INNOVATION-LED TRANSITIONS IN ENERGY SUPPLY

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ABSTRACT

I reconcile a benchmark model of directed technical change with the historical experience of energy transitions by allowing for a non-unitary elasticity of substitution between machines and the other factor of production, interpreted here as energy resources. I show that the economy becomes increasingly locked-in to the dominant sector when machines and resources are gross substitutes, but a transition from the dominant sector to the other is possible when machines and resources are gross complements. Consistent with history, a transition in research activity leads the transition in resource supply. A calibrated numerical implementation shows that innovation is critical for climate change policy. A policymaker would use a U-shaped emission tax trajectory so as to immediately transition innovation away from the fossil sector, wait for clean technology to improve, and then hasten a transition in resource supply later in the century.

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Energy historians have emphasized the dramatic transformations in energy use that accompanied industrialization. According to Smil (2010, 2), the preindustrial era saw "only very slow changes" in energy use, "but the last two centuries have seen a series of remarkable energy transitions." Rosenberg (1994, 169) notes that "the diversity of energy inputs and the changing usage of those inputs over time is a central feature of the historical record." And in their seminal analysis, Marchetti and Nakicenovic (1979, 15) observe that the transitions have been so regular that "it is as though the system had a schedule, a will, and a clock." It is important to understand the economic drivers of these transitions. First, energy use is closely linked to the First Industrial Revolution (via coal), to the Second Industrial Revolution (via electricity and oil), and to the distribution of output across countries. Yet growth theory has largely abstracted from energy. Second, policymakers around the world are currently attempting to induce a new transition to low-carbon resources in order to avoid dangerous climate change. Understanding the drivers of past transitions should improve policies that aim to stimulate and sustain a new transition.

Resource economists have long focused on how depletion or exhaustion can induce transitions between resources (e.g., Nordhaus, 1973; Chakravorty and Krulce, 1994; Chakravorty et al., 1997). For example, the Herfindahl (1967) rule holds that resources should be exploited in order of increasing cost. In contrast, energy and economic historians have argued that technological change, not depletion, has been critical to past transitions between different types of resources (e.g., Marchetti, 1977; Marchetti and Nakicenovic, 1979; Rosenberg, 1983; Grübler, 2004; Fouquet, 2010; Wilson and Grubler, 2011).[1] The British transition from biomass to coal was driven by technologies such as the steam engine, not by changes in the relative abundance of timber and coal. The later transition from coal to oil was driven by the technology of the internal combustion engine, not by a lack of coal. Resource economists' emphasis on depletion may explain the development of a given type of resource, but standard models cannot capture historians' understanding of transitions between types of resources.[2]

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[1]I give five examples. Marchetti and Nakicenovic (1979, 7–8) argue, "The causal importance of resource availability is weakened by the fact that oil successfully penetrated the energy market when coal still had an enormous potential, just as coal had previously penetrated the market when wood still had an enormous potential." Fouquet (2010, 6591) observes, "In all cases, cheaper or better services were the key to the switch [between sources of energy]. In a majority of cases, the driver was better or different services." Rosenberg (1994, 169) observes that "technological innovations are often not neutral with respect to their energy requirements." Flinn (1959) emphasizes that the surmounting of "technological barriers," not the scarcity of timber, drove the British to shift towards coal. Finally, Grübler (2004, 170) writes, "It is important to recognize that these two major historical shifts [from biomass to coal, and then from coal to oil and natural gas] were not driven by resource scarcity or by direct economic signals such as prices, even if these exerted an influence at various times. Put simply, it was not the scarcity of coal that led to the introduction of more expensive oil. Instead, these major historical shifts were, first of all, technology shifts, particularly at the level of energy end use. Thus, the diffusion of steam engines, gasoline engines, and electric motors and appliances can be considered the ultimate driver, triggering important innovation responses in the energy sector and leading to profound structural change."

[2]The overemphasis on depletion at the expense of innovation dates back to Jevons (1865), who underes-

I develop a model of directed technical change in which innovation-led transitions occur endogenously, even without any type of depletion.[3] A final good is produced from two types of energy services, which are gross substitutes. Each type of energy service is produced by combining an energy resource with specialized machines. For instance, coal is combined with steam engines to produce mechanical motion or electricity. A fixed measure of scientists works to improve these machines. Each scientist targets whichever type of machine provides a more valuable patent. Scientists' efforts change the quality of machines from period to period, which in turn changes equilibrium use of each energy resource from period to period.

I show that the elasticity of substitution between resources and machines determines whether a transition in energy supply can occur in the absence of policy and of depletion. Imagine that one type of energy initially attracts the majority of scientists and also uses more raw resources. The technology in the sector that dominates research is improving relative to the other sector's technology. I show that three forces determine how each sector's share of research and resource extraction changes in the following period. First, as the dominant sector becomes more advanced, *market size effects* increase that sector's share of research and of extraction. The improvement in the dominant sector's quality of machines expands the market for the energy resource, and the resulting increase in resource extraction raises the value of a patent by expanding the market for machines. This positive feedback between extraction and research works to lock in whichever sector is already dominant. Second, a *patent quality effect* drives scientists to the sector where their patent will cover a higher quality machine. This effect draws additional scientists to the sector that dominated research in the previous period, which again works to lock in whichever sector is already dominant. Third, a *supply expansion effect* reduces the value of a patent as the average quality of a sector's machines increases. An improvement in the quality of a sector's machines shifts out the supply of machine services, which reduces the price of machine services and thus reduces the value of a patent. This force pushes scientists away from the sector that dominated research effort in the previous period. It is the only force that works against lock-in and in favor of a transition away from the dominant sector.

The elasticity of substitution between resources and machines determines the relative strengths of the patent quality and supply expansion effects. When that elasticity is strictly greater than 1 (machines are "energy-saving"), demand for machine services is elastic and

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timated the scope for innovation in his famous analysis of the advancing depletion of British coal reserves (Madureira, 2012).

[3]Formally, I analyze directed technical change (Acemoglu, 2002) when final good production has a nested constant elasticity of substitution structure that allows innovation and other inputs to be complements. A prominent strand of literature argues that complementarities have been a critical—and often overlooked— element of economic growth (Rosenberg, 1976; Matsuyama, 1995, 1999; Evans et al., 1998). Milgrom et al. (1991) show how complementarities between techniques and inputs can generate persistent patterns of technical change without needing to assume increasing returns. In the present setting, increasing returns to innovation can work to lock in the dominant technology. I here use complementarities to explain changes in energy technologies and supply without needing to impose decreasing returns to innovation.

the price of machine services does not fall by much as technology improves. The patent quality effect dominates the supply expansion effect. Whichever sector dominates research and extraction in some period then does so to an increasing degree in all later periods.[4] However, when that elasticity is strictly less than 1 (machines are "energy-using"), demand for machine services is inelastic and the price of machine services falls by a lot as technology improves. The supply expansion effect dominates the patent quality effect. In that case, as the dominant sector becomes more advanced, scientists can begin switching to the other sector. Eventually, their research efforts raise the quality of technology in the dominated sector, which begins increasing that sector's share of extraction via market size effects. The shift in scientists away from the dominant sector can thereby generate a transition in energy supply.

Machines must be energy-using if the theoretical model is to be consistent with the observed history of energy transitions. Much empirical literature indeed suggests that machines are energy-using (i.e., that capital and energy are gross complements). Such evidence comes from industries in the U.S. (Berndt and Wood, 1975; Pindyck and Rotemberg, 1983; Prywes, 1986), the United Kingdom (Hunt, 1984), and a set of OECD countries (van der Werf, 2008). Hassler et al. (2012) estimate that the aggregate elasticity of substitution between energy and a capital-labor composite is very close to zero in U.S. data. Stern and Kander (2012) also find an aggregate elasticity of substitution that is smaller than unity in 200 years of data from Sweden. These empirical results support parameter values that the present setting suggests are necessary to explain historians' observations of innovation-led transitions.[5]

In order to analyze the implications of energy-using machines for policies to address climate change, I specialize the setting to the case of a fossil and a renewable resource, where using the fossil resource generates greenhouse gas emissions that eventually warm the climate and harm production of the final good. A policymaker can design sequences of emission taxes and/or research subsidies to maximize intertemporal welfare. The numerical calibration matches recent patterns in fossil and renewable resource consumption, innovation, and economic growth.

I show that innovation is critical to both laissez-faire outcomes and to welfare-maximizing policy. First, in laissez faire, an endogenous shift in innovation towards the renewable resource eventually works to redirect resource supply towards the renewable resource. However, this shift in supply is too slow to avoid substantial warming. Second, if the policymaker's only instrument is an emission tax, then the tax's trajectory is U-shaped: the tax starts

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[4]The forces generating lock-in are similar to those explored in a related literature on path dependency in technology adoption (e.g., David, 1985; Arthur, 1989; Cowan, 1990). That literature focuses on "dynamic increasing returns" as the source of path dependency, where the likelihood of using a technology increases in the number of times it was used in the past (perhaps through learning-by-doing or network effects). In the present setting, market size and patent quality effects both act like dynamic increasing returns.

[5]This theory of innovation-led transitions will apply to other settings with complementarities between machines and other factors of productions. The required complementarities may be common. For instance, Grossman et al. (2017) summarize evidence that labor and capital are complements.

high enough to shift most scientists to the renewable sector, drops to a very low level once scientists will continue working in the renewable sector anyway, and rises late in the century to hasten a transition towards use of the renewable resource. The tax therefore plays two roles, as it is first used to shift near-term innovation and is later used to shift resource consumption. The first role is quantitatively more important. A policymaker who has access to a research subsidy but not to an emission tax can achieve greater welfare because she can more directly incentivize the early transition in innovation, even though she lacks a policy tool that she can use in later periods to hasten the transition in resource supply. Finally, I find that welfare-maximizing policies to manage climate change are more sensitive to the degree of substitutability between types of energy than to commonly studied parameters such as the discount rate. When renewable and fossil energy are strong substitutes, the policymaker limits climate change to a level consistent with recent international agreements, but when the renewable and fossil resource are only weak substitutes, the policymaker finds it more costly to reduce emissions and thus allows temperature to increase along a trajectory consistent with conventional climate-economy models (e.g., Nordhaus, 2008).

My theoretical setting generalizes Acemoglu et al. (2012). Their economy demonstrates a high degree of lock-in or path dependency: whichever sector initially dominates extraction and research effort will increase its dominance as time passes.[6] This result is not consistent with the history of energy transitions. I show that their high degree of lock-in results from their use of a Cobb-Douglas aggregator to combine resources and machines, which fixes the elasticity of substitution between resources and machines at unity. I show that a unit elasticity is the knife-edge case in which the patent quality and supply expansion effects exactly offset each other. The evolution of research and extraction in Acemoglu et al. (2012) is therefore determined entirely by market size effects (demonstrated in Section 2 below), which generate positive feedbacks between research and extraction that lock in the dominant sector. The assumption of Cobb-Douglas production has qualitatively important implications for their economy's dynamics.[7]

The assumption of Cobb-Douglas production also has important implications for their policy conclusions. First, in Acemoglu et al. (2012), the fossil resource is initially locked-in

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[6]An exception is when they model resources as exhaustible or depletable. Thus, when transitions arise in their setting, these transitions are driven by the same forces explored in the resource economics literature.

[7]Most analyses that combine directed technical change and energy have divided technologies between those that augment resources and those that augment other factors such as labor (Smulders and de Nooij, 2003; Di Maria and Valente, 2008; Grimaud and Rouge, 2008; Pittel and Bretschger, 2010; André and Smulders, 2012; Hassler et al., 2012). These studies have focused on the potential for technical change to enable long-run growth even when an exhaustible resource is essential to production. In contrast, the present paper and Acemoglu et al. (2012) both allow research effort to be directed between multiple types of resources in order to study questions about energy transitions. Acemoglu et al. (2016) develop a related setting in which two types of energy technologies compete in each of many product lines. Each product line's production function is Cobb-Douglas. As a result, their setting again generates strong path dependency or lock-in.

and, in the absence of policy, becomes more locked-in over time. An "environmental disaster" therefore inevitably occurs in the absence of policy. In contrast, the present calibrated model suggests that a laissez-faire transition in both innovation and resource supply would occur over the next century, so that the policy challenge becomes one of timing. Whereas optimal policy is, in effect, infinitely valuable in Acemoglu et al. (2012), we here see more limited benefits.[8] Second, the cost of delaying policy differs between the settings. In Acemoglu et al. (2012), the fossil resource becomes more locked-in as time passes in the laissez-faire economy. Delaying policy is costly because it becomes harder to redirect innovation as lock-in progresses. Lock-in is less problematic in our calibrated economy. In fact, we will see that the policymaker would prefer to allow fossil-using technologies to advance relative to renewable-using technologies over a 50-year interval of delay rather than fix both types of technologies at their initial qualities. The benefits of technological change here outweigh the costs of temporarily misdirecting innovation towards the fossil sector.

The next section describes the theoretical setting. Section 2 analyzes the relative incentive to research technologies in each sector. Section 3 describes the economy's laissez-faire dynamics. Section 4 numerically explores the implications for climate change policies that aim to induce a transition to renewable energy. The final section concludes. The appendix contains additional formal analysis as well as proofs.

1   Setting

Consider a discrete-time economy in which final-good production uses two types of energy intermediates. These energy intermediates are generated by combining energy resources with machines. Resources are supplied isoelastically. A fixed measure of households works as scientists, trying to improve the quality of machines used in producing the energy intermediates. Scientists decide which type of machine to work on. The equilibrium allocation of resources and scientists changes over time as technologies improve. Figure 1 illustrates the model setup, which we now formalize.

Begin with final good production. The time t final good Y_t is produced competitively from two energy intermediates Y_{jt} and Y_{kt}. We take the final good as the numeraire in each period. The representative firm's production function takes the familiar constant elasticity

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[8]The settings also generate different conclusions about the long-run emission tax. In Acemoglu et al. (2012), the policymaker may no longer need an emission tax once the renewable resource's technologies have advanced far enough to become locked-in. Path dependency increases the hurdle that near-term policies must overcome but reduces the role for long-term policy. In contrast, we here see that the policymaker wants to use an emission tax to redirect resource supply even long after the renewable sector has begun to attract all research effort. Note, though, that one should beware of comparing the level of optimal policy between the two papers. First, the tax in Acemoglu et al. (2012) does not have a clear interpretation as an emission tax. Second, the current setting uses more realistic models of climate change and resource supply as well as a different calibrational approach.
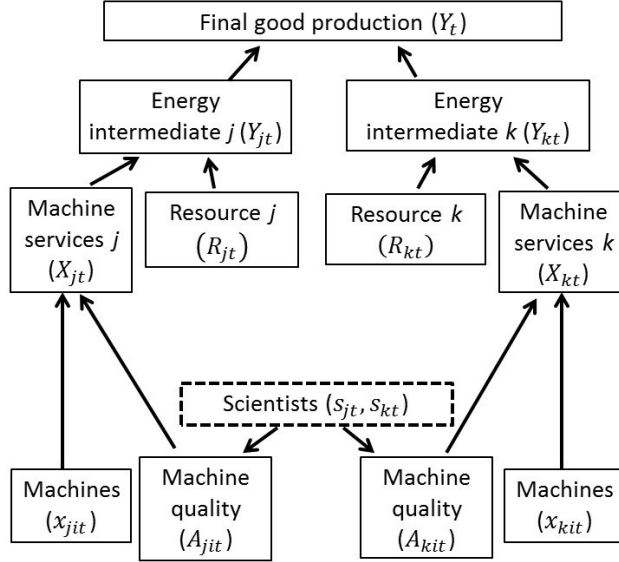
Figure 1: Overview of the theoretical setting.

of substitution (CES) form:

$$Y_t = A_Y \left(\nu\, Y_{jt}^{\frac{\epsilon-1}{\epsilon}} + (1-\nu) Y_{kt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

The parameter $\nu \in (0,1)$ is the distribution (or share) parameter, and $A_Y > 0$ is a productivity parameter. We say that resource j is *higher quality* than resource k if and only if $\nu > 0.5$. The parameter ϵ is the elasticity of substitution. The two energy intermediates are gross substitutes ($\epsilon > 1$).[9]

The energy intermediates Y_{jt} and Y_{kt} are the energy services produced by combining resource inputs with machines. Production of energy intermediates has the following CES forms:

$$Y_{jt} = \left(\kappa R_{jt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad Y_{kt} = \left(\kappa R_{kt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa) X_{kt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

The parameter $\kappa \in (0,1)$ is the distribution (or share) parameter. I describe the resource inputs R and machine service inputs X below. The elasticity of substitution between these resource and machine inputs is σ. We call machines *energy-using* when resources and machines are gross complements ($\sigma < 1$), and we call machines *energy-saving* when resources and machines are gross substitutes ($\sigma > 1$). Resources and machines are less substitutable than are different types of energy intermediates ($\sigma < \epsilon$).

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[9]The restriction that $\epsilon > 1$ is consistent with evidence in Papageorgiou et al. (2017).

Machine services X_{jt} and X_{kt} are produced in a Dixit-Stiglitz environment of monopolistic competition from machines of varying qualities:

$$X_{jt} = \int_0^1 A_{jit}^{1-\alpha} x_{jit}^\alpha \, \mathrm{d}i \quad \text{and} \quad X_{kt} = \int_0^1 A_{kit}^{1-\alpha} x_{kit}^\alpha \, \mathrm{d}i,$$

where $\alpha \in (0,1)$. The machines x_{jit} and x_{kit} that work with a given resource at time t are divided into a continuum of types, indexed by i. The quality (or efficiency) of machine x_{jit} (or x_{kit}) is then given by A_{jit} (or A_{kit}). Machines of type i are produced by monopolists who each take the price of machine services (p_{jXt}, p_{kXt}) as given (each is small) but recognize their ability to influence the price (p_{jxit}, p_{kxit}) of machines of type i. The cost of producing a machine is $a > 0$ units of the final good, which we normalize to $a = \alpha^2$. The first-order condition for a producer of machine services yields the following demand curve for machines of type i in sector j (with analogous results for sector k):

$$x_{jit} = \left(\frac{p_{jXt}}{p_{jxit}} \alpha \right)^{\frac{1}{1-\alpha}} A_{jit}. \tag{1}$$

The monopolist producer of x_{jit} therefore faces an isoelastic demand curve and accordingly marks up its price by a constant fraction over marginal cost: $p_{jxit} = a/\alpha = \alpha$. In equilibrium, the producer of machine type i for use with resource j earns profits of:

$$\pi_{jxit} = (p_{jxit} - a)x_{jit} = \alpha(1-\alpha)p_{jXt}^{\frac{1}{1-\alpha}} A_{jit},$$

with analogous results for π_{kxit}.

