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ABSTRACT

This paper measures the output costs of sovereign risk by combining a sovereign debt model with firm- and bank-level data. An increase in sovereign risk lowers the price of government debt and has an adverse impact on banks' balance sheets, disrupting their ability to finance firms. The resulting fall in credit supply impacts firms directly, as they need to borrow at higher interest rates, and indirectly through general equilibrium effects on the price of inputs and other goods. Importantly, firms are not equally affected by these developments: those that have greater financing needs and that borrow from banks that hold more government debt are mostly affected by the change in borrowing rates, while firms that do not borrow are only impacted indirectly. We show that these direct and indirect effects can be recovered using a firm-level regression, which we estimate using Italian data. We calibrate our model to match the measured firm-level elasticities and find that heightened sovereign risk was responsible for one-third of the observed output decline during the Italian debt crisis.

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1 Introduction

During the European sovereign debt crisis, as in many emerging markets crises, many countries experienced large declines in real economic activity. An active research agenda has put forth various explanations for this negative association between sovereign risk and output. Sovereign default models in the tradition of Arellano (2008) and Aguiar and Gopinath (2006) predict that governments have greater temptation to default when the economy is in a recession, potentially explaining why sovereign risk increases when economic conditions deteriorate. Several other papers (Gennaioli, Martin, and Rossi, 2014; Perez, 2015; Bocola, 2016), however, have emphasized that sovereign risk may also have negative spillovers on the economy: because financial intermediaries hold large quantities of domestic government debt, a fall in the value of these securities can disrupt their balance sheets and induce a tightening of credit supply to the private sector.

Quantifying the two-way feedback between sovereign risk and output is a challenging open question in macroeconomics, yet it is relevant for policymakers dealing with sovereign debt crises. The challenge arises because debt crises and economic outcomes are jointly determined, which makes it hard to disentangle to what extent sovereign risk rises in response to deteriorating economic conditions and to what extent it causes them. Researchers have tackled this challenge with two main methodologies. One approach consists of fitting structural models to aggregate data and using them to measure the macroeconomic consequences of sovereign risk. This approach suffers from the criticism that the identification of the relevant effects relies partly on ancillary assumptions, as aggregate data alone provide little information about the direction of causality. A second approach uses firm-bank datasets and various empirical methods to estimate the impact that sovereign risk has on bank credit and on firms' performance. While these methods offer a more transparent identification of the firm-level responses to sovereign risk, they are not designed to capture the aggregate effects.

The contribution of this paper is to combine these two approaches by building a model of sovereign debt with heterogeneous firms to measure the feedback between sovereign risk and output. We show that in our framework, the response of output to an increase in sovereign risk depends on a set of firm-level elasticities. We estimate these elasticities using Italian micro data, and use them as empirical targets to estimate the model structural parameters. In our main counterfactual, we find that spillovers from the government to the private sector were sizable and accounted for about one-third of the output decline observed during the Italian debt crisis.

Our framework incorporates financial intermediaries and heterogeneous firms into an otherwise canonical general equilibrium model of sovereign debt and default. The economy consists of islands populated by firms, financial intermediaries, and households, and a central government. Firms differ in their productivity, and they borrow from intermediaries to finance payments of labor and capital services, factors that are used to produce a differentiated good. These working capital needs are also heterogeneous: some firms need to advance a greater fraction of their payments than other firms. Intermediaries borrow from households and use their own net worth to purchase long-term government debt and extend loans to firms. These credit markets are local in that firms borrow exclusively from intermediaries operating on their island, and intermediaries across islands are heterogeneous in their holdings of government debt. Financial intermediaries face occasionally binding leverage constraints, as the amount they borrow cannot exceed a multiple of their net worth. The government funds public consumption by collecting taxes and issuing long-term bonds, and chooses whether to default on its debt.

The model is perturbed by two aggregate shocks: a shock that moves the productivity process of firms and a shock to the value of default for the government. In response to these shocks, our environment features a two-way feedback loop between the government and the private sector. Importantly, different from standard sovereign default models, our environment does not feature exogenous output costs from default, but instead the output losses from sovereign risk potentially arise from the balance sheet effects of financial intermediaries who hold sovereign debt.

The first side of this loop reflects the endogeneity of government default risk as changes in aggregate productivity and enforcement affect the values for the government of repaying versus defaulting, thereby inducing time variation in sovereign default probabilities and hence interest rate spreads of government securities. The second side of this loop is that fluctuations in government default risk affect production through their impact on financial intermediation. When sovereign risk increases, the market value of government debt on the balance sheet of financial intermediaries falls, leading to a decline in their net worth. This decline tightens the banks' leverage constraint, which leads intermediaries to restrict credit supply. The reduction in credit supply raises the interest rates at which firms finance their working capital, and these higher costs induce firms to reduce their demand for inputs. Alongside this *direct effect* that sovereign risk has on economic activity through its impact on firms' borrowing costs, the model features additional general equilibrium mechanisms: as firms that are exposed to higher borrowing costs cut their production, the demand for other intermediate goods and for labor falls, affecting in equilibrium the prices faced by all other firms. We refer to these as the *indirect effects* of sovereign risk.

An important result in our paper is that, up to a first-order approximation, the direct and indirect effects of sovereign risk map into the parameters of a reduced-form linear regression of firms' sales. Specifically, we show that the direct effect is identified by comparing the output changes of firms with different working capital needs during a sovereign debt crisis across locations with different sovereign debt exposure of banks. This result is intuitive: comparing firms with different working capital needs *within an island* nets out the indirect effects of sovereign risk because firms in the same location share the same factor markets; comparing this differential *across islands* with different sovereign debt exposure nets out other confounding factors, such as the differential impact that aggregate productivity shocks have on firms with different working capital needs. We similarly show that the indirect effects of sovereign risk can be identified from the relative response of firms that do not borrow across locations with differential exposure to sovereign debt. These "zero-leverage" firms are not affected by fluctuations in borrowing rates, so their performance during a debt crisis is informative about the spillovers that sovereign risk has on firms through its impact on goods and labor markets.

Given this result, our empirical strategy consists of two steps. In the first step, we use Italian firm- and bank-level data to estimate the model derived empirical specification and recover the relevant elasticities. Our analysis focuses on the 2008-2015 period and links three main datasets: firm-level balance sheet data from ORBIS-AMADEUS, balance sheet information of Italian banks from Bankscope, and reports from the Bank of Italy on the geographical location of banks' branches. Using this dataset, we find evidence of a negative direct effect and positive indirect effects of sovereign risk. Specifically, we show that during the Italian sovereign debt crisis, highly levered firms contracted more than firms with low leverage, and this differential was larger in regions where banks were highly exposed to government debt. Moreover, zero leverage firms expanded relatively more in regions with high exposure to sovereign risk. While these results are derived for the model-based specification, we show that they are robust to a wide range of sensitivity checks.

In the second step of our analysis, we calibrate the structural parameters of the model to match this firm-level evidence along with other firm, bank, and aggregate statistics. We then use the calibrated model to assess the output losses due to sovereign risk during the Italian debt crisis. We find that sovereign risk has significant effects on firms' borrowing rates: holding other things constant, a 100 basis point increase in sovereign interest rates has on average a pass-through of 64 basis points on firms' interest rates. We also find sizable spillovers on output: absent the increase in sovereign risk, output in 2012 would have declined only 3.2%, instead of the observed 6.4%. More generally, our analysis suggests that the government debt crisis accounted for roughly one-third of the output losses observed in Italy during the 2011-2013 period.

Related Literature. Our paper combines elements of the sovereign default literature with those of the literature on the effect of financial imperfections on firms. We also contribute to the growing literature that combines structural models with micro data to infer aggregate elasticities.

Several papers in the sovereign debt literature study how sovereign defaults and the private sector are linked through financial intermediation. Mendoza and Yue (2012) propose a model in which firms lose access to external financing conditional on a government default, and they show that such a mechanism can generate substantial output costs in a sovereign default. Similar dynamics are present in the quantitative models of Sosa-Padilla (2018) and Perez (2015) and in the more stylized frameworks of Farhi and Tirole (2018) and Gennaioli, Martin, and Rossi (2014). We share with these papers the emphasis on financial intermediation, but we depart from their analysis by focusing on this feedback in periods in which the government is not in default: in our model, an increase in the likelihood of a future default—even when the government keeps repaying—propagates to the real sector because of its effect on firms' interest rates. Many debt crises, and in particular the one that we are studying, are characterized by rising sovereign spreads but no actual default.

In this respect, our paper is closer to Neumeyer and Perri (2005), Uribe and Yue (2006), Corsetti, Kuester, Meier, and Müller (2013), Gourinchas, Philippon, and Vayanos (2017), and Bocola (2016), who measure the macroeconomic effects of sovereign risk by estimating or calibrating structural models, and the reduced form approach in Hébert and Schreger (2017) and Bahaj (2020).[1] Compared with the above papers, a main contribution of our approach is to show that cross-sectional moments are informative about the propagation of sovereign risk on real economic activity and to use micro data and a model to carry out the measurement. In doing so, we rely on a model with firm heterogeneity, in the tradition of the literature of firm dynamics.[2]

Our empirical findings relate to an extensive literature that uses micro data to measure the effect of banks' balance sheet shocks on firms' outcomes. In the context of the European debt crisis, Bottero, Lenzu, and Mezzanotti (2020) use the Italian credit registry and show that banks with more exposure to sovereign debt decreased their lending after the Greek

[1] Hébert and Schreger (2017) cleverly exploit the rulings in the case *Republic of Argentina v. NML Capital* as exogenous variation of sovereign risk and document large negative effects of sovereign risk on Argentinian stock returns. Bahaj (2020) uses a narrative high-frequency approach to identify plausibly exogenous variation in sovereign risk.

[2] Starting from the work of Cooley and Quadrini (2001), Arellano, Bai, and Zhang (2012), Kahn and Thomas (2013), Midrigan and Xu (2014), and others develop models with firm heterogeneity and financial frictions to study aggregate implications for business cycles and misallocation.

bailout. Their approach controls for possible shifts in credit demand induced by the debt crises by focusing on firms that borrow from multiple banks, as in Khwaja and Mian (2008). In addition, the authors assess the effect of the sovereign crisis on firms' outcomes by exploiting differential exposure of their lenders to the Italian government. The main finding is that firms borrow from more exposed lenders experienced a reduction in output, employment and investment relative to firms borrowing from less exposed lenders. See also Kalemli-Ozcan, Laeven, and Moreno (2018), Bofondi, Carpinelli, and Sette (2018), Acharya, Eisert, Eufinger, and Hirsch (2018), Balduzzi, Brancati, and Schiantarelli (2018), Altavilla, Pagano, and Simonelli (2017), De Marco (2019) and Manaresi and Pierri (2018) for related findings. Our estimate for the direct effect of sovereign risk is related to the reduced-form specifications estimated in this literature. As a result, one contribution of our paper is to clarify the interpretation of these estimates and highlight that variations across firms that are differentially exposed to sovereign risk isolate only part of the macroeconomic effects.

A number of recent papers share the emphasis on using firm and bank-level data to measure the aggregate effects of credit shocks. Chodorow-Reich (2014) shows that firms related to lenders that were exposed to the Lehman bankruptcy cut their employment more than firms related to healthier lenders. His analysis clarifies that these firm-level estimates are not sufficient to measure the aggregate effects of a credit shock as the comparison across firms nets out certain general equilibrium effects—what we label as indirect effects in our analysis. Sraer and Thesmar (2018) derive explicit formulas for these indirect effects that are valid in a large class of models and show how to use them to aggregate firm-level estimates. Their approach requires taking a stand on key macro elasticities—for example, the Frisch elasticity of labor supply and the elasticity of substitution across goods.[3] Our paper shows theoretically that these indirect effects can also be identified from micro data by comparing the behavior of the zero-leverage firms across locations where banks have different sovereign debt exposure. This is because zero-leverage firms are not affected by fluctuations in borrowing rates, so their performance during a debt crisis is informative about the spillovers that sovereign risk has on firms through its effect on goods and labor markets.[4] In our application, we find that these indirect effects dampen the overall effect of the debt crisis on output. Orchard, Ramey, and Wieland (2023) also document dampening general equilibrium forces in a different application.

An active research agenda centers on using micro data to inform aggregate structural models. Researchers have used related "micro-to-macro" approaches in a variety of settings: see, for example, Nakamura and Steinsson (2014), Beraja, Hurst, and Ospina (2019),

[3] See also Blattner, Farinha, and Rebelo (2019) for a similar approach in the context of the European sovereign debt crisis.

[4] Huber (2018) exploits a similar insight for estimating local general equilibrium effects of financial shocks.

Kaplan, Moll, and Violante (2018), Hagedorn, Karahan, Manovskii, and Mitman (2013), Chodorow-Reich, Coglianese, and Karabarbounis (2019), Lyon and Waugh (2018), Arellano, Bai, and Kehoe (2019), and Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017). To the best of our knowledge, our paper is the first to apply a similar set of tools to study the macroeconomic consequences of sovereign debt crises.

Overview. The paper is organized as follows. We present the model in Section 2. Section 3 discusses the main mechanisms and our empirical strategy. Section 4 presents our data sources and the empirical results. In Section 5, we use the model to measure the macroeconomic effects of sovereign risk and perform a sensitivity analysis of our results. Section 6 concludes.

2 Model

The economy is composed of a central government and J islands where final goods firms, intermediate goods firms, financial intermediaries, and families interact.

The central government collects tax revenues from final goods firms and borrows from financial intermediaries to finance public goods and service outstanding debt. The government can default on its debt, and the rate at which it borrows reflects the risk of default.

Each island has two types of firms. Final goods firms are competitive, and they have a technology that converts intermediate goods into a final good. Intermediate goods firms operate under monopolistic competition, and they use capital and labor to produce differentiated goods. They borrow from financial intermediaries to finance a portion of their input costs, and they differ in their productivity and financing needs.

Families are composed of workers and bankers. They have preferences over consumption and labor, and they own intermediate goods firms. Families decide on labor for workers and investment, and they rent out their capital to firms. They can also deposit savings in financial intermediaries. Financial intermediaries are run by bankers who borrow from families to lend to intermediate goods firms and the central government.

The economy is perturbed by two aggregate shocks. The first shock, A_t, is an aggregate shock to the firms' productivity. The second shock, ν_t, affects the utility of the government in case of a default. The timing of events within the period is as follows. First, all aggregate and idiosyncratic shocks are realized, and the government chooses whether to default and how much to borrow. After that, given shocks and government policies, all private decisions are made, and goods, labor, and credit markets clear.

We start with the description of the problem of the central government and the agents on each island. We then define the equilibrium for this economy and conclude the section with a discussion of the key simplifying assumptions.

2.1 The government

The central government decides the level of public goods G_t to provide to its citizens. It finances these expenditures by levying a constant tax rate τ on final goods firms and by issuing debt to financial intermediaries. The debt instrument is a perpetuity that specifies a price q_t and a quantity M_t such that the government receives $q_t M_t$ units of final goods in period t. The following period, a fraction ϑ of outstanding debt matures. Let B_t be the stock of debt at the beginning of period t. Conditional on not defaulting, the government's debt in $t+1$ is the sum of non-matured debt $(1-\vartheta)B_t$ and the new issuance M_t, such that $B_{t+1} = (1-\vartheta)B_t + M_t$.

The time t budget constraint, conditional on not defaulting, is

$$\vartheta B_t + G_t = q_t\left[B_{t+1} - (1-\vartheta)B_t\right] + \tau \sum_j Y_t^j, \tag{1}$$

where Y_t^j are the aggregate final goods on island j.

Every period, the government chooses G_t and B_{t+1} and decides whether to repay its outstanding debt, $D_t = 0$, or default, $D_t = 1$. A default eliminates the government's debt obligations, but it also induces a utility cost ν_t. When in default, the government can still issue new bonds. Its budget constraint is as it is in equation (1) with $B_t = 0$. We assume that ν_t shocks follow an autoregressive process,

$$\nu_t = \bar{\nu}(1-\rho_\nu) + \rho_\nu \nu_{t-1} + \varepsilon_{\nu t},$$

with $\varepsilon_{\nu t} \sim \mathcal{N}(0, \sigma_\nu)$. These shocks generate fluctuations in the value of default for the government, inducing shifts in the bond price schedule for a given level of debt and productivity.[5] We will refer to these as *enforcement shocks*, but they can also be interpreted as financial shocks that affect the terms at which the government can borrow—for example, a pure change in investors' expectations about the government's ability to service its debt.[6]

[5] The quantitative sovereign debt literature finds that changes in the value of default for the government are necessary to fit the data on government spreads (Arellano, 2008). In most papers, fluctuations in default values are generated by assuming that the cost of default depends on income. In our model, as in Aguiar and Amador (2013) and Muller, Storesletten, and Zilibotti (2019), these fluctuations are directly induced by ν_t shocks.

[6] In the context of the Italian sovereign debt crisis, these fluctuations in default risk that are orthogonal to

The government's objective is to maximize the present discounted value of the utility derived from public goods net of any default costs,

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_g^t \left(\frac{G_t^{1-\sigma} - 1}{1-\sigma} - D_t \nu_t \right) \right].$$

2.2 The private sector

The private sector consists of J islands, with firms, families, and financial intermediaries operating on each island.

Final goods firms. The final good Y_{jt} is traded across islands, and its price is normalized to 1. Final goods are produced from a fixed variety of intermediate goods $i \in [0, 1]$ via the technology

$$Y_{jt} \leq \left[\int (y_{ijt})^{\eta} \, di \right]^{\frac{1}{\eta}}, \tag{2}$$

where the elasticity of demand is $1/(1-\eta) > 1$. Final goods firms also pay a proportional tax from their revenue with tax rate τ. They choose the intermediate goods $\{y_{ijt}\}$ to solve

$$\max_{\{y_{ijt}\}} (1-\tau) Y_{jt} - \int p_{ijt} y_{ijt} \, di$$

subject to (2), where p_{ijt} is the price of good i on island j relative to the price of the final good. This problem yields that the demand y_{ijt} for good i is

$$y_{ijt} = \left(\frac{1-\tau}{p_{ijt}} \right)^{\frac{1}{1-\eta}} Y_{jt}. \tag{3}$$

This standard demand function for good i depends negatively on the relative price p_{ijt} and positively on the island output Y_{jt}.

Intermediate goods firms. A measure of intermediate goods firms produce differentiated goods in this economy. Each firm i combines capital k_{ijt} and labor ℓ_{ijt} to produce output y_{ijt} using a constant returns to scale technology. Production is affected by productivity shocks $\tilde{z}_{ijt}$. The output produced by firm i on island j at time t is

$$y_{ijt} = \tilde{z}_{ijt} \ell_{ijt}^{1-\alpha} k_{ijt}^{\alpha}. \tag{4}$$

the country's debt and output dynamics could proxy for contagion effects from other countries (Bahaj, 2020) or sunspots (Bocola and Dovis, 2019).

Firms' productivity $\tilde{z}_{ijt}$ is affected by idiosyncratic productivity shocks z_{ijt} and by the aggregate productivity shock A_t, with $\tilde{z}_{ijt} = \exp\{A_t + z_{ijt}\}$. The processes for idiosyncratic and aggregate productivity are

$$z_{ijt} = \rho_z z_{ijt-1} + \sigma_z \varepsilon_{ijt} \tag{5}$$

$$A_t = \rho_A A_{t-1} + \sigma_A \epsilon_t, \tag{6}$$

where ε_{ijt} and ϵ_t are standard normal random processes. This formulation implies that the distribution of firms' idiosyncratic productivity is the same across islands given symmetric initial conditions.

