

NBER WORKING PAPER SERIES

FUND TRADEOFFS

Lubos Pastor
Robert F. Stambaugh
Lucian A. Taylor

Working Paper 23670
<http://www.nber.org/papers/w23670>

NATIONAL BUREAU OF ECONOMIC RESEARCH

1050 Massachusetts Avenue
Cambridge, MA 02138
August 2017, Revised October 2018

This paper previously circulated under the title “Portfolio Liquidity and Diversification: Theory and Evidence.” The views in this paper are the responsibility of the authors, not the institutions they are affiliated with, nor are the necessarily those of the National Bureau of Economic Research. We are grateful for comments from our conference discussants Daniel Andrei, Wayne Ferson, Marcin Kacperczyk, Clemens Sialm, Ashish Tiwari, and Mungo Wilson. We are also grateful for comments from Jonathan Berk, Gene Fama, Laszlo Jakab, Don Keim, Rob Vishny, and Yao Zeng, conference participants at the Western Finance Association, European Finance Association, Duke/UNC Asset Pricing Conference, ESSFM Gerzensee, FARFE (MIT), GSU-CEAR Finance Conference, Conference on Professional Asset Management (Erasmus), Q Group, and seminar participants at Boston University, University of Chicago, University of Illinois at Urbana-Champaign, University of Pennsylvania, University of Utah, and WU Vienna. We are also grateful to Will Cassidy, Yeguang Chi, and Pierre Jaffard for superb research assistance. This research was funded in part by the Fama-Miller Center for Research in Finance and the Center for Research in Security Prices at Chicago Booth.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at <http://www.nber.org/papers/w23670.ack>

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2017 by Lubos Pastor, Robert F. Stambaugh, and Lucian A. Taylor. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Fund Tradeoffs

Lubos Pastor, Robert F. Stambaugh, and Lucian A. Taylor

NBER Working Paper No. 23670

August 2017, Revised October 2018

JEL No. G11,G23

ABSTRACT

We derive equilibrium relations among active mutual funds' key characteristics: fund size, expense ratio, turnover, and portfolio liquidity. Portfolio liquidity, a concept introduced here, depends not only on the liquidity of the portfolio's holdings but also on the portfolio's diversification. As our model predicts, funds with smaller size, higher expense ratios, and lower turnover hold less-liquid portfolios. Additional model predictions are also supported empirically: Larger funds are cheaper. Larger and cheaper funds trade less and are less active, based on our novel measure of activeness. Better-diversified funds hold less-liquid stocks; they are also larger, cheaper, and trade more.

Lubos Pastor
University of Chicago
Booth School of Business
5807 South Woodlawn Ave
Chicago, IL 60637
and NBER
lubos.pastor@chicagobooth.edu

Lucian A. Taylor
Finance Department
The Wharton School
University of Pennsylvania
2300 Steinberg Hall - Dietrich Hall
3620 Locust Walk
Philadelphia, PA 19104-6367
luket@wharton.upenn.edu

Robert F. Stambaugh
Finance Department
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367
and NBER
stambaugh@wharton.upenn.edu

1. Introduction

Mutual funds manage tens of trillions of dollars. Through their investment decisions, these funds play a major role in allocating capital in the economy. Active mutual funds differ in numerous respects, including their size, expense ratio, turnover, and the liquidity of their investments. Are there any tradeoffs among these characteristics? For example, do funds with higher expense ratios trade more heavily, or less? Do funds that trade more heavily have more liquid investments, or less? Are larger funds more expensive, or less? We attempt to answer such questions, both theoretically and empirically.

The most popular topic in the mutual fund literature is fund performance. Our focus on the tradeoffs among fund characteristics can be motivated by Berk and Green (2004), who argue that each fund's expected performance—return relative to a passive benchmark—is zero in equilibrium. If expected performance is zero, then realized performance is rather uninformative. Our study therefore turns to fund characteristics as a potentially richer source of insights into the economics of mutual funds.

We derive equilibrium relations among four key fund characteristics: fund size, expense ratio, turnover, and portfolio liquidity. This last characteristic is novel. While the literature presents a variety of liquidity measures for individual securities, it offers little guidance for assessing liquidity at the portfolio level. We introduce the concept of portfolio liquidity and show that funds trade off this characteristic against others in important ways. Our measure of portfolio liquidity is derived theoretically based on the simple idea that a portfolio is more liquid if it has lower trading costs. Specifically, if one trades equal dollar amounts of two portfolios, the portfolio with lower trading costs has greater liquidity.

We develop an equilibrium model relating portfolio liquidity to fund size, expense ratio, and turnover. When choosing its characteristics, a fund recognizes that lower liquidity and higher turnover raise expected gross profits but also raise transaction costs. Those costs increase in the fund's size as well. This role of fund size is recognized by investors when they decide how much capital to allocate to the fund, as in Berk and Green (2004).

The model implies a novel link between the four key mutual fund characteristics. Funds whose portfolios are less liquid should have smaller size, higher expense ratios, and lower turnover. We investigate these equilibrium tradeoffs in a sample of 2,789 active U.S. equity mutual funds from 1979 through 2014. When we estimate the cross-sectional regression of portfolio liquidity on fund size, expense ratio, and turnover in our panel dataset, we find strong support for the model. All three slopes have their predicted signs and are highly significant, both economically and statistically, with t -statistics ranging from 4.9 to 13.8.

Funds that are smaller, more expensive, and trade less tend to hold less-liquid portfolios, as the model predicts.

The model also makes predictions for correlations among fund characteristics. First, larger funds should be cheaper. In the data, the correlation between fund size and expense ratio is indeed negative, both in the cross section (-32%) and in the time series (-25%). The model also predicts that fund turnover should be negatively related to fund size and positively related to expense ratio. These relations hold strongly in the data as well: funds that trade less are larger and cheaper, both across funds and over time.

We also offer new insights into fund activeness. First, in gauging a fund's activeness, the fund's turnover should be included. Our model delivers a novel measure of activeness that combines turnover with portfolio liquidity. The latter characteristic depends on the portfolio's weights versus the benchmark, with a less-liquid portfolio being more active. In that respect our measure of activeness resembles the popular active share measure of Cremers and Petajisto (2009). Portfolio holdings are only part of the story, though. In our model, a fund is also more active if it trades more. The model implies that more active funds should be smaller and more expensive, and we find evidence of both tradeoffs in the data.

A fund's scale is typically equated to its size. Our study implies a new concept of scale, which depends not only on the fund's size but also on its activeness. In our model, funds face decreasing returns to scale, but the implied measure of scale is size times activeness, not simply size. This idea makes intuitive sense. If two funds manage equal amounts of money, but one of them deploys its money more actively, it seems reasonable to view that fund as operating at a larger scale, essentially leaving a bigger footprint in the market.

In deriving our measure of portfolio liquidity, we apply a familiar concept: less-liquid assets are costlier to trade. We extend this concept to portfolios, viewing a portfolio as an asset and thereby considering the cost of trading the portfolio as a given basket of securities. When assessing portfolio liquidity, it seems natural to consider the average liquidity of the portfolio's constituents. For example, portfolios of small-cap stocks tend to be less liquid than portfolios of large-cap stocks. While this assessment is a useful starting point, it is incomplete. We show that a portfolio's liquidity depends not only on the liquidity of the stocks held in the portfolio but also on the degree to which the portfolio is diversified:

$$\text{Portfolio Liquidity} = \text{Stock Liquidity} \times \text{Diversification} . \tag{1}$$

The more diversified a portfolio, the less costly is trading a given fraction of it. For example, a fund trading just 1 stock will incur higher costs than a fund spreading the same dollar amount of trading over 100 stocks, even if all of the stocks are equally liquid. Throughout,

we focus on equity portfolios, but our ideas are more general.

Our measure of portfolio liquidity is easy to calculate from the portfolio’s composition. Following equation (1), our measure has two components. The first, stock liquidity, reflects the average market capitalization of the portfolio’s holdings. The second component, diversification, has its own intuitive decomposition:

$$\text{Diversification} = \text{Coverage} \times \text{Balance} . \tag{2}$$

Coverage reflects the number of stocks in the portfolio. Portfolios holding more stocks have greater coverage. Balance reflects how the portfolio weights the stocks it holds. Portfolios with weights closer to market-cap weights have greater balance.

Diversification’s role in portfolio liquidity is important empirically. We compute our measures of portfolio liquidity and diversification, relative to the value-weighted market benchmark, for the mutual funds in our sample. We find that fund portfolios have become more liquid over time, from 1979 through 2014. Average portfolio liquidity almost doubled over the sample period, driven by diversification. Diversification quadrupled, as both of its components in equation (2) rose steadily. Coverage rose because the number of stocks held by the average fund grew from 54 to 126. Balance rose because funds’ portfolio weights increasingly resembled market-cap weights.¹

Our model predicts tradeoffs between diversification and other fund characteristics. In equilibrium, funds with more-diversified portfolios should be larger and cheaper, they should trade more, and their stock holdings should be less liquid. We find strong empirical support for all four predictions. The negative relation between diversification and stock liquidity implies that these components of portfolio liquidity are substitutes: funds holding less-liquid stocks make up for it by diversifying more, and vice versa. The components of diversification, coverage and balance, are also substitutes: portfolios with lower coverage tend to be better balanced, and vice versa. Both substitution effects are predicted by our model.

Our focus on fund characteristics, and the tradeoffs among them, seems novel. Some of the concepts we address are familiar, however. One strand of related literature studies returns to scale in active management. This literature explores the hypothesis that as a fund’s size increases, its ability to outperform its benchmark declines (Berk and Green, 2004).² This hypothesis is motivated by liquidity constraints. Being larger erodes performance because

¹The increased resemblance of active funds’ portfolios to the market benchmark is also apparent from measures such as active share and tracking error (e.g., Cremers and Petajisto, 2009, and Stambaugh, 2014).

²This is the hypothesis of fund-level decreasing returns to scale. A complementary hypothesis of industry-level decreasing returns to scale is that as the size of the active mutual fund industry increases, the ability of any given fund to outperform declines (see Pástor and Stambaugh, 2012, and Pástor, Stambaugh, and Taylor, 2015). In this paper, we focus on the fund-level hypothesis.

a larger fund trades larger dollar amounts, and trading larger dollar amounts incurs higher proportional trading costs. The hypothesis has received some empirical support. Fund size appears to negatively predict fund performance, especially among funds holding small-cap stocks (Chen et al., 2004) and less-liquid stocks (Yan, 2008), suggesting that the adverse effects of scale are related to liquidity. However, the evidence on the size-performance relation is fairly sensitive to the methodological approach, as discussed, for example, by Pástor, Stambaugh, and Taylor (2015).³ We do not examine fund performance. Our analysis of tradeoffs among fund characteristics reveals different evidence of decreasing returns to scale. We find that larger funds tend to have lower turnover and higher portfolio liquidity. This evidence is in line with our model, in which diseconomies of scale lead larger funds to trade less and hold more-liquid portfolios, either by holding more-liquid stocks or by diversifying more. Our results represent strong evidence of decreasing returns to scale, with a refined notion of scale, as explained earlier. It is not clear what mechanism other than decreasing returns to scale could explain why larger funds trade less and hold more-liquid portfolios.

Two other studies provide related evidence on returns to scale. Pollet and Wilson (2008) find that mutual funds respond to asset growth mostly by scaling up existing holdings rather than by increasing the number of stocks held. But the authors also find that larger funds and small-cap funds are less reluctant to diversify in response to growth, exactly as our theory predicts. In their comprehensive analysis of mutual fund trading costs, Busse et al. (2017) report that larger funds trade less and hold more-liquid stocks. This evidence, which overlaps with our findings, also supports our model. In the language of equation (1), Busse et al. show that larger funds have higher stock liquidity; we show they also have higher diversification. The evidence of Busse et al. is based on a sample much smaller than ours (583 funds in 1999 through 2011), dictated by their focus on trading costs. Neither Busse et al. nor Pollet and Wilson do any theoretical analysis.

Our study is also related to the literature on portfolio diversification. We propose a new measure of diversification that has strong theoretical motivation. Our measure blends features of two ad-hoc measures, the number of stocks held and the Herfindahl index of portfolio weights. By using our measure, we show that mutual funds have become substantially more diversified over time, yet their diversification remains relatively low.⁴ We also derive predictions for the determinants of diversification. Funds with more-diversified portfolios should be larger and cheaper, they should trade more, and their holdings should be less liquid, on

³For additional evidence on returns to scale in mutual funds, see, for example, Bris et al. (2007), Pollet and Wilson (2008), Reuter and Zitzewitz (2015), and Harvey and Liu (2017).