Scientists choose which resource they want to study (j or k) and are then randomly allocated to a machine type i. Each scientist succeeds in innovating with probability $\eta \in (0,1]$. If they fail, scientists earn nothing and the quality of that type of machine is unchanged. Following Acemoglu et al. (2012), successful scientists receive a one-period patent to produce their type of machine, and they improve the quality of their machine type to $A_{jit} = A_{ji(t-1)} + \gamma A_{ji(t-1)}$ (using resource j as an example), where $\gamma > 0$.[10] If a scientist succeeds in innovating at time t, she exercises her patent to obtain the monopoly profit π_{jxit}. Her expected reward to choosing to research machines that work with resource type j is therefore

$$\Pi_{jt} = \eta \, \alpha \, (1-\alpha) p_{jXt}^{\frac{1}{1-\alpha}} (1+\gamma) A_{j(t-1)}, \tag{2}$$

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[10]We here follow the literature in using an increasing returns representation of innovation. We will see that an innovation-led transition is possible even though increasing returns push scientists toward the more advanced sector. If we were to interpret the quality of machines as their efficiency of energy conversion, then a decreasing returns representation would have merit. See Hart (2015) for an analysis of that case in a setting closely related to Acemoglu et al. (2012).

where $A_{j(t-1)}$ is the average quality of machines in sector j. This average quality evolves as

$$A_{jt} = \int_0^1 \left[\eta s_{jt}(1+\gamma)A_{ji(t-1)} + (1 - \eta s_{jt})A_{ji(t-1)} \right] \mathrm{d}i = (1 + \eta\gamma s_{jt})A_{j(t-1)}, \qquad (3)$$

where s_{jt} is the measure of scientists working on resource j. All relationships for resource k are analogous. Scientists are of fixed measure, normalized to 1:

$$1 = s_{jt} + s_{kt}.$$

Producing the next unit of resource j or k requires $(R_{jt}/\Psi_j)^{1/\psi}$ or $(R_{kt}/\Psi_k)^{1/\psi}$ units of the final good. The competitive resource extraction sectors therefore supply resources isoelastically:

$$R_{jt} = \Psi_j p_{jRt}^{\psi} \quad \text{and} \quad R_{kt} = \Psi_k p_{kRt}^{\psi}, \qquad (4)$$

where p_{jRt}, p_{kRt} are the prices received for each type of resource, $\psi > \alpha/(1-\alpha)$ is the price elasticity of resource supply, and $\Psi_j, \Psi_k > 0$ are supply shifters. We say that resource j is *more abundant* than resource k if and only if $\Psi_j > \Psi_k$. Requiring $\psi > \alpha/(1-\alpha)$ ensures that the own-price elasticity of resource supply is greater than the elasticity of machine services with respect to the resource price.

The economy's time t resource constraint is

$$Y_t \geq c_t + a \left[\int_0^1 x_{jit} \, \mathrm{d}i + \int_0^1 x_{kit} \, \mathrm{d}i \right] + \frac{\psi}{1+\psi} \left[(R_{jt}/\Psi_j)^{1/\psi} R_{jt} + (R_{kt}/\Psi_k)^{1/\psi} R_{kt} \right],$$

where $c_t \geq 0$ is the composite consumption good and where the final term arises from integrating the marginal cost of resource provision. Households have strictly increasing utility for the consumption good. Scientists therefore each choose their resource type so as to maximize expected earnings.

We study equilibrium outcomes.

Definition 1. *An* equilibrium *is given by sequences of prices for energy intermediates (p_{jt}^*, p_{kt}^*), prices for machine services (p_{jXt}^*, p_{kXt}^*), prices for machines (p_{jxit}^*, p_{kxit}^*), prices for resources (p_{jRt}^*, p_{kRt}^*), demands for inputs $(Y_{jt}^*, Y_{kt}^*, R_{jt}^*, R_{kt}^*, X_{jt}^*, X_{kt}^*, x_{jit}^*, x_{kit}^*)$, and factor allocations (s_{jt}^*, s_{kt}^*) such that, in each period t: (i) (Y_{jt}^*, Y_{kt}^*) maximizes profits of final good producers, (ii) $(R_{jt}^*, R_{kt}^*, X_{jt}^*, X_{kt}^*)$ maximizes profits of energy intermediate producers, (iii) (p_{jxit}^*, x_{jit}^*) and (p_{kxit}^*, x_{kit}^*) maximize profits of the producers of machine i in sectors j and k, respectively, (iv) (s_{jt}^*, s_{kt}^*) maximizes expected earnings of scientists, (v) prices clear the factor and input markets, and (vi) average technologies evolve as in equation (3).*

The equilibrium prices clear all factor markets and all firms maximize profits. If scientists are employed in both sectors, they receive the same expected reward from both, and if they are employed in only one, they receive a greater expected reward in the sector with nonzero scientists. The first appendix establishes that the equilibrium is stable in a tâtonnement sense. Throughout, I drop the asterisks when clear.

2   The Direction of Research

We now consider the relative incentive to research technologies that work with resource j rather than technologies that work with resource k. From equation (2), we have

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)} + \gamma A_{j(t-1)}}{A_{k(t-1)} + \gamma A_{k(t-1)}} \left[\frac{p_{jXt}}{p_{kXt}} \right]^{\frac{1}{1-\alpha}}. \tag{5}$$

The intermediate-good producer's first-order conditions for profit-maximization yield

$$p_{jXt} = (1 - \kappa) p_{jt} \left[\frac{X_{jt}}{Y_{jt}} \right]^{-1/\sigma} \quad \text{and} \quad p_{jRt} = \kappa \, p_{jt} \left[\frac{R_{jt}}{Y_{jt}} \right]^{-1/\sigma}.$$

We see that the relative incentive to research technologies for use in sector j increases in the relative price of the intermediates and decreases in the machine-intensity of sector j's output. Combining the first-order conditions, we have

$$p_{jXt} = \frac{1 - \kappa}{\kappa} \left[\frac{R_{jt}}{X_{jt}} \right]^{1/\sigma} p_{jRt}. \tag{6}$$

From equation (1) and the monopolist's markup, we have

$$x_{jit} = p_{jXt}^{\frac{1}{1-\alpha}} A_{jit}.$$

Substituting into the definition of X_{jt} and using the definition of A_{jt}, we have

$$X_{jt} = p_{jXt}^{\frac{\alpha}{1-\alpha}} A_{jt}. \tag{7}$$

Substitute into equation (6) and solve for equilibrium machine prices:

$$p_{jXt} = \left[p_{jRt} \frac{1 - \kappa}{\kappa} \right]^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{1-\alpha}{\sigma(1-\alpha)+\alpha}}. \tag{8}$$

This yields

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \underbrace{\frac{A_{j(t-1)} + \gamma A_{j(t-1)}}{A_{k(t-1)} + \gamma A_{k(t-1)}}}_{\text{patent quality effect}} \underbrace{\left(\frac{A_{jt}}{A_{kt}} \right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}}}_{\text{supply expansion effect}} \underbrace{\left(\frac{R_{jt}}{R_{kt}} \right)^{\frac{1}{\sigma+\alpha(1-\sigma)}}}_{\text{market size effect}} \underbrace{\left(\frac{p_{jRt}}{p_{kRt}} \right)^{\frac{\sigma}{\sigma+\alpha(1-\sigma)}}}_{\text{machine substitution effect}}. \tag{9}$$

We see four channels determining scientists' relative incentive to research machines. The first term directs research effort to the sector in which scientists will end up with the patent to better technology. This *patent quality effect* depends on the realized technology, not
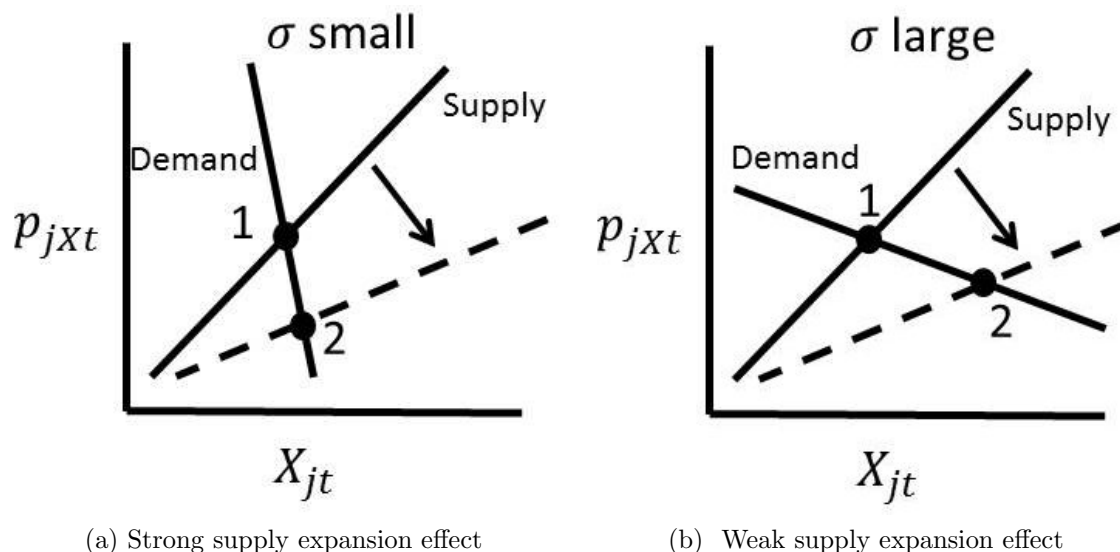
(a) Strong supply expansion effect   (b) Weak supply expansion effect

Figure 2: Improving the average quality of the machines in sector j shifts the supply of machine services outward and changes the equilibrium from point 1 to point 2.

solely on the increment to technology produced by a scientist's efforts, which introduces a type of business-stealing distortion. Obtaining a patent to a sufficiently advanced technology is valuable even if the scientist does not improve the technology. If γ differed by sector and were very small in the more advanced sector, scientists could nonetheless have a stronger incentive to research machines in the more advanced sector even though their efforts would not improve these machines. However, this business-stealing distortion vanishes under the standard assumption of identical γ because the ratio of the increments to technology $(\gamma A_{ji(t-1)}/\gamma A_{ki(t-1)})$ is identical to the ratio of the realized technologies (A_{jit}/A_{kit}). By attracting scientists to the more advanced sector, the patent quality effect here also attracts them to the sector where they make the greatest advance.

The other channels derive from the relative price of machine services: $(p_{jXt}/p_{kXt})^{1/(1-\alpha)}$ in equation (5). Figure 2 plots supply and demand for machine services X_{jt}, conditional on R_{jt}. The intersection of these supply and demand curves determines the equilibrium machine price p_{jXt}. The supply of X_{jt} follows from equation (7). It is steeper for smaller α. Demand for X_{jt} (conditional on R_{jt}) is given by equation (6), from the first-order conditions for the intermediate-good producers. The left panel of Figure 2 plots a case with less elastic demand (σ small), and the right panel of Figure 2 plots a case with more elastic demand (σ large).

Now consider the three machine price channels. We begin with the *supply expansion effect*, which pushes scientists away from the more advanced sector. From equation (7), the supply of X_{jt} shifts out when its machines' average quality A_{jt} increases, and it shifts out to

an especially large degree when α is small. The dashed lines in Figure 2 plot the consequence of an increase in A_{jt}. When σ is small (machines are energy-using), the demand curve is steep because the marginal product of additional machines is constrained by the supply of R_{jt}. By shifting out supply, the increase in A_{jt} induces a relatively large decline in the equilibrium price p_{jXt}, from point 1 to point 2 in the left panel. However, when σ is large, machine are energy-saving and the demand curve is relatively flat. The increase in A_{jt} then induces a relatively small decline in the equilibrium price p_{jXt}. Improving technology therefore pushes scientists away to a greater degree when the demand curve is steep (σ is small) or the shift in supply is large (α is small) because it then reduces p_{jXt} more strongly.

Now consider the net effect of a relative improvement in sector j's average technology. We have seen that this relative improvement attracts scientists through the patent quality effect and repels scientists through the supply expansion effect. From equations (3) and (9), the supply expansion effect dominates the patent quality effect if and only if $\sigma < 1$. As $\sigma \to 0$, demand for machines becomes perfectly inelastic and the supply expansion effect becomes large. As $\sigma \to \infty$, demand for machines becomes perfectly elastic and the supply expansion effect vanishes. As $\sigma \to 1$, the two effects exactly cancel, so that the incentives to research machines in one sector or the other do not directly depend on the relative quality of technology in each sector. This result explains the absence of relative technology from the research incentives in Acemoglu et al. (2012): technology matters in their equation (17) via the same patent quality effect seen here (which they call a "direct productivity effect") and also through their "price effect", but substituting in for relative output prices from their equation (A.3) shows that these two effects exactly cancel. Conditional on market size, relative technology plays no role in steering research activity in their setting.[11] We will see that whether improving technology attracts or repels scientists determines whether a transition in energy supply is possible.

The final two machine price channels in equation (9) both make the relative incentive to research machines in sector j increase in sector j's share of resource production. The first of these two channels is a *market size effect*. From equation (6), an increase in R_{jt} shifts out demand for X_{jt}, and does so to an especially large degree as σ becomes small. Increasing the supply of one factor makes the other factor relatively scarce and thus increases demand for that other factor, and does so to an especially strong degree when the two factors are complements ($\sigma < 1$). The second of these channels is a *machine substitution effect*. It increases demand for X_{jt} when the price of resources increases. This channel is especially strong when the elasticity of substitution between resources and machines is large, and it vanishes as the elasticity goes to zero. Substituting for resource prices from (4), we see that

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[11]Relative technology ends up playing a role in their setting's equilibrium (see their equation (18)) because relative market size is proportional to the relative quality of technology (see their equation (A.5)). As we will discuss, this channel for relative technologies appears in our setting as well: our equation (14) will show that relative market size increases in the relative quality of technology. However, our use of a more general CES aggregator means that the relationship between market size and relative technology is no longer linear.

the machine substitution effect amplifies the market size effect, and that it vanishes as the supply of resources becomes perfectly elastic.

3   Laissez-Faire Dynamics

We now consider how each sector's share of research activity and resource extraction evolves over time. We then analyze transitions, lock-in, and long-run outcomes before providing a numerical example.

3.1   Evolution of Research and Extraction

Begin by considering how the equilibrium allocation of scientists changes over time. The appendix shows that, at an interior allocation of scientists,

$$s_{j(t+1)} - s_{jt} \propto \underbrace{\frac{\psi + \sigma}{\psi} \frac{R_{kt}}{R_{jt}} \left(\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}} \right)}_{\text{resource channel}} + \underbrace{2(\sigma - 1)(1 - \alpha) \frac{\eta\gamma}{1 + \eta\gamma s_{kt}} \left(s_{jt} - \frac{1}{2} \right)}_{\text{innovation channel}}. \quad (10)$$

We see that the evolution of research effort depends on the evolution of extraction and of technology. The first term is a *resource channel*: scientists tend to move towards whichever sector sees its share of total resource extraction increase. This channel operates through the market size and machine substitution effects in equation (9). The second term is an *innovation channel*. If $s_{jt} > 0.5$, then sector j is becoming relatively more advanced as a result of time t research activity, and if $s_{jt} < 0.5$, then sector k is becoming relatively more advanced as a result of time t research activity. This channel pushes scientists towards whichever sector is becoming relatively more advanced if and only if $\sigma > 1$. Advancing technology affects relative research incentives through the patent quality and supply expansion effects in equation (9), where the patent quality effect attracts scientists to the more advanced sector and the supply expansion effect repels scientists from the more advanced sector. We saw that the patent quality effect dominates the supply expansion effect if and only if $\sigma > 1$.

Now consider how sector j's share of extraction changes from time t to $t + 1$. Combining the intermediate good producers' first-order condition for resources with the final good producers' first-order conditions, we find demand for each resource:

$$p_{jRt} = \kappa \nu A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{jt}}{Y_t} \right]^{-1/\epsilon} \left[\frac{R_{jt}}{Y_{jt}} \right]^{-1/\sigma} \quad \text{and} \quad p_{kRt} = \kappa (1 - \nu) A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{kt}}{Y_t} \right]^{-1/\epsilon} \left[\frac{R_{kt}}{Y_{kt}} \right]^{-1/\sigma}. \quad (11)$$

Market-clearing for each resource then implies

$$\left[\frac{R_{jt}}{\Psi_j}\right]^{1/\psi} = \kappa\,\nu A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{jt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{jt}}{Y_{jt}}\right]^{-1/\sigma}, \tag{12}$$

$$\left[\frac{R_{kt}}{\Psi_k}\right]^{1/\psi} = \kappa\,(1-\nu) A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_{kt}}{Y_t}\right]^{-1/\epsilon} \left[\frac{R_{kt}}{Y_{kt}}\right]^{-1/\sigma}. \tag{13}$$

Demand for sector j's resources (for example) shifts inward as the share of those resources in the production of intermediate good j increases and also shifts inward as the share of intermediate good j in production of the final good increases.

Rearranging equations (12) and (13) and then dividing, we have:

$$\left[\frac{R_{jt}}{R_{kt}}\right]^{\frac{1}{\sigma}+\frac{1}{\psi}} = \frac{\nu}{1-\nu}\left[\frac{\Psi_j}{\Psi_k}\right]^{1/\psi}\left[\frac{Y_{jt}}{Y_{kt}}\right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}}. \tag{14}$$

The change in sector j's share of resource extraction from time t to time $t+1$ therefore has the same sign as the change in sector j's share of intermediate good production. For any given quantity of resource extraction R_{jt}, increasing the average quality of technology A_{jt} increases production of the intermediate good Y_{jt}. The share of resources in production of intermediate good j falls as machine quality improves, which shifts demand for the resource R_{jt} outward in proportion to $1/\sigma$. However, intermediate good j's share of final good production increases, which shifts demand for R_{jt} inward in proportion to $1/\epsilon$. Because $\sigma < \epsilon$, the first effect dominates, so that increasing A_{jt} increases demand for R_{jt} (and does so more strongly when resources and machines are stronger complements). Thus, sector j's share of resource extraction increases from time t to $t+1$ when $s_{j(t+1)} = 1$, and it decreases from time t to $t+1$ when $s_{j(t+1)} = 0$.

When $s_{j(t+1)} \in (0,1)$, the average quality of technology improves in both sectors. The following proposition describes what happens in these intermediate cases, where we call a sector intensive in factor z if that factor's share of production in greater than half.

Proposition 1. *There exists a unique $\hat{s}_{t+1}$ such that sector j's share of resource extraction increases from time t to $t+1$ if and only if $s_{j(t+1)} \geq \hat{s}_{t+1}$. Let sector j be machine-intensive and sector k be resource-intensive. Then: $\hat{s}_{t+1}$ decreases in σ; as $\sigma \to 0$, $\hat{s}_{t+1} \to 1$; as $\sigma \to 1$, $\hat{s}_{t+1} \to 0.5$; and as $\sigma \to \infty$, $\hat{s}_{t+1} \to 0$.*

Proof. See appendix.   □

There exists an intermediate value of $s_{j(t+1)}$, labeled $\hat{s}_{t+1}$, such that each resource's share of extraction is constant over time if and only if $s_{j(t+1)} = \hat{s}_{t+1}$.

