### NBER WORKING PAPER SERIES

### PATERNALISM AND PSEUDO-RATIONALITY

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Working Paper 23620 http://www.nber.org/papers/w23620

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 July 2017, Revised July 2019

We acknowledge very helpful feedback from Youssef Benzarti, Raj Chetty, David Cutler, Emir Kamenica, Scott Kominers, Benjamin Lockwood, and participants at the ASSA and National Tax Association annual meetings. We thank Edward Kong for excellent research assistance. We are grateful for funding from the Eric Mindich fund for Research in the Foundations of Human Behavior. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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Paternalism and Pseudo-Rationality Itzik Fadlon and David Laibson NBER Working Paper No. 23620 July 2017, Revised July 2019 JEL No. D61,E70,H00

### ABSTRACT

Resource allocations are jointly determined by the actions of social planners and households. In this paper, we highlight the distinction between planner optimization and household optimization. We show that planner optimization is a partial substitute for household optimization and that this is still true when households have private in- formation about their preferences. We illustrate a novel identification problem: some aggregate equilibrium properties that are implied by household optimization are also implied by planner optimization. A rational utilitarian planner enacts policies that cause the Euler equation to hold on average in a population of non-optimizing households, which we refer to as pseudo-rationality. Hence, widely studied allocative optimality conditions also arise in an allocatively inefficient economy with a rational planner, who uses tools such as default savings and Social Security to influence the consumption choices of myopic and passive households. Accordingly, aggregate classical optimization tests implemented with data from economies with paternalistic governments cannot reveal whether households would make optimal decisions on their own.

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### 1 Introduction

Resource allocations are jointly determined by many actors, including households, firms, and governments. In this paper, we study the interaction between households and a well-intentioned government, which we will refer to as the social planner.<sup>1</sup> We analyze the case in which (i) the social planner is a (benevolent) rational utilitarian, (ii) households are hetero-geneous in their degree of rationality, and (iii) the social planner has some scope to intervene – e.g., default savings and mandatory savings, which mirror institutions like 401(k) auto-enrollment and Social Security.<sup>2</sup> We show that in equilibrium, planner optimization is a (close) substitute for household optimization. This is true even when there are information asymmetries, so that households know more about their preferences than planners.

Our analysis illustrates a novel identification problem. Are seemingly optimal allocations caused by optimizing households, or are such allocations caused by planners who paternalistically influence household behavior? We find that some widely studied classical tests for household optimization, which rely on the Euler equation and its variants, do not identify the causal source of the (average) allocative optimality. Equilibrium properties that are implied by household optimization are also implied by planner optimization. The actions of rational planners may result in pseudo-rational allocations by endogenously causing the Euler equation to hold on *average* in a population of non-optimizing households, including myopic and passive households, even though the Euler equation does *not* hold for each individual household.

To demonstrate these mechanisms, we study a classic life-cycle model in which agents earn labor income during working life and can save a fraction of their earnings for retirement consumption. The planner has two policy levers: mandatory retirement savings (similar to Social Security or defined benefit pensions), and voluntary retirement savings with a default

<sup>&</sup>lt;sup>1</sup>Naturally, many governments are not (even approximately) well-intentioned. They are not trying to maximize the well-being of their citizens. These cases can also be studied using the framework in this paper, but we leave those cases for future work.

<sup>&</sup>lt;sup>2</sup>Defined benefit (DB) pensions are another example of a mandatory savings scheme. DB pensions have fallen in popularity in the U.S., though they remain commonplace in other developed economies.

savings rate (similar to 401(k) accounts).<sup>3</sup> We consider an economy with three types of households: optimizing households, who behave optimally throughout their life-cycle, myopic households, who opt out of the default and consume their *entire* disposable income in each period, and passive households, who accept the planner's default and consume their residual income. We include myopes – an extreme type – to emphasize that our results hold even when the deck is stacked against social efficiency. Additionally, we allow for agents to have privately observed preference parameters of arbitrary structure. Our planner jointly chooses a default level of savings within the system of voluntary savings accounts and designs a Social Security system in order to maximize total social welfare, taking into account the behavior of optimizing households and the suboptimal behavior of myopic and passive households.

We show that, in equilibrium, average marginal utility before retirement is equal to average marginal utility after retirement for *any* distribution of optimizing, myopic, and passive households. Such marginal utility smoothing applies to all concave utility functions with a general class of taste shocks. Accordingly, Euler equations, which characterize an economy with only optimizing households, also arise in an economy with institutions that are optimally designed by a planner for non-optimizing households. As such, aggregate Euler equations do not differentiate between household optimization and planner optimization.

Marginal utility smoothing is closely related to consumption smoothing, which is among the most common tests for household optimization. The precise form of consumption smoothing depends on the curvature of the utility function and the structure of taste shocks. We show that when exact aggregate consumption smoothing arises in a world comprised only of optimizing households, it also arises in our economy with an optimizing planner and any distribution of optimizing, myopic, and passive households. Moreover, in some leading cases, *exact* consumption smoothing does *not* arise with optimizing households, but does arise with an optimizing planner and non-optimizing households. These results imply that exact aggregate consumption smoothing is a more robust property of the model *without* optimizing households, in contrast to the common view.

<sup>&</sup>lt;sup>3</sup>Later in the paper we show that this restriction to only two policy levers is without loss of generality. This generalization occurs in Section 6.

Although we show that the average Euler equation and average consumption smoothing are not generally diagnostic of household optimization, we demonstrate that other types of economic information – both from the cross section and from time series – identify the extent of household optimization. For example, bunching (i.e., a mass point) at the default savings level identifies passive households. In the time series, only passive households change their consumption when the default savings rate changes. Such tests are not confounded by the equilibrium link between household and planner optimization. As such, they provide ways to overcome the mis-attribution problems that we highlight. We provide a range of methods for identifying the mass of optimizing, myopic, and passive households.

Our choice of the particular framework for illustrating our arguments – namely, savings over the life-cycle – is motivated by the seemingly contradictory findings of the recent research on household savings. Some papers find evidence that is consistent with optimal savings while others highlight savings anomalies. See Skinner (2007) and Poterba (2014) for reviews of this literature and Section 7 of our paper for a specific discussion of how our model reconciles the different sets of findings. For example, our model implies that household-level sub-optimization, arising from passivity and myopia, will be partially offset by paternalistic policies, like defaults and Social Security. Hence, the economy that our model describes will simultaneously feature both household-level mistakes and many characteristics of "aggregate" optimality. In equilibrium, the Euler equation is satisfied on average across all households, though it is not satisfied for some, or potentially even all, individual households.

Our paper is also related to the behavioral economics literature on optimal paternalism. In our setting the planner chooses policies that dramatically improve the welfare of myopic and passive agents, while relatively weakly distorting the choices and welfare of rational agents (whose behavior would be optimal under laissez faire policies). The social desirability of policies that disproportionately affect non-rational consumers was first highlighted by Camerer, Issacharoff, Loewenstein, O'Donoghue, and Rabin (2003) in their paper on asymmetric paternalism. Our analysis incorporates defaults, which preserve freedom of choice while still influencing behavior. Sunstein and Thaler (2003) and Thaler and Sunstein (2003, 2008) refer to such choice-preserving nudges as libertarian paternalism. Our analytic framework also includes mandates (like Social Security), which fall outside the domain of libertarian paternalism. Our paper derives socially optimal paternalistic policies for households with behavioral biases, which follows a line of related papers: Diamond (1977), Kotlikoff, Spivak, and Summers (1982), Feldstein (1985), Gruber and Köszegi (2001), Choi et al. (2003), O'Donoghue and Rabin (2003, 2006), Carroll et al. (2009), Fang and Silverman (2009), Loginova and Persson (2012), Bubb and Pildes (2014), Alcott and Taubinsky (2015), Bubb, Corrigan and Warren (2015), Chetty (2015), Moser and de Souza e Silva (2015), Farhi and Gabaix (2015), Bubb and Warren (2016), and Lockwood (2017). Another closely related literature studies *self*-binding policies, in other words, self-directed paternalism: e.g., Ashraf, Karlan and Yin (2006), Gine, Karlan, and Zinman (2010), Bryan, Karlan, and Nelson (2010), Amador, Werning, and Angeletos (2006), Augenblick, Niederle, and Sprenger (2015), Kaur, Kremer and Mullainathan (2015), and Schilbach (2015).

