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ABSTRACT

Generalizing models of directed technical change, I show that complementarities between innovations and factors of production (here energy resources) can drive transitions away from a dominant sector. In a calibrated numerical implementation, the economy gradually transitions energy supply from coal to gas and then to renewable energy even in the absence of policy. The welfare-maximizing tax on carbon emissions is J-shaped, immediately redirects most research to renewables, and rapidly transitions energy supply directly to renewables. The emission tax is twice as valuable as either the welfare-maximizing research subsidy or the welfare-maximizing mandate to use renewable resources.

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A online appendix is available at http://www.nber.org/data-appendix/w23420

1 Introduction

Economists have long recognized that the best climate change policy combines emission taxes with subsidies for research into clean technologies. If political considerations prevent policymakers from deploying both instruments, which should they emphasize? To what degree can one instrument substitute for the other? Some models of climate policy that endogenize innovation have found that emission taxes are far more valuable (Popp, 2006; Fischer and Newell, 2008; Hart, 2019) and others have found that research subsidies are critical (Acemoglu et al., 2016; Greaker et al., 2018).[1] In the recent models, market incentives direct innovation to either fossil or renewable resources. These incentives act to "lock-in" the initially dominant fossil resource. The only drivers of long-run change are resource depletion and policy. Because we are going to run out of atmosphere before we run out of fossil fuels, policy aims to escape fossil lock-in and create clean energy lock-in.

I here show that directed technical change does not imply a lock-in framing. In fact, historical experience suggests that technological change, not depletion, has been critical to past transitions between different types of resources (e.g., Flinn, 1959; Marchetti, 1977; Marchetti and Nakicenovic, 1979; Rosenberg, 1983; Grübler, 2004; Fouquet, 2010; Wilson and Grubler, 2011).[2] A model used to study a future transition to renewable energy should allow innovation dynamics thought to drive past transitions in energy supply. If these dynamics might also drive a transition to renewable energy, policy would focus on accelerating and steering that transition rather than on changing which resource is locked-in.

I develop the first model with laissez-faire transitions driven by endogenous innovation decisions. I show that an empirically relevant generalization of past models is critical to the possibility of innovation-led transitions. A final good is produced from labor, capital, and several imperfectly substitutable types of energy. Each type of energy is produced by combining an energy resource with specialized machines. For instance, coal is combined with steam engines to produce mechanical motion or electricity. A fixed measure of scientists works to improve these machines. Each scientist targets whichever type of machine provides a more valuable patent. Scientists' efforts change the quality of machines from period to period, which in turn changes equilibrium use of each energy resource from period to period. In Acemoglu et al. (2012), the elasticity of substitution between resources and machines is fixed at unity. Relaxing this restriction, I analytically demonstrate that innovation-led

[1] Historically, economists prioritized emission taxes. Schneider and Goulder (1997) favor emission taxes because they are closer to the primary market failure. Nordhaus (2008, 22) says that proposals to address climate change by providing research support instead of pricing emissions are "not really serious" and fail to recognize "the central economic question about how to slow climate change". Temporary research subsidies can be sufficient to manage climate change within the models of Acemoglu et al. (2012, 2016).

[2] Resource economists have long focused on how depletion or exhaustion can induce transitions between resources (e.g., Nordhaus, 1973; Chakravorty and Krulce, 1994; Chakravorty et al., 1997). The emphasis on depletion at the expense of innovation dates back to Jevons (1865), who underestimated the scope for innovation in his famous analysis of the advancing depletion of British coal reserves (Madureira, 2012).

transitions occur only if that elasticity of substitution is strictly less than unity.

Imagine that there are only two types of energy and that one type of energy initially attracts the majority of scientists and uses more raw resources. I show that several forces determine how each sector's share of research and resource use changes in the following period. First, *market size* and *resource cost effects* attract scientists to whichever sector is increasing its share of energy resource use. By drawing scientists in, these effects increase that sector's share of resource use in subsequent periods, thereby attracting even more scientists. This positive feedback between resource use and research works to lock in whichever sector is already dominant. Second, a *direct productivity effect* pulls scientists to the sector where their patent will cover a higher quality machine. This effect draws additional scientists to the sector that dominated research in the previous period, which again works to lock in whichever sector is already dominant.[3]

Third, a *supply expansion effect* drives scientists away from the sector with higher quality machines. Advancing technology shifts out the supply of a sector's machine services and thus reduces their price. The reduced value of machines' output pushes scientists away from the sector that dominated research effort in the previous period. As an example of this story, Nordhaus (1996) documents both that the price of light fell dramatically over time and that the efficiency of incandescent lightbulbs barely improved after 1920 even as other lighting technologies advanced. The present model's explanation would be that researchers shifted to lighting types whose relative backwardness made their output scarcer and improvements more valuable. In the absence of resource depletion, the supply expansion effect is the only one of the forces that works against lock-in and in favor of a transition away from the dominant sector.

The elasticity of substitution between resources and machines determines the relative strengths of the direct productivity and supply expansion effects. When that elasticity of substitution is equal to 1, we have a knife-edge case in which the direct productivity and supply expansion effects exactly offset each other. The research allocation is entirely determined by market size and resource cost effects, so whichever sector initially dominates research and resource supply does so forever. The dominant sector is locked-in, as in Acemoglu et al. (2012) and related literature.[4]

[3]The forces generating lock-in are similar to those explored in a related literature on path dependence in technology adoption (e.g., David, 1985; Arthur, 1989; Cowan, 1990). That literature focuses on "dynamic increasing returns" as the source of path dependence, where the likelihood of using a technology increases in the number of times it was used in the past (perhaps through learning-by-doing or network effects). In the present setting, market size, resource cost, and direct productivity effects all act like dynamic increasing returns.

[4]The Cobb-Douglas assumption dates to early models of directed technical change (Acemoglu, 2002, 2007). Work on climate and directed technical change has used variants of the Cobb-Douglas assumption (Acemoglu et al., 2012; Hémous, 2016; van den Bijgaart, 2017; Fried, 2018; Greaker et al., 2018). Acemoglu et al. (2016) develop a setting in which two types of energy technologies compete in each of many product lines. Each product line's production function is Cobb-Douglas. As a result, their setting again generates

When that elasticity of substitution is strictly greater than 1 (machines are "resource-saving"), demand for machine services is elastic and the price of machine services does not fall by much as technology improves. The direct productivity effect dominates the supply expansion effect. Whichever sector dominates research and resource use in some period then does so to an increasing degree in all later periods. The dominant sector is again locked-in.

However, when that elasticity of substitution is strictly less than 1 (machines are "resource-using"), demand for machine services is inelastic and the price of machine services falls by a lot as technology improves. The supply expansion effect dominates the direct productivity effect. In that case, as the dominant sector becomes more advanced, scientists can begin switching to the other sector. Eventually, their research efforts raise the quality of technology in the dominated sector, which begins increasing that sector's share of resource use via market size and resource cost effects. The transition in research away from the dominant sector can thereby induce a subsequent transition in energy supply.

To explore the implications for climate change policy, I integrate the model of directed technical change with the benchmark DICE climate-economy model (Nordhaus, 2017) and obtain new quantitative estimates of emission taxes under endogenous innovation. Use of coal and gas resources generates carbon dioxide emissions whereas use of renewables does not. Carbon emissions eventually raise global temperature and thereby reduce the quantity of final goods produced. Unlike in DICE, the emission intensity of output and the cost of reducing emissions are here endogenous. I calibrate the model to match market data. Outside estimates suggest that the elasticity of substitution between resources and machines is around 0.4. Internal model dynamics also better match history and projections with this value than with alternatives of 1 or 1.5.

Consistent with dominant popular narratives, I find that a transition from coal to gas is underway and that a transition from gas to renewables will eventually follow. As described analytically, both transitions are innovation-led, eventually occurring even if fossil resources were not depletable. Transitions in research proceed swiftly and to completion, but the subsequent transitions in resource use proceed much more slowly and do not eliminate use of other resources. The slowness and incompleteness of transitions in energy supply align well with historical evidence (Smil, 2010, Chapter 2). Policy is critical to limiting warming.

A welfare-maximizing policymaker would design an emission tax to begin at \$132 per tCO_2, dip shallowly for a few periods, and then increase steadily. The initial emission tax immediately redirects most research to the renewable sector.[5] The subsequent dip partly

strong path dependence or lock-in. Subsequent to the present paper, Acemoglu et al. (2019) use a Leontief production function and describe a transition when extraction technologies are fixed. In contrast to the present paper, they predict that renewable resources fully crowd out fossil resources after the transition, which occurs because renewable energy does not require a costly resource input in their setting. Hart (2019) mechanically weakens path dependence by modeling knowledge spillovers between sectors. Innovation interacts multiplicatively with other factors of production, and laissez-faire transitions are still driven by exhaustion.

[5] I show that an emission tax has an analytically ambiguous effect on the direction of research, as it

reflects scientists' increased willingness to work on renewables. The emission tax makes the economy transition directly to renewables, two centuries faster than in laissez-faire and skipping the transition from coal to gas.

A policymaker who can only subsidize research into renewables immediately shifts all research to renewables. As in Acemoglu et al. (2012, 2016), this subsidy need only be temporary because scientists soon want to work in the renewable sector as its technology improves, but in contrast to Acemoglu et al. (2012, 2016), the research subsidy cannot drive fossil resource use to trivial levels and thus cannot fully control long-run warming. The emission tax is twice as valuable a policy instrument because it faces no such limit. In fact, a policymaker achieves only small benefits by adding a research subsidy to the emission tax: using the research subsidy to direct innovation lowers the optimal initial emission tax only to \$122 per tCO_2 and affects later taxes even less, so that economic and climatic trajectories are similar with or without the research subsidy available as a second instrument.

I also explore the implications of directed technical change for a different type of policy instrument: a mandate to use a minimum share of renewable resources. Similar mandates are common. For instance, around 30 U.S. states and the European Union each mandate a minimum share of renewable electricity and the U.S. Congress has considered analogous policies. I show that accounting for endogenous innovation responses is critical to the evaluation of a mandate. Sufficiently large mandates (of 20% or more) ignite an energy transition by redirecting research effort. Further, welfare is neither monotonic nor concave in the stringency of the mandate: mandates that are large enough to ignite a transition can provide substantial net benefits even though smaller mandates provide only small net benefits. Accounting for endogenous innovation responses is of first-order importance when evaluating mandates, as these responses are the difference between concluding that large mandates are costly and concluding that large mandates are beneficial.[6] Nonetheless, and broadly consistent with stylized examples in Fischer and Newell (2008), a mandate is only as valuable as a research subsidy and half as valuable as an emission tax.

Modeling resource-using machines is critical to the quantitative results. Laissez-faire dynamics are less sensible if I calibrate an economy with resource-saving machines, as they predict that gas dominated the past but is of minimal importance to the future. A research subsidy would become necessary to redirect scientists to the renewable sector. An emission tax would still be more valuable than a research subsidy, but because the emission tax would no longer shift much research to the renewable sector, this alternative model would

both increases the market share of renewables and increases the price of the fossil fuel-using machines that researchers could sell. In the calibrated model, higher emission taxes do direct researchers to renewables.

[6]Previously, some have informally argued that such mandates might allow the energy sector to escape lock-in (e.g., Lehmann and Gawel, 2013). Formal analyses of this channel typically focus on learning-by-doing as the mechanism for technological change (Gerlagh and van der Zwaan, 2006; Kalkuhl et al., 2012), which means that technology matures jointly with energy production. In contrast, here renewable technology begins maturing *before* renewables begin "escaping" lock-in. Other theoretical (Clancy and Moschini, 2018) and empirical (Johnstone et al., 2010) work reports that mandates can induce innovation.

predict greater benefits from adding a research subsidy to an emission tax.[7] The stickiness of research would also change the evaluation of mandates. Large mandates would impose large costs that are not much affected by endogenizing innovation.

This paper makes a theoretical contribution to understanding directed technical change and a quantitative contribution to analysis of climate change policy. Formally, I analyze directed technical change when final good production has a nested constant elasticity of substitution structure that allows innovation to complement other inputs. The use of a Cobb-Douglas aggregator for innovators' machines and factors of production dates to the earliest models of directed technical change (Acemoglu, 2002, 2007). This paper joins other recent work in noting that Cobb-Douglas assumptions are knife-edge cases with qualitatively special results (e.g., Alvarez-Cuadrado et al., 2017; Baqaee and Farhi, 2019). Complementarities between innovation and factors of production may be common. For instance, Grossman et al. (2017) summarize evidence that capital and labor are complements. The present framework provides one mechanism through which the economy could have transitioned from an era in which unskilled labor was dominant to the modern era in which skilled labor is dominant.[8] Baqaee and Farhi (2019) summarize evidence of complementarities across intermediates throughout supply chains. The present theory of innovation-led transitions could thus explain dynamics in manufacturing activity.

Quantitatively, this paper is the first to integrate a calibrated model of directed technical change with the benchmark DICE integrated assessment model. The DICE model predicts a steadily increasing emission tax (e.g., Nordhaus, 2017), as occurs here after the first few periods. The needs to direct innovation and resource depletion both raise the tax in the earliest period.[9] Some previous models with directed technical change have had the tax decline either forever (Greaker et al., 2018) or decline once lock-in begins working in renewables' favor to crowd out use of fossil resources (Acemoglu et al., 2016). Here complementarities limit production from high-quality machines for using renewables. Substantial fossil resource use can persist even after a transition to renewable energy, so an emission tax remains important even post-transition. In Hart (2019), the optimal emission tax increases monotonically. The difference in the first few periods may arise because innovation has less of an effect on resource use in that model (as is also true here if machines were resource-saving

[7]A case with Cobb-Douglas (i.e., resource-neutral) machines is similar to the case with resource-saving machines, with the exception that depletion does eventually drive a laissez-faire shift in research to the renewable sector.

[8]Acemoglu (2002) explains how shocks to the relative supply of skilled labor interact with the skill premium through endogenous innovation, but his model does not explain how the economy could have transitioned away from an era in which unskilled labor was dominant. I thank Greg Casey for raising this point.

[9]Goulder and Mathai (2000) show that endogenizing innovation could either increase or decrease the optimal emission tax. Here endogenizing innovation increases the optimal tax. If I fix the trajectory of technology to its laissez-faire level, the optimal initial emission tax falls to \$122 per tCO_2, which is also the optimal initial tax when the policymaker uses a research subsidy to direct research.

or Cobb-Douglas) and so its initial tax is less concerned with redirecting innovation.

The present paper clarifies the relative value of emission taxes and research subsidies. It favors the former because the optimal emission tax redirects research nearly as effectively as a research subsidy while also redirecting resource use in ways a research subsidy cannot. This result extends recent empirical evidence that endogenous innovation can increase the emission reductions from a given emission tax (Aghion et al., 2016; Fried, 2018) or reduce the cost of a given emission cap (Calel, 2020). In Hart (2019), laissez-faire transitions are driven by depletion and research subsidies are not very valuable because they have difficulty affecting resource use. Here, the research subsidy does greatly accelerate a transition. Its drawback is its inability to fully clean up supply post-transition. My counterfactuals suggest that research subsidies have a harder time steering resource use when machines are resource-saving or Cobb-Douglas. Acemoglu et al. (2016) find that a portfolio with a research subsidy and an emission tax is appreciably more valuable than a standalone emission tax. My base case does not support this conclusion, and my counterfactuals suggest their result may be due to their use of Cobb-Douglas machines.

The next section describes the theoretical setting. Section 3 analyzes the relative incentive to research technologies in each sector. Section 4 theoretically describes the economy's laissez-faire dynamics. Section 5 numerically explores the implications for policies that aim to control future climate change. The final section concludes. The appendix contains calibration details, robustness checks, a numerical example, and proofs.

2 Setting

I study a discrete-time economy in which final good production uses multiple types of energy intermediates and scientists can work to improve production of any of these intermediates. Figure 1 illustrates the key relationships.

The time t final good Y_t is produced competitively from L_t units of labor, K_t units of capital, and an energy aggregate E_t. The representative firm's production function is Cobb-Douglas:[10]

$$Y_t = D(T_t)\, A_{Yt}\, L_t^{1-\beta_K-\beta_E} K_t^{\beta_K} E_t^{\beta_E}.$$

$D(\cdot) \in (0, 1]$ gives damages from time t surface warming T_t, $A_{Yt} > 0$ is an exogenous productivity parameter, and $\beta_K \in [0, 1)$ and $\beta_E \in (0, 1]$ are the factor shares of capital and energy, with $\beta_K + \beta_E \leq 1$.

Labor inputs L_t to final good production are exogenous. Capital K_t increases through investment, as determined by a fixed savings rate $\Upsilon \in [0, 1)$, and depreciates at rate $\delta \in [0, 1]$:

$$K_{t+1} = (1 - \delta)K_t + \Upsilon\, Y_t.$$

[10]Hassler et al. (2021) analyze innovation in energy technologies when the final good production function is not Cobb-Douglas.

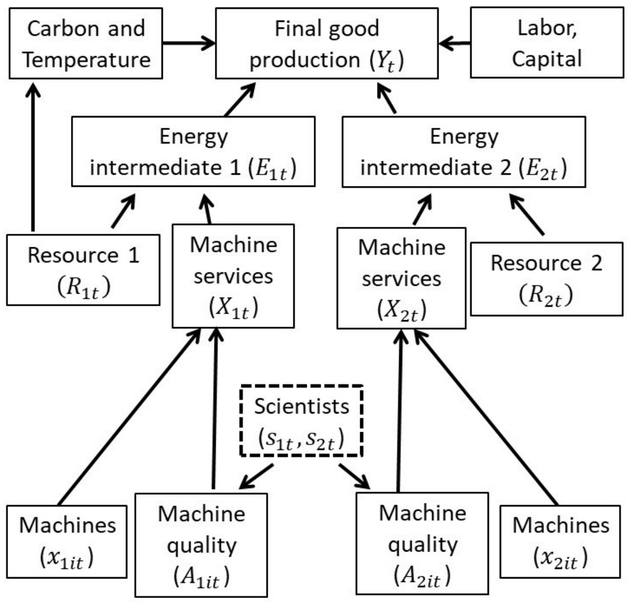


Figure 1: Overview of the setting, with $N = 2$ and resource 1 generating carbon emissions.

The energy aggregate is produced from N intermediates E_{jt}, where $j \in \{1, ..., N\}$. Production of the energy aggregate has a constant elasticity of substitution (CES) form:

$$E_t = \left(\sum_{j=1}^{N} \nu_j E_{jt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The parameters $\nu_j \in (0, 1)$ are the distribution (or share) parameters, with $\sum_{j=1}^{N} \nu_j = 1$. The parameter ϵ is the elasticity of substitution. The energy intermediates are gross substitutes ($\epsilon > 1$), consistent with intuition and with evidence in Papageorgiou et al. (2017). The final good is the numeraire in each period.

The energy intermediates E_{jt} are the energy services produced by combining resource inputs R_{jt} with machine inputs X_{jt}.[11] Production of energy intermediate j has the following CES form:

$$E_{jt} = \left(\kappa R_{jt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

The parameter $\kappa \in (0, 1)$ is the distribution (or share) parameter. The elasticity of substitution between the resource and machine inputs is σ. I call machines *resource-using* when

[11] I interpret the machines as devices for converting resources to useful energy, but one can also interpret them as machines for accessing and extracting resources. I avoid referring to the machines as capital in order to avoid confusion with K_t, but they can be interpreted as a form of capital used in energy production that depreciates fully over the timestep.

resources and machines are gross complements ($\sigma < 1$), and I call machines *resource-saving* when resources and machines are gross substitutes ($\sigma > 1$). Resources and machines are less substitutable than are different types of energy intermediates ($\sigma < \epsilon$).

Machine services X_{jt} are produced in a Dixit-Stiglitz environment of monopolistic competition from machines of varying qualities:

$$X_{jt} = \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \, \mathrm{d}i,$$

where $\alpha \in (0, 1)$ and where an implicit fixed factor of production ensures constant returns to scale. The machines x_{jit} that work with resource j at time t are divided into a continuum of types, indexed by i. The quality (or efficiency) of machine x_{jit} is given by A_{jit}. Machines of type i are produced by monopolists who each take the price (p_{jXt}) of machine services as given (each is small) but recognize their ability to influence the price p_{jxit} of machines of type i. The cost of producing a machine is $a > 0$ units of the final good, normalized to $a = \alpha^2$ (e.g., Acemoglu et al., 2012).

Following Acemoglu et al. (2012), scientists choose which resource they want to study and are then randomly allocated to a machine type i. Scientists working on resource j are of measure s_{jt}. Each scientist succeeds in innovating with probability $\eta \in (0, 1]$. If they fail, scientists earn nothing and the quality of that type of machine is unchanged. As in Acemoglu et al. (2012) and Hart (2019), among others, successful scientists receive a one-period patent to produce their type of machine. In the numerical implementation, each period will be ten years. Using resource j as an example, successful scientists improve the quality of their machine type to

$$A_{jit} = A_{ji(t-1)} + \gamma A_{ji(t-1)}, \tag{1}$$

where $\gamma > 0$. In each period, households supply a fixed measure of research effort in aggregate, normalized to 1:[12]

$$1 = \sum_{j=1}^{N} s_{jt}.$$

Firms that enter into production of resource j find a deposit containing one unit of the resource, which they sell for p_{jRt}. Firms must pay a fixed cost (in units of the final good) to develop the nth deposit. In equilibrium, all deposits with fixed costs less than p_{jRt} either get developed in period t or were already developed in some earlier period. Order the continuum of deposits by fixed cost. The fixed cost of the nth deposit is $F_j(n) = (n/\Psi_j)^{1/\psi_j}$

[12] Hart (2004, 2019) allow an extensive margin in research. Following Acemoglu et al. (2012, 2016), I keep the total pool of researchers exogenous so that I can focus on implications of the directedness of technical change rather than on well-known externalities in the quantity of research undertaken. See page 55 of Acemoglu et al. (2016) for a discussion.

for $\psi_j, \Psi_j > 0$. Define $Q_{jt} = \sum_{s=1}^{t-1} R_{js}$ as cumulative use of resource j prior to t. $\zeta_j \in \{0,1\}$ indicates whether resource j is depletable: when $\zeta_j = 1$, deposits are permanently exhausted once they are developed, but when $\zeta_j = 0$, the cost profile is constant over time, as when deposits regenerate between periods. In equilibrium, $F_j(R_{jt} + \zeta_j Q_{jt}) = p_{jRt}$. As a result,

$$R_{jt} = \Psi_j p_{jRt}^{\psi_j} - \zeta_j Q_{jt}. \tag{2}$$

I impose $\psi_j \geq \alpha/(1-\alpha)$, which ensures that the resource price affects resource supply at least as much as it affects machine services.

Resource use generates carbon dioxide emissions that eventually cause warming. Time t emissions e_t are

$$e_t = \bar{e} + \sum_{j=1}^{3} \xi_j R_{jt},$$

with ξ_j the emission intensity of resource j and $\bar{e}$ exogenous emissions from other sources.[13] Emissions increase atmospheric carbon stocks, modeled as four reservoirs. Stacking the four reservoirs in a vector $\boldsymbol{M}_t$, atmospheric carbon evolves as

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Lambda}\, \boldsymbol{M}_t + \boldsymbol{b}\, e_t$$

where $\boldsymbol{\Lambda}$ is a 4×4 matrix of transfer coefficients and $\boldsymbol{b}$ is a 4×1 vector controlling how emissions are allocated to atmospheric reservoirs. Surface temperature T_t evolves as $T_{t+1}(\boldsymbol{M}_{t+1}, T_t, T_t^o)$, and ocean temperature T_t^o evolves as $T_{t+1}^o(T_t, T_t^o)$. Appendix B gives the specific equations and parameterizations.

The economy's time t resource constraint is

$$(1-\Upsilon)Y_t \geq c_t + a \sum_{j=1}^{N} \int_0^1 x_{jit}\,\mathrm{d}i + \sum_{j=1}^{N} \int_{\zeta_j Q_{jt}}^{R_{jt}+\zeta_j Q_{jt}} F_j(n)\,\mathrm{d}n,$$

where $c_t \geq 0$ is the composite consumption good. Households have strictly increasing utility for the consumption good. Scientists therefore each choose their resource type so as to maximize expected earnings.

I study equilibrium outcomes.

Definition 1. *An* equilibrium *is given by sequences of prices for energy intermediates ($\{p_{jt}^*\}_{j=1}^N$), prices for machine services ($\{p_{jXt}^*\}_{j=1}^N$), prices for machines ($\{p_{jxit}^*\}_{j=1}^N$), prices for resources ($\{p_{jRt}^*\}_{j=1}^N$), demands for inputs ($\{E_{jt}^*\}_{j=1}^N, \{R_{jt}^*\}_{j=1}^N, \{X_{jt}^*\}_{j=1}^N, \{x_{jit}^*\}_{j=1}^N$), factor allocations ($\{s_{jt}^*\}_{j=1}^N$), and consumption ($c_t^*$) such that, in each period t: (i) $\{E_{jt}^*\}_{j=1}^N$*

[13] In Acemoglu et al. (2012) and Greaker et al. (2018), emissions from sector j are proportional to production of the intermediate, so that technical change directly increases emissions. I here distinguish between the raw resources that are converted to emissions and the useful work to which they are put.

maximizes profits of final good producers, (ii) $(\{R^*_{jt}\}_{j=1}^N, \{X^*_{jt}\}_{j=1}^N)$ *maximizes profits of energy intermediate producers, (iii)* $(\{p^*_{jxit}\}_{j=1}^N, \{x^*_{jit}\}_{j=1}^N)$ *maximize profits of the producers of each machine* i *in each sector* j*, (iv) resource producers enter until they earn zero profits, (v)* $\{s^*_{jt}\}_{j=1}^N$ *maximizes expected earnings of scientists, (vi) prices clear the factor and input markets, (vii) technologies evolve as in equation* (1)*, and (viii) the economy's resource constraint holds with equality.*

The equilibrium prices clear all factor markets and all firms maximize profits. If scientists are employed in any two sectors, they receive the same expected reward from both, and if they are not employed in some sector, they receive a greater expected reward in some other sector that has nonzero scientists. Throughout, I drop the asterisks when clear.

2.1 Policy

A policymaker seeks to maximize utilitarian welfare over $\hat{t}$ periods. Welfare W is

$$W = \sum_{t=1}^{\hat{t}} \frac{L_t\, u(c_t/L_t)}{(1+\rho)^{t-1}},$$

where ρ is the discount rate per timestep and $u(\cdot)$ is the period utility function. The policymaker has some combination of three tools at their disposal. First, they can tax emissions at rate τ_t, so that equation (2) becomes

$$R_{jt} = \Psi_j [p_{jRt} - \tau_t \xi_j]^{\psi_j} - \zeta_j Q_{jt}.$$

Second, they can subsidize research into some sector j at rate ω_{jt}. Third, they can require that sector j supply at least a fraction Σ_j of the resources used by the economy. Such a production mandate imposes $R_{jt}/\sum_{k=1}^N R_{kt} \geq \Sigma_j$.

3 The Equilibrium Direction of Research

I now tease apart the forces determining the equilibrium allocation of scientists at some time t.

