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INCENTIVES, CONNECTIVITY, AND SYSTEMIC STABILITY

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ABSTRACT

This paper develops a framework to analyze the consequences of alternative designs for interbank networks, in which a failure of one bank may lead to others. Earlier work had suggested that, provided shocks were not too large (or too correlated), denser networks were preferred to more sparsely connected networks because they were better able to absorb shocks. With large shocks, especially when systems are non-conservative, the likelihood of costly bankruptcy cascades increases with dense networks. Governments, worried about the cost of bailouts, have proposed bail-ins, where banks contribute. We analyze the conditions under which governments can credibly implement a bail-in strategy, showing that this depends on the network structure as well. With bail-ins, government intervention becomes desirable even for relatively small shocks, but the critical shock size above which sparser networks perform better is decreased; with sparser networks, a bail-in strategy is more credible.

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Bail-ins and Bail-outs: Incentives, Connectivity, and Systemic Stability

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This paper develops a framework to analyze the consequences of alternative designs for interbank networks, in which a failure of one bank may lead to others. Earlier work had suggested that, provided shocks were not too large (or too correlated), denser networks were preferred to more sparsely connected networks because they were better able to absorb shocks. With large shocks, especially when systems are non-conservative, the likelihood of costly bankruptcy cascades increases with dense networks. Governments, worried about the cost of bailouts, have proposed bail-ins, where banks contribute. We analyze the conditions under which governments can credibly implement a bail-in strategy, showing that this depends on the network structure as well. With bail-ins, government intervention becomes desirable even for relatively small shocks, but the critical shock size above which sparser networks perform better is decreased; with sparser networks, a bail-in strategy is more credible.

Financial institutions are linked to each other via bilateral contractual obligations and are thus exposed to counterparty risk of their obligors. If one institution is in distress, it will default on its agreements, thereby affecting the solvency of its creditors. Since the creditors are also borrowers, they may not be able to repay what they owe and default themselves — problems in one financial institution spread to others in what is called financial contagion.

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Large shocks can trigger a cascade of defaults with potentially devastating effects for the economy. The government is thus forced to intervene in some way and stop the cascade to reduce the negative externalities imposed on the economy. The extent of these cascades — the magnitude of the systemic risk — depends on the nature of the linkages, i.e., the topology of the financial system. In the 2008 crisis, it became apparent that the financial system had evolved in a way which enhanced its ability to absorb small shocks but made it more fragile in the face of a large shock. While a few studies called attention to these issues before the crisis, it was only after the crisis that the impact of the network structure on systemic risk became a major object of analysis.[1] Most of the existing studies analyze the systemic risk implications of a default cascade, taking into account network topology, asset liquidation costs, and different forms of inefficiencies that arise at default. Many of these models, however, do not account for the possibility of intervention by social planner/banks in an attempt to stop the cascade. There is either no rescue of insolvent banks or the social planner and/or financial institutions intervene by following an exogenously specified protocol. The objective of our paper is to investigate the impact of the network structure on social welfare when the rescue of insolvent banks results from the strategic interaction between social planner and financial institutions. The possibility of government interventions and strategic responses to these interventions leads to striking differences with the no-intervention case: while densely connected networks are more stable in models without intervention, in our model of strategic intervention, more sparsely connected networks are preferable in scenarios of distress.

We develop a theoretical framework for studying the economic incentives behind the determination of intervention plans. We consider an ex-post scenario, that is, when banks have already observed the realization of (non-interbank) asset returns and need to simultaneously clear their liabilities. The

[1]Most notably, Allen and Gale (2000) and Greenwald and Stiglitz (2003). See also Boissay (2006), Boss et al. (2004), Castiglionesi (2007), May, Levin and Sugihara (2008), and Nier et al. (2007). One of the reasons for the limited study is the scarce availability of data on interbank linkages. An early construction of Japan's interbank network, done before the crisis but published afterwards, is De Masi et al. (2011). With the exception of Haldane at the Bank of England, remarkably, central bankers paid little attention to the interplay of systemic risk and network topology; see Haldane (2009).

starting point of our analysis is the network contagion framework proposed by Eisenberg and Noe (2001), who develop an algorithm to determine the set of payments that simultaneously clear the banks' liabilities. Unlike Eisenberg and Noe, we consider the effect of bankruptcy losses as in Rogers and Veraart (2013) and Battison et al. (2016). When there are no bankruptcy costs, the system is "conservative" and the analysis simply reduces to a redistribution of wealth in the network. In the presence of bankruptcy costs, there are losses and the total dissipation of resources caused by bankruptcy depends on the network topology, also referred to as the architecture of the financial system in Stiglitz (2010*a*,*b*). While some topologies perform better (lead to smaller welfare losses) for small uncorrelated shocks, others perform better for large or highly correlated small shocks. The central results of this paper show that the network topology affects the set of credible government policies: a commitment not to intervene may be credible under some topologies but not under others. For a given prior distribution of shock sizes, our analysis reveals which network architecture is socially preferable ex ante. Different from the model of Rogers and Veraart (2013), in which a defaulting bank can either liquidate its assets in full or not liquidate them at all, in our framework banks can partially liquidate their assets to the extent they need to service their liabilities, similarly to Acemoglu, Ozdaglar and Tahbaz-Salehi (2015). Unlike that paper, there is a cost to early liquidation of interbank claims.

We consider three forms of intervention: (i) bailouts, (ii) bail-ins, and (iii) subsidized bail-ins. The most prevalent assistance plan up until the global financial crisis of 2007–2008 has been the *bailout*. During a bailout, the government injects liquidity to help distressed banks service their debt. For example, during the economic downturn, capital was injected into banks to prevent fire sale losses, including the intervention of the Bank of England and the U.S. Treasury Department's Asset Relief Program (TARP); see also Duffie (2010) for a related discussion.[2] When banks are *bailed in*, they reduce payments to their creditors, which enables them to stay solvent. In exchange, the creditors

[2]The Bush administration bailed out large financial institutions (AIG insurance, Bank of America and Citigroup) and government sponsored entities (Fannie Mae, Freddie Mac) at the heart of the crisis. The European Commission intervened to bail out financial institutions in Greece and Spain.

receive equity in the reorganized company. A prominent example of a bail-in is the consortium organized by the Federal Reserve Bank of New York to rescue the hedge fund Long-Term Capital Management.[3] The bail-in approach alleviates the burden for taxpayers and places it on creditors of the distressed banks instead. We also consider a third form of liquidity assistance, called *subsidized bail-in*. This corresponds to a mix of bail-in and bailout strategies, where the social planner provides liquidity assistance to incentivize the formation of a bail-in consortium. Such a strategy strikes a balance between the contributions of private creditors and taxpayers. These forms of resolution plans have been implemented during the global financial crisis.[4]

We model the provision of liquidity assistance as a sequential game consisting of three stages. In the first stage, the social planner proposes a subsidized bail-in allocation policy, specifying the quantity of debt of insolvent institutions that should be purchased by each solvent bank, as well as the additional liquidity injections that he wishes to provide to each bank (subsidies). In the second stage, each bank decides whether or not to accept the proposal of the social planner. If all banks accept, the game ends with the proposed rescue consortium and financial contagion is stopped; otherwise it moves to the third stage where the social planner is confronted with three choices: (i) purchase the debt that was supposed to be bought by the banks which rejected the proposal, (ii) purchase the entire debt, i.e., resort to a public bailout, or (iii) avoid any rescue and let the default cascade occur. The social planner's option of playing this last action is what we call the social planner's *threat of no intervention*. The threat, however, may not be credible if walking away from the proposal

[3] Long-Term Capital Portfolio collapsed in the late 1990s. On September 23, 1998, a recapitalization plan of \$3.6 billion was coordinated under the supervision of the Federal Reserve Bank of New York. A total of sixteen banks, including Bankers Trust, Barclays, Chase, Credit Suisse, Deutsche Bank, Goldman Sachs, Merrill Lynch, Morgan Stanley, Salomon Smith Barney, UBS, Société Général, Paribas, Crédit Agricole, Bear Stearns, and Lehman Brothers originally agreed to participate. However, Bear Stearns and Lehman Brothers later declined to participate and their agreed-upon contributions was instead provided by the remaining 14 banks.

[4] A noticeable example of subsidized bail-in is Bear Stearns. JPMorgan Chase (JPM) and the New York Federal Reserve stepped in with an emergency cash bailout on March, 2008. The provision of liquidity by the Federal Reserve was taken to avoid a potential fire sale of nearly U.S. \$210 billion of Bear Stearns' assets. The Chairman of the Fed, Ben Bernanke, defended the bailout by stating that Bear Stearns' bankruptcy would have affected the economy, causing a "chaotic unwinding" of investments across the U.S. markets and a further devaluation of other securities across the banking system.

decreases the social planner's welfare function. We show that the threat is credible if and only if the amplification of the shock through the financial system is sufficiently small. A large initial shock, low recovery rates of liquidated assets, and a high degree of interconnectedness between defaulting banks all contribute positively to the amplification of the shock. If the amplification is large, the threat is not credible and leaves a public bailout as the only possible rescue option. If, in contrast, the amplification is small, the social planner's threat is credible and a subsidized bail-in can be organized. We characterize the welfare-maximizing subsidized bail-in that arises as the generically unique subgame perfect equilibrium (up to equivalent proposals). Our analysis shows that the banks' equilibrium contributions tend to be larger in more sparsely connected networks. This may be understood as follows. An individual bank is willing to contribute to a bail-in up to the amount it would lose in a default cascade. This amount is greater in a sparsely connected network, where the losses of a defaulted debtor are borne by a small set of creditors, than in a densely connected network, where the shock is spread among a large number of banks. As a result, the primary creditors of the defaulting banks can be incentivized to make larger contributions in a more sparsely connected network.

We compare the credibility of the social planner's threat between the ring network and the complete network as representative structures of sparsely and densely connected networks, respectively. We show that the credibility of the social planner's threat exhibits a form of phase transition as the size of the initial shock grows larger. If the size of the initial shock is small, the threat is more credible in the complete network because a more diversified financial network behaves as a better shock absorber; see also Allen and Gale (2000) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015). If the magnitude of the initial shock exceeds a certain level, the threat is more credible in the ring network because dense interconnections serve as a mechanism for the shock's amplification. The threshold of shock sizes, above which the threat is more credible in the ring network, is increasing in the recovery rate of interbanking claims. For a fixed shock size, the threat is more credible in the complete network if the recovery rate is sufficiently large, whereas it is more credible in

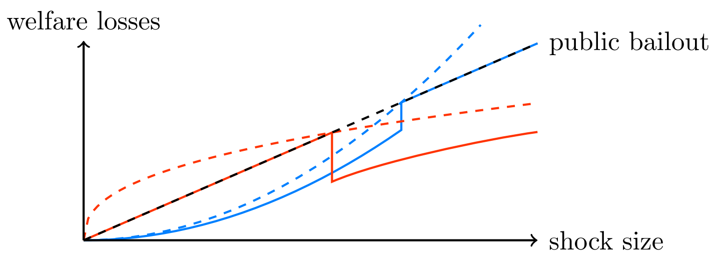


Figure 1: The figure compares equilibrium welfare losses in the complete (blue) and the ring network (red) in the presence (solid lines) and absence (dashed lines) of intervention.[5] When the no-intervention losses exceed the costs of a public bailout (black dashed line), the government's threat to not intervene is not credible and a public bailout is the only possible equilibrium rescue. Banks' contributions are larger in a the ring network.

the ring network for smaller recovery rates.

Our findings reverse the presumptions in earlier work without intervention, which indicate that welfare losses in response to a shock are higher in more sparsely connected networks unless the shock is large enough to cause a systemic default; see Allen and Gale (2000) and Acemoglu, Ozdaglar and Tahbaz-Salehi (2015). The intuitions behind our findings are twofold: (i) the no-intervention threat is more credible in sparsely connected networks when the shock is large or interbank recovery rates are low and (ii) banks can be incentivized to make larger contributions to a subsidized bail-in if the network is more sparsely connected. When the shock is large, the welfare losses under the equilibrium bail-in plan are lower in a more sparsely connected network. Although our analysis does not provide a clear-cut statement in the case of small shocks, welfare losses may still be lower in the more sparsely connected network because of the banks' increased contributions to the equilibrium bail-in plan; see Figure 1 for a graphical visualization and Section 5 for a numerical illustration of our results. The threshold of shock sizes, beyond which the more sparsely connected network is preferable, is thus lowered with intervention. Our conclusions that sparsely connected networks are socially preferable thus remain valid (and would even be strengthened) if the social planner were to be risk averse instead of risk neutral.

[5]The figure displays our results in the stylized case of a continuum of banks to highlight the key differences between welfare losses in a ring and a complete network. In our model, the financial network consists of a

Our analysis offers an explanation for some of the decisions made by the sovereign authorities during distress scenarios. For instance, a private bail-in was coordinated to rescue the Long-Term Capital Management hedge fund in 1998. In contrast, the government of the United States rescued Citigroup through a public bailout in November 2008. Because several default events occurred prior to Citigroup's bailout, the financial system was much more lowly capitalized during the global financial crisis than when Long-Term Capital Management was rescued.[6] Since the amplification of the shock is larger in a lowly capitalized system, the government's threat to not intervene and not bailout Citigroup, the largest bank in the world at the time, might not have been credible. As a result, a public bailout was the only option for rescuing Citigroup, whereas it was possible to secure a bail-in for LTCM.

The remainder of the paper is organized as follows. In Section 1, we provide a review of the existing literature, before developing the details of our model in Section 2. We characterize the optimal bail-in plan and the equilibrium outcome for a fixed network topology in Section 3. We analyze the impact of the network topology on the credibility of the social planner's threat in Section 4. Section 5 illustrates and interprets our main results with examples. Section 6 provides concluding remarks and Section 7 discusses avenues for future research. All technical proofs are delegated to the appendices.

1 Literature Review

Our paper is related to a vast branch of literature on financial contagion in interbank networks. The work by Allen and Gale (2000) relates the network structure to the fragility of the financial system. Their model of payment flows captures propagation of financial crises in an environment where both liquidity and solvency shocks affect financial intermediaries. They show that interbank lending, while allowing for risk sharing, can also create financial fragility. They show that a ring network, where each bank has exactly one creditor, is less

finite number of banks, which leads to additional discontinuities in welfare losses. We refer to Section 5 for a numerical example consisting of a finite number of banks.

[6] We recall the seven credit events occurred in the month of September 2008, involving Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir and Kaupthing.

resilient to financial shocks than a densely interconnected network because in the latter, the impact of the first default is diluted among a larger set of banks. Gai, Haldane and Kapadia (2011) show, via numerical simulations, how the concentration and complexity of financial linkages can endanger the resilience of the network. They demonstrate that macro-prudential policies, imposing tough liquidity requirements, can improve the stability of the system. A related study by Gai and Kapadia (2010) uses statistical network theory to analyze how knock-on effects of distress can lead to write down the value of institutional assets. Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) analyze the interplay of network topology and shock size from a social welfare perspective. Their analysis confirms the findings of Allen and Gale (2000) for small shocks and extends them for large shocks, showing that the optimal network configuration is a weakly connected financial network, in which different subsets of banks have minimal claims on one another, with complete diversification within each subset. The findings of Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) also rationalize the "robust-yet-fragile" property of highly interconnected financial networks evidenced by Gai and Kapadia (2010) through numerical simulations. Capponi, Chen and Yao (2016) generalize the analysis of Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) from regular networks to a wider class. They show that a more diversified structure of interbank liabilities is socially desirable if the financial network is highly capitalized, whereas higher concentration of liabilities is socially preferable if the network is lowly capitalized. In the absence of intervention, our findings corroborate those of Allen and Gale (2000); Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) and Capponi, Chen and Yao (2016). High interconnectivity provides a potential for absorption of small shocks and implies stability. For large enough shocks causing financial distress to a significant fraction of the system, dense connections between the distressed banks amplify the shock.

Glasserman and Young (2015) demonstrate in an Eisenberg-Noe type model that contagion effects coming from direct counterparty exposures may not be as strong as fire sales and other related mechanisms in determining losses. Elliott, Golub and Jackson (2014) study the trade-off of diversification, spread of cross-holdings of equity shares, and integration on the levels of interbank exposure. They find that at extreme levels of integration and diversification, the risk of far-reaching default cascades of financial failures is the lowest. Battiston et al. (2012*a*) demonstrate that diversification prevents systemic defaults if the system is in good financial conditions. If, in contrast, many banks are already fragile, initial defaults can trigger a systemic default when diversification is high. Battiston et al. (2012*b*) study the time evolution of a credit network using a system of coupled stochastic processes, each describing the robustness dynamics of a financial institution. They show that there is an optimal degree of diversification so that beyond a point, there is a trade-off between decreasing individual risk due to risk sharing, and increasing systemic risk due to propagation of systemic distress.[7] We refer to Glasserman and Young (2016) for a thorough survey on financial contagion, discussing the interplay of network topology with balance sheet variables. Other related contributions include Cifuentes, Ferrucci and Shin (2005) which analyze the impact of fire sales on financial network contagion; Zawadowski (2013) which consider the network of derivative exposures and its role in hedging risks; Cabrales, Gottardi and Vega-Redondo (2014) which study the trade-off between the risk-sharing generated by more dense interconnections and the greater potential for default cascades; Elsinger, Lehar and Summer (2006) which study transmission of contagion in the Austrian banking system; Cont, Santos and Moussa (2013) which perform a similar analysis on the Brazilian interbank system; Craig and Von Peter (2014) which uncover a core-periphery structure of the interbank network using bilateral interbank data of German banks.