The remainder of the paper is organized as follows. In Section 2, we describe our baseline model in which the population is comprised of three types of households: optimizers, myopes, and passives. We describe the general (privately observed) taste shocks that these households experience. We also describe the policy levers available to the government. Finally, we characterize the equilibrium behavior of the households (holding fixed the behavior of the social planner). Section 3 derives the equilibrium behavior of the social planner. In this section we show that the classical aggregate Euler equation holds, regardless of the proportions of optimizers, myopes, and passives (as long as the social planner is a rational utilitarian). In Section 4, we study several special cases – multiplicative taste shocks, quadratic utility, and constant relative risk aversion. Using these special cases and our earlier Euler equation results, we derive equilibrium consumption dynamics, which are related to but distinct from the dynamics of marginal utility. We show that consumption smoothing is a robust feature of our model. Indeed, the forces that produce consumption smoothing only grow stronger as the fraction of the population that is myopic and passive increases. In Section 5, we discuss the issue of identification – how can the distribution of optimizing, myopic, and passive households be

identified? In Section 6, we derive an isomorphism between our analysis and the associated mechanism design problem. Specifically, we show that the equilibrium of our model, with a restricted policy space using only default savings and mandatory savings, exactly matches the equilibrium that arises when the government's policy tools are maximally generalized and the problem is treated as a mechanism design problem. Section 7 explains why our model reconciles several tensions in the empirical consumption literature. Our framework explains why a population that is comprised of a mix of optimizing, myopic, and passive households will have relatively smooth average consumption dynamics, although significant deviations from optimality will arise at the level of individual households, including sensitivity to default savings. Section 8 concludes.

## 2 Baseline Model

Setup. We consider a two-period model, where period t = 1 is working life and period t = 2 is retirement. Real output during working life is  $y_1 = y$  and real output during retirement is 0. Real consumption is denoted  $c_1$  and  $c_2$ . We assume that the real interest rate, r, is fixed and let  $R \equiv 1 + r$ . The life-time budget constraint household i faces is

$$c_1 + \frac{c_2}{R} \le y.$$

Preferences. Consider a household with consumption  $\{c_1, c_2\}$  and a general taste shifter, vector  $\theta$ . Total life-time utility is given by

$$U(c_1, c_2; \theta) = u_1(c_1; \theta) + \delta u_2(c_2; \theta),$$

where  $\delta$  is a discount factor.<sup>4</sup> The taste parameter  $\theta \in \Theta$  varies across households and is independently drawn from a common cumulative distribution function  $F(\theta)$  over a compact

$$U(c_1, c_2; \theta) = u(c_1) + \delta u(c_2),$$

where  $\hat{\delta} = \delta \theta$ .

<sup>&</sup>lt;sup>4</sup>The case in which  $\delta$  is heterogeneous across households is embedded in our model. To illustrate this point, a special case of our model is

space  $\Theta$ .<sup>5</sup> We assume that  $\theta$  is known to the households at time 0, but not known to the government. Finally, we assume that  $u'_t(\cdot; \theta) > 0$  and  $u''_t(\cdot; \theta) < 0$  for all  $\theta$  and for  $t = \{1, 2\}$ .

To simplify notation, we suppress the *i* index, and simply refer to households by their taste shock  $\theta$ .

Institutions. There are two kinds of institutions: a voluntary savings account and a forced retirement savings account. We assume that the planner sets a default level of savings in the voluntary savings account,  $s_D$ . The households are able to opt-out of this default at zero cost. In addition, the planner sets forced retirement savings (a minimum level of savings from labor income) of  $s_F$ , which is deposited into the forced savings account during working life.<sup>6</sup> Because  $s_F$  is defined as the minimum level of savings, passively following the default must engender a level of savings at least as great as  $s_F$ . In notation,  $s_F + s_D \ge s_F$ . Hence, an operational default requires

$$s_D \ge 0. \tag{1}$$

In section 6, we show that the institutional restrictions described in the previous paragraph are made without loss of generality. In other words, even if the planner solves a completely general mechanism design problem (for this economy), the solution is the same as the solution that we characterize for the institutionally restricted model.

Household types. There are three types of households (indexed on a continuum from 0 to1): Optimizers, Myopes, and Passives. We explain each of these in turn.

Optimizers (notated O) choose the optimal level of consumption in all time periods, taking into account their private information about  $\theta$  and the institutional constraints that they face. Optimizing households may not be able to achieve their first best allocation if their optimal level of savings is lower than the forced level of savings. Formally, optimizing

<sup>&</sup>lt;sup>5</sup>This common distribution assumption for  $\theta$  can be relaxed. As long as we maintain the assumption that the government holds rational expectations, we can allow households to have preference parameters that are drawn from (ex-ante) heterogeneous distributions. To simplify notation, we adopt the assumption of a common iid CDF,  $F(\theta)$ .

<sup>&</sup>lt;sup>6</sup>This is essentially what has been adopted by Australia, Israel, and Singapore, and has similarities to Social Security in the US.

households choose the life-time consumption path  $\{c_1, c_2\}$  that solves

$$\max_{\{c_1, c_2\}} u_1(c_1; \theta) + \delta u_2(c_2; \theta)$$

subject to two constraints

$$c_1 + \frac{c_2}{R} \le y,$$
$$c_1 \le y - s_F \equiv \bar{c}_1$$

The first constraint is the budget constraint. The second constraint is the period-one liquidity constraint because  $s_F$  is the level of mandatory savings – i.e., the minimum level of savings.

To characterize the equilibrium behavior of optimizers,  $\{c_1(\theta), c_2(\theta)\}$ , we first consider the unconstrained problem – that it, we focus on optimizing households that are unconstrained during their working life. In this case, the standard Euler equation holds for each household:

$$u_1'(c_1(\theta);\theta) = \delta R u_2'(c_2(\theta);\theta).$$

By contrast, for the constrained optimizing households  $c_1(\theta) = y - s_F = \bar{c}_1$  and  $c_2(\theta) = R \times s_F$ .

This completes our discussion of optimizers. We now turn to the second and third types of households.

Myopes (notated M) opt out of the default and consume as much as possible in every period. Hence, myopes are constrained only by the forced savings, so that they consume  $c_1^M = y - s_F = \bar{c}_1$  and  $c_2^M = R \times s_F$ , which is the same as the constrained optimizers.

Passives (notated P) accept the default and consume the residual income flow. That is, for them  $c_1^P = y - s_F - s_D \equiv c_1^D$  and  $c_2^P = R \times (s_F + s_D)$ .

The rational planner's problem. The shares of optimizing, myopic, and passive households are  $\mu_O$ ,  $\mu_M$ , and  $\mu_P$ , respectively, where  $0 \le \mu_O, \mu_M, \mu_P \le 1$  and  $\mu_O + \mu_M + \mu_P = 1$ . We denote the distribution of these "decision" types by  $\mu \equiv (\mu_O, \mu_M, \mu_P)$ . The utilitarian social planner's objective is to choose the policy tools  $\{s_D, s_F\}$  that maximize total utility. Note that any pair of values  $\{s_D, s_F\}$  generates equilibrium values for period-one consumption by optimizers,  $c_1(\theta)$ , myopes,  $c_1^M$ , and passives,  $c_1^P$ . Because of the household budget constraint, these consumption levels imply  $c_2(\theta) = R \times (y - c_1(\theta))$ ,  $c_2^M = R \times (y - c_1^M)$ , and  $c_2^P = R \times (y - c_1^P)$ . Accordingly, the planner chooses  $\{s_D, s_F\}$  to maximize

$$W \equiv \mu_O \int_{\Theta} [u_1(c_1(\theta); \theta) + \delta u_2(c_2(\theta); \theta)] dF(\theta) + \mu_M \int_{\Theta} [u_1(c_1^M; \theta) + \delta u_2(c_2^M; \theta)] dF(\theta) + \mu_P \int_{\Theta} [u_1(c_1^P; \theta) + \delta u_2(c_2^P; \theta)] dF(\theta).$$
(2)

Recall that this expression is maximized subject to the constraint  $s_D \ge 0$  (equation (1)).

We will make frequent use of the expectation operator,  $E[\cdot]$ , which is the expectation taken over the entire population. Specifically, for any random variable  $x(\theta)$ , this expectation operator integrates jointly over decision types (optimizers, myopes, and passives) and over taste shocks ( $\theta$ ):

$$E[x(\theta)] \equiv \mu_O \int_{\Theta} x^O(\theta) dF(\theta) + \mu_P \int_{\Theta} x^M(\theta) dF(\theta) + \mu_M \int_{\Theta} x^P(\theta) dF(\theta).$$

## 3 Equilibrium with Optimal Institutions

We begin by analyzing the basic Euler equation. First, consider the benchmark of an economy in which *all* households are optimizers, so that  $u'_1(c_1(\theta); \theta) = \delta R u'_2(c_2(\theta); \theta)$  for each household. In principle, if an econometrician knew each household's value of  $\theta$ , then it would be possible to test this equation at the household level. However,  $\theta$ , which is a taste shock, is not observable. Moreover, in practice most variables are measured with noise, which prevents equations from holding exactly. Accordingly, empirical analysis tends to focus on whether the Euler equation is satisfied on average, that is, whether:

$$E\left[u_1'(c_1(\theta);\theta)\right] = \delta RE\left[u_2'(c_2(\theta);\theta)\right].$$

Naturally, this holds in the benchmark economy of optimizing households. Our first proposition proves that this last equation *also* holds in our economy, in which optimizing households can represent any fraction of the economy (i.e.,  $\mu_O \in [0, 1]$ ).

We now characterize the equilibrium allocation in our economy with a *rational* social planner. We prove that some basic equilibrium features that are commonly attributed to household optimization also appear in our partially passive and partially myopic economy as a result of the planner's intervention.

