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Characterizing Global Value Chains: Production Length and Upstreamness
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ABSTRACT

We develop a new set of country-sector level indicators of Global Value Chains (GVCs) characteristics in terms of average production length, and relative "upstreamness" on a production network, which we argue are better than the existing ones in the literature. We distinguish production activities into four types: those whose value added is both generated and absorbed within the country, those whose value-added crosses borders only once for consumption, those whose value added crosses borders only once for production, and those whose value added crosses borders more than once. Based on such an accounting framework, we further decompose total production length into different segments. Using these measures, we characterize cross-country production sharing patterns and their evolution for 56 sectors and 44 countries over 2000-2014. While the production chain has become longer for the world as a whole, there are interesting variations across countries and sectors.

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1. Introduction

With falling trade barriers and communication costs, production has become more modularized or "longer," and has often involved different stages in different countries. The changing patterns of international trade and production correspondingly demand new statistical indicators to capture the new features. This paper proposes measures of production length and "upstreamness" associated with a country-sector. We argue these measures have more desirable properties than the existing ones in the literature.

A "value chain" represents value added at various stages of production, which runs from the initial phase such as R&D and design to the delivery of the final product to consumers. A value chain can be national if all stages of production occur within a country, or regional or global if different stages take place in different countries. In practice, most products or services are produced by a regional or global value chain. We will label all production processes that involve international trade in intermediate inputs as global value chains (GVCs).

Production length, as a basic measure of GVCs, is defined as the number of stages in a value chain, reflecting the complexity of the production process. Such measures are necessary to assess specialization patterns of countries in relatively upstream versus downstream stages of global production processes (Antras et al., 2012). Based on the production length, the upstreamness and downstreamness indexes are proposed in the recent literature (see Antras et al., 2012; and Miller and Temurshoev, 2015) to measure a sector/country's position in a global production process.

The recent work in the production length measures for GVCs started with Fally (2012), who proposed two measures, "distance to final demand," or "upstreamness," i.e., the average number of stages between production and final consumption, and "the average number of production stages embodied in each product" or "downstreamness" to quantify the length of production chains and a sector's position in the chain simultaneously. These two measures are further explored in Antras et al. (2012) and Antras and Chor (2013), respectively. Curiously, sector rankings by "upstreamness" and "downstreamness" measures do not coincide with each other. This implies certain inconsistency in the way that these measures are defined. As we will argue, a key source of the problem is that the existing measures start from a sector's gross output and have been defined as absolute measures.

As argued by Erik (2005, 2007), a production chain starts from the sector's primary inputs (or value added) such as labor and capital, not its gross output.[1] By defining production length as the number of stages between primary inputs in one country/sector to final products in another country/sector, our new measure provides better internal consistency and easier economic interpretations. For example, in our framework, the average production length of a value chain is the average number of times that the value-added created by the production factors in a country-sector has been counted as gross output in the sequential production process; it equals the ratio of the accumulated gross outputs to the corresponding value-added that induces the output.

Following the production activity accounting framework proposed by Wang et al. (2017a), we further split the total production length into a pure domestic segment, a segment related to "traditional" trade, and a segment related to GVCs that involve production sharing activities crossing national borders.

While "production length" counts the number of production stages, the "production position" of a country-sector on a value chain is a relative concept. The relative "upstreamness" or "downstreamness" in a global production network for a particular country-sector can only be determined by comparing production length measured by forward and backward inter-industry linkages. We propose a new production position measure as the relative distance of a particular production stage (country-sector) to the both ends of a value chain. Using our definitions, the

[1] It is important to bear in mind that gross outputs are endogenous variables, while primary inputs and final demand are exogenous variables in the standard Leontief model. Converting gross output (gross exports are part of it) into final demand is the key technical step to establishing their gross trade accounting framework in both Koopman, Wang, and Wei (2014) and Wang, Wei, and Zhu (2013).

sector ranking by upstreamness and downstreamness would be exactly inversely related. This removes one inconsistency with the existing measures in the literature.

The inconsistence of the existing measures has been recognized in the literature. For example, Antras et al. (2016) has defined an “upstreamness” index between any pair of sectors based on the “average propagation length" (APL) proposed by Dietzenbacher et al. (2005,2008). This is also invariant to whether one adopts a forward or backward industrial linkage perspective when computing the average number of stages between a pair of industries at the most detailed bilateral sector level. Escaith and Inomata (2016) have proposed similar ideas that use the ratio of forward and backward linkage based aggregate APL measure to identify the relative position of economies within regional and global supply chains, and applied such measures to study the changes in relative positions of East Asian economies between 1985 and 2005.

However, our measure of production length is different from the APL measure in two important ways. First, the economic interpretation is different. Production length measures the average number of times that value-added associated with certain primary factors in a country sector is counted as gross output along a production chain, until it is embodied in final products. It is the footprint of value-added created from a particular country/sector pair in the whole economy. APL is defined as the average number of stages that an exogenous shock starting in one industry has to go through before it has impacts on another industry, measuring the average distance of inter-industrial linkages between two industries. It focuses on propagation transmission of gross output across sectors, and has no relation with the magnitude of value-added in the economy.

Second, the computation is different. Production length is the ratio of gross output to related value-added or final products. Its denominator is value-added or final products generated from a value chain, its nominator is cumulative gross output of the value chain. APL can be computed by the Ghosh or Leontief inverse alone without involving sector value-added. The diagonal elements of the Ghosh/Leontief inverse are subtracted for APL to take out initial cost shock/demand injection, because such exogenous changes do not depend on the economy's industrial linkage and hence are not relevant to how far the "distance" is between two industries. The diagonal elements of Ghosh /Leontief inverse need to be kept for average production length, because the direct value-added created by primary factor inputs in the first stage of production matters for average production length. Without taking it into account, we cannot tell where the production line starts. Both measures are useful for some research questions. However, as we will show, the numerical results of production length are relatively robust. For example, the total production length will not change as the number of sector classification increase as long as the total gross output and GDP keep constant, while the numerical estimates of APL will change as the number of sector classifications changes.[2] More details on the differences and their aggregation properties are provided in Appendix A.

We apply these new measures to the recently updated Inter-Country Input Output database (2016 version of WIOD, Timmer et al., 2016) and obtain some interesting results. We show that Fally's (2012) result that the production length has become shorter in the United States (based on the US IO table) is not globally representative. Consequently, his main hypothesis that value-added has gradually shifted towards the downstream stage, closer to the final consumers, may only apply to some high income countries such as Japan and the United States.

Our empirical results differ from the existing literature in a number of ways. First, we show that emerging economies such as China experience a lengthening of the overall production chains over time. Because the lengthening by these countries dominates shortening of production by others, for the world as a whole, the production line has become longer over time. Second, we decompose changes in total production length into changes in the pure domestic segment, changes in the segment related to traditional trade, and changes in the segment related to global value chains. With such decomposition, we show that the international production length has progressively become a rising part of the overall production length, although this trend is stalled

[2] Appendix A also provides a numerical example to show the differences.

after 2011. Finally, we analyze the role GVCs have played in transmitting economic shocks in the recent global financial crisis and find that a country/sector's GVC participation intensity has significant effects. The deeper and more intense a country-sector's participation in GVCs, the stronger the impact of the global economic shock. In addition, the effect of a global crisis increases with the relative length of the international portion of the relevant global value chains.

This paper builds on but also goes beyond the production activity accounting framework developed by Wang et al.(2017). The two papers collectively form a set of GVC indicators, which could facilitate future empirical work on GVC-related topics. The rest of the paper is organized as follows: Section 2 formally defines total production length measure and how it can be decomposed into domestic and GVC production length based on the production activity account framework proposed by Wang et al. (2017); Section 3 discusses the relation between production length and production line position measure and formally define the GVC position index; Section 4 reports major empirical results based on the 2016 version of WIOD; and Section 5 concludes.

2. The length of production

We define the length of production as the average number of production stages between the primary inputs in a country-sector to final products in another country/sector: it is the average number of times that value-added created by the prime factors employed in the country/sector pair has been counted as gross output in the production process until it is embodied in final products.

Without loss generality, let's consider an Inter-Country Input-Output (ICIO) model for *G* countries and *N* sectors. Its structure can be described by Table 1:

Table 1 General Inter-Country Input-Output table

Outputs / Inputs		Intermediate Use				Final Demand				Total Output
		1	2	⋯	G	1	2	⋯	G	
Intermediate Inputs	1	Z^{11}	Z^{12}	⋯	Z^{1g}	Y^{11}	Y^{12}	⋯	Y^{1g}	X^{1}
	2	Z^{21}	Z^{22}	⋯	Z^{2g}	Y^{21}	Y^{22}	⋯	Y^{2g}	X^{2}
	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮	⋮
	G	Z^{g1}	Z^{g2}	⋯	Z^{gg}	Y^{g1}	Y^{g2}	⋯	Y^{gg}	X^{g}
Value-added		Va^{1}	Va^{2}	⋯	Va^{g}					
Total input		$(X^{1})'$	$(X^{2})'$	⋯	$(X^{g})'$					

where Z^{sr} is an N×N matrix of intermediate input flows that are produced in country s and used in country r; Y^{sr} is an N×1 vector giving final products produced in country s and consumed in country r; X^{s} is also an N×1 vector giving gross outputs in country s; and VA^{s} denotes a 1×N vector of direct value added in country s. In this ICIO model, the input coefficient matrix can be defined as $A = Z\hat{X}^{-1}$, where $\hat{X}$ denotes a diagonal matrix with the output vector X in its diagonal. The value added coefficient vector can be defined as $V = Va\hat{X}^{-1}$. Gross outputs X can be split into intermediate and final products, $AX + Y = X$. Rearranging terms, we can reach the classical Leontief (1936) equation, $X = BY$, where $B = (I - A)^{-1}$ is the well-known (global) Leontief inverse matrix.

2.1 Length of total production

Based on Leontief (1936), value added and final products in the global ICIO model specified in Table 1 are linked by the following equation: $Va' = \hat{V}X = \hat{V}BY$. It is obvious that primary inputs (value added) of sector *i* only can be directly embodied in final products of sector *j* if sector *i* and sector *j* are the same. Therefore, in the first stage of any production process, the value added of sector *i of* country s embodied in final products of sector *j of* country r can be quantified as $\delta_{ij}^{sr} v_i^s y_j^r$, where δ_{ij}^{sr} is a dummy variable. If sector *i* and sector *j*, country s and country r are the same, δ_{ij}^{sr} equals 1, otherwise it equals 0. At this stage, the length of the production chain is 1, and the output in this production chain is $\delta_{ij}^{sr} v_i^s y_j^r$.

In the second stage, the value added of sector *i* of country s directly embodied in its gross output that is used as intermediates to produce final products of sector *j* of country r can be measured as $v_i^s a_{ij}^{sr} y_j^r$, which is the value added of sector *i of* country s indirectly (first round) embodied in final products of sector *j* of country r. Up to this stage, the length of the production chain is 2, and the output induced by this production chain is $2v_i^s a_{ij}^{sr} y_j^r$, which accounts for value-added $v_i^s a_{ij}^{sr} y_j^r$ as output twice, once for sector i of country s, once for sector j of country r.

In the third stage, indirect value added from sector *i* of country s can be embodied in intermediate goods from any sector and countries, which are used as intermediates to produce final products in sector j of country r. Domestic value added from sector i of country s in this stage can be measured as$v_i^s \sum_{t,k}^{G,N} a_{ik}^{st} a_{kj}^{tr} y_j^r$. This is the second round indirect value-added from sector *i* of country s embodied in intermediate goods used by any sector k of country t and absorbed by final goods in sector *j* of country r. At this stage, the length of the production chain is 3, and the output induced by this production chain is $3 v_i^s \sum_{t,k}^{G,N} a_{ik}^{st} a_{kj}^{tr} y_j^r$. The same value-added originally produced from sector *i* of country s is counted as output three times, once in sector i of country s, once in sector k of country t, and once in sector *j* of country r.

The same goes on for the succeeding stages.

Generalizing the above process to include all rounds of value-added in sector *i* of country s directly and indirectly embodied in final goods of sector *j* in country r, we obtain the following:

$$\delta_{ij}^{sr} v_i^s y_j^r + v_i^s a_{ij}^{sr} y_j^r + v_i^s \sum_{t,k}^{G,N} a_{ik}^{st} a_{kj}^{tr} y_j^r + \cdots = v_i^s b_{ij}^{sr} y_j^r$$

$$\delta_{ij}^{sr} = \begin{cases} 1, & i = j \text{ and } s = r \\ 0, & i \neq j \text{ or } s \neq r \end{cases} \tag{1a}$$

Expressing (1a) in matrix notation

$$\hat{V}\hat{Y} + \hat{V}A\hat{Y} + \hat{V}AA\hat{Y} + \cdots = \hat{V}(I + A + AA + \cdots)\hat{Y}$$

$$= \hat{V}(I-A)^{-1}\hat{Y} = \hat{V}B\hat{Y} \quad (1b)$$

Each element in the $\hat{V}B\hat{Y}$matrix represents the value added from a source country/sector directly or indirectly used in the production of final goods and services in a particular country/sector. The element of row (*s*, *i)* and column (*r, j)* in the matrix, $v_i^s b_{ij}^{sr} y_j^r$, is the total value added (direct and indirect) of sector *i* in country s embodied in the final products produced by sector *j* of country r.

Using the length of each stage as weights and summing across all production stages, we obtain the following equation that gives the total output in (induced by) a particular production chain (sector *i* in source country s to sector *j* in final production country r):

$$\delta_{ij}^{sr} v_i^s y_j^r + 2v_i^s a_{ij}^{sr} y_j^r + 3v_i^s \sum_{t,k}^{G,N} a_{ik}^{st} a_{kj}^{tr} y_j^r + \cdots = v_i^s \sum_{t,k}^{G,N} b_{ik}^{st} b_{kj}^{tr} y_j^r \quad (2a)$$

It captures the footprint of sector value added in each production stage. Expressing in matrix notation

$$\hat{V}\hat{Y} + 2\hat{V}A\hat{Y} + 3\hat{V}AA\hat{Y} + \cdots = \hat{V}(I + 2A + 3AA + \cdots)\hat{Y}$$

$$= \hat{V}(B + AB + AAB + \cdots)\hat{Y} = \hat{V}BB\hat{Y} \quad (2b)$$

The element of row (*s,i)* and column (*r,j)* in the matrix at the right side of equation (2b), $v_i^s \sum_{t,k}^{G,N} b_{ik}^{st} b_{kj}^{tr} y_j^r$, is the total output induced by the production chain from sector i's value added in country s and finally absorbed by sector j's final products of country r. Dividing by $v_i^s b_{ij}^{sr} y_j^r$, the total value-added of sector i of country s embodied in the final product of sector j of country r, the average production length of value added from sector *i* of country s to final products of sector *j in* country r can be computed as:

$$plvy_{ij}^{sr} = \frac{v_i^s \sum_{t,k}^{G,N} b_{ik}^{st} b_{kj}^{tr} y_j^r}{v_i^s b_{ij}^{sr} y_j^r} \quad (3a)$$

Expressing in matrix notation

$$\text{PLvy} = \frac{\hat{V}BB\hat{Y}}{\hat{V}B\hat{Y}} \quad (3b)$$

The denominator is the total value added from sector i of country s contributing to the final product in sector j of country r, and the numerator is the total output accumulated along the production chain induced by the value added. When value added is used as input in a production stage, either as primary input or embodied in intermediate inputs, it will be counted as output where it is used. Therefore, the length of a production chain is the number of times of value added counted as output in the production chain, from the first time it is used as the primary input until it absorbed by a final product.

Aggregating equation (3) over all products j of country r, we obtain the total average production length of value added generated in sector i of country s, i.e., the average production length measure of sector value-added in country s based on forward industrial linkage:

$$plv_i^s = \frac{Xv_i^s}{Va_i^s} = \frac{v_i^s \sum_{t,k}^{G,N} b_{ik}^{st} \sum_{r,j}^{G,N} b_{kj}^{tr} y_j^r}{v_i^s \sum_{r,j}^{G,N} b_{ij}^{sr} y_j^r} = (x_i^s)^{-1} \sum_{t,k}^{G,N} b_{ik}^{st}\, x_k^t = \sum_{t,k}^{G,N} h_{ik}^{st} \tag{4a}$$

where $\sum_{r,j}^{G,N} b_{ij}^{sr}\, y_j^r y_j^r = x_i^s$ and $\sum_{r,j}^{G,N} b_{kj}^{tr}\, y_j^r y_j^r = x_k^t$. Expressing in matrix notation gives:

$$PLv = \frac{\text{Xv}}{\text{Va}} = \frac{\hat{V}BB\hat{Y}u'}{\hat{V}B\hat{Y}u'} = \frac{\hat{V}BBY}{\hat{V}BY} = \frac{\hat{V}BX}{\hat{V}X} = \hat{X}^{-1}BX = \hat{X}^{-1}B\hat{X}u' = Hu' \tag{4b}$$

where u is a 1×GN unit vector with all its elements equal to 1, and H is the Ghosh inverse matrix[3].

It is the sum along the rows of the Ghosh inverse matrix, which equals the total value of gross outputs that are related to one unit of value added created by primary inputs from a particular sector. Therefore, equation (4) measures total gross outputs induced by one unit of value added at the sector level, which are the footprints of each sector's value added in the economy as a whole. The longer the production chain, the greater the number of downstream production stages a sector's value added is counted as gross output in the economy. Xv, the nominator in (4b) equals average production length multiplied by sector value added, which is total gross output induced by sector value added. It can be seen as follows:

[3] The definition of Ghosh model and the linkage with Leontief model can be expressed in Appendix B.

$$Xv = \hat{V}BB\hat{Y}u' = \hat{V}BBY = \hat{V}BX = \hat{V}X + \hat{V}AX + \hat{V}AAX + \hat{V}AAAX + \cdots \quad (5)$$

The first term on the right side of equation (5) is the value added directly embodied in its own sector's output, and we may name it as the first footprint of the sector value added in its own sector gross output; the second term is the value added embodied in its own sector's gross output used by all sectors as intermediates to produce outputs, and we may name it as the second footprint of the sector value added directly and indirectly embodied in total gross outputs of this second stage production process. Summing up all terms on the right hand side of (5), we obtain all footprints of sector value added in the whole economy, which equals the total value of gross outputs that relates to the sector value added created by primary inputs from a particular sector. Therefore, the average production length of a particular sector based on forward industrial linkages equals the ratio of total gross output induced by the value added in the sector and the sector value-added.

Using the shares of sectoral value added in GDP as weights to aggregate equation (5) over all sectors, we obtain:

$$PLvw = \left(Va\hat{X}^{-1}B\hat{X}u'\right)/(uVa) = (VBX)/GDP = (uX)/GDP \quad (6)$$

where $Va\hat{X}^{-1} = V$, $\hat{X}u' = X$ and $VB = u$. Equation (6) indicates that the average length of the production chain in the world economy equals the ratio of total gross outputs to GDP,[4] which can be regarded as an index of complexity of the production process in the global economy, i.e., the higher this ratio, the more complex the production process in the global economy.