The first-order condition for a producer of machine services yields demand for machines of type i in sector j:

$$x_{jit} = \left(\frac{p_{jXt}}{p_{jxit}}\alpha\right)^{\frac{1}{1-\alpha}} A_{jit}. \tag{3}$$

The monopolist producer of x_{jit} therefore faces an isoelastic demand curve and accordingly marks up its price by a constant fraction over marginal cost: $p_{jxit} = a/\alpha = \alpha$. In equilibrium, the producer of machine type i for use with resource j earns profits of:

$$\pi_{jxit} = (p_{jxit} - a)x_{jit} = \alpha(1-\alpha)p_{jXt}^{\frac{1}{1-\alpha}} A_{jit}.$$

If a scientist succeeds in innovating at time t, she exercises her patent to obtain the monopoly profit π_{jxit}. Her expected reward to choosing to research machines that work with resource type j is therefore

$$\Pi_{jt} = (1+\omega_{jt})\eta\,\alpha\,(1-\alpha)p_{jXt}^{\frac{1}{1-\alpha}}(1+\gamma)A_{j(t-1)}, \tag{4}$$

where $A_{j(t-1)}$ is the average quality of machines in sector j. This average quality evolves as

$$A_{jt} = \int_0^1 \left[\eta s_{jt}(1+\gamma)A_{ji(t-1)} + (1-\eta s_{jt})A_{ji(t-1)}\right] \mathrm{d}i = (1+\eta\gamma s_{jt})A_{j(t-1)}. \tag{5}$$

Consider the relative incentive to research technologies that work with resource j rather than technologies that work with resource k. Begin with a laissez-faire economy that lacks emission taxes or research subsidies. From equation (4),

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{(1+\gamma)A_{j(t-1)}}{(1+\gamma)A_{k(t-1)}}}_{\text{direct productivity effect}} \underbrace{\left[\frac{p_{jXt}}{p_{kXt}}\right]^{\frac{1}{1-\alpha}}}_{\text{price effect}}. \tag{6}$$

These terms and their labels are familiar from Acemoglu et al. (2012).[14] The price effect reflects that scientists prefer patents on more valuable machines. Appendix A shows that

$$p_{jXt} = \left[p_{jRt}\frac{1-\kappa}{\kappa}\right]^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}}\right]^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}}. \tag{7}$$

[14]A market size effect would appear here if the quantity of the implicit fixed factor differed by sector or if, as in Acemoglu et al. (2012), a scarce factor were allocated between sectors. I maintain identical fixed factors in order to focus on market size effects that evolve with resource use (see equation (8) below). The calibration will absorb differences in fixed factors into the estimated technology parameters.

Substituting (7) into (6) and then using (2),

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{(1+\gamma)A_{j(t-1)}}{(1+\gamma)A_{k(t-1)}}}_{\text{direct productivity effect}} \overbrace{\underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}}\right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}}}_{\text{supply expansion effect}} \underbrace{\left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}}}_{\text{market size effect}} \underbrace{\left(\frac{\left[\frac{R_{jt}+\zeta_j Q_{jt}}{\Psi_j}\right]^{1/\psi_j}}{\left[\frac{R_{kt}+\zeta_k Q_{kt}}{\Psi_k}\right]^{1/\psi_k}}\right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}}}_{\text{resource cost effect}}}^{\text{price effect}}. \tag{8}$$

Four terms determine scientists' relative incentive to research machines. The first term is a *direct productivity effect* that directs research effort to the sector in which scientists will end up with the patent to better technology, which is the sector with better incoming technology given the standing-on-shoulders representation of innovation used here and in many related papers.[15] The other channels derive from the price effect in equation (6). The *supply expansion effect* pushes scientists away from the more advanced sector. From equation (A-2), the supply of X_{jt} shifts out when its machines' average quality A_{jt} increases, and it shifts out to an especially large degree when α is small. When σ is small (machines are resource-using), the demand curve is steep because the marginal product of additional machines is constrained by the supply of R_{jt}. By shifting out supply, the increase in A_{jt} induces a relatively large decline in the equilibrium price p_{jXt}. However, when σ is large, machines are resource-saving and the demand curve is relatively flat. The increase in A_{jt} then induces a relatively small decline in the equilibrium price p_{jXt}. Improving technology therefore pushes scientists away to a greater degree when σ and α are small.

Pause to consider the net effect of a relative improvement in sector j's average technology. We have seen that this relative improvement attracts scientists through the direct productivity effect and repels scientists through the supply expansion effect. Combining these channels, the exponent on relative technology is proportional to $(\sigma-1)(1-\alpha)$. The supply expansion effect dominates the direct productivity effect if and only if $\sigma < 1$. As $\sigma \to 1$, the two effects exactly cancel, so that the incentives to research machines in one sector or the other do not directly depend on the relative quality of technology in each sector. As $\sigma \to 0$, demand for machines becomes perfectly inelastic and the supply expansion effect becomes large. As $\sigma \to \infty$, demand for machines becomes perfectly elastic and the supply

[15] The direct productivity effect depends on realized technology $(1+\gamma)A_{j(t-1)}$, not solely on the increment to technology $\gamma A_{j(t-1)}$ produced by a scientist's efforts. If γ differed by sector and were very small in the more advanced sector, then scientists could have a stronger incentive to research machines in the more advanced sector even though their efforts would not improve these machines. This business-stealing distortion vanishes under the assumption of identical γ: by attracting scientists to the more advanced sector, the direct productivity effect here also attracts them to the sector where they make the largest advance.

expansion effect vanishes.

The remaining price effect channels in equation (8) connect research incentives to resource supply. They direct research towards the sector with greater resource use. First, a *market size effect* makes innovation more attractive in the sector with greater resource use. From equation (A-1), an increase in R_{jt} shifts out demand for X_{jt}, and it does so to an especially large degree when machines and resources are stronger complements (i.e., as σ becomes small). Second, a *resource cost effect* makes innovation more attractive in the sector with higher resource prices. From equation (A-1), an increase in p_{jRt} (for given R_{jt}) shifts out demand for X_{jt} as firms substitute machines for resources. This channel is especially strong when machines are more substitutable for resources (i.e., σ is large). These two effects draw scientists towards whichever sector is increasing its share of resource supply over time.

Depletion affects innovation incentives through the resource cost effect. As cumulative resource use increases, the price of depletable resources goes up. Innovation becomes attractive as a means of reducing input costs to intermediate production. For given resource supply decisions, depletion plays a larger role in steering innovation when machines are more substitutable for resources (i.e., σ is large).

Now consider how the combination of the market size and resource cost effects evolves over time. Using first-order conditions and resource market-clearing (see Appendix A), we find:

$$\left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}} \left(\frac{\left[\frac{R_{jt}+\zeta_j Q_{jt}}{\Psi_j}\right]^{1/\psi_j}}{\left[\frac{R_{kt}+\zeta_k Q_{kt}}{\Psi_k}\right]^{1/\psi_k}}\right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}} = \left(\frac{\nu_j}{\nu_k}\left[\frac{E_{jt}}{E_{kt}}\right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}\right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}}. \tag{9}$$

Because $\epsilon > \sigma$, the change in sector j's share of resource use from time t to time $t+1$ has the same sign as the change in sector j's share of energy intermediate production.[16] Observe that increasing the average quality of technology A_{jt} increases production of the energy intermediate E_{jt}. Thus, sector j's share of resource use tends to increase when the average quality of its technology is advancing relative to sector k. The sector that is advancing more rapidly tends to attract even more scientists in later periods through market size and resource cost effects, which works to lock in that sector's technological advantage.

[16] The elasticity of substitution (σ) between resources and machines controls how resource demand shifts out as energy intermediate production increases, with greater substitutability allowing machines to drive more production of the intermediate and thus limiting the shift in resource demand. But the elasticity of substitution (ϵ) between energy intermediates controls how resource demand shifts in as other intermediates become relatively scarce, with greater substitutability reducing the value of having multiple types of intermediates and thus limiting the shift in resource demand. For $\sigma < \epsilon$, the first shift dominates.

Policy can alter equilibrium incentives. Equation (9) becomes:

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{1}{\sigma}} \frac{\left[\frac{R_{jt}+\zeta_j Q_{jt}}{\Psi_j}\right]^{1/\psi_j} + \tau_t \xi_j}{\left[\frac{R_{kt}+\zeta_k Q_{kt}}{\Psi_k}\right]^{1/\psi_k} + \tau_t \xi_k} = \frac{\nu_j}{\nu_k} \left[\frac{E_{jt}}{E_{kt}}\right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}. \tag{10}$$

For given technologies, increasing τ_t reduces equilibrium R_{jt}/R_{kt} if and only if $\xi_j > \xi_k$. As is intuitive, emission-intensive sectors are especially penalized by higher emission taxes. Equation (8) becomes:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{1+\omega_{jt}}{1+\omega_{kt}} \underbrace{\frac{(1+\gamma)A_{j(t-1)}}{(1+\gamma)A_{k(t-1)}}}_{\text{direct productivity effect}} \overbrace{\underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}}\right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}}}_{\text{supply expansion effect}} \underbrace{\left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}}}_{\text{market size effect}} \underbrace{\left(\frac{\left[\frac{R_{jt}+\zeta_j Q_{jt}}{\Psi_j}\right]^{1/\psi_j} + \tau_t \xi_j}{\left[\frac{R_{kt}+\zeta_k Q_{kt}}{\Psi_k}\right]^{1/\psi_k} + \tau_t \xi_k}\right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}}}_{\text{resource cost effect}}}^{\text{price effect}}. \tag{11}$$

Increasing τ_t has ambiguous effects on the allocation of research. By increasing the consumer price of resources, a higher tax works to direct research towards the more emission-intensive sector through the resource cost effect, but by reducing relative use of the emission-intensive resource, a higher tax works to direct research away from the emission-intensive sector through the market size and resource cost effects.[17] In contrast, increasing ω_{jt} explicitly directs scientists towards sector j. Through the right-hand side of equation (10), this effect works to increase R_{jt} and to decrease R_{kt}, which further drives scientists to sector j through the market size and resource cost effects. Finally, imposing a mandate Σ_j also works to increase R_{jt} and decrease R_{kt}, which again drives scientists to sector j through the market size and resource cost effects.

4 The Equilibrium Evolution of Resource Use and Technology in Laissez-Faire

I now study the evolution of the economy. I analytically show that the possibility of a transition and the nature of long-run outcomes are sensitive to whether machines are resource-using

[17] If larger τ_t does increase the cleaner sector's share of research, then the change in technologies further increases the cleaner sector's share of resource use through the right-hand side of equation (10). Also, observe that the effect of the tax on research is unambiguous as $\sigma \to 0$ because the resource cost effect vanishes.

or resource-saving. Three special cases of σ highlight the relevant dynamics. Appendix D illustrates the main ideas with a numerical example.

Let $N = 2$ and label the two sectors j and k. In order to isolate whether innovation can drive transitions, set $\zeta_j = \zeta_k = 0$. And in order to isolate dynamics internal to the energy sector, fix $\psi_j, \psi_k = \psi$, $A_{Yt} = A_Y$, $D(\cdot) = 1$, and $\beta_E = 1$.[18] Appendix E.1 establishes that the equilibrium is stable in a tâtonnement sense. Equation (8) becomes

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{(1+\gamma)A_{j(t-1)}}{(1+\gamma)A_{k(t-1)}}}_{\text{direct productivity effect}} \overbrace{\underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}}\right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}}}_{\text{supply expansion effect}} \underbrace{\left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}}}_{\text{market size effect}} \underbrace{\left(\frac{R_{jt}/\Psi_j}{R_{kt}/\Psi_k}\right)^{\frac{1}{\sigma+\alpha(1-\sigma)}\frac{\sigma}{\psi}}}_{\text{resource cost effect}}}^{\text{price effect}}, \tag{12}$$

and equation (9) becomes

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{1}{\sigma}+\frac{1}{\psi}} = \frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{1/\psi}\left[\frac{E_{jt}}{E_{kt}}\right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}. \tag{13}$$

The following assumption will be useful for studying transitions. It describes a time t_0 at which sector j dominates research activity with technology that is more advanced than (or not too much less advanced than) sector k's technology:

Assumption 1. *$A_{j(t_0-1)}/A_{k(t_0-1)} > [\Psi_j/\Psi_k]^\theta$ and $s^*_{jt_0} > 0.5$ for some time t_0, where $\theta \triangleq 1/[(1-\alpha)(1+\psi)] \in (0,1]$.*

The next lemma establishes one set of structural conditions under which Assumption 1 holds:

Lemma 1. *If $\nu_j = \nu_k$ and $\Psi_j = \Psi_k$, then Assumption 1 holds if (i) $A_{j(t_0-1)} > A_{k(t_0-1)}$ and (ii) either $\sigma > 1$ or σ is not too much smaller than 1.*

Proof. See Appendix E.5. □

The *steady state* for this economy has the research allocation fixed forever, so each type of technology improves at a constant rate. Define a *transition in research* as occurring at the first time $t \geq t_0$ at which s_{jt} begins declining, a *transition in resource use* as occurring at the first time $t \geq t_0$ at which R_{jt}/R_{kt} begins declining, and a *transition in technology* as occurring at the first time $t \geq t_0$ at which A_{jt}/A_{kt} begins declining. Finally, define the dominant resource as being *locked-in* from time t_0 when no type of transition occurs after t_0.

Begin by considering the case with $\sigma > 1$:

[18] For this analysis, a model with $\beta_E = 1$ is equivalent to one with $\beta_E < 1$ and labor and capital fixed over time.

Proposition 2. *Let $\sigma > 1$.*

1. *If Assumption 1 holds, then resource j is locked-in from time t_0.*
2. *If $s^*_{jt} \in (0.5, 1)$, then $s^*_{j(t+1)} > s^*_{jt}$. If $s^*_{jt} \in (0, 0.5)$, then $s^*_{j(t+1)} < s^*_{jt}$.*
3. *The only stable steady states are at $s_{jt} = 0$ and $s_{jt} = 1$.*

Proof. See Appendix E.6. □

If machines are resource-saving, then a transition cannot happen. The economy is locked-in to the dominant sector. The proof shows that sector j increases its share of resource supply whenever it dominates research effort. And when sector j is both increasing its share of resource supply and dominating research effort, the market size, resource cost, and direct productivity channels in equation (12) pull even more scientists towards sector j. Sector j therefore increases its dominance of research effort over time and continually increases its technological advantage over sector k. Sector j's increasing share of resource supply and its increasing share of research activity form a positive feedback loop that prevents sector k from ever catching up: sector j's increasingly improved technology and increasing share of resource use both work to attract ever more scientists to sector j, and the improving relative quality of technology in sector j works to increase its share of resource use over time. The economy therefore approaches a corner allocation in research effort.[19]

The dynamics are qualitatively different if $\sigma < 1$. First consider the steady-state research allocation:

Proposition 3. *Let $\sigma < 1$. Then the only steady-state research allocation has $s_{jt} = 0.5$ and the following are true as $t \to \infty$:*

1. *$s^*_{jt} \to 0.5$ (i.e., the steady state is stable).*
2. *If $\nu_j = \nu_k$ and $\Psi_j = \Psi_k$, then $R^*_{jt} = R^*_{kt}$ and $A_{jt} = A_{kt}$.*
3. *If $\nu_j \geq \nu_k$ and $\Psi_j \geq \Psi_k$ with strict inequality for at least one, then $R^*_{jt} > R^*_{kt}$ and $A_{jt} > A_{kt}$.*
4. *R^*_{jt} and R^*_{kt} become constant, and R^*_{jt}/R^*_{kt} approaches $\left[\left(\frac{\nu_j}{\nu_k}\right)^{\psi} \frac{\Psi_j}{\Psi_k}\right]^{\frac{\epsilon}{\epsilon+\psi}}$.*

Proof. See Appendix E.7. □

[19]The only exception is a knife-edge case in which the initial period's equilibrium has scientists equally allocated between the two sectors.

The proposition gives four results. First, the economy approaches a steady-state research allocation in which the average quality of each technology improves at the same rate. The steady state is both unique and stable.[20] Second, if the two resources are of the same quality and accessibility, then the steady state has identical technology and use of each. Third, if one sector's resource is of higher quality (with larger ν) and more accessible (with larger Ψ), then that sector dominates resource use and has better technology. Fourth, resource use eventually approaches a constant value in each sector. Resource supply becomes less sensitive to further advances in machine quality as machines becomes more advanced, so resource use cannot grow at a nonzero constant rate for all time. Observe that the long-run share of each resource is not sensitive to the magnitude of σ. These shares are instead completely determined by the characteristics of each resource (specifically, Ψ_j, Ψ_k, ν_j, ν_k, and ψ) and by the elasticity of substitution between the two types of energy (ϵ).

I now analyze the possibility of transitions with resource-using machines:

Proposition 4. *Let $\sigma < 1$, and let Assumption 1 hold.*

1. *A transition in resource use occurs only after a transition in research, and a transition in technology occurs only after a transition in resource use.*
2. *If $\Psi_j \geq \Psi_k$, then a transition in technology occurs while sector j still provides the larger share of resource supply.*
3. *If $\nu_j = \nu_k$ and $\Psi_j = \Psi_k$, then a transition in research and a transition in resource use both occur before reaching the steady-state research allocation.*

Proof. See Appendix E.8. □

If machines are resource-using, then sector j's dominant share of research activity works to push scientists away from sector j through the supply expansion effect in equation (12) even as sector j's improving relative technology works to increase its share of resource supply and thus strengthens the market size and resource cost effects that pull scientists towards sector j. The change in the market size and resource cost effects is especially significant when sector j's technology is still immature, so that sector j can increasingly dominate research effort over time. However, the market size and resource cost effects become less and less sensitive to the quality of sector j's technology as that technology becomes more advanced. The supply expansion effect eventually dominates these effects, which pushes

[20] A corner allocation cannot persist when $\sigma < 1$. The proof shows that as the average quality of technology in sector j improves, changes in the market size and resource cost effects become negligibly small: resources are not constrained by the availability of machines when machines become very advanced, so further improvements in their average quality do not affect resource use very much. Eventually the supply expansion effect dominates not just the direct productivity effect but also the market size and resource cost effects. Π_{jt}/Π_{kt} then begins to decline. If the allocation of scientists is held fixed at the corner, Π_{jt}/Π_{kt} eventually falls below unity, at which point the corner allocation can no longer be an equilibrium.

scientists back towards sector k. At this point a transition in research occurs. As sector j's share of research continues to fall, a transition in resource use can occur. The transition in resource use is *innovation-led*: it can occur only after the transition in research. Even though research transitions before resource use, sector k does not begin to dominate research effort (triggering a transition in technology) until sometime after the transition in resource use, when the market size, resource cost, and supply expansions effect all push scientists towards sector k. Finally, if resource j is relatively accessible (i.e., $\Psi_j \geq \Psi_k$), then a transition in technology must happen while sector j still dominates resource supply. Just as the transition in resource use must follow a transition in research, so too a change in the sector that dominates resource supply must follow a change in the sector that dominates research.

The first two parts of the proposition establish that a transition *can* happen when machines are resource-using. The final part of the proposition describes conditions under which a transition *must* happen. The sector with more advanced technology can attract the majority of researchers when neither technology is very advanced. However, the relatively backward sector must eventually dominate the research allocation because the steady state has both sectors being equally advanced (see Proposition 3). As the technologies improve, scientists must eventually start switching towards the relatively backward sector, and we already saw that resource use must also start switching towards the relatively backward sector sometime before the relatively backward sector begins to dominate the research allocation. Transitions in research, resource use, and technology must occur sometime before reaching a steady-state research allocation.

I next explore three special cases that highlight the competing effects that drive the evolution of the economy. I structurally ground Assumption 1 in each case. In Section 5, I explore dynamics with three resources in a calibrated quantitative application. The forces in play are the same, except operating between every pair of resources.

4.1 Special Case With Resource Markets Directing Innovation Independently of Technology: $\sigma = 1$

Begin by considering the Cobb-Douglas case studied in previous literature (see footnote 4), which arises as $\sigma \to 1$. This special case allows for especially tractable solutions.

Let $E_{jt} = R_{jt}^{\kappa} X_{jt}^{1-\kappa}$ and $E_{kt} = R_{kt}^{\kappa} X_{kt}^{1-\kappa}$. Equation (12) becomes:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \overbrace{\left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}}\right)^{-1} \underbrace{\frac{R_{jt}}{R_{kt}}}_{\text{market size}} \underbrace{\left(\frac{R_{jt}/\Psi_j}{R_{kt}/\Psi_k}\right)^{\frac{1}{\psi}}}_{\text{resource cost}}}^{\text{price effect}}. \tag{14}$$

As previously discussed, the technology terms in the direct productivity and supply expansion effects exactly cancel, so that relative technology ceases to directly affect the direction of innovation. Resource use shares completely determine the evolution of the research allocation.[21]

How do resource use shares evolve over time? Appendix E.9 shows that

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\Gamma} = \frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{\frac{1}{\psi}\left[1-\alpha(1-\kappa)\frac{\epsilon-1}{\epsilon}\right]}\left[\frac{A_{jt}}{A_{kt}}\right]^{(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}}, \tag{15}$$

where $\Gamma \triangleq \frac{\psi+1}{\psi} - \frac{\epsilon-1}{\epsilon}\left(\kappa + (1-\kappa)\alpha\frac{\psi+1}{\psi}\right) > 0$. Sector j's share of resource use increases in the relative quality of sector j's technology.[22] The more that sector j advances relative to sector k, the more that R_{jt}/R_{kt} grows, and the more that R_{jt}/R_{kt} grows, the more that Π_{jt}/Π_{kt} shifts up for any given s_{jt}. The equilibrium s_{jt} must therefore increase as $A_{j(t-1)}/A_{k(t-1)}$ increases.

Plugging (14) into (15), we obtain:

$$\frac{\Pi_{jt}}{\Pi_{kt}} \propto \left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}}\right)^{-1+(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}\frac{\psi+1}{\psi}\frac{1}{\Gamma}}\left[\frac{A_{j(t-1)}}{A_{k(t-1)}}\right]^{(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}\frac{\psi+1}{\psi}\frac{1}{\Gamma}}.$$

This expression is directly analogous to the central equation (18) in Acemoglu et al. (2012).[23] Because Π_{jt}/Π_{kt} decreases in s_{jt}, we have $s_{jt} > 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} > \left[\frac{\nu_j}{\nu_k}\right]^{-\frac{1}{(1-\alpha)(1-\kappa)\frac{\epsilon-1}{\epsilon}}}\left[\frac{\Psi_j}{\Psi_k}\right]^{-\frac{\kappa}{(1-\alpha)(1-\kappa)(\psi+1)}}.$$

If $\Psi_j < \Psi_k$ and $\nu_j < \nu_k$, then Assumption 1 holds when this inequality holds. We now see how lock-in arises: $s_{jt} > 0.5$ implies that $A_{jt}/A_{kt} > A_{j(t-1)}/A_{k(t-1)}$, which ensures that $s_{j(t+1)} > s_{jt} > 0.5$, which implies that $A_{j(t+1)}/A_{k(t+1)} > A_{jt}/A_{kt}$, and so on. There is a knife-edge case in which $s_{jt} = 0.5$ for all time, but if equilibrium s_{jt} ever takes on any other value, then the economy progresses to a corner allocation in research.

[21] As in (14), the direction of research in Acemoglu et al. (2012) is affected by technology only insofar as technology affects relative use of non-machine factors of production: substituting for relative output prices from their equation (A.3) into their equation (17) shows that the direct productivity effect and price effect of technology exactly cancel.

[22] Again, this special case largely recovers results familiar from Acemoglu et al. (2012): relative technology ends up playing a role in their setting's equilibrium (see their equation (18)) because relative market size increases in the relative quality of technology (see their equation (A.5)).

[23] This expression is not identical to equation (18) in Acemoglu et al. (2012) for two reasons. First, whereas ψ here captures the elasticity of resource supply in each sector, the analogue of resources in the base model of Acemoglu et al. (2012) is a fixed supply of labor that allocates itself between sectors (their ψ merely denotes $(1-\alpha)(1-\epsilon)$). Second, the intermediate-good production function that would be directly analogous to Acemoglu et al. (2012) would set $E_{jt} = R_{jt}^{1-\alpha} X_{jt} = R_{jt}^{1-\alpha}\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha}\,\mathrm{d}i$, but I here have diminishing returns to machine services X_{jt}, with $E_{jt} = R_{jt}^{\kappa} X_{jt}^{1-\kappa} = R_{jt}^{\kappa}\left(\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha}\,\mathrm{d}i\right)^{1-\kappa}$. No permissible values of κ and α directly recover the production function of Acemoglu et al. (2012).

4.2 Special Case With Technology Driving Innovation Independently of Resource Use: $\sigma = \epsilon$

Now consider a special case with $\sigma > 1$. In particular, let σ be as large as permitted, so that $\sigma = \epsilon$. Because resources and machines are just as substitutable as different types of energy, total energy can be thought of as mixing and matching all types of resources and machines:

$$E_t = \left(\kappa \sum_{j=1}^{N} \nu_j R_{jt}^{\frac{\epsilon-1}{\epsilon}} + (1-\kappa) \sum_{j=1}^{N} \nu_j X_{jt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Energy intermediates are now arbitrary concepts. Their effect on relative resource demand in (13) vanishes, making relative resource use independent of technology. Thus, as $\sigma \to \epsilon$, the shares of resource use are fixed over time:

$$\frac{R_{jt}}{R_{kt}} \to \left(\frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right)^{\frac{\sigma\psi}{\sigma+\psi}}.$$

Because R_{jt}/R_{kt} is fixed over time, market size and resource cost effects cease to steer the evolution of research activity. Substituting for R_{jt}/R_{kt} in equation (12), we have:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{A_{j(t-1)}}{A_{k(t-1)}}}_{\text{direct productivity}} \overbrace{\underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}} \right)^{\frac{-1}{(1-\alpha)\epsilon+\alpha}}}_{\text{supply expansion}} \left(\frac{\nu_j}{\nu_k} \right)^{\frac{\epsilon}{(1-\alpha)\epsilon+\alpha}}}^{\text{price effect}}.$$

As the average quality of technology in sector j improves, the direct productivity effect shifts Π_{jt}/Π_{kt} upward and so increases the share of scientists working in sector j. It dominates the supply expansion effect because $\sigma = \epsilon > 1$. If

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} > \left(\frac{\nu_j}{\nu_k} \right)^{-\frac{\epsilon}{(1-\alpha)(\epsilon-1)}},$$

then $s_{jt} > 0.5$. If, in addition, $\nu_j < \nu_k$, then Assumption 1 holds. In that case, sector j is locked-in insofar as its share of research increases towards a corner allocation in which sector j attracts all scientists, but this increasing dominance of research activity does not affect sector j's share of resource use. There is a knife-edge case in which $s_{jt} = 0.5$ for all time, but as with the Cobb-Douglas case analyzed above, if equilibrium s_{jt} ever takes on any other value, then the economy progresses to a corner allocation in research.

4.3 Special Case With Technology Directing Innovation Towards Less Advanced Sectors: $\sigma = 0$

Finally, consider the special case of a Leontief production function for each intermediate good, which arises as $\sigma \to 0$.[24] In order to aid exposition, fix $\psi = \alpha/(1-\alpha)$. Let $E_{jt} = \min\{R_{jt}, X_{jt}\}$ and $E_{kt} = \min\{R_{kt}, X_{kt}\}$. In equilibrium, $R_{jt} = X_{jt}$ and $R_{kt} = X_{kt}$. Appendix E.10 shows that

$$\frac{R_{jt}}{R_{kt}} = \left(\frac{\nu_j}{\nu_k} \frac{\left[\Psi_k^{-\frac{1-\alpha}{\alpha}} + A_{kt}^{-\frac{1-\alpha}{\alpha}} \right]}{\left[\Psi_j^{-\frac{1-\alpha}{\alpha}} + A_{jt}^{-\frac{1-\alpha}{\alpha}} \right]} \right)^{\frac{\epsilon\alpha}{\alpha+(1-\alpha)\epsilon}} \tag{16}$$

and

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{A_{j(t-1)}}{A_{k(t-1)}}}_{\text{direct productivity}} \overbrace{\underbrace{\left(\frac{(1+\eta\gamma s_{jt})A_{j(t-1)}}{(1+\eta\gamma s_{kt})A_{k(t-1)}} \right)^{-1/\alpha}}_{\text{supply expansion}} \underbrace{\left(\frac{R_{jt}}{R_{kt}} \right)^{1/\alpha}}_{\text{market size}}}^{\text{price effect}}. \tag{17}$$

The resource cost effect vanishes because machines cannot substitute for resources.