At the beginning of the period, idiosyncratic and aggregate shocks are realized. Firms make input choices for capital k_{ijt} and labor ℓ_{ijt} to be used in production. We assume that firms need to borrow a fraction of their input costs before production, and they borrow from financial intermediaries by issuing bonds b^f_{ijt} at interest rate R_{jt}. These working capital needs, which we denote by λ_i, are firm specific and time invariant. Accordingly, the financing requirement for firm i is

$$b^f_{ijt} = \lambda_i (r^k_{jt} k_{ijt} + w_{jt} \ell_{ijt}), \tag{7}$$

where r^k_{jt} is the rental rate for capital and w_{jt} is the wage rate on island j at period t. We assume that the cross-sectional distribution of working capital needs is constant across time and space, and it is denoted by $\Lambda_\lambda(\lambda)$.

At the end of the period, production takes place; firms decide on the price p_{ijt} for their product, taking as given their demand schedule (3); and they repay their debt $R_{jt} b^f_{ijt}$ and the remainder of their input costs. Firms' profits, which are rebated to families, are

$$\Pi_{ijt} = p_{ijt} y_{ijt} - (1 - \lambda_i)(r^k_{jt} k_{ijt} + w_{jt} \ell_{ijt}) - R_{jt} b^f_{ijt}. \tag{8}$$

In our baseline model, firms always repay their debt. In an extension, discussed in Section 5.3, we allow for the possibility of firm default by introducing an idiosyncratic shock that affects firms' revenues after their input choices are made.

Families. Each island has a representative family composed of workers and bankers. Each period, the family sends out the workers to provide L_{jt} labor to firms. It also sends out the bankers to run financial intermediaries for one period, providing them with net worth N_{jt}. At the end of the period, workers and bankers return the proceeds of their operations to the family, which then decides how to allocate these resources. The family has preferences over consumption C_{jt} and labor L_{jt} given by

$$U_j = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(C_{jt} - \chi \frac{L_{jt}^{1+\gamma}}{1+\gamma} \right) \right].$$

Preferences over consumption are linear and decreasing and convex over labor, with $1/\gamma > 0$ being the Frisch elasticity of labor supply. As we will show below, the linearity of preferences over consumption simplifies the characterization of the equilibrium and reduces the number of aggregate state variables.

Families own capital K_{jt-1}, which depreciates at rate δ, and they rent it to intermediate goods firms at the rental rate r^k_{jt}. They can save by accumulating new capital and by saving in one-period deposits a_{jt} with financial intermediaries at the price q^a_{jt}. They receive the profits from the intermediate goods producers, Π_{jt}, the wages from the workers, $w_{jt}L_{jt}$, and the returns from the operations of the bankers, F_{jt}. As we discuss later, the payment from bankers includes the returns from the bonds issued by the firms and from the island's holdings of government debt B_{jt}.

The family also endows bankers with net worth N_{jt}, which consists of the value of government bonds that did not mature held in region j, as well as a constant transfer $\bar{n}_j$:

$$N_{jt} = \bar{n}_j + (1 - D_t)(1 - \vartheta) q_t B_{jt}. \tag{9}$$

The dynamics of bond prices that reflect default risk q_t, actual defaults D_t, and government debt holdings B_{jt} will induce variation in the net worth of financial intermediaries.

The budget constraint of the representative family is

$$C_{jt} + K_{jt} - (1 - \delta)K_{jt-1} + q^a_{jt} a_{jt} + N_{jt} = w_{jt} L_{jt} + r^k_{jt} K_{jt} + a_{jt-1} + F_{jt} + \Pi_{jt}. \tag{10}$$

The optimality conditions for families imply that the deposit rate and the rental rate of capital are constant over time, $q^a_{jt} = \beta$ and $r^k_{jt} = 1 - \beta(1 - \delta)$. In contrast, the wage rate is time varying and island specific, and it equals the marginal disutility of labor,

$$w_{jt} = \chi L^{\gamma}_{jt}.$$

Financial intermediaries. Financial intermediaries in each island use their net worth and the deposits of the family to purchase debt issued by the government and the firms. Financial intermediaries are competitive and take all prices as given. The beginning-of-the-period budget constraint for an intermediary is

$$q_t B_{jt+1} + \int b^f_{ijt} di \leq N_{jt} + q^a_{jt} a_{jt}. \tag{11}$$

Financial intermediaries face a standard leverage constraint that limits their ability to raise deposits,

$$q^a_{jt} a_{jt} \leq q_t B_{jt+1} + \theta \int b^f_{ijt} di. \tag{12}$$

That is, the amount that intermediaries can borrow from households is bounded by the value of their collateral. We assume that intermediaries can fully pledge their holdings of government debt but can pledge only a fraction θ of the firms' loans.[7] Combining the budget constraint (11) and leverage constraint (12) implies that the amount that a bank can lend to firms is bounded by a proportion $1/(1-\theta)$ of its net worth,

$$\frac{N_{jt}}{1-\theta} \geq \int b^f_{ijt} di. \tag{13}$$

At the end of the period, each financial intermediary receives the payment from firms and the government and pays back deposits. The end-of-the-period returns depend on whether the government defaults, and they equal

$$F_{jt+1} = (1 - D_{t+1}) \left[\vartheta B_{jt+1} + q_{t+1}(1-\vartheta) B_{jt+1} \right] + R_{jt} \int b^f_{ijt} di - a_{jt}.$$

These returns are distributed back to the family. The objective of an intermediary is to choose $\{a_{jt}, B_{jt+1}, b^f_{ijt}\}$ to maximize the expected return $\mathbb{E}_t[\beta F_{jt+1}]$ subject to (11) and (12).

The financial intermediaries' problem gives rise to the following pricing condition for firm loans:

$$R_{jt} = \frac{1+\zeta_{jt}}{\beta}, \tag{14}$$

where ζ_{jt} is the Lagrange multiplier on constraint (12). Condition (14) implies that firms pay a premium ζ_{jt}/β over the risk-free rate on their loans when the leverage constraint of banks binds: in this case, banks cannot further increase the supply of funds, so the interest rate rises in order to clear the credit market.

The decision problem of financial intermediaries also gives rise to the following pricing

[7] The assumption that government debt can be pledged fully captures the fact that these securities are effectively the best collateral for financial institutions in, for example, refinancing operations with the European Central Bank. This restriction can easily be relaxed by introducing a discount θ^g in equation (12).

condition for government securities:

$$q_t = \mathbb{E}_t \beta \left[(1 - D_{t+1}) \left(\vartheta + q_{t+1}(1 - \vartheta) \right) \right]. \tag{15}$$

The price of long-term government bonds compensates for default risk. In no-default states, each unit of a discount bond pays the maturing fraction ϑ and the value of the non-maturing fraction $q_{t+1}(1-\vartheta)$. The Lagrange multiplier does not appear in the pricing equation for government bonds, because they are fully pledgeable.

The government interest rate spread, spr_t, is the difference between yield to maturity of government bonds relative to the risk free rate. The yield to maturity is the constant interest rate that the promised flow payments are discounted by such that they equate the market price per unit of bond q_t. Given the structure of our perpetuity contracts and a risk free rate of $1/\beta$ this implies the following mapping between bond prices and government spread

$$q_t = \sum_{j=0}^{\infty} \frac{\vartheta(1-\vartheta)^j}{(1/\beta + \text{spr}_t)^{j+1}} = \frac{\vartheta}{1/\beta + \text{spr}_t - (1-\vartheta)}. \tag{16}$$

2.3 Equilibrium

We can now formally define a Markov equilibrium for this economy. We characterize the equilibrium conditions for the private sector in each island, taking the government policies as given. We then describe the recursive problem of the government.

We first describe our state variables and switch to recursive notation. The linearity in preferences for private consumption implies that we do not need to record the distribution of capital and deposits across islands as aggregate state variables because the wealth of families does not matter for the choices of labor, capital, and deposits. In addition, because of the linearity of households' preferences, financial intermediaries are indifferent about the amount of government bonds they hold in their balance sheet, so the holdings of government debt across regions are indeterminate in our economy. We will focus on an equilibrium in which financial intermediaries in region j hold a constant fraction φ_j of the issued debt—$B'_j = \varphi_j B'$, with $\sum_j \varphi_j = 1$. We will treat $\{\varphi_j\}$ as parameters in our model and use bank-level data on holdings of government debt to discipline them empirically. The aggregate state of the economy includes the aggregate shocks for productivity and enforcement, $S = \{A, \nu\}$, and the initial level of government debt B.

We describe the equilibrium of each island, leveraging the convenient feature of our model that government policies affect the private equilibrium mainly through their impact on intermediaries' net worth N_j, as seen in equation (9). Specifically, the firms' and families' choices of labor, capital, borrowing, and deposits, depend only on the islands' prices, which include wages, firm's borrowing rates, rental rate of capital and the deposit prices. These prices themselves only depend on government policies through their effect on net worth. Government policies for default D and debt B do not affect these variables through other channels because in our set up the risk free rate is a constant. With these considerations, we define an island state variable $X_j = \{A, N_j\}$ that includes the aggregate productivity shock and the intermediaries' net worth, and define the island equilibrium as a function of this island state. Subsequently, we define the recursive of equilibrium for the entire economy, where the government understands the impact of its policies on the island equilibrium through their effect on net worth.

Definition 1. *Given an island state $X_j = \{A, N_j\}$, the island j equilibrium consists of*

- *intermediate goods firms' policies for labor $\ell(z, \lambda, X_j)$, capital $k(z, \lambda, X_j)$, and borrowing $b^f(z, \lambda, X_j)$, given idiosyncratic states $\{z, \lambda\}$, and final goods firms' output $Y(X_j)$;*
- *policies for labor $L(X_j)$, capital $K(X_j)$, and deposits $a(X_j)$*
- *price functions for wages $w(X_j)$ and firm borrowing rates $R(X_j)$, and the constant capital rental rate r^k and deposit price q^a; and*
- *the distribution of firms over idiosyncratic productivity and working capital needs $\Lambda(z, \lambda)$;*

such that (i) *the policy functions of intermediate and final goods firms satisfy their optimization problem;* (ii) *the policies for labor, capital, and deposit satisfy the families' optimality conditions;* (iii) *firm borrowing rates satisfy equation (14), and the leverage constraint (13) is satisfied;* (iv) *labor, capital, and firm bond markets clear; and* (v) *the distribution of firms is consistent with idiosyncratic shocks*

Using the optimization problems for firms and families as well as the the market clearing conditions for firm labor, capital, deposits and borrowing, we can characterize more sharply the island equilibrium. In the next proposition, we provide explicit solutions for the three conditions that determine the equilibrium level of firms' borrowing rates $R(X_j)$, wages $w(X_j)$, and output $Y(X_j)$ in each island.

Proposition 1. *In the equilibrium for island j given state X_j, firms' borrowing rates $R(X_j)$, wages $w(X_j)$, and output $Y(X_j)$ satisfy the following conditions:*

$$\frac{N_j}{1-\theta} \geq M_n \overline{\lambda}(X_j) \left[\exp\{A\}^{\frac{\eta}{1-\eta}} / R_w(X_j)\right]^{\frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma}} \tag{17}$$

$$w(X_j) = M_w \left[\exp\{A\}^{\frac{\eta}{1-\eta}} / R_w(X_j)\right]^{\frac{(1-\eta)}{\eta(1-\alpha)}} \tag{18}$$

$$Y(X_j) = M_y \frac{\left[\exp\{A\}^{\frac{\eta}{1-\eta}} / R_w(X_j)\right]^{\frac{1-\eta+(1-\alpha\eta)\gamma}{\eta(1-\alpha)\gamma}}}{\exp\{A\}^{\frac{\eta}{1-\eta}} / R_y(X_j)}, \tag{19}$$

where $R(X_j) = 1/\beta$ if condition (17) is a strict inequality; $R_w(X_j)$, $R_y(X_j)$, and $\overline{\lambda}(X_j)$ are functions of the island's interest rate $R(X_j)$ and Λ_λ, $R_w(X_j)^{-1} = \int_\lambda r_\lambda(X_j)^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda$, $R_y(X_j)^{-1} = \int_\lambda r_\lambda(X_j)^{-\frac{1}{1-\eta}} d\Lambda_\lambda$, and $\overline{\lambda}(X_j) = \int_\lambda \lambda \pi_\lambda(X_j) d\Lambda_\lambda$, where $r_\lambda(X_j) = 1 + \lambda(R(X_j) - 1)$ and $\pi_\lambda(X_j) = r_\lambda(X_j)^{-\frac{1}{1-\eta}} / \int r_\lambda(X_j)^{-\frac{1}{1-\eta}} d\Lambda_\lambda$; and the constants $\{M_n, M_w, M_y\}$ are functions of the model parameters.

The proof of this proposition is in Appendix A. The inequality in (17) is the equilibrium condition of the credit market: credit supply by financial intermediaries cannot exceed $N_j/(1-\theta)$ because of the leverage constraint, while credit demand—the sum of the loans issued by all firms in a given region—is given by the expression on the right-hand side in (17). Credit demand increases with the weighted average of firms' working capital needs $\overline{\lambda}(X_j)$ and with aggregate productivity A, and decreases with average interest rate paid by firms $R_w(X_j)$. Given X_j, condition (17) can be used to determine the interest rate faced by firms in a given island. If inequality (17) is satisfied at $R(X_j) = 1/\beta$, the intermediaries have enough funds to finance the demand of credit by firms, so the equilibrium interest rate equals $1/\beta$. If (17) is not satisfied at $R(X_j) = 1/\beta$, then interest rates need to increase so that (17) holds with equality. Once the local interest rate is determined, equations (18) and (19) determine the equilibrium level of wages and output in an island.

In the next corollary, we use equation (17) to show how changes in banks' net worth affect firms' borrowing rates.

Corollary 1. *Borrowing rates weakly decrease with net worth N_j, $\partial R/\partial N_j \leq 0$.*

When the leverage constraint binds, a decline in bank net worth reduces credit supply, and it leads to an increase in the interest rate that firms in the island pay. These movements in the borrowing rate affect firms' demand for inputs and production decisions, which then affect wages and output in the island.

Having characterized the island equilibrium, we can now describe the recursive problem of the economy which includes the government. The government collects as tax revenues T a fraction τ of each island's final goods output, $T = \tau \sum_j Y(X_j)$. It understands that the island net worth, which is part of the state X_j, depends on its states and choices with $N_j = \bar{n}_j + \varphi_j(1-D)(1-\vartheta)q(S, B')B$. This mapping makes tax revenues a function that depends on the aggregate shocks and the states and choices of the government, $T(S, B, D, B')$, because the aggregate output of each island depends on these variables through its effect on net worth.

The recursive problem of the government follows the quantitative sovereign default literature. Given a bond price function, $q(S, B')$, the government makes its choices to maximize its value. Let $W(S, B)$ be the value of the option to default such that

$$W(S,B) = \max_{D=\{0,1\}}\{(1-D)V(S,B) + D\,[V(S,0)-\nu]\},$$

where $V(S, B)$ is the value of repaying debt B and is given by

$$V(S,B) = \max_{B'} u_g(G) + \beta_g \mathbb{E}\, W(S',B'),$$

subject to the budget constraint

$$G + \vartheta B = T(S,B,D,B') + q(S,B')\left[B' - (1-\vartheta)B\right]$$

and the evolution of aggregate shocks. The value of default is $V(S, 0) - \nu$, because with default, the debt B is written off and the government experiences the default cost ν.

Importantly, the government internalizes the feedback that its choices have on the private sector behavior. By affecting default risk and the price of its debt, government actions affect the net worth of financial intermediaries via equation (9), and these changes in net worth affect the island equilibrium characterized in Proposition 1. This feedback matters for the government because the private sector determines current and future tax revenues $T(S, B, D, B')$, which the government views as a schedule that depend on its choices. This problem gives decision rules for default $D(S, B)$, borrowing $B'(S, B)$, and public consumption $G(S, B)$.

We can now define the recursive equilibrium of this economy.

Definition 2. *The Markov recursive equilibrium consists of government policy functions for default $D(S, B)$, borrowing $B'(S, B)$, public consumption $G(S, B)$, and value functions $V(S, B)$ and $W(S, B)$, and the bond price function $q(s, B')$ such that* (i) *taking as given the bond price function, the policy and value functions for the government satisfy its optimization problem;* (ii) *the island equilibrium is satisfied; and* (iii) *the bond price function is consistent with future policies and satisfy the condition $q(S,B') = \mathbb{E}\beta\left[(1-D'(S',B'))\left(\vartheta + q(S',B''(S',B'))(1-\vartheta)\right)\right]$.*

2.4 Discussion

Before moving forward, we discuss some key elements of the model. As explained in the previous section, in our model the government affects the private sector only through its effect on the net worth of financial intermediaries. The literature has identified other channels through which public sector strains can be transmitted to the real economy, such as incentives for indebted governments to raise corporate taxes (Aguiar, Amador, and Gopinath, 2009; Acharya, Drechsler, and Schnabl, 2014) or to more generally interfere with the private sector (Arellano, Atkeson, and Wright, 2015). Our analysis is silent about the quantitative importance of these other mechanisms.

Our modeling of the financial sector borrows from a recent literature that introduced financially constrained intermediaries in otherwise standard business cycle models, such as Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).[8] The key difference with these papers is that the bankers in our framework exit after one period, whereas in these other models, they can operate for more than one period. Technically, this implies that in our framework, the evolution of net worth, governed by the transfer rule in equation (9), depends only on the dynamics of government debt and sovereign risk, whereas in these other papers, net worth also depends on the savings of financial intermediaries. Our restriction is motivated mostly by tractability, because the numerical solution of our model with one additional state variable per island, while feasible, is substantially more involved. It is important to emphasize, however, that this modification would not alter the mapping between the output effects of sovereign risk and the cross-sectional moments that we will discuss in the next section. Thus, given the measurement strategy we pursue in the paper, we expect the results to be robust to introducing savings for financial intermediaries.

In our framework, firms differ in their leverage because of exogenous differences in the working capital requirements λ.[9] This is clearly a crude way of modeling leverage, because in reality, firms borrow for reasons besides financing their working capital expenses. The advantage of this formulation is tractability: as firms do not choose their leverage, their decision problem remains static. This greatly helps in characterizing the private sector equilibrium, as we saw in Proposition 1. We do not believe that the transmission of sovereign risk to the private sector would change substantially if we did model the borrowing decisions of firms: irrespective of why firms decide to borrow, the fact that they are

[8]An alternative modeling of the links between sovereign spreads and banks' credit supply would be to allow for the possibility of banks' default. In Ari (2018), for example, the deterioration of banks' balance sheet after an increase in sovereign risk raises the prospect of a bank's default, increasing in equilibrium banks' funding costs.

[9]Indeed, the debt-to-asset ratio for firm i in our model is equal to $\lambda_i(r^k/\alpha)$. For this reason, we will interchangeably use the terms *leverage* and *working capital requirement* in the rest of the paper.

levered exposes them to changes in the interest rate. While we do not have a deep theory that explains differences in leverage across firms, our flexible formulation allows the model to fit the distribution of firms' leverage that we observe in the data.

In addition, sovereign risk in our model affects firms' behavior through changes in firms' borrowing rates. There is however empirical evidence that sovereign risk impacted firms by changing the quantity of credit available, see the evidence in Bottero, Lenzu, and Mezzanotti (2020) for credit lines. In Section 5.3, we present a version of our model in which sovereign risk affects both the price and quantity of credit for firms and show that the results are quantitatively comparable to those of our baseline model.

Finally, the islands in our model are regions in which credit, intermediate goods, and labor markets are local, while final goods, produced with local intermediate goods, are perfectly substitutable and traded across islands. These assumptions on markets and the input-output structure are clearly stylized, but we think that they are more reasonable for our data than an assumption in which all markets are national. First, the majority of Italian firms in our dataset are small enterprises, and their predominant form of external finance is local banks. Second, most firms in our dataset operate in non-tradable sectors. Third, our analysis is conducted over a fairly short period of time, during which it is reasonable to assume that labor is not perfectly mobile across regions.