The proof of Proposition 1 shows that $\hat{s}_{t+1}$ is

$$\hat{s}_{t+1} = \frac{\Sigma_{Y_{kt},X_{kt}} C_{jt}}{\Sigma_{Y_{jt},X_{jt}} C_{kt} + \Sigma_{Y_{kt},X_{kt}} C_{jt}}, \tag{15}$$

where $\Sigma_{w,z}$ is the elasticity of w with respect to z and where $C_{jt}, C_{kt} > 0$. $\Sigma_{Y_{jt},X_{jt}}$ large relative to $\Sigma_{Y_{kt},X_{kt}}$ means that increasing machine services in each sector has an especially strong effect on production of intermediate j. In this case, if scientists are divided equally between the two sectors at time $t + 1$, then Y_j/Y_k increases from time t to $t + 1$ and, from equation (14), R_j/R_k also increases from time t to $t + 1$. Therefore, when $\Sigma_{Y_{jt},X_{jt}}$ is relatively large, the research allocation that holds R_j/R_k constant from time t to $t + 1$ must have $s_{j(t+1)} < 0.5$.

Now consider what makes $\Sigma_{Y_{jt},X_{jt}}$ large or small relative to $\Sigma_{Y_{kt},X_{kt}}$. Note that $\Sigma_{Y_{jt},X_{jt}} = (1 - \kappa)(X_{jt}/Y_{jt})^{\frac{\sigma-1}{\sigma}}$. Increasing machine services in each sector by the same percentage has an especially strong effect on intermediate good production in the relatively advanced, machine-intensive sector if and only if $\sigma > 1$. Intuitively, if machines substitute for resources ($\sigma > 1$), then intermediate good producers respond more strongly to improving machines when machines are relatively abundant, but if machines complement resources ($\sigma < 1$), then intermediate good producers respond more strongly to improving machines when machines are relatively scarce. Thus, if sector j is relatively machine-intensive, then the allocation of scientists that holds extraction constant must have $s_{j(t+1)} < 0.5$ if and only if $\sigma > 1$.

Finally, note that as $\sigma \to 1$, the elasticity of intermediate good production with respect to machines is constant. Each sector responds to improved technology in the same way, regardless of how advanced it is. In this special case, $\hat{s}_{t+1} = 0.5$. Extraction then shifts towards whichever sector is advancing more rapidly.

3.2   Transitions and Lock-In

Now consider a case in which, without loss of generality, sector j is relatively advanced and is increasing its share of extraction and research over time.

Assumption 1. *Sector j is relatively more advanced than it is abundant $(A_{j(t-1)}/A_{k(t-1)} \geq \Psi_j/\Psi_k)$, and sector j increasingly dominates technology ($s_{jt} \geq 0.5$), extraction ($R_{j(t+1)}/R_{k(t+1)} \geq R_{jt}/R_{kt}$), and research ($s_{j(t+1)} \geq s_{jt}$).*

Define a *transition in research* as occurring at the first time that s_{jt} begins declining, a *transition in extraction* as occurring at the first time that R_{jt}/R_{kt} begins declining, and a *transition in technology* as occurring at the first time that A_{jt}/A_{kt} begins declining. Finally, define resource j as being *locked-in* from time t when no type of transition occurs after t. We have the following result:

Proposition 2. *Let Assumption 1 hold at some time t. If $\sigma > 1$, then resource j is locked-in from time t. If $\sigma < 1$, then a transition in extraction occurs only after a transition in research and a transition in technology occurs only after a transition in extraction. If resource j is relatively abundant ($\Psi_j \geq \Psi_k$), then a transition in technology occurs while sector j still provides the larger share of resource supply.*

We see two cases. First, if machines are energy-saving ($\sigma > 1$), then a transition cannot happen. The economy is locked-in to the dominant sector. The condition that $A_{j(t-1)}/A_{k(t-1)} \geq \Psi_j/\Psi_k$ ensures that $\hat{s}_{t+1} \leq 0.5$. Thus, sector j increases its share of resource supply whenever it dominates research effort. And when sector j is both increasing its share of resource supply and dominating research effort, the resource channel and the innovation channel in equation (10) are both positive. Sector j therefore increases its dominance of research effort over time and further increases its technological advantage over sector k. Sector j's increasing share of resource supply and its increasing share of research activity form a positive feedback loop that prevents sector k from ever catching up: sector j's increasingly improved technology and increasing share of resource extraction both work to attract ever more scientists to sector j, and the improving relative quality of technology in sector j works to increase its share of extraction over time.

The dynamics are qualitatively different if machines are energy-using ($\sigma < 1$). Now sector j's dominant share of research activity works to push scientists away from sector j by making the innovation channel negative in equation (10). As long as sector j's share of research effort is increasing, its improving relative technology works to increase its share of resource supply, but eventually the strengthening supply expansion effect succeeds in pushing scientists back towards sector k. At this point a transition in research occurs. Eventually, sector j's share of research effort can fall below $\hat{s}$, at which point a transition in extraction occurs. The transition in extraction is *innovation-led*: it can occur only after the transition in research. Even though research transitions before extraction, sector k does not begin to dominate research effort (triggering a transition in technology) until sometime after the transition in extraction, when both the resource channel and the innovation channel in equation (10) work to push scientists towards sector k. If resource j is relatively abundant, then a transition in technology must happen while sector j still dominates resource supply. So just as the transition in extraction must follow a transition in research, so too a change in the sector that dominates resource supply must follow a change in the sector that dominates research.

3.3   Long-Run Outcomes

Now consider long-run outcomes. We focus on a path along which the average quality of each type of technology improves at the same rate. Begin by studying energy-using machines ($\sigma < 1$):

Proposition 3. *Assume $\sigma < 1$. The following are true along a path with identical growth in each type of technology:*

  1. $s_{jt} = 0.5$.

  2. R_{jt}/R_{kt} is constant.

  3. $\Psi_j > \Psi_k$ if and only if $A_{jt} > A_{kt}$.

4. $A_{jt} > A_{kt}$ if and only if $R_{jt} > R_{kt}$.

5. If $\Psi_j = \Psi_k$ and $\nu = 0.5$, then $A_{jt} = A_{kt}$ and $R_{jt} = R_{kt}$.

6. $R_{j(t+1)}/R_{jt}$ and $R_{k(t+1)}/R_{kt}$ each equal $\left(1 + \frac{1}{2}\eta\gamma\right)^{\frac{\psi}{\psi - \alpha/(1-\alpha)}}$, which increases in α and decreases in ψ.

Proof. See appendix. $\square$

The condition of identical growth rates for each type of technology implies that A_{jt}/A_{kt} is constant over time. A_{jt}/A_{kt} is constant over time if and only if $s_{jt} = 0.5$ (result 1), and via equation (A-1), any equilibrium that keeps $s_{jt} = 0.5$ for multiple periods must have R_{jt}/R_{kt} constant over those periods (result 2). Thus, identical growth in each type of technology implies identical growth in extraction of each type of resource. R_{jt}/R_{kt} constant over time requires $s_{j(t+1)} = \hat{s}_{t+1}$, as defined in Proposition 1. For this condition to hold with $\sigma < 1$ along the path with A_{jt}/A_{kt} constant, the sector with the more abundant resource must have more advanced technology and a greater share of extraction (results 3 and 4). Extraction of each type of resource is increasing over time (result 6, using $\psi > \alpha/(1-\alpha)$).

Now consider long-run outcomes when machines are energy-saving ($\sigma > 1$). There is a knife-edge case in which research is divided equally between the two sectors so that the relative quality of technology remains constant. However, in general, this economy will not approach a path with identical growth in each type of technology. Instead, it tends to be locked-in to the more advanced sector, in which case the quality of technology and the quantity of resource extraction always grow faster in the locked-in sector.

3.4   Numerical Example

In order to make these ideas more concrete, Figure 3 plots the evolution of sector j's share of extraction and of research activity, starting from a point at which sector j is more advanced.[12] Sector j begins with the majority of extraction and research activity, and its share of each is initially increasing. In the case of energy-saving technologies (left panel, $\sigma = 2$), research activity and extraction are locked-in to sector j, which attracts all research effort in all periods and increases its share of resource extraction over time. In the case of energy-using technologies (right panel, $\sigma = 0.5$), we see the type of innovation-led transition described above. A transition in research occurs after 10 periods, a transition in extraction occurs 3 periods later, a transition in technology occurs 5 periods after that, and a change in the sector that dominates resource supply occurs only after another 18 periods. After that point, the supply expansion effect works to push scientists back towards sector j. The economy then approaches a long-run allocation with research activity and extraction divided

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[12]The example's parameters are $\epsilon = 3$, $\nu = 0.45$, $\alpha = 0.5$, $\kappa = 0.5$, $\psi = 3$, $\Psi_j = \Psi_k = 1$, $\eta = 1$, $\gamma = 0.5$, $A_{j0} = 0.5$, and $A_{k0} = 0.005$. The qualitative results are not sensitive to the choice of parameters.

(a) $\sigma = 2$ (b) $\sigma = 0.5$
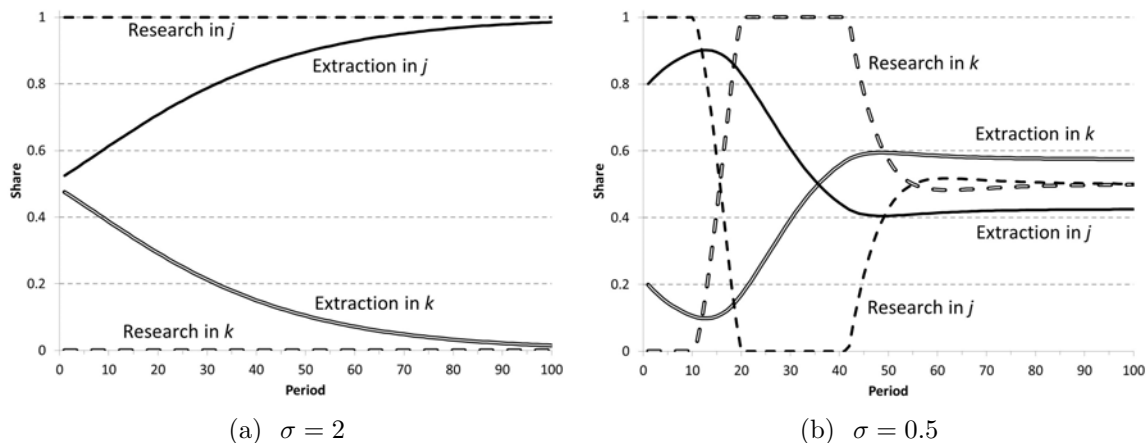
Figure 3: An example of lock-in (left, with $\sigma = 2$), and an example of an innovation-led transition (right, with $\sigma = 0.5$).

equally between the two sectors (producing identical growth in the average quality of each technology) and with extraction also divided equally between the two sectors (because $\Psi_j = \Psi_k$).

The endogenous dynamics of our setting with energy-using machines are qualitatively similar to historical patterns. Figure 4 plots resource shares since 1800. The historical patterns in these shares are similar to the patterns that emerge from our numerical simulations with energy-using machines: resource shares change slowly at first, change faster as a transition occurs, and stabilize at some nonzero long-run level after a transition has occurred. The historical patterns are nothing like the patterns that emerge from our simulations with energy-saving machines. This long-run evidence coheres with the shorter-run empirical evidence (described in the introduction) that energy and non-energy factors of production tend to be gross complements.

4   Climate Change Policy

We now consider the implications of the present model for policies to address climate change. Let resource j be a fossil fuel resource and resource k be a clean, renewable resource. Consuming resource j generates greenhouse gas emissions that eventually warm the climate and thereby damage the economy. These emissions' consequences are external to actors in the economy. The policymaker can use emission taxes and/or research subsidies to steer the economy towards a trajectory that provides greater welfare. In contrast to standard climate-economy models (e.g., Nordhaus, 2008), the cost of reducing emissions at time t is endogenous: this cost depends on the supply of each resource, on the time t quality of the
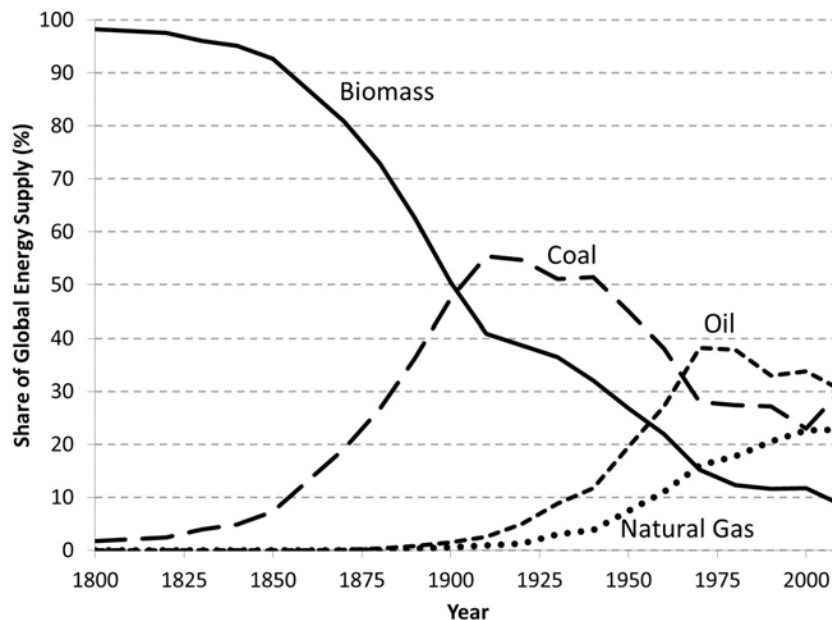
Figure 4: Shares of global energy supply, from Smil (2010). This plot excludes hydroelectricity and nuclear, whose shares never exceed 7%.

machines for using each type of resource, and on the substitutability of each type of energy for the other. I first describe the calibration before presenting laissez-faire trajectories and analyzing optimal policy within this model.

4.1   Calibration

Begin by considering the supply of each type of resource, where we now allow ψ to differ by sector. McCollum et al. (2014) develop supply curves for coal, oil, and gas for the MESSAGE energy model. Combining these resources into a single supply curve for energy, I estimate $\psi_j = 1.58$. I calculate the emission intensity of the fossil resource from world carbon dioxide emissions from fossil fuels in 2011 and from initial consumption of the fossil resource (described below). Drawing in part on the work of others, Johnson et al. (2017) describe the supply of power from solar photovoltaics, concentrating solar power, onshore wind, and offshore wind available by region of the world and by resource quality.[13] Aggregating across

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[13]Costs are reported in dollars per unit power and resource potential is reported in units of energy. I convert costs to dollars per unit electrical energy by using the capacity factor reported for each resource quality bin in each region. This capacity factor adjusts for the fact that the power producible from renewable resources is not available throughout the day or throughout the year. And I convert dollars per unit of electrical energy to dollars per units of energy in the resource by using the efficiency of each type of generator. From the Energy Information Administration's Annual Energy Review 2011, the efficiencies are 12% for solar

resource types and regions, I estimate $\psi_k = 3.00$. I calibrate Ψ_j and Ψ_k so that the initial price of each resource is consistent with Energy Information Administration forecasts for 2018, with natural gas generation being the marginal use of the fossil resource and solar photovoltaics being the marginal use of the renewable resource. Finally, I allow the fossil resource to be depletable. I model depletion by replacing R_{jt} in equation (4) with $\sum_{i=0}^{t} R_{ji}$ and adjusting the economy's aggregate resource constraint appropriately.[14]

I fix σ at 0.1, in line with evidence in Hassler et al. (2012) that σ is near 0.[15] I fix α at 0.5 and η at 0.02. I calibrate $1 - \kappa$ to match the share of machines in intermediate-good production in Acemoglu et al. (2012), which yields $\kappa = 2/3$. I explore cases in which fossil and renewable energy are stronger ($\epsilon = 10$) and weaker ($\epsilon = 2$) substitutes.

We have five remaining parameters: A_{j0}, A_{k0}, A_Y, ν, and γ. I calibrate these so that the first period's equilibrium matches conditions on R_{j0}, R_{k0}, Y_0, and s_{k0} as well as a condition on the growth rate of final-good production. Initial fossil resource consumption R_{j0} comes from summing the consumption of oil, gas, and coal from 2011–2015, as reported in the BP Statistical Review of World Energy. Doubling this value yields $R_{j0} = 4666$ EJ over the first ten-year timestep. Using the analogous values for non-hydro renewables yields $R_{k0} = 224$ EJ.[16] The NSF's Business Research and Development and Innovation 2014 gives worldwide employment in fossil R&D as 19,000 people and in renewable R&D as 2,000 people.[17] This implies $s_{k0} = 0.0952$. World Bank data for global output from 2011–2015 implies $Y_0 = 765$ trillion year 2014 dollars over the first ten-year timestep. I require Y_t to grow at an annual rate of 2% from the first to the second timestep, which is consistent with the benchmark DICE-2007 integrated assessment model (Nordhaus, 2008). Matching these five conditions yields $A_{j0} > A_{k0}$, which is consistent with the intuition that fossil resources currently have better technology.[18] A tax of \$10 (\$250) per tCO_2 reduces emissions in the initial period by nearly 3% (44%) when $\epsilon = 10$ and by around 1% (27%) when $\epsilon = 2$. The emission reductions from a tax of \$250 per tCO_2 in the case with $\epsilon = 10$ are consistent with estimates in Calvin et al. (2017, Figure S9).

The evolution of carbon dioxide (CO_2) and temperature follow Lemoine and Rudik (2014). Warming of T_t degrees Celsius reduces time t output to $Y_t/(1 + D(T_t))$, where

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photovoltaics, 21% for solar thermal, and 26% for wind.

[14]When modeling depletion, I assume that fossil resource owners have a one-period property right to a unit of the resource, which is consistent with the model of patenting.

[15]I was unable to successfully calibrate the model with σ near 1 or greater than 1, which I take as further evidence that machines are energy-using.

[16]To obtain the energetic content of renewables from the reported tonnes of oil equivalent, use BP's assumed thermal efficiency of 38% to obtain the equivalent electrical energy and then use a 20% generator efficiency to convert electrical energy to energy in the renewable resource (see footnote 13).

[17]For fossil R&D, I use NAICS 21 ("mining, extraction, and support activities"), and for renewable R&D, I use NAICS 22 ("utilities").

[18]The calibrated initial technologies are not sensitive to choosing $\epsilon = 2$ or $\epsilon = 10$. The effect of ϵ on the results will therefore not be driven by differences in initial technologies.

$D(T_t), D'(T_t) > 0$. The calibration of $D(T_t)$ follows DICE-2007 (Nordhaus, 2008). Intertemporal welfare is a function of final-good production and takes the standard power utility form. Consistent with Nordhaus (2008), the utility discount rate is 1.5% per year and the elasticity of intertemporal substitution is 0.5.[19] I use a ten-year timestep and a horizon of 400 years.