⁴Low diversification by institutional investors is also reported by Kacperczyk, Sialm, and Zheng (2005), Pollet and Wilson (2008), and others. Household portfolios also exhibit low diversification, as shown by Blume and Friend (1975), Polkovnichenko (2005), Goetzmann and Kumar (2008), and others.

average. We find strong empirical support for all of these predictions.

The rest of the paper is organized as follows. Section 2 introduces our measure of portfolio liquidity. Section 3 examines the tradeoffs among fund characteristics. Section 4 analyzes tradeoffs that involve the components of portfolio liquidity, including diversification. Section 5 addresses tradeoffs that involve fund activeness. Section 6 explores fund tradeoffs in the form of simple correlations. Section 7 rethinks the concept of scale. Section 8 contrasts our setting with that of Berk and Green (2004). Section 9 concludes. Additional material is in Appendices A through C. Further empirical results are in the Internet Appendix, which is available on the authors' websites.

2. Introducing Portfolio Liquidity

The definition of portfolio liquidity is based on trading costs: If one trades the same dollar amounts of two portfolios, the portfolio generating lower trading costs has greater liquidity. We show that this fundamental concept is captured by the following measure:

$$L = \left(\sum_{i=1}^N \frac{w_i^2}{m_i} \right)^{-1}, \quad (3)$$

where N is the number of stocks in the portfolio, w_i is the portfolio's weight on stock i , and m_i denotes the weight on stock i in a market-cap-weighted benchmark portfolio. The latter portfolio can be the overall market, the most familiar benchmark, or it can be the portfolio of all stocks in the sector in which the portfolio is focused, such as large-cap growth. We apply both choices in our empirical analysis of active mutual funds.

To derive our measure, we begin with the familiar concept that less-liquid assets are costlier to trade. We apply this concept to portfolios by considering the cost of trading the portfolio as a whole, as if it were just another asset. That is, if D is the total dollar amount traded of the portfolio, the dollar amount traded of stock i is

$$D_i = Dw_i. \quad (4)$$

The corresponding total trading cost is

$$C = \sum_{i=1}^N D_i C_i, \quad (5)$$

where C_i is the cost per dollar traded of stock i . We assume C_i is larger when trading a larger fraction of M_i , the market capitalization of stock i . Specifically,

$$C_i = c \frac{D_i}{M_i}, \quad (6)$$

where the positive constant c is identical across the stocks in the benchmark. Equation (6) reflects the basic idea that larger trades have higher proportional trading costs, such as price impact. This idea has strong empirical support (e.g., Keim and Madhavan, 1997). The linearity of equation (6) implies that trading, say, 1% of a stock’s market capitalization costs twice as much per dollar traded compared to trading 0.5% of the stock’s capitalization.⁵

Combining equations (4) through (6), we can rewrite the total trading cost as

$$C = (c/M) D^2 \underbrace{\left(\sum_{i=1}^N \frac{w_i^2}{m_i} \right)}_{L^{-1}}, \quad (7)$$

where $m_i = M_i/M$. We define M as the market capitalization of all stocks in the benchmark portfolio, which allows L to be compared across portfolios having the same benchmark. Equation (7) shows that the expression for portfolio liquidity, given by equation (3), arises from trading costs. Trading a given dollar amount, D , of a portfolio with lower liquidity, L , incurs a greater total cost, C .

Portfolio liquidity is a characteristic that does not hinge on the trading behavior of whoever might hold the portfolio. For example, we do not assume any mutual fund actually trades each stock according to equation (4). The same portfolio can be held by two different funds trading in different ways, yet there is only one portfolio-specific value of L . In this respect we maintain the perspective on liquidity that is widely accepted for individual stocks. Measures of a stock’s liquidity, such as its bid-ask spread or its turnover, do not hinge on the behavior of whoever is trading the stock. Two investors trading equal amounts of the same stock often incur different costs, depending on how patiently they trade, how they execute their trades, etc. Nevertheless, less-liquid stocks are generally assumed to be costlier to trade. We simply assume the same about portfolios. In our analysis of fund tradeoffs in the following section, we assume that funds holding portfolios with lower L incur higher trading costs, *ceteris paribus*, regardless of how these funds trade.

Our measure of portfolio liquidity takes values between 0 and 1. The least liquid portfolio is fully invested in a single stock, the one with the smallest market capitalization among stocks in the benchmark. The liquidity of this portfolio is equal to the benchmark’s market-cap weight on that smallest stock, so L can be nearly 0. A portfolio can be no more liquid than its benchmark, for which $L = 1$. This statement is proven in Appendix A, but its simple

⁵A linear function for the proportional trading cost in a given stock is entertained, for example, by Kyle and Obizhaeva (2016). That study examines portfolio transition trades and concludes that a linear function fits the data only slightly less well than a nonlinear square-root specification. The assumption of linearity substantially simplifies our theoretical analysis, but we also consider nonlinear trading costs in Appendix B. We show that our main empirical results are similar for a wide range of nonlinearities.

intuition follows from the trading-cost assumption in equation (6). When trading a given dollar amount of the benchmark portfolio, which has market-cap weights, the proportional cost of trading each stock is equal across stocks. With this cost denoted by κ , the proportional cost of the overall trade is also κ . If the benchmark portfolio is perturbed by buying one stock and selling another, then more weight is put on a stock whose proportional cost is now greater than κ , and less weight is put on a stock whose proportional cost is now smaller than κ . Therefore, the proportional cost of trading the same dollar amount of this alternative portfolio exceeds κ .

Another important property of our portfolio liquidity measure L in equation (3) is that it is increasing in the portfolio's diversification, as indicated in equation (1). Better-diversified portfolios are more liquid. We clarify the role of diversification in Section 4.

3. Tradeoffs Among Fund Characteristics

In this section, we examine the relations among key characteristics of active funds: portfolio liquidity, fund size, expense ratio, and turnover. We first derive such relations theoretically, from optimizing behavior of fund managers and investors. We then verify these relations empirically.

3.1. Fund Characteristics in Equilibrium

An actively managed fund chooses its turnover, T , portfolio liquidity, L , and expense ratio, f , which we treat as its fee rate. The fund's objective is to maximize its total fee revenue,

$$F = fA , \tag{8}$$

where A is the fund's equilibrium "size," or assets under management. Following Berk and Green (2004), we assume that competing investors allocate the amount A to the fund such that, in equilibrium, the fund's expected return net of fees and trading costs is zero:

$$\alpha = 0 . \tag{9}$$

As explained below, however, this assumption can be relaxed to some extent. The equilibrium is partial in the same sense as, for example, the equilibrium in Berk and Green (2004) (e.g., we do not model the sources of funds' profit opportunities). Throughout, fund returns are benchmark-adjusted.

The fund's expected gross return, before fees and costs, depends on the fund's skill as well as how actively that skill is applied. To capture this interaction, we model the expected gross return as

$$a = \mu g(T, L) , \quad (10)$$

where μ is a fund-specific positive constant reflecting skill in identifying profitable trading opportunities. How actively the fund applies that skill is represented by the function $g(T, L)$, which is increasing in T and decreasing in L . A fund is more active if it trades more and if it holds a less-liquid portfolio. The roles of T and L in equation (10) are discussed in more detail in Section 5, which analyzes fund activeness and the related tradeoffs.

The fund's expected total trading cost is given by the function

$$C(A, T, L) = \theta A^\gamma T^\lambda L^{-\phi} , \quad (11)$$

where θ , γ , λ , and ϕ are positive constants, with $\gamma > 1$. Given T and L , trading costs are increasing and convex in A , capturing this familiar role of fund size modeled by Berk and Green (2004). In contrast to that study, we also let trading costs depend on turnover and portfolio liquidity. The relevance of T for equation (11) is clear because, holding A and L constant, a fund that trades more incurs higher trading costs.

The inclusion of L in equation (11) is motivated by the arguments from Section 2. Recall from equation (7) that a portfolio with a lower L incurs higher trading costs, holding A and T fixed. Saying that it is costlier to trade a portfolio with a lower L is like saying it is costlier to trade a stock with a higher bid-ask spread or any other illiquidity measure. Saying that such a stock is costlier to trade does not assume one trades the stock in a particular way, e.g., trading at the bid and ask quotes. In the same vein, saying that a fund with a lower L incurs higher trading costs does not assume the fund trades its portfolio in a particular way, e.g., trading stocks in proportion to portfolio weights. We assign a negative but flexible role to L in determining trading costs (as well as flexible positive roles to A and T) by treating the exponents in equation (11) as free parameters. That equation does nest equation (7) as a special case when $\theta = c/M$, $\gamma = 2$, $\lambda = 2$, and $\phi = 1$, with the fund's traded dollar amount recognized as $D = AT$.

Given the specifications of expected gross return and costs from equations (10) and (11), a fund's expected return net of costs and fees equals

$$\begin{aligned} \alpha &= a - C(A, T, L)/A - f \\ &= \mu g(T, L) - \theta A^{\gamma-1} T^\lambda L^{-\phi} - f . \end{aligned} \quad (12)$$

Equations (9) and (12) imply that the fund's equilibrium size satisfies

$$A = \left(\frac{1}{\theta} T^{-\lambda} L^{\phi} [\mu g(T, L) - f] \right)^{\frac{1}{\gamma-1}} . \quad (13)$$

Multiplying both sides of equation (13) by f implies the equilibrium fee revenue equal to

$$F = \left(\frac{1}{\theta} T^{-\lambda} L^{\phi} [\mu g(T, L) - f] f^{\gamma-1} \right)^{\frac{1}{\gamma-1}} . \quad (14)$$

This fee revenue is maximized by the fund's choices of T , L , and f . The first-order condition $\partial F / \partial f = 0$ implies

$$\frac{1}{(\gamma-1)\theta} F^{2-\gamma} T^{-\lambda} L^{\phi} [(\gamma-1)f^{\gamma-2} \mu g(T, L) - \gamma f^{\gamma-1}] = 0 . \quad (15)$$

Because F , T , and L are all positive for an active fund, the bracketed term in equation (15) must be zero, which implies

$$\mu g(T, L) = \frac{\gamma}{\gamma-1} f . \quad (16)$$

Substituting the right-hand side of equation (16) for $\mu g(T, L)$ in equation (12), imposing equation (9), and taking logs, we obtain our key equilibrium relation:

$$\ln L = \left(\frac{\gamma-1}{\phi} \right) \ln A - \frac{1}{\phi} \ln f + \left(\frac{\lambda}{\phi} \right) \ln T + \ln[\theta(\gamma-1)] , \quad (17)$$

or

$$\ln L = b_1 \ln A - b_2 \ln f + b_3 \ln T + \text{constant} , \quad (18)$$

where b_1 , b_2 , and b_3 are positive and the constant equals $\ln[\theta(\gamma-1)]$.

Equation (18) captures the equilibrium tradeoffs among the four fund characteristics: portfolio liquidity L , fund size A , expense ratio f , and turnover T . A fund with a more liquid portfolio is larger and cheaper, and it trades more. To understand these novel predictions, consider the implication of changing one variable on the right-hand side of equation (18) while holding the other two variables constant. Holding f and T constant, a larger fund size A dictates a more liquid portfolio to offset the costs of trading larger amounts. Holding A and T constant, a higher f implies a less-liquid portfolio because, as our model also implies, greater fee revenue (Af) corresponds to greater skill.⁶ A more skilled fund can more effectively offset the higher trading costs associated with a less-liquid portfolio. For example, it can afford to concentrate its portfolio on its best ideas or to trade in less-liquid stocks, which are more susceptible to mispricing. Finally, holding A and f constant, the fund's fee

⁶This correspondence between fee revenue and skill, expected in a competitive market, is proven in Appendix A.

revenue is fixed, and so is its skill. Therefore, the greater cost of heavier trading (i.e., larger T) must be offset by holding a more liquid portfolio.

Equation (18) is quite general. Its derivation does not require any assumptions about the functional form of $g(T, L)$, or about how $g(T, L)$ is chosen by the fund. In Section 5.1, we do take a stand on $g(T, L)$ and let the fund choose its optimal level. Doing so allows us to derive additional fund tradeoffs that involve the fund’s activeness.