3 The propagation of sovereign risk

We now study how sovereign risk propagates to the private sector in the model and discuss our strategy to use firm-level data to empirically discipline this mechanism. In Section 3.1, we use the properties of the regional equilibrium to show that an increase in sovereign risk reduces firms' output via two channels. First, by reducing banks' net worth, higher sovereign risk leads to higher borrowing costs for firms. As noted earlier, we label this mechanism the *direct effect* of sovereign risk. Second, as firms that are directly exposed to the sovereign shock cut their production, they set in motion general equilibrium spillovers on all the other firms in the economy. We refer to this second mechanism as the *indirect effects* of sovereign risk. Section 3.2 discuss to what extent firm-level data can be used to measure the strength of these two channels, and Section 3.3 outlines our empirical strategy.

3.1 Direct and indirect effects of sovereign risk

Taking prices as given, firms maximize their profit (8) subject to their demand schedule (3) and the financing requirement (7). Given state X_j, optimal firm capital satisfies

$$k = C_0\left(\exp\{A+z\}\right)^{\frac{\eta}{1-\eta}} Y(X_j) w(X_j)^{-\frac{(1-\alpha)\eta}{1-\eta}} \left(1+\lambda(R(X_j)-1)\right)^{-\frac{1}{1-\eta}}.$$

Firms choose a higher capital when productivity $A+z$ is high, when input prices, both borrowing rates and wages, are low, and when the demand for their product is high. Firm labor is proportional to capital and decreases with the wage $\ell = \frac{1-\alpha}{\alpha}\frac{r_k}{w(X_j)}k$. Using these optimality conditions, as well as the individual firm's demand (3), we can express the sales of a firm with idiosyncratic state (z, λ) operating in an island with state X_j as

$$\hat{py}(z,\lambda,X_j) = C_1 + \frac{\eta}{1-\eta}(A+z) - \frac{\eta}{1-\eta}\lambda R(X_j) + \hat{Y}(X_j) - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}(X_j), \tag{20}$$

where $\hat{x}$ denotes the log of x.[10] Recall that the equilibrium borrowing rates $R(X_j)$, demand $Y(X_j)$, and wages $w(X_j)$ are determined by the expressions in Proposition 1 and only depend on the island state X_j.

We can now use equation (20) to analyze the response of firms' output to an increase in sovereign spreads. Sovereign spreads affect firms' behavior through their impact on borrowing rates R, wages w, and demand Y. As we explained in Section 2, the channel by which the price of government debt affects these prices is through their impact on the net worth of financial intermediaries. Using equation (20), we can therefore write the response of firms' sales to an increase in sovereign spreads as follows:[11]

$$\frac{\partial \hat{py}}{\partial spr} = \underbrace{-\frac{\eta}{1-\eta}\lambda\left(\frac{\partial R}{\partial N_j}\frac{\partial N_j}{\partial spr}\right)}_{\text{direct effect}} + \underbrace{\left(\frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\partial\hat{w}}{\partial N_j}\right)\frac{\partial N_j}{\partial spr}}_{\text{indirect effects}}. \tag{21}$$

This elasticity can be decomposed into a direct effect on firms' borrowing rates and indirect effects that operate through aggregate demand and wages.

[10] The constants $C_0 = \left((1-\tau)\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}\right)^{\frac{1}{1-\eta}} r_k^{-\frac{1-(1-\alpha)\eta}{1-\eta}}$ and $C_1 = \frac{1}{1-\eta}\log(1-\tau) + \frac{\eta}{1-\eta}\log\{\eta\alpha\} + \frac{(1-\alpha)\eta}{1-\eta}\log\left\{\frac{1-\alpha}{\alpha}\right\} - \frac{\eta\alpha}{1-\eta}\log r_k$.

[11] There is some abuse of notation here because sovereign spreads are endogenous in our model. Yet, this experiment is well defined. One can think of equation (21) as the response of firms' sales to an enforcement shock that marginally raises sovereign spreads, as this shock affects firms on impact only via movements in sovereign spreads.

The direct effect arises because financial intermediaries hold legacy government debt and face a potentially binding leverage constraint. An increase in sovereign spreads is equivalent to a fall in the value of government bonds in the balance sheet of banks. This translates into a fall of their net worth—see equation (9). From Corollary 1, we know that a fall in banks' net worth leads to an increase in firms' borrowing rate when the leverage constraint binds, $(\partial R/\partial N_j) \times (\partial N_j/\partial spr) \geq 0$. Hence, this direct effect (weakly) reduces sales for every firm with $\lambda > 0$. The magnitude of the direct effect is heterogeneous across islands and firms because islands differ in the degree of balance sheet exposure to government debt and firms differ in their leverage.

The indirect effect arises because of the equilibrium responses of demand and wages to the changes that the direct effect induces. Because borrowing rates rise with sovereign risk, firms that need to borrow cut on their production and reduce their demand for labor. These responses lead to a decline in wages and in the demand of all other intermediate goods on the island because of complementarities in the production of final goods. These general equilibrium effects in demand and wages further influence the production decisions of firms. Specifically, the fall in demand depresses firms' output, while the decrease in wages incentivizes firms to produce more, as it reduces their marginal costs. The overall indirect effect, therefore, could be recessionary or expansionary, depending on which of these two channels dominates. These indirect effects are heterogeneous across islands, but they affect all firms within an island in the same fashion.

The sign and magnitudes of the direct and indirect effects, along with the distribution of firms' leverage and banks' exposures, shape the strength of the aggregate response of output to an increase in sovereign risk. Thus, being able to match these elasticities and distributions with data counterparts is a natural way to discipline the output costs of sovereign risk in the model. We now turn to discuss how we can use micro data to identify the direct and indirect effects.

3.2 Measuring the propagation of sovereign risk

The following proposition shows that, up to a first-order approximation, the direct and indirect effects of equation (21) map into the coefficients of a linear relation that can be estimated using firm-level data. With a slight abuse of notation, we denote a firm i at time t by (ι, j, k, t), where ι stands for the working capital need of firm i, λ_ι; j stands for the island in which firm i is located; k for the idiosyncratic productivity state of firm i; and t for time.

Proposition 2. *Let $\hat{py}_{\iota,j,k,t}$ be the log of sales for firm i with leverage λ_ι, operating in island j with* *exposure φ_j and with idiosyncratic productivity $z_{k,t}$ at time t. Let the point $x = [z, A, \nu, B]$ and assume that $N_j = N \,\forall\, j$ given x. Up to a first-order approximation around x, we have that*

$$\begin{aligned} \hat{py}_{\iota,j,k,t} &= \alpha_\iota + \beta_1(spr_t \times \varphi_j \times \lambda_\iota) + \beta_2(spr_t \times \varphi_j) + \beta_3 A_t + \beta_4(A_t \times \lambda_\iota) + \beta_5(B_t \times \varphi_j) \\ &+ \beta_6(B_t \times \varphi_j \times \lambda_\iota) + \frac{\eta}{1-\eta} z_{k,t}, \end{aligned} \tag{22}$$

where β_1 and β_2 are given by

$$\begin{aligned} \beta_1 &= -\frac{\eta}{1-\eta}\frac{\partial R}{\partial N} M \\ \beta_2 &= \frac{\partial \hat{Y} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right]\partial \hat{w}}{\partial N} M, \end{aligned}$$

with $M = -\frac{(1-\vartheta)B\vartheta}{(\vartheta + 1/\beta - 1 + spr(A,\nu,B))^2}$.

The proof of this result is in Appendix A, along with explicit expressions for the remaining coefficients $\{\beta_k\}_{k=3}^{6}$. Given empirical counterparts to λ_ι and φ_j, we can estimate equation (22) using panel data on firms' sales and aggregate data on productivity, sovereign spreads, and government debt—leaving idiosyncratic productivity $z_{k,t}$ as the residual. The coefficients of this regression would be identified because the distribution of idiosyncratic productivity shocks is independent from the other covariates. Importantly, these coefficients are informative about the direct and indirect effects of sovereign risk. Indeed, looking at the expression for β_1 and β_2 in Proposition 2 and recognizing that $\partial N/\partial spr = M \times \varphi_j$, we can see that $\beta_1 \varphi_j \lambda_\iota$ equals the direct effect while $\beta_2 \varphi_j$ equals the indirect effects defined in equation (21).

Let us provide some intuition on why it is possible to estimate the direct and indirect effect of sovereign risk using firm-level data. For that purpose, consider a special case with two groups of firms, two islands, and two periods $t = 1, 2$. We can think of period 1 as a situation in which sovereign spreads are close to zero, and period 2 as a sovereign debt crisis, so $\Delta \text{spr}_t > 0$. Firms in each island have financing needs $\lambda_\iota = \{\lambda_L, \lambda_H\}$, with $\lambda_L = 0$, and experience idiosyncratic shocks $z_{k,t}$. The two islands vary in their banks' exposure to sovereign debt, $\varphi_j = \{\varphi_L, \varphi_H\}$ with $\varphi_H > \varphi_L$. For simplicity, we further assume that government debt B does not change between period 1 and 2. We denote by $\hat{py}_{\lambda_\iota,\varphi_j,k,t}$ the log-sales in period t for a firm with leverage λ_ι, idiosyncratic productivity $z_{k,t}$, in an island with exposure φ_j.

Given these assumptions, we can show that β_1 and β_2 are identified using two *difference-in-differences* estimators. The coefficient β_2 is identified by comparing average sales growth for the "zero-leverage" firms ($\lambda_\iota = 0$) across the two regions. Specifically, using equation (22), we have:

$$\mathbb{E}\left[\Delta\left(\hat{py}_{\lambda_L,\varphi_H,k,t} - \hat{py}_{\lambda_L,\varphi_L,k,t}\right)\right] = \beta_2\left([\varphi_H - \varphi_L] \times \Delta spr_t\right), \tag{23}$$

where the expectation is taken over the idiosyncratic shocks $z_{k,t}$. This result is intuitive. Zero-leverage firms are not exposed to fluctuations in borrowing rates because these firms do not borrow. Yet, they are exposed to the general equilibrium changes in demand and wages. Therefore, the differential behavior of sales growth of this type of firm across the two islands is informative about the relative importance of the indirect effects of sovereign risk in the two regions. Because the indirect effect is locally linear in φ_j, the left hand side of equation (23) can be used, along with knowledge of $\{\varphi_L, \varphi_H\}$, to identify the indirect effect of sovereign risk—the coefficient β_2.

Similarly, the coefficient β_1 is identified by comparing sales growth between the high-λ and the low-λ firms, differenced out across the two islands. To understand why we need the triple differencing, consider the difference-in-differences between high and low-λ firms in the high exposure region,

$$\mathbb{E}\left[\Delta\left(\hat{py}_{\lambda_H,\varphi_H,k,t} - \hat{py}_{\lambda_L,\varphi_H,k,t}\right)\right] = \beta_1\left(\varphi_H \times \lambda_H \times \Delta spr_t\right) + \beta_4\left(\lambda_H \times \Delta A_t\right).$$

We can see that the above statistic does not depend on β_2, the measure of the indirect effect: because the indirect effects affect equally all firms in the same islands, they cancel out when we difference sales growth between high and low leverage firms within an island. However, this expression also clarifies that a simple difference-in-differences estimator is not enough to identify the direct effect of sovereign risk because of the presence of aggregate productivity shocks—which also induce differential effects on firms depending on firms' working capital needs λ_i. To control for those, we need to difference out the above expression once more, this time across regions:

$$\begin{aligned}\mathbb{E}\left[\Delta\left(\hat{py}_{\lambda_H,\varphi_H,k,t} - \hat{py}_{\lambda_L,\varphi_H,k,t}\right)\right] \;-\; &\mathbb{E}\left[\Delta\left(\hat{py}_{\lambda_H,\varphi_L,k,t} - \hat{py}_{\lambda_L,\varphi_L,k,t}\right)\right] = \\ = \;&\beta_1\left([\varphi_H - \varphi_L] \times \lambda_H \times \Delta spr_t\right).\end{aligned} \tag{24}$$

Again, because the direct effect is locally linear in φ_j and λ_ι, the left-hand side of equation (24) can be used to identify the direct effect β_1.

3.3 Empirical strategy

In view of this discussion, our approach to measure the output effects of sovereign risk proceeds in two steps. First, we estimate equation (22) using Italian firm-level data to measure the direct and indirect effects of sovereign risk. Second, we use these moments—along with other aggregate, regional, and firm-level statistics—as an empirical target when calibrating the structural model. We will then use the parametrized model to compute the overall effect of sovereign risk on output and perform counterfactual experiments.

For this purpose, we construct a panel dataset that merges firm, bank and aggregate data for Italy for the period 2007-2015. We first construct empirical counterparts to λ_ι and φ_j. We partition firms in two groups based on their financial leverage—assigning $lev_i = 1$ to firms with relatively high leverage and zero otherwise—and divide the Italian regions in two groups based on banks' exposure to government debt—assigning $exp_j = 1$ to a region where banks are highly exposed to the government and zero otherwise.[12] In order to minimize endogeneity concerns, we perform this partition using 2007, which preceeds the global financial crisis and the Italian sovereign debt crisis.

The baseline panel regression that we will estimate for the 2008-2015 period is *identical* to equation (22)—up to the fact that exp_j and lev_i are dummy variables:

$$\hat{py}_{i,j,t} = \alpha_i + \hat{\beta}_1 \left(\text{spr}_t \times \text{lev}_i \times \text{exp}_j\right) + \hat{\beta}_2 \left(\text{spr}_t \times \text{exp}_j\right) + \delta' \Gamma_{i,j,t} + \varepsilon_{i,j,t}, \tag{25}$$

where α_i is a firm fixed-effect and $\Gamma_{i,j,t}$ is a vector of controls that includes the level of sovereign spreads spr_t and its interaction with lev_i, the level of aggregate TFP at time t and its interaction with lev_i, and the debt-to-gdp ratio at time t and its interaction with exp_j, lev_i, and with $\text{exp}_j \times \text{lev}_i$.

In equation (25), the coefficient $\hat{\beta}_1$ represents the differential response of high and low leverage firms to a change in sovereign risk, differenced out between the high and the low exposure regions, which maps to the left hand side of equation (24). Similarly, the coefficient $\hat{\beta}_2$ represents the response of low leverage firms to a change in sovereign risk, differenced out between the high and the low exposure region. This statistic maps to the left hand side of equation (23). Given the discussion in the previous section, $\hat{\beta}_1$ and $\hat{\beta}_2$ will allow us to identify from the micro data the direct and indirect effects of sovereign risk.

[12] We choose these partitions because in the numerical analysis of our model we will consider two λ-type firms and two islands. As we will show, however, the empirical results are robust to using a continuous measure of firms' leverage and banks' exposure to government debt.

4 Empirical analysis

We now turn to the estimation of equation (25). Section 4.1 describes the data used in the analysis, and Section 4.2 reports the main empirical results.

4.1 Data

Firm-level data. We obtain yearly firm-level balance sheet data from the AMADEUS dataset. Our sample covers the period 2007-2015 and provides detailed information on publicly and privately held Italian firms. The core variables in our analysis are indicators of firm performance (operating revenues and profits), key balance sheet indicators (total assets, short- and long-term loans, and account receivables), and additional firm-level information regarding the location of the firm's headquarters and its sector. We perform standard steps to guarantee the quality of the data and scale all nominal variables by the consumer price index. We further restrict the sample by considering a balanced panel of firms operating continuously between 2007 and 2015 and by excluding firms that operate in the financial industry or sectors with a strong government presence. In Appendix B, we provide details on variables' definitions and sample selection.

We define *leverage* as the ratio of a firm's debt to total assets, our proxy for λ_i. Our baseline measure of debt is broad, and it includes short-term loans, long-term loans, and accounts receivable.[13] In view of the estimation of equation (25), we partition firms in two groups based on their leverage in 2007. We assign $lev_i = 1$ to a firm whose leverage in 2007 was above the 25^{th} percentile of the leverage distribution in that year and zero otherwise. We use the 25^{th} percentile as a cutoff because it roughly corresponds to the fraction of firms with a leverage ratio of zero whose behavior during the crisis is critical to identify the indirect effect. The average leverage for the firms with $lev_i = 1$ equals 0.51, and it equals 0.01 for those with $lev_i = 0$.

Table 1 reports a set of summary statistics for the firm-level data in 2007. The median firm in our sample is privately held, has seven employees, has operating revenues of roughly 5 million euros, and has a leverage ratio of 37%.[14]

[13] We include accounts receivable in order to proxy for accounts receivable financing loans, see Bocola and Bornstein (2023). These instruments represent a large portion of the debt liabilities of Italian non-financial firms, but they are not reported as "debt" by firms when compiling their annual balance sheet. In addition, we include long-term loans, because, as documented in Chodorow-Reich and Falato (2017), repayments of this type of debt are also sensitive to interest rate changes during financial crises because of frequent renegotiations.

[14] A median leverage of 37% is comparable to that reported by other papers that use the Italian credit registry rather than firms' balance sheets. For example, Schivardi, Sette, and Tabellini (2017) report a median debt-to-asset ratio of 40% in 2005.

Table 1: Summary statistics for the firm panel, 2007

	Obs.	Mean	P25	P50	P75
Number of employees	123,514	27	3	7	18
Operating revenues	336,047	40543	1118	5083	17972
Total assets	336,047	44273	2635	7465	21239
Debt	336,047	8680	0	342	3623
Accounts receivable	336,047	7842	35	657	3518
Leverage	336,047	0.38	0.07	0.37	0.63

Note: Monetary values are reported in thousands of euros and deflated using the consumer price index (2010 base year). See Appendix B for a definition of the variables.

Bank-level data. We obtain from Bankscope balance sheet information for banks headquartered in Italy.[15] The variables we use in our analysis are total equity, banks' holdings of government debt, and the ZIP code of the banks' headquarters.[16] We use this dataset to construct government debt holdings as a fraction of total equity at the regional level, a proxy for φ_j in the model. From Bank of Italy reports, we obtain data on the distribution of bank branches across Italian regions as of December 31, 2007. We use this information to group the banks in the sample in two categories, *local* and *national* banks. National banks are the five largest banks in our dataset by total assets in 2007, and their operations are distributed throughout the country.[17] For these banks, we geographically allocate their holdings of government debt using their network of branches. Local banks are smaller, and we assume that they operate exclusively in the region in which their headquarters are located.

Our measure of bank exposure is the 2007 ratio of government debt holdings to equity for the banks that operate in a given region j. Letting M_{nj} be the number of branches of national bank n in region j, and M_n being the total number of branches of bank n, we have

$$\text{exposure}_j = \frac{\sum_i B_i^{loc,j} + \sum_n \frac{M_{nj}}{M_n} B_n^{nat}}{\sum_i E_i^{loc,j} + \sum_n \frac{M_{nj}}{M_n} E_n^{nat}}, \tag{26}$$

[15]This sample is representative for the whole Italian banking sector. Total assets for the banks in our sample were 2,985 billion euros at the end of 2007. The corresponding statistic for *all* monetary and financial institutions (banks and money market funds) in Italy was 3,289 billion euros at the end of 2007.

[16]While Bankscope does not provide a breakdown of government bond holdings by nationality, Gennaioli, Martin, and Rossi (2018) document that this indicator captures mainly the holdings of banks to domestic government debt. This reflects the high degree of home bias in international financial portfolios. See also Table 5 in Kalemli-Ozcan, Laeven, and Moreno (2018).

[17]The national banks are Unicredit, Intesa-Sanpaolo, Monte dei Paschi di Siena, Banca Nazionale del Lavoro, and Banco Popolare.