4.2   Laissez-Faire

Figure 5 plots laissez-faire outcomes, for cases with a larger ($\epsilon = 10$, filled) and a smaller ($\epsilon = 2$, hollow) elasticity of substitution between fossil and renewable energy and for cases in which the fossil resource is (connected) and is not (dashed) vulnerable to depletion. It also plots cases (dotted) in which the quality of each technology is held fixed at its initial level. The top left panel shows that, when the fossil resource is vulnerable to depletion, research activity has already transitioned from the fossil sector to the renewable sector if the two types of energy are weak substitutes and transitions later in the century if the two types of energy are strong substitutes. The transition is completed around the same time in either case, so that the renewable sector attracts all research activity from 2100–2200. However, when the fossil resource is not vulnerable to depletion, research activity does not transition towards the renewable sector within the next 100 years. Depletion therefore plays a critical role in speeding up a transition in innovation.

The top right panel shows that a transition in resource supply begins a few decades later than the transition in research, but the transition is slow: the renewable resource only begins to dominate supply around the middle of the next century.[20] The middle and bottom panels of Figure 5 show that the laissez-faire transition in resource supply would happen too late to avoid substantial climate change. The middle panels show that fossil resource use (and thus CO_2 emissions) remains greater than its year 2015 value for at least the next two centuries. As a result, CO_2 increases substantially from its present-day level (bottom left panel). Total warming exceeds the internationally agreed limit of 2°C late in the present century and exceeds 4°C around 2200 (bottom right panel). Policy will be required to avoid the rather large degree of climate change seen in laissez-faire.[21]

Innovation and depletion are both critical to the possibility of a timely laissez-faire transition in resource supply. The renewable resource's share of supply remains below 10% for much of the next 200 years if either depletion or innovation is not possible (top right panel). Innovation and depletion are also both critical to the climate challenge, but in different

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[19]I also analyze cases with an annual utility discount rate of 0.01%, consistent with Stern (2007). As I will describe below, the results are not sensitive to this change in discount rate.

[20]The renewable resource's share of supply comes close to 100% around 2300. The renewable technology becomes better than the fossil technology around 2225 when $\epsilon = 2$ and around 2300 when $\epsilon = 10$.

[21]The next 200 years' temperature trajectories (and, in the case of weak substitutes, also the CO_2 trajectory) are broadly consistent with the laissez-faire results of the DICE-2007 integrated assessment model (Nordhaus, 2008), even though the modeled economies are quite different.

ways. First, if the fossil resource were not vulnerable to depletion (dashed lines), then its equilibrium use would grow rapidly for at least 200 years (middle left panel), and its emissions would bring CO_2 and temperature to remarkably high levels (bottom panels). The possibility of depletion is critical to avoiding laissez-faire scenarios with an extreme degree of climate change.

Now consider what happens when the fossil resource is depletable but innovation is not possible (dotted lines).[22] When the quality of each technology is fixed at its initial level, use of each resource declines over time (middle panels), so that emissions also fall over time. CO_2 increases for the next few decades before beginning to decline (bottom left panel). Temperature increases for a longer period due to the climate system's inertia, but it remains below 2°C (bottom right panel). Thus, without innovation, even the laissez-faire economy would keep temperature within the level targeted by recent international agreements. Innovation in fossil technologies creates the climate challenge by increasing laissez-faire demand for fossil resources over time. We will next see how redirecting innovation can help to address the climate challenge.
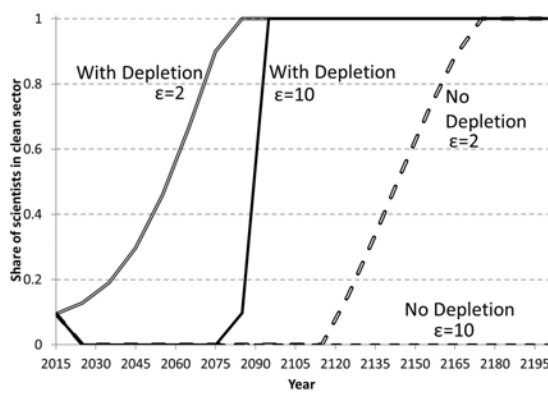
4.3   Policy

We now consider how a policymaker would steer the economy to maximize intertemporal welfare. Begin by considering a case in which the policymaker only has access to an emission tax. Figure 6 plots the evolution of key variables when the fossil resource is depletable, for the case of strong substitutes ($\epsilon = 10$, solid) and weak substitutes ($\epsilon = 2$, dashed). For reference, it also plots the evolution of the laissez-faire economy (dotted).

The top left panel plots the optimal emission tax for this model.[23] The tax starts at a high level (\$150–300 per tCO_2) in order to incentivize scientists to shift to the renewable sector. In the case of strong substitutes, all scientists work in the renewable sector from the first period on, and in the case of weak substitutes, more than half of the scientists work in the renewable sector from the first period on (with all scientists working in the renewable sector by 2075). Once the renewable sector's technology has improved sufficiently, all scientists would remain in the renewable sector even in the absence of an emission tax. At that point, the regulator drops the emission tax to a very low level and waits for the renewable resource's technology to improve. After a few decades, the regulator begins raising the emission tax again in order to hasten the transition in resource supply.[24]
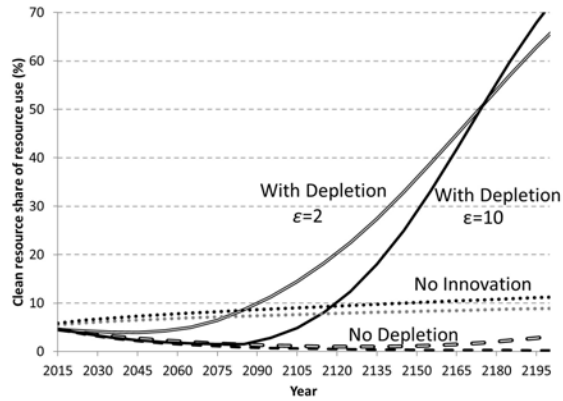
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[22]The results without innovation are nearly identical for either weak or strong substitutability between the two types of energy.

[23]Technically, it plots the constrained-optimal emission tax for this model. The fully optimal emission tax would match marginal damage along a pathway in which all of the distortions in the economy were corrected. These distortions include market power in machine production, externalities in innovation, externalities from emissions, and intertemporal market failures in supply of the depletable resource. See Acemoglu et al. (2012).
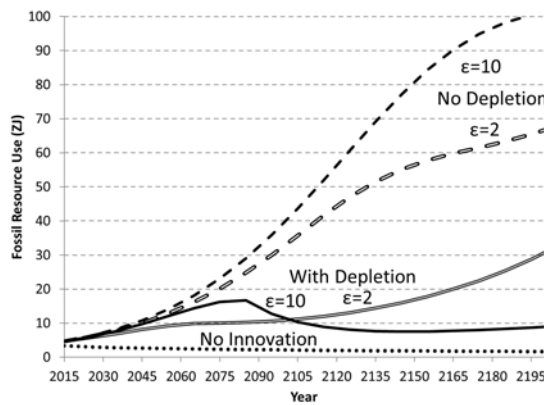
[24]The results are not sensitive to using a much smaller annual utility discount rate of 0.1%. In that case, the most notable difference is that the initial emission tax is much higher in the case of weak substitutes so
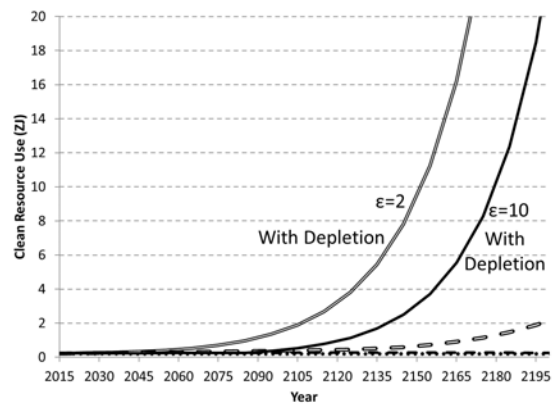
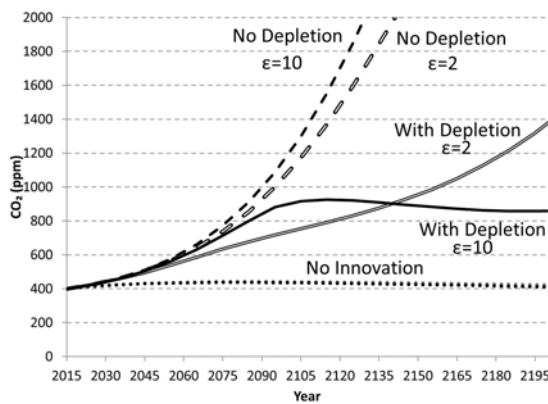(a) Renewable Resource's Share of Research

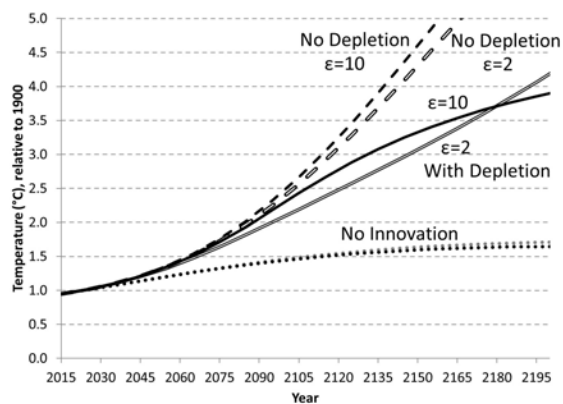(b) Renewable Resource's Share of Supply

(c) Fossil Resource Use
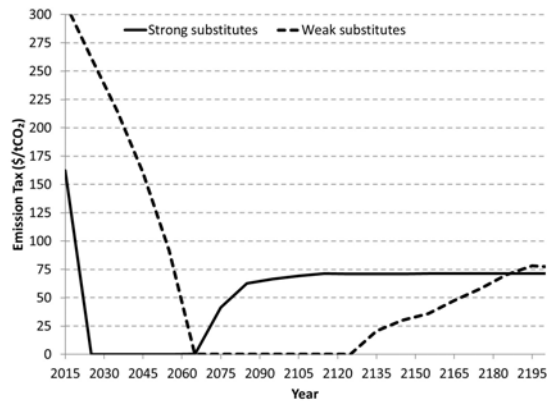
(d) Renewable Resource Use
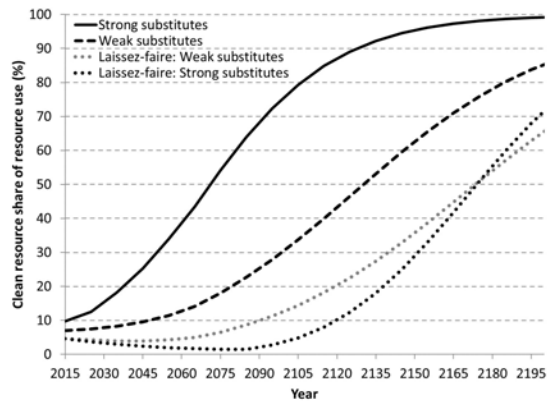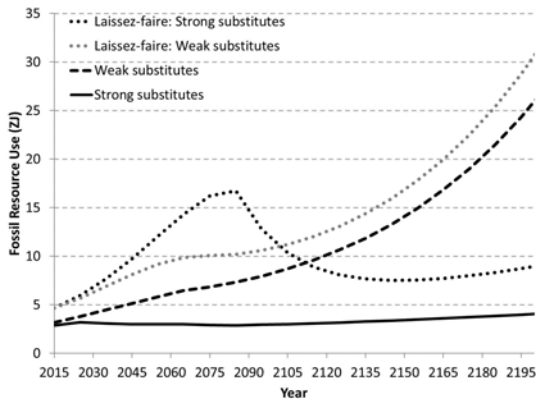
(e) Carbon Dioxide

(f) Temperature

Figure 5: The evolution of the laissez-faire economy. Dashed lines do not allow for depletion in the fossil resource, and dotted lines hold technology fixed for both resources. Filled lines use $\epsilon = 10$ ("strong substitutes"), and hollow lines use $\epsilon = 2$ ("weak substitutes").
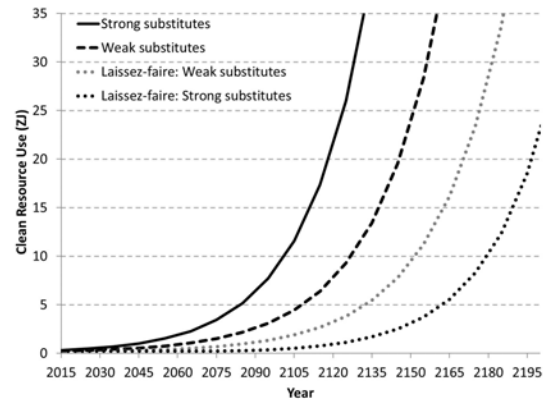
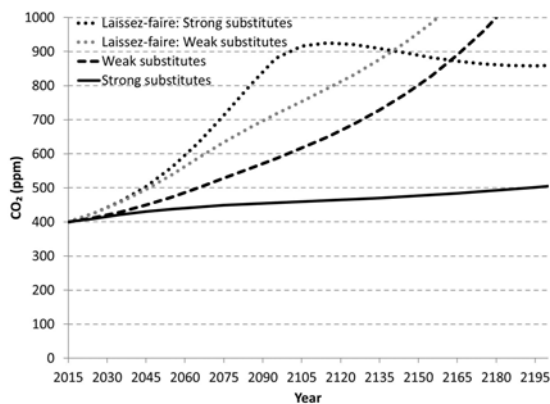(a) Emission Tax

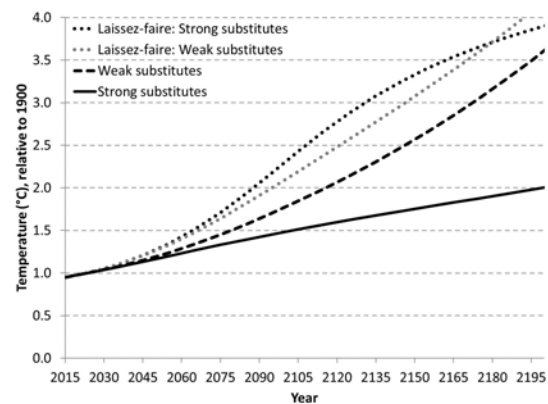(b) Renewable Resource's Share of Supply

(c) Fossil Resource Use

(d) Renewable Resource Use

(e) Carbon Dioxide

(f) Temperature

Figure 6: The evolution of the economy when the policymaker has access to an emission tax (solid: $\epsilon = 10$; dashed: $\epsilon = 2$) and under laissez-faire (dotted).

The top right panel and middle panels describe this transition in resource supply. In every period, the regulated economy uses more of the renewable resource and less of the fossil resource than did the unregulated economy. When the two types of energy are strong substitutes, optimal policy for this model does not let fossil resource use increase appreciably from current levels, so that the renewable resource soon comes to dominate supply. In contrast, fossil resource use increases over time when the two types of energy are only weak substitutes, even though the fossil resource's share of total supply falls over time. These different resource trajectories arise because, as described in Section 4.1, the endogenous cost of reducing emissions is sensitive to the substitutability of renewable energy for fossil energy. These different trajectories of fossil resource use have severe implications for the optimized evolution of the climate (bottom panels). Policy substantially constrains the CO_2 and temperature trajectories when the two types of energy are strong substitutes, even preventing temperature from exceeding the benchmark level of 2°C by 2200. In contrast, when the two types of energy are weak substitutes, policy restrains both CO_2 and temperature relative to laissez-faire, but both environmental indicators nonetheless continue increasing into the distant future.[25]

I also solve for optimal policy when the modeled regulator can use both a tax on emissions and a subsidy for research into technologies that use the renewable resource. In this case, the initial emission tax is very small because the policymaker uses the research subsidy, not the emission tax, to shift scientists to the renewable sector in the first periods. The renewable resource's technology soon advances sufficiently to allow the policymaker to eliminate the research subsidy. The evolution of resource supply and of the climate are not notably different from the case with only an emission tax. In fact, the evolution of these variables is also not notably different when the policymaker has access to a research subsidy but not an emission tax. The most important policy objective is to shift scientists quickly towards the renewable sector, and the policymaker can use either an emission tax or a research subsidy to achieve this goal. The later use of the emission tax to hasten the transition in resource supply is secondary.

The policymaker achieves similar outcomes with either instrument, but which instrument is more important? Removing access to the research subsidy reduces the stationary consumption-equivalent to welfare by around 1.5% (8.2%) when the two types of energy are strong (weak) substitutes, but removing access to the emission tax has only a negligible effect on welfare.[26] Both instruments can shift innovation to the desired trajectory and thereby redirect resource supply in later decades, but the emission tax imposes extra costs because

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as to shift scientists to the renewable sector faster. The evolution of CO_2 and temperature are fairly similar. Experiments in which the fossil resource is not vulnerable to depletion also yield qualitatively similar results.

[25]The optimized temperature trajectory in the case with weak substitutes is broadly consistent with the next 200 years' optimized path in DICE-2007 (Nordhaus, 2008).

[26]The stationary-equivalent consumption to welfare w is the constant consumption level c that would provide welfare w if maintained forever.

it redirects innovation by constraining near-term resource supply. In this model, the most effective way to control climate change is to rapidly transition scientists towards research into technologies for using the renewable resource, and the research subsidy is a more efficient way of accomplishing that transition.

Finally, Table 1 describes the cost of delaying policy by 50 years, measured as the percentage change in stationary-equivalent consumption relative to a case in which both an emission tax and a research subsidy were available from the first period on. Comparing the rows shows that at this late date, the regulator would slightly prefer to have access to an emission tax rather than a research subsidy. As seen in Figure 5, the renewable sector will soon dominate innovation with or without policy, so the emission tax's role in redirecting resource supply becomes more important. Comparing the rows also shows that delay is more costly in the case with strong substitutes, largely because the difference between the direction of research in the regulated and laissez-faire economies is greater.

The columns undertake experiments wherein some state variables are reset to their initial values at the end of the 50-year interval of delay. These experiments reveal the drivers of the cost of delay. Comparing the first and the second columns, we see that the policymaker can achieve greater welfare if, after the forced 50 years of delay, she begins to optimize policy with the evolved technologies rather than with the initial technologies, even though the majority of scientists work in the fossil sector throughout the delay interval and thus will have advanced fossil technology relative to renewable technology. The interval of delay does impose substantial costs on the policymaker (reducing stationary-equivalent consumption by 10–30% when technology evolves) because scientists suboptimally direct technology towards the fossil resource, but the benefits of technological change nonetheless outweigh the costs of seeming to lock in fossil technology even further.[27] In contrast, comparing the third and the first columns (and also the fourth and the second columns) shows that the policymaker would prefer to keep the initial CO_2 and temperature levels rather than have to manage the higher levels arising from the next 50 years' emissions, but the benefits of resetting CO_2 and temperature are small (reducing the cost of delay by only 0.2 percentage points). Environmental lock-in is not as costly as technological lock-in, which is not as costly as preventing technology from advancing at all.

