SOVEREIGN DEFAULT RISK AND FIRM HETEROGENEITY

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ABSTRACT

This paper measures the output costs of sovereign risk by combining a sovereign debt model with firm- and bank-level data. In our framework, an increase in sovereign risk lowers the price of government debt and has an adverse impact on banks' balance sheets, disrupting their ability to finance firms. Importantly, firms are not equally affected by these developments: those that have greater financing needs and borrow from banks that are more exposed to government debt cut their production the most in a debt crisis. We measure the extent of this heterogeneity using Italian data and parameterize the model to match these cross-sectional facts. In counterfactual analysis, we find that heightened sovereign risk was responsible for one-third of the observed output decline during the 2011-2012 crisis in Italy.

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1 Introduction

As the recent experience of southern European countries has shown once more, sovereign debt crises are often associated with a tightening of credit for the private sector and large declines in real economic activity. An active research agenda has emphasized various explanations for this negative association between sovereign risk and aggregate output. One explanation for these patterns, developed in sovereign default models in the tradition of Eaton and Gersovitz (1981), Arellano (2008), and Aguiar and Gopinath (2006), argues that governments have greater temptation to default when economic conditions deteriorate. Another popular explanation for this association highlights that sovereign debt crises have disruptive effects on financial intermediation and real economic activity because banks are often the main creditor of their own government (Gennaioli, Martin, and Rossi, 2014; Bocola, 2016; Perez, 2015).

Quantifying this two-way feedback between sovereign risk and output is a challenging open question in macroeconomics yet relevant for policymakers dealing with sovereign debt crises.[1] The challenge arises because debt crises and economic outcomes are jointly determined, which makes it hard to disentangle to what extent sovereign risk rises in response to deteriorating economic conditions and to what extent it causes them. Researchers have approached this challenge mainly with two methodologies. Some studies fit structural models to aggregate data and use them to measure the macroeconomic consequences of sovereign risk. This approach suffers from the criticism that the identification of the relevant effects partly relies on ancillary assumptions, as aggregate data alone provide little information about the direction of causality. A different approach in some recent studies, uses micro firm-bank datasets and difference-in-difference methodologies to estimate the impact that sovereign risk has on credit to firms and their performance.[2] While the identification of these micro elasticities is often more transparent in this approach, it is not designed to capture the general equilibrium effects needed to measure aggregate responses.

The contribution of this paper is to combine these two approaches by building a model of sovereign debt with heterogeneous firms to measure the feedback between sovereign risk and output. We show that in our framework, the response of output to an increase in

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[1] For instance, proponents of the fiscal austerity measures implemented in Europe over the past few years have often emphasized how these policies—by reducing sovereign risk— could have positive spillovers into the private sector. Clearly, assessing the importance of these "non-Keynesian" effects of fiscal policy requires a quantification of the recessionary effects of sovereign risk.

[2] Bottero, Lenzu, and Mezzanotti (2017) use the Italian credit registry and firms' balance sheet data to gauge the impact of the government debt crisis on lending behavior. In some recent work, Kalemli-Ozcan, Laeven, and Moreno (2018) also use matched firm-bank data and find that investment falls for firms that borrow from banks with high exposure to government debt. See also Bofondi, Carpinelli, and Sette (2017), Altavilla, Pagano, and Simonelli (2017) and De Marco (2017).

sovereign risk depends on a set of cross-sectional firm-level elasticities. We fit the model to these moments using Italian micro data as well as aggregate data and use the model to measure the impact that sovereign risk has on the economy. In our main counterfactual, we find that spillovers from the government into the private sector were sizable and accounted for one-third of the output decline observed during the Italian debt crisis of 2011-2012.

Our framework incorporates financial intermediaries and heterogeneous firms into an otherwise canonical general equilibrium model of sovereign debt and default. The economy is composed by islands populated by firms, financial intermediaries, and households, and by a central government. Firms differ in their productivity, and they borrow from intermediaries to finance payments of labor and capital services, factors that are used to produce a differentiated good. These working capital needs are also heterogeneous, with some firms needing to advance a greater fraction of their payments than others. Intermediaries borrow from households and use their own net worth to purchase long-term government debt and extend loans to firms. These credit markets are local as firms borrow exclusively from intermediaries operating on their island, and intermediaries across islands are heterogeneous in their holdings of government debt. Importantly, financial intermediaries face occasionally binding leverage constraints, as the amount they borrow cannot exceed a proportion of their net worth. The government collects taxes and issues long-term bonds to fund public consumption, and chooses whether or not to default on its debt.

The model is perturbed by two aggregate shocks: a shock that moves the productivity process of firms and a shock to the value of default for the government, which can be interpreted as capturing time variation in the enforcement of sovereign debt.[3] In response to these shocks, our environment features a two-way feedback loop between the government and the private sector.

The first side of this loop reflects the endogeneity of government default risk as changes in aggregate productivity and enforcement affect the values of repaying versus defaulting for the government, thereby inducing time variation in sovereign default probabilities and hence interest rate spreads of government securities. The second side of this loop is that fluctuations in government default risk can affect production through its impact on financial intermediation. When sovereign risk increases, the market value of government debt on the balance sheet of financial intermediaries falls, leading to a decline in their net worth. A large enough decline in net worth triggers a binding leverage constraint, which leads intermediaries to tighten credit supplied to firms. These effects increase borrowing costs

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[3]As discussed in the quantitative literature of sovereign debt, variation in the default value gives the model flexibility to fit the behavior of interest rate spreads; see Arellano (2008) and Chatterjee and Eyigungor (2012).

for firms, which then reduce their production. Alongside this *direct effect* that sovereign risk has on firms' borrowing costs, the model features additional general equilibrium mechanisms: as firms that are exposed to higher borrowing costs cut their production, aggregate demand for labor and goods produced in the economy falls, affecting the prices faced by all other firms. We refer to these latter effects as the *indirect effects* of sovereign risk on real economic activity.

These mechanisms affect firms in different ways. Consider the direct effect of sovereign risk. We show that this channel has more adverse effects for firms that have higher borrowing needs, and more so if they are located on islands in which financial intermediaries are more exposed to government debt. Therefore, when the direct effect of sovereign risk to the private sector is sizable, we should observe large differences in production across firms during a sovereign debt crisis, depending on firms' financial needs and on whether they borrow from intermediaries that hold a sizable fraction of government debt. Importantly, these cross-sectional comparisons do not provide information on the indirect effects of sovereign risk because firms with high and low borrowing needs share the same labor and goods markets. We show, however, that we can learn about the magnitude of the indirect effects by looking at the behavior of firms with no borrowing needs. These firms are not affected by fluctuations in borrowing rates, so their performance during a debt crisis is informative about the spillovers that sovereign risk has on firms through its impact on labor and goods markets.

We formalize these insights in two steps. We first derive a model implied linear relation where firms' output are a function of aggregate productivity and sovereign interest rate spreads, with the coefficients varying across firms depending on their borrowing needs and location, as well as the deep structural parameters. These coefficients can be mapped into the direct and indirect effects and they are sufficient to construct the elasticity of firm sales with respect to sovereign risk. We then derive a correspondence between these micro elasticities and the response of aggregate output to sovereign risk. These relationships are the basis of our methodology of using cross-sectional information to empirically discipline the aggregate effect of sovereign risk.

We apply our framework to Italian data during the 2007-2015 period. We link three datasets for our analysis: firm-level balance sheet data from ORBIS-AMADEUS, balance sheet information of Italian banks from Bankscope, and reports on the geographical location of bank branches from the Bank of Italy. We use these data to estimate the linear firm-level relation implied by our model in order to recover the elasticity of firms' output to changes in sovereign spreads and how they vary across firms and regions. Using dummy variables, we classify firms into four groups, depending on whether their leverage is "high"

or "low" or whether they operate in locations where banks' exposure to government debt is "high" or "low." The main empirical finding, robust to a wide range of specifications, is that highly levered firms in regions where banks are highly exposed to government debt experienced the largest contraction in their output in periods of high sovereign spreads. These results confirm the empirical predictions of our theory that sovereign risk has negative direct effects on the private sector. By analyzing the behavior of firms with zero leverage, we also find that the indirect effects of sovereign risk are negative, and more so in regions with high exposure to government debt.

We use these regression coefficients, along with the standard empirical targets considered in the sovereign debt literature, to estimate the model. We then use the model to measure the macroeconomic effects of sovereign risk. To this end, we feed into the model the aggregate productivity series measured in the data, and we retrieve the path for enforcement shocks so that the model reproduces the sovereign spreads in the data. We then use the model to construct counterfactual series for output and firms' interest rates that would have emerged in Italy if the level of sovereign debt enforcement were held constant at its pre-crisis level. We can then net out the effects of sovereign risk on these variables by comparing their benchmark and counterfactual time paths. Our main findings indicate a sizable propagation of sovereign risk to real economic activity. Specifically, we find that on average, a 100 basis point increase in sovereign interest rate spreads increases average firms' interest rates by about 70 basis points and induces a 0.67% decline in aggregate output, of which 0.45% is due to the direct effect on firms' borrowing costs. These numbers imply that the government debt crisis accounted for roughly one-third of the output losses observed during the 2011-2012 crisis episode in Italy.

Related Literature. Our paper combines elements of the sovereign default literature with the literature on the impact of financial imperfections on firms. We also contribute to the growing literature that combines structural models with micro data to infer aggregate elasticities.

Several papers in the sovereign debt literature study the links between sovereign defaults and the private sector through financial intermediation. Mendoza and Yue (2012) propose a model in which firms lose access to external financing conditional on a government default, and they show that such a mechanism can generate substantial output costs in a sovereign default. Similar dynamics are present in the quantitative models of Sosa Padilla (2013) and Perez (2015), and in the more stylized frameworks of Fahri and Tirole (2014) and Gennaioli, Martin, and Rossi (2014). We share with these papers the emphasis on financial intermediation, but we depart from their analysis by focusing on this feedback in periods

in which the government is not in default: in our model, an increase in the likelihood of a future default—even when the government keeps repaying— propagates to the real sector because of its impact on firms' interest rates. Many debt crises, and in particular the one that we are studying, are characterized by rising sovereign spreads but no actual default.

In this respect, our paper is closer to Neumeyer and Perri (2005), Uribe and Yue (2006), Corsetti, Kuester, Meier, and Müller (2013), and Bocola (2016), who measure the macroeconomic effects of sovereign risk by estimating or calibrating structural models, and the reduced form approach in Hebert and Schreger (2016) and Bahaj (2019).[4] A main contribution of our approach relative to all of the above-mentioned papers is to show that cross-sectional moments are informative about the propagation of sovereign risk on real economic activity and to use micro data and a model to carry out the measurement.

Our emphasis on informing macro elasticities from micro data is shared by a number of recent papers. Nakamura and Steinsson (2014) use regional variation in military build-ups to provide an estimate of the aggregate spending multiplier. Beraja, Hurst, and Ospina (2018) use regional fluctuations in employment and wages to estimate the aggregate effects of demand shocks and wage stickiness for the Great Recession. These two papers illustrate important methodological challenges faced by this type of analysis from important differences between the "regional" elasticities recovered using cross-sectional variation and the "aggregate" elasticities that macroeconomists are interested in. In these papers, monetary policy is common across states and reacts to aggregate and not regional conditions, blurring the mapping between regional and aggregate elasticities. A similar issue arises in our context, as factor prices are common across firms.

An active research agenda centers on using micro data to inform aggregate structural models. Researchers have used related "micro-to-macro" approaches to understand the mechanisms from monetary policy to consumption (Kaplan, Moll, and Violante, 2018), unemployment benefit extensions on labor market outcomes (Hagedorn, Karahan, Manovskii, and Mitman, 2013; Chodorow-Reich, Coglianese, and Karabarbounis, 2018), quantifying the losses from international trade (Lyon and Waugh, 2018; Caliendo, Dvorkin, and Parro, forthcoming), measuring the effects from volatility shocks at the firm level on aggregates during the Great Recession (Arellano, Bai, and Kehoe, 2016), or gauging the impact of declining real interest rates on input misallocation and aggregate productivity (Gopinath, Kalemli-Ozcan, Karababounis, and Villegas-Sanchez, 2015). To the best of our knowledge, our paper is the first to apply a similar set of tools to study the macroeconomic conse-

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[4]Hebert and Schreger (2016) cleverly exploit the rulings in the case *Republic of Argentina v. NML Capital* as exogenous variation of sovereign risk and document large negative effects of sovereign risk on Argentinian stock returns. Bahaj (2019) uses a narrative high-frequency approach to identify plausibly exogenous variation in sovereign risk.

quences of sovereign debt crises.

Our heterogeneous firm model builds on the literature of firm dynamics with financial frictions. Cooley and Quadrini (2001) develop a workhorse model of heterogeneous firms with incomplete financial markets and default risk. Kahn and Thomas (2013) focus on aggregate fluctuations in a model with heterogeneous firms facing financial frictions and financial shocks. In their work, shocks to the collateral constraint can generate long-lasting recessions. Buera and Moll (2015), Buera, Kaboski, and Shin (2011), Arellano, Bai, and Zhang (2012), and Midrigan and Xu (2014) also develop models with firm heterogeneity and financial frictions and compare the misallocation costs across economies with varying degrees of financial development. In contrast to these papers, we focus on the interaction between firms' financial frictions and sovereign default risk while simplifying the decision problem of firms. Our paper shares this emphasis with the recent work by de Ferra (2016) and Kaas, Mellert, and Scholl (2016).

Layout. The paper is organized as follows. We present the model in Section 2. Section 3 discusses the main mechanisms and our empirical strategy. Section 4 presents our data sources and the results of the firm-level regressions. In Section 5 we use the model to measure the macroeconomic effects of sovereign risk. Section 6 concludes.

2   Model

The economy is composed of a central government and J islands where a continuum of final goods firms, intermediate goods firms, financial intermediaries, and families interact.

The central government collects tax revenues from final goods firms and borrows from financial intermediaries to finance public goods and service outstanding debt. The government can default on its debt, and the rate at which it borrows reflects the risk of default.

Each island has two types of firms. Final goods firms are competitive, and they have a technology that converts intermediate goods into a final good. Intermediate goods firms operate under monopolistic competition, and they use capital and labor to produce differentiated goods. They borrow from financial intermediaries to finance a portion of their input costs, and they differ in their productivity and financing needs.

Families are composed of workers and bankers. They have preferences over consumption and labor, and they own intermediate goods firms. Families decide on labor for workers and investment, and they rent out their capital to firms. They can also deposit savings in financial intermediaries. Financial intermediaries are run by bankers who use net worth

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and the savings of families to lend to intermediate goods firms and the central government.

The economy is perturbed by two aggregate shocks. The first shock, p_t^d, is an aggregate shock to the firms' productivity. The second shock, v_t, affects the utility of the government in case of a default. The timing of events within the period is as follows. First, all aggregate and idiosyncratic shocks are realized and the government chooses whether to default and how much to borrow. Given shocks and government policies, all private decisions are made, and goods, labor, and credit markets clear.

We start with the description of the problem of the central government and the agents on each island. We then define the equilibrium for this economy and conclude the section with a discussion of the key simplifying assumptions.

2.1 The government

The central government decides the level of public goods G_t to provide to its citizens. It finances these expenditures by levying a constant tax rate τ on final goods firms and by issuing debt to financial intermediaries. The debt instrument is a perpetuity that specifies a price q_t and a quantity M_t such that the government receives $q_t M_t$ units of final goods in period t. The following period a fraction ϑ of outstanding debt matures. Let B_t be the stock of debt at the beginning of period t. Conditional on not defaulting, the government's debt in $t + 1$ is the sum of non-matured debt $(1 - \vartheta)B_t$ and the new issuance M_t, such that $B_{t+1} = (1 - \vartheta)B_t + M_t$.

The time t budget constraint, conditional on not defaulting, is

$$\vartheta B_t + G_t = q_t \left[B_{t+1} - (1 - \vartheta)B_t\right] + \tau \sum_j Y_t^j, \tag{1}$$

where Y_t^j are the aggregate final goods on island j.

