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THE SOURCES OF CAPITAL MISALLOCATION

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ABSTRACT

We develop and implement a methodology to disentangle various sources of capital misallocation, i.e., dispersion in static marginal products. Our strategy uses readily observable moments in firm-level data, e.g., capital and revenues, to measure the contributions of technological and informational frictions as well as a rich class of (potentially distortionary) firm-specific factors. Applying this method to manufacturing firms in China reveals a modest role for adjustment costs and uncertainty. A substantial fraction of the observed misallocation comes from other idiosyncratic factors, both productivity/size-dependent as well as permanent. Adjustment costs are relatively more salient for large US firms, though permanent firm-specific factors remain important. We bound the effects of unobserved heterogeneity in technologies/markups – our results suggest they account for a limited fraction of observed misallocation in China, but a potentially large share for US firms.

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1 Introduction

A large and growing body of work analyzes the 'misallocation' of productive resources across firms, i.e., dispersion in static marginal products, and the resulting adverse effects on aggregate productivity and output. A number of recent studies examine the role of specific factors hindering period-by-period marginal product equalization. Examples of such factors include adjustment costs, imperfect information, financial frictions, as well as firm-specific 'distortions' (e.g., due to economic policies or other institutional features). A common methodological approach in this work is to focus on one particular source of misallocation (and in a few instances, on a small subset of potential sources) while abstracting from others. However, this approach is potentially problematic – because the data reflect the influence of all of these factors, examining them in isolation runs the risk of reaching biased conclusions of their severity and the resulting contribution to observed misallocation.

In this paper, we develop a methodology to distinguish various sources of measured capital misallocation. We augment a standard general equilibrium model of firm dynamics with a number of forces that contribute to *ex-post* dispersion in static marginal products.[1] These include (1) capital adjustment costs, (2) informational frictions, in the form of imperfect signals about future fundamentals and (3) a rich class of firm-specific factors, meant to capture all other forces influencing a firm's investment choice, including, but not limited to, financial frictions, unobserved heterogeneity in markups and/or production technologies, or institutional/policy-related distortions. We use a flexible specification for these factors, which allows for time-variation and correlation with firm characteristics.

Our main contribution is an empirical strategy designed to precisely measure the contribution of each of these factors. The method uses observable moments from widely available firm-level data, specifically, elements from the covariance matrix of firm-level capital and revenues. The key insight behind our approach is that while each moment is a complicated function of multiple factors, making any single moment insufficient to identify a particular factor, combining the information in a larger set of moments can be extremely helpful in disentangling these factors. Indeed, we show that allowing these forces to act in tandem is essential to reconcile a broad set of moments in firm-level investment dynamics.

To understand the difficulty in separating these factors, consider, as an example, the impact of convex adjustment costs. A common approach to gauge the severity of these costs is to examine the variability of firm-level investment. When adjustment costs are the only force present, this moment has an intuitive, one-to-one mapping with their magnitude – the lower is invest-

[1] Throughout the paper we use the term misallocation to refer to dispersion in static marginal products, whether stemming from distortionary factors or efficient ones, for example, adjustment costs.

ment volatility relative to fundamentals, the greater the adjustment cost. However, suppose that there are other firm-specific factors that influence investment decisions (e.g., idiosyncratic distortions). Now, depending on the correlation of those factors with firm-level fundamentals (either demand- or supply-side), they can serve to either increase or dampen investment volatility. As a result, using this particular moment to make inferences regarding the extent of adjustment costs leads to a biased estimate of their severity. The empirically relevant case turns out to be one where these other factors are negatively correlated with fundamentals (so that they tend to disincentivize investment by firms with better fundamentals), which implies a positive bias, i.e., a model with only adjustment costs will overstate their importance. As a second example, consider the effects of firm-level uncertainty. If fundamentals are revealed only slowly through time, imperfect information reduces the contemporaneous correlation between investment and fundamentals. However, a low correlation could also be the result of factors orthogonal to fundamentals that enter the firm's investment problem (e.g., uncorrelated, transitory distortions). Again, using this moment in isolation to measure uncertainty runs the risk of overstatement by incorrectly attributing these distortions to lower quality information on the part of firms.

We use a salient special case of our model – when firm-level fundamentals follow a random walk – to develop an identification strategy that combines the information in a broad set of data moments. The tractability of this special case allows us to prove that a set of four moments uniquely identify the underlying structural parameters that determine the contribution of each factor. Specifically, (1) the variance of investment, (2) the autocorrelation of investment, (3) the correlation of investment with past fundamentals, and (4) the covariance of the marginal (revenue) product of capital (*mrpk*) with fundamentals combine to identify the severity of adjustment costs, the extent of uncertainty and the magnitudes of the correlated and transitory/uncorrelated components of other firm-specific factors.

Our choice of moments is guided by a key insight: taken in pairs, they have opposing effects on a corresponding pair of parameters. To see this, consider the challenge described earlier of disentangling adjustment costs from other factors that are negatively correlated with firm fundamentals. Both dampen the firm's incentives to respond to changing fundamentals and so depress the volatility of investment. However, they have opposing effects on the autocorrelation of investment - convex adjustment costs create incentives to smooth investment over time and so tend to make investment more serially correlated. A distortion that directly reduces the response to fundamentals, on the other hand, increases the relative importance of transitory factors in investment, reducing the autocorrelation. Holding all else fixed, these two moments allow us to separate the two forces. Similar arguments can be developed for the remaining factors as well. In our quantitative work, where we depart from the polar random walk case, we follow an estimation strategy guided by these findings and demonstrate numerically that the same logic carries through.

We apply our methodology to data on manufacturing firms in China from the Annual Surveys of Industrial Production over the period 1998-2009. These data represent a census of all state and non-state manufacturing firms above a certain size threshold. Our results show that adjustment and informational frictions account for a relatively modest share of misallocation among Chinese firms, composing about 1% and 10% of overall dispersion in the marginal product of capital, respectively. Losses in aggregate total factor productivity (TFP) from these two sources (relative to the undistorted first-best) are 1% and 8%. These findings suggest that a substantial portion of observed misallocation in China is due to other firm-specific factors, both correlated with fundamentals (and therefore, vary over time with the fortunes of the firm) and ones that are essentially permanent. These lead to TFP losses of 38% and 36%, respectively.[2]

We also apply the methodology to data on publicly traded firms in the US. Although the two sets of firms are not directly comparable, the results for firms in a developed economy such as the US serve as a useful benchmark to put our results for China in context. As one would expect, the overall degree of misallocation is considerably smaller for these US firms. More interestingly, adjustment costs account for a larger share (about 11%) of observed *mrpk* dispersion than in China, though their overall magnitude remains modest, especially relative to earlier estimates in the literature. Uncertainty and other correlated factors play a smaller role than among Chinese firms, reducing aggregate TFP by 1% and 3%, respectively. However, other firm-specific fixed factors, although considerably smaller in absolute magnitude than in China, also seem to be quite significant as a share of total *mrpk* dispersion, even among large firms in the US. Our estimates suggest eliminating them could increase TFP by as much as 13%. In sum, even for the US, technological and informational frictions alone cannot account for the majority of observed marginal product dispersion and leave an important role for other (permanent) firm-specific factors.[3]

These patterns are robust to a number of alternative variants of our baseline setup. For example, they are largely unchanged when we allow for non-convex adjustment costs. Assuming that labor is subject to the same frictions and distortions as capital also leads to similar conclusions on the relative importance of the various forces – specifically, adjustment costs and uncertainty still account for only about 13% and 11% of *mrpk* dispersion, respectively. However, since both inputs are affected by each of the forces, the *absolute* importance of all factors

[2] Our analysis allows for distortions that are transitory and uncorrelated with firm characteristics. However, our estimation finds them to be negligible.

[3] We also report results for Chinese publicly traded firms as well as Colombian and Mexican manufacturing firms. The results regarding the role of various factors in driving misallocation are quite similar to our baseline findings for Chinese manufacturers.

– i.e., the impact on aggregate TFP and output – is much higher. For example, correlated and permanent factors are estimated to lead to TFP drops of 144% and 90%, respectively. We interpret these estimates as an upper bound, with reality likely falling somewhere in between this and the baseline version with frictionless labor. Importantly, capital misallocation is still largely driven by factors other than adjustment and information frictions.

Finally, we investigate the potential for these factors to be driven by misspecified production/demand systems – specifically, heterogeneity in capital elasticities and markups across firms. We derive an upper bound for their contributions to marginal product dispersion. This requires an extension of our baseline model and additional data on labor and intermediate inputs. Our results reveal a modest scope for these factors in China – together, they account for at most about 27% of *mrpk* dispersion. In contrast, for US publicly traded firms, they can explain as much as 90%. These findings suggest that this form of unobserved heterogeneity seems like a promising explanation for the observed 'misallocation' in the US, but the predominant drivers among Chinese firms lie elsewhere e.g., institutional/policy-related distortions.

The paper is organized as follows. Section 2 describes our model of production and frictional investment. Section 3 spells out our approach to identifying these frictions using the analytically tractable random walk case, while Section 4 details our numerical analysis and presents our quantitative results. We summarize our findings and discuss directions for future research in Section 5. Details of derivations and data work are provided in the Appendix.

Related literature. Our paper relates to several branches of literature. We bear a direct connection to the growing body of work focused on measuring and quantifying the effects of resource misallocation.[4] Following the seminal contributions of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), recent attention has shifted toward analyzing the roles of specific factors in generating misallocation. Contributions include work by Asker et al. (2014) on adjustment costs, Buera et al. (2011), Moll (2014) and Midrigan and Xu (2014) on financial frictions, David et al. (2016) on uncertainty and Peters (2016) on markup dispersion. A few papers study subsets of these factors in combination, for example, Gopinath et al. (2017) and Kehrig and Vincent (2017) combine financial and adjustment frictions, while Song and Wu (2015) estimate a model with adjustment costs, permanent distortions and heterogeneity in markups/technologies.

Our primary contribution is to develop a unified framework that encompasses many of these factors and devise an empirical strategy based on observable firm-level data to disentangle them. We augment a standard adjustment cost model with information frictions and a flexible class of additional, potentially distortionary, factors. Our modeling of these factors as implicit taxes

[4]Restuccia and Rogerson (2017) and Hopenhayn (2014) provide recent overviews of this line of work.

that can be correlated with fundamentals follows the approach taken by, e.g., Restuccia and Rogerson (2008), Guner et al. (2008), Bartelsman et al. (2013), Buera et al. (2013), Buera and Fattal-Jaef (2016) and Hsieh and Klenow (2014). An analytically tractable special case of our model allows us to prove identification in an intuitive and transparent fashion. Our findings underscore the importance of studying such a broad set of forces in tandem. This breadth is partly what distinguishes us from the work of Song and Wu (2015), who abstract from time-variation in firm-level distortions (as well as in firm-specific markups/technologies), ruling out, by assumption, any role for so-called 'correlated' or size-dependent distortions.[5] Many papers in the literature – e.g., Restuccia and Rogerson (2008), Bartelsman et al. (2013), Hsieh and Klenow (2014) and Bento and Restuccia (2016) – emphasize the need to distinguish such factors from those that are orthogonal to fundamentals. This message is reinforced by our quantitative findings, which reveal a significant role for correlated factors (in addition to uncorrelated, permanent ones), particularly in developing countries such as China.

Our methodology and findings also have relevance beyond the misallocation context, perhaps most notably for studies of adjustment and informational frictions. A large literature has examined the implications of adjustment costs, examples of which include Cooper and Haltiwanger (2006), Khan and Thomas (2008) and Bloom (2009). Our analysis shows that accounting for other firm-specific factors acting on firms' investment decisions is potentially crucial in order to accurately estimate the severity of these frictions and reconcile a broader set of micro-level moments. A similar point applies to recent work on quantifying firm-level uncertainty, for example, Bloom (2009), Bachmann and Elstner (2015) and Jurado et al. (2015).

2 The Model

We consider a discrete time, infinite-horizon economy, populated by a representative household. The household inelastically supplies a fixed quantity of labor N and has preferences over consumption of a final good. The household discounts time at rate β. The household side of the economy is deliberately kept simple as it plays a limited role in our study. Throughout the analysis, we focus on a stationary equilibrium in which all aggregate variables remain constant.

Production. A continuum of firms of fixed measure one, indexed by i, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\hat{\alpha}_1} N_{it}^{\hat{\alpha}_2}, \quad \hat{\alpha}_1 + \hat{\alpha}_2 \leq 1 \ . \tag{1}$$

[5] We also differ from Song and Wu (2015) in our explicit modeling (and measurement) of information frictions and in our approach to quantifying heterogeneity in markups/technologies.

These intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left(\int \hat{A}_{it} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} ,$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between intermediate goods and $\hat{A}_{it}$ represents an idiosyncratic demand shifter. This is the only source of fundamental uncertainty in the economy (i.e., we abstract from aggregate risk).

Market structure and revenue. The final good is produced frictionlessly by a representative competitive firm. This yields a standard demand function for intermediate good i:

$$Y_{it} = P_{it}^{-\theta} \hat{A}_{it}^{\theta} Y_t \quad \Rightarrow \quad P_{it} = \left(\frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\theta}} \hat{A}_{it} ,$$

where P_{it} denotes the relative price of good i in terms of the final good, which serves as numeraire. Revenues for firm i at time t are

$$P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} \hat{A}_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} ,$$

where

$$\alpha_j = \left(1 - \frac{1}{\theta} \right) \hat{\alpha}_j, \quad j = 1, 2 .$$

This framework accommodates two alternative interpretations of the idiosyncratic component $\hat{A}_{it}$: as a firm-specific shifter of either demand or productive efficiency, and so we simply refer to $\hat{A}_{it}$ as a firm-specific fundamental.

Input choices. In our baseline analysis, we assume that firms hire labor period-by-period under full information at a competitive wage W_t.[6] At the end of each period, firms choose investment in new capital, which becomes available for production in the following period. Investment is subject to quadratic adjustment costs, given by

$$\Phi \left(K_{it+1}, K_{it} \right) = \frac{\hat{\xi}}{2} \left(\frac{K_{it+1}}{K_{it}} - (1 - \delta) \right)^2 K_{it} , \tag{2}$$

where $\hat{\xi}$ parameterizes the severity of the adjustment cost and δ is the rate of depreciation.[7]

Investment decisions are likely to be affected by a number of additional factors (other than

[6]We relax this assumption in Section 4.6.2.

[7]We generalize this specification to include non-convex costs in Section 4.6.1. Our quantitative results change little.

productivity/demand and the level of installed capital). These could originate, for example, from distortionary government policies – e.g., taxes, size restrictions or regulations, or other features of the institutional environment - from other market frictions that are not explicitly modeled, e.g., financial frictions, or from unmodeled heterogeneity in markups/production technologies. For now, we do not take a stand on the precise nature of these additional factors. To capture them, we follow, e.g., Hsieh and Klenow (2009), and introduce a class of idiosyncratic 'wedges' that appear in the firm's optimization problem as proportional taxes on the flow cost of capital. We denote these wedges by T^K_{it+1} and, in a slight abuse of terminology, refer to them as 'distortions' or wedges throughout the paper, even though they may partly reflect sources that are efficient (for example, production function heterogeneity). Later, in Section 4.5, we demonstrate how progress can be made in further disentangling some of these sources.

The firm's problem in a stationary equilibrium can be represented in recursive form as (we suppress the time subscript on all aggregate variables)

$$\begin{aligned}\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right) = \max_{N_{it},K_{it+1}} \quad & \mathbb{E}_{it}\left[Y^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2} - WN_{it} - T^K_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad & \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] \ ,\end{aligned}$$

where $\mathbb{E}_{it}\left[\cdot\right]$ denotes the firm's expectations conditional on $\mathcal{I}_{it}$, the information set of the firm at the time of making its period t investment choice. We describe this set explicitly below. The term $1-\beta(1-\delta)$ is the user cost per unit of capital.

After maximizing over N_{it}, this becomes

$$\begin{aligned}\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right) = \max_{K_{it+1}} \quad & \mathbb{E}_{it}\left[GA_{it}K_{it}^{\alpha} - T^K_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad & \mathbb{E}_{it}\beta\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] \ ,\end{aligned} \tag{3}$$

where $G \equiv \left(1-\alpha_2\right)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}$, $A_{it} \equiv \hat{A}_{it}^{\frac{1}{1-\alpha_2}}$ and $\alpha \equiv \frac{\alpha_1}{1-\alpha_2}$ is the curvature of operating profits (revenues net of wages).[8]

Equilibrium. We can now define a *stationary equilibrium* in this economy as (i) a set of value and policy functions for the firm, $\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right)$, $N_{it}\left(K_{it},\mathcal{I}_{it}\right)$ and $K_{it+1}\left(K_{it},\mathcal{I}_{it}\right)$, (ii) a wage W and (iii) a joint distribution over $\left(K_{it},\mathcal{I}_{it}\right)$ such that (a) taking as given wages and the law of motion for $\mathcal{I}_{it}$, the value and policy functions solve the firm's optimization problem, (b) the

[8] Allowing for labor market distortions that manifest themselves in firm-specific wages has no effect on our identification strategy or our results about the sources of *mrpk* dispersion – see Appendix A.2. In Section 4.6.2, we subject the firm's labor choice to the same frictions – whether due to adjustment costs, informational or distortionary factors – as its capital investment decision and show that this setup leads to a very similar specification with suitably re-defined fundamentals and curvature.

labor market clears and (c) the joint distribution remains constant through time.

Characterization. We solve the model using perturbation methods. In particular, we log-linearize the firm's optimality conditions and laws of motion around the undistorted non-stochastic steady state, where $A_{it} = \bar{A}$ and $T_{it}^K = 1$. Appendix A.1.1 derives the following log-linearized Euler equation:[9]

$$k_{it+1}\left((1+\beta)\xi + 1 - \alpha\right) = \mathbb{E}_{it}\left[a_{it+1} + \tau_{it+1}\right] + \beta\xi\mathbb{E}_{it}\left[k_{it+2}\right] + \xi k_{it} \ , \tag{4}$$

where ξ is a composite parameter that captures the degree of adjustment costs and τ_{it+1} summarizes the effect of T_{it+1}^K on the firm's investment decision.