2 Model

We consider an interbank network with simultaneous clearing in the spirit of Eisenberg and Noe (2001). Banks $i = 1, \dots, n$ are connected through interbank liabilities $\mathbf{L} = (L^{ij})_{i,j=1,\dots,n}$, where L^{ij} denotes the liability of bank j

[7] Greenwald and Stiglitz (1986) show that because of the pervasive externalities which arise when there are incomplete risk markets, contracts (linkages) which are individually rational may not be welfare maximizing. For further discussion of optimal diversification, see Battiston et al. (2012*b*).

to bank i. We denote by $L^j := \sum_{i=1}^n L^{ij}$ the total liability of bank j to other banks in the network. Define the relative liability matrix $\pi = (\pi^{ij})$ by setting $\pi^{ij} = L^{ij}/L^j$ if $L^j \neq 0$ and $\pi^{ij} = 0$ otherwise. Our framework can accommodate lending from the private sector by adding a "sink node" $n+1$ that has only interbank assets but no interbank liabilities. Banks have investments in outside assets with values $e = (e^1, \ldots, e^n)$, cash holdings $c_h = (c_h^1, \ldots, c_h^n)$ and financial commitments $c_f = (c_f^1, \ldots, c_f^n)$ with a higher seniority than the interbank liabilities. These commitments include wages and other operating expenses. If a bank i is not able to meet its liabilities $L^i + c_f^i$ out of current income, it will liquidate a part $\ell^i \in [0, e^i]$ of its outside investments, but will recover only a fraction $\alpha \in (0, 1]$ of its value.[8] If a bank i cannot meet its liabilities even after liquidating all of its outside assets e^i, it will default. As in Rogers and Veraart (2013) and Battison et al. (2016), the default of a bank is costly and only a fraction $\beta \in (0, 1]$ can be recovered from the interbank assets. We use $A = \pi L$ to denote the vector of book values of interbank assets with $A^i = (\pi L)^i = \sum_{j=1}^n \pi^{ij} L^j$. Because the operating expenses c_f have higher seniority than the interbank liabilities, the model depends on c_h and c_f only through the net cash balance $c = c_h - c_f$.

We denote by (L, π, e, c) the financial system after risks have been taken and after an exogenous shock has hit the system. The shock may lower the value of banks' outside assets or the net cash holdings of the banks. This may result in a negative net cash balance if, for example, a bank intended to use the returns from an investment to cover the operating expenses, but the returns turned out to be lower than expected. We refer to the defaults that occur as an immediate consequence of the shock as the *fundamental defaults* and denote their index set by $\mathcal{F} := \{i \mid L^i > c^i + \alpha e^i + A^i\}$. These are banks which cannot meet their obligations even if every other bank repays its liabilities in full. Because fundamentally defaulting banks are able to only partially repay their creditors, their defaults may lead to additional defaults in the system, resulting in a default cascade. If, however, banks in $\mathcal{F}$ receive a liquidity injection so

[8] In reality, recovery rates are asset-specific and some assets may directly be transferred to the creditors of defaulting institutions without liquidation. The parameter α is to be understood as an average recovery rate across all assets. It is equal to 1 if all assets are transferred to the creditors.

that they can meet their obligations, the financial system is stabilized. In this section, we first characterize the outcome of a default cascade and then discuss in detail the different types of rescue under consideration.

2.1 Default cascade

A defaulting bank will recall its assets and repay its creditors according to their seniority. Depositors are the most senior creditors, hence they are given priority over lenders from the interbank network and the private sector, to whom we refer as junior creditors. Creditors with the same seniority are repaid proportionally to their claim sizes. How much a bank is able to recall from its interbank assets depends on the solvency of the other banks in the system. A clearing equilibrium is a set of repayments, simultaneously executed by all banks, for which every solvent bank repays its liabilities in full and every insolvent bank precisely repays its total value after liquidation.[9]

Definition 2.1. A *clearing equilibrium* (ℓ, p) of a network (L, π, e, c) consists of a liquidation decision ℓ^i of every player i and a clearing payment vector $p = (p^1, \dots, p^n)$, which constitute a fixed point of the following set of equations

$$\ell^i = \min\left(\frac{1}{\alpha}\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} p^j\right)^+, e^i\right),$$

$$p^i = \begin{cases} L^i & \text{if } c^i + \alpha\ell^i + \sum_{j=1}^{n} \pi^{ij} p^j \geq L^i, \\ \left(c^i + \alpha\ell^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j\right)^+ & \text{otherwise.}^{10} \end{cases}$$

In a clearing equilibrium, bank i either remains solvent by liquidating an amount $\ell^i = \frac{1}{\alpha}\left(L^i - c^i - (\pi p)^i\right)^+$ of its outside assets, in which case bank i repays its liabilities L^i in full, or bank i cannot meet its liabilities even when liquidating all of its outside assets. In the latter case, the entire value of bank i

[9] In practice, liabilities may be cleared sequentially rather than simultaneously and the order of clearing may impact the outcome. This method of simultaneous clearing is standard in the literature and may represent the fact that clearing of liabilities occurs on a much smaller time scale than the formation of rescue consortia.

[10] We use $x^+ = \max(x, 0)$ and $x^- = \max(-x, 0)$ to denote the positive and negative part of x, respectively.

after liquidation is transferred to its creditors. If the payment p^i is positive, it is divided pro-rata among bank i's junior creditors and the senior creditors (e.g., the depositors) are paid in full. If $p^i = 0$, the junior creditors do not receive anything and the senior creditors suffer a loss of

$$\delta^i := \left(c^i + \alpha \ell^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j \right)^{-}.$$

Our definition of a clearing equilibrium extends the corresponding notion in Rogers and Veraart (2013), by allowing banks to partially liquidate their outside assets, and the corresponding notion in Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), by allowing for partial recovery of interbank claims at a rate $\beta < 1$. The value of bank i's equity in a clearing equilibrium (ℓ, p) equals

$$V^i(\ell, p) := (\pi p + c + e - (1-\alpha)\ell - p)^i 1_{\{p^i = L^i\}}.$$

Let $\mathcal{D}(\ell, p) := \{i \mid p^i < L^i\}$ denote the set of banks which default in the clearing equilibrium (ℓ, p). For any clearing equilibrium (ℓ, p), we define the welfare losses $w(\ell, p)$ as the weighted sum of the losses due to default costs, that is,

$$w(\ell, p) = (1-\alpha) \sum_{i=1}^{n} \ell^i + (1-\beta) \sum_{i \in \mathcal{D}(\ell,p)} (\pi p)^i + \lambda \sum_{i \in \mathcal{D}(\ell,p)} \delta^i. \tag{1}$$

The first two terms in (1) are deadweight losses due to inefficient asset liquidation: If bank i decides to liquidate a positive amount ℓ^i, only $\alpha \ell^i$ is recovered and $(1-\alpha)\ell^i$ is lost. A defaulting bank $i \in \mathcal{D}$ also recalls its interbank assets at a rate β, leading to losses of $(1-\beta)(\pi p)^i$, where $(\pi p)^i = \sum_{j=1}^{n} \pi^{ij} p^j$ are the payments made to bank i in the clearing equilibrium (ℓ, p). The last term in (1) are the depositors' losses δ of the defaulting banks, weighted by a constant $\lambda \geq 0$. The weight λ captures the importance that the social planner assigns to the depositor's losses relative to the deadweight losses from asset liquidation and bankruptcy proceedings. We will weigh taxpayer contributions to a public bailout and a subsidized bail-in with the same factor λ; see

Sections 2.2 and 2.3.[11] The social planner's goal is to minimize welfare losses and the parameter λ captures his priorities in doing so. A social planner with $\lambda = 0$ views government subsidies and depositors' losses merely as transfers of wealth and not as losses to the economy. A higher value of λ indicates a higher priority to the welfare of the economy exclusive of the banking sector. The coefficient λ may also be interpreted as a measure of political pressure on the social planner: government subsidies and a haircut on deposits make, respectively, taxpayers and depositors unhappy. Because λ is presumed to be constant, we omit the dependence of welfare losses on λ for ease of notation.

We obtain the following existence result for clearing equilibria. Similarly to Rogers and Veraart (2013), clearing equilibria need not be unique if $\beta < 1$.

Lemma 2.1. *There exist clearing equilibria* $(\underline{\ell}, \underline{p})$ *and* $(\hat{\ell}, \hat{p})$ *such that for any clearing equilibrium* (ℓ, p)*, we have* $\underline{\ell} \geq \ell \geq \hat{\ell}$ *and* $\underline{p} \leq p \leq \hat{p}$*, as well as*

$$V^i(\underline{\ell}, \underline{p}) \leq V^i(\ell, p) \leq V^i(\hat{\ell}, \hat{p}), \qquad \text{and} \qquad w(\hat{\ell}, \hat{p}) \leq w(\ell, p) \leq w(\underline{\ell}, \underline{p}).$$

In particular, $(\hat{\ell}, \hat{p})$ *Pareto dominates any other clearing equilibrium.*

Because $(\hat{\ell}, \hat{p})$ Pareto dominates any other clearing equilibrium, we assume that all parties agree to clear the liabilities with $(\hat{\ell}, \hat{p})$, making it the unique outcome in a default cascade. For the sake of brevity, we omit the argument when referring to the set of defaulting banks $\mathcal{D} := \mathcal{D}\big(\hat{\ell}, \hat{p}\big)$ in the clearing equilibrium $\big(\hat{\ell}, \hat{p}\big)$. We denote by $\mathcal{C} := \mathcal{D} \setminus \mathcal{F}$ the set of contagious defaults and by $\mathcal{S} := \{i \mid \hat{p}^i = L^i\}$ the set of banks which remain solvent. For two sets $S, I \subseteq \{1, \ldots, n\}$, denote by $\pi^{S,I}$ the submatrix of π given by the rows in S and the columns in I. Similarly, given a vector x, we use x^S to denote the subvector of x with entries in S. We will often use the 1-norm to denote the sum over the absolute entries of a vector, i.e.,

$$\|x\|_1 = \sum_{i=1}^{n} \big|x^i\big|, \qquad \big\|x^S\big\|_1 = \sum_{i \in S} \big|x^i\big|.$$

[11] In reality, the social planner may assign a weight λ_1 to depositor's losses and a different weight λ_2 to taxpayer contributions. In many situations, $\lambda_1 > \lambda_2$ so that the depositors are bailed out. However, the crises in Cyprus and Iceland have shown that this is not always the case. Our results do not crucially depend on $\lambda_1 = \lambda_2$, and we make this assumption for purely aesthetic purposes.

2.2 Public bailout

In a public bailout, the social planner makes up for the shortfall of the fundamentally defaulting banks. He decides whether or not the outside assets of these banks should be liquidated. If banks liquidate their outside assets $e^{\mathcal{F}}$, a fraction α of their value $\left\|e^{\mathcal{F}}\right\|_1$ is recovered and the social planner covers the remaining shortfall $B := \left\|L^{\mathcal{F}} - c^{\mathcal{F}} - A^{\mathcal{F}} - \alpha e^{\mathcal{F}}\right\|_1$. Since the social planner assigns a welfare loss of λ to a taxpayer dollar, the resulting welfare loss is $\lambda B + (1-\alpha)\left\|e^{\mathcal{F}}\right\|_1$. The social planner may also choose to cover the entire shortfall $B + \alpha\left\|e^{\mathcal{F}}\right\|_1$ instead, leading to a welfare loss of $\lambda B + \alpha\lambda\left\|e^{\mathcal{F}}\right\|_1$. Because the social planner aims to minimize welfare losses, it follows that it is optimal to require the liquidation of outside assets if $\alpha \geq 1/(1+\lambda)$.

We analyze a financial network after it has been hit by a shock. While only banks in $\mathcal{F}$ are fundamentally defaulting as a result of the shock, other banks may also be in need of some additional liquidity to cover their liabilities. Specifically, any bank $i \in \mathcal{C} \cup \mathcal{S}$ has to raise additional liquidity to cover its shortfall $(L^i - c^i - A^i)^+$, even if the fundamentally defaulting banks are being rescued. This may happen by liquidating an amount $\ell_*^i := \frac{1}{\alpha}(L^i - c^i - A^i)^+$ of bank i's outside asset, leading to deadweight losses $(1-\alpha)\ell_*^i$ or by receiving a liquidity injection of $\alpha\ell_*^i$ from the social planner. For the same reasons as above, the social planner prefers to provide a liquidity injection to non-fundamentally defaulting banks if $\alpha < 1/(1+\lambda)$. For ease of reference, we summarize this discussion in a lemma, where we use $V_0^i := c^i + e^i + A^i - L^i$ to denote the value of bank i after the realization of the shock but before the liquidation of its outside assets.

Lemma 2.2. *The equity value of each non-fundamentally defaulting bank $i \in \mathcal{C} \cup \mathcal{S}$ in a public bailout is equal to*

$$V_P^i := \begin{cases} V_0^i - (1-\alpha)\ell_*^i & \text{if } \alpha \geq \frac{1}{1+\lambda}, \\ V_0^i + \alpha\ell_*^i & \text{if } \alpha < \frac{1}{1+\lambda}. \end{cases} \tag{2}$$

The welfare losses are equal to $w_P = \lambda B + \min(\lambda\alpha, 1-\alpha)\|\ell_*\|_1$.

2.3 Subsidized bail-ins

In a bail-in, the banks buy up the debt of the fundamentally defaulting banks in order to ensure the stability of the financial system. In a subsidized bail-in, the government may buy up part of the debt and it may provide subsidies to participating banks.

Definition 2.2.

1. A *bail-in allocation* $b = (b^0, b^1, \ldots, b^n)$ assigns to each bank $i = 1, \ldots, n$ an amount of debt b^i that bank i has to purchase, as well as an amount b^0 that is to be covered by the social planner.

2. A *subsidized bail-in* (b, s) consists of a bail-in allocation b and a vector of subsidies $s = (s^1, \ldots, s^n)$. In a subsidized bail-in, each bank i receives a cash subsidy s^i from the social planner for participating in the rescue. The cash subsidy is paid before any liquidation decision is made. Each bank i thus has to liquidate an amount of assets equal to $\ell^i(b - s)$, where

$$\ell^i(x) = \frac{1}{\alpha}(x^i + e^i - V_0^i)^+. \tag{3}$$

3. A subsidized bail-in is *feasible* if it covers the entire shortfall, that is, $\|b\|_1 \geq B$, if the fundamentally defaulting banks are not part of the rescue, i.e., $b^i = s^i = 0$ for every $i \in \mathcal{F}$, and if every bank $i \in \mathcal{C} \cup \mathcal{S}$ remains within its budget limit $c^i + \alpha e^i + A^i - L^i = V_0^i - (1 - \alpha)e^i$, that is, $b^i - s^i \leq V_0^i - (1 - \alpha)e^i$.

Remark 2.1. Similarly to a public bailout, the social planner wishes to raise an amount equal to $\|b\|_1 = B + \alpha\left\|e^{\mathcal{F}}\right\|_1$ if $\alpha < 1/(1 + \lambda)$ to avoid liquidation costs, whereas for $\alpha \geq 1/(1 + \lambda)$, the social planner aims to raise an amount equal to $\|b\|_1 = B$. Observe that subsidized bail-ins contain both a public bailout and a privately backed bail-in as special cases. A public bailout is a subsidized bail-in, in which the banks' contributions are equal to zero, that is, $b^1 = \ldots = b^n = 0$. In a private bail-in, the government contributions are equal to 0, i.e., $b^0 = 0$ and $s^1 = \ldots = s^n = 0$.

In our model, the social planner takes the main role in organizing a subsidized bail-in: he may propose a bail-in strategy and try to convince banks to join, but he cannot force banks to participate.[12] The social planner is thus restricted to proposals that are incentive compatible for the individual banks. We achieve this by considering subgame perfect equilibria in the following three-stage game.

1. The social planner proposes a subsidized bail-in (b, s).

2. Each bank $i \in \mathcal{C} \cup \mathcal{S}$ chooses a binary action $a^i \in \{0, 1\}$, indicating whether or not it accepts the social planner's proposal. Let $\mathcal{A} := \{i \in \mathcal{C} \cup \mathcal{S} \mid a^i = 0\}$ denote the set of banks which reject the proposal.

3. If $\mathcal{A} = \emptyset$, the consortium buys up the debt of banks in $\mathcal{F}$ as planned and the game ends. If $\mathcal{A} \neq \emptyset$, the social planner has three options:

   (a) $a^0 = R$: The social planner proceeds with the proposed bail-in, but covers an amount equal to $b^0 + \left\| b^{\mathcal{A}} \right\|_1$. Banks in $\mathcal{A}$ do not participate in the rescue and do not receive a subsidy. The equity value of each bank $i \in \mathcal{C} \cup \mathcal{S}$ equals

$$V_R^i(b, s, a) = \begin{cases} V_0^i - b^i + s^i - (1-\alpha)\ell^i(b - s) & \text{if } i \in \mathcal{A}^c, \\ V_0^i - (1-\alpha)\ell_*^i & \text{if } i \in \mathcal{A}. \end{cases} \tag{4}$$

   We refer to $\mathcal{R}_{b,s}(a) := \{i \in \mathcal{A}^c \mid b^i > s^i\}$ as the *rescue consortium* associated with the bail-in. It is composed of the banks which contribute a positive net amount to the rescue. Expression (4) indicates that the equity of a bank i in the consortium is reduced by its net cash contribution $b^i - s^i$ and the liquidation costs incurred to retrieve this cash amount. A bank $i \in \mathcal{A}$ that rejects the proposal has to liquidate an amount ℓ_*^i of its outside assets to remain solvent,

[12] Duffie and Wang (2017) consider bail-in strategies which are done contractually, rather than by a central planner. In their model, prioritization of bail-ins is viewed as based on the type of the instrument, and not as one based on mitigating failure contagion through the network. Under strong axioms and assumptions on bilateral bargaining conditions, they show that the efficient choice of bail-in arrangements is made voluntarily.

which induces losses in the size of $(1-\alpha)\ell_*^i$. The resulting welfare losses are equal to

$$w_R(b,s,a) = \lambda\big(b^0 + \big\|b^{\mathcal{A}}\big\|_1 + \big\|s^{\mathcal{A}^c}\big\|_1\big) + (1-\alpha)\big(\big\|\ell^{\mathcal{A}^c}(b-s)\big\|_1 + \big\|\ell_*^{\mathcal{F}\cup\mathcal{A}}\big\|_1\big).$$

These losses include the government's contributions to the bail-in as well as those generated by the inefficient liquidation of the assets.