Aggregating equation (3) over value-added from all sectors i of country s that have contributed to the final goods and services produced by sector j of country r, we obtain the production length measure based on backward industrial linkages as:

$$ply_j^r = \frac{Xy_j^r}{y_j^r} = \frac{\sum_{s,i}^{G,N} v_i^s \sum_{r,j}^{G,N} b_{ik}^{st} b_{kj}^{tr} y_j^r}{\sum_{s,i}^{G,N} v_i^s b_{ij}^{sr} y_j^r} = \sum_{r,j}^{G,N} b_{kj}^{tr} \quad (7a)$$

[4] This is also recognized by Fally (2012).

where $\sum_{s,i}^{G,N} v_i^s\, b_{ij}^{sr} = \sum_{s,i}^{G,N} v_i^s\, b_{ik}^{sr} = 1$. Expressing in matrix notation

$$PLy = \frac{Xy}{Y} = \frac{u\hat{V}BB\hat{Y}}{u\hat{V}B\hat{Y}} = \frac{VBB\hat{Y}}{VB\hat{Y}} = uB \qquad (7b)$$

It is the sum along the column of the Leontief inverse matrix, which equals the total value of inputs induced by a unit of final product produced in a particular sector. Therefore, equation (15) measures total intermediate inputs induced by a unit value of a particular final product throughout all upstream sectors in the economy, which is called the footprint of final goods and services in the literature. The longer the production chain, the greater the number of upstream production stages a particular final product has in the economy. Using the sectoral ratio of final goods to GDP as a weight to aggregate equation (15) over all sectors, we obtain:

$$PLyw = \left(VBB\hat{Y}u'\right)/\left(VB\hat{Y}u'\right) = (uBY)/GDP = (uX)/GDP \qquad (8)$$

which gives the same gross output to GDP ratio as equation (6) and therefore has the same economic interpretation.

It is worth noting that the average length of a production chain based on forward industrial linkages as expressed in equation (4) is mathematically equivalent to the upstreamness index defined by Fally (2012a, 2012b, 2013) and Antras et al. (2012, 2013).[5] On the other hand, the average length of a production chain based on backward industrial linkages expressed in equation (7) is mathematically equivalent to the downstreamness index defined by Antras and Chor (2013). However, there are two notable differences. First, similar to Miller and Temurshoev (2013), our indexes are obtained by the sum of the rows/columns of the Ghosh/Leontief inverse matrices respectively, which are simpler in mathematics and are part of the classic input-output literature; Second, we measure a production chain length from primary inputs in sector *i* of country s to final products of sector *j* in country r, starting from primary inputs (value added), not gross outputs (as Fally and Antras did), and provide clear economic interpretations for both the numerator and denominator in the production length indexes

[5] The proof is provided in Appendix C.

discussed above. Most important, such concepts of production length allow us to decompose the total production length in the world economy into different segments.

According to the value-added and final goods production decomposition framework proposed by Wang et al. (2017), production activities can be divided into four parts based on whether there are cross border activities for production as follows:

$$\hat{V}B\hat{Y} = \hat{V}L\hat{Y}^D + \hat{V}L\hat{Y}^F + \hat{V}LA^F L\hat{Y}^D + \hat{V}LA^F(B\hat{Y} - L\hat{Y}^D) \quad (9)$$

Parts 1 and 2 involve no cross-country production sharing activities, and satisfy domestic and foreign demand respectively. Value-added in Part 2 crosses boarders once, but only for consumption activities; all value-added embodied in its intermediate inputs comes from domestic sources, so it can be considered as "traditional trade" in value-added. Value added in Parts 3 and 4 are embodied in trade of intermediate products: part 3 is value-added embedded in intermediate products absorbed by direct importers; part 4 is value-added crossing boarders at least twice to satisfy domestic and/or foreign final demand. These two parts measure GVC production activities. In the same logic, total production length also can be decomposed into 4 segments: the first and second segments measure length of pure domestic production and traditional trade; the third and fourth segments measure production length of simple and complex GVC activities.

2.2 Length of pure domestic production

Let us first consider the segment of domestic value added that is generated by production activities entirely within a country at each stage of production.

As pointed out by Wang et al. (2017), in an infinite sequence of production process, domestic value added of the source country embodied in its final products for domestic final demand equals $\hat{V}L\hat{Y}^D$. Following a similar logic as equation (1), using the length of each production stage as weights and summing up all production stages, we obtain an equation that gives the gross output induced by $\hat{V}L\hat{Y}^D$ as follows:

$$X\_D = \hat{V}\hat{Y}^D + 2\hat{V}A^D\hat{Y}^D + 3\hat{V}A^D A^D\hat{Y}^D + \cdots$$

$$= \hat{V}(I - A^D)^{-1}(I - A^D)^{-1}\hat{Y}^D = \hat{V}LL\hat{Y}^D \quad (10)$$[6]

where $I + A^D + A^D A^D + \cdots = L$

Because production activities that generate this part of domestic value-added have no relation with cross border trade, we define its production length as that of pure domestic production. It equals the portion of gross output of a particular country generated by the production of the country's GDP without any cross-border production and trade activities (how many times $\hat{V}L\hat{Y}^D$ has been counted as gross output in the economy). Therefore, the average pure domestic production length of a particular country equals the ratio of this portion of gross output to the corresponding domestic value added, and can be expressed as[7]

$$PL\_D = \frac{\hat{V}LL\hat{Y}^D}{\hat{V}L\hat{Y}^D} \quad (11)$$

Similarly, production of "traditional trade" ($\hat{V}L\hat{Y}^F$) also takes place entirely domestically. The gross output it induced can be expressed as

$$X\_RT = \hat{V}\hat{Y}^F + 2\hat{V}A^D\hat{Y}^F + 3\hat{V}A^D A^D\hat{Y}^F + \cdots$$
$$= \hat{V}(I - A^D)^{-1}(I - A^D)^{-1}\hat{Y}^F = \hat{V}LL\hat{Y}^F \quad (12)$$

And its production length equals the average times $\hat{V}L\hat{Y}^F$ has been counted as gross output in the economy:

$$PL\_RT = \frac{\hat{V}LL\hat{Y}^F}{\hat{V}L\hat{Y}^F} \quad (13)$$

2.3. Length of Global Value Chain production

The production process of GVC trade is more complicated than the previous two segments. To better understand such a process, let us start from considering the segment of domestic value added that is generated by production activities related to a country's bilateral intermediate exports at each stage of production.

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[6] A detailed mathematical proof of equations (9) is provided in Appendix D.

[7] A division symbol below denotes elements-wide divisions.

Obviously, intermediate exports only occur in a cross country production process that has at least two stages. In such a two-stage production process, domestic value added generated from the source country will be first embodied in its gross output that is used as intermediate exports to other countries and used by these countries to produce final products consumed there or exported. It can be measured as $\hat{V}A^F\hat{Y}$. Both its domestic and international production length equal 1. The output induced by this production chain is $\hat{V}A^F\hat{Y}$, which account value-added $\hat{V}A^F\hat{Y}$ as output twice, once in the source country, once in the importing country.

In a three-stage production process, the domestic value added generated from a particular country will be embodied in the final products produced from the third stage and consumed in all possible destination counties. It can be measured as $\hat{V}A^DA^F\hat{Y}+\hat{V}A^FA\hat{Y}$ and can be decomposed into two parts: $\hat{V}A^DA^F\hat{Y}$, and $\hat{V}A^FA\hat{Y}$. Their domestic production lengths equal 2, and 1, respectively, and their international production lengths equal 1, and 2, respectively. The output induced by such a production chain is $2\hat{V}A^DA^F\hat{Y}+\hat{V}A^FA\hat{Y}$ and $\hat{V}A^DA^F\hat{Y}+2\hat{V}A^FA\hat{Y}$, respectively. The same value-added originally produced from the source country is counted as output three times, either twice in the source country, once in the importing country, or once in the source country, once in importing country, and once in the other country.

The same goes on for an *n*-stage production process.

Summing over the production stages in an infinite stage production process, we have

$$VY\_GVC=\hat{V}A^F\hat{Y}+\hat{V}A^DA^F\hat{Y}+\hat{V}A^F\mathrm{A}\hat{Y}+\hat{V}A^DA^DA^F\hat{Y}+\hat{V}A^DA^F\mathrm{A}\hat{Y}+\hat{V}A^F\mathrm{AA}\hat{Y}+\cdots$$
$$=\hat{V}LA^F\hat{Y}+\hat{V}LA^F\mathrm{A}\hat{Y}+\hat{V}LA^F\mathrm{AA}\hat{Y}+\cdots=\hat{V}LA^F\mathrm{B}\hat{Y} \tag{14}$$

It measures the amount of domestic value added generated from the production of gross intermediate exports and can be further decomposed into two parts according to equation (3) in Wang et al. (2017a) as follows:

$$VY\_GVC=\underbrace{\hat{V}LA^FL\hat{Y}^D}_{VY\_GVC\_S}+\underbrace{\hat{V}LA^F(B\hat{Y}-L\hat{Y}^D)}_{VY\_GVC\_C} \tag{15}$$

They are the source country's DVA in bilateral intermediate exports directly absorbed in the importing country (VY_GVC_S), and used by the importing country to produce the final or intermediate exports (indirect absorbed by direct importing country or re-exported to other countries, VY_GVC_C), respectively. All of them are involved in production sharing activities with foreign countries, so we label them together as GVC production activities.

Following the same logic to derive equations (1), i.e., using the domestic or international production length of each stage of intermediate exports production discussed above as weights and summing across all production stages, we can obtain the domestic gross output generated by GVC production activities as:

$$Xd\_GVC = \hat{V}A^F\hat{Y} + 2\hat{V}A^DA^F\hat{Y} + \hat{V}A^F\mathrm{A}\hat{Y} + 3\hat{V}A^DA^DA^F\hat{Y} + 2\hat{V}A^DA^F\mathrm{A}\hat{Y}$$
$$+\hat{V}A^F\mathrm{AA}\hat{Y} + \cdots = \hat{V}LLA^F\mathrm{B}\hat{Y} = \underbrace{\hat{V}LLA^FL\hat{Y}^D}_{Xd\_GVC\_S} + \underbrace{\hat{V}LLA^F(B\hat{Y} - L\hat{Y}^D)}_{Xd\_GVC\_C} \qquad (16)$$

Term 1 is the source country's domestic gross outputs generated by the production of simple GVC exports, in which the source country's domestic value added in intermediate exports is directly consumed by its trading partners. We label it as Xd_GVC_S. Term 2 is the source country's domestic gross outputs induced by production of complex GVC exports, in which the source country's value added is used by its partner country to produce exports. We label it as Xd_GVC_C. All of those gross outputs are associated with domestic value-added in source country's intermediate exports before it leaves the country through forward domestic inter-industrial linkage.

Therefore, the average domestic length of GVC production can be computed as

$$PLd\_GVC = \frac{Xd\_GVC}{VY\_GVC} = \frac{Xd\_GVC\_S + Xd\_GVC\_C}{VY\_GVC\_S + VY\_GVC\_C} = \frac{\hat{V}LLA^F\mathrm{B}\hat{Y}}{\hat{V}LA^F\mathrm{B}\hat{Y}} \qquad (17)$$

The average domestic production length of the two components are labeled as PLd_GVC_S and PLd_GVC_C respectively.

Similarly, the total international gross outputs induced by domestic value-added of source country embodied in its intermediate exports can be expressed as:

$$Xi\_GVC = \hat{V}A^F\hat{Y} + \hat{V}A^DA^F\hat{Y} + 2\hat{V}A^F\mathrm{A}\hat{Y} \ \ldots$$
$$= \hat{V}LA^F\mathrm{BB}\hat{Y} = \underbrace{\hat{V}LA^FLL\hat{Y}^D}_{Xvi\_GVC\_S} + \underbrace{\hat{V}LA^F(BB\hat{Y} - LL\hat{Y}^D)}_{Xvi\_GVC\_C} \qquad (18)$$

Term 1 represents international gross outputs generated in the process between domestic value-added of the source country embodied in its intermediate exports arriving at the importing country, and the value-added absorbed by final products consumed in the importing country without further border crossing. We label it as Xi_GVC_S. Term 2 represents international gross outputs generated in the process, starting from the domestic value-added of the source country embodied in its intermediate exports and arriving at importing country until it is used in final goods production for the source country or third countries. We label it as $Xi\_GVC\_C^s$. All of those gross outputs are associated with the production of intermediate exports of the source country after it leaves the country through forward inter-industrial inter-country linkages. Therefore, the average international length of the source country's GVC production can be computed as:

$$PLi\_GVC = \frac{Xi\_GVC}{VY\_GVC} = \frac{Xi\_GVC\_S + Xi\_GVC\_C}{VY\_GVC\_S + VY\_GVC\_C} = \frac{\hat{V}LA^F \mathrm{BB}\hat{Y}}{\hat{V}LA^F \mathrm{B}\hat{Y}} \tag{19}$$

The average international production length of the two components are labeled as PLi_GVC_S and PLi_GVC_C respectively.

Summing equations (16) and (18), we obtain the total average production length of domestic value-added of the source country embodied in its bilateral intermediate exports as follows:

$$PL\_GVC = PLd\_GVC + PLi\_GVC = \frac{Xd\_GVC}{VY\_GVC} + \frac{Xi\_GVC}{VY\_GVC} = \frac{X\_GVC}{VY\_GVC} \tag{20}$$

Obviously, X_GVC measures total gross outputs generated by GVC production activities in the world economy. Intermediate exports used by direct importers in their production of domestically consumed final products are involved in the production process only within the direct importing country, therefore, the international production length of the source countries' domestic value-added embodied in such intermediate exports equals their production length in the direct importing country. The international production length of the remaining part of gross intermediate exports can be different from their domestic production length. Since this embodied domestic value-added crosses national borders at least twice, it represents relatively complex cross-country production sharing arrangements.

There is a nice symmetry among the terms in equations (16)–(19): all of them are based on the measurement and decomposition of both domestic value-added in intermediate exports and their induced gross outputs. It is consistent with the gross trade accounting framework proposed in KWW (2014). Using corresponding components of domestic value-added embodied in intermediate trade in equation (15) as the denominators to divide equations (16) and (18) (i.e., the corresponding part of value-added induced gross outputs as numerators), we can obtain the average length of production of each segment and their weighted average in a particular global value chain (equations 17 and 19). It measures the amount of global gross output that can be generated by one unit of domestic value-added in intermediate exports from source country and its total subsequent utilization in the global production network.

Summing the numerator of equations (11), (13), Xd_GVC and Xi_GVC defined in equations (16) and (18) respectively, we obtain

$$X\_D + X\_RT + Xd\_GVC + Xi\_GVC = \hat{V}LL\hat{Y}^D + \hat{V}LL\hat{Y}^F + \hat{V}BB\hat{Y} - \hat{V}LL\hat{Y}$$
$$= \hat{V}BB\hat{Y} = Xvy \qquad (21)$$

Equation (21) shows clearly that the sum of gross output induced by traditional and GVC exports (equals global total output induced by domestic value-added in gross exports of source country to the world) plus the gross output induced by pure domestic production defined in equation (12) equals $Xvy = \hat{V}BB\hat{Y}$, the total gross output induced by production of sector value added used in final goods in the whole world economy as defined in equation (2b).

2.4 GVC production length and number of border crossing

International length of GVC production specified above can be further decomposed into number of border crossings of intermediate trade flows (border crossing for production) and domestic production length in all countries involved in the global value chain after intermediate exports leaving the source country.

It can be shown that Xi_GVCin equation (25) can also be decomposed into the following 3 terms:

$$Xi\_GVC = \hat{V}LA^F BB\hat{Y} \ = \hat{V}LA^F B\hat{Y} + \hat{V}LA^F BA^F B\hat{Y} + \hat{V}LA^F BA^D B\hat{Y} \qquad (22)$$

The first term accounts for intermediate exports crossing borders for the first time, equals to domestic value-added of source Country (s) embodied in its intermediate exports to partner Country (r) to produce final products for domestic consumption or exports；the second term accounts for domestic value-added in intermediate exports from the home country that is embodied in the importer’s intermediate products that are exported to the world；the last term accounts for domestic value-added in intermediate exports from Country (s) that is embodied in intermediate inputs used by all countries’ domestic production. Therefore, the sum of the first and second terms equals the total amount of domestic value-added from the source country that has been accounted for its intermediate exports. It can be expressed as

$$E\_GVC = \hat{V}LA^F B\hat{Y} + \hat{V}(B-L)A^F B\hat{Y} = \hat{V}BA^F B\hat{Y} \qquad (23a)$$

The last term in the right hand of equation (22a) is total intermediate exports induced by domestic value added in intermediate exports of the source country.

E_GVC can be further decomposed into gross exports induced by simple and complex GVC production activities in the source country as follows:

$$E\_GVC = E\_GVC\_S + E\_GVC\_C = \hat{V}BA^F B\hat{Y}$$

$$= \hat{V}LA^F L\hat{Y}^D + (\hat{V}BA^F B\hat{Y} - \hat{V}LA^F L\hat{Y}^D) \qquad (23b)$$

Similarly, the last term in equation (21) is the total amount of intermediate inputs that has been accounted for after it crosses national borders and is used in domestic production within all countries involved in the global value chain:

$$Xf\_GVC = \hat{V}LA^F BA^F B\hat{Y} \qquad (24a)$$

Xf_GVC can also be further decomposed into gross outputs induced by VY_GVC_S and VY_GVC_C .

$$Xf\_GVC = Xf\_GVC\_S + Xf\_GVC\_C$$

$$= \hat{V}LA^F(L-I)L\hat{Y}^D + \hat{V}LA^F[BA^F B\hat{Y} - (L-I)L\hat{Y}^D] \qquad (24b)$$

Dividing equations (23a) and (24a) by VY_GVC, we decompose international production length of global value chain into two segments: (1) the average number of border crossings for production activities; (2) the average domestic production length of GVC activities within all countries involved in the GVCs after intermediate exports leave the source country. Adding the average domestic production length PLd_GVC defined in equation (17), we decompose the total average production length of GVC activities into three segments:

$$PL\_GVC = PLd\_GVC + CB\_GVC + PLf\_GVC$$
$$= \frac{Xd\_GVC}{VY\_GVC} + \frac{E\_GVC}{VY\_GVC} + \frac{Xf\_GVC}{VY\_GVC} \quad (25)$$

The structure and internal linkage of our production length and border crossing index system can be represented as a tree diagram, as shown in Figure 1.

Muradov (2016) has proposed a measure of the average number of border crossings. Different from the measure for the number of border crossings defined in this paper, his measure includes not only border crossings for production, but also border crossings for consumption (it also accounts for border crossings of the final goods trade). Both measures are useful and can be used in different settings. A detailed derivation of equations (21) to (23) and the relationship between Muradov's (2016) border crossing measure of and what is defined in this paper is provided in appendix E for interested readers.

