\right)^{1-\sigma}} \int_0^{\omega_m^x} \alpha\left(\omega\right)^{1-\sigma} d\omega \frac{\alpha'\left(\omega_m^x\right)\omega_m^x}{\alpha\left(\omega_m^x\right)} \frac{f^x}{f} d\ln\omega_m^x \\
= & \frac{f_m}{f} d\ln\left(\frac{f_m}{f}\right) - \left(\frac{1}{\alpha\left(\omega_m^x\right)^{1-\sigma}} \int_0^{\omega_m^x} \alpha\left(\omega\right)^{1-\sigma} d\omega - \omega_m^x\right) \frac{f^x}{f} d\ln\left(\frac{f^x}{f}\right)
\end{aligned}$$

The system can be written in matrix format:

$$\underline{\Delta}\begin{pmatrix} (\sigma-1)\frac{\alpha'\left(\omega_m^d\right)\omega_m^d}{\alpha\left(\omega_m^d\right)}d\ln\omega_m^d \\ (\sigma-1)\frac{\alpha'\left(\omega_m^x\right)\omega_m^x}{\alpha\left(\omega_m^x\right)}d\ln\omega_m^x \end{pmatrix} = \begin{pmatrix} d\ln\left(\frac{f^x}{f}\right)+(\sigma-1)\,d\ln\tau \\ \frac{f_m}{f}d\ln\left(\frac{f_m}{f}\right)-\left(\frac{1}{\alpha(\omega_m^x)^{1-\sigma}}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega-\omega_m^x\right)\frac{f^x}{f}d\ln\left(\frac{f^x}{f}\right) \end{pmatrix}$$

where $\underline{\Delta}= \begin{bmatrix} 1 & -1 \\ \frac{1}{\alpha\left(\omega_m^d\right)^{1-\sigma}}\int_0^{\omega_m^d}\alpha\left(\omega\right)^{1-\sigma}d\omega & \frac{1}{\alpha(\omega_m^x)^{1-\sigma}}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega\frac{f^x}{f} \end{bmatrix}$, with $|\underline{\Delta}|>0$.

Then, the solution can be expressed as

$$\begin{aligned} |\underline{\Delta}|\,(\sigma-1)\,\frac{\alpha'\left(\omega_m^d\right)\omega_m^d}{\alpha\left(\omega_m^d\right)}d\ln\omega_m^d &= \frac{f_m}{f}d\ln f_m-\left(\omega_m^x\frac{f^x}{f}+\frac{f_m}{f}\right)d\ln f+\omega_m^x\frac{f^x}{f}d\ln f^x \\ &\quad +\frac{1}{\alpha\left(\omega_m^x\right)^{1-\sigma}}\int_0^{\omega_m^x}\alpha\left(\omega\right)^{1-\sigma}d\omega\frac{f^x}{f}\,(\sigma-1)\,d\ln\tau \end{aligned}$$

and

$$\begin{aligned} |\underline{\Delta}|\,(\sigma-1)\,\frac{\alpha'\left(\omega_m^x\right)\omega_m^x}{\alpha\left(\omega_m^x\right)}d\ln\omega_m^x &= \frac{f_m}{f}d\ln f_m+\omega_m^d d\ln f-\left(\omega_m^d+\frac{f_m}{f}\right)d\ln f^x \\ &\quad -\frac{1}{\alpha\left(\omega_m^d\right)^{1-\sigma}}\int_0^{\omega_m^d}\alpha\left(\omega\right)^{1-\sigma}d\omega\,(\sigma-1)\,d\ln\tau \end{aligned}$$

7.2 Alternative Setting: Fixed Number of Firms

7.2.1 The environment

Suppose that the number of low-tech, non-screening firms, n_s, and the number of high-tech, screening firms, n_m, are fixed (no free entry). For simplicity we assume that $f^m=0$ as it plays no role here. Firms continue to need managerial labor, however, to fund their product lines. The economy is closed.

We consider the planner's problem first and then show that it deviates from the market equilibrium in a way that is similar to that of the free entry case. Note that as we will show below, all firms will produce multiple products so we will refer to type M and type S firms instead of multiproduct and single product firms. We also demonstrate that the inefficiency here intuitively is manifested in too many product lines of type M firms and not enough product lines at type S firms. We conclude by deriving the optimal subsidy that corrects the market inefficiency.