(b) $a^0 = P$: The social planner resorts to a public bailout, which results in equity values given in (2).

(c) $a^0 = N$: The social planner abandons the rescue, which results in a default cascade as in Section 2.1. We denote by $V_N^i := V^i\big(\hat{\ell}, \hat{p}\big)$ the equity value of bank i and by $w_N := w\big(\hat{\ell}, \hat{p}\big)$ the welfare losses.

The restriction to subgame perfect equilibria eliminates the non-credible threat of the social planner to abandon the rescue in the third stage when, in fact, he prefers a public bailout over a default cascade. This makes it impossible to incentivize banks to participate in a bail-in if $w_P < w_N$. Indeed, every bank is aware that the social planner will inevitably resort to a public bailout after a rejection of his proposal. Since the banks prefer a public bailout over a bail-in, they will reject any proposal when $w_P < w_N$. The amount of the welfare losses that is due to the social planner's lack of commitment power can be characterized as a consequence of our main theorem; see Remark 3.1.

3 Optimal proposal of the social planner

We begin this section by characterizing the banks' equilibrium response to any given proposal (b, s) of the social planner. When a bank considers rejecting the proposal, it has to take into account what reaction this will trigger from the social planner. If either the welfare losses w_P in a public bailout, or the welfare losses w_R in a bail-in without bank i, are lower than the welfare losses in a default cascade, then bank i knows that it needs not fear a default cascade. Such an outcome would involve a suboptimal response by the social planner. If, however, a rejection of bank i makes $a^0 = N$ the best response for the social planner, bank i is better off accepting any proposal, in which its net contributions to the bail-in are lower than its losses in a default cascade.

Let $\zeta^i := \pi^{i,\mathcal{D}}(L^{\mathcal{D}} - \hat{p}^{\mathcal{D}})$ denote the cumulative losses in bank i's interbank assets that arise as the result of a default cascade. If this shock ζ^i is larger than the total amount $c^i + \alpha e^i + A^i - L^i = V_0^i - (1-\alpha)e^i$ that bank i can procure by liquidating its outside assets, then bank i will default in the absence of intervention. The losses of any bank $i \in \mathcal{C} \cup \mathcal{S}$ in a default cascade are thus given by $\xi^i + \ell^i(\xi)$, where $\xi^i := \min\bigl(\zeta^i, V_0^i - (1-\alpha)e^i\bigr)$ and $\ell^i(x)$ is defined in (3). Since ℓ^i is increasing, it follows from (4) that bank i is better off in the default cascade than in the subsidized bail-in (b, s) if and only if $\xi^i \le b^i - s^i$. The following lemma characterizes the banks' equilibrium responses depending on the credibility of the social planner's threat.

Lemma 3.1. *Let (b, s) be a proposed bail-in with equilibrium response a.*

1. *If $w_P < w_N$, then $a^i = 1$ if and only if either $s^i - b^i \ge \alpha \ell_*^i 1_{\{\alpha < 1/(1+\lambda)\}}$ or*
   - *(a) $s^i - b^i \ge 0$, and*
   - *(b) $w_R\bigl(b, s, (0, a^{-i})\bigr) \le w_P$.*
2. *If $w_P \ge w_N$, then $a^i = 1$ if and only if either $s^i - b^i \ge 0$ or*
   - *(a) $b^i - s^i \le \xi^i$, and*
   - *(b) $w_R\bigl(b, s, (0, a^{-i})\bigr) \ge w_N$.*

In the first case, the social planner's threat of abandoning the rescue is not credible. A rejection of bank i will thus never lead to a default cascade as the social planner will resort to a public bailout instead. If $w_P < w_R\bigl(b, s, (0, a^{-i})\bigr)$, the social planner prefers a public bailout over a bail-in without bank i. Bank i will thus accept the proposal only if its net gains in the proposal are higher than in a public bailout. If the social planner prefers a bail-in without bank i over a public bailout, a bail-in will be coordinated regardless of whether or not bank i participates. Bank i thus accepts if and only if it receives a positive net amount. Neither of these scenarios are attractive for the social planner.

In the second case, the social planner's threat is credible, hence the social planner will never resort to a public bailout. A bank i will thus certainly accept a net subsidy, as it dominates any alternative outcome. If the social planner asks a bank for a net contribution, condition 2.(b) implies that a rejection of bank i will lead to a default cascade and because of condition 2.(a), this makes bank i worse off than accepting the proposal. The two conditions together are thus sufficient for bank i to accept the proposal.

Because the social planner can anticipate the banks' response in equilibrium, it is unnecessary for him to propose a bail-in that is rejected by any bank. It is thus enough to model the negotiation between social planner and the banks as a single interaction. In reality, there might be several rounds of negotiation.[13] If some bank rejects the social planner's proposal, the social planner might revise it to convince other banks to join the consortium. In a game-theoretic model with complete information, the entire negotiation process is collapsed into a single stage by proposing only bail-in plans that are incentive compatible for all members of the consortium.

Condition 2.(b) of Lemma 3.1 implies that the welfare loss $w_R\big(b, s, (0, 1^{-i})\big)$ after the proposal's rejection of a single bank i are bounded from below by w_N. Since $w_R\big(b, s, (0, 1^{-i})\big) = w_R(b, s, 1) + \lambda b^i - (1 - \alpha)\big(\ell^i(b) - \ell^i_*\big)$ for any bank $i \in \mathcal{R}_{b,s}(1)$ by (4), welfare losses in a bail-in (b, s) admit the lower bound

$$w_R(b, s, 1) \geq w_N - \min_{i \in \mathcal{R}_{b,s}(1)} \big(\lambda b^i - (1 - \alpha)\big(\ell^i(b) - \ell^i_*\big)\big). \tag{5}$$

The social planner will thus strive to include banks in the bail-in which offer a high contribution to the rescue consortium and generate low deadweight losses when they liquidate their outside assets to retrieve the contributed amount, i.e., banks for which $\lambda b^i - (1 - \alpha)\big(\ell^i(b) - \ell^i_*\big)$ is as large as possible. Which choice of b is optimal for the social planner depends on the value of α: if the recovery rate is high ($\alpha \geq 1/(1 + \lambda)$), the social planner prefers that banks liquidate their outside assets to buy up a larger amount of debt, whereas for

[13] When the bail-in consortium for the rescue of Long Term Capital Management was coordinated in the late 90s, Bear Stearns and Lehman Brothers rejected the bail-in proposed by the Federal Reserve Bank of New York. Their share of the bail-in was then redistributed among the remaining 14 banks.

low recovery rates ($\alpha < 1/(1+\lambda)$), the social planner prefers to buy more debt himself so as to avoid the liquidation of banks' outside assets. Observe that the maximum amount a bank $i \in \mathcal{C} \cup \mathcal{S}$ is willing/able to contribute without liquidating its outside asset is $\eta^i := \min(\zeta^i, (V_0^i - e^i)^+)$.

For a fixed set of banks $\mathcal{R} \subseteq \mathcal{C} \cup \mathcal{S}$, define the following bail-in plan that asks every bank in $\mathcal{R}$ to contribute the maximal incentive-compatible amount.

Definition 3.1. For a consortium $\mathcal{R} \subseteq \mathcal{C} \cup \mathcal{S}$, let $(b_{\mathcal{R}}, s_{\mathcal{R}})$ denote the subsidized bail-in defined by

$$b^i_{\mathcal{R}} = \begin{cases} \xi^i & \text{if } i \in \mathcal{R}, \alpha \geq \frac{1}{1+\lambda}, \\ \eta^i & \text{if } i \in \mathcal{R}, \alpha < \frac{1}{1+\lambda}, \\ 0 & \text{if } i \notin \mathcal{R}, \end{cases} \qquad s^i_{\mathcal{R}} = \begin{cases} 0 & \text{if } \alpha \geq \frac{1}{1+\lambda}, \\ \alpha \ell^i_* & \text{if } \alpha < \frac{1}{1+\lambda}, \end{cases}$$

for $i \in \mathcal{C} \cup \mathcal{S}$ and $b^0_{\mathcal{R}} = B + \alpha \big\| e^{\mathcal{F}} \big\|_1 1_{\{\alpha < 1/(1+\lambda)\}} - \big\| b^{\mathcal{C} \cup \mathcal{S}}_{\mathcal{R}} \big\|_1$. The net contribution of bank $i \in \mathcal{R}$ to the bail-in $(b_{\mathcal{R}}, s_{\mathcal{R}})$ increases social welfare by an amount

$$\nu^i := \begin{cases} \lambda \xi^i - (1-\alpha)\big(\hat{\ell}^i - \ell^i_*\big) & \text{if } \alpha \geq \frac{1}{1+\lambda}, \\ \lambda \eta^i & \text{if } \alpha < \frac{1}{1+\lambda}. \end{cases}$$

Let $w_{\mathcal{R}} := w_R(b_{\mathcal{R}}, s_{\mathcal{R}}, 1)$ denote the welfare losses after acceptance by all banks.

We are now ready to state our main result, which characterizes the optimal proposal of the social planner and its equilibrium welfare losses.

Theorem 3.2. *Let $i_1, \ldots, i_{|\mathcal{C} \cup \mathcal{S}|}$ be a non-increasing ordering of banks in $\mathcal{C} \cup \mathcal{S}$ according to ν^i so that $\nu^{i_1} \geq \nu^{i_2} \geq \ldots, \geq \nu^{i_{|\mathcal{C} \cup \mathcal{S}|}}$. If $w_P < w_N$, then the unique equilibrium outcome is a public bailout by the social planner. If $w_N \leq w_P$, then the unique equilibrium outcome is a subsidized bail-in with welfare losses amounting to*

$$w_* = \min\big(w_{\{i_1, \ldots, i_m\}}, w_N - \nu^{i_{m+1}}\big),$$

where $m := \min\big(k \mid w_{\{i_1, \ldots, i_k\}} < w_N\big)$ and $w_{\{i_1, \ldots, i_k\}}$ is the welfare loss associated with the rescue consortium $\{i_1, \ldots, i_k\}$ as in Definition 3.1.

The intuition behind Theorem 3.2 is the following. Because of the lower bound (5) on welfare losses that can be achieved in a bail-in, the social planner wishes to include banks in the bail-in consortium with a large potential for contribution. He will thus add banks to the bail-in consortium in the decreasing order $i_1, i_2, \ldots$, until the welfare losses become smaller than in the case of no intervention. This ensures that no bank has an incentive to decline the proposal: because $w_{\{i_1,\ldots,i_m\}} + \nu^{i_k} \geq w_N$ for any $k = 1, \ldots, m$, the social planner will choose not to cover bank i_k's part of the bail-in if bank i_k should reject it. A rejection thus leads to a default cascade, which is not profitable for bank i_k. By definition of m, a bail-in executed by the consortium $\{i_1, \ldots, i_m\}$ will always lead to lower welfare losses than not intervening. If the difference $w_N - w_{\{i_1,\ldots,i_m\}}$ is smaller than the possible contribution $\nu^{i_{m+1}}$ of bank i_{m+1}, the social planner can reduce the welfare loss by including bank i_{m+1} into the consortium and giving away subsidies in the amount $w_N - w_{\{i_1,\ldots,i_m\}}$ to banks outside of the consortium. This way, the social planner can ensure that no bank $i_1, \ldots, i_{m+1}$ has an incentive to deviate.

Remark 3.1. If the social planner had the power to commit to playing N in the third stage, the equilibrium outcome would improve to w_* even if $w_P < w_N$. Commitment power would thus improve social welfare by $(w_P - w_*)1_{\{w_P<w_N\}}$.

4 Credibility of the social planner's threat

In this section we identify conditions under which the government threat is credible. These results complement Theorem 3.2, which characterizes the equilibrium outcome of the game up to the credibility of the social planner's threat. The first subsection states conditions for a given network, illustrating how the credibility of the social planner's threat depends on the network topology and the recovery rates. In the second subsection, we compare the credibility of the social planner's threat between two networks. Because the credibility depends on the network topology in a highly nonlinear way, we provide a comparison between two specific network structures that leads to analytically tractable results. We choose the ring network, as a representative structure for sparsely connected networks, and the complete network in representation of densely connected networks.

4.1 Absolute credibility analysis

The main component of welfare losses in a public bailout is the aggregate shortfall B of fundamentally defaulting banks. It can be understood as a measure of the size of the exogenous shock hitting the financial system. The welfare losses in a default cascade w_N, on the other hand, is a measure of the shock size after the shock propagates through the financial system. A relevant determinant for the credibility of the social planner's threat is thus the amplification of the shock through the interbank network. For this analysis, we split the shortfall B into the two components $\left\|\delta^{\mathcal{D}}\right\|_1$ and $B - \left\|\delta^{\mathcal{D}}\right\|_1$. Note that only the latter component is amplified through the system as the former component hits the depositors of the banks. We define the size of the shock after the amplification as $\left\|V_0^{\mathcal{C}\cup\mathcal{S}} - V_N^{\mathcal{C}\cup\mathcal{S}}\right\|_1 + (1-\alpha)\left\|e^{\mathcal{F}}\right\|_1$, the sum of aggregate losses of non-fundamentally defaulting banks and the liquidation losses of the fundamentally defaulting banks. Our first result states that the social planner's threat is credible if and only if the amplification of the shock without intervention is smaller than a certain threshold.

Lemma 4.1. *Let* $\chi := \left\|V_0^{\mathcal{C}\cup\mathcal{S}} - V_N^{\mathcal{C}\cup\mathcal{S}}\right\|_1 + (1-\alpha)\left\|e^{\mathcal{F}}\right\|_1 - \left(B - \left\|\delta^{\mathcal{D}}\right\|_1\right)$ *denote the amplification of the shock through the interbank network. The social planner's threat is credible and* $w_N \leq w_P$ *if and only if*

$$\chi \leq \lambda\left(B - \left\|\delta^{\mathcal{D}}\right\|_1\right) + \min(\lambda\alpha, 1-\alpha)\|\ell_*\|_1. \tag{6}$$

The amplification of the shock depends only on the financial network and it is independent of the social planner's preference parameter λ. Lemma 4.1 thus states that the social planner's threat becomes more credible if he assigns a larger weight λ to taxpayers' and depositors' money. Indeed, as λ increases, a bailout is perceived as more costly and the threat to not bail out the fundamentally defaulting banks is more credible.

The amplification of the shock can be decomposed into two components: The first component measures how the initial shock $B - \|\delta^{\mathcal{D}}\|_1$ spreads through the network and causes losses in interbank assets in the size of $\|\xi^{\mathcal{C}\cup\mathcal{S}}\|_1$. The second component consists of the deadweight losses $(1-\alpha)\|\hat{\ell}\|_1$ associated with the liquidation of outside assets after the banks' interbank assets have been hit by the shock ξ. The next lemma describes the first component and how it depends on the network topology.

Lemma 4.2. *In any financial system, the aggregate amplification due to network effects equals* $\|\xi^{\mathcal{C}\cup\mathcal{S}}\|_1 - \left(B - \|\delta^{\mathcal{D}}\|_1\right) = (1-\beta)\|(\pi\hat{p})^{\mathcal{D}}\|_1$. *Moreover, if* $\beta < 1$, *then for any set* $S \subseteq \mathcal{C}\cup\mathcal{S}$ *of banks, we have*

$$\zeta^S := \pi^{S,\mathcal{D}}\left(I - \beta\pi^{\mathcal{D},\mathcal{D}}\right)^{-1}\left((1-\alpha)e^{\mathcal{D}} + (1-\beta)A^{\mathcal{D}} - V_0^{\mathcal{D}} + \delta^{\mathcal{D}}\right). \tag{7}$$

The first statement of Lemma 4.2 characterizes the total size of the amplification due to network effects and it gives an easy interpretation of the second term in (1). The second statement helps in obtaining a qualitative understanding for the role of the network topology in the shock's amplification. If $\beta < 1$, one can solve for ζ explicitly as in (7) and obtain direct interpretations for the contributing terms. It follows from the definition of B that $\sum_{i\in\mathcal{D}}(1-\alpha)e^i - V_0^i = B - \|V_0^{\mathcal{C}} - (1-\alpha)e^{\mathcal{C}}\|_1$. Equation (7) can thus be understood as follows:

1. The initial shock B is increased by the bankruptcy costs $(1-\beta)\|A^{\mathcal{D}}\|_1$ and the depositors' losses $\|\delta^{\mathcal{D}}\|_1$, and it is dampened by the available equity that banks in $\mathcal{C}$ have after liquidating their outside assets, given by $\|V_0^{\mathcal{C}} - (1-\alpha)e^{\mathcal{C}}\|_1$.

2. The shock is non-linearly amplified by the Leontief matrix $(I - \beta\pi^{\mathcal{D},\mathcal{D}})^{-1}$ of the subnetwork of defaulting banks $\pi^{\mathcal{D},\mathcal{D}}$. A high density of liabilities between banks in $\mathcal{D}$ and a low value of β make this amplification large.

3. The shock is dispersed among banks in S according to $\pi^{S,\mathcal{D}}$. For $S \subseteq \mathcal{S}$, a more diversified distribution of liabilities from defaulting to solvent banks reduces deadweight losses from inefficient liquidation of outside assets.

If the interbank asset recovery rate β is close to 1, the amplification effects due to the network are small by the first statement of Lemma 4.2. Therefore, the main component of the amplification is due to inefficient liquidation of the outside assets. The following lemma provides a sufficient condition for the social planner's threat to be credible: if the recovery rate α is sufficiently high ($\alpha > 1/(1+\lambda)$) or the value of banks' outside assets make up a sufficiently small proportion of the total value, then the threat is credible for all interbank recovery rates β above some threshold.