As mentioned earlier, the zero-alpha assumption in equation (9) can be relaxed somewhat, allowing, for example, for some forms of capital misallocation. Let $\alpha = \nu$, where ν is a non-zero quantity known to the fund. The fund’s optimal choices of its characteristics then in general depend on ν . However, it is easy to show that equation (17) still obtains. The derivation of equation (17) is essentially unchanged, with $\mu g(T, L)$ replaced by $\mu g(T, L) - \nu$ in equations (13) through (16). The main tradeoff implications of our model thus do not hinge on the zero-alpha assumption. Instead, they arise from the fund’s facing a known equilibrium α , unaffected by the fund’s choices. We assume $\alpha = 0$ simply because that condition seems easiest to motivate a priori, using the reasoning of Berk and Green (2004).

3.2. Empirical Evidence

We analyze a sample of 2,789 actively managed U.S. domestic equity mutual funds covering the 1979–2014 period. To construct this sample, we begin with the dataset constructed by Pástor, Stambaugh, and Taylor (2015, 2017), which combines data from the Center for Research in Securities Prices (CRSP) and Morningstar. We add three years of data and merge in the Thomson Reuters dataset of fund holdings. We restrict the sample to fund-month observations whose Morningstar category falls within the traditional 3×3 style box (small/mid/large-cap interacted with growth/blend/value). This restriction excludes non-equity funds, international funds, and industry-sector funds. We exclude index funds because our model is designed for active funds trying to outperform a benchmark. We also exclude funds of funds and funds smaller than \$15 million. We classify each fund into one of nine sectors corresponding to Morningstar’s 3×3 style box.⁷ A detailed description of our sample, including the variable definitions and their summary statistics, is in Appendix C.

For each fund and quarter-end, we compute portfolio liquidity from the fund’s quarterly holdings data. Initially, we compute portfolio liquidity by using the market portfolio as the

⁷Morningstar assigns funds to style categories based on the funds’ reported portfolio holdings, and it updates these assignments over time. Since the assignments are made by Morningstar rather than the funds themselves, there is no room for benchmark manipulation of the kind documented by Sensoy (2009). The benchmark assigned by Morningstar can differ from that reported in the fund’s prospectus.

benchmark. Our definition of the market portfolio includes ordinary common shares (CRSP share code with first digit equal to 1) and REIT shares of beneficial interest (CRSP share code of 48). This definition is guided by the end-of-sample holdings of the world’s largest mutual fund, Vanguard’s Total Stock Market Index fund, as we explain in Appendix C.

To test the predictions from equation (18), we estimate this equation as a regression of $\ln(L)$ on the other fund characteristics. Equation (18) does not represent a causal relation. Instead, it captures the equilibrium relation among the jointly determined, endogenous fund characteristics. Testing our model does not require that we estimate any causal relations.

We estimate the regression corresponding to equation (18) using our mutual fund dataset. The unit of observation is the fund/quarter. We include sector-quarter fixed effects in the regression, which offers three important benefits. First, we treat our model’s predictions as cross-sectional, and the fixed effects isolate variation across funds. In principle, one could also view our model as describing a given fund solving a series of single-period problems. However, applying the model to a fund’s time series would confront the problem that two fund characteristics, expense ratio and turnover, are measured in a way that is poorly suited for time-series analysis: turnover is measured only annually and expense ratios vary little over time. Second, by including sector-quarter fixed effects, we effectively use L defined with respect to a sector-specific benchmark rather than the market. Sector-benchmarked L is equal to market-benchmarked L divided by the fraction of the total stock market capitalization accounted for by the sector. Since that fraction is sector-specific within a given quarter, sector-benchmarked $\ln(L)$ is equal to market-benchmarked $\ln(L)$ minus a sector-quarter-specific constant that is absorbed by our fixed effects. Third, our model assumes c is constant, and this assumption is more likely to hold across funds within a given sector and quarter. The sector-quarter fixed effects absorb variation in $\ln(c/M)$, the constant in equation (18), both across sectors and over time. Our specification therefore allows liquidity conditions to vary over time and across sectors. Estimates of equation (18) that use only quarter fixed effects, equivalent to using market-benchmarked L , are quite similar (see the Internet Appendix).

Table 1 provides strong support for the model’s predictions in equation (18). The slope coefficients on all three regressors have their predicted signs, not only for the multiple regression, which is implied by the model, but also for simple regressions. Moreover, all three slopes are highly significant in the multiple regression. The slope on fund size ($t = 13.76$) shows that larger funds tend to have more-liquid portfolios. A one-standard-deviation increase in the logarithm of fund size is associated with a 0.22 standard-deviation increase in $\ln(L)$ (sector- and quarter-adjusted). The slope on expense ratio ($t = -11.26$) shows that cheaper funds tend to have more-liquid portfolios. The economic significance of expense ratio is com-

parable to that of fund size: a one-standard-deviation increase in $\ln(f)$ is associated with a 0.24 standard-deviation decrease in $\ln(L)$. Finally, the slope on turnover ($t = 4.93$) shows that funds that trade more tend to have more-liquid portfolios. A one-standard-deviation increase in $\ln(T)$ is associated with a 0.10 standard-deviation increase in $\ln(L)$. We conclude that funds with less-liquid portfolios trade less and are smaller and more expensive, fully in line with our theory.

4. Liquidity Tradeoffs

In this section, we examine fund tradeoffs that involve components of portfolio liquidity. After identifying these components in Section 4.1, we present empirical evidence of their tradeoffs in Section 4.2.

4.1. Components of Portfolio Liquidity

We show in Appendix A that portfolio liquidity from equation (3) can be decomposed as

$$L = \underbrace{\frac{1}{N} \sum_{i=1}^N \mathcal{L}_i}_{\text{Stock Liquidity}} \times \underbrace{\left(\frac{N}{N_M}\right) \left[1 + \text{Var}^* \left(\frac{w_i}{m_i^*}\right)\right]^{-1}}_{\text{Diversification}}. \quad (19)$$

The first component of L is the equal-weighted average of $\mathcal{L}_i = M_i/\bar{M}$, with \bar{M} denoting the average market capitalization of stocks in the benchmark. That is, $\bar{M} = \frac{1}{N_M} \sum_{j=1}^{N_M} M_j$, where N_M is the number of stocks in the benchmark. We label this component “stock liquidity” because \mathcal{L}_i captures the liquidity of stock i relative to all stocks in the benchmark. Stock liquidity is larger (smaller) than 1 if the portfolio’s holdings have a larger (smaller) average market capitalization than the average stock in the benchmark.

Using a stock’s market capitalization to measure its liquidity follows from our assumption (6), which implies that trading \$1 of stock i incurs a cost proportional to $1/m_i$. This implication is intuitive—trading a given dollar amount of a small-cap stock (whose m_i is small) incurs a larger price impact than trading the same dollar amount of a large-cap stock (whose m_i is large). Moreover, market capitalization is closely related to other measures of stock liquidity in the data. For example, we calculate the correlations between the log of market capitalization and the logs of two popular measures, the Amihud (2002) measure of illiquidity and dollar volume, across all common stocks. The two correlations average -0.91 and 0.90, respectively, across all years in our sample period. In a robustness analysis,

we show that alternative measures of stock liquidity, namely the Amihud measure, dollar volume, and the bid-ask spread, produce similar tradeoff results, especially with quarter fixed effects (see the Internet Appendix). Also, it makes little difference whether market capitalization is float-adjusted or not: the correlation between the logs of float-adjusted and unadjusted market capitalization is 0.98.⁸ We use unadjusted market capitalization in our empirical analysis to maximize data coverage.

The second component of L is “diversification.” We choose this label for the second factor in equation (19) because that factor includes several elements that are commonly used to judge the extent to which a portfolio is diversified, as explained below.

Broadly speaking, diversification refers to spreading one’s wealth across many assets in a balanced fashion. The implications of diversification for portfolio risk are well understood. We show that diversification also has implications for transaction costs: better-diversified portfolios are cheaper to trade. Such portfolios are more liquid because they incur lower trading costs than more concentrated portfolios with the same size and turnover.

Diversification is a foundational concept in finance, yet there is no accepted standard for measuring it. In an important early contribution, Blume and Friend (1975) use two measures. The first one is the number of stocks in the portfolio. This measure is also used by Goetzmann and Kumar (2008), Ivkovich, Sialm, and Weisbenner (2008), Pollet and Wilson (2008), and others. The idea is that portfolios holding more stocks are better diversified. While this idea is sound, the measure is far from perfect. Consider two portfolios holding the same set of 500 stocks. The first portfolio weights the stocks in proportion to their market capitalization. The second portfolio is 99.9% invested in a single stock while the remaining 0.1% is spread across the remaining 499 stocks. Even though both portfolios hold the same number of stocks, the first portfolio is clearly better diversified.

The second measure of diversification used by Blume and Friend is the sum of squared deviations of portfolio weights from market weights, essentially a market-adjusted Herfindahl index. The Herfindahl index measures portfolio concentration, the inverse of diversification. Studies using versions of this measure include Kacperczyk, Sialm, and Zheng (2005), Goetzmann and Kumar (2008), and Cremers and Petajisto (2009), among others.

Our measure of portfolio diversification blends the ideas from both of the above measures.

⁸We compute this correlation using data on the Russell 3000 stocks from 2011 to 2014. Data on stocks’ shares outstanding are from CRSP. Data on float-adjusted shares outstanding are from Russell.

As one can see from equation (19), our measure can be further decomposed as

$$\text{Diversification} = \underbrace{\left(\frac{N}{N_M}\right)}_{\text{Coverage}} \times \underbrace{\left[1 + \text{Var}^*\left(\frac{w_i}{m_i^*}\right)\right]^{-1}}_{\text{Balance}}. \quad (20)$$

The first component, “coverage,” is the number of stocks in the portfolio (N) divided by the total number of stocks in the benchmark (N_M). Dividing by the latter number makes sense. If all firms in the benchmark were to merge into one conglomerate, a portfolio holding only the conglomerate’s stock would be perfectly diversified despite holding only a single stock. Given N_M , portfolios holding more stocks have larger coverage. The value of coverage is always between 0 and 1, with the maximum value reached if the portfolio holds every stock in the benchmark.

The second component of diversification, “balance,” reflects the portfolio’s allocations across its holdings, regardless of their number. A portfolio is highly balanced if its weights are close to market-cap weights. The degree to which a portfolio’s weights are close to market-cap weights is captured by the term $\text{Var}^*(w_i/m_i^*)$, which is the variance of w_i/m_i^* with respect to the probability measure defined by scaled market-cap weights $m_i^* = m_i / \sum_{i=1}^N m_i$, so that $\sum_{i=1}^N m_i^* = 1$.⁹ If portfolio weights equal market-cap weights, so $w_i/m_i^* = 1$, then $\text{Var}^*(w_i/m_i^*) = 0$ and balance equals 1. Like coverage, balance is always between 0 and 1.

Equation (20) shows that a portfolio is well diversified if it holds a large fraction of the benchmark’s stocks and if its weights are close to market-cap weights. Given the ranges of coverage and balance, diversification is always between 0 and 1. The benchmark portfolio has coverage and balance both equal to 1.¹⁰

Figure 1 provides some history of L and its components for active mutual funds. Panel A plots the time series of the cross-sectional means of L across all funds, relative to the market benchmark. Average L doubled between 1980 and 2000, indicating that fund portfolios became substantially more liquid relative to the market benchmark. To understand this pattern, we plot in Panel B the time series of the two components of L : stock liquidity and diversification. Stock liquidity rose sharply in the late 1990s, explaining the contemporaneous increase in L observed in Panel A, but it declined steadily in the 21st century.¹¹ Judging by this large decline, one might expect fund portfolios to have become less liquid in the 21st

⁹Note that $\sum_{i=1}^{N_M} m_i = 1$, but $\sum_{i=1}^N m_i \leq 1$, because $N \leq N_M$. $\text{Var}^*(\cdot)$ can be easily computed using the expression $\text{Var}^*(w_i/m_i^*) = \sum_{i=1}^N w_i^2/m_i^* - 1$. Details are in Appendix A.

¹⁰Our measure of diversification is easy to calculate from equation (20). Those wishing to circumvent the calculation of variance with respect to the m^* probability measure can follow a simple two-step approach: first compute L from equation (3) and then divide it by stock liquidity, following equation (19).