Consider how Π_{jt}/Π_{kt} evolves when $s_{jt} = 1$. If this allocation were to last forever, then research incentives eventually become

$$\lim_{A_{j(t-1)} \to \infty} \frac{\Pi_{jt}}{\Pi_{kt}} = 0.$$

However, this condition is incompatible with $s_{jt} = 1$, as scientists should prefer to work in sector k. A corner allocation can persist for some finite interval when A_{jt} is not too large, but over time the weakening market size effect leads Π_{jt}/Π_{kt} to decrease as A_{jt} continues to grow. As established by Proposition 3, a corner allocation in research cannot persist indefinitely.

Next consider a steady-state research allocation, with $s_{jt} = s$ for all $t \geq t_0$. Because a corner allocation cannot persist, s must be strictly greater than 0 and strictly less than 1. Appendix E.10 shows that

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)} \right)^{-\Delta \frac{1-\alpha}{\alpha}} = 1 \tag{18}$$

[24] In subsequent work, Acemoglu et al. (2019) analyze a Leontief production function.

for all $\Delta \geq 0$. This holds if and only if $s = 0.5$, so the steady state research allocation must have $s = 0.5$, as shown in Proposition 3 for $\sigma < 1$.[25]

Finally, consider an early time t_0 at which $A_{j(t_0-1)}$ and $A_{k(t_0-1)}$ are much smaller than Ψ_j and Ψ_k, respectively, and the economy is not yet at a steady-state research allocation. Equation (17) becomes:

$$\frac{\Pi_{jt_0}}{\Pi_{kt_0}} \approx \left[\left(\frac{A_{j(t_0-1)}}{A_{k(t_0-1)}} \right)^{(1-\alpha)(\epsilon-1)} \left(\frac{1+\eta\gamma s_{jt_0}}{1+\eta\gamma(1-s_{jt_0})} \right)^{-1} \left(\frac{\nu_j}{\nu_k} \right)^{\epsilon} \right]^{\frac{1}{\alpha+(1-\alpha)\epsilon}} . \tag{19}$$

The right-hand side increases in $A_{j(t_0-1)}/A_{k(t_0-1)}$ and decreases in s_{jt_0}. We have that $s_{jt_0} > 0.5$ if and only if[26]

$$\frac{A_{j(t_0-1)}}{A_{k(t_0-1)}} > \left(\frac{\nu_j}{\nu_k} \right)^{\frac{-\epsilon}{(1-\alpha)(\epsilon-1)}} . \tag{20}$$

If $s_{jt_0} > 0.5$, then $A_{j(t_0-1)}/A_{k(t_0-1)}$ increases over time and the right-hand side of equation (19) shifts up over time. As a result, $s_{j(t_0+1)} > s_{jt_0}$. Therefore, sector j can increase its share of research effort over an interval of time with not-too-advanced technology. The reason is that the market size effect increasingly incentivizes scientists to work in sector j (see equation (16)). Eventually sector j's technology becomes sufficiently advanced that this effect weakens and the supply expansion effect pushes scientists back towards sector k (see equation (E-26)). This derivation shows how complementarities can drive energy transitions: when $\sigma < 1$, the sensitivity of R_{jt}/R_{kt} to technological quality diminishes as technology advances, which eventually makes the supply expansion effect the primary determinant of research activity for some length of time and thereby ignites a transition in research that can turn into a transition in resource use.

5 Climate Change Policy

I now quantitatively assess the implications of innovation-led transitions in energy supply for climate policy. I model coal, natural gas, and renewables ($N = 3$), which compete in electricity and heating. Renewable resources are calibrated to wind and solar. Coal and gas are depletable and generate carbon dioxide emissions, whereas renewables are not depletable and do not generate carbon dioxide emissions.

[25] We also see the second through fourth parts of Proposition 3 emerge in this special case with $\sigma = 0$. From equation (E-26), $\nu_j \geq \nu_k$ and $\Psi_j \geq \Psi_k$ imply $A_{j(t-1)} \geq A_{k(t-1)}$ in the steady-state research allocation, with $A_{j(t-1)} > A_{k(t-1)}$ if in addition either $\nu_j > 0.5$ or $\Psi_j > \Psi_k$. Further, from equation (16), R_{jt}/R_{kt} approaches a constant value as t becomes large and $\nu_j \geq \nu_k$ with $\Psi_j \geq \Psi_k$ imply $R_{jt} \geq R_{kt}$, with $R_{jt} > R_{kt}$ if either $\nu_j > \nu_k$ or $\Psi_j > \Psi_k$.

[26] If $\nu_j \leq \nu_k$ and $\Psi_j \leq \Psi_k$, then inequality (20) implies that Assumption 1 holds at t_0.

Appendix B details the calibration, which uses a 10-year timestep. I set ϵ to 1.8 based on evidence in Papageorgiou et al. (2017) and Stern (2012), and I set σ to 0.4 based on estimates in Koesler and Schymura (2015) for the elasticity of substitution between energy and value-added in a panel of countries.[27] The value of 0.4 is in the ballpark of elasticities of substitution used by computable general equilibrium models of energy use (see Appendix B). I also report results for counterfactual calibrations with $\sigma = 1.5$ (resource-saving machines) and $\sigma = 1$ (as in previous literature). As we will see, the implied model dynamics turn out to support the base calibration.

The parameter γ determines the timing of laissez-faire transitions. In the base case, allocating all scientific research to a single type of energy doubles the quality of its technology over a decade ($\gamma = 1$). I also explore a large advances case in which allocating all scientific research to a single type of energy septuples that quality in a decade ($\gamma = 6$). These two values of γ will in practice generate reasonable bounds on laissez-faire dynamics, and they are consistent with the range of values implied by calibrations in Acemoglu et al. (2019) and Fried (2018), as calculated in Appendix B.

The welfare parameters follow DICE-2016R (Nordhaus, 2017). In particular, the pure rate of time preference is 1.5% per year in the base case and period utility takes the conventional power form with an elasticity of intertemporal substitution of 1/1.45. The policymaker optimizes over a 400-year horizon beginning in 2015.

The evolution of labor and total factor productivity and the transition equation for capital follow DICE-2016R, as do the effects of climate change on the economy. Climate and carbon dynamics follow recommendations in Dietz et al. (2021).

I calibrate the resource supply elasticities to a combination of outside data and outside estimates. I choose the remaining parameters so that the initial period's equilibrium matches market data. In particular, I match resource use, shares of R&D spending, levelized costs for each type of energy, and total economic output. When exploring different values for σ, I recalibrate these parameters to preserve the match to market data. Appendix B reports that the initial period's emission reductions from relevant emission taxes, a non-targeted moment, are broadly consistent with the DICE-2016R calibration.

5.1 Laissez-Faire Outcomes

Figure 2 plots the laissez-faire trajectories of resource use (top and middle) and research (bottom). It plots simulated trajectories both backward in time for 50 years from the calibration year 2015 and also forward in time over the four centuries from 2015.

Begin by considering the left column, which is the base calibration with $\sigma = 0.4$. The model predicts, correctly, that coal dominated resource use in the late twentieth century. It also predicts that the world is currently beginning a transition from a dominant coal resource

[27] Marten and Garbaccio (2018) map the estimates of Koesler and Schymura (2015) into energy supply sectors, and Lemoine (2020) aggregates them into a single energy supply sector.

to a dominant gas resource and will subsequently transition from gas to renewables. These future dynamics are broadly consistent with standard views of energy market dynamics in the absence of aggressive climate policy (e.g., EIA, 2021; IEA, 2021). In line with the theory, research activity drives these transitions. Gas and renewables each come to dominate the research allocation before they come to dominate resource supply. Additional experiments (not depicted) confirm that innovation, not depletion, is the critical element for endogenous transitions in resource supply: if coal and gas are non-depletable, then the transitions to renewables in research and resource use still do arise (albeit delayed by over a century), whereas allowing depletion but fixing technology at its initial level leads coal to dominate resource use throughout the 400-year horizon.

Calculating the channels from equation (8), the transition to renewable research is ignited by the supply expansion effect becoming large enough (because gas technology becomes advanced enough) to overwhelm the other effects. The direct productivity effect increasingly pushes scientists away from renewables over the next two centuries, but the supply expansion effect always (and increasingly) dominates it because σ is less than 1. The market size and resource cost effects push scientists away from renewables over at least the next three centuries, although they begin weakening as resource supply begins to transition around 200 years from now. As renewables gain market share, these effects tilt just enough to attract more scientists to renewables. This shift in the research allocation further increases renewables' share of supply, compounding the change in the market size and resource cost effects and generating a self-reinforcing process that culminates in renewable resources dominating both research and supply.[28,29]

The optimistic message is that an innovation-led transition to renewables does occur in laissez-faire. The pessimistic message is that it is not sufficient to avoid dangerous and costly levels of climate change. First, the transition does not occur for a couple of centuries. Second, fossil resource use remains substantial even after the transition. The middle panels of Figure 2 show that improvements in each resource's technologies and in total factor productivity drive a large expansion in total resource use, so that coal and gas consumption can be substantial even when these resources provide only a small share of total supply.[30] The drawn-out nature and incompleteness of the transitions from coal to gas and from gas to renewables accord with

[28]There is a slight difference relative to Proposition 4: the transition in resource use begins just before the transition in research, although it does not really get going until after the transition in research. This difference is driven by the depletability of the dominant gas resource. It does not occur in experiments that match the theoretical analysis by turning off depletion.

[29]Coal prices grow at less than 3% per year and, in all but the first period, gas prices grow at less than 6% per year. If property rights to resources extended over multiple periods, then resource owners may have an incentive to shift resource use to near-term periods. The market size and resource cost effects would favor fossil resource use even more strongly in the near term but would favor renewables more strongly in the longer-term.

[30]The middle left panel truncates the vertical axis for legibility. Renewable resource use continues increasing, reaching 250 ZJ per decade by the end of the 400-year horizon.

evidence from history (Smil, 2010, Chapter 2). The level of fossil resource use determines emissions and global climate change. The left panel of Figure 3 shows that warming exceeds 2°C in less than a century and continues over the subsequent centuries. Speeding up the transition to renewables and controlling long-run fossil resource use both require policy.

The right panels of Figure 2 show that the key theoretical innovation of the paper is consequential for laissez-faire dynamics. These panels calibrate σ to 1.5, so that machines are now resource-saving. The theory predicts that a dominant resource should now be locked-in, with resource depletion the only force that could prevent one sector from increasingly dominating research activity forever. Indeed, this model predicts that coal will dominate resource use for centuries. Depletion erodes its dominance only slowly. As the theoretical analysis leads us to expect, the resource that is calibrated to dominate research in the first period (gas) continues to do so over the next centuries.[31] Nonetheless, the share of resources supplied by gas rapidly becomes quite small because gas supply is much less elastic than is the supply of coal ($\psi_1 > \psi_2$) and thus is more affected by depletion.[32] The prediction of a coal-dominated, gas-scarce future is less consistent with dominant narratives (e.g., EIA, 2021; IEA, 2021). And the backward simulation is also less consistent with history, as it now predicts that gas formerly dominated resource use. Projections and history thus both favor the calibration with resource-using machines.[33] Finally, observe that future resource use is less than in the case with $\sigma = 0.4$, primarily because resource use is here not essential for energy production.[34] As a result, Figure 3 shows that the planet warms less than in the case with $\sigma = 0.4$. It also shows that consumption per capita is eventually larger with $\sigma = 1.5$, both because energy supply is not as constrained by resource supply and because warming is not as severe.

[31]Calculating the channels from equation (8), the gas resource increases its dominance of the research allocation over the first periods because the initial allocation has it attracting more than half of scientists. Its technology therefore improves relatively more quickly, which reinforces its direct productivity effect and thus its share of research. After fifty years, depletion makes its strengthening resource cost effect also come to dominate its small market size effect (relative to either coal or renewables).

[32]In experiments without depletion, coal continues to dominate resource supply over the entire 400-year horizon, but its share does peak after 150 years or so. Its subsequent slow decline is driven by an increase in the share of gas, as the research effort in gas improves its technology.

[33]The calibration with $\sigma = 1.5$ is also less intuitively appealing. In order to justify the target research allocation at the relatively small level of renewable resource use, the calibration requires renewables to have the best technology (so as to drive a pro-renewable direct productivity effect). The same type of calibration arises for $\sigma = 1$. In contrast, when $\sigma = 0.4$, renewables must have the worst initial technology (so as to drive a pro-renewable supply expansion effect). In reality, renewable technologies are widely perceived as less mature than coal and gas technologies. This view most straightforwardly supports the $\sigma = 0.4$ calibration.

[34]Secondarily, research is now directed towards gas technologies that are not used as intensively.

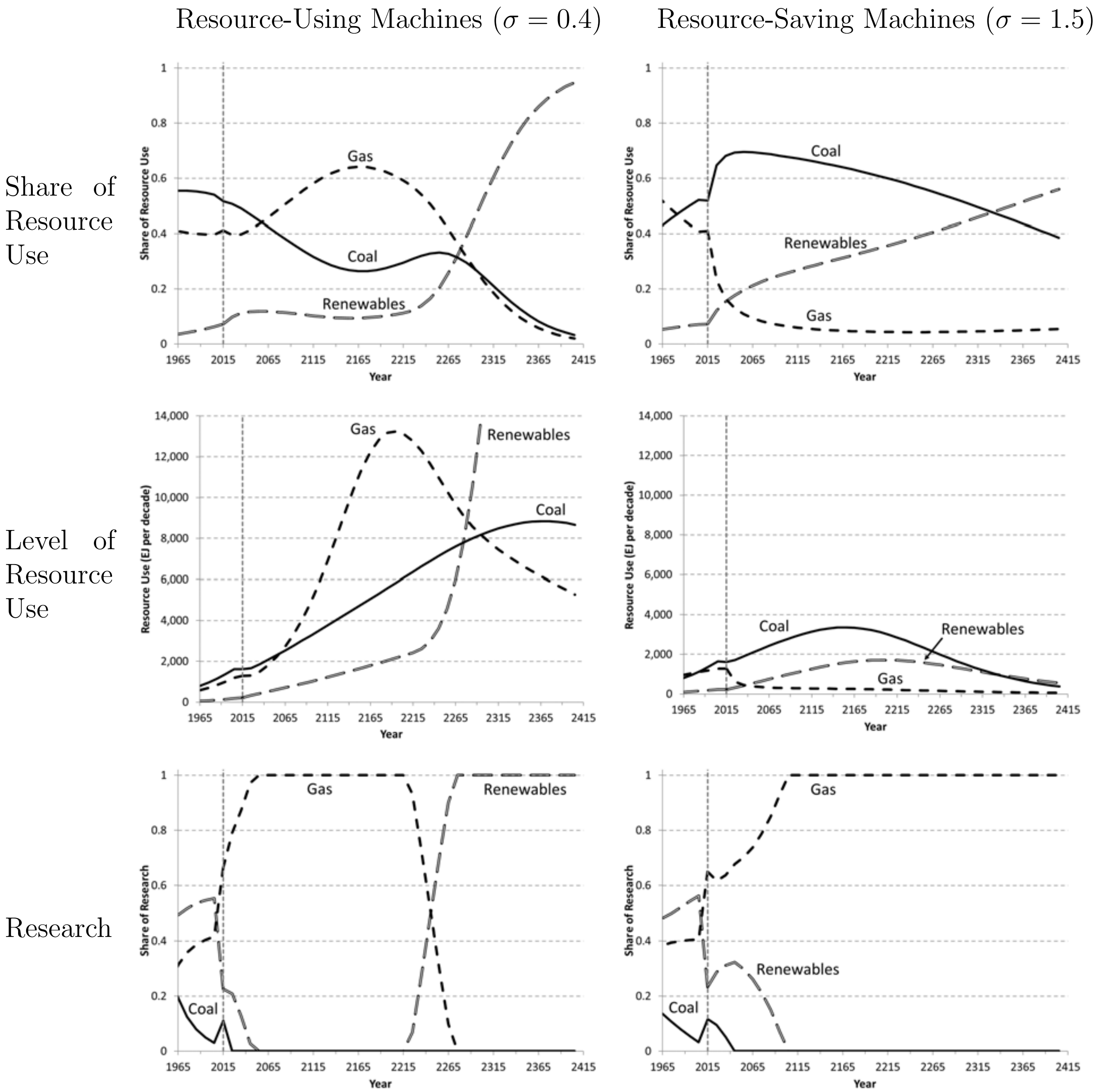
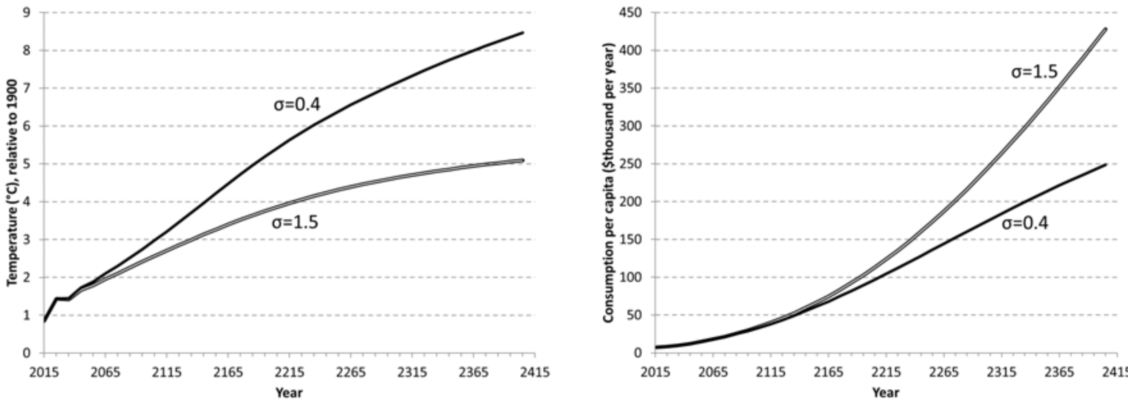


Figure 2: Laissez-faire resource use and research allocation for the base case (left, with $\sigma = 0.4$) and an alternate case with $\sigma = 1.5$ (right). The dotted vertical lines indicate the year 2015, which is the year to which the model is calibrated.



(a) Temperature (b) Consumption per capita

Figure 3: Temperature and consumption per capita in laissez-faire.

5.2 Welfare-Maximizing Tax and Research Policies

We have seen that a transition to renewable resources does eventually occur in laissez-faire. However, that transition is too late to avoid substantial warming, and residual fossil fuel use after the transition drives further warming. Now consider how a policymaker would direct resource use and innovation to maximize welfare. Depending on the scenario, the policymaker can tax carbon emissions, subsidize research into renewable technologies, or both tax carbon emissions and subsidize research into renewable technologies.

Figure 4 plots policy choices and outcomes. When the tax is the only instrument, its trajectory is J-shaped, with an initial value of \$132 per tCO_2. The tax dips slightly after the first period, bottoming out at \$114 per tCO_2 in the third period (2035). It then increases steadily, as is familiar from benchmark climate-economy models that lack endogenous innovation. Section 3 showed that an emission tax could in theory direct research towards either more or less emission-intensive sectors. The middle left panel shows that the emission tax in fact redirects nearly all research towards the renewable sector, speeding up the transition in research by 200 years. All scientists work on renewables in the third period despite the smaller tax, suggesting that the initial dip in the standalone emission tax is largely driven by scientists' increasing willingness to work in the renewable sector as its technology improves. The middle right panel shows that the emission tax triples renewables' initial share of resource use and also dramatically speeds up the transition in resource use. The bottom panels show that the welfare-maximizing tax reduces emissions throughout the policy horizon and reduces long-run warming by nearly 3°C.

In contrast to the tax trajectory, the top right panel of Figure 4 shows that the welfare-maximizing research subsidy starts high but declines rapidly. The policymaker completely phases out the subsidy within fifty years and never revives it. The policymaker sets the subsidy to ensure that all scientists work in the renewable sector (middle left panel), and after fifty years of advances, no further subsidy is required to sustain this outcome. Improved technology leads the renewable resource to dominate supply within the century (middle right panel). However, even though the research subsidy is set to its maximally effective level, it cannot reduce emissions as strongly as did the optimal emission tax, in part because both policies soon have all scientists working in the renewable sector (at which point further research subsidies are ineffective). As a result, long-run warming is about 1.2°C greater under the optimal standalone research subsidy than under the optimal standalone emission tax. To control warming most effectively, policy must make fossil resources more expensive, not just make clean energy cheaper (see also Hassler et al., 2020).

A policymaker who can use both policies at once can reduce the initial emission tax even while incentivizing all scientists to work in the renewable sector. However, the top left panel of Figure 4 shows that the initial emission tax is only slightly lower than in the case with a standalone emission tax ($122 vs $132 per t$CO_2$), demonstrating that the initial emission tax is primarily driven by the value of reducing near-term emissions and not by the objective of redirecting near-term research. The emission tax trajectory now dips only between the first and second periods.[35] Because the initial emission tax is still substantial, the policymaker can shift all scientists to the renewable sector with a smaller and shorter-lived subsidy than in the case with a standalone research subsidy (top right panel). Finally, the trajectories of resource use, emissions, and temperature are nearly identical whether or not a policymaker's toolkit includes a research subsidy in addition to an emission tax. This close similarity arises because the emission tax and research trajectories are very similar in either case.

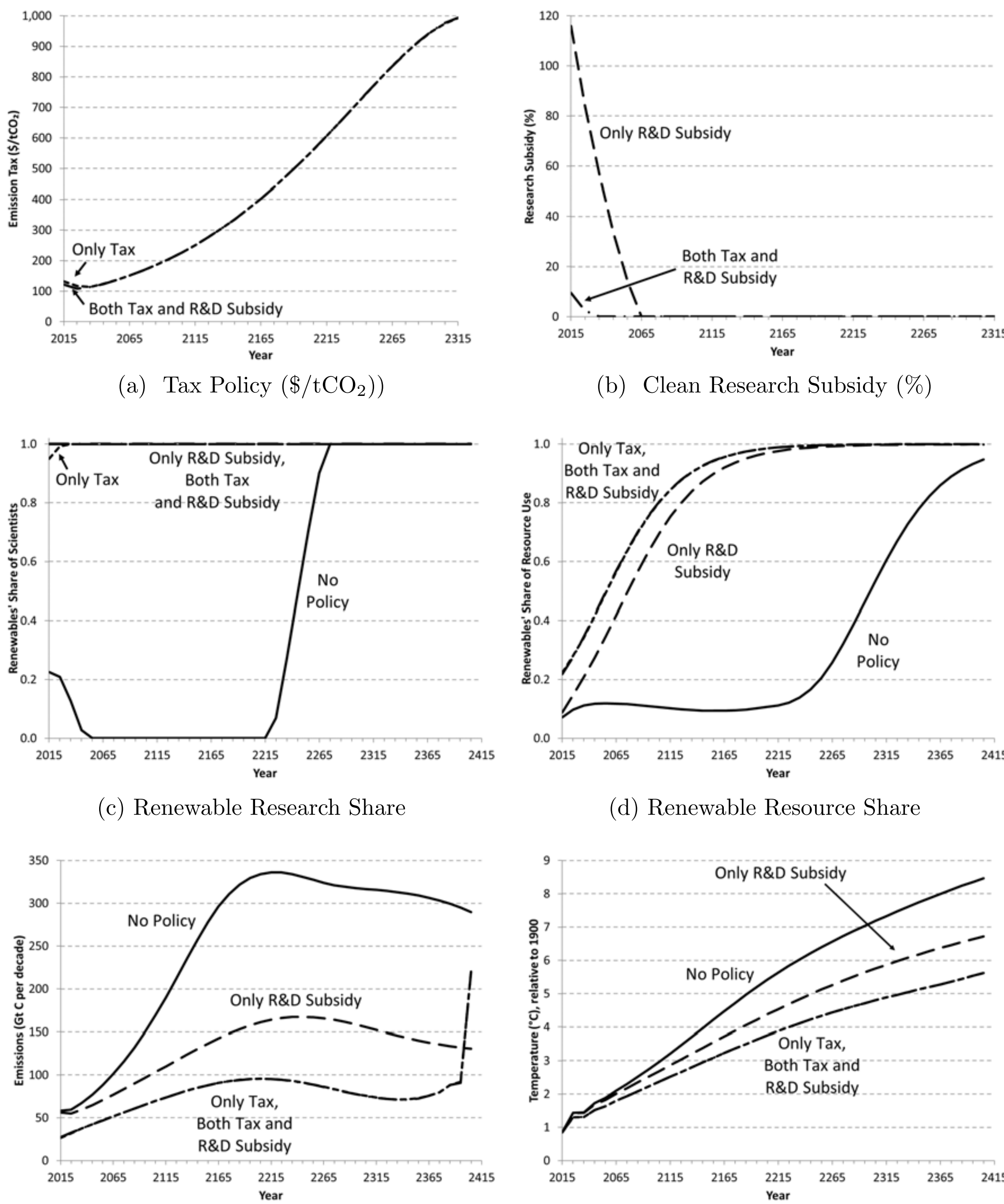
Figure 5 depicts policy choices and outcomes if machines were resource-saving (with $\sigma = 1.5$). As before, the emission tax trajectory is J-shaped and is similar whether or not the policymaker can also use a research subsidy. Also as before, requiring the emission tax to both control emissions and redirect research only slightly increases its initial level relative to a case in which it is designed purely to control emissions (here $99 vs $90 per t$CO_2$). However, as in Acemoglu et al. (2012, 2016) and Greaker et al. (2018), the research subsidy is now critical to overcoming lock-in and redirecting research to renewables: scientists slowly depart the renewable sector under the optimal emission tax in nearly the same way as in laissez-faire.[36] While it is plausible that a research subsidy could here be more effective at limiting warming, Figure 5 shows that a research subsidy has trouble affecting resource use, as in Hart (2019).[37] Because each policy here has different strengths, combining policies is

[35]The remaining J-shape is due to an attempt to internalize depletion externalities. In the absence of depletion, the optimal emission tax is J-shaped only when the policymaker cannot also use a research subsidy (see footnote 39 below).

[36]Equation (11) showed that an emission tax favors research into renewables by increasing renewable resource use but discourages that research by raising the cost of fossil resources. Two factors can explain the importance of σ. First, the increase in renewable resource use is a bit larger when $\sigma = 0.4$ (see Table C-1). Second, and perhaps more importantly, the discouraging effect in equation (11) is stronger when σ is larger.

[37]In equation (9), producing more energy intermediate j shifts out demand for resource j more strongly

Base Case, with Resource-Using Machines ($\sigma = 0.4$)



(a) Tax Policy ($/tCO$_2$)) (b) Clean Research Subsidy (%)

(c) Renewable Research Share (d) Renewable Resource Share

(e) Emissions (f) Temperature

Figure 4: Welfare-maximizing policies and outcomes for the base calibration ($\sigma = 0.4$).

more valuable than in the base case: a policymaker who can combine a research subsidy with an emission tax can attain a discernibly lower temperature trajectory.

5.3 The Value of Each Policy Instrument, with Robustness Checks

The foregoing results should make us suspect that a standalone emission tax is more valuable than a standalone research subsidy. Table 1 quantifies this value and assesses its robustness to alternate model specifications.[38] It reports the benefit from policy in terms of balanced growth equivalent (BGE) changes in consumption. The BGE translates changes in welfare into the constant relative difference in consumption between two counterfactual consumption trajectories that grow at the same constant rate (Mirrlees and Stern, 1972; Anthoff and Tol, 2009). Table C-1 in Appendix C summarizes policy choices, economic outcomes, and warming in each alternate specification.[39]

The top row reports BGE for the base model. The standalone emission tax is around twice as valuable as the standalone research subsidy. Moreover, adding the research subsidy to the emission tax increases the BGE by only a small amount. The second row considers the case of resource-saving machines ($\sigma = 1.5$). The standalone emission tax is still more valuable than the standalone research subsidy, but now adding a research subsidy to the emission tax increases value by nearly half. In the base case, the emission tax redirects research with or without the subsidy, so that the benefits from adding the subsidy arise purely from the ability to use a smaller emission tax. However, in the case with resource-saving machines, the renewable sector is soon excluded from research unless the policymaker can use a research subsidy to redirect research. Even though Figure 5 showed that redirecting research had only a small effect on warming, it still creates substantial value in an economy that increasingly relies on renewable resources. The research subsidy's ability to more directly control the evolution of technology did not matter in the base case of resource-using machines because an emission tax sufficed but is critical to breaking lock-in if machines are resource-saving.