Every period the government chooses G_t and B_{t+1} and decides whether to repay its outstanding debt, $D_t = 0$, or default, $D_t = 1$. A default eliminates the government's debt obligations, but it also induces a utility cost v_t, which follows a Markov process with transition probabilities $\pi^v(v_{t+1}, v_t)$.[5] When in default, the government can still issue new bonds. Its budget constraint is as in equation (1) with $B_t = 0$.

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[5]The shock v_t generates fluctuations in the value of default for the government, which the quantitative sovereign debt literature has found necessary to fit the data on government spreads (Arellano, 2008). In most of this literature, fluctuations in default values are generated by assuming that the cost of default depends on income. In our model, as in Aguiar and Amador (2013) and Muller, Storesletten, and Zilibotti (2018), such fluctuations are directly induced by v_t shocks.

The government's objective is to maximize the present discounted value of the utility derived from public goods net of any default costs,

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_g^t (u_g(G_t) - D_t v_t) \right].$$

2.2 The private sector

The private sector consists of J islands with firms, families, and financial intermediaries operating on each island. We assume that islands differ only in the exposure that their financial intermediaries have to government debt. We present the problems of agents in island j next.

Final goods firms. The final good Y_{jt} is produced from a fixed variety of intermediate goods $i \in [0, 1]$ via the technology

$$Y_{jt} \leq \left[\int (y_{ijt})^\eta \, di \right]^{\frac{1}{\eta}}, \tag{2}$$

where the elasticity of demand is $1/(1 - \eta) > 1$. Final goods firms also pay a proportional tax from their revenue with tax rate τ. They choose the intermediate goods $\{y_{ijt}\}$ to solve

$$\max_{\{y_{ijt}\}} (1 - \tau) Y_{jt} - \int p_{ijt} y_{ijt} \, di$$

subject to (2), where p_{ijt} is the price of good i relative to the price of the final good on island j. This problem yields that the demand y_{ijt} for good i is

$$y_{ijt} = \left(\frac{1 - \tau}{p_{ijt}} \right)^{\frac{1}{1-\eta}} Y_{jt}. \tag{3}$$

Intermediate goods firms. A measure of intermediate goods firms produce differentiated goods in this economy. Each firm i combines capital k_{ijt} and labor ℓ_{ijt} to produce output y_{ijt} using a constant returns to scale technology. Production is affected by productivity shocks z_{ijt}. The output produced by firm i on island j at time t is

$$y_{ijt} = z_{ijt} \ell_{ijt}^{1-\alpha} k_{ijt}^{\alpha}. \tag{4}$$

Firms' productivity z_{ijt} is affected by an aggregate and a firm specific component. We

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model the aggregate shock following the literature on "disaster risk" as in Gourio (2012): every period with probability p_t^d, a firm's productivity declines by μ. This probability is common across firms and is drawn from a distribution $\Pi^p(p^d)$. The process for firms' productivity is

$$\log(z_{ijt}) = \rho_z \log(z_{ijt-1}) - I_{ijt}\mu + \sigma_z \varepsilon_{ijt}. \tag{5}$$

The variable I_{ijt} follows a Bernoulli distribution with $\Pr(I_{ijt} = 1) = p_t^d$. The idiosyncratic shock is persistent with autocorrelation ρ_z and is subject to an innovation ε_{ijt}, which follows a standard normal random process.

At the beginning of the period, aggregate and idiosyncratic shocks are realized. Firms make input choices for capital k_{ijt} and labor ℓ_{ijt} to be used in production.[6] We assume that firms need to borrow a fraction of their input costs before production, and they borrow from financial intermediaries by issuing bonds b_{ijt}^f at interest rate R_{jt}. These working capital needs are firm-specific and time-invariant, and we denote them by λ_i. Accordingly, the financing requirement for firm i is

$$b_{ijt}^f = \lambda_i(r_{jt}^k k_{ijt} + w_{jt}\ell_{ijt}), \tag{6}$$

where r_{jt}^k is the rental rate for capital and w_{jt} is the wage rate on island j at period t.

At the end of the period, production takes place, firms decide on the price p_{ijt} for their product taking as given their demand schedule (3), and repay their debt $R_{jt}b_{ijt}^f$ and the remainder of their input costs. Firms' profits, which are rebated to families, are

$$\Pi_{ijt} = p_{ijt}y_{ijt} - (1 - \lambda_i)(r_{jt}^k k_{ijt} + w_{jt}\ell_{ijt}) - R_{jt}b_{ijt}^f. \tag{7}$$

Families. Each island has a representative family composed of an equal mass of workers and bankers. Each period, the family sends out a mass of workers to provide L_{jt} labor to firms. It also sends out bankers to run financial intermediaries for one period providing them with net worth N_{jt}. At the end of the period, workers and bankers return the proceeds of their operations to the family, which then decides how to allocate these resources. The family has preferences over consumption C_{jt} and labor L_{jt} and discounts the future at rate β,

$$U_j = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \left(C_{jt} - \chi\frac{L_{jt}^{1+\gamma}}{1+\gamma}\right)\right].$$

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[6]In an earlier version of the paper, we assumed that firms choose inputs before observing the idiosyncratic shock, as in Arellano, Bai, and Kehoe (2016). In that environment, the problem of the firm was dynamic, and it could generate firms' default in equilibrium. The current formulation is more tractable and gives very similar quantitative results.

Preferences over consumption are linear and decreasing and convex over labor, with $1/\gamma > 0$ being the Frisch elasticity of labor supply. As we will show below, the linearity of preferences over consumption greatly simplifies the characterization of the equilibrium because it fixes the risk-free rate and reduces the number of aggregate state variables.[7]

Families own capital K_{jt-1}, which depreciates at rate δ, and they rent it to intermediate goods firms at the rental rate r_{jt}^k. They can save by accumulating new capital and by saving in one-period deposits A_{jt} with financial intermediaries at the price q_{jt}^a. They receive the profits from the intermediate goods producers, Π_{jt}, the wages from the workers, $w_{jt}L_{jt}$, and the returns from the operations of the bankers, F_{jt}. As we discuss later, the payment from bankers includes the returns from the bonds issued by the firms and from the island's holdings of government debt B_{jt}.

The family also endows bankers with net worth N_{jt} that consists of a fraction ω_j of the value of government bonds that did not mature as well as a constant transfer $\bar{n}_j$,

$$N_{jt} = \bar{n}_j + \omega_j(1 - D_t)(1 - \vartheta)q_t B_{jt}. \tag{8}$$

The only parameters that differ across regions are $(\bar{n}_j, \omega_j)$.

The budget constraint of the representative family is

$$C_{jt} + K_{jt} - (1 - \delta)K_{jt-1} + q_{jt}^a A_{jt} + N_{jt} = w_{jt}L_{jt} + r_{jt}^k K_{jt} + A_{jt-1} + F_{jt} + \Pi_{jt}. \tag{9}$$

The optimality conditions for families imply that the deposit rate and the rental rate of capital are constant over time, $q_{jt}^a = \beta$ and $r_{jt}^k = 1 - \beta(1 - \delta)$. In contrast, the wage rate is time-varying and island-specific, and it equals the marginal disutility of labor,

$$w_{jt} = \chi L_{jt}^\gamma.$$

Financial intermediaries. A continuum of financial intermediaries in each island with mass $1/J$ use their net worth and the deposits of the family to purchase debt issued by the government and the firms. Financial intermediaries are competitive and take all prices as given. The beginning-of-the-period budget constraint for an intermediary is

$$q_t B_{jt+1} + \int b_{ijt}^f di \leq N_{jt} + q_{jt}^a A_{jt}. \tag{10}$$

Financial intermediaries face a standard financial constraint that limits their ability to

raise deposits, which implies that variations in their net worth potentially affect their ability to lend. The financial constraint we consider is given by the following *leverage constraint*:

$$q_{jt}^a A_{jt} \leq q_t B_{jt+1} + \theta \int b_{ijt}^f di, \tag{11}$$

which specifies that the amount of deposits that the intermediaries can borrow from households is bounded by the value of their collateral. We assume that government debt can be fully pledged, while intermediaries can pledge only a fraction θ of the firms' debt.[8] Combining the budget constraint (10) and leverage constraint (11) implies that the amount that a bank can lend to firms is bounded by a proportion $1/(1 - \theta)$ of their net worth,

$$\frac{N_{jt}}{1 - \theta} \geq \int b_{ijt}^f di. \tag{12}$$

At the end of the period, each financial intermediary receives the payment from firms and the government and pays back deposits. The end-of-the-period returns depend on whether or not the government defaults and they equal

$$F_{jt+1} = (1 - D_{t+1}) \left[\vartheta B_{jt+1} + q_{t+1}(1 - \vartheta) B_{jt+1} \right] + R_{jt} \int b_{ijt}^f di - A_{jt}. \tag{13}$$

Intermediaries distribute back to the family their end-of-the-period return. Their objective is to choose $\{A_{jt}, B_{jt+1}, b_{ijt}^f\}$ to maximize the expected return $\mathbb{E}_t[\beta F_{jt+1}]$ subject to (10) and (11).

The financial intermediaries' problem gives rise to the following pricing condition for firm loans:

$$R_{jt} = \frac{1 + \zeta_{jt}}{\beta}, \tag{14}$$

where ζ_{jt} is the Lagrange multiplier on constraint (11). Condition (14) implies that firms pay a premium ζ_{jt}/β over the risk-free rate on their loans when the leverage constraint of banks binds. This premium reflects a standard balance sheet mechanism. When the constraint binds, a reduction in net worth reduces the supply of credit by financial intermediaries. In equilibrium, the interest rate that firms pay must rise to clear the credit market.

The decision problem of financial intermediaries also gives rise to the following pricing

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[8]The assumption that government debt can be pledged fully captures the fact that these securities are effectively the best collateral for financial institutions, for example, in refinancing operations with the European Central Bank. This restriction can easily be relaxed by introducing a discount θ^g in equation (11).

condition for government securities:

$$q_t = \mathbb{E}_t \beta \left[(1 - D_{t+1}) \left(\vartheta + q_{t+1}(1 - \vartheta) \right) \right]. \tag{15}$$

The price of long-term government bonds compensates for default risk. In no default states, each unit of a discount bond pays the maturing fraction ϑ and the value of the non-maturing fraction $q_{t+1}(1 - \vartheta)$. The Lagrange multiplier does not appear in this pricing equation for government bonds because they are fully pledgeable.

The bond price q_t maps into the government interest rate spread, spr_t, through the standard yield to maturity formulation such that

$$q_t = \frac{\vartheta}{\vartheta + 1/\beta - 1 + \text{spr}_t}. \tag{16}$$

2.3 Equilibrium

We can now formally define a Markov equilibrium for this economy. We characterize the equilibrium conditions for the private sector, taking the government policies as given. We then describe the recursive problem of the government.

We first describe our state variables and switch to recursive notation. The linearity in preferences for private consumption implies that we do not need to record the distribution of capital and deposits across islands as aggregate state variables because the wealth of families does not matter for the choices of labor, capital, and deposits. Moreover, this linearity also gives a symmetric pricing condition for government bonds across all islands, seen in (15), which implies that intermediaries are indifferent about the amount they lend to the government. For simplicity, we assume that intermediaries lend equal amounts of government securities across islands such that $B'_j = B'$ and restrict the heterogeneity across islands to the exposure of government debt through the net worth of financial intermediaries in (8).[9] The aggregate state of the economy, hence, includes the aggregate shocks $\{v, p^d\}$, the distribution of firms across idiosyncratic productivity and borrowing needs, Λ, and the initial level of government debt B. We express the aggregate state by $\{S, B\}$, with $S = \{v, p^d, \Lambda\}$. Given the aggregate state, the government makes choices for default and borrowing, with decision rules given by $B' = H_B(S, B)$ and $D = H_D(S, B)$.

These public sector states and choices for default D and borrowing B' are relevant for the firms' and the family's choices of labor, capital, and deposits on each island only because

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[9]Alternative assumptions for islands' lending portfolios would not change our results. As we explain below, we target ω_j to the observed levels of government debt exposure; a different portfolio B_j would simply change the parameter ω_j.

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they affect the net worth of financial intermediaries N_j. It is therefore useful to define an island state X_j that includes the aggregate productivity shock, the distribution of firms, and the intermediaries' net worth $X_j = (p^d, \Lambda, N_j)$. These variables, along with the idiosyncratic states $\{z, \lambda\}$, are sufficient to determine the decisions of the firms and the family's choices of labor, capital, and deposits. The consumption of the family, however, depends on the state $\{S, B\}$ as well as on the government's default and borrowing choices $\{D, B'\}$.

We now formally define the private sector equilibrium.

Definition 1. *Given an aggregate state $\{S, B\}$, arbitrary government policies for default and borrowing $\{D, B'\}$, future government decision rules $H_B = B''(S', B')$ and $H_D = D'(S', B')$, and the associated island state $X_j = (p^d, \Lambda, N_j)$, the private equilibrium for island j consists of*

- *Intermediate goods firms' policies for labor $\ell(z, \lambda, X_j)$, capital $k(z, \lambda, X_j)$, and borrowing $b(z, \lambda, X_j)$, and final goods firms' output $Y(X_j)$,*
- *Policies for labor $L(X_j)$, capital $K(X_j)$, deposits $A(X_j)$, and consumption $C(S, B, D, B')$,*
- *Price functions for wages $w(X_j)$ and firm borrowing rates $R(X_j)$, and the constant capital rental rate r^k and deposit price q^a,*
- *The transition function for the distribution of firms $H_\Lambda(\Lambda(z, \lambda), p^{d\prime})$,*
- *The government bond price function $q(S, B')$,*

such that: (i) the policy functions of intermediate and final goods firms satisfy their optimization problem; (ii) the policies for families satisfy their optimality conditions; (iii) firm borrowing rates satisfy equation (14) and the leverage constraint (12) is satisfied; (iv) labor, capital, and firm bond markets clear, (v) the evolution of the distribution of firms is consistent with the equilibrium behavior of firms, (vi) the government bond price schedule satisfies equation (15), and (vii) net worth is $N_j = \bar{n}_j + \omega_j(1 - D)(1 - \vartheta)q(S, B')B$.

The labor market clearing conditions require that labor demanded by firms equals the labor supplied by families for each island j,

$$L(X_j) = \int_{\{z, \lambda\}} \ell(z, \lambda, X_j) d\Lambda(z, \lambda).$$

Likewise, the capital rental market clearing condition implies that the capital demanded by firms equal the capital rented by families,

$$K(X_j) = \int_{\{z, \lambda\}} k(z, \lambda, X_j) d\Lambda(z, \lambda).$$

Market clearing requires that the loans demanded by all firms equal all the funds supplied

by financial intermediaries with their leverage constraints satisfied,

$$\frac{N_j}{1-\theta} \geq \int_{\{z,\lambda\}} b^f(z,\lambda,X_j)d\Lambda(z,\lambda).$$

The productivity process follows equation (5), and it is independent from the invariant distribution of λ across firms, so that $\Lambda(z,\lambda) = \Lambda_z(z)\Lambda_\lambda(\lambda)$. Note that the evolution of the distribution of firms over $\{z,\lambda\}$ depends on the current distribution of firms, $\Lambda(z,\lambda)$, and the productivity shock $p^{d\prime}$ such that

$$\Lambda'_z(z') = H_\Lambda(\Lambda(z,\lambda), p^{d\prime}).$$

Next we characterize the equilibrium on each island. The following lemma derives the three conditions that determine island-level wages $w(X_j)$, output $Y(X_j)$, and firms' borrowing rates $R(X_j)$.

Lemma 1. *In the private equilibrium for island j, wages $w(X_j)$, output $Y(X_j)$, and firms' borrowing rates $R(X_j)$ satisfy the following conditions:*

$$\frac{N_j}{1-\theta} \geq M_n \left[Z(X_j)R_w(X_j)\right]^{\frac{(1-\eta)(1+\gamma)}{\eta(1-\alpha)\gamma}} \tag{17}$$

$$w(X_j) = M_w \left[Z(X_j)R_w(X_j)\right]^{\frac{(1-\eta)}{\eta(1-\alpha)}} \tag{18}$$

$$Y(X_j) = M_y \frac{\left[Z(X_j)R_w(X_j)\right]^{\frac{1-\eta+(1-\alpha\eta)\gamma}{\eta(1-\alpha)\gamma}}}{Z(X_j)R_y(X_j)}, \tag{19}$$

where $R(X_j) = 1/\beta$ if condition (17) is an inequality. $R_w(X_j)$ and $R_y(X_j)$ are weighted averages of firms' borrowing rates: $R_w(X_j) = \int_\lambda \left(1 + \lambda[R(X_j) - 1]\right)^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda$ and $R_y(X_j) = \int_\lambda \left(1 + \lambda[R(X_j) - 1]\right)^{-\frac{1}{1-\eta}} d\Lambda_\lambda$, $Z(X_j)$ is the average productivity: $Z(X_j) = \int_z z^{\frac{\eta}{1-\eta}} d\Lambda_z$, and the constants $\{M_n, M_w, M_y\}$ are functions of the model parameters.