Stochastic processes. We assume that A_{it} follows an AR(1) process in logs with normally distributed i.i.d. innovations σ_μ^2, i.e.,

$$a_{it} = \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \ , \tag{5}$$

where the parameter ρ is the persistence of firm-level fundamentals.

For the distortion, τ_{it} , we adopt a specification that allows for a rich correlation structure, both over time as well as with firm fundamentals. Specifically, τ_{it} is assumed to have the following representation:

$$\tau_{it} = \gamma a_{it} + \varepsilon_{it} + \chi_i, \quad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \chi_i \sim \mathcal{N}\left(0, \sigma_\chi^2\right) \ , \tag{6}$$

where the parameter γ controls the extent to which τ_{it} co-moves with fundamentals. If $\gamma < 0$, the distortion discourages (encourages) investment by firms with stronger (weaker) fundamentals – arguably, the empirically relevant case. The opposite is true if $\gamma > 0$. The uncorrelated component of τ_{it} has an element, ε_{it}, that is i.i.d. over time and a permanent term, denoted χ_i. Thus, the severity of these factors is summarized by 3 parameters: $(\gamma, \sigma_\varepsilon^2, \sigma_\chi^2)$.

Information. Next, we spell out $\mathcal{I}_{it}$, the information set of the firm at the time of choosing period t investment, i.e., K_{it+1}. This includes the entire history of its fundamental shock realizations through period t, i.e., $\{a_{it-s}\}_{s=0}^{\infty}$. Given the AR(1) structure of uncertainty, this history can be summarized by the most recent observation, namely a_{it}. The firm also observes

[9] We use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $x_{it} = \log X_{it}$.

a noisy signal of the following period's innovation in fundamentals:

$$s_{it+1} = \mu_{it+1} + e_{it+1}, \qquad e_{it+1} \sim \mathcal{N}\left(0, \sigma_e^2\right) \ ,$$

where e_{it+1} is an i.i.d., mean-zero and normally distributed noise term. This is in essence an idiosyncratic 'news shock,' since it contains information about future fundamentals. Finally, firms also perfectly observe the uncorrelated transitory component of distortions, ε_{it+1} (as well as the fixed component, χ_i) at the time of choosing period t investment. They do not see the correlated component but are aware of its structure, i.e., they know γ.

Thus, the firm's information set is given by $\mathcal{I}_{it} = (a_{it}, s_{it+1}, \varepsilon_{it+1}, \chi_i)$. Direct application of Bayes' rule yields the conditional expectation of the fundamental a_{it+1}:

$$\begin{aligned} a_{it+1}|\mathcal{I}_{it} \quad &\sim \quad N\left(\mathbb{E}_{it}\left[a_{it+1}\right], \mathbb{V}\right) \qquad \text{where} \\ \mathbb{E}_{it}\left[a_{it+1}\right] \quad &= \quad \rho a_{it} + \frac{\mathbb{V}}{\sigma_e^2} s_{it+1}, \quad \mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2}\right)^{-1} \ . \end{aligned}$$

There is a one-to-one mapping between the posterior variance $\mathbb{V}$ and the noisiness of the signal, σ_e^2 (given the volatility of fundamentals, σ_μ^2). In the absence of any learning (or 'news'), i.e., when σ_e^2 approaches infinity, $\mathbb{V} = \sigma_\mu^2$, that is, all uncertainty regarding the realization of the fundamental shock μ_{it+1} remains unresolved at the time of investment. In this case, we have a standard one period time-to-build structure with $\mathbb{E}_{it}\left[a_{it+1}\right] = \rho a_{it}$. At the other extreme, when σ_e^2 approaches zero, $\mathbb{V} = 0$ and the firm becomes perfectly informed about μ_{it+1} so that $\mathbb{E}_{it}\left[a_{it+1}\right] = a_{it+1}$. It turns out to be more convenient to work directly with the posterior variance, $\mathbb{V}$, and so, for the remainder of the analysis, we will use $\mathbb{V}$ as our measure of uncertainty.

Optimal investment. Appendix A.1.1 derives the log-linearized version of the firm's optimal investment policy:

$$k_{it+1} = \psi_1 k_{it} + \psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i \tag{7}$$

where

$$\begin{aligned} \xi\left(\beta\psi_1^2 + 1\right) \quad &= \quad \psi_1\left((1+\beta)\xi + 1 - \alpha\right) \qquad (8) \\ \psi_2 = \frac{\psi_1}{\xi\left(1-\beta\rho\psi_1\right)}, \qquad \psi_3 \quad &= \quad \frac{\psi_1}{\xi}, \qquad \psi_4 = \frac{1-\psi_1}{1-\alpha} \ . \end{aligned}$$

The coefficients ψ_1–ψ_4 depend only on production (and preference) parameters, including the adjustment cost, and are independent of assumptions about information and distortions. The coefficient ψ_1 is increasing and ψ_2-ψ_4 decreasing in the severity of adjustment costs, ξ. If there are no adjustment costs (i.e., $\xi = 0$) , $\psi_1 = 0$ and $\psi_2 = \psi_3 = \psi_4 = \frac{1}{1-\alpha}$. At the other extreme, as ξ tends to infinity, ψ_1 approaches one and ψ_2-ψ_4 go to zero. Intuitively, as adjustment costs become large, the firm's choice of capital becomes more autocorrelated and less responsive to fundamentals and distortions. Our empirical strategy essentially relies on identifying the coefficients in the policy function, ψ_1 and $\psi_2(1+\gamma)$, from observable moments. Expression (8) shows that, for given values of α and β, we can use the estimate of ψ_1 to compute ξ. Next, we can use that value, along with the estimate of $\psi_2(1+\gamma)$ to compute γ.

Aggregation. We now turn to the aggregate economy, and in particular, measures of aggregate output and TFP. In Appendix A.1.2, we show that aggregate output can be expressed as

$$\log Y \equiv y = a + \hat{\alpha}_1 k + \hat{\alpha}_2 n \ ,$$

where k and n denote the (logs of the) aggregate stock of capital and labor inputs, respectively, and aggregate TFP, denoted by a, is given by

$$a = a^* - \frac{(\theta\hat{\alpha}_1 + \hat{\alpha}_2)\,\hat{\alpha}_1}{2}\sigma^2_{mrpk} \qquad \frac{da}{d\sigma^2_{mrpk}} = -\frac{(\theta\hat{\alpha}_1 + \hat{\alpha}_2)\,\hat{\alpha}_1}{2} \ , \tag{9}$$

where a^* is the level of TFP in the absence of all frictions (i.e., where static marginal products are equalized) and σ^2_{mrpk} is the cross-sectional dispersion in (the log of) the marginal product of capital ($mrpk_{it} = p_{it}y_{it} - k_{it}$). Thus, aggregate TFP monotonically decreases in the extent of capital misallocation, which in this log-normal world is summarized by σ^2_{mrpk}. The effect of σ^2_{mrpk} on aggregate TFP depends on the elasticity of substitution, θ, and the relative shares of capital and labor in production. The higher is θ, that is, the closer we are to perfect substitutability, the more severe the losses from mis-allocated resources. Similarly, fixing the degree of overall returns to scale in production, for a larger capital share, $\hat{\alpha}_1$, a given degree of misallocation has larger effects on aggregate outcomes.[10]

In our framework, a number of forces – adjustment costs, information frictions, and distortions – will lead to *mrpk* dispersion. Once we quantify their contributions to σ^2_{mrpk}, equation (9) allows us to directly map those contributions to their aggregate implications.

Measuring the contribution of each factor is a challenging task, since all the data moments confound all the factors (i.e., each moment reflects the influence of more than one factor). As a result, there is no one-to-one mapping between moments and parameters – to accurately identify the contribution of any factor, we need to explicitly control for the others. In the

[10] Aggregate output effects are larger than TFP losses by a factor $\frac{1}{1-\hat{\alpha}_1}$. This is because misallocation also reduces the incentives for capital accumulation and therefore, the steady-state capital stock.

following section, we overcome this challenge by exploiting the fact that these forces have different implications for different moments.[11]

3 Identification

In this section, we lay out our strategy to identify the key parameters of the model using observable moments from firm-level data on revenues and investment. We use a tractable special case – when firm-level shocks follow a random walk, i.e., $\rho = 1$ – to derive analytic expressions for key moments, allowing us to prove our identification result formally and demonstrate the underlying intuition. When we return to our general model in the following section, we will demonstrate numerically that this intuition extends to the case with $\rho < 1$.

We assume that the preference and technology parameters – the discount factor, β, the curvature of the profit function, α, and the depreciation rate, δ – are known to the econometrician (e.g., calibrated using aggregate data). The remaining parameters of interest are the costs of capital adjustment, ξ, the quality of firm-level information (summarized by $\mathbb{V}$), and the severity of distortions, parameterized by γ, σ^2_ε and σ^2_χ.

Our methodology uses a set of carefully chosen elements from the covariance matrix of firm-level capital and fundamentals (since α is assumed known, the latter can be directly measured using data on revenues and capital). Note that $\rho = 1$ implies non-stationarity in levels and so we work with moments of (log) changes. This means that we cannot identify σ^2_χ, the variance of the fixed component.[12] Here, we focus on the four remaining parameters, namely ξ, γ, $\mathbb{V}$ and σ^2_ε. Our main result is to show that these are exactly identified by the following four moments: (1) the autocorrelation of investment, denoted $\rho_{k,k_{-1}}$, (2) the variance of investment, σ^2_k, (3) the correlation of period t investment with the innovations in fundamentals in period $t-1$, denoted $\rho_{k,a_{-1}}$ and (4) the coefficient from a regression of $\Delta mrpk_{it}$ on Δa_{it}, which we denote $\lambda_{mrpk,a}$.

Several of these moments have been used in the literature to quantify the various factors in isolation. For example, $\rho_{k,k_{-1}}$ and σ^2_k are standard targets in the literature on adjustment costs – see, e.g., Asker et al. (2014) and Cooper and Haltiwanger (2006). The lagged responsiveness to fundamentals, $\rho_{k,a_{-1}}$, is used by Klenow and Willis (2007) in a price setting model to quantify information frictions. The covariance of $mrpk$ with fundamentals – which we proxy with $\lambda_{mrpk,a}$ – is often interpreted as indicative of correlated distortions, e.g., Bartelsman et al. (2013) and

[11] Asker et al. (2014) make a similar observation – they find that a one period time-to-build model (but no adjustment costs) produces very similar patterns in σ^2_{mrpk} compared to a model with a rich structure of adjustment costs. But, the implications for other moments (e.g. the variability of investment) are quite different across these two specifications – see columns (3) and (5) of Table 9 in that paper, along with the accompanying discussion and footnote 37.

[12] For our numerical analysis in Section 4, we use a stationary model (i.e., with $\rho < 1$) and use σ^2_{mrpk}, a moment computed using levels of capital and fundamentals, to pin down σ^2_χ.

Buera and Fattal-Jaef (2016). We will use the tractability of this special case to shed light on the necessity of analyzing these moments/factors in tandem (and the potential biases from doing so in isolation).

Our main result is stated formally in the following proposition:

Proposition 1. *The parameters ξ, γ, $\mathbb{V}$ and σ_ε^2 are uniquely identified by the moments $\rho_{k,k_{-1}}$, σ_k^2, $\rho_{k,a_{-1}}$ and $\lambda_{mrpk,a}$.*

3.1 Intuition

The proof of Proposition 1 (in Appendix A.3) involves tedious, if straightforward, algebra. Here, we provide a more heuristic argument, based on a pairwise analysis of the parameters of interest, which highlights the intuition behind the result. Specifically, we analyze parameters in pairs and show that they can be uniquely identified by a pair of moments, holding the other parameters fixed. To be clear, this is a local identification argument – our goal here is simply to provide intuition about how the different moments can be combined to disentangle the different forces. The identification result in Proposition 1 is a global one and shows that the four moments uniquely pin down all four parameters.

Adjustment costs and correlated distortions. We begin with adjustment costs, parameterized by ξ and correlated distortions, γ. The relevant moment pair is the variance and autocorrelation of investment, σ_k^2 and $\rho_{k,k_{-1}}$. Both of these moments are commonly used to estimate quadratic adjustment costs – for example, Asker et al. (2014) target the former and Cooper and Haltiwanger (2006) (among other moments), the latter. In our setting, these moments are given by:

$$\sigma_k^2 = \left(\frac{\psi_2^2}{1-\psi_1^2}\right)(1+\gamma)^2\sigma_\mu^2 + \frac{2\psi_3^2}{1+\psi_1}\sigma_\varepsilon^2 \quad (10)$$

$$\rho_{k,k_{-1}} = \psi_1 - \psi_3^2\frac{\sigma_\varepsilon^2}{\sigma_k^2}\,, \quad (11)$$

where the ψ's are defined in equation (8). Our argument rests on the fact that the two forces have similar effects on the variability of investment, but opposing effects on the autocorrelation. To see this, recall that ψ_1 is increasing and ψ_2 and ψ_3 decreasing in the size of adjustment costs, but all three are independent of γ. Then, holding all other parameters fixed, σ_k^2 is decreasing in both the severity of adjustment costs (higher ξ) and correlated factors (more negative γ).[13] The autocorrelation, $\rho_{k,k_{-1}}$, on the other hand, increases with ξ but decreases as γ becomes

[13] The latter is true only for $\gamma > -1$, which is the empirically relevant region.

more negative (through its effect on σ_k^2). Intuitively, while both factors dampen the volatility of investment, they do so for different reasons – adjustment costs make it optimal to smooth investment over time (increasing its autocorrelation) while correlated factors reduce sensitivity to the serially correlated fundamental (reducing the autocorrelation of investment).

The top left panel of Figure 1 shows how these properties help identify the two parameters. The panel plots a pair of 'isomoment' curves: each curve traces out combinations of the two parameters that give rise to a given value of the relevant moment, holding the other parameters fixed. Take the σ_k^2 curve: it slopes upward because higher ξ and lower γ have similar effects on σ_k^2 – if γ is relatively small (in absolute value), adjustment costs must be high in order to maintain a given level of σ_k^2. Conversely, a low ξ is consistent with a given value of σ_k^2 only if γ is very negative. An analogous argument applies to the $\rho_{k,k_{-1}}$ isomoment curve: since higher ξ and more negative γ have opposite effects on $\rho_{k,k_{-1}}$, the curve slopes downward. As a result, the two curves cross only once, yielding the unique combination of the parameters that is consistent with both moments. By plotting curves corresponding to the empirical values of these moments, we can uniquely pin down the pair (ξ, γ) (holding all other parameters fixed).

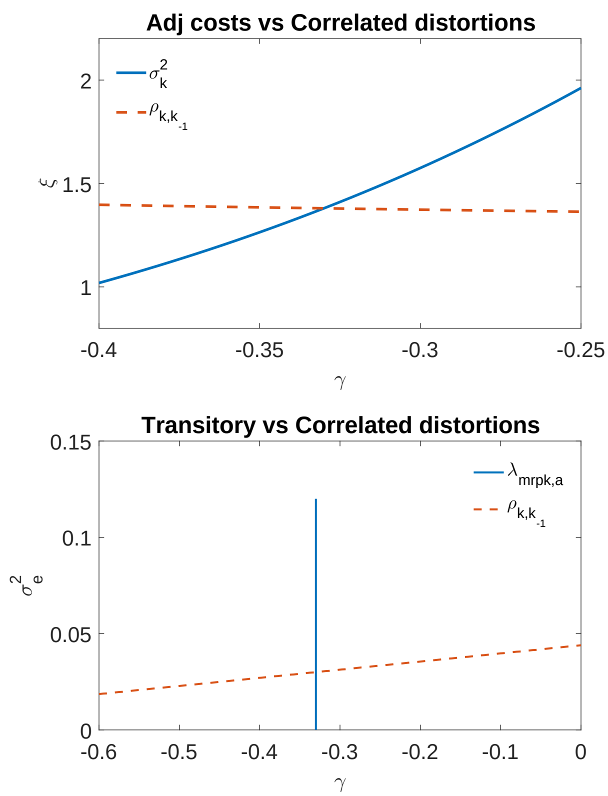


Figure 1: Pairwise Identification - Isomoment Curves

The graph also illustrates the potential bias introduced when examining these forces in isolation. For example, estimating adjustment costs while ignoring correlated distortions (i.e., imposing $\gamma = 0$) puts the estimate on the very right-hand side of the horizontal axis. The estimate for ξ can be read off the vertical height of the isomoment curve corresponding to the targeted moment. Because the σ_k^2 curve is upward sloping, targeting this moment alone leads to an overestimate of adjustment costs (at the very right of the horizontal axis, the curve is above the point of intersection, which corresponds to the true value of the parameters).[14] Targeting $\rho_{k,k_{-1}}$ alone leads to a bias in the opposite direction – since the $\rho_{k,k_{-1}}$ curve is downward sloping, imposing $\gamma = 0$ yields an underestimate of adjustment costs.

The remaining panels in Figure 1 repeat this analysis for other combinations of parameters. Each relies on the same logic as shown in the top left panel.