The third row considers the case of Cobb-Douglas production from machines and resources, as in previous literature. From Section 4.1, technology now has no effect on research incentives for a given allocation of resource use. In laissez-faire, coal dominates resource supply over the next two centuries, as in the case with resource-saving machines. Depletion drives an eventual shift towards renewables.[40] But also as in the case with resource-saving

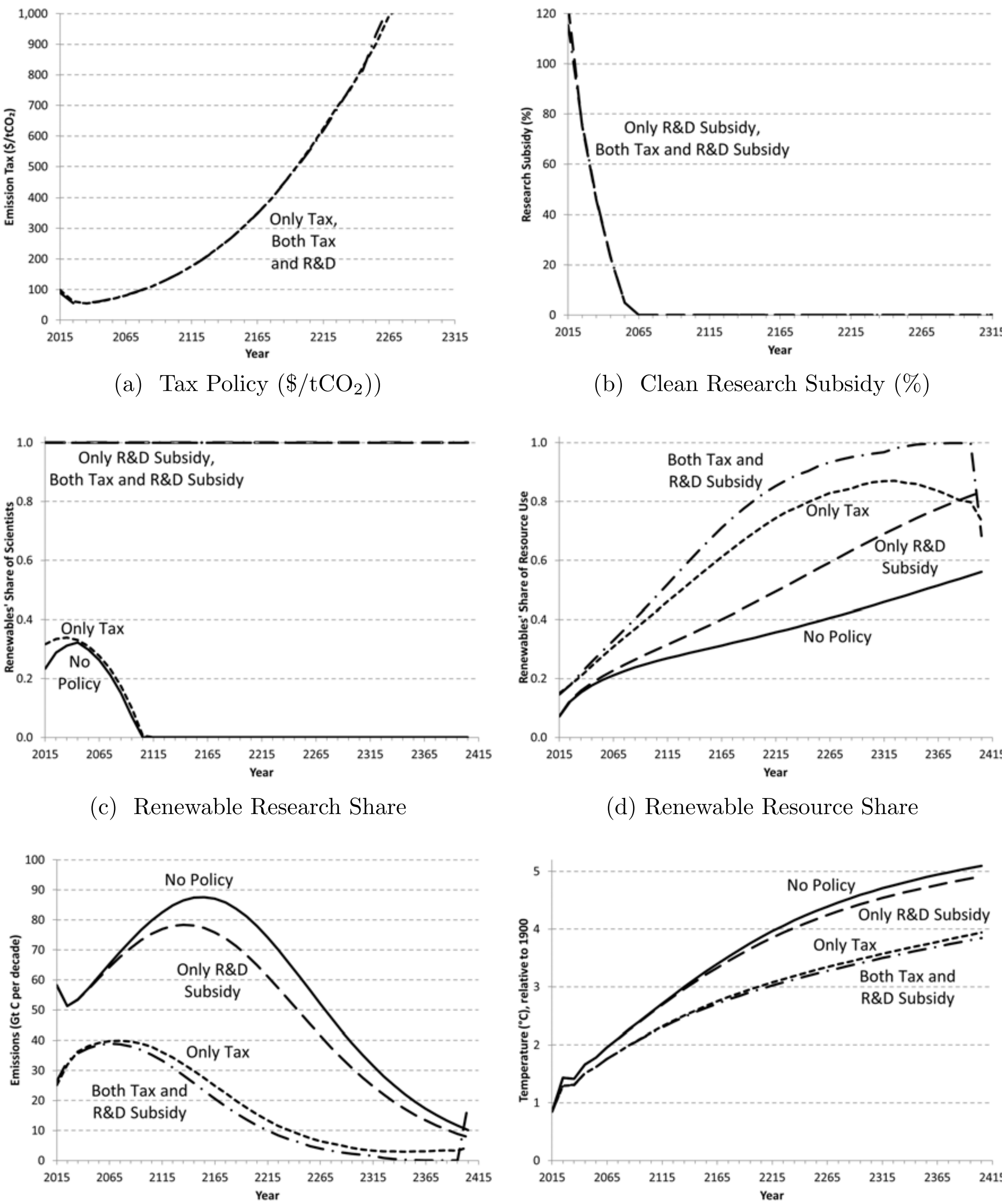
when σ is smaller.

[38] The model is recalibrated in each case, so that the initial conditions are the same across all rows, and the model is also re-optimized in each case, so that policies differ by row.

[39] The optimal emission tax is J-shaped in all cases except in the case of delay (where policy begins on the right-hand side of the J), in the case of high damages (where it increases monotonically because the very large initial tax already takes care of innovation incentives and depletion concerns), and when combined with the optimal research subsidy in the absence of depletion (see footnote 35).

[40] As we should expect from year 2015 market shares and the theory in Section 4.1, renewables get crowded out of research and maintain only a small share of resource use in the absence of depletion. In contrast to

Counterfactual with Resource-Saving Machines ($\sigma = 1.5$)



(a) Tax Policy ($\$/tCO_2$))

(b) Clean Research Subsidy (%)

(c) Renewable Research Share

(d) Renewable Resource Share

(e) Emissions

(f) Temperature

Figure 5: Welfare-maximizing policies and outcomes for the case with resource-saving machines ($\sigma = 1.5$).

machines, the transition to renewables is only gradual, is limited, and is not accelerated much by a research subsidy. Unlike the case with resource-saving machines, research does transition to renewables in laissez-faire, but like the case with resource-saving machines, the emission tax does not affect research by much. As a result, the policy story is similar to the case with resource-saving machines.

Appendix C describes the remaining rows in more detail. I here summarize the takeaways for the relative value of the policy options. First, a standalone emission tax is more valuable than a standalone research subsidy in all cases. The gap is smaller when either the research subsidy is more effective at shifting supply (as in the cases with larger advances or more substitutable energy intermediates) or the optimal tax redirects less research to renewables (as in the cases with larger advances or no depletion). The gap is larger when the policy portfolio includes a subsidy that corrects for market power in machine production: in a demonstration of the theory of the second-best (Lipsey and Lancaster, 1956), correcting the market failure in machine production actually reduces welfare when the policymaker can use only a research subsidy.

Second, adding a research subsidy does not create much additional value when machines are resource-using. Adding a research subsidy to the emission tax is most important when the optimal emission tax shifts fewer researchers to renewables, as in the cases with large advances and without depletion, but even then the value-added is small. Adding a research subsidy to an emission tax creates no additional value when the optimal initial emission tax is large enough to shift all research to renewables, as in cases with low discount rates, high climate damages, more substitutable energy intermediates, and the optimal machine subsidy.

5.4 Mandates to use Renewable Resources

I have thus far examined emission taxes and research subsidies, but a more politically feasible policy may simply mandate a minimum share of renewable resource use. We will see that accounting for endogenous innovation is critical to properly evaluating such policies.

A binding mandate pushes scientists towards the clean sector through the market size and resource cost effects. The top row of Figure 6 plots the share of renewable resource use under mandates ranging from 10% to 50%, for the base calibration with resource-using machines (left) and for the alternate calibration with resource-saving machines (right). All mandates hasten a shift in resource use towards the renewable sector. Moreover, the mandates may not bind for very long: by redirecting scientists to the clean sector (middle row), mandates make themselves nonbinding within decades (in the case of resource-using machines) or within two centuries (in the case of resource-saving machines).

The most striking effect of mandates arises in the case of resource-using machines. If mandates are sufficiently large (requiring 20% or more renewable resource use), then they

the case with $\sigma = 0.4$, the transition to renewables is depletion-led, not innovation-led.

Table 1: Balanced growth equivalent gain relative to laissez-faire.

	Policy Tools Available		
Specification	**Emission tax**	**Research subsidy**	**Both instruments**
Base	0.86	0.44	0.87
Resource-Saving Machines[a]	0.35	0.2	0.52
Cobb-Douglas Machines[b]	0.38	0.27	0.57
50-Year Delay	0.44	0.24	0.45
Less Discounting[c]	5.44	3.86	5.44
Higher Damages[d]	11.43	5.93	11.43
More Substitutable Energy Types[e]	2.26	1.89	2.26
Larger Scientific Advances[f]	1.53	1.32	1.67
Optimal Machine Subsidy[g]	0.89	0.43	0.89
No Depletion[h]	1.1	0.75	1.15

[a] σ increased from 0.4 to 1.5.
[b] σ increased from 0.4 to 1.
[c] ρ reduced from 1.5% to 0.01% per year, as in Stern (2007).
[d] Damages increased to calibration of Lemoine (2021), from survey evidence in Pindyck (2019).
[e] ϵ increased from 1.8 to 5.
[f] Innovation step size increased from $\gamma = 1$ to $\gamma = 6$.
[g] p_{jxit} reduced from α to α^2 in policy scenarios but not in laissez-faire.
[h] Each ζ_j set to zero.

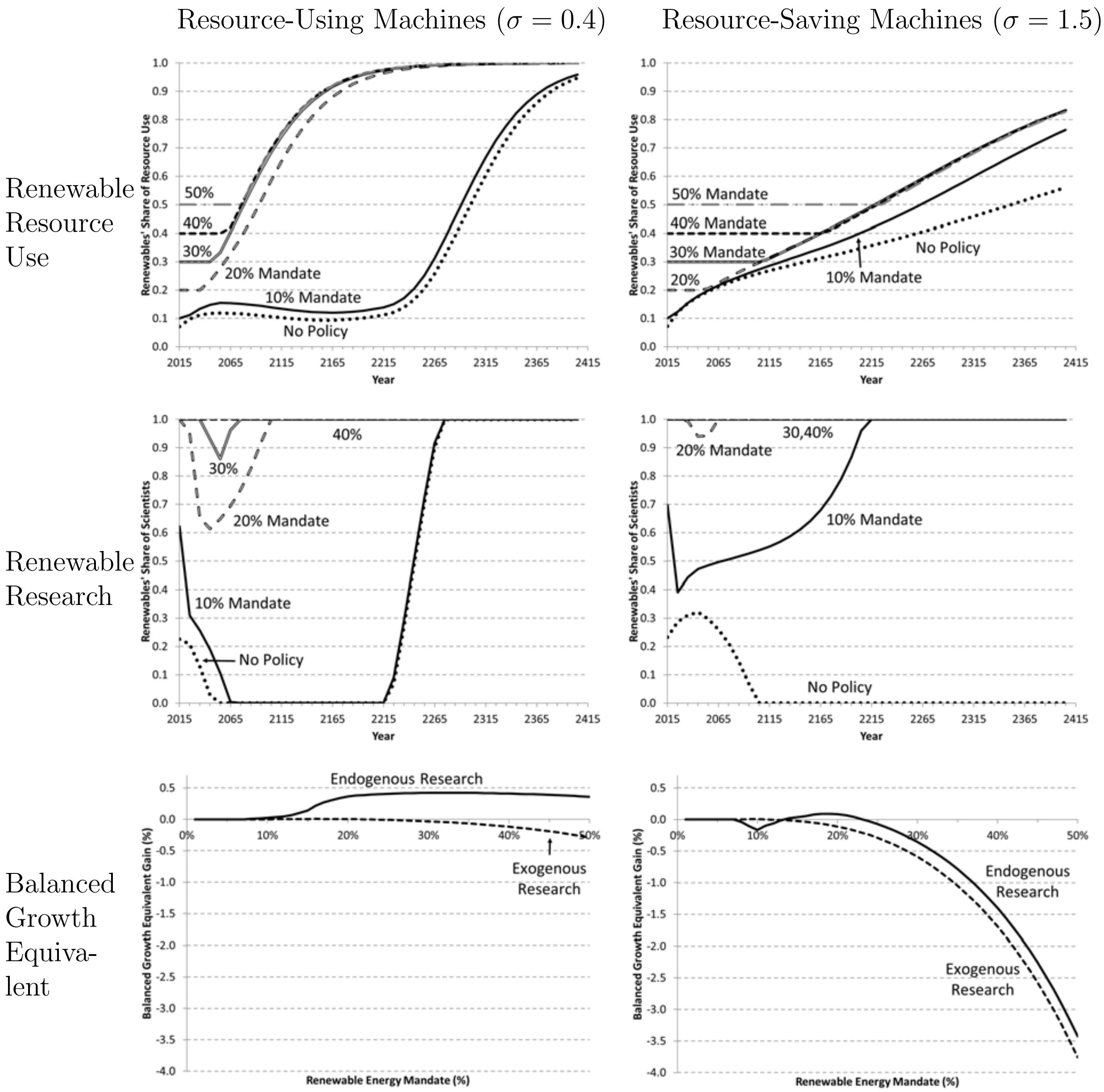


Figure 6: The effects of mandating that renewable resources provide a minimum share of resource use.

not only make themselves nonbinding but ignite a transition to renewables centuries earlier than would occur in laissez-faire. The middle left panel shows that the mandates that ignite a transition are also ones that succeed in shifting all scientists to the renewable sector in the first period. These scientists' advances lead scientists to want to keep working in that sector, and their efforts eventually generate the dramatic shift in resource use. Large mandates thus lead the economy to skip the initial transition to gas seen in Figure 2. Mandates can also shift all scientists to the renewable sector when machines are resource-saving, but as discussed in Section 5.2, the shift in research efforts does not dramatically affect the trajectory of renewable resource use when machines are resource-saving.

The bottom row of Figure 6 compares the benefits from various mandates. The solid lines indicate cases in which scientists respond to the mandate (as in the top and middle rows), and the dashed lines indicate cases in which the allocation of scientists is held fixed at the laissez-faire trajectory (plotted as the dotted "no policy" curves in the middle row). In the base case of resource-using machines (left), small mandates of 10–15% can provide small benefits and larger mandates that ignite a transition can provide substantial benefits. The best mandate (of 32%) provides benefits of 0.42% relative to no policy, which is comparable to the benefits achievable from a standalone research subsidy in Table 1 but less than half the value of the standalone emission tax. Mandates are better than no policy, but they are inferior to emission pricing, even when standalone emission pricing is second-best.

Conventional cost-benefit analyses take a static perspective on mandates, analogous to the experiments with research fixed to the laissez-faire trajectory. We see that such analyses can be highly misleading. An analysis that ignored endogenous research responses (as in the dashed lines) would assess mandates larger than 22% to be costly, but we instead see that they can be more beneficial than smaller mandates. For instance, an analysis that ignored endogenous research responses would predict that mandates of 50% would generate BGE losses of 0.29% whereas the correct analysis would predict BGE benefits of 0.36%. The endogenous response of research to mandates dominates the welfare evaluation when machines are resource-using, even changing the sign of the welfare effect.

However, matters are once again rather different when machines are resource-saving. In this case, mandates never deliver more than tiny benefits, large mandates are costly even when research is endogenous, and those costs are rather large. Large mandates can succeed in redirecting research, but because they bind for so long, they distort resource supply for a long time. These distortions generate costs that accumulate. The elasticity of substitution between resources and machines is thus critical to evaluations of mandates as policy tools.

6 Conclusions

We have seen that complementarities between innovation and factors of production are critical to the possibility of innovation-led transitions in factor use. These complementarities eventually push scientists away from the more advanced sector, and the redirection of scientific effort eventually redirects factor use away from the dominant sector. In a calibrated numerical implementation, I find that laissez-faire use of energy resources eventually does transition towards clean renewable resources from emission-intensive coal and gas and that pricing emissions is more important than directly subsidizing clean research.

I want to highlight two simplifications in my modeling of innovation, both of which follow much other literature on directed technical change and the environment. First, I have treated patents as lasting only for a single period. If patents lasted longer, then researchers would anticipate future resource supply and future emission taxes when choosing the types of energy to work on. I conjecture that this extension would not eliminate lock-in when machines are resource-saving, although it could generate additional equilibria that depend on agents' expectations (see Smulders and Zhou, 2020). I expect that this extension would increase the effectiveness of emission taxes at redirecting research and thereby further increase the advantage of emission taxes over research subsidies.

Second, I have assumed that the pool of researchers is fixed. Endogenizing the number of researchers should increase the potency of research subsidies, which here quickly reach the limit of their possible effects on resource use and warming. However, this change could also make emission taxes more effective, so the implications for policies' relative value are unclear.

Future work should assess the importance of complementarities in driving innovation-led transitions in non-energy sectors of the economy. Evidence suggests that complementarities may be widespread. And many sectors have undergone substantial change over time. Benchmark models of directed technical change may be able to rationalize these changes once generalized to permit complementarities.

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Appendices to "Innovation-Led Transitions in Energy Supply"

Appendix A derives some expressions useful elsewhere in the main text and appendix. Appendix B details the calibration and solution method. Appendix C reports further robustness checks. Appendix D contains a numerical example of the main analytic results. Appendix E contains additional theoretical results and proofs.

A Derivations of Useful Expression

The intermediate-good producer's first-order conditions for profit-maximization yield

$$p_{jXt} = (1-\kappa)p_{jt}\left[\frac{X_{jt}}{E_{jt}}\right]^{-1/\sigma} \quad \text{and} \quad p_{jRt} = \kappa\, p_{jt}\left[\frac{R_{jt}}{E_{jt}}\right]^{-1/\sigma}.$$

The relative incentive to research technologies for use in sector j increases in the relative price of the intermediates and decreases in the machine-intensity of sector j's output. Combining the first-order conditions, we have

$$p_{jXt} = \frac{1-\kappa}{\kappa}\left[\frac{R_{jt}}{X_{jt}}\right]^{1/\sigma} p_{jRt}. \tag{A-1}$$

From equation (3) and the monopolist's markup, we have

$$x_{jit} = p_{jXt}^{\frac{1}{1-\alpha}} A_{jit}.$$

Substituting into the definition of X_{jt} and using the definition of A_{jt}, we have

$$X_{jt} = p_{jXt}^{\frac{\alpha}{1-\alpha}} A_{jt}. \tag{A-2}$$

Substituting into equation (A-1) and solving for equilibrium machine prices yields (7) and (11) in the main text.

The final-good producer's first-order condition for intermediates j is:

$$p_{jt} = \beta_E\, \nu_j \frac{Y_t}{\sum_{j=1}^N \nu_j E_{jt}^{\frac{\epsilon-1}{\epsilon}}} E_{jt}^{-\frac{1}{\epsilon}}. \tag{A-3}$$

Combining the intermediate-good producers' first-order condition for resources with the final-good producers' first-order conditions, we find demand for resource j:

$$p_{jRt} = \kappa\, \beta_E\, \nu_j \frac{Y_t^{\frac{\epsilon-1}{\epsilon}}}{\sum_{j=1}^N \nu_j E_{jt}^{\frac{\epsilon-1}{\epsilon}}}\left[\frac{E_{jt}}{Y_t}\right]^{-1/\epsilon}\left[\frac{R_{jt}}{E_{jt}}\right]^{-1/\sigma}.$$

Market-clearing for resource j then implies

$$\left[\frac{R_{jt}+\zeta_j Q_{jt}}{\Psi_j}\right]^{1/\psi_j}+\tau_t\xi_j=\kappa\,\beta_E\,\nu_j\frac{Y_t^{\frac{\epsilon-1}{\epsilon}}}{\sum_{j=1}^N \nu_j E_{jt}^{\frac{\epsilon-1}{\epsilon}}}\left[\frac{E_{jt}}{Y_t}\right]^{-1/\epsilon}\left[\frac{R_{jt}}{E_{jt}}\right]^{-1/\sigma}. \tag{A-4}$$

Demand for sector j's resources (for example) shifts inward as the share of those resources in the production of intermediate good j increases and also shifts inward as the share of intermediate good j in production of the final good increases. Equations (9) and (10) in the main text follow from dividing by the analogous equation for resource k.

Final-good producers' zero-profit condition is

$$Y_t=w_t L_t+r_t K_t+\sum_{j=1}^N p_{jt}E_{jt}, \tag{A-5}$$

where w_t is the wage paid to labor and r_t is the rental rate of capital. From final-good producers' first-order conditions, these are:

$$w_t=(1-\beta_K-\beta_E)\frac{Y_t}{L_t},$$
$$r_t=\beta_K\frac{Y_t}{K_t}.$$

B Calibration, Climate Change Modeling, and Solution Method

Table B-1 reports parameter values that are fixed across all specifications. Table B-2 reports market data used to calculate remaining parameters. I use a 10-year timestep and a policy horizon of 400 years. Let resources 1, 2, and 3 represent coal, natural gas, and renewables, respectively. I model coal and natural gas as depletable ($\zeta_1, \zeta_2 = 1$) and renewables as non-depletable ($\zeta_3 = 0$), as if renewable energy installations must be rebuilt every ten years. I set $Q_{j1} = 0$ for each j.

Begin by considering the supply of each type of resource. Marten et al. (2019) follow, among others, Haggerty et al. (2015) in using a long-run supply elasticity of 2.4 for coal. Marten et al. (2019) follow Arora (2014) in using a long-run supply elasticity of 0.5 for natural gas. Based on these, I use $\psi_1 = 2.4$ and $\psi_2 = 0.5$. The price-responsiveness of wind and solar derives from heterogeneity in resource sites' quality. Drawing in part on the work of others, Johnson et al. (2017) describe the supply of power from solar photovoltaics, concentrating solar power, onshore wind, and offshore wind available by region of the world and by resource quality. Costs are reported in dollars per unit power and resource potential is reported in units of energy. I convert costs to dollars per unit electrical energy by using the capacity factor reported for each resource quality bin in each region. This capacity factor adjusts for the fact that the power producible from renewable resources is not available throughout the day or throughout the year.[1] I then convert dollars per unit of electrical energy to dollars per units of energy in the resource by using the efficiency of each type of generator. From the Energy Information Administration's Annual Energy Review 2011, the efficiencies are 12% for solar photovoltaics, 21% for solar thermal, and 26% for wind. Aggregating across resource types and regions, I estimate $\psi_3 = 3.00$.

Next consider the elasticities of substitution in the final-good and intermediate-good production functions. Papageorgiou et al. (2017) estimate an elasticity of substitution between clean and dirty energy capacity of around 1.8, and Stern (2012) estimates an elasticity of substitution between coal and gas of 1.426, with a standard error of 0.387. Version 6 of the EPPA model uses an elasticity of substitution of 1.5 (Chen et al., 2016), and the ADAGE model uses an elasticity of substitution of 1.25 (Ross, 2009). In line with these, I fix $\epsilon = 1.8$.

Much literature has estimated the elasticity of substitution between energy and other inputs, but there is not much literature on the elasticity of substitution between resources and other inputs in the production of energy. I fix $\sigma = 0.4$ based on several lines of evidence. The most directly relevant calibration is the calibration of the energy supply sector's production function in Lemoine (2020). This calibration assigns an elasticity of substitution of 0.42 to the energy supply sector, based on estimates in Koesler and Schymura (2015) implemented by Marten and Garbaccio (2018).[2] As further evidence, some computable general equilibrium models of energy use assign an elasticity of substitution of 0.3 to nearly all sectors (see Turner, 2009), version 6 of the EPPA model uses an elasticity of substitution of 0.1 between resources and a capital-labor composite in electricity production (Chen et al., 2016), and ADAGE uses an elasticity of substitution of 0.6 between resources and a materials-value-added composite (Ross, 2009).

The inverse of α is the markup over marginal cost charged by machine producers. The average markup in 2016 was around 1.6 both in the U.S. (De Loecker et al., 2020) and globally (De Loecker and Eeckhout, 2018). I therefore fix $\alpha = 1/1.6 = 0.625$.

I fix $\kappa = 0.5$ and, following Golosov et al. (2014), fix $\beta_K = 0.3$ and $\beta_E = 0.04$. The theory showed that the critical share parameters were the ν_j, not the β or κ, and sensitivity tests support this conclusion.

[1] In my setting, capacity factors are implicitly captured by the calibration of the technology variables and the share parameters. Further, the elasticity of substitution σ can be interpreted as imposing a larger capacity factor penalty at higher penetrations.

[2] Koesler and Schymura (2015) use a nonlinear least squares estimator of a CES production function with a panel of countries. Marten and Garbaccio (2018) report those elasticities of substitution along with NAICS codes. Using these, Lemoine (2020) reports the average elasticity of substitution in a combined energy supply sector, weighted by gross output from the Bureau of Economic Analysis. The underlying elasticities are all similar.

Population L_t evolves as in DICE-2016R:

$$L_t = L_\infty \left(\frac{L_1}{L_\infty}\right)^{e^{-g_L(t-1)}},$$

where I convert the DICE-2016R equation into a differential equation (with time in decades) and solve it. The capital stock follows DICE-2016R. The initial value K_1 uses World Bank GDP deflators to change the DICE-2016R initial value of 223 trillion year 2010 dollars to trillion year 2014 dollars. DICE uses an annual depreciation rate of 0.1. Converting to the decadal timestep yields

$$\delta = 1 - (1 - 0.1)^{10} = 0.6513.$$

The savings rate is endogenous in DICE-2016R but varies only between 0.24 and 0.26 over the 500-year horizon. I therefore fix $\Upsilon = 0.25$.

Now consider climate damages. The climate-economy integrated assessment literature typically models climate change as reducing total production. Letting T_t be surface temperature relative to 1900, we have, adapting Nordhaus (2017),

$$D(T_t) = 1 - d\, T_t^2$$

with $d = 0.00236$. The robustness check with higher damages increases d to 0.0228, from the mean of the calibration to Pindyck (2019) in Appendix C.1 of Lemoine (2021).

The evolution of total factor productivity A_{Yt} follows DICE-2016R (Nordhaus, 2017). It grows initially at 1.48% annually, with the growth rate declining at a rate of 0.5% annually:

$$A_{Y(t+1)} = A_{Yt} \prod_{s=0}^{9} \left[1 + (0.0148)e^{-0.005*(10*(t-1)+s)}\right].$$

Now consider the innovation function. Only the product of η and γ is important for improvements in technology over time. I therefore fix η at 1. Changes in γ do not affect the realized first-period technology, as the calibration of the A_{j0} (described below) adjusts to offset γ. Instead, changes in γ affect how rapidly technology evolves after the first period. Different values of γ can be interpreted as different step sizes for research advances, as different probabilities of research successes, and/or as different sizes for the population of researchers. I choose values of γ for the base case and the robustness check to generate a range of plausible futures, from relatively slow transitions in the base case ($\gamma = 1$) to relatively fast transitions in the "larger scientific advances" robustness check ($\gamma = 6$).

These two values for γ are consistent with the range of values implied by prior literature. In the calibration of Acemoglu et al. (2019), each scientist expects to advance technology by 11% over 5 years at the initial level of renewable scientists used here, implying a γ of around 0.2 for our 10-year timestep.[3] This value is close to the base case. Ignoring spillovers between sectors, Fried (2018) estimates that marginally increasing the share of scientists improves technology by 426% over 5 years at the initial level of renewable scientists used here, implying a γ of around 8 for our 10-year timestep.[4] This estimate is close to the "larger scientific advances" case. (Acemoglu et al. (2016) also estimate an innovation production function, but the mapping to the present paper is less clear.)