5   Conclusion

We have seen that complementarities between resources and machines are critical to the possibility of innovation-led transitions in factor use, such as was seen in the history of

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[27]The stationary-equivalent consumption loss from delaying policy for the entire 400-year horizon is 39.0% in the case of strong substitutes and 12.4% in the case of weak substitutes. The 50-year delay therefore eats up most of the potential gains from optimizing policy. Further, the losses from optimizing policy after a 50-year interval with technology reset to its initial value are greater than the losses from following the laissez-faire path forever.

Table 1: Stationary-equivalent consumption loss (%) from delaying policy for 50 years, relative to having both instruments and no delay in implementation.

	Variables that evolve over the delay interval			
	Depletion	Depletion+Tech	Depletion+Climate	Depletion+Tech+Climate
Strong substitutes				
Emission tax only	40.5	31.4	40.7	31.6
Research subsidy only	40.5	30.5	40.7	30.7
Weak substitutes				
Emission tax only	33.8	12.0	34.1	12.2
Research subsidy only	33.7	11.5	33.9	11.7

Notes: The columns vary which state variables are reset to their initial values after the interval of delay and which continue evolving. The evolution of the state variables during the interval of delay is unchanged across columns. For example, the third column fixes $A_{j5} = A_{j0}$ and $A_{k5} = A_{k0}$ (where $t = 5$ corresponds to 50 years), but it does allow $A_{jt} > A_{j0}$ and $A_{kt} > A_{k0}$ for $t \in \{1, 2, 3, 4\}$.

energy supply. These complementarities can push scientists away from the more advanced sector, and the redirection of scientific effort can eventually redirect factor use away from the dominant sector. We have also seen that the current economy may be transitioning away from fossil fuels in laissez-faire, driven by the combination of endogenous innovation and the depletability of the fossil resource. A transition in research effort would lead the transition in resource supply. However, the transition in resource supply would occur too slowly to limit climate change to levels consistent with recent international agreements. A primary role for policy is to shift near-term research effort towards renewable resources so as to slow the increase in fossil resource use and speed the transition to renewable resource use.

Future work should explore three extensions. First, we have abstracted from questions about the durability of capital investments and infrastructure. Complementarities may be present between the existing stock of machines and particular resources. For instance, coal power plants have lifetimes of fifty years or more. Future work should analyze the interaction between durable capital stocks and the direction of innovation.

Second, we have followed previous literature in abstracting from expectations. The assumption that patents last for only a single period means that agents solve static problems. This assumption is defensible when the model is calibrated with long timesteps. However, some policymakers may commit to future policies so as to influence the current direction of research. Allowing expectations to matter could increase an emission tax's ability to direct research without constraining current resource supply. Future work should relax the assumption of single-period patent lifetimes in order to explore the interaction between expectations and policy.

Third, we have used a standard increasing returns model of innovation. However, en-

ergy sector innovation may be subject to particular forms of diminishing returns, as when technologies approach thermodynamic limits. Future work should estimate the returns to energy sector innovation in order to further understand the implications of directed technical change for climate policy.

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Appendices

The first appendix considers the stability of each period's equilibrium, and the second appendix contains proofs and derivations.

First Appendix: Tâtonnement Stability

One may be concerned that interior equilibria are not "natural" equilibria in the presence of positive feedbacks from resource extraction to innovation and of potential complementarities. Indeed, Acemoglu (2002) and Hart (2012) have emphasized the role of knowledge spillovers in allowing interior research allocations to be stable in the long run. This appendix shows that interior equilibria are in fact "natural" equilibria in the present setting.

Substituting from the resource supply functions, we can rewrite equation (9) as

$$
\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)} + \gamma A_{j(t-1)}}{A_{k(t-1)} + \gamma A_{k(t-1)}} \left(\frac{A_{j(t-1)} + \eta\gamma s_{jt} A_{j(t-1)}}{A_{k(t-1)} + \eta\gamma s_{kt} A_{k(t-1)}} \right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}} \left(\frac{R_{jt}}{R_{kt}} \right)^{\frac{1+\sigma/\psi}{\sigma+\alpha(1-\sigma)}} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{-\sigma/\psi}{\sigma+\alpha(1-\sigma)}}.
$$

(A-1)

Rearranging and using $s_{jt} + s_{kt} = 1$, we obtain s_{jt} as an explicit function of $A_{j(t-1)}/A_{k(t-1)}$ and of R_{jt}/R_{kt} at an interior allocation.[28] Substituting into equations (12) and (13) then gives us two equations in two unknowns, which define the equilibrium R_{jt} and R_{kt} that clear the markets for each resource.

Define the tâtonnement adjustment process and stability as follows:

Definition A-1. *A tâtonnement adjustment process increases R_{jt} if equation (12) is not satisfied and its right-hand side is greater, decreases R_{jt} if equation (12) is not satisfied and its left-hand side is greater, and obeys analogous rules for R_{kt} using equation (13). We say that an equilibrium (R_{jt}^*, R_{kt}^*) is tâtonnement-stable if and only if the tâtonnement adjustment process leads to (R_{jt}^*, R_{kt}^*) from (R_{jt}, R_{kt}) sufficiently close to (R_{jt}^*, R_{kt}^*).*

The tâtonnement process changes R_{jt} and R_{kt} so as to eliminate excess supply or demand, and tâtonnement stability requires that this adjustment process converge to an equilibrium point from values close to the equilibrium. We can show that our equilibrium is tâtonnement-stable:

Proposition A-1. *The equilibrium is tâtonnement-stable.*

Proof. See appendix.   □

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[28]Technically, this function should be written to allow for corner solutions in the research allocation. The proof of stability will account for corner solutions.

A Walrasian auctioneer would find our equilibrium at any time t.

Now use equations (12) and (13) to define R_{jt} and R_{kt} as functions of s_{jt},[29] and then restate equation (A-1) as a function only of s_{jt}:

$$\frac{\Pi_{jt}}{\Pi_{kt}} = \frac{A_{j(t-1)} + \gamma A_{j(t-1)}}{A_{k(t-1)} + \gamma A_{k(t-1)}} \left(\frac{A_{j(t-1)} + \eta\gamma s_{jt} A_{j(t-1)}}{A_{k(t-1)} + \eta\gamma(1 - s_{jt}) A_{k(t-1)}} \right)^{\frac{-1}{\sigma+\alpha(1-\sigma)}} \left(\frac{R_{jt}(s_{jt})}{R_{kt}(s_{jt})} \right)^{\frac{1+\sigma/\psi}{\sigma+\alpha(1-\sigma)}} \left[\frac{\Psi_j}{\Psi_k} \right]^{\frac{-\sigma/\psi}{\sigma+\alpha(1-\sigma)}}.$$
$$(\text{A-2})$$

The following corollary gives us the derivative of Π_{jt}/Π_{kt} with respect to s_{jt}:

Corollary A-2. *The right-hand side of equation* (A-2) *strictly decreases in s_{jt}.*

Proof. See appendix. □

The supply expansion effect makes the relative incentive to research in sector j decline in the number of scientists working in sector j. However, when sector j's share of resource extraction increases in the relative quality of its technology, we have a positive feedback between research and extraction that maintains sector j's research incentives even as more scientists move to sector j. The proof shows, as is intuitive, that whether the relative incentive to research in sector j declines in the number of scientists working in sector j is identical to whether the equilibrium is tâtonnement-stable: tâtonnement-stability is not consistent with positive feedbacks that are strong enough to overwhelm the supply expansion effect. And we have already seen that interior equilibria are in fact tâtonnement-stable.

Second Appendix: Proofs and Derivations

This second appendix derives useful intermediate results before providing proofs and derivations omitted from the main text.

Useful Lemmas

First, note that equations (7) and (8) imply

$$X_{jt} = \left[\frac{1 - \kappa}{\kappa} p_{jRt} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} A_{jt}. \tag{A-3}$$

---

[29]Rearrange equations (12) and (13) to put all terms on the right-hand side. For given s_{jt}, the Jacobian of this system in R_{jt} and R_{kt} is negative definite.

Rearranging equation (A-1) and using $s_{jt} + s_{kt} = 1$, we obtain s_{jt} as an explicit function of $A_{j(t-1)}/A_{k(t-1)}$ and of R_{jt}/R_{kt} at an interior allocation:

$$s_{jt}\left(\frac{R_{jt}}{R_{kt}}, \frac{A_{j(t-1)}}{A_{k(t-1)}}\right) = \frac{(1+\eta\gamma)\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)}\frac{R_{jt}}{R_{kt}}\left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma} - 1}{\eta\gamma + \eta\gamma\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)}\frac{R_{jt}}{R_{kt}}\left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma}}. \quad (A-4)$$

Let $\Sigma_{x,y}$ represent the elasticity of x with respect to y, and let $\Sigma_{x,y|z}$ represent the elasticity of x with respect to y holding z constant. The following lemma establishes signs and bounds for key elasticities:

Lemma A-3. *The following hold, with analogous results for sector k:*

1. $\Sigma_{Y_t,Y_{jt}}, \Sigma_{Y_t,Y_{kt}} \in [0,1]$ *and* $\Sigma_{Y_t,Y_{jt}} + \Sigma_{Y_t,Y_{kt}} = 1$.

2. $\Sigma_{Y_{jt},R_{jt}|X_{jt}}, \Sigma_{Y_{jt},X_{jt}} \in [0,1]$ *and* $\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}} = 1$.

3. $\Sigma_{X_{jt},A_{jt}} = \frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha} \in (0,1)$

4. $\Sigma_{X_{jt},R_{jt}} = \frac{\alpha\sigma/\psi+\alpha}{\sigma(1-\alpha)+\alpha} \in (0,1)$ *if and only if* $\psi > \frac{\alpha}{1-\alpha}$

5. $\Sigma_{A_{jt},s_{jt}} = \frac{\eta\gamma s_{jt}}{1+\eta\gamma s_{jt}} \in [0,1)$

6. $\Sigma_{s_{jt},R_{jt}} = \frac{\psi+\sigma}{\psi}\frac{2+\eta\gamma}{\eta\gamma s_{jt}}Z_t > 0$, *where* $Z_t \in \left[\frac{1+\eta\gamma}{(2+\eta\gamma)^2}, \frac{1}{4}\right]$. $\Sigma_{s_{jt},R_{kt}} = -\Sigma_{s_{jt},R_{jt}}$.

7. $\Sigma_{s_{jt},A_{j(t-1)}} = -\frac{(1-\sigma)(1-\alpha)}{A_{j(t-1)}}\frac{(2+\eta\gamma)}{\eta\gamma}Z_t$, *which is* < 0 *if and only if* $\sigma < 1$. Z_t *is as above.*
$\Sigma_{s_{jt},A_{k(t-1)}} = -\Sigma_{s_{jt},A_{j(t-1)}}$.

8. $\Sigma_{s_{jt},s_{kt}} = -s_{kt}/s_{jt} \leq 0$

Proof. All of the results follow by differentiation and the definition of an elasticity. #1 follows from differentiating the final-good production function $Y_t(Y_{jt}, Y_{kt})$, #2 follows from differentiating the intermediate-good production function $Y_{jt}(R_{jt}, X_{jt})$, #3 and #4 follow from differentiating equation (A-3), #5 follows from differentiating equation (3), #6 and #7 follow from differentiating equation (A-4), and #8 follows from the research constraint.

To derive #6 and #7, define

$$Z_t \triangleq \frac{\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)}\frac{R_{jt}}{R_{kt}}\left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma}}{\left[1+\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)}\frac{R_{jt}}{R_{kt}}\left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma}\right]^2}$$

and recognize that $s_{jt} \in (0, 1)$ implies

$$\left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{-(1-\sigma)(1-\alpha)} \frac{R_{jt}}{R_{kt}} \left[\frac{[R_{jt}/\Psi_j]^{1/\psi}}{[R_{kt}/\Psi_k]^{1/\psi}}\right]^{\sigma} \in \left(\frac{1}{1+\eta\gamma}, \ 1+\eta\gamma\right)$$

from equation (A-1).

$\square$

Note that $\Sigma_{X,A}$ and $\Sigma_{X,R}$ are the same in each sector. We therefore often omit the sector subscripts on these terms.

Using $s_{jt}\left(\frac{R_{jt}}{R_{kt}}, \frac{A_{j(t-1)}}{A_{k(t-1)}}\right)$, the equilibrium is defined by equations (12) and (13), which are functions only of R_{jt} and R_{kt}. Rewrite these equations as (suppressing the technology arguments)

$$1 = \kappa\nu A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t(R_{jt}, R_{kt}, s_{jt}(R_{jt}/R_{kt}))}{Y_{jt}(R_{jt}, s_{jt}(R_{jt}/R_{kt}))}\right]^{1/\epsilon} \left[\frac{Y_{jt}(R_{jt}, s_{jt}(R_{jt}/R_{kt}))}{R_{jt}}\right]^{1/\sigma} \left[\frac{R_{jt}}{\Psi_j}\right]^{-1/\psi} \triangleq G_j(R_{jt}, R_{kt}),$$

$$1 = \kappa(1-\nu)A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t(R_{jt}, R_{kt}, s_{jt}(R_{jt}/R_{kt}))}{Y_{kt}(R_{kt}, s_{jt}(R_{jt}/R_{kt}))}\right]^{1/\epsilon} \left[\frac{Y_{kt}(R_{kt}, s_{jt}(R_{jt}/R_{kt}))}{R_{kt}}\right]^{1/\sigma} \left[\frac{R_{kt}}{\Psi_k}\right]^{-1/\psi} \triangleq G_k(R_{jt}, R_{kt}).$$

Then we have:

Lemma A-4. $\partial G_j(R_{jt}, R_{kt})/\partial R_{jt} < 0$ and $\partial G_k(R_{jt}, R_{kt})/\partial R_{kt} < 0$.

Proof. Differentiating, we have:

$$\frac{\partial G_j(R_{jt}, R_{kt})}{\partial R_{jt}} = G_j \left\{ -\left(\frac{1}{\psi}+\frac{1}{\sigma}\right)\frac{1}{R_{jt}} + \left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)\frac{1}{Y_{jt}}\left[\frac{\partial Y_{jt}}{\partial R_{jt}}+\frac{\partial Y_{jt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\right] \right.$$

$$\left. + \frac{1}{\epsilon}\frac{1}{Y_t}\left[\frac{\partial Y_t}{\partial Y_{jt}}\frac{\partial Y_{jt}}{\partial R_{jt}}+\frac{\partial Y_t}{\partial Y_{jt}}\frac{\partial Y_{jt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}+\frac{\partial Y_t}{\partial Y_{kt}}\frac{\partial Y_{kt}}{\partial s_{kt}}\frac{\partial s_{kt}}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\right] \right\}$$

$$= \frac{G_j}{R_{jt}} \left\{ -\frac{1}{\psi} - \frac{1}{\sigma}\left[1 - \Sigma_{Y_{jt},R_{jt}|X_{jt}} - \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right] \right.$$

$$- \frac{1}{\epsilon}\left[\left(1-\Sigma_{Y_t,Y_{jt}}\right)\left(\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right.$$

$$\left.\left. - \Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right] \right\}.$$

If we are at a corner in s_{jt}, then $\Sigma_{s_{jt},R_{jt}} = 0$ and, using Lemma A-3, the above expression is clearly negative. So consider a case with interior s_{jt}. The final two lines are negative. So

the overall expression is negative if the third-to-last line is negative, which is the case if and only if

$$
\begin{aligned}
0 \geq & -\frac{1}{\psi} + \frac{1}{\sigma}\left[-1 + \Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right] \\
= & -\frac{1}{\psi} + \frac{1}{\sigma}\left[-1 + \Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\left(\frac{\sigma + \psi}{\psi}\frac{\alpha + \sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha) + \alpha}\right)\right] \\
= & -\frac{1}{\psi} + \frac{1}{\sigma}\Sigma_{Y_{jt},X_{jt}}\left[-1 + \frac{\sigma+\psi}{\psi}\frac{\alpha + \sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha) + \alpha}\right],
\end{aligned} \tag{A-5}
$$

where we use results from Lemma A-3. Note that $\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t \leq 3/4$, which implies that $\Sigma_{Y_{jt},X_{jt}}\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\sigma(1-\alpha)+\alpha} < 1$. Using this, inequality (A-5) holds if and only if

$$
\frac{\sigma}{\psi} \geq \Sigma_{Y_{jt},X_{jt}}\frac{-1 + \frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)}}{1 - \Sigma_{Y_{jt},X_{jt}}\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)}}. \tag{A-6}
$$

$\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t \leq 3/4$ implies that $\frac{\alpha+\sigma(1-\alpha)\frac{2+\eta\gamma}{1+\eta\gamma s_{jt}}Z_t}{\alpha+\sigma(1-\alpha)} < 1$, which implies that the right-hand side of inequality (A-6) is negative. Thus, inequality (A-6) always holds and $\partial G_j(R_{jt}, R_{kt})/\partial R_{jt} < 0$.

The analysis of $\partial G_k(R_{jt}, R_{kt})/\partial R_{kt}$ is virtually identical.