**Proposition 1** Assume a rational planner. Then for any distribution of optimizing, myopic, and passive households, a classical Euler equation will hold on average in the population:

$$E\left[u_1'(c_1;\theta)\right] = \delta R E\left[u_2'(c_2;\theta)\right].$$
(3)

Proposition 1 establishes that the aggregate Euler equation (3) holds for any mass vector  $\mu$  characterizing the fraction of optimizing, myopic, and passive households. The results in the rest of the paper all have the property that a classical optimality condition holds on average in the population regardless of the fraction of optimizing households.

The proof of this first proposition uses three steps that correspond to the following three lemmas.

**Lemma 1**  $(c_1^D < \bar{c}_1)$  At the planner's optimum, the default consumption in period 1 is strictly less than maximal consumption in period 1:

$$c_1^D \equiv y - s_F - s_D < y - s_F \equiv \bar{c}_1.$$

Equivalently,  $s_D > 0$ .

The social planner uses the sum  $s_F + s_D$  to pin down total savings for passives. The social planner recognizes that mandatory savings,  $s_F$ , has a negative externality on optimizers, some of whom are constrained by  $s_F$ . Note that  $s_D$  has no negative externality because it only affects the choices of passives. Accordingly, the social planner sets the default savings rate,  $s_D$ , strictly greater than zero, which implies that myopes save less than passives:  $s_F < s_F + s_D$ . This first lemma ( $s_D > 0$ ) is proved in the appendix.

Our second lemma describes a change of variables for the planner's optimization problem.

Lemma 2 (Change of Variables) With the institutions we have assumed, the planner's problem is isomorphic to jointly choosing the optimal level of two variables: (i)  $c_1^D \equiv y - s_F - s_D$  (default consumption in period 1), (ii)  $\bar{c}_1 \equiv y - s_F$  (maximal consumption in period 1, given the level of mandatory savings), subject to the constraint that  $c_1^D \leq \bar{c}_1$  (implied by the original constraint  $s_D \geq 0$ ).

To see this, let  $\Gamma(\bar{c}_1) \subset \Theta$  denote the set of  $\theta$  values that would induce an optimizer to be strictly constrained if period-one consumption were bounded above by  $\bar{c}_1$ . Then, we can re-write the planner's optimization problem (2) as choosing  $c_1^D$  and  $\bar{c}_1$  to maximize

$$W \equiv \mu_O \int_{\Theta} [u_1(c_1(\theta); \theta) + \delta u_2(R(y - c_1(\theta)); \theta)] dF(\theta)$$

$$+ \mu_M \int_{\Theta} [u_1(\bar{c}_1; \theta) + \delta u_2(R(y - \bar{c}_1); \theta)] dF(\theta)$$

$$+ \mu_P \int_{\Theta} [u_1(c_1^D; \theta) + \delta u_2(R(y - c_2^D); \theta)] dF(\theta),$$
(4)

subject to the constraint

$$c_1^D \le \bar{c}_1,\tag{5}$$

where  $c_1(\theta)$  solves  $u'_1(c_1(\theta); \theta) = \delta R u'_2(R(y - c_1(\theta)); \theta)$  for  $\theta \notin \Gamma(\bar{c}_1)$  and  $c_1(\theta) = \bar{c}_1$  for  $\theta \in \Gamma(\bar{c}_1)$ .

**Lemma 3 (Euler Equation)** If social welfare is maximized with respect to  $c_1^D$  and  $\bar{c}_1$ , then the associated first order conditions imply

$$E\left[u_1'(c_1;\theta)\right] = \delta RE\left[u_2'(c_2;\theta)\right].$$

**Proof.** Lemma 1 establishes that at an optimum  $\bar{c}_1 > c_1^D$ . In other words, myopes

consume in period 1 *strictly* more than passives at an optimum. Because  $\bar{c}_1 > c_1^D$ , (local) perturbations of  $c_1^D$  do not affect the socially optimal value of  $\bar{c}_1$ , and vice versa. Accordingly, the constraint in Lemma 2 (equation (5)) can be ignored when taking first order conditions. Exploiting the change of variables in Lemma 2 and the irrelevance of the constraint, we know that at the optimum

$$\frac{\partial W}{\partial c_1^D} = \frac{\partial W}{\partial \bar{c}_1} = 0.$$

Note that perturbations of  $c_1^D$  will only affect passives, a property we will exploit in the next paragraph. Likewise, perturbations of  $\bar{c}_1$  will only affect myopes and (some) optimizers, which we will also exploit in the next paragraph.

Recall that  $c_1^P = c_1^D$  and  $c_2^P = R \times (y - c_1^D)$ . The planner's choice of  $c_1^D$  establishes an average Euler equation for passives. Specifically,  $\frac{\partial W}{\partial c_1^D} = 0$  implies that

$$\int_{\Theta} u_1'(c_1^P; \theta) dF(\theta) = \delta R \int_{\Theta} u_2'(c_2^P; \theta) dF(\theta)$$
(6)
where  $c_2^P = R \times (y - c_1^P).$ 

The first order condition for  $\bar{c}_1$  is

$$\int_{\Theta} \left[ \mu_O u_1'(c_1(\theta);\theta) \frac{dc_1(\theta)}{d\bar{c}_1} + \mu_M u_1'(c_1^M;\theta) \frac{dc_1^M(\theta)}{d\bar{c}_1} \right] dF(\theta) \\
+ \delta \int_{\Theta} \left[ \mu_O u_2'(c_2(\theta);\theta) \frac{dc_2(\theta)}{d\bar{c}_1} + \mu_M u_2'(c_2^M;\theta) \frac{dc_2^M}{d\bar{c}_1} \right] dF(\theta) = 0. \quad (7)$$

For myopes,  $\frac{dc_1^M}{d\bar{c}_1}R + \frac{dc_2^M}{d\bar{c}_1} = 0$ . This is a direct consequence of the household budget constraint. Moreover,  $\frac{dc_1^M}{d\bar{c}_1} = 1$  so that  $\frac{dc_2^M}{d\bar{c}_1} = -R$ . For optimizers  $\frac{dc_1(\theta)}{d\bar{c}_1} \in \{0,1\}$ , where  $\frac{dc_1(\theta)}{d\bar{c}_1} = 1$  iff  $\theta \in \Gamma(\bar{c}_1)$ .<sup>7</sup> This implies that

$$\int_{\theta \in \Theta} u_1'(c_1(\theta); \theta) \frac{dc_1(\theta)}{d\bar{c}_1} dF(\theta) = \int_{\theta \notin \Gamma(\bar{c}_1)} \left[ u_1'(c_1(\theta); \theta) \times 0 \right] dF(\theta) + \int_{\theta \in \Gamma(\bar{c}_1)} \left[ u_1'(\bar{c}_1; \theta) \times 1 \right] dF(\theta)$$
$$= \int_{\theta \in \Gamma(\bar{c}_1)} u_1'(c_1(\theta); \theta) dF(\theta).$$

Likewise,  $\frac{dc_2(\theta)}{d\bar{c}_1} \in \{0, -R\}$ , where  $\frac{dc_2(\theta)}{d\bar{c}_1} = -R$  iff  $\theta \in \Gamma(\bar{c}_1)$ . This implies that

$$\int_{\theta \in \Theta} u_2'(c_2(\theta);\theta) \frac{dc_2(\theta)}{d\bar{c}_1} dF(\theta) = \int_{\theta \notin \Gamma(\bar{c}_1)} \left[ u_2'(c_2(\theta);\theta) \times 0 \right] dF(\theta) + \int_{\theta \in \Gamma(\bar{c}_1)} \left[ u_2'(\bar{c}_2;\theta) \times (-R) \right] dF(\theta)$$
$$= -R \int_{\theta \in \Gamma(\bar{c}_1)} u_2'(c_2(\theta);\theta) dF(\theta).$$

Equation (7) therefore reduces to:

$$\int_{\theta \in \Gamma(\bar{c}_{1})} \mu_{O} u_{1}'(c_{1}(\theta);\theta) dF(\theta) + \int_{\theta \in \Theta} \mu_{M} u_{1}'(c_{1}^{M};\theta) dF(\theta)$$
$$= \delta R \int_{\theta \in \Gamma(\bar{c}_{1})} \mu_{O} u_{2}'(c_{2}(\theta);\theta) dF(\theta) + \delta R \int_{\theta \in \Theta} \mu_{M} u_{2}'(c_{2}^{M};\theta) dF(\theta).$$
(8)

We also have for all  $\theta \notin \Gamma(\bar{c}_1)$ :

$$u_1'(c_1(\theta);\theta) = \delta R u_2'(c_2(\theta);\theta).$$
(9)

Combining equations (6), (8), and (9), we have

$$E\left[u_1'(c_1;\theta)\right] = \delta RE\left[u_2'(c_2;\theta)\right].$$

Proposition 1 follows immediately by combining Lemmas 1, 2, and 3.