The remaining parameters are each A_{j0}, each Ψ_j, each ν_j, and A_{Y1}. I calibrate these ten parameters so that the first period's equilibrium Y_1, R_{j1}, s_{j1}, and p_{j1} match data (see Table B-2). World Bank data for global output from 2011–2015 imply that the value of the final good produced over the first ten-year timestep is 765 trillion year 2014 dollars. Initial resource consumption comes from summing consumption from 2011–2015, as reported in the BP Statistical Review of World Energy.[5] The International Energy Agency's World Energy Investment 2017 gives R&D spending on clean energy, on thermal generation, on coal production, and on oil and gas production. I divide thermal expenditures equally between coal and gas and attribute all oil and gas spending to gas. The first period must therefore have 12% of scientists working on coal, 65% of them working on gas, and 23% of them working on renewables. I calibrate each p_{j1} to be consistent with levelized costs from IEA (2015). Using the market discount rate of 7%, the median cost for coal is around 80 \$/MWh, for natural gas combined cycle plants is around 100 \$/MWh, and for solar photovoltaics is around 150 \$/MWh.[6]

The initial conditions on the R_{j1} and the s_{j1} and the guesses for the A_{j0} and the Ψ_j combine to yield the E_{j1}. I then use the ratio of the final-good firms' first-order conditions (see equation (A-3)) and the adding-up constraint on the share parameters to solve for the

[3] In their paper, scientists improve technology by a factor γ: $A_{t+1} = \gamma * A_t$. The probability of success is $\eta s_t^{-\psi}$ (in practice, they fix their $\zeta = 0$). So the expected breakthrough per scientist is, in their notation, $\eta s_t^{-\psi}(\gamma - 1)$. Using their values of $\eta = 0.598$, $\psi = 0.67$, and $\gamma = 1.07$ yields an expected breakthrough per scientist of 0.1105.

[4] The increase in next period's technology A_{t+1} due to a marginal increase in scientists s_t is, using equation (4) in Fried (2018) and adjusting for the population of scientists being 1 for me and 0.01 for Fried (2018), $dA_{t+1}/ds_t = \gamma\eta(100\rho)(s_t/(100\rho))^{\eta-1}A_t$ (in her notation). Her Table 1 gives $\rho = 0.01$, $\gamma = 3.96$, and $\eta = 0.79$, implying that $dA_{t+1}/ds_t = 4.26\, A_t$.

[5] Natural gas and coal are used for electricity generation, heating, and industrial processes. I here abstract from these differences. To obtain the energetic content of renewables from the reported tonnes of oil equivalent, use BP's assumed thermal efficiency of 38% to obtain the equivalent electrical energy and then use a 20% generator efficiency to convert electrical energy to energy in the renewable resource.

[6] These costs have changed over time and can be affected by pollution regulations. Further, costs for heating applications may be different from costs for electricity. Experiments suggest that results are not highly sensitive to these choices.

ν_j:

$$\nu_3 = \frac{1}{1 + \frac{p_{2,1}}{p_{3,1}} \left(\frac{E_{2,1}}{E_{3,1}}\right)^{1/\epsilon} \left(1 + \frac{p_{1,1}}{p_{2,1}} \left(\frac{E_{1,1}}{E_{2,1}}\right)^{1/\epsilon}\right)},$$

$$\nu_2 = \frac{(1 - \nu_3)}{1 + \frac{p_{1,1}}{p_{2,1}} \left(\frac{E_{1,1}}{E_{2,1}}\right)^{1/\epsilon}},$$

$$\nu_1 = 1 - \nu_2 - \nu_3.$$

For the initial conditions and any given guesses for the A_{j0} and Ψ_j, I set A_{Y1} to ensure that initial final good production matches Y_0.[7]

We now have the ν_j, A_{Y1}, the initial conditions, and the guesses for the A_{j0} and the Ψ_j. The levels of the intermediate goods' prices then follow from the final-good firms' first-order conditions. We require six conditions to pin down the A_{j0} and the Ψ_j. The zero-profit conditions for intermediate-good firms provide three conditions. The conditions on the initial research allocation provide two more conditions, as $\Pi_{1,1}/\Pi_{2,1} = 1$ and $\Pi_{1,1}/\Pi_{3,1} = 1$. These two conditions can be thought of as defining $A_{2,0}$ and $A_{3,0}$ as functions of $A_{1,0}$ and the Ψ_j. Final-good firms' zero-profit condition (equation (A-5)) provides the remaining condition. This zero-profit condition uses the calibrated intermediate prices, not the price implied by the final-good firm's first-order conditions (which would trivially satisfy the zero-profit condition by Euler's Homogeneous Function Theorem). This last condition can be thought of as pinning down the level of the final-good firms' first-order conditions. I solve for the A_{j0} and the Ψ_j via an optimizer that seeks to satisfy the nonlinear equality constraints subject to the implied share parameters being positive and summing to a value less than 1.[8]

Resource use generates carbon dioxide emissions that eventually cause warming. Time t emissions are

$$e_t = \bar{e} + \sum_{j=1}^{3} \xi_j R_{jt}.$$

I calculate the emission intensities of coal and gas by dividing emissions for each resource from 2010–2014 (from the Carbon Dioxide Information Analysis Center) by resource consumption over the initial timestep. Other emissions $\bar{e}$ come from summing emissions from all other reported categories, which includes emissions from oil.[9] The renewable resource does not generate emissions ($\xi_3 = 0$).

[7]Note that A_{Y1} absorbs any unit conversions between energy, other inputs, and output.

[8]The optimizer succeeds in satisfying the constraints to within 1% for all parameterizations used in the paper.

[9]In the base case's laissez-fair scenario, eliminating the (mostly oil) emissions $\bar{e}$ reduces global temperature by around 0.4°C in 400 years. In fact, projected oil use is not so far from constant in the base scenario of IEA (2021) and only slowly increasing in the reference case of EIA (2021). Fixing $\bar{e}$ may slightly understate future warming under laissez-faire but overstate future warming under optimal policy.

The carbon cycle and climate model update those in DICE-2016R. The carbon cycle follows Joos et al. (2013, Table 5), as recommended and compiled by Dietz et al. (2021). That carbon cycle has

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Lambda}\boldsymbol{M}_t + \boldsymbol{b}e_t$$

where $\boldsymbol{M}$ is a 4×1 vector of atmospheric carbon reservoirs. The coefficient matrices are:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix}^{10}$$

and

$$\boldsymbol{b} = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix}.$$

The year 2015 values (in Gt C) are

$$\boldsymbol{M}_1 = \begin{bmatrix} 588 + 139.1 \\ 90.2 \\ 29.2 \\ 4.2 \end{bmatrix},$$

where 588 Gt C is the stock of preindustrial carbon.

The parameters of the climate model come from Geoffroy et al. (2013), as compiled by Dietz et al. (2021). Additional atmospheric carbon increases radiative forcing to $F_t(\boldsymbol{M}_t)$, which measures additional energy at the earth's surface due to CO_2 in the atmosphere. Forcing is

$$F_t(\boldsymbol{M}_t) = f_{2x} \frac{\ln\left(\sum_{i=1}^{4} M_t^i/588\right)}{\ln(2)},$$

where M_t^i indicates element i of $\boldsymbol{M}_t$ and f_{2x} is forcing induced by doubling CO_2. Surface temperature evolves as

$$T_{t+1} = T_t + \frac{10}{5}\phi_1 \left[F_{t+1}(\boldsymbol{M}_{t+1}) - \lambda T_t - \phi_3 \left(T_t - T_t^o\right)\right].$$

Ocean temperature evolves as

$$T_{t+1}^o = T_t^o + \frac{10}{5}\phi_4 \left[T_t - T_t^o\right].$$

Steady-state warming from doubled carbon dioxide ("climate sensitivity") is $f_{2x}/\lambda = 3.1°$C.

The base specification's preferences follow DICE-2016R. Period utility takes the familiar power form in per-capita consumption, with elasticity of intertemporal substitution EIS. Converting a 1.5% per year utility discount rate to a per-decade rate yields:

$$\rho = (1 + 0.015)^{10} - 1 = 0.1605.$$

The policymaker seeks to maximize utilitarian welfare W:

$$W = \sum_{t=1}^{\hat{t}} \frac{L_t}{(1+\rho)^{t-1}} \frac{(c_t/L_t)^{1-1/EIS}}{1 - 1/EIS}.$$

I set $\hat{t} = 40$, implying a 400-year horizon.

In contrast to the DICE climate-economy model, abatement cost emerges endogenously within a period from the tradeoffs between fuels and evolves endogenously as technologies and resource depletion change over time. In the initial period, a tax of 1 \$/tCO_2 reduces emissions by 16%, a tax of 10 \$/tCO_2 reduces emissions by 19%, a tax of 50 \$/tCO_2 reduces emissions by 25%, and a tax of 100 \$/tCO_2 reduces emissions by 30%. In DICE-2016R, emission reductions of 25% require a tax of 59 \$/tCO_2 and emission reductions of 30% require a tax of 80 \$/tCO_2. These values are in the same ballpark as the present model even though there is nothing in the calibration that requires them to be.

In the no-policy simulations, I solve each period's equilibrium by solving for the research allocation that maximizes scientists' expected profits (using equations (4) and (7)) within a search for the resource allocation that clears the market for resources (as in equation (A-4)). For any given resource allocation, I first check whether a case with all scientists in the renewable sector generates greater expected profits in that sector than in any other. If it does, the corner allocation is an equilibrium, but if it does not, I solve for the research allocation between the coal and gas sectors conditional on no scientists working in the renewable sector. If this allocation is also not an equilibrium, I solve for the equilibrium allocation between coal and gas conditional on any number of scientists in renewables and search for the number of scientists in working in renewables that equalizes that sector's expected profit to the expected profit from the other sectors that have nonzero scientists.

When working backwards in time from the year 2015, I solve for the time t equilibrium as follows. First, I guess a time t research allocation and a time t capital stock. Then I solve for the time t incoming technology implied by this allocation and the known time $t+1$ technology. The time t technology in turn implies a time t equilibrium, which includes the time t equilibrium research allocation and implies the time $t+1$ equilibrium capital stock. I search for the time t research allocation and time t capital stock at which the implied time t equilibrium research allocation matches the guess and the implied time $t+1$ equilibrium capital stock matches the known time $t+1$ capital stock. I simulate backwards with resource depletion fixed at its year 2015 value and with the realized history of global surface temperature from Zhang et al. (2021), adjusted slightly to ensure a match with T_1. I use the fitted population growth representation from Lemoine (2021, pgs A-7–A-8) to project population backwards, and I maintain the present calibration of growth in A_{Yt} when projecting total factor productivity backwards.

To optimize policy, I search for the policy and resource use trajectories that maximize welfare while clearing the market. This is a mathematical program with equilibrium constraints, which can be quite difficult to solve. There are 12 state variables: the capital stock, the two cumulative resource use trackers, the three average technology levels, the four carbon stocks, and the two temperature variables. The key to solving the model is to convert it to a form that allows for an analytic gradient. The trick is to have the solver guess not only the trajectories of the tax and/or research subsidy but also the trajectories of the 12 state variables and the three resource use trajectories, imposing constraints that the resource markets clear in every period (equation (A-4)) and the transition equations hold in every period.[10] For any given guess, I solve for each period's equilibrium allocation of scientists using equations (4) and (7)) and the algorithm described above. At a solution, the state variables' trajectories are as if the model were simulated forward with the chosen policies.[11]

This problem is still a difficult bilevel programming problem, with the lower level programming problem often finding corner solutions (i.e., it is often true that some sector has no scientists). But this form of the problem allows for the provision of analytic gradients for the objective and constraints: we essentially have a series of static problems once we condition on the full set of state variables, because the partial derivative of the objective (and also of the constraints) with respect to any element of the solver's guess needs to account only for effects on same-period payoffs and on the same-period transition equations (observing that the partial derivative holds later states fixed because they are also elements of the solver's guess). Within those analytic gradients, I obtain the derivatives of equilibrium scientists by applying the implicit function theorem to the system of equations defined by equalized expected profits (for those sectors for which scientists are interior) and by the constraint on total scientists. I solve the model using the active-set algorithm in the Knitro solver for Matlab (Byrd et al., 2006).

[10] In the cases with the research subsidy, the solver chooses the number of clean scientists directly, with the other two types of scientists clearing their markets conditional on this choice. The level of the subsidy is implied by the resulting research allocation. At a corner allocation with all scientists in the renewable sector, I define the subsidy as the smallest value compatible with the corner allocation.

[11] In effect, the policymaker gets to simultaneously choose the trajectories of all states and all policy controls subject to constraints imposed by the market and by physical laws. If I did not impose the market constraints, then I would have the social planner's problem.

Table B-1: Parameters fixed across specifications.

Parameter	Value	Description
Market parameters		
ϵ	1.8	Elasticity of substitution in final-good production
σ	0.4	Elasticity of substitution in intermediate-good production
β_K	0.3	Factor share of capital in final-good production
β_E	0.04	Factor share of energy in final-good production
κ	0.5	Share parameter in intermediate-good production
α	0.625	Inverse of machine producers' markup
ψ_1, ψ_2, ψ_3	2.4, 0.5, 3	Resource supply elasticities
$\zeta_1, \zeta_2, \zeta_3$	1, 1, 0	Indicators for resource depletion
$Q_{1,1}, Q_{2,1}, Q_{3,1}$	0, 0, 0	Year 2015 depletion adjustment
η	1	Probability of research success
γ	1	Innovation step size
L_1	7403	Year 2015 population (millions)
L_∞	11500	Asymptotic population (millions)
g_L	0.7	Rate of approach to asymptotic population level
δ	0.6513	Depreciation rate of capital per decade
Υ	0.25	Capital savings rate
K_1	238.6	Year 2015 capital (trillion year 2014 dollars)
Welfare parameters		
ρ	0.1605	Utility discount rate per decade
EIS	1/1.45	Elasticity of intertemporal substitution
$\hat{t}$	40	Horizon (decades)
Climate parameters		
d	0.00236	Damage parameter
ξ_1, ξ_2, ξ_3	0.0250, 0.0139, 0	Emission intensity of resources (Gt C per EJ)
$\bar{e}$	37.7	Exogenous emissions per timestep (Gt C per decade)
ϕ_1	0.386	Warming delay parameter
ϕ_3	0.73	Parameter governing transfer of heat from ocean to surface
ϕ_4	0.034	Parameter governing transfer of heat from surface to ocean
f_{2x}	3.503	Forcing from doubling CO_2 (W/m^2)
λ	1.13	Forcing per degree warming ($[W/m^2]/°C$)
$\boldsymbol{M}_1$	see text	Year 2015 carbon reservoirs (Gt C)
T_1	0.85	Year 2015 surface temperature (°C, relative to 1900)
T_1^o	0.0068	Year 2015 lower ocean temperature (°C, relative to 1900)

Table B-2: Market data matched by the first period's equilibrium (2011–2020). Resources are ordered as coal, gas, renewable.

Endogenous Outcome	Target	Description
Y_1	765	Global output in trillion year 2014 dollars
$\{R_{1,1}, R_{2,1}, R_{3,1}\}$	$\{1617, 1278, 224\}$	Resource consumption in EJ
$\{p_{1,1}, p_{2,1}, p_{3,1}\}$	$\{80, 100, 150\}$	Energy prices in \$/MWh
$\{s_{1,1}, s_{2,1}, s_{3,1}\}$	$\{0.12, 0.65, 0.23\}$	Shares of research

C Additional Robustness Results

Table C-1 reports the data underlying Table 1 in the main text. The first rows in each panel of Table C-1 repeat results familiar from the main text. I here discuss the fourth through final rows in more detail than in the main text.

The fourth row delays policy by 50 years. Whereas a policymaker again uses a research subsidy to shift all scientists to the renewable sector as soon as she can, the optimal emission tax is actually less effective at redirecting scientists to the renewable sector than in the base case. The delay reduces the benefits of each type of policy by around half, but the policies' relative value is largely unchanged.

The fifth row applies a lower utility discount rate. Each policy is now nearly ten times more valuable than before because the present-day policymaker is more sensitive to future damages from warming. The level of the standalone research subsidy is unchanged because the policymaker maxed it out even in the base case, but the initial emission tax increases to \$188 per tCO_2. The magnitude of the standalone emission tax's advantage over the standalone research subsidy is now larger than in the base case, but its relative benefit is now smaller. Adding a research subsidy to an emission tax does not generate any additional value because the emission tax is large enough to switch all scientists to the renewable sector with or without the complementary research subsidy.

The sixth row considers a case in which each unit of climate change reduces output to a larger degree. The initial emission tax is now much higher, increasing from \$132 to \$261 per tCO_2. It is also insensitive to the presence of the research subsidy: when emission reduction motivations justify a tax so large as to immediately shift all researchers to the renewable sector, the policymaker does not care whether she also has access to a research subsidy or not. The emission tax is again around twice as valuable as the research subsidy, and now the optimal portfolio of the two policies provides exactly the same value as the optimal standalone emission tax.

The seventh row studies a case in which energy intermediates are more substitutable for each other, as with an improved electric grid or improved battery technology. Laissez-faire is qualitatively consistent with the base case. Because policy now more quickly shifts resource supply towards renewables, it limits warming to lower levels and provides greater value than in the base case. The increased ease of shifting resource supply narrows the wedge (as a percentage of policy value) between the emission tax and the research subsidy, and the optimal portfolio of the two policies provides exactly the same value as the optimal standalone emission tax because the standalone emission tax shifts all research to renewables in the first period.

The eighth row reports an alternate parameterization of the research process, increasing the innovation step size γ from 1 to 6 (discussed in Appendix B). The laissez-faire transition to renewable resource use occurs around a century earlier than in the base case because innovation is so much more effective (in particular, the supply expansion effect pushes researchers to renewables sooner), and a standalone research subsidy advances that by another eighty years. Renewables now dominate resource supply by midcentury whether or not the policymaker can also use an emission tax. The standalone emission tax is still more valuable than the standalone research subsidy, but the gap is narrower than in the base case. Further, the standalone emission tax does not shift researchers towards the renewable resource as effectively as in the base case. As a result, the benefits from combining the two policies are larger than in the base case.

The ninth row considers a policymaker who optimally subsidizes production of machines in order to overcome market power.[12] Correcting this additional market failure increases welfare when the policymaker can use an emission tax, but the aditional value created is only a tiny fraction of the value created by the emission tax. Further, an initial emission tax of \$122 per tCO_2 now suffices to redirect all research to the renewable sector, which eliminates the gap in value between the standalone emission tax and the portfolio of the two instruments. However, in a demonstration of the theory of the second-best (Lipsey and Lancaster, 1956), correcting the market failure in machine production actually reduces welfare when the policymaker can use only a research subsidy. Allowing the policymaker to subsidize machine production strengthens the importance of the emission tax, and this machine production subsidy is itself far less important than either the emission tax or the research subsidy.

The final row assesses the importance of resource depletion. Now a laissez-faire transition to renewable research occurs only near the end of the policy horizon and a laissez-faire transition to renewable resources happens just after the policy horizon. Relative to the base model, turning off depletion increases laissez-faire temperature in 2115 (2415) from 3.2°C (8.5°C) to 3.9°C (12.9°C). The optimal year 2015 emission tax falls from \$132 to \$74 per tCO_2. Instead of shifting 95% of scientists to the renewable sector, this tax shifts only 75% of scientists. As a result, the wedge between the value of the standalone emission tax and the standalone research subsidy is narrower than in the base model, and adding a research subsidy to the emission tax now creates more value (and substantially lowers the optimal initial emission tax, to \$13 per tCO_2).[13] However, the main story is unchanged, as the emission tax is still more valuable than the research subsidy and still provides nearly as much value as the portfolio of the two.

[12]This subsidy reduces the consumer price p_{jxit} of machines from α to α^2. It is not applied when calibrating the model. It is also not applied in laissez-faire, so the reported balanced growth equivalent benefit of policy includes the benefits of the machine subsidy.

[13]The much smaller emission tax in the absence of depletion likely reflects two factors. First, consumption per capita reaches extraordinary levels, which leads to very high long-run consumption discount rates via Ramsey discounting intuition. Second, the marginal effect of emissions on long-run warming is smaller in the absence of depletion because the "forcing" that determines warming is concave in the stock of atmospheric carbon (see Appendix B). This concavity becomes especially relevant because laissez-faire carbon dioxide increases from 394 ppm in 2015 to a staggering 8730 ppm in 2415, as opposed to "only" 2500 ppm under laissez-faire in the base case.

Table C-1: Additional results for alternate model versions.

	Policy Tools Available			
Specification	**No policy**	**Emission tax**	**Research subsidy**	**Both instruments**
Emission Tax in 2015 (\$ per tCO_2)				
Base	-	131.8	-	122.3
Resource-Saving Machines[a]	-	98.6	-	90.4
Cobb-Douglas Machines[b]	-	99	-	73.1
50-Year Delay	-	0	-	0
Less Discounting[c]	-	188.2	-	188.2
Higher Damages[d]	-	260.9	-	260.9
More Substitutable Energy Types[e]	-	117	-	117
Larger Scientific Advances[f]	-	163	-	118.3
Optimal Machine Subsidy[g]	-	121.9	-	121.8
No Depletion[h]	-	74.4	-	12.9
Renewables' Share of Resources in 2015 (%)				
Base	7.2	22.3	8.8	21.8
Resource-Saving Machines[a]	7.2	15.1	7.3	14.7
Cobb-Douglas Machines[b]	7.2	17.6	7.6	15.8
50-Year Delay	7.2	7.2	7.2	7.2
Less Discounting[c]	7.2	26.2	8.8	26.2
Higher Damages[d]	7.2	31	8.8	31
More Substitutable Energy Types[e]	7.2	28.4	10.1	28.4
Larger Scientific Advances[f]	7.2	25.3	11.5	25.3
Optimal Machine Subsidy[g]	7.2	26.8	10.5	26.8
No Depletion[h]	7.2	17.9	8.8	14.6
Renewables' Share of Scientists in 2015 (%)				
Base	22.6	95	100	100
Resource-Saving Machines[a]	23.2	38.3	100	99.8
Cobb-Douglas Machines[b]	23.3	31.7	100	100
50-Year Delay	22.6	22.6	22.6	22.6
Less Discounting[c]	22.6	100	100	100
Higher Damages[d]	23	100	100	100
More Substitutable Energy Types[e]	23.4	100	100	100
Larger Scientific Advances[f]	23.3	56.4	100	100
Optimal Machine Subsidy[g]	22.6	100	100	100
No Depletion[h]	22.6	75.1	100	100
Temperature in 2115 (°C, relative to 1900)				
Base	3.2	2.5	2.9	2.5
Resource-Saving Machines[a]	2.7	2.3	2.7	2.3
Cobb-Douglas Machines[b]	2.7	2.2	2.7	2.2
50-Year Delay	3.2	2.7	3	2.7
Less Discounting[c]	3.2	2.4	2.9	2.4
Higher Damages[d]	3.1	2.2	2.8	2.2
More Substitutable Energy Types[e]	3.7	2.3	2.7	2.3
Larger Scientific Advances[f]	3.8	2.5	2.7	2.4
Optimal Machine Subsidy[g]	3.2	2.5	2.9	2.5
No Depletion[h]	3.9	3	3.4	3

[a] σ increased from 0.4 to 1.5.
[b] σ increased from 0.4 to 1.
[c] ρ reduced from 1.5% to 0.01% per year, as in Stern (2007).
[d] Damages increased to calibration of Lemoine (2021), from survey evidence in Pindyck (2019).
[e] ϵ increased from 1.8 to 5.
[f] Innovation step size increased from $\gamma = 1$ to $\gamma = 6$.
[g] p_{jxit} reduced from α to α^2 in policy scenarios but not in laissez-faire.
[h] Each ζ_j set to zero.

D Numerical Example

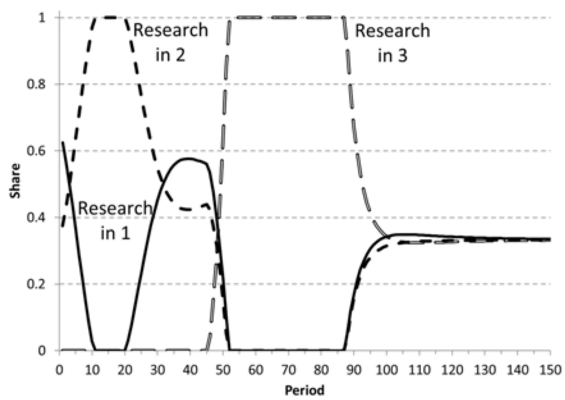
A numerical example will make the analytic results more concrete. Ignore climate damages and growth in productivity, and set $\beta_E = 1$ so that energy is the only input to final-good production (or, equivalently, capital and labor are fixed over time). Let there be three types of energy ($N = 3$), which differ only in their quality ν and in their initial technology. Let the first type of energy represent coal, the second represent oil, and the third represent gas. Looking back two hundred years, technologies for using coal were far more advanced than technologies for using oil, which in turn were more developed than technologies for using gas. I therefore fix the initial average quality of technology at 0.05 for coal, at 1% of this value for oil, and at 0.1% of this value for gas. We can think of the quality of fossil fuel resources as largely determined by the ratio of carbon to hydrogen bonds.[14] Energy derives from breaking hydrogen bonds. Fuels with a lot of carbon and little hydrogen are considered to be of lower quality because they are bulkier and more polluting. Coal is mostly carbon, oil has more hydrogen bonds per unit carbon, and natural gas has the most hydrogen bonds per unit carbon. I therefore set $\nu_1 = 0.27$ (for coal), $\nu_2 = 0.34$ (for oil), and $\nu_3 = 0.40$ (for gas).[15]

The top panels of Figure D-1 plot a case with $\sigma = 0.5$, and the lower left panel plots a case with $\sigma = 1.5$. The "coal" sector 1 begins with the majority of resource use and research activity. In the case of resource-saving technologies (bottom left), research activity and resource use are locked-in to the "coal" sector 1, which attracts all research effort in all periods and increases its share of resource use over time. In the case of resource-using technologies, we see innovation-led transitions. Research begins transitioning immediately towards the "oil" sector 2 (top left panel), and resource use eventually follows (top right panel). The "gas" sector 3 does not attract any research effort for a while and maintains a very small share of resource use even as oil displaces coal. However, after 20 periods, research effort shifts strongly towards the gas sector, and resource use shifts towards the gas sector after 60 periods. In the long run, all sectors attract identical shares of research effort and maintain stable shares of resource use, with their ordering determined by the quality ν of each resource.

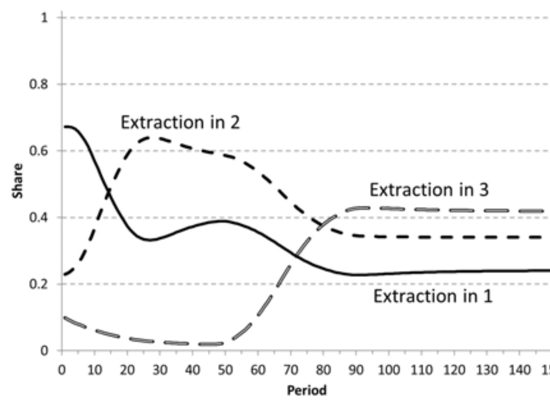
The endogenous dynamics of our setting with resource-using machines are qualitatively similar to historical patterns. The bottom right panel of Figure D-1 plots resource shares since 1800. The historical patterns in these shares are similar to the patterns that emerge from our numerical simulations with resource-using machines: resource shares change rapidly

[14] Smil (2017, 245) describes how oil is of higher quality than coal because it has higher energy density, is cleaner, and is more transportable and storable. On page 270, he writes: "There has been a clear secular shift toward higher-quality fuels, that is, from coals to crude oil and natural gas, a process that has resulted in relative decarbonization (a rising H:C ratio) of global fossil fuel extraction…"

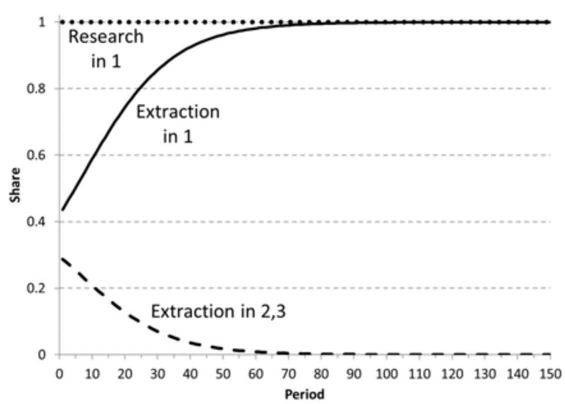
[15] The remaining parameters are $D(\cdot) = 0$, $A_{Y1} = 1$, $\epsilon = 3$, $\alpha = 0.5$, $\kappa = 0.5$, $\psi_1 = \psi_2 = \psi_3 = 3$, $\Psi_1 = \Psi_2 = \Psi_3 = 1$, $\eta = 1$, and $\gamma = 0.5$. The qualitative results are not sensitive to the choice of these parameters.