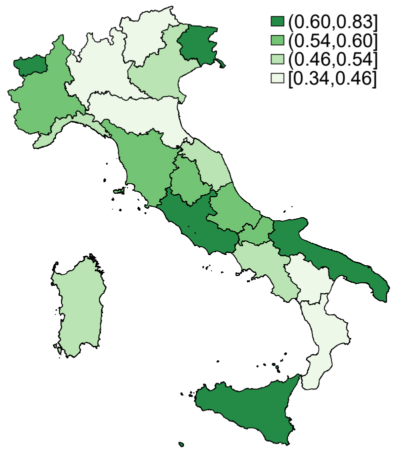

Figure 1: Banks' exposure to government debt by region in 2007

where $B_i^{loc,j}$ denotes the holdings of government debt by a local bank i operating in region j, $E_i^{loc,j}$ denotes bank equity for the same banks, and B_n^{nat} and E_n^{nat} report the same information for a national bank n.

Figure 1 reports this indicator for the 20 Italian regions. Light colors indicate locations where banks have lower exposure to government debt, while dark colors represent regions with higher exposure. There is substantial variation across regions. In Calabria, the region where banks are the least exposed to government debt, banks' holdings of government debt are equivalent to 34% of their total equity. Conversely, in Lazio, the region where banks are most exposed, this number equals 83%. We partition these regions into two groups for the empirical and quantitative analysis. The *high-exposure* regions ($\text{exp}_j = 1$) are those with the exposure indicator above the median, and they have an average exposure of 62%. The *low-exposure* ($\text{exp}_j = 0$) regions are those with a value below the median, and have an average exposure of 45%.

Appendix B provides an analysis of how firms and regional characteristics vary by our leverage and exposure groups. High leverage firms tend to be on average larger, more productive, more profitable, and less volatile than low leverage firms. Importantly, however, these differences between high and low leverage firms are remarkably similar across the two groups of regions. In addition, we show that along a wide range of characteristics, regions where banks have above-median sovereign debt exposure are not systematically different from those with below-median exposure.

Aggregate data. We now report some aggregate and regional statistics for the Italian economy. Figure 2 plots the time series of linearly detrended real GDP and total factor productivity (TFP); sovereign interest rate spreads; two indicators of the health of the Italian

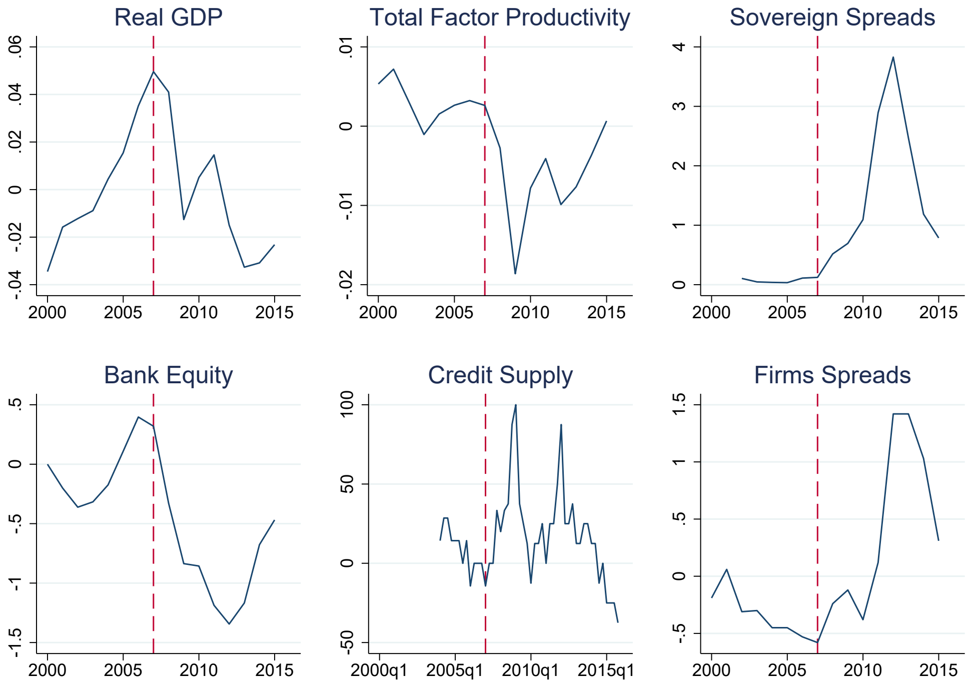

Figure 2: Aggregate time series

Note: Real GDP and total factor productivity are in logs and linearly detrended. *Sovereign spreads* are the difference in percentages between the yields on Italian and German government bonds with five-year maturities. *Bank equity* is the market value of equity of Italian monetary and financial institutions in log deviations from the 2000 value. *Credit supply* is the difference in percentage of loan officers reporting a "tightening" relative to "easing" in the credit standards from the Italian Bank Lending Survey. *Firms spreads* are the difference in percentages between the interest rate on loans up to one year for Italian and German non-financial corporations. See Appendix B for detailed definitions and data sources.

banking sector, the market value of equity issued by monetary and financial institutions and information on bank credit supply from the Italian Bank Lending Survey; and an indicator of firms' intererest rate spreads: the difference between the average interest rate on short-term loans faced by Italian and German non-financial firms.[18] These series are collected from different sources detailed in Appendix B.

The Italian economy was hit by the global financial crisis of 2008-2009 and started recovering in 2010, but it eventually experienced a second deep recession. These two crises share some similarities, as they were both associated with a strong deterioration in financial conditions. An important difference between the two episodes is the behavior of sovereign interest rate spreads. The first recession was not associated with a sovereign debt crisis, as

[18]The differential between interest rates on Italian firms and German government securities follows a very similar pattern, although the spread is on average 200 basis points higher, reflecting the presence of credit risk for firms. We focus on the spread relative to German firms to net out such credit risk, because in our baseline model, firms do not default.

sovereign interest rate spreads remained close to zero. The second recession, on the other hand, was characterized by turbulence in sovereign debt markets. After the Greek request in April 2010 for an EU/IMF bailout package, interest rate spreads on several southern European government bonds, including Italian ones, increased sharply. These tensions intensified dramatically in 2011 and eventually resolved with the introduction during the summer of 2012 of the Outright Monetary Transaction program by the European Central Bank. Note that the aggregate data show patterns consistent with the key mechanism in our model: the rise in sovereign risk was associated with a drop in banks' equity, a tightening of bank credit supply, and an increase in firms' interest rate spreads.

4.2 Results

Column (1) of Table 2 reports the estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ in equation (25) along with standard errors clustered at the regions and time level. This specification is identical to the model implied relationship of Proposition 2, and it will be our baseline.

Our results suggest a negative direct effect: holding everything else constant, after a 100 basis points increase in sovereign spreads, the differential in sales between high and low leverage firms is 0.72% larger for firms located in high-exposure regions compared with the same differential for firms located in the low-exposure regions. In addition, we find evidence for a positive and significant indirect effect: *ceteris paribus*, low leverage firms in regions that were more exposed to the sovereign debt crisis expanded more than low leverage firms in the less exposed regions. In Appendix B.4, we report the estimates for all the regressors in this baseline specification.

We perform a battery of robustness checks that we report in the remaining columns of Table 2. First, we check whether our results are robust to different samples. In column (2), we drop from the dataset firms that belong to certain industries for which output is challenging to measure.[19] In column (3), we estimate the baseline specification using an unbalanced panel in which firms are included as long as they report balance sheet information in 2007. In both of these cases, we estimate a negative direct and a positive indirect effect, quantitatively in line with the baseline estimates. In column (4) we report the estimates of the baseline specification when restricting the sample to 2008-2011. This sub-sample excludes the periods following the policy responses of the European authorities to the sovereign debt crisis (e.g., the longer-term refinancing operations), which could act

[19] Specifically, we exclude firms that operate in agriculture (NACE 1-3), mining and quarrying (NACE 5-9), utilities (NACE 35-39), postal service and courier activities (NACE 53), sports, arts entertainment activities, and activities of membership organizations (NACE 90-94), activities of households as employers (NACE 97-98), activities of extraterritorial organizations and bodies (NACE 99), and real estate activities (NACE 68).

Table 2: Estimation of equation (25): direct and indirect effects

	Model implied (1)	Excluding industries (2)	Unbalanced panel (3)	2008-2011 sample (4)	No long-term debt (5)	RZ index (6)	Continuous variables (7)
$\hat{\beta}_1$	-0.717^{***}	-0.838^{***}	-0.651^{**}	-1.531^{*}	-0.482^{***}	-0.433^{***}	-0.761^{***}
	(0.129)	(0.126)	(0.222)	(0.598)	(0.109)	(0.038)	(0.130)
$\hat{\beta}_2$	1.230^{***}	1.451^{***}	1.122^{**}	0.571^{*}	1.048^{**}	0.959^{**}	1.293^{***}
	(0.341)	(0.238)	(0.456)	(0.312)	(0.395)	(0.364)	(0.307)
R^2	0.87	0.88	0.87	0.91	0.87	0.87	0.87
Obs.	2,589,772	2,003,831	3,014,016	1,291,672	2,589,772	2,589,772	2,589,772

Note: This table reports the estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ in equation (25). The set of controls $X_{i,j,t}$ in all specifications includes: spr_t, $\text{spr}_t \times \text{lev}_i$, TFP_t, $\text{TFP}_t \times \text{lev}_i$, debt_t, $\text{debt}_t \times \text{lev}_i$, $\text{debt}_t \times \text{exp}_j$, $\text{debt}_t \times \text{lev}_i \times \text{exp}_j$. Standard errors in parenthesis are clustered two ways at the regions/time level. Point estimates and standard errors are multiplied by 100. $***$-significant at 1%, $**$-significant at 5%, $*$-significant at 10%.

as a confounding factor for the identification of the direct effect. Also in this case, we can see a negative direct and a positive indirect effect, although the estimates in this case are less precise.

Second, we perform robustness concerning how we measure firms' reliance on external finance. In column (5) of Table 2, we report a specification in which we exclude long-term loans from the computation of financial leverage, to be more consistent with the model. Next, we estimate the baseline specification using a different proxy for λ_i: rather than using debt over assets, we sort firms based on their value for the Rajan and Zingales (1998) index of dependence on external finance, assigning a value of $\text{lev}_i = 1$ for firms with an indicator above the 25^{th} percentile and zero otherwise.[20] This indicator measures the extent to which a given industry requires external finance for running operations, and thus it is less affected by the endogeneity concern that surrounds the use of financial leverage in our regression. Finally, we estimate the baseline specification with lev_i being the leverage indicator (debt over assets) of firm i rather than being a dummy variable.[21] We can see that in all those cases, we find a significantly negative direct effect and significantly positive

[20]The indicator is constructed by taking the difference between capital expenditures and cash flows and scaling it by capital expenditure. We use the updated index from Tong and Wei (2008) that uses Compustat firms in the US and construct it at the 3-digit industry level.

[21]To enhance the comparison to the baseline specification, $\hat{\beta}_1$ in the "Continous variables" specification reports the coefficient of the interaction between leverage and exposure for firms with a leverage ratio of 0.51, the average leverage for firms with $\text{lev}_i = 1$, and differenced it out with the one for firms with a leverage ratio of 0.01, the average leverage for firms with $\text{lev}_i = 1$.

indirect effects.

Misspecification. Under the assumptions we made in our model, we can estimate equation (22) using panel data and identify the direct and indirect effects of sovereign risk. This is possible because the distribution of the error term in that equation, the idiosyncratic productivity shock, is independent from the covariates. In practice, however, our model abstracts from a number of realistic mechanisms that, while not relevant to the question we are studying in this paper, could pose a threat to the identification of the direct and indirect effects. In what follows, we discuss this issue in more detail and assess the robustness of our estimates to these concerns.

To explore this issue, we model misspecification of the structural model by introducing in equation (22) an additional error term that takes the following form:

$$\varepsilon_{\iota,j,t} = \gamma_\iota \xi_t + \eta_j \xi_t + \zeta_{\iota,j} \xi_t. \tag{27}$$

The variable ξ_t represents an aggregate shock (for instance, aggregate TFP or some other shock not modeled explicitly in our framework), and the parameters $\{\gamma_\iota, \eta_j, \zeta_{\iota,j}\}$ are firm-specific loadings that can vary by leverage group and location.

Equation (27) is a flexible way of modeling shocks and propagation mechanisms that are not present in our model—thus omitted from equation (22). The term $\gamma_i \xi_t$ allows for differential sensitivity of firms to the aggregate shock based on firms' characteristics that are independent of where these firms are located. For example, this term could capture a differential sensitivity of firms to ξ_t based on their size or on the sector in which they operate. The term $\eta_j \xi_t$ captures a region-specific response that has uniform effects on firms within that region, such as a region-specific shock that affects all firms equally. Finally, the term $\zeta_{i,j} \xi_t$ allows for further flexibility—for example, region-specific shocks that have heterogeneous effects across firms.

In this context, the main threat to the identification of direct and indirect effects comes from a systematic relation between the firm/regional characteristics we are focusing on (firms' leverage and banks' exposure to sovereign debt), and the firm-region specific loadings $\{\gamma_\iota, \eta_j, \zeta_{\iota,j}\}$. This systematic relationship can arise for a variety of reasons, most notably because a firm's leverage and a bank's exposure to sovereign debt are not randomly assigned, but they are themselves the result of explicit decisions. For example, regions where there are few investment opportunities may feature banks holding large quantities of government debt and, at the same time, an outsized presence of the government aiming at making up regional disparities—something that would lead to a systematic relation

Table 3: Robustness to misspecification

	Model implied (1)	Add time-firm fixed effects (2)	Add time-region fixed effects (3)	Add time-firm-region fixed effects (4)
$\hat{\beta}_1$	-0.717^{***}	-0.638^{***}	-0.640^{***}	-0.685^{***}
	(0.129)	(0.084)	(0.059)	(0.074)
$\hat{\beta}_2$	1.230^{***}	0.958^{***}	1.104^{***}	1.166^{***}
	(0.341)	(0.146)	(0.187)	(0.212)
Fixed effects				
Firm	Yes	Yes	Yes	Yes
Time-firm group	No	Yes	Yes	No
Time-region	No	No	Yes	No
Time-firm group-region	No	No	No	Yes
R^2	0.87	0.88	0.88	0.88
Obs.	2,589,772	2,589,424	2,589,424	2,587,850

Note: This table reports the estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ in equation (25). See the notes to Table 2 for a definition of the firms' controls in the model implied baseline, column (1). Column (2) adds time fixed effects that vary with firms characteristics–the sector of operations and bins for total assets, operating profits, and standard deviation of sales growth. Column (3) adds time fixed effects that vary with regional characteristics–regional GDP, GDP per capital, and public spending to GDP. Column (4) adds time fixed effects that vary with the interaction of firm and regional characteristics. Standard errors in parenthesis are clustered two ways at the regions/time level. Point estimates and standard errors are multiplied by 100. $***$-significant at 1%.

between φ_j and η_j. If that were the case, our estimation of the indirect effect would be polluted by the omitted factor $\eta_j \xi_t$ if we do not properly control for it.

We perform additional sensitivities to address these important concerns. Our strategy consists of saturating the baseline specification of equation (25) with fixed effects to control for these omitted factors. First, we control for the term $\gamma_\iota \xi_t$ by introducing in the baseline specification a set of time fixed effects that can vary with certain firms' characteristics: sector, size, profitability, and volatility.[22] Column (2) of Table 3 reports the estimates of the direct and indirect effect. We can see that we still estimate a significantly negative direct effect and a positive indirect effect, with size being remarkably comparable to our baseline estimates (reported in column (1) for convenience). Next, we control for the term $\eta_j \xi_t$ by introducing, in addition to the time-firm fixed effects, a set of time fixed effects that vary

[22]The sector of operation is defined at the two-digit NACE level. For the other firms' characteristics, we construct dummy variables that equal 1 if a given firm's characteristic is above the 50th percentile of its cross-sectional distribution. The variables used are total assets in 2007, operating profits in 2007, and the standard deviation of sales growth over the sample period.

with some regional characteristics: a region's GDP, GDP per capita, and public spending relative to GDP. As for the previous experiments, these are constructed by interacting time fixed effects with dummy variables equal to 1 if a region lies above the median for that characteristic and zero otherwise.[23] This specification allows us to control for spurious results driven by an association between banks' exposure to sovereign debt and other regional characteristics—such as size, level of development, and relative importance of the central government. We can see from column (3) of Table 3 that our results survive this specification. Finally, column (4) of the Table interacts the time-firm fixed effects with the time-region fixed effects, to control for $\zeta_{i,j}\xi_t$. We can see that we still estimate a significantly negative direct effect and a positive indirect effect, with their size being remarkably comparable to our baseline estimates.

5 Quantitative analysis

We now use our model to measure the propagation of sovereign risk to the Italian economy. We start in Section 5.1 by parametrizing the model and assessing its fit. Section 5.2 reports the results of our main quantitative experiment, in which we use the calibrated economy to measure the output effects of sovereign risk during the Italian debt crisis. Section 5.3 presents a sensitivity analysis for our results.

5.1 Calibration and model fit

A period in the model is a year. We consider two islands ($J = 2$) of equal measure and two leverage types for firms—a mass of measure 0.75 with $\lambda = \lambda_{\text{high}}$ and a mass of measure 0.25 with $\lambda = \lambda_{\text{low}}$. We collect all model parameters in the vector Θ. It is useful to partition Θ in two groups of parameters, $\Theta = [\Theta_1, \Theta_2]$. The parameters in Θ_1 are set externally, while the parameters in Θ_2 are jointly chosen so that the model can match a set of firm, regional, and aggregate moments for the Italian economy.

Parameterization of Θ_1. The parameters in $\Theta_1 = [\alpha, \eta, \delta, \rho_A, \sigma_A, \rho_z, \beta, \sigma, \chi, \gamma, \tau, \vartheta]$ include technological parameters, preference parameters, the tax rate, and the fraction of debt maturing each year. The parameters α and η determine the shape of the production function of intermediate and final goods firms. We set α to 0.30 and η to 0.75, both of which are conventional values in the literature. We set the depreciation rate δ to 0.10. The persistence

[23] For regional GDP and GDP per capita, we are able to use 2007 data to sort regions into the two groups. For public expenditure over GDP, we sort regions using 2008, our first available data point.

parameters for the aggregate and idiosyncratic productivity shocks follow the business cycle literature, and we set both ρ_A and ρ_z to 0.81. The standard deviation of the innovation of the aggregate productivity shock σ_A is set to 0.004 to match the volatility of Italian TFP series of Section 4.1.[24] The discount factor of households β is set to match an annual risk-free rate of 2%, while χ is set to obtain average worked hours equal to 0.3. We specify a government utility function of the CRRA form and set its parameter σ to 2. The inverse Frisch elasticity of labor supply, γ, is set to 1.33, which implies a Frisch elasticity of 0.75, consistent with the macro estimate in Chetty, Guren, Manoli, and Weber (2011). Regarding the public finance parameters, we set ϑ to 0.05, a conventional value in the literature and we let $\tau = 0.20$, so to obtain a ratio of government consumption to output that is close to 20% on average, the estimate for Italy in Mendoza, Tesar, and Zhang (2014).

Parameterization of Θ_2. The remaining model parameters include those that govern the balance sheet of intermediaries, $[\bar{n}_j/(1-\theta), \varphi_j/(1-\theta)]$ in each island j;[25] the leverage and idiosyncratic productivity of firms, $[\lambda_{\text{low}}, \lambda_{\text{high}}, \sigma_z]$; the stochastic process for ν_t, $[\bar{\nu}, \rho_\nu, \sigma_\nu]$; and the government discount factor β_g.