<sup>&</sup>lt;sup>7</sup>This follows from the following argument. For unconstrained households whose choice of  $c_1(\theta)$  is strictly interior to their choice set, we have  $\frac{dc_1(\theta)}{d\bar{c}_1} = 0$ . For households who are strictly constrained, we have  $\frac{dc_1(\theta)}{d\bar{c}_1} = 1$ . Lastly, for households who are weakly constrained, we have  $\frac{dc_1(\theta)}{d\bar{c}_1} = 0$ .

In summary, Proposition 1 implies that a standard classical Euler equation characterizes equilibrium allocations in the economy, regardless of the proportions of optimizers, myopes, and passives. Note that the Euler equation holds in expectation across all households, but it does not hold at the level of each individual household. Some households consume too little in period 1 (optimizers and myopes who have a taste shifter,  $\theta$ , that would imply an optimal level of  $c_1 > \bar{c}_1$ , and passives who have a taste shifter,  $\theta$ , that would imply an optimal level of  $c_1 > c_1^D$ ). Some households consume too much (myopes who have a taste shifter,  $\theta$ , that would imply an optimal level of  $c_1 < \bar{c}_1$ , and passives who have a taste shifter,  $\theta$ , that would imply an optimal level of  $c_1 < \bar{c}_1$ . The Euler equation is only satisfied on *average* across the population of households.

### 3.1 When could an econometrician reject the Euler equation?

Proposition 1 establishes that an Euler equation is satisfied on *average* in the economy that we study despite the existence of non-optimizing households. However, as we have explained, an analogous Euler equation will not be satisfied household-by-household (though it will be satisfied for all unconstrained optimizers). An econometrician with full information – i.e., an econometrician who knows household preferences, including each household's taste shifter,  $\theta$ – would be able to test the Euler equation household-by-household,

$$u_1'(c_1;\theta) = \delta R u_2'(c_2;\theta) \quad \text{for } c_1 < \bar{c}_1$$
$$u_1'(c_1;\theta) \ge \delta R u_2'(c_2;\theta) \quad \text{for } c_1 = \bar{c}_1,$$

and would find that it is not universally satisfied. More generally, the econometrician will find that the Euler equation holds when she uses the same coarse information that the planner has, and the econometrician will find that the Euler equation does not hold when she uses private, household-level information about the value of  $\theta$  (which is not directly available to the planner).

### 4 Special Cases

We now study the special case in which household life-time utility is given by

$$u_1(c_1;\theta) + \delta u_2(c_2;\theta) = u(c_1) + \delta \theta u(c_2).$$

This is a commonly studied case of multiplicative taste shocks (c.f., Atkeson and Lucas 1992; Amador, Werning, and Angeletos 2006; and Beshears et al. 2015). Without loss of generality and to simplify notation, we assume that  $\delta R = 1$ . We maintain these assumptions throughout this section.

We first study the mean of the ratio of marginal utilities before and after retirement,  $E\left[\frac{u'(c_1)}{u'(c_2)}\right]$ . Note that in a fully optimizing economy, for any given value of  $\theta$  we have

$$\frac{u'(c_1(\theta))}{u'(c_2(\theta))} = \theta$$

Accordingly, averaging across all households yields

$$E\left[\frac{u'(c_1)}{u'(c_2)}\right] = E\left[\theta\right].$$

When  $E[\theta] = 1$  (a natural benchmark), marginal utility is smoothed such that the mean ratio of marginal utilities equals one, i.e.,  $E\left[\frac{u'(c_1)}{u'(c_2)}\right] = 1.$ 

In the next proposition we prove that these relationships hold in our economy as well.

**Proposition 2 (Multiplicative Taste Shocks)** Assume a rational planner. Then for any distribution of optimizing, myopic, and passive households, a classical Euler equation ratio will hold on average in the population:

$$E\left[\frac{u'(c_1)}{u'(c_2)}\right] = E\left[\theta\right].$$

**Proof.** Equation (6) implies that  $\int_{\Theta} \left( -\frac{u'(c_1^P)}{u'(c_2^P)} + \theta \right) dF(\theta) = 0.$  Since  $c_2(\theta) = R(y - \bar{c}_1)$  for

$$\theta \in \Gamma(\bar{c}_1) \text{ and } c_2^M = R(y - \bar{c}_1), \text{ we can also re-write equation (8) as } \int_{\theta \in \Gamma(\bar{c}_1)} \mu_O\left(-\frac{u'(c_1(\theta))}{u'(c_2(\theta))} + \theta\right) dF(\theta) + \int_{\Theta} \mu_M\left(-\frac{u'(c_1^M)}{u'(c_2^M)} + \theta\right) dF(\theta) = 0. \text{ Lastly, for } \theta \notin \Gamma(\bar{c}_1), \ u'_1(c_1(\theta); \theta) = \delta Ru'_2(c_2(\theta); \theta) \text{ implies }$$

$$\text{that } -\frac{u'(c_1(\theta))}{u'(c_2(\theta))} + \theta = 0 \text{ and hence } \int_{\theta \notin \Gamma(\bar{c}_1)} \mu_O\left(-\frac{u'(c_1(\theta))}{u'(c_2(\theta))} + \theta\right) dF(\theta) = 0. \text{ Put together, we have }$$

unat

$$E\left[\frac{u'(c_1)}{u'(c_2)}\right] = \mu_O\left[\int\limits_{\theta\in\Gamma(\bar{c}_1)} \frac{u'(c_1\left(\theta\right))}{u'(c_2\left(\theta\right))} dF(\theta) + \int\limits_{\theta\notin\Gamma(\bar{c}_1)} \frac{u'(c_1\left(\theta\right))}{u'(c_2\left(\theta\right))} dF(\theta)\right] + \mu_P \int\limits_{\Theta} \frac{u'(c_1^P)}{u'(c_2^P)} dF(\theta) + \mu_M \int\limits_{\Theta} \frac{u'(c_1^M)}{u'(c_2^M)} dF(\theta) = E\left[\theta\right].$$

This completes the proof.  $\blacksquare$ 

#### 4.1 Consumption Smoothing

The degree of consumption smoothing is widely used as an indicator of household optimization. The mapping from marginal-utility smoothing to consumption smoothing depends on the curvature of the utility function. In this subsection, we work out this mapping for quadratic utility and constant relative risk aversion.

For quadratic utility we get exact results. When there are no taste shocks ( $\theta = 1$  for all households), we show that  $E[c_2 - c_1] = 0$  regardless of the distribution of optimizing, myopic, and passive households. On the other hand, when there are household-level taste shocks but no taste shocks on average  $(E[\theta] = 1)$ , we show that  $E[c_2 - c_1] = 0$  only if there are no optimizing households. Surprisingly, the presence of optimizing households causes average consumption to fall at retirement:  $E[c_2 - c_1] < 0$ .

For the case of constant relative risk aversion, we get analogous results studying the rate of change of consumption at retirement (using a first-order approximation).

Together, these results imply that *consumption* smoothing is a more robust property of the model without optimizing households. In other words, there are leading preference and taste-shock cases in which consumption smoothing arises without optimizing households but

does not arise with optimizing households (and not vice versa).

We now introduce our notation. We study the cases of: (i) quadratic utility, i.e.,  $u(c) = c - \frac{b}{2}c^2$ , where b > 0, and (ii) constant relative risk aversion, i.e.,  $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ , where  $\gamma > 0.^8$ For each case, we analyze consumption moments under two assumptions: first, when  $\theta = 1$ for every household as implicitly assumed in most papers, and, second, when there are no taste shocks on average at retirement, i.e., when  $E[\theta] = 1$ . The results are summarized in the following lemma.

Lemma 4 (Parametric Utility Functions) Assume any distribution of optimizing, myopic, and passive households.

- (A) Quadratic utility implies:
- 1. With no taste shocks at retirement ( $\theta = 1$  for all households),

$$E[c_2-c_1]=0;$$

2. With no taste shocks on average at retirement  $(E [\theta] = 1)$ ,

$$E\left[c_2 - c_1\right] = -cov\left(\theta, c_2\right) \le 0.$$

(B) Constant relative risk aversion utility implies (to a first-order approximation):

1. With no taste shocks at retirement ( $\theta = 1$  for all households),

$$E\left[\frac{c_2-c_1}{c_1}\right] \cong 0;$$

2. With no taste shocks on average at retirement  $(E[\theta] = 1)$ ,

$$E\left[\frac{c_2-c_1}{c_1}\right] \cong -cov\left(\theta,\frac{c_2}{c_1}\right) \le 0.$$

**Proof.** (A) Plugging  $u(c) = c - \frac{b}{2}c^2$  into Proposition 1 implies that  $E[c_1 - \theta c_2] = \frac{E[1-\theta]/b}{b}$ . Assuming that  $\theta = 1$  for every household yields  $E[c_2 - c_1] = 0$ , and assuming  $\frac{1}{8} \text{As } \gamma \to 1$ , this function converges to  $\ln(c)$ .