Lemma 4.3. *Let $\mathcal{B} := \{i \in \mathcal{S} \mid (1+\lambda)(1-\alpha)e^i > \lambda V_0^i\}$ denote the set of solvent banks, whose value of outside assets accounts for a fraction larger than $\frac{\lambda}{(1+\lambda)(1-\alpha)}$ of its total value. Suppose that*

$$\big(1-(1+\lambda)\alpha\big)\big\|e^{\mathcal{D}\cup\mathcal{B}}\big\|_1 < \lambda \sum_{i\in\mathcal{B}\cup\mathcal{C}} (c^i + A^i - L^i). \tag{8}$$

Then there exists $\beta^* < 1$ such that for all $\beta \geq \beta^*$, the government threat is credible.

The complement of the set $\mathcal{B}$ consists of the banks which satisfy (8) individually. The sufficient condition in Lemma 4.3 can thus be rephrased as follows: All solvent banks which do not satisfy (8) individually have to satisfy (8) in aggregate, when combined with the defaulting banks.

4.2 Relative credibility analysis

In this section, we compare the credibility of the threat between two network topologies and study how this affects the resulting welfare losses. Our first lemma states that equilibrium welfare losses are always lower in networks, in which the social planner's threat is credible.

Lemma 4.4. *For fixed L, c, e, α, β, the equilibrium welfare losses after intervention are smaller in network π_1 than in network π_2 if the social planer's threat is credible in network π_1 but not in network π_2.*

Indeed, since the social planner's threat is credible in network π_1 but not in network π_2, Theorem 3.2 implies that $w_{*,1} < w_{N,1} \leq w_P = w_{*,2}$. The next result states that the difference between no-intervention losses and losses in a public bailout are monotonic in the asset recovery rates α and β. As a consequence, for fixed β and B, there exists a unique threshold value α^* such that the social planner's threat is credible for all $\alpha > \alpha^*$. A comparison of credibility between two networks can thus be obtained by comparing the respective values for α^*.[14]

Lemma 4.5. *For any financial system (L, π, c, e), the quantity $w_N - w_P$ is monotonically decreasing in α and β.*

Definition 4.1.

1. We say that the social planner's threat is *more credible* in network π_1 than in network π_2 for fixed values of β and B if $\alpha_1^* < \alpha_2^*$.
2. We say that the social planner's threat is *uniformly more credible* in network π_1 than in network π_2 for fixed values of B if, for every pair (α, β) such that the social planner's threat is credible in π_2, the threat is also credible in π_1.

To obtain a more quantitative comparison, we choose two specific topologies, which are representative of sparsely and densely connected networks, respectively. We consider the ring and the complete network in our analysis, which is a standard choice in the literature; see also Acemoglu, Ozdaglar and Tahbaz-Salehi (2015). We will assume that aside from the network topology, all parameters are identical so that the difference in the credibility really stems from the pattern of interconnectedness. Specifically, we consider a financial network with n banks such that after the arrival of the shock, there is precisely one fundamentally defaulting bank, there are n_l lowly capitalized banks with outside assets e_l and n_h highly capitalized banks with outside assets $e_h > e_l$.[15]

[14] A similar comparison could be obtained by comparing β^*, the smallest value of the interbank recovery rate such that, for fixed α and B, the social planner's threat is credible for any $\beta > \beta^*$.

[15] The assumption that there is precisely one fundamentally defaulting bank is not crucial for our results, but greatly simplifies the presentation of our results.

Without loss of generality, we assume that bank 1 is the fundamentally defaulting bank, characterized by the value of its outside asset e^1 and the bailout cost B, which implicitly determine bank 1's net cash balance c^1. The value of its outside assets e^1 may be different from e_l and e_h. We assume further that all non-fundamentally defaulting banks have an identical net cash balance, that is, $c^i = c$ for every $i \neq 1$ and some constant c, and that all banks have the same total interbank liability, that is, $L^i = L$ for every bank i and some constant L. We denote by $V_{0,l}$ and $V_{0,h}$ the initial value of a lowly and a highly capitalized bank, respectively.

In the complete network π_C, every bank is equally liable to every other bank in the system, i.e., $\pi_C^{ij} = \frac{1}{n-1}$ for every pair i, j. Interbank liabilities are thus maximally diversified. In a ring network π_R, each bank is liable to exactly one other bank so that $\pi_R^{ij} = 1$ if and only if $i = j+1$ modulo n. While the complete network is unique, the ring network depends on the labeling of banks. If bank 2 is a highly capitalized bank, it can absorb a larger part of the shock and the resulting welfare losses will be smaller than if bank 2 is a lowly capitalized bank. From an ex-post perspective, the best-possible ring network is the network where banks $2, \ldots, n_h + 1$ are highly capitalized and the worst-possible ring network is the ring where banks $2, \ldots, n_l + 1$ are lowly capitalized. In this comparison, we will focus on the latter network, where all lowly capitalized banks are hit before the highly capitalized banks. We find that even the worst-possible ring network may outperform the complete network. See panels (b) and (c) in Figure 2 for a graphical representation of the two networks.

Let B_* and B^* denote the thresholds above which all lowly capitalized banks and all highly capitalized banks default in the complete network, respectively. Our next result states that for small shocks $B \leq B^*$ or very large shocks $B > B^*$, the social planner's threat is uniformly more credible in one network over the other.

Proposition 4.6. *Fix a shortfall B and suppose that $L \geq \frac{1+\rho}{\beta} B$, where $\rho = n_l/n_h$. Then the social planner's threat is uniformly more credible in the complete network if $0 \leq B \leq B_*$ and it is uniformly more credible in the ring network if $B > B^*$.*

The condition $L \geq \frac{1+\rho}{\beta} B$ simply guarantees that interbank liabilities are sufficiently large for the shortfall B to propagate through the system in its entirety. If interbank liabilities were lower, the effects of an increased shortfall B would be felt by the senior creditors but not by the interbank system, thereby not affecting the credibility of the social planner's threat. Note that the second statement is not vacuous: as illustrated in the examples of the next section, larger shocks may be necessary to cause a systemic default in the ring network than in the complete network.

Proposition 4.6 states that the relative credibility of the social planner's threat does not depend on the game parameters if the shock size is sufficiently small or sufficiently large. This is different for intermediate shock sizes, where the interbank recovery rate β and the size of interbank liabilities L are crucial determinants for the credibility of the threat. We perform a comparative statics analysis for the credibility of the threat along these two dimensions. In both cases, the intuition is the same: if the bankruptcy costs $(1-\beta)L$ are sufficiently large, the threat is more credible in the ring network.

Proposition 4.7. *For $\beta = 1$, there exists L' such that for any $L \geq L'$, the social planner's threat is more credible in the complete network than in the ring network for any $B \in (B_*, B^*]$. For any $\beta < 1$, there exists L^* such that for any $L \geq L^*$, the social planner's threat is more credible in the ring network for any $B \in (B_*, B^*]$.*

Proposition 4.7 states that unless interbank assets are recovered in full, the social planner's threat is more credible in the ring network for sufficiently large interbank liabilities. The next proposition shows that if we fix the size of interbank liabilities, the credibility threshold rises from B_* to B^* as the recovery rate increases from a small value to $\beta = 1$.

Proposition 4.8. *Suppose that $\beta^{n_l} < \rho$ and $\frac{c+\alpha e_l}{c+\alpha e_h} > \frac{1}{n_h}$ and let $L \geq \frac{1+\rho}{\beta} B^*$. Then there exist $B'(\beta)$ and $B''(\beta)$ with $B_* \leq B' \leq B'' \leq B^*$ such that the following statements hold:*

1. *The threat is more credible in the complete network for any $B < B'$.*

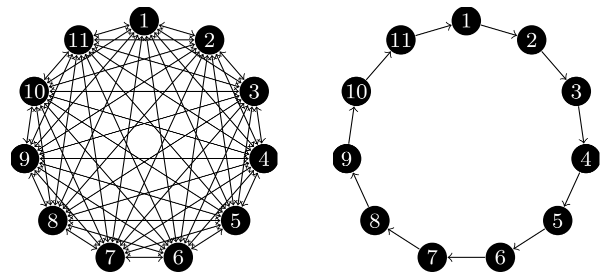
2. *The threat is more credible in the ring network for any $B > B''$.*

3. *B' and B'' are monotonically increasing in β with $B'(1) = B''(1) = B^*$.*

The condition $\beta^{n_l} < \rho$ requires that either the number n_l of lowly capitalized banks in the system is not too small or that the interbank recovery rate is not too high. The condition $\frac{c+\alpha e_l}{c+\alpha e_h} > \frac{1}{n_h}$ says that the capital of lowly capitalized banks cannot be too small compared to that of highly capitalized banks. If either condition is violated, the amplification is small in both networks and there may not be a single phase transition. If the conditions are met, there is a nonnegligible amplification of the shock and a phase transition is observed. While it is possible that $B' = B''$, this is not generally true. In the ring network, defaults occur sequentially and the amplification of the shock exhibits discontinuities at shock sizes B_i, for which bank i defaults. As a result, the more credible network may switch from the ring network for $B \leq B_i$ to the complete network for $B > B_i$ at these discontinuities. Proposition 4.8 states that any such reversions to the complete network being the more credible network are local phenomena due to the unequal distribution of the shock's amplification over the interval $(B_i, B_{i+1}]$. As the shock grows larger, these local effects are dominated by the negative feedback effects of dense interconnections among defaulting banks, making the ring network the more credible network for $B > B''$. We may interpret the region $B' < B \leq B''$ as the region where the threat is approximately equally credible in both networks, whereas the complete network is the more credible for $B < B'$.

Comparing our results to Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), we find that imperfect asset recovery and intervention policies promote sparsely connected networks. While in their model, the complete network is socially preferable for any $B < B^*$, our analysis shows that for shock sizes $B \in (B'', B^*]$, more credible bail-in policies are available in the ring network. Moreover, Theorem 3.2 shows that an individual bank's contribution is larger in the ring network, suggesting that the ring network may be socially preferable where the threat is credible in both networks, even if welfare losses are lower in the complete network without intervention. We illustrate this by means of two

Bank	L	c	e
1	1	-1	0.3
$2, \ldots, 6$	1	0	0.1
$7, \ldots, 11$	1	0	0.8



(a) Banks' assets and liabilities. **(b)** The complete network. **(c)** The ring network.

Figure 2: We compare the densely connected complete network (b) to the sparsely connected ring network (c) for banks with assets and liabilities given in (a). Bank 1 is insolvent, banks $2, \ldots, 6$ are lowly capitalized, and banks $7, \ldots, 11$ are highly capitalized.

examples in the next section.

If we consider an ex-ante perspective where there is uncertainty regarding which bank is going to be hit by a shock, then Proposition 4.8 compares the complete network to the ring architecture for which the shock has the worst impact on the system. Any other ring architecture performs better than the configuration considered. Proposition 4.8 thus provides a lower bound for the credibility of the social planner's threat in a ring network.

5 The impact of network topology on welfare losses

In this section we present numerical examples to illustrate the results stated in our Theorem 3.2 and Propositions 4.6–4.8. In both examples, we compare the ring network to the complete network in a financial system of $n = 11$ banks with $\lambda = 1$. An exogenous shock renders bank 1 insolvent and divides the remaining banks into two types: lowly capitalized banks $2, \ldots, 6$ and highly capitalized banks $7, \ldots, 11$. We focus mainly on the comparison between the complete network C and the ring network R, where all lowly capitalized banks are hit before any of the highly capitalized banks. In Example 1 we will also briefly discuss the ring network R', where highly capitalized banks are hit first. Note that the ring architecture R is the one treated in Propositions 4.6–4.8.

Example 1. Consider a financial network, where banks' assets and liabilities after the shock are given in the left panel of Figure 2. For the sake of presen-

Table 1: Shown are the clearing payments, the number of defaulting banks and the welfare losses under no intervention, as well as the equilibrium welfare loss for the three networks.

Network	$\hat{p}$	$\lvert\mathcal{D}\rvert$	w_N	w_*
Complete	$(0.05, 0.83, 0.83, 0.83, 0.83, 0.83, 1.00, 1.00, 1.00, 1.00, 1.00)$	6	1.01	0.85
Ring R	$(0.13, 0.19, 0.24, 0.29, 0.34, 0.38, 0.94, 1.00, 1.00, 1.00, 1.00)$	7	0.68	0.65
Ring R'	$(0.13, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 0.71)$	2	0.48	0.26

tation, all interbank liabilities are normalized to 1.[16] In this system, bank 1 is the only fundamentally defaulting bank with a bailout cost of $B = 0.775$. Outside investments can be recovered at $\alpha = 75\%$ of their face value, whereas interbank liabilities are recovered almost in full at $\beta = 90\%$. The three networks considered have relative liability matrices given by $\pi_C^{ij} = \frac{1}{n-1} = 0.1$ for every $i \neq j$, $\pi_R^{ij} = 1$ if and only if $i = j + 1$ (modulo n) and $\pi_{R'}^{ij} = 1$ if and only if $i = j - 1$ (modulo n).

Observe first that a public bailout is independent of the network topology. In all three networks, a public bailout leads to a welfare loss of $w_P = B + (1 - \alpha)e^1 = 0.85$. The no-intervention outcomes of the three networks are summarized in Table 1, together with the equilibrium welfare losses after intervention. In the complete network, the shock is spread equally among all banks. This causes the default of lowly capitalized banks, but leaves the highly capitalized banks solvent. In order to remain solvent, however, each highly capitalized bank i has to liquidate an amount $\hat{\ell}^i = 0.24$ of its outside assets. This leads to a welfare loss of 1.01 without intervention, which renders the government threat of no intervention non-credible. As a result, the equilibrium outcome is a public bailout with a welfare loss of 0.85.

In the ring network, the no-intervention outcome depends heavily on the configuration. If the lowly capitalized banks are hit first, the shock tears through their outside assets fairly quickly, leading to their defaults as well

[16] In comparing ring and complete network, we are assuming that banks are charging the same interest rates to their creditors under both ring and complete network. In reality, the interest rates charged would be based on the banks' beliefs about the probability of bankruptcy, which in turn dependent on the topology of the network and the degree to which it amplifies losses. Greenwald and Stiglitz (2003), see Chapter 7 therein, analyze this situation in a model of credit interlinkages consisting of three banks. They show the existence of multiple equilibria, in one of which bad outcomes lead to a systemic default, and another in which systemic defaults do not occur.

as the default of the highly capitalized bank 7. Even though there is one additional default when compared to the complete network, the welfare losses without intervention are smaller in the ring R: because of the low capitalization of banks $2, \ldots, 6$, the resulting liquidation losses are small, and because of the linear network structure, there are no negative feedback effects that cause a decrease in the clearing payments. Even smaller are the welfare losses in the ring configuration R', where only one highly capitalized bank defaults. For both configurations, the social planner's threat is credible and hence a subsidized bail-in is formed according to Theorem 3.2. The highest-possible incentive-compatible contributions come from the direct creditors of bank 1. In the network R, banks 2–5 are each willing to contribute an amount $\nu^i = 0.05$ to a subsidized bail-in, leading to welfare loss of 0.65 after a bail-in. In the network R', banks 10 and 11 contribute $\nu^{10} = 0.19$ and $\nu^{11} = 0.4$, respectively. The resulting welfare loss is 0.26.

Example 2. Consider the same financial network as in Example 1 with the exception that lowly capitalized banks have a higher amount of outside assets, equal to $e_l = 0.5$. In the complete network, there are no contagious defaults even when there is no intervention. The deadweight losses in the no-intervention case are equal to $w_{N,C} = 0.4667$. In comparison, the ring R leads to the defaults of banks $1, \ldots, 3$ with a welfare loss of $w_{N,R} = 0.5483$ under no intervention. Nevertheless, the ring network outperforms the complete network under the optimal subsidized bail-in strategy: The maximal incentive-compatible contribution of any bank $i = 2, \ldots, 11$ in the complete network is $\xi^i - (1-\alpha)\big(\hat{\ell}^i - \ell^i_*\big) = 0.0583$. The optimal bail-in in the complete network thus involves any seven of the ten banks and results in a welfare loss of $w_{*,C} = 0.4417$. The optimal subsidized bail-in in the ring network involves only banks 2 and 3, but each of them is willing to contribute 0.25, leading to a welfare loss of $w_{*,R} = 0.35$. Because these contributions are much larger than the contribution of any individual bank in the complete network, the resulting welfare losses in a bail-in are smaller than in the complete network, despite the fact that the welfare losses without intervention are smaller in the complete network. This is illustrated graphically in Figure 3.

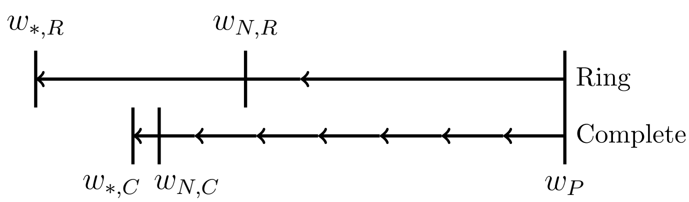
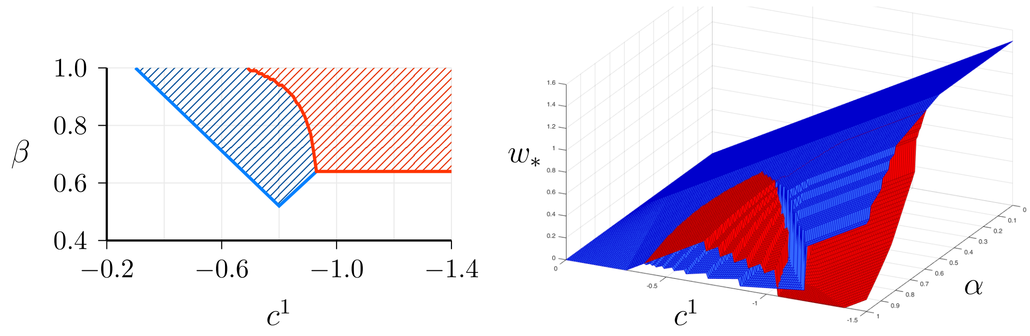


Figure 3: Welfare losses under public bailout (w_P), subsidized bail-in ($w_{*,R}, w_{*,C}$) and without intervention ($w_{N,R}, w_{N,C}$) for the two networks in Example 2. Each arrow indicates the reduction of welfare losses by the incentive compatible contribution of an individual bank.