We target 12 sample moments that include firms, banks, and aggregate statistics. The firms' statistics include the baseline estimates of $\hat{\beta}_1$ and $\hat{\beta}_2$ of equation (25) reported in column 1 of Table 3, the average leverage for firms with $\text{lev}_i = 0$ and $\text{lev}_i = 1$ in 2007, and the standard deviation of firms' log sales. The banks' statistics are the ratios of government bond holdings to banks' equity for the low- and high-exposure regions in 2007. The aggregate statistics include the mean, standard deviation, autocorrelation, and skewness of sovereign interest rate spreads, as well as the correlation between sovereign spreads and output.

We solve the model using global methods; see Appendix D for details. Given the model policy functions, we perform simulations to obtain the model-implied counterparts of our targets. To replicate as closely as possible our empirical analysis, we estimate equation (25) using a small "T," large "N" simulation from the model. Specifically, we choose the sequence of A_t and ν_t shocks to match the observed path of sovereign spreads and output during the 2008-2015 period and simulate idiosyncratic productivity shocks for 100,000 firms over this period. We then use this panel to estimate equation (25) and obtain a model counterpart for $\hat{\beta}_1$ and $\hat{\beta}_2$, and compute the other firms' and banks' statistics that we target

[24] These values are in line with Foster, Haltiwanger, and Syverson (2008) who estimate idiosyncratic productivity shocks using a long panel of US firms and the annualized persistence estimates for aggregate productivity in Chari, Kehoe, and McGrattan (2007).

[25] The linearity of the leverage constraint (9) implies that only the ratios of $\bar{n}_1, \bar{n}_2, \varphi_1, \varphi_2$ relative to the pledgeability parameter $(1-\theta)$ matter for the equilibrium. Therefore, we can recover only the ratios $\{\bar{n}_j/(1-\theta), \varphi_j/(1-\theta)\}$ in each region j.

Table 4: Parameter values

Parameters set externally, Θ_1	
Capital share	$\alpha = 0.30$
Markup parameter	$\eta = 0.75$
Depreciation rate	$\delta = 0.10$
Aggregate productivity process	$[\rho_A, \sigma_A] = [0.81, 0.004]$
Idiosyncratic productivity shocks, persistence z	$\rho_z = 0.81$
Households discount factor	$\beta = 0.98$
Government risk aversion	$\sigma = 2.00$
Labor disutility	$\chi = 5$
Frisch elasticity	$1/\gamma = 0.75$
Fraction of bonds maturing	$\vartheta = 0.05$
Tax rate	$\tau = 0.20$
Internally calibrated parameters, Θ_2	
Banks' net worth	$[\bar{n}_1/(1-\theta), \bar{n}_2/(1-\theta)] = [0.44, 0.32]$
	$[\varphi_1/(1-\theta), \varphi_2/(1-\theta)] = [0.83, 1.36]$
Working capital requirements	$[\lambda_{\text{low}}, \lambda_{\text{high}}] = [0.00, 1.27]$
Idiosyncratic productivity shocks, volatility z	$\sigma_z = 0.36$
Enforcement shock process	$[\bar{v}, \rho_v, \sigma_v] = [1.82, 0.97, 0.14]$
Government discount factor	$\beta_g = 0.979$

in the calibration. The aggregate moments regarding the behavior of sovereign spreads and output are computed on a long simulation of our model. The parameters in Θ_2 are then chosen to minimize a weighted distance between the moments in the model and their corresponding counterparts in the data. Table 4 reports the numerical values for the model parameters.

Even though the parameters in Θ_2 are jointly chosen, we can give a heuristic description of how the sample moments that we target inform specific parameters. First, in our model, the leverage ratio for a firm with λ_ι equals $\lambda_\iota r^k/\alpha$. So, given $r^k = 1 - \beta(1-\delta)$ and α, the average leverage of the two groups of firms pins down λ_{low} and λ_{high}. Similarly, there is a tight relation between banks' holdings of government bonds as a fraction of their equity and φ_j. Indeed, this statistic in the model equals $\varphi_j q_t(1-\vartheta)B_t/(\bar{n}_j + \varphi_j q_t(1-\vartheta)B_t)$: given $\bar{n}_j$ and the market value of debt, these moments pin down φ_j for the two regions. The coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ in equation (25) provide information on $\bar{n}_1$ and $\bar{n}_2$. To see why, suppose $\bar{n}_j$'s are so large that the intermediaries' leverage constraints are always slack in both islands. Then, the model predicts the direct and indirect effects to be equal to zero. As $\bar{n}_j$'s decrease, the constraints start to bind and the coefficients deviate from zero. The strength of the direct and indirect effects, also depends heavily on the regional exposures to government bonds. The standard deviation of firms' sales provides information about the size of idiosyncratic productivity shocks. The mean, standard deviation, autocorrelation, and skewness of sovereign spreads have a tight connection with the process for the enforcement shock ν_t and with the government discount factor β_g.[26]

Model fit. Table 5 reports the target moments in the model and the data. The model fits well the empirical targets. In the aggregate, the model fits well the time series properties of sovereign spreads and their correlation with output. The mean sovereign interest rate spreads on government debt is 1.0% in both the model and data. The standard deviation in the model is 1.1%, very close to the data counterparts of 1.2%. Sovereign spreads have an autocorrelation of 0.8, a skewness of 1.0, and a correlation with output of -0.6—moments that are similar to their data counterparts. The model also matches leverage for the two groups of firms, the cross-sectional standard deviations of firms' log sales, and banks' exposure to government debt in 2007 for high- and low-exposure regions. Crucially, the model does a good job of fitting the direct and indirect effects estimated using firm-level data: the estimated coefficient $\hat{\beta}_1$ and $\hat{\beta}_2$ in model-simulated data of -0.8 and 1.1 are close and well within the confidence bands of the empirical coefficients -0.7 and 1.2.

Table 5 also reports out of sample statistics in the model and in the data. First, we check whether our model captures the behavior of firms' interest rate spreads in the data—an important test because, as we have seen, sovereign risk in our economy affects the private sector through its effect on firms' borrowing rates. Table 5 reports several moments for our series of average interest rate spreads of Italian non-financial firms (firm spr_t) and compares them with the model. In the model, firm spreads are defined as the difference in firms' average borrowing rate relative to the deposit rate. The model generates empirically plausible fluctuations in firms' interest rate spreads and captures the positive association between sovereign and private sector interest rate spreads.

Second, we verify whether the geographic implications of our model are empirically plausible. We can see that our model reproduces the high correlation of output between the high- and low-exposure regions, $Y_{H,t}$ and $Y_{L,t}$, and it fits the relative fall in output between the high- and low-exposure regions during the sovereign debt crisis. Output in the high-exposure regions fell on average 0.6% more than output in the low-exposure regions between 2011 and 2013 relative to 2007 in the data, while it fell 0.7 in the model.[27]

[26] See Bocola, Bornstein, and Dovis (2019) for a discussion on how the skewness of sovereign spreads is informative of the government discount factor in sovereign default models.

[27] We average the relative decline in output across the regions for 2011, 2012, and 2013 because these are the three years when sovereign risk is highest and affects output in our model, as we show in Section 5.2.

Table 5: Moments in model and data

	Data	Model
Targeted moments		
Stdev$(\hat{py}_{it})$	0.52	0.52
Firms' leverage	[.0 .51]	[.0 .51]
Banks' exposure	[.45 .62]	[.45 .62]
$\hat{\beta}_1$ in equation (25)	-0.7	-0.8
$\hat{\beta}_2$ in equation (25)	1.2	1.1
Mean(spr_t)	1.0	1.0
Stdev(spr_t)	1.2	1.1
Acorr(spr_t)	0.8	0.8
Skewness(spr_t)	1.2	1.0
Corr$(\text{spr}_t, \hat{Y}_t)$	-0.4	-0.6
Out of sample moments		
Mean(firm spr_t)	0.3	0.4
Stdev(firm spr_t)	0.8	0.8
Acorr(firm spr_t)	0.5	0.3
Skewness(firm spr_t)	0.7	2.2
Corr$(\text{spr}_t$, firm $\text{spr}_t)$	0.9	0.70
Corr$(\hat{Y}_{L,t}, \hat{Y}_{H,t})$	0.98	0.99
Mean$_{\text{crisis}}(\hat{Y}_{H,t} - \hat{Y}_{L,t})$	-0.6	-0.7

Note: In the table Stdev$(\hat{py}_{it})$ denotes the average across firms of the standard deviation of firms' log real sales. The table reports the mean (Mean), standard deviation (Stdev), autocorrelation (Acorr) and skewness (Skewness) of sovereign spreads (spr_t), and firm spreads (firm spr_t). $\hat{Y}_{L,t}$ and $\hat{Y}_{H,t}$ are log regional output relative to 2007. Mean$_{\text{crisis}}(\hat{Y}_{H,t} - \hat{Y}_{L,t})$ is the difference in output across the two regions averaged over the crisis period (2011-2013). See Appendix B for data sources and definitions.

This last result summarizes how the direct and indirect effects of sovereign risk map into aggregate regional differences.

5.2 Dissecting the Italian debt crisis

We now use the calibrated model to measure the propagation of sovereign risk to the Italian economy over the 2008-2015 period. We proceed in two steps. In the first step, as we explained above, we choose the time path for the aggregate shocks so that our model reproduces the observed time series for output and sovereign interest rate spreads over the period of analysis. In the second step, we use the model to construct the path of macroeconomic aggregates in a counterfactual scenario in which the Italian economy, while facing the same sequence of aggregate productivity shocks, does not experience a sovereign debt crisis. Operationally, this *no debt crisis* counterfactual is constructed by feeding the model

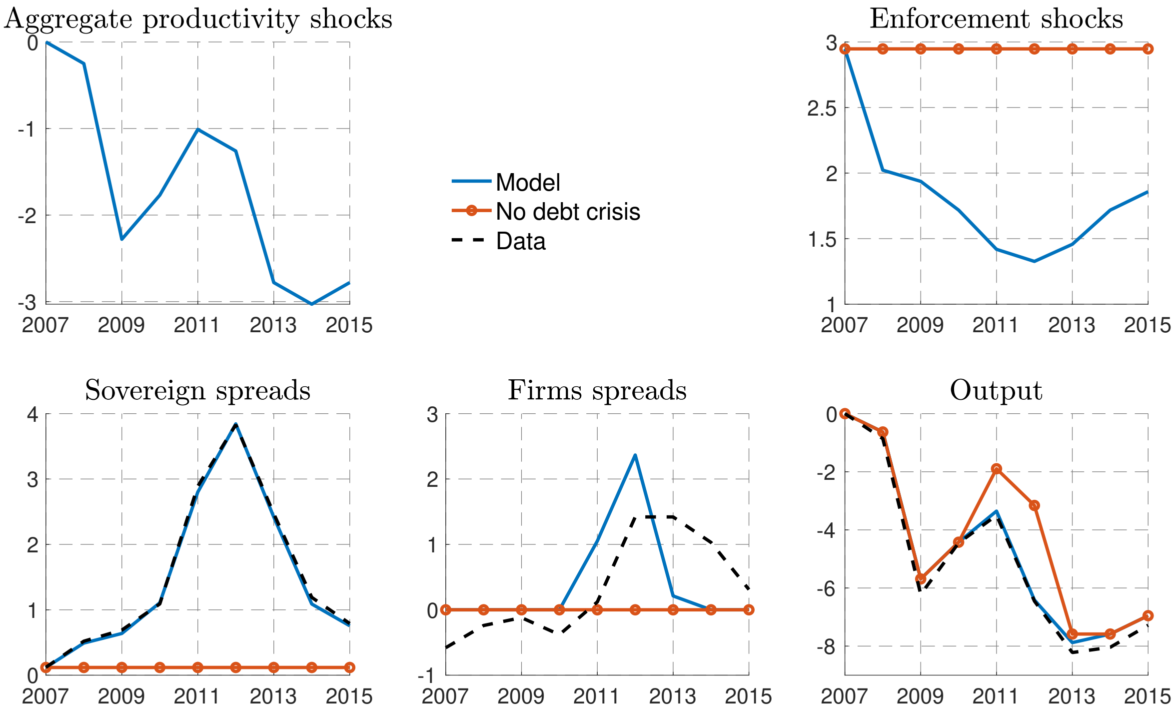

Figure 3: Measuring the output costs of sovereign default risk

an alternative sequence of enforcement shocks that guarantees that sovereign spreads remain at their 2007 value during the event. The difference between the path of output in the data and in the no debt crisis counterfactual isolates the output effect of the sovereign crisis for the Italian economy.

The solid lines in Figure 3 report the time path for the aggregate shocks, sovereign and firm interest rate spreads, and aggregate output. The model needs an overall decline in aggregate productivity and a progressive deterioration of enforcement to reproduce the dynamics of output and sovereign spreads observed in the data. By construction, the model fits almost perfectly these latter two series. The figure also shows that the model fits well the path for firms' interest rate spreads, confirming the findings of Table 5.

The circled lines in the figure report the trajectories for these variables in the no-debt crisis counterfactual. By construction, sovereign interest rate spreads are constant in this experiment, so banks do not experience losses in their sovereign bond portfolio. As a result, credit supply does not fall as much as it does in the baseline. Indeed, we can see that the counterfactual economy does not experience the rise in firms' interest rate spreads observed at the height of the debt crisis. Our results imply that the passthrough from sovereign spreads to firms' interest rate spreads during the Italian debt crisis is 64%—an increase in sovereign spread of 3.7% in 2012 leads to an average increase in firms' interest rate spread

Table 6: Output losses in the Italian debt crisis

	2011	2012	2013	Average (11-13)
Output, baseline	-3.4	-6.4	-7.9	-5.9
Output, no debt crisis	-1.9	-3.2	-7.6	-4.2
Output losses from sovereign risk				
Total	-1.5	-3.3	-0.3	-1.7
Direct effect	-2.9	-6.3	0.6	-3.3
Indirect effect	1.4	3.1	0.3	1.6

Note: Output is reported as the percentage deviation from its 2007 value.

of 2.4%. Because of the lower firms' interest rate spreads, output in the counterfactual economy lies above the one in the baseline over the 2011-2013 period. This experiment suggests that the output losses associated with sovereign default risk are sizable: output would have declined only 3.2% in 2012 without the sovereign debt crisis, instead of the 6.4% decline observed in the data.

Table 6 centers on the output dynamics over the 2011-2013 period to highlight the years with output losses from sovereign risk. Output was on average 5.9% below trend during this period, while it would have been 4.2% below trend without the sovereign debt crisis. Therefore, our model predicts that sovereign risk was responsible for roughly 30% of the output losses observed in Italy over the episode. The table also reports a decomposition of these output losses into those that are due to the direct effect on firms' borrowing rates and those that are due to the indirect effects that work via aggregate demand and wages.[28] We can see that the average decline in output attributed to the direct effect is 3.3%, while the total is 1.7%. Therefore, in our calibration, the general equilibrium mechanisms working through wages and aggregate demand dampen part of the decline in output that is due to the effects of sovereign risk on firms' borrowing rates.[29]

[28]To do so, we use equation (20), which expresses log sales for a firm as a function of interest rates, wages, and aggregate demand. Specifically, we use this expression to evaluate the firms' sales that would have been realized if interest rates on firms followed the same path as in the baseline event of Figure 3, while wages and aggregate demand follow the path of the no debt crisis counterfactual economy. We aggregate across firms to generate a path for aggregate output with only the direct effect operating. By comparing this path of output with the counterfactual output path, we can evaluate the output losses arising because of the increase in firms' borrowing rates alone.

[29]Interestingly, this is achieved with a path for wages that is empirically plausible. In the event study, wages fall by 2.2% between 2011 and 2013, while in the data average wages in real terms declined by 2.8% (OECD).

5.3 Sensitivity analysis

We now perform three experiments to quantify the importance of some key model mechanisms and assess the robustness of our result. First, we feed the path of the shocks estimated in the previous section in an economy that is identical to the one we consider, except that there is no pass-through of sovereign risk to the private sector. This experiment allows us to isolate the importance of the "doom-loop", that is the extent to which declining economic activity induced by sovereign risk feeds into the decision problem of the sovereign, leading to an even higher probability of a government default. Second, we parameterize the model to fit different estimates for the direct and indirect effects of sovereign risk in an empirically plausible range, to obtain a range of empirically plausible measures of the output effects of sovereign risk. Third, we consider an extension of our model that allows firms to default on their debt, and use this extended model to compute the output costs of the sovereign debt crisis.

The event and counterfactual analyses of the previous section point toward significant output losses from sovereign risk during the Italian debt crisis. As summarized in Column (1) of Table 7, about 30% of the output declines in the 2011-2013 period are due to the output losses of sovereign risk. In our model, these effects in turn matter for the government's incentives to borrow and default, affecting the debt crisis itself. In the baseline, sovereign spreads increased 3% on average for the 2011-2013 period; this increase is due to changes in output, which depends on both productivity changes and spread changes, and directly on enforcement shocks, even when these do not affect output.

We isolate the importance of the endogenous response of output for the behavior of sovereign spreads by performing an event study in an economy that is identical to our baseline, with the exception that sovereign risk has no effects on output. We do so by increasing enough the pledgeability parameter θ, which in the baseline is 0.9, to completely relax the leverage constraint of banks (12) in all regions. We keep all other parameters as in the baseline and conduct the event analysis by feeding into the model the same aggregate productivity and enforcement shocks from the baseline and presented in Figure 3. Column (2) of Table 7 summarizes these results. Because output does not respond to the increase in sovereign risk, the decline of output in this "No passthrough" model is smaller than in the baseline. In addition, we can see that this version of the model has much less responsive sovereign spreads: the average spreads from 2011 to 2013 increases by only 160 basis points, about half of the increase in the baseline (310 basis points). This exercise suggests that there was a significant feedback loop between output and sovereign risk during the Italian debt crisis.

In the second exercise, we explore how the magnitude of the estimated regression coef-

Table 7: Sensitivity Analysis

	Baseline	No passthrough	High $\lvert\hat{\beta}_1\rvert$	Low $\lvert\hat{\beta}_1\rvert$	Firms' default
	(1)	(2)	(3)	(4)	(5)
$\hat{\beta}_1$	−0.8	0	−1.0	−0.5	−0.7
$\hat{\beta}_2$	1.1	0	1.3	0.6	0.9
Output decline	−5.9	−4.2	−9.4	−4.8	−6.1
Output losses (% of decline)					
Total	0.3	0	0.5	0.1	0.3
Direct effect	0.6	0	1.1	0.2	0.6
Sovereign spreads	3.0	1.6	4.3	2.6	3.0

Note: The table reports sensitivity results for estimated direct and indirect effects, total output decline, output losses from sovereign risk along with decomposition from direct effects, and mean levels of spreads along with decomposition from only enforcement shocks. Statistics for output decline, losses, and sovereign spreads are over the period 2011-2013. See Section 5.3 for details on the construction of these sensitivity results.

ficients affects the measured output loss of sovereign risk. To this end, we consider more modest comparative statics with respect to the parameter θ and vary it to marginally affect the tightness of the leverage constraint of banks relative to the baseline. Specifically, we vary this parameter to obtain an estimate for $\hat{\beta}_1$ that is 2 standard deviations below and above our point estimate. The other parameters and shocks are kept as in the baseline. Columns (3) and (4) of Table 7 report these results. When the estimated $\hat{\beta}_1$ is more negative, about 2 standard deviations lower, the output losses of sovereign risk are higher, in absolute values and also as a contribution of the output decline. The increase in spreads is also sharper because, although the shocks that we feed in are the same as in the baseline, the higher the output loss of sovereign risk, the more severe the debt crisis. The comparative static for $\hat{\beta}_1$ less negative gives the opposite results: smaller output losses, and less severe debt crisis. Interestingly, we find that more (less) negative $\hat{\beta}_1$, tends to result also in more (less) positive $\hat{\beta}_2$, which offset the output declines from sovereign risk. These exercises suggest that the output losses from sovereign risk vary between 10% and 50% of the overall output losses for a plausible range of estimates.