The proof of this lemma is in Appendix B. Equation (17) can be interpreted as an equilibrium condition of the credit market: credit supply cannot exceed $N_j/(1-\theta)$ because of the leverage constraint, while credit demand is given by expression on the right hand side in (17). This condition determines the level of interest rates that financial intermediaries charge to firms. If at $R(X_j) = 1/\beta$ the inequality is satisfied, then we have that equilibrium interest rates equal $1/\beta$. On the other hand, if at those interest rates the demand for credit exceeds the maximal amount that can be supplied by financial intermediaries, then interest rates need to increase so that (17) holds with equality. Equations (18) and (19) determine

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the equilibrium level of wages and output given the equilibrium level of interest rates.

Lemma 1 also clarifies the mechanisms through which government policies affect the private sector. By affecting default risk and the price of its debt, the actions of the government have an impact on the net worth of financial intermediaries through equation (8). These changes in net worth affect equilibrium interest rates via equation (17), and changes in interest rates affect the demand for labor and output by the firms, affecting equilibrium wages and aggregate output. These effects are summarized in equations (18) and (19). Shortly, we will discuss these mechanisms in details.

Having defined a private sector equilibrium, we can now describe the recursive problem of the government. The government collects as tax revenues a fraction τ of the final goods output of each island $Y(X_j)$. Tax revenues are a function $T(S, B, D, B')$ that depends on the aggregate shocks, the distribution of firms, and the states and choices of the government because the aggregate output of each island depends on these variables. Tax revenues are $T(S, B, D, B') = \tau \sum_j Y(X_j(p^d, \Lambda, N_j))$, with $N_j = N_j(S, B, D, B')$ as specified in Definition 1.

The recursive problem of the government follows the quantitative sovereign default literature. Let $W(S, B)$ be the value of the option to default such that

$$W(S, B) = \max_{D=\{0,1\}} \{(1-D)V(S, B) + D\, [V(S, 0) - \nu]\},$$

where $V(S, B)$ is the value of repaying debt B and is given by

$$V(S, B) = \max_{B'} u_g(G) + \beta_g \mathbb{E}\, W(S', B'),$$

subject to the budget constraint

$$G + \vartheta B = T(S, B, D, B') + q(S, B') \left[B' - (1-\vartheta)B \right],$$

and the evolution of aggregate shocks and firm distributions. The value of default is $V(S, 0) - \nu$ because with default the debt B is written off and the government experiences the default cost ν.

Importantly, the government internalizes the feedback that its choices have on the private equilibrium. This feedback matters for the government because the private equilibrium determines current and future tax revenues $T(S, B, D, B')$ and bond prices $q(S, B')$. The government views tax revenues and bond prices as schedules that depend on borrowing B'. This problem gives decision rules for default $D(S, B)$, borrowing $B'(S, B)$, and public consumption $G(S, B)$.

We can now define the recursive equilibrium of this economy.

Definition 2. *The Markov recursive equilibrium consists of government policy functions for default $D(S, B)$, borrowing $B'(S, B)$, public consumption $G(S, B)$, and value functions $V(S, B)$ and $W(S, B)$ such that: (i) the policy and value functions for the government satisfy its optimization problem; (ii) the private equilibrium is satisfied; and (iii) the functions H_B and H_D are consistent with the government policies.*

2.4 Discussion

Before moving forward, we discuss some key elements of the model. As explained in the previous section, in our model the government affects the private sector only through its impact on the net worth of financial intermediaries. The literature has identified other channels through which public sector strains can be transmitted to the real economy, such as incentives for indebted governments to raise corporate taxes (Aguiar, Amador, and Gopinath, 2009) or, more generally, to interfere with the private sector (Arellano, Atkeson, and Wright, 2016). Our analysis is silent about the quantitative importance of these other mechanisms.

Our modeling of the financial sector borrows from a recent literature that introduced financial intermediation in otherwise standard business cycle models, such as Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). The key difference with these papers is that the bankers in our framework exit after one period whereas in these other models they can operate for more than one period. Technically, this implies that in our framework the evolution of net worth is governed entirely by the "transfer rule" in equation (8), whereas in these other papers, net worth also has an endogenous component. Our restriction is motivated mostly by tractability because the numerical solution of our model with one additional state variable per island, while potentially feasible, is substantially more involved. While such an addition will likely have an impact on our quantitative results, the main economic mechanisms emphasized here will be unchanged (see Bocola (2016)).

Firms' heterogeneity in our framework is introduced in two ways: first, by assuming that firms differ in their borrowing needs (λ), and second by assuming that they borrow from banks that potentially have different exposures to government debt (ω_j). This structure implies that the effects of sovereign risk on firms are heterogeneous across the population. As we will see shortly, these cross-sectional implications will be at the core of our measurement strategy. While we do not have a deep theory that explains these differences, our formulation is flexible enough to reproduce the observed degree of heterogeneity across firms and financial intermediaries in these dimensions.

Finally, note that the islands in our model are regions in which credit, goods, and labor markets are local. This assumption is clearly a stretch, but we think that the alternative

assumption in which these markets are national would be even more extreme. First, the majority of Italian firms in our dataset are small enterprises, and their predominant form of external finance is local banks. Second, most firms in our dataset operate in non-tradable sectors. Third, our analysis is conducted over a fairly short period of time, during which it is reasonable to assume that labor is not perfectly mobile across regions.

3 The propagation of sovereign risk

Having presented the model, we now analyze the mechanisms that govern the propagation of sovereign risk to the private sector, and we discuss our empirical strategy to discipline these mechanisms using firm-level data. Section 3.1 discusses the aggregate effects of an increase in sovereign interest rate spreads. A change in spreads has an impact on the output produced by firms because it affects their borrowing costs, which we label the *direct effect*, and because it affects the wages and the demand for their product, which we label the *indirect effect*. In Section 3.2, we provide two main results. First, we establish a mapping between these direct and indirect effects and the coefficients of a linear relation between firms' sales, sovereign spreads, and aggregate productivity. Second, we derive a relation between these coefficients and the response of aggregate output to a change in sovereign spreads. These results motivate our quantitative strategy, which consists of estimating the coefficients of this linear relation using Italian data, and we subsequently parameterize the model such that it reproduces these coefficients in simulated data.

3.1 Direct and indirect effects of sovereign risk

Taking as given prices and aggregate demand, firms in each region maximize their profit (7) subject to their demand schedule (3) and the financing requirement (6). We can express the optimal sales of a firm with idiosyncratic state (z, λ) operating in region j as

$$\log(py(z, \lambda, X_j)) = C_1 + \frac{\eta}{1-\eta} \log z - \frac{\eta}{1-\eta} \lambda R(X_j) + \log Y(X_j) - \frac{\eta(1-\alpha)}{1-\eta} \log w(X_j), \quad (20)$$

where equilibrium borrowing rates, wages, and demand are determined by the expressions in Lemma 1. Holding everything else constant, a firm facing higher borrowing interest rates $R(X_j)$ has lower sales, especially if the firm has a higher λ. Firms borrow a fraction λ of their input costs for working capital needs, and so higher borrowing rates translate into higher effective input costs for firms with high λ. Similarly, higher wages $w(X_j)$ and lower demand for the firm's output $Y(X_j)$ contribute to lower sales.

We can use equation (20) to explain the mechanisms through which sovereign risk affects firms. As explained in Lemma 1, changes in interest rate spreads on government debt affect firms only by their impact on the net worth of financial intermediaries. Moreover, the marginal effect of net worth with respect to a given change in spreads is the same regardless of whether the spread change is driven by aggregate shocks or borrowing choices. Therefore, we can directly consider a marginal response of firms sales, equation (20), to a change in spr as follows:

$$\frac{\partial \log(py)}{\partial spr} = \underbrace{-\frac{\eta}{1-\eta}\lambda\left(\frac{\partial R(X_j)}{\partial N_j}\frac{\partial N_j}{\partial spr}\right)}_{\text{direct effect}} + \underbrace{\left(\frac{\partial \log Y(X_j) - \frac{\eta(1-\alpha)}{1-\eta}\partial \log w(X_j)}{\partial N_j}\right)\frac{\partial N_j}{\partial spr}}_{\text{indirect effects}}. \quad (21)$$

The effect of a change in sovereign spreads on firms' sales can be decomposed into a direct effect on firms' borrowing rates and indirect effects that operate through aggregate demand and wages.

The direct effect arises because financial intermediaries hold legacy government debt and face a potentially binding leverage constraint. When sovereign risk increases, the value of government bonds declines, as does financial intermediaries' net worth, which is related to the value of government debt according to equation (8). If the leverage constraint binds, the decline in net worth translates into a decline in credit supply, which in equilibrium is met by an increase in firms' borrowing rates. This direct effect is heterogeneous across islands and firms because islands differ in ω—the degree of balance sheet exposure to government debt, and firms differ in λ—their borrowing needs.

Along with this direct effect, our model features two additional channels through which sovereign risk affects the behavior of firms that operate through the equilibrium change in demand and wages, shown in Lemma 1. Consider these additional general equilibrium forces that arise, for example, from an increase in sovereign spreads that lead to an increase in firms' borrowing rates. High borrowing rates depress the production of firms, as does the demand for all the other intermediate goods on the island because of complementarities in the production of aggregate output. This aggregate demand channel further depresses firms' production. High borrowing rates also depress the demand for labor by firms, which leads to a decline in wages. Lower wages reduce the marginal cost of production for firms, which on the margin boost their production. Thus, the overall indirect effect could be expansionary or recessionary, depending on which of these two channels dominates.

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3.2 Measuring the propagation of sovereign risk

We now show that these direct and indirect effects map into the coefficients of a linear relation that can be recovered using firm-level data. For the following results, we work with first order approximations around a state in which leverage constraints bind and consider responses to shocks that are small enough such that no default occurs. Appendix B contains all the proofs.

Proposition 1. *To first order, the response of the sales of firm i operating in region j to idiosyncratic shock z_{ijt} and aggregate shocks p_t^d and v_t is a linear function of the sovereign spread spr_t and the productivity shocks $\{z_{ijt}, p_t^d\}$*

$$\log py_{ijt} = \delta_i + \beta_{s,j} spr_t + \gamma_{s,j}(\lambda_i \times spr_t) + \beta_{p,j} p_t^d + \gamma_{p,j}(\lambda_i \times p_t^d) + \frac{\eta}{1-\eta}\log(z_{ijt}), \quad (22)$$

with the coefficients given by

$$\beta_{s,j} = -\frac{\partial \log Y(X_j^0) - \left[\frac{\eta(1-\alpha)}{1-\eta}\right]\partial \log w(X_j^0)}{\partial N}M_q\omega_j \quad \gamma_{s,j} = \frac{\eta}{1-\eta}\frac{\partial R(X_j^0)}{\partial N}M_q\omega_j$$

$$\beta_{p,j} = \frac{\partial \log Y(X_j^0) - \left[\frac{\eta(1-\alpha)}{1-\eta}\right]\partial \log w(X_j^0)}{\partial p^d} \quad \gamma_{p,j} = -\frac{\eta}{1-\eta}\frac{\partial R(X_j^0)}{\partial p^d}, \quad (23)$$

where we used the derivative $\frac{\partial N_j}{\partial spr} = -M_q\omega_j$ and $M_q = \frac{B_0\vartheta}{(\vartheta+1/\beta-1+spr_0)^2}$.

Equation (22) recovers the response of the sales of firm i of type (λ, z) to idiosyncratic and aggregate shocks. The response is a function of the government spread because in our model spreads are endogenous to aggregate shocks $\{p_t^d, v_t\}$ and government borrowing choices B', which themselves depend on aggregate shocks and the initial state. Moreover, the micro elasticities with respect to spreads are the same regardless of the source of the spread fluctuations. A comparison between the expressions in (23) and equation (21) reveals that $\beta_{s,j}$ and $\gamma_{s,j}\lambda_i$ capture, respectively, the indirect and direct effects of sovereign risk defined in the earlier section. This relation is the basis of our quantitative strategy, which consists of estimating an empirical version of equation (22) using micro data and subsequently using these estimated coefficients as empirical targets when parameterizing the model.

Why is it important that our model reproduces these firms' responses accurately? It is important because, as the next result shows, these moments are closely related to the response of aggregate output to sovereign risk.

Corollary 1. *To first order, the marginal response of aggregate output to a change in sovereign spreads is a weighted average of the firm responses*

$$\frac{\partial \log Y_t}{\partial spr_t} = \sum_j \pi_j \left[\beta_{s,j} + \gamma_{s,j} m_j \right],$$ (24)

where $m_j = \dfrac{\int_\lambda \lambda \left[1+\lambda(R(X_j^0)-1) \right]^{-\frac{1}{1-\eta}} d\Lambda_\lambda}{\int_\lambda \left[1+\lambda(R(X_j^0)-1) \right]^{-\frac{\eta}{1-\eta}} d\Lambda_\lambda}$ *and* $\pi_j = Y_j/Y_0$.

This result asserts that by ensuring that the model reproduces the firm-level elasticities summarized by $\gamma_{s,j}$ and $\beta_{s,j}$, we are disciplining empirically the response of aggregate output to a change in sovereign risk.

The next result further characterizes the expressions in (23) and derives some empirical predictions for the micro elasticities.

Proposition 2. *The direct effect on firms' sales from increases in sovereign spreads is negative, $\gamma_{s,j} \leq 0 \ \forall j$. Across two islands with $N_1^0 = N_2^0$, the direct effect is more negative in islands with higher exposure to government debt, $\gamma_{s,1} \geq \gamma_{s,2}$ if $\omega_1 \leq \omega_2$.*

Our theory predicts that an increase in sovereign risk is associated with a larger decline in sales for firms that borrow more. Our theory also predicts that such differential effect is stronger in islands where financial intermediaries have a higher ratio of government bond holdings to net worth: in those regions, a given increase in sovereign spreads implies a deeper decline in net worth for financial intermediaries, and so a larger increase in interest rates charged to firms. The sign of the indirect effects in our model, however, is ambiguous but it is stronger for islands with higher exposure to government debt, $|\beta_{s,1}| \leq |\beta_{s,2}|$ if $\omega_1 \leq \omega_2$. As seen in the appendix, the magnitudes of both sets of coefficients, $\gamma_{s,j}$ and $\beta_{s,j}$ across j, provide information of the parameters of the model. Below we use firm and bank data to test the model's empirical predictions and use the recovered coefficients for direct and indirect effects in Italy to parameterize the model.

3.3 Discussion

Before moving forward, we further discuss the role of micro data in our approach. First, it is useful to point out which feature of the data informs the direct and indirect effects of sovereign risk in our approach. The indirect effect is captured by $\beta_{s,j}$, which in equation (22) represents the response of firms that do not borrow ($\lambda_i = 0$) to an increase in sovereign spreads, controlling for idiosyncratic and aggregate productivity. The reason

why the behavior of "zero-leverage" firms is informative about the indirect effects is intuitive: these firms are not affected by fluctuations in borrowing rates, so any change in their performance must come from the general equilibrium effects that work through wages and aggregate demand. The direct effect is, instead, identified by exploiting cross-sectional variation in λ_i. If, conditional on an increase in spreads, we see sales dropping substantially more for firms with high borrowing relative to firms with low borrowing, we will infer from the data a more negative $\gamma_{s,j}$.