Uncertainty and correlated distortions. To disentangle information frictions from correlated factors (the top right panel), we use the correlation of investment with past innovations in fundamentals, $\rho_{k,a_{-1}}$, and the regression coefficient $\lambda_{mrpk,a}$. These moments can be written as:

$$\rho_{k,a_{-1}} = \left[\frac{\mathbb{V}}{\sigma_\mu^2}(1-\psi_1)+\psi_1\right]\frac{\sigma_\mu\psi_2(1+\gamma)}{\sigma_k} \tag{12}$$

$$\lambda_{mrpk,a} = 1-(1-\alpha)(1+\gamma)\psi_2\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right). \tag{13}$$

A higher $\mathbb{V}$ implies a higher correlation of investment with lagged fundamental innovations. Intuitively, the more uncertain is the firm, the greater the tendency for its actions to reflect fundamentals with a 1-period lag. In contrast, a higher (more negative) γ increases the relative importance of transitory factors in the firm's investment decision, reducing its correlation with fundamentals. Therefore, to maintain a given level of $\rho_{k,a_{-1}}$, a decrease in $\mathbb{V}$ must be accompanied by a less negative γ, i.e., the isomoment curve slopes downward. On the other hand, higher uncertainty and a more negative gamma both cause *mrpk* to covary more positively with contemporaneous fundamentals, a, leading to an upward sloping $\lambda_{mrpk,a}$ curve. Together, these two curves pin down $\mathbb{V}$ and γ, holding other parameters fixed.

As before, the graph also reveals the direction of bias when estimating these factors in isolation. Assuming full information ($\mathbb{V} = 0$) and using $\lambda_{mrpk,a}$ to discipline the strength of correlated distortions – e.g. as in Bartelsman et al. (2013) and Buera and Fattal-Jaef (2016) – overstates their importance. Using the lagged responsiveness to fundamentals to discipline information frictions while abstracting from correlated factors understates uncertainty.

[14]This approach would also predict a counter-factually high level of the autocorrelation of investment.

Transitory and correlated distortions. To disentangle correlated from uncorrelated transitory factors, consider $\lambda_{mrpk,a}$ and $\rho_{k,k_{-1}}$. The former is increasing in the severity of correlated distortions, but independent of transitory ones, implying a vertical isomoment curve. The latter is decreasing in both types of distortions – a more negative γ and higher σ_{ε}^2 both increase the importance of the transitory determinants of investment, yielding an upward sloping isomoment curve.

Uncertainty and adjustment costs. Finally, the bottom right panel shows the intuition for disentangling uncertainty from adjustment costs. An increase in the severity of either of these factors contributes to sluggishness in the response of actions to fundamentals, i.e., raises the correlation of investment with past fundamental shocks $\rho_{k,a_{-1}}$. However, the autocorrelation of investment $\rho_{k,k_{-1}}$ is independent of uncertainty and determined only by adjustment costs (and other factors). Thus, holding those other factors fixed, the autocorrelation of investment in combination with the correlation of investment with lagged shocks jointly pin down the magnitude of adjustment frictions and the extent of uncertainty.

4 Quantitative Analysis

The analytical results in the previous section showed a tight relationship between the moments $\left(\rho_{k,a_{-1}}, \rho_{k,k_{-1}}, \sigma_k^2, \lambda_{mrpk,a}\right)$ and the parameters $(\mathbb{V}, \xi, \sigma_{\varepsilon}^2, \gamma)$ for the special case of $\rho = 1$. In this section, we use this insight to develop a empirical strategy for the more general case where fundamentals follow a stationary AR(1) process and apply it to data on Chinese manufacturing firms. This allows us to quantify the severity of the various forces and their impact on misallocation and economic aggregates. For purposes of comparison, we also provide results for publicly traded firms in the US.[15]

4.1 Parameterization

We begin by assigning values to the more standard preference and production parameters of our model. We assume a period length of one year and accordingly set the discount factor $\beta = 0.95$. We keep the elasticity of substitution θ common across countries and set its value to 6, roughly in the middle of the range of values in the literature. We assume constant returns to

[15]The two sets of firms are not directly comparable due to their differing coverage (for example, the Chinese data include many more small firms). To address this concern, in Appendix D, we repeat the analysis on the set of Chinese publicly-traded firms and find a pattern that is quite similar to our results for Chinese manufacturing firms, suggesting that the cross-country differences in the importance of different factors are a robust feature of the data. This is further supported by results for two additional developing countries, Colombia and Mexico, also presented in Appendix D.

scale in production, but allow the individual production parameters $\hat{\alpha}_1$ and $\hat{\alpha}_2$ to vary across countries. In the US, we set these to standard values of 0.33 and 0.67, respectively. A number of recent papers, for example, Bai et al. (2006), have found that capital share's of value-added is about one-half in China and so we set $\hat{\alpha}_1 = \hat{\alpha}_2 = 0.5$ in that country. These values imply an α equal to 0.62 in the US and 0.71 in China.[16]

Next, we turn to the parameters governing the process for fundamentals, a_{it}: the persistence, ρ, and the variance of the innovations, σ_μ^2. Under our assumptions, the fundamental is directly given by (up to an additive constant) $a_{it} = va_{it} - \alpha k_{it}$ where va_{it} denotes the log of value-added. Controlling for industry-year fixed effects to isolate the firm-specific idiosyncratic component of fundamentals, we use a standard autoregression to estimate the parameters ρ and σ_μ^2.

To pin down the remaining parameters – the adjustment cost, ξ, the quality of firm information, $\mathbb{V}$, and the size of other factors, summarized by γ and σ_ε^2 – we follow a strategy informed by the results in the previous section. Specifically, we target the correlation of investment growth with lagged shocks to fundamentals ($\rho_{\iota,a_{-1}}$), the autocorrelation of investment growth ($\rho_{\iota,\iota_{-1}}$), the variance of investment growth (σ_ι^2) and the correlation of the marginal product of capital with fundamentals ($\rho_{mrpk,a}$).[17] Finally, to infer σ_χ^2, the fixed component of distortions in equation (6), we match the overall dispersion in the marginal product of capital, σ_{mrpk}^2, which is clearly increasing in σ_χ^2. Thus, by construction, our parameterized model will generate the amount of misallocation observed in the data, allowing us to decompose the role of each factor in contributing to the total. We summarize our empirical approach in Table 1.

4.2 Data

The data on Chinese manufacturing firms are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The surveys include all industrial firms (both state-owned and non-state owned) with sales above 5 million RMB (about $600,000).[18] We use data spanning the period 1998-2009. The original data come as a repeated cross-section. A panel is constructed following almost directly the method outlined in Brandt et al. (2014), which also contains an excellent overview of the data for the interested reader. The Chinese data have been used multiple times and are by now familiar in the misallocation literature –

[16]The US value is close to the estimate of 0.59 in Cooper and Haltiwanger (2006). Our findings on the relative contributions of various sources of misallocation are not sensitive to the value of α, see, e.g., the results in Section 4.6.2. We have also analyzed the case where the capital share in China is set equal to that in the US. The results are very similar to the baseline case.

[17]We follow the literature by working with the growth rate of investment (in the analytical cases studied earlier, we used investment, i.e., the growth rate of capital). See, for example, Morck et al. (1990) and David et al. (2016).

[18]Industrial firms correspond to Chinese Industrial Classification codes 0610-1220, 1311-4392 and 4411-4620, which includes mining, manufacturing and utilities.

Table 1: Parameterization - Summary

Parameter	Description	Target/Value
Preferences/production		
θ	Elasticity of substitution	6
β	Discount rate	0.95
$\hat{\alpha}_1$	Capital share	0.33 US/0.50 China
$\hat{\alpha}_2$	Labor share	0.67 US/0.50 China
Fundamentals/frictions		
ρ	Persistence of fundamentals	$\rho_{a,a_{-1}}$
σ_μ^2	Shocks to fundamentals	σ_a^2
$\mathbb{V}$	Signal precision	$\rho_{\iota,a_{-1}}$
ξ	Adjustment costs	$\rho_{\iota,\iota_{-1}}$
γ	Correlated factors	$\rho_{mrpk,a}$
σ_ε^2	Transitory factors	σ_ι^2
σ_χ^2	Permanent factors	σ_{mrpk}^2

for example, Hsieh and Klenow (2009) – although our use of the panel dimension is rather new. The data on US publicly traded firms comes from Compustat North America. We use data covering the same period as for the Chinese firms.

We measure the firm's capital stock, k_{it}, in each period as the value of fixed assets in China and of property, plant and equipment (PP&E) in the US, and investment as the change in the capital stock relative to the preceding period. We construct the fundamental as $a_{it} = va_{it} - \alpha k_{it}$, where we compute value-added from revenues using a share of intermediates of 0.5. Ignoring constant terms that do not affect our calculations, we measure the marginal product of capital as $mrpk_{it} = va_{it} - k_{it}$. First differencing k_{it} and a_{it} gives investment and changes in fundamentals between periods. To isolate the firm-specific variation in our data series, we extract a time-by-industry fixed-effect from each and use the residual as the component that is idiosyncratic to the firm. In both countries, industries are classified at the 4-digit level. This is equivalent to deviating each firm from the unweighted average within its industry in each time period and serves to eliminate any aggregate components, as well as render our calculations to be within-industry, which is a standard approach in the literature. After eliminating duplicates and problematic observations (for example, firms reporting in foreign currencies), outliers, observations with missing data etc., our final sample consists of 797,047 firm-year observations in China and 34,260 in the US. Appendix B provides further details on how we build our sample and construct the moments, as well as summary statistics from one year of our data, 2009.[19]

[19] We have also examined moments year-by-year. They are reasonably stable over time.

Table 2 reports the target moments for both countries. The first two columns show the fundamental processes, which have similar persistence but higher volatility in China. The remaining columns show that investment growth in China is more correlated with past shocks, is more volatile and less autocorrelated, that there is a higher correlation between firm fundamentals and *mrpk*, and that the overall dispersion in the *mrpk* is substantially higher than among publicly traded US firms. This variation will lead us to find significant differences in the severity of investment frictions and distortions across the two sets of firms.

Table 2: Target Moments

	ρ	σ_μ^2	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ_ι^2	σ_{mrpk}^2
China	0.91	0.15	0.29	−0.36	0.76	0.14	0.92
US	0.93	0.08	0.13	−0.30	0.55	0.06	0.45

4.3 Identification

Before turning to the estimation results, we revisit the issue of identification. Although we no longer have analytical expressions for the mapping between moments and parameters, we use a numerical experiment to show that the intuition developed in Section 3 for the random walk case applies here as well. In that section, we used a pairwise analysis to demonstrate how various moments combine to help disentangle the various sources of observed misallocation. Here, we repeat that analysis by plotting numeric isomoment curves in Figure 2, using the moments and parameter values for US firms (from Tables 2 and 3, respectively). The graph reveals the same broad patterns as Figure 1, indicating that the logic of that special case goes through here as well.[20]

4.4 Results

Table 3 contains our baseline results. In the top panel we display the parameter estimates. In the second panel, we report the contribution of each factor to dispersion in the *mrpk*, which we denote $\Delta\sigma^2_{mrpk}$.[21] These are calculated under the assumption that only the factor of interest is operational, i.e., in the absence of the others, so that the contribution of each one

[20]The differences in the precise shape of some of the curves in the two figures come partly from the departure from the random walk case and also from the fact that they use slightly different moments (Figure 2 works with changes in investment and $\rho_{mrpk,a}$ while Figure 1 used changes in k and $\lambda_{mrpk,a}$).

[21]For adjustment costs, we do not have an analytic mapping between the severity of these costs and σ^2_{mrpk}, but this is a straightforward calculation to make numerically; for each of the other factors, we can compute their contributions to misallocation analytically.

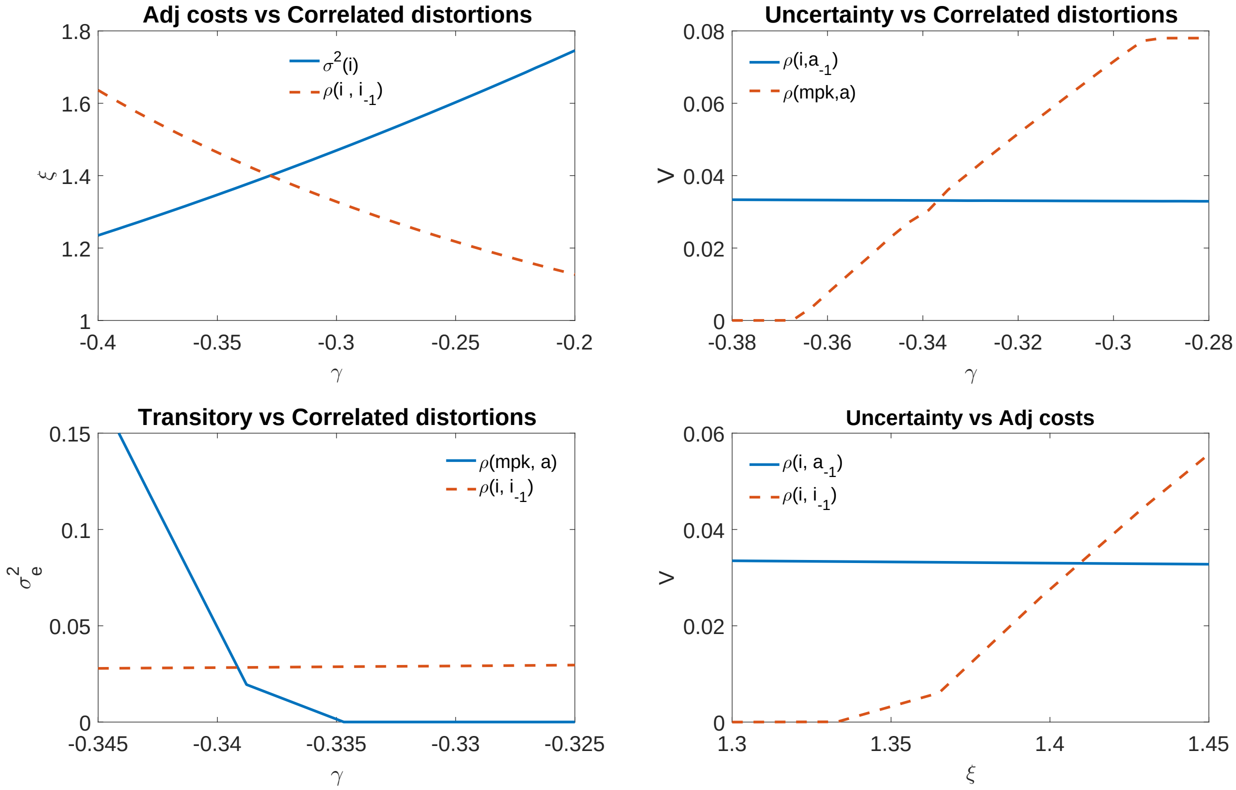


Figure 2: Isomoment Curves - Quantitative Model

is measured relative to the undistorted first-best.[22] The third panel expresses this contribution as a percentage of the total *mrpk* dispersion measured in the data, denoted $\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$. Because of interactions between the factors, there is no *a priori* reason to expect these relative contributions to sum to one. In practice, however, we find that the total is reasonably close to one, allowing us to interpret this exercise as a decomposition of total observed misallocation. In the bottom panel of the table, we compute the implied losses in aggregate TFP, again relative to the undistorted first-best level, i.e., $\Delta a = a^* - a$. Once we have the contribution of each factor to *mrpk* dispersion, computing these values is simply an application of expression (9).

Adjustment costs. One of our key quantitative findings is a relatively modest degree of adjustment frictions in both countries. For example, the estimate of $\xi = 1.38$ for the US in Table 3 implies a value of 0.2 for the primitive parameter $\hat{\xi}$ in the adjustment cost function (2). [23] This is generally lower than existing estimates in the literature. Asker et al. (2014),

[22] An alternative would be to calculate the contribution of each factor holding the others constant at their estimated values. It turns out that the interactions between the factors are small at the estimated parameter values, so the two approaches yield similar results. Table 9 in Appendix C shows that the effects of each factor on *mrpk* dispersion in the US are close under either approach. Interactions effects are even smaller in China.

[23] The mapping between ξ and $\hat{\xi}$ is derived in equation (21) in Appendix A.1.1. We use an annual depreciation rate of $\delta = 0.10$.

Table 3: Contributions to Misallocation

	Adjustment Costs	Uncertainty	Other Factors		
			Correlated	Transitory	Permanent
Parameters	ξ	$\mathbb{V}$	γ	σ_{ε}^2	σ_{χ}^2
China	0.13	0.10	−0.70	0.00	0.41
US	1.38	0.03	−0.33	0.03	0.29
$\Delta\sigma_{mrpk}^2$					
China	0.01	0.10	0.44	0.00	0.41
US	0.05	0.03	0.06	0.03	0.29
$\frac{\Delta\sigma_{mrpk}^2}{\sigma_{mrpk}^2}$					
China	1.3%	10.3%	47.4%	0.0%	44.4%
US	10.8%	7.3%	14.4%	6.3%	64.7%
Δa					
China	0.01	0.08	0.38	0.00	0.36
US	0.02	0.01	0.03	0.01	0.13

for example, report an estimate of 8.8 for their convex adjustment cost parameter estimated using data on US manufacturing firms. To interpret this difference, consider a firm that doubles its capital stock in a year. Our estimates imply that the firm would incur adjustment costs equal to about 11% of the value of the investment, whereas the corresponding number using the Asker et al. (2014) estimate would be 60%.[24] The level of costs is estimated to be even lower in China.

These results imply a limited role for adjustment costs in generating misallocation, particularly so in China – if this were the only friction, *mrpk* dispersion in China arising from this channel would be 0.01, representing about 1% of total σ_{mrpk}^2. The contribution of adjustment costs in the US is significantly higher, where they lead to *mrpk* dispersion of 0.05, about 11% of the total. The corresponding losses in aggregate TFP are about 1% and 2% in the two countries, respectively. Thus, adjustment frictions are relatively more important in the US compared to China. However, these estimates imply that adjustment costs alone cannot account for the majority of marginal product dispersion in either country. While their effect is not trivial, it is quite modest compared to the effect of the remaining factors described below. Later, we show that this finding remains robust to a number of variants of our baseline model, e.g., non-convex components to these costs, frictions/distortions in labor and alternative specifications of distortions, including financial constraints.