<sup>17</sup> 

that  $E[\theta] = 1$  yields  $E[c_2 - c_1] = -cov(\theta, c_2).$ 

(B) Similar to the proof of Proposition 2, one can show that  $E\left[\frac{\theta u'(c_2)}{u'(c_1)}\right] = 1$ . This is because equation (6) implies that  $\int_{\Theta} \left(-1 + \frac{\theta u'(c_2^P)}{u'(c_1^P)}\right) dF(\theta_i) = 0$ , equation (8) combined with  $c_1(\theta) = \bar{c}_1$  for  $\theta \in \Gamma(\bar{c}_1)$  and  $c_1^M = \bar{c}_1$  implies that  $\int_{\theta \in \Gamma(\bar{c}_1)} \mu_O\left(-1 + \frac{\theta u'(c_2(\theta))}{u'(c_1(\theta))}\right) dF(\theta) + \int_{\Theta} \mu_M\left(-1 + \frac{\theta u'(c_2^M)}{u'(c_1^M)}\right) dF(\theta) = 0$ , and  $u'_1(c_1(\theta); \theta) = \delta R u'_2(c_2(\theta); \theta)$  for  $\theta \notin \Gamma(\bar{c}_1)$  implies that  $\frac{\theta u'(c_2(\theta))}{u'(c_1(\theta))} = 1$  and hence  $\int_{\theta \notin \Gamma(\bar{c}_1)} \mu_O\left(-1 + \frac{\theta u'(c_2(\theta))}{u'(c_1(\theta))}\right) dF(\theta) = 0$ , which together yield  $E\left[\frac{\theta u'(c_2)}{u'(c_1)}\right] = 1$ . Linearizing u'(c) around  $c = c_1$  results in  $E\left[-\theta \gamma \frac{c_2-c_1}{c_1}\right] \cong 1 - E[\theta]$ . Assuming that  $\theta = 1$  for every household yields  $E\left[\frac{c_2-c_1}{c_1}\right] \cong 0$ , and assuming that  $E[\theta] = 1$  yields  $E\left[\frac{c_2-c_1}{c_1}\right] \cong -cov\left(\theta, \frac{c_2}{c_1}\right)$ .

Lemma 4 demonstrates that when there are no taste shocks, our economy is characterized by mean consumption smoothing for any distribution of optimizing, myopic, and passive households. Therefore, in this commonly studied case, consumption smoothing is not a diagnostic test for household optimization. In addition, with no *average* taste shocks, the covariance expressions in Lemma 4 are weakly positive and proportional to the mass of optimizing households. This introduces some counter-intuitive properties. First, the average change in consumption (entering retirement) will be exactly zero only when the fraction of optimizing households is zero (since  $cov(\theta, c_2) = cov(\theta, \frac{c_2}{c_1}) = 0$  if and only if  $\mu_O = 0$ ). Second, with a positive share of optimizing households, the consumption drop at retirement *increases* (since  $\mu_O > 0$  implies that  $cov(\theta, c_2) > 0$  and  $cov(\theta, \frac{c_2}{c_1}) > 0$ ). That is, not only are consumption smoothing tests not conclusive for analyzing household optimization, but in the commonly studied case of  $E[\theta] = 1$ , *exact* aggregate consumption smoothing is actually inconsistent with the null hypothesis of household optimization.

However, the latter qualitative observation turns out to be quantitatively small in the sense that (calibrated) variance in the taste shock ( $\theta$ ) produces relatively little *aggregate* decline in consumption. To see an example of the magnitude of this effect, assume that all agents are optimizers, have constant relative risk aversion, and that  $\theta = \exp(\varepsilon - \sigma^2/2)$ , where

 $\varepsilon$  is distributed with a mean of zero and a variance of  $\sigma^2$ . Then we have

$$u'(c_1) = \theta u'(c_2),$$

for all households, which implies that average consumption growth between working life  $(c_1)$ and retirement  $(c_2)$  will be

$$E\ln\frac{c_2}{c_1} = -\frac{\sigma^2}{2\gamma}.$$

If we assume that  $\sigma = 0.25$  and  $\gamma = 2$ , then the average decline in consumption between working life and retirement is 1.6%. To gain intuition for this calibration, a one-standarddeviation realization causes a household to prefer a

$$\ln \frac{c_2}{c_1} = \frac{\pm \sigma - \sigma^2/2}{\gamma}$$

growth rate in consumption between working life and retirement. With  $\sigma = 0.25$ , a onestandard-deviation realization implies either an 11% desired growth rate in consumption or a -14% desired growth rate in consumption, depending on the sign of the preference shock.

## 5 Identification of Optimizers, Myopes, and Passives

While the average Euler equation and average consumption smoothing cannot generally reveal the distribution of household types, tests that rely on other moments of households' economic behavior do reveal the mass of optimizing and non-optimizing households.

Consider the cross-sectional distribution of savings. We characterize the distribution of overall savings assuming that the distribution of taste shocks,  $F(\theta)$ , is continuous.

The first type of test for household optimization relies on the cross section. One indicator is bunching (excess mass) around the combined levels of forced and default savings,  $s_F + s_D$ . A discrete jump in the savings' cumulative distribution function at this point identifies the mass of passive households.

Another type of test employs quasi-experiments and relies on behavioral responses to

policy variations,<sup>9</sup> which can be measured by changes in the distribution of savings (or by household-level elasticities). First, if there are passive households, a change in the default level of savings from  $s_D$  to  $s'_D$  will engender (new) bunching around  $s_F + s'_D$ . A different measure for testing the same hypothesis is the change in average (or overall) savings. If all households are optimizers, then a change in defaults will keep the distribution of savings, and hence average savings, unchanged. Any change in average savings would reject the hypothesis of perfect household optimization, and in our world, would reveal the share of passive households.

Second, changes to the level of forced savings can shed light on the presence and fraction of myopic households and of optimizing households. To see this, consider a change in the level of mandatory savings and evaluate the change in the mass of households that continue to save no more than the mandatory savings level,  $s_F$ :

$$\frac{d\Pr(s=s_F)}{ds_F} = \frac{d}{ds_F} \left[ \mu_O \times \Pr(\theta \in \Gamma(\bar{c}_1)) + \mu_M \right] = \mu_O \times \frac{d\Pr(\theta \in \Gamma(\bar{c}_1))}{ds_F}.$$

This *change* in the mass of households at the forced savings level is positive if and only if there are optimizing households on the boundary. Indeed, if the observer knows the utility function and the distribution of  $\theta$ ,  $F(\theta)$ , then the derivative  $\frac{d \Pr(\theta \in \Gamma(\bar{c}_1))}{ds_F}$  can be calculated. Hence, in such a case, observing  $\frac{d \Pr(s=s_F)}{ds_F}$  enables the econometrician to calculate the mass of optimizing agents,  $\mu_0$ . Once  $\mu_0$  is known, the econometrician can also calculate the mass of myopes:

$$\mu_M = \Pr(s = s_F) - \mu_O \times \Pr(\theta \in \Gamma(\bar{c}_1)).$$

A related global test can be developed by studying average savings. Consider a decrease in the forced savings level. If this engenders a one-for-one decline in average economy-wide savings, then all households are either myopic or passive. However, if the decline is less than one-for-one, then there is a strictly positive measure of optimizing households: parametric assumptions on the utility function (and the distribution of taste shifters  $\theta$ ) would be needed

<sup>&</sup>lt;sup>9</sup>For instance, policy changes might arise as the government learns about the efficacy of new policies (e.g., the research that led to passage of the Pension Protection Act of 2006).

to impute the population mass of optimizers.

The tests that we highlight here are derived from our positive model. However, the broader point is that an analysis of cross-sectional distributions of economic outcomes and how they change in response to variation in the economic environment can provide tests for household optimization that are not confounded by the presence of a rational social planner.

## 6 Generalization as a Mechanism Design Problem

In this section, we show that the equilibrium of our model above (which has a restricted policy space) exactly matches the equilibrium that arises when the government's policy tools are maximally generalized and the problem is treated as a mechanism design problem. To recap, the problem posed in Sections 2 and 3 is to choose the two policy variables  $s_F$  (mandatory savings) and  $s_D$  (additional default savings) to maximize the social planner's objective

$$W\left(c_{1}(\theta), c_{2}(\theta), c_{1}^{M}, c_{2}^{M}, c_{1}^{P}, c_{2}^{P}\right) \equiv \mu_{O} \int_{\Theta} \left[u_{1}(c_{1}(\theta); \theta) + \delta u_{2}(c_{2}(\theta); \theta)\right] dF(\theta) + \mu_{M} \int_{\Theta} \left[u_{1}(c_{1}^{M}; \theta) + \delta u_{2}(c_{2}^{M}; \theta)\right] dF(\theta) + \mu_{P} \int_{\Theta} \left[u_{1}(c_{1}^{P}; \theta) + \delta u_{2}(c_{2}^{P}; \theta)\right] dF(\theta),$$
(10)

subject to the household behavioral models summarized in Section 2.