We can also use our model to contrast our procedure to the more standard approach in the literature that uses only aggregate data. Integrating both sides of equation (22) across firms and regions, one is left with an equation linking aggregate output, sovereign spreads, and productivity p_t^d,

$$Y_t = a_0 + a_1 spr_t + a_2 p_t^d.$$

Thus, rather than using micro data as we do in our paper, one could in principle estimate the above expression using only aggregate data, and use these coefficients as empirical targets in the estimation of the model parameters.

We believe, however, that our approach is superior in at least two dimensions. First, the mechanism studied in this paper has a number of cross-sectional predictions that cannot be verified using aggregate data exclusively. As we have seen in Proposition 2, the sovereign risk channel studied in this paper predicts that $\gamma_{s,j} < 0$, more so in regions where banks are highly exposed to government debt. In our approach, with firm- and bank-level data, we can verify whether these predictions hold, and impose more empirical discipline on measurement. Second, the heterogeneity in behavior across firms provides information on the different mechanisms through which sovereign risk affects the real economy. As we explained earlier, we can measure the direct effect via cross-sectional comparisons and the indirect effects by looking at the behavior of zero-leverage firms. This is something one cannot do when using aggregate data exclusively.

4 Empirical analysis

This section uses Italian data to test the empirical predictions of our model and provide estimates for the micro elasticities in equation (22). The results will be used in the subsequent sections to estimate the parameters of the structural model. Section 4.1 describes the data and Section 4.2 reports the main empirical results.

4.1 Data

We merge three main datasets for our analysis. First, we obtain yearly firm-level data from the ORBIS-AMADEUS dataset. The dataset covers the period 2007-2015 and provides detailed information on publicly and privately held Italian firms. The core variables used in our analysis are indicators of a firm's performance (operating revenues, operating profits, and number of employees), key balance sheet indicators (total assets and short-term loans), and additional firm-level information regarding the location of a firm's headquarters and the sector in which the firm operates. We define firm leverage as the ratio of short-term loans to total assets. We perform standard steps to guarantee the quality of the data, we scale all nominal variables by the consumer price index, and we eliminate outliers by dropping the 1st and 99th percentile of all variables used in the analysis. We further restrict the sample by considering a balanced panel of firms operating continuously between 2007 and 2015, and by excluding firms that operate in the financial industry or in sectors with a strong government presence.[10] In Appendix A, we provide details on our sample selection and variables. Table 1 reports a set of summary statistics for the main variables used in the analysis for the year 2007. The median firm in our sample is privately held, has seven employees, has operating revenues of roughly one million euros, and has little debt, with average leverage equal to 3%.

Table 1: Summary statistics for the firm panel

	Mean	Standard deviation	P25	P50	P75
Number of employees	16	54	3	7	16
Operating revenues	2320	3830	355	912	2443
Total assets	2326	3706	397	994	2528
Short term debt	2952	8359	0.00	20	206
Leverage	0.11	0.14	0.00	0.03	0.17

Note: The data are from a panel of 224,359 firms for the year 2007. Monetary values are reported in thousands of euros and deflated using the consumer price index (2010 base year). See Appendix A for a definition of the variables.

The second dataset is Bankscope, from which we obtain balance sheet information for banks headquartered in Italy.[11] The main variables in our analysis are total assets, total equity, banks' holdings of government debt, and the ZIP code of the banks' headquarters. Unfortunately, Bankscope does not provide a breakdown of government bond holdings

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[10] We exclude firms that operate in public administration and defense (NACE 84), education (NACE 85), and health care (NACE 86-88). Our results are robust to including these firms in the sample.

[11] This sample is highly representative for the whole Italian banking sector. Total assets for the banks in our sample were 2,985 billion euros at the end of 2007. The corresponding statistic for *all* monetary and financial institutions (banks and money market funds) in Italy was 3,289 billion euros at the end of 2007.

by nationality, which means that in principle a large value for this indicator might reflect exposure to foreign governments rather than the Italian one. However, Gennaioli, Martin, and Rossi (2018) document that this is a minor concern, as the Bankscope indicator captures mainly banks' exposure to domestic government debt. This reflects the high degree of home bias in international financial portfolios.[12]

The third dataset consists of Bank of Italy reports, which collects information on the distribution of bank branches across Italian regions as of December 31, 2007. The 20 regions in Italy are the first-level administrative divisions.

Before turning to test the empirical predictions of our model and estimating the micro elasticities in equation (22), we perform two preliminary steps that are necessary to obtain some of the covariates for our analysis. First, we estimate a firm-specific productivity process $z_{i,t}$ and use it to obtain the time path for p_t^d. Second, we use the Bankscope data together with the data on the geographical distribution of bank branches to construct an indicator of banks' exposure to government debt at the regional level.

Production function estimation. To estimate firm productivity, we use the two-step generalized method of moments implementation of Levinsohn and Petrin (2003) developed in Wooldridge (2009). As is standard, we allow the elasticities of value added with respect to inputs to vary at the two-digit industry level and consider a sample of firms that operate in manufacturing (NACE codes 10-33). Specifically, for each industry n, we have the following equation:

$$\log(y_{it}) = \alpha + \beta_t(n) + \beta_1(n)\log(\ell_{it}) + \beta_2(n)\log(k_{it}) + \epsilon_{it}, \tag{25}$$

where y_{it} is the value added for firm i at time t, $\ell_{i,t}$ is its cost of labor inputs, and k_{it} is capital. In the above equation, $\beta_1(n)$ and $\beta_2(n)$ are sector-specific factor shares, and $\beta_t(n)$ is a sector-specific time effect. The level n is defined at the two-digit NACE level. Given the estimates for the coefficients in equation (25), we can compute for each firm the implied (log) of revenue total factor productivity,

$$\text{TFPR}_{it} = \log(y_{it}) - [\alpha + \beta_t(n) + \beta_1(n)\log(\ell_{it}) + \beta_2(n)\log(k_{it})].$$

[12]See also Kalemli-Ozcan, Laeven, and Moreno (2018). They have access to proprietary data from the European Central Bank on the holdings of domestic government bonds by banks. They compare this indicator to the one constructed using Bankscope and find minimal differences in their analysis (see their Table 5).

We can then use the estimated TFPR$_{it}$ to retrieve the time path for the aggregate shock p_t^d.[13] From the productivity process in equation (5) and the law of large numbers, we have that

$$p_t^d = -\max\left\{\frac{\bar{z}_t - \rho_z \bar{z}_{t-1}}{\mu}, 0\right\}, \tag{26}$$

where $\bar{z}_t$ is the cross-sectional average of the log of the firm's productivity at date t. Because of the short dimension of our panel, we do not directly estimate ρ_z but set it to 0.9, in line with the results in Foster, Haltiwanger, and Syverson (2008), which use a longer panel of U.S. firms. Moreover, we normalize μ to 0.3, which corresponds to the 5th percentile of the panel data for $\log(z_{it})$. Given ρ_z and μ, we can compute p_t^d using equation (26).

Figure 1: The cross section of firms' productivity in Italy, 2007-2015

(a) Firms' productivity percentiles   (b) Aggregate productivity shock
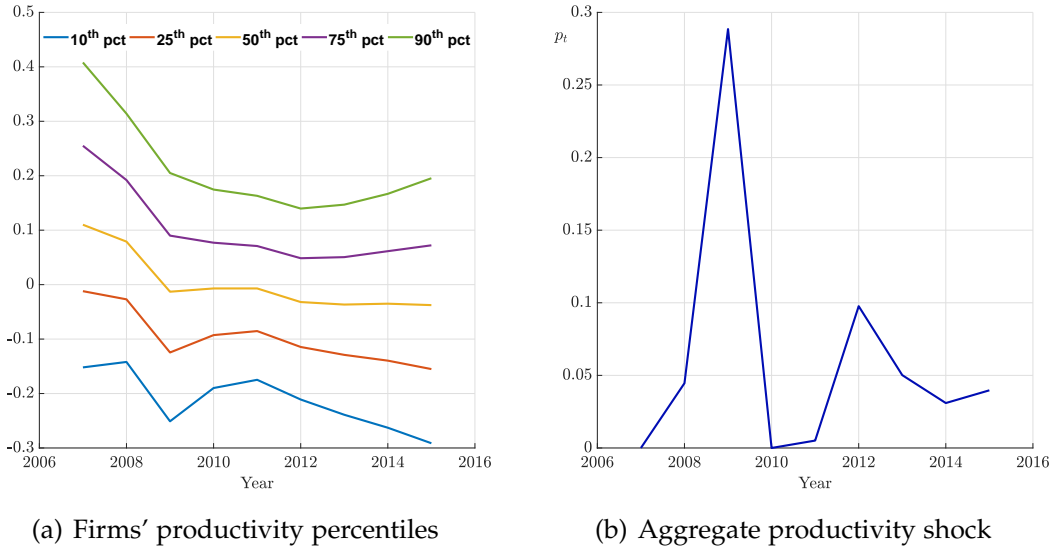
The plots in Figure 1 illustrate the behavior of firms' productivity in our sample. Panel (a) reports percentiles of the cross-sectional distribution of $z_{i,t}$ for each year t. We can see that average productivity fell sharply in 2008-2009 and recovered little after that.[14] Panel (b) in the figure plots the p_t^d process that we recover from the data. Consistent with

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[13]In our model, TFPR$_{it}$ is not equal to physical productivity. The relation between the two is given by

$$\log(z_{i,t}) = \frac{1}{\eta} TFPR_{it} - \frac{(1-\eta)}{\eta} \log(Y_t).$$

So, conditional on η and regional aggregate output Y_t, we could use the above expression to correct for the discrepancy between TFPR$_{it}$ and z_{it}. In practice, we have verified that this correction does not affect our results very much given the value of η that we will adopt in the quantitative analysis.

[14]In our estimates, the average decline in TFP between 2008 and 2009 is on the order of 10%. This is consistent with OECD data (Productivity and ULC by main economic activity) showing a decline in value added per hours worked of 7.1% in manufacturing during the same period.

the distributional plot, p_t^d displays a sharp increase in 2008-2009 and a somewhat smaller increase in 2012. To place these data into context, the dynamics of p_t^d closely mirror those of aggregate real GDP growth, which display a "double-dip" recession over the sample.

Banks' holdings of government debt by region. We now construct an indicator of banks' exposure to government debt at the regional level. A limitation of the Bankscope dataset is that it does not contain bank-level information on the geographical distribution of their operations. We overcome this issue by using the location of a bank's headquarters and the geographical distribution of its branches as a proxy for the size of a bank's operations in a given region.

Specifically, we group the banks in the sample into two categories: *local* and *national* banks. National banks are the five largest banks in our dataset by total assets in 2007, and their operations are distributed throughout the national territory.[15] Local banks are smaller, and we assume that they operate exclusively in the region in which their headquarters are located.

The indicator is constructed in two steps. In the first step, we compute the ratio of the government's bond holdings to the bank's equity for the five national banks,

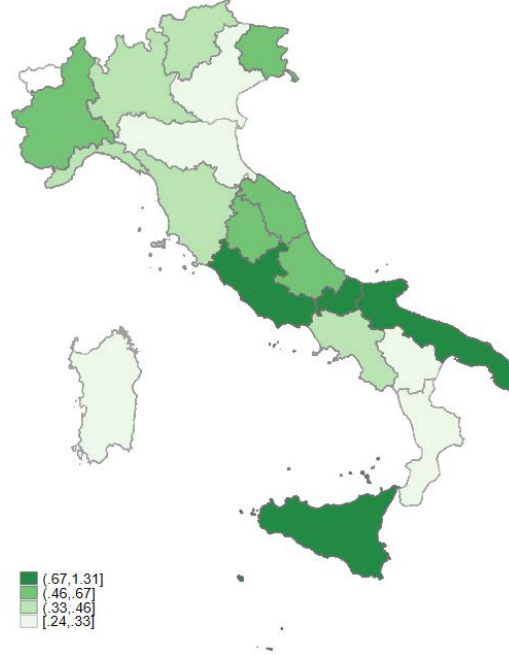$$\text{exposure}_i^{\text{nat}} = \frac{B_i}{E_i},$$

where B_i is the book value of government debt held by national banks i at the end of 2007, and E_i is the book value of the bank's equity in the same year. This indicator measures the size of these financial positions as a fraction of a bank's capital: a large value for the indicator means that the bank is particularly exposed to domestic sovereign risk. We also construct this indicator for the local banks operating in region j,

$$\text{exposure}_j^{\text{loc}} = \frac{\sum_i B_{i,j}}{\sum_i E_{i,j}}.$$

The second step consists of constructing region-specific weights for these two variables. For this purpose, we use the information on the geographical location of branches provided by the Bank of Italy. Let M_j^{br} be the total number of bank branches in region j at the end of 2007, and let $M_{i,j}^{\text{br}}$ be the number of branches for national bank i in that region. We construct the variable $\alpha_{i,j}^{\text{nat}} = \frac{M_{i,j}^{\text{br}}}{M_j^{\text{br}}}$, which denotes the proportion of bank branches in a given region j

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[15]The *national* banks are Unicredit, Intesa-Sanpaolo, Monte dei Paschi di Siena, Banca Nazionale del Lavoro, and Banco Popolare.

Figure 2: Banks' exposure to government debt by region in 2007

that belong to national bank i.

Given these definitions, we define the indicator of exposure for region j as

$$\text{exposure}_j = \left(1 - \sum_i \alpha_{i,j}^{\text{nat}}\right) \text{exposure}_j^{\text{loc}} + \sum_i \alpha_{i,j}^{\text{nat}} \text{exposure}_i^{\text{nat}}. \tag{27}$$

A high value of exposure_j indicates that in 2007, banks operating in that region had large quantities of debt issued by the government in their portfolio as a fraction of their capital. This can happen for two reasons. First, it could be that local banks in region j were highly exposed to government debt. Second, it could be that a national bank that was particularly exposed to government debt also had a strong presence in region j, as proxied by the number of its bank branches.

Figure 2 is a map of the exposure measure for the 20 Italian regions. Light colors indicate regions where banks have relatively low values for this indicator, while dark colors denote regions characterized by higher values for exposure_j. We can verify that there is substantial variation across regions. In Calabria, the region where banks are the least exposed to government debt, banks' holdings of government debt are equivalent to 24% of their total equity, while in Molise, the region where banks are most exposed, this number equals 131%.

We partition these regions into two groups for the empirical and quantitative analysis.

The *high-exposure* regions are those with exposure$_j$ above the median, while *low-exposure* regions are those with a value of exposure$_j$ below the median. The high-exposure regions consist of two from Northern Italy (Piemonte and Friuli Venezia Giulia), two from the South (Puglia and Sicilia), and five from Central Italy (Molise, Lazio, Abruzzo, Marche, and Umbria).

In the model, we have assumed that regions differ in their exposure to government debt but share the same distribution of firm productivity and leverage. We now show that this assumption is a good description of Italian data. Table 2 reports the firms' summary statistics in 2007 for the high- and low-exposure regions. We can verify that firms in the two regions are very similar: they have comparable numbers of employees and operating revenues. Low-exposure regions have slightly more total assets and leverage, most likely reflecting the fact that these regions are also more manufacturing-intensive, with 26% of firms operating in manufacturing relative to 20% in the high-exposure regions.

Table 2: Distribution of firms in high- and low-exposure regions

	High exposure			Low exposure		
	P25	P50	P75	P25	P50	P75
Number of employees	3	7	15	3	8	17
Operating revenues	338	846	2210	373	980	2643
Total assets	372	926	2335	430	1066	2688
Short-term debt	0.00	13	173	0.00	27	233
Leverage	0.00	0.02	0.15	0.00	0.03	0.19

Note: The data are from a panel of 224,359 firms for the year 2007. Monetary values are reported in thousands of euros and deflated using the consumer price index (2010 base year). See Appendix A for a definition of the variables.

Figure 3 reports the time series of real income per capita averaged across regions for the two groups of regions during the 1995-2016 period. In both panels, the solid vertical line denotes 2007, the year in which we partition the regions. We can see from panel (b) of the figure that the two groups have remarkably similar dynamics prior to 2007, as the ratio of income per capita is stable at 0.935 throughout the 1995-2007 period. During this period government spreads for Italian government bonds were close to zero. After 2007, and in conjunction with the sovereign debt crisis, we can see that regions where banks are more exposed to government debt suffer a much deeper decline in income per capita. This pattern is consistent with the mechanism emphasized in this paper and also important for the difference-in-differences methodology employed in the next subsection. The similar trends in output across the two regions prior to the debt crisis and the diverging performance during the crisis are in line with the parallel trend assumption embedded in

the methodology.