□

Now define the matrix G:

$$
G \triangleq \begin{bmatrix} \frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_j(R_{jt},R_{kt})}{\partial R_{kt}} \\ \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{jt}} & \frac{\partial G_k(R_{jt},R_{kt})}{\partial R_{kt}} \end{bmatrix}.
$$

Then we have:

Lemma A-5. *The determinant of G is positive.*

Proof. Analyze $det(G)$:

$$det(G) \propto \left\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \left[\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right]\right\}$$

$$\left\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \left[\Sigma_{Y_{kt},R_{kt}|X_{kt}} + \Sigma_{Y_{kt},X_{kt}}\left(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\right]\right\}$$

$$+ \left\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \left[\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right.\right.$$

$$\left.\left. - \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right]\right\}$$

$$\left\{ \frac{1}{\epsilon} \left[\Sigma_{Y_t,Y_{kt}}\left(\Sigma_{Y_{kt},R_{kt}|X_{kt}} + \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},R_{kt}} + \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\right.\right.$$

$$\left.\left. + \Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right]\right\}$$

$$+ \left\{ -\frac{1}{\psi} - \frac{1}{\sigma} + \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right) \left[\Sigma_{Y_{kt},R_{kt}|X_{kt}} + \Sigma_{Y_{kt},X_{kt}}\left(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\right.\right.$$

$$\left.\left. - \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right]\right\}$$

$$\left\{ \frac{1}{\epsilon} \left[\Sigma_{Y_t,Y_{jt}}\left(\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right)\right.\right.$$

$$\left.\left. + \Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right]\right\}$$

$$- \left(\frac{1}{\sigma} - \frac{1}{\epsilon}\right)^2 \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}},$$

where we factored $G_j G_k / R_{jt} R_{kt}$. Use $\Sigma_{Y_t,Y_{jt}} + \Sigma_{Y_t,Y_{kt}} = 1$ from Lemma A-3 and cancel terms

with $1/\epsilon^2$ to obtain:

$$det(G) \propto \left\{ -\frac{1}{\psi} - \frac{1}{\sigma}\left[1 - \Sigma_{Y_{jt},R_{jt}|X_{jt}} - \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right) \right] \right\}$$

$$\left\{ -\frac{1}{\psi} - \frac{1}{\sigma}\left[1 - \Sigma_{Y_{kt},R_{kt}|X_{kt}} - \Sigma_{Y_{kt},X_{kt}}\left(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right) \right] \right\}$$

$$-\frac{1}{\sigma}\left(\frac{1}{\sigma} - \frac{1}{\epsilon} \right)\left(\Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right)\left(\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right)$$

$$+\left\{ -\frac{1}{\psi} - \frac{1}{\sigma} \right\}\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}$$

$$\left[-\left(\Sigma_{Y_{kt},R_{kt}|X_{kt}} + \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},R_{kt}} + \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right) \right.$$

$$\left. + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right]$$

$$+\left\{ -\frac{1}{\psi} - \frac{1}{\sigma} \right\}\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}$$

$$\left[-\left(\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}} + \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right) \right.$$

$$\left. + \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right]$$

$$+\frac{1}{\epsilon}\frac{1}{\sigma}\left[\Sigma_{Y_{jt},R_{jt}|X_{jt}} + \Sigma_{Y_{jt},X_{jt}}\left(\Sigma_{X_{jt},R_{jt}} + \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right) \right]$$

$$\left[\Sigma_{Y_{kt},R_{kt}|X_{kt}} + \Sigma_{Y_{kt},X_{kt}}\left(\Sigma_{X_{kt},R_{kt}} + \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right) \right]. \qquad \text{(A-7)}$$

All lines after the first three are positive by results from Lemma A-3. Expanding the products in those first three lines and rearranging, those first three lines become:

$$\frac{1}{\psi^2}$$

$$+\frac{1}{\sigma^2}\left[1 - \Sigma_{X,R} \right]\Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\left(1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right)$$

$$+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{Y_{kt},X_{kt}}\left[1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right]$$

$$+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{Y_{jt},X_{jt}}\left[1 - \Sigma_{X,R} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right]$$

$$+\frac{1}{\sigma}\frac{1}{\epsilon}\left(\Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \right)\left(\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \right), \qquad \text{(A-8)}$$

where we write $\Sigma_{X,R}$ because this elasticity is the same in each sector. The term in parentheses on the second line becomes

$$
\begin{aligned}
&1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} \\
&= \frac{1}{\psi}\frac{\sigma}{\sigma(1-\alpha)+\alpha}\left\{\psi[1-\alpha] - \alpha - (1-\alpha)[\sigma+\psi]\frac{(2+\eta\gamma)^2}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})}Z_t\right\}.
\end{aligned}
\tag{A-9}
$$

Substituting for Z_t and using equation (A-1) at $\Pi_{jt}/\Pi_{kt} = 1$, we have

$$
\frac{Z_t}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})} = \frac{1}{[2+\eta\gamma]^2}.
$$

Equation (A-9) then becomes

$$
1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} = -\frac{\sigma}{\psi}.
$$

Substituting into (A-8), we have that the first three lines of (A-7) are equal to

$$
\begin{aligned}
&\frac{1}{\psi^2} \\
&-\frac{1}{\psi}\frac{1}{\sigma}\left[1 - \Sigma_{X,R}\right]\Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}} \\
&+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{Y_{kt},X_{kt}}\left[1 - \Sigma_{X,R} - \Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right] \\
&+\frac{1}{\psi}\frac{1}{\sigma}\Sigma_{Y_{jt},X_{jt}}\left[1 - \Sigma_{X,R} - \Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right] \\
&+\frac{1}{\sigma}\frac{1}{\epsilon}\left(\Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{kt}}\right)\left(\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{jt}}\right).
\end{aligned}
$$

The final line is positive. If the first through fourth lines are positive with nonzero $\Sigma_{s_{jt},R_{jt}}$ and nonzero $\Sigma_{s_{jt},R_{kt}}$, then they are also positive if s_{jt} is fixed (as at a corner solution). So $det(G) > 0$ if we are at a corner allocation in research. Factoring $1/\psi$, the first through

fourth lines are positive if and only if:

$$
\begin{aligned}
0 \leq & \frac{1}{\psi} + \frac{1}{\sigma}\Bigg[\left(1 - \Sigma_{X,R}\right) \left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right) \\
& \qquad - \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}\Sigma_{A_{jt},s_{jt}}\Sigma_{s_{jt},R_{jt}} - \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}\Sigma_{A_{kt},s_{kt}}\Sigma_{s_{kt},s_{jt}}\Sigma_{s_{jt},R_{kt}} \Bigg] \\
= & \frac{1}{\psi} + \frac{1}{\sigma}\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right) \\
& - \frac{1}{\sigma}\frac{\sigma+\psi}{\psi}\frac{1}{\sigma(1-\alpha)+\alpha}\Bigg[\alpha\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right) \\
& \qquad + \sigma(1-\alpha)\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma} \Bigg],
\end{aligned}
$$
$$(\text{A-10})$$

where we use $\frac{Z_t}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})} = \frac{1}{[2+\eta\gamma]^2}$. Note that $\Sigma_{Y_{jt},X_{jt}}+\Sigma_{Y_{kt},X_{kt}}-\Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}$ increases in $\Sigma_{Y_{jt},X_{jt}}$ and thus reaches a maximum at $\Sigma_{Y_{jt},X_{jt}} = 1$.

$$\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}} \leq 1 + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{kt},X_{kt}} = 1.$$

Also note that $\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})$ increases in each elasticity, and each elasticity is ≤ 1. Thus,

$$\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt}) \leq (1+\eta\gamma s_{kt}) + (1+\eta\gamma s_{jt}) = 2 + \eta\gamma,$$

which implies

$$\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma} \leq 1.$$

These results together imply that

$$
\begin{aligned}
& \alpha + \sigma(1-\alpha) \\
\geq & \alpha\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right) + \sigma(1-\alpha)\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}.
\end{aligned}
$$
$$(\text{A-11})$$

Using this, we have that inequality (A-10) holds if and only if

$$\frac{\sigma}{\psi} \geq \Bigg\{ -\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right) + \frac{1}{\sigma(1-\alpha)+\alpha}\bigg[\alpha\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right)$$

$$+ \sigma(1-\alpha)\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}\bigg]\Bigg\}$$

$$\Bigg\{ 1 - \frac{1}{\sigma(1-\alpha)+\alpha}\bigg[\alpha\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right)$$

$$+ \sigma(1-\alpha)\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)\frac{1}{2+\eta\gamma}\bigg]\Bigg\}^{-1}.$$

$$(A\text{-}12)$$

The denominator on the right-hand side is positive via inequality (A-11). The numerator on the right-hand side is equal to:

$$\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right)$$

$$\Bigg\{ -1 + \frac{1}{\sigma(1-\alpha)+\alpha}\bigg[\alpha + \sigma(1-\alpha)\frac{\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right)}{(2+\eta\gamma)\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right)}\bigg]\Bigg\}$$

$$(A\text{-}13)$$

Consider the fraction in brackets. If that fraction is ≤ 1, then the whole expression is negative. Assume that the fraction is > 1. Then:

$$\left(\Sigma_{Y_{jt},X_{jt}}(1+\eta\gamma s_{kt}) + \Sigma_{Y_{kt},X_{kt}}(1+\eta\gamma s_{jt})\right) > (2+\eta\gamma)\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}\right)$$

$$\Leftrightarrow \eta\gamma s_{kt}\Sigma_{Y_{jt},X_{jt}} + \eta\gamma s_{jt}\Sigma_{Y_{kt},X_{kt}} \geq (1+\eta\gamma)\left(\Sigma_{Y_{jt},X_{jt}} + \Sigma_{Y_{kt},X_{kt}}\right) - (2+\eta\gamma)\Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}$$

Assume without loss of generality that $\Sigma_{Y_{jt},X_{jt}} > \Sigma_{Y_{kt},X_{kt}}$. Then the left-hand side of the last line attains its largest possible value when $s_{kt} = 1$. The inequality on the last line is then satisfied only if

$$0 > \Sigma_{Y_{jt},X_{jt}} + (1+\eta\gamma)\Sigma_{Y_{kt},X_{kt}} - (2+\eta\gamma)\Sigma_{Y_{jt},X_{jt}}\Sigma_{Y_{kt},X_{kt}}. \qquad (A\text{-}14)$$

The right-hand side is monotonic in $\Sigma_{Y_{jt},X_{jt}}$. At $\Sigma_{Y_{jt},X_{jt}} = 1$, the right-hand side is

$$1 + (1+\eta\gamma)\Sigma_{Y_{kt},X_{kt}} - (2+\eta\gamma)\Sigma_{Y_{kt},X_{kt}} = 1 - \Sigma_{Y_{kt},X_{kt}} \geq 0.$$

But this contradicts inequality (A-14). Now consider the other extremum: $\Sigma_{Y_{jt},X_{jt}} = 0$. The right-hand side of inequality (A-14) becomes:

$$(1+\eta\gamma)\Sigma_{Y_{kt},X_{kt}} \geq 0,$$

which again contradicts inequality (A-14). Because the right-hand side of inequality (A-14) was monotonic in $\Sigma_{Y_{jt},X_{jt}}$ and was not satisfied for either the greatest or smallest possible values for $\Sigma_{Y_{jt},X_{jt}}$, the inequality is not satisfied for any values of $\Sigma_{Y_{jt},X_{jt}}$. Thus, the fraction in brackets in (A-13) is ≤ 1, which means that the right-hand side of inequality (A-12) is ≤ 0 and inequality (A-12) is satisfied. As a result, the first three lines of (A-7) are positive, which means that $det(G) > 0$.

$\square$

Derivation of Equation (10)

Equation (A-1) implicitly defines s_{jt} as a function of R_{jt}/R_{kt} and $A_{j(t-1)}/A_{k(t-1)}$ (for interior s_{jt}). To a first-order approximation, the total change in s_{jt} is

$$
\begin{aligned}
s_{j(t+1)} - s_{jt} =& \frac{\mathrm{d}s_{jt}}{\mathrm{d}[R_{jt}/R_{kt}]}\left[\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}}\right] + \frac{\mathrm{d}s_{jt}}{\mathrm{d}[A_{j(t-1)}/A_{k(t-1)}]}\left[\frac{A_{jt}}{A_{kt}} - \frac{A_{j(t-1)}}{A_{k(t-1)}}\right] \\
=& -\frac{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial[R_{jt}/R_{kt}]}}{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}}\left[\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}}\right] - \frac{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial[A_{j(t-1)}/A_{k(t-1)}]}}{\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}}\left[\frac{A_{jt}}{A_{kt}} - \frac{A_{j(t-1)}}{A_{k(t-1)}}\right] \\
\propto& \frac{1+\sigma/\psi}{R_{jt}/R_{kt}}\left[\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}}\right] - \frac{(1-\sigma)(1-\alpha)}{A_{j(t-1)}/A_{k(t-1)}}\left[\frac{A_{jt}}{A_{kt}} - \frac{A_{j(t-1)}}{A_{k(t-1)}}\right] \\
=& \frac{1+\sigma/\psi}{R_{jt}/R_{kt}}\left[\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}}\right] - 2(1-\sigma)(1-\alpha)\frac{\eta\gamma}{1+\eta\gamma s_{kt}}\left(s_{jt} - \frac{1}{2}\right),
\end{aligned}
$$

where the second line uses the implicit function theorem and the third line factors $-\{\partial[\Pi_{jt}/\Pi_{kt}]/\partial s_{jt}\}^{-1}$, factors $[\sigma + \alpha(1-\sigma)]^{-1}$, and uses $\Pi_{jt} = \Pi_{kt}$ at an interior equilibrium.

From this we have the following lemma, which will be useful in proving Proposition 2:

Lemma A-6. *If $s_{jt} \geq 0.5$, $R_{jt}/R_{kt} \leq R_{j(t+1)}/R_{k(t+1)}$, and $(1-\sigma)(1-\alpha) \leq 0.5$, then $s_{j(t+1)} \geq 0.5$.*

Proof. Note that

$$
\begin{aligned}
-\frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} =& \frac{1}{\sigma + \alpha(1-\sigma)}\frac{\Pi_{jt}}{\Pi_{kt}}\left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}}\right)^{-1}\left(\frac{\eta\gamma}{1+\eta\gamma s_{kt}} + \frac{\eta\gamma(1+\eta\gamma s_{jt})}{(1+\eta\gamma s_{kt})^2}\right) \\
=& \frac{\eta\gamma}{\sigma + \alpha(1-\sigma)}\frac{\Pi_{jt}}{\Pi_{kt}}\frac{2+\eta\gamma}{(1+\eta\gamma s_{jt})(1+\eta\gamma s_{kt})}.
\end{aligned}
$$

Restoring the factored terms in equation (10) and using $R_{jt}/R_{kt} \leq R_{j(t+1)}/R_{k(t+1)}$, we have

$$
s_{j(t+1)} - s_{jt} \geq -2(1-\sigma)(1-\alpha)\frac{1+\eta\gamma s_{jt}}{2+\eta\gamma}\left(s_{jt} - \frac{1}{2}\right) \geq -2(1-\sigma)(1-\alpha)\left(s_{jt} - \frac{1}{2}\right).
$$

Using the assumption that $(1 - \sigma)(1 - \alpha) \leq 0.5$, we have:

$$s_{j(t+1)} - s_{jt} \geq - \left(s_{jt} - \frac{1}{2} \right),$$

which implies $s_{j(t+1)} \geq 0.5$.

$\square$

Proof of Proposition 1

The change in R_{jt}/R_{kt} from time t to $t+1$ is

$$\frac{R_{j(t+1)}}{R_{k(t+1)}} - \frac{R_{jt}}{R_{kt}} = \frac{(R_{j(t+1)} - R_{jt})R_{kt} - (R_{k(t+1)} - R_{kt})R_{jt}}{R_{k(t+1)}R_{kt}}$$

$$\propto \frac{R_{j(t+1)} - R_{jt}}{R_{jt}} - \frac{R_{k(t+1)} - R_{kt}}{R_{kt}},$$

where the first equality adds and subtracts $R_{jt}R_{kt}$ in the numerator and the second line factors $R_{jt}/R_{k(t+1)}$. To a first-order approximation, this is

$$\frac{1}{R_{jt}} \left(\frac{\mathrm{d}R_{jt}}{\mathrm{d}A_{jt}} \left[A_{j(t+1)} - A_{jt} \right] + \frac{\mathrm{d}R_{jt}}{\mathrm{d}A_{kt}} \left[A_{k(t+1)} - A_{kt} \right] \right) - \frac{1}{R_{kt}} \left(\frac{\mathrm{d}R_{kt}}{\mathrm{d}A_{jt}} \left[A_{j(t+1)} - A_{jt} \right] + \frac{\mathrm{d}R_{kt}}{\mathrm{d}A_{kt}} \left[A_{k(t+1)} - A_{kt} \right] \right).$$

The derivatives follow from applying the implicit function theorem to the system of equations defining $G_j(R_{jt}, R_{kt})$ and $G_k(R_{jt}, R_{kt})$. Doing this yields:

$$\frac{1}{R_{jt}} \left(\frac{-\frac{\partial G_j}{\partial A_{jt}} \frac{\partial G_k}{\partial R_{kt}} + \frac{\partial G_j}{\partial R_{kt}} \frac{\partial G_k}{\partial A_{jt}}}{det(G)} \left[A_{j(t+1)} - A_{jt} \right] + \frac{-\frac{\partial G_j}{\partial A_{kt}} \frac{\partial G_k}{\partial R_{kt}} + \frac{\partial G_j}{\partial R_{kt}} \frac{\partial G_k}{\partial A_{kt}}}{det(G)} \left[A_{k(t+1)} - A_{kt} \right] \right)$$

$$- \frac{1}{R_{kt}} \left(\frac{-\frac{\partial G_k}{\partial A_{jt}} \frac{\partial G_j}{\partial R_{jt}} + \frac{\partial G_k}{\partial R_{jt}} \frac{\partial G_j}{\partial A_{jt}}}{det(G)} \left[A_{j(t+1)} - A_{jt} \right] + \frac{-\frac{\partial G_k}{\partial A_{kt}} \frac{\partial G_j}{\partial R_{jt}} + \frac{\partial G_k}{\partial R_{jt}} \frac{\partial G_j}{\partial A_{kt}}}{det(G)} \left[A_{k(t+1)} - A_{kt} \right] \right)$$

$$\propto \left[-\frac{\partial G_j}{\partial A_{jt}} s_{j(t+1)} A_{jt} - \frac{\partial G_j}{\partial A_{kt}} s_{k(t+1)} A_{kt} \right] \left[\frac{1}{R_{jt}} \frac{\partial G_k}{\partial R_{kt}} + \frac{1}{R_{kt}} \frac{\partial G_k}{\partial R_{jt}} \right]$$

$$+ \left[\frac{\partial G_k}{\partial A_{jt}} s_{j(t+1)} A_{jt} + \frac{\partial G_k}{\partial A_{kt}} s_{k(t+1)} A_{kt} \right] \left[\frac{1}{R_{jt}} \frac{\partial G_j}{\partial R_{kt}} + \frac{1}{R_{kt}} \frac{\partial G_j}{\partial R_{jt}} \right], \quad \text{(A-15)}$$

where the first expression factors R_{jt}/R_{kt} and the second expression factors $\eta\gamma/det(G)$, which is positive by Lemma A-5. Differentiation and algebraic manipulations (including applying relationships from Lemma A-3) yield:

$$-\frac{\partial G_j}{\partial A_{jt}} s_{j(t+1)} A_{jt} - \frac{\partial G_j}{\partial A_{kt}} s_{k(t+1)} A_{kt} = -G_j \left\{ \frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,Y_{kt}} \right\} \Sigma_{Y_{jt},X_{jt}} \Sigma_{X_{jt},A_{jt}} s_{j(t+1)}$$

$$- G_j \frac{1}{\epsilon} \Sigma_{Y_t,Y_{kt}} \Sigma_{Y_{kt},X_{kt}} \Sigma_{X_{kt},A_{kt}} (1 - s_{j(t+1)}),$$

$$\frac{\partial G_k}{\partial A_{jt}}s_{j(t+1)}A_{jt} + \frac{\partial G_k}{\partial A_{kt}}s_{k(t+1)}A_{kt} = G_k\left\{\frac{1}{\sigma} - \frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\right\}\Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},A_{kt}}(1 - s_{j(t+1)})$$
$$+ G_k\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},A_{jt}}s_{j(t+1)},$$

$$\frac{1}{R_{jt}}\frac{\partial G_k}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial G_k}{\partial R_{jt}} = \frac{G_k}{R_{jt}R_{kt}}\left\{-\frac{1}{\psi} - \frac{1}{\sigma}\left[1 - \Sigma_{Y_{kt},R_{kt}|X_{kt}} - \Sigma_{Y_{kt},X_{kt}}\Sigma_{X_{kt},R_{kt}}\right]\right.$$
$$\left.+ \frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\left[\Sigma_{X,R} - 1\right]\left[\Sigma_{Y_{jt},X_{jt}} - \Sigma_{Y_{kt},X_{kt}}\right]\right\},$$

$$\frac{1}{R_{jt}}\frac{\partial G_j}{\partial R_{kt}} + \frac{1}{R_{kt}}\frac{\partial G_j}{\partial R_{jt}} = \frac{G_j}{R_{jt}R_{kt}}\left\{-\frac{1}{\psi} - \frac{1}{\sigma}\left(1 - \Sigma_{Y_{jt},R_{jt}|X_{jt}} - \Sigma_{Y_{jt},X_{jt}}\Sigma_{X_{jt},R_{jt}}\right)\right.$$
$$\left.+ \frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\left[\Sigma_{X,R} - 1\right]\left[\Sigma_{Y_{kt},X_{kt}} - \Sigma_{Y_{jt},X_{jt}}\right]\right\}.$$