Observe that in both examples, the systemic default is triggered by a smaller shock in the complete network than in the ring network. In Example 1, all banks default in the complete network for shortfalls larger than $B^* = 3.075$, whereas it takes a shortfall as large as $B_n = 5.7137$ for the last bank in the ring network to default. In the second example, $B^* = 4.575$ and $B_n = 7.7942$. This shows how large the potential for amplification is in a network with dense connections, even at an interbank recovery rate as high as 90%. We re-iterate the two trade-offs between densely and sparsely connected networks that are illustrated by these examples:

1. Even without any methods of intervention, a higher density of the interbank network does not necessarily lead to a reduction of welfare losses. If all banks have a reasonably high level of capitalization as in Example 2, densely connected networks have a potential for absorption of the shock if the shock size is small. If, however, the level of capitalization is low for a large enough fraction of the system as in Example 1, then the dense connections of the defaulting banks cause a negative feedback effect on the available capital for repayments, resulting in an amplification of the shock.

2. In sparsely connected networks, each immediate creditor of the fundamentally defaulting banks is hit by a large part of the shock. The threat of not intervening is thus very severe, creating incentives to contribute large amounts to a bail-in. Figure 3 illustrates that this may lead to lower equilibrium welfare losses, even if the more densely connected network was preferable without intervention.



(a) Comparison of credibility.

(b) Equilibrium welfare losses.

Figure 4: The left panel illustrates the regions in the (c^1, β)-plane, where the social planner's threat is more credible in the ring (red) and the complete network (blue), respectively. The border between the two regions is characterized by $B(c^1) = B_*(\beta)$, that is, the more credible network switches at B_* in this case. The threat fails to be credible in either network outside of these regions. The right panel shows equilibrium welfare losses for different recovery rates α and different net cash holdings c^1. The network with lower welfare losses is socially preferable. The "steps" illustrate the contributions of banks to the bail-in.

We conclude this section by showing that these findings are robust to changes in the interbank recovery rate β and the size of the ex-post net cash holdings c^1 of the fundamentally defaulting bank. All other parameters are taken as in Example 1. A negative value of c^1 signifies a net senior debt and $-c^1$ can be interpreted as a measure of the size of the shock.[17] Figure 4a shows the areas in the (c^1, β)-plane, where the social planner's threat is more credible in the ring and the complete network, respectively. For the chosen parameters in Example 1, there is a single threshold, where the network with higher credibility switches from the complete to the ring topology.

Figure 4b compares the equilibrium welfare losses in the two networks for a fixed value $\beta = 0.9$ and different values of α and c^1. If the recovery rate α is low, the threat fails to be credible in either network and a public bailout is the only option. For high recovery rates, the threat is credible and a subsidized bail-in can be organized. The steps indicate the contributions of banks to the subsidized bail-in. It is clearly visible that the size of the contributions are much larger in the ring network: we observe 5 small steps, indicating the contributions of lowly capitalized banks to the subsidized bail-in, followed by

[17] In financial systems with $A^1 = L^1$, such as the systems in Examples 1 and 2, the bailout cost is of the form $B = -c^1 - \alpha e^1$. Thus, $-c^1$ is simply a reparametrization of the bailout cost B that is independent of α.

one large step, the contribution of the first highly capitalized bank in the ring network. For high recovery rates, banks can be incentivized to form a privately-backed bail-in without any contribution of the social planner. In the complete network, banks can be motivated to contribute to a bail-in for smaller shocks than in the ring network, because the threat is more credible for small shocks. The size of the contributions, however, is small.

6 Concluding Remarks

Government support of financial institutions designated to be too big or too important to fail has been costly during the global financial crisis. Various initiatives have been undertaken by central governments and monetary authorities to expand resolution plans and tools. We provide a tractable framework for analyzing the economic forces behind the determination of the two most common default resolution procedures, bail-ins and bailouts. Our analysis indicates that the interplay of network topology, shock sizes, and liquidation costs play a critical role in determining the optimal rescue plan: when the government's threat of no intervention switches from being credible to non-credible, the equilibrium rescue switches from a subsidized bail-in plan to a public bailout. If the amplification of the shock is large due to high liquidation costs or a high degree of interconnectivity in the network, the social planner cannot credibly threaten the banks not to intervene himself and a public bailout is the only incentive-compatible rescue option. If the threat is credible, we find that the optimal intervention plan is a subsidized bail-in with contributions from both the government and the creditors of the defaulting banks. This discontinuity in the equilibrium rescue plan may have severe implications on the banks' ex-ante behavior, leading to important future research questions as discussed in the next section.

While the current analysis indicates that, regardless of the network structure, the threat is credible if the recovery rates on external and interbanking assets of a defaulted institution are high enough, the network structure plays an important role if recovery rates are low. We show that the no-intervention threat switches from being more credible in the complete network to being more credible in the ring network as the magnitude of the initial shock increases. Together with the fact that the creditors of fundamentally defaulting banks are willing to contribute a larger amount to rescue insolvent banks in a more sparsely connected network, this leads to the surprising finding that even in the "small shock regime" of Acemoglu, Ozdaglar and Tahbaz-Salehi (2015)[18], sparse networks may lead to lower welfare losses. This happens despite the fact that welfare losses are lower in a more densely connected network in the absence of intervention.

7 Future Research

In the present study, we characterize the equilibrium bail-in consortium that arises after the realization of an exogenous shock. By assuming a prior distribution over shock sizes, our analysis can be extended in a straightforward manner to identify the ex-ante preferred network topology from the social planner's perspective. However, since it is the banks which choose the network topology, a more important (but more difficult) extension is to analyze how the equilibrium bail-in procedure affects the banks' trading and investment decisions. By anticipating which bail-in consortia are credible for which network topologies, banks choose their counterparties so that the resulting interbank network minimizes their ex-ante expected contributions to the equilibrium bail-in plan. This adds an important layer to the moral hazard literature: in addition to maximizing the value of their bailout option through excessive risk taking, banks can affect the likelihood of a government bailout in bad states of the world through their interbank lending decisions. While the collective moral hazard problem of banks being "too correlated to fail" has been thoroughly investigated in the literature, many of these studies, with the exception of a recent study by Erol (2016), do not account for the prominent role played

[18]In Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), the "small shock regime" is the range of shocks for which at least one bank survives. The "large shock regime", instead, is where all banks default. Acemoglu, Ozdaglar and Tahbaz-Salehi (2015) restrict their attention to regular interbank networks, where every banks has identical interbank and outside assets.

by the interbank network structure.[19] The government would initially commit to a regulatory structure by imposing structural constraints on the set of banks' interlinkages (e.g., restricting the number of possible counterparties or by specifying exposure limits). Such an endogenous network formation model would inform the design of structural policies aimed at preventing banks from reaching a network architecture, in which the government threat fails to be credible. Existing models for optimal regulatory structures do not take into account the possibility of bailouts or of credible bail-in policies.

Our focus has been on a specific form of network interaction among financial institutions; namely, the spread of counterparty risk via unsecured debt contracts. In practice, financial institutions are exposed to counterparty risk via a larger class of contracts, including credit derivatives. Collateral posting can be used to mitigate counterparty credit risk, but the risk cannot be fully eliminated because of the wrong-way risk, which occurs when the joint default risk of the reference entity and the counterparty leads to a substantial upward jump in the price of the contract. When a bank defaults, even institutions which have not directly traded with the defaulting bank may experience losses if they happen to have sold protection on that bank. Under adverse circumstances, the payments that they have to make to the protection buyers may be high and possibly wipe out the entire equity capital of the bank. Roukny, Battiston and Stiglitz (2017) consider a financial network with partially collateralized over-the-counter credit contracts, which determine the recovery rate for the lender after the borrower defaults.[20] The construction of a framework with intervention accounting for this class of derivatives would result in a richer framework, capturing the full set of interbank trading activities.

[19] Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) show that banks may find it optimal to invest in highly correlated assets, in anticipation of a bailout triggered by the occurrence of many simultaneous failures. Keister (2016) finds that bailouts lead intermediaries to excessively invest in illiquid assets, but prohibiting them lowers aggregate welfare. Chari and Kehoe (2016) show that if the social planner cannot commit to avoid bailouts ex-post, then overborrowing may happen ex-ante. Acharya, Shin and Yorulmazer (2011) study how policy interventions targeting the resolution of bank failures affect the ex-ante choice of banks to hold liquid assets. Jeanne and Korinek (2017) contrast the merits of bailouts vs. regulations.

[20] Roukny, Battiston and Stiglitz (2017) show that in presence of cyclical credit relations, there exist a range of external shocks under which multiple equilibria coexist, including one in which no bank defaults and another in which all banks default. They investigate how such an equilibrium uncertainty depends on leverage, volatility, network topology, and correlation across shocks hitting external banks' assets.

Another interesting research avenue is to study the nature of the equilibrium of the dynamic regulatory game arising in a contestable democracy. Two rival parties may have different preferences over intervention plans and thus implement different strategies when they are in power. A conservative political regime, for example, will put higher weight on taxpayers' money than a more capitalist regime, which views taxpayer contributions as a mere redistribution of wealth. Since the capitalist regime has a lower loss threshold for the implementation of a public bailout relative to the conservative regime, it will implement a distorted policy when it is in power. The capitalist regime will loosen regulations and promote risk taking activities beyond the level that it would set if it were in power permanently. By doing so, the capitalist regime will the force the hands of the conservative regime, making it much more difficult for the latter to implement private bail-ins when it takes office. In other words, the capitalist regime is imposing externalities on the conservative regime, forcing it to operate closer to the capitalist regime and further away from how the conservative regime would have acted if it were the only governing party.[21]

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[21] Korinek and Stiglitz (2009) study the non-cooperative equilibrium of a dynamic political game in a contestable democracy with two rival parties. These parties extract different utility from macroeconomic outcomes and implement different dividend tax policies when they are in power. Under a conservative regime, when dividend taxes are low, firms pay out higher dividends while taxes are still low in anticipation that the next move will be a tax increase. Under the social democratic rule, by contrast, firms expect that the next change in dividend taxes will be a cut; they reduce dividend payments so as to postpone some of their distributions until after the expected dividend tax cut. Such an action raises investment and output.

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A Proof of Theorem 3.2

We first prove the following preliminary result for the proof of Lemma 3.1.

Lemma A.1. *Without intervention, the equity value of each bank is* $V_N^i = 0$ *for* $i \in \mathcal{F}$ *and* $V_N^i = V_0^i - \xi^i - (1-\alpha)\hat{\ell}^i$ *for* $i \in \mathcal{C} \cup \mathcal{S}$*, where* $\hat{\ell}^i = \ell^i(\xi)$*.*

Proof. Observe that $\xi^i = \zeta^i$ for $i \in \mathcal{S}$ and $\xi^i = V_0^i - (1-\alpha)e^i$ for $i \in \mathcal{C}$. We distinguish the two cases. Consider first a bank $i \in \mathcal{S}$, liquidating an amount $\hat{\ell}^i = \frac{1}{\alpha}\big(L^i - z^i - (\pi\hat{p})^i\big)^+ = \frac{1}{\alpha}\big(e^i - V_0^i + A^i - (\pi\hat{p})\big)$. The equality $\hat{\ell}^i = \ell^i(\zeta)$ thus follows from $\zeta = \pi(L-\hat{p}) = A - \pi\hat{p}$. Since $\hat{p}^i = L^i$ for $i \in \mathcal{S}$, we obtain

$$V^i(\hat{\ell}, \hat{p}) = V_0^i - A^i + (\pi\hat{p})^i - (1-\alpha)\hat{\ell}^i = V_0^i - \zeta^i - (1-\alpha)\ell^i(\zeta).$$

A bank $i \in \mathcal{C}$ liquidates $e^i = \ell^i\big(V_0 - (1-\alpha)e\big)$ and has a value of

$$V_0^i - \big(V_0^i - (1-\alpha)e^i\big) - (1-\alpha)e^i = 0. \qquad \square$$

Proof of Lemma 3.1. Suppose first that $w_P < w_N$. If $s^i - b^i \geq \alpha\ell_*^i 1_{\{\alpha<1/(1+\lambda)\}}$, then accepting the proposal is better for bank i than even a public bailout. It is thus both optimal to accept the proposal and suboptimal to refuse it. If $0 \leq s^i - b^i < \alpha\ell_*^i 1_{\{\alpha<1/(1+\lambda)\}}$ and Condition 1.(b) is satisfied for bank i, then bank i's action will determine between a bail-in of banks in $\{j \mid a^j = 1\}$ and a bail-in of banks in $\{j \neq i \mid a^j = 1\}$. Because $0 \leq s^i - b^i$, bank i prefers to be part of the bail-in.

For the converse, suppose towards a contradiction that a is an equilibrium response to (b, s) with $a^i = 1$ for some banks i but that Condition 1 is violated for bank i. That is, either $s^i - b^i < 0$ or $0 \leq s^i - b^i < \alpha\ell_*^i 1_{\{\alpha<1/(1+\lambda)\}}$

and Condition 1.(b) is violated. In the former case, bank i is worse off than in both a public bailout or in a bail-in by $\{j \neq i \mid a^j = 1\}$. Rejecting the proposal is thus a profitable deviation of bank i. In the latter case, a unilateral deviation of bank i will lead to a public bailout as that is the social planner's preferred outcome. Therefore, bank i should not have accepted the proposal as a deviation leads to the higher value $V_P^i = V_0^i + \alpha \ell_*^i$.

If $w_P \geq w_N$, the social planner will not pay for a public bailout, regardless of any bank's action. For some bank i, the alternatives to accepting the proposal are thus (a) a rescue by other banks or (b) a default cascade. If $s^i \geq b^i$, then the bail-in is a net gain for bank i and hence a better outcome than any of its alternatives. It is thus optimal to accept the proposal. If $s^i < b^i$ instead, then Condition 2.(b) implies that a deviation of player i would lead to a default cascade. By Condition 2.(a), this is worse for bank i than participating in the bail-in. Again, it is optimal to accept the proposal.

For the converse, suppose towards a contradiction that a is an equilibrium response to (b, s) with $a^i = 1$ for some i but that Condition 2 is violated for bank i. That is, either 2.(a) holds, but $b^i - s^i > \xi^i$, or $0 < b^i - s^i \leq \xi^i$ and 2.(a) is violated. In the first case where $b^i - s^i > \xi^i$, being part of the bail-in leads to a lower value for bank i than it receives in the default cascade. Rejecting the proposal is thus a profitable deviation, which means that a is not an equilibrium response. If 2.(a) is violated, then a unilateral deviation of bank i will result in a bail-in by the banks in $\{j \neq i \mid a^j = 1\}$, where the social planner buys up bank i's part of the debt. Since $0 < b^i - s^i$, this deviation is profitable for player i, contradicting the fact that a is an equilibrium response. □

The following lemma formalizes that the social planner cannot gain anything by proposing bail-in plans that will be rejected by any bank. The lemma also rules out the necessity of proposals that award a direct subsidy to banks which have to buy up a positive amount of debt.

Lemma A.2. *Suppose that $w_P \geq w_N$. For any proposed bailout (b, s) with equilibrium response a, there exists a proposal $(\tilde{b}, \tilde{s})$ with $w\big(\tilde{b}, \tilde{s}, 1\big) = w(b, s, a)$ and $\tilde{b}^i \tilde{s}^i = 0$ for every bank i such that 1 is an equilibrium response to $(\tilde{b}, \tilde{s})$.*

Proof. Let $\mathcal{A} := \{i \in \mathcal{C} \cup \mathcal{S} \mid a^i = 0\}$ denote the set of banks, for which the equilibrium response to (b, s) is to reject the proposal. We define the modified bail-in proposal $(\tilde{b}, \tilde{s})$ as follows: For any bank $i \in \mathcal{A}$, set $\tilde{b}^i = \tilde{s}^i = 0$. For any other bank i, set $\tilde{b}^i := (b^i - s^i)^+$ and $\tilde{s}^i := (s^i - b^i)^+$. Finally, choose $\tilde{b}^0$ such that $\|b\|_1 = \|\tilde{b}\|_1$. The choice of $\tilde{b}^0$ ensures that $(\tilde{b}, \tilde{s})$ is feasible. Since $\tilde{b}^i - \tilde{s}^i = b^i - s^i$ for $i \in \mathcal{A}^c$, it follows that

$$\tilde{b}^0 + \|\tilde{s}\|_1 = b^0 + \sum_{i=1}^{n} \left(b^i - \tilde{b}^i + \tilde{s}^i\right) = b^0 + \sum_{i \in \mathcal{A}} b^i + \sum_{i \in \mathcal{A}^c} s^i.$$

Similarly, $\ell_*^i = \ell^i(0)$ implies $\left\|\ell^{\mathcal{C}\cup\mathcal{S}}(\tilde{b} - \tilde{s})\right\|_1 + \left\|\ell_*^{\mathcal{F}}\right\|_1 = \left\|\ell^{\mathcal{A}^c}(b - s)\right\|_1 + \left\|\ell_*^{\mathcal{F}\cup\mathcal{A}}\right\|_1$ and hence $w\left(\tilde{b}, \tilde{s}, 1\right) = w(b, s, a)$. It remains to check that 1 is an equilibrium response. Let $\mathcal{R}_{\tilde{b},\tilde{s}} := \left\{i \mid \tilde{b}^i - \tilde{s}^i > 0\right\}$ denote the set of banks which have to buy up a positive net amount in $(\tilde{b}, \tilde{s})$. By Lemma 3.1, any bank $i \notin \mathcal{R}_{\tilde{b},\tilde{s}}$ accepts $(\tilde{b}, \tilde{s})$. Moreover, $\tilde{b}^i - \tilde{s}^i = b^i - s^i \leq \xi^i$ for any $i \in \mathcal{R}_{\tilde{b},\tilde{s}}$ because these banks accepted the original proposal (b, s). For any $i \in \mathcal{R}_{\tilde{b},\tilde{s}}$, we compute

$$\begin{aligned}
w\left(\tilde{b}, \tilde{s}, (0, 1^{-i})\right) &= w\left(\tilde{b}, \tilde{s}, 1\right) + \lambda\left(\tilde{b}^i - \tilde{s}^i\right) - (1 - \alpha)\left(\ell^i(\tilde{b} - \tilde{s}) - \ell_*^i\right) \\
&= w(b, s, a) + \lambda(b^i - s^i) - (1 - \alpha)\left(\ell^i(b - s) - \ell_*^i\right) \\
&= w\left(b, s, (0, a^{-i})\right).
\end{aligned}$$

Since a is an equilibrium response to (b, s), it follows that $w\left(b, s, (0, a^{-i})\right) \geq w_N$ from Lemma 3.1. Thus, again by Lemma 3.1, 1 is an □

As a consequence of Lemma A.2, we may assume that the optimal bail-in will result from a proposal that is accepted by all banks. By Lemma 3.1, this is the case only if the net contribution of any bank i is smaller than ξ^i. The welfare-minimizing bail-in of a fixed consortium $\mathcal{R} \subseteq \mathcal{C} \cup \mathcal{S}$, subject to incentive-compatibility constraints, is thus given by $(b_{\mathcal{R}}, s_{\mathcal{R}})$ of Definition 3.1 as formalized in the following lemma.