Figure 3: Income per capita in high and low exposure regions

(a) Income per capita

(b) Ratio of high exposure to low exposure
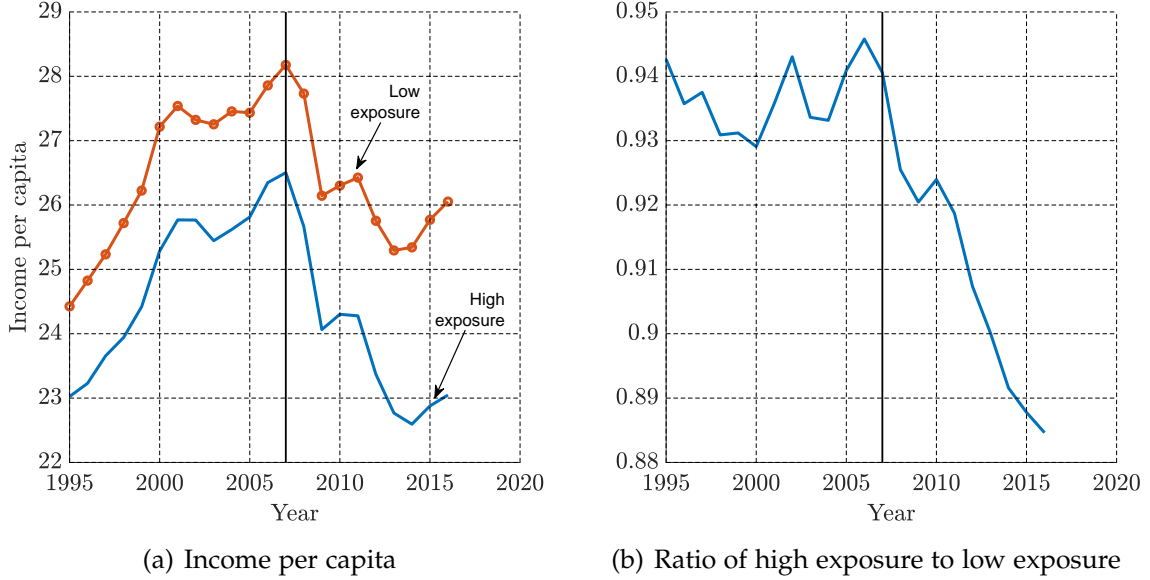
Note: The line with circles in panel (a) reports the average income per capita in the regions categorized as "low-exposure." The solid line reports the same object for "high-exposure" regions. Income per capita is expressed in thousands of euros at 2009 prices. The data are from the Istat Regional Accounts. Panel (b) reports the ratio between the solid line and the line with circles in panel (a).

4.2 Firm-level regressions

We now turn to estimating firm level regressions to test the empirical predictions of our model and measure the micro elasticities in equation (22). The two aggregate time series we use are taken directly from the data. Sovereign spreads, spr_t, are the difference in 10-year yields between Italian bonds relative to German bonds. The aggregate productivity series p_t^d is the one constructed in the previous section. For regional exposure, we use a dummy variable $\exp_i$ which equals 1 if firm i is headquartered in a high-exposure region and zero otherwise. Firm sales y_{it} equals the firm's operating revenue. The last variable we need to measure is λ_i—the fraction of input costs that must be paid in advance by firm i. In our model, there is a mapping between this indicator and the leverage of the firm, that is, the fraction of short-term debt over total assets, as the latter equals $\lambda_i r^k / \alpha$. We exploit this mapping and proxy for λ_i using a firm's leverage. Specifically, we construct a dummy variable lev_i that takes the value of 1 if firm i was in the upper 75th percentile of the leverage distribution and zero otherwise. To minimize concerns about the endogeneity of

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leverage, we sort firms according to the leverage distribution using only 2007 data, the first year in our panel. In our sample, firms with $\text{lev}_i = 1$ had, on average, a leverage of 0.241 in 2007, while firms with $\text{lev}_i = 0$ were borrowing very little, with an average leverage ratio of 0.038.

The benchmark equation that we estimate is

$$
\begin{aligned}
\Delta y_{it} \;=\; & \delta_i + \alpha_1 \text{spr}_t + \alpha_2(\text{lev}_i \times \text{spr}_t) + \alpha_3(\exp_i \times \text{spr}_t) + \alpha_4(\text{lev}_i \times \exp_i \times \text{spr}_t) \\
& + \; \alpha_5 p_t^d + \alpha_6(\text{lev}_i \times p_t^d) + \alpha_7(\exp_i \times p_t^d) + \alpha_8(\text{lev}_i \times \exp_i \times p_t^d) + \varepsilon_{it},
\end{aligned}
\tag{28}
$$

where Δy_{it} is the growth rate of sales for firm i between time t and $t-1$, δ_i are the firm's fixed effects, spr_t are interest rate differentials between Italian and German 5-year government bonds, and p_t^d is aggregate productivity.

There are a number of differences between equation (22) and equation (28). First, in equation (28) we consider the growth rate of sales rather than its level. We do so to deal with potential trends in the data. The second difference is that we do not control for idiosyncratic productivity in equation (28), and we absorb it in the residual ε_{it}. We do so because the productivity process is estimated only for firms in manufacturing, while equation (28) is estimated for all firms in our panel. Third, in equation (28) we use a dummy for leverage, while in the model λ_i is potentially continuous. In the numerical solution of the model, however, we restrict λ_i to take only two values, so the dummy variable approach is without loss of generality. Finally, note that equation (22) is only an approximation to time path of firms' sales and does not hold exactly in our model: the true relationship is nonlinear and also depends on the dynamics of the other state variables, B_t and Λ_t. To deal with this issue, we will estimate equation (28) on simulated data from our model when estimating the structural parameters via indirect inference.

Despite these differences, the coefficients in equation (28) can easily be mapped into the indirect and direct effects described in Section 3. The indirect effects in the two regions are captured by α_1 and $\alpha_1 + \alpha_3$. The coefficient α_1 in equation (28) approximates the response of zero-leverage firms conditional on an increase in sovereign spreads in low-exposure regions because firms with $\text{lev}_i = 0$ are essentially not borrowing in our sample. The sum of the coefficients $\alpha_1 + \alpha_3$ approximates this response for the high-exposure regions. The direct effects, instead, are captured by the interactions between the leverage dummy and sovereign spreads: in the low-exposure regions, that would be equal to α_2, while in the high-exposure regions, the direct effect is represented by $\alpha_2 + \alpha_4$.

Table 3 reports the results of the estimation of equation (28). Standard errors in every

Table 3: Firms sales on sovereign risk, productivity, leverage, and banks' exposure

	(1)	(2)	(3)	(4)	(5)
spr_t	-1.56	-1.49	-1.35		-0.90
	(-4.03)	(-4.06)	(-3.82)		(-4.13)
$\text{spr}_t \times \text{lev}_i$		-0.28	-0.26	-0.24	-0.17
		(-2.53)	(-2.34)	(-2.26)	(-2.19)
$\text{spr}_t \times \text{exp}_i$			-0.44		-0.34
			(-2.44)		(-1.82)
$\text{spr}_t \times \text{lev}_i \times \text{exp}_i$			-0.13	-0.07	-0.13
			(-9.27)	(-2.58)	(-3.43)
p_t^d	-0.42	-0.38	-0.43		-0.37
	(-12.98)	(-12.52)	(-14.82)		(-17.70)
$p_t^d \times \text{lev}_i$		-0.16	-0.16	-0.04	-0.17
		(-10.45)	(-10.77)	(-4.99)	(-13.44)
$p_t^d \times \text{exp}_i$			0.14		0.13
			(10.78)		(10.23)
$p_t^d \times \text{lev}_i \times \text{exp}_i$			0.03	0.01	0.03
			(3.70)	(1.28)	(3.28)
Firm fixed effects	yes	yes	yes	yes	yes
Time/region/sector fixed effects	no	no	no	yes	no
Time-varying firms' controls	no	no	no	no	yes
R^2	0.10	0.10	0.10	0.11	0.13
Number of observations	1941426	1941426	1937619	1936935	1722328

Note: This table reports the estimation of equation (28) and robustness specifications. Standard errors are clustered two ways, across firm and time. t-statistics are reported in the parentheses. See Appendix A for data sources and variable definitions.

specification are clustered two ways, across firm and time. In column (1) we report a version of equation (28) that includes only sovereign spreads spr_t and the productivity series p_t^d as covariates. The table shows that both variables are negatively related to firms' sales. An increase of 100 basis points in interest rate spreads is associated with a decline of of 1.56% in firms' sales growth, while a 1% increase in p_t^d is associated with a decline of 0.42%. Both coefficients are statistically different from zero at the 99% level.

In column (2) we introduce the interactions between these two variables and our leverage dummy. Both interactions are negative and statistically significant at the 95% level. On average, firms belonging to the low-leverage group experience a decline in sales growth of 1.49% after a 100 basis point increase in spreads, while for high-leverage firms, sales

growth falls by 1.77% (1.49+0.28). This result is consistent with the idea that sovereign risk has recessionary effects because of its impact on the financial sector, as firms that borrow are more exposed to sovereign risk relative to firms that do not borrow.

Column (3) of Table 3 reports the estimation of equation (28), our benchmark specification. To summarize the results, let's first consider firms that are headquartered in regions where banks have a low exposure to government debt. In those regions, a 100 basis point increase in spreads is associated with a 1.35% and 1.61% (1.35+0.26) decline in sales growth for the low- and high-leverage groups, respectively. Now consider the regions in which banks have high exposure to government debt. These elasticities become −1.79% (1.35+0.44) and −2.18% (1.35+0.26+0.44+0.13), respectively. These results are in accordance with the empirical predictions in our model from Proposition 2. In the data, as predicted in the model, the sales for high-leverage firms decline more than for low-leverage firms when sovereign spreads increase, and this differential response is stronger in regions where banks are more exposed to government debt. These results are also consistent with the empirical implication of our model that the indirect effects of sovereign risk, captured here by the effects of low-leverage firms, are stronger in regions with higher exposure to government debt.

The remaining specifications in Table 3 report sensitivity analysis of the results. Column (4) introduces interactions of time, region, and two-digit sector fixed effects. These fixed effects absorb all the shocks common to firms in each region and sector, such as changes in the demand for their goods and labor, and in credit conditions including those arising from changes in sovereign risk and productivity. Although we cannot identify in this specification the indirect effects in both regions, we confirm that the benchmark results for the direct effects are qualitatively and quantitatively robust. Column (5) introduces in the regression some time-varying firm controls: lagged leverage, lagged firms' size and lagged profitability. Again, we observe little variation in the coefficients when comparing columns (3) and (5).

The empirical specification of this panel regression is guided by our model, and hence through the lens of our theory, it recovers the response of firms' output to sovereign risk. In our model, however, firms' leverage and banks' exposure to government debt are exogenously given. If we were to relax these assumptions, the difference-in-differences methodology employed here might not recover the micro elasticities of interest because of endogeneity. We discuss next some of these concerns and our approach to handling them.

One potential problem arises because time-varying determinants of firms' leverage and banks' exposure to government debt might be systematically related to the error term in equation (28). For example, a negative idiosyncratic productivity shock might reduce firms'

output and at the same time reduce its incentive to borrow. To address these concerns, in our baseline specification we use measures of firm leverage and regional banks' exposure to government debt using dummy variables, and classify firms and regions into groups based on the ex ante distributions of these variables in 2007, before the debt crisis.

A related concern is the possibility that even these pre-determined firms and regional groups are systematically correlated due to the error term in equation (28). For example, one might be worried that banks with a large exposure to government debt might systematically lend more to certain type of firms (ex. government's contractors) that might be more sensitive to sovereign risk than others. The recovered regression coefficients could then reflect such sorting rather than the mechanism discussed in this paper. While properly addressing this issue would require a different dataset than the one used here, note that specification (4) controls for this issue if the omitted firms' characteristics are invariant across firms within the same region, sector and time.[16] Our results are reassuring in that this specification continues to display significant robust patterns for the direct effects of sovereign risk similar to the benchmark specification.

A distinct concern is that these regression results arise because of an omitted aggregate shock that affects the performance of high-leverage firms and sovereign spreads at the same time.[17] Changes in global appetite for risk, for example, could have an impact on all financial contracts, which would affect more firms with high leverage and, at the same time, the market for government bonds. We think that the results on the direct effects of sovereign risk are largely robust to these considerations. First in the baseline specification, we find that the coefficient on the triple interaction between sovereign spreads, firm leverage, and banks' exposure – the direct effect– is negative and significant: we believe that a change in global risk would not generate such pattern because it is unlikely to vary across regions based on banks' government bond holdings. Second, in specification (4), the negative direct effect is robust when we control for *any* aggregate shock that is potentially correlated with sovereign risk and that can differentially impact firms based on their region/sector of operation. The inference on the indirect effects, instead, relies more on the structure of our model.

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[16]With the credit registry, for example, one could control for this issue by considering firms with multiple lending relationship. See Khwaja and Mian (2008).

[17]In a recent empirical study, Bottero, Lenzu, and Mezzanotti (2015) actually argue that the increase in Italian spreads following the Greek bailout of 2010 can be considered a "natural experiment" to assess the impact of sovereign risk on banks' lending behavior. They document, consistent with our theory, that banks that were holding more Italian government securities prior to such event significantly tightened credit relative to less exposed banks.

5 Quantitative analysis

This section performs a quantitative evaluation of our model to study the evolution of the Italian economy and to measure the propagation of sovereign risk to the real economy. We first use the regression results of the previous section as well as aggregate Italian data to parameterize the model. We then compare the model implications against micro and aggregate data and perform an event analysis and counterfactual to assess the macroeconomic implications of the Italian debt crisis in 2011 and 2012.

5.1 Parameterization

We start with some functional forms. The preferences of the government are given by the standard utility function $u_g(G) = \frac{G^{1-\sigma}-1}{1-\sigma}$, where σ is the risk aversion parameter. The default cost shocks are assumed to follow a autoregressive process $v_t = \bar{v} + \rho_v v_{t-1} + \varepsilon_{vt}$, with an autocorrelation parameter ρ_v and where $\varepsilon_{vt} \sim \mathcal{N}(0, \sigma_\varepsilon^v)$. We consider two regions, $J = 2$, and a period in the model is a year.

We classify the structural parameters into three groups of vectors $\theta = [\theta_1, \theta_2, \theta_3]$. The parameters in θ_1 govern the idiosyncratic and aggregate productivity process $\theta_1 = [\rho_z, \sigma_z, \mu, \Pi^p(p)]$. These parameters are chosen to fit the process for firm-specific productivity described in Section 4.1. The parameters $\theta_2 = [\alpha, \eta, \delta, \beta, \sigma, \chi, \tau]$ are set outside the model and are based on other studies. The remaining parameters, $\theta_3 = [\bar{n}_1, \bar{n}_2, \omega_1, \omega_2, \theta, \Pi^\lambda(\lambda), \bar{v}, \sigma_v, \rho_v, \beta_g, \gamma]$, are chosen so that our model matches a set of micro and macro moments for the Italian economy.

Parameterization of θ_2. The parameters in $\theta_2 = [\alpha, \eta, \delta, \beta, \sigma, \chi, \tau]$ include technological parameters, preference parameters, the disutility of labor, and the tax rate. The parameters α and η determine the shape of the production function of intermediate goods firms and final goods firms. We set the labor share α to 0.67, consistent with the labor share in Italy. The parameter η controls the elasticity of substitution across intermediate goods, $1/(1-\eta)$, and the markup. In setting this parameter, we use markup estimates for Italy in Christopoulou and Vermeulen (2012). Using data from 1981 to 2004, they find that markup estimates for Italy range from 0.23 to 0.87 across industries. We take a median level of 0.5 implying an elasticity of substitution of 3. We set the depreciation rate δ to 0.10 and the discount factor of households β to get a risk-free rate of 2%. We set the risk aversion parameter in the utility to a standard value, $\sigma = 2$. The parameter χ normalizes the average level of labor. We set it such that labor equals 0.3 when $p^d = 0$ and leverage constraints

are not binding. Finally, we set the tax rate to 0.2, which obtains a ratio of government consumption to output that is close to 20%, the estimate for Italy in Mendoza, Tesar, and Zhang (2014).