Using these in (A-15) and factoring $\Sigma_{X,A}G_jG_k/[R_{jt}R_{kt}]$ yields:

$$\left\{ -s_{j(t+1)}\left\{\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\right\}\Sigma_{Y_{jt},X_{jt}}-(1-s_{j(t+1)})\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\right\}$$

$$\left\{-\frac{1}{\psi}-\frac{1}{\sigma}\Sigma_{Y_{kt},X_{kt}}\left[1-\Sigma_{X,R}\right]\right\}$$

$$+\left\{(1-s_{j(t+1)})\left\{\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\right\}\Sigma_{Y_{kt},X_{kt}}+s_{j(t+1)}\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}\right\}$$

$$\left\{-\frac{1}{\psi}-\frac{1}{\sigma}\Sigma_{Y_{jt},X_{jt}}\left(1-\Sigma_{X,R}\right)\right\}$$

$$+\frac{1}{\epsilon}\left[1-\Sigma_{X,R}\right]\left[\Sigma_{Y_{kt},X_{kt}}-\Sigma_{Y_{jt},X_{jt}}\right]$$

$$\left\{-s_{j(t+1)}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\right]\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}-(1-s_{j(t+1)})\left\{\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\right\}\Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\right\}$$

$$-\frac{1}{\epsilon^2}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_t,Y_{kt}}\left[1-\Sigma_{X,R}\right]\left[\Sigma_{Y_{kt},X_{kt}}-\Sigma_{Y_{jt},X_{jt}}\right]\left\{(1-s_{j(t+1)})\Sigma_{Y_{kt},X_{kt}}+s_{j(t+1)}\Sigma_{Y_{jt},X_{jt}}\right\}$$

$$=s_{j(t+1)}\Sigma_{Y_{jt},X_{jt}}\left\{\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\right]\right.$$

$$\left.+\frac{1}{\sigma}\left(1-\Sigma_{X,R}\right)\left[\frac{1}{\sigma}\Sigma_{Y_{kt},X_{kt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}\right]\right\}$$

$$-(1-s_{j(t+1)})\Sigma_{Y_{kt},X_{kt}}\left\{\frac{1}{\psi}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\right]\right.$$

$$\left.+\frac{1}{\sigma}\left(1-\Sigma_{X,R}\right)\left[\frac{1}{\sigma}\Sigma_{Y_{jt},X_{jt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\right]\right\}$$

$$+\frac{1}{\epsilon}\left[1-\Sigma_{X,R}\right]\left[\Sigma_{Y_{kt},X_{kt}}-\Sigma_{Y_{jt},X_{jt}}\right]$$

$$\left\{-s_{j(t+1)}\left[\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{kt}}\right]\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_{jt},X_{jt}}-(1-s_{j(t+1)})\left\{\frac{1}{\sigma}-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\right\}\Sigma_{Y_t,Y_{kt}}\Sigma_{Y_{kt},X_{kt}}\right.$$

$$\left.-\frac{1}{\epsilon}\Sigma_{Y_t,Y_{jt}}\Sigma_{Y_t,Y_{kt}}\left[(1-s_{j(t+1)})\Sigma_{Y_{kt},X_{kt}}+s_{j(t+1)}\Sigma_{Y_{jt},X_{jt}}\right]\right\}$$

$$= s_{j(t+1)} \Sigma_{Y_{jt},X_{jt}} \left\{ \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \right] + \frac{1}{\sigma} \left(1 - \Sigma_{X,R} \right) \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,Y_{kt}} \right] \Sigma_{Y_{kt},X_{kt}} \right\}$$

$$- (1 - s_{j(t+1)}) \Sigma_{Y_{kt},X_{kt}} \left\{ \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \right] + \frac{1}{\sigma} \left(1 - \Sigma_{X,R} \right) \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \Sigma_{Y_t,Y_{jt}} \right] \Sigma_{Y_{jt},X_{jt}} \right\}$$

$$- s_{j(t+1)} \frac{1}{\sigma} \frac{1}{\epsilon} \left[1 - \Sigma_{X,R} \right] \Sigma_{Y_t,Y_{jt}} \Sigma_{Y_{jt},X_{jt}} \Sigma_{Y_{kt},X_{kt}} + (1 - s_{j(t+1)}) \frac{1}{\sigma} \frac{1}{\epsilon} \left[1 - \Sigma_{X,R} \right] \Sigma_{Y_t,Y_{kt}} \Sigma_{Y_{kt},X_{kt}} \Sigma_{Y_{jt},X_{jt}}$$

$$= \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \right] \left[s_{j(t+1)} \Sigma_{Y_{jt},X_{jt}} - (1 - s_{j(t+1)}) \Sigma_{Y_{kt},X_{kt}} \right]$$

$$+ \frac{1}{\sigma^2} \left(1 - \Sigma_{X,R} \right) \Sigma_{Y_{kt},X_{kt}} \Sigma_{Y_{jt},X_{jt}} \left(2 s_{j(t+1)} - 1 \right) - \frac{1}{\sigma} \frac{1}{\epsilon} \left(1 - \Sigma_{X,R} \right) \Sigma_{Y_{jt},X_{jt}} \Sigma_{Y_{kt},X_{kt}} \left(2 s_{j(t+1)} - 1 \right)$$

$$= \frac{1}{\psi} \left[\frac{1}{\sigma} - \frac{1}{\epsilon} \right] \left[s_{j(t+1)} \Sigma_{Y_{jt},X_{jt}} - (1 - s_{j(t+1)}) \Sigma_{Y_{kt},X_{kt}} \right] + \frac{1}{\sigma} \left(\frac{1}{\sigma} - \frac{1}{\epsilon} \right) \left(1 - \Sigma_{X,R} \right) \Sigma_{Y_{kt},X_{kt}} \Sigma_{Y_{jt},X_{jt}} \left(2 s_{j(t+1)} - 1 \right)$$

Substituting for $\Sigma_{X,R}$ and rearranging, we obtain

$$\frac{1}{\psi} \left(\frac{1}{\sigma} - \frac{1}{\epsilon} \right) \left[s_{j(t+1)} \Sigma_{Y_{jt},X_{jt}} \left(1 + \frac{\psi[1 - \alpha] - \alpha}{\sigma(1 - \alpha) + \alpha} \Sigma_{Y_{kt},X_{kt}} \right) \right.$$
$$\left. - (1 - s_{j(t+1)}) \Sigma_{Y_{kt},X_{kt}} \left(1 + \frac{\psi[1 - \alpha] - \alpha}{\sigma(1 - \alpha) + \alpha} \Sigma_{Y_{jt},X_{jt}} \right) \right]. \tag{A-16}$$

This expression is positive if and only if the term in brackets is positive. Define $\hat{s}_{t+1}$ as the $s_{j(t+1)}$ such that $R_{jt}/R_{kt} = R_{j(t+1)}/R_{k(t+1)}$. Then $\hat{s}_{t+1}$ is the root of the term in brackets. Solving for that root yields equation (15).

We now consider how $\hat{s}_{t+1}$ changes in σ. Holding the time t allocation of resource extraction and research fixed, the derivative of the right-hand side of equation (15) with respect to σ is proportional to

$$- \left. \frac{\partial \Sigma_{Y_{jt},X_{jt}}}{\partial \sigma} \right|_{R_{jt},s_{jt} \text{ fixed}} \Sigma_{Y_{kt},X_{kt}} \left[1 + \frac{\psi[1 - \alpha] - \alpha}{\sigma(1 - \alpha) + \alpha} \Sigma_{Y_{kt},X_{kt}} \right]$$
$$+ \left. \frac{\partial \Sigma_{Y_{kt},X_{kt}}}{\partial \sigma} \right|_{R_{kt},s_{jt} \text{ fixed}} \Sigma_{Y_{jt},X_{jt}} \left[1 + \frac{\psi[1 - \alpha] - \alpha}{\sigma(1 - \alpha) + \alpha} \Sigma_{Y_{jt},X_{jt}} \right].$$

Note that

$$\left. \frac{\partial \Sigma_{Y_{jt},X_{jt}}}{\partial \sigma} \right|_{R_{jt},s_{jt} \text{ fixed}} \propto \frac{1}{\sigma^2} \kappa(1 - \kappa) X_{jt}^{\frac{\sigma-1}{\sigma}} R_{jt}^{\frac{\sigma-1}{\sigma}} \left[\ln X_{jt} - \ln R_{jt} \right]$$
$$> 0 \text{ iff } X_{jt} > R_{jt}.$$

The results for sector k are analogous. $X_{jt} > R_{jt}$ when A_{jt} is sufficiently large. We have that the right-hand side of equation (15) (and thus also $\hat{s}_{t+1}$) decreases in σ if $X_{jt} > R_{jt}$ and $X_{kt} < R_{kt}$.

Consider what happens to the right-hand side of equation (15) as $\sigma \to 0$. The elasticity $\Sigma_{Y_{jt},X_{jt}}$ becomes $(X_{jt}/\min\{R_{jt}, X_{jt}\})^{\frac{\sigma-1}{\sigma}}$, and analogously for sector k. Thus, under the assumption that $X_{jt} > R_{jt}$ and $X_{kt} < R_{kt}$, $\Sigma_{Y_{jt},X_{jt}} \to 0$ as $\sigma \to 0$ and $\Sigma_{Y_{kt},X_{kt}} \to 1$ as $\sigma \to 0$. These imply that the right-hand side of equation (15) goes to 1 as $\sigma \to 0$.

As $\sigma \to 1$, the right-hand side of equation (15) goes to $1/2$.

Finally, as $\sigma \to \infty$, the elasticities $\Sigma_{Y_{jt},X_{jt}}$ and $\Sigma_{Y_{kt},X_{kt}}$ each go to either 1 or 0. Under the assumption that $X_{jt} > R_{jt}$ and $X_{kt} < R_{kt}$, $\Sigma_{Y_{jt},X_{jt}} \to 1$ while $\Sigma_{Y_{kt},X_{kt}} \to 0$. The right-hand side of equation (15) thus goes to 0 as $\sigma \to \infty$.

Proof of Proposition 2

The following lemma relates $\hat{s}_{t+1}$ and 0.5.

Lemma A-7. *If $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if $A_{j(t-1)}/A_{k(t-1)} \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$. If $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if $A_{j(t-1)}/A_{k(t-1)} \leq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$.*

Proof. From equation (15),

$$\left\{\hat{s}_{t+1} \geq \frac{1}{2}\right\} \Leftrightarrow \left\{\Sigma_{Y_{kt},X_{kt}} \geq \Sigma_{Y_{jt},X_{jt}}\right\},$$

where the right-hand side is evaluated at $\hat{s}_{t+1}$. Using the explicit expressions for the elasticities, for intermediate-good production, and for X_{jt} and X_{kt} from equation (A-3), we

have:

$$\Sigma_{Y_{kt},X_{kt}} \geq \Sigma_{Y_{jt},X_{jt}}$$

$$\Leftrightarrow 0 \leq \frac{(1-\kappa)X_{kt}^{\frac{\sigma-1}{\sigma}}Y_{jt}^{\frac{\sigma-1}{\sigma}} - (1-\kappa)X_{jt}^{\frac{\sigma-1}{\sigma}}Y_{kt}^{\frac{\sigma-1}{\sigma}}}{Y_{kt}^{\frac{\sigma-1}{\sigma}}Y_{jt}^{\frac{\sigma-1}{\sigma}}}$$

$$\Leftrightarrow 0 \leq X_{kt}^{\frac{\sigma-1}{\sigma}}Y_{jt}^{\frac{\sigma-1}{\sigma}} - X_{jt}^{\frac{\sigma-1}{\sigma}}Y_{kt}^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow 0 \leq \kappa R_{jt}^{\frac{\sigma-1}{\sigma}}X_{kt}^{\frac{\sigma-1}{\sigma}} + (1-\kappa)X_{jt}^{\frac{\sigma-1}{\sigma}}X_{kt}^{\frac{\sigma-1}{\sigma}} - \kappa R_{kt}^{\frac{\sigma-1}{\sigma}}X_{jt}^{\frac{\sigma-1}{\sigma}} - (1-\kappa)X_{kt}^{\frac{\sigma-1}{\sigma}}X_{jt}^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow 1 \leq \left(\frac{R_{jt}\left[\frac{1-\kappa}{\kappa}\left(\frac{R_{kt}}{\Psi_k}\right)^{1/\psi}\right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}}\left[\frac{R_{kt}}{A_{kt}}\right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}}A_{kt}}{R_{kt}\left[\frac{1-\kappa}{\kappa}\left(\frac{R_{jt}}{\Psi_j}\right)^{1/\psi}\right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}}\left[\frac{R_{jt}}{A_{jt}}\right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}}A_{jt}} \right)^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow 1 \leq \left[\left(\frac{\Psi_j}{\Psi_k}\right)^{\frac{\alpha\sigma/\psi}{\sigma(1-\alpha)+\alpha}} \left(\frac{R_{jt}}{R_{kt}}\right)^{\frac{\sigma(1-\alpha-\alpha/\psi)}{\sigma(1-\alpha)+\alpha}} \left(\frac{A_{kt}}{A_{jt}}\right)^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} \right]^{\frac{\sigma-1}{\sigma}}$$

$$\Leftrightarrow 1 \leq \left(\frac{\Psi_j}{\Psi_k}\right)^{\chi\frac{1}{\psi}[\alpha+\sigma(1-\alpha)]} \left(\frac{1+\eta\gamma s_{jt}}{1+\eta\gamma s_{kt}}\right)^{-\chi\frac{1}{\psi}[\alpha+\sigma(1-\alpha)]} \left(\frac{A_{j(t-1)}}{A_{k(t-1)}}\right)^{\chi(1-\alpha)[(1-\sigma)(1-\alpha-\alpha/\psi)-(1+\sigma/\psi)]},$$

$$(A\text{-}17)$$

where the final line substitutes for R_{jt}/R_{kt} from equation (A-1) (which must hold for $\hat{s}_{t+1}$ interior) and where

$$\chi \triangleq \frac{\sigma-1}{[\sigma(1-\alpha)+\alpha][1+\sigma/\psi]} < 0 \text{ iff } \sigma < 1.$$

The right-hand side of inequality (A-17) is increasing in s_{jt} if and only if $\sigma < 1$. Therefore, if $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if the strict version of the inequality does not hold at $s_{jt} = 0.5$, and if $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if the inequality holds at $s_{jt} = 0.5$. If $\sigma < 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} \geq \left[\frac{\Psi_j}{\Psi_k}\right]^\theta,$$

and if $\sigma > 1$, then $\hat{s}_{t+1} \geq 0.5$ if and only if

$$\frac{A_{j(t-1)}}{A_{k(t-1)}} \leq \left[\frac{\Psi_j}{\Psi_k}\right]^\theta,$$

where

$$\theta \triangleq \frac{-\frac{1}{\psi}[\alpha+\sigma(1-\alpha)]}{(1-\alpha)[(1-\sigma)(1-\alpha-\alpha/\psi)-(1+\sigma/\psi)]} = \frac{1}{(1-\alpha)(1+\psi)} > 0.$$

□

Note that $\psi > \alpha/(1-\alpha)$ implies $(1-\alpha)(1+\psi) > 1$, in which case $\Psi_j/\Psi_k \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$ and thus $A_{j(t-1)}/A_{k(t-1)} \geq \Psi_j/\Psi_k$ implies $A_{j(t-1)}/A_{k(t-1)} \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$. Thus Assumption 1 implies $\hat{s}_{t+1} \leq 0.5$ if $\sigma > 1$ and $\hat{s}_{t+1} \geq 0.5$ if $\sigma < 1$.

Begin with a case in which $\sigma > 1$ and in which Assumption 1 holds at time t. We will show that Assumption 1 holding at time t implies that Assumption 1 holds at time $t+1$. We immediately have $s_{j(t+1)} \geq 0.5$ from $s_{jt} \geq 0.5$ and $s_{j(t+1)} \geq s_{jt}$. It remains to show that (i) $A_{jt}/A_{kt} \geq \Psi_j/\Psi_k$, (ii) $s_{j(t+2)} \geq s_{j(t+1)}$, and (iii) $R_{j(t+2)}/R_{k(t+2)} \geq R_{j(t+1)}/R_{k(t+1)}$.

Note that $s_{jt} \geq 0.5$ implies $A_{jt}/A_{kt} \geq A_{j(t-1)}/A_{k(t-1)}$, and $A_{j(t-1)}/A_{k(t-1)} \geq \Psi_j/\Psi_k$ then implies $A_{jt}/A_{kt} \geq \Psi_j/\Psi_k$. This proves (i).

Now note that $s_{j(t+1)} \geq 0.5$ implies $A_{j(t+1)}/A_{k(t+1)} \geq A_{jt}/A_{kt}$. From equations (A-1) and (14), Π_{jt}/Π_{kt} increases in $A_{j(t-1)}/A_{k(t-1)}$ for any given s_{jt} if $\sigma > 1$. Therefore, $A_{j(t+1)}/A_{k(t+1)} \geq A_{jt}/A_{kt}$ implies $\Pi_{j(t+2)}/\Pi_{k(t+2)} \geq \Pi_{j(t+1)}/\Pi_{k(t+1)}$ for any given x such that $s_{j(t+2)} = s_{j(t+1)} = x$. By Corollary A-2, we have $s_{j(t+2)} \geq s_{j(t+1)}$. This proves (ii).