¹¹This decline indicates that the average stock held by mutual funds became smaller relative to the average stock in the benchmark. Either funds tilted their portfolios toward smaller stocks or the average benchmark

century, but that is not the case, as shown in Panel A. The reason is that fund portfolios have become much more diversified, with diversification almost tripling between 2000 and 2014. The two opposing effects, the decrease in stock liquidity and the increase in diversification, roughly cancel out, resulting in a flat pattern in L since 2000.

The sharp increase in diversification after 2000 is striking. To shed more light on it, we plot in Panel C the components of diversification: balance and coverage. Both components rise steadily, especially after 2000.¹² The increase in coverage, equal to N/N_M , is dissected in Panel D. The average N rises essentially linearly from 54 in 1980 to 126 in 2014, indicating that funds hold an increasingly large number of stocks. In contrast, the number of stocks in the market plummets from about 8,600 in the late 1990s to fewer than 5,000 in 2014. The observed increase in coverage is thus driven by a combination of a rising N and falling N_M . Together, Panels C and D show that the portfolios of active mutual funds have become more index-like. In addition to this time-series evidence, we provide cross-sectional descriptive evidence on L and its components in the Internet Appendix.

4.2. Empirical Evidence of Tradeoffs

In addition to the main fund tradeoffs implied by equation (18), our model also implies tradeoffs that involve the components of portfolio liquidity. Equation (19) implies that

$$\ln(L) = \ln(\text{Stock Liquidity}) + \ln(\text{Diversification}) . \quad (21)$$

Combined with equation (18), this equation implies

$$\ln(\text{Diversification}) = b_1 \ln A - b_2 \ln f + b_3 \ln T - \ln(\text{Stock Liquidity}) + \text{constant} , \quad (22)$$

where b_1 , b_2 , b_3 , and ‘constant’ are the same positive constants as before. Equation (22) makes strong predictions about the determinants of portfolio diversification. In equilibrium, funds with more-diversified portfolios should be larger and cheaper, they should trade more, and their stock holdings should be less liquid, on average.

Column 1 of Table 2 provides strong support for all of these predictions. Fund size, expense ratio, and turnover help explain diversification with the predicted signs, and the slopes have magnitudes similar to those in column 4 of Table 1. The new regressor, stock liquidity, also enters with the right sign and is highly significant, both statistically ($t = -21.61$) and

stock increased in size. Evidence of the former effect is provided by Blume and Keim (2017), who show that institutional investors increased their holdings of smaller stocks in recent decades.

¹²The upward trends in both components of diversification, as well as the resulting upward trend in portfolio liquidity, are statistically significant, as we show in the Internet Appendix.

economically. A one-standard-deviation increase in $\ln(\text{Stock Liquidity})$ is associated with a 0.95 decrease in $\ln(\text{Diversification})$, for example, a decrease in diversification from 0.26 to 0.10. Stock liquidity and diversification are thus substitutes: funds tend to make up for the low liquidity of their holdings by diversifying more. This evidence fits our model.

The tradeoffs involving diversification are very robust. They obtain not only for our theoretically motivated measure of diversification from equation (20) but also for three ad-hoc measures: the Herfindahl index of portfolio weights, the number of stocks in the portfolio, and the R -squared from the regression of fund returns on benchmark returns. Moreover, the tradeoffs obtain not only with sector-quarter fixed effects, as in Table 2, but also with quarter fixed effects. Finally, the tradeoffs also emerge from simple correlations. For example, the within-sector cross-sectional correlation between diversification and stock liquidity is -41%. See the Internet Appendix for details.

Next, we drill deeper by decomposing diversification following equation (2):

$$\ln(\text{Diversification}) = \ln(\text{Coverage}) + \ln(\text{Balance}) . \quad (23)$$

Combined with equation (22), this equation implies

$$\ln(\text{Coverage}) = b_1 \ln A - b_2 \ln f + b_3 \ln T - \ln(\text{Stock Liq.}) - \ln(\text{Balance}) + \text{constant} \quad (24)$$

and

$$\ln(\text{Balance}) = b_1 \ln A - b_2 \ln f + b_3 \ln T - \ln(\text{Stock Liq.}) - \ln(\text{Coverage}) + \text{constant} . \quad (25)$$

These equations make predictions about the determinants of portfolio coverage and balance.

Columns 2 and 3 of Table 2 support those predictions. In both regressions, all the variables enter with the predicted signs. Most of the variables are highly significant; only turnover in column 3 is marginally significant. The slopes on balance in column 2 and coverage in column 3 are both negative, indicating that coverage and balance are substitutes. Funds that are less diversified in terms of coverage tend to be more diversified in terms of balance, and vice versa.

Finally, column 4 of Table 2 tests the prediction analogous to that in equation (22), except that diversification and stock liquidity switch sides: the former appears on the right-hand side and the latter on the left-hand side of the regression. The evidence again supports the model, though a bit less strongly than the first four columns. Three of the four slopes have the right sign and are all significant (the t -statistic on stock liquidity is -24.49). The slope on turnover is negative but not significantly different from 0.

In a robustness exercise, we split the sample into two subsamples, 1979 through 2004 and 2005 through 2014, which contain roughly the same number of fund-quarter observations. The counterparts of Tables 1 and 2 for both subsamples look very similar to the originals, leading to the same conclusions. We show these tables in the Internet Appendix.

5. Fund Activeness

Funds actively apply their skill in an effort to reap profits. Recall that the function $g(T, L)$ in equation (10) captures how actively skill is applied. This function, which we refer to as “activeness,” is increasing in turnover, T , and decreasing in portfolio liquidity, L .

The role of T in activeness is consistent with the theory and empirical evidence of Pástor, Stambaugh, and Taylor (2017), who establish a positive link between a fund’s turnover and its performance. Intuitively, higher turnover means the fund is more frequently applying its skill in identifying profit opportunities.

Recall from equation (19) that L is the product of stock liquidity and diversification, so a fund’s activeness is decreasing in both of those quantities. The role of stock liquidity in activeness reflects evidence that mispricing is greater among less-liquid and smaller stocks (e.g., Sadka and Scherbina, 2007, and Stambaugh, Yu, and Yuan, 2015), consistent with arguments that arbitrage is deterred by higher trading costs and greater volatility (e.g., Shleifer and Vishny, 1997, Pontiff, 2006). A fund tilting toward such stocks is more actively pursuing mispricing where it is most prevalent.

Both components of diversification—coverage and balance—explain diversification’s role in activeness. By holding fewer stocks (i.e., lower coverage), a fund can focus on its best trading ideas, leading to higher expected gross profits. By deviating more from market-cap weights (i.e., lower balance), a fund can place larger bets on its better ideas, again boosting performance. Theoretical settings in which portfolio concentration (lower diversification) arises optimally include Merton (1987), van Nieuwerburgh and Veldkamp (2010), and Kacperczyk, van Nieuwerburgh, and Veldkamp (2016). Empirical evidence linking portfolio concentration to performance includes results in Kacperczyk, Sialm, and Zheng (2005), Ivkovich, Sialm, and Weisbenner (2008), and Choi et al. (2017).

5.1. Choosing Activeness: Implications

In the setting presented in Section 3.1, the fund chooses T , L , and f separately when maximizing its fee revenue. To derive additional insights, we consider a simplified version of that general setting in which the fund's choices of T and L collapse to the choice of a single quantity, activeness. In this simplification, $g(T, L)$ takes the form

$$g(T, L) = T^{\lambda/\gamma} L^{-\phi/\gamma} . \quad (26)$$

With this specification, T and L interact in the same manner as in the cost function in equation (11), where the quantity $T^\lambda L^{-\phi}$ also appears. Raising that quantity to the power $1/\gamma$, to give the right-hand side of equation (26), provides a specification of g for which the first-order condition $\partial F/\partial g = 0$ implies equation (16), the same condition implied by $\partial F/\partial f = 0$. The fund's fee revenue, F , thus achieves the same maximum for any choice of the fee rate, f , with the accompanying choice of activeness, g , satisfying equation (16). A higher fee rate dictates greater activeness but does not produce greater fee revenue. To see this, we substitute from equations (16) and (26) into equation (14) to obtain

$$F = \mu^{\frac{\gamma}{\gamma-1}} (\gamma - 1) \left(\frac{1}{\theta \gamma^\gamma} \right)^{\frac{1}{\gamma-1}} . \quad (27)$$

A fund's equilibrium fee revenue is pinned down by the fund's skill μ , holding the cost parameters θ and γ constant. It makes sense for more skilled funds to earn higher fee revenue, and this prediction is not unique to our model.¹³ The fee revenue, $F = Af$, does not depend on f because when f changes, A adjusts in the opposite direction to keep F constant. Irrelevance of f for F also occurs, for example, in the equilibrium models of Berk and Green (2004), Hugonnier and Kaniel (2010), and Stambaugh (2014).

The data confirm that when $g(T, L)$ is computed as in equation (26), it depends significantly on both T and L in the correct directions. Pairing the regression estimates of b_1 , b_2 , and b_3 in equation (18) with their corresponding functions of γ , λ , and ϕ in equation (17) delivers implied estimates of 0.138 and -1.367 for the exponents of T and L , respectively, in equation (26). Dividing those values by their standard errors, computed via the delta method, gives t -statistics of 4.58 and -13.98 , confirming that both T and L enter $g(T, L)$ significantly.

We obtain two implications for activeness: (i) larger funds choose to be less active, controlling for f , and (ii) higher-fee funds choose to be more active, controlling for A . To see

¹³Skill also determines fee revenue in the model of Berk and Green (2004). See Section 8 for further discussion. Berk and van Binsbergen (2015) also discuss the relation between skill and fee revenue.

these, substitute the right-hand side of equation (16) for $\mu g(T, L)$ in equation (13), giving

$$A = g^{-\frac{\gamma}{\gamma-1}} \left[\frac{f}{\theta(\gamma-1)} \right]^{\frac{1}{\gamma-1}}, \quad (28)$$

where activeness, g , obeys equation (26). Taking logs and rearranging, we obtain

$$\ln(g) = d_1 \ln(f) - d_2 \ln(A) + \text{constant}, \quad (29)$$

where d_1 and d_2 are positive and the constant equals $-(1/\gamma) \ln[\theta(\gamma-1)]$. Thus, g is increasing in f and decreasing in A . Intuitively, larger funds, facing diseconomies of scale, optimally reduce their trading costs by reducing their activeness. Lower-fee funds also choose to be less active because they are less skilled, holding size constant.

Suppose we were to compute observations of g from equation (26), with the exponents on T and L implied by the coefficients from the regression corresponding to equation (18), as discussed above. Then the regression corresponding to equation (29) would deliver estimates of d_1 and d_2 that are simple transformations of our previously reported estimates of the coefficients in equation (18). In other words, the data would supply no additional information about the implied fund tradeoffs involving activeness. Therefore, we instead look for empirical confirmation of these tradeoffs when g is computed in a simpler way.

5.2. Computing Activeness: Evidence

Our simplified calculation of activeness is motivated by the same setting in which we derive our portfolio liquidity measure. In that setting, a portfolio is viewed as just another asset, effectively traded as such, with each stock's traded amount being proportional to its portfolio weight. Recall that if a fund trades its portfolio that way, its cost function is given by equation (11) with $\gamma = 2$, $\lambda = 2$, and $\phi = 1$. Applying those parameter values to equation (26) gives

$$g(T, L) = TL^{-1/2}, \quad (30)$$

our empirical measure of activeness.

With g computed as in equation (30), we estimate the regression corresponding to equation (29). The results are reported in Table 3. As our model predicts, activeness is related negatively to fund size and positively to expense ratio. Both relations are very strong, with t -statistics of about 10 in magnitude. These relations obtain not only in the multiple regression but also in simple regressions, with t -statistics exceeding 13 in magnitude. Like Tables 1 and 2, Table 3 reports results with sector-quarter fixed effects, but including just

quarter fixed effects produces very similar results. We also find very similar results in two subsamples, 1979 through 2004 and 2005 through 2014. See the Internet Appendix.

Our activeness measure from equation (30) has a correlation of 55% with the popular active share measure of Cremers and Petajisto (2009), in logs. Active share is computed by using only portfolio weights of the fund and the benchmark, as is our portfolio liquidity measure, L . Both active share and L capture deviations of portfolio weights from benchmark weights, so it is not surprising that the correlation between active share and L is high, -79% , in logs. But our measure of activeness incorporates not only L but also T . This inclusion of turnover captures the intuitive notion that a fund is more active if it trades more. The presence of turnover in activeness, and its absence from active share, is the main difference between the two measures. Yet when we replace activeness by active share in Table 3, we obtain the same conclusions: smaller funds and higher-fee funds tend to be more active. We also obtain the same conclusions when replacing activeness by another proxy, the inverse of the R -squared from the regression of fund returns on benchmark returns. See the Internet Appendix.