7.2.2 The planner's problem

The social planner wishes to maximize the utility of the representative consumers subject to the resource constraints of the economy. Specifically, the planner solves the following program:

$$\max \left(n_s \int_0^{\omega_s^d} x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + n_m \int_0^{\omega_m^d} x_m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$

such that

$$n_m \omega_m^d f + n_s \omega_s^d f \leq M \tag{60}$$

$$n_s \int_0^{\omega_s^d} x_s(\omega) \alpha(\omega) d\omega \leq L \int_{\underline{z}}^{\widetilde{z}} z dG(z) \tag{61}$$

$$n_m \int_0^{\omega_m^d} x_m(\omega) \alpha(\omega) d\omega \leq L \int_{\widetilde{z}}^{\infty} a(z) dG(z) \tag{62}$$

Note that we have ignored the complementary slackness condition.

The Lagrangian is given by

$$\left(n_s \int_0^{\omega_s^d} x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + n_m \int_0^{\omega_m^d} x_m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} + \lambda_s \left[L \int_{\underline{z}}^{\widetilde{z}} z dG(z) - n_s \int_0^{\omega_s^d} x_s(\omega) \alpha(\omega) d\omega \right]$$
$$+ \lambda_m \left[L \int_{\widetilde{z}}^{\infty} a(z) dG(z) - n_m \int_0^{\omega_d} x_m(\omega) \alpha(\omega) d\omega \right] + \chi \left[M - n_s \omega_s^d f - n_m \omega_m^d f \right],$$

where λ_s, λ_m, and χ are the multipliers on the three resource constraints. The first order conditions are given by

$$\frac{\sigma}{\sigma-1} x_s(\omega_s^d)^{\frac{\sigma-1}{\sigma}} Q^{\frac{1}{\sigma}} = \lambda_s x_s\left(\omega_s^d\right) \alpha(\omega_s^d) + \chi f \tag{63}$$

$$\frac{\sigma}{\sigma-1} x_m(\omega_m^d)^{\frac{\sigma-1}{\sigma}} Q^{\frac{1}{\sigma}} = \lambda_m x_m(\omega_m^d) \alpha(\omega_m^d) + \chi f \tag{64}$$

$$x_m(\omega)^{-\frac{1}{\sigma}} Q^{\frac{1}{\sigma}} = \lambda_m \alpha(\omega) \tag{65}$$

$$x_s(\omega)^{-\frac{1}{\sigma}} Q^{\frac{1}{\sigma}} = \lambda_s \alpha(\omega) \tag{66}$$

$$\lambda_m a(\widetilde{z}) = \lambda_s \widetilde{z} \tag{67}$$

where

$$Q = \left(n_s \int_0^{\omega_s^d} x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + n_m \int_0^{\omega_m^d} x_m(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

In addition, we have the three binding resource constraints. From (65), we have

$$x_m(\omega)^{\frac{\sigma-1}{\sigma}} Q^{\frac{1}{\sigma}} = \lambda_m \alpha(\omega) x_m(\omega)$$

and substituting this into (64), we have for M type firms, we have

$$\frac{1}{\sigma - 1} x_m(\omega_m^d)^{\frac{\sigma-1}{\sigma}} Q^{\frac{1}{\sigma}} = \chi f,$$

and for S type firms, we have

$$\frac{1}{\sigma - 1} x_s(\omega_s^d)^{\frac{\sigma-1}{\sigma}} Q^{\frac{1}{\sigma}} = \chi f$$

These expressions tell us that $x_m(\omega_m^d) = x_s(\omega_s^d)$. Another implication is that all firms here are multiproduct.