Lemma A.3. *Fix a rescue consortium $\mathcal{R} \subseteq \mathcal{C} \cup \mathcal{S}$. Among all admissible subsidized bail-ins (b, s) that satisfy $b^i - s^i \leq \xi^i$ as well as $\{i \mid b^i > 0\} \subseteq \mathcal{R}$, the bail-in proposal $(b_{\mathcal{R}}, s_{\mathcal{R}})$ minimizes the welfare losses.*

Proof. If everybody accepts a proposal (b, s), the subsidized bail-in results in a welfare loss of

$$w(b,s,1) = \lambda\left(B + \alpha\left\|e^{\mathcal{F}}\right\|_1 1_{\{\alpha<1/(1+\lambda)\}} - \sum_{i\in\mathcal{R}_b} b^i + \sum_{i\notin\mathcal{R}_b} s^i\right) + \frac{1-\alpha}{\alpha}(b^i + e^i - V_0^i)^+ + \frac{1-\alpha}{\alpha}(e^i - V_0^i - s^i)^+.$$

If $\alpha \geq 1/(1+\lambda)$, then $(1-\alpha)/\alpha \leq \lambda$ and hence $w(b,s,1)$ is non-increasing in b^i, $i \in \mathcal{R}_b$ and non-decreasing in s^i, $i \notin \mathcal{R}_b$. It follows that the negative welfare impact is minimized at $s = 0$ and the maximum possible b^i, for which (b, s) can be accepted, which is $b^i = \xi^i$ for $i \in \mathcal{R}_b$ by Lemma 3.1. If $\alpha < 1/(1+\lambda)$, then $(1-\alpha)/\alpha > \lambda$. Therefore, $w(b,s,1)$ is decreasing in b^i on $[0, V_0^i - e^i)$ and increasing in b^i on $(V_0^i - e^i, \xi^i]$. It follows that $w(b,s,1)$ is minimized at $b^i = \min(\xi^i, (V_0^i - e^i)^+) = \eta^i$ for $i \in \mathcal{R}_b$, where we used that $(V_0^i - e^i)^+ < V_0^i - (1-\alpha)e^i$. Similarly, $w(b,s,1)$ is decreasing in s^i on $[0, e^i - V_0^i)$ and increasing for $s^i > e^i - V_0^i$. It follows that $w(b,s,1)$ is minimized at $s^i = (e^i - V_0^i)^+ = \alpha\ell_*^i$. □

The proof of Theorem 3.2 is concluded by showing that $(b_{\mathcal{R}}, s_{\mathcal{R}})$ for $\mathcal{R} = \{i_1, \ldots, i_m\}$ also satisfies condition 2.(b) of Lemma 3.1.

Proof of Theorem 3.2. Consider first the case where $w_P < w_N$ and the social planner's threat is not credible. If $\alpha \geq 1/(1+\lambda)$, then Lemma 3.1 implies that banks will accept a proposal if and only if they receive a net positive amount, i.e., if $s^i - b^i \geq 0$. In any such proposal, the social planner has to buy up the entire debt B, and hence such a proposal is dominated by a public bailout. The social planner will thus resort to a public bailout. If $\alpha < 1/(1+\lambda)$, then Lemma A.3 shows that it is optimal for the social planner to provide a subsidy of $\alpha\ell_*^i$ to every bank $i \in \mathcal{C} \cup \mathcal{S}$. Since it is necessary that $s^i \geq b^i$ for bank i to accept the proposal, it follows that the social planner has to cover $B + \alpha\left\|e^{\mathcal{F}}\right\|_1$ himself. Any such subsidized bail-in leads to a welfare loss of at least $B + \alpha\|\ell_*\|_1 = w_P$. Thus, it is optimal for the social planner to resort to a public bailout.

If $w_N \leq w_P$, then the social planner's threat is credible. We first show that $w_{\mathcal{C}\cup\mathcal{S}} \leq w_N$ so that a subsidized bail-in is in the interest of the social planner. The optimal proposal of Lemma A.3 leads to deadweight losses of

$$w_{\mathcal{R}} = \begin{cases} \lambda\big(B - \big\|\xi^{\mathcal{R}}\big\|_1\big) + (1-\alpha)\big(\big\|\hat{\ell}^{\mathcal{R}}\big\|_1 + \big\|\ell_*^{\mathcal{R}^c}\big\|_1\big) & \alpha \geq \frac{1}{1+\lambda}, \\ \lambda\big(B - \big\|\eta^{\mathcal{R}}\big\|_1\big) + \alpha\|\ell_*\|_1 & \alpha < \frac{1}{1+\lambda}. \end{cases}$$

Thus, for $\mathcal{R} = \mathcal{C} \cup \mathcal{S}$ we obtain $w_{\mathcal{C}\cup\mathcal{S}} \leq \lambda\big(B - \big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1\big) + (1-\alpha)\big\|\hat{\ell}\big\|_1$ with equality if and only if $\alpha \geq 1/(1+\lambda)$. Together with the first identity in Lemma 4.2, this implies $w_{\mathcal{C}\cup\mathcal{S}} \leq w_N - (1+\lambda)(1-\beta)\big\|A^{\mathcal{D}} - \zeta^{\mathcal{D}}\big\|_1 \leq w_N$.

We proceed to characterize the optimal proposal of the social planner. Suppose first that $\alpha \geq 1/(1+\lambda)$. Lemma 3.1 states that a bank i accepts a proposal (b,s) if and only if $b^i - s^i \leq \xi^i$ and $w\big(b,s,(0,1^{-i})\big) \geq w_N$. Since the optimal choice of (b,s) is $(\xi,0)$ by Lemma A.3, the latter condition is equivalent to

$$w(b,s,1) \geq w_N - \nu^i$$

for $\nu^i = \lambda\xi^i - (1-\alpha)\big(\hat{\ell}^i - \ell_*^i\big)$. In any proposal, the social planner can thus achieve a welfare loss of $w_N - \min_{i\in\mathcal{R}_{b,s}} \nu^i$ at best, where $\mathcal{R}_{b,s} = \{i \mid b^i - s^i > 0\}$. It is thus in the interest of the social planner to include banks, for which ν^i is as high as possible. Consider first the proposal (b_*, s_*) with $b_*^i = \nu^i 1_{\{i\in\{i_1,\ldots,i_k\}\}}$ and $s_* = 0$. The definition of k together with Lemma 3.1 imply that this proposal will be accepted by all banks. Since $i_1, i_2, \ldots$ are non-increasingly ordered according to ν^i, it follows that $w_{\mathcal{R}} > w_N$ for any $\mathcal{R} \subseteq \mathcal{C}\cup\mathcal{S}$ with $|\mathcal{R}| < k$. Thus, the social planner will not want to propose such a bail-in. Moreover, any $\mathcal{R}$ with $|\mathcal{R}| = k$ satisfies $w_{\mathcal{R}} \geq w_{\{i_1,\ldots,i_k\}}$, hence the social planner cannot reduce the welfare loss below $w_{\{i_1,\ldots,i_k\}}$ with any such proposal.

If $w_N - w_{\{i_1,\ldots,i_k\}} < \nu^{i_{k+1}}$, then the social planner can improve the proposal (b_*, s_*) to the proposal $(b_\dagger, s_\dagger)$ with $b_\dagger^i = \nu^i 1_{\{i\in\{i_1,\ldots,i_{k+1}\}\}}$ and $\|s_\dagger\|_1 = w_N - w_{\{i_1,\ldots,i_k\}}$. By giving away subsidies in the amount of $w_N - w_{\{i_1,\ldots,i_k\}}$, the social planner is able to include bank i_{k+1} into the proposal without violating the condition that $w\big(b,s,(0,1^{-i})\big) \geq w_N$ for any bank i of the consortium.

Therefore, Lemma 3.1 implies that $(b_\dagger, s_\dagger)$ will be accepted by all banks. Finally, any bail-in (b, s) with $|\mathcal{R}_{b,s}| > k$ has a lower bound on the welfare loss of $w_N - \min_{i\in\mathcal{R}_{b,s}} \nu^i$ by Lemma 3.1. Since $w_N - \min_{i\in\mathcal{R}_b} \nu^i \geq w_N - \nu^{i_{k+1}}$ for any such proposal, it follows that the only reasonable proposals are (b_*, s_*) and $(b_\dagger, s_\dagger)$, of which the social planner will choose whichever minimizes the welfare loss. The proof for $\alpha < 1/(1+\lambda)$ works analogously with $\nu^i = \lambda\eta^i$. $\square$

B Proof of Propositions 4.6–4.8

For Online Publication. The social planner's threat is more credible in the ring network than in the complete network if the welfare losses without intervention are smaller in the ring network than in the complete network. The first identity in Lemma 4.2 shows that the welfare losses without intervention are equal to

$$w_N = (1-\alpha)\big\|\hat{\ell}\big\|_1 + \big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1 - B + (1+\lambda)\big\|\delta^{\mathcal{D}}\big\|_1. \tag{9}$$

Equation (9) gives a convenient way to compute welfare losses once we show that $L > \frac{1+\rho}{\beta}B$ implies $\big\|\delta^{\mathcal{D}}\big\|_1 = 0$. We start by characterizing the losses ζ to interbank assets in the two networks and the resulting welfare losses. For the sake of brevity, we use $z_l = V_{0,l} - (1-\alpha)e_l$ and $z_h = V_{0,h} - (1-\alpha)e_h$ to denote the wealth of lowly and highly capitalized banks, respectively, after accounting for the loss due to inefficient liquidation of the assets.

Lemma B.1. *Fix a shortfall B of bank 1 and suppose that $L \geq \frac{1+\rho}{\beta}B$. The shock size $\zeta_C(B)$ to any bank's interbank assets in the complete network equals*

$$\zeta_C(B) = \begin{cases} \frac{B+(1-\beta)L}{n-1} & \text{if } B \leq B_*, \\ \frac{B+(n_l+1)(1-\beta)L - n_l z_l}{n_h + n_l(1-\beta)} & \text{otherwise.} \end{cases}$$

where $B_* = (n-1)z_l - (1-\beta)L$. All lowly capitalized banks default if and only if $B > B_*$ and all banks default if and only if $B > B^*$, where

$$B^* = \big(n_h + n_l(1-\beta)\big)z_h + n_l\big(z_l - (1-\beta)L\big) - (1-\beta)L.$$

For $B \le B_*$, the welfare losses without intervention are equal to

$$w_{N,C}(B) = (1-\alpha)e^1 + (1-\beta)L + \frac{1-\alpha}{\alpha}\big(B + (1-\beta)L - (n-1)c\big)^+$$

and for $B_* < B \le B^*$, they are equal to

$$w_{N,C}(B) = (1-\alpha)e^1 + n_l V_{0,l} + n_h\left(\zeta_C(B) + \frac{1-\alpha}{\alpha}\big(\zeta_C(B) - c\big)\right) - B.$$

We will be using the following auxiliary result in the proof.

Lemma B.2. *Let π be an $m \times m$-dimensional matrix with diagonal entries of 1 and off-diagonal entries $a \in \big(\frac{-1}{m-1}, 0\big)$. The inverse of π has diagonal entries b and off-diagonal entries c, where*

$$b = \frac{(m-2)a+1}{(1-a)((m-1)a+1)}, \qquad c = \frac{-a}{(1-a)((m-1)a+1)}. \tag{10}$$

Proof. Let B denote the $m \times m$ dimensional matrix with b on the diagonal and c elsewhere. Then πB and $B\pi$ have diagonal entries $b + (m-1)ac$ and off-diagonal entries $c + ab + (m-2)ac$. It is easy to check that this coincides with the identity matrix if and only if b and c are given by (10). □

Proof of Lemma B.1. Because bank 1 is equally liable to every other bank in the complete network, there are no contagious defaults if and only if the shock ζ^i to every bank $i \neq 1$ is smaller than the capitalization level z_l of the lowly capitalized banks. We show that this is precisely the case if $B \le B_*$. Indeed, if there are no contagious defaults, every bank $i \neq 1$ repays its liabilities in full and bank 1 makes an equilibrium payment in the size of $\hat{p}^1 = (c^1 + \alpha e^1 + \beta L)^+ = \beta L - B$. Every bank $i \neq 1$ is thus hit by a shock to its interbank assets in the size of

$$\zeta^i|_{\mathcal{C}=\emptyset} = \frac{L - \hat{p}^1}{n-1} = \frac{\min(L, B + (1-\beta)L)}{n-1} = \frac{B + (1-\beta)L}{n-1}. \tag{11}$$

Since we have assumed that there are no contagious defaults, $\zeta^i|_{\mathcal{C}=\emptyset} \le z_l$ has to hold, which is the case if and only if $B \le B_*$. Let $\mathcal{I}_L$ denote the index set of all lowly capitalized banks. Since $\zeta^i|_{\mathcal{C}\supseteq\mathcal{I}_L} \geq \zeta^i|_{\mathcal{C}=\emptyset}$ for any values of L and B, it follows that $\zeta^i > z_l$ if and only if $B > B_*$.

Highly capitalized banks default only if also the lowly capitalized banks default. It is thus necessary that $B > B_*$. The shock ζ^i to interbank assets of a highly capitalized bank is thus given by $\zeta^i|_{\mathcal{C}=\mathcal{I}_L} = \frac{1}{n-1}\big(L - \hat{p}^1 + n_l(L - \hat{p}_L)\big)$ for

$$L - \hat{p}^1 = \min\biggl(L, B + (1-\beta)L + \frac{\beta n_l}{n-1}(L - \hat{p}_L)\biggr),$$
$$L - \hat{p}_L = \min\biggl(L, (1-\beta)L - z_l + \frac{\beta}{n-1}\big(L - \hat{p}^1 + (n_l - 1)(L - \hat{p}_L)\big)\biggr).$$

For the sake of brevity, denote by X^1 and X_L the values of $L - \hat{p}^1$ and $L - \hat{p}_L$, respectively, when these values are strictly smaller than L. We show first that $\hat{p}_L \geq \hat{p}^1$ with equality if and only if $\hat{p}^1 = \hat{p}_L = 0$. Suppose that $\hat{p}^1 \geq \hat{p}_L$, which implies that

$$X^1 = X_L + B + z_l + \frac{\beta}{n-1}(\hat{p}^1 - \hat{p}_L) > X_L. \tag{12}$$

This leads to a contradiction if $L > X^1$. Indeed, (12) implies that $L > X_L$ and hence $\hat{p}^1 = L - X^1 < L - X_L = \hat{p}_L$, a contradiction. It is thus necessary that $L - \hat{p}^1 = L$ and hence $\hat{p}^1 = 0$ and $\hat{p}_L \leq \hat{p}^1 = 0$. We have thus shown that either $L - \hat{p}_L < L - \hat{p}^1$ or $L - \hat{p}_L = L - \hat{p}^1 = L$.