6. Tradeoffs: Simple Correlations

The tradeoffs implied by our model take the form of multiple regressions (e.g., equation (18)), but they emerge also from simple correlations. In this section, we analyze the correlations among the four main fund characteristics. To understand these correlations in the context of our model, we need additional assumptions.

6.1. Larger Funds Are Cheaper

Our model predicts a negative correlation between fund size, A , and expense ratio, f . We derive this prediction from equation (27), in which a fund's equilibrium fee revenue, $F = Af$, is determined by the fund's skill, μ . Holding μ constant, A and f are perfectly negatively correlated across funds. If μ varies across funds, the correlation between A and f is no longer perfect, but it remains negative as long as μ is not highly correlated with f across funds. Specifically, let $\beta_{\mu,f}$ denote the slope from the cross-sectional regression of $\ln(\mu)$ on $\ln(f)$. Our model implies a negative cross-sectional correlation between A and f as long as $\beta_{\mu,f} < (\gamma - 1)/\gamma$ (see Appendix A for the proof). It makes sense for $\beta_{\mu,f}$ to be positive, in that more skilled funds should be able to charge higher fee rates. Nonetheless, it seems plausible for $\beta_{\mu,f}$ to be small enough to satisfy the assumption because in practice, expense

ratios have a variety of determinants beyond skill (marketing, distribution, etc.).

Empirical evidence strongly supports the prediction that larger funds are cheaper. Table 4 reports correlations between fund characteristics, again measured in logs. In our mutual fund dataset, the cross-sectional within-sector correlation between fund size and expense ratio is -31.5% ($t = -15.27$). Larger funds clearly charge lower expense ratios. This evidence is consistent with our model. Other studies have already reported a negative correlation between fund size and expense ratio (e.g., Ferris and Chance, 1987). But we appear to be the first to provide a theoretical justification for this strong stylized fact.

The correlation between fund size and expense ratio is also strongly negative in the time series for the typical fund, -25.1% ($t = -17.54$). In computing the time-series correlations in Panel B of Table 4, we need to account for the substantial growth in the dollar values of stocks that renders dollar assets under management (AUM) unappealing as a time-series measure of fund size: AUM values in the 1980s are not comparable to those today. To address this fact, we divide each fund’s AUM by the contemporaneous total stock market capitalization. Pástor, Stambaugh, and Taylor (2015) also deflate fund size by stock-market value when analyzing a time series of fund size.

6.2. Funds That Trade Less Are Larger and Cheaper

Recall from equation (29) that the fund’s activeness, $g(T, L)$, is positively correlated with f , controlling for A , and negatively correlated with A , controlling for f . These correlations obtain also without controls, under additional assumptions. If skill (μ) is constant across funds, both simple correlations are perfect. The positive correlation between g and f follows directly from equation (16). The negative correlation between g and A obtains when we substitute for f from equation (16) into equation (28), yielding

$$A = \frac{1}{g} \mu^{\frac{1}{\gamma-1}} (\gamma\theta)^{-\frac{1}{\gamma-1}} . \quad (31)$$

The product of fund size and activeness, Ag , is determined by μ . Holding μ constant, A is perfectly negatively correlated with g . If μ varies across funds, both correlations retain their signs as long as μ is not too highly correlated with f or A . Specifically, let $\beta_{\mu,A}$ denote the slope from the regression of $\ln(\mu)$ on $\ln(A)$. The model implies a negative correlation between g and A as long as $\beta_{\mu,A} < \gamma - 1$ and a positive correlation between g and f as long as $\beta_{\mu,f} < 1$ (see Appendix A for the proof). Empirical evidence strongly supports both of these predictions, as shown in columns 1 and 2 of Table 3. Funds that are more active tend to be smaller and more expensive, as the model predicts.

Besides activeness, we also consider T and L individually. Our predictions for $g(T, L)$ imply that, controlling for L , T should be negatively related to fund size and positively related to expense ratio. This is indeed true in the data, and the relations hold even without controlling for L . In Table 4, T is negatively correlated with fund size, both in the cross section and in the time series: the correlations are -10.5% ($t = -6.00$) and -14.7% ($t = -12.11$), respectively. In addition, T is positively correlated with expense ratio: the correlation is 13.0% ($t = 6.34$) in the cross section and 10.5% ($t = 7.54$) in the time series. In short, funds that trade less are larger and cheaper, as predicted by our model.

6.3. Funds with More-Liquid Portfolios Are Larger and Cheaper

Our predictions for $g(T, L)$ also imply that, controlling for T , L should be positively related to fund size and negatively related to expense ratio. Again, both relations hold strongly even in simple correlations, as shown in Table 4. The correlations between L and A are 28.5% ($t = 17.77$) and 30.8% ($t = 18.00$) in the cross section and time series, respectively. The correlations between L and f are -29.1% ($t = -13.29$) and -11.8% ($t = -6.78$). These correlations also emerge from the simple-regression results reported in Table 1. In short, funds with more-liquid portfolios are larger and cheaper, as predicted by our model.

The cross-sectional correlations that involve L are extremely robust. The correlations in Panel A of Table 4 are computed from panel regressions with quarter-sector fixed effects, which isolate cross-sectional correlations within sectors.¹⁴ Those correlations are therefore weighted averages of cross-sectional correlations, where the averaging is across all quarters in our sample. It turns out that the cross-sectional relations involving L hold not only on average, but also in every single quarter in our sample. This stunning fact is plotted in Figure 2. Both correlations involving L retain the same sign in every quarter between 1980 and 2014. In fact, in each quarter, their magnitudes exceed 20% in absolute value.

Two other cross-sectional correlations discussed earlier are similarly strong, which is why we plot their time series in Figure 2. The correlation between fund size and expense ratio, analyzed in Section 6.1, is negative in every single quarter, varying between -0.74 and -0.23 across quarters. The correlation between turnover and expense ratio, analyzed in Section 6.2, is positive in every quarter, varying between 0.10 and 0.36. It is rare to see a model's theoretical predictions hold so consistently in the data.

¹⁴We also compute plain cross-sectional correlations (i.e., including quarter fixed effects instead of sector-quarter fixed effects). The results are very similar to those in Panel A of Table 4 so we report them only in the Internet Appendix. In that Appendix, we also show the results from another robustness exercise, in which we recompute Table 4 for two subperiods containing roughly the same number of fund-month observations. The results in both subsamples look very similar to the full-sample ones.

While Figure 2 plots cross-sectional correlations, the time-series correlations reported in Table 4 are of similar magnitudes. The time-series correlation between L and fund size, 30.8%, is particularly strong. It shows that when a fund gets larger, its portfolio becomes more liquid. This fact is easily interpreted in the context of our theory. Consider a fund that receives a large inflow. Cognizant of decreasing returns to scale, the fund’s manager makes the fund’s portfolio more liquid. And vice versa—after a large outflow, a fund can afford to make its portfolio less liquid.

To illustrate these effects, we pick the example of Fidelity Magellan, the largest mutual fund at the turn of the millenium. Figure 3 plots the time series of Magellan’s AUM and its portfolio liquidity. The comovement between the two series is striking. Between 1980 and 2000, Magellan’s assets grew rapidly, in large part due to the fund’s stellar performance under Peter Lynch in 1977 through 1990. Over the same period, and especially after 1993, the liquidity of Magellan’s portfolio also grew rapidly. From 1993 to 2001, Magellan’s L grew from 0.1 to 0.4, a remarkable increase equal to nearly five standard deviations of the sample distribution of L . After 2000, though, Magellan’s assets shrank steadily, and by 2014, they were down by almost 90%. Over the same period, Magellan’s L was down also, back to about 0.1. A natural interpretation is that Magellan’s large size around 2000 forced the fund’s managers to increase the liquidity of Magellan’s portfolio to shelter the fund from the pernicious effects of decreasing returns to scale.

7. Rethinking Scale

What is a fund’s scale? Following Berk and Green (2004), active funds are typically viewed as facing decreasing returns to scale, with scale given by fund size, i.e., AUM. Our framework offers a new perspective on scale. In our setting, funds face decreasing returns to scale, but scale depends not only on size but also on activeness.

Let Π denote the fund’s expected dollar profit net of trading costs (but before fees). In an equilibrium satisfying the zero-net-alpha condition (9), Π is equal to the fund’s fee revenue, F . Therefore, the fund’s objective of maximizing F is equivalent to maximizing Π . With expected gross return and trading costs given by equations (10) and (11), we have

$$\begin{aligned}\Pi &= aA - C(A, T, L) \\ &= \mu g(T, L)A - \theta A^\gamma T^\lambda L^{-\phi} .\end{aligned}\tag{32}$$

When the fund chooses activeness, so that $g(T, L)$ is given by equation (26), then

$$\Pi = \mu T^{\lambda/\gamma} L^{-\phi/\gamma} A - \theta A^\gamma T^\lambda L^{-\phi}$$

$$= \mu S - \theta S^\gamma, \quad (33)$$

where

$$\begin{aligned} S &= T^{\lambda/\gamma} L^{-\phi/\gamma} A \\ &= g(T, L) A. \end{aligned} \quad (34)$$

The net profit function given by equation (33) is hump-shaped with respect to S (recall that $\gamma > 1$). That is, as the fund seeks the greatest equilibrium Π , it faces decreasing returns to scale with respect to S .

The fund's scale, S , is activeness times size, not just size. This concept of fund scale makes intuitive sense. If two funds manage equal amounts of money, but one fund deploys its money more actively, that fund leaves a bigger footprint in the market.

8. Relation to Berk and Green (2004)

Besides delivering a different concept of scale, our setting departs from that of Berk and Green (2004), hereafter BG, in other key respects. First, we incorporate both turnover and portfolio liquidity. The fund's choices of those characteristics, absent from BG, enter the fund's trading costs as well as its activeness. Our setting considers four fund characteristics: size, expense ratio, turnover, and portfolio liquidity. Only two of them, size and expense ratio, appear in the BG setting. Our richer setting allows us to obtain new insights into the tradeoffs involving turnover and portfolio liquidity, as well as the tradeoffs involving the components of portfolio liquidity: stock liquidity, diversification, coverage, and balance.

The BG setting can be shown to imply tradeoffs between fund characteristics, but in a more limited way than ours. The most straightforward is the tradeoff between size and expense ratio. In the BG setting, a fund that cuts its fee rate attracts additional capital, which is indexed at low cost. The fund's size is thus inversely related to its expense ratio.

By adding mild assumptions to the BG setting, we can also derive tradeoffs between size and the two fund characteristics that do not explicitly appear in that setting. Assuming the fund's indexed portion has zero turnover, the BG setting implies a negative relation between a fund's size and its turnover. It also implies a positive relation between size and portfolio liquidity, our newly introduced measure.¹⁵ Thus, although not discussed by BG,

¹⁵This relation follows from the following result, which we derive in Appendix A: If a portfolio with liquidity L is combined with the benchmark (index fund), the liquidity of the resulting combination, \tilde{L} , obeys $\tilde{L}^{-1} = 1 + \omega^2(L^{-1} - 1)$, where ω is the non-indexed (active) fraction of the fund.

it is possible to derive some relations among the four fund characteristics in their setting. Importantly, however, in the BG setting all four characteristics have a single quantity driving those relations—the fraction of the fund that is indexed. So, for example, one cannot consider the implications for size and expense ratio if the fund were to increase its turnover but not change portfolio liquidity. In the BG setting, an increase in turnover would have to reflect a lower fraction indexed, so it would have to be accompanied by a decrease in portfolio liquidity.

In our setting, each fund characteristic trades off against independent variation in the other three. Such independent variation is clearly present in the data, as the correlations among fund characteristics are well below one (see Table 4). Our equation (18) and the accompanying regression in Table 1 allow for this independence, whereas the BG indexing scenario, with its single underlying driver of all fund characteristics, does not. Finally, by providing a more complete specification of trading costs that incorporates turnover and liquidity, we are able to derive equation (18) and apply it cross-sectionally, whereas BG do not make cross-sectional predictions about fund characteristics.