Other results can be obtained by substituting (65) and (66) into these expressions

$$\begin{aligned}
\frac{1}{\sigma - 1} \lambda_m \alpha(\omega_m^d) x_m(\omega_m^d) &= \chi f \\
\frac{1}{\sigma - 1} \lambda_s \alpha(\omega_s^d) x_s(\omega_s^d) &= \chi f
\end{aligned}$$

which together imply

$$\begin{aligned}
\lambda_m \alpha(\omega_m^d) &= \lambda_s \alpha(\omega_s^d) \\
\frac{\alpha(\omega_s^d)}{\alpha(\omega_m^d)} &= \frac{\lambda_m}{\lambda_s}
\end{aligned}$$

And from (67), we have

$$\frac{\lambda_m}{\lambda_s} = \frac{\widetilde{z}}{a(\widetilde{z})}.$$

This leads us to an important implication from the planner's problem:

$$\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)} = \frac{a(\widetilde{z})}{\widetilde{z}}. \tag{68}$$

Because (65) implies

$$\frac{x_m(\omega)}{x_m(\omega_m^d)} = \left(\frac{\alpha(\omega)}{\alpha(\omega_m^d)} \right)^{-\sigma},$$

the labor resource constraint for type M firms can be written

$$\begin{aligned} n_m \int_0^{\omega_m^d} x_m(\omega)\,\alpha(\omega) d\omega &= L \int_{\widetilde{z}}^{\infty} a(z) dG(z) \\ x_m(\omega_m^d)\alpha(\omega_m^d)^\sigma \int_0^{\omega_m^d} \alpha(\omega)^{1-\sigma} d\omega &= \frac{L}{n_m} \int_{\widetilde{z}}^{\infty} a(z) dG(z), \end{aligned}$$

and for type S firms the equivalent condition is

$$x_m(\omega_s^d)\alpha(\omega_s^d)^\sigma \int_0^{\omega_s^d} \alpha(\omega)^{1-\sigma} d\omega = \frac{L}{n_s} \int_{\underline{z}}^{\widetilde{z}} z dG(z).$$

Combining the two equations, we have a second relationship between scope cutoffs and the skill cutoff:

$$\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^\sigma \frac{\int_0^{\omega_m^d} \alpha(\omega)^{1-\sigma} d\omega}{\int_0^{\omega_s^d} \alpha(\omega)^{1-\sigma} d\omega} = \frac{n_s}{n_m} \frac{\int_{\widetilde{z}}^{\infty} a(z) dG(z)}{\int_{\underline{z}}^{\widetilde{z}} z dG(z)}. \tag{69}$$

Finally, to pin down the three cutoffs, we have the managerial resource constraint:

$$\omega_m^d = \frac{M - n_s \omega_s^d f}{f n_m}.$$

7.2.3 Market Equilibrium in the Closed Economy

We now demonstrate how the misallocation of skill in the market economy manifests itself in the fixed entry environment. As in the free entry case, which imply

$$\frac{\alpha(\omega_s^d)}{\alpha(\omega_m^d)} = \frac{c_m}{c_s}. \tag{70}$$

Further, we have from sorting

$$c_m a(\widetilde{z}) = c_s \overline{z}\,(\widetilde{z})$$

and so in equilibrium, we must have

$$\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)} = \frac{a(\widetilde{z})}{\overline{z}\,(\widetilde{z})} \tag{71}$$

which suggests the misallocation as sorting occurs on the basis of the average skill level in the non-screened labor pool rather than the marginal skill level as it appears in (68).

Given the optimal firm pricing rule, we continue to have

$$\frac{x_m(\omega)}{x_m(\omega_m^d)} = \left(\frac{\alpha(\omega)}{\alpha(\omega_m^d)}\right)^{-\sigma}$$
$$\frac{x_s(\omega)}{x_s(\omega_s^d)} = \left(\frac{\alpha(\omega)}{\alpha(\omega_s^d)}\right)^{-\sigma}$$

which is identical to the planner problem. Hence, these conditions combined with the labor market clearing condition yields the same condition (69) as it appears in the planner's problem.

Note that (69) and managerial clearing imply a strictly decreasing relationship between ω_m^d and $\widetilde{z}$. So assuming that $a(\widetilde{z})/\widetilde{z}$ and $a(\widetilde{z})/\overline{z}(\widetilde{z})$ are strictly increasing, then the market equilibrium features a lower $\widetilde{z}$ than the social planner would choose and it implies that the relative scope of MPF is too large relative to SPF. So, when the number of firms is fixed, the market inefficiency manifests itself in the scope of the different firm types: large firms have too broad a scope while small firms have too small a scope.