Since X^1 is bounded above by $B + \Bigl(1 - \frac{\beta}{1+\rho}\Bigr)L$, the assumption $L \geq \frac{1+\rho}{\beta}B$ implies that $X^1 \leq L$. In particular, $L - \hat{p}^1 = X^1$ and $L - \hat{p}_L = X_L$, which implies that $\left\|\delta^{\mathcal{D}}\right\|_1 = 0$. We now use Lemma 4.2 to compute $\zeta^i|_{\mathcal{C}=\mathcal{I}_L}$. Since $\pi^{i,\mathcal{D}}$ is an $n_l + 1$-dimensional row vector with identical entries $1/(n-1)$, it follows that $(\pi^{i,\mathcal{D}}\chi)^j = \sum_k \chi^{kj}/(n-1)$ for any matrix χ. Using the notation of Lemma B.2 for $a = -\beta/(n-1)$ and $m = |\mathcal{D}| = n_l + 1$, each column of $\big(I - \beta\pi^{\mathcal{D},\mathcal{D}}\big)^{-1}$ sums up to

$$b + n_l c = \frac{1-a}{(1-a)\big(n_l a + 1\big)} = \frac{1}{1 + n_l a} = \frac{n-1}{n-1-\beta n_l}.$$

Since $n-1 = n_h + n_l$, each entry of $\pi^{i,\mathcal{D}}\big(I - \beta\pi^{\mathcal{D},\mathcal{D}}\big)^{-1}$ equals $1/\big(n_h + n_l(1-\beta)\big)$ and hence

$$\zeta^i|_{\mathcal{C}=\mathcal{I}_L} = \frac{B + (n_l+1)(1-\beta)L - n_l z_l}{n_h + n_l(1-\beta)}. \tag{13}$$

This shows that the highly capitalized banks default if and only if $B > B^*$. Welfare losses are now computed easily with (9), recalling that $\xi^i = \min(\zeta^i, z^i)$ as well as $V_0^i = (1-\alpha)e^i + z^i$. □

Lemma B.3. *Fix a shortfall B of bank 1 and suppose that $L \geq \frac{1}{\beta}B$. Bank $i+1$ defaults in the ring network if and only if $B > B_{i+1}$, where $B_2 = z_l - (1-\beta)L$ and*

$$B_{i+1} = \max\left(B_2, \frac{2-\beta-\beta^{i-1}}{\beta^{i-1}(1-\beta)} z_l + \frac{2-\beta-\beta^{i-n_l-1}}{\beta^{i-1}(1-\beta)}(z_h - z_l)1_{\{i>n_l\}} - \frac{1-\beta^i}{\beta^{i-1}} L\right)$$

for $i > 2$. The welfare losses without intervention are equal to

$$w_{N,R}(B_{i+1}) = (1-\alpha)e^1 + iV_{0,l} + (i-n_l)^+(e_h - e_l) - B_{i+1}.$$

Proof. Since $L \geq B + (1-\beta)L$ by assumption, the shock size ζ^2 to bank 2's interbank assets is equal to $\zeta^2 = B + (1-\beta)L$, which implies $B_2 = z_l - (1-\beta)L$. We will derive B_{i+1} for $i > 1$ recursively. Any bank $i+1$ defaults if and only if all banks $2, \ldots, i$ default and the shock to bank $i+1$'s interbank assets exceeds z^{i+1}, that is, $\zeta^{k+1} > z^{k+1}$ for $k = 1, \ldots, i$. In particular, this shows that $B_{i+1} \geq B_2$ for any $i > 1$. The recursive relation

$$\zeta^{k+1} = \min\Big(L, \big((1-\beta)L + \beta\zeta^k - z^k\big)^+\Big) \tag{14}$$

implies that bank k's default leads to a minimal shock to bank $k+1$'s interbank assets of size $(1-\beta)(L - z^k)$. If this minimal shock size is larger than z^{k+1}, bank $k+1$ defaults at the same time as bank k and $\zeta^{k+1} \geq \zeta^k$. If $(1-\beta)(L - z^k) < z_{k+1}$, then $\zeta^{k+1} < \zeta^k$ and hence $B_{k+1} \geq B_k$.

Consider first the case where $i \leq n_l$. If $\beta \leq 1 - \frac{z_l}{L - z_l}$, then $(1-\beta)(L - z_l) \geq z_l$ and hence the default of any lowly capitalized bank $k > 2$ occurs at the same time as the default of bank k. It follows that $B_{i+1} = B_2$ in that case.

If $\beta > 1 - \frac{z_l}{L-z_l}$, then $(\zeta^k)_{k\geq 2}$ is decreasing in k. In particular, $L \geq \frac{1}{\beta}B$ implies that $L \geq \zeta^2 \geq \zeta^k$ and hence we can find ζ^{i+1} with the recursion $\zeta^{k+1} = \big((1-\beta)L + \beta\zeta^k - z_l\big)^+$, starting from $\zeta^2 = \min(L, B + (1-\beta)L)$. A short calculation shows that

$$\zeta^{i+1} = \left(\beta^{i-1}B + (1-\beta^i)L - \frac{1-\beta^{i-1}}{1-\beta}z_l\right)^+ \tag{15}$$

for $i \leq n_l + 1$. Bank $i+1 \leq n_l + 1$ defaults if and only if $\zeta^{i+1} > z_l$. Solving $\zeta^{i+1} = z_l$ for B yields B_{i+1} in the case $i \leq n_l$.

Consider now the case where $i > n_l$. Similarly as before, if $\beta \leq 1 - \frac{z_h}{L-z_h}$, then $B_{i+1} = B_2$ for every bank $i+1$. Suppose, therefore, that $\beta > 1 - \frac{z_h}{L-z_h}$ and hence $(\zeta^k)_{k\geq n_l+2}$ is decreasing in k. For $k \leq n_l + 2$, equation (15) implies that $\zeta^k < L$ if and only if $L > \frac{1}{\beta} - \frac{1-\beta^{k-2}}{1-\beta}z_l$. In particular, $\zeta^k \leq L$ for any k. It is thus sufficient to consider the recursion $\zeta^{k+1} = \big((1-\beta)L + \beta\zeta^k - z_l\big)^+$ with the explicit solution

$$\zeta^{i+1} = \left(\beta^{i-1}B + (1-\beta^i)L - \frac{\beta^{i-n_l-1} - \beta^{i-1}}{1-\beta}z_l - \frac{1-\beta^{i-n_l-1}}{1-\beta}z_h\right)^+. \tag{16}$$

Solving for $\zeta^{i+1} = z_h$ yields B_{i+1} in the case $i > n_l$. The welfare losses without intervention follow directly from (9). □

We next present two auxiliary lemmas that will be needed in the proof of Proposition 4.8.

Lemma B.4. *Suppose that $i > j \geq 0$ and $i \geq 2$. Then*

$$g_{i,j}(\beta) := \frac{2-\beta}{1-\beta} - \frac{\beta + j(1-\beta)\beta^{i-j-1} - \beta^{i-j}}{(i-1)(1-\beta)^2}$$

is increasing in β with $\lim_{\beta\to 1} g_{i,j}(\beta) = 1 + \frac{(i-j-1)(i+j)}{2(i-1)}$.

Proof. We show monotonicity by induction on $i = j + k$. The result is trivial for $i = j + 1$ since $g_{i,j} \equiv 1$ in that case. Observe that $g_{j+k,j}$ satisfies the recursive identity

$$g_{j+k+1,j}(\beta) = 2 + \frac{j+k-1}{j+k}\beta\big(g_{j+k,j}(\beta) - 1\big).$$

Taking the derivative with respect to β we obtain

$$\frac{\partial g_{j+k+1,j}(\beta)}{\partial \beta} = \frac{j+k-1}{j+k}\left(g_{j+k,j}(\beta) - 1 + \beta\frac{\partial g_{j+k,j}(\beta)}{\partial \beta}\right).$$

Inductively, we derive that $g_{j+k+1,j}(\beta) \geq 1$ for any β and hence $\frac{\partial g_{j+k+1,j}(\beta)}{\partial \beta} \geq 0$. The limit result is a straight-forward application of L'Hôpital's rule,

$$\begin{aligned}\lim_{\beta\to 1} g_{i,j}(\beta) &= \lim_{\beta\to 1}\frac{(i-1)(2\beta-3) - 1 + (j+1)(i-j)\beta^{i-j-1} - j(i-j-1)\beta^{i-j-2}}{-2(i-1)(1-\beta)} \\ &= 1 + \frac{(i-j-1)(i+j)}{2(i-1)}.\end{aligned}$$

□

Lemma B.5. *If $L \geq iz_l + (i-n_l)^+(z_h - z_l)$, then $B_{i+1} + (1-\beta)L$ is non-decreasing in β, where B_{i+1} is given in Lemma B.3.*

Proof. The statement holds trivially where $B_{i+1} + (1-\beta)L = z_l$. To see that the quantity $B_{i+1} + (1-\beta)L$ is non-decreasing everywhere, suppose first that $i \leq n_l$ and take the derivative to obtain

$$\frac{\partial\big(B_{i+1} + (1-\beta)L\big)}{\partial \beta} = \frac{i-1}{\beta^i}\big(L - g_{i,0}(\beta)z_l\big).$$

Since $g_{i,0}$ is increasing by Lemma B.4 with $g_{i,0}(1) = 1 + \frac{i}{2}$, the assumption $L \geq iz_l$ implies that $L \geq g_{i,0}(1)z_l \geq g_{i,0}(\beta)z_l$ and hence the derivative is positive everywhere. For $i > n_l$, a straight-forward computation shows that

$$\frac{\partial B_{i+1} + (1-\beta)L}{\partial \beta} = \frac{i-1}{\beta^i}\big(L - g_{i,0}(\beta)z_l - g_{i,n_l}(\beta)(z_h - z_l)\big).$$

The condition on L now implies that the derivative is positive everywhere.in the same way as before. □

Proof of Proposition 4.6. We consider first the case where $B \leq B_*$. We will show that $w_{N,R}(B) \geq w_{N,C}(B)$ for any values of α and β, which shows that the threat is uniformly more credible in the complete network than in the ring network. If bank 1's shortfall B is lower than $(n-1)c - (1-\beta)L$, then no banks other than bank 1 have to liquidate anything in the complete network. Thus, $w_{N,C}(B) = (1-\alpha)e^1 + (1-\beta)L$ attains the minimum possible value for welfare losses and hence $w_{N,C}(B) \leq w_{N,R}(B)$. Suppose next that $B > (n-1)c - (1-\beta)L$ and that $B = B_{i+1}$ for some bank i. Lemmas B.1 and B.3 imply that $w_{N,R}(B_{i+1}) \geq w_{N,C}(B_{i+1})$ if and only if

$$iz_l + (i-n_l)^+(z_h - z_l) + (n-i-1)(1-\alpha)c \geq B_{i+1} + (1-\beta)L. \tag{17}$$

This inequality is clearly satisfied if $B_{i+1} + (1-\beta)L \leq iz_l + (i-n_l)^+(z_h - z_l)$. Suppose, therefore, that $B_{i+1} + (1-\beta)L > iz_l + (i-n_l)^+(z_h - z_l)$ holds. The condition $L > \frac{1+\rho}{\beta}B_{i+1}$ implies $L > B_{i+1} + (1-\beta)L > iz_l + (i-n_l)^+(z_h - z_l)$, hence Lemma B.5 shows that the right-hand side of (17) is increasing in β. The maximum is thus attained at $\beta = 1$. Lemma B.3 shows that B_{i+1} converges to $iz_l + (i-n_l)^+(z_h - z_l)$ as $\beta \to 1$, which satisfies (17). It follows that $w_{N,R}(B_{i+1}) \geq w_{N,C}(B_{i+1})$ for any values of α, β and i. To see that the statement also holds for $B \in (B_i, B_{i+1}]$, note that

$$w_{N,R}(B) = w_{N,R}(B_i) + (1-\alpha)\hat{\ell}^{i+1} + \zeta^{i+1} - (B - B_i).$$

It follows from (16) that

$$\frac{\partial w_{N,R}(B)}{\partial B} = \begin{cases} \beta^{i-1} - 1 & \text{if } B_{i+1} < B \leq B_i + \big(c - (1-\beta)(L - z_l)\big)^+, \\ \frac{\beta^{i-1}-\alpha}{\alpha} & \text{otherwise.} \end{cases} \tag{18}$$

Lemma B.1 implies that $\frac{\partial w_{N,C}(B)}{\partial B} = \frac{1-\alpha}{\alpha}$, showing that the rate of increase of welfare losses is larger in the complete network than in the ring network. Since we have shown $w_{N,C}(B_{i+1}) \leq w_{N,R}(B_{i+1})$ already, this shows that also $w_{N,C}(B) \leq w_{N,R}(B)$ for any $B \in (B_i, B_{i+1}]$. Finally, for any $B > B^*$, all banks default in the complete network, implying that $w_{N,C}(B) \geq w_{N,R}(B)$. $\square$

Lemma B.6. *If $L \geq \frac{1}{\beta} B_*$, then $B_{n_l+1} < B_*$.*

Proof. Suppose towards a contradiction that $B_{n_l+1} \geq B_*$. Then $L \geq \frac{1}{\beta} B_*$ implies that $L \geq n_l z_l$, hence Lemma B.5 shows that the maximum of $B_{n_l+1} + (1-\beta)L$ is attained at $\beta = 1$. Since $B_{n_l+1} \to n_l z_l$ as $\beta \to 1$, this yields

$$B_{n_l+1} + (1-\beta)L \leq n_l z_l < (n-1) z_l = B_* + (1-\beta)L,$$

which is a contradiction. □

Proof of Proposition 4.7. Consider first the case where $\beta = 1$. A straightforward application of Lemmas B.1 and B.3 yields that

$$\begin{aligned} w_{N,R}(B_{i+1}) - w_{N,C}(B_{i+1}) &= (i - n_l) V_{0,h} + \frac{1-\alpha}{\alpha} n_h c - \frac{n_h}{\alpha} \zeta_C(B_{i+1}) \\ &= \frac{1-\alpha}{\alpha}(n-1-i)c \end{aligned}$$

for any i with $B_{i+1} \geq B_*$. This implies that $w_{N,R}(B_{i+1}) \geq w_{N,C}(B_{i+1})$ in the same way as in the proof of Proposition 4.6. For the second statement where $\beta < 1$, we will show that $w_{N,C}(B) \geq w_{N,R}(B)$ holds for L above the threshold

$$L^* := z_h + \frac{z_h - z_l}{\rho(1-\beta)}. \tag{19}$$

Let $B \in (B_i, B_{i+1}]$ for some i. Since $B > B_*$, Lemma B.6 implies that $i > n_l$. Monotonicity of $w_{N,R}(B)$ and $w_{N,C}(B)$ in B together with Lemmas B.1 and B.3 show that

$$\begin{aligned} w_{N,R}(B) - w_{N,C}(B) &\leq w_{N,R}(B) - \lim_{B \searrow B_*} w_{N,C}(B) \\ &= (i - n_l) V_{0,h} + \frac{n_h}{\alpha d}\big((1-\alpha) dc - n_h z_l - n_l (1-\beta) L\big) \\ &\leq \frac{n_h}{\alpha d}\big(d z_h - n_h z_l - n_l(1-\beta)L\big) \\ &\leq \frac{n_h}{\alpha d}\big(n_h (z_h - z_l) - n_l (1-\beta)(L - z_h)\big). \end{aligned}$$

This bound is smaller or equal to 0 if $L \geq L^*$, which shows the claim. □

Proof of Proposition 4.8. Observe first that the result holds true by Propositions 4.6 and 4.7 if $\beta = 1$, stating that $B' = B'' = B^*$, or if $\beta < 1$ and $L \geq L^*$, stating that $B' = B'' = B_*$, where L^* is given in (19). Suppose, therefore, that $\beta < 1$ and $L < L^*$. Let i_0 denote the smallest integer i such that $B_{i+1} > B_*$ and note that $i_0 > n_l$ by Lemma B.6. Consider the sequence $Z = (Z_{i+1})_{i \geq i_0}$ of differences $Z_{i+1} := w_{N,R}(B_{i+1}) - w_{N,C}(B_{i+1})$ between welfare losses in the two networks. We will show that $Z = (Z_{i+1})_{i \geq i_0}$ is non-increasing under the stated assumptions. Lemmas B.1 and B.3 imply that

$$\begin{aligned} w_{N,R}(B_{i+1}) - w_{N,R}(B_i) &= V_{0,h} - (B_{i+1} - B_i), \\ w_{N,C}(B_{i+1}) - w_{N,C}(B_i) &= \Big(\frac{n_h}{\alpha d} - 1\Big)(B_{i+1} - B_i), \end{aligned}$$

where we denote $d = n_h + n_l(1-\beta)$ for the sake of brevity. Let $\hat{\beta}_i$ be the solution to $V_{0,h} = \frac{n_h}{\alpha d}(B_{i+1} - B_i)$, that is, where $Z_{i+1} - Z_i = 0$. We first show that $\big(\hat{\beta}_i\big)_{i \geq i_0}$ is decreasing. Indeed, observe that

$$B_{i+1} - B_i = \frac{1}{\beta^{i-1}}\big(z_h - (1-\beta)(L - z_h)\big).$$

Since $B_{i+1} \geq B_*$, it is necessary that $z_h > (1-\beta)(L - z_h)$ as otherwise B_{i+1} would equal $B_2 < B_*$ by Lemma B.3. It follows that $B_{i+1} - B_i$ is increasing in i. Taking the derivative of $B_{i+1} - B_i$ with respect to β yields

$$\frac{\partial(B_{i+1} - B_i)}{\partial \beta} = \frac{\beta + (i-1)(1-\beta)}{\beta^i}\left(L - z_h - \frac{i-1}{\beta + (i-1)(1-\beta)} z_h\right).$$

Since $L > B^* > (n-1)z_h$, this shows that $B_{i+1} - B_i$ is increasing in β for any $i > n_l$. Using these monotonicity properties and the definition of $\hat{\beta}_i$, we obtain

$$V_{0,h} = \frac{n_h}{\alpha d}\Big(B_{i+1}\big(\hat{\beta}_i\big) - B_i\big(\hat{\beta}_i\big)\Big) < \frac{n_h}{\alpha d}\Big(B_{i+2}\big(\hat{\beta}_i\big) - B_{i+1}\big(\hat{\beta}_i\big)\Big).$$

Since $B_{i+2} - B_{i+1}$ is increasing in β, this shows that $\hat{\beta}_{i+1} < \hat{\beta}_i$. For a fixed β, it follows that $Z_{i+1} - Z_i$ is positive if and only if $\beta < \hat{\beta}_i$. Since $\big(\hat{\beta}_i\big)_{i \geq i_0}$ is decreasing in i, it follows that $Z_{i+1} - Z_i$ is positive for all $i \leq i_*$, where

$i_* = \max\{k \mid \beta < \hat{\beta}_k\}$, and non-positive for all $i > i_*$. It remains to show that $i_* < i_0$. The indices i_* and i_0 are the unique integers i and j, respectively, for which $\hat{\beta}_{i+1} \leq \beta < \hat{\beta}_i$ and $B_j \leq B_* < B_{j+1}$. Thus, the definition of $\hat{\beta}_i$ and the expression for B_j in Lemma B.3 imply that i_* and i_0 are defined by

$$\frac{1}{\beta^{i_*-1}} < \frac{\alpha d V_{0,h}}{n_h\big(z_h - (1-\beta)(L - z_h)\big)} \leq \frac{1}{\beta^{i_*}}, \qquad \frac{1}{\beta^{i_0-2}} \leq h(\beta) < \frac{1}{\beta^{i_0-1}}, \quad (20)$$

where

$$h(\beta) = \frac{z_h - (1-\beta^{n_l})z_l + (1-\beta)\beta^{n_l}\big((n-1)z_l - L\big)}{\beta^{n_l}\big(z_h - (1-\beta)L - z_h)\big)}.$$

Because $\alpha V_{0,h} = \alpha c + \alpha e_h \leq c + \alpha e_h = z_h$, it is sufficient to show that

$$h(\beta) \geq \frac{d z_h}{n_h\big(z_h - (1-\beta)(L - z_h)\big)} \quad (21)$$

as this readily implies $\frac{1}{\beta^{i_*-1}} < \frac{1}{\beta^{i_0-1}}$ by (20), which is equivalent to $i_* < i_0$. A short computation shows that (21) is equivalent to

$$L \leq \frac{z_h - z_l}{\beta^{n_l}(1-\beta)} + (n-1)z_l - \rho z_h + \frac{n_h z_l - z_h}{n_h(1-\beta)}. \quad (22)$$

The conditions $\beta^{n_l} < \rho$ and $\frac{z_l}{z_h} > \frac{\rho}{n-1}$ together with $L < L^*$ imply that

$$L < \frac{z_h - z_l}{\rho(1-\beta)} + z_h \leq \frac{z_h - z_l}{\beta^{n_l}(1-\beta)} + (n-1)z_l - \rho z_h.$$

This implies (22) because the last term in (22) is non-negative by assumption.