9. Conclusions

We model and document strong tradeoffs among the most salient characteristics of active mutual funds: fund size, expense ratio, turnover, and portfolio liquidity. We find empirically that funds with smaller size, higher expense ratios, and lower turnover tend to hold less-liquid portfolios. They also hold less-diversified portfolios. All of these findings are predicted by our equilibrium model, in which the key fund characteristics are jointly determined. Additional model predictions also hold in the data. For example, larger funds are cheaper, funds that trade less are larger and cheaper, and funds that are less active are larger and cheaper. These results provide strong new evidence of decreasing returns to scale in active management. A fund’s scale is captured by its activeness times AUM, not just AUM.

Another contribution of our study is to introduce the concept of portfolio liquidity. We show that a portfolio’s liquidity depends not only on the liquidity of its holdings but also on its diversification. We derive simple measures of portfolio liquidity and diversification. Based on these measures, we find that active mutual funds’ portfolios have become relatively more liquid over time, mostly as a result of becoming more diversified. We also find that the components of portfolio liquidity are substitutes: funds holding less-liquid stocks tend to diversify more, and funds holding fewer stocks choose portfolio weights closer to market-cap weights.

Our empirical analysis focuses on U.S. equity mutual funds. Future research can apply our concepts and measures to portfolios held by other types of institutions, such as hedge funds, private equity funds, fixed income mutual funds, and pension funds. More research into relations among fund characteristics also seems warranted.

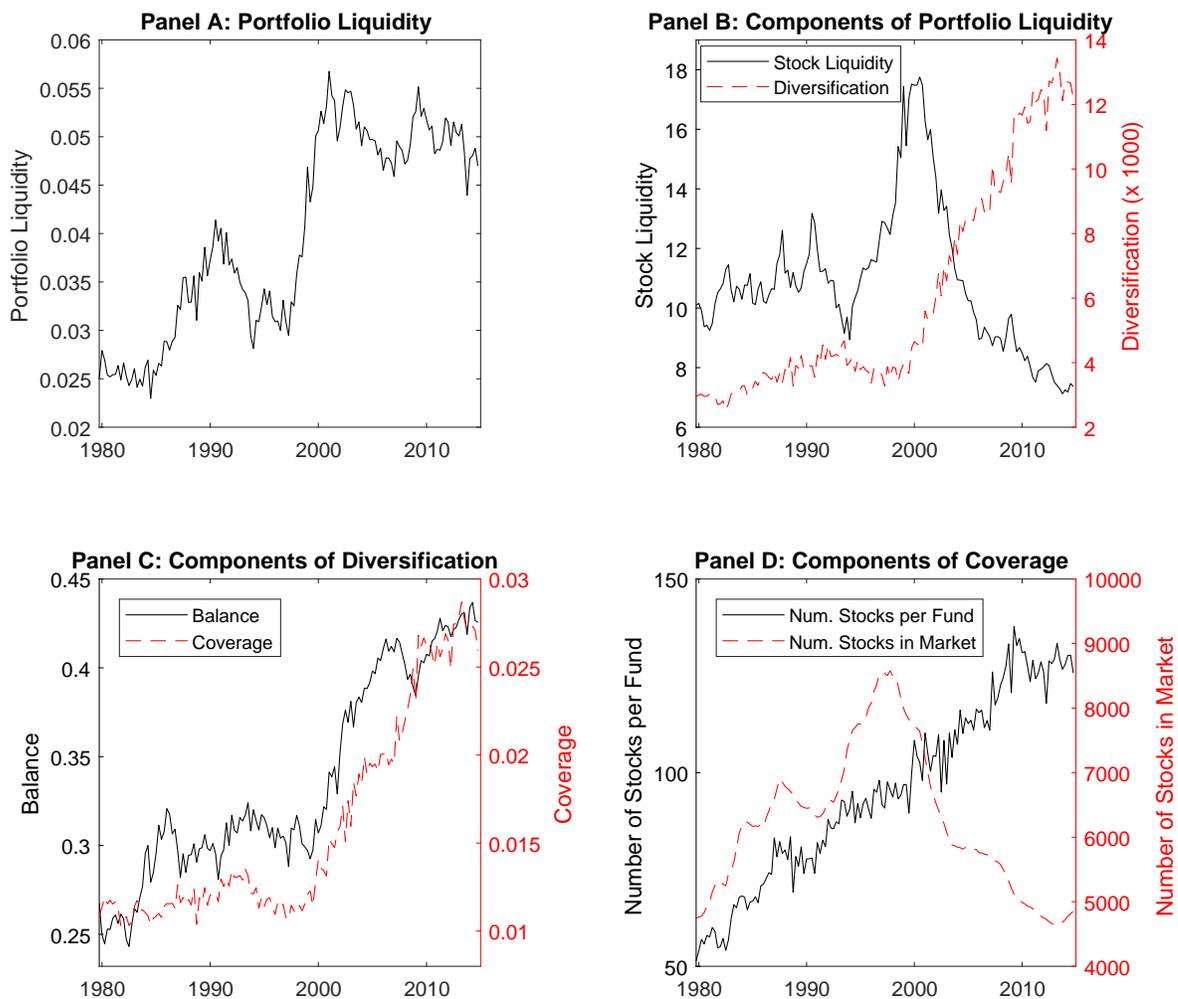


Figure 1. Time Series of Average Portfolio Liquidity and Its Components. This figure plots the quarterly time series of the cross-sectional means of portfolio liquidity, stock liquidity, diversification, coverage, balance, and the number of stocks held by each fund. Liquidity, diversification, and its components are computed with respect to the market benchmark. In Panel D we also plot the number of stocks in the market portfolio.

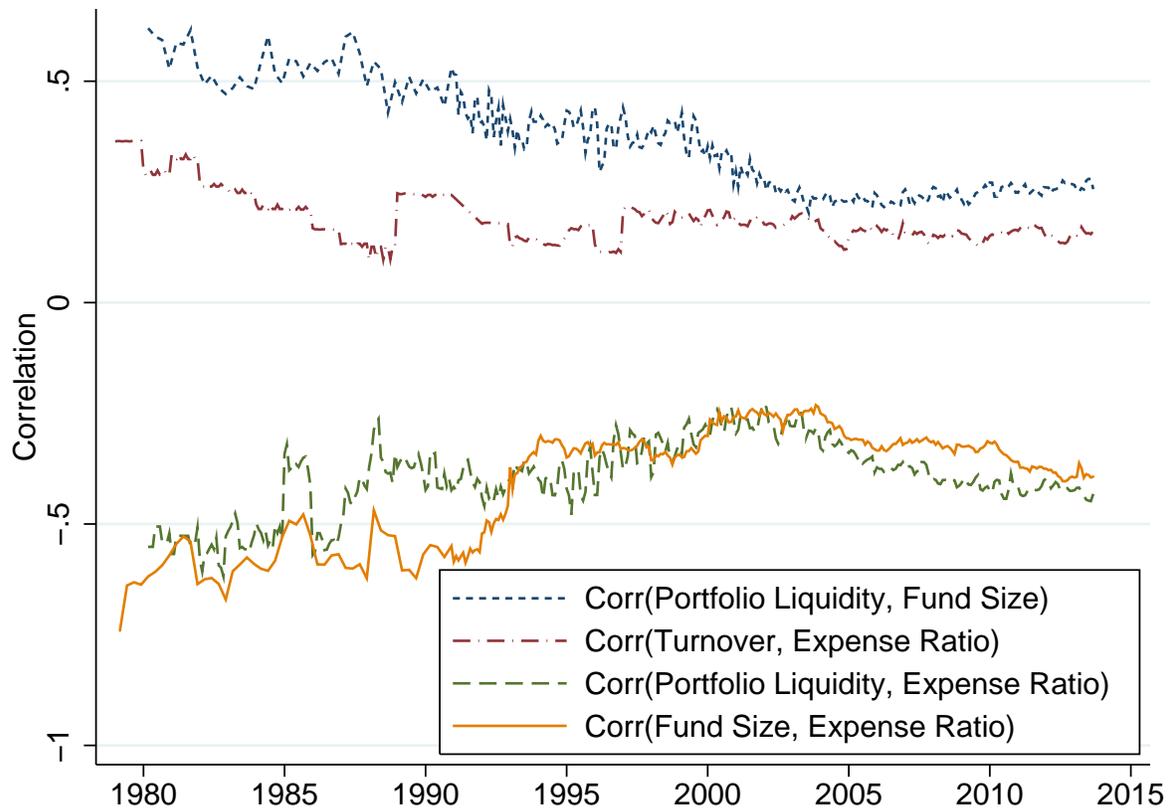


Figure 2. Cross-Sectional Correlations Over Time. This figure plots monthly time series of the cross-sectional correlation between the two variables noted in the legend. All variables are measured in logs. For each correlation, we drop months with fewer than 30 observations. To convert portfolio liquidity from a quarterly to a monthly variable, we take portfolio liquidity from the current month or, if missing, from the previous two months. Portfolio liquidity is computed with respect to the market benchmark.

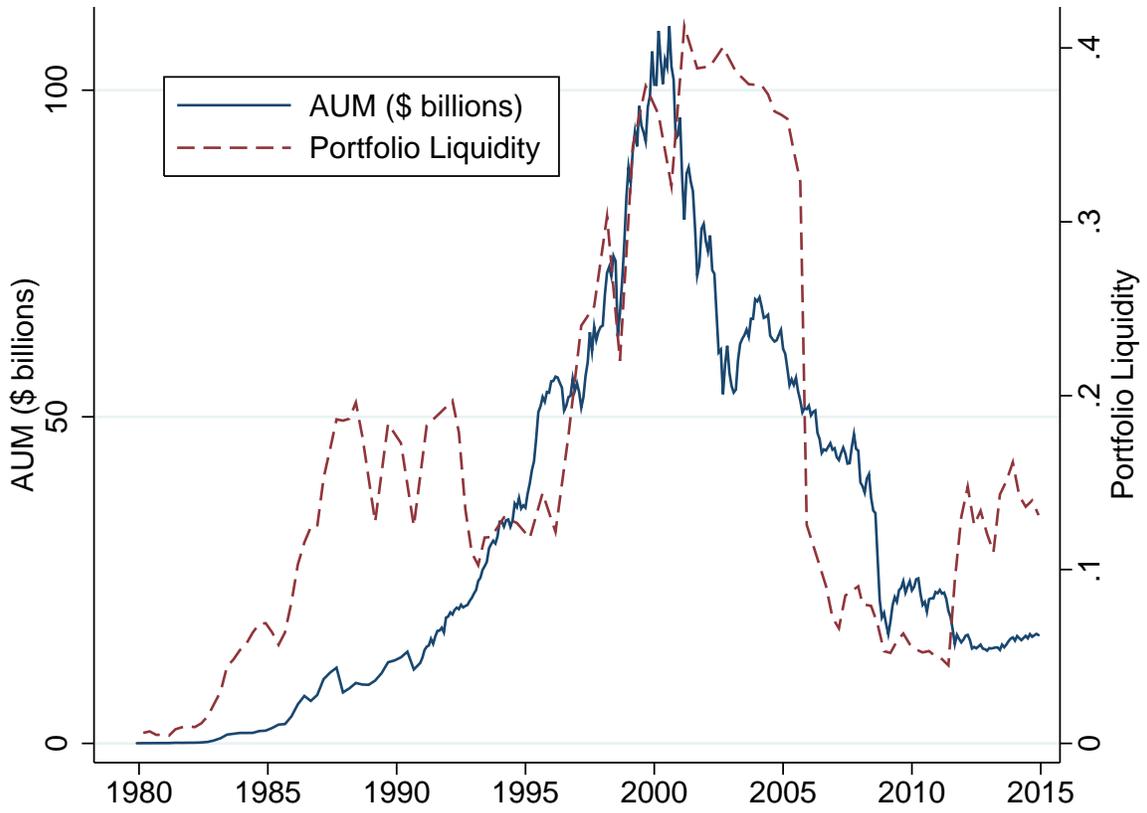


Figure 3. Fidelity Magellan Fund. This figure plots Magellan’s assets under management (AUM) and portfolio liquidity, computed with respect to the market benchmark.