[24]For the US, our estimates are closer to (and slightly higher than) Cooper and Haltiwanger (2006) and Bloom (2009).

Importantly, one would reach very different conclusions from examining adjustment costs in isolation. To show this, we also estimated a version of our model in which we abstract from the other forces and parameterize those costs to match a single moment in the data. If we use the volatility of investment growth, σ_ι^2, we get considerably larger estimates of ξ, about 1.6 times higher in the US and almost 10 times higher in China. The resulting *mrpk* dispersion increases by a similar proportion, so that this approach would suggest a substantially larger role for adjustment frictions. However, the implied autocorrelation of investment growth from this approach is much higher than that observed in the data, -0.17 vs a true value of -0.30 in the US and -0.20 vs -0.36 in China. These are precisely the patterns predicted by our identification arguments in Sections 3 and 4.3 – a strategy that parameterizes adjustment costs by targeting the variability of investment alone overestimates those costs and thus overstates the autocorrelation of investment. Allowing for other factors enables the model to reconcile both moments with a smaller degree of adjustment frictions. This is the main source of the difference between our findings and papers like Asker et al. (2014), which estimate adjustment costs in isolation. Similarly, a strategy targeting only the autocorrelation of investment leads to the opposite conclusion – here, we would have understated the magnitude of adjustment costs and predicted too high a variability of investment compared to the data.

Uncertainty. Table 3 shows that firms in both countries make investment decisions under considerable uncertainty, with the information friction more severe for Chinese firms. As a share of the prior uncertainty, σ_μ^2, residual uncertainty, $\frac{\mathbb{V}}{\sigma_\mu^2}$, is 0.42 in the US and 0.63 in China. In other words, imposing a one period time-to-build assumption without additional information, i.e. setting $\mathbb{V} = \sigma_\mu^2$, overstates the role of uncertainty (and biases the estimates of adjustment costs and other parameters). It is straightforward to show that in an environment where imperfect information is the only friction, $\sigma_{mrpk}^2 = \mathbb{V}$, so that the contribution of uncertainty alone to observed misallocation can be directly read off the second column in Table 3 - namely 0.10 in China and 0.03 in the US. These represent about 10% and 7% of total *mrpk* dispersion in the two countries, respectively. The implications for aggregate TFP are substantial in China – losses are about 8% – and are lower in the US, about 1%.[25]

[25] Our values for $\frac{\mathbb{V}}{\sigma_\mu^2}$ are similar to those in David et al. (2016), who find 0.41 and 0.63 for publicly traded firms in the US and China, respectively. The absolute values of $\mathbb{V}$ are different but are not directly comparable – David et al. (2016) focus on longer time horizons (they analyze 3-year time intervals). This might lead one to conclude that ignoring other factors – as David et al. (2016) do – leads to negligible bias when estimating uncertainty (using an appropriate moment), but it must be noted that this is not a general result, but a quantitative one and rests on the fact that adjustment costs and uncorrelated distortions are modest. Then, as Figure 2 shows, the sensitivity of actions to signals turns out to be a very good indicator of uncertainty. It is easy to come up with realistic parameter combinations where the bias from estimating $\mathcal{V}$ alone can be quite significant.

'Distortions'. The last three columns of Table 3 show that other, potentially distortionary, factors play a significant role in generating the observed *mrpk* dispersion in both countries. Turning first to the correlated component, the negative values of γ suggest that they act to disincentivize investment by more productive firms and especially so in China. The contribution of these distortions to *mrpk* dispersion is given by $\gamma^2\sigma_a^2$, which amounts to 0.44 in China, or 47% of total misallocation. The associated aggregate consequences are also quite sizable – TFP losses from these sources are 38%. In contrast, the estimate of γ in the US is significantly less negative than in China, suggesting that these types of correlated factors are less of an issue for firms in the US, both in an absolute sense – the *mrpk* dispersion from these factors in the US is 0.06, less than one-seventh that in China – and in relative terms – they account for only 14% of total observed *mrpk* dispersion in the US. The corresponding TFP effects are also considerably smaller for the US - losses from correlated sources are only about 3%.

Next, we consider the role of distortions that are uncorrelated with firm fundamentals. Table 3 shows that purely transitory factors (measured by σ_ε^2) are negligible in both countries, but permanent firm-specific factors (measured by σ_χ^2) play a prominent role. Their contribution to *mrpk* dispersion, which is also given by σ_χ^2, amounts to 0.41 in China and 0.29 in the US. Thus, their absolute magnitude in the US is considerably below that in China, but in relative terms, these factors seem to account for a substantial portion of measured misallocation in both countries. The aggregate consequences of these types of distortions are also significant, with TFP losses of 36% in China and about 13% in the US.

In sum, the estimation results point to the presence of substantial distortions to investment, especially in China, where they disproportionately disincentivize investment by more productive firms. What patterns in the data lead us to this conclusion? The *mrpk* in both countries shows significant dispersion and a high correlation with fundamentals, indicating a dampened response of investment to fundamentals. In principle, this pattern could emerge from adjustment costs, imperfect information or correlated distortions. However, the autocorrelation of investment growth, $\rho_{\iota,\iota_{-1}}$, in the data is relatively low, which bounds the severity of adjustment frictions. Similarly, the response of investment to past shocks, $\rho_{\iota,a_{-1}}$, is also modest, limiting the role of the informational friction. Hence, the estimation assigns a substantial role to correlated distortions, particularly in China, as well as fixed distortions, in order to generate the observed patterns in the *mrpk*.[26] As we show in Section 4.6, this result is robust to a number of modifications to our baseline setup, e.g. allowing for non-convex adjustment costs, frictional labor choice, size-dependent (as opposed to productivity-dependent) distortionary factors and additive measurement error. Further, we have applied the methodology to data on Colombian

[26] A high value for $\rho_{mrpk,a}$ also limits the scope for uncorrelated transitory distortions as an important driver of investment decisions.

and Mexican firms (in addition to the set of publicly traded firms in China) - the results resemble those for Chinese manufacturing firms, in that they point to a substantial role for correlated factors, as well as fixed ones (details are in Appendix D).

In the next section, we investigate the extent to which heterogeneity in markups and production technologies can account for these 'distortions'.

4.5 Heterogeneity in Markups and Technologies

In our baseline setup, we assumed all firms within an industry (1) have homogeneous production technologies and (2) are monopolistically competitive facing CES demand curves and therefore, have identical markups. Under this structure, measured *mrpk* deviations could, in part, reflect firm-level heterogeneity in technologies and/or markups. In this section, we explore this possibility using a modified version of our baseline model which allows for heterogeneous markups and capital intensities. Quantifying these forces requires more assumptions as well as additional data. Using these, we are able to provide an upper bound on the contribution of these elements to observed misallocation.

We begin by generalizing the production function from Section 2 to include intermediate inputs and to allow for (potentially time-varying) heterogeneity in capital intensities. Specifically, the output of firm i is now given by

$$Y_{it} = K_{it}^{\hat{\alpha}_{it}} N_{it}^{\hat{\zeta}-\hat{\alpha}_{it}} M_{it}^{1-\hat{\zeta}} ,$$

where M_{it} denotes intermediate or materials input. For simplicity, we abstract from adjustment/information frictions in firms' input decisions. Our decompositions above reveal a relatively minor role for these forces in leading to *mrpk* dispersion.[27] Capital and labor choices are each subject to a factor-specific 'distortion' (in addition to the markup), denoted T_{it}^{K} and T_{it}^{N}, respectively. The choice of intermediates is distorted by the firm-specific markup. Under this structure, we can put to use the powerful methodology pioneered by De Loecker and Warzynski (2012) to measure markups at the firm level without taking a stand on the nature of competition/demand.[28]

[27]It is possible to extend the identification methodology from Section 3 to explicitly include heterogeneity in $\hat{\alpha}$ and markups. Although this would require more assumptions (e.g., on the correlation structure of markups/technologies with fundamentals and over time) and make the intuition more complicated, the basic insights should still go through.

[28]The method is also robust to the presence of distortions in the market for intermediate inputs, so long as they are reflected in the price that the firm pays. In other words, even if firms pay idiosyncratic prices for intermediate inputs, the method accurately identifies markup dispersion.

The contribution of markup dispersion. Identification of markup dispersion makes use of the following optimality condition from the firm's cost minimization problem:

$$P_t^M = MC_{it}\left(1-\hat{\zeta}\right)\frac{Y_{it}}{M_{it}} \quad \Rightarrow \quad \frac{P_t^M M_{it}}{P_{it}Y_{it}} = (1-\hat{\zeta})\frac{MC_{it}}{P_{it}}\ , \tag{14}$$

where P_t^M is the price of materials and MC_{it} is the marginal cost of the firm. This condition states that, at the optimum, the firm sets the materials share in gross output equal to the inverse of the markup, $\frac{MC_{it}}{P_{it}}$, multiplied by the materials elasticity $1-\hat{\zeta}$.

Expression (14) suggests a simple way to estimate the cross-sectional dispersion in markups. The left-hand side is the materials' share of revenue – the dispersion in this object (in logs) maps one-for-one into (log) markup dispersion across firms.[29] The results of applying this procedure are reported in the first rows of the two panels in Table 4. The variance of the share of materials in revenue is about 0.09 in the US Compustat data and 0.05 in China, accounting for about 28% of σ^2_{mrpk} among the US firms, but only about 4% of σ^2_{mrpk} among Chinese manufacturing firms. Thus, markup heterogeneity composes a significant fraction of observed misallocation among US publicly-traded firms but seems to be an almost negligible force in China.

The contribution of heterogeneity in technology. Cost minimization also implies that the average revenue products of capital and labor are given by:[30]

$$\log\left(\frac{P_{it}Y_{it}}{K_{it}}\right) = \log\frac{P_{it}}{MC_{it}} - \log\hat{\alpha}_{it} + \tau_{it}^K + \text{Constant} \tag{15}$$

$$\log\left(\frac{P_{it}Y_{it}}{N_{it}}\right) = \log\frac{P_{it}}{MC_{it}} - \log(\hat{\zeta}-\hat{\alpha}_{it}) + \tau_{it}^N + \text{Constant} \tag{16}$$

$$\approx \log\frac{P_{it}}{MC_{it}} + \frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\log\hat{\alpha}_{it} + \tau_{it}^N + \text{Constant}\ , \tag{17}$$

where τ_{it}^K and τ_{it}^N are the logs of the capital and labor wedges T_{it}^K and T_{it}^N, respectively, and $\bar{\alpha}$ is the average capital elasticity across firms.[31] Observed average revenue products are combinations of the firm-specific production elasticities as well as markups and distortionary factors. Importantly, the expressions reveal that the capital elasticity, $\hat{\alpha}_{it}$ has opposing effects on the average products of capital and labor. Specifically, firms with a high $\hat{\alpha}_{it}$ will, *ceteris paribus*, tend to have a low average product of capital and a high average product of labor. This property enables us to use the observed covariance of the average products to bound the extent of

[29] Note that this assumes no heterogeneity in the materials elasticity, $1-\hat{\zeta}$. To the extent there is heterogeneity in $1-\hat{\zeta}$ that is uncorrelated (or negatively correlated) with markups, the strategy would overestimate markup variation.

[30] See Appendix A.4 for details.

[31] The third equation is derived by log-linearizing (16) around $\hat{\alpha}_{it} = \bar{\alpha}$.

variation in $\hat{\alpha}_i$. Let

$$\begin{aligned} arpk_{it} &\equiv \log\left(\frac{P_{it}Y_{it}}{K_{it}}\right) - \log\left(\frac{P_{it}}{MC_{it}}\right) \\ arpn_{it} &\equiv \log\left(\frac{P_{it}Y_{it}}{N_{it}}\right) - \log\left(\frac{P_{it}}{MC_{it}}\right) \end{aligned}$$

denote the markup-adjusted revenue products of capital and labor. Appendix A.4 proves the following result:

Proposition 2. *Suppose* $\log \hat{\alpha}_{it}$ *is uncorrelated with the distortions* τ_{it}^K *and* τ_{it}^N. *Then, the cross-sectional dispersion in* $\log \hat{\alpha}_{it}$ *satisfies*

$$\sigma^2(\log \hat{\alpha}_{it}) \leq \frac{\sigma_{arpk}^2 \sigma_{arpn}^2 - cov(arpk, arpn)^2}{2\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}} cov(arpk, arpn) + \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2 \sigma_{arpk}^2 + \sigma_{arpn}^2} . \quad (18)$$

The bound in (18) is obtained by setting the correlation between the distortionary factors τ_{it}^K and τ_{it}^N to 1. Given the observed second moments of $(arpk_{it}, arpn_{it})$, this maximizes the potential for variation in $\hat{\alpha}_{it}$, which, as noted earlier, is a source of negative correlation between $arpk_{it}$ and $arpn_{it}$. The expression for the bound reveals the main insight: the more positive the covariance between $(arpk_{it}, arpn_{it})$, the lower is the scope for heterogeneity in $\hat{\alpha}_{it}$.

To compute this bound for the two countries, we set $\hat{\zeta}$, the share of materials in gross output, to 0.5. The results, along with the moments, are reported in Table 4. Heterogeneous technologies can potentially account for a substantial portion of σ_{mrpk}^2 in the US - as much as 62% - and a more modest, though still significant, fraction in China, about 23%.[32] The last row of Table 4 shows that in total, unobserved heterogeneity in markups and technologies can potentially explain as much as 90% of measured misallocation in the US and at most about 27% in China.

Hsieh and Klenow (2009) perform an alternative experiment to bound the role of technological heterogeneity: they attribute all the variation in firm-level capital-labor ratios to heterogeneity in $\hat{\alpha}_i$. In our setting, this amounts to assuming that $\tau_{it}^K = \tau_{it}^N$, which implies:

$$k_{it} - n_{it} = arpn_{it} - arpk_{it} \approx \frac{\hat{\zeta}}{\hat{\zeta} - \bar{\alpha}} \log \hat{\alpha}_{it} \quad \Rightarrow \quad \sigma^2 (k_{it} - n_{it}) = \left(\frac{\hat{\zeta}}{\hat{\zeta} - \bar{\alpha}}\right)^2 \sigma^2(\log \hat{\alpha}_{it}) .$$

[32] There is some evidence that the share of intermediates may be higher in China than the US, see, e.g., Table 1 in Brandt et al. (2014). We re-computed the bound with $\hat{\zeta} = 0.25$ and obtained very similar results. We also verified the quality of the approximation by working directly with (16) instead of the log-linearized version in (17). This yielded bounds that were slightly lower for both countries: 53% and 17% of σ_{mrpk}^2 in the US and China, respectively.

Table 4: Heterogeneous Markups and Technologies

	China		US	
Moments				
$\sigma^2\left(\log \frac{P_{it}Y_{it}}{P_t^M M_{it}}\right)$	0.05		0.09	
$\text{cov}\left(arpk_{it}, arpn_{it}\right)$	0.41		0.12	
$\sigma^2\left(arpk_{it}\right)$	1.37		0.41	
$\sigma^2\left(arpn_{it}\right)$	0.76		0.25	
Estimated $\Delta\sigma^2_{mrpk}$				
Dispersion in Markups	0.05	(3.8%)	0.09	(28.3%)
Dispersion in $\log\hat{\alpha}_{it}$	0.30	(23.1%)	0.19	(62.2%)
Total	0.35	(26.9%)	0.28	(90.5%)

Notes: The values in parentheses in the bottom panel are the contributions to *mrpk* dispersion expressed as a fraction of total σ^2_{mrpk}.

This procedure yields estimates for $\sigma^2\left(\log\hat{\alpha}_{it}\right)$ that are quite close to those in Table 4: 0.27 (compared to 0.30) for China and 0.16 (compared to 0.19) in the US.

4.6 Robustness

In this section, we explore a number of variants on our baseline approach. We generalize our specification of adjustment costs to include a non-convex component. We also use this exercise to assess the accuracy of the log-linearized solution, since this case requires nonlinear solution techniques. We consider the implications of a frictional labor choice. We explore the potential for measurement error. Our main conclusions about the relative contribution of various factors to observed misallocation is robust across these exercises.

4.6.1 Non-Convex Adjustment Costs

Our baseline model only allowed for convex adjustment costs. This allowed us to use perturbation techniques which yielded both analytical tractability for our identification arguments as well as computational efficiency. However, it raises two questions: one, is the log-linearization a sufficiently good approximation for the true non-linear solution? And two, are the results robust to allowing for non-convex adjustment costs? Here, we address both of these concerns by extending our baseline setup to include non-convex costs and solving the model without linearization. Specifically, the adjustment cost function now takes the form:

$$\Phi\left(K_{it+1}, K_{it}\right) = \frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^2 K_{it} + \hat{\xi}_f \mathbb{I}\left\{I_{it} \neq 0\right\}\pi\left(A_{it}, K_{it}\right) \ ,$$

where $I_{it} = K_{it+1} - (1-\delta) K_{it}$ denotes period t investment and $\mathbb{I}\{\cdot\}$ the indicator function. Capital adjustment costs are composed of two components: the first is the quadratic cost, the same as before. The second is a fixed component that must be incurred if the firm undertakes any non-zero investment. This component is parameterized by $\hat{\xi}_f$ and scales with profits (so that it does not become negligible for large firms), a common formulation in the literature, see, e.g., Asker et al. (2014).

Because of the fixed component, we cannot use perturbation methods to solve this version of the model. Therefore, we do so non-linearly using a standard value function iteration and re-estimate the parameters. We now have an additional parameter, $\hat{\xi}_f$. To pin this down, we add a new moment: the share of 'non-adjusters,' i.e., firms that make very small adjustments to their capital stock in a particular period. Specifically, we match the share of firms with net investment rates of less than 5% in absolute value, which is 14% of firms in China and 27% of firms in the US.