Parameterization of θ_3. We set the parameters in θ_3 to match moments in Italian data. These parameters include those controlling the balance sheet of financial intermediaries, the distribution of firms' financing needs, the stochastic process for enforcement shocks, and preference parameters for the government discount factor and labor supply elasticity.

The parameters that determine the balance sheet of intermediaries, $\{\bar{n}_1, \bar{n}_2, \omega_1, \omega_2, \theta\}$, affect the equilibrium only through the leverage constraint in equation (8). The linearity of the constraint implies that only the ratios of $\bar{n}_1, \bar{n}_2, \omega_1, \omega_2$ relative to the pledgeability parameter $(1 - \theta)$ matter for the equilibrium. Therefore, we can only recover the ratios $\{\bar{n}_j/(1-\theta), \omega_j/(1-\theta)\}$ in each region j. We discretize the distribution of firms' financing needs into two points $\lambda = \{\lambda_{\text{low}}, \lambda_{\text{high}}\}$, with probabilities equal to 0.75 and 0.25, in line with the groups in the estimated firm-level regressions of Section 4. We discretize the enforcement shocks ν into 16 points following the Tauchen (1986) method.

We target 12 sample moments that include cross-sectional, regional, and aggregate statistics. The cross-sectional statistics include the distribution of leverage and the regression coefficients of the firm-level regressions reported in specification (3) of Table 3. Specifically, we target the average short-term debt-to-asset ratio for the firms with $\text{lev}_i = 0$ and those with $\text{lev}_i = 1$, as well as $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ in equation (28). The regional statistics are the average ratio of government bond holdings to banks' equity for the low- and high-exposure regions. The aggregate statistics include the mean, standard deviation, and autocorrelation of sovereign interest rate spreads, as well as the correlation between sovereign spreads and aggregate output.

We solve the model using global methods (see Appendix C for details). Given the model policy functions, we compute aggregate moments in the model by simulating a long realization of a panel of 100,000 firms and calculating statistics on the simulated aggregated data. For recovering the model implied regression coefficients we estimate the exact specification of equation (28), used in our baseline empirical regression, for a large panel of firms in a short simulation. Specifically, the sequence of aggregate shocks for this simulation consist of the observed productivity shocks and a constructed sequence for enforcement shocks to match the observed path of sovereign spreads during the 2007-2012 period, while the initial conditions for government debt and the distribution of firms are their ergodic mean. The parameters in θ_3 are then chosen to minimize a weighted distance between the moments in the model and their corresponding counterparts in the data. Table 4 summarizes all values

for the parameters $\theta = \{\theta_1, \theta_2, \theta_3\}$.

Table 4: Parameter Values

Productivity Parameters θ_1		
Firm persistence z	$\rho_z = 0.9$	Foster, Haltiwanger, and Syverson (2008)
Firm volatility z	$\sigma_z = 0.05$	AMADEUS dataset, Italy
Aggregate p^d shocks	$p_t^d = \{0.0, 0.04, 0.1, 0.29\}$	AMADEUS dataset, Italy
	$\pi^p = \{3/9, 4/9, 1/9, 1/9\}$	AMADEUS dataset, Italy
Productivity decline	$\mu = 0.3$	AMADEUS dataset, Italy
Assigned Parameters θ_2		
Labor share	$\alpha = 0.67$	National accounts Italy
Markup parameter	$\eta = 0.67$	Christopoulou and Vermeulen (2012)
Discount factor	$\beta = 0.98$	Annual risk-free rate
Depreciation rate	$\delta = 0.1$	National accounts Italy
Risk aversion	$\sigma = 2$	Standard value in the literature
Labor disutility	$\chi = 11$	Normalization of labor
Tax rate	$\tau = 0.2$	Italian government to output
Parameters from Moment Matching θ_3		
Frisch elasticity	$1/\gamma = 2.8$	Method of simulated moments
Leverage constraint	$\bar{n}_1/(1-\theta) = 0.09$	Method of simulated moments
	$\bar{n}_2/(1-\theta) = 0.06$	Method of simulated moments
Regional exposure	$\omega_1/(1-\theta) = 0.3, \omega_2/(1-\theta) = 0.6$	Method of simulated moments
Financing requirement	$\lambda_{low} = 0.1, \lambda_{high} = 0.6$	Method of simulated moments
Process for ν_t	$\bar{\nu} = 0.6, \sigma_\nu = 0.05, \rho_\nu = 0.9$	Method of simulated moments
Government discount factor	$\beta_g = 0.9$	Method of simulated moments

Even though the parameters in θ_3 are chosen jointly, we can give a heuristic description of how the sample moments included in the estimation inform specific parameters. First, in our model, the leverage ratio for a firm of type i equals $\lambda_i r^k / \alpha$. Using the parameters in θ_2 and the equilibrium level for $r^k = 1 - \beta(1-\delta)$, the empirical measure for leverage for firms with leverage below and above the 75th percentile pins down λ_{low} and λ_{high}. Similarly, there is a tight relation between the government-bonds-to-equity ratio of banks in a given region and ω_j. Indeed, such statistic in the model equals $\omega_j q_t (1-\vartheta) B_t / (\bar{n}_j + \omega_j q_t (1-\vartheta) B_t)$: given $\bar{n}_j$ and the market value of debt, these moments pin down ω_j for the two regions. The coefficients associated with the direct effects of sovereign risk (α_2, α_4) provide information on $\bar{n}_j$. To see why, suppose $\bar{n}_j$ is so large that the intermediaries' leverage constraints are slack in the simulation. Then, the model predicts these coefficients to be equal to zero. As $\bar{n}_j$ decreases, the coefficients become negative. The coefficients associated with the indirect effects (α_1, α_3), instead, provide information on the Frisch elasticity of labor supply: a

higher Frisch elasticity is associated with a lower sensitivity of wages to movements in labor demand, which dampens the general equilibrium effects that are working through wages, as discussed in Section 3. Thus, a high $1/\gamma$ is associated with more negative α_1 and α_3 in model-simulated data. The mean, standard deviation, and persistence of sovereign spreads have a tight connection with the process for the enforcement shock ν_t and with the government discount factor β_g.

Table 5 reports the target moments in the model and the data. Overall, the model produces similar statistics to the ones in the data. The model average and standard deviation of interest rate spreads on government debt of 2.6% and 1.1% are similar to the data counterparts of 1.8% and 1.3%, respectively. The persistence of the spread in the model of 0.8 is slightly higher than the one in the data of 0.6. The mean leverage for firms in two leverage group of 4% and 24% are exactly matched. The mean exposure to government debt across the two regions in the model of 0.31 and 0.55 are similar to those in the data of 0.34 and 0.58. The model generates a correlation between sovereign spreads and output of -0.68, similar to the data one of -0.54. In terms of the regression coefficients, the model reproduces their signs, with magnitudes in line with the data for all coefficients, with the exception of α_1, which is 40% of his corresponding target in the data. This means that the indirect effects of sovereign spreads on output in the low-exposure region are somewhat smaller in the model relative to the data. However, the estimated coefficient in the data also has a wide confidence interval that ranges from -2.2 to -0.58, and the model-implied statistics fall almost within this confidence interval.

In Table 5, we also report some additional implications of our model for aggregate and regional moments and compare them with the data. In the data, the high-exposure region is more negatively correlated with government spreads, as in our model. Our model also successfully replicates the positive co-movement of output across the two regions, 0.96 for both the model and the data.

An important implication of our model is that borrowing rates for firms positively co-move with the sovereign spread, as they do in the data. In the model, firms' borrowing rates co-move positively with government spreads because both respond to the aggregate productivity shock and also because sovereign spreads affect financial intermediaries' balance sheets. The correlation between sovereign and average firms' spreads in the model, defined as $R_{jt} - 1/\beta$, equals 0.70. Although our dataset does not contain firms' interest rates, we construct an indicator of average firms' spreads using data from the European Central Bank for comparisons with our model. Specifically, we obtain the average interest rate charged on short-term loans by Italian non-financial corporations below 1 million euros at the monthly level and average it at the annual level. We construct the same average

Table 5: Moments in Model and Data

	Data	Model
Targeted moments		
Govt spread mean	1.8	2.6
Govt spread volatility	1.3	1.1
Govt spread persistence	0.8	0.6
Firms' leverage	[.04 .24]	[.04 .24]
Banks' exposure to government debt	[.34 .58]	[.31 .55]
Corr(output, spread)	-.54	-.68
Regression coefficients:		
$\alpha_1 : \text{spr}_t$	-1.35	-.53
$\alpha_2 : \text{spr}_t \times \text{lev}_i$	-.26	-.30
$\alpha_3 : \text{spr}_t \times \text{exp}_j$	-.44	-.35
$\alpha_4 : \text{spr}_t \times \text{lev}_i \times \text{exp}_j$	-.13	-.12
$\alpha_5 : p_t^d$	-.43	- .34
Other Moments		
Corr(output$_L$, spread)	-.47	-.41
Corr(output$_H$, spread)	-.49	-.54
Corr(output$_L$, output$_H$)	.96	.96
Corr(spread, firm spreads)	.89	.70

Note: output_L and output_H are the output of low-exposure and high-exposure region respectively.

interest rate for German non-financial corporations and take the difference between the two.[18] Between 2007 and 2012, the correlation between this interest rate differential and sovereign spreads equals 0.89.

5.2 Dissecting the Italian debt crisis

We now use the model to measure the propagation of sovereign risk to the real economy during the Italian debt crisis. As described above, in our event study, we feed the observed productivity shocks into the model and construct a sequence of enforcement shocks to produce the observed rise in sovereign spreads during the debt crisis. We assess the implications of our model for the paths of firms' borrowing rates and output. We then conduct counterfactual experiments to measure the impact of the sovereign debt crisis on these variables.

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[18]The differential between interest rates on Italian firms and German government securities follows a very similar pattern, although the spread is on average 200 basis points higher, which reflects credit risk. We focus on the spread relative to German firms to net out such credit risk that we abstract from in our model.

Figure 4: Event study: 2007-2012

(a) Productivity shock

(b) Enforcement shock

(c) Sovereign spreads

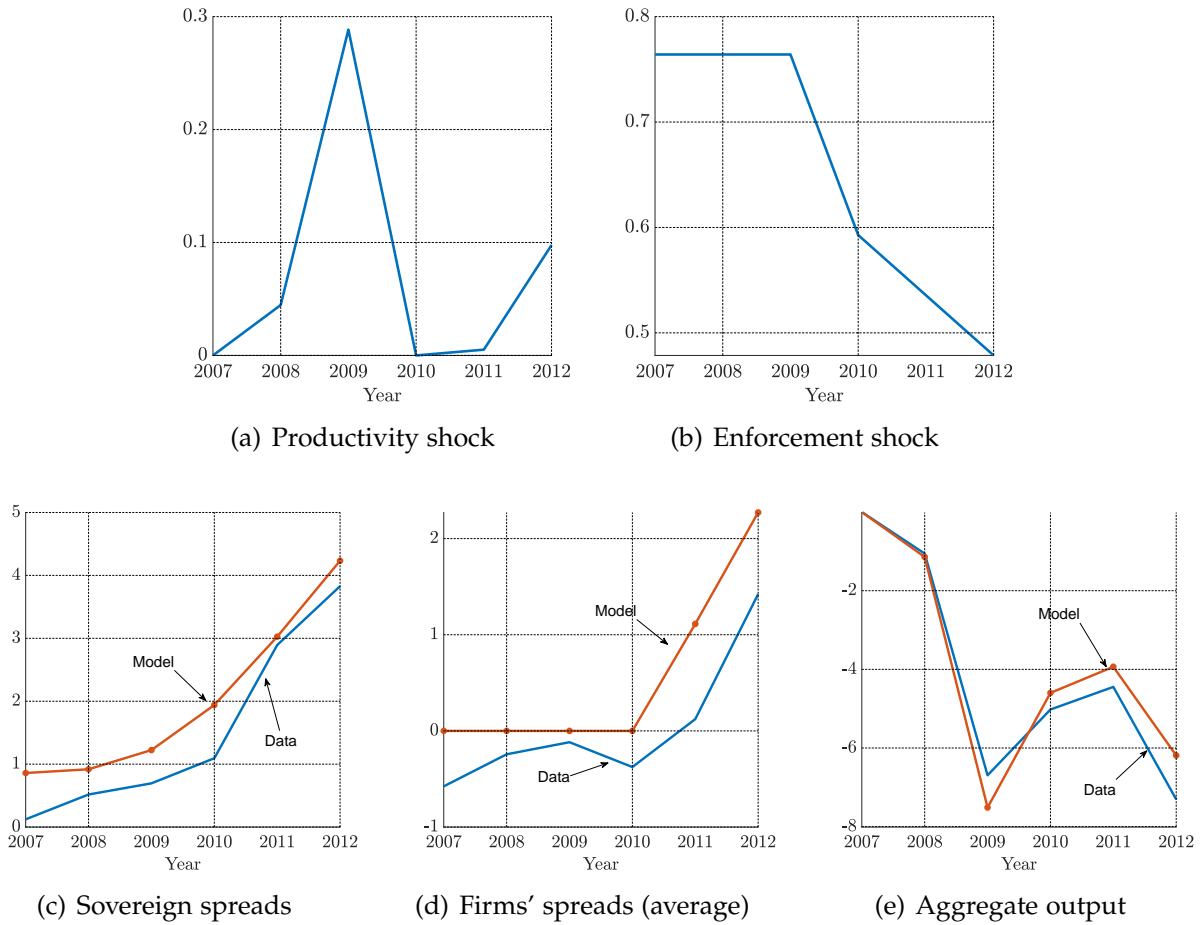(d) Firms' spreads (average)

(e) Aggregate output

Figure 4 reports the time path for the aggregate shocks, firms' and sovereign spreads, and aggregate output. The productivity shock is the one we measured using the firm-level productivity estimates, while the time path for the enforcement shock is chosen to minimize a weighted distance between the observed and model-implied sovereign spreads.[19] From panel (b), we can see that the model calls for a progressive deterioration of enforcement after 2009 in order to reproduce the rise in sovereign interest rate spreads observed in the data.

Panels (d) and (e) in the figure report the time path for firms' spreads and aggregate output in the model and data. The model predicts slack leverage constraints for financial intermediaries until 2010, implying zero interest rate spreads on firms. After 2010, these leverage constraints start to bind, and firms' spreads increase by about 225 basis points.

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[19]We initialize the other state variables as follows. We fix p^d at its 2007 level and ν at the highest level in the grid and simulate a long time-series from the model. We then select the ergodic level of government debt and Λ as the initial values in our event study.

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Figure 5: Counterfactual path for sovereign spreads, firms' spreads, and aggregate output

(a) Sovereign spreads   (b) Firms' spreads (average)   (c) Aggregate output
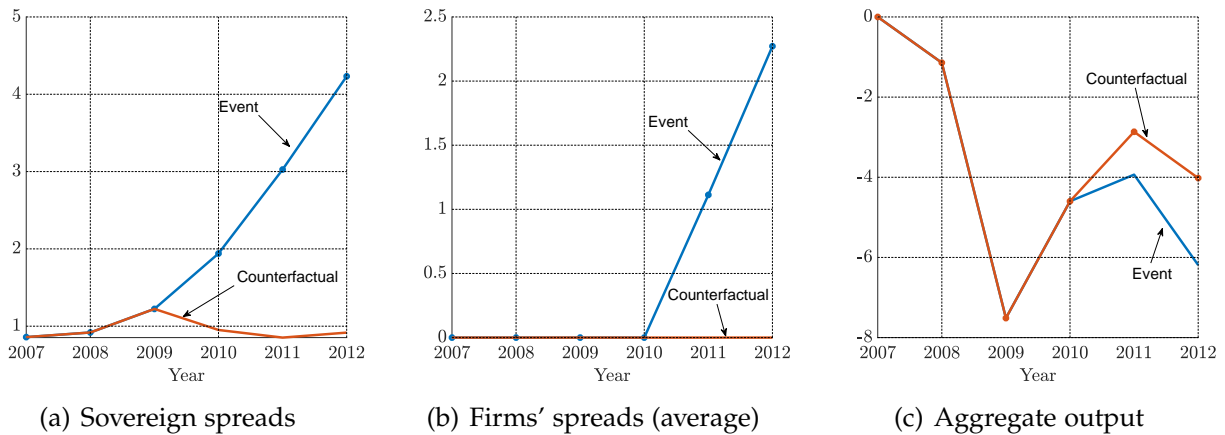
Such dynamics are consistent with the behavior of firms' interest rate spreads in the data, shown in the solid line in panel (d) of the figure. As for output (panel (e)), we can observe two large declines in this episode in the data: 6.2% in 2009 and 7.5% in 2012. Our model successfully produces these sizable declines, and it also replicates the short recovery during 2010 and 2011. These results indicate a good out-of-sample fit for our model because we did not include the volatility of aggregate output or of firms' interest rate spreads in the moment-matching exercise.