7.2.4 Optimal Subsidy

In this section, we show how a carefully chosen subsidy can correct the labor market inefficiency. As in the text, we consider an advalorem subsidy, s, to employment at type S firms. From the planner problem we have the optimal condition for sorting is

$$\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)} = \frac{a(\widetilde{z})}{\widetilde{z}}.$$

To obtain this relationship (and the values of the variables in the planner's problem), we construct the optimal subsidy. In the presence of this subsidy we have from the optimal scope conditions

$$c_s(1-s)\alpha(\omega_s^d) = c_m\alpha(\omega_m^d)$$

and from the sorting condition we have

$$c_s\overline{z}(\widetilde{z}) = c_m a(\widetilde{z}).$$

Combining the two expressions, we have

$$(1-s)\frac{a(\widetilde{z})}{\overline{z}(\widetilde{z})} = \frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)},$$

which can be reorganized to demonstrate that the optimal subsidy must satisfy

$$1-s=\frac{\overline{z}\left(\widetilde{z}\right)}{\widetilde{z}}.$$

We now confirm that a government induced wedge of this size will maximize welfare. It can be shown that the real national expenditure (net of the subsidy) is given by

$$\frac{E}{P}=L\frac{\int_{\underline{z}}^{\widetilde{z}}zdG(z)+\frac{\overline{z}(\widetilde{z})}{a(\widetilde{z})(1-s)}\int_{\widetilde{z}}^{\infty}a(z)dG(z)}{\left(n_s\int_0^{\omega_s^d}\alpha(\omega)^{1-\sigma}d\omega+n_m\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}}.$$

The first order condition for maximizing real national expenditure is

$$\begin{aligned}
&\frac{\frac{\overline{z}(\widetilde{z})}{a(\widetilde{z})(1-s)^2}\int_{\widetilde{z}}^{\infty}a(z)dG(z)}{\int_{\underline{z}}^{\widetilde{z}}zdG(z)+\frac{\overline{z}(\widetilde{z})}{a(\widetilde{z})(1-s)}\int_{\widetilde{z}}^{\infty}a(z)dG(z)}\\
&+\left(\frac{\widetilde{z}g(\widetilde{z})+\frac{1}{(1-s)}\left(\frac{\overline{z}'(\widetilde{z})}{\overline{z}(\widetilde{z})}-\frac{a'(\widetilde{z})}{a(\widetilde{z})}\right)\frac{\overline{z}(\widetilde{z})}{a(\widetilde{z})}\int_{\widetilde{z}}^{\infty}a(z)dG(z)-\frac{\overline{z}(\widetilde{z})g(\widetilde{z})}{1-s}}{\int_{\underline{z}}^{\widetilde{z}}zdG(z)+\frac{\overline{z}(\widetilde{z})}{a(\widetilde{z})(1-s)}\int_{\widetilde{z}}^{\infty}a(z)dG(z)}\right)\frac{d\widetilde{z}}{ds}\\
&+\frac{1}{\sigma-1}\frac{n_s\alpha(\omega_s^d)^{1-\sigma}-n_m\left(\sigma-1\right)\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega\frac{\alpha'(\omega_s^d)}{\alpha(\omega_s^d)}}{n_s\int_0^{\omega_s^d}\alpha(\omega)^{1-\sigma}d\omega+n_m\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega}\frac{d\omega_s^d}{ds}\\
&+\frac{1}{\sigma-1}\frac{n_m\alpha(\omega_s^d)^{1-\sigma}+n_m\left(\sigma-1\right)\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega\frac{\alpha'(\omega_m^d)}{\alpha(\omega_m^d)}}{n_s\int_0^{\omega_s^d}\alpha(\omega)^{1-\sigma}d\omega+n_m\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega}\frac{d\omega_m^d}{ds}\\
=\;&0
\end{aligned}$$