We report the results in Table 5. The estimated value for $\hat{\xi}_f$ is quite small in both countries. The value in the US, which implies a cost of about 0.2% of annual profits, is lower than previous estimates in the literature, underscoring the importance of controlling for other factors when estimating adjustment frictions.[33] The remaining parameters and their relative contributions to σ^2_{mrpk} are quite close to their values in the baseline analysis. These results demonstrate that (1) non-convex costs play a negligible role in determining *mrpk* dispersion, (2) abstracting from them does not significantly bias our estimates of other parameters and their contributions to misallocation and (3) the perturbation approach used for our baseline results is quite accurate.

Table 5: Non-Convex Adjustment Costs

Parameters	$\hat{\xi}$	(ξ)	$\hat{\xi}_f$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
China	0.034	(0.23)	0.000	0.09	−0.635	0.00	0.45
US	0.135	(0.92)	0.002	0.03	−0.320	0.02	0.29
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$							
China	4.3%		0.0%	10.0%	38.1%	0.0%	48.8%
US	11.1%		0.4%	7.5%	13.5%	4.4%	64.4%

Notes: The second column (in parentheses) reports the value of the normalized adjustment cost parameter, ξ, for purposes of comparison to Table 3. The mapping between ξ and $\hat{\xi}$ is given in expression (21).

[33] For example, Bloom (2009) estimates a fixed adjustment cost of 1% of annual sales for US Compustat firms. Asker et al. (2014) and Cooper and Haltiwanger (2006) work with data on US manufacturing firms and estimate this parameter at 9% of annual sales and 4% of the capital stock, respectively.

4.6.2 Frictional Labor

Our baseline analysis makes the rather stark assumption that there are no adjustment or information frictions in labor decisions, rendering the labor input a static choice made under full information. Although this is not an uncommon assumption in the literature, it may not be an apt description of labor markets in developing economies such as China. In this section, we extend our analysis to allow for frictions in the adjustment of labor. In particular, we show in Appendix A.5.1 that when labor is subject to the same forces as capital - adjustment and informational frictions and distortions - the firm's intertemporal investment problem takes the same form as in expression (3), but where the degree of curvature is equal to $\alpha = \alpha_1 + \alpha_2$ (and with slightly modified versions of the G and A_{it} terms). Thus, the qualitative analysis of the model is unchanged, although the quantitative results will differ since we now have $\alpha = \alpha_1 + \alpha_2 = 0.83$. Table 6 reports results from this specification for the Chinese firms. The top panel of the table displays the target moments recomputed under this scenario (recall that a number of the moments depend on the value of α). A comparison to the baseline moments in Table 2 shows that under the assumption of frictional labor, the correlation of investment with lagged shocks increases, as does the correlation of the *mrpk* with fundamentals. The second panel of the table reports the associated parameter estimates. The new values imply a higher level of adjustment costs, greater uncertainty and more severe correlated distortions. As a result, a lower level of the permanent component of uncorrelated distortions, σ_χ^2, is needed to match the dispersion in the *mrpk*.

Aggregate TFP in this case is equal to[34]

$$a = a^* - \frac{1}{2}\theta\sigma_{mrpk}^2 \ , \qquad (19)$$

where a^* is again TFP in the frictionless, undistorted benchmark.

Expression (19) shows that the relative shares of capital and labor in production – a key determinant of the aggregate ramifications of misallocation in the baseline case with frictionless labor - no longer play a role. Only the overall curvature, θ, matters and, as in the baseline case, the smaller the amount of curvature (i.e., the larger is θ), the greater the losses from misallocation. Further, it is straightforward to see that for a fixed set of parameters, the cost of misallocation is larger here than in the baseline case.

The bottom panel of Table 6 reports the contribution of each factor to total misallocation and computes the implications for aggregate TFP. There is a noticeable increase in the impact of adjustment costs from the baseline case – in this specification, these costs account for almost 13% of *mrpk* dispersion in China (compared to 1% above). There is also a slight increase in

[34] The derivation is in Appendix A.5.2.

the impact of uncertainty (from 10% to 11%). However, both of these forces remain muted compared to other correlated and permanent factors, which continue to account for the largest portion of observed misallocation. The effects on aggregate productivity are much larger than in the baseline scenario – here, these forces distort both inputs into production. For example, adjustment costs and imperfect information now lead to TFP losses of about 36% and 32%, respectively. The corresponding figures for correlated and permanent distortions imply gaps relative to the first-best of about 144% and 90%. While the main message of our analysis remains – much of observed misallocation stems from other correlated and permanent factors – this version illustrates the potential for large aggregate consequences of adjustment/information frictions.

Table 6: Frictional Labor - China

Moments	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
	0.92	0.16	0.33	−0.36	0.81	0.14	0.94
Parameters			ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
			0.78	0.11	−0.68	0.04	0.30
Aggregate Effects							
$\Delta\sigma^2_{mpk}$			0.12	0.11	0.48	0.04	0.30
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$			12.8%	11.3%	51.2%	4.0%	32.2%
Δa			0.36	0.32	1.44	0.11	0.90

4.6.3 Size vs. Productivity-Dependent Factors

Our baseline specification considered wedges that are correlated with firm fundamentals. Here, we generalize that specification to include additional factors that are correlated with firm size, measured by the level of installed capital. We formulate these factors as

$$\tau_{it} = \gamma_k k_{it} + \gamma a_{it} + \varepsilon_{it} + \chi_i \ ,$$

where the parameter γ_k indexes the degree of size-dependence. The log-linearized Euler equation now takes the form

$$k_{it+1}\left((1+\beta)\xi + 1 - \alpha - \gamma_k\right) = (1+\gamma)\,\mathbb{E}_{it}\left[a_{it+1}\right] + \varepsilon_{it+1} + \chi_i + \beta\xi\mathbb{E}_{it}\left[k_{it+2}\right] + \xi k_{it} \ . \tag{20}$$

Expression (20) is identical to expression to (4), but with $\alpha + \gamma_k$ taking the place of α. It is straightforward to derive the firm's investment policy function and verify that the same adjustment goes through, i.e., expressions (7) and (8) hold, with α everywhere replaced by

$\alpha + \gamma_k$.

Intuitively, the size-dependent component, γ_k, changes the effective degree of curvature in the firm's investment problem – although the true extent of curvature remains α, the firm acts as if it is $\alpha + \gamma_k$. The effects are broadly similar to those coming from γ: if $\gamma_k < 0$, the distortion dampens the responsiveness of investment to shocks (in contrast to γ, γ_k affects the response of investment to all shocks, whether from fundamentals or distortions). If $\gamma_k > 0$, the responsiveness of investment is amplified.

If γ_k were the only factor distorting investment choices, the implied law of motion for k_{it} is identical (up to a first-order) to the baseline one with only productivity-dependent factors, where $\gamma = \frac{\gamma_k}{1-\alpha-\gamma_k}$. This isomorphism implies that we cannot distinguish the two factors using observed series of capital and fundamentals. This intuition carries through to the case when other factors are present, though the mapping between the two specifications is more complicated (and requires changes to other parameters as well). To see this, note that our empirical strategy can be thought of as essentially recovering the law of motion for k_{it} – in particular, the coefficients ψ_1, $\psi_2 (1 + \gamma)$, ψ_3 and ψ_4. Importantly, these estimates are invariant to assumptions about γ_k, which only affects the mapping from these coefficients to the underlying structural parameters. For example, suppose we assume $\gamma_k = 0$. Then, given our values for (α, β, δ), the estimated ψ_1 identifies the adjustment cost parameter ξ. Next, the value of ξ can be used to pin down ψ_2, allowing us to recover γ from the estimated $\psi_2 (1 + \gamma)$. This procedure can be applied for any non-zero γ_k as well. Since the estimated ψ_1 and $\psi_2 (1 + \gamma)$ do not change, for any γ_k, the adjustment cost parameter becomes, from (8),

$$\xi = \psi_1 \frac{1 - \alpha - \gamma_k}{\beta \psi_1^2 + 1 - \psi_1 (1 + \beta)} \ .$$

The next step is the same as before: the estimated ξ implies a value for ψ_2, which then allows us to back out γ from the estimated $\psi_2 (1 + \gamma)$. Applying this procedure for different values of γ_k traces out a set of parameters that are observationally equivalent, i.e., that cannot be distinguished using only data on capital and revenues.

What about the contribution to misallocation? Table 7 reports the results for Chinese firms for two different values of γ_k, namely -0.18 and -0.36. These imply that the effective curvature $\alpha + \gamma_k$ is one-quarter and one-half of the true α, respectively. The table shows, first, that larger (i.e., more negative) values of γ_k reduce the estimated γ (i.e., make it less negative). Thus, the total contribution of both types of correlated distortions remains quite stable, ranging between 40% and 47%. The estimates of adjustment costs remain quite modest over this wide range of effective curvature, as does their contribution to *mrpk* dispersion, which is between 1% and 3%. In this sense, our baseline results appear to be robust to the precise structure of correlation between distortions and firm characteristics, be it with firm size or underlying fundamentals.

Table 7: Size vs Productivity-Dependent Factors

	Correlated Factors			
	Size-Dependent γ_k	Prod.-Dependent γ	Total	Adj. Costs ξ
$\alpha+\gamma_k=0.71$ (baseline)				
Parameters	0.00	-0.70		0.13
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$	0.0%	47.4%	47.4%	1.3%
$\alpha+\gamma_k=0.54$				
Parameters	-0.18	-0.51		0.21
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$	14.2%	25.4%	39.6%	2.3%
$\alpha+\gamma_k=0.36$				
Parameters	-0.36	-0.33		0.29
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$	29.6%	10.2%	39.8%	3.2%

Financial frictions. Thus far, we have not explicitly modeled financial frictions. In Appendix A.6, we lay out a tractable modification to our baseline setup that subjects firms to liquidity costs and show that they are isomorphic to a size-dependent wedge. In other words, this form of financial friction can be mapped to a non-zero γ_k (which is determined by the parameters of the underlying liquidity cost). Thus, the results above on size-dependent distortionary factors directly apply. This suggests that (1) our estimates for correlated factors, γ, may include financial considerations and (ii) disentangling their role using only production-side data may be difficult and would require additional financial data, ideally at the firm-level.

4.6.4 Measurement Error

Measurement error is an important and challenging concern, not just for our analysis but for the misallocation literature more broadly. In an important recent contribution, Bils et al. (2017) propose a method to identify additive measurement error. Here, we apply their methodology to our data. The Bils et al. (2017) approach essentially involves estimating the following regression:

$$\Delta rev_{it} = \Phi mrpk_{it} + \Psi \Delta k_{it} - \Psi\left(1-\lambda\right) mrpk_{it}\cdot\Delta k_{it} + D_{jt} + \epsilon_{it} \ ,$$

where Δrev_{it} and Δk_{it} denote changes in (log) revenues and capital respectively, D_{jt} is a full set of industry-year fixed effects and $mrpk_{it}$ is (the log of the) marginal revenue product of capital.

The key object is the coefficient on the interaction term. Bils et al. (2017) show that, under certain assumptions, λ equals the ratio of the true dispersion in the *mrpk* to its measured counterpart (and inversely, $1 - \lambda$ is the contribution of measurement error to the observed σ^2_{mrpk}). Intuitively, to the extent measured *mrpk* deviations are due to additive measurement error, revenues of firms with high observed *mrpk* will display a lower elasticity with respect to capital.

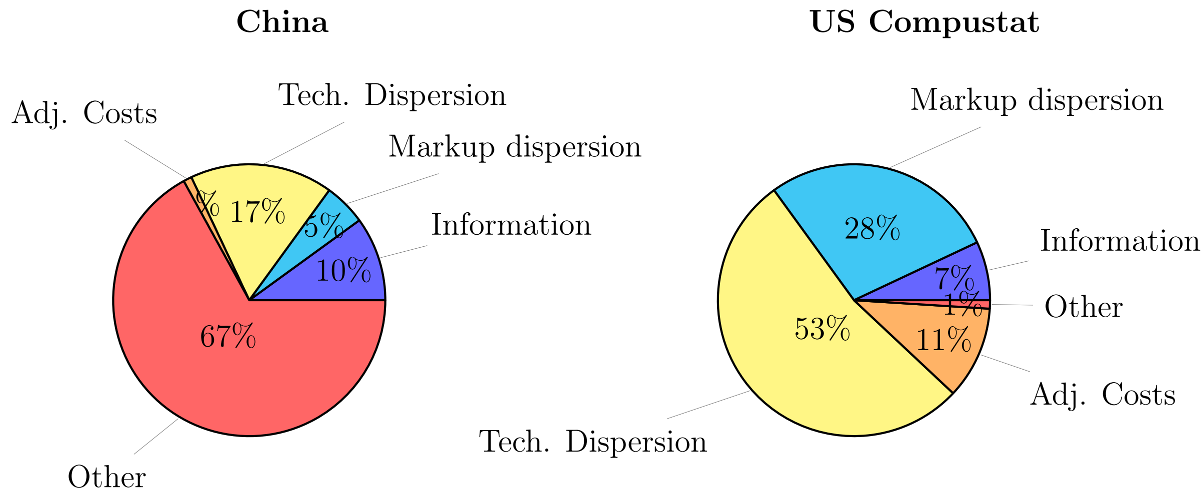
Estimating this regression in our data yields estimates for λ of 0.92 in China and 0.88 in the US. These values suggest that, in both countries, only about 10% of the observed σ^2_{mrpk} can be accounted for by additive measurement error. Of course, it must be pointed out that this method is silent about other forms of measurement error (e.g., multiplicative).[35]

5 Conclusion

In this paper, we have laid out a model of investment featuring multiple factors that interfere with static marginal product equalization, along with an empirical strategy to disentangle them using widely available firm-level production data. Figure 3 summarizes our main results on the sources of misallocation in China (left panel) and the US (right panel). They suggest that much of the observed misallocation in emerging economies like China stems not from technological and informational frictions but from other firm-specific factors. They also show that misspecification of demand and production technologies can potentially account for a significant portion of observed misallocation, though less so in China. Crucially, analyzing these factors in isolation would have led to very different conclusions, highlighting the value of using a unified framework and empirical approach like ours.

There are several promising directions for future work. Our findings suggest that misallocation of productive resources, particularly in countries like China, are largely driven by factors that systematically disincentivize investment by larger/more productive firms or are uncorrelated, permanent firm-specific distortions. They provide a guide for future research linking these factors, for example, to specific policies and/or features of the institutional environment. A straightforward first step would be to analyze subsamples of firms – e.g., small vs. large, state-owned vs. private in China, etc. – to gain a deeper understanding of what these factors may be. Applying our methodology on a more disaggregated sectoral level might also be helpful in identifying segments of the economy that are more 'distorted' than others and the underlying sources. Hopenhayn (2014) reviews a number of recent papers that have investigated specific

[35] There are a few approaches in the literature to deal with multiplicative measurement error, e.g. Collard-Wexler and De Loecker (2016) and Song and Wu (2015), who have to impose additional structure in order to make some progress on this dimension.



Notes: The numbers for the contribution of technological dispersion denote the upper bound as calculated in footnote 32.

Figure 3: The Sources of Misallocation

size-dependent policies. Buera et al. (2013) show how irreversibility in government policy can result in fixed distortions at the firm-level. Our identification results show that accurately quantifying the effects of observed policies and/or additional frictions in input choices is likely to require additional data (e.g., as our analysis of financial frictions in Appendix A.6 reveals). It also seems reasonable to conjecture that observed misallocation is the combined effect of a number of policies, so the main message of this paper – the need to use a broad set of data moments to discipline the effects of individual factors – is relevant for this line of work as well.

Our analysis has abstracted from the implications of frictions and distortions for a number of other phenomena studied by the firm dynamics literature. Midrigan and Xu (2014) show that the same factors behind static misallocation can have even larger effects on aggregate outcomes by influencing entry and exit decisions. Similarly, a number of recent papers examine the impact of distortions on the life-cycle of the firm and the distribution of productivity itself, e.g., Hsieh and Klenow (2014), Bento and Restuccia (2016) and Da-Rocha et al. (2017). An important insight from these papers is that the exact nature of the underlying distortions (e.g., the extent to which they are correlated with fundamentals) is key to understanding their dynamic implications. An ambitious and important next step would be to use an empirical strategy like the one in this paper to analyze richer environments featuring some of these elements. Finally, our methodology can be easily applied to disentangle the causes of labor misallocation by using detailed data on labor inputs.

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Appendix

A Derivations

A.1 Baseline Model

This appendix provides detailed derivations for our baseline analysis.