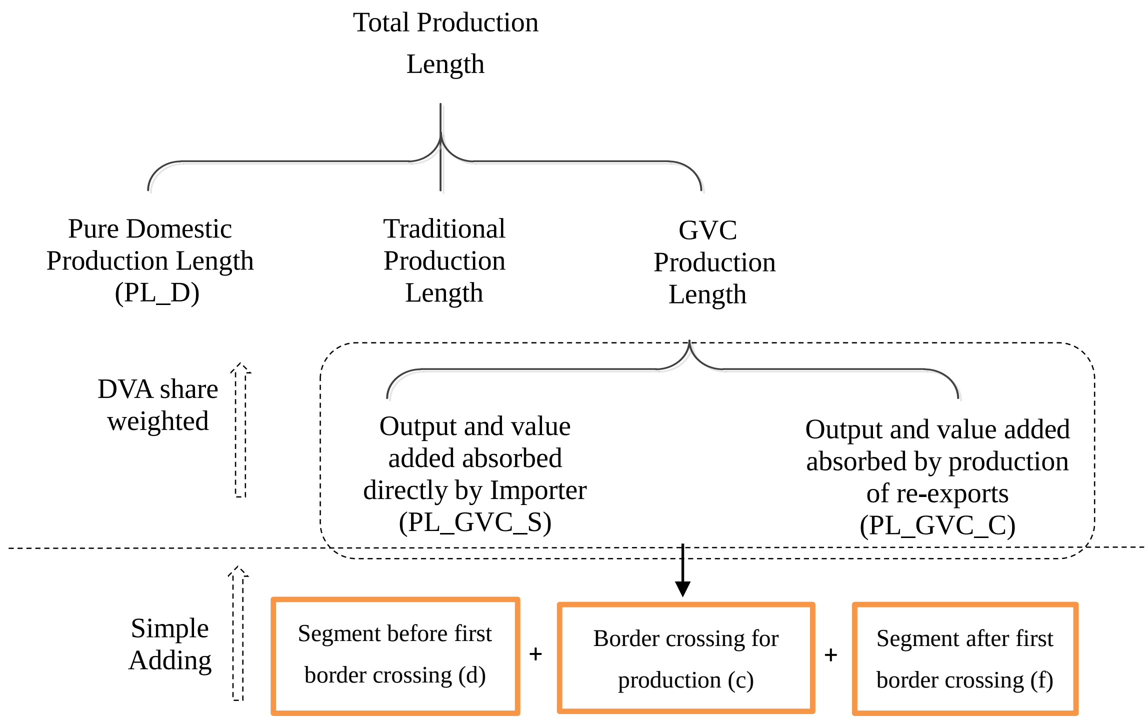


Figure 1 Index System for Production Length

2.5 Production length based on forward and backward inter-industry cross country linkage

Based on the decomposition of value added and its induced gross output in sections 2.2 to 2.4, following the same logic of equations (4) to sum each component of those GN by GN decomposition matrixes along the row across columns (horizontally), we can obtain decomposition of domestic value-added for each country/sector pair (GDP by industry) and their induced gross output (Xv). The corresponding domestic and international gross outputs induced by different parts of value added production can by computed as:

$$Xv = Xv\_D + Xv\_RT + Xv\_GVC = X\_Du' + X\_RTu' + X\_GVCu' = \hat{V}BB\hat{Y}u' \tag{26a}$$

$$Xv\_GVC = Xv\_GVC\_S + Xv\_GVC\_C = X\_GVC\_Su' + X\_GVC\_Cu' = Xvd\_GVC\_S + Ev\_GVC\_S + Xvf\_GVC\_S + Xvd\_GVC\_C + Ev\_GVC\_C + Xvf\_GVC\_C \tag{26b}$$

The ratio of these gross output components to those corresponding components of value added production decomposition in the source country, is the average domestic or international production length based on forward inter-industry and cross-country linkage.

$$PLv = PLv\_D + PLv\_RT + PLv\_GVC = \frac{Xv\_D}{V\_D} + \frac{Xv\_RT}{V\_RT} + \frac{Xv\_GVC}{V\_GVC} \quad (27a)$$

$$PLv\_GVC = PLv\_GVC\_S + PLv\_GVC\_C = \frac{Xv\_GVC\_S}{V\_GVC\_S} + \frac{Xv\_GVC\_C}{V\_GVC\_C}$$

$$= PLvd\_GVC\_S + CBv\_GVC\_S + PLvf\_GVC\_S + PLvd\_GVC\_C + CBv\_GVC\_CPLvf\_GVC\_C$$

$$= \frac{Xvd\_GVC\_S}{V\_GVC\_S} + \frac{Ev\_GVC\_S}{V\_GVC\_S} + \frac{Xvf\_GVC\_S}{V\_GVC\_S} + \frac{Xvd\_GVC\_C}{V\_GVC\_C} + \frac{Ev\_GVC\_C}{V\_GVC\_C} + \frac{Xvf\_GVC\_C}{V\_GVC\_C} \quad (27b)$$

Based on the decomposition of value added and its induced gross output in section 2.2 to 2.4, following the same logic of equations (7) to sum each component of those GN by GN decomposition matrixes along the column across rows (vertically), we can obtain decomposition of final goods and services production for each country/sector pair and their induced gross output (Xy). The corresponding domestic and international gross outputs induced by different parts of final goods and services production can by computed as:

$$Xy = Xy\_D + Xy\_RT + Xy\_GVC = uX\_D + uX\_RT + uX\_GVC = u\hat{V}BB\hat{Y} \quad (28a)$$

$$Xy\_GVC = Xy\_GVC\_S + Xy\_GVC\_C = uX\_GVC\_S + uX\_GVC\_C$$

$$= Xyd\_GVC\_S + Ey\_GVC\_S + Xyf\_GVC\_S + Xyd\_GVC\_C + Ey\_GVC\_C + Xyf\_GVC\_C$$

(28b)

Therefore, the ratio of these gross output components to those corresponding components of final products produced in their completion location, is the average domestic and international production length based on backward inter-industry and cross-country linkage.

$$PLy = PLy\_D + PLy\_RT + PLy\_GVC = \frac{Xy\_D}{V\_D} + \frac{Xy\_RT}{V\_RT} + \frac{Xy\_GVC}{V\_GVC} \quad (29a)$$

$$PLy\_GVC = PLy\_GVC\_S + PLy\_GVC\_C = \frac{Xy\_GVC\_S}{Y\_GVC\_S} + \frac{Xy\_GVC\_C}{Y\_GVC\_C}$$

$$= PLyd\_GVC\_S + CBy\_GVC\_S + PLyf\_GVC\_S + PLyd\_GVC\_C + CBy\_GVC\_C + PLyf\_GVC\_C$$

$$= \frac{Xyd\_GVC\_S}{Y\_GVC\_S} + \frac{Ey\_GVC\_S}{Y\_GVC\_S} + \frac{Xyf\_GVC\_S}{Y\_GVC\_S} + \frac{Xyd\_GVC\_C}{Y\_GVC\_C} + \frac{Ey\_GVC\_C}{Y\_GVC\_C} + \frac{Xyf\_GVC\_C}{Y\_GVC\_C} \quad (29b)$$

Detailed derivations can be found in Appendix F.

Because global final demand always sums to global value-added, the forward and backward linkage based production lengths and each of their segments is equal to each other at the global level. However, they are not equal to each other at the country or country/sector level due to international trade and cross border production activities. This naturally raises the questions: What is the relation between production length measure and production line position? Can we use production length measure directly to infer upstreamness or downstreamness of a country or a country/sector pair? Current literature is not clear on such important questions and often uses production length measures to infer production line position directly. This is the topic we will address in the next section.

3. From production length to production line positions

As we have defined GVC related production and trade activities earlier, it is easy to see that a GVC production line has not only a starting and an ending stage, but also potentially many middle stages – since value-added in global production chains needs to have production activities cross national borders. Therefore, GVC position index is a relative measure. If a country/sector pair participates in the GVC at a particular production stage, the fewer production stages occurring before, the relatively more upstream the country-sector's position is in the particular GVC. On the other hand, the fewer the number of production stages following the country-sector in question, the more downstream the country-sector is in the GVC. This suggests that a meaningful production line position index needs to take into account the length stage to both ends of the global value chain.

Let us consider a particular country-sector. "The length to the end" measures the average production length of domestic value-added embodied in intermediate products s from its first use as a primary input until its final absorption in final goods and services.

Based on equation (26), the average production length forward (to the end of the chain) is the ratio of GVC related domestic value-added and its induced gross output:

$$PLv\_GVC = \frac{Xv\_GVC}{V\_GVC} \tag{30}$$

Based on equation (28), we can obtain the average production length backward (to the starting point of the chain) as the ratio of GVC related foreign value-added and its induced gross output：

$$PLy\_GVC = \frac{Xy\_GVC}{Y\_GVC} \tag{31}$$

It measures the average production length of foreign value-added embodied in intermediate imports from their first use as primary inputs until their final absorption into a certain country's production of final products (for its domestic use or exports).

As a node in the global production network, the longer is a particular country-sector's forward linkage, the more upstream is the country-sector. Conversely, the longer is a particular country-sector's backward linkage, the more downstream is the country/sector.

The average production line position in a global value chain can be defined as the ratio of the two production lengths:

$$GVCPs = \frac{PLv\_GVC}{[PLy\_GVC]'} \tag{32}$$

The greater the value of the index, the more upstream is the country-sector. Equation (32) indicates that the production line position index is closely related to the measure of production length, but the production length measure may not directly imply production line position. Only through aggregation, considering both forward and backward linkage based production length measures of a particular country/sector pair, by first determining its "distance" to both the starting and ending stages of all related production lines, the relative "upstreamness" or "downstreamness" in global production for a particular country/sector pair can be correctly determined.

Most importantly, under definition of (32), if country-sector A is more upstream than country-sector B, then country-sector B must be more downstream than country-sector A. In other words, the relative rankings of the country-sectors by these two measures are consistent with each other. This solves the consistency problem of the production position indexes used in the current literature, such as the N* and D* indexes proposed by Fally (2012) and the Down measure proposed by Antras and Chor (2013). In addition, such a GVC position index has a nice numerical property: because at the global aggregation level, the forward and backward linkage based GVC production lengths are the same and this index equals to one.

4. Numerical Results

We now apply these new measures to the recently updated WIOD data for 44 countries and 56 industries over 2000–2014.

Since all the indexes can be computed at both the most aggregated "world" and the more disaggregated "bilateral-sector" levels, we obtain a large amount of numerical results. To illustrate the computation outcomes in a manageable manner, we first report a series of examples at various disaggregated levels to highlight the stylized facts based on our new GVC index system and demonstrate their advantages compared to the existing indexes in the literature; we then conduct econometric analysis on the role of GVCs in the economic shocks brought by the recent global financial crisis as a more comprehensive application of these newly developed GVC measures.

4.1 Production length index

4.1.1 Emperical results

We take the "Computer, Electronic and Optical Products" sector as an example. Figure 2 reports the basic results for China and the US, at the "Country-Sector" level in 2014. The results are:

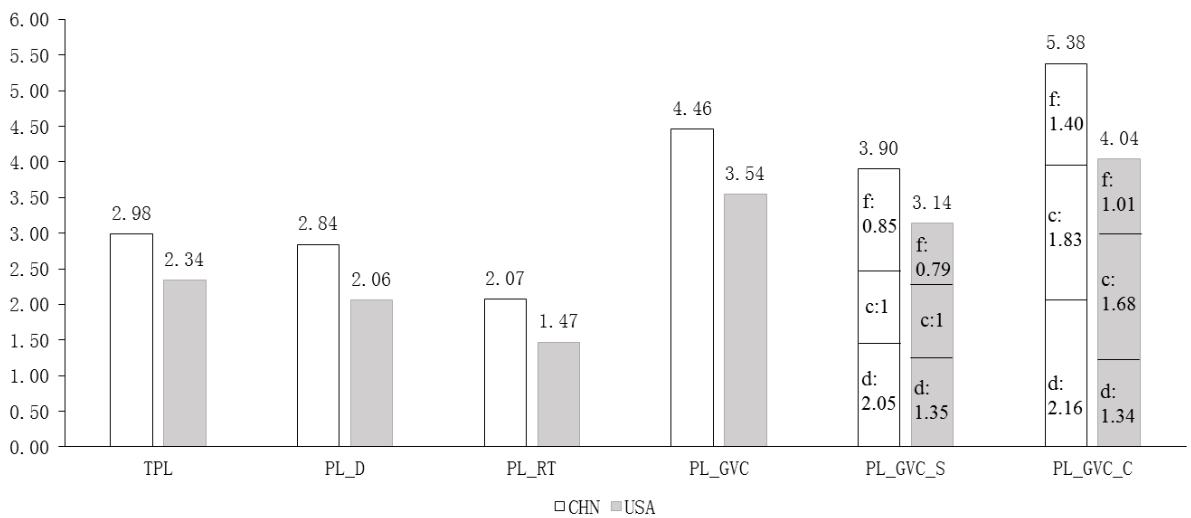
(1) The values of production length index are always higher for China than those for the US, which means the value added created by China (forward linkage) has to go through more steps before reaching its final uses, or final products produced by China (backward linkage) has more stages in its upstream production process.

(2) Compared with the pure domestic and the traditional final goods trade, value added created along the GVCs has the longest production length (PL_GVC). This result appears intuitive and reasonable as more participants and production steps are involved in the GVC production process. When further dividing up GVC production into simple and complex cross

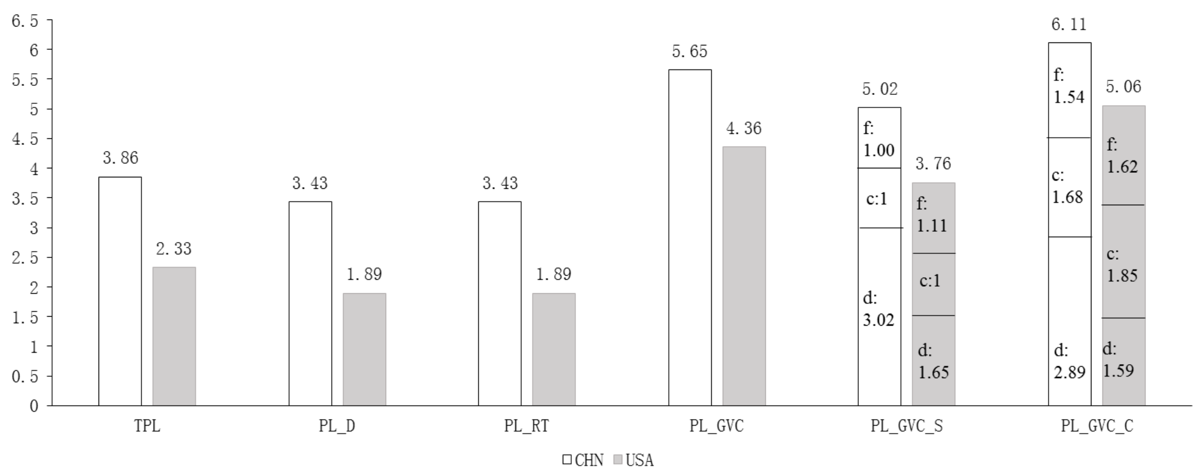
country production sharing activities, the latter (PL_GVC_C) is longer. In such a case, value added flowing back to the global production network from the direct importing country may go through several more production stages, possibly in other countries or back to the source country, before being finally embodied in final products.

Figure 2 Production Length of Electrical and Optical Equipment Sector, 2014

Forward Linkage



Backward Linkage



(3) Comparing the three portions of length between simple and complex GVCs, there is only one time border crossing for production in simple GVC activities by definition, but also less than twice in complex GVC activities for both US and China. This indicates that a large amount of

intermediate imports used by the importing country is absorbed by exports of final products produced from the importing country. When these final products cross national border again to third countries, it is no longer a border crossing for production, thus reducing the size of the average number of border crossings for production. Comparing the three portions of GVC length between the US and China, China seems have a longer domestic portion than that of the US, indicating that China engages in more domestic production stages while the US tends to offshore its production activities in the global production sharing network.

Table 2a and 2b report forward/backward linkage based production length in four aggregate industries: agriculture, mining, manufacture and services for the 5 largest economies in terms of the highest GDP/final products output by industries.

Comparing industries, agriculture, and especially mining, tend to have greater forward-linkage-based length, but shorter backward-linkage-based length. Manufacturing sectors, on the other hand, can have greater length based on both forward and backward linkages.

Comparing across countries, China has the longest production length across most industries in both forward and backward linkage based measures. As the world factory and second largest world economy, more production stages of GVCs are take place within China, as measured by the domestic portion of both simple and complex GVC production activities listed in table 2a and 2b. Generally speaking, emerging economies have a longer length in both total and GVC production than that of advanced economies due to the longer domestic portion of their production chain.

Comparing different portions of the production length, we see a similar pattern to that shown in Figure 2: GVC production is significantly longer than pure domestic and traditional trade production in all countries and industries, and complex GVC production activities is significantly longer than simple GVC production activities.

Table 2a Forward-Linkage Production Length, Country-Sector Level, 2014

Sector	Country	TPL	PL_D	PL_RT	PL_GVC	PL_GVC_S	=	d	+	c	+	F	PL_GVC_C	=	d	+	c	+	f
Agriculture	CHN	3.05	2.79	3.66	6.35	5.79		4.03		1		0.75	7.43		4.32		1.77		1.34
	USA	2.73	2.58	2.11	3.88	3.54		1.66		1		0.88	4.83		1.82		1.67		1.33
	BRA	2.35	1.89	2.09	3.64	3.39		1.32		1		1.07	4.36		1.39		1.55		1.42
	IDN	2.09	1.78	2.13	4.23	3.87		2.08		1		0.79	5.27		2.15		1.70		1.43
	IND	1.71	1.59	2.05	4.02	3.64		1.82		1		0.83	4.94		2.00		1.66		1.29
Manufacturing	CHN	3.18	3.00	2.46	4.92	4.38		2.56		1		0.82	5.77		2.56		1.84		1.38
	KOR	3.00	2.38	1.81	4.41	3.98		1.77		1		1.21	5.03		1.75		1.80		1.49
	JPN	2.57	2.14	1.78	4.29	3.79		1.76		1		1.04	5.05		1.73		1.83		1.49
	DEU	2.31	1.67	1.30	3.63	3.12		1.29		1		0.83	4.19		1.28		1.83		1.08
	USA	2.21	1.95	1.49	3.81	3.36		1.52		1		0.84	4.38		1.48		1.76		1.14
Mining	CHN	5.06	4.75	4.97	6.66	6.05		4.16		1		0.89	7.71		4.22		1.91		1.59
	RUS	4.29	3.53	2.18	4.51	3.82		1.25		1		1.57	5.37		1.23		2.11		2.02
	NOR	3.43	1.44	1.83	3.99	3.25		1.10		1		1.16	4.86		1.10		2.12		1.63
	CAN	3.23	2.14	2.40	3.67	3.28		1.22		1		1.06	4.91		1.22		2.02		1.67
	USA	2.74	2.34	2.74	4.62	4.02		2.00		1		1.02	5.56		1.95		1.98		1.63
Service	CHN	2.49	2.17	3.38	5.34	4.71		2.88		1		0.83	6.49		3.11		1.87		1.51
	DEU	1.97	1.52	2.32	4.54	3.95		2.11		1		0.84	5.36		2.25		1.87		1.25
	GBR	1.85	1.53	1.85	4.12	3.52		1.77		1		0.75	5.02		1.81		1.88		1.32
	USA	1.68	1.55	1.97	4.27	3.72		1.93		1		0.79	5.07		1.94		1.84		1.28
	JPN	1.66	1.48	2.39	4.97	4.37		2.40		1		0.96	6.04		2.56		1.86		1.61

Table 2b Backward-Linkage Production Length, Country-Sector Level, 2014

Sector	Country	TPL	PL_D	PL_RT	PL_GVC	PL_GVC_S	=	d	+	c	+	F	PL_GVC_C	=	d	+	c	+	f
	USA	2.20	1.95	1.92	4.27	3.73		1.89		1		0.84	5.24		1.88		1.88		1.48
	RUS	2.12	1.86	1.86	4.18	3.54		1.62		1		0.93	5.72		1.72		2.25		1.75
Agriculture	CHN	2.08	1.90	1.90	5.51	4.90		2.99		1		0.91	7.26		3.02		2.26		1.98
	IND	1.44	1.33	1.28	4.92	4.39		2.24		1		1.15	6.40		2.11		2.26		2.03
	IDN	1.34	1.22	1.20	4.41	3.87		1.63		1		1.25	5.92		1.66		2.24		2.02
	CHN	3.49	3.09	3.17	5.59	5.00		3.04		1		0.96	6.06		2.76		1.72		1.58
	IND	2.66	2.19	2.16	4.34	3.80		1.76		1		1.04	5.33		1.73		1.88		1.73
Manufacturing	JPN	2.66	2.03	2.21	4.63	3.93		1.92		1		1.01	5.41		2.12		1.64		1.64
	DEU	2.52	1.89	1.79	4.00	3.22		1.43		1		0.80	4.32		1.43		1.66		1.24
	USA	2.44	2.07	1.98	4.20	3.58		1.63		1		0.95	5.19		1.61		1.94		1.64
	CHN	2.59	2.27	2.27	5.07	4.44		2.59		1		0.85	6.08		2.70		1.80		1.57
	AUS	1.96	1.64	1.64	4.36	3.79		1.71		1		1.09	5.67		1.70		2.17		1.80
Mining	CAN	1.63	1.35	1.35	3.94	3.49		1.59		1		0.90	5.18		1.58		2.01		1.59
	USA	1.61	1.42	1.42	3.76	3.23		1.48		1		0.75	5.66		1.57		2.28		1.82
	NOR	1.41	1.20	1.20	3.92	3.30		1.54		1		0.77	5.18		1.51		2.23		1.44
	CHN	2.75	2.46	2.09	5.68	5.06		3.14		1		0.93	7.31		3.05		2.28		1.98
	GBR	1.86	1.57	1.66	4.16	3.59		1.79		1		0.79	5.50		1.80		2.18		1.52
Service	JPN	1.77	1.53	1.55	4.52	4.01		2.08		1		0.94	6.23		2.06		2.24		1.93
	DEU	1.75	1.49	1.65	4.18	3.56		1.75		1		0.81	5.44		1.75		2.21		1.49
	USA	1.71	1.57	1.60	4.38	3.81		1.90		1		0.91	5.91		1.86		2.28		1.78

4.1.2 Has the length of Global Value Chains become longer or shorter over time?

One important question addressed in the recent GVC measurement literature is: Has the global production chain become less or more fragmented?