From Lemma A-7, $A_{jt}/A_{kt} \geq \Psi_j/\Psi_k$ with $\sigma > 1$ implies that $\hat{s}_{t+2} \leq 0.5$. Because we have shown $s_{j(t+2)} \geq s_{j(t+1)}$ and $s_{j(t+1)} \geq 0.5$, we have $s_{j(t+2)} \geq \hat{s}_{t+2}$. Therefore, $R_{j(t+1)}/R_{k(t+1)} \leq R_{j(t+2)}/R_{k(t+2)}$. This proves (iii).

We have thus seen that Assumption 1 holds at time $t+1$ as well. Proceeding by induction, Assumption 1 must hold at all times later than t, so that sector j's shares of research and extraction increase forever. Resource j is locked-in from time t if $\sigma > 1$ and Assumption 1 holds at time t.

Now consider a case in which $\sigma < 1$ and in which Assumption 1 holds at time t. Let time w be the first time at which sector j's share of extraction begins decreasing, so that $R_{jx}/R_{kx} \leq R_{j(x+1)}/R_{k(x+1)}$ for all $x \in [t, w-1]$ and $R_{jw}/R_{kw} > R_{j(w+1)}/R_{k(w+1)}$, which in turn requires $s_{jx} \geq \hat{s}_x$ for all $x \in [t+1, w]$ and $s_{j(w+1)} < \hat{s}_{w+1}$. Note that $s_{jt} \geq 0.5$ implies that $A_{jt}/A_{kt} \geq A_{j(t-1)}/A_{k(t-1)}$. Assume that sector j's share of research begins declining sometime after its share of extraction does, so that $s_{jx} \leq s_{j(x+1)}$ for all $x \in [t, w]$. Then we have $A_{jx}/A_{kx} \geq A_{j(x-1)}/A_{k(x-1)}$ for all $x \in [t, w+1]$, and thus $A_{jx}/A_{kx} \geq \Psi_j/\Psi_k$ for all $x \in [t, w+1]$. Using this with Lemma A-7 and $\sigma < 1$ then implies $\hat{s}_{x+1} \geq 0.5$ for all $x \in [t, w+2]$. Combining this with the requirement that $s_{jw} \geq \hat{s}_w$, we have $s_{jw} \geq 0.5$. From equation (10) and $\sigma < 1$, we then have $s_{j(w+1)} \geq s_{jw}$ only if $R_{jw}/R_{kw} \leq R_{j(w+1)}/R_{k(w+1)}$. But that contradicts the definition of w, which required $R_{jw}/R_{kw} > R_{j(w+1)}/R_{k(w+1)}$. Sector j's share of research must have begun declining no later than time w. We have shown that a transition in extraction occurs only after a transition in research.

We now have two possibilities. We will see that the first one implies that $s_{jx} \geq 0.5$ at all times $x \in [t+1, w]$ and the second one generates a contradiction.

First, we could have $A_{j(x-2)}/A_{k(x-2)} \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$ at all times $x \in [t+1, w]$. Then by Lemma A-7, $\hat{s}_x \geq 0.5$ at all times $x \in [t+1, w]$. The definition of time w then requires $s_{jx} \geq 0.5$ at all times $x \in [t+1, w]$.

Second, we could have $A_{j(x-2)}/A_{k(x-2)} < [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$ at some time $x \in [t+1, w]$.

In order for this to happen, it must be true that $s_{jx} < 0.5$ at some times $x \in [t+2, w]$.[30] Let z be the first time at which $s_{jx} < 0.5$. $A_{j(t-1)}/A_{k(t-1)} \geq \Psi_j/\Psi_k$ and $s_{jx} \geq 0.5$ for all $x \in [t, z-1]$ imply that $A_{j(z-2)}/A_{k(z-2)} \geq \Psi_j/\Psi_k$, which implies by Lemma A-7 and $\sigma < 1$ that $\hat{s}_z \geq 0.5$. So we have $s_{jz} < \hat{s}_z$, which means that $R_{j(z-1)}/R_{k(z-1)} > R_{jz}/R_{kz}$. But this contradicts the definition of time w as the first time at which sector j's share of extraction begins decreasing.

Therefore, we must have $A_{j(x-2)}/A_{k(x-2)} \geq [\Psi_j/\Psi_k]^{1/[(1-\alpha)(1+\psi)]}$ and $s_{jx} \geq 0.5$ at all times $x \in [t+1, w]$. Observe that $s_{jx} \geq 0.5$ at all times $x \in [t, w]$ implies $A_{jx}/A_{kx} \geq A_{j(x-1)}/A_{k(x-1)}$ at all times $x \in [t, w]$. We have shown that a transition in technology happens only after a transition in extraction.

Finally, consider the first time $z > t$ at which $R_{jz} < R_{kz}$. Assume that $\Psi_j \geq \Psi_k$ and that $s_{jx} \geq 0.5$ for $x \in [t, z]$. Assumption 1, $\Psi_j \geq \Psi_k$, and $s_{jx} \geq 0.5$ imply $A_{jx} \geq A_{kx}$ for $x \in [t, z]$. Using $\sigma < 1$, we see that $A_{j(z-1)} \geq A_{k(z-1)}$, $\Psi_j \geq \Psi_k$, and $R_{jz} < R_{kz}$ imply that the right-hand side of equation (A-2) is < 1 when evaluated at $s_{jz} = 0.5$. So by Corollary A-2, time z equilibrium scientists must be less than 0.5. But $s_{jz} < 0.5$ contradicts $s_{jx} \geq 0.5$ for $x \in [t, z]$. Therefore, if $\Psi_j \geq \Psi_k$, then there must be some time $x \in [t, z]$ at which $s_{jx} < 0.5$. We have shown that if $\Psi_j \geq \Psi_k$, then sector k must begin dominating research before it begins dominating extraction.

Proof of Proposition 3

Along a path with identical growth in each type of technology, $A_{j(t+1)}/A_{k(t+1)} = A_{jt}/A_{kt}$. From equation (3), this holds if and only if $s_{jt} = 0.5$. For equation (A-1) to hold at $s_{jt} = 0.5$ along a path with identical growth in each type of technology, it must be the case that R_{jt}/R_{kt} is constant along this path. This, in turn, implies that $s_{j(t+1)} = \hat{s}_{t+1}$, which means that $\hat{s}_{t+1} = 0.5$.

Imposing equality in inequality (A-17) and using $\sigma < 1$ and $\hat{s}_{t+1} = 0.5$, we see that $\Psi_j > \Psi_k$ if and only if $A_{j(t-1)} > A_{k(t-1)}$ at all times along this path. Using $\Psi_j > \Psi_k$, $A_{j(t-1)} > A_{k(t-1)}$, and $s_{jt} = 0.5$ in equation (A-1) with $\sigma < 1$, we have that $R_{jt} > R_{kt}$. By similar logic, if $\Psi_j = \Psi_k$, then $A_{j(t-1)} = A_{k(t-1)}$ and $R_{jt} = R_{kt}$ along this path. Equation (14) then implies $\nu = 0.5$.

Now consider the growth rate of R_{jt} and R_{kt} along the path with identical growth in each

---

[30]Recall that $s_{jt} \geq 0.5$ and $s_{j(t+1)} \geq s_{jt}$ imply $s_{j(t+1)} \geq 0.5$.

type of technology. Note that

$$
Y_{jt} = \left(\kappa \left[R_{jt} \right]^{\frac{\sigma-1}{\sigma}} + (1-\kappa) \left\{ \left[\frac{1-\kappa}{\kappa} p_{jRt} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} A_{jt} \right\}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
$$

$$
= A_{jt} \left(\kappa \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\sigma-1}{\sigma}} + (1-\kappa) \left\{ \left[\frac{1-\kappa}{\kappa} \left(\frac{R_{jt}}{\Psi_j} \right)^{1/\psi} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} \right\}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
$$

$$
\triangleq A_{jt} \tilde{Y}_{jt}.
$$

Substituting into equation (14), we have:

$$
1 = \frac{\nu}{1-\nu} \left[\frac{\tilde{Y}_{jt}}{\tilde{Y}_{kt}} \right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}} \left[\frac{A_{jt}}{A_{kt}} \right]^{-\frac{1}{\epsilon}} \left[\frac{R_{jt}/A_{jt}}{R_{kt}/A_{kt}} \right]^{-1/\sigma} \left[\frac{p_{jRt}}{p_{kRt}} \right]^{-1}.
$$

Equate to time $t+1$ variables:

$$
\left[\frac{\tilde{Y}_{jt}}{\tilde{Y}_{kt}} \right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}} \left[\frac{A_{jt}}{A_{kt}} \right]^{-\frac{1}{\epsilon}} \left[\frac{R_{jt}/A_{jt}}{R_{kt}/A_{kt}} \right]^{-1/\sigma} \left(\frac{p_{jRt}}{p_{kRt}} \right)^{-1}
$$

$$
= \left[\frac{\tilde{Y}_{j(t+1)}}{\tilde{Y}_{k(t+1)}} \right]^{\frac{1}{\sigma}-\frac{1}{\epsilon}} \left[\frac{A_{j(t+1)}}{A_{k(t+1)}} \right]^{-\frac{1}{\epsilon}} \left[\frac{R_{j(t+1)}/A_{j(t+1)}}{R_{k(t+1)}/A_{k(t+1)}} \right]^{-1/\sigma} \left(\frac{p_{jR(t+1)}}{p_{kR(t+1)}} \right)^{-1}.
$$

Recognizing that relative technology and relative resource extraction are constant along this path, we have:

$$
\frac{\tilde{Y}_{jt}}{\tilde{Y}_{kt}} = \frac{\tilde{Y}_{j(t+1)}}{\tilde{Y}_{k(t+1)}},
$$

which implies

$$
\frac{\tilde{Y}_{j(t+1)}}{\tilde{Y}_{jt}} = \frac{\tilde{Y}_{k(t+1)}}{\tilde{Y}_{kt}}.
$$

Because this must hold for all time intervals once we reach the path with identical growth in each technology, each ratio must equal some constant, which we label χ. So we seek a

constant χ such that $\tilde{Y}_{j(t+1)} = \chi \tilde{Y}_{jt}$. Analyze:

$$
\tilde{Y}_{j(t+1)} = \left(\kappa \left[\frac{R_{j(t+1)}}{A_{j(t+1)}} \right]^{\frac{\sigma-1}{\sigma}} + (1-\kappa) \left\{ \left[\frac{1-\kappa}{\kappa} \left(\frac{R_{j(t+1)}}{\Psi_j} \right)^{1/\psi} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{j(t+1)}}{A_{j(t+1)}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} \right\}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
$$

$$
= \left(\kappa \left[\frac{R_{jt}}{A_{jt}} \frac{R_{j(t+1)}/R_{jt}}{A_{j(t+1)}/A_{jt}} \right]^{\frac{\sigma-1}{\sigma}} \right.
$$

$$
\left. + (1-\kappa) \left\{ \left[\frac{1-\kappa}{\kappa} \left(\frac{R_{jt}}{\Psi_j} \right)^{1/\psi} \left(\frac{R_{j(t+1)}}{R_{jt}} \right)^{1/\psi} \right]^{\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left[\frac{R_{jt}}{A_{jt}} \frac{R_{j(t+1)}/R_{jt}}{A_{j(t+1)}/A_{jt}} \right]^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}} \right\}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.
$$

There exists χ such that this equals $\chi \tilde{Y}_{jt}$ if and only if

$$
\frac{R_{j(t+1)}/R_{jt}}{A_{j(t+1)}/A_{jt}} = \left(\frac{R_{j(t+1)}}{R_{jt}} \right)^{\frac{1}{\psi}\frac{\alpha\sigma}{\sigma(1-\alpha)+\alpha}} \left(\frac{R_{j(t+1)}/R_{jt}}{A_{j(t+1)}/A_{jt}} \right)^{\frac{\alpha}{\sigma(1-\alpha)+\alpha}}
$$

$$
\Leftrightarrow \left(\frac{1}{1+\eta s_{jt}\gamma} \right)^{\frac{\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}} = \left(\frac{R_{j(t+1)}}{R_{jt}} \right)^{\frac{\frac{1}{\psi}\alpha\sigma-\sigma(1-\alpha)}{\sigma(1-\alpha)+\alpha}}
$$

$$
\Leftrightarrow \frac{R_{j(t+1)}}{R_{jt}} = (1+0.5\eta\gamma)^{\frac{\psi}{\psi-\frac{\alpha}{1-\alpha}}},
$$

where the last line recognizes that $s_{jt} = 0.5$ along a path with identical growth in each type of technology. The same condition must hold for $R_{k(t+1)}/R_{kt}$.

Proof of Proposition A-1

The tâtonnement adjustment process generates, to constants of proportionality, the following system for finding the equilibrium within period t:

$$
\dot{R}_{jt} = H_j \left(G_j(R_{jt}, R_{kt}) - 1 \right),
$$

$$
\dot{R}_{kt} = H_k \left(G_k(R_{jt}, R_{kt}) - 1 \right),
$$

where dots indicate time derivatives (where the fictional time for finding an equilibrium here flows within a period t), $H_i(0) = 0$, and $H_i'(\cdot) > 0$, for $i \in \{j, k\}$. The system's steady state occurs at the equilibrium values, which I denote with stars. Linearizing around the steady state, we have

$$
\begin{bmatrix} \dot{R}_{jt} \\ \dot{R}_{kt} \end{bmatrix} \approx \begin{bmatrix} \frac{\partial G_j(R_{jt}, R_{kt})}{\partial R_{jt}} H_j' & \frac{\partial G_j(R_{jt}, R_{kt})}{\partial R_{kt}} H_j' \\ \frac{\partial G_k(R_{jt}, R_{kt})}{\partial R_{jt}} H_k' & \frac{\partial G_k(R_{jt}, R_{kt})}{\partial R_{kt}} H_k' \end{bmatrix} \begin{bmatrix} R_{jt} - R_{jt}^* \\ R_{kt} - R_{kt}^* \end{bmatrix} = G \begin{bmatrix} R_{jt} - R_{jt}^* \\ R_{kt} - R_{kt}^* \end{bmatrix},
$$

where G is the 2×2 matrix of derivatives, each evaluated at (R^*_{jt}, R^*_{kt}). The results of Lemma A-4 imply that the trace of G is negative, in which case at least one of the two eigenvalues must be negative. Lemma A-5 shows that $det(G) > 0$, which means that both eigenvalues must be strictly negative. The linearized system is therefore globally asymptotically stable, and, by Lyapunov's Theorem of the First Approximation, the full nonlinear system is locally asymptotically stable around the equilibrium.

Proof of Corollary A-2

Now treat equations (12) and (13) as functions of R_{jt}, R_{kt}, and s_{jt} (recognizing that $s_{kt} = 1 - s_{jt}$):

$$1 = \kappa \nu A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t(R_{jt}, R_{kt}, s_{jt})}{Y_{jt}(R_{jt}, s_{jt})} \right]^{1/\epsilon} \left[\frac{Y_{jt}(R_{jt}, s_{jt})}{R_{jt}} \right]^{1/\sigma} \left[\frac{R_{jt}}{\Psi_j} \right]^{-1/\psi} \quad \triangleq \hat{G}_j(R_{jt}, R_{kt}; s_{jt}),$$

$$1 = \kappa (1 - \nu) A_Y^{\frac{\epsilon-1}{\epsilon}} \left[\frac{Y_t(R_{jt}, R_{kt}, s_{jt})}{Y_{kt}(R_{kt}, s_{jt})} \right]^{1/\epsilon} \left[\frac{Y_{kt}(R_{kt}, s_{jt})}{R_{kt}} \right]^{1/\sigma} \left[\frac{R_{kt}}{\Psi_k} \right]^{-1/\psi} \quad \triangleq \hat{G}_k(R_{jt}, R_{kt}; s_{jt}).$$

This system of equations implicitly defines R_{jt} and R_{kt} as functions of the parameter s_{jt}. Define the matrix $\hat{G}$ analogously to the matrix G. Using the implicit function theorem, we have

$$\frac{\partial R_{jt}}{\partial s_{jt}} = \frac{-\frac{\partial \hat{G}_j}{\partial s_{jt}} \frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}} \frac{\partial \hat{G}_k}{\partial s_{jt}}}{det(\hat{G})} \quad \text{and} \quad \frac{\partial R_{kt}}{\partial s_{jt}} = \frac{-\frac{\partial \hat{G}_k}{\partial s_{jt}} \frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial \hat{G}_k}{\partial R_{jt}} \frac{\partial \hat{G}_j}{\partial s_{jt}}}{det(\hat{G})}.$$

Interpreting equation (A-1) as implicitly defining s_{jt} as a function of R_{jt} and R_{kt}, we have:

$$\frac{\partial s_{jt}}{\partial R_{jt}} = -\frac{\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}}}{\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}} \quad \text{and} \quad \frac{\partial s_{jt}}{\partial R_{kt}} = -\frac{\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}}}{\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}},$$

and thus

$$\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}} = -\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} \frac{\partial s_{jt}}{\partial R_{jt}} \quad \text{and} \quad \frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}} = -\frac{\partial [\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} \frac{\partial s_{jt}}{\partial R_{kt}}.$$

Using these expressions, consider how the right-hand side of equation (A-2) changes in s_{jt}:

$$
\begin{aligned}
\frac{\mathrm{d}[\Pi_{jt}/\Pi_{kt}]}{\mathrm{d}s_{jt}} =& \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} + \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{jt}}\frac{\partial R_{jt}}{\partial s_{jt}} + \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial R_{kt}}\frac{\partial R_{kt}}{\partial s_{jt}} \\
=& \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}} \\
& - \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\frac{-\frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}}{det(\hat{G})} - \frac{\partial[\Pi_{jt}/\Pi_{kt}]}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}\frac{-\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial \hat{G}_k}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}}}{det(\hat{G})} \\
\propto& - \frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} \\
& - \frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}} - \frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial \hat{G}_j}{\partial R_{jt}} + \frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}}\frac{\partial \hat{G}_j}{\partial s_{jt}} \\
=& - \left(\frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial R_{kt}} + \frac{\partial \hat{G}_j}{\partial R_{jt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}} \right) \\
& + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} + \frac{\partial \hat{G}_j}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial R_{jt}} + \frac{\partial \hat{G}_j}{\partial R_{kt}}\frac{\partial \hat{G}_k}{\partial s_{jt}}\frac{\partial s_{jt}}{\partial R_{jt}} \\
=& - det(G)/H'.
\end{aligned}
$$

The third expression factored $det(\hat{G})$, which is positive by the proof of Proposition A-1 for a corner solution in s_{jt}, and it also factored $\partial[\Pi_{jt}/\Pi_{kt}]/\partial s_{jt}$, which is negative. The final equality recognizes that the only difference between the equations with a hat and the equations without a hat are that the equations without a hat allow s_{jt} to vary with R_{jt} and R_{kt}. Lemma A-5 showed that $det(G) > 0$. Thus the right-hand side of equation (A-2) strictly decreases in s_{jt}.