A.1.1 Model Solution

The first order condition and envelope conditions associated with (3) are, respectively,

$$\begin{aligned} T^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1},K_{it}\right) &= \beta\mathbb{E}_{it}\left[\mathcal{V}_1\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] \\ \mathcal{V}_1\left(K_{it},\mathcal{I}_{it}\right) &= \Pi_1\left(K_{it},A_{it}\right)-\Phi_2\left(K_{it+1},K_{it}\right) \end{aligned}$$

and combining yields the Euler equation

$$\mathbb{E}_{it}\left[\beta\Pi_1\left(K_{it+1},A_{it+1}\right)-\beta\Phi_2\left(K_{it+2},K_{it+1}\right)-T^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)-\Phi_1\left(K_{it+1},K_{it}\right)\right]=0$$

where

$$\begin{aligned} \Pi_1\left(K_{it+1},A_{it+1}\right) &= \alpha G A_{it+1}K^{\alpha-1}_{it+1} \\ \Phi_1\left(K_{it+1},K_{it}\right) &= \hat{\xi}\left(\frac{K_{it+1}}{K_{it}}-\left(1-\delta\right)\right) \\ \Phi_2\left(K_{it+1},K_{it}\right) &= -\hat{\xi}\left(\frac{K_{it+1}}{K_{it}}-\left(1-\delta\right)\right)\frac{K_{it+1}}{K_{it}}+\frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}}-\left(1-\delta\right)\right)^2 \\ &= \frac{\hat{\xi}}{2}\left(1-\delta\right)^2-\frac{\hat{\xi}}{2}\left(\frac{K_{it+1}}{K_{it}}\right)^2 \end{aligned}$$

In the undistorted $\left(\bar{T}^K=1\right)$ non-stochastic steady state, these are equal to

$$\begin{aligned} \bar{\Phi}_1 &= \hat{\xi}\delta \\ \bar{\Phi}_2 &= \frac{\hat{\xi}}{2}\left(1-\delta\right)^2-\frac{\hat{\xi}}{2} \\ \bar{\Pi}_1 &= \alpha\bar{G}\bar{A}\bar{K}^{\alpha-1} \end{aligned}$$

Log-linearizing the Euler equation around this point yields

$$\mathbb{E}_{it}\left[\beta\bar{\Pi}_1\pi_{1,it+1}-\beta\bar{\Phi}_2\phi_{2,it+1}-\tau^K_{it+1}\left(1-\beta\left(1-\delta\right)\right)-\bar{\Phi}_1\phi_{1,it}\right]=0$$

where $\tau^K_{it+1}=\log T^K_{it+1}$ and

$$\begin{aligned} \bar{\Pi}_1\pi_{1,it+1} &\approx \alpha\bar{G}\bar{A}\bar{K}^{\alpha-1}\left(a_{it+1}+\left(\alpha-1\right)k_{it+1}\right) \\ \bar{\Phi}_1\phi_{1,it} &\approx \hat{\xi}\left(k_{it+1}-k_{it}\right) \\ \bar{\Phi}_2\phi_{2,it+1} &\approx -\hat{\xi}\left(k_{it+2}-k_{it+1}\right) \end{aligned}$$

Rearranging gives

$$k_{it+1}\left((1+\beta)\xi+1-\alpha\right)=\mathbb{E}_{it}\left[a_{it+1}+\tau_{it+1}\right]+\beta\xi\mathbb{E}_{it}\left[k_{it+2}\right]+\xi k_{it}$$

where

$$\xi=\frac{\hat{\xi}}{\beta\bar{\Pi}_1},\qquad \tau_{it+1}=-\frac{1-\beta\left(1-\delta\right)}{\beta\bar{\Pi}_1}\tau_{it+1}^{K}$$

which is expression (4) in the text. Using the steady state Euler equation,

$$\beta\bar{\Pi}_1-\beta\bar{\Phi}_2=1+\bar{\Phi}_1\quad\Rightarrow\quad\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}=1-\beta\left(1-\delta\right)+\hat{\xi}\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)$$

we have

$$\begin{aligned}\xi &= \frac{\hat{\xi}}{1-\beta\left(1-\delta\right)+\hat{\xi}\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)} \\ \tau_{it+1} &= -\frac{1-\beta\left(1-\delta\right)}{1-\beta\left(1-\delta\right)+\hat{\xi}\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)}\tau_{it+1}^{K}\end{aligned} \tag{21}$$

To derive the investment policy function, we conjecture that it takes the form in expression (7). Then,

$$\begin{aligned}k_{it+2} &= \psi_1 k_{it+1}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it+1}a_{it+2}+\psi_3\varepsilon_{it+2}+\psi_4\chi_i \\ \mathbb{E}_{it}\left[k_{it+2}\right] &= \psi_1 k_{it+1}+\psi_2\left(1+\gamma\right)\rho\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_4\chi_i \\ &= \psi_1\left(\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i\right)+\psi_2\left(1+\gamma\right)\rho\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_4\chi_i \\ &= \psi_1^2 k_{it}+\left(\psi_1+\rho\right)\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_1\psi_3\varepsilon_{it+1}+\psi_4\left(1+\psi_1\right)\chi_i\end{aligned}$$

where we have used $\mathbb{E}_{it}\left[\varepsilon_{it+2}\right]=0$ and $\mathbb{E}_{it}\left[\mathbb{E}_{it+1}\left[a_{it+2}\right]\right]=\rho\mathbb{E}_{it}\left[a_{it+1}\right]$. Substituting and rearranging,

$$\begin{aligned}&\left(1+\beta\xi\psi_4\left(1+\psi_1\right)\right)\chi_i+\left(1+\beta\xi\psi_1\psi_3\right)\varepsilon_{it+1} \\ +\ &\left(1+\beta\xi\left(\psi_1+\rho\right)\psi_2\right)\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\xi\left(1+\beta\psi_1^2\right)k_{it} \\ =\ &\left(\left(1+\beta\right)\xi+1-\alpha\right)\left(\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i\right)\end{aligned}$$

Finally, matching coefficients gives

$$\begin{aligned}
\xi\left(\beta\psi_1^2+1\right) &= \psi_1\left((1+\beta)\xi+1-\alpha\right) \\
1+\beta\xi\left(\psi_1+\rho\right)\psi_2 &= \psi_2\left((1+\beta)\xi+1-\alpha\right) \Rightarrow \psi_2=\frac{1}{1-\alpha+\beta\xi\left(1-\psi_1-\rho\right)+\xi} \\
1+\beta\xi\psi_1\psi_3 &= \psi_3\left((1+\beta)\xi+1-\alpha\right) \Rightarrow \psi_3=\frac{1}{1-\alpha+\left(1-\psi_1\right)\beta\xi+\xi} \\
1+\beta\xi\psi_4\left(1+\psi_1\right) &= \psi_4\left((1+\beta)\xi+1-\alpha\right) \Rightarrow \psi_4=\frac{1}{1-\alpha+\xi\left(1-\beta\psi_1\right)}
\end{aligned}$$

A few lines of algebra yields the expressions in (8).

A.1.2 Aggregation

To derive aggregate TFP and output, substitute the firm's optimality condition for labor

$$N_{it}=\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\hat{A}_{it}K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}$$

into the production function (1) to get

$$Y_{it}=\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\right)^{\frac{\hat{\alpha}_2}{1-\alpha_2}}\hat{A}_{it}^{\frac{\hat{\alpha}_2}{1-\alpha_2}}K_{it}^{\frac{\hat{\alpha}_1}{1-\alpha_2}}$$

and using the demand function, revenues are

$$P_{it}Y_{it}=Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}A_{it}K_{it}^{\alpha}$$

Labor market clearing implies

$$\int N_{it}di=\int\left(\frac{\alpha_2 Y^{\frac{1}{\theta}}}{W}\right)^{\frac{1}{1-\alpha_2}}A_{it}K_{it}^{\alpha}di=N$$

so that

$$\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}=\left(\frac{N}{\int A_{it}K_{it}^{\alpha}di}\frac{1}{Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}}\right)^{\alpha_2}\Rightarrow P_{it}Y_{it}=Y^{\frac{1}{\theta}}\frac{A_{it}K_{it}^{\alpha}}{\left(\int A_{it}K_{it}^{\alpha}di\right)^{\alpha_2}}N^{\alpha_2}$$

By definition,

$$MRPK_{it}=\alpha\frac{A_{it}K_{it}^{\alpha-1}}{\left(\int A_{it}K_{it}^{\alpha}di\right)^{\alpha_2}}Y^{\frac{1}{\theta}}N^{\alpha_2}$$

so that

$$K_{it} = \left(\frac{\alpha Y^{\frac{1}{\theta}} A_{it}}{MRPK_{it}}\right)^{\frac{1}{1-\alpha}} \left(\frac{N}{\int A_{it} K_{it}^{\alpha} di}\right)^{\frac{\alpha_2}{1-\alpha}}$$

and capital market clearing implies

$$K = \int K_{it} di = \left(\alpha Y^{\frac{1}{\theta}}\right)^{\frac{1}{1-\alpha}} \left(\frac{N}{\int A_{it} K_{it}^{\alpha} di}\right)^{\frac{\alpha_2}{1-\alpha}} \int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di$$

The latter two equations give

$$K_{it}^{\alpha} = \left(\frac{A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}}}{\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di} K\right)^{\alpha}$$

Substituting into the expression for $P_{it}Y_{it}$ and rearranging, we can derive

$$P_{it}Y_{it} = \frac{\frac{A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}}}{\left(\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\alpha}}}{\left(\frac{\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} di}{\left(\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\alpha}}\right)^{\alpha_2}} Y^{\frac{1}{\theta}} K^{\alpha_1} N^{\alpha_2}$$

Using the fact that $Y = \int P_{it}Y_{it} di$, we can derive

$$Y = \int P_{it}Y_{it} di = Y^{\frac{1}{\theta}} A K^{\alpha_1} N^{\alpha_2}$$

where

$$A = \left(\frac{\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}} di}{\left(\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\alpha}}\right)^{1-\alpha_2}$$

or in logs,

$$a = (1-\alpha_2)\left[\log\left(\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{\alpha}{1-\alpha}}\right) - \alpha \log\left(\int A_{it}^{\frac{1}{1-\alpha}} MRPK_{it}^{-\frac{1}{1-\alpha}}\right)\right]$$

The first term inside brackets is equal to

$$\frac{1}{1-\alpha}\overline{a} - \frac{\alpha}{1-\alpha}\overline{mrpk} + \frac{1}{2}\left(\frac{1}{1-\alpha}\right)^2 \sigma_a^2 + \frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^2 \sigma_{mrpk}^2 - \frac{\alpha}{(1-\alpha)^2}\sigma_{mrpk,a}$$

and the second,

$$\frac{\alpha}{1-\alpha}\overline{a}-\frac{\alpha}{1-\alpha}\overline{mrpk}+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_a^2+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_{mrpk}^2-\frac{\alpha}{(1-\alpha)^2}\sigma_{mrpk,a}$$

Combining,

$$a=(1-\alpha_2)\left[\overline{a}+\frac{1}{2}\frac{1}{1-\alpha}\sigma_a^2-\frac{1}{2}\frac{\alpha}{1-\alpha}\sigma_{mrpk}^2\right]$$

and

$$\begin{aligned} y &= \frac{1}{\theta}y+(1-\alpha_2)\,\overline{a}+\frac{1}{2}\frac{1-\alpha_2}{1-\alpha}\sigma_a^2-\frac{1}{2}\alpha\frac{1-\alpha_2}{1-\alpha}\sigma_{mrpk}^2+\alpha_1 k+\alpha_2 n \\ &= \frac{\theta}{\theta-1}(1-\alpha_2)\,\overline{a}+\frac{\theta}{\theta-1}\frac{1}{2}\frac{1-\alpha_2}{1-\alpha}\sigma_a^2-\frac{\theta}{\theta-1}\frac{1}{2}\alpha\frac{1-\alpha_2}{1-\alpha}\sigma_{mrpk}^2+\hat{\alpha}_1 k+\hat{\alpha}_2 n \\ &= a+\hat{\alpha}_1 k+\hat{\alpha}_2 n \end{aligned}$$

where, using $a_{it}=\frac{1}{1-\alpha_2}\hat{a}_{it}$, $\sigma_a^2=\left(\frac{1}{1-\alpha_2}\right)^2\sigma_{\hat{a}}^2$ and $\alpha=\frac{\alpha_1}{1-\alpha_2}$,

$$\begin{aligned} a &= \frac{\theta}{\theta-1}\bar{\hat{a}}+\frac{1}{2}\frac{\theta}{\theta-1}\frac{1}{1-\alpha_1-\alpha_2}\sigma_{\hat{a}}^2-\frac{1}{2}\left(\theta\hat{\alpha}_1+\hat{\alpha}_2\right)\hat{\alpha}_1\sigma_{mrpk}^2 \\ &= a^*-\frac{1}{2}\left(\theta\hat{\alpha}_1+\hat{\alpha}_2\right)\hat{\alpha}_1\sigma_{mrpk}^2 \end{aligned}$$

which is equation (9) in the text.

To compute the effect on output, notice that the aggregate production function is

$$y=\hat{\alpha}_1 k+\hat{\alpha}_2 n+a$$

so that

$$\begin{aligned} \frac{dy}{d\sigma_{mrpk}^2} &= \hat{\alpha}_1\frac{dk}{da}\frac{da}{d\sigma_{mrpk}^2}+\frac{da}{d\sigma_{mrpk}^2} \\ &= \frac{da}{d\sigma_{mrpk}^2}\left(1+\hat{\alpha}_1\frac{dk}{da}\right) \end{aligned}$$

In the stationary equilibrium, the aggregate marginal product of capital must be a constant, denote it by $\bar{R}$, i.e., $\log\hat{\alpha}_1+y-k=\bar{r}$ so that

$$k=\frac{1}{1-\hat{\alpha}_1}\left(\log\hat{\alpha}_1+\hat{\alpha}_2 n+a-\bar{r}\right)$$

and

$$\frac{dk}{da} = \frac{1}{1-\hat{\alpha}_1}$$

Combining,

$$\frac{dy}{d\sigma^2_{mrpk}} = \frac{da}{d\sigma^2_{mrpk}}\left(1+\frac{\hat{\alpha}_1}{1-\hat{\alpha}_1}\right) = \frac{da}{d\sigma^2_{mrpk}}\frac{1}{1-\hat{\alpha}_1}$$

A.2 Firm-Specific Wages

In this appendix, we show that to the extent distortions to the labor choice are reflected in firm-specific wages, they change the interpretation of fundamentals but otherwise do not affect our analysis of capital misallocation. In particular, they do not contribute to measured *mrpk* dispersion and so our strategy for disentangling the various sources of capital misallocation and our estimates for their magnitudes go through unchanged.

We allow wages to vary at the firm level due to distortions, i.e., introduce $W_{it} \equiv WT^N_{it}$ into the firm's problem, which becomes

$$\begin{aligned}\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right) = \max_{N_{it},K_{it+1}} \quad & \mathbb{E}_{it}\left[Y^{\frac{1}{\theta}}\hat{A}_{it}K^{\alpha_1}_{it}N^{\alpha_2}_{it} - WT^N_{it}N_{it} - T^K_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right) - \Phi\left(K_{it+1},K_{it}\right)\right] \\ + \quad & \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right]\end{aligned}$$

The labor choice satisfies the first order condition

$$N_{it} = \left(\alpha_2\frac{Y^{\frac{1}{\theta}}\hat{A}_{it}K^{\alpha_1}_{it}}{WT^N_{it}}\right)^{\frac{1}{1-\alpha_2}}$$

Substituting, we can derive operating profits (revenues net of total wages) as

$$\begin{aligned}P_{it}Y_{it} - WT^N_{it}N_{it} &= Y_t^{\frac{1}{\theta}}\hat{A}_{it}K^{\alpha_1}_{it}\left(\alpha_2 Y_t^{\frac{1}{\theta}}\frac{\hat{A}_{it}K^{\alpha_1}_{it}}{WT^N_{it}}\right)^{\frac{\alpha_2}{1-\alpha_2}} - WT^N_{it}\left(\alpha_2 Y_t^{\frac{1}{\theta}}\frac{\hat{A}_{it}K^{\alpha_1}_{it}}{WT^N_{it}}\right)^{\frac{1}{1-\alpha_2}} \\ &= \left(1-\alpha_2\right)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\frac{\hat{A}_{it}^{\frac{1}{1-\alpha_2}}}{(T^N_{it})^{\frac{\alpha_2}{1-\alpha_2}}}K_{it}^{\frac{\alpha_1}{1-\alpha_2}} \\ &= GA_{it}K^{\alpha}_{it}\end{aligned}$$

which is the same form as in the baseline version, except now the fundamental A_{it} also incorporates the effect of the labor distortion:[36]

$$A_{it} \equiv \left(\frac{\hat{A}_{it}}{\left(T_{it}^{N}\right)^{\alpha_2}}\right)^{\frac{1}{1-\alpha_2}}$$

With this re-interpretation, the firm's dynamic investment decision is still given by (3). To see that these labor taxes do not contribute to *mrpk* dispersion, assume that they are the only friction, i.e., the capital choice is made under full information with no adjustment costs or uncertainty. The capital choice is then static and given by

$$K_{it} = \left(\frac{\alpha G \hat{A}_{it}^{\frac{1}{1-\alpha_2}}}{\left(T_{it}^{N}\right)^{\frac{\alpha_2}{1-\alpha_2}}}\right)^{\frac{1}{1-\alpha}}$$

Combining this with the expression for revenues, the measured *mrpk* is equal to

$$\begin{aligned} mrpk_{it} &= \text{Const.} + p_{it} + y_{it} - k_{it} \\ &= \text{Const.} + \frac{-\alpha_2}{1-\alpha_2}\tau_{it}^{N} + \frac{1}{1-\alpha_2}\hat{a}_{it} + (\alpha-1)\frac{-\alpha_2}{1-\alpha_2}\frac{1}{1-\alpha}\tau_{it}^{N} + (\alpha-1)\frac{1}{1-\alpha_2}\frac{1}{1-\alpha}\hat{a}_{it} \\ &= \text{Const.} \end{aligned}$$

So, T_{it}^{N} does lead to any measured dispersion in the *mrpk*.

A.3 Identification

In this appendix we derive analytic expressions for the four moments in the random walk case, i.e., when $\rho = 1$, and prove Proposition 1.

Moments. From expression (7), we have the firm's investment policy function

$$k_{it+1} = \psi_1 k_{it} + \psi_2 (1+\gamma)\mathbb{E}_{it}\left[a_{it+1}\right] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

and substituting for the expectation,

$$k_{it+1} = \psi_1 k_{it} + \psi_2 (1+\gamma)\left(a_{it} + \phi\left(\mu_{it+1} + e_{it+1}\right)\right) + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

where $\phi = \frac{\mathbb{V}}{\sigma_e^2}$ so that $1-\phi = \frac{\mathbb{V}}{\sigma_\mu^2}$. Then,

$$\Delta k_{it+1} = \psi_1 \Delta k_{it} + \psi_2 (1+\gamma)\left(\left(1-\phi\right)\mu_{it} + \phi\mu_{it+1} + \phi\left(e_{it+1} - e_{it}\right)\right) + \psi_3\left(\varepsilon_{it+1} - \varepsilon_{it}\right)$$

[36] Note that this is also the a_{it} we would measure from the data using the definition $a_{it} = va_{it} - \alpha k_{it}$.