We can also study the generalized version of this problem using a mechanism design framework. Now the planner chooses  $\{c_1(\theta)\}_{\theta\in\Theta}, \{c_2(\theta)\}_{\theta\in\Theta}, c_1^P$ , and  $c_2^P$  to maximize equation (10) subject to the within-household budget constraints,

$$c_1(\theta) + \frac{c_2(\theta)}{R} \le y \text{ for all } \theta,$$
 (11)

$$c_1^M + \frac{c_2^M}{R} \le y,\tag{12}$$

$$c_1^P + \frac{c_2^P}{R} \le y,\tag{13}$$

incentive compatibility constraints for optimizers,

$$u_1(c_1(\theta);\theta) + \delta u_2(c_2(\theta);\theta) \ge u_1(c_1(\theta');\theta) + \delta u_2(c_2(\theta');\theta), \quad \forall \ \theta, \theta'$$
(14)

$$u_1(c_1(\theta);\theta) + \delta u_2(c_2(\theta);\theta) \ge u_1(c_1^P;\theta) + \delta u_2(c_2^P;\theta), \quad \forall \ \theta$$
(15)

and a maximally impatient reporting rule for myopes,

$$c_1^M = \max\left\{\sup_{\theta\in\Theta} c_1(\theta), c_1^P\right\}.$$
(16)

We now summarize the mechanism design problem: the planner chooses  $\{c_1(\theta)\}_{\theta\in\Theta}$ ,  $\{c_2(\theta)\}_{\theta\in\Theta}$ ,  $c_1^P$ , and  $c_2^P$  to maximize the objective in equation (10), subject to the budget constraints and incentive compatibility constraints in equations (11)-(16). In this mechanism design problem, optimizers report their type (truthfully in equilibrium), myopes always report the type that has the highest immediate consumption in the mechanism, and passives follow the defaults  $c_1^P$  and  $c_2^P$ . With this set-up, we can now present our third proposition.

**Proposition 3 (Characterization as a Mechanism Design Problem)** The mechanism design problem and the constrained problem (i.e., maximizing equation (10) by choosing arguments  $s_F$  and  $s_D$ ) generate the same equilibrium allocation.

This implies an immediate corollary.

**Corollary 1** The mechanism design problem and the constrained problem (maximizing equation (10) by choosing arguments  $s_F$  and  $s_D$ ) generate the same aggregate Euler equation:

$$E\left[u_1'(c_1;\theta)\right] = \delta RE\left[u_2'(c_2;\theta)\right].$$

Accordingly, the institutional assumptions that are made in Section 2 are made without loss of generality.

**Proof (Characterization as a Mechanism Design Problem).** We first show that the (second-best) optimal allocation is characterized by a *maximum consumption rule* for

optimizers, which we will define after describing an unconstrained allocation. For every type  $\theta$ , there exists a (full-information) unconstrained allocation,  $(c_1^*(\theta), c_2^*(\theta))$ , which satisfies budget balance

$$c_2^*(\theta) = R(y - c_1^*(\theta)).$$

and the first order condition

$$u'(c_1^*(\theta);\theta) = \delta Ru'(c_2^*(\theta);\theta).$$

Next, consider a *different* allocation that assigns consumption  $(c_1(\theta), c_2(\theta))$ , for all  $\theta \in \Theta$ . We now define a *maximum consumption rule*.

**Definition:** Consider an unconstrained allocation  $(c_1^*(\theta), c_2^*(\theta))$ , for all  $\theta \in \Theta$ . Now consider an alternative allocation  $(c_1(\theta), c_2(\theta))$ , for all  $\theta \in \Theta$ . This alternative allocation is a 'maximum consumption rule' if and only if two conditions are both satisfied:

(i) every type with  $c_1^*(\theta) \leq \sup_{\theta \in \Theta} c_1(\theta)$  obtains  $c_1(\theta) = c_1^*(\theta)$  and  $c_2(\theta) = c_2^*(\theta)$  in the allocation; and

(ii) every type with  $c_1^*(\theta) > \sup_{\theta \in \Theta} c_1(\theta)$  obtains  $c_1(\theta) = \sup_{\theta \in \Theta} c_1(\theta)$  and  $c_2(\theta) = R(y - \sup_{\theta \in \Theta} c_1(\theta))$  in the allocation.

To prove that an optimal mechanism generates an allocation that is a maximum consumption rule for optimizers, consider a candidate allocation given by  $\{\hat{c}_1(\theta)\}_{\theta\in\Theta}, \{\hat{c}_2(\theta)\}_{\theta\in\Theta}, \hat{c}_1^M, \hat{c}_2^M, \hat{c}_1^P, \text{ and } \hat{c}_2^P$  that is incentive compatible (i.e., satisfying equations (11)-(16)), and is *not* a maximum consumption rule for the optimizers. Because the candidate allocation is incentive compatible, it follows that  $\sup_{\theta\in\Theta} \hat{c}_1(\theta) \leq \hat{c}_1^M$  and  $\hat{c}_1^P \leq \hat{c}_1^M$ .

Now perturb  $\{\hat{c}_1(\theta)\}_{\theta\in\Theta}, \{\hat{c}_2(\theta)\}_{\theta\in\Theta}, \hat{c}_1^M, \hat{c}_2^M, \hat{c}_1^P, \text{ and } \hat{c}_2^P$  in the following way. Let  $\bar{c}_1 = \sup_{\theta\in\Theta} \hat{c}_1(\theta)$  and construct a new allocation such that (i) every optimizer with  $c_1^*(\theta) \leq \bar{c}_1$  obtains  $c_1(\theta) = c_1^*(\theta)$  and  $c_2(\theta) = c_2^*(\theta)$  in the mechanism; (ii) every optimizer with  $c_1^*(\theta) > \bar{c}_1$  achieves  $c_1(\theta) = \bar{c}_1$  and  $c_2(\theta) = R(y - \bar{c}_1(\theta))$  in the mechanism; and (iii)  $\hat{c}_1^M, \hat{c}_2^M, \hat{c}_1^P$ , and  $\hat{c}_2^P$  stay the same.

Note that this new allocation is a maximum consumption rule for the optimizers. This

new allocation is incentive compatible for optimizers and myopes: households either achieve their first-best allocation or they obtain  $(\bar{c}_1, R(y - \bar{c}_1))$ , which is their most preferred consumption pair in the set of all offered consumption pairs (because of the concavity of u). The new allocation improves welfare weakly for every agent compared to their welfare in the candidate allocation: agents that achieve their first-best allocation in the new allocation obtain a weak improvement in welfare (because they are now at their unconstrained optimum), and agents that obtain the maximum level of consumption obtain a weak improvement in welfare (because they previously had  $\hat{c}_1(\theta) \leq \bar{c}_1$  and they now also have an allocation that uses their entire endowment). Because the original allocation was not a maximum consumption rule for optimizers, the new allocation generates a strict improvement in welfare for at least one type of optimizer.<sup>10</sup>

Hence, only allocations that are maximum consumption rules for optimizers can be solutions to the mechanism design problem. It follows that the planner's problem can be reduced to the choice of a maximum consumption rule for optimizers – i.e., the choice of  $\bar{c}_1$  – and the choice of  $c_1^P$  (note that  $c_1^P = c_1^D$ ). By Lemma 2, the mechanism design problem (choosing  $\{c_1(\theta)\}_{\theta\in\Theta}, \{c_2(\theta)\}_{\theta\in\Theta}, c_1^P$ , and  $c_2^P$  to maximize the objective in equation (10) subject to the budget constraints and incentive compatibility constraints in equations (11)-(16)) and the constrained problem (maximizing equation (10) by choosing  $s_F$  and  $s_D$ ) are isomorphic optimization problems. Accordingly, they have the same equilibrium allocation.

## 7 Related Empirical Literature

Our choice of the particular framework for illustrating our ideas – namely, consumption dynamics over the life-cycle – is motivated by the tension between research that finds relatively smooth average life-cycle consumption dynamics (consistent with rational consumer behavior) and research that finds that individual households are highly sensitive to savings defaults

<sup>&</sup>lt;sup>10</sup>There are two ways for an allocation to fail to be a maximum consumption rule. Either (i) there is a type with  $c_1^*(\theta) \leq \bar{c}_1$ , and that type fails to obtain  $c_1(\theta) = c_1^*(\theta)$  and  $c_2(\theta) = c_2^*(\theta)$  in the mechanism; or (ii) there is a type with  $c_1^*(\theta) > \bar{c}_1$ , and that type fails to obtain  $c_1(\theta) = \bar{c}_1$  and  $c_2(\theta) = R(y - \bar{c}_1(\theta))$  in the mechanism. If case (i) applies, the type is made strictly better off by construction. If case (ii) applies, the type is made strictly concave.

(which is inconsistent with rational consumer behavior).

Analyzing life-cycle consumption dynamics, many papers find evidence that is consistent with optimal retirement savings. For example, using a structural life-cycle analysis and accounting for government transfers and Social Security benefits, Scholz et al. (2006) estimate that less than twenty percent of households in the Health and Retirement Study under-save for retirement. Moreover, Scholz et al. (2006) find that the wealth deficits are generally a small fraction of life-time wealth. Bernheim, Skinner, and Weinberg (2001) report that expenditure on food drops at retirement, but Aguiar and Hurst (2005) report that the drop in food expenditure at retirement is illusory, in the sense that caloric consumption and other measures of meal quality do not change when households enter retirement.<sup>11</sup> Studying overall nondurable expenditure, Aguila et al. (2011) do not find a statistically significant drop in consumption at retirement.