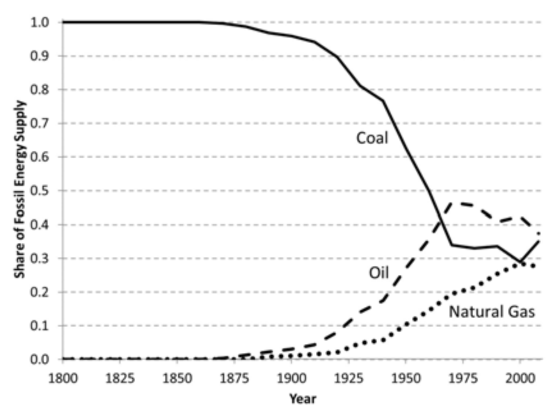
(a) Research Shares with $\sigma = 0.5$



(b) Resource Use Shares with $\sigma = 0.5$



(c) Research and Resource Use Shares with $\sigma = 1.5$



(d) Historical Resource Use Shares

Figure D-1: Top: An example of an innovation-led transition, with $\sigma = 0.5$. Bottom left: An example of lock-in, with $\sigma = 1.5$. Resources 2 and 3 have nearly identical resource use shares. Bottom right: Shares of global fossil energy supply, from Smil (2010).

as a transition occurs, and transitions do not drive formerly dominant resources out of the market. In fact, resource shares have been fairly stable since 1970. The historical patterns are nothing like the patterns that emerge from our simulations with resource-saving machines.

E Proofs and Derivations for Section 4

This appendix derives useful intermediate results before providing proofs and derivations omitted from the main text.

E.1 Tâtonnement Stability

One may be concerned that interior equilibria are not "natural" equilibria in the presence of positive feedbacks from resource use to innovation and of potential complementarities. Indeed, Acemoglu (2002) and Hart (2012) have emphasized the role of knowledge spillovers in allowing interior research allocations to be stable in the long run. This appendix shows that interior equilibria are in fact "natural" equilibria in the present setting.

Rearranging equation (12) and using $s_{jt} + s_{kt} = 1$, we obtain s_{jt} as an explicit function of $A_{j(t-1)}/A_{k(t-1)}$ and of R_{jt}/R_{kt} at an interior allocation.[16] Substituting into the versions of equation (A-4) corresponding to each resource then gives us two equations in two unknowns. This system defines the equilibrium R_{jt} and R_{kt} that clear the markets for each resource.

Define the tâtonnement adjustment process and stability as follows:

Definition E-1. *A* tâtonnement adjustment process *increases R_{jt} if equation* (A-4) *is not satisfied and its right-hand side is greater, decreases R_{jt} if equation* (A-4) *is not satisfied and its left-hand side is greater, and obeys analogous rules for R_{kt}. I say that an equilibrium (R^*_{jt}, R^*_{kt}) is* tâtonnement-stable *if and only if the tâtonnement adjustment process leads to (R^*_{jt}, R^*_{kt}) from (R_{jt}, R_{kt}) sufficiently close to (R^*_{jt}, R^*_{kt}).*

The tâtonnement process changes R_{jt} and R_{kt} so as to eliminate excess supply or demand, and tâtonnement stability requires that this adjustment process converge to an equilibrium point from values close to the equilibrium. This process is the same as that in Samuelson (1941) and Arrow and Hurwicz (1958), except expressed in quantities rather than prices. The following proposition shows that our equilibrium is tâtonnement-stable:

Proposition E-1. *The equilibrium is tâtonnement-stable.*

Proof. See Appendix E.3. □

[16] Technically, this function should be written to allow for corner solutions in the research allocation. The proof of stability will account for corner solutions.

Now use the versions of equation (A-4) corresponding to each resource to define R_{jt} and R_{kt} as functions of s_{jt},[17] and then restate equation (12) as a function only of s_{jt}:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)}}{A_{k(t-1)}} \left(\frac{A_{j(t-1)} + \eta\gamma s_{jt} A_{j(t-1)}}{A_{k(t-1)} + \eta\gamma(1-s_{jt}) A_{k(t-1)}} \right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}} \left(\frac{R_{jt}(s_{jt})}{R_{kt}(s_{jt})} \right)^{\frac{1+\sigma/\psi}{\sigma+\alpha(1-\sigma)}} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{-\sigma/\psi}{\sigma+\alpha(1-\sigma)}} . \tag{E-1}$$

The following corollary gives us the total derivative of Π_{jt}/Π_{kt} with respect to s_{jt}:

Corollary E-2. *The right-hand side of equation* (E-1) *strictly decreases in* s_{jt}.

Proof. See Appendix E.4 □

The supply expansion effect makes the relative incentive to research in sector j decline in the number of scientists working in sector j. However, when sector j's share of resource use increases in the relative quality of its technology, a positive feedback between research and resource use maintains sector j's research incentives even as more scientists move to sector j. The proof shows, as is intuitive, that whether the relative incentive to research in sector j declines in the number of scientists working in sector j is identical to whether the equilibrium is tâtonnement-stable: tâtonnement-stability is not consistent with positive feedbacks that are strong enough to overwhelm the supply expansion effect. And we have already seen that interior equilibria are in fact tâtonnement-stable.

E.2 Useful Lemmas

First, note that equations (A-2) and (7) imply

$$X_{jt} = \left[\frac{1-\kappa}{\kappa} p_{jRt} \right]^{\frac{\alpha\,\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} A_{jt}. \tag{E-2}$$

Rearranging equation (12) and using $s_{jt} + s_{kt} = 1$, we obtain s_{jt} as an explicit function of $A_{j(t-1)}/A_{k(t-1)}$ and of R_{jt}/R_{kt} at an interior allocation:

$$s_{jt}\left(\frac{R_{jt}}{R_{kt}}, \frac{A_{j(t-1)}}{A_{k(t-1)}} \right) = \frac{(1+\eta\gamma) \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}} \right]^{\sigma} - 1}{\eta\gamma + \eta\gamma \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}} \right]^{\sigma}}. \tag{E-3}$$

Let $\Sigma_{x,y}$ represent the elasticity of x with respect to y, and let $\Sigma_{x,y|z}$ represent the elasticity of x with respect to y holding z constant. The following lemma establishes signs and bounds for elasticities that will prove useful:

[17] Rearrange the versions of equation (A-4) corresponding to each resource to put all terms on the right-hand side. For given s_{jt}, the Jacobian of this system in R_{jt} and R_{kt} is negative definite.

Lemma E-3. *The following hold, with analogous results for sector k:*

1. $\Sigma_{Y_t,E_{jt}}, \Sigma_{Y_t,E_{kt}} \in [0,1]$ *and* $\Sigma_{Y_t,E_{jt}} + \Sigma_{Y_t,E_{kt}} = 1$.
2. $\Sigma_{E_{jt},R_{jt}|X_{jt}}, \Sigma_{E_{jt},X_{jt}} \in [0,1]$ *and* $\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} = 1$.
3. *If* $\sigma < 1$, *then* $\Sigma_{E_{jt},X_{jt}} \to 0$ *as* $A_{j(t-1)} \to \infty$ *and* $\Sigma_{E_{kt},X_{kt}} \to 0$ *as* $A_{k(t-1)} \to \infty$.
4. $\Sigma_{X_{jt},A_{jt}} = \frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha} \in (0,1)$
5. $\Sigma_{X_{jt},R_{jt}} = \frac{\alpha\sigma/\psi+\alpha}{\sigma(1-\alpha)+\alpha} \in (0,1]$
6. $\Sigma_{A_{jt},s_{jt}} = \frac{\eta\gamma s_{jt}}{1+\eta\gamma s_{jt}} \in [0,1)$
7. $\Sigma_{s_{jt},R_{jt}} = \frac{\psi+\sigma}{\psi}\frac{2+\eta\gamma}{\eta\gamma s_{jt}} Z_t > 0$, *where* $Z_t \in \left[\frac{1+\eta\gamma}{(2+\eta\gamma)^2}, \frac{1}{4}\right]$. $\Sigma_{s_{jt},R_{kt}} = -\Sigma_{s_{jt},R_{jt}}$.
8. $\Sigma_{s_{jt},A_{j(t-1)}} = -\frac{(1-\sigma)(1-\alpha)}{A_{j(t-1)}}\frac{(2+\eta\gamma)}{\eta\gamma} Z_t$, *which is* < 0 *if and only if* $\sigma < 1$. Z_t *is as above.* $\Sigma_{s_{jt},A_{k(t-1)}} = -\Sigma_{s_{jt},A_{j(t-1)}}$.
9. $\Sigma_{s_{jt},s_{kt}} = -s_{kt}/s_{jt} \leq 0$

Proof. Most of the results follow by differentiation and the definition of an elasticity. #1 follows from differentiating the final-good production function $Y_t(E_{jt}, E_{kt})$; #2 follows from differentiating the intermediate-good production function $E_{jt}(R_{jt}, X_{jt})$; #4 follows from differentiating equation (E-2); #5 follows from differentiating equation (E-2) after using equation (2) to substitute for p_{jRt} and using $\psi \geq \alpha/(1-\alpha)$; #6 follows from differentiating equation (5); #7 and #8 follow from differentiating equation (E-3); and #9 follows from the research constraint.

To derive #3, note that

$$\Sigma_{E_{jt},X_{jt}} = \frac{(1-\kappa)X_{jt}^{\frac{\sigma-1}{\sigma}}}{\kappa R_{jt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa)X_{jt}^{\frac{\sigma-1}{\sigma}}}.$$

From (A-1), (A-2), and (2), we have:

$$\begin{aligned} X_{jt} =& A_{jt}\left(\frac{1-\kappa}{\kappa}\left[\frac{R_{jt}}{X_{jt}}\right]^{1/\sigma}\Psi_j^{-1/\psi}R_{jt}^{1/\psi}\right)^{\frac{\alpha}{1-\alpha}} \\ =& A_{jt}\left(\frac{1-\kappa}{\kappa}\Psi_j^{-1/\psi}R_{jt}^{\frac{1}{\psi}+\frac{1}{\sigma}}\right)^{\frac{\sigma\alpha}{\sigma(1-\alpha)+\alpha}}. \end{aligned}$$

$X_{jt} \to \infty$ as $A_{j(t-1)} \to \infty$, which implies with $\sigma < 1$ that $\Sigma_{E_{jt},X_{jt}} \to 0$ as $A_{j(t-1)} \to \infty$. Analogous results hold for sector k.

To derive #7 and #8, define

$$Z_t \triangleq \frac{\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma}}{\left[1+\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma}\right]^2}$$

and recognize that $s_{jt} \in (0,1)$ implies

$$\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma} \in \left(\frac{1}{1+\eta\gamma},\ 1+\eta\gamma\right)$$

from equation (12).

□

Note that $\Sigma_{X,A}$ and $\Sigma_{X,R}$ are the same in each sector. I therefore often omit the sector subscripts on these terms.

Using $s_{jt}\left(\frac{R_{jt}}{R_{kt}}, \frac{A_{j(t-1)}}{A_{k(t-1)}}\right)$, the equilibrium is defined by the versions of equation (A-4) corresponding to each resource, which are functions only of R_{jt} and R_{kt}. Rewrite these equations as (suppressing the predetermined technology arguments in s_{jt}):

$$1 = \kappa\,\nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t\left(R_{jt}, R_{kt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}{E_{jt}\left(R_{jt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}\right]^{1/\epsilon} \left[\frac{E_{jt}\left(R_{jt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}{R_{jt}}\right]^{1/\sigma} \left[\frac{R_{jt}}{\Psi_j}\right]^{-1/\psi} \triangleq G_j(R_{jt}, R_{kt}),$$

$$1 = \kappa\,(1-\nu_j) A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t\left(R_{jt}, R_{kt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}{E_{kt}\left(R_{kt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}\right]^{1/\epsilon} \left[\frac{E_{kt}\left(R_{kt}, s_{jt}\left(R_{jt}/R_{kt}\right)\right)}{R_{kt}}\right]^{1/\sigma} \left[\frac{R_{kt}}{\Psi_k}\right]^{-1/\psi} \triangleq G_k(R_{jt}, R_{kt}).$$

We have:

Lemma E-4. *$\partial G_j(R_{jt}, R_{kt})/\partial R_{jt} < 0$ and $\partial G_k(R_{jt}, R_{kt})/\partial R_{kt} < 0$.*

Proof. Differentiating yields:

$$\begin{aligned}
\frac{\partial G_j(R_{jt}, R_{kt})}{\partial R_{jt}} =& G_j \Bigg\{ -\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\frac{1}{R_{jt}} + \left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)\frac{1}{E_{jt}}\left[\frac{\partial E_{jt}}{\partial R_{jt}} + \frac{\partial E_{jt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\right] \\
&+ \frac{1}{\epsilon}\frac{1}{Y_t}\left[\frac{\partial Y_t}{\partial E_{jt}}\frac{\partial E_{jt}}{\partial R_{jt}} + \frac{\partial Y_t}{\partial E_{jt}}\frac{\partial E_{jt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}} + \frac{\partial Y_t}{\partial E_{kt}}\frac{\partial E_{kt}}{\partial s_{kt}}\frac{\partial s_{kt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\right]\Bigg\} \\
=& \frac{G_j}{R_{jt}}\Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\left[1 - \Sigma_{E_{jt},R_{jt}|X_{jt}} - \Sigma_{E_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right] \\
&- \frac{1}{\epsilon}\Bigg[\left(1-\Sigma_{Y_t,E_{jt}}\right)\left(\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}} + \Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right) \\
&- \Sigma_{Y_t,E_{kt}}\Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg]\Bigg\}.
\end{aligned}$$

If the economy is at a corner in s_{jt}, then $\Sigma_{s_{jt},R_{jt}} = 0$ and, using Lemma E-3, the above expression is clearly negative. So consider a case with interior s_{jt}. The final two lines are negative. So the overall expression is negative if the third-to-last line is negative, which is the case if and only if

$$\begin{aligned}
0 \geq& -\frac{1}{\psi} + \frac{1}{\sigma}\left[-1 + \Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right] \\
=& -\frac{1}{\psi} + \frac{1}{\sigma}\left[-1 + \Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}}\left(\frac{\sigma+\psi}{\psi}\frac{\alpha + \sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha)+\alpha}\right)\right] \\
=& -\frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\left[-1 + \frac{\sigma+\psi}{\psi}\frac{\alpha + \sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha)+\alpha}\right],
\end{aligned} \tag{E-4}$$

where I use results from Lemma E-3. Note that $\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t \leq 3/4$, which implies that $\Sigma_{E_{jt},X_{jt}}\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha)+\alpha} < 1$. Using this, inequality (E-4) holds if and only if

$$\frac{\sigma}{\psi} \geq \Sigma_{E_{jt},X_{jt}}\frac{-1 + \frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)}}{1 - \Sigma_{E_{jt},X_{jt}}\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)}}. \tag{E-5}$$

$\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t \leq 3/4$ implies that $\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)} < 1$, which implies that the right-hand side of inequality (E-5) is negative. Thus, inequality (E-5) always holds and $\partial G_j(R_{jt}, R_{kt})/\partial R_{jt} < 0$.

The analysis of $\partial G_k(R_{jt}, R_{kt})/\partial R_{kt}$ is virtually identical.

□

Now define the matrix G:

$$G \triangleq \begin{bmatrix} \frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{kt}} \\ \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{kt}} \end{bmatrix}.$$

We have:

Lemma E-5. *The determinant of G is positive.*

Proof. Analyze $det(G)$:

$$
\begin{aligned}
det(G) \propto & \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \Bigg[\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Bigg(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{jt}}\Bigg)\Bigg]\Bigg\} \\
& \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \Bigg[\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Bigg(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{kt}}\Bigg)\Bigg]\Bigg\} \\
& + \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \Bigg[\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Bigg(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{jt}}\Bigg) \\
& \qquad - \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{jt}}\Bigg]\Bigg\} \\
& \Bigg\{ \frac{1}{\epsilon} \Bigg[\Sigma_{Y_t,E_{kt}} \Bigg(\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},R_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{kt}}\Bigg) \\
& \qquad + \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{kt}}\Bigg]\Bigg\} \\
& + \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \Bigg[\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Bigg(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{kt}}\Bigg) \\
& \qquad - \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{kt}}\Bigg]\Bigg\} \\
& \Bigg\{ \frac{1}{\epsilon} \Bigg[\Sigma_{Y_t,E_{jt}} \Bigg(\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},R_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{jt}}\Bigg) \\
& \qquad + \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{jt}}\Bigg]\Bigg\} \\
& - \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right)^2 \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} \Sigma_{A_{jt},s_{jt}} \Sigma_{s_{jt},R_{kt}} \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} \Sigma_{A_{kt},s_{kt}} \Sigma_{s_{kt},s_{jt}} \Sigma_{s_{jt},R_{jt}},
\end{aligned}
$$

where I factored $G_j G_k / R_{jt} R_{kt}$. Use $\Sigma_{Y_t,Yjt} + \Sigma_{Y_t,E_{kt}} = 1$ from Lemma E-3 and cancel terms with $1/\epsilon^2$ to obtain:

$$\begin{aligned}
det(G) \propto & \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg[1 - \Sigma_{E_{jt},R_{jt}|X_{jt}} - \Sigma_{E_{jt},X_{jt}}\Bigg(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg)\Bigg]\Bigg\} \\
& \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg[1 - \Sigma_{E_{kt},R_{kt}|X_{kt}} - \Sigma_{E_{kt},X_{kt}}\Bigg(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg)\Bigg]\Bigg\} \\
& - \frac{1}{\sigma}\left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right)\left(\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\left(\Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right) \\
& + \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg\}\frac{1}{\epsilon}\Sigma_{Y_t,E_{jt}} \\
& \quad \Bigg[- \Bigg(\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},R_{kt}} + \Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg) \\
& \qquad + \Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg] \\
& + \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg\}\frac{1}{\epsilon}\Sigma_{Y_t,E_{kt}} \\
& \quad \Bigg[- \Bigg(\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}} + \Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg) \\
& \qquad + \Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg] \\
& + \frac{1}{\epsilon}\frac{1}{\sigma}\Bigg[\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}}\Bigg(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg)\Bigg] \\
& \quad \Bigg[\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}}\Bigg(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg)\Bigg].
\end{aligned} \tag{E-6}$$

All lines after the first three are positive by results from Lemma E-3. Expanding the products in those first three lines and rearranging, those first three lines become:

$$\begin{aligned}
& \frac{1}{\psi^2} \\
& + \frac{1}{\sigma^2}\Bigg[1 - \Sigma_{X,R}\Bigg]\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\Bigg(1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg) \\
& + \frac{1}{\psi}\frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\Bigg[1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg] \\
& + \frac{1}{\psi}\frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\Bigg[1 - \Sigma_{X,R} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\Bigg] \\
& + \frac{1}{\sigma}\frac{1}{\epsilon}\left(\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\left(\Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right),
\end{aligned} \tag{E-7}$$

where I write $\Sigma_{X,R}$ because this elasticity is the same in each sector. At corner allocations of research, $\Sigma_{s_{jt},R_{jt}} = \Sigma_{s_{jt},R_{kt}} = 0$. In this case, (E-7) is clearly positive. Now assume an interior allocation of research, so that $\Pi_{jt} = \Pi_{kt}$. Note that

$$\begin{aligned}
&1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\\
=&\frac{1}{\psi}\frac{\sigma}{\sigma(1-\alpha)+\alpha}\left\{\psi[1-\alpha] - \alpha - (1-\alpha)[\sigma+\psi]\frac{(2+\eta\gamma)^2}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})}Z_t\right\}. \qquad \text{(E-8)}
\end{aligned}$$

Substituting for Z_t and using equation (12) at $\Pi_{jt}/\Pi_{kt} = 1$, we have

$$\frac{Z_t}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})} = \frac{1}{[2+\eta\gamma]^2}.$$

Equation (E-8) then becomes

$$1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} = -\frac{\sigma}{\psi}.$$

Substituting into (E-7), the first three lines of (E-6) are equal to

$$\begin{aligned}
&\frac{1}{\psi^2}\\
&-\frac{1}{\psi}\frac{1}{\sigma}\left[1-\Sigma_{X,R}\right]\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\\
&+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\left[1-\Sigma_{X,R}-\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right]\\
&+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\left[1-\Sigma_{X,R}-\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right]\\
&+\frac{1}{\sigma}\frac{1}{\epsilon}\left(\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\left(\Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right). \qquad \text{(E-9)}
\end{aligned}$$

The final line is positive. Factoring $1/\psi$, the first four lines are jointly positive if and only if:

$$\begin{aligned}
0 \leq&\frac{1}{\psi}+\frac{1}{\sigma}\Bigg[(1-\Sigma_{X,R})\left(\Sigma_{E_{jt},X_{jt}}+\Sigma_{E_{kt},X_{kt}}-\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\
&\qquad -\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}-\Sigma_{E_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Bigg]\\
=&\frac{1}{\psi}+\frac{1}{\sigma}\left(\Sigma_{E_{jt},X_{jt}}+\Sigma_{E_{kt},X_{kt}}-\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\
&-\frac{1}{\sigma}\frac{\sigma+\psi}{\psi}\frac{1}{\sigma(1-\alpha)+\alpha}\Bigg[\alpha\left(\Sigma_{E_{jt},X_{jt}}+\Sigma_{E_{kt},X_{kt}}-\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\
&\qquad +\sigma(1-\alpha)\Big(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt})+\Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\Big)\frac{1}{2+\eta\gamma}\Bigg], \qquad \text{(E-10)}
\end{aligned}$$

where we use $\frac{Z_t}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})} = \frac{1}{[2+\eta\gamma]^2}$. Note that $\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}$ increases in $\Sigma_{E_{jt},X_{jt}}$ and thus reaches a maximum at $\Sigma_{E_{jt},X_{jt}} = 1$. Therefore,

$$\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}} \leq 1 + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{kt},X_{kt}} = 1.$$

Also note that $\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})$ increases in each elasticity, and each elasticity is ≤ 1. Thus,

$$\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt}) \leq (1+\eta\gamma s_{kt}) + (1+\eta\gamma s_{jt}) = 2+\eta\gamma,$$

which implies

$$\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma} \leq 1.$$

These results together imply that

$$\begin{aligned}&\alpha + \sigma(1-\alpha)\\ \geq&\alpha\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right) + \sigma(1-\alpha)\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}.\end{aligned} \tag{E-11}$$

Using this, we have that inequality (E-10) holds if and only if

$$\begin{aligned}\frac{\sigma}{\psi} \geq&\Bigg\{-\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right) + \frac{1}{\sigma(1-\alpha)+\alpha}\Bigg[\alpha\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\ &\qquad + \sigma(1-\alpha)\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}\Bigg]\Bigg\}\\ &\Bigg\{1 - \frac{1}{\sigma(1-\alpha)+\alpha}\Bigg[\alpha\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\ &\qquad + \sigma(1-\alpha)\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}\Bigg]\Bigg\}^{-1}.\end{aligned} \tag{E-12}$$

The denominator on the right-hand side is positive via inequality (E-11). The numerator on the right-hand side is equal to:

$$\begin{aligned}&\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)\\ &\left\{-1 + \frac{1}{\sigma(1-\alpha)+\alpha}\left[\alpha + \sigma(1-\alpha)\frac{\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)}{(2+\eta\gamma)\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)}\right]\right\}.\end{aligned} \tag{E-13}$$

Consider the fraction in brackets. If that fraction is ≤ 1, then the whole expression is negative and we are done. I will now prove that the fraction cannot be > 1. Assume that the fraction is > 1. Then:

$$\left(\Sigma_{E_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{E_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right) > (2+\eta\gamma)\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\right)$$
$$\Leftrightarrow \eta\gamma s_{kt}\Sigma_{E_{jt},X_{jt}} + \eta\gamma s_{jt}\Sigma_{E_{kt},X_{kt}} \geq (1+\eta\gamma)\left(\Sigma_{E_{jt},X_{jt}} + \Sigma_{E_{kt},X_{kt}}\right) - (2+\eta\gamma)\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}.$$

Assume without loss of generality that $\Sigma_{E_{jt},X_{jt}} > \Sigma_{E_{kt},X_{kt}}$. Then the left-hand side of the last line attains its largest possible value when $s_{kt} = 1$. The inequality on the last line is then satisfied only if

$$0 > \Sigma_{E_{jt},X_{jt}} + (1+\eta\gamma)\Sigma_{E_{kt},X_{kt}} - (2+\eta\gamma)\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}. \tag{E-14}$$

The right-hand side is monotonic in $\Sigma_{E_{jt},X_{jt}}$. At $\Sigma_{E_{jt},X_{jt}} = 1$, the right-hand side is

$$1 + (1+\eta\gamma)\Sigma_{E_{kt},X_{kt}} - (2+\eta\gamma)\Sigma_{E_{kt},X_{kt}} = 1 - \Sigma_{E_{kt},X_{kt}} \geq 0.$$

But this contradicts inequality (E-14). Now consider the other extremum: $\Sigma_{E_{jt},X_{jt}} = 0$. The right-hand side of inequality (E-14) becomes:

$$(1+\eta\gamma)\Sigma_{E_{kt},X_{kt}} \geq 0,$$

which again contradicts inequality (E-14). Because the right-hand side of inequality (E-14) was monotonic in $\Sigma_{E_{jt},X_{jt}}$ and was not satisfied for either the greatest or smallest possible values for $\Sigma_{E_{jt},X_{jt}}$, the inequality is not satisfied for any values of $\Sigma_{E_{jt},X_{jt}}$. Thus, the fraction in brackets in (E-13) is ≤ 1, which means that the right-hand side of inequality (E-12) is ≤ 0 and inequality (E-12) is satisfied. As a result, the first three lines of (E-6) are positive, which means that $det(G) > 0$.

□

The next two lemmas establish how relative resource use and relative profit change with the average quality of technology in sector j:

Lemma E-6. *Define* $\mathbf{R}(A_{jt}, A_{kt}) \triangleq [R_{jt}(A_{jt}, A_{kt})/R_{kt}(A_{jt}, A_{kt})]$. *Then (i)* $\partial\mathbf{R}/\partial A_{jt} > 0$ *and (ii)* $\partial\mathbf{R}/\partial A_{jt} \to 0$ *as* $A_{jt} \to \infty$.