We will use the fact that

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \mu_{it+1}\right) &= \psi_2 \left(1+\gamma\right) \phi \sigma_\mu^2 \\
\text{cov}\left(\Delta k_{it+1}, e_{it+1}\right) &= \psi_2 \left(1+\gamma\right) \phi \sigma_e^2 \\
\text{cov}\left(\Delta k_{it+1}, \varepsilon_{it+1}\right) &= \psi_3 \sigma_\varepsilon^2
\end{aligned}$$

Now,

$$\begin{aligned}
\text{var}\left(\Delta k_{it+1}\right) &= \psi_1^2 \text{var}\left(\Delta k_{it}\right) + \psi_2^2 \left(1+\gamma\right)^2 \left(1-\phi\right)^2 \sigma_\mu^2 \\
&+ \psi_2^2 \left(1+\gamma\right)^2 \phi^2 \sigma_\mu^2 + 2\psi_2^2 \left(1+\gamma\right)^2 \phi^2 \sigma_e^2 + 2\psi_3^2 \sigma_\varepsilon^2 \\
&+ 2\psi_1 \psi_2 \left(1+\gamma\right) \left(1-\phi\right) \text{cov}\left(\Delta k_{it}, \mu_{it}\right) - 2\psi_1 \psi_2 \left(1+\gamma\right) \phi \text{cov}\left(\Delta k_{it}, e_{it}\right) \\
&- 2\psi_1 \psi_3 \text{cov}\left(\Delta k_{it}, \varepsilon_{it}\right)
\end{aligned}$$

where substituting, rearranging and using the fact that the moments are stationary gives

$$\sigma_k^2 \equiv \text{var}\left(\Delta k_{it}\right) = \frac{\left(1+\gamma\right)^2 \psi_2^2 \sigma_\mu^2 + 2\left(1-\psi_1\right) \psi_3^2 \sigma_\varepsilon^2}{1-\psi_1^2}$$

which can be rearranged to yield expression (10).

Next,

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \Delta k_{it}\right) &= \psi_1 \text{var}\left(\Delta k_{it}\right) + \psi_2 \left(1+\gamma\right) \left(1-\phi\right) \text{cov}\left(\Delta k_{it}, \mu_{it}\right) \\
&- \psi_2 \left(1+\gamma\right) \phi \text{cov}\left(\Delta k_{it}, e_{it}\right) - \psi_3 \text{cov}\left(\Delta k_{it}, \varepsilon_{it}\right) \\
&= \psi_1 \text{var}\left(\Delta k_{it}\right) - \psi_3 \text{cov}\left(\Delta k_{it}, \varepsilon_{it}\right) \\
&= \psi_1 \sigma_k^2 - \psi_3^2 \sigma_\varepsilon^2
\end{aligned}$$

so that

$$\rho_{k,k_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta k_{it-1}\right) = \psi_1 - \psi_3^2 \frac{\sigma_\varepsilon^2}{\sigma_k^2}$$

which is expression (11).

Similarly,

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \Delta a_{it}\right) &= \text{cov}\left(\Delta k_{it+1}, \mu_{it}\right) \\
&= \psi_1 \text{cov}\left(\Delta k_{it}, \mu_{it}\right) + \psi_2 \left(1+\gamma\right) \left(1-\phi\right) \sigma_\mu^2 \\
&= \psi_1 \psi_2 \left(1+\gamma\right) \phi \sigma_\mu^2 + \psi_2 \left(1+\gamma\right) \left(1-\phi\right) \sigma_\mu^2 \\
&= \left(1-\phi\left(1-\psi_1\right)\right) \psi_2 \left(1+\gamma\right) \sigma_\mu^2
\end{aligned}$$

and from here it is straightforward to derive

$$\rho_{k,a_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta a_{it-1}\right) = \left[\frac{\mathbb{V}}{\sigma_\mu^2}\left(1-\psi_1\right)+\psi_1\right]\frac{\sigma_\mu \psi_2\left(1+\gamma\right)}{\sigma_k}$$

as in expression (12).

Finally,

$$mrpk_{it} = \text{Const} + p_{it} + y_{it} - k_{it} = \text{Const} + a_{it} + \alpha k_{it} - k_{it} = \text{Const} + a_{it} - \left(1-\alpha\right)k_{it}$$

so that

$$\Delta mrpk_{it} = \Delta a_{it} - \left(1-\alpha\right)\Delta k_{it} = \mu_{it} - \left(1-\alpha\right)\Delta k_{it}$$

which implies

$$\text{cov}\left(\Delta mrpk_{it}, \mu_{it}\right) = \left(1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)\sigma_\mu^2$$

and

$$\begin{aligned}\lambda_{mrpk,a} \equiv \frac{\text{cov}\left(\Delta mrpk_{it}, \mu_{it}\right)}{\sigma_\mu^2} &= 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi \\ &= 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right)\end{aligned}$$

which is expression (13).

To see that the correlation $\rho_{mrpk,a}$ is decreasing in σ_ε^2, we derive

$$\begin{aligned}\text{var}\left(\Delta mrpk_{it}\right) &= \sigma_\mu^2 + \left(1-\alpha\right)^2\sigma_k^2 - 2\left(1-\alpha\right)\text{cov}\left(\Delta k_{it}, \mu_{it}\right) \\ &= \sigma_\mu^2 + \left(1-\alpha\right)^2\left(\frac{\psi_2^2\left(1+\gamma\right)^2\sigma_\mu^2 + 2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2}{1-\psi_1^2}\right) - 2\left(1-\alpha\right)\psi_2\left(1+\gamma\right)\phi\sigma_\mu^2 \\ &= \frac{1}{1-\psi_1^2}\left(\left(\left(1-\psi_1^2\right)\left(1-2\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)+\left(1-\alpha\right)^2\left(1+\gamma\right)^2\psi_2^2\right)\sigma_\mu^2\right) \\ &+ \frac{1}{1-\psi_1^2}\left(2\left(1-\alpha\right)^2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2\right)\end{aligned}$$

so

$$\rho_{mrpk,a} = \frac{\left(1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)\sigma_\mu\sqrt{1-\psi_1^2}}{\sqrt{\left(\left(1-\psi_1^2\right)\left(1-2\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\phi\right)+\left(1-\alpha\right)^2\left(1+\gamma\right)^2\psi_2^2\right)\sigma_\mu^2 + 2\left(1-\alpha\right)^2\left(1-\psi_1\right)\psi_3^2\sigma_\varepsilon^2}}$$

Proof of Proposition 1. Write the variance of investment as

$$\sigma_k^2 = \psi_1^2 \sigma_k^2 + (1+\gamma)^2 \psi_2^2 \sigma_\mu^2 + 2(1-\psi_1)\psi_3^2 \sigma_\varepsilon^2$$

To rewrite the last term as a function of an observable moment, use the autocovariance of investment,

$$\sigma_{k,k_{-1}} = \psi_1 \sigma_k^2 - \psi_3^2 \sigma_\varepsilon^2 \tag{22}$$

and substitution yields

$$\sigma_k^2 = \psi_1^2 \sigma_k^2 + (1+\gamma)^2 \psi_2^2 \sigma_\mu^2 + 2(1-\psi_1)\left(\psi_1 \sigma_k^2 - \sigma_{k,k_{-1}}\right) \tag{23}$$

To eliminate the second term, use the equation for $\lambda_{mrpk,a}$ to solve for

$$(1+\gamma)\psi_2 \phi = \frac{1-\lambda_{mrpk,a}}{1-\alpha} = \tilde{\lambda} \tag{24}$$

where $\tilde{\lambda}$ is a decreasing function of $\lambda_{mrpk,a}$ that depends only on the known parameter α. Substituting into the expression for the covariance of investment with the lagged shock, $\sigma_{k,a_{-1}}$, and rearranging yields

$$(1+\gamma)\psi_2 = \frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}(1-\psi_1) \tag{25}$$

which is an equation in ψ_1 and observable moments. Substituting into (23) gives

$$\sigma_k^2 = \psi_1^2 \sigma_k^2 + \left(\frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}(1-\psi_1)\right)^2 \sigma_\mu^2 + 2(1-\psi_1)\left(\psi_1 \sigma_k^2 - \sigma_{k,k_{-1}}\right)$$

and rearranging, we can derive

$$0 = \left(\hat{\lambda}^2 - 1\right)(1-\psi_1)^2 + 2\left(\hat{\lambda}\rho_{k,a_{-1}} - \rho_{k,k_{-1}}\right)(1-\psi_1) + \rho_{k,a_{-1}}^2 \tag{26}$$

where

$$\hat{\lambda} = \frac{\sigma_\mu}{\sigma_k}\tilde{\lambda} = \frac{\sigma_\mu}{\sigma_k}\left(\frac{1-\lambda_{mrpk,a}}{1-\alpha}\right)$$

Equation (26) represents a quadratic equation in a single unknown, $1-\psi_1$, or equivalently, in ψ_1. The solution features two positive roots, one greater than one and one less. The smaller root corresponds to the true ψ_1 that represents the solution to the firm's investment policy. The value of ψ_1 pins down the adjustment cost parameter ξ as well as ψ_2 and ψ_3. We can then back out γ from (25), ϕ (and so $\mathbb{V}$) from (24) and finally, σ_ε^2 from (22).

□

A.4 Heterogeneity in Markups/Technologies

The firm's cost minimization problem is

$$\min_{K_{it},N_{it},M_{it}} \quad R_t T^K_{it} K_{it} + W_t T^N_{it} N_{it} + P^M_t M_{it} \quad s.t. \quad Y_{it} \leq K^{\hat{\alpha}_{it}}_{it} N^{\hat{\zeta}-\hat{\alpha}_{it}}_{it} M^{1-\hat{\zeta}}_{it}$$

The first order condition on M_{it} gives

$$P^M_t = \left(1-\hat{\zeta}\right)\frac{Y_{it}}{M_{it}} MC_{it} \quad \Rightarrow \quad \frac{P^M_t M_{it}}{P_{it}Y_{it}} = \left(1-\hat{\zeta}\right)\frac{MC_{it}}{P_{it}}$$

where MC_{it} is the Lagrange multiplier on the constraint (i.e., the marginal cost). Rearranging gives expression (14). In logs,

$$\log\frac{P_{it}}{MC_{it}} = \log\left(1-\hat{\zeta}\right) + \log\frac{P_{it}Y_{it}}{P^M_t M_{it}} \quad \Rightarrow \quad \sigma^2\left(\log\frac{P_{it}}{MC_{it}}\right) = \sigma^2\left(\log\frac{P_{it}Y_{it}}{P^M_t M_{it}}\right)$$

Similarly, the optimality conditions for K_{it} and N_{it} yield:

$$\begin{aligned} \log\frac{P_{it}Y_{it}}{K_{it}} &= \log\frac{P_{it}}{MC_{it}} - \log\hat{\alpha}_{it} + \tau^K_{it} + \text{Constant} \\ \log\frac{P_{it}Y_{it}}{N_{it}} &= \log\frac{P_{it}}{MC_{it}} - \log\left(\hat{\zeta}-\hat{\alpha}_{it}\right) + \tau^N_{it} + \text{Constant} \end{aligned}$$

Log-linearizing around the average $\hat{\alpha}_{it}$, denote it $\bar{\alpha}$, and ignoring constants yields $\log\left(\hat{\zeta}-\hat{\alpha}_{it}\right) \approx -\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\log\hat{\alpha}_{it}$. Substituting gives expression (17).

Proof of Proposition 2. Assuming $\log\hat{\alpha}_{it}$ is uncorrelated with τ^K_{it} and τ^N_{it},

$$\begin{aligned} cov\left(arpk_{it}, arpn_{it}\right) &= -\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\sigma^2_{\log\hat{\alpha}} + \text{cov}\left(\tau^k_{it}, \tau^n_{it}\right) \\ \sigma^2_{arpk} &= \sigma^2_{\log\hat{\alpha}} + \sigma^2_{\tau^k} \\ \sigma^2_{arpn} &= \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2\sigma^2_{\log\hat{\alpha}} + \sigma^2_{\tau^n} \end{aligned}$$

From here, we can solve for the correlation of the distortions:

$$\rho\left(\tau^K_{it}, \tau^N_{it}\right) = \frac{\text{cov}\left(arpk_{it}, arpn_{it}\right) + \frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\sigma^2_{\log\hat{\alpha}}}{\sqrt{\sigma^2_{arpk} - \sigma^2_{\log\hat{\alpha}}}\sqrt{\sigma^2_{arpn} - \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2\sigma^2_{\log\hat{\alpha}}}}$$

which is increasing in $\sigma^2_{\log\hat{\alpha}}$. An upper bound for $\sigma^2_{\log\hat{\alpha}}$, denoted $\bar{\sigma}^2_{\log\hat{\alpha}}$, is where $\rho\left(\tau^K_{it}, \tau^N_{it}\right) = 1$, and substituting and rearranging gives

$$\bar{\sigma}_{\hat{\alpha}}^2 = \frac{\sigma_{arpk}^2 \sigma_{arpn}^2 - \text{cov}\left(arpk_{it}, arpn_{it}\right)^2}{2\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\text{cov}\left(arpk_{it}, arpn_{it}\right) + \left(\frac{\bar{\alpha}}{\hat{\zeta}-\bar{\alpha}}\right)^2 \sigma_{arpk}^2 + \sigma_{arpn}^2}$$

□

A.5 Frictional Labor

In this appendix, we provide detailed derivations for the case of frictional labor.

A.5.1 Model Solution

When labor is chosen under the same frictions as capital, the firm's value function takes the form

$$\begin{aligned} \mathcal{V}\left(K_{it}, N_{it}, \mathcal{I}_{it}\right) = \max_{K_{it+1}, N_{it+1}} \quad & \mathbb{E}_{it}\left[Y^{\frac{1}{\theta}} \hat{A}_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2}\right] \\ - \quad & \mathbb{E}_{it}\left[T_{it+1} K_{it+1}\left(1-\beta\left(1-\delta\right)\right) + \Phi\left(K_{it+1}, K_{it}\right)\right] \\ - \quad & \mathbb{E}_{it}\left[T_{it+1} W N_{it+1}\left(1-\beta\left(1-\delta\right)\right) + W\Phi\left(N_{it+1}, N_{it}\right)\right] \\ + \quad & \mathbb{E}_{it}\left[\beta \mathcal{V}\left(K_{it+1}, N_{it+1}, \mathcal{I}_{it+1}\right)\right] \end{aligned} \tag{27}$$

where the adjustment cost function $\Phi(\cdot)$ is as defined in expression (2). Because the firm makes a one-time payment to hire incremental labor, the cost of labor W is now to be interpreted as the present discounted value of wages. Capital and labor are both subject to the same adjustment frictions, the same distortions, denoted T_{it+1}, and are chosen under the same information set, though the cost of labor adjustment is denominated in labor units.

The first order and envelope conditions yield two Euler equations:

$$\begin{aligned} \mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right) + \Phi_1\left(K_{it+1}, K_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta \alpha_1 Y^{\frac{1}{\theta}} \hat{A}_{it+1} K_{it+1}^{\alpha_1 - 1} N_{it+1}^{\alpha_2} - \beta \Phi_2\left(K_{it+2}, K_{it+1}\right)\right] \\ \mathbb{E}_{it}\left[W T_{it+1}\left(1-\beta\left(1-\delta\right)\right) + \Phi_1\left(N_{it+1}, N_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta \alpha_2 Y^{\frac{1}{\theta}} \hat{A}_{it+1} K_{it+1}^{\alpha_1} N_{it+1}^{\alpha_2 - 1} - \beta W \Phi_2\left(N_{it+2}, N_{it+1}\right)\right] \end{aligned}$$

To show that this setup leads to an intertemporal investment problem that takes the same form as (3), we prove that there exists a constant η such that $N_{it+1} = \eta K_{it+1}$ which leads to the same solution as if the firm were choosing only capital facing a degree of curvature $\alpha = \alpha_1 + \alpha_2$.

Under this conjecture, we can rewrite the firm's problem in (27) as

$$\begin{aligned}\tilde{\mathcal{V}}\left(K_{it},\mathcal{I}_{it}\right)=\max_{K_{it+1}} \quad & \mathbb{E}_{it}\left[\frac{\eta^{\alpha_2}}{1+W\eta}Y^{\frac{1}{\theta}}\hat{A}_{it}K_{it}^{\alpha_1+\alpha_2}-T_{it+1}K_{it+1}\left(1-\beta\left(1-\delta\right)\right)\right]\\ + \quad & \mathbb{E}_{it}\left[-\Phi\left(K_{it+1},K_{it}\right)+\beta\tilde{\mathcal{V}}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right]\end{aligned}$$

Let $\{K_{it}^{*}\}$ be the solution to this problem. By definition, it must satisfy the following optimality condition

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^{*},K_{it}^{*}\right)\right] &= \mathbb{E}_{it}\left[\beta\frac{\left(\alpha_1+\alpha_2\right)Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{1+W\eta}\right] \\ &- \mathbb{E}_{it}\left[\beta\Phi_2\left(K_{it+2}^{*},K_{it+1}^{*}\right)\right]\end{aligned} \tag{28}$$

Now substitute the conjecture that $N_{it}^{*}=\eta K_{it}^{*}$ into the optimality condition for labor from the original problem and rearrange to get:

$$\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^{*},K_{it}^{*}\right)\right]=\mathbb{E}_{it}\left[\beta\frac{\alpha_2Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{W\eta}-\beta\Phi_2\left(K_{it+2}^{*},K_{it+1}^{*}\right)\right] \tag{29}$$

If η satisfies

$$\frac{\alpha_1+\alpha_2}{1+W\eta}=\frac{\alpha_2}{W\eta} \qquad \Rightarrow \qquad W\eta=\frac{\alpha_2}{\alpha_1} \tag{30}$$

then (29) is identical to (28). In other words, under (30), the sequence $\{K_{it}^{*},N_{it}^{*}\}$ satisfies the optimality condition for labor from the original problem. It is straightforward to verify that this also implies that $\{K_{it}^{*},N_{it}^{*}\}$ satisfy the optimality condition for capital from the original problem:

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\left(1-\beta\left(1-\delta\right)\right)+\Phi_1\left(K_{it+1}^{*},K_{it}^{*}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_1Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}-\beta\Phi_2\left(K_{it+2}^{*},K_{it+1}^{*}\right)\right]\\ &= \mathbb{E}_{it}\left[\beta\frac{\alpha_2Y^{\frac{1}{\theta}}\hat{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{W\eta}-\beta\Phi_2\left(K_{it+2}^{*},K_{it+1}^{*}\right)\right]\end{aligned}$$

Thus, we can analyze this environment in an analogous fashion to the baseline setup, where the firm's intertemporal optimization problem takes the same form as expression (3), with $\alpha=\alpha_1+\alpha_2$, $G=\frac{\eta^{\alpha_2}Y^{\frac{1}{\theta}}}{1+W\eta}$ and $A_{it}=\hat{A}_{it}$.