Another body of research studies employer retirement savings plans, and shows that automatic enrollment and other institutional nudges have a large effect on household savings. Many households do not deviate from the default contribution rate as well as the default asset allocation, and are largely unresponsive to government- or employer-subsidies for retirement savings contributions (Madrian and Shea 2001, Choi et al. 2004, Beshears et al. 2009, and Chetty et al. 2014). A related body of research finds that consumers have a high marginal propensity to consume out of both anticipated and unanticipated windfalls (e.g., Soulelos 1999, Parker 1999, 2017, Stephens 2003, Stephens and Unayama 2011, Parker et al. 2013), which is consistent with the existence of subpopulations of myopic (e.g., highly impatient) and passive households.

Our model is consistent with these seemingly contradictory sets of findings. Our framework explains why a population that is comprised of a mix of optimizing, passive, and myopic households will have relatively smooth average consumption dynamics around retirement and will satisfy an aggregate Euler equation. We show that a benevolent government will set pol-

<sup>&</sup>lt;sup>11</sup>However, in a recent working paper, Stephens and Toohey (2018) revisit the analysis in Aguiar and Hurst with a broader set of data sources and methodologies and report that caloric consumption does decline on average around the time of retirement.

icy that elicits this aggregate (second-best) efficiency property, although significant deviations from optimality will arise at the level of individual households, including sensitivity to default savings.

## 8 Conclusion

We study a simple setting that illustrates the interactions between optimizing social planners and heterogeneous households, some of whom are optimizers, some of whom are myopic, and some of whom are passive. In this setting, planner optimization is a partial substitute for household optimization. This substitution arises because the social planner has the ability to design institutions – e.g., default savings and Social Security – that influence the consumption profiles of households. In equilibrium, classical Euler equations hold on average in the crosssection of households (but not for each household). These Euler equation properties arise generally, whether or not households are optimizers.

These results imply that Euler equation tests and related consumption-smoothing tests – e.g., the lack of an average drop in consumption at retirement – do not differentiate between an optimizing social planner and optimizing households. However, even in the economy that we have studied, planner optimization is distinguishable, in principle, from household optimization. Under the assumption of household optimization, the Euler equation will hold for *each* household, but, with planner optimization (and without universal household optimization), it will only hold on *average* in the cross section. Although household-by-household tests are theoretically determinative, such fine-grained analysis is practically problematic if data are measured with noise or if some variables are unobservable (e.g., household-level taste shocks). Still, other tests do distinguish between planner optimization and household optimization. Exogenous changes in policy (e.g., a default change at the level of a firm, or a natural experiment in the Social Security system), reveal more about household rationality than averages of observational data in the cross section.

Our conclusions depend upon the assumption that the government is a fully rational utilitarian. It is likely that flesh-and-blood governments fall short of this benevolent benchmark, despite (or because of) the pressures that they face to get re-elected. This leads to a natural follow-up question: how would our results change if the government is not utilitarian, but is instead a self-interested political party? The answer depends on two key considerations: what is the voting frequency of different types of households and to what degree do altruistic motives influence voting? These are important extensions, which we leave for future research.

## References

- Aguiar, Mark, and Erik Hurst (2005): "Consumption versus Expenditure," Journal of Political Economy, 113(5): pp. 919-948.
- [2] Aguila, Emma, Orazio Attanasio, and Costas Meghir (2011): "Changes in Consumption at Retirement: Evidence from Panel Data," *Review of Economics and Statistics*, 93(3): pp. 1094-1099.
- [3] Allcott, Hunt, and Dmitry Taubinsky (2015): "Evaluating Behaviorally Motivated Policy: Experimental Evidence from the Lightbulb Market," *American Economic Review*, 105(8): pp. 2501-2538.
- [4] Amador, Manuel, Iván Werning, and George-Marios Angeletos (2006): "Commitment vs. Flexibility," *Econometrica*, 74(2): pp. 365-396.
- [5] Ashraf, Nava, Dean Karlan, and Wesley Yin (2006): "Tying Odysseus to The Mast: Evidence from a Commitment Savings Product in the Philippines," *Quarterly Journal* of Economics, 121(2): pp. 635-672.
- [6] Atkeson, Andrew, and Robert E. Lucas (1992): "On Efficient Distribution with Private Information," *Review of Economic Studies*, 59(3): pp. 427-453.
- [7] Augenblick, Ned, Muriel Niederle, and Charles Sprenger (2015): "Working Over Time: Dynamic Inconsistency in Real Effort Tasks," *Quarterly Journal of Economics*, 130(3): pp. 1067-1115.
- [8] Bernheim, B. Douglas, Jonathan Skinner, and Steven Weinberg (2001): "What Accounts for the Variation in Retirement Wealth among US Households?" *American Economic Review*, pp. 832-857.
- [9] Beshears, John, James J. Choi, Christopher Harris, David Laibson, Brigitte C. Madrian, and Jung Sakong (2015): "Self Control and Commitment: Can Decreasing the Liquidity

of a Savings Account Increase Deposits?," Working Paper 21474, National Bureau of Economic Research.

- [10] Beshears, John, James J. Choi, David Laibson, and Brigitte C. Madrian (2009): "The Importance of Default Options for Retirement Saving Outcomes: Evidence from the United States," in Jeffrey Brown, Jeffrey Liebman, and David A. Wise, eds. Social Security Policy in a Changing Environment, pp. 167-195. University of Chicago Press.
- [11] Bryan, Gharad, Dean Karlan, and Scott Nelson (2010): "Commitment Devices," Annual Review of Economics, 2(1): pp. 671-698.
- [12] Bubb, Ryan, Patrick Corrigan, and Patrick L. Warren (2015): "A Behavioral Contract Theory Perspective on Retirement Savings." *Connecticut Law Review*, Commentary Issue of Volume 47.
- [13] Bubb, Ryan, and Richard Pildes (2014): "How Behavioral Economics Trims Its Sails and Why," *Harvard Law Review*, Vol 127.
- [14] Bubb, Ryan, and Patrick L. Warren (2016): "An Equilibrium Theory of Retirement Plan Design." NYU Law and Economics Research Paper No. 16-28.
- [15] Camerer, Colin, Samuel Issacharoff, George Loewenstein, Ted O'Donoghue, and Matthew Rabin (2003): "Regulation for Conservatives: Behavioral Economics and the Case for 'Asymmetric Paternalism'," University of Pennsylvania Law Review 151(3): pp. 1211–1254.
- [16] Carroll, Gabriel D., James J. Choi, David Laibson, Brigitte Madrian, and Andrew Metrick (2009): "Optimal Defaults and Active Decisions," *Quarterly Journal of Economics* 124(4): pp. 1639-1674.
- [17] Chetty, Raj (2015): "Behavioral Economics and Public Policy: A Pragmatic Perspective," American Economic Review 105(5): pp. 1-33

- [18] Chetty, Raj, John N. Friedman, Søren Leth-Petersen, Torben Heien Nielsen, and Tore Olsen (2014): "Active vs. Passive Decisions and Crowd-Out in Retirement Savings Accounts: Evidence from Denmark," *Quarterly Journal of Economics*, 129(3): pp. 1141-1219.
- [19] Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick (2003): "Optimal Defaults." American Economic Review 93(2): pp. 180-185
- [20] Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick (2004): "For Better or for Worse: Default Effects and 401 (k) Savings Behavior," in David A. Wise, ed. *Perspectives on the Economics of Aging*, pp. 81-126. University of Chicago Press.
- [21] Diamond, Peter A. (1977): "A Framework for Social Security Analysis," Journal of Public Economics, 8(3): pp. 275–298.
- [22] Fang, Hanming, and Dan Silverman (2009): "Time-Inconsistency and Welfare Program Participation: Evidence from the NLSY," *International Economic Review* 50(4): pp. 1043-1077.
- [23] Farhi, Emmanuel, and Xavier Gabaix (2015): "Optimal Taxation with Behavioral Agents," Working Paper No. w21524, National Bureau of Economic Research.
- [24] Feldstein, M. (1985): "The Optimal Level of Social Security Benefits," Quarterly Journal of Economics, 100(2): pp. 303-320.
- [25] Giné, Xavier, Dean Karlan, and Jonathan Zinman (2010): "Put Your Money Where Your Butt Is: A Commitment Contract for Smoking Cessation," American Economic Journal: Applied Economics 2(4): pp. 213-235.
- [26] Gruber, Jonathan, and Botond Köszegi (2001): "Is Addiction 'Rational'? Theory and Evidence," Quarterly Journal of Economics, 116(4): pp. 1261-1303.
- [27] Kaur, Supreet, Michael Kremer, and Sendhil Mullainathan (2015): "Self-Control at Work," Journal of Political Economy 123(6): pp. 1227-1277.