Using

$$\frac{d\omega_s^d}{ds}=-\frac{n_m}{n_s}\frac{d\omega_m^d}{ds}$$

and

$$\frac{n_m}{n_s}\left(\frac{\alpha(\omega_m^d)}{\alpha(\omega_s^d)}\right)^{\sigma-1}\frac{\int_0^{\omega_m^d}\alpha(\omega)^{1-\sigma}d\omega}{\int_0^{\omega_s^d}\alpha(\omega)^{1-\sigma}d\omega}=\frac{\overline{z}\left(\widetilde{z}\right)}{a(\widetilde{z})(1-s)}\frac{\int_{\widetilde{z}}^{\infty}a(z)dG(z)}{\int_{\underline{z}}^{\widetilde{z}}zdG(z)}$$

The first order condition becomes

$$\begin{aligned}
& \frac{\overline{z}\left(\widetilde{z}\right)}{a(\widetilde{z})(1-s)^{2}} \\
& +\left(\frac{\widetilde{z}g(\widetilde{z})}{\int_{\widetilde{z}}^{\infty}a(z)dG(z)}+\left(\frac{\overline{z}'\left(\widetilde{z}\right)}{\overline{z}\left(\widetilde{z}\right)}-\frac{a'(\widetilde{z})}{a(\widetilde{z})}\right)\frac{\overline{z}\left(\widetilde{z}\right)}{a(\widetilde{z})}\frac{1}{(1-s)}-\frac{\overline{z}\left(\widetilde{z}\right)}{1-s}\frac{g(\widetilde{z})}{\int_{\widetilde{z}}^{\infty}a(z)dG(z)}\right)\frac{d\widetilde{z}}{ds} \\
& +\frac{d\omega_{m}^{d}}{ds}\frac{\overline{z}\left(\widetilde{z}\right)}{a(\widetilde{z})(1-s)}\left(\frac{n_{m}}{n_{s}}\frac{\alpha'(\omega_{s}^{d})}{\alpha(\omega_{s}^{d})}+\frac{\alpha'(\omega_{m}^{d})}{\alpha(\omega_{m}^{d})}\right) \\
= & \; 0
\end{aligned}$$

Now, imposing our subsidy, we obtain

$$\frac{\widetilde{z}}{\overline{z}\left(\widetilde{z}\right)}+\left(\frac{\overline{z}'\left(\widetilde{z}\right)}{\overline{z}\left(\widetilde{z}\right)}-\frac{a'(\widetilde{z})}{a(\widetilde{z})}\right)\frac{d\widetilde{z}}{ds}+\frac{d\omega_{m}^{d}}{ds}\left(\frac{n_{m}}{n_{s}}\frac{\alpha'(\omega_{s}^{d})}{\alpha(\omega_{s}^{d})}+\frac{\alpha'(\omega_{m}^{d})}{\alpha(\omega_{m}^{d})}\right)=0.$$

We now show from the first order conditions that this expression must hold in an equilibrium in which the optimal subsidy is imposed. Using the sorting and optimal scope equations, we have

$$(1-s)\frac{a(\widetilde{z})}{\overline{z}\left(\widetilde{z}\right)}-\frac{\alpha(\omega_{m}^{d})}{\alpha(\omega_{s}^{d})}=0,$$

which when differentiated becomes

$$-\frac{a(\widetilde{z})}{\overline{z}\left(\widetilde{z}\right)}+(1-s)\frac{a(\widetilde{z})}{\overline{z}\left(\widetilde{z}\right)}\left(\frac{a'(\widetilde{z})}{a(\widetilde{z})}-\frac{\overline{z}'\left(\widetilde{z}\right)}{\overline{z}\left(\widetilde{z}\right)}\right)\frac{d\widetilde{z}}{ds}-\frac{\alpha(\omega_{m}^{d})}{\alpha(\omega_{s}^{d})}\left(\frac{\alpha'(\omega_{m}^{d})}{\alpha(\omega_{s}^{d})}+\frac{n_{m}}{n_{s}}\frac{\alpha'(\omega_{s}^{d})}{\alpha(\omega_{s}^{d})}\right)\frac{d\omega_{m}^{d}}{ds}=0$$

Simplifying and imposing the optimal subsidy, this expression becomes identical to the first order condition for the planner's problem, i.e.