Table 1
Explaining Mutual Funds' Portfolio Liquidity

This table presents results from OLS panel regressions in which the dependent variable is a mutual fund's portfolio liquidity, L . The regressors—fund size, A , expense ratio, f , and fund turnover, T —are measured contemporaneously with the dependent variable. All variables are measured in logs. The unit of observation is the fund/quarter. All regressions include sector \times quarter fixed effects (FEs) and cluster by fund. The R^2 values in the penultimate row include the FEs' contribution. The last row contains the R^2 values from the regression of the dependent variable on the FEs alone. t -statistics are in parentheses.

	(1)	(2)	(3)	(4)
Fund Size	0.157 (17.77)			0.124 (13.76)
Expense Ratio		-0.766 (-13.29)		-0.608 (-11.26)
Turnover			0.0408 (1.93)	0.101 (4.93)
Observations	88925	89017	81892	76928
R^2	0.627	0.623	0.591	0.652
R^2 (FEs only)	0.594	0.588	0.591	0.598

Table 2
Explaining the Components of Portfolio Liquidity

This table presents results from four OLS panel regressions with dependent variables noted in the column headers. All regressors are measured contemporaneously with the dependent variable. All variables are measured in logs. The unit of observation is the fund/quarter. All regressions include sector \times quarter fixed effects (FEs) and cluster by fund. The R^2 values in the penultimate row include the FEs' contribution. The last row contains the R^2 values from the regression of the dependent variable on the FEs alone. t -statistics are in parentheses.

	(1)	(2)	(3)	(4)
	Diversification	Coverage	Balance	Stock Liquidity
Fund Size	0.134 (15.00)	0.0940 (12.08)	0.0452 (7.54)	0.0122 (2.35)
Expense Ratio	-0.622 (-11.00)	-0.408 (-9.33)	-0.238 (-6.95)	-0.132 (-5.26)
Turnover	0.122 (5.96)	0.102 (6.37)	0.0247 (1.92)	-0.0146 (-1.32)
Stock Liquidity	-0.621 (-21.61)	-0.337 (-14.21)	-0.308 (-14.90)	
Balance		-0.0447 (-2.08)		
Coverage			-0.0343 (-2.09)	
Diversification				-0.264 (-24.49)
Observations	76928	76928	76928	76928
R^2	0.465	0.336	0.286	0.882
R^2 (FEs only)	0.240	0.163	0.172	0.857

Table 3
Explaining Fund Activeness

This table presents results from OLS panel regressions with the dependent variable equal to Activeness. Activeness equals $TL^{-1/2}$, where T is turnover and L is portfolio liquidity. All regressors are measured contemporaneously with the dependent variable. All variables are measured in logs. The unit of observation is the fund/quarter. All regressions include sector \times quarter fixed effects (FEs) and cluster by fund. The last row contains the R^2 values from the regression of the dependent variable on the FEs alone. The R^2 values in the penultimate row include the FEs' contribution. t -statistics are in parentheses.

	(1)	(2)	(3)
Fund Size	-0.138 (-13.23)		-0.100 (-9.53)
Expense Ratio		0.712 (13.14)	0.558 (10.12)
Observations	76928	76928	76928
R^2	0.392	0.398	0.415
R^2 (FEs only)	0.356	0.356	0.356

Table 4
Correlations Among Fund Characteristics

This table reports correlations among the given fund characteristics, all measured in logs. Panel A reports correlations across funds within sector-quarters. Starting with our full panel dataset, we first de-mean each variable using the mean across all observations in the same sector and quarter, then we compute the full-sample correlation between the two de-measured variables. Panel B reports time-series correlations within funds, which we compute analogously except that we de-mean each variable using each fund's time-series mean. Fund size is scaled by total stock market capitalization. Portfolio liquidity is defined with respect to the market benchmark. *t*-statistics are computed clustering by fund and adjusting for de-meaning.

	Fund Size	Expense Ratio	Portfolio Liquidity	Turnover
Panel A: Cross-Sectional Correlations Within Sectors				
Fund Size	1			
Expense Ratio	-0.315 (-15.27)	1		
Portfolio Liquidity	0.285 (17.77)	-0.291 (-13.29)	1	
Turnover	-0.105 (-6.00)	0.130 (6.34)	0.039 (1.93)	1
Panel B: Time-Series Correlations				
Fund Size	1			
Expense Ratio	-0.251 (-17.54)	1		
Portfolio Liquidity	0.308 (18.00)	-0.118 (-6.78)	1	
Turnover	-0.147 (-12.11)	0.105 (7.54)	-0.109 (-6.76)	1

REFERENCES

- Amihud, Yakov, 2002, Illiquidity and stock returns: cross-section and time-series effects, *Journal of Financial Markets* 5, 31–56.
- Berk, Jonathan B., and Richard C. Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.
- Berk, Jonathan B., and Jules H. Van Binsbergen, 2015, Measuring skill in the mutual fund industry, *Journal of Financial Economics* 118, 1–20.
- Blume, Marshall E., and Irwin Friend, 1975, The asset structure of individual portfolios and some implications for utility functions, *Journal of Finance* 30, 585–603.
- Blume, Marshall E., and Donald B. Keim, 2014, The changing nature of institutional stock investing, *Critical Finance Review* 6, 1–41.
- Bris, Arturo, Huseyin Gulen, Padma Kadiyala, and P. Raghavendra Rau, 2007, Good stewards, cheap talkers, or family men? The impact of mutual fund closures on fund managers, flows, fees, and performance, *Review of Financial Studies* 20, 953–982.
- Busse, Jeffrey A., Tarun Chordia, Lei Jiang, and Yuehua Tang, 2017, Mutual fund trading costs, Working paper, Emory University.
- Chen, Joseph, Harrison Hong, Ming Huang, and Jeffrey Kubik, 2004, Does fund size erode mutual fund performance? *American Economic Review*, 94: 1276–1302.
- Choi, Nicole, Mark Fedenia, Hilla Skiba, and Tatyana Sokolyk, 2017, Portfolio concentration and performance of institutional investors worldwide, *Journal of Financial Economics* 123, 189–208.
- Cremers, Martijn, and Antti Petajisto, 2009, How active is your fund manager? A new measure that predicts performance, *Review of Financial Studies* 22, 3329–3365.
- Ferris, Stephen P., and Don M. Chance, 1987, The effect of 12b-1 plans on mutual fund expense ratios: A note, *Journal of Finance* 42, 1077–1082.
- Goetzmann, William N., and Alok Kumar, 2008, Equity portfolio diversification, *Review of Finance* 12, 433–463.
- Goyal, Amit, and Ivo Welch, 2003, Predicting the equity premium with dividend ratios, *Management Science* 49, 639–654.
- Harvey, Campbell R., and Yan Liu, 2017, Decreasing returns to scale, fund flows, and performance, Working paper, Duke University.
- Hugonnier, Julien, and Ron Kaniel, 2010, Mutual fund portfolio choice in the presence of dynamic flows, *Mathematical Finance* 20, 187–227.
- Ivkovic, Zoran, Clemens Sialm, and Scott Weisbenner, 2008, Portfolio concentration and the performance of individual investors, *Journal of Financial and Quantitative Analysis* 43, 613–655.
- Kacperczyk, Marcin, Clemens Sialm, and Lu Zheng, 2005, On the industry concentration of actively managed equity mutual funds, *Journal of Finance* 60, 1983–2011.
- Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp, 2016, A rational theory of mutual funds’ attention allocation, *Econometrica* 84, 571–626.

- Keim, Donald, and Ananth Madhavan, 1997, Transaction costs and investment style: An inter-exchange analysis of institutional equity trades, *Journal of Financial Economics* 46, 265–292.
- Kyle, Albert S., and Anna A. Obizhaeva, 2016, Market microstructure invariance: Empirical hypotheses, *Econometrica* 84, 1345–1404.
- Merton, Robert C., 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483–510.
- Pástor, Ľuboš, and Robert F. Stambaugh, 2012, On the size of the active management industry, *Journal of Political Economy* 120, 740–781.
- Pástor, Ľuboš, Robert F. Stambaugh, and Lucian A. Taylor, 2015, Scale and skill in active management, *Journal of Financial Economics* 116, 23–45.
- Pástor, Ľuboš, Robert F. Stambaugh, and Lucian A. Taylor, 2017, Do funds make more when they trade more?, *Journal of Finance* 72, 1483–1528.
- Polkovichenko, Valery, 2005, Household portfolio diversification: A case for rank-dependent preferences, *Review of Financial Studies* 18, 1467–1502.
- Pollet, Joshua, and Mungo Wilson, 2008, How does size affect mutual fund behavior? *Journal of Finance* 63, 2941–2969.
- Pontiff, Jeffrey, 2006, Costly arbitrage and the myth of idiosyncratic risk, *Journal of Accounting and Economics* 42, 35–52.
- Reuter, Jonathan, and Eric Zitzewitz, 2015, How much does size erode mutual fund performance? A regression discontinuity approach, Working paper, Boston College.
- Sadka, Ronnie, and Anna Scherbina, 2007, Analyst disagreement, mispricing, and liquidity, *Journal of Finance* 62, 2367–2403.
- Sensoy, Berk A., 2009, Performance evaluation and self-designated benchmark indexes in the mutual fund industry, *Journal of Financial Economics* 92, 25–39.
- Shleifer, Andrei, and Robert W. Vishny, 1997, The limits of arbitrage, *Journal of Finance* 52, 35–55.
- Stambaugh, Robert F., 2014, Presidential address: Investment noise and trends, *Journal of Finance* 69, 1415–1453.
- Stambaugh, Robert F., Jianfeng Yu, and Yu Yuan, 2015, Arbitrage asymmetry and the idiosyncratic volatility puzzle, *Journal of Finance* 70, 1903–1948.
- Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2010, Information acquisition and under-diversification, *Review of Economic Studies* 77, 779–805.
- Yan, Xuemin, 2008, Liquidity, investment style, and the relation between fund size and fund performance, *Journal of Financial and Quantitative Analysis* 43, 741–768.

Appendix A. Proofs

Proof that the most liquid portfolio is the benchmark portfolio:

Starting from equation (3), we solve the following constrained minimization problem:

$$\min_{\{w_i\}} \sum_{i=1}^{N_M} \frac{w_i^2}{m_i} \quad \text{subject to} \quad \sum_{i=1}^{N_M} w_i = 1, \quad (\text{A1})$$

where N_M is the number of stocks in the benchmark. The problem is convex, so the first-order conditions describe the minimum. Denoting the optimal portfolio weights by \tilde{w}_i and the Lagrange multiplier by ζ , the first-order conditions are $\frac{2\tilde{w}_i}{m_i} - \zeta = 0$, so that $\tilde{w}_i = \frac{\zeta m_i}{2}$. Substituting into the constraint yields $\sum_{i=1}^{N_M} \frac{\zeta m_i}{2} = 1$, which implies $\frac{\zeta}{2} \sum_{i=1}^{N_M} m_i = 1$, which in turn implies $\zeta = 2$, which then gives $\tilde{w}_i = m_i$.

A different proof, which is instructive in its own right, relies on a perturbation argument. Consider a portfolio with liquidity L . We perturb this portfolio by buying a bit of stock i and selling a bit of stock j , so the new portfolio weights are $w_i^* = w_i + u$ and $w_j^* = w_j - u$, where $u > 0$ and all other weights remain the same. The portfolio's illiquidity changes to

$$\begin{aligned} (L^{-1})^* &= \sum_{n \notin \{i,j\}} \frac{w_n^2}{m_n} + \frac{(w_i + u)^2}{m_i} + \frac{(w_j - u)^2}{m_j} \\ &= L^{-1} + 2u \left(\frac{w_i}{m_i} - \frac{w_j}{m_j} \right) + u^2 \left(\frac{1}{m_i} + \frac{1}{m_j} \right). \end{aligned} \quad (\text{A2})$$

If the original portfolio is the benchmark portfolio, for which $w_i/m_i = w_j/m_j = 1$, it follows immediately that any perturbation increases portfolio illiquidity: $(L^{-1})^* > L^{-1}$.