Proof. I begin by using the implicit function theorem on the two-dimensional system obtained from the versions of equation (A-4) corresponding to each resource. Rewriting previous expressions for G_j and G_k to hold s_{jt} fixed at some value s, the two-dimensional system becomes:

$$1 = \kappa\,\nu_j A_Y^{\frac{\epsilon-1}{\epsilon}}\left[\frac{Y_t\left(R_{jt}, R_{kt}, s_{jt}=s\right)}{E_{jt}\left(R_{jt}, s_{jt}=s\right)}\right]^{1/\epsilon}\left[\frac{E_{jt}\left(R_{jt}, s_{jt}=s\right)}{R_{jt}}\right]^{1/\sigma}\left[\frac{R_{jt}}{\Psi_j}\right]^{-1/\psi} \triangleq H_j(R_{jt}, R_{kt}; s_{jt}=s),$$
$$1 = \kappa\,(1-\nu_j) A_Y^{\frac{\epsilon-1}{\epsilon}}\left[\frac{Y_t\left(R_{jt}, R_{kt}, s_{jt}=s\right)}{E_{kt}\left(R_{kt}, s_{jt}=s\right)}\right]^{1/\epsilon}\left[\frac{E_{kt}\left(R_{kt}, s_{jt}=s\right)}{R_{kt}}\right]^{1/\sigma}\left[\frac{R_{kt}}{\Psi_k}\right]^{-1/\psi} \triangleq H_k(R_{jt}, R_{kt}; s_{jt}=s).$$

Fixing $s_{jt} = s$ makes A_{jt} a parameter. I analyze the following:

$$\begin{aligned}\frac{\partial \mathbf{R}(A_{jt}, A_{kt})}{\partial A_{jt}} =& \frac{R_{jt}}{R_{kt}}\left\{\frac{\partial R_{jt}}{\partial A_{jt}}\frac{1}{R_{jt}} - \frac{\partial R_{kt}}{\partial A_{jt}}\frac{1}{R_{kt}}\right\}\\ =& \frac{R_{jt}}{R_{kt}}\left\{\frac{1}{R_{jt}}\frac{-\frac{\partial H_j}{\partial A_{jt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{\partial H_j}{\partial R_{kt}}\frac{\partial H_k}{\partial A_{jt}}}{det(H)} - \frac{1}{R_{kt}}\frac{-\frac{\partial H_k}{\partial A_{jt}}\frac{\partial H_j}{\partial R_{jt}} + \frac{\partial H_k}{\partial R_{jt}}\frac{\partial H_j}{\partial A_{jt}}}{det(H)}\right\}\\ =& \frac{R_{jt}}{R_{kt}}\frac{1}{det(H)}\left\{-\frac{\partial H_j}{\partial A_{jt}}\left[\frac{1}{R_{jt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_k}{\partial R_{jt}}\right] + \frac{\partial H_k}{\partial A_{jt}}\left[\frac{1}{R_{jt}}\frac{\partial H_j}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_j}{\partial R_{jt}}\right]\right\}.\end{aligned} \tag{E-15}$$

Differentiation and algebraic manipulations (including applying relationships from Lemma E-3) yield:

$$-\frac{\partial H_j}{\partial A_{jt}} = -H_j\left\{\frac{1}{\sigma} - \frac{1}{\epsilon}\Sigma_{Y_t,E_{kt}}\right\}\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\frac{1}{A_{jt}},$$

$$\frac{\partial H_k}{\partial A_{jt}} = H_k\frac{1}{\epsilon}\Sigma_{Y_t,E_{jt}}\Sigma_{E_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\frac{1}{A_{jt}},$$

$$\begin{aligned}\frac{1}{R_{jt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_k}{\partial R_{jt}} =& \frac{H_k}{R_{jt}R_{kt}}\left\{-\frac{1}{\psi} - \frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\left[1 - \Sigma_{X,R}\right]\right.\\ &\left. + \frac{1}{\epsilon}\Sigma_{Y_t,E_{jt}}\left[\Sigma_{X,R} - 1\right]\left[\Sigma_{E_{jt},X_{jt}} - \Sigma_{E_{kt},X_{kt}}\right]\right\},\end{aligned}$$

$$\begin{aligned}\frac{1}{R_{jt}}\frac{\partial H_j}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_j}{\partial R_{jt}} =& \frac{H_j}{R_{jt}R_{kt}}\left\{-\frac{1}{\psi} - \frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\left[1 - \Sigma_{X,R}\right]\right.\\ &\left. + \frac{1}{\epsilon}\Sigma_{Y_t,E_{kt}}\left[\Sigma_{X,R} - 1\right]\left[\Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}}\right]\right\}.\end{aligned}$$

Using these in equation (E-15), we obtain:

$$\frac{\partial \mathbf{R}(A_{jt}, A_{kt})}{\partial A_{jt}} = \frac{1}{A_{jt}}\frac{1}{det(H)}\frac{R_{jt}}{R_{kt}}\frac{H_j H_k}{R_{jt}R_{kt}}\Sigma_{X,A}\left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right)\Sigma_{E_{jt},X_{jt}}\left(\frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}[1 - \Sigma_{X,R}]\right). \tag{E-16}$$

Now consider $det(H)$. It follows from our analysis of $det(G)$ with $\Sigma_{s,R} = 0$. Make this change in equation (E-6):

$$det(H) = \frac{H_j H_k}{R_{jt} R_{kt}} \Bigg(\Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg[1 - \Sigma_{E_{jt},R_{jt}|X_{jt}} - \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},R_{jt}}\Bigg]\Bigg\}$$
$$\Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg[1 - \Sigma_{E_{kt},R_{kt}|X_{kt}} - \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},R_{kt}}\Bigg]\Bigg\}$$
$$+ \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Bigg[- \Bigg(\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},R_{kt}} \Bigg) \Bigg]$$
$$+ \Bigg\{ -\frac{1}{\psi} - \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Bigg[- \Bigg(\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},R_{jt}} \Bigg) \Bigg]$$
$$+ \frac{1}{\epsilon}\frac{1}{\sigma}\Bigg[\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},R_{jt}}\Bigg]\Bigg[\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},R_{kt}}\Bigg]\Bigg).$$

Now analyze, using relations in Lemma E-3:

$$det(H) = \frac{H_j H_k}{R_{jt} R_{kt}} \Bigg(\Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\Bigg[1 - \Sigma_{X,R}\Bigg]\Bigg\}\Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\Bigg[1 - \Sigma_{X,R}\Bigg]\Bigg\}$$
$$+ \Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Bigg(\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X,R} \Bigg)$$
$$+ \Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Bigg(\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X,R} \Bigg)$$
$$+ \frac{1}{\epsilon}\frac{1}{\sigma}\Bigg[\Sigma_{E_{jt},R_{jt}|X_{jt}} + \Sigma_{E_{jt},X_{jt}} \Sigma_{X,R}\Bigg]\Bigg[\Sigma_{E_{kt},R_{kt}|X_{kt}} + \Sigma_{E_{kt},X_{kt}} \Sigma_{X,R}\Bigg]\Bigg)$$
$$= \frac{H_j H_k}{R_{jt} R_{kt}} \Bigg(\Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\Bigg[1 - \Sigma_{X,R}\Bigg]\Bigg\}\Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\Bigg[1 - \Sigma_{X,R}\Bigg]\Bigg\}$$
$$+ \Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Bigg[1 - \Sigma_{E_{kt},X_{kt}} (1 - \Sigma_{X,R}) \Bigg]$$
$$+ \Bigg\{ \frac{1}{\psi} + \frac{1}{\sigma}\Bigg\} \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Bigg[1 - \Sigma_{E_{jt},X_{jt}} (1 - \Sigma_{X,R}) \Bigg]$$
$$+ \frac{1}{\epsilon}\frac{1}{\sigma}\Bigg[1 - \Sigma_{E_{jt},X_{jt}} (1 - \Sigma_{X,R})\Bigg]\Bigg[1 - \Sigma_{E_{kt},X_{kt}} (1 - \Sigma_{X,R})\Bigg]\Bigg).$$

From Lemma E-3, $1 - \Sigma_{X,R} = \frac{\sigma}{\psi}\frac{\psi[1-\alpha]-\alpha}{\sigma(1-\alpha)+\alpha}$. Substituting $det(H)$ into equation (E-16), we have:

$$\begin{aligned}\frac{\partial \mathbf{R}(A_{jt},A_{kt})}{\partial A_{jt}} =& \frac{1}{A_{jt}}\frac{R_{jt}}{R_{kt}}\Sigma_{X,A}\left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)\Sigma_{E_{jt},X_{jt}}\left(\frac{1}{\psi}+\frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}[1-\Sigma_{X,R}]\right)\\ &\left(\left\{\frac{1}{\psi}+\frac{1}{\sigma}\Sigma_{E_{jt},X_{jt}}\left[1-\Sigma_{X,R}\right]\right\}\left\{\frac{1}{\psi}+\frac{1}{\sigma}\Sigma_{E_{kt},X_{kt}}\left[1-\Sigma_{X,R}\right]\right\}\right.\\ &+\left\{\frac{1}{\psi}+\frac{1}{\sigma}\right\}\frac{1}{\epsilon}\Sigma_{Y_t,E_{jt}}\left[1-\Sigma_{E_{kt},X_{kt}}(1-\Sigma_{X,R})\right]\\ &+\left\{\frac{1}{\psi}+\frac{1}{\sigma}\right\}\frac{1}{\epsilon}\Sigma_{Y_t,E_{kt}}\left[1-\Sigma_{E_{jt},X_{jt}}(1-\Sigma_{X,R})\right]\\ &\left.+\frac{1}{\epsilon}\frac{1}{\sigma}\left[1-\Sigma_{E_{jt},X_{jt}}(1-\Sigma_{X,R})\right]\left[1-\Sigma_{E_{kt},X_{kt}}(1-\Sigma_{X,R})\right]\right)^{-1} \\ >&0.\end{aligned} \tag{E-17}$$

We have established the first part of the lemma. To establish the second part, use Lemma E-3 in equation (E-17).

□

Lemma E-7. *Fix $s_{jt} = s$. If $\sigma > 1$ or σ is not too much smaller than 1, then Π_{jt}/Π_{kt} increases in $A_{j(t-1)}$. As $A_{j(t-1)} \to \infty$, Π_{jt}/Π_{kt} decreases in $A_{j(t-1)}$ for all $\sigma < 1$.*

Proof. To a first-order approximation, we have, with s_{jt} fixed at s,

$$\begin{aligned}&\frac{\mathrm{d}\ln[\Pi_{jt}/\Pi_{kt}]}{\mathrm{d}A_{j(t-1)}}\\ \approx&\frac{1}{A_{j(t-1)}}\left[1-\frac{1}{\sigma+\alpha(1-\sigma)}\right]+\frac{1+\sigma/\psi}{\sigma+\alpha(1-\sigma)}\frac{\partial A_{jt}}{\partial A_{j(t-1)}}\frac{\partial[R_{jt}/R_{kt}]}{\partial A_{jt}}\frac{R_{kt}}{R_{jt}}\\ =&\frac{1}{A_{j(t-1)}}\left[1-\frac{1}{\sigma+\alpha(1-\sigma)}\right]+\frac{1}{\psi}\frac{\psi+\sigma}{\sigma+\alpha(1-\sigma)}(1+\eta\gamma s)\frac{\partial[R_{jt}/R_{kt}]}{\partial A_{jt}}\frac{R_{kt}}{R_{jt}}\\ =&\frac{1}{A_{j(t-1)}}\frac{(1-\alpha)(\sigma-1)}{\sigma+\alpha(1-\sigma)}+\frac{1}{\psi}\frac{\psi+\sigma}{\sigma+\alpha(1-\sigma)}(1+\eta\gamma s)\frac{\partial[R_{jt}/R_{kt}]}{\partial A_{jt}}\frac{R_{kt}}{R_{jt}}.\end{aligned}$$

The first term is positive if and only if $\sigma > 1$ and, using Lemma E-6, the second term is positive. Therefore the whole expression is positive if $\sigma > 1$. The first term becomes small for σ close to 1. Therefore the second term dominates (and the whole expression is positive) for σ not too much smaller than 1. Finally, Lemma E-6 shows that the second term goes to 0 as $A_{j(t-1)} \to \infty$ if $\sigma < 1$. Therefore the whole expression is negative if $\sigma < 1$ and $A_{j(t-1)} \to \infty$.

□

Finally, consider the evolution of relative resource use and thus of market size and resource cost effects. From equation (13), R_{jt}/R_{kt} increases in s_{jt}. Define $\hat{s}_{t+1}$ as the unique value of $s_{j(t+1)}$ such that sector j's share of resource resource use increases from time t to $t+1$ if and only if $s_{j(t+1)} \geq \hat{s}_{t+1}$. Lemma E-6 implies that $\hat{s}_{t+1} \in (0,1)$.

Lemma E-8. *If $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if $A_{j(t-1)}/A_{k(t-1)} \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$. If $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if $A_{j(t-1)}/A_{k(t-1)} \leq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$.*

Proof. The change in R_{jt}/R_{kt} from time t to $t+1$ is

$$\begin{aligned}\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}} =& \frac{(R_{j(t+1)} - R_{jt})R_{kt} - (R_{k(t+1)} - R_{kt})R_{jt}}{R_{k(t+1)}R_{kt}} \\ \propto& \frac{R_{j(t+1)} - R_{jt}}{R_{jt}} - \frac{R_{k(t+1)} - R_{kt}}{R_{kt}},\end{aligned}$$

where the first equality adds and subtracts $R_{jt}R_{kt}$ in the numerator and the second line factors $R_{jt}/R_{k(t+1)}$. To a first-order approximation, this is proportional to

$$\frac{1}{R_{jt}}\left(\frac{\mathrm{d}R_{jt}}{\mathrm{d}A_{jt}}\left[A_{j(t+1)} - A_{jt}\right] + \frac{\mathrm{d}R_{jt}}{\mathrm{d}A_{kt}}\left[A_{k(t+1)} - A_{kt}\right]\right) - \frac{1}{R_{kt}}\left(\frac{\mathrm{d}R_{kt}}{\mathrm{d}A_{jt}}\left[A_{j(t+1)} - A_{jt}\right] + \frac{\mathrm{d}R_{kt}}{\mathrm{d}A_{kt}}\left[A_{k(t+1)} - A_{kt}\right]\right),$$

with the derivatives evaluated at the time t allocation. Note that s_{jt} is included in A_{jt} when differentiating with respect to A_{jt}, which reflects that we will seek the allocation of scientists that holds R_{jt}/R_{kt} constant. Defining $H_j(R_{jt}, R_{kt}; s_{jt} = s)$ and $H_k(R_{jt}, R_{kt}; s_{jt} = s)$ as in the proof of Lemma E-6 and using the implicit function theorem, the previous expression becomes:

$$\begin{aligned}&\frac{1}{R_{jt}}\left(\frac{-\frac{\partial H_j}{\partial A_{jt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{\partial H_j}{\partial R_{kt}}\frac{\partial H_k}{\partial A_{jt}}}{det(H)}\left[A_{j(t+1)} - A_{jt}\right] + \frac{-\frac{\partial H_j}{\partial A_{kt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{\partial H_j}{\partial R_{kt}}\frac{\partial H_k}{\partial A_{kt}}}{det(H)}\left[A_{k(t+1)} - A_{kt}\right]\right) \\ &- \frac{1}{R_{kt}}\left(\frac{-\frac{\partial H_k}{\partial A_{jt}}\frac{\partial H_j}{\partial R_{jt}} + \frac{\partial H_k}{\partial R_{jt}}\frac{\partial H_j}{\partial A_{jt}}}{det(H)}\left[A_{j(t+1)} - A_{jt}\right] + \frac{-\frac{\partial H_k}{\partial A_{kt}}\frac{\partial H_j}{\partial R_{jt}} + \frac{\partial H_k}{\partial R_{jt}}\frac{\partial H_j}{\partial A_{kt}}}{det(H)}\left[A_{k(t+1)} - A_{kt}\right]\right) \\ \propto& \left[-\frac{\partial H_j}{\partial A_{jt}} s_{j(t+1)} A_{jt} - \frac{\partial H_j}{\partial A_{kt}} s_{k(t+1)} A_{kt}\right]\left[\frac{1}{R_{jt}}\frac{\partial H_k}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_k}{\partial R_{jt}}\right] \\ &+ \left[\frac{\partial H_k}{\partial A_{jt}} s_{j(t+1)} A_{jt} + \frac{\partial H_k}{\partial A_{kt}} s_{k(t+1)} A_{kt}\right]\left[\frac{1}{R_{jt}}\frac{\partial H_j}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial H_j}{\partial R_{jt}}\right],\end{aligned} \tag{E-18}$$

where the second expression factors $\eta\gamma/det(H)$, which is readily seen to be positive by altering the proof of Lemma E-5 to set the $\Sigma_{s,R}$ terms to zero. Differentiation and algebraic manipulations (including applying relationships from Lemma E-3) yield:

$$-\frac{\partial H_j}{\partial A_{jt}} s_{j(t+1)} A_{jt} - \frac{\partial H_j}{\partial A_{kt}} s_{k(t+1)} A_{kt} = - H_j \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \right\} \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} s_{j(t+1)} \\ - H_j \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} (1 - s_{j(t+1)}),$$

$$\frac{\partial H_k}{\partial A_{jt}} s_{j(t+1)} A_{jt} + \frac{\partial H_k}{\partial A_{kt}} s_{k(t+1)} A_{kt} = H_k \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \right\} \Sigma_{E_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} (1 - s_{j(t+1)}) \\ + H_k \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} s_{j(t+1)}.$$

Substitute these and expressions derived in the proof of Lemma E-6 into (E-18) and factor

$\Sigma_{X,A} H_j H_k / [R_{jt} R_{kt}]$:

$$
\begin{aligned}
&\left\{ - s_{j(t+1)} \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \right\} \Sigma_{E_{jt},X_{jt}} - (1 - s_{j(t+1)}) \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \right\} \\
&\qquad \left\{ - \frac{1}{\psi} - \frac{1}{\sigma} \Sigma_{E_{kt},X_{kt}} \left[1 - \Sigma_{X,R} \right] \right\} \\
&+ \left\{ (1 - s_{j(t+1)}) \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \right\} \Sigma_{E_{kt},X_{kt}} + s_{j(t+1)} \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} \right\} \\
&\qquad \left\{ - \frac{1}{\psi} - \frac{1}{\sigma} \Sigma_{E_{jt},X_{jt}} \left(1 - \Sigma_{X,R} \right) \right\} \\
&+ \frac{1}{\epsilon} \left[1 - \Sigma_{X,R} \right] \left[\Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}} \right] \\
&\qquad \left\{ - s_{j(t+1)} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \right] \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} - (1 - s_{j(t+1)}) \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \right\} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \right\} \\
&- \frac{1}{\epsilon^2} \Sigma_{Y_t,E_{jt}} \Sigma_{Y_t,E_{kt}} \left[1 - \Sigma_{X,R} \right] \left[\Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}} \right] \left\{ (1 - s_{j(t+1)}) \Sigma_{E_{kt},X_{kt}} + s_{j(t+1)} \Sigma_{E_{jt},X_{jt}} \right\} \\
=& s_{j(t+1)} \Sigma_{E_{jt},X_{jt}} \left\{ \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \right] \right. \\
&\qquad \left. + \frac{1}{\sigma} \left(1 - \Sigma_{X,R} \right) \left[\frac{1}{\sigma} \Sigma_{E_{kt},X_{kt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} \right] \right\} \\
&- (1 - s_{j(t+1)}) \Sigma_{E_{kt},X_{kt}} \left\{ \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \right] \right. \\
&\qquad \left. + \frac{1}{\sigma} \left(1 - \Sigma_{X,R} \right) \left[\frac{1}{\sigma} \Sigma_{E_{jt},X_{jt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \right] \right\} \\
&+ \frac{1}{\epsilon} \left[1 - \Sigma_{X,R} \right] \left[\Sigma_{E_{kt},X_{kt}} - \Sigma_{E_{jt},X_{jt}} \right] \\
&\qquad \left\{ - s_{j(t+1)} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{kt}} \right] \Sigma_{Y_t,E_{jt}} \Sigma_{E_{jt},X_{jt}} - (1 - s_{j(t+1)}) \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \right\} \Sigma_{Y_t,E_{kt}} \Sigma_{E_{kt},X_{kt}} \right. \\
&\qquad \left. - \frac{1}{\epsilon} \Sigma_{Y_t,E_{jt}} \Sigma_{Y_t,E_{kt}} \left[(1 - s_{j(t+1)}) \Sigma_{E_{kt},X_{kt}} + s_{j(t+1)} \Sigma_{E_{jt},X_{jt}} \right] \right\}
\end{aligned}
$$

$$
\begin{aligned}
=& s_{j(t+1)}\Sigma_{E_{jt},X_{jt}}\left\{\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\right]+\frac{1}{\sigma}\left(1-\Sigma_{X,R}\right)\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,E_{kt}}\right]\Sigma_{E_{kt},X_{kt}}\right\}\\
&-(1-s_{j(t+1)})\Sigma_{E_{kt},X_{kt}}\left\{\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\right]+\frac{1}{\sigma}\left(1-\Sigma_{X,R}\right)\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,E_{jt}}\right]\Sigma_{E_{jt},X_{jt}}\right\}\\
&-s_{j(t+1)}\frac{1}{\sigma}\frac{1}{\epsilon}\left[1-\Sigma_{X,R}\right]\Sigma_{Y_t,E_{jt}}\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}+(1-s_{j(t+1)})\frac{1}{\sigma}\frac{1}{\epsilon}\left[1-\Sigma_{X,R}\right]\Sigma_{Y_t,E_{kt}}\Sigma_{E_{kt},X_{kt}}\Sigma_{E_{jt},X_{jt}}\\
=&\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\right]\left[s_{j(t+1)}\Sigma_{E_{jt},X_{jt}}-(1-s_{j(t+1)})\Sigma_{E_{kt},X_{kt}}\right]\\
&+\frac{1}{\sigma^2}\left(1-\Sigma_{X,R}\right)\Sigma_{E_{kt},X_{kt}}\Sigma_{E_{jt},X_{jt}}\left(2s_{j(t+1)}-1\right)-\frac{1}{\sigma}\frac{1}{\epsilon}\left(1-\Sigma_{X,R}\right)\Sigma_{E_{jt},X_{jt}}\Sigma_{E_{kt},X_{kt}}\left(2s_{j(t+1)}-1\right)\\
=&\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\right]\left[s_{j(t+1)}\Sigma_{E_{jt},X_{jt}}-(1-s_{j(t+1)})\Sigma_{E_{kt},X_{kt}}\right]+\frac{1}{\sigma}\left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)\left(1-\Sigma_{X,R}\right)\Sigma_{E_{kt},X_{kt}}\Sigma_{E_{jt},X_{jt}}\left(2s_{j(t+1)}-1\right)
\end{aligned}
$$

Substituting for $\Sigma_{X,R}$ and rearranging, we obtain

$$
\begin{aligned}
\frac{1}{\psi}\left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)\Bigg[&s_{j(t+1)}\Sigma_{E_{jt},X_{jt}}\left(1+\frac{\psi[1-\alpha]-\alpha}{\sigma(1-\alpha)+\alpha}\Sigma_{E_{kt},X_{kt}}\right)\\
&-(1-s_{j(t+1)})\Sigma_{E_{kt},X_{kt}}\left(1+\frac{\psi[1-\alpha]-\alpha}{\sigma(1-\alpha)+\alpha}\Sigma_{E_{jt},X_{jt}}\right)\Bigg].
\end{aligned}
\tag{E-19}
$$

This expression is positive if and only if the term in brackets is positive. Define $\hat{s}_{t+1}$ as the $s_{j(t+1)}$ such that $R_{jt}/R_{kt} = R_{j(t+1)}/R_{k(t+1)}$. Then $\hat{s}_{t+1}$ is the root of the term in brackets. Solving for that root, we have:

$$
\hat{s}_{t+1}=\frac{\Sigma_{E_{kt},X_{kt}}C_{jt}}{\Sigma_{E_{jt},X_{jt}}C_{kt}+\Sigma_{E_{kt},X_{kt}}C_{jt}},
\tag{E-20}
$$

where $\Sigma_{w,z}$ is the elasticity of w with respect to z and where

$$
\begin{aligned}
C_{jt}&\triangleq 1+\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}\left[\psi-\frac{\alpha}{1-\alpha}\right]\Sigma_{E_{jt},X_{jt}}>0,\\
C_{kt}&\triangleq 1+\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}\left[\psi-\frac{\alpha}{1-\alpha}\right]\Sigma_{E_{kt},X_{kt}}>0.
\end{aligned}
$$

Thus,

$$
\left\{\hat{s}_{t+1}\geq\frac{1}{2}\right\}\Leftrightarrow\left\{\Sigma_{E_{kt},X_{kt}}\geq\Sigma_{E_{jt},X_{jt}}\right\},
$$

where the right-hand side is evaluated at $\hat{s}_{t+1}$. Using the explicit expressions for the elasticities, for intermediate-good production, and for X_{jt} and X_{kt} (see equation (E-2)), we have:

$$\begin{aligned}
\Sigma_{E_{kt},X_{kt}} \geq & \Sigma_{E_{jt},X_{jt}} \\
\Leftrightarrow 0 \leq & \frac{(1-\kappa) X_{kt}^{\frac{\sigma-1}{\sigma}} E_{jt}^{\frac{\sigma-1}{\sigma}} - (1-\kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} E_{kt}^{\frac{\sigma-1}{\sigma}}}{E_{kt}^{\frac{\sigma-1}{\sigma}} E_{jt}^{\frac{\sigma-1}{\sigma}}} \qquad \text{(E-21)} \\
\Leftrightarrow 0 \leq & X_{kt}^{\frac{\sigma-1}{\sigma}} E_{jt}^{\frac{\sigma-1}{\sigma}} - X_{jt}^{\frac{\sigma-1}{\sigma}} E_{kt}^{\frac{\sigma-1}{\sigma}} \\
\Leftrightarrow 0 \leq & \kappa R_{jt}^{\frac{\sigma-1}{\sigma}} X_{kt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} X_{kt}^{\frac{\sigma-1}{\sigma}} - \kappa R_{kt}^{\frac{\sigma-1}{\sigma}} X_{jt}^{\frac{\sigma-1}{\sigma}} - (1-\kappa) X_{kt}^{\frac{\sigma-1}{\sigma}} X_{jt}^{\frac{\sigma-1}{\sigma}} \\
\Leftrightarrow 1 \leq & \left(\frac{R_{jt} \left[\frac{1-\kappa}{\kappa} \left(\frac{R_{kt}}{\Psi_k} \right)^{1/\psi} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{kt}}{A_{kt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} A_{kt}}{R_{kt} \left[\frac{1-\kappa}{\kappa} \left(\frac{R_{jt}}{\Psi_j} \right)^{1/\psi} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} A_{jt}} \right)^{\frac{\sigma-1}{\sigma}} \\
\Leftrightarrow 1 \leq & \left[\left(\frac{\Psi_j}{\Psi_k} \right)^{\frac{\alpha\sigma/\psi}{\sigma(1-\alpha)+\alpha}} \left(\frac{R_{jt}}{R_{kt}} \right)^{\frac{\sigma(1-\alpha-\alpha/\psi)}{\sigma(1-\alpha)+\alpha}} \left(\frac{A_{kt}}{A_{jt}} \right)^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \right]^{\frac{\sigma-1}{\sigma}} \\
\Leftrightarrow 1 \leq & \left(\frac{\Psi_j}{\Psi_k} \right)^{\chi \frac{1}{\psi}[\alpha+\sigma(1-\alpha)]} \left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}} \right)^{-\chi \frac{1}{\psi}[\alpha+\sigma(1-\alpha)]} \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{\chi(1-\alpha)[(1-\sigma)(1-\alpha-\alpha/\psi)-(1+\sigma/\psi)]}, \qquad \text{(E-22)}
\end{aligned}$$

where the final line substitutes for R_{jt}/R_{kt} from equation (12) (which must hold for $\hat{s}_{t+1}$ interior) and where

$$\chi \triangleq \frac{\sigma - 1}{[\sigma(1-\alpha)+\alpha][1+\sigma/\psi]} < 0 \text{ iff } \sigma < 1.$$

The right-hand side of inequality (E-22) is increasing in s_{jt} if and only if $\sigma < 1$. Therefore, if $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if the strict version of the inequality does not hold at $s_{jt} = 0.5$, and if $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if the inequality holds at $s_{jt} = 0.5$. If $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} \geq \left[\frac{\Psi_j}{\Psi_k} \right]^{\theta},$$

and if $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} \leq \left[\frac{\Psi_j}{\Psi_k} \right]^{\theta},$$

where

$$\theta \triangleq \frac{-\frac{1}{\psi}[\alpha+\sigma(1-\alpha)]}{(1-\alpha)[(1-\sigma)(1-\alpha-\alpha/\psi)-(1+\sigma/\psi)]} = \frac{1}{(1-\alpha)(1+\psi)} > 0.$$

□

E.3 Proof of Proposition E-1

The tâtonnement adjustment process generates, to constants of proportionality, the following system for finding the equilibrium within period t:

$$\begin{aligned}\dot{R}_{jt} =& h\Big(G_j(R_{jt}, R_{kt}) - 1\Big),\\ \dot{R}_{kt} =& h\Big(G_k(R_{jt}, R_{kt}) - 1\Big),\end{aligned}$$

where dots indicate time derivatives (with the fictional time for finding an equilibrium here flowing within a period t), $h(0) = 0$, and $h'(\cdot) > 0$. The system's steady state occurs at the equilibrium values, which I denote with stars. Linearizing around the steady state, we have

$$\begin{bmatrix}\dot{R}_{jt}\\ \dot{R}_{kt}\end{bmatrix} \approx h'(0)\begin{bmatrix}\frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{kt}}\\ \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{kt}}\end{bmatrix}\begin{bmatrix}R_{jt}-R^*_{jt}\\ R_{kt}-R^*_{kt}\end{bmatrix} = h'(0)\, G\begin{bmatrix}R_{jt}-R^*_{jt}\\ R_{kt}-R^*_{kt}\end{bmatrix},$$

where G is the 2×2 matrix of derivatives, each evaluated at (R^*_{jt}, R^*_{kt}). Lemma E-4 implies that the trace of G is strictly negative, in which case at least one of the two eigenvalues must be strictly negative. Lemma E-5 shows that $det(G) > 0$, which means that both eigenvalues must have the same sign. Therefore both eigenvalues are strictly negative. The linearized system is therefore globally asymptotically stable, and, by Lyapunov's Theorem of the First Approximation, the full nonlinear system is locally asymptotically stable around the equilibrium.