Having chosen a sequence of enforcement shocks, we next proceed to measure the macroeconomic implications of the sovereign debt crisis. To do so, we construct the time path for the model's endogenous variables in a counterfactual in which the enforcement shock is fixed at its 2007 level throughout this episode while aggregate productivity follows the path observed in the data. Figure 5 reports the time path of sovereign spreads, firms' borrowing rates, and aggregate output in this counterfactual, and we compare it to the one we obtained in the event study of Figure 4.

From panel (a), we can see that sovereign spreads increase very modestly in this counter-factual. This muted deterioration implies that financial intermediaries' leverage constraints remain slack, as net worth does not suffer losses. The slackness in these constraints implies no rise in firms' borrowing costs over the sample (see panel (b)). Without such increase in firms' borrowing costs, the recession in Italy would have continued to be sizable but sub-stantially milder as seen in panel (c) of Figure 5. By 2012, output would have been about 4% below trend, whereas in the benchmark it was 6.2%. Thus, our model predicts a sizable propagation of sovereign risk to real economic activity, contributing to 2.2% of the output decline in 2012.

We can further decompose the output losses associated with sovereign risk into those that are due to the direct effects on firms' borrowing rates and those that are due to the additional general equilibrium mechanisms working through aggregate demand and wages. To do so, we use equation (20), which expresses log-sales for a firm as a function of interest rates, wages, and aggregate demand. We use this expression to evaluate the firms' sales that would have been realized if interest rates on firms follow the same path as in the event of Figure 4 while wages and aggregate demand follow the path of the counterfactual economy. We aggregate across firms to generate a path for aggregate output with only the direct effect operating. By comparing this path of output to the counterfactual output path, we can evaluate the output losses arising because of the increase in firms' borrowing rates rates alone. This decomposition is reported in Table 6. We can see that out of the 2.2% decline in output attributed to the sovereign debt crisis, 1.5% is due to the direct effects of sovereign risk. The indirect effects of sovereign risk that depressed aggregate demand and wages are also negative and account for an additional 0.7% decline.

Table 6: The Italian debt crisis, 2007-2012

	Output	Govt spreads
Data	-7.3	3.7
Benchmark Model	-6.2	3.4
Counterfactual: No Govt Debt Crisis	-4.0	.1
Counterfactual Decomposition		
Crisis total effect	-2.2	3.3
Crisis direct effect	-1.5	

Note: Output is reported as the percentage deviation between 2012 and 2007. Spreads are measured in percentage points.

Using the results from this counterfactual, we can compute the passthrough from sovereign spreads to firms borrowing rates and the elasticity of aggregate output with respect to changes in sovereign spreads. Our results imply that on average during the Italian debt crisis, a 100 basis point increase in sovereign spreads led to an increase in firm borrowing rates of 68 basis points (225/330×100) and a decline of 0.67% (-2.2/3.3) in aggregate output. The macro elasticity in our full quantitative model is state dependent. As seen from the event paths, the elasticity is lower earlier in the event and highest at the height of the crisis in 2012. The state dependency in our full nonlinear model arises because the dynamics of the state variables dictate when the leverage constraint for intermediaries is binding. As implied from the paths of firms' borrowing costs in panel (d) in Figure 4, earlier in the event the leverage constraint for intermediaries is not binding (which leads to zero firm spreads), and it becomes binding later in the event, with firm spreads rising on average as

the sovereign debt crisis becomes prolonged and sovereign spreads rise substantially.

6 Conclusion

We have developed a framework that combines a structural model of sovereign debt with financial intermediaries and heterogeneous firms with micro data to study the macroeconomic implications of sovereign risk. We showed that firm-level data can be useful for measuring the macroeconomic implications of sovereign risk and the different transmission mechanisms. In our application, we find that the effect of sovereign risk on the private sector is sizable, accounting for about one-third of the observed decline in output during the Italian debt crisis.

Our approach could be generalized along other dimensions. The sovereign debt literature has suggested several mechanisms through which sovereign risk affects the economy, for example, by disrupting international trade or by hindering firms' investment plans because of increased uncertainty about fiscal policy. We believe that a fruitful avenue for future research would be to exploit the cross-sectional variation that is present in firm-level datasets to test and quantify these theories. We leave these applications to future research.

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Online Appendix to "Sovereign Default Risk and Firm Heterogeneity"

by Cristina Arellano, Yan Bai, and Luigi Bocola

A Data sources

A.1 ORBIS-AMADEUS

The construction of the firm-level dataset follows closely the work of Gopinath et al. (2015). Here we report some basic information, and we refer the reader to that paper for additional details. We use firm-level data on Italian firms from ORBIS-AMADEUS, accessed online through Wharton Research Data Services (WRDS). The data set has detailed balance sheet information for public and privately held firms, and use only the unconsolidated data on active firms.

We clean this dataset in a series of steps. First, we control for basic reporting mistakes by dropping firm-year observations with negative values for total assets, tangible fixed assets, number of employees, and operating revenues. Second, we drop firms that have missing values for the variables of interest over the 2007-2015 period. Finally, we winsorize variables at the 1st and 99th percentile and deflate monetary values using the Italian consumer price index (CPI) obtained from FRED. The data used in our benchmark specifications therefore comprise a balanced panel of approximately 225,000 firms operating continuously between 2007 and 2015.

In the production function estimation, we restrict the analysis to all the firms in our sample that operate in manufacturing (NACE codes 10-33). We correct further reporting mistakes by dropping firm-year observations with missing or negative values for material costs and wage bill, and with missing values for value added, where value added is calculated as the difference between operating revenue and material costs. As a result of these additional steps, we are left with a balanced panel of firms with 425,168 firm-year observations over the 2007-2015 period.

For the firm-level regressions in Table 3, we use the following variables (in emphasis the AMADEUS mnemonics)

SALES GROWTH: We compute the symmetric percentage changes, defined as the ratio of change in operating revenues (*opre*) relative to the previous year to the average operating

revenues in those years,

$$growth_t = 200 \frac{opre_t - opre_{t-1}}{opre_t + opre_{t-1}}.$$

LEVERAGE: Ratio of short term debt (*loan*) relative to total assets (*toas*).

PROFITABILITY: Ratio of profit (*plbt*) to sales (*opre*).

SIZE: Log of total assets (*toas*).

A.2 Bankscope and Bank of Italy reports

From Bankscope, we extract balance sheet data for banks headquartered in Italy. We keep data only for 2007, and drop observations with no information on total assets (*totalassets*), total equity (*totalequity*) and holdings of government bonds (*memogovernmentsecuritiesincluded*). We then map the city of incorporation (*city*) to one of the twenty Italian regions: Abruzzo, Basilicata, Calabria, Campania, Emilia-Romagna, Friuli-Venezia Giulia, Lazio, Liguria, Lombardia, Marche, Molise, Piemonte, Puglia, Sardegna, Sicilia, Toscana, Trentino-Alto Adige, Umbria, Valle d'Aosta, and Veneto. We use these data to construct $\text{exposure}_j^{\text{loc}}$ for each region j defined in the main text, and $\text{exposure}_{i,j}^{\text{nat}}$ for the five largest banks by total assets in 2007.

We obtain the number of banks' branches for the national banks using Bank of Italy *Albi and Elenchi Vigilanza* which can be accessed at https://www.bancaditalia.it/compiti/vigilanza/albi-elenchi/. We manually use the website query to obtain the geographic distribution of bank branches for UNICREDIT, Intesa-Sanpaolo, Monte dei Paschi di Siena, Banca Nazionale del Lavoro and Banco Popolare. The branches are reported at the city level as of December 31st 2007, and we use ZIP codes to aggregate branches at the regional level. The total number of banks' branches as of December 31st 2007 for each region is obtained from the Bank of Italy *Base Dati Statistica*. The series name is *TDB20207*.

B Proofs

Proof of Lemma 1 Taking as given the aggregate demand and wage, a firm (z, λ) in region j with state $X_j = (p^d, \nu, N_j)$ chooses capital and labor to maximize its profit (7) subject to the demand schedule (3) and financing requirement (6). In equilibrium the optimal capital satisfies

$$k(z, \lambda, X_j) = M_k z^{\frac{\eta}{1-\eta}} Y(X_j) w(X_j)^{\frac{(1-\alpha)\eta}{\eta-1}} \left[1 - \lambda + \lambda R(X_j) \right]^{\frac{1}{\eta-1}},$$

$$\ell(z, \lambda, X_j) = \frac{1 - \alpha}{\alpha} \frac{r_k}{w(X_j)} k(z, \lambda, X_j),$$

$$b(z, \lambda, X_j) = \lambda \frac{r_k}{\alpha} k(z, \lambda, X_j),$$

$$y(z, \lambda, X_j) = z k(z, \lambda, X_j)^\alpha \ell(z, \lambda, X_j)^{1-\alpha},$$

where the constant M_k is given by

$$M_k = (1 - \tau)^{\frac{1}{1-\eta}} \left(\eta \alpha^{1-(1-\alpha)\eta} (1 - \alpha)^{(1-\alpha)\eta} \right)^{\frac{1}{1-\eta}} (r_k)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}.$$

Aggregating up $y(z, \lambda, X_j)$ and $\ell(z, \lambda, X_j)$ and applying the market clearing conditions and family's optimal condition $w(X_j) = \chi L(X_j)^\gamma$, we get the equilibrium wage and output as in equation (18) and (19) with the constants $M_w = M_y^{(1-\eta)/(1-\alpha)}$ and M_y as

$$M_y = (1 - \tau)^{\frac{1}{1-\eta}} \left(\frac{1 - \alpha}{\alpha} \right)^{1-\alpha} \left(\eta \alpha^{1-(1-\alpha)\eta} (1 - \alpha)^{(1-\alpha)\eta} \right)^{\frac{1}{1-\eta}} (r_k)^{-\frac{\alpha}{1-\eta}}.$$

Summing over the loan demand $b(z, \lambda, X_j)$ over (z, λ) and using the equilibrium (18) and (19), we get the total loan demand of firms and the loan market condition (17) where $M_n = \alpha/(\chi(1-\alpha)r_k) M_y^{(1+\gamma)(1-\eta)/(\gamma(1-\alpha))}$. *Q.E.D*

Proof of Proposition 1   Recall that the state of region j is given by

$$X_{jt} = \left[p_t^d, \Lambda_t, N_{jt} \left(S_t, B_t, D_t, B_{t+1} \right) \right] \tag{A.1}$$

with $S_t = \{v_t, p_t^d, \Lambda_t\}$. Here we consider small shocks so that there are no default in equilibrium, and the net worth of each region N_{jt} is a function of $(p_t^d, v_t, \Lambda_t, B_t, B_{t+1})$. Using the definition of spread from equation (16), we can rewrite the net worth equation (8) with spread,

$$N_{jt} = \bar{n}_j + \omega_j (1 - \vartheta) \frac{\vartheta}{\vartheta + 1/\beta - 1 + spr_t} B_t. \tag{A.2}$$

Given that $p_t^d, v_t, , \Lambda_t,$ and B_{t+1} only affect N_{jt} through their impacts on spread spr_t, we can define a function of net worth on spread and B as $\tilde{N}_j(spr_t, B_t) = N_{jt} \left(p_t^d, v_t, \Lambda_t, B_t, B_{t+1} \right)$ where the spread spr_t is the evaluation of the spread function $spr(p^d, v, \Lambda, B)$ at period t's state, i.e.

$$spr_t = spr(p_t^d, v_t, \Lambda_t, B_t) = H_S(p_t^d, v_t, B_{t+1}(p_t^d, v_t, \Lambda_t, B_t)). \tag{A.3}$$

Consider approximating linearly $\log py_{ijt} = f(\lambda_i, z_{ijt}; X_{jt})$ around a point $(\lambda_i, z_0; X_j^0)$. We

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can follow standard steps and consider a first order Taylor expansion,

$$\log py_{i,j,t} - f(\lambda_i, z_0; X_j^0) \approx \frac{\eta}{1-\eta}(\log z_{ijt} - \log z_0) + f_p(\lambda_i, z_0; X_j^0)(p_t^d - p_0^d) + f_v(\lambda_i, z_0; X_j^0)(v_t - v_0),$$
(A.4)

and use equation (20) to obtain these derivatives:

$$f_p(\lambda_i, z_{ijt}; X_{jt}) = \frac{\partial \log Y(X_{jt}) - \left[\frac{(\eta(1-\alpha))}{(1-\eta)}\right] \log w(X_{jt})}{\partial p_t^d} - \frac{1-\eta}{\eta}\lambda_i \frac{\partial R(X_{jt})}{\partial p_t^d}$$
(A.5)
$$+ \frac{\partial \log Y(X_{jt}) - \left[\frac{(\eta(1-\alpha))}{(1-\eta)}\right] \log w(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \frac{\partial spr_t}{\partial p_t^d} - \frac{1-\eta}{\eta}\lambda_i \frac{\partial R(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \frac{\partial spr_t}{\partial p_t^d}$$

$$f_v(\lambda_i, z_{ijt}; X_{jt}) = \frac{\partial \log Y(X_{jt}) - \left[\frac{(\eta(1-\alpha))}{(1-\eta)}\right] \log w(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \frac{\partial spr_t}{\partial v_t} - \frac{1-\eta}{\eta}\lambda_i \frac{\partial R(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \frac{\partial spr_t}{\partial v_t}.$$
(A.6)

Note that from (A.1), p_t^d enters the state of the region in two ways. First, it affects the private economy directly. Second, it affects the net worth of banks since the productivity shock affects the government's default incentive and hence spread of the government. Hence f_p includes the derivatives of prices and output on p_t^d itself and the derivatives through net worth.