Finally, we conduct a sensitivity check, where we repeat our analysis in a version of the model in which firms can default in equilibrium. We do so for two reasons. First, in a model with firm default, the transmission of sovereign risk to firms' performance occurs not only because of changes in borrowing rates (as in our baseline model) but also because of secondary effects that may affect firms' borrowing and default incentives. Second, over the period of analysis, firms' default rates increased sharply in Italy (see Figure A-3 in

Appendix C), so it is reasonable to ask whether accounting for this development affects the measurement of the output effects of sovereign risk.

We discuss the details of this extension of the model in Appendix C. We introduce firm default as in Arellano, Bai, and Kehoe (2019) by assuming that firms face an idiosyncratic revenue shock after they have made their input choices but before servicing their debt. For sufficiently large realizations of this shock, firms default on their loans. In addition, we assume that the pledgeability parameter θ now depends on firm default and is given by $\theta_{jt} = \theta[1 - \bar{d}_{jt}]$, where $\bar{d}_{jt}$ is the weighted average of firms' defaults in region j. This assumption implies that banks can pledge a lower fraction of their loan portfolio when firms are more likely to default.

The presence of firm default changes borrowing incentives of firms in response to changes in sovereign risk. An increase in sovereign risk, increases firms' interest rates and reduces their profits. These effects tend to increase firm default risk. In response to these changes, however, firms tend to reduce their borrowing to not only avoid the higher interest rates, but also to avoid increased default risk. A reduction in firm demand for loans tends to relax the leverage constraint of banks, which in turn reduces the output effect of sovereign risk. The net effect on output depends on the sensitivity of firm borrowing relative to changes in the leverage constraint.

We perform a quantification of these effects in a model with firm default. As we explain in Appendix C, we performed this exercise under the simplifying assumption that the properties for sovereign debt and spreads during the event remain as in the baseline model, and we allow the standard deviation of the idiosyncratic revenue shock to vary over time so that the model can match the time path for firm default rates in the data. For the event analysis, we follow the procedure of the baseline and choose the productivity shock to fit the path of output in the data. From Column 5 of Table 7, we can see that the output loss from sovereign risk is on average 1.7% over the 2011-2013 period in the model with firm default, or about 30% of the output decline. These results tell us that the introduction of firm default in the model does not substantially change the inference about the output losses of sovereign risk. Moreover the estimated regression coefficients that measure the direct and indirect effect of sovereign risk, are well within the confidence bands of the empirical coefficients. We do find, however, interesting interactions between sovereign and firm default. In the model and the data, spreads during the crisis increase by 3.6%. Our counterfactual suggests that sovereign risk was responsible for 28% of the firms' default rates observed during the Italian debt crisis.

5.4 Assessing the role of heterogeneity

In this final section, we assess the role of heterogeneity in our analysis. We do so by conducting our measurement exercise in a model with a representative firm and region, fitting it to match some moments of aggregate variables commonly used in the literature. We will show that this model can match the same moment with two very different parametrizations, one characterized by a strong pass-through of sovereign risk to the rest of the economy and one with no passthrough at all. This shows the importance of using micro data in a model with heterogeneity to measure the output costs of sovereign risk.

We consider a counterfactual economy where we shut down heterogeneity, parameterize it to aggregate moments, and perform our measurement exercise. Specifically, we eliminate idiosyncratic productivity shocks, $\sigma_z = 0$, consider a representative firm with working capital requirement λ, and a representative region with bank net worth parameters given by $\bar{n}/(1-\theta), \varphi/(1-\theta)$. We parameterize the no-heterogeneity model, recalibrating the firm with working capital requirement λ, bank net worth parameters given by $\bar{n}/(1-\theta), \varphi/(1-\theta)$, and standard deviation of the enforcement shock σ_v to match 5 target aggregate moments generated by our model with heterogeneity, namely the average firm leverage, bank exposure, the mean and standard deviation of sovereign spreads, and the correlation between spreads an output. All other parameters are as Table 4.

Table 8 presents two parameterizations of the model without heterogeneity that can generate a very similar fit for the targeted moments considered. We perform the event studies for both economies, following the same procedure as the one described earlier: we feed in a sequence of productivity and enforcement shocks such that each model replicates the paths of output and spreads observed in the data. We then compute the output losses by considering the counterfactual economy without the increase in sovereign risk. We can see from the table that the two parameterizations give wildly different output costs of sovereign risk. Under the parameterization in column (1), about 40% of the fall in output is explained by the feedback of sovereign risk on the rest of the economy, while in the parameterization in column (2) the output costs are zero. In other words, the aggregate data are not enough to discipline the output costs of sovereign risk in this model. This exercise highlights the importance of introducing heterogeneity in this class of models: as we showed in our analysis, certain firm-level elasticities contain lots of information about the overall output costs of sovereign risk.

Table 8: Counterfactual: No-Heterogeneity

	Baseline Model	No-Heterogeneity	
		(1)	(2)
Aggregate effects			
Output decline	5.9	5.9	5.9
Output losses (% of decline)	0.3	0.4	0.0
Targeted moments			
Firms' leverage	0.4	0.4	0.4
Banks' exposure	0.5	0.5	0.5
Mean(spr_t)	1.0	1.0	1.0
Stdev(spr_t)	1.1	1.0	1.2
Corr(spr_t, $\hat{Y}_t$)	−0.6	−0.6	−0.5

Note: This table compares the baseline results to counterfactual economies with no heterogeneity under two different calibrations. See Section 5.4 for a detailed description of the experiment.

6 Conclusion

We have developed a framework that combines a structural model of sovereign debt with financial intermediaries and heterogeneous firms with micro data to study the macroeconomic implications of sovereign risk. We showed that firm-level data can be useful for measuring the macroeconomic implications of sovereign risk and the different transmission mechanisms. In our application, we find that the effect of sovereign risk on the private sector is sizable, accounting for about one-third of the observed decline in output during the Italian debt crisis.

Our approach could be generalized along other dimensions. The sovereign debt literature has suggested several mechanisms through which sovereign risk affects the economy—for example, by disrupting international trade or by hindering firms' investment plans because of increased uncertainty about fiscal policy. We believe that a fruitful avenue for future research would be to exploit the cross-sectional variation that is present in firm-level datasets to test and quantify these theories.

Our work used micro data to measure the extent to which collateral constraints of financial intermediaries responded to sovereign risk and affected private lending during the Italian debt crisis and, as such, purposefully abstracted from analyzing counterfactual financial regulation policy. We think an additional useful generalization of our framework would be to directly measure how financial regulation policies, such as those in Basel III, can alter the feedback mechanisms that we identify by exploiting bank- and firm-level data.[30] We leave these applications to future research.

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[30] See Chari, Dovis, and Kehoe (2020) who study theoretically the interactions between financial regulatory constraints, sovereign default, and financial repression.

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Online Appendix to "Sovereign Default Risk and Firm Heterogeneity"

by Cristina Arellano, Yan Bai, and Luigi Bocola

A Proofs

Proof of Proposition 1 Taking as given the aggregate demand and wage, a firm (z, λ) in region j with state $X_j = \{A, N_j\}$ chooses capital and labor to maximize its profit (8) subject to the demand schedule (3) and financing requirement (7). In equilibrium, the optimal capital satisfies

$$k(z, \lambda, X_j) = M_k \left(\exp\{A + z\}\right)^{\frac{\eta}{1-\eta}} Y(X_j) w(X_j)^{-\frac{(1-\alpha)\eta}{1-\eta}} r_\lambda(X_j)^{-\frac{1}{1-\eta}}, \tag{A.1}$$

$$\ell(z, \lambda, X_j) = \frac{1-\alpha}{\alpha} \frac{r_k}{w(X_j)} k(z, \lambda, X_j), \tag{A.2}$$

$$b^f(z, \lambda, X_j) = \lambda \frac{r_k}{\alpha} k(z, \lambda, X_j), \tag{A.3}$$

$$y(z, \lambda, X_j) = \exp\{A + z\} k(z, \lambda, X_j)^\alpha \ell(z, \lambda, X_j)^{1-\alpha},$$

with $r_\lambda(X_j) = 1 + \lambda(R(X_j) - 1)$ and $M_k = \left((1-\tau)\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}\right)^{\frac{1}{1-\eta}} r_k^{-\frac{1-(1-\alpha)\eta}{1-\eta}}$. Aggregating up individual firms' output $y(z, \lambda, X_j)$ and labor $\ell(z, \lambda, X_j)$ and applying the market clearing conditions and the family's optimal condition $w(X_j) = \chi L(X_j)^\gamma$, we get the equilibrium wage and output functions as in equations (18) and (19) with the constants M_w and M_y as

$$M_w = \left(\frac{1-\alpha}{\alpha}\right)^{1-\eta} \left[(1-\tau)\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}\right]^{\frac{1}{1-\alpha}} r_k^{-\frac{\alpha}{1-\alpha}} (\bar{z})^{\frac{1-\eta}{\eta(1-\alpha)}},$$

$$M_y = \frac{\alpha}{1-\alpha} \left[(1-\tau)\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}\right]^{-\frac{1}{1-\eta}} r_k^{\frac{\eta\alpha}{1-\eta}} (\bar{z})^{-1} \chi^{-\frac{1}{\gamma}} M_w^{\frac{1-\eta+(1-\alpha\eta)\gamma}{(1-\eta)\gamma}},$$

where the weighted average productivity $\bar{z}$ is given by $\bar{z} = \exp\left\{\frac{\eta^2\sigma_z^2}{2(1-\eta)^2(1-\rho_z)^2}\right\}$. Summing over the loan demand $b^f(z, \lambda, X_j)$ over (z, λ) and using the equilibrium (18) and (19), we get the total loan demand $B^d(A,R)$:

$$B^d(A,R) = M_n \left[\int_\lambda \frac{\lambda(1+\lambda(R-1))^{-\frac{1}{1-\eta}}}{\int (1+\lambda(R-1))^{-\frac{1}{1-\eta}} d\Lambda_\lambda} d\Lambda_\lambda \right] \left[\exp\{A\}^{\frac{\eta}{1-\eta}} \int_\lambda (1+\lambda(R-1))^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda \right]^{\frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma}} . \tag{A.4}$$

This gives rise a loan demand function B^d that only depends on the aggregate productivity A and the borrowing rate R. The constant M_n is given by

$$M_n = \frac{1}{1-\alpha} \chi^{-\frac{1}{\gamma}} M_w^{1+\frac{1}{\gamma}}.$$

If this loan demand is less than the loan supply $N_j/(1-\theta)$, the borrowing rates $R(A,N_j) = 1/\beta$; otherwise, $R(A,N_j)$ solve $N_j/(1-\theta) = B^d(A,R)$. Using the definitions of $r_\lambda(X_j)$, $\bar{\lambda}(X_j)$, and $R_w(X_j)$, we get the loan market condition (17). *Q.E.D*

Proof of Corollary 1 Here, we prove the borrowing rates weakly decrease with net worth N_j; that is, $\partial R/\partial N_j \leq 0$. We first show the loan demand function B^d decreases with the borrowing rate R. Define $H_1 = \int_\lambda \lambda \frac{(1+\lambda(R-1))^{-\frac{1}{1-\eta}}}{\int (1+\lambda(R-1))^{-\frac{1}{1-\eta}} d\Lambda_\lambda} d\Lambda_\lambda$, $H_2 = exp\{A\}^{\frac{\eta}{1-\eta}} \int_\lambda (1+\lambda(R-1))^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda \Big]^{\frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma}}$, and $r_\lambda \equiv 1+\lambda(R-1)$. Note that both H_1 and H_2 are positive. The partial derivative of the total loan demand B^d over R is as follows:

$$\frac{\partial B^d}{\partial R} = M_n \left[\frac{\partial H_1}{\partial R} H_2 + \frac{\partial H_2}{\partial R} H_1 \right],$$

where

$$\begin{aligned} \frac{\partial H_1}{\partial R} &= -\frac{1}{1-\eta} \int_\lambda \lambda \frac{\lambda r_\lambda^{\frac{\eta-2}{1-\eta}} \int r_\lambda^{-\frac{1}{1-\eta}} d\Lambda_\lambda - r_\lambda^{-\frac{1}{1-\eta}} \int \lambda r_\lambda^{\frac{\eta-2}{1-\eta}} d\Lambda_\lambda}{(\int r_\lambda^{-\frac{1}{1-\eta}} d\Lambda_\lambda)^2} d\Lambda_\lambda \\ &= -\frac{1}{1-\eta} \int_\lambda \lambda \frac{r_\lambda^{\frac{\eta-2}{1-\eta}} \int (\int r_\lambda^{\frac{\eta-2}{1-\eta}} d\Lambda_\lambda) d\Lambda_\lambda}{(\int r_\lambda^{-\frac{1}{1-\eta}} d\Lambda_\lambda)^2} d\Lambda_\lambda, \end{aligned}$$

where the second equation uses integration by parts, $\int xf(x)dx = x\int f(x)dx - \int(\int f(x)dx)dx$. It is clear that $\partial H_1/\partial R < 0$. Furthermore,

$$\frac{\partial H_2}{\partial R} = \frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma} \left[\exp\{A\}^{\frac{\eta}{1-\eta}} \int_\lambda r_\lambda^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda \right]^{\frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma}-1} \left[-\frac{\eta}{1-\eta} \exp\{A\}^{\frac{\eta}{1-\eta}} \int_\lambda r_\lambda^{-\frac{1}{1-\eta}} d\Lambda_\lambda \right],$$

which is also negative. Hence, we show $\frac{\partial B^d}{\partial R} < 0$; the loan demand decreases with R.

Next, we show that when the net worth decreases, the borrowing rates weakly increase. There are two cases. First, when the loan supply $N_j/(1-\theta)$ is large enough and higher than the loan demand at the borrowing rate $1/\beta$— that is, $N_j/(1-\theta) \geq B^d(A, 1/\beta)$— the equilibrium borrowing rate $R(A, N_j) = 1/\beta$. Second, when the loan supply decreases such that $N_j/(1-\theta) < B^d(A, 1/\beta)$, the borrowing rates $R(A, N_j)$ equate loan demand and supply,

$$\frac{N_j}{1-\theta} = B^d(A, R).$$

In this case, the borrowing rate has to be higher than $1/\beta$ since $\partial B^d/\partial R < 0$. Furthermore,

$$\frac{\partial R}{\partial N} = \frac{1}{(1-\theta)\partial B^d/\partial R} < 0.$$

Hence the borrowing rates R weakly decrease with net worth N_j. *Q.E.D*

Proof of Proposition 2 Recall that the state of region j is given by

$$X_{jt} = \left[A_t, N_{jt}\left(S_t, B_t, D_t, B_{t+1}\right)\right], \tag{A.5}$$

with $S_t = \{A_t, \nu_t\}$. Here, we consider small shocks so that there is no default in equilibrium, and the net worth of each region N_{jt} is a function of $(A_t, \nu_t, B_t, B_{t+1})$. Using the definition of spread from equation (16), we can rewrite the net worth equation (9) with spread:

$$N_{jt} = \bar{n}_j + \varphi_j(1-\vartheta)\frac{\vartheta}{\vartheta + 1/\beta - 1 + \text{spr}_t}B_t. \tag{A.6}$$

Given that A_t, ν_t, and B_{t+1} affect N_{jt} only through their effects on spread spr_t, we can define a function of net worth on spread and B as $\tilde{N}_j(\text{spr}_t, B_t) = N_{jt}\left(A_t, \nu_t, B_t, B_{t+1}\right)$, where the spread spr_t is the evaluation of the spread function $\text{spr}(A, \nu, B)$ at period t's state, that is,

$$\text{spr}_t = \text{spr}(A_t, \nu_t, B_t) = H_S(A_t, \nu_t, B_{t+1}(A_t, \nu_t, B_t)). \tag{A.7}$$

Consider approximating log of sales $\hat{py}_{ijt} \equiv \log py_{ijt} = f(\lambda_i, x_{ijt})$ around a point $x = [z, A, \nu, B]$. We can follow standard steps and consider a first-order Taylor expansion,

$$\hat{py}_{i,j,t} - f(\lambda_i, x) \approx \frac{\eta}{1-\eta}(z_{it} - z) + f_A(\lambda_i, x)(A_t - A) + f_\nu(\lambda_i, x)(\nu_t - \nu) + f_B(\lambda_i, x)(B_t - B),$$

and use equation (20) to obtain these derivatives:

$$f_A(\lambda_i, x) = \frac{\eta}{1-\eta} + \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial A} - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial A}$$
$$+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial A} - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial A}, \tag{A.8}$$

$$f_\nu(\lambda_i, x) = \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial \nu} - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial \nu}, \tag{A.9}$$

$$f_B(\lambda_i, x) = \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial B} - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial \text{spr}}\frac{\partial \text{spr}}{\partial B}$$
$$+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial B} - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial B}. \tag{A.10}$$

Note that from (A.5), A_t enters the state of the region in two ways. First, it directly affects the private economy. Second, it affects the net worth of banks, since the productivity shock affects the government's default incentive and hence spread of the government. Hence, f_A includes the derivatives of prices and output on A_t itself and the derivatives through net worth.

Plugging the derivatives (A.8)-(A.10) into the Taylor expansion and combining terms, we have

$$\hat{py}_{i,j,t} \approx f(\lambda_i, x) + \frac{\eta}{1-\eta}(z_{it} - z) + \frac{\eta}{1-\eta}(A_t - A)$$
$$+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial A}(A_t - A) - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial A}(A_t - A)$$
$$+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial \text{spr}}\left[\frac{\partial \text{spr}}{\partial A}(A_t - A) + \frac{\partial \text{spr}}{\partial \nu}(\nu_t - \nu) + \frac{\partial \text{spr}}{\partial B}(B_t - B)\right]$$
$$- \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial \text{spr}}\left[\frac{\partial \text{spr}}{\partial A}(A_t - A) + \frac{\partial \text{spr}}{\partial \nu}(\nu_t - \nu) + \frac{\partial \text{spr}}{\partial B}(B_t - B)\right]$$
$$+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}\frac{\partial N}{\partial B}(B_t - B) - \frac{\eta}{1-\eta}\lambda_i \frac{\partial R}{\partial N}\frac{\partial N}{\partial B}(B_t - B). \tag{A.11}$$

The government's spread varies with productivity shock A_t, the default cost shock ν_t, and its debt holding B_t. The first-order Taylor expansion over the spread function (A.7) implies

$$
\begin{aligned}
&\mathrm{spr}_t - \mathrm{spr}(A,\nu,B) \\
&= \left[H_{S1} + H_{S3}\frac{\partial B'}{\partial A}\right](A_t - A) + \left[H_{S2} + H_{S3}\frac{\partial B'}{\partial \nu}\right](\nu_t - \nu) + H_{S3}\frac{\partial B'}{\partial B}(B_t - B) \\
&= \frac{\partial \mathrm{spr}}{\partial A}(A_t - A) + \frac{\partial \mathrm{spr}}{\partial \nu}(\nu_t - \nu) + \frac{\partial \mathrm{spr}}{\partial B}(B_t - B),
\end{aligned} \tag{A.12}
$$

where H_{Si} is the derivative of function H_S over its *ith* argument and the borrowing function B' depends on (A,ν,B). Note that the second equation holds because of the derivatives of equation (A.7): $\frac{\partial \mathrm{spr}}{\partial A} = H_{S1} + H_{S3}\frac{\partial B'}{\partial A}$, $\frac{\partial \mathrm{spr}}{\partial \nu} = \left[H_{S2} + H_{S3}\frac{\partial B'}{\partial \nu}\right]$, and $\frac{\partial \mathrm{spr}}{\partial B} = H_{S3}\frac{\partial B'}{\partial B}$. We can replace $\frac{\partial \mathrm{spr}}{\partial A}(A_t - A) + \frac{\partial \mathrm{spr}}{\partial \nu}(\nu_t - \nu) + \frac{\partial \mathrm{spr}}{\partial B}(B_t - B)$ in equation (A.11) with $\mathrm{spr}_t - \mathrm{spr}$.