Proof of equation (19):

First, define $m = \sum_{i=1}^N m_i$ and note that

$$m = \sum_{i=1}^N m_i = \sum_{i=1}^N \frac{M_i}{M} = \frac{\sum_{i=1}^N M_i}{\sum_{i=1}^{N_M} M_i} = \frac{N}{N_M} \times \frac{1}{N} \sum_{i=1}^N \frac{M_i}{\frac{1}{N_M} \sum_{j=1}^{N_M} M_j}. \quad (\text{A3})$$

Second, rearrange the inverse of portfolio liquidity from equation (3) as follows:

$$\begin{aligned} L^{-1} &= \sum_{i=1}^N \frac{w_i^2}{m_i} = \frac{1}{m} \sum_{i=1}^N \frac{w_i^2}{m_i^*} = \frac{1}{m} \sum_{i=1}^N m_i^* \left(\frac{w_i}{m_i^*} \right)^2 = \frac{1}{m} \mathbf{E}^* \left\{ \left(\frac{w_i}{m_i^*} \right)^2 \right\} \\ &= \frac{1}{m} \left[\left(\mathbf{E}^* \left\{ \frac{w_i}{m_i^*} \right\} \right)^2 + \text{Var}^* \left(\frac{w_i}{m_i^*} \right) \right] = \frac{1}{m} \left[1 + \text{Var}^* \left(\frac{w_i}{m_i^*} \right) \right], \end{aligned} \quad (\text{A4})$$

where E^* is the expectation with respect to the m^* measure. Combining equations (A3) and (A4) yields equation (19).

Proof of the statement from Section 3.1 that fee revenue is increasing in skill:

Let A_i , f_i , T_i , and L_i be the values of fund characteristics that maximize fund i 's fee revenue under the equilibrium condition $\alpha_i = 0$. Consider two funds, where fund 2 is more skilled than fund 1: $\mu_1 < \mu_2$. Then we show below that fund 2's equilibrium fee revenue is greater than fund 1's revenue: $F_1 < F_2$. Suppose fund 2 makes the (suboptimal) choices $\tilde{T}_2 = T_1$, $\tilde{L}_2 = L_1$, and $\tilde{f}_2 = f_1 + (\mu_2 - \mu_1)g(T_1, L_1)$. Then investors allocate capital to fund 2 until its size is $\tilde{A}_2 = A_1$, because under that size, fund 2's net alpha is zero:

$$\tilde{\alpha}_2 = \mu_2 g(\tilde{T}_2, \tilde{L}_2) - \theta \tilde{A}_2^{\gamma-1} \tilde{T}_2^\lambda \tilde{L}_2^{-\phi} - \tilde{f}_2 = \mu_1 g(T_1, L_1) - \theta A_1^{\gamma-1} T_1^\lambda L_1^{-\phi} - f_1 = \alpha_1 = 0 .$$

In other words, fund 2's size of $\tilde{A}_2 = A_1$ satisfies the equilibrium condition under these choices of T , L , and f . Fund 2's fee revenue with these choices, \tilde{F}_2 , can be no greater than its maximum equilibrium fee revenue, F_2 , and

$$\tilde{F}_2 = \tilde{f}_2 \tilde{A}_2 = f_1 A_1 + (\mu_2 - \mu_1)g(T_1, L_1)A_1 = F_1 + (\mu_2 - \mu_1)g(T_1, L_1)A_1 > F_1 .$$

Proofs of statements from Section 6:

First, we prove that the cross-sectional correlation between fund size and expense ratio is negative as long as $\beta_{\mu,f} < (\gamma - 1)/\gamma$. Take logs in equation (27), so that

$$\ln(A) = -\ln(f) + \frac{\gamma}{\gamma - 1} \ln(\mu) + \text{constant} , \quad (\text{A5})$$

and note that

$$\begin{aligned} \text{Cov}(\ln(A), \ln(f)) &= \text{Cov}\left(-\ln(f) + \frac{\gamma}{\gamma - 1} \ln(\mu), \ln(f)\right) \\ &= \frac{\gamma}{\gamma - 1} \text{Cov}(\ln(\mu), \ln(f)) - \text{Var}(\ln(f)) . \end{aligned} \quad (\text{A6})$$

This covariance is negative if $\text{Cov}(\ln(\mu), \ln(f))/\text{Var}(\ln(f)) < \frac{\gamma-1}{\gamma}$, or $\beta_{\mu,f} < \frac{\gamma-1}{\gamma}$.

Second, we prove that the correlation between g and f is positive as long as $\beta_{\mu,f} < 1$. Take logs in equation (16), so that

$$\ln(g) = \ln(f) - \ln(\mu) + \ln\left(\frac{\gamma}{\gamma - 1}\right) , \quad (\text{A7})$$

and note that

$$\begin{aligned} \text{Cov}(\ln(g), \ln(f)) &= \text{Cov}(\ln(f) - \ln(\mu), \ln(f)) \\ &= \text{Var}(\ln(f)) - \text{Cov}(\ln(\mu), \ln(f)) . \end{aligned} \quad (\text{A8})$$

This covariance is positive if $\text{Cov}(\ln(\mu), \ln(f))/\text{Var}(\ln(f)) < 1$, or $\beta_{\mu,f} < 1$.

Finally, we prove that the correlation between g and A is negative as long as $\beta_{\mu,A} < \gamma - 1$. Take logs in equation (31), so that

$$\ln(g) = \frac{1}{\gamma - 1} \ln(\mu) - \ln(A) + \text{constant} , \quad (\text{A9})$$

and note that

$$\begin{aligned}\text{Cov}(\ln(g), \ln(A)) &= \text{Cov}\left(\frac{1}{\gamma-1} \ln(\mu) - \ln(A), \ln(A)\right) \\ &= \frac{1}{\gamma-1} \text{Cov}(\ln(\mu), \ln(A)) - \text{Var}(\ln(A)) .\end{aligned}\quad (\text{A10})$$

This covariance is negative if $\text{Cov}(\ln(\mu), \ln(A))/\text{Var}(\ln(A)) < \gamma - 1$, or $\beta_{\mu,A} < \gamma - 1$.

Proof of the statement from footnote 15:

Suppose an active portfolio is blended with a passive benchmark so that $\omega \in [0, 1]$ is the weight on the active portfolio and $1 - \omega$ is the weight on the benchmark. The active portfolio has liquidity L and weights w_i ; the benchmark has liquidity of one and weights m_i . The blended portfolio's weights are $\tilde{w}_i = \omega w_i + (1 - \omega) m_i$. Its illiquidity is

$$\begin{aligned}\tilde{L}^{-1} &= \sum_i \frac{\tilde{w}_i^2}{m_i} \\ &= \sum_i \frac{\omega^2 w_i^2 + 2\omega(1-\omega)w_i m_i + (1-\omega)^2 m_i^2}{m_i} \\ &= \omega^2 L^{-1} + 2\omega(1-\omega) \left(\sum_i \frac{w_i m_i}{m_i} \right) + (1-\omega)^2 \\ &= \omega^2 L^{-1} + 1 - \omega^2.\end{aligned}\quad (\text{A11})$$

In words, the blended portfolio's illiquidity is a weighted average of the illiquidities of the active portfolio and the benchmark, where the weights are ω^2 and $1 - \omega^2$. Also note that $\tilde{L}^{-1} \leq L^{-1}$: indexing a part of the portfolio reduces the portfolio's illiquidity.

Appendix B. Nonlinear Trading Cost Function

We now generalize the trading cost function underlying our portfolio liquidity measure. In equation (6), the cost per dollar traded increases linearly with the ratio of the dollar amount traded to market capitalization. We replace this linearity by nonlinearity:

$$C_i = c \left(\frac{D_i}{M_i} \right)^\eta , \quad (\text{A12})$$

where $\eta > 0$. The trading cost function then becomes

$$C = \left(\frac{c}{M^\eta} \right) D^{1+\eta} \underbrace{\left(\sum_{i=1}^N \frac{w_i^{1+\eta}}{m_i^\eta} \right)}_{L^{-1}} , \quad (\text{A13})$$

so that portfolio liquidity is given by

$$L = \left(\sum_{i=1}^N \frac{w_i^{1+\eta}}{m_i^\eta} \right)^{-1} . \quad (\text{A14})$$

For the baseline case of $\eta = 1$, which we use throughout the paper, equations (A12), (A13), and (A14) simplify to equations (6), (7), and (3), respectively. Under this alternative definition of L , we still have $L \in (0, 1]$, and the maximum value of $L = 1$ is still achieved by the benchmark portfolio.

This alternative measure of portfolio liquidity can also be decomposed into stock liquidity and diversification, as in equation (19), but the formulas are a bit more complicated:

$$L = \underbrace{\left(\frac{1}{N} \sum_{i=1}^N \mathcal{L}_i \right)^\eta}_{\text{Stock Liquidity}} \times \underbrace{\left(\frac{N}{N_M} \right)^\eta \left[\mathbb{E}^* \left\{ \left(\frac{w_i}{m_i^*} \right)^{1+\eta} \right\} \right]^{-1}}_{\text{Diversification}}. \quad (\text{A15})$$

When we reestimate our main specification from Table 1 for the alternative measure of L with values of η ranging from 0.1 to 0.9, we find similar results. See the Internet Appendix.

Appendix C. Data

To construct our sample of actively managed U.S. domestic equity mutual funds, we begin with the 1979–2011 dataset constructed by Pástor, Stambaugh, and Taylor (2015), which combines and cross-validates data from CRSP and Morningstar. A detailed description of the dataset is in the online Data Appendix to that paper. We expand the dataset by merging it with the Thomson Reuters dataset of fund holdings and adding data from 2012 through 2014. We restrict the sample to include fund-month observations whose historical Morningstar category falls within the traditional 3×3 style box (small-cap, mid-cap, large-cap interacted with growth, blend, and value). This restriction excludes non-equity funds, international funds, and industry-specific funds. We also exclude funds identified by CRSP or Morningstar as index funds, funds whose name contains the word “index,” and funds classified by Morningstar as funds of funds. We exclude fund-month observations with expense ratios below 0.1% per year since they are extremely unlikely to belong to actively managed funds. Finally, we exclude fund-month observations with lagged fund size below \$15 million in 2011 dollars. We aggregate share classes belonging to the same fund.¹⁶

When computing portfolio weights w , we drop all fund holdings that are not included in our definition of the market portfolio, which is guided by the holdings of Vanguard’s Total Stock Market Index fund. This fund tracks the CRSP US Total Market Index, which is designed to track the entire U.S. equity market. We find that 98.9% of the fund’s holdings are either ordinary common shares (CRSP share code, *shrcd*, with first digit equal to 1) or REIT shares of beneficial interest (*shrcd* = 48). We therefore define the market as all CRSP securities with these share codes. This definition includes foreign-incorporated firms (*shrcd* = 12), many of which are deemed domestic by CRSP (they make up 1.4% of the Vanguard fund’s holdings), but it excludes securities such as ADRs (*shrcd* = 31) and units or limited partnerships (*shrcd* first digit equal to 7). A fund’s holding can fall outside the market if its CUSIP cannot be linked to the CRSP database (1.0% of the Vanguard fund’s holdings), or if the security is in CRSP but outside our definition of the market (0.1% of the

¹⁶Many mutual funds offer multiple share classes, which represent claims on the same underlying assets but have different fee structures. Different share classes of the same fund have the same Morningstar FundID. We aggregate all share classes of the same fund. Specifically, we compute a fund’s size by summing AUM across the fund’s share classes, and we compute the fund’s expense ratio, returns, and other variables by asset-weighting across share classes.

Vanguard fund's holdings). These holdings mainly represent cash, bonds, and other non-equity securities. For the median (average) fund/month observation in our sample, 2.3% (3.5%) of holding names and 1.9% (3.1%) of holding dollars are outside the market.

When computing fund size, we cross-verify monthly AUM between CRSP and Morningstar as described in Pástor, Stambaugh, and Taylor (2015). Annual data on expense ratios and turnover of mutual funds are from CRSP. Following Pástor, Stambaugh, and Taylor (2017), we winsorize turnover at the 1st and 99th percentiles. Monthly fund returns, net of expense ratio, are from CRSP and Morningstar. Following Pástor, Stambaugh, and Taylor (2015), we require that CRSP and Morningstar agree closely on a fund's return; otherwise we set it to missing.

For any fund-level variable requiring holdings data, we set the variable to missing if there is a large discrepancy in a fund's AUM between our CRSP/Morningstar database and the Thomson Reuters holdings database. We compute the ratio of the fund's AUM according to CRSP/Morningstar to the fund's AUM obtained by adding up all the fund's holdings from Thomson Reuters. If this ratio exceeds 2.0 or is less than 0.5, we set all holdings-based measures to missing. This filter drops the holdings-based variables for 3.4% of fund/quarter observations. We suspect that some of these large discrepancies are due to poor links between Thomson Reuters and CRSP/Morningstar.