$$\frac{\widetilde{z}}{\overline{z}\left(\widetilde{z}\right)}+\left(\frac{\overline{z}'\left(\widetilde{z}\right)}{\overline{z}\left(\widetilde{z}\right)}-\frac{a'(\widetilde{z})}{a(\widetilde{z})}\right)\frac{d\widetilde{z}}{ds}+\left(\frac{n_{m}}{n_{s}}\frac{\alpha'(\omega_{s}^{d})}{\alpha(\omega_{s}^{d})}+\frac{\alpha'(\omega_{m}^{d})}{\alpha(\omega_{s}^{d})}\right)\frac{d\omega_{m}^{d}}{ds}=0,$$

which confirms our conjecture that this is the optimal subsidy.

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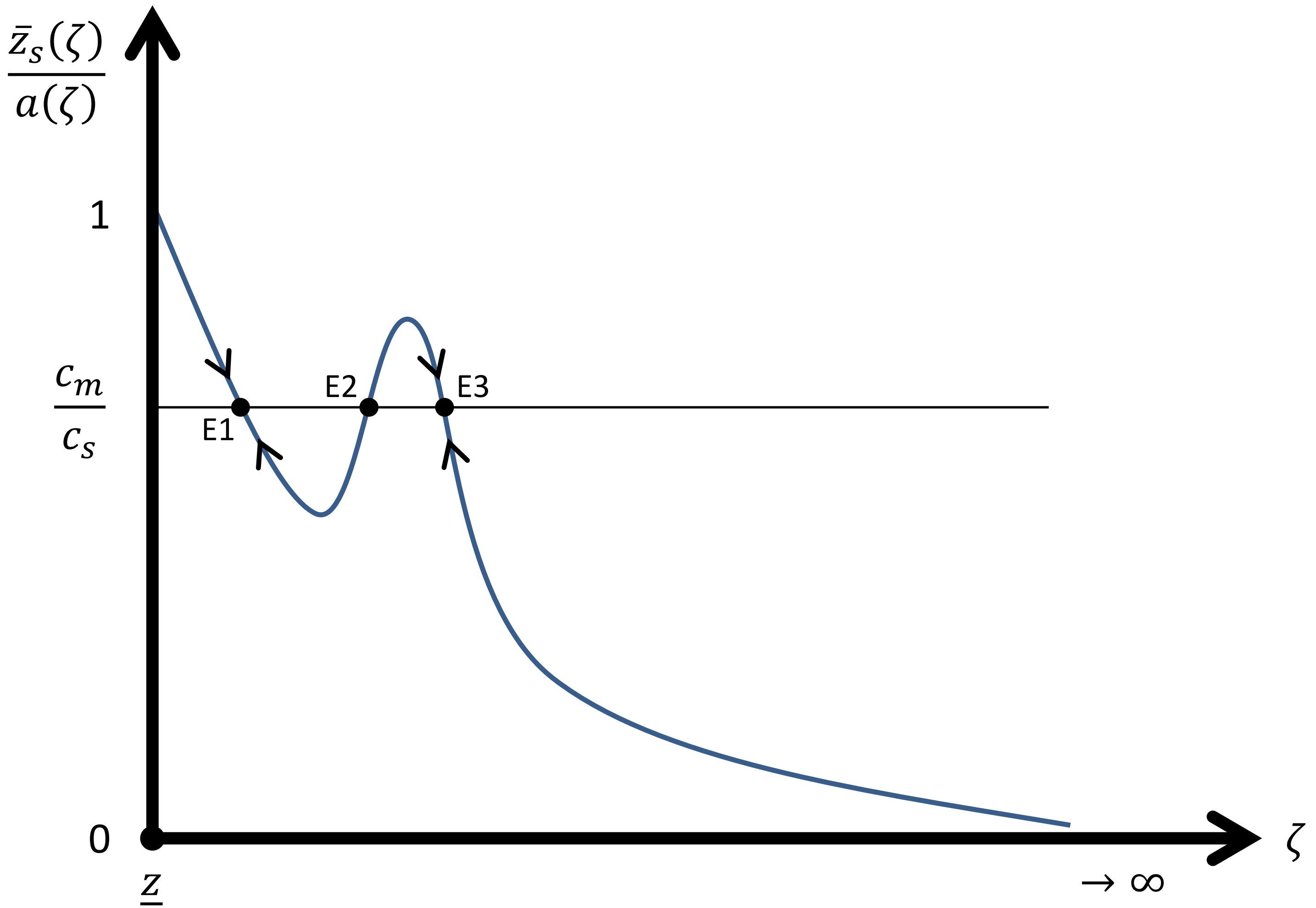