For the $\partial N/\partial \mathrm{spr}$ and $\partial N/\partial B$ terms in equation (A.11), we can use the partial derivative of N over spr and B in equation (A.6),

$$
\begin{aligned}
\partial N/\partial \mathrm{spr} &= -\varphi_j(1-\vartheta)B\vartheta/(\vartheta + 1/\beta - 1 + \mathrm{spr}(A,\nu,B))^2 \equiv M\varphi_j \\
\partial N/\partial B &= \varphi_j(1-\vartheta)\vartheta/(\vartheta + 1/\beta - 1 + \mathrm{spr}(A,\nu,B)) \equiv M_b\varphi_j.
\end{aligned}
$$

Plugging these equations and (A.12) in (A.11), collecting terms, and rearranging gives

$$
\begin{aligned}
\hat{py}_{i,j,t} &= \alpha_i + \frac{\eta}{1-\eta}(z_{it}) + \frac{\eta}{1-\eta}A_t \\
&+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial A}A_t - \frac{\eta}{1-\eta}\lambda_i\frac{\partial R}{\partial A}A_t \\
&+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}M\varphi_j\mathrm{spr}_t - \frac{\eta}{1-\eta}\lambda_i\frac{\partial R}{\partial N}M\varphi_j\mathrm{spr}_t \\
&+ \frac{\partial \hat{Y} - \frac{\eta(1-\alpha)}{1-\eta}\hat{w}}{\partial N}M_b\varphi_jB_t - \frac{\eta}{1-\eta}\lambda_i\frac{\partial R}{\partial N}M_b\varphi_jB_t.
\end{aligned} \tag{A.13}
$$

where α_i collects all the time invariant terms and is defined below. This equation is (22) as follows

$$
\begin{aligned}
\hat{py}_{ijt} &= \alpha_i + \beta_1(\mathrm{spr}_t \times \varphi_j) + \beta_2(\mathrm{spr}_t \times \varphi_j \times \lambda_i) + \beta_3A_t + \beta_4(A_t \times \lambda_i) + \beta_5(B_t \times \varphi_j) \\
&+ \beta_6(B_t \times \varphi_j \times \lambda_i) + \frac{\eta}{1-\eta}z_{it},
\end{aligned} \tag{A.14}
$$

with

$$\beta_1 = \frac{\partial \hat{Y} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right] \partial \hat{w}}{\partial N} M, \qquad \beta_2 = -\frac{\eta}{1-\eta}\frac{\partial R}{\partial N} M,$$

$$\beta_3 = \frac{\eta}{1-\eta} + \frac{\partial \hat{Y} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right] \partial \hat{w}}{\partial A}, \quad \beta_4 = -\frac{\eta}{1-\eta}\frac{\partial R}{\partial A},$$

$$\beta_5 = \frac{\partial \hat{Y} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right] \partial \hat{w}}{\partial N} M_b, \qquad \beta_6 = -\frac{\eta}{1-\eta}\frac{\partial R}{\partial N} M_b,$$

where α_i is given by

$$\alpha_i = f(\lambda_i, x) - \frac{\eta}{1-\eta} z - (\beta_3 + \lambda_i \beta_4)\, A - (\beta_1 + \lambda_i \beta_2)\varphi_j \text{spr} - (\beta_5 + \lambda_i \beta_6)\varphi_j B.$$

Q.E.D.

B Data sources

We document the data sources for the aggregate, regional, firm, and bank data we use in the paper.

B.1 ORBIS-AMADEUS

The construction of the firm-level dataset follows closely the work of Gopinath et al. (2017). Here, we report some basic information, and we refer the reader to that paper for additional details. We use firm-level data on Italian firms from ORBIS-AMADEUS, accessed online through Wharton Research Data Services (WRDS). The dataset has detailed balance sheet information for public and privately held firms; we use only the unconsolidated data on active firms.

We clean this dataset in a series of steps. We start from an initial panel that has 467,063 firm-level observations for operating revenues in 2007. We control for basic reporting mistakes by dropping firm-year observations with negative values for total assets, tangible fixed assets, number of employees, wage bill, and operating revenues. This reduces to 455,564 the number of observations on operating revenues in 2007. We next drop firm-year observations that have missing values for operating revenues, total assets, short-term debt, long-term debt and accounts receivable. This restriction does not reduce the number of observations in 2007. We deflate monetary values using the Italian consumer price index (CPI) obtained from FRED and drop firm-year observation for which our indicator of leverage is above the 99th percentile or below the 1st percentile. This reduces to 441,873 the number of observations in 2007. In most of our analysis, we require firms to have observations on operating revenues for every year between 2007 and 2015. Given this restriction, the number of observations in 2007 drops to 349,687. Finally, we drop firms that operate in the financial industry (NACE 64, 65 and 66) or in sectors with a strong government presence, which are public administration and defense (NACE 84), education (NACE 85), and health care (NACE 86-88). The final sample for the balanced panel has 336,047 firms in 2007.

For the firm-level regressions in Table 2, we use the following variables (the AMADEUS abbreviations are in italics).

SALES: log of operating revenues (*OPRE*) deflated with the Consumer Price Index.

LEVERAGE: ratio of the sum of short-term loans (*LOAN*), long-term loans (*LTDB*), and accounts receivable (*DEBT*) to total assets (*TOAS*) in 2007:

$$\text{leverage}_i = \frac{LOAN_{i,2007} + LTDB_{i,2007} + DEBT_{i,2007}}{TOAS_{i,2007}}.$$

SIZE: Log of total assets (*TOAS*) deflated with the Consumer Price Index.

PROFITABILITY: Ratio of profit (*PLBT*) to total assets.

VOLATILITY: Standard deviation of firm's sales growth, $(OPRE_t - OPRE_{t-1})/(0.5(OPRE_t + OPRE_{t-1}))$, from 2008 to 2015.

B.2 Bankscope and Bank of Italy reports

From Bankscope, we extract balance sheet data for banks headquartered in Italy. We keep data only for 2007 and drop observations with no information on total assets (*totalassets*), total equity (*totalequity*), and holdings of government bonds (*memogovernmentsecuritiesincluded*). We then map the city of incorporation (*city*) to one of the 20 Italian regions: Abruzzo, Basilicata, Calabria, Campania, Emilia-Romagna, Friuli-Venezia Giulia, Lazio, Liguria, Lombardia, Marche, Molise, Piemonte, Puglia, Sardegna, Sicilia, Toscana, Trentino-Alto Adige, Umbria, Valle d'Aosta, and Veneto. We use these data to construct exposure_j as in equation (26) in the main text.

We obtain the number of national banks' branches using Bank of Italy's *Albi and Elenchi Vigilanza*, which can be accessed at `https://www.bancaditalia.it/compiti/vigilanza/`

albi-elenchi/. We manually use the website query to obtain the geographic distribution of bank branches for UNICREDIT, Intesa-Sanpaolo, Monte dei Paschi di Siena, Banca Nazionale del Lavoro, and Banco Popolare. The branches are reported at the city level as of December 31st, 2007, and we use ZIP codes to aggregate branches at the regional level. The total number of banks' branches for each region as of December 31st, 2007, is obtained from the Bank of Italy's *Base Dati Statistica*. The series name is *TDB20207*.

B.3 Aggregate and regional data

Real GDP: Real GDP is obtained from the OECD national accounts and equal to gross domestic product at market prices and deflated by the GDP deflator (PGDP). The series are log and linearly detrended with data from 2000 to 2015.

Total Factor Productivity: Total factor productivity (TFP_t) is obtained as the residual of the Cobb-Douglas technology $Y_t = TFP_t\ K_t^{\alpha} H_t^{1-\alpha}$, where Y_t is real GDP; H_t is total hours worked; and K_t is capital in year t, with α set to the standard capital share of 0.3. Hours are measured as the product of the number of hours worked per employee for the total economy times total employment, both taken from the OECD database. We use investment series to construct the capital stock using standard methods. Investment I_t is gross capital formation at current prices deflated by PGDP$_t$. The capital series is pinned down by the initial stock of capital, K_0, and the standard law of motion $K_{t+1} = I_t + (1-\delta)K_t$, where the depreciation δ is set to 10%. We use investment data from 1960 to 2015 and estimate K_0 such that $\frac{K_0}{Y_0} = \frac{1}{56}\sum_{t=1}^{56}\frac{K_t}{Y_t}$. Total factor productivity is reported from 2000 to 2015 in log-linear detrended values.

Sovereign spreads: Sovereign spreads are the difference between the yields on Italian and German government bonds with a five-year maturity, obtained from the Global Financial Database. The values are annual averages and reported in percentages.

Bank equity: Bank equity is the market value of total shares and other equity issued by Italian monetary and financial institutions other than the central bank, averaged over the year. The series is obtained from the financial accounts published by the Bank of Italy and can be downloaded from the Bank of Italy's *Base Dati Statistica*.

Credit supply: Credit supply is the percentage of loan officers, who report a "tightening" in the standards for approving loans or credit lines to enterprises less the percentage of loan officers who report an "easing." This information is obtained from the Bank Lending Survey published quarterly since 2003 by the Bank of Italy.

Firms spreads: Firms' interest rate spread is the difference between the average interest rate on loans up to one year and below one million euros for Italian and German non-financial firms, reported in percentages. Interest rates are obtained from the European Central Bank statistical data warehouse, MFI interest rate statistics (MIR statistics). We filter the dataset by reference area (Italy and Germany), maturity (up to one year), and amount category (up to and including one million euros). We obtain interest rates at a monthly frequency and average them at the annual level for each country. The firms' spread series is the difference between the two averages for each year.

Regional data: Real GDP and real GDP per capita at the regional level for Italy are obtained from ISTAT, the Italian National Institute of Statistics, from the regional accounts within the national accounts. Public sector expenditure at the regional level is obtained from "*La spesa statale regionalizzata*", an annual publication by the Ministry of Economics and Finance that can be accessed at https://www.rgs.mef.gov.it/VERSIONE-I/pubblicazioni/pubblicazioni_statistiche/la_spesa_statale_regionalizzata.

B.4 Additional summary statistics

In this section, we report additional summary statistics of our data. Specifically, we show how firms' and regions' characteristics vary by firms' leverage and banks' sovereign debt exposure. We also report the complete set of regression coefficients for the baseline model implied empirical specification.

The firms' characteristics we consider are firms' total assets, debt, leverage, profitability, productivity, and the standard deviation of their sales growth.[31] All these characteristics are standardized, and with the exception of the last of them, they are measured in 2007. In Figure A-1 "leverage" reports the difference in mean of a given characteristic between firms with $lev_i = 1$ and $lev_i = 0$; "exposure" reports the difference in mean between firms located in high-exposure regions and those located in low-exposure regions; "interaction" reports the difference in mean across locations for the difference in a given characteristic between the two leverage groups.

We can see that high leverage firms in our dataset are larger on average, more profitable, and more productive; in addition, their sales are less volatile compared with to low leverage firms. Differences in these firms' characteristics across regions are less striking, with the

[31]We estimate firm-level revenue total factor productivity (TFPR) using the two-step generalized method of moments implementation of Levinsohn and Petrin (2003) developed in Wooldridge (2009). Because of the difficulties inherent in estimating TFPR for the service sector, we consider only firms that operate in manufacturing (NACE codes 1100-3999).

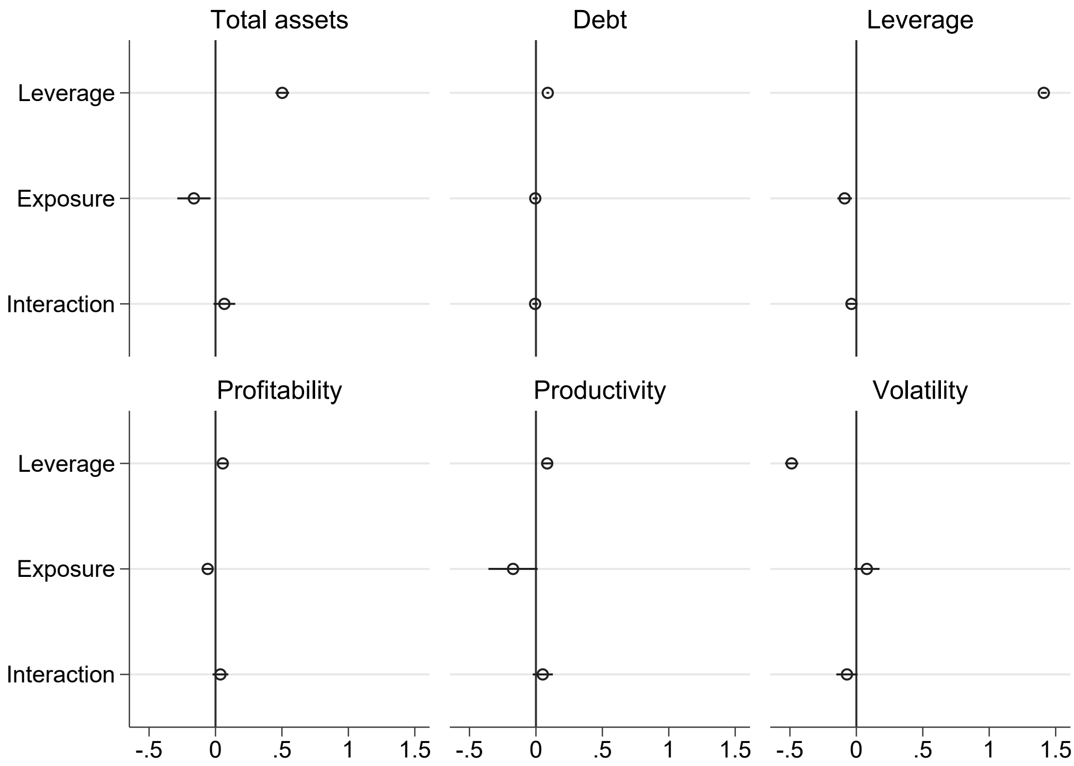


Figure A-1: Firms' characteristics by leverage and exposure

Note: Total assets, debt, leverage, profitability, and productivity are measured in 2007, while volatility is computed as the standard deviation of sales growth for each firm from 2008 to 2015. Each variable is de-meaned and scaled by its cross-sectional standard deviation. The figure reports the point estimate of the difference in means for each of these characteristics, along with the 90% confidence interval. Mean tests are across leverage groups, exposure groups, and for the interaction of leverage and exposure. Standard errors are clustered at the regions and firms level.

high-exposure regions having somewhat smaller, less profitable, and less levered firms. Importantly, we can see that for any of these characteristics, the differences in mean between high and low leverage firms are homogeneous across regions, as the coefficient "Interaction" is always a well identified zero. This latter result shows that firms' characteristics are balanced as far are the estimation of the direct effect in equation (25) is concerned.

Figure A-2 reports how regional characteristic vary across the two groups of regions. We use the firm-level data to construct the manufacturing share in each region in 2007 and to construct region-specific changes in average firm productivity between 2007 and 2009 (Dtfp 09-07) and between 2010 and 2012 (Dtfp 12-10). The figure reports the difference in mean for these statistics across the two groups of regions—those with high banks' sovereign debt exposure and those with low banks' sovereign debt exposure—along with a 90% confidence interval. We can see from the figure that these statistics are not significantly different across

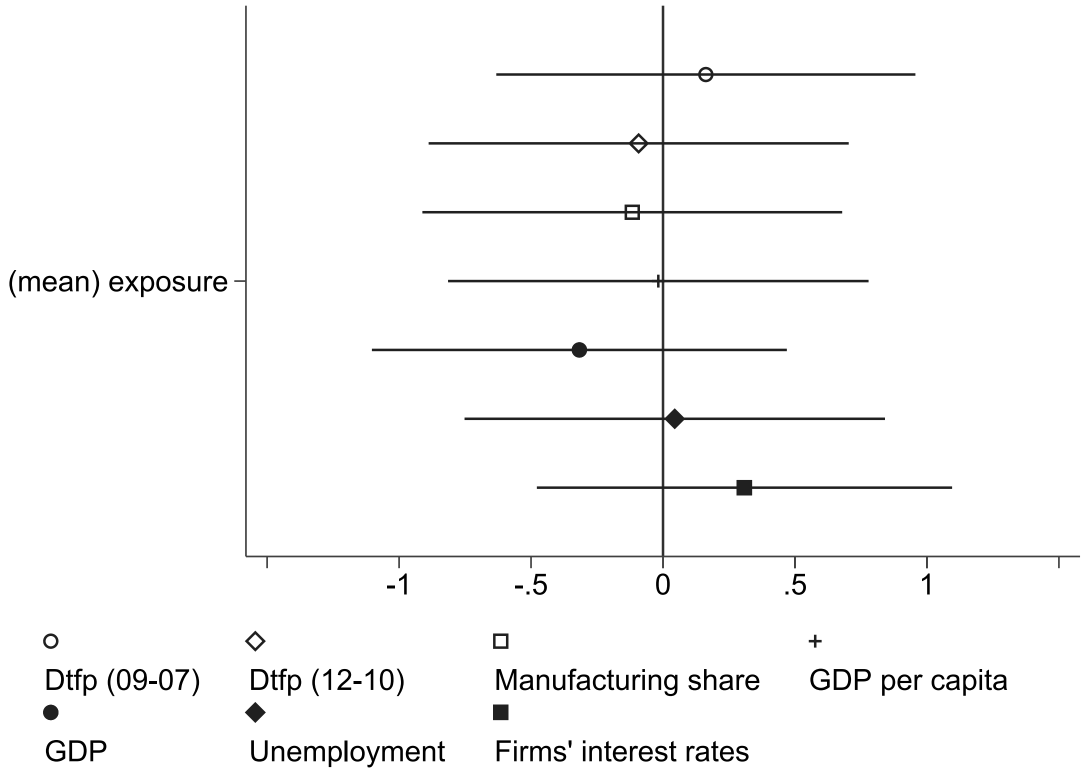


Figure A-2: Regional characteristics by exposure

Note: Each variable is de-meaned and scaled by its cross-sectional standard deviation. The figure reports the point estimate of the difference in means across the two groups of regions for each of these characteristics, along with the 90% confidence interval.

the two groups of regions. The figure reports also the difference in mean across the two groups of regions in the level of GDP per capita, GDP, unemployment, and firm interest rates in 2007. We can verify again that there are no statistically significant differences in regional characteristics across the two groups of regions.

Next, we analyze in more detail the results from the baseline model implied empirical specification. In Table A-1, we report the complete set of regression coefficients for the baseline regression, namely the model implied regression of Column 1 in Table 2. These results reiterate that firm leverage matters for the sensitivity of sales to shocks. High leverage firms perform worse in low government exposure regions when spreads increase and also when government debt increases, but do not appear differentially sensitive to aggregate TPF shocks.

Table A-1: Estimates of Model Implied Regression

	Coefficient	Std Error
$\hat{\beta}_1$	-0.717^{***}	0.129
$\hat{\beta}_2$	1.230^{***}	0.341
spr_t	1.409^{**}	0.579
$\text{spr}_t \times \text{lev}_i$	-0.631^{**}	0.230
TFP_t	0.598	0.963
$\text{TFP}_t \times \text{lev}_i$	-0.147	0.535
debt_t	-0.533^{***}	0.132
$\text{debt}_t \times \text{lev}_i$	-0.571^{***}	0.090
$\text{debt}_t \times \text{exp}_j$	0.011	0.051
$\text{debt}_t \times \text{lev}_i \times \text{exp}_j$	-0.037	0.108
Firm fixed effects	Yes	
R^2	0.87	
Obs.	2,589,772	

Note: The table contains the complete regression results for the model implied regression of Column 1, Table 2. Standard errors are clustered two ways at the regions/time level. Point estimates and standard errors are multiplied by 100. $***$-significant at 1%, $**$-significant at 5%.