A.5.2 Aggregation

To derive aggregate output and TFP for this case, we use the fact that, as shown above, $N_{it} = \eta K_{it}$ where $\eta = \frac{\alpha_2}{\alpha_1 W}$. Substituting into the revenue function gives

$$P_{it}Y_{it} = Y^{\frac{1}{\theta}}\hat{A}_{it}\eta^{\alpha_2}K_{it}^{\alpha_1+\alpha_2} = Y^{\frac{1}{\theta}}\hat{A}_{it}\eta^{\alpha_2}K_{it}^{\alpha}$$

By definition,

$$MPRK_{it} = \alpha Y^{\frac{1}{\theta}}\hat{A}_{it}\eta^{\alpha_2}K_{it}^{\alpha-1}$$

so that

$$K_{it} = \left(\frac{\alpha Y^{\frac{1}{\theta}}\hat{A}_{it}\eta^{\alpha_2}}{MPRK_{it}}\right)^{\frac{1}{1-\alpha}}$$

so that

$$\begin{aligned} P_{it}Y_{it} &= Y^{\frac{1}{\theta}}\eta^{\alpha_2}\hat{A}_{it}\left(\frac{\alpha Y^{\frac{1}{\theta}}\eta^{\alpha_2}\hat{A}_{it}}{MRPK_{it}}\right)^{\frac{\alpha}{1-\alpha}} \\ &= \alpha^{\frac{\alpha}{1-\alpha}}Y^{\frac{1}{\theta}\frac{1}{1-\alpha}}\eta^{\frac{\alpha_2}{1-\alpha}}\hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}} \end{aligned}$$

and

$$Y = \int P_{it}Y_{it}di = \alpha^{\frac{\alpha}{1-\alpha}}Y^{\frac{1}{\theta}\frac{1}{1-\alpha}}\eta^{\frac{\alpha_2}{1-\alpha}}\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}di$$

or, rearranging,

$$Y = \alpha^{\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}}Y^{\frac{1}{\theta}\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}}\eta^{\frac{\hat{\alpha}_2}{1-\alpha}}\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}di\right)^{\frac{\theta}{\theta-1}}$$

Capital market clearing implies

$$K = \int K_{it}di = \alpha^{\frac{1}{1-\alpha}}Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha}}\eta^{\frac{\alpha_2}{1-\alpha}}\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di$$

so that

$$K^{\hat{\alpha}_1}N^{\hat{\alpha}_2} = \alpha^{\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}}Y_t^{\frac{1}{\theta}\frac{\hat{\alpha}_1+\hat{\alpha}_2}{1-\alpha}}\eta^{\hat{\alpha}_2+\frac{\alpha_2}{1-\alpha}(\hat{\alpha}_1+\hat{\alpha}_2)}\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\hat{\alpha}_1+\hat{\alpha}_2}$$

Aggregate TFP is

$$A = \frac{Y}{K^{\hat{\alpha}_1}N^{\hat{\alpha}_2}} = \frac{\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{\alpha}{1-\alpha}}di\right)^{\frac{\theta}{\theta-1}}}{\left(\int \hat{A}_{it}^{\frac{1}{1-\alpha}}MRPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\hat{\alpha}_1+\hat{\alpha}_2}}$$

Following similar steps as in the baseline case, we can derive

$$a = a^* - \frac{1}{2}\frac{\theta}{\theta - 1}\frac{\alpha}{1-\alpha}\sigma^2_{mrpk}$$

Under constant returns to scale in production, this simplifies to

$$a = a^* - \frac{1}{2}\theta\sigma^2_{mrpk}$$

The output effects are the same as in the baseline case.

A.6 Financial Frictions

In this appendix, we show how financial frictions/liquidity constraints can give rise to a size-dependent distortion.

We modify our baseline setup by assuming that firms face a liquidity cost $\Upsilon\left(K_{it+1}, B_{it+1}\right)$, where B_{it+1} denotes holdings of liquid assets. The cost is increasing (decreasing) in K_{it+1} (B_{it+1}). These assets are assumed to earn an exogenous rate of return $R < \frac{1}{\beta}$. This specification captures the idea that firms need costly liquidity in order to operate (e.g., as working capital). Using a continuous penalty function rather than an occasionally binding constraint allows us to continue using perturbation methods. Note also that this is subtly different from the standard borrowing constraint used widely in the literature.[37] Our firms are not constrained in terms of their ability to raise funds. This implies that self-financing, which often significantly weakens the bite of borrowing constraints, plays no role here.

We use the following flexible parameterization of the liquidity cost function:

$$\Upsilon\left(K_{it+1}, B_{it+1}\right) = \hat{\nu}\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2}}$$

where $\hat{\nu}$, ω_1 and ω_2 are parameters.

Including the liquidity cost (we abstract from other distortions T^K_{it} for ease of notation, but they are easy to include), the firm's recursive problem can be written as

$$\begin{aligned}\mathcal{V}\left(K_{it}, B_{it}, \mathcal{I}_{it}\right) = \max_{B_{it+1}, K_{it+1}} \quad & \mathbb{E}_{it}\left[\Pi\left(K_{it}, A_{it}\right) + RB_{it} - B_{it+1} - K_{it+1}\left(1 - \beta\left(1-\delta\right)\right)\right] \\ - \quad & \Phi\left(K_{it+1}, K_{it}\right) - \Upsilon\left(K_{it+1}, B_{it+1}\right) + \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1}, B_{it+1}, \mathcal{I}_{it+1}\right)\right]\end{aligned}$$

[37] For example, Midrigan and Xu (2014), Moll (2014) and Gopinath et al. (2017).

The first order conditions are given by

$$\begin{aligned}\mathbb{E}_{it}\left[\beta\Pi_1\left(K_{it+1},A_{it+1}\right)-\beta\Phi_2\left(K_{it+2},K_{it+1}\right)\right] &= 1-\beta\left(1-\delta\right)+\Phi_1\left(K_{it+1},K_{it}\right)+\Upsilon_1\left(K_{it+1},B_{it+1}\right)\\ -\Upsilon_2\left(K_{it+1},B_{it+1}\right)+\beta R &= 1\end{aligned}$$

Note that

$$\Upsilon_2\left(K_{it+1},B_{it+1}\right)=-\hat{\nu}\omega_2\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2+1}},\qquad \Upsilon_1\left(K_{it+1},B_{it+1}\right)=\hat{\nu}\omega_1\frac{K_{it+1}^{\omega_1-1}}{B_{it+1}^{\omega_2}}$$

Using the FOC for B_{it+1}

$$\begin{aligned}1 &= \hat{\nu}\omega_2\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2+1}}+\beta R \quad\Rightarrow\quad B_{it+1}=\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{1}{\omega_2+1}}K_{it+1}^{\frac{\omega_1}{\omega_2+1}}\\ \Upsilon_1\left(K_{it+1},B_{it+1}\right) &= \hat{\nu}\omega_1\frac{K_{it+1}^{\omega_1-1}}{B_{it+1}^{\omega_2}}=\hat{\nu}\omega_1\frac{K_{it+1}^{\omega_1-1}}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}}K_{it+1}^{\frac{\omega_2\omega_1}{\omega_2+1}}}=\frac{\hat{\nu}\omega_1}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}}}\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}K_{it+1}^{\frac{\omega_1-(\omega_2+1)}{\omega_2+1}}\\ &= \nu K_{it+1}^{\omega}\ ,\end{aligned}$$

where

$$\begin{aligned}\nu &\equiv \frac{\hat{\nu}\omega_1}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}}}\left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}}\\ \omega &\equiv \frac{\omega_1-(\omega_2+1)}{\omega_2+1}.\end{aligned}$$

Log-linearizing,

$$\begin{aligned}\bar{\Upsilon}_1+\bar{\Upsilon}_1\upsilon_{1t+1} &\approx \nu\bar{K}^{\omega}+\nu\bar{K}^{\omega}\omega k_{it+1}\\ \bar{\Upsilon}_1\upsilon_{1t+1} &\approx \nu\bar{K}^{\omega}\omega k_{it+1}\ .\end{aligned}$$

Substituting into the FOC,

$$\mathbb{E}_{it}\left[\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}\left(a_{it+1}+\left(\alpha-1\right)k_{it+1}\right)+\beta\hat{\xi}\left(k_{it+2}-k_{it+1}\right)\right]=\hat{\xi}\left(k_{it+1}-k_{it}\right)+\nu\bar{K}^{\omega}\omega k_{it+1}\ ,$$

or

$$k_{it+1}\left(\left(1+\beta\right)\xi+1-\tilde{\alpha}\right)=\mathbb{E}_{it}\left[a_{it+1}\right]+\beta\xi\mathbb{E}_{it}\left[k_{it+2}\right]+\xi k_{it}\ ,$$

where

$$\tilde{\alpha}\equiv\alpha-\frac{\nu\omega\bar{K}^{\omega}}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}$$

Thus, this formulation gives rise to an Euler equation of exactly the same form in (20) with

$$\gamma_k = -\left(\frac{\nu\omega\bar{K}^{\omega}}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}\right)$$

The effect of the liquidity cost is captured by the two composite parameters ν and ω. The former is positive, while the latter can be positive or negative. If $\omega < 0$, then the marginal cost of liquidity is decreasing in K_{it+1}. The opposite is true if $\omega > 0$ – the arguably more intuitive case, where the need to hold costly liquidity dampens incentives to respond to fundamentals.

Because this form of financial frictions manifest themselves exactly as a size-dependent distortion, γ_k, all results from Section 4.6.3 go through. This analysis indicates that while our estimates for correlated distortions might be picking up the effect of financial considerations, disentangling them from from other factors using production-side data alone may be difficult. Further progress on quantifying these frictions would likely require very detailed financial data at the firm-level.

B Data

As described in the text, our Chinese data are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The data span the period 1998-2009 and are built into a panel following quite closely the method outlined in Brandt et al. (2014). We measure the capital stock as the value of fixed assets and calculate investment as the change in the capital stock relative to the preceding period. We construct firm fundamentals, a_{it}, as the log of value-added less α multiplied by the log of the capital stock and (the log of) the marginal product of capital, $mrpk_{it}$ (up to an additive constant), as the log of value-added less the log of the capital stock. We compute value-added from revenues using a share of intermediates of 0.5 (our data does not include a direct measure of value-added in all years). We first difference the investment and fundamental series to compute investment growth and changes in fundamentals. To extract the firm-specific variation in our variables, we regress each on a year by time fixed-effect and work with the residual. Industries are defined at the 4-digit level. This eliminates the industry-wide component of each series common to all firms in an industry and time period (as well the aggregate component common across all firms) and leaves only the idiosyncratic variation. To estimate the parameters governing firm fundamentals, i.e., the persistence ρ and variance of the innovations σ_{μ}^2, we perform the autoregression implied by (5), again including industry by year controls. We eliminate duplicate observations (firms with multiple observations within a single year) and trim the 3% tails of each series. We additionally exclude observations with excessively high variability in investment (investment rates over 100%). Our final sample in China consists of 797,047 firm-year observations.

Our US data are from Compustat North America and also spans the period 1998-2009. We measure the capital stock using gross property, plant and equipment. We treat the data in exactly the same manner as just described for the set of Chinese firms. We additionally eliminate firms that are not incorporated in the US and/or do not report in US dollars. Our final sample in the US consists of 34,260 firm-year observations.

Table 8 reports a number of summary statistics from one year of our data, 2009: the number of firms (with available data on sales), the share of GDP they account for, and average sales and capital.

Table 8: Sample Statistics 2009

	No. of Firms	Share of GDP	Avg. Sales ($M)	Avg. Capital ($M)
China	303623	0.65	21.51	8.08
US	6177	0.45	2099.33	1811.35

For the analyses in Section 4.5, labor is measured as the number of employees in the US Compustat data and wage bill in the Chinese data. Expenditures on intermediate inputs are reported in the Chinese data. In the US, we construct a measure of intermediates following the method outlined in İmrohoroğlu and Tüzel (2014), i.e., as total expenses less labor expenses, where total expenses are calculated as sales less operating income (before depreciation and amortization, Compustat series OIBDP). From here we can calculate materials' share and the markup-adjusted revenue products of capital and labor. We isolate the firm-specific variation in these series following a similar procedure as described above, i.e., by extracting a full set of industry by time fixed effects and working with the residual. We trim the 1% tails of each series.

C Interaction between factors

In the main text (specifically, Table 3), we measured the contribution of each factor in isolation, i.e., setting all other forces to zero. The top panel of Table 9 reproduces those estimates (labeled 'In isolation') and compares them to the case where all the other factors are held fixed at their estimated levels (labeled 'Joint'). The table shows some evidence of interactions, but since adjustment and informational frictions are modest, the numbers are quite similar under both approaches.

Table 9: Interactions Between Factors - US

	Adj Costs	Uncertainty	Other Factors: Correlated	Other Factors: Transitory	Other Factors: Permanent
In isolation					
$\Delta\sigma^2_{mrpk}$	0.05	0.03	0.06	0.03	0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	10.8%	7.3%	14.4%	6.3%	64.7%
Joint					
$\Delta\sigma^2_{mrpk}$	0.04	0.03	0.08	0.00	0.29
$\frac{\Delta\sigma^2_{mrpk}}{\sigma^2_{mrpk}}$	8.0%	5.7%	17.4%	0.3%	64.7%

D Estimates for Other Countries/Firms

In this appendix, we apply our empirical methodology to two additional countries for which we have firm-level data - Colombia and Mexico - as well as to publicly traded firms in China.

The Colombian data come from the Annual Manufacturers Survey (AMS) and span the years 1982-1998. The AMS contains plant-level data and covers plants with more than 10 employees, or sales above a certain threshold (around $35,000 in 1998, the last year of the data). We use data on output and capital, which includes buildings, structures, machinery and equipment. The construction of these variables is described in detail in Eslava et al. (2004). Plants are classified into industries defined at a 4-digit level. The Mexican data are from the Annual Industrial Survey over the years 1984-1990, which covers plants of the 3200 largest manufacturing firms. They are also at the plant-level. We use data on output and capital, which includes machinery and equipment, the value of current construction, land, transportation equipment and other fixed capital assets. A detailed description is in Tybout and Westbrook (1995). Plants are again classified into industries defined at a 4-digit level. Data on publicly traded Chinese firms are from Compustat Global. Due to a lack of a sufficient time-series for most firms, we focus on single cross-section for 2015 (the moments use data going back to 2012). Similarly, due to the sparse representation of many industries, we focus on those with at least 20 firms. For all the datasets, we compute the target moments following the same methodology as outlined in the main text of the paper. Our final samples consist of 44,909 and 3,208 plant-year observations for Colombia and Mexico, respectively, and 1,055 firms in China.

Table 10 reports the moments and estimated parameter values for these sets of firms, as well as the share of *mrpk* dispersion arising from each factor and the effects on aggregate productivity. The results are quite similar to those for Chinese manufacturing firms in Table 3 in the main text. The contribution of adjustment costs and uncertainty to misallocation is rather limited, and that of uncorrelated transitory factors negligible - across these sets of firms, a large portion of misallocation stems from correlated and permanent firm-specific factors.

Table 10: Additional Countries/Firms

Moments	ρ	σ^2_μ	$\rho_{\iota,a_{-1}}$	$\rho_{\iota,\iota_{-1}}$	$\rho_{mrpk,a}$	σ^2_ι	σ^2_{mrpk}
Colombia	0.95	0.09	0.28	-0.35	0.61	0.07	0.98
Mexico	0.93	0.07	0.17	-0.39	0.69	0.02	0.79
China Compustat	0.96	0.04	0.30	-0.42	0.76	0.04	0.41
Parameters			ξ	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Colombia			0.54	0.05	-0.55	0.01	0.60
Mexico			0.13	0.04	-0.82	0.00	0.42
China Compustat			0.15	0.03	-0.69	0.00	0.18
$\Delta\sigma^2_{mpk}$							
Colombia			0.02	0.05	0.30	0.01	0.60
Mexico			0.00	0.04	0.36	0.00	0.42
China Compustat			0.00	0.03	0.22	0.00	0.18
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$							
Colombia			2.5%	5.6%	30.9%	0.7%	61.3%
Mexico			0.5%	4.9%	44.9%	0.0%	52.8%
China Compustat			0.8%	6.3%	54.0%	0.2%	43.7%
Δa							
Colombia			0.01	0.02	0.13	0.00	0.26
Mexico			0.00	0.02	0.16	0.00	0.18
China Compustat			0.00	0.02	0.19	0.00	0.16