E.4 Proof of Corollary E-2

Now treat the versions of equation (A-4) corresponding to each resource as functions of R_{jt}, R_{kt}, and s_{jt} (recognizing that $s_{kt} = 1 - s_{jt}$):

$$\begin{aligned}1 = \kappa\,\nu_j A_Y^{\frac{\epsilon-1}{\epsilon}}\left[\frac{Y_t(R_{jt},R_{kt},s_{jt})}{E_{jt}(R_{jt},s_{jt})}\right]^{1/\epsilon}\left[\frac{E_{jt}(R_{jt},s_{jt})}{R_{jt}}\right]^{1/\sigma}\left[\frac{R_{jt}}{\Psi_j}\right]^{-1/\psi} &\triangleq \hat{G}_j(R_{jt},R_{kt};s_{jt}),\\ 1 = \kappa\,(1-\nu_j) A_Y^{\frac{\epsilon-1}{\epsilon}}\left[\frac{Y_t(R_{jt},R_{kt},s_{jt})}{E_{kt}(R_{kt},s_{jt})}\right]^{1/\epsilon}\left[\frac{E_{kt}(R_{kt},s_{jt})}{R_{kt}}\right]^{1/\sigma}\left[\frac{R_{kt}}{\Psi_k}\right]^{-1/\psi} &\triangleq \hat{G}_k(R_{jt},R_{kt};s_{jt}).\end{aligned}$$

This system of equations implicitly defines R_{jt} and R_{kt} as functions of the parameter s_{jt}. Define the matrix $\hat{G}$ analogously to the matrix G. Using the implicit function theorem, we have

$$\frac{\partial R_{jt}}{\partial s_{jt}} = \frac{-\frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}}{det(\hat{G})} \quad \text{and} \quad \frac{\partial R_{kt}}{\partial s_{jt}} = \frac{-\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial \hat{G}_k}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}}}{det(\hat{G})}.$$

Interpreting equation (12) as implicitly defining s_{jt} as a function of R_{jt} and R_{kt}, we have:

$$\frac{\partial s_{jt}}{\partial R_{jt}} = -\frac{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}}}{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}} \quad \text{and} \quad \frac{\partial s_{jt}}{\partial R_{kt}} = -\frac{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}}}{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}},$$

and thus

$$\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}} = -\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}} \quad \text{and} \quad \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}} = -\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}.$$

Using these expressions, consider how the right-hand side of equation (E-1) changes in s_{jt}:

$$\begin{aligned}
\frac{\mathrm{d}[\Pi_{jt}/\Pi_{kt}]}{\mathrm{d}s_{jt}} =& \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} + \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}}\frac{\partial R_{jt}}{\partial s_{jt}} + \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}}\frac{\partial R_{kt}}{\partial s_{jt}} \\
=& \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} \\
& - \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\frac{-\frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}}{det(\hat{G})} - \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}\frac{-\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial \hat{G}_k}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}}}{det(\hat{G})} \\
\propto& - \frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} \\
& - \frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}} - \frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}} \\
=& - \left(\frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}\right) \\
& + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} + \frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}} \\
=& - det(G).
\end{aligned}$$

The third expression factored $det(\hat{G})$, which is positive by the proof of Proposition E-1 for a corner solution in s_{jt}, and it also factored $\partial[\Pi_{jt}/\Pi_{kt}]/\partial s_{jt}$, which is negative. The final equality recognizes that the only difference between the equations with a hat and the equations without a hat are that the equations without a hat allow s_{jt} to vary with R_{jt} and R_{kt}. Lemma E-5 showed that $det(G) > 0$. Thus the right-hand side of equation (E-1) strictly decreases in s_{jt}.

E.5 Proof of Lemma 1

Under the given assumption that $\nu = 0.5$ and $\Psi_j = \Psi_k$, we have $R_{jt} = R_{kt}$ when $A_{j(t-1)} = A_{k(t-1)}$ and $s_{jt} = 0.5$. Therefore, it is easy to see that $\Pi_{jt}/\Pi_{kt} = 1$ at $s_{jt} = 0.5$ when $A_{j(t-1)} = A_{k(t-1)}$. By Lemma E-7, increasing $A_{j(t-1)}$ increases Π_{jt}/Π_{kt} if either $\sigma > 1$ or σ is not too much smaller than 1. In those cases, Corollary E-2 gives us that $A_{j(t-1)} > A_{k(t-1)}$ implies $s^*_{jt} > 0.5$. The lemma follows from observing that $A_{j(t-1)} > A_{k(t-1)}$ and $\Psi_j = \Psi_k$ imply that $A_{j(t-1)}/A_{k(t-1)} > (\Psi_j/\Psi_k)^{1/[(1-\alpha)(1+\psi)]}$.

E.6 Proof of Proposition 2

To start, let Assumption 1 hold. From Lemma E-8, $\hat{s}_{t+1} < 0.5$. Therefore $s_{jt_0} > \hat{s}_{t+1}$. Assume that $s_{j(t_0+1)} < s_{jt_0}$. From equation (12), $\Pi_{j(t_0+1)}/\Pi_{k(t_0+1)}$ increases in A_{jt_0}/A_{kt_0} for any given $s_{j(t_0+1)}$ if $\sigma > 1$. Therefore, for the equilibrium to have $s_{j(t_0+1)} < s_{jt_0}$, it must be true that $R_{jt_0}/R_{kt_0} > R_{j(t_0+1)}/R_{k(t_0+1)}$ and thus $s_{j(t_0+1)} < \hat{s}_{t_0+1}$. From Corollary E-2 and $s_{jt_0} > \hat{s}_{t_0+1}$, it must be true that $\Pi_{jt_0}/\Pi_{kt_0} > 1$ when evaluated at $\hat{s}_{t_0+1}$. Because $R_{jt_0}/R_{kt_0} = R_{j(t_0+1)}/R_{k(t_0+1)}$ if $s_{j(t_0+1)} = \hat{s}_{t_0+1}$ and $A_{jt_0}/A_{kt_0} > A_{j(t_0-1)}/A_{k(t_0-1)}$ by $s_{jt_0} > 0.5$, it therefore must be true that $\Pi_{j(t_0+1)}/\Pi_{k(t_0+1)} > 1$ when evaluated at $\hat{s}_{t_0+1}$. By Corollary E-2, it then must be true that $s_{j(t_0+1)} > \hat{s}_{t_0+1}$. We have a contradiction. It must be true that $s_{j(t_0+1)} \geq s_{jt_0}$.

Because $s_{j(t_0+1)} \geq s_{jt_0} > 0.5 > \hat{s}_{t+1}$, it follows that $R_{jt_0}/R_{kt_0} \leq R_{j(t_0+1)}/R_{k(t_0+1)}$ and $A_{jt_0}/A_{kt_0} > A_{j(t_0-1)}/A_{k(t_0-1)}$. Therefore Assumption 1 still holds at time $t_0 + 1$. Proceeding by induction, sector j's shares of research and resource use increase forever: resource j is locked-in from time t_0 if $\sigma > 1$ and Assumption 1 holds at time t_0. We have established the first part of the proposition.

Now consider the remaining parts of the proposition, no longer imposing Assumption 1. We know that $\Pi^*_{jt}/\Pi^*_{kt} = 1$ when $s^*_{jt} \in (0, 1)$. Assume that $s^*_{jt} \in (0.5, 1)$. By Lemma E-7, $\Pi_{j(t+1)}/\Pi_{k(t+1)} > 1$ when evaluated at s^*_{jt}. Therefore, by Corollary E-2, $s^*_{j(t+1)} > s^*_{jt}$. Analogous arguments apply when $s^*_{jt} \in (0, 0.5)$. We have established the second part of the proposition.

By the foregoing, the only possible steady states are at $s^*_{jt} = 0.5$, $s^*_{jt} = 0$, and $s^*_{jt} = 1$. We just saw that a steady state at $s^*_{jt} = 0.5$ cannot be stable (should it even exist). When $s^*_{jt} = 1$, only $A_{j(t-1)}$ changes over time, increasing by $\eta\gamma A_{j(t-1)}$ at each time t. By Lemma E-7, $\Pi_{j(t_0+1)}/\Pi_{k(t_0+1)} > \Pi_{jt_0}/\Pi_{kt_0}$ if $s_{j(t_0+1)} \geq s_{jt_0}$. If $s_{jt_0} = 1$, then $\Pi_{jt_0} > \Pi_{kt_0}$, in which case $\Pi_{j(t_0+1)} > \Pi_{k(t_0+1)}$ if $s_{j(t_0+1)} = s_{jt_0}$. It is then an equilibrium for s^*_{jt} to equal 1 for all $t \geq t_0$. An analogous proof covers the case where $s^*_{jt} = 0$.

E.7 Proof of Proposition 3

First consider whether a corner allocation can persist indefinitely. If $s^*_{jt} = 1$ for all $t \geq t_0$, then $A_{j(t-1)} \to \infty$ as $t \to \infty$ and, by Lemma E-6, R_{jt}/R_{kt} goes to a constant. In that case, from equation (12), Π_{jt}/Π_{kt} goes to zero for all s_{jt}. But Π_{jt}/Π_{kt} cannot be zero if $s^*_{jt} = 1$ because $s^*_{jt} = 1$ implies that $\Pi_{jt}/\Pi_{kt} \geq 1$. We have contradicted the assumption that $s^*_{jt} = 1$ for all $t \geq t_0$. Analogous arguments show that it cannot be true that $s^*_{kt} = 1$ for all $t \geq t_0$. It therefore must be true that, for all t_0, there exists some $t > t_0$ such that $s^*_{jt} \in (0, 1)$.

Because a corner research allocation cannot persist indefinitely, A_{jt} and A_{kt} both become arbitrarily large as t becomes large. From equations (A-2), (7), and (2), we have

$$\begin{aligned} X_{jt} &= \left\{ \left[\left(\frac{R_{jt}}{\Psi_j} \right)^{1/\psi} \frac{1-\kappa}{\kappa} \right]^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}} \right\}^{\frac{\alpha}{1-\alpha}} A_{jt} \\ &= \left[\Psi_j^{-1/\psi} \frac{1-\kappa}{\kappa} \right]^{\frac{\sigma\alpha}{\sigma(1-\alpha)+\alpha}} A_{jt}^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} R_{jt}^{\frac{\alpha(1+\sigma/\psi)}{\sigma(1-\alpha)+\alpha}}. \end{aligned}$$

X_{jt} and X_{kt} thus also become arbitrarily large as t becomes large. This in turn implies that $E_{jt} \to \kappa^{\frac{\sigma}{\sigma-1}} R_{jt}$ and $E_{kt} \to \kappa^{\frac{\sigma}{\sigma-1}} R_{kt}$ as t becomes large. From equation (13), we have:

$$\left[\frac{R_{jt}}{R_{kt}} \right]^{\frac{1}{\sigma}+\frac{1}{\psi}} \to \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \left[\frac{R_{jt}}{R_{kt}} \right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}$$

as t becomes large. Therefore, as $t \to \infty$,

$$\frac{R_{jt}}{R_{kt}} \to \left\{ \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right\}^{\frac{\epsilon\psi}{\epsilon+\psi}}. \tag{E-23}$$

Define $\Omega_t \triangleq A_{jt}/A_{kt}$, so that

$$\Omega_t = \frac{1+\eta\gamma s_{jt}}{1+\eta\gamma(1-s_{jt})} \Omega_{t-1}. \tag{E-24}$$

Because a corner allocation cannot persist indefinitely, $\Pi^*_{jt}/\Pi^*_{kt} = 1$ for some t sufficiently large. Using this and equation (E-23) in equation (12), we have:

$$\frac{1+\eta\gamma s^*_{jt}}{1+\eta\gamma(1-s^*_{jt})} = \Omega_{t-1}^{-(1-\sigma)(1-\alpha)} \left(\left\{ \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right\}^{\frac{\epsilon\psi}{\epsilon+\psi}} \right)^{1+\sigma/\psi} \left[\frac{\Psi_j}{\Psi_k} \right]^{-\sigma/\psi}.$$

Therefore, from equation (E-24),

$$\Omega_t = \Omega_{t-1}^{1-(1-\sigma)(1-\alpha)} \left(\left\{ \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right\}^{\frac{\epsilon\psi}{\epsilon+\psi}} \right)^{1+\sigma/\psi} \left[\frac{\Psi_j}{\Psi_k} \right]^{-\sigma/\psi}.$$

Define $\tilde{\Omega}_t \triangleq \ln[\Omega_t]$. We then have:

$$\tilde{\Omega}_t = [1-(1-\sigma)(1-\alpha)]\tilde{\Omega}_{t-1} + \ln \left[\left(\left\{ \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right\}^{\frac{\epsilon\psi}{\epsilon+\psi}} \right)^{1+\sigma/\psi} \left[\frac{\Psi_j}{\Psi_k} \right]^{-\sigma/\psi} \right].$$

This is a linear difference equation. For $\sigma < 1$, the coefficient on $\tilde{\Omega}_{t-1}$ is strictly between 0 and 1. The linear difference equation is therefore stable. The system approaches a steady state in $\tilde{\Omega}_t$ and therefore in Ω_t. From equation (E-24), any steady state in Ω_t must have $s^*_{jt} = 0.5$. Therefore as $t \to \infty$, $s^*_{jt} \to 0.5$. We have established the first result.

Equation (E-23) implies that if $\nu_j = 0.5$ and $\Psi_j = \Psi_k$ then $R^*_{jt} = R^*_{kt}$. Further, if $\nu_j \geq 0.5$ and $\Psi_j \geq \Psi_k$ with at least one inequality being strict, then $R^*_{jt} > R^*_{kt}$. Now substitute into equation (12) and use $s_{jt} = 0.5$:

$$\begin{aligned}
\frac{\Pi_{jt}}{\Pi_{kt}} &\to \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{\frac{-(1-\sigma)(1-\alpha)}{\sigma+\alpha(1-\sigma)}} \left(\left\{ \frac{\nu}{1-\nu} \left[\frac{\Psi_j}{\Psi_k} \right]^{1/\psi} \right\}^{\frac{\epsilon\psi}{\epsilon+\psi}} \right)^{\frac{1+\sigma/\psi}{\sigma+\alpha(1-\sigma)}} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{-\sigma/\psi}{\sigma+\alpha(1-\sigma)}} \\
&= \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{\frac{-(1-\sigma)(1-\alpha)}{\sigma+\alpha(1-\sigma)}} \left(\frac{\nu_j}{1-\nu_j} \right)^{\frac{\sigma+\psi}{\sigma+\alpha(1-\sigma)} \frac{\epsilon}{\epsilon+\psi}} \left(\frac{\Psi_j}{\Psi_k} \right)^{\frac{\epsilon-\sigma}{\sigma+\alpha(1-\sigma)} \frac{1}{\epsilon+\psi}},
\end{aligned}$$

and this must equal 1 because $s^*_{jt} = 0.5$. Therefore, if $\nu_j = 0.5$ and $\Psi_j = \Psi_k$ then $A_{jt} = A_{kt}$, and if $\nu_j \geq 0.5$ and $\Psi_j \geq \Psi_k$ with at least one inequality being strict, then $A_{jt} > A_{kt}$. We have established the second and third results.

Finally, as t becomes large along a path with $s^*_{jt} = 0.5$, using previous results in equation (A-4) yields:

$$
\begin{aligned}
\left[\frac{R_{jt}}{\Psi_j}\right]^{1/\psi} \rightarrow & \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{E_{jt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{jt}}{E_{jt}}\right]^{-1/\sigma} \\
= & \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{\kappa^{\frac{\sigma}{\sigma-1}} R_{jt}}{Y_t}\right]^{-1/\epsilon} \left[\kappa^{\frac{\sigma}{\sigma-1}}\right]^{1/\sigma} \\
= & \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{\kappa^{\frac{\sigma}{\sigma-1}} R_{jt}}{A_Y E_{jt}\left(\nu_j + (1-\nu_j)\left(\frac{E_{kt}}{E_{jt}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}\right]^{-1/\epsilon} \left[\kappa^{\frac{\sigma}{\sigma-1}}\right]^{1/\sigma} \\
= & \kappa\, \nu_j A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{1}{A_Y \left(\nu_j + (1-\nu_j)\left(\frac{R_{kt}}{R_{jt}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}}\right]^{-1/\epsilon} \left[\kappa^{\frac{\sigma}{\sigma-1}}\right]^{1/\sigma} \\
= & \nu_j \kappa^{\frac{\sigma}{\sigma-1}} A_Y \left[\nu_j + (1-\nu_j)\left(\frac{R_{kt}}{R_{jt}}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}.
\end{aligned}
\tag{E-25}
$$

From equation (E-23), R^*_{jt}/R^*_{kt} becomes constant as t becomes large. Then from (E-25), R^*_{jt} approaches a constant. An analogous derivation establishes that R^*_{kt} approaches a constant. We have established the final result.

E.8 Proof of Proposition 4

Let time $w \geq t_0$ be the first time after t_0 at which sector j's share of resource use begins decreasing, so that $R_{jx}/R_{kx} \leq R_{j(x+1)}/R_{k(x+1)}$ for all $x \in [t_0, w-1]$ and $R_{jw}/R_{kw} > R_{j(w+1)}/R_{k(w+1)}$, which in turn requires $s_{jx} \geq \hat{s}_x$ for all $x \in [t_0+1, w]$ and $s_{j(w+1)} < \hat{s}_{w+1}$. Note that $s_{jt_0} > 0.5$ implies that $A_{jt_0}/A_{kt_0} > A_{j(t_0-1)}/A_{k(t_0-1)}$. Assume that sector j's share of research begins declining sometime after its share of resource use does, so that $s_{jx} \leq s_{j(x+1)}$ for all $x \in [t_0, w]$. Then we have $A_{jx}/A_{kx} > A_{j(x-1)}/A_{k(x-1)}$ for all $x \in [t_0, w+1]$, and thus $A_{jx}/A_{kx} > [\Psi_j/\Psi_k]^\theta$ for all $x \in [t_0, w+1]$. Using this with Lemma E-8 and $\sigma < 1$ then implies $\hat{s}_{x+1} \geq 0.5$ for all $x \in [t_0, w+2]$. Combining this with the requirement that $s_{jw} \geq \hat{s}_w$, we have $s_{jw} \geq 0.5$. From equation (12) and $\sigma < 1$, we then have $s_{j(w+1)} \geq s_{jw}$ only if $R_{jw}/R_{kw} \leq R_{j(w+1)}/R_{k(w+1)}$. But that contradicts the definition of w, which required $R_{jw}/R_{kw} > R_{j(w+1)}/R_{k(w+1)}$. Sector j's share of research must have begun declining no later than time w. We have shown that a transition in resource use occurs only after a transition in research.

We now have two possibilities. We will see that the first one implies that $s_{jx} \geq 0.5$ at all times $x \in [t+1, w]$ and the second one generates a contradiction.

First, we could have $A_{j(x-2)}/A_{k(x-2)} \geq [\Psi_j/\Psi_k]^\theta$ at all times $x \in [t_0+1, w]$. Then by Lemma E-8, $\hat{s}_x \geq 0.5$ at all times $x \in [t_0+1, w]$. The definition of time w then requires $s_{jx} \geq 0.5$ at all times $x \in [t_0+1, w]$.

Second, we could have $A_{j(x-2)}/A_{k(x-2)} < [\Psi_j/\Psi_k]^\theta$ at some time $x \in [t_0+1, w]$. In order for this to happen, it must be true that $s_{jx} < 0.5$ at some times $x \in [t_0+2, w]$.[18] Let z be the first time at which $s_{jx} < 0.5$. $A_{j(t_0-1)}/A_{k(t_0-1)} > [\Psi_j/\Psi_k]^\theta$ and $s_{jx} \geq 0.5$ for all $x \in [t_0, z-1]$ imply that $A_{j(z-2)}/A_{k(z-2)} > [\Psi_j/\Psi_k]^\theta$, which implies by Lemma E-8 and $\sigma < 1$ that $\hat{s}_z \geq 0.5$. So we have $s_{jz} < \hat{s}_z$, which means that $R_{j(z-1)}/R_{k(z-1)} > R_{jz}/R_{kz}$. But this contradicts the definition of time w as the first time at which sector j's share of resource use begins decreasing.

Therefore, we must have $A_{j(x-2)}/A_{k(x-2)} \geq [\Psi_j/\Psi_k]^\theta$ and $s_{jx} \geq 0.5$ at all times $x \in [t_0+1, w]$. Observe that $s_{jx} \geq 0.5$ at all times $x \in [t_0, w]$ implies $A_{jx}/A_{kx} \geq A_{j(x-1)}/A_{k(x-1)}$ at all times $x \in [t_0, w]$. We have shown that a transition in technology happens only after a transition in resource use. We have established the first part of the proposition.

Now consider the first time $z > t_0$ at which $R_{jz} < R_{kz}$. Assume that $\Psi_j \geq \Psi_k$ and that $s_{jx} \geq 0.5$ for $x \in [t_0, z]$. Assumption 1, $\Psi_j \geq \Psi_k$, and $s_{jx} \geq 0.5$ imply $A_{jx} \geq A_{kx}$ for $x \in [t_0, z]$. Using $\sigma < 1$, we see that $A_{j(z-1)} \geq A_{k(z-1)}$, $\Psi_j \geq \Psi_k$, and $R_{jz} < R_{kz}$ imply that the right-hand side of equation (E-1) is < 1 when evaluated at $s_{jz} = 0.5$. So by Corollary E-2, time z equilibrium scientists must be less than 0.5. But $s_{jz} < 0.5$ contradicts $s_{jx} \geq 0.5$ for $x \in [t_0, z]$. Therefore, if $\Psi_j \geq \Psi_k$, then there must be some time $x \in [t_0, z]$ at which $s_{jx} < 0.5$. We have shown that if $\Psi_j \geq \Psi_k$, then sector k must begin dominating research before it begins dominating resource use. We have established the second part of the proposition.

Finally, let $\nu_j = \nu_k$ and $\Psi_j = \Psi_k$. By Proposition 3, $A_{jt} = A_{kt}$ in the steady-state research allocation. But Assumption 1 ensures that $A_{jt_0} > A_{kt_0}$. Thus there exists $t_1 > t_0$ such that $s_{jt_1} < 0.5$. By the foregoing parts of this proposition, a transition in research, a transition in resource use, and a transition in technology must happen between t_0 and t_1. We have established the third part of the proposition.

E.9 Intermediate steps for Cobb-Douglas special case

Substituting the Cobb-Douglas forms, equation (13) becomes

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{\psi+1}{\psi}-\kappa\frac{\epsilon-1}{\epsilon}} = \frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{1/\psi}\left[\frac{X_{jt}}{X_{kt}}\right]^{(1-\kappa)\frac{\epsilon-1}{\epsilon}}.$$

[18]Recall that $s_{jt} \geq 0.5$ and $s_{j(t+1)} \geq s_{jt}$ imply $s_{j(t+1)} \geq 0.5$.

Substituting equation (A-1) into equation (A-2) and then using equation (2), we have:

$$X_{jt} = \left[\frac{1-\kappa}{\kappa} R_{jt}^{\frac{\psi+1}{\psi}} \Psi_j^{-1/\psi} \right]^{\alpha} A_{jt}^{1-\alpha}.$$

We then have equation (15).

E.10 Intermediate steps for Leontief special case

From equation (A-2) and $R_{jt} = X_{jt}$,

$$p_{jXt} = \left(\frac{R_{jt}}{A_{jt}} \right)^{\frac{1-\alpha}{\alpha}}.$$

Equation (17) follows from equation (6).

From equation (A-2),

$$p_{jXt} X_{jt} = X_{jt}^{1/\alpha} A_{jt}^{-\frac{1-\alpha}{\alpha}}.$$

And from equation (2),

$$p_{jRt} R_{jt} = \Psi_j^{-1/\psi} R_{jt}^{\frac{1+\psi}{\psi}}.$$

Intermediate good producers' zero-profit condition is

$$p_{jt} E_{jt} = \Psi_j^{-1/\psi} R_{jt}^{\frac{1+\psi}{\psi}} + X_{jt}^{1/\alpha} A_{jt}^{-\frac{1-\alpha}{\alpha}}.$$

Substituting for p_{jt} from the final good producers' first-order condition and then setting $X_{jt} = R_{jt}$ and $E_{jt} = R_{jt}$, we have:

$$\nu_j Y_t^{1/\epsilon} = A_Y^{\frac{1-\epsilon}{\epsilon}} R_{jt}^{\frac{1-\epsilon}{\epsilon}} \left[\Psi_j^{-1/\psi} R_{jt}^{\frac{1+\psi}{\psi}} + R_{jt}^{1/\alpha} A_{jt}^{-\frac{1-\alpha}{\alpha}} \right].$$

Using $\psi = \alpha/(1-\alpha)$, we have:

$$\nu_j Y_t^{1/\epsilon} = A_Y^{\frac{1-\epsilon}{\epsilon}} R_{jt}^{\frac{1-\epsilon}{\epsilon}+\frac{1}{\alpha}} \left[\Psi_j^{-\frac{1-\alpha}{\alpha}} + A_{jt}^{-\frac{1-\alpha}{\alpha}} \right].$$

An analogous result holds for sector k. Equation (16) follows.

Now consider the steady-state research allocation. For $s \in (0,1)$, $A_{j(t-1)}$ and $A_{k(t-1)}$ become arbitrarily large as t increases. From equations (16) and (17), we have:

$$\lim_{t\to\infty} \frac{\Pi_{jt}}{\Pi_{kt}} \to \left(\frac{A_{j(t-1)}}{A_{k(t-1)}} \right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)} \right)^{-\frac{1}{\alpha}} \left(\frac{\nu_j}{\nu_k} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{1-\alpha}{\alpha}} \right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}}. \tag{E-26}$$

At an equilibrium with $s \in (0,1)$, $\Pi_{jt} = \Pi_{kt}$. Then, for t sufficiently large,

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{\frac{1}{\alpha}} = \left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}}.$$

At a steady state, $A_{j(t-1)} = (1+\eta\gamma s)^{\Delta} A_{j(t-1-\Delta)}$ and $A_{k(t-1)} = (1+\eta\gamma(1-s))^{\Delta} A_{k(t-1-\Delta)}$. Therefore the following must hold for all $\Delta \geq 0$:

$$\left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{\frac{1}{\alpha}} = \left(\frac{1+\eta\gamma s}{1+\eta\gamma(1-s)}\right)^{-\Delta\frac{1-\alpha}{\alpha}} \left(\frac{A_{j(t-1-\Delta)}}{A_{k(t-1-\Delta)}}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\nu_j}{\nu_k}\left[\frac{\Psi_j}{\Psi_k}\right]^{\frac{1-\alpha}{\alpha}}\right)^{\frac{\epsilon}{\alpha+(1-\alpha)\epsilon}}.$$

This implies equation (18).

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