C Model with firm default

In this appendix, we present the extended model with firm default. In this extension, intermediate goods firms are affected by a second idiosyncratic shock ξ_{ijt} that affects their revenue. These shocks are realized at the end of the period, after firms produce. Revenue shocks ξ_{ijt} are i.i.d. across firms and follow a normal distribution Φ with mean zero and volatility $\sigma_{\xi,t}$. This volatility is an i.i.d aggregate shock that is realized in the beginning of the period and that is common across all firms in all regions. We allow the volatility to change over time to better match the observed firms' default during Italian debt crisis.

As in the baseline model, at the beginning of the period after aggregate and idiosyncratic productivity shocks are realized, each firm makes choices for capital k_{ijt} and labor ℓ_{ijt} and borrows from financial intermediaries for its working capital needs. Firms borrow by issuing discount bonds that are defaultable. In the beginning of the period, they get $q^f_{ijt} b^f_{ijt}$ to finance their working capital,

$$q^f_{ijt} b^f_{ijt} = \lambda_i (r^k_{jt} k_{ijt} + w_{jt} \ell_{ijt}),$$

and promise to repay at the end of the period b^f_{ijt}, conditional on not defaulting. The price of the bond q^f_{ijt} is a function that compensates financial intermediaries for default risk. At the end of the period after the idiosyncratic revenue shock is realized, firms decide whether to repay their debt. Firms distribute back their profits as equity payouts, which are required to be non-negative:

$$\Pi_{ijt} = p_{ijt}y_{ijt} - (1-\lambda_i)(r_{kt}k_{ijt} + w_t\ell_{ijt}) - b^f_{ijt} - \xi_{ijt} \geq 0. \tag{A.15}$$

Firms can also choose to default on their debt b^f_{ijt}; if firms default, $d_{ijt} = 1$; otherwise, $d_{ijt} = 0$. Defaulting firms sell their output and use these resources to pay for the residual input costs $(1-\lambda)(r_{kt}k_{ijt} + w_t\ell_{ijt})$, before exiting with a firm value of zero.

Financial intermediaries The financial intermediaries face a similar problem to that in the baseline model. The main modification is that firm default affects financial intermediaries' balance sheets by changing their pledgeable net worth.

They use net worth and deposits to fund borrowing for the government and intermediate goods firms such that their budget constraint is

$$q_t B_{jt+1} + \int q^f_{ijt} b^f_{ijt} di \leq N_{jt} + q^a_{jt} A^d_{jt}, \tag{A.16}$$

and they face a leverage constraint that bounds the borrowing from household to the value of their collateral. Intermediaries in region j can pledge a fraction θ_{jt} of the value of firms' bonds,

$$q^a_{jt} A^d_{jt} \leq q_t B_{jt+1} + \theta_{jt} \int q^f_{ijt} b^f_{ijt} di. \tag{A.17}$$

We assume that the pledgeability of firm loans depends on the default rates of firms, such that

$$\theta_{jt} = \theta[1 - \bar{d}_{jt}], \tag{A.18}$$

where $\bar{d}_{jt} = \int d_{ijt}\kappa_{ijt} di$ is the weighted average of firms' defaults in region j, with the weights κ_{ijt} corresponding to the firm weight in the financial intermediaries' portfolios of private loans, $\bar{\kappa}_{ijt} = b^f_{ijt} / \int b^f_{ijt} di$.

Combining the budget and leverage constraints leads to the collateral constraint that bounds firms loans to be a proportion $1/(1-\theta_{jt})$ of their net worth,

$$\int q^f_{ijt} b^f_{ijt} di \leq \frac{1}{(1-\theta_{jt})} N_{jt}. \tag{A.19}$$

Elevated firm default affects financial intermediaries' balance sheets by reducing the effective net worth that can be used for private lending.

Financial intermediary returns at the end of the period depend now not only on government default but also on firm default. They equal

$$F_{jt+1} = (1 - D_{t+1}) \left[\vartheta B_{jt+1} + q_{t+1}(1-\vartheta) B_{jt+1}\right] + \int (1 - d_{ijt}) b^f_{ijt} di - A^d_{jt}. \tag{A.20}$$

With the net worth N_{jt}, a banker chooses lending to the government B_{jt+1}, lending to firms $\left\{b^f_{ijt}\right\}$, and deposits A^d_{jt} to maximize expected return. This problem gives the same government pricing condition as in the baseline model (15). The firm pricing condition with firm default is

$$q^f_{ijt} = \frac{\beta \mathbb{E}(1 - d_{ijt})}{1 + \zeta_{jt}} \equiv \frac{\mathbb{E}(1 - d_{ijt})}{R_{jt}}, \tag{A.21}$$

where ζ_{jt} is the Lagrange multiplier on constraint (A.19). We define the regional interest rate R_{jt} as the borrowing rate without any default, which reflects the risk-free rate β and how binding the collateral constraint $\zeta_{jt} > 0$ is. The firm-specific bond price reflects this regional interest rate and the firm repayment probability $\mathbb{E}(1 - d_{ijt})$.

Characterization of firms' problem We now set up the intermediate goods firms' program in recursive notation. The regional state variable X_j now includes the volatility shock, $X_j = (A, \sigma_\xi, N_j)$. A firm in region j with idiosyncratic and regional state $x = (z, \lambda, X_j)$ chooses capital, labor, and borrowing to maximize its value, such that

$$v(x) = \max_{p,k,\ell,b^f} E_\xi \max\left\{py - (1-\lambda)(r_k k + w(X_j)\ell) - b^f - \xi + \beta E v(x'), 0\right\}, \tag{A.22}$$

subject to the demand function (3), the production function (4), and the following financing constraint, non-negative equity payouts condition, and evolution of the aggregate state:

$$\begin{aligned} q^f(k,\ell,b,x) b^f &= \lambda(r_k k + w(X_j)\ell) \\ py - (1-\lambda)(r_k k + w(X_j)\ell) - b^f - \xi &\geq 0 \\ X'_j &= H(X_j). \end{aligned}$$

The price of the bond is a function $q^f(b^f, k, \ell, x)$ that compensates financial intermediaries for firm default risk and changes default free borrowing rates R, which in turn reflect the collateral constraints of financial intermediaries, as seen in equation (A.21). The bond price function therefore depend on firms' choices $\{b^f, k, \ell\}$ and all the idiosyncratic and aggregate states $x = (z, \lambda, X_j)$. The bond price schedule arising from default risk gives rise to a Laffer curve for firm credit which increases with borrowing and reaches a peak. This Laffer curve is defined by $q^f(b^f, k, \ell, x) b^f$, and it maps b^f to the resources raised by the firms to these issuances of debt. In general, as borrowing increases, the marginal resources the firms is able to obtain decreases—as $q^f(k, b^f, s)$ decreases with b^f—and this acts implicitly as a "borrowing constraint" for firms.

In this model with firm default, increase in sovereign default risk reduces the net worth of financial intermediaries, which not only increases R, but it also increases firms' default risk. As a result an increase in sovereign risk shifts in the credit Laffer curve, effectively a tightening in the borrowing constraints faced by firms. This credit Laffer curve can be interpreted as a credit line, where the amount that firms can draw declines when sovereign risk increases.

From the problem of intermediate goods firms, it is easy to see that the default decision satisfies a cutoff rule with respect to the revenue shock ξ: there exists a cutoff revenue shock $\xi_j^*(x; k, \ell, b^f)$ such that firms default if and only if $\xi \geq \xi_j^*(x; k, \ell, b^f)$ where ξ_j^* satisfies

$$\xi_j^* = py - (1-\lambda)(r_k k + w(X_j)\ell) - b^f.$$

The default cutoff implies that the default probability for this firm ex-ante is given by the cumulative distribution function evaluated at the cutoff, $\Phi(\xi_j^*; \sigma_\xi)$, where the volatility of the distribution depends on the state σ_ξ. The firm-specific bond price is then $q^f(k, \ell, b^f, x) = \Phi(\xi_j^*(x; k, \ell, b^f); \sigma_\xi)/R(X_j)$. As in the baseline model, intermediate goods firms make optimal choices, taking as given the evolution of the aggregate state and mapping between prices and the state. Taking as given the aggregate prices $\{Y(X_j), w_j(X_j), R(X_j)\}$, each firm (z, λ) chooses $\{k, \ell, \xi^*, b^f\}(x)$ to satisfy the following conditions:

$$\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}[Y(X_j)Az]^{1-\eta}k^{\eta-1} = \left[1-\lambda+\lambda\frac{R(X_j)}{\Phi(\xi^*;\sigma_\xi)}\right](r_k)^{1-(1-\alpha)\eta}w(X_j)^{(1-\alpha)\eta}$$

$$\ell = \frac{1-\alpha}{\alpha}\frac{r_k}{w(X_j)}k$$

$$\xi^* = (1-\eta)[Y(X_j)Az]^{1-\eta}(k^\alpha\ell^{1-\alpha})^\eta$$

$$\frac{\Phi(\xi^*;\sigma_\xi)}{R(X_j)}b^f = \lambda(r_k k + w(X_j)\ell) = \lambda\frac{1}{\alpha}r_k k$$

The model with firm default is more difficult to sharply characterize because of the nonlinearity of default. Firms' choices are distorted by firms' incentives to avoid default risk to reduce the interest rate they pay on their loans.

The market clearing conditions for the regional private equilibrium are similar to those in the baseline model. The credit market clearing condition is modified to account for firm default risk, with the total regional firm borrowing in discount bonds being less than or equal to the regional pledgeable net worth:

$$\int \frac{\Phi(\xi^*;\sigma^{\xi})}{R(X_j)} b_j^f \, d\Lambda(z,\lambda) \leq \frac{1}{1-\theta_j(X_j)} N_j.$$

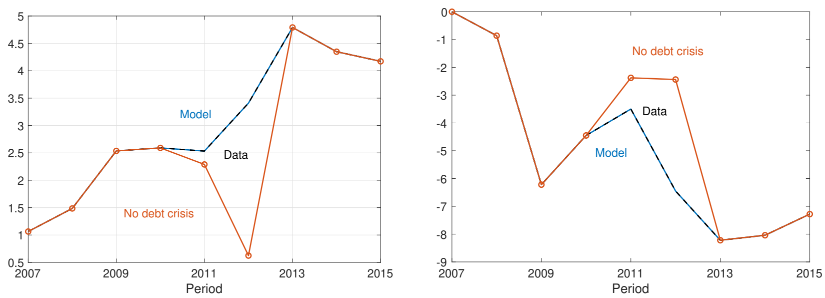
Parameterization. Our goal with this extension is to assess whether firm default alters our measured output costs from sovereign risk during the Italian debt crisis. For this exercise, we assume that all the parameters remain as in the baseline model and make a simplifying assumption for computational tractability that the path for sovereign debt and spreads during our event window (from 2007 to 2015) follows the same path as in the baseline model.[32] The model, nevertheless, contains additional parameters, namely, the volatility of the revenue shock, $\sigma_{\xi,t}$. For the 2008-2015 event analysis, we choose a sequence of aggregate productivity shocks and $\sigma_{\xi t}$ to match paths for Italian aggregate output and the observed firm default rates. Firm default rates for Italy—taken from the Bank of Italy and based on reports to the Central Credit Registry—range from 1% to 4.8% between 2008 to 2015. Our counterfactual experiment follows the procedure we use in the benchmark model. To the calibrated model, we feed a constant sovereign spread at the 2007 level, the same sequence of A_t, and also the same sequence of $\sigma_{\xi,t}$.

Figure A-3 plots firm default rates and output for the data, the firm default model, and the counterfactual experiment using the firm default model. Firm default rates rise from 1% in 2007 to a peak of 4.8% in 2013. The counterfactual without a sovereign debt crisis results in lower paths for firm default rates and, as in the benchmark, higher paths for aggregate output. Without the sovereign debt crisis, the firm default rates would be 1.5% lower during 2011 and 2012. These effects are sizable, implying that a large part of the firm defaults in 2012 in Italy were driven by the sovereign debt crisis. Figure A-3(b) shows the path for output in the data/model and in the no debt crisis counterfactual. As in the benchmark, this analysis is consistent with the findings that the output loss of sovereign risk was sizable during 2011, 2012, and 2013.

D Numerical solution

We solve the model in two steps. The first step solves a pseudo private equilibrium. The second step solves the Markov equilibrium in which the government takes as given the

[32]We could also calibrate the enforcement shock ν_t in this experiment to get the observed sovereign spread path by considering the default decision of the government in the model with firm default. This, however, is a cumbersome procedure, and it would result in identical outcomes.



(a) Firm Default Rate (b) Output

Figure A-3: Model with firm default

private responses over its default and debt choices.

We have already shown in the main text that the government's decisions affect the private economy through their effects on banks' net worth, which in turn determines firms' borrowing rates. Furthermore, Corollary 1 shows that firms' borrowing rate R weakly decreases with banks' net worth. In the private equilibrium, under a given shock and firm distribution, there must be a level of net worth associated with a firms' borrowing rate $R \geq 1/\beta$. This motivates us to solve a pseudo private equilibrium in the first step. For each state $\hat{X} = (A, R)$, we compute the private equilibrium of $\{Y(\hat{X}), w(\hat{X}), T(\hat{X}), L(\hat{X}), B^f(\hat{X}), k(z, \lambda; \hat{X}), \ell(z, \lambda; \hat{X})\}$, where $B^d(\hat{X})$ is the aggregate loan demand of the firms in region $\hat{X}$; that is,

$$B^d(\hat{X}) = \int_{(z,\lambda)} \lambda \frac{1}{\alpha} r_k k(z, \lambda; \hat{X}) d\Lambda(z, \lambda).$$

In the second step, we solve the government's problem taking as given the private equilibrium. In particular, for any state (A, ν, B) and the government's choice (D, B'), the state for the private economy becomes $X = (S, B, D, B')$, with $S = (A, \nu)$. The implied banks' net worth $N_j(X)$ in region j is given by

$$N_j(X) = \bar{n}_j + \varphi_j (1 - D(S, B)) q(S, B'(B))(1 - \vartheta) B.$$

The pseudo private state is $\hat{X}_j(X) = (A, R_j(X))$, with $R_j(X) = 1/\beta$ if $B^d(A, 1/\beta) \leq N_j(X)/(1 - \theta)$; otherwise, $R_j(X)$ is given by the inverse of the aggregate private loan demand; that is,

$$R_j(X) = \left(B_j^d\right)^{-1} (N_j(X)/(1 - \theta), A).$$

We now describe in details the computation algorithm.

D.1 Step 1: Computation for private equilibrium

1. Construct grid points for A and R.
2. Compute the equilibrium prices $\{Y, w\}(\hat{X})$ for each grid of (A, R):

$$w(\hat{X}) = M_w \left[\exp\{A\}^{\frac{\eta}{1-\eta}} / R_w(X_j)\right]^{\frac{(1-\eta)}{\eta(1-\alpha)}}$$

$$Y(\hat{X}) = M_y \frac{\left[\exp\{A\}^{\frac{\eta}{1-\eta}} / R_w(X_j)\right]^{\frac{1-\eta+(1-\alpha\eta)\gamma}{\eta(1-\alpha)\gamma}}}{\exp\{A\}^{\frac{\eta}{1-\eta}} / R_y(X_j)},$$

where $R_w(R)^{-1} = \int_\lambda r_\lambda(R)^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda$, $R_y(R)^{-1} = \int_\lambda r_\lambda(R)^{-\frac{1}{1-\eta}} d\Lambda_\lambda$, $r_\lambda(R) = 1 + \lambda(R - 1)$, and the constants $\{M_w, M_y\}$ are functions of the model parameters, given in the proof of Proposition 1 in Appendix A.
3. Construct total tax $T(\hat{X}) = \tau Y(\hat{X})$ and aggregate loan demand function $B^d(\hat{X})$ using the optimal capital decision $k(z, \lambda; \hat{X})$:

$$k(z, \lambda, \hat{X}) = M_k \left(\exp\{A + z\}\right)^{\frac{\eta}{1-\eta}} Y(\hat{X}) w(\hat{X})^{-\frac{(1-\alpha)\eta}{1-\eta}} r_\lambda(R)^{-\frac{1}{1-\eta}},$$

where $M_k = \left((1-\tau)\eta\alpha^{1-(1-\alpha)\eta}(1-\alpha)^{(1-\alpha)\eta}\right)^{\frac{1}{1-\eta}} r_k^{-\frac{1-(1-\alpha)\eta}{1-\eta}}$.

D.2 Step 2: Computation for the Markov equilibrium

Taking given the functions of $B^d(A, R)$ and $T(A, R)$, the government solves its problem. Let Ψ be the conditional CDF of default cost shock ν. We solve the following problem:

Define the expected value H_V as follows:

$$H_V(S, B') = \beta_g E_S \left\{ V(S', B') + \Psi(\nu^*(S', B')|\nu)\nu^*(S', B') - \int_{\nu^*(S', B')} \nu' d\Psi(\nu'|\nu) \right\}.$$

1. Construct a large set of grids for ν.
2. Guess $H_V^{\{0\}}(S, B')$, $q^{\{0\}}(S, B') = \frac{\beta\vartheta}{1-\beta(1-\vartheta)}$, and tax revenue $T_{x,j}^{\{0\}}(S, B, D, B')$ as follows. Let

$$N_j^{\{n\}}(S, B, D, B') = \bar{n}_j + \varphi_j(1 - D)\left[q^{\{n\}}(S, B'(B))(1-\vartheta)B\right].$$

If $N_j^{\{n\}}(S, B, D, B')/(1-\theta) \geq B^d(A, 1/\beta)$ for region j, $R_j^{\{n\}} = 1/\beta$ and $T_{x,j}^{\{n\}}(S, B, D, B') =$

$T(A, 1/\beta)$; otherwise,

$$R_j^{\{n\}}(S, B, D, B') = \left(B^d\right)^{-1} \left(N_j^{\{n\}}(S, B, D, B')/(1-\theta), A\right)$$

and

$$T_{x,j}^{\{n\}}(S, B, D, B') = T\left(A, R_j^{\{n\}}(S, B, D, B')\right).$$

3. Solve the government's problem

$$V^{\{n+1\}}(S, B) = \max_{G, B'} u_g(G) + \beta_g H_V^{\{n\}}(S, B'),$$

subject to

$$G + \vartheta B = \sum_j T_{x,j}^{\{n\}}(S, B, D, B') + q^{\{n\}}(S, B')\left[B' - (1-\vartheta)B\right].$$

4. We update the default cutoff ν^*

$$\nu^*(S, B) = V^{\{n+1\}}(S, 0) - V^{\{n+1\}}(S, B),$$

H_V function

$$H_V^{\{n+1\}}(S, B') = \beta_g E_S\left\{V^{\{n+1\}}(S', B') + \Psi(\nu^*(S', B')|\nu)\nu^*(S', B') - \int_{\nu^*(S', B')} \nu' d\Psi(\nu'|\nu)\right\},$$

and q schedule

$$q^{\{n+1\}}(S, B') = \beta E_{A,\nu}\left\{[1 - \Psi(\nu^*(S', B'))]\left(\vartheta + (1-\vartheta)q^{\{n\}}(S', B''(S, B'))\right)\right\}.$$

5. Iterate procedure 3 and 4 until q and H_V function converge.