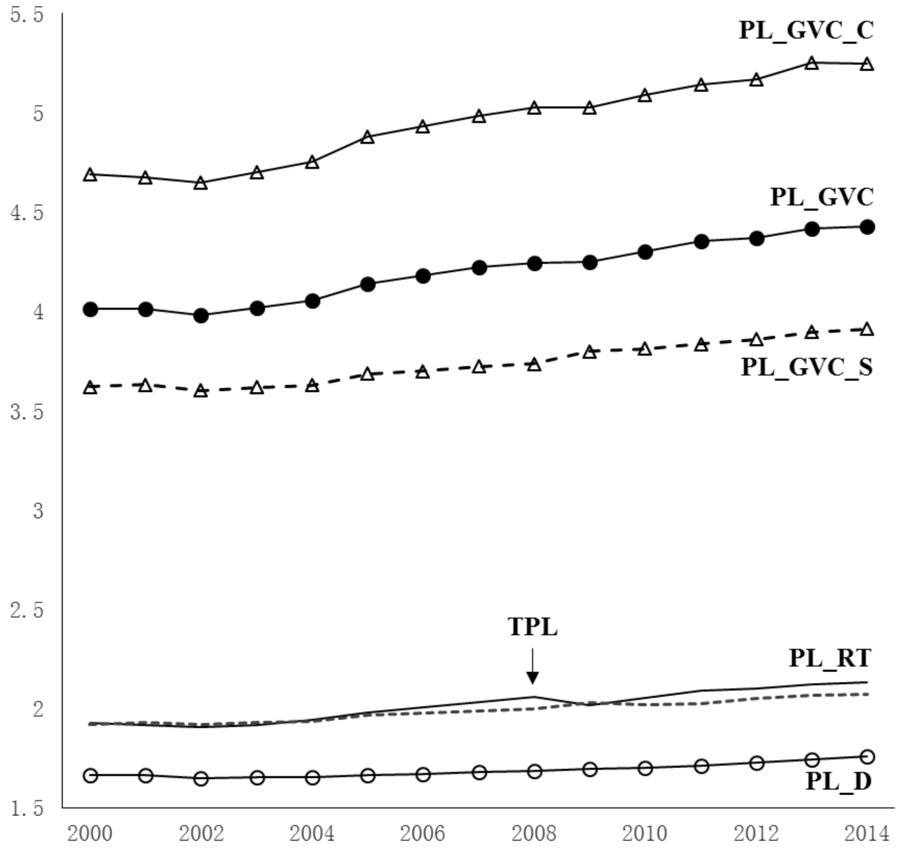
Most studies conclude that global production has become more fragmented today than decades ago. As shown in Feenstra and Hanson (1996), imported intermediate inputs in the US have increased from 5.3% to 11.6% between 1972 and 1990. Similarly, Hummels et al. (2001) find that the world VS (Vertical specialization) share of exports has grown almost 30% between 1970 and 1990, which accounts for more than 30% of overall export growth.[8]

Our numerical results also clearly show that the Global Value Chain is getting longer, which reflects the increasing fragmentation of GVC production and trade activities. Moreover, the distinction between different types of production and trade activities enable us to further investigate the major drivers behind the lengthening of GVCs.

As shown in Figure 2, the world average "Total Production Length" has a clearly upward trend, especially after year 2002 (this trend was temporarily interrupted by the global financial crisis during 2008 to 2009). Furthermore, the average production length of GVCs has increased by 0.42 from 2000 to 2014, which is much faster than traditional exports (PL_RT) and pure domestic production length (PL_D). The lengthening of GVC is reflected in both simple and complex GVC production activities, but lengthening of complex GVC is more dramatic; for example, from 2000 to 2014, the length of complex GVC has increased 0.55, almost doubling the growth of the length of simple GVCs.

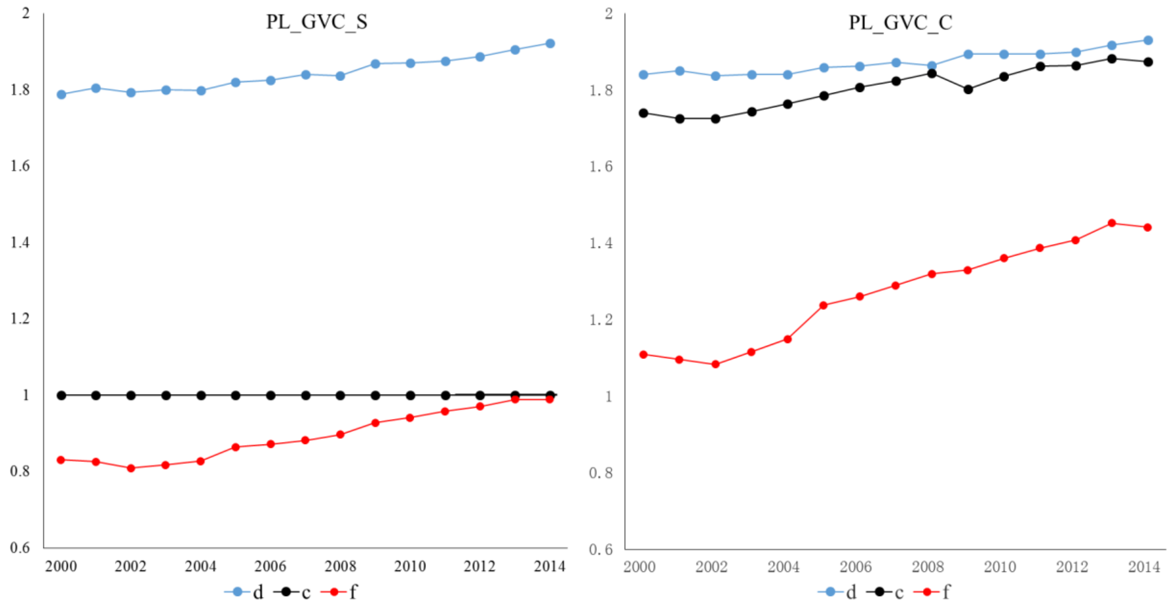
[8] Fally (2011) indicates that the production chain (or the distance to final demand) in the US appears to have shortened over time and concludes that such a trend is also a global phenomenon. Consistent with Fally, our calculation also shows that the production length of the US is getting shorter. But this finding is reversed at the global level. In Appendix G, we show that the strong assumption "The same industries have the same production length across countries" is the main factor that leads to the puzzling finding by Fally.

Figure 3 The Upward Trend of Production Length, World Average



In Figure 3, we focus on GVC production activities to investigate the changes of its domestic and international portions. We find that the increasing length of GVCs is primarily driven by two factors: 1).The increasing number of border crossings for production；2). The lengthening of GVC production after its first border crossing. Because the number of border crossings for production is constant in simple GVC activities, the lengthening of complex GVC activities is the major driven force.

Figure 4 Changes of GVC Production Length: Border Crossing for Production, before and after first border crossing, World Average, 2000 to 2014



To ensure robustness of results, we further investigate the changes of production length at the country and industry level. In Figure 5, we select the four largest countries ranked by GDP -- the US, China, Japan and Germany -- to compare the changes of major portions of both forward and backward linkage based production length.

For China, the total average production length, as well as all of its segments, is longer in 2014 than in 2000. This holds for both forward and backward linkage based measures. For Germany, Japan, and the US, the production length for pure domestic (D) and traditional exports (RT) has decreased during the sample period. But the average GVC production length, especially the complex GVC segment, has increased considerably for all countries over this period, even when the total average production length became shorter for Japan and the US in the forward linkage based measures.[9]

[9] This may reflect the phenomenon of "offshoring" production activities abroad in these developed economies. When more production activities go abroad, the international portion of GVCs gets longer while its domestic portion becomes shorter.

Figure 5a Changes of Major Segments of Production Length for Major Economies, Forward-Linkage based, 2000-2014

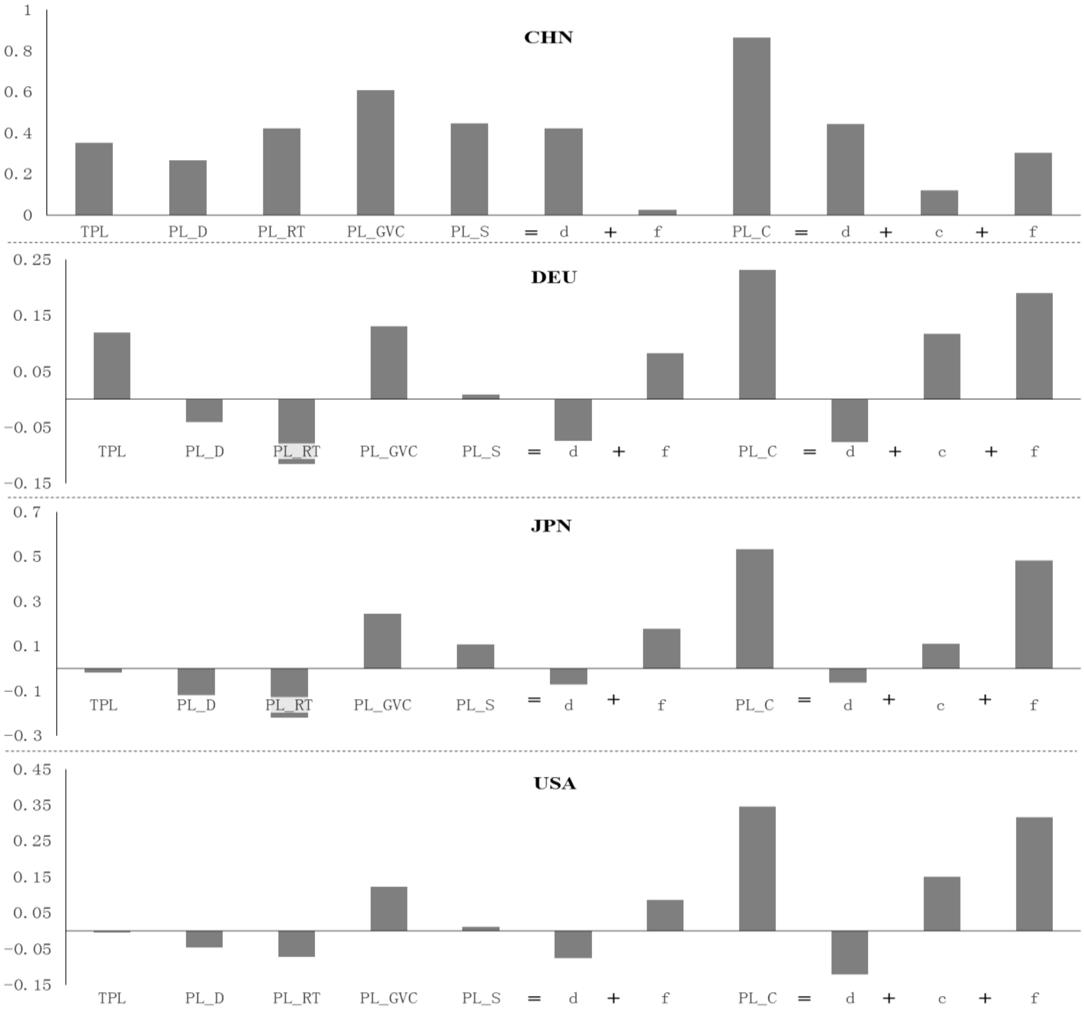
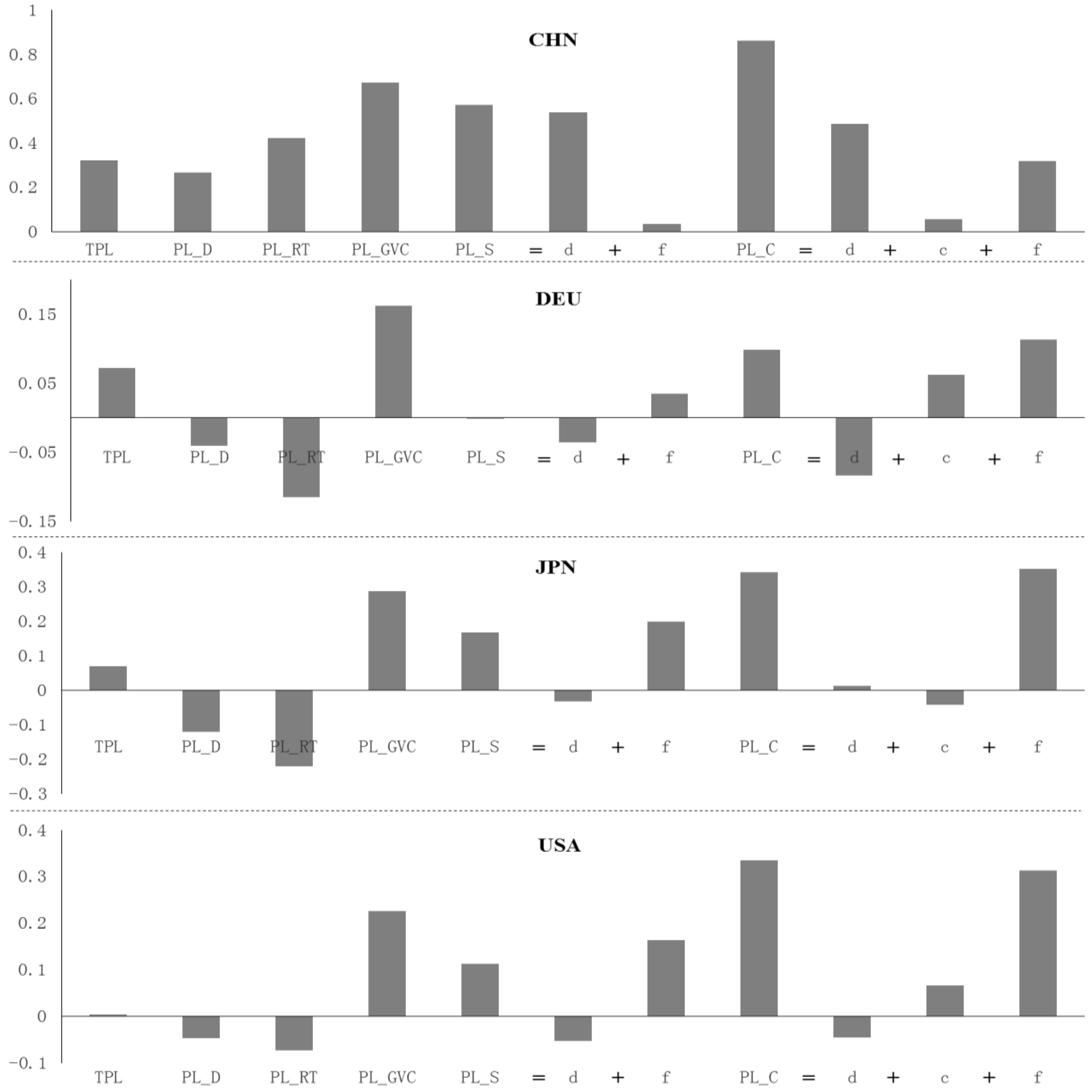


Figure 5b Changes of Major Segments of Production Length for Major Economies, Backward-Linkage based, 2000-2014



Although GVC production, especially the complex GVC activities, have become longer during 2000 to 2014, the driving forces are different for the four selected large economies. For China, the lengthening of the domestic portion is a dominant feature, reflecting China's deeper and finer divisions of labor through developing more production stages at home. For the three advanced industrial economies, the situation is opposite: the lengthening of their GVC production line is driven by the increase of the number of border crossings and production stages within other countries, while the domestic production length even become shorter. This is consistent with the fact that advanced countries rely heavily on offshoring to organize their global production network.

In Figure 6, we report the changes of production length of the four aggregate sectors during 2000 to 2014.