Plugging the derivatives (A.5) and (A.6) into the Taylor expansion (A.4) and combining terms, we have

$$\log py_{i,j,t} \approx f(\lambda_i, z_0; X_j^0) + \frac{\eta}{1-\eta}(\log z_{ijt} - \log z_0)$$
$$+ \frac{\partial \log Y(X_{jt}) - \left[\frac{(\eta(1-\alpha))}{(1-\eta)}\right] \log w(X_{jt})}{\partial p_t^d}(p_t^d - p_0^d) - \frac{1-\eta}{\eta}\lambda_i \frac{\partial R(X_{jt})}{\partial p_t^d}(p_t^d - p_0^d)$$
$$+ \frac{\partial \log Y(X_{jt}) - \left[\frac{(\eta(1-\alpha))}{(1-\eta)}\right] \log w(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \left[\frac{\partial spr_t}{\partial p_t^d}(p_t^d - p_0^d) + \frac{\partial spr_t}{\partial v_t}(v_t - v_0)\right]$$
$$- \frac{1-\eta}{\eta}\lambda_i \frac{\partial R(X_{jt})}{\partial N_{jt}} \frac{\partial N_{jt}}{\partial spr_t} \left[\frac{\partial spr_t}{\partial p_t^d}(p_t^d - p_0^d) + \frac{\partial spr_t}{\partial v_t}(v_t - v_0)\right].$$
(A.7)

The government's spread varies with both productivity shock p_t^d and the default cost shock v_t. Assuming $B_t = B_0$, the first order Talyor expansion over the spread function (A.3)

implies

$$spr_t - spr_0 = \left[H_{S,1,t} + H_{S,3,t}\frac{\partial B_{t+1}}{\partial p_t^d}\right](p_t^d - p_0^d) + \left[H_{S,2,t} + H_{S,3,t}\frac{\partial B_{t+1}}{\partial v_t}\right](v_t - v_0)$$

$$= \frac{\partial spr}{\partial p_t^d}(p_t^d - p_0^d) + \frac{\partial spr}{\partial v_t}(v_t - v_0). \tag{A.8}$$

where $H_{S,i,t}$ is the derivative of function H_S over its *ith* argument at period t. Note that the second equation holds because the derivatives of equation (A.3) shows $\frac{\partial spr}{\partial p_t^d} = H_{S,1,t} + H_{S,3,t}\frac{\partial B_{t+1}}{\partial p_t^d}$ and $\frac{\partial spr}{\partial v_t} = \left[H_{S,2,t} + H_{S,3,t}\frac{\partial B_{t+1}}{\partial v_t}\right]$. We can replace $\frac{\partial spr}{\partial p_t^d}(p_t^d - p_0^d) + \frac{\partial spr}{\partial v_t}(v_t - v_0)$ in equation (A.7) with $spr_t - spr_0$. For $\partial N_j/\partial spr$ in equation (A.7), we can take the partial derivative of N_{jt} over spr_t in equation (A.2) and evaluate it at B_0, which ends up with $\partial N_j/\partial spr = -B_0\vartheta/(\vartheta + 1/\beta - 1 + spr_0)^2\omega_j \equiv -M_q\omega_j$. Collecting the constant terms together as δ_i and using the definition of $\beta_{s,j}$, $\beta_{p,j}$, $\gamma_{s,j}$, and $\gamma_{p,j}$ in (23), we have equation (22) in Proposition 1. *Q.E.D*

Proof of Corollary 1 Aggregate output is the sum of the output in each region,

$$Y_t = \sum_j Y_{jt}\left(p_t^d, \Lambda_t, \tilde{N}_j\left(spr(p_t^d, v_t, \Lambda_t, B_t), B_t\right)\right),$$

where $\tilde{N}_j(spr_t, B_t)$ and $spr(p_t^d, v_t, \Lambda_t, B_t)$ are the same as in the Proof of Proposition 1. Let $\pi_j = Y_j/Y$ be the output share of region j at $\{X_j^0\}$. Using the first order approximation, we have

$$\log Y_t - \log Y_0 = \sum_j \pi_j \frac{\partial \log Y_{jt}}{\partial N_{jt}}\frac{\partial N_{jt}}{\partial spr_t}(spr_t - spr_0).$$

Recall that the coefficients of $\gamma_{s,j}$ and $\beta_{s,j}$ in equation (22):

$$\gamma_{s,j} = -\frac{\eta}{1-\eta}\frac{\partial R_{jt}}{\partial N_{jt}}\frac{\partial N_{jt}}{\partial spr_t}\bigg|_{X_j^0} \qquad \beta_{s,j} = \frac{\partial \log Y_{jt} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right]\partial \log w_{jt}}{\partial N_{jt}}\frac{\partial N_{jt}}{\partial spr_t}\bigg|_{X_j^0}$$

Applying the equilibrium condition (18) and (19), we have

$$\log Y_t - \log Y_0 = \sum_j \pi_j\left(\frac{\partial \log Y_{jt} - \left[\frac{\eta(1-\alpha)}{1-\eta}\right]\partial \log w_{jt}}{\partial N_{jt}}\frac{\partial N_{jt}}{\partial spr_t} + \frac{\gamma\eta(1-\alpha)}{(1+\gamma)(1-\eta)}\frac{1}{N_j^0}\frac{dN_{jt}}{\partial spr_t}\right)(spr_t - spr_0)$$

$$= \sum_j \pi_j\left(\beta_{s,j} + \frac{\gamma\eta(1-\alpha)}{(1+\gamma)(1-\eta)}\frac{1}{N_j}\frac{dN_{jt}}{\partial spr_t}\right)(spr_t - spr_0) \tag{A.9}$$

Under the assumption of binding leverage constraint, the loan market clearing condition for each region j implies

$$\frac{1}{N_j}\frac{\partial N_{jt}}{\partial R_{jt}} = -\frac{(1+\gamma)(1-\eta)}{\gamma\eta(1-\alpha)}\frac{\eta}{1-\eta}\frac{1}{R_w}\sum_\lambda \pi(\lambda)\lambda\left[1-\lambda+\lambda R^f\right]^{-\frac{1}{1-\eta}}$$

Applying the equation for $\gamma_{s,j}$, we have

$$\frac{\partial N_{jt}}{\partial spr_t} = -\gamma_{s,j}\frac{1-\eta}{\eta}\frac{\partial N_{jt}}{\partial R_{jt}}.$$

Hence

$$\frac{\gamma\eta(1-\alpha)}{(1+\gamma)(1-\eta)}\frac{1}{N_j}\frac{\partial N_{jt}}{\partial sprt} = \gamma_{s,j}\frac{\int_\lambda \lambda\left[1+\lambda(R(X_j^0)-1)\right]^{-\frac{1}{1-\eta}}d\Lambda_\lambda}{\int_\lambda \left[1+\lambda(R(X_j^0)-1)\right]^{-\frac{\eta}{1-\eta}}d\Lambda_\lambda} \tag{A.10}$$

Plugging equation (A.10) into (A.9), we get Corollary 1. *Q.E.D*

Proof of Proposition 2 Assume banks' leverage constraint bind at X_j^0. Loan market clearing condition (17) holds with equality. Taking first order condition over (17), we can get

$$\gamma_{s,j} = -\frac{\eta}{1-\eta}\frac{\partial R_{jt}}{\partial N_{jt}}\frac{\partial N_{jt}}{\partial spr_t}\bigg|_{X_j^0}$$

$$= -\frac{\eta}{1-\eta}\frac{\gamma(1-\alpha)}{(1+\gamma)}\frac{\sum_\lambda \pi(\lambda)\left[1-\lambda+\lambda R(X_j^0)\right]^{-\frac{\eta}{1-\eta}}}{\sum_\lambda \pi(\lambda)\lambda\left[1-\lambda+\lambda R(X_j^0)\right]^{-\frac{1}{1-\eta}}}\frac{1}{N_j^0}\frac{B_0\vartheta}{(\vartheta+1/\beta-1+spr_0)^2}\omega_j.$$

It is easy to see that $\gamma_{s,j} < 0$. For the two regions with the same net worth, $N_1^0 = N_2^0$, they will have the same firm borrowing rate, $R(X_1^0) = R(X_2^0)$. Hence, $\omega_1 \leq \omega_2$ implies $\gamma_{s,1} > \gamma_{s,2}$.

The indirect effect is captured by $\beta_{s,j}$, which takes the following form

$$\beta_{s,j} = -\left[1 - \frac{\nu(1-\alpha)}{(1-\eta)(1+\nu)}\frac{\sum_\lambda \pi(\lambda)\lambda\left[R_\lambda(X_j^0)\right]^{-\frac{2-\eta}{1-\eta}}}{\sum_\lambda \pi(\lambda)\lambda\left[R_\lambda(X_j^0)\right]^{-\frac{1}{1-\eta}}}\frac{\sum_\lambda \pi(\lambda)\left[R_\lambda(X_j^0)\right]^{-\frac{\eta}{1-\eta}}}{\sum_\lambda \pi(\lambda)\left[R_\lambda(X_j^0)\right]^{-\frac{1}{1-\eta}}}\right]\frac{1}{N_j^0}\frac{B_0\vartheta}{(\vartheta+1/\beta-1+spr_0)^2}\omega_j$$

where $R_\lambda(X_j^0) = 1-\lambda+\lambda R(X_j^0)$. *Q.E.D*

C   Numerical solution

We solve the model in two steps. The first step solves a pseudo private equilibrium. The second step solves the Markov equilibrium where the government takes as given the private responses over its default and debt choices.

We have already shown in the main text that the government's decisions affect the private economy through banks' net worth, which in terms determines firms' borrowing rate. Furthermore, it is also easy to show that firms' borrowing rate R decreases monotonically with banks' net worth. In the private equilibrium, under a given shock and firm distribution, there must be a level of net worth associated with a firms' borrowing rate $R \geq 1/\beta$. This motives us to solve a pseudo private equilibrium in the first step. For each state $\hat{X} = (\Lambda, R)$, we compute the private equilibrium of $\{Y(\hat{X}), w(\hat{X}), T(\hat{X}), L(\hat{X}), B^f(\hat{X}), k(z, \lambda; \hat{X}), \ell(z, \lambda; \hat{X})$, where $B^f(\hat{X})$ is the aggregate loan demand of the firms in region (Λ, R^f), i.e.

$$B^f(\hat{X}) = \int_{(z,\lambda)} \lambda \frac{1}{\alpha} r_k k(z, \lambda; \hat{X}) d\Lambda.$$

In the second step, we solve the government's problem taking as given the private equilibrium. In particular, for any state (p, v, Λ, B) and the government's choice (D, B'), the state for the private economy becomes $X = (S, \Lambda, B, D, B')$ with $S = (v, p^d, \Lambda)$. The implied banks' net worth $N_j(X)$ in region j is given by

$$N_j(X) = \bar{N}_j + \omega_j \left[(1 - D(S, B)) q(S, B'(B))(1 - \vartheta)B + D(S, B) q(S, B'(R))(1 - \vartheta)R \right].$$

The pseudo private state is $\hat{X}_j(X) = (\Lambda_j, R_j(X))$ with $R_j(X) = \beta$ if $(1 - \theta) B^f(\Lambda_j, \beta) \leq N_j(X)$, otherwise $R_j(X)$ is given by the inverse of the aggregate private loan demand, i.e.

$$R_j(X) = \left(B_j^f \right)^{-1} (N_j(X); \Lambda_j).$$

We now describe in details the computation algorithm.

C.1   Step 1: computation for private equilibrium

We compute the private equilibrium by iterating over the market prices $\{Y, w\}$ for any given private state $\hat{X} = (\Lambda, R)$.

1. We discretize the idiosyncratic productivity shock into two points (z_L, z_H). With two points, we only need to track the fraction of z_L in each period. We abuse notation and use Λ_z for the fraction of z_L in the firms' distribution. We construct the state space

for $\Lambda_z \in (0,1)$. The finance requirements λ also takes two values λ_L and λ_H. The probabilities are given by Λ_λ, which are time invariant.

2. Construct Λ_z-specific state space for interest rate $R(\Lambda_z) \in [1/\beta, R_{\max}(\Lambda_z)]$.

3. Compute the equilibrium $\{Y, w\}(\hat{X})$ for each grid of (Λ_z, R):

$$w(\hat{X}) = \left(M_y^\eta \left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{\frac{1-\eta}{\eta(1-\alpha)}}$$

$$Y(\hat{X}) = \frac{\left(M_y^\eta \left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{\frac{1-\eta+(1-\alpha\eta)\nu}{\nu\eta(1-\alpha)}}}{M_\ell \chi \left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{1}{1-\eta}} \right]}$$

$$K(\hat{X}) = \frac{M_k}{\chi M_\ell} \left(M_y^\eta \left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{\frac{1+\nu}{\nu} \frac{(1-\eta)}{\eta(1-\alpha)}}$$

$$L(\hat{X}) = \frac{1}{\chi} \left(M_y^\eta \left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R(X) \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{\frac{1-\eta}{\nu\eta(1-\alpha)}}$$

$$k(z, \lambda; \hat{X}) = \frac{M_k}{\chi M_\ell} \left(\left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{\frac{1+\nu}{\nu} \frac{(1-\eta)}{\eta(1-\alpha)}} \times$$

$$\frac{z \left[1 - \lambda + \lambda R \right]^{-\frac{1}{1-\eta}}}{\left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{1}{1-\eta}} \right]}$$

$$y(z, \lambda; \hat{X}) = Y(\hat{X}) \left(\left[\sum \pi(z; \Lambda_z) z \right] \left[\sum_\lambda \Lambda_\lambda(\lambda) \left[1 - \lambda + \lambda R \right]^{-\frac{\eta}{1-\eta}} \right] \right)^{-\frac{1}{\eta}} z^{\frac{1}{\eta}} \left[1 - \lambda + \lambda R \right]^{-\frac{1}{1-\eta}}$$

where

$$M_k = \left(\eta \alpha^{1-(1-\alpha)\eta} (1-\alpha)^{(1-\alpha)\eta} \right)^{\frac{1}{1-\eta}} (r_k)^{-\frac{1-(1-\alpha)\eta}{1-\eta}}$$

$$M_\ell = \frac{1-\alpha}{\alpha} \left(\eta \alpha^{1-(1-\alpha)\eta} (1-\alpha)^{(1-\alpha)\eta} \right)^{\frac{1}{1-\eta}} (r_k)^{-\frac{\alpha\eta}{1-\eta}}$$

$$M_y = M_k^\alpha M_\ell^{1-\alpha} = \left(\frac{1-\alpha}{\alpha} \right)^{1-\alpha} \left(\eta \alpha^{1-(1-\alpha)\eta} (1-\alpha)^{(1-\alpha)\eta} \right)^{\frac{1}{1-\eta}} (r_k)^{-\frac{\alpha}{1-\eta}}$$

4. Construct total tax $T(\hat{X}) = \tau Y(\hat{X})$ and aggregate loan demand function $B^f(\hat{X})$

$$B^f(\hat{X}) = \sum_{(z,\lambda)} \left[\lambda \frac{1}{\alpha} r_k k(z,\lambda;\hat{X}) \pi(z;\Lambda_z) \Lambda_\lambda(\lambda) \right].$$

C.2 Step 2: computation for the Markov equilibrium

Taking given the functions of $B^f(\Lambda, R)$ and $T(\Lambda, R)$, the government solve its problem. Let Ψ be the conditional CDF of default cost shock v. We solve the following problem:

Define the expected value H_V as follows

$$H_V(S, B') = \beta E_S \left\{ V(S', B') + \Psi(v^*(S', B')|v)v^*(S', B') - \int_{v^*(S',B')} v' d\Psi(v'|v) \right\}$$

1. Construct a large set of grids for v.
2. Guess $H_V^{\{0\}}(S, B')$, $q^{\{0\}}(S, B') = \frac{\beta\vartheta}{1-\beta(1-\vartheta)}$, and tax revenue $T_{x,j}^{\{0\}}(S, B, D, B')$ as follows. Let

$$N_j^{\{n\}}(S, B, D, B') = \bar{N}_j + \omega_j(1 - D) \left[q^{\{n\}}(S, B'(B))(1 - \vartheta)B \right].$$

If $N_j^{\{n\}}(S, B, D, B')/(1 - \theta) \geq B^f(\Lambda_j, 1/\beta)$ for region j

$$T_{x,j}^{\{n\}}(S, B, D, B') = T(\Lambda_j, 1/\beta);$$

otherwise

$$R_j^{\{n\}}(S, B, D, B') = \left(B^f\right)^{-1} \left(N_j^{\{n\}}(S, B, D, B')/(1 - \theta), \Lambda_j\right)$$

and

$$T_{x,j}^{\{n\}}(S, B, D, B') = T\left(\Lambda_j, R_j^{\{n\}}(S, B, D, B')\right).$$

3. Solve the government's problem

$$V^{\{n+1\}}(S, B) = \max_{G, B'} u_g(G) + \beta H_V^{\{n\}}(S, B')$$

subject to

$$G + \vartheta B = \sum_j T_{x,j}^{\{n\}}(S, B, D, B') + q^{\{n\}}(S, B') \left[B' - (1 - \vartheta)B\right].$$

4. We update the default cutoff v^*

$$v^*(S, B) = V^{\{n+1\}}(S, 0) - V^{\{n+1\}}(S, B),$$

H_V function

$$H_V^{\{n+1\}}(S, B') = \beta E_S \left\{ V^{\{n+1\}}(S', B') + \Psi(v^*(S', B')|v)v^*(S', B') - \int_{v^*(S', B')} v' d\Psi(v'|v) \right\},$$

q schedule

$$q^{\{n+1\}}(S, B') = \beta E_S \left\{ [1 - \Psi(v^*(S', B'))] \left(\vartheta + (1 - \vartheta)q^{\{n\}}(S', B''(S, B')) \right) \right\}.$$

5. Iterate procedure 3 to 5 until q and H_V function converge.