Figure 1: Stability of Sorting Equilibrium

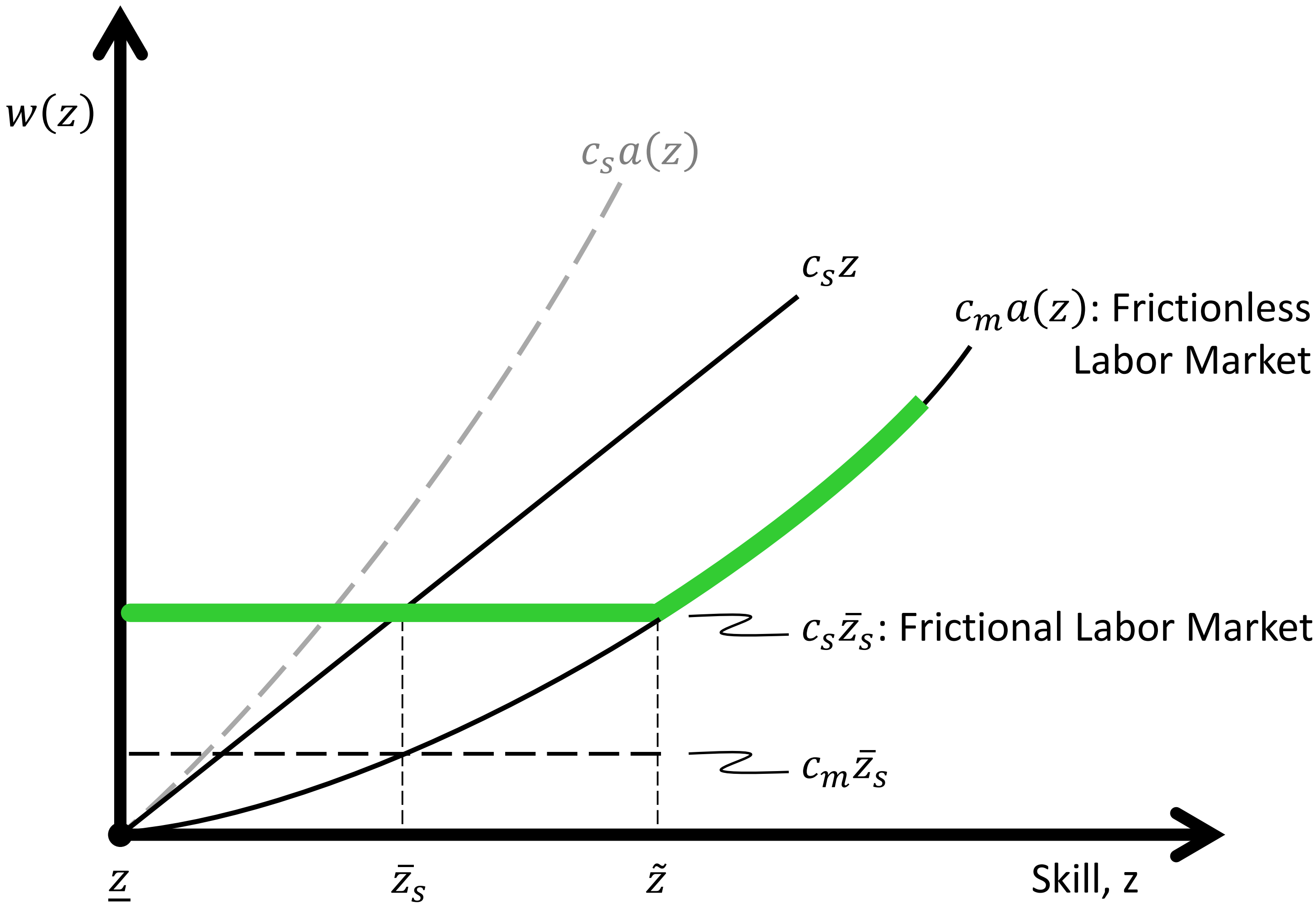