Figure 6a Changes of Production Length at Sectoral Level, 2000-2014, Forward-Linkage based

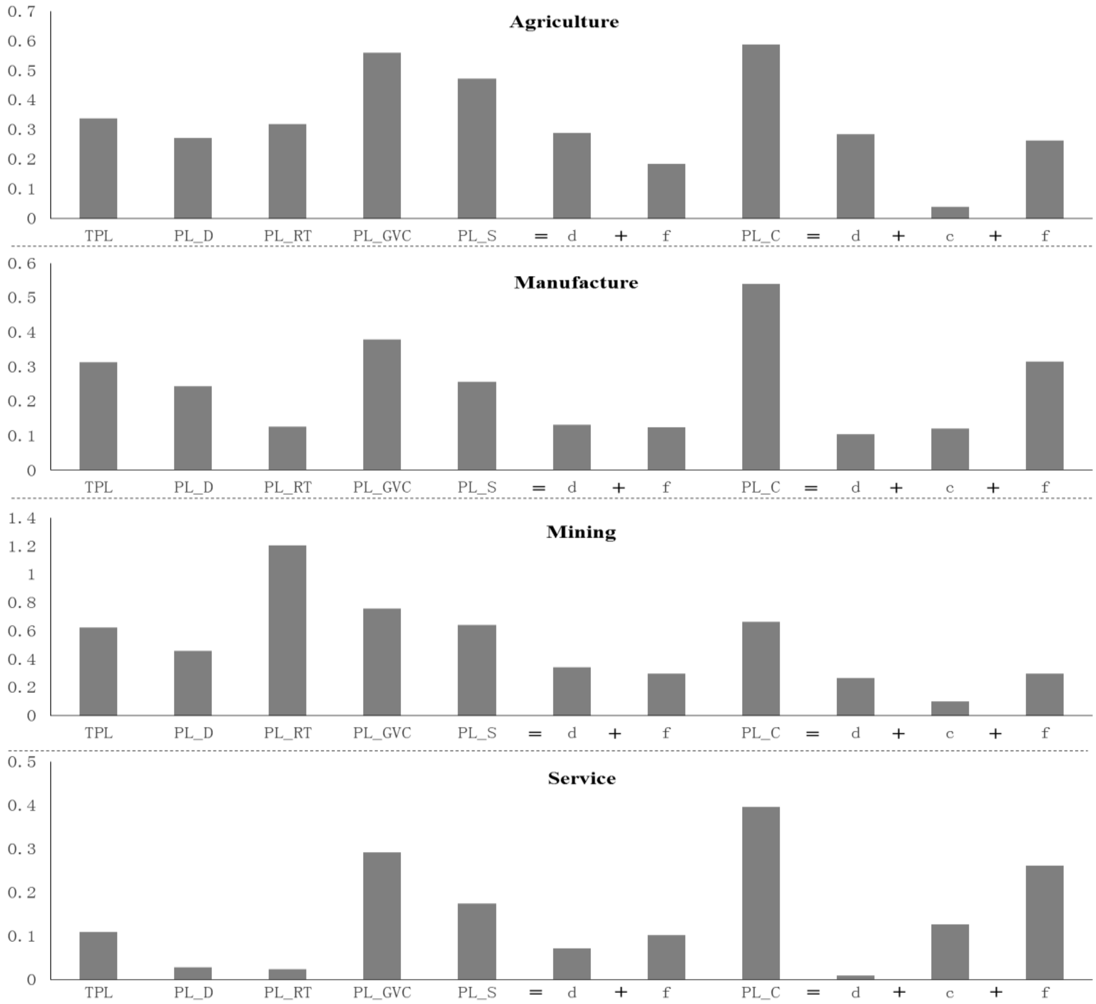
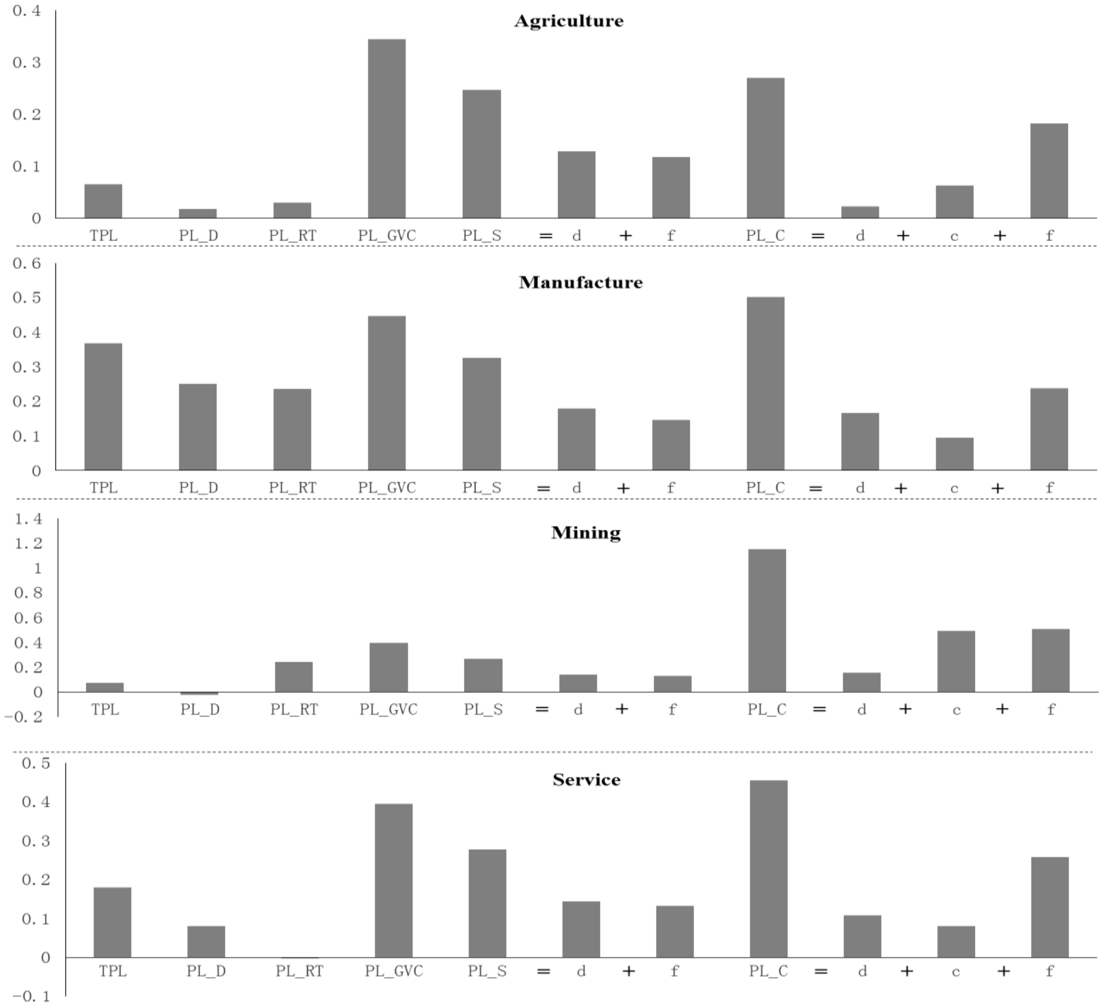


Figure 6b Changes of Production Length at Sectoral Level, 2000-2014, Backward-Linkage based



Although GVC production length in all of the four aggregate sectors has become longer during this period, the underlying driving forces are different. In agriculture, manufacturing and service sectors, the length of the complex GVC segment increases much faster than that of the simple GVC segment, but the major driving factor in agriculture is the lengthening of the domestic segment of the production chain from finer divisions of labor, while the dominate force in manufacture and service sectors is the increase of the number of border crossings and production stages in foreign countries. On the contrary, in mining sector, the length of simple GVC segment increases faster than its complex GVC segment.

In conclusion, using the production length indexes newly defined in this paper, we have observed an increasing trend of fragmentation in production, especially in GVC production activities.

4.1.3 A more robust measure of production length than APL

APL has been used to measure length in production (Dietzenbacher, and Romero, 2007). A potential shortcoming is it changes as the aggregation level of industrial classification changes. As we discussed in Section 2, production length defined in this paper equals the number of times of value-added is counted as gross output from it is first used as primary factors to its embodiment in final products. This ratio is invariant to the disaggregation level of the sector classification. This is an advantage of the production length defined in this paper over APL.

To show this, we compute both the production length and APL based on original WIOD data, aggregate the original table from 56 sectors into 10 and 3 sectors respectively, and report our computation results of the average global production length in Figure 7. The results show clearly that the size of the APL index increases as the aggregation level increases, while our production length measure keeps constant across the 3 different sector aggregations.

At the country and sector levels, the new production length measure is also more robust than APL under different sector aggregations. Using the 2014 WIOD table at 10 and 56 sector as example, Figures 8 show that there is a systemic upward bias for the APL measure as sector classification become more disaggregated, while the changes of our production length measure due to aggregation are much smaller and nearly zero. Similar situation can be find at the country and country/sector level.

Figure 7 Global Average APL, Calculated from IO Tables at Different Aggregation Levels

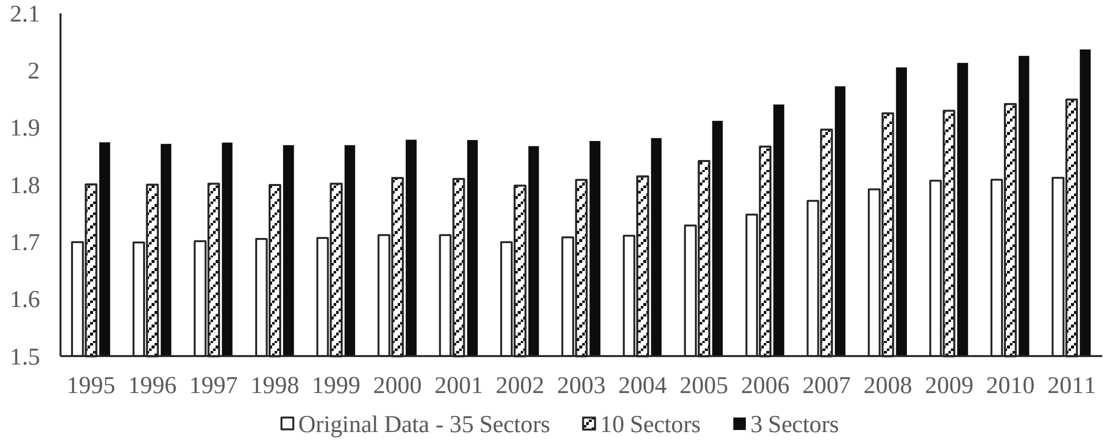
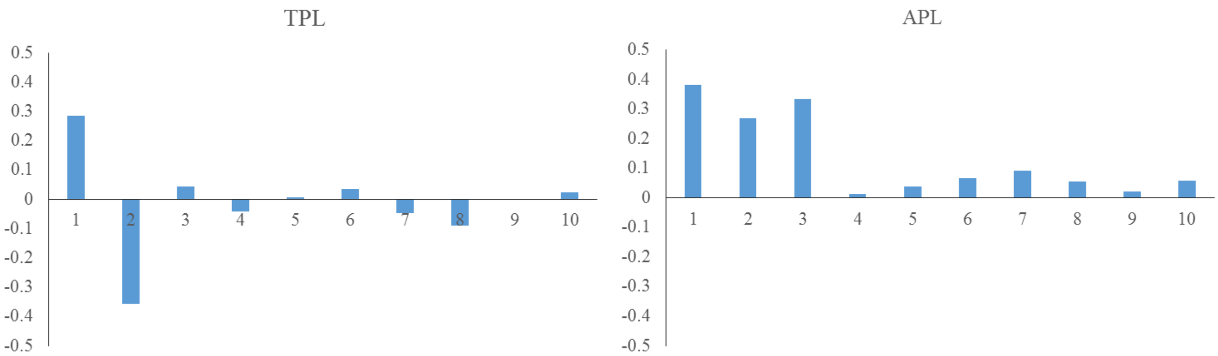
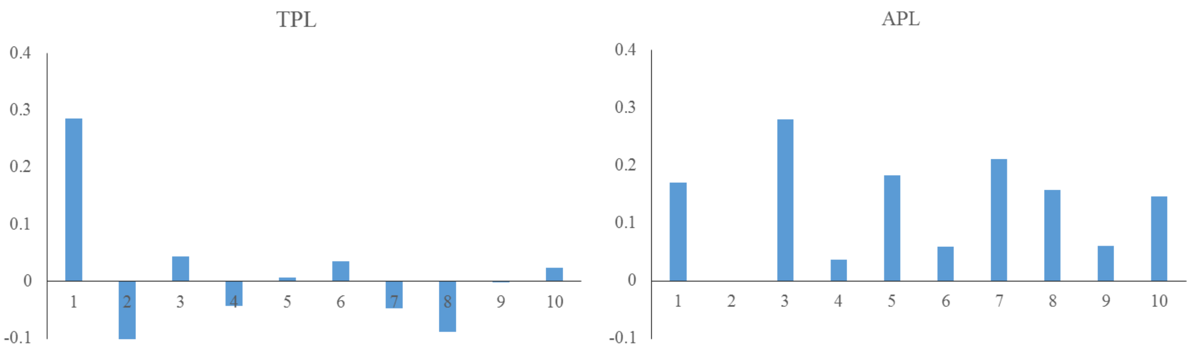


Figure 8 The Changes of TPL and APL at Sectoral Level, 2014

Forward-Linkage



Backward-Linkage



4.2 Position Index

In section 2.5, we have discussed why the ratio of forward and backward industrial linkage based production length can been used as measure of the production line position in global value chains. Now we report numerical results of such GVC position indexes in this subsection.

4.2.1 Country level

We compute the ratios of both forward and backward linkage based total and GVC production length in 2014, obtaining two types of production line position indexes, and report them in Table 3. Both types of indexes indicate that China, India and Mexico were located relatively closer to the downstream side among the 20 economies reported. However, the two types of position indexes give very different rankings for upstream countries. For instance, Canada is ranked on the upstream side according to the total production length based position index, while its ranking moves down to the bottom by the GVC production length based position index. In the meanwhile, the ranking of some other countries, such as Japan, French and Belgium, has moved up.

Such differences may come from the structure difference between an economy's total production and GVC production activities. The position index based on total production length measures a country's production activities as a whole, including its pure domestic production and production of tradition trade that is unrelated to cross country production activities (this part is often dominant in many economies), while the position index based on GVC production length only focuses on the position of a country in cross country production sharing activities, so we define it as the "GVC Position Index".

It is worth to point out that our numerical results of countries' production line position seems contradictory to Miller et al. (2015). Their results show that, compared with other countries, China is the most upstream country in the world, far away from the final consumption end; but in fact, our results are not actually contradictory with

Miller's findings if carefully looked at through the numerical results. The reasons for the inconsistency are as follows:

When the "Upstreamness" (OU) and "Downstreamness" (ID) indexes of a country/sector pair computed by Miller et al. are high, it means that the distance between the country/sector pair to the factor input/final consumption end is longer. However, as we pointed out earlier, using backward or forward linkage based production lengths alone cannot tell the country/sector pair's relative position in a production line because the country/sector pair as a middle stage of production process, its forward and backward length to each end of the production line could be relatively shorter or longer. Just as Table 3 shows, both the forward and backward total and GVC production of China are significantly longer than that of other countries. This means that China would have been placed in the upstream if we use the conventional Upstreamness or Downstreamness measure as Miller et al (2015).

Table 3 Country Level Position Index, 2014

Country	Production Position Index (Forward / Backward)	TPL Forward	TPL Backward	Country	GVC Position Index (Forward / Backward)	PL_GVC Forward	PL_GVC Backward
RUS	1.226	2.503	2.041	AUS	1.048	4.865	4.645
NLD	1.125	2.270	2.017	RUS	1.036	4.688	4.525
AUS	1.091	2.212	2.027	BEL	1.018	3.977	3.905
SWE	1.065	2.034	1.911	SWE	1.016	4.091	4.027
DEU	1.043	2.054	1.970	FRA	1.013	4.170	4.114
CAN	1.017	2.004	1.972	BRA	1.011	4.410	4.361
GBR	1.013	1.946	1.922	JPN	1.011	4.608	4.560
BEL	1.003	2.140	2.134	DEU	1.005	4.096	4.077
IDN	1.001	2.094	2.091	NLD	1.002	3.976	3.968
BRA	0.992	1.844	1.859	ESP	0.992	4.243	4.275
ITA	0.985	1.985	2.015	ITA	0.989	4.330	4.376
USA	0.984	1.785	1.814	IDN	0.989	4.391	4.441
KOR	0.979	2.348	2.399	GBR	0.986	4.066	4.122
FRA	0.979	1.874	1.915	TUR	0.975	4.238	4.347
CHN	0.976	2.891	2.963	MEX	0.967	3.838	3.970
ESP	0.965	1.890	1.958	KOR	0.962	4.625	4.810
JPN	0.965	1.857	1.925	USA	0.961	4.136	4.304
TUR	0.957	1.970	2.059	CAN	0.950	3.923	4.130
IND	0.934	1.813	1.942	CHN	0.945	5.322	5.634
MEX	0.930	1.704	1.833	IND	0.937	4.190	4.472

4.2.2 Sector level results

Table 4 reports average production line position for global industries in 2014. Similar to Table 3, we compute the ratios of both forward and backward linkage based total and GVC production length to obtain the two types of production line position indexes.

There are differences in ranking global industries by the two types of position indexes. As we discussed, the position index based on total production length measures a country's production as a whole, where pure domestic production activities are dominant, while the GVC position index only concerns cross country production sharing activities in the global production network. As we showed earlier, the two type production activities are quite different even within the same industry.

Taking the construction sector as an example, when considering pure domestic production and cross country production as a whole, construction is often located at the bottom of the industrial production chain because it uses large amounts of intermediate inputs from other sectors and its products after completion will enter the consumer market immediately; therefore it has a very short distance to final demand. However, when only cross border production activities are considered, products from the construction sector may be difficult to export directly. Its factor content is often embodied in other sectors' exports involving international production sharing indirectly. As a consequence, its position in the GVC production network will move relatively upstream. Similar phenomena exist in many services sectors such as transportation and public services.

In summary, by excluding pure domestic production/consumption activities, our GVC position index ranks traditional non-tradeable sector such as utility and services as upstream stages in the value chains, and most manufacturing sectors such as textile, leather and apparel, electronics and machinery as downstream stages in the value chains. These seem sensible and intuitive.

Table 4 Sectoral Level Positions Index, World Average, 2014

Sector	Production Position Index	Sector	GVC Position Index
Mining and quarrying	2.221	Electricity and gas supply	1.368
Forestry and logging	1.919	Coke and refined petroleum products	1.285
Financial service activities	1.609	Financial service activities	1.228
Activities of head offices, Legal and accounting	1.592	Real estate activities	1.140
Administrative and support service	1.519	Mining and quarrying	1.137
Advertising and market research	1.474	Other service activities	1.134
Postal and courier	1.455	Forestry and logging	1.126
Support activities for transportation	1.427	Motion picture, video and television programme	1.123
Activities auxiliary to financial services	1.411	Water supply	1.119
Wholesale trade	1.314	Telecommunications	1.105
Accommodation and food service	0.768	Machinery and equipment	0.879
Textiles and leather products	0.766	Computer, electronic and optical products	0.876
Food, beverages and tobacco	0.736	Furniture	0.876
Education	0.718	Publishing	0.856
Public administration and defense	0.686	Textiles and leather products	0.846
Other transport equipment	0.666	Electrical equipment	0.816
Motor vehicles, trailers and semi-trailers	0.663	Motor vehicles, trailers and semi-trailers	0.803
Human health and social work	0.574	Basic pharmaceutical products	0.795
Construction	0.505	Other transport equipment	0.779
Extraterritorial organizations and bodies	0.362	Activities of households	0.743

4.2.3 Country-sector level

Our numerical results also show that the GVC position for a sector may vary considerably across countries, which reflects the differences in location by each country along a particular production network. Three typical sectors in 20 largest Economies in terms of GDP are shown in Table 5.

Table 5 Sectoral Level:
A comparison of GVC positions across Economies, 2014

Computer, Electronic and Optical Products		**Telecommunications**		**Textiles and Leather Products**	
Country	Position	Country	Position	Country	Position
RUS	1.188	BRA	1.307	RUS	0.988
AUS	1.019	MEX	1.235	JPN	0.900
NLD	0.965	AUS	1.233	GBR	0.870
BRA	0.940	CAN	1.196	MEX	0.850
TUR	0.911	RUS	1.191	CHE	0.844
CHE	0.910	JPN	1.157	NLD	0.835
TWN	0.896	DEU	1.155	USA	0.825
ESP	0.893	TUR	1.147	AUS	0.825
IND	0.874	NLD	1.098	ESP	0.800
ITA	0.873	FRA	1.097	DEU	0.789
GBR	0.872	KOR	1.097	ITA	0.785
KOR	0.864	TWN	1.092	TUR	0.783
CHN	0.858	ESP	1.084	FRA	0.782
JPN	0.856	CHN	1.066	KOR	0.776
IDN	0.848	CHE	1.062	BRA	0.775
CAN	0.833	GBR	1.034	CHN	0.770
USA	0.830	IDN	1.027	CAN	0.768
FRA	0.826	USA	0.978	TWN	0.767
DEU	0.820	ITA	0.936	IDN	0.733
MEX	0.806	IND	0.849	IND	0.707

In the “Computer, Electronic and Optical Products” sector, countries that specialize in assembling and processing activities, such as China, Indonesia and Mexico, and developed countries such as Germany, French and United States, are located on the most downstream end, as they are placed at the final stage of the production chain. In contrast, two natural-resources-abundant countries, Australia and Russia, are positioned in the most upstream end to provide energy and mining needs for the whole value chain.