Let j denote the largest integer k such that $w_{N,R}(B_{k+1}) \geq w_{N,C}(B_{k+1})$. Fix a shortfall $B \in (B_i, B_{i+1}]$ for some i with $i_0 \leq i \leq j$. It follows from (18) and

$$\frac{\partial w_{N,C}(B)}{\partial B} = \frac{n_h}{\alpha d} - 1$$

that $\frac{\partial w_{N,C}(B)}{\partial B} \geq \frac{\partial w_{N,R}(B)}{\partial B}$ if and only if $f_i(\beta) \geq \frac{n_l}{n_h}$, where $f_i(\beta) = \frac{1-\beta^{i-1}}{\beta^{i-1}(1-\beta)}$. A short computation shows that f_i is decreasing in β with $\lim_{\beta \to 1} f_i(\beta) = i-1$. It follows that $f_i(\beta) > f_i(1) = i-1 \geq n_l$, which implies $\frac{\partial w_{N,C}(B)}{\partial B} - \frac{\partial w_{N,R}(B)}{\partial B} > 0$.

Since $w_{N,R}(B_{i+1}) \geq w_{N,C}(B_{i+1})$ by the choice of i and the infinitesimal increase of $w_{N,C}$ is larger than that of $w_{N,R}$ on the entire interval $(B_i, B_{i+1}]$, this shows that $w_{N,R}(B) \geq w_{N,C}(B)$ for any $B \in (B_i, B_{i+1}]$. In particular, we have shown that $w_{N,R}(B) \geq w_{N,C}(B)$ for any $B \in (B_*, B_{j+1}]$ and hence B_{j+1} serves as B'.

Similarly, we observe that $w_{N,C}(B) - w_{N,R}(B)$ remains positive for $B > B_k$ where B_k is the smallest integer i, for which $\lim_{\tilde{B} \searrow B_i} w_{N,C}\big(\tilde{B}\big) - w_{N,R}\big(\tilde{B}\big) \geq 0$. Indeed, on $(B_*, B^*]$, $w_{N,C}$ is continuous and $w_{N,R}$ has discontinuities only at B_i with a jump size $\Delta w_{N,R}(B_{i+1}) = (1-\beta)(L - z_h)$ for any $i > n_l + 1$. Since the jump sizes are constant and $(Z_{i+1})_{i \geq i_*}$ is decreasing, it follows that $\lim_{\tilde{B} \searrow B_i} w_{N,C}\big(\tilde{B}\big) - w_{N,R}\big(\tilde{B}\big) \geq 0$ for all $i \geq k$. Since $\frac{\partial w_{N,C}(B)}{\partial B} - \frac{\partial w_{N,R}(B)}{\partial B} > 0$ on (B_i, B_{i+1}) for any $i \geq k$ and $w_{N,C} - w_{N,R}$ is positive at left limits of these intervals, it follows that $w_{N,C} \geq w_{N,R}$ on $[B_k, B^*]$ and hence B_k serves as B''.

Finally, let j_0 denote the largest integer such that $B_{j_0+1} \leq B^*$. It follows from the definition of B^* and Lemma B.1 that $w_{N,C}(B^*) + B^*$ is constant in β. Equation (18) shows that $\frac{\partial w_{N,R}(B)+B}{\partial B}$ is positive on intervals $(B_i, B_{i+1}]$. Since $\frac{\partial B^*}{\partial \beta}$ is positive as well, it follows that $w_{N,R}(B^*) - w_{N,C}(B^*) = w_{N,R}(B^*) + B^* - \big(w_{N,C}(B^*) + B^*\big)$ is increasing in β, where B_{j_0+1} is constant. Because $Z_{i+1} - Z_i = V_{0,h} - \frac{n_h}{\alpha d}(B_{i+1} - B_i)$ is decreasing in β, we deduce by backward induction that Z_{i+1} is increasing in β for any i with $i_0 \leq i \leq j_0$. This implies that the sequence $(Z_i)_{i \geq i_0}$ crosses the thresholds 0 and $-(1-\beta)(L - z_h)$ later as β increases and hence B_{j+1} and B_k are increasing in β. $\square$

C Proofs of auxiliarly reults

Lemma 2.1 follows as a consequence of the following result, which is a straightforward adaptation of Theorem 1 in Rogers and Veraart (2013) to our setting.

Lemma C.1. *Let $\varphi^{(k)}$ denote the k-fold application of the operator*

$$\varphi^i(p) := \begin{cases} L^i & \text{if } c^i + \alpha e^i + \sum_{j=1}^n \pi^{ij} p^j \geq L^i, \\ \Big(c^i + \alpha e^i + \beta \sum_{j=1}^n \pi^{ij} p^j\Big)^+ & \text{otherwise.} \end{cases}$$

Then $\hat{p} = \lim_{k \to \infty} \varphi^{(k)}(L)$ and $\hat{\ell} = \min\Big(\frac{1}{\alpha}(L - c - \pi\hat{p})^+, e\Big)$.

Simultaneous proofs of Lemma 2.1 and C.1. Note first that the payment vector p of any clearing equilibrium (ℓ, p) is a fixed point of the operator φ because a bank i defaults if and only if it cannot repay its liabilities after the liquidation of the entire outside asset. A fixed point p of φ can be completed to a clearing equilibrium by assigning $\ell(p) = \min\big(\frac{1}{\alpha}(L - z - \pi p)^+, e\big)$, where ℓ is defined in (3). Conversely, any clearing equilibrium (ℓ, p) is of the form $\big(\ell(p), p\big)$.

The same arguments as in the proof of Theorem 1 in Rogers and Veraart (2013) show that there exist $\underline{p}$ and $\hat{p}$ such that $\underline{p} \le p \le \hat{p}$ for any fixed point p of φ. Moreover, $(\hat{\ell}, \hat{p})$ can be found with the algorithm in Lemma C.1. Since $\ell(p)$ non-increasing in p, it follows that $\hat{\ell} := \ell(\hat{p}) \le \ell \le \ell(\underline{p})$ for any clearing equilibrium (ℓ, p). The final statement follows from monotonicity of the banks' value of equity and the deadweight loss in the clearing equilibrium (ℓ, p). $\square$

Proof of Lemma 4.1. The value of recalled interbank assets of any defaulting bank $i \in \mathcal{D}$ equals $\beta(\pi\hat{p})^i = \beta\big(\pi^{i,\mathcal{S}} L^{\mathcal{S}} + \pi^{i,\mathcal{D}} L^{\mathcal{D}} - \pi^{i,\mathcal{D}} L^{\mathcal{D}} + \pi^{i,\mathcal{D}} \hat{p}^{\mathcal{D}}\big) = \beta(A^i - \zeta^i)$. By the definition of a clearing equilibrium, we have $\hat{p}^i - \delta^i = c^i + \alpha e^i + \beta(\pi\hat{p})^i$ for any $i \in \mathcal{D}$, which can be rewritten as

$$V_0^i - (1-\alpha)e^i - (1-\beta)A^i - \beta\zeta^i + L^i - \hat{p}^i + \delta^i = 0, \qquad i \in \mathcal{D}. \tag{23}$$

Since $\hat{p}^{\mathcal{S}} = L^{\mathcal{S}}$, summing $L^i - \hat{p}^i$ over $i \in \mathcal{D}$ yields $\|L - \hat{p}\|_1$. Because π is doubly stochastic, this is equal to $\|\zeta\|_1$. Summing (23) over $i \in \mathcal{D}$ thus yields

$$\big\|V_0^{\mathcal{C}} - (1-\alpha)e^{\mathcal{C}}\big\|_1 - B - (1-\beta)\big\|A^{\mathcal{D}} - \zeta^{\mathcal{D}}\big\|_1 + \big\|\zeta^{\mathcal{S}}\big\|_1 + \big\|\delta^{\mathcal{D}}\big\|_1 = 0.$$

Using the definition of ξ, we can rewrite this identity as

$$\big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1 - \big(B - \big\|\delta^{\mathcal{D}}\big\|_1\big) = (1-\beta)\big\|(\pi\hat{p})^{\mathcal{D}}\big\|_1 = (1-\beta)\big\|A^{\mathcal{D}} - \zeta^{\mathcal{D}}\big\|_1, \tag{24}$$

hence $w_N = (1-\alpha)\big\|\hat{\ell}\big\|_1 + \big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1 - B + (1+\lambda)\big\|\delta^{\mathcal{D}}\big\|_1$. For $i \in \mathcal{C}\cup\mathcal{S}$, we have $\xi^i + (1-\alpha)\hat{\ell}^i = V_0^i - V_N^i$ by Lemma A.1. Now $B = \sum_{i\in\mathcal{F}}(1-\alpha)e^i - V_0^i$ implies

$$w_N = \big\|V_0^{\mathcal{C}\cup\mathcal{S}} - V_N^{\mathcal{C}\cup\mathcal{S}}\big\|_1 + (1-\alpha)\big\|e^{\mathcal{F}}\big\|_1 - B + (1+\lambda)\big\|\delta^{\mathcal{D}}\big\|_1,$$

Together with Lemma 2.2 this shows that $w_N - w_P \le 0$ is equivalent to (6). $\square$

Proof of Lemma 4.2. We have shown the first statement in (24) already. For the second statement, observe that the clearing payment $\hat{p}^{\mathcal{D}}$ is a solution to

$$\hat{p}^{\mathcal{D}} - \delta^{\mathcal{D}} = c^{\mathcal{D}} + \alpha e^{\mathcal{D}} + \beta\pi^{\mathcal{D},\mathcal{S}} L^{\mathcal{S}} + \beta\pi^{\mathcal{D},\mathcal{D}}\hat{p}^{\mathcal{D}}. \tag{25}$$

For $\beta < 1$, the spectral radius of $I - \beta\pi^{\mathcal{D},\mathcal{D}}$ is strictly smaller than 1 and hence $I - \beta\pi^{\mathcal{D},\mathcal{D}}$ is invertible. The system (25) thus admits the explicit solution

$$\hat{p}^{\mathcal{D}} = \big(I - \beta\pi^{\mathcal{D},\mathcal{D}}\big)^{-1}\big(c^{\mathcal{D}} + \alpha e^{\mathcal{D}} + \beta\pi^{\mathcal{D},\mathcal{S}} L^{\mathcal{S}} + \delta^{\mathcal{D}}\big).$$

Using that $L^{\mathcal{D}} = \big(I - \beta\pi^{\mathcal{D},\mathcal{D}}\big)^{-1}\big(L^{\mathcal{D}} - \beta\pi^{\mathcal{D},\mathcal{D}} L^{\mathcal{D}}\big)$ and $A^{\mathcal{D}} = \pi^{\mathcal{D},\mathcal{D}} L^{\mathcal{D}} + \pi^{\mathcal{D},\mathcal{S}} L^{\mathcal{S}}$, we obtain

$$L^{\mathcal{D}} - \hat{p}^{\mathcal{D}} = \big(I - \beta\pi^{\mathcal{D},\mathcal{D}}\big)^{-1}\big(L^{\mathcal{D}} - c^{\mathcal{D}} - \alpha e^{\mathcal{D}} - \beta A^{\mathcal{D}} + \delta^{\mathcal{D}}\big).$$

This readily implies the statement. □

Proof of Lemma 4.3. Equation (9) implies that $w_N \leq w_P$ if and only if

$$(1-\beta)(1+\lambda)\big\|A^{\mathcal{D}} - \zeta^{\mathcal{D}}\big\|_1 \leq \lambda\big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1 - (1-\alpha)\big\|\hat{\ell}\big\|_1 + \min(\lambda\alpha, 1-\alpha)\|\ell_*\|_1. \tag{26}$$

We will show that for $\beta = 1$, the right-hand side of (26) is strictly positive. This implies that the social planner's threat is credible for $\beta = 1$. Moreover, since the left-hand side of (26) is continuous in β, this implies that the threat is credible also for $\beta < 1$ sufficiently large.

For $\alpha \geq 1/(1+\lambda)$, the welfare loss $(1-\alpha)\ell^i(\xi)$ due to liquidation of bank i's outside asset is smaller than or equal to $\lambda\xi^i$, hence the right-hand side of (26) is strictly positive as long as $\alpha < 1$. Suppose now that $\alpha < 1/(1+\lambda)$ and that (8) is satisfied instead. A quick calculation shows that (8) is equivalent to

$$(1-\alpha)\big\|e^{\mathcal{D}\cup\mathcal{B}}\big\|_1 < \lambda\alpha\big\|e^{\mathcal{F}}\big\|_1 + \lambda\big\|V_0^{\mathcal{C}\cup\mathcal{B}}\big\|_1 - \lambda(1-\alpha)\big\|e^{\mathcal{C}\cup\mathcal{B}}\big\|_1. \tag{27}$$

For any bank $i \in \mathcal{S}$, define the function $f^i(x) = (1-\alpha)\ell^i(x) - \lambda x^i$ and observe that f^i is decreasing until $x^i = V_0^i - e^i$ and increasing afterwards. Since

$f^i(0)=0$ for any $i\in\mathcal{S}$ and $f^i\big(V_0-(1-\alpha)e\big)>0$ if and only if $i\in\mathcal{B}$, this shows that $f^i(\zeta)\le f^i\big(V_0-(1-\alpha)e\big)$ for any $i\in\mathcal{B}$ and any value of ζ^i. We obtain the following auxiliary inequality

$$\lambda\zeta^i=(1-\alpha)\ell^i(\zeta)-f^i(\zeta)\ge(1-\alpha)\ell^i(\zeta)-f^i\big(V_0-(1-\alpha)e\big),\qquad i\in\mathcal{B},\ (28)$$

that we will use later. Next, we show that $(1-\alpha)\hat\ell^i\le\lambda\zeta^i$ for a bank $i\in\mathcal{S}\setminus\mathcal{B}$. Indeed, if $\zeta^i\le V_0^i-e^i$, then bank i does not need to liquidate anything and hence $\hat\ell^i=0\le\lambda\zeta^i$ is trivially satisfied. For $\zeta^i>V_0^i-e^i$, the function f^i is increasing and hence $f^i(\zeta)\le f^i\big(V_0-(1-\alpha)e\big)\le 0$, where the latter inequality holds since $i\notin\mathcal{B}$. We have thus shown that $(1-\alpha)\hat\ell^i\le\lambda\zeta^i$ for any $i\in\mathcal{S}\setminus\mathcal{B}$ and any ζ^i. Together with (27) and (28), thus implies that

$$\begin{aligned}(1-\alpha)\big\|\hat\ell\big\|_1&\le(1-\alpha)\big\|e^{\mathcal{D}\cup\mathcal{B}}\big\|_1+(1-\alpha)\big(\big\|\hat\ell^{\mathcal{B}}\big\|_1-\big\|e^{\mathcal{B}}\big\|_1\big)+\lambda\big\|\xi^{\mathcal{S}\setminus\mathcal{B}}\big\|_1\\&<\lambda\big\|\xi^{\mathcal{C}\cup\mathcal{S}\setminus\mathcal{B}}\big\|_1+\lambda\alpha\big\|e^{\mathcal{F}}\big\|_1+(1-\alpha)\big\|\hat\ell^{\mathcal{B}}\big\|_1-\sum_{i\in\mathcal{B}}f^i\big(V_0-(1-\alpha)e\big)\\&\le\lambda\big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1+\lambda\alpha\big\|e^{\mathcal{F}}\big\|_1.\end{aligned}$$

Since $\big\|e^{\mathcal{F}}\big\|_1\le\|\ell_*\|_1$, this shows that the right-hand side of (26) is positive, thereby concluding the proof. □

Proof of Lemma 4.5. Note first that $w_P=\lambda B+(1-\alpha)\|\ell_*\|_1-\big(1-(1+\lambda)\alpha\big)^+\|\ell_*\|_1$. Together with (9), this shows that

$$w_N-w_P=(1-\alpha)\big\|\hat\ell-\ell_*\big\|_1+\big(1-(1+\lambda)\alpha\big)^+\|\ell_*\|_1+\big\|\xi^{\mathcal{C}\cup\mathcal{S}}\big\|_1-(1+\lambda)\big(B-\big\|\delta^{\mathcal{D}}\big\|_1\big).$$

For fixed B, the interbank repayments $\hat p$ are increasing in α and β since a larger amount can be recovered and $\big\|\delta^{\mathcal{D}}\big\|_1$ is decreasing in α and β for the same reasons. Therefore, $\zeta^i=\big(\pi(L-\hat p)\big)^i$ is decreasing and so is $\hat\ell^i=\frac{1}{\alpha}(\zeta^i+e^i-V_0^i)^+$. Since ℓ_*^i is constant for fixed B and smaller or equal to $\hat\ell^i$, the statement follows. □