- [28] Kotlikoff, Laurence J., Avia Spivak, and Lawrence H. Summers (1982): "The Adequacy of Savings," *American Economic Review*, 2(5): pp. 1056–1069.
- [29] Lockwood, Benjamin (2016): "Optimal Income Taxation with Present Bias," Working paper.
- [30] Loginova, Uliana, and Petra Persson (2012): "Paternalism, Libertarianism, and the Nature of Disagreement," Working paper.
- [31] Madrian, Brigitte C., and Dennis F. Shea (2001): "The Power of Suggestion: Inertia in 401 (k) Participation and Savings Behavior," *Quarterly Journal of Economics*, 116(4): pp. 1149-1187.
- [32] Moser, Christian, and Pedro Olea de Souza e Silva (2015): "Paternalism vs Redistribution: Designing Retirement Savings Policies with Behavioral Agents," Working paper Columbia University.
- [33] O'Donoghue, Ted, and Matthew Rabin (2003): "Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes," American Economic Review, 93(2): pp. 186-191.
- [34] O'Donoghue, Ted, and Matthew Rabin (2006): "Optimal Sin Taxes," Journal of Public Economics 90(10): pp. 1825-1849.
- [35] Parker, Jonathan A. (1999): "The Reaction of Household Consumption to Predictable Changes in Social Security Ttaxes," *American Economic Review* 89(4): pp. 959-973.
- [36] Parker, Jonathan A. (2017): "Why Don't Households Smooth Consumption? Evidence from a \$25 Million Experiment," *American Economic Journal: Macroeconomics* 9(4): pp.153-183.
- [37] Parker, Jonathan A., Nicholas S. Souleles, David S. Johnson, and Robert McClelland (2013): "Consumer Spending and The Economic Stimulus Payments of 2008," American Economic Review 103(6): pp. 2530-2553.

- [38] Poterba, James M. (2014): "Retirement Security in an Aging Population," American Economic Review 104(5): pp. 1-30.
- [39] Schilbach, Frank (2015): "Alcohol and Self-Control: A Field Experiment in India," Working paper.
- [40] Scholz, John Karl, Ananth Seshadri, and Surachai Khitatrakun (2006): "Are Americans Saving 'Optimally' for Retirement?," *Journal of Political Economy*, 114(4): pp. 607-643.
- [41] Skinner, Jonathan (2007): "Are You Sure You're Saving Enough for Retirement?" Journal of Economic Perspectives 21(3): pp. 59-80.
- [42] Souleles, Nicholas S. (1999) "The Response of House hold Consumption to Income Tax Refunds." American Economic Review, September, 89(4), pp. 947-58.
- [43] Stephens, Melvin (2003) "3rd of tha Month': Do Social Security Recipients Smooth Consumption Between Checks?" The American Economic Review, March, vol.93, n.1, pp. 406–422.
- [44] Stephens, Melvin, and Takashi Unayama (2011). "The Consumption Response to Seasonal Income: Evidence from Japanese Public Pension Benefits." American Economic Journal: Applied Economics, 3(4): 86-118.
- [45] Stephens, Melvin, and Desmond Toohey (2018) "Changes in Nutrient Intake at Retirement," working paper.
- [46] Sunstein, Cass R., and Richard H. Thaler (2003): "Libertarian Paternalism is not an Oxymoron," University of Chicago Law Review: pp. 1159-1202.
- [47] Thaler, Richard H., and Cass R. Sunstein (2003): "Libertarian Paternalism," American Economic Review, 93(2): pp.175-179.
- [48] Thaler, Richard H., and Cass R. Sunstein (2008): "Nudge: Improving Decisions about Health, Wealth, and Happiness" Yale University Press.

## 9 Appendix

# **9.1** Proof of Lemma 1 $(c_1^D < \bar{c}_1; \text{ i.e., } s_D > 0).$

First, we break the planner's problem down into two separable sub-problems. In the first sub-problem, we solve for  $s_F = y - \bar{c}_1$  and *ignore* the passives. In the second sub-problem, we solve for  $s_F + s_D$  (holding  $s_F$  fixed from the first sub-problem) and *ignore* both the myopes and the optimizers. This separation of the two problems is only admissible if the resulting optima,  $s_F$  and  $s_D$ , satisfy an incentive compatibility (IC) constraint,

$$c_1^D \le \bar{c}_1,\tag{IC}$$

which can also be expressed as

$$y - s_F - s_D \le y - s_F.$$

This constraint follows from the constraint  $s_D \ge 0$  (equation (1)). At the end of this proof we confirm that an even stronger condition applies:  $s_D > 0$ , which is the claim in the lemma.

To summarize our strategy, we characterize optimal policy for the myopes and optimizers – the minimum savings level  $s_F$  – without taking account of optimal policy for the passives. Then we characterize optimal policy for the passives – the sum  $s_F + s_D$  – holding fixed the policy parameter that influences the myopes and optimizers,  $s_F$ . At the end of the analysis we verify that the constraint  $s_D \ge 0$  is satisfied at the solution.

In the separated problem, the optimal level of  $\bar{c}_1$  is given by the following Euler equation:

$$\begin{split} \int_{\Theta} \left[ \mu_O u_1'(c_1\left(\theta\right);\theta) \frac{dc_1\left(\theta\right)}{d\bar{c}_1} + \mu_M u_1'(c_1^M;\theta) \frac{dc_1^M\left(\theta\right)}{d\bar{c}_1} \right] dF(\theta) \\ &+ \delta \int_{\Theta} \left[ \mu_O u_2'(c_2\left(\theta\right);\theta) \frac{dc_2\left(\theta\right)}{d\bar{c}_1} + \mu_M u_2'(c_2^M;\theta) \frac{dc_2^M}{d\bar{c}_1} \right] dF(\theta) = 0. \end{split}$$

We can rearrange this by grouping together the optimizer terms and the myope terms:

$$\begin{split} \mu_O &\int\limits_{\Theta} \left[ u_1'(c_1\left(\theta\right);\theta) \frac{dc_1\left(\theta\right)}{d\bar{c}_1} + \delta u_2'(c_2\left(\theta\right);\theta) \frac{dc_2\left(\theta\right)}{d\bar{c}_1} \right] dF(\theta) \\ &+ \mu_M \int\limits_{\Theta} \left[ u_1'(c_1^M;\theta) \frac{dc_1^M\left(\theta\right)}{d\bar{c}_1} + \delta u_2'(c_2^M;\theta) \frac{dc_2^M}{d\bar{c}_1} \right] dF(\theta) = 0. \end{split}$$

Further simplifying, note that  $u'_1(c_1(\theta); \theta) \frac{dc_1(\theta)}{d\bar{c}_1} + \delta u'_2(c_2(\theta); \theta) \frac{dc_2(\theta)}{d\bar{c}_1} = 0$  for all  $\theta \notin \Gamma(\bar{c}_1)$ . Consequently, we have

$$\begin{split} \mu_O & \int_{\theta \in \Gamma(\bar{c}_1)} \left[ u_1'(c_1(\theta);\theta) - \delta R u_2'(c_2(\theta);\theta) \right] dF(\theta) \\ &+ \mu_M \int_{\Theta} \left[ u_1'(c_1^M;\theta) - \delta R u_2'(c_2^M;\theta) \right] dF(\theta) = 0. \end{split}$$

Note that  $u'_1(c_1(\theta); \theta) - \delta R u'_2(c_2(\theta); \theta) > 0$  for all  $\theta \in \Gamma(\bar{c}_1)$ , where  $\Gamma(\bar{c}_1) \subset \Theta$  denotes the set of  $\theta$  values that would induce an optimizer to be strictly constrained if period-one consumption were bounded above by  $\bar{c}_1$ . Therefore, for the leading case in which the mass of optimizers is non-zero and the mass of myopes is non-zero, then

$$\int_{\theta \in \Gamma(\bar{c}_1)} \left[ u_1'(c_1(\theta);\theta) - \delta R u_2'(c_2(\theta);\theta) \right] dF(\theta) > 0.$$

and accordingly

$$\int_{\Theta} \left[ u_1'(c_1^M; \theta) - \delta R u_2'(c_2^M; \theta) \right] dF(\theta) < 0$$

At  $s_D = 0$ ,

$$\begin{split} \frac{dW}{ds_D} &= -\int\limits_{\Theta} \left[ u_1'(c_1^P;\theta) - \delta R u_2'(c_2^P;\theta) \right] dF(\theta) \\ &= -\int\limits_{\Theta} \left[ u_1'(y - s_F;\theta) - \delta R u_2'(Rs_F;\theta) \right] dF(\theta) \\ &= -\int\limits_{\Theta} \left[ u_1'(c_1^M;\theta) - \delta R u_2'(c_2^M;\theta) \right] dF(\theta) \\ &> 0. \end{split}$$

This shows that  $\frac{dW}{ds_D} > 0$  at  $s_D = 0$ . Because the optimization with respect to  $s_D$  is globally concave,<sup>12</sup> it follows that  $s_D > 0$ . This proves the lemma *and* confirms that the IC constraint is satisfied by the solutions of the separated problems.

$$\int\limits_{\Theta} \left[ u_1''(y - s_F - s_D; \theta) + \delta R^2 u_2''(R(s_F + s_D); \theta) \right] dF(\theta) < 0,$$

implying that the objective is concave.

<sup>&</sup>lt;sup>12</sup>The second derivative of the objective is