In the "Textiles and Leather" sector, India, Indonesia and Taiwan, China are located in the final product end of the value chain, while Russia is still positioned on the top of the value chain by providing natural resource based intermediate inputs. Japan's position is also more upstream by providing more manufactured intermediate inputs into the production chain. In the "Telecommunications" sector, India is located at the end of the value chain by providing direct services such as calling centers and clinic record keeping. Countries located upstream are those countries where business services are an important intermediate input for their manufacturing industry, such as Mexico and Germany, and natural resource providing countries such as Australia and Russia.

4.2.4 Time Trend: selected industries

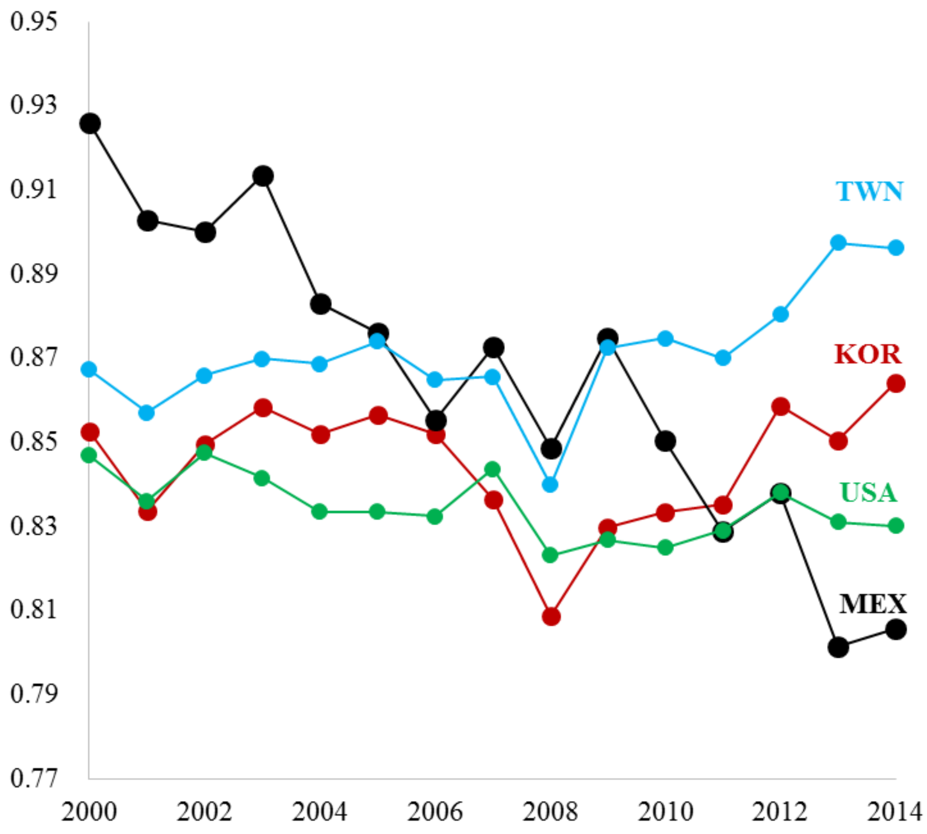
Analyzing changes of GVC position index over time may allow us quantify the evolution path of each country's role and position along a particular production chain. Here we use typical GVC industries - Computer, Electronic and Optical Products (WIOD sector 17) as examples. Figure 9 plots out the time trend of the GVC position index for the United States, Mexico, Korea and Taiwan during 2000 to 2014.

As a member of NAFTA since 1994, Mexico gradually become a processing and assembling center of electronic and optical equipment in the western hemisphere due to its low cost of production (such as labor cost), proximity and duty free access to the world largest consumer electronic market. As our GVC position index indicates, its forward production linkage based GVC production length become shorter and shorter, its backward industrial linkage based production length become longer and longer, and its production line position on the value chain has been moved from relatively upstream in 2000, to the most downstream in 2014.

Similar to Mexico, the U.S. position in this global production chain also moved relatively downstream. However, the driven force of such a move is different from Mexico. Both forward and backward linkage based U.S production length has become longer since U.S. has offshored a large part of its middle production stages and also imports a large amount of parts and components from other countries in its final goods production. The relatively faster lengthening of backward linkage based production length than that of forward linkage based production length lowers the US GVC position index.

In 2000, Korea and Taiwan were located relatively downstream, but in the rapidly developing electronic supply chain cluster in East Asia, particular in China's south coast area, Korea and Taiwan have become major suppliers of electronic parts and components in the world. Their position in the computer, electronic and optical products production chain increased quickly. Similar to the US, both forward and backward linkage based production lengths have become longer for Korea and Taiwan, but their forward linkage based production length grew much faster, so their production line position moved upstream.

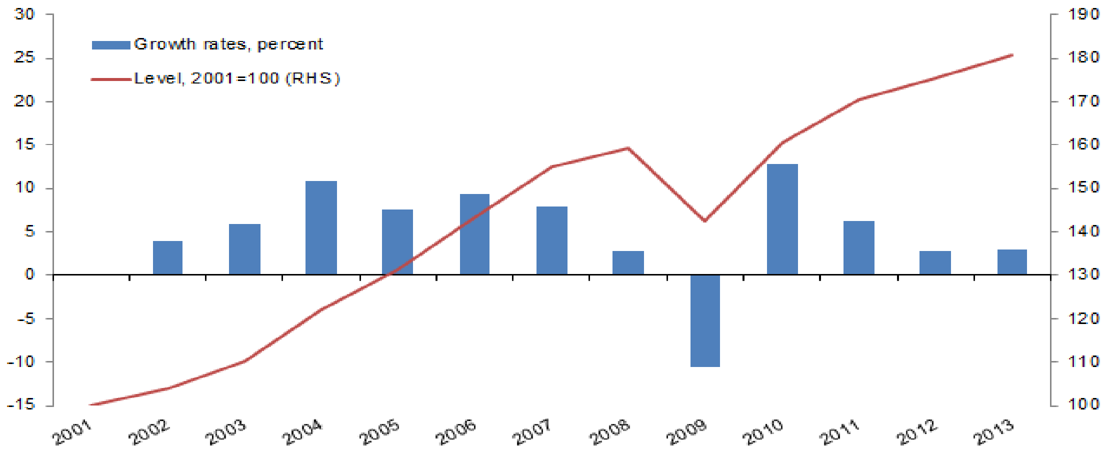
Figure 9 Time Trend of GVC Position Index, Electrical and Optical Equipment



4.3 Index application: participation intensity, production line positions, GVC length, and the economic shocks of the recent global financial crisis

In the aftermath of the Global Financial Crisis, as shown in Figure 10, world trade grew by 6.2% in 2011, 2.8% in 2012, and 3.0% in 2013. This growth in trade volumes is substantially lower than the pre-crisis average of 7.1% (1987–2007), and is slightly below the growth rate of world GDP in real terms.

Figure 10 The Growth of World Trade before and after the Financial Crisis



Following the production activity accounting framework proposed by Wang et al. (2017), value-added creation activities by a country can be decomposed into four parts: pure domestic production, production of traditional trade, simple, and complex cross country production sharing activities. Then, in financial crises, are there differences in the degree of effects on the four types of value added?

Figure 11 Different Effects of the Same Economic Shock to Different Value Added Creating Activities – Impact of global Financial Crisis

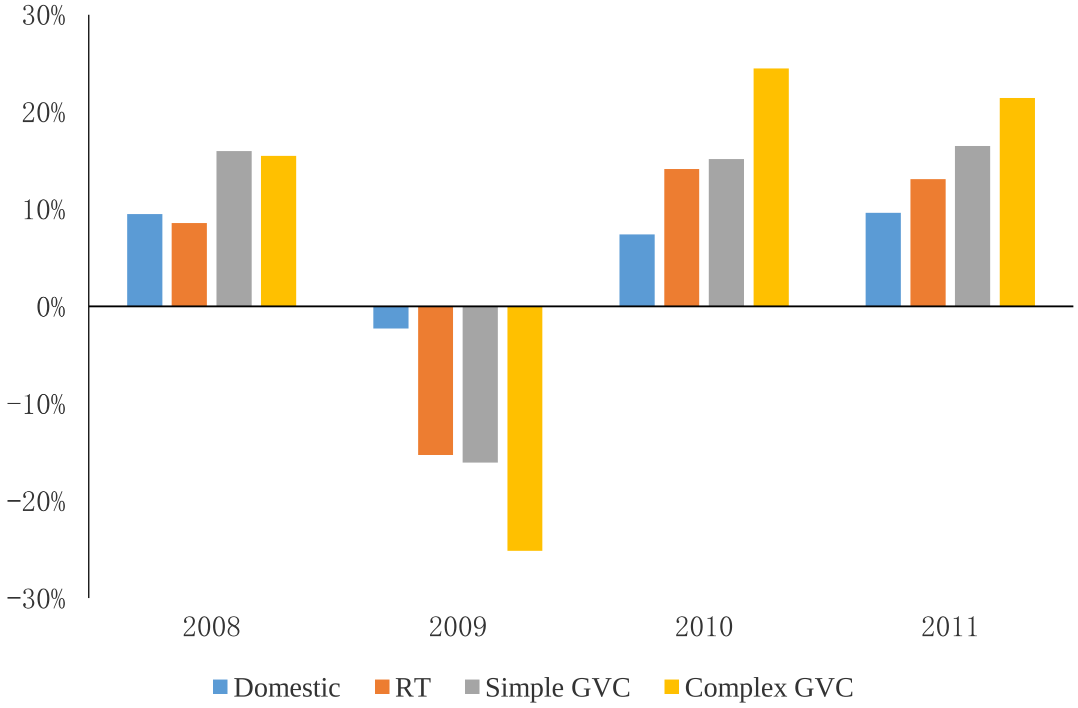


Figure 11 shows the result at the global level. During the financial crisis in 2009, pure domestic production activities were least affected (in comparison with 2008, the fall was only 2.27%), the impact on production of traditional trade rank next, while the cross country GVC production activities were most affected, as the fall reached 16.05% in its simple portion and 25.14% in its complex portion. However, it is also observed that the two portions of GVC production activities had the fastest after-crisis-recovery.

Divided among different countries and sectors, the above phenomenon also holds. Table 6 shows that pure domestic production is least affected by the financial crisis (China even continued a positive growth). For most sectors, GVC production and trade activities were most affected. The second issue is this: Are the GVC participation intensity, GVC length and production line positions related to the degree of effects of the financial crisis? To test this, we estimate the following regression model:

$$\Delta\ln(Va_{ic}) = \beta_0 + \beta_1 \times Position_{ic} + \beta_2 \times PLv\_GVC\_International_{ic} + \beta_3 \times PLv\_GVC\_Domestic_{ic} + \beta_4 \times GVCPt_{ic} + \beta_5 \times W_{ic} + \beta_6 \times Z_c + \gamma_i + u_{ic}$$

where

$\Delta\ln(Va_{ic})$ equals to the change of sectoral GDP, $\ln(Va_{ic})$ in year 2009 minus $\ln(Va_{ic})$ in year 2008, which quantifies the degree of effects on this industry/country pair during the financial crisis;

$Position_{ic}$ is the production line position Index, calculated as TPLv/TPLy. When the value is high, it means that this sector is relatively further from the final consumption end;

$PLv\_GVC\_International_{ic}$ and $PLv\_GVC\_domestic_{ic}$ is the International portion (f + c) and domestic portion (d) of the forward linkage production length;

$GVCPt_{ic}$ is the forward (or backward) GVC participation ratio. It can be divided into simple and complex portions (Simple GVCP and Complex GVCP);

W_{ic} represents the country-sector level control variables, including the labor productivity defined as value added per worker, and hours worked by high-skilled workers (share in total hours);

Z_c represents the country level control variable. In the regressions, we use a dummy variable to indicate whether this is a mature economy (=1).

We also control for sector fixed effects by including a sector dummy γ_i in the model.

Table 6 The Effects of Financial Crisis to Different Value Added Creating Activities (Sectoral Level)

China					USA				
Sector	**Domestic**	**RT**	**Simple GVC**	**Complex GVC**	**Sector**	**Domestic**	**RT**	**Simple GVC**	**Complex GVC**
Agriculture	9.0%	-3.2%	-8.3%	-14.3%	Agriculture	-13.9%	-21.5%	-27.4%	-30.8%
Mining	18.0%	-9.4%	-20.5%	-28.6%	Mining	-23.5%	-33.2%	-19.6%	-38.0%
Food	7.9%	-3.7%	-10.4%	-16.4%	Food	16.0%	8.0%	-2.2%	-12.1%
Textiles Products	23.7%	-6.0%	-5.8%	-11.9%	Textiles Products	-19.9%	-7.9%	-18.5%	-22.1%
Leather and Footwear	18.3%	-6.5%	-6.0%	-10.0%	Leather and Footwear	-19.8%	14.4%	-10.0%	-14.3%
Wood Products	15.4%	-13.4%	-18.5%	-23.9%	Wood Products	-15.8%	-22.0%	-20.7%	-30.3%
Paper and Printing	13.5%	-8.7%	-12.0%	-19.6%	Paper and Printing	-1.7%	-10.5%	-5.1%	-18.2%
Refined Petroleum	16.5%	-11.3%	-21.6%	-23.5%	Refined Petroleum	-21.4%	-25.7%	-24.5%	-37.8%
Chemical Products	17.9%	-6.0%	-14.3%	-22.7%	Chemical Products	10.8%	15.9%	4.7%	-8.3%
Rubber and Plastics	20.3%	-7.2%	-9.2%	-18.3%	Rubber and Plastics	-3.1%	-8.2%	-2.3%	-14.7%
Other Non-Metal	10.4%	-17.5%	-17.9%	-28.5%	Other Non-Metal	-2.5%	-11.5%	0.6%	-18.5%
Basic Metals	22.8%	-10.5%	-22.3%	-33.2%	Basic Metals	-15.6%	-16.9%	-11.6%	-28.1%
Machinery	20.3%	-16.8%	-22.0%	-28.6%	Machinery	-10.6%	-8.6%	2.3%	-15.1%
Electrical Equipment	28.6%	-8.6%	-5.2%	-16.1%	Electrical Equipment	1.1%	4.4%	5.9%	-11.1%
Transport Equipment	14.0%	-12.3%	-16.6%	-25.1%	Transport Equipment	-1.6%	-8.3%	-4.1%	-27.1%
Recycling	41.1%	-5.9%	-10.4%	0.3%	Recycling	-8.3%	-12.6%	3.3%	-0.5%

Summary statistics for key variables are provided in Table 7:

Table 7 Summary Statistics for Key Variables

	Variable	Observations	Mean	SD	Min	Max
2008	Va	1400	37009	114850	0	1972298
	Position	1382	1.01	0.31	0.32	2.85
	GVCPt (forward)	1382	0.25	0.20	0	0.94
	Simple GVCPt (forward)	1382	0.14	0.12	0	0.67
	Complex GVCPt (forward)	1382	0.10	0.09	0	0.59
	GVCPt (backward)	1382	0.24	0.15	0	0.85
	PLv_GVC: International Portion	1361	2.15	0.18	1.44	3.21
	PLv_GVC: Domestic Portion	1361	1.84	0.61	1.00	4.17
2009	Va	1400	35170	114725	0	1902096
	Position	1379	1.03	0.33	0.32	3.41
	GVCPt (forward)	1379	0.24	0.20	0	1.10
	Simple GVCPt (forward)	1379	0.15	0.13	0	0.86
	Complex GVCPt (forward)	1379	0.09	0.09	0	0.51
	GVCPt (backward)	1379	0.21	0.14	0	0.84
	PLv_GVC: International Portion	1358	2.13	0.18	1.43	3.16
	PLv_GVC: Domestic Portion	1358	1.86	0.62	1.00	4.23

The regression results are shown in Table 8. In all regressions we find that production line position has significant impact on the degree of effect of the global financial crisis. The further is the position from the final consumption end, the less affected the node would be by the financial crisis. In the meanwhile, as shown in regressions (6)-(9), the influence of financial crises tend to be more severe for countries with a longer international portion and shorter domestic portion of forward linkage based GVC production length.

Besides that, regressions (1), (2), (4), (6) and (8) indicate that the forward linkage based GVC participation intensity (GVCPr) also has significant impact on the degree of effect of the global financial crisis. The higher the ratio, the greater the degree of negative impact. And as show in regression (2), the impact of backward linkage based GVC participation ratio is not very significant.

In regression (3), (5), (7), (9), we further differentiate the GVC participation into two categories to represent the simple and complex production sharing activities, respectively. Regression results clearly show that the impact of GVC participation on sectoral GDP during the financial crisis mainly comes from its complex portion, while the coefficients of its simple portion are not significant.

Furthermore, as shown in Regression (4), (5), (8) and (9), sectors in mature economies are less affected, while the negative shocks on sectors with a higher ratio of high-skill labor are more severe and the impact of sectoral level labor productivity is not significant.

5. Conclusions

In this paper, we have developed a GVC index system that includes three types of indexes based on both forward and backward inter-industry and cross-country linkages: a participation index for the intensity of a country-sector's engagement in global value chains; a production length index for the average number of production stages and complexity of the global value chain; and a position index for the location of a country sector on a global value chain, or the relative distance of a particular production stage to both ends of a global value chain. While the existing literature has proposed similar measures, our indices contain improvements that we argue are desirable and sensible from the viewpoint of economic intuition.

We thus can provide a comprehensive picture of each country/sector pair's GVC activities from multiple dimensions. All these indexes are built at the decomposition of statistics of production activities (such as GDP by industry) and can be further divided into different components with clear economic interpretations. By estimating these indexes according to the real world data, we produce a large set of indicators. These indexes could be used in advancing understanding of global supply chains.