Figure 2: Hockey Stick Wage Profile

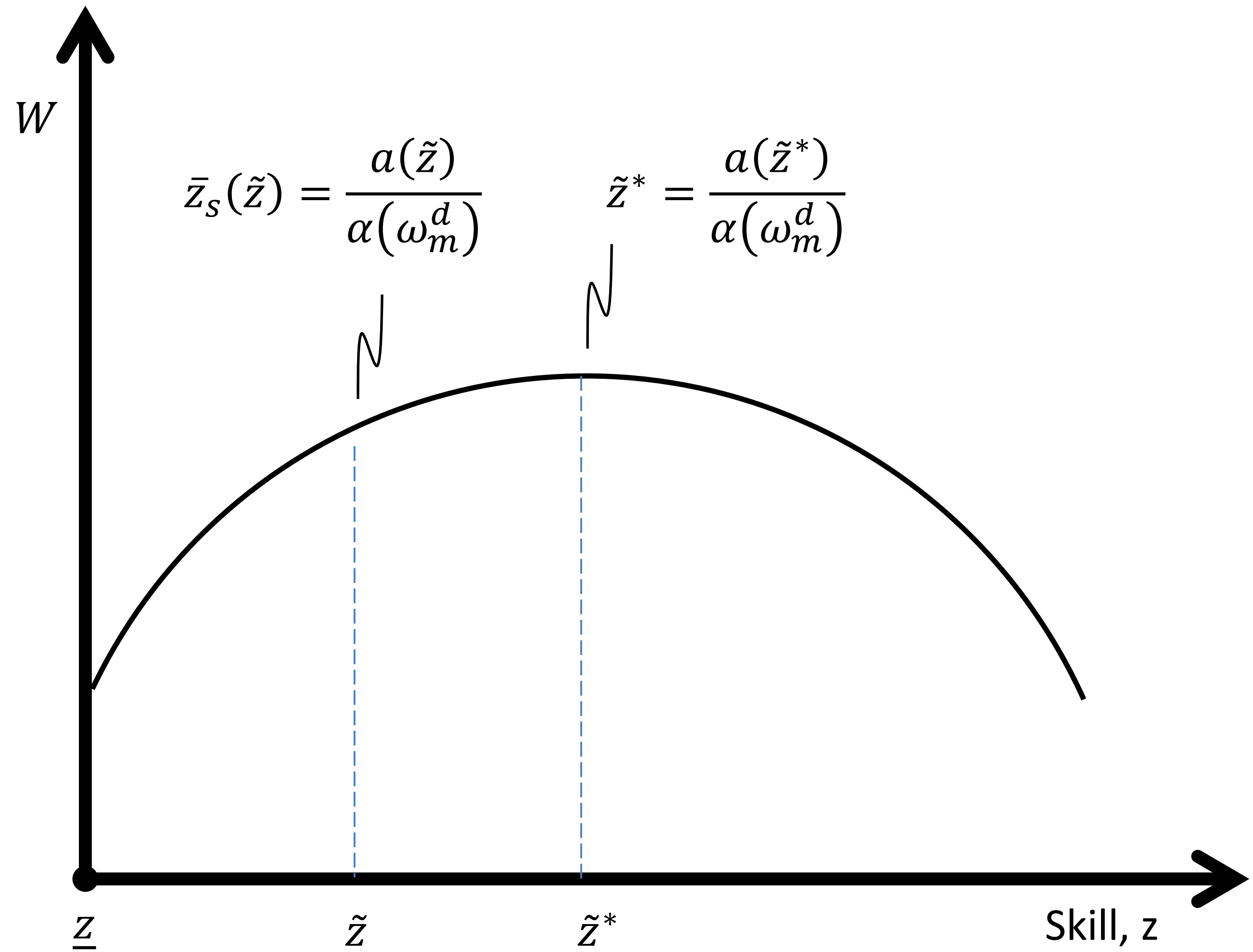

Figure 3: Sorting and Welfare