Table 8 Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Position	8.303**	8.231*	8.633**	8.544**	8.871**	7.744*	7.596*	8.077*	7.931*
	(4.223)	(4.992)	(4.162)	(4.254)	(4.211)	(4.455)	(4.403)	(4.508)	(4.476)
International Portion						-9.240**	-7.108	-10.26**	-8.319*
						(4.608)	(4.427)	(4.660)	(4.481)
Domestic Portion						3.942***	3.890***	4.112***	4.086***
						(1.048)	(1.039)	(1.049)	(1.040)
GVCP (forward)	-26.94***	-26.79***		-25.64***		-20.14***		-18.29***	
	(3.430)	(5.206)		(3.462)		(3.830)		(3.945)	
GVCP (backward)		-0.302							
		(8.278)							
Simple GVCP (forward)			-4.909		-5.483		-0.228		-0.561
			(8.348)		(8.354)		(8.400)		(8.426)
Complex GVCP (forward)			-61.26***		-56.95***		-51.20***		-45.72***
			(13.41)		(13.33)		(12.77)		(12.51)
Mature（=1）				3.402***	3.125***			3.324***	3.063***
				(0.789)	(0.799)			(0.796)	(0.804)
Labor Productivity				-0.00450*	-0.00355			-0.00419	-0.00332
				(0.00272)	(0.00254)			(0.00272)	(0.00257)
High Skill				-17.02***	-17.27***			-17.82***	-18.02***
				(3.707)	(3.692)			(3.770)	(3.751)
Constant	-18.75***	-18.64***	-19.75***	-19.15***	-19.93***	-8.983	-13.40*	-7.893	-11.79
	(4.627)	(5.856)	(4.616)	(4.718)	(4.708)	(8.224)	(8.042)	(8.264)	(8.062)
Sector Fixed Effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,379	1,379	1,379	1,343	1,343	1,358	1,358	1,322	1,322
R-squared	0.229	0.229	0.236	0.245	0.251	0.236	0.241	0.253	0.258

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Definition of major variables

Label	Description
X	GN by 1 Gross outputs vector
$\boldsymbol{Y}$	GN by 1 Final products vector
$\boldsymbol{Z}$	GN by GN Intermediate flow matrix
Va	1 by GN Value added vector
A	GN by GN Input coefficient matrix
V	1 by GN Value added coefficient vector
B	GN by GN Leontief inverse matrix (Global)
G	GN by GN Ghosh inverse matrix
δ	dummy variable
$plvy_{ij}$	Production length of value added from sector i embodied in the final products of sector j
Xv	GN by 1 vector of accumulated value of gross outputs that induced by value added
PLv	Forward linkage based production length
PLy	Backward linkage based production length
L^{ss}	N by N Local Leontief inverse of country s
GVCPs	Position of the particular sector in GVC
GVCPt	Participation of the particular sector in GVC
APL	The Average Propagation Length (APL)

Definition of major label

Label	Description
_D	Pure domestic production
_RT	Production of traditional trade
_GVC	global value chains or cross country production sharing activities
_ GVC_S	Simple GVC activities, factor content cross national border for production only once
_GVC_C	Complex GVC activities, factor content cross national border at least twice
D	Segment of production length before first border crossing
C	Number of border crossing for production
F	Segment of production length after first border crossing

Appendix

Appendix A Difference between production length and APL in mathematical terms

Production length has some similarities to the Average Propagation Length (APL) proposed by Erik et al. (2005), but the two are different in both economic interpretation and mathematical expression. The APL is used to measure the distance between two sectors, which is defined as the average number of steps that it takes an exogenous change in one sector to affect the value of production in another sector. Based on equation 11 of the Erik et al, paper, the APL can be defined as

$$\mathrm{APL} = \frac{\mathrm{G(G-I)}}{\mathrm{G-I}} = \frac{B(B-I)}{B-I} \quad \text{(A1)}$$

And the APL from sector i to sector j can be expressed as

$$apl_{ij} = \frac{1}{g_{ij}-\delta_{ij}}\left[\sum_k^n g_{ik}g_{kj} - g_{ij}\right] = \frac{1}{b_{ij}-\delta_{ij}}\left[\sum_k^n b_{ik}b_{kj} - b_{ij}\right] \quad \text{(A2)}$$

The average production length we defined in the main text,

$$PL = \frac{\hat{V}BB\hat{Y}}{\hat{V}B\hat{Y}} = \frac{BB}{B} \quad \text{(A3)}$$

$$pl_{ij} = \frac{v_i \sum_k^n b_{ik}b_{kj}y_j}{v_i b_{ij} y_j} = \frac{\sum_k^n b_{ik}b_{kj}}{b_{ij}} \quad \text{(A4)}$$

If sector i ≠ sector j, $\delta_{ij} = 0$, therefore

$$apl_{ij} = \frac{\sum_k^n b_{ik}b_{kj} - b_{ij}}{b_{ij}} = \frac{\sum_k^n b_{ik}b_{kj}}{b_{ij}} - 1 = pl_{ij} - 1 \quad \text{(A5)}$$

If sector i = sector j, $\delta_{ij} = 1$, therefore

$$apl_{ii} = \frac{\sum_k^n b_{ik}b_{ki} - b_{ii}}{b_{ii}-1} = \frac{pl_{ii}b_{ii} - b_{ii}}{b_{ii}-1} = pl_{ii} + \frac{pl_{ii} - b_{ii}}{b_{ii}-1} \quad \text{(A6)}$$

From the definition of Leontief Inverse, $b_{ii} - 1 > 0$. From the definition of production length, $pl_{ii} - b_{ii} > b_{ii}b_{ii} - b_{ii} > 0$.

Therefore, in the off-diagonals, APL are smaller than production length, but in diagonal elements, APL are larger than production length.

We defined the average production length as total output value induced by an unit of particular value added or final products, which equals total gross output to GDP ratio. Therefore, if a closed economy's total output and GDP are stable, it's average production length is also robust. However, the APL is the average number of production stages that it takes an exogenous change in one sector to affect the value of production in another sector.

APL will change as the number of sector classifications changes.

Let's use a simple example to illustrate the relationship between APL and PL

Table J1 An **ICIO Table** with country S and R

	S1	S2	R	S	R	TO
S1	3	1	2	3	1	10
S2	1	2	1	1	0	5
R	1	1	4	2	2	10
VA	5	1	3			
TI	10	5	10			

Table J2 The **input coefficients** matrix

A	S1	S2	R
S1	0.3	0.2	0.2
S2	0.1	0.4	0.1
R	0.1	0.2	0.4

Table J3 The **Leontief Inverse** matrix

B	S1	S2	R
S1	1.63	0.77	0.67
S2	0.34	1.92	0.43
R	0.38	0.77	1.92

Table J4 The Square of Leontief Inverse matrix

BB	S1	S2	R
S1	3.19	3.25	2.73
S2	1.36	4.29	1.89
R	1.63	3.25	4.29

Table J5 The average production length (PL)

PL	S1	S2	R
S1	1.95	4.23	4.05
S2	4.05	2.23	4.37
R	4.23	4.23	2.23

Table J6 Aggregating the average production length (PL)

	S1	S2	R	WLD
PLy	2.36	3.46	3.03	**2.78**
Y	4.00	1.00	4.00	9.00
PLv	2.69	3.5	2.69	**2.78**
VA	5	1	3	9.00

Table J7 Combining the ICIO table to a sector (World level)

	W	Y	TO
W	16	9	25
V	9		

TI	25

And A=0.64， B=2.78，BB=7.72，PL=2.78

The average production length in the ICIO table and a sector model are the same.

Table J8 The indirect input coefficients matrix

B-I	S1	S2	R
S1	0.63	0.77	0.67
S2	0.34	0.92	0.43
R	0.38	0.77	0.92

Table J9 The matrix of indirect input coefficients and Leontief Inverse matrix

B(B-I)	S1	S2	R
S1	1.56	2.49	2.05
S2	1.03	2.37	1.46
R	1.24	2.49	2.37

Table J10 The APL

APL	S1	S2	R
S1	2.45	3.23	3.05
S2	3.05	2.56	3.37
R	3.23	3.23	2.56

Table J11 The Aggregating APL

	S1	S2	R	WLD
APL_b	2.73	2.9	2.82	**2.81**
Z_b	5	4	7	16
APL_f	2.78	2.9	2.78	**2.81**
Z_f	6	4	6	16

Combine the ICIO table to a sector (World level)

And A=0.64， B-I=1.78，B(B-I)=4.94，APL=2.78

When sectors in ICIO are aggregated, the APL changes, while PL stays the same.

Appendix B: the Ghosh input-output model and its linkage with the Leontief model

We define the output coefficient matrix as $D = \hat{X}^{-1} Z$, and the final products coefficient vector as $F = \hat{X}^{-1} Y$ as in Ghosh (1958). From the input side, gross inputs can be split into intermediate inputs and value added, $X'D + Va = X'$. Rearranging terms, we can reach the classical Ghosh inverse equation, $X' = VaH$, where $H = (I - D)^{-1}$ is the Ghosh inverse matrix. The linkage between value added and final products can also be expressed as: $Y' = X'\hat{F} = VaH\hat{F}$.

It is easy to derive the linkage between the input and output coefficient matrices as: $\hat{X}^{-1}A\hat{X} = \hat{X}^{-1}Z$. Similarly, the linkage between the Leontief inverse and the Ghosh inverse matrices are:

$$\hat{X}^{-1}B\hat{X} = \hat{X}^{-1}(I-A)^{-1}\hat{X} = \left[\hat{X}^{-1}(I-A)\hat{X}\right]^{-1}$$

$$= \left(I - \hat{X}^{-1}A\hat{X}\right)^{-1} = \left(I - \hat{X}^{-1}Z\right)^{-1} = (I-D)^{-1} = H \quad \text{(B1)}$$

Appendix C Derivation of Upstreamness

As defined in Fally (2012a, 2012b, 2013) and Antras et al (2012, 2013), the Upstreamness of an industry's output in the value chain can be measured as

$$U = \frac{Y}{X} + 2\frac{AY}{X} + 3\frac{AAY}{X} + \cdots = \frac{Y+2AY+3AAY+\cdots}{X} \quad \text{(C1)}$$

The numerator of equation C1 can be expressed in matrix notation as

$$Y + 2AY + 3AAY + \cdots = (Y + AY + AAY + \cdots)$$

$$+A(Y + AY + AAY + \cdots) + AA(Y + AY + AAY + \cdots) + \cdots$$

$$= BY + ABY + AABY + \cdots = BB = BX$$

Therefore, Upstreamness of an industry's output can be measured as

$$U = \frac{BX}{X} = Hu' \quad \text{(C2)}$$

The right side of equation C2 is the same as equation (12b) of the main text.

As defined in Antras and Chor (2013), the Downstreamness of an industry's output in the value chain can be measured as

$$D = \frac{V\hat{X}}{X'} + 2\frac{VA\hat{X}}{X'} + 3\frac{VAA\hat{X}}{X'} + \cdots = V + 2VA + 3VAA + \cdots = VBB = uB \quad \text{(C3)}$$

Therefore, Downstreamness of an industry's output can be measured as

$$D = uB \quad \text{(C4)}$$

The right side of equation C4 is the same as equation (15b) of the main text.

Appendix D: Proof of Equations (9) and (11)

Multiplying domestic value-added generated from each production stage of section 2.3.2 with production length of that stage and summing all production stages in an infinite stage production process, we can obtain the pure domestic value-added induced gross output as

$$Xv\_D = \hat{V}Y^D\mu + 2\hat{V}A^DY^D + 3\hat{V}A^DA^DY^D + \cdots$$
$$= \hat{V}(I + A^D + A^DA^D + \cdots)Y^D + \hat{V}(A^D + A^DA^D + \cdots)Y^D + \cdots$$
$$= \hat{V}(L + A^DL + A^DA^DL + \cdots)Y^D = \hat{V}LLY^D \quad \text{(D1)}$$

Where $I + A^D + A^DA^D + \cdots = L$

Similarly, where production of value-added in "traditional trade" is also entirely taking place domestically, the gross output it induced can be expressed as

$$Xv\_RT = \hat{V}Y^F + 2\hat{V}A^DY^F + 3\hat{V}A^DA^DY^F + \cdots$$
$$= \hat{V}(L + A^DL + A^DA^DL + \cdots)Y^F = \hat{V}LLY^F \quad \text{(D2)}$$

Appendix E Derivation of Equations (21) to (23)

Using the domestic or international production length of each stage of intermediate exports production discussed earlier as weights and summing across all production stages, we can obtain the global gross output generated by GVC related trade as well as its 3 components in any particular bilateral route.

$$Xd\_GVC = \hat{V}A^F\hat{Y} + 2\hat{V}A^DA^F\hat{Y} + \hat{V}A^FA\hat{Y} + 3\hat{V}A^DA^DA^F\hat{Y}$$
$$+2\hat{V}A^DA^FA\hat{Y} + \hat{V}A^FAA\hat{Y} + \cdots$$
$$= \left[\hat{V}A^F\hat{Y} + 2\hat{V}A^DA^F\hat{Y} + 3\hat{V}A^DA^DA^F\hat{Y} + \cdots\right]$$
$$+\left[\hat{V}A^FA\hat{Y} + 2\hat{V}A^DA^FA\hat{Y} + 3\hat{V}A^DA^DA^FA\hat{Y} + \cdots\right] + \cdots$$
$$= \hat{V}LLA^F\hat{Y} + \hat{V}LLA^FA\hat{Y} + \hat{V}LLA^FAA\hat{Y} + \cdots = \hat{V}LLA^FB\hat{Y} \quad \text{(E1)}$$

Similarly, the total international (foreign) gross outputs induced by value-added of source country embodied in its GVC related intermediate exports can be expressed as:

$$Xi\_GVC = \hat{V}A^F\hat{Y} + \hat{V}A^DA^F\hat{Y} + 2\hat{V}A^FA\hat{Y} + \hat{V}A^DA^DA^F\hat{Y}$$
$$+2\hat{V}A^DA^FA\hat{Y} + 3\hat{V}A^FAA\hat{Y} + \cdots$$
$$= \left[\hat{V}A^F\hat{Y} + \hat{V}A^DA^F\hat{Y} + \hat{V}A^DA^DA^F\hat{Y} + \cdots\right]$$
$$+2\left[\hat{V}A^FA\hat{Y} + \hat{V}A^DA^FA\hat{Y} + \hat{V}A^DA^DA^FA\hat{Y} + \cdots\right] + \cdots$$
$$= \hat{V}LA^F\hat{Y} + 2\hat{V}LA^FA\hat{Y} + 3\hat{V}LA^FAA\hat{Y} + \cdots = \hat{V}LA^FBB\hat{Y} \quad \text{(E2)}$$

Adding equation (E1) and (E2), the total outputs induced by value-added of source country embodied in its GVC related intermediate exports can be expressed as:

$$X\_GVC = Xd\_GVC + Xi\_GVC = \hat{V}LLA^F B\hat{Y} + \hat{V}LA^F BB\hat{Y}$$
$$= \hat{V}L(B-L)\hat{Y} + \hat{V}(B-L)B\hat{Y} = \hat{V}BB\hat{Y} - \hat{V}LL\hat{Y} \quad \text{(E3)}$$

Summing the numerator of equations (9), (11), Xd_GVC and Xi_GVC defined in equations (15) and (17) of the main text, we obtain

$$X\_D + X\_RT + X\_GVC$$
$$= \hat{V}LL\hat{Y}^D + \hat{V}LL\hat{Y}^F + \hat{V}BB\hat{Y} - \hat{V}LL\hat{Y} = \hat{V}BB\hat{Y} = Xvy \quad \text{(E4)}$$

As we discussed in section 2 of the main text, V_GVC measures domestic value-added embodied in intermediate exports. Based on equation (20) in the main text, it is easy to see that the part of $\mathrm{V\_GVC}$ which crosses border for production only once can be measured as $\hat{\mathrm{V}}\mathrm{LA^F L}\hat{Y}$, where $\mathrm{A^F}$ is the import input using coefficient matrix (with zero block matrix in the diagonal block in the GN by GN input using coefficient matrix A). The part of $\mathrm{V\_GVC}$ which crosses the border twice for production can be quantified as $\hat{V}LA^F LA^F L\hat{Y}$, and the part of $\mathrm{V\_GVC}$ which crosses the border three times for production can be quantified as $\hat{\mathrm{V}}\mathrm{LA^F LA^F LA^F L}\hat{Y}$. The same goes for successive border crossing.

Summing up all the above production stages, we obtain the total intermediate exports induced by $\mathrm{V\_GVC}$ as follow

$$\mathrm{E\_GVC} = \hat{\mathrm{V}}\mathrm{LA^F L}(\mathrm{I} + 2\mathrm{A^F L} + 3\mathrm{A^F LA^F L} + 4\mathrm{A^F LA^F LA^F L} + \cdots)\hat{Y}$$
$$= \hat{\mathrm{V}}\mathrm{LA^F L}(\mathrm{I} - \mathrm{A^F L})^{-1}(\mathrm{I} - \mathrm{A^F L})^{-1}\hat{Y} \quad \text{(E5)}$$

Based on the definition of the Leontief Inverse, $\mathrm{B} - \mathrm{L} = \mathrm{BA^F L}$, rearranging as

$$\mathrm{B} = \mathrm{L}(\mathrm{I} - \mathrm{A^F L})^{-1} = \mathrm{L}(\mathrm{I} + \mathrm{A^F L} + \mathrm{A^F LA^F L} + \mathrm{A^F LA^F LA^F L} + \cdots)$$

And $(\mathrm{I} - \mathrm{A^D})\mathrm{B} = (\mathrm{I} - \mathrm{A^F L})^{-1}$

Inserting these two equation into the equation (E5),

$$\mathrm{E\_GVC} = \hat{\mathrm{V}}\mathrm{LA^F B}(\mathrm{I} - \mathrm{A^D})\mathrm{B}\hat{Y} = \hat{\mathrm{V}}\mathrm{BA^F L}(\mathrm{I} - \mathrm{A^D})\mathrm{B}\hat{Y} = \hat{\mathrm{V}}\mathrm{BA^F B}\hat{Y} \quad \text{(E6)}$$

Where $\mathrm{LA^F B} = \mathrm{B} - \mathrm{L} = \mathrm{BA^F L}$, and $\mathrm{BY} = \mathrm{X}$.

The total domestic output of foreign countries induced by GVC production is the rest of output which equals that of the total international (foreign) gross outputs induced by GVC Production minus the total intermediate exports induced by GVC Production. It can be measured as

$$\mathrm{Xf\_GVC} = \mathrm{Xi\_}GVC - E\_GVC = \hat{V}LA^F BB\hat{Y} - \hat{V}LA^F B(I - A^D)B\hat{Y}$$
$$= \hat{V}LA^F BA^D B\hat{Y} = \hat{V}LA^F BA^D B\hat{Y} = \hat{V}BA^F B\hat{Y} - \hat{V}LA^F B\hat{Y} \quad \text{(E7)}$$

Summing up the total domestic output of foreign countries induced by GVC production and the total domestic output of source countries induced by GVC production, we obtain the total domestic output induced by GVC production.

$$Xd\_GVC + Xf\_GVC = \hat{V}LA^F BA^D B\hat{Y} + \hat{V}LLA^F B\hat{Y}$$
$$= \hat{V}BA^D B\hat{Y} - \hat{V}LB\hat{Y} + \hat{V}B\hat{Y} + \hat{V}BB\hat{Y} - \hat{V}BB\hat{Y} + \hat{V}LB\hat{Y} - \hat{V}LL\hat{Y}$$
$$= \hat{V}BA^D B\hat{Y} + \hat{V}LB\hat{Y} - \hat{V}LL\hat{Y} = \hat{V}BB\hat{Y} - \hat{V}LL\hat{Y} - \hat{V}BA^F B\hat{Y} \quad \text{(E8)}$$

Summing up the equation (D7) and (D9), we obtain the total output induced by V_GVC.

$$X\_GVC = \text{E\_GVC} + Xd\_GVC + Xf\_GVC = \hat{V}BB\hat{Y} - \hat{V}LL\hat{Y}$$
$$= Xd\_GVC + Xi\_GVC \quad \text{(E9)}$$

Muradov (2016) has proposed a measure of the average number of border crossings:

$$C = \frac{(I-A^F L)^{-2} Y^F \mu + [(I-A^F L)^{-1} - I](I-A^F L)^{-1} Y^D}{(I-A^F L)^{-1} Y^F \mu + [(I-A^F L)^{-1} - I] Y^D} \quad \text{(E10)}$$

He names the denominator as accumulated exports or $Ecum$, and where $L = (I - A^D)^{-1}$. From the definition of the Leontief Inverse, we have $BA^F L = B - L$. Rearranging:

$$(I - A^F L)^{-1} = (I - A^D)B = I + AB - A^D B = I + A^F B \quad \text{(E11)}$$

Inserting (G12) into the numerator and the denominator of C respectively, we obtain:

$$(I - A^F L)^{-2} Y^F + [(I - A^F L)^{-1} - I](I - A^F L)^{-1} Y^D$$
$$= (I + A^F B)^2 Y^F + A^F B(I + A^F B) Y^D$$
$$= (I + A^F B)^2 Y^F + (I + A^F B) A^F B Y^D$$
$$= (I + A^F B)(Y^F + A^F BY) \quad \text{(E12)}$$
$$Ecum = (I - A^F L)^{-1} Y^F + [(I - A^F L)^{-1} - I] Y^D$$
$$= (I + A^F B) Y^F + A^F B Y^D = Y^F + A^F BY \quad \text{(E13)}$$

It is easy to see that the 1st terms in both the nominator (E12) and the denominator (E13) are final exports, the 2nd terms are intermediate exports induced by various final products. Therefore, the definition of C can be can be rearranging as

$$C = \frac{(I+A^F B)(Y^F + A^F BY)}{Y^F + A^F BY} = \frac{(I+A^F B)Ecum}{Ecum} \quad \text{(E14)}$$

Multiplying $\widehat{VL}$, the diagonal local value added multiplier (VL) matrix to both the nominator and denominator, we have

$$C = \frac{\widehat{VL}(I+A^F B)Ecum}{\widehat{VL}Ecum} \quad \text{(E15)}$$

Aggregating both nominator and denominator across the column and along the row, of this GN by G matrix, we obtain a GN by 1 vector of gross exports at country/sector level.

$$Ecum\ \mu = (I + A^F B)Y^F + A^F BY^D = Y^F + A^F BY = E \quad \text{(E16)}$$

$$C = \frac{(I+A^F B)E}{E} = \frac{\widehat{VL}(I+A^F B)E}{\widehat{VL}E} \quad \text{(E17)}$$

The average number of border crossing of country s can be measured as

$$C^s = \frac{\sum_{t\neq s}^{G} E^{st} + \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{\sum_{t\neq s}^{G} E^{st}} = \frac{[V^s L^{ss}]' \# [\sum_{t\neq s}^{G} E^{st} + \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}]}{[V^s L^{ss}]' \# [\sum_{t\neq s}^{G} E^{st}]} \quad \text{(E18)}$$

The average number of border crossing based on forward industrial linkage in our method

$$CBv\_T = \frac{\hat{V}BE}{\hat{V}LE} = \frac{\hat{V}LE\mu + \hat{V}LA^F BE}{\hat{V}LE} \quad \text{(E19)}$$

$$CBv\_T^s = \frac{\hat{V}^s L^{ss} \sum_{t\neq s}^{G} E^{st} + \hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{\hat{V}^s L^{ss} \sum_{t\neq s}^{G} E^{st}} \quad \text{(E20)}$$

Comparing equations (E18) and (E20), the two methods seem different. The denominator of Muradov (2016) is a country-sector's total gross exports to the world, and the nominator is the total gross exports plus the sum of the intermediate exports used to produce exports (repeated counting). While the denominator in our method is domestic value-added embodied in gross exports， and the nominator is the part of value-added induced gross exports by all countries, they are not equal each other at the country-sector level. Once we aggregate them to either the country or the global level, these two methods become the same.

$$c^s = \frac{V^s L^{ss} \sum_{t\neq s}^{G} E^{st} + V^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{V^s L^{ss} \sum_{t\neq s}^{G} E^{st}} \quad \text{(E21)}$$

$$cbv\_t^s = \frac{V^s L^{ss} \sum_{t\neq s}^{G} E^{st} + V^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{V^s L^{ss} \sum_{t\neq s}^{G} E^{st}} \quad \text{(E22)}$$

$$C^w = \frac{u(I+A^F B)E}{uE} = \frac{u\widehat{VL}(I+A^F B)E}{u\widehat{VL}E} = \frac{VLE\mu + VLA^F BE}{VLE} = \frac{VBE}{VLE} \quad \text{(E23)}$$

$$CBv\_T^w = \frac{u\hat{V}BE}{u\hat{V}LE} = \frac{VBE}{VLE} \quad \text{(E24)}$$

Appendix F: Derivation of equations (27) and (28)

Similarly, as we discussed in section 2 of the main text, Y_GVC measure both domestic and foreign value-added in intermediate imports. It can be seen that the part of Y_GVC which crosses border for production only once can be measured by $VLA^F L\hat{Y}$. The part of Y_GVC which crosses border 2 times for production can be quantified as $VLA^F LA^F L\hat{Y}$, and the part of Y_GVC which cross border 3 times for production can be quantified as

$VLA^F LA^F LA^F L\hat{Y}$. The same goes on for successive border crossing for production.

Summing up all the above stages, we obtain the total intermediate exports induced by Y_GVC as follow

$$Ey\_GVC = VLA^F L(I + 2A^F L + 3A^F LA^F L + 4A^F LA^F LA^F L + \cdots)\hat{Y}$$

$$= VLA^F L(I - A^F L)^{-1}(I - A^F L)^{-1}\hat{Y} = VBA^F B\hat{Y} = uA^F B\hat{Y} \quad \text{(F1)}$$

Aggregating equation (F1) over all countries and sectors, we obtain total global intermediate imports.

The total domestic output of foreign countries induced by Y_GVC can be measured as

$$Xyf\_GVC = Xyi\_GVC - Ey\_GVC = uBA^F L\hat{Y} - uA^F B\hat{Y}$$

$$= uLA^F B\hat{Y} - uA^F B\hat{Y} = uA^D LA^F B\hat{Y} = uA^D BA^F L\hat{Y} \quad \text{(F2)}$$

Dividing equations (F1) and (F2) by Y_GVC, we can obtain (1) the average number of border crossings of intermediate imports used in the source country final product production activities; (2) the average domestic production length of Y_GVC within countries involved in the GVCs after entering the importing country as

$$PLyi\_GVC = CBy\_GVC + PLyf\_GVC$$

$$= \frac{Ey\_GVC}{Y\_GVC} + \frac{Xyf\_GVC}{Y\_GVC} \quad \text{(F3)}$$

The total domestic output induced by Y_GVC can be measured as

$$Xyf\_GVC + Xyd\_GVC = uA^D BA^F L\hat{Y} + uA^F LL\hat{Y} \quad \text{(F4)}$$

Summing up the equation (F1) and (F4), we obtain the total output induced by Y_GVC.

$$Xy\_GVC = Ev\_GVC + Xyf\_GVC + Xyd\_GVC = Xvd\_GVC + Xvi\_GVC$$

$$= uA^F B\hat{Y} + uA^D BA^F L\hat{Y} + uA^F LL\hat{Y}$$

$$= u(I + A^D L)A^F B\hat{Y} + uA^F LL\hat{Y} = uLA^F B\hat{Y} + uL\hat{Y} - VLL\hat{Y} \quad \text{(F5)}$$

$$= uB\hat{Y} - uL\hat{Y} + uL\hat{Y} - VLL\hat{Y} = VBB\hat{Y} - VLL\hat{Y}$$

Adding final product exports, we obtain the total cross country exports induced by gross exports

$$Ey\_T = Ey\_GVC + Y^F = uA^F B\hat{Y} + Y^F \quad \text{(F6)}$$

And dividing Ey_T by the import value of total final goods and services production, we can obtain the average number of border crossings of intermediate imports used in a particular country/sector final product production activities

$$CBy = \frac{Ey\_T}{Y\_RT + Y\_GVC} = \frac{uA^F B\hat{Y} + u\hat{Y}^F}{\mathrm{VL}\hat{Y}^F + \mathrm{VL}A^F \mathrm{B}\hat{Y}} \quad \text{(F7)}$$

Appendix G：Has US production become longer or shorter?

Fally (2012) showed a somewhat puzzling finding that the production chain (or the distance to the final demand) appears to have shortened over time in the United States, and he conjectures this to be a global phenomenon.

Fally's definition of "production length" (or "Upstreamness") is the average number of production stages from a sector's gross output to the final users. His results rely on the US IO tables, which covers 85 industries from 1947 to 2002, or 540 product categories from 1967 to 1992. To estimate the global production length, Fally made an assumption that "same industries have the same production length across countries". In this part, we argue that this assumption may be responsible for the puzzling finding.

First, for the United States, we also find that the production has become shorter. This is consistent with Fally. Table G1 reports the overall production length for US sectors. The production length has decreased for 26 out of 35 sectors from 1995 to 2011.

Table G1 Production Length (Forward Linkage) of US Sectors, 2011

Sector	Year 1995	Year 2011	Become shorter?
Agriculture	2.677	2.583	√
Mining	2.918	2.487	√
Food	1.679	1.688	
Textiles Products	2.227	2.112	√
Leather and Footwear	1.632	1.252	√
Wood Products	2.531	2.597	
Paper and Printing	2.581	2.306	√
Refined Petroleum	2.375	2.305	√
Chemical Products	2.665	2.468	√
Rubber and Plastics	2.659	2.509	√
Other Non-Metal	2.615	2.563	√
Basic Metals	3.025	3.027	
Machinery	1.834	1.784	√
Electrical Equipment	2.187	2.016	√
Transport Equipment	1.802	1.672	√
Recycling	1.570	1.588	
Electricity, Gas and Water	2.061	1.820	√
Construction	1.246	1.295	
Sale of Vehicles and Fuel	1.386	1.324	√
Wholesale Trade	2.154	1.937	√
Retail Trade	1.321	1.204	√
Hotels and Restaurants	1.446	1.435	√
Inland Transport	2.429	2.289	√
Water Transport	2.298	1.740	√
Air Transport	1.806	1.654	√
Other Transport	2.805	2.693	√
Post and Telecommunications	2.266	2.115	√
Financial Intermediation	2.187	2.311	
Real Estate	1.472	1.429	√
Business Activities	2.590	2.453	√
Public Admin	1.103	1.110	
Education	1.254	1.097	√
Health and Social Work	1.036	1.029	
Other Services	1.764	1.785	
Private Households	1.386	1.324	√

At the country level, the average production length for US decreased during 1995-2003, but has increased during 2003-2008, before resuming a decline again.

Figure G1 Average Production Length for US

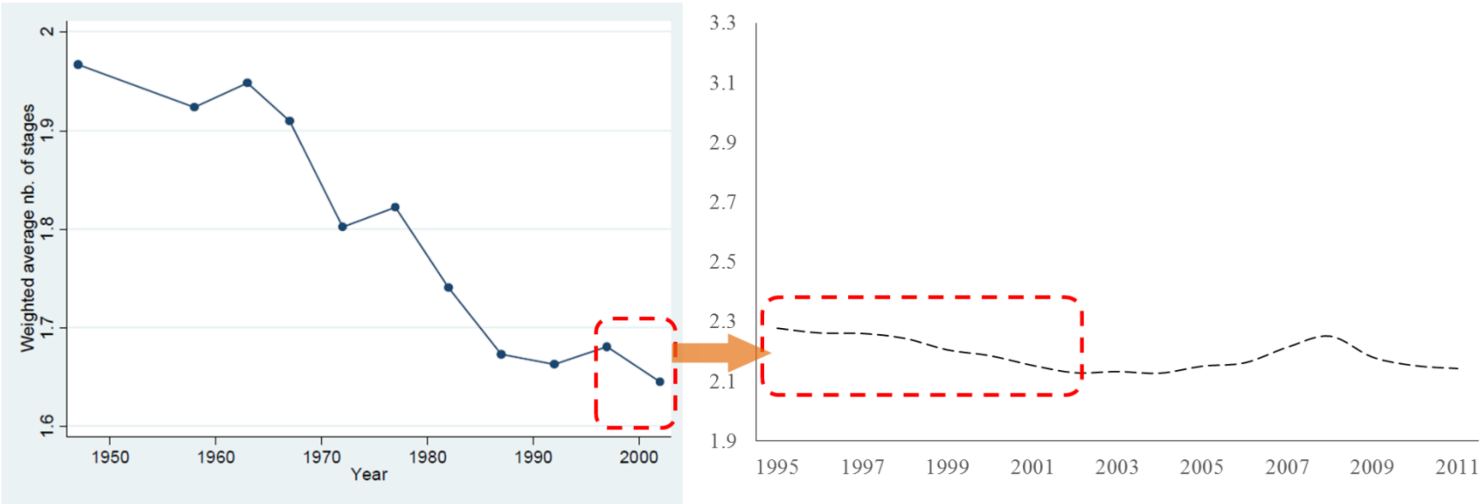

Figure 1 in Fally (2012) "Production Staging: Measurement and Facts"
Aggregate measure of vertical fragmentation (tradable goods excluding petroleum)

However, this pattern does not hold for the world as a whole. As shown in Figure J2, the production length for a given industry varies considerably across countries. In particular, production has become longer for China and for the world as a whole.

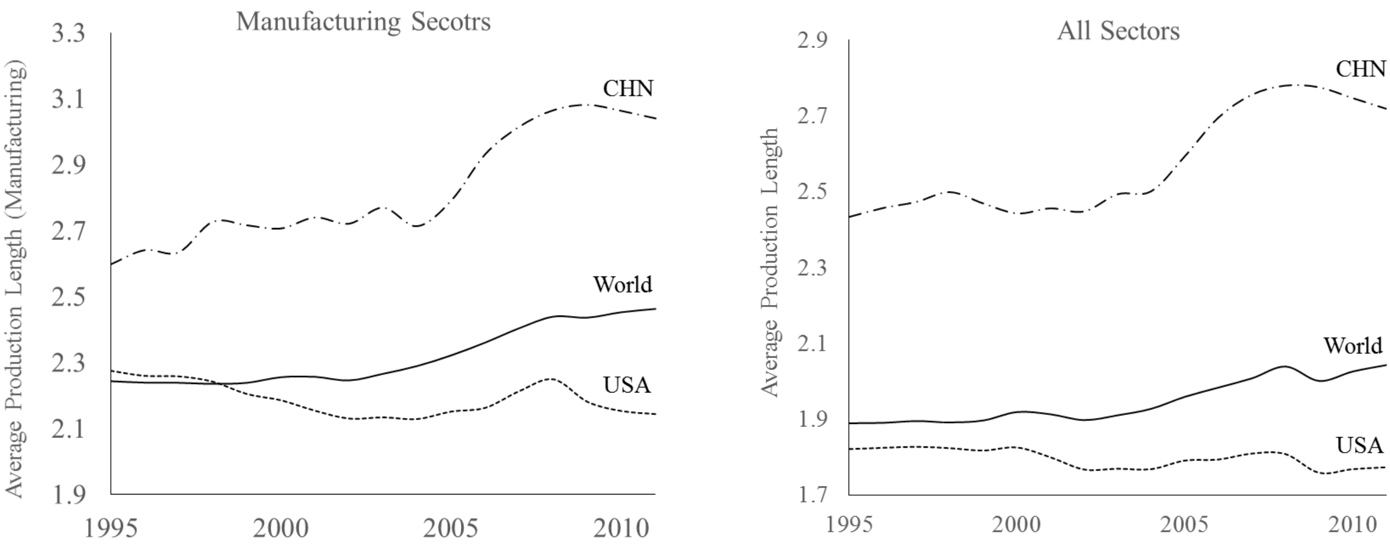

Figure G2 Average Production Length, China, US and the World

As a counterfactual thought experiment, let us assume that the production length found in the United States is applicable to all other countries in the world. We re-estimate the weighted average global production length and find that the upward trend of the global production length in Figure G2 has disappeared. Instead, we see a downward trend in Figure G3.

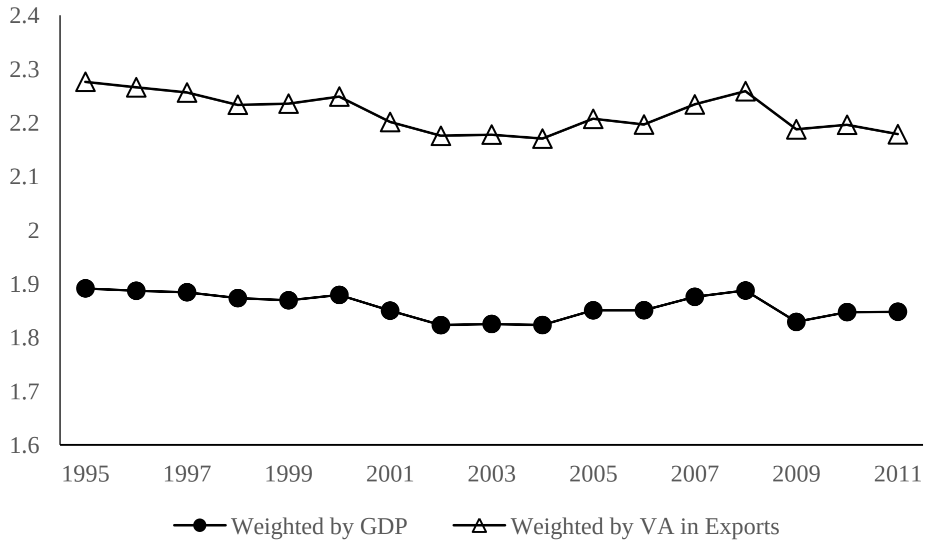

Figure G3 Global Average Production Length under the "Equal Length Assumption"

Table A1 WIOD Sectors

Code	NACE	Industry
r1	A01	Crop and animal production, hunting and related service activities
r2	A02	Forestry and logging
r3	A03	Fishing and aquaculture
r4	B	Mining and quarrying
r5	C10-C12	Manufacture of food products, beverages and tobacco products
r6	C13-C15	Manufacture of textiles, wearing apparel and leather products
r7	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
r8	C17	Manufacture of paper and paper products
r9	C18	Printing and reproduction of recorded media
r10	C19	Manufacture of coke and refined petroleum products
r11	C20	Manufacture of chemicals and chemical products
r12	C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
r13	C22	Manufacture of rubber and plastic products
r14	C23	Manufacture of other non-metallic mineral products
r15	C24	Manufacture of basic metals
r16	C25	Manufacture of fabricated metal products, except machinery and equipment
r17	C26	Manufacture of computer, electronic and optical products
r18	C27	Manufacture of electrical equipment
r19	C28	Manufacture of machinery and equipment n.e.c.
r20	C29	Manufacture of motor vehicles, trailers and semi-trailers
r21	C30	Manufacture of other transport equipment
r22	C31_C32	Manufacture of furniture; other manufacturing
r23	C33	Repair and installation of machinery and equipment
r24	D35	Electricity, gas, steam and air conditioning supply
r25	E36	Water collection, treatment and supply
r26	E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
r27	F	Construction
r28	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
r29	G46	Wholesale trade, except of motor vehicles and motorcycles
r30	G47	Retail trade, except of motor vehicles and motorcycles
r31	H49	Land transport and transport via pipelines
r32	H50	Water transport
r33	H51	Air transport
r34	H52	Warehousing and support activities for transportation
r35	H53	Postal and courier activities
r36	I	Accommodation and food service activities
r37	J58	Publishing activities
r38	J59_J60	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities

r39	J61	Telecommunications
r40	J62_J63	Computer programming, consultancy and related activities; information service activities
r41	K64	Financial service activities, except insurance and pension funding
r42	K65	Insurance, reinsurance and pension funding, except compulsory social security
r43	K66	Activities auxiliary to financial services and insurance activities
r44	L68	Real estate activities
r45	M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities
r46	M71	Architectural and engineering activities; technical testing and analysis
r47	M72	Scientific research and development
r48	M73	Advertising and market research
r49	M74_M75	Other professional, scientific and technical activities; veterinary activities
r50	N	Administrative and support service activities
r51	O84	Public administration and defence; compulsory social security
r52	P85	Education
r53	Q	Human health and social work activities
r54	R_S	Other service activities
r55	T	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
r56	U	Activities of extraterritorial organizations and bodies