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FRICTIONS OR DISTORTIONS?

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ABSTRACT

We study a model of investment in which both technological and informational frictions as well as institutional/policy distortions lead to capital misallocation, i.e., static marginal products are not equalized. We devise an empirical strategy to disentangle these forces using readily observable moments in firm-level data. Applying this methodology to manufacturing firms in China reveals that adjustment costs and uncertainty have significant aggregate consequences but account for only a modest share of the observed dispersion in the marginal product of capital. A substantial fraction of misallocation stems from firm-specific distortions, both productivity/size-dependent as well as permanent. For large US firms, adjustment costs are relatively more salient, though permanent firm-level factors remain important. These results are robust to the presence of liquidity/financial constraints.

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1 Introduction

A large and growing body of work has investigated the 'misallocation' of factors of production across firms - measured by the extent to which period-by-period marginal products are not equalized - and the resulting adverse effects on macroeconomic outcomes such as aggregate productivity and output. In addition to empirically documenting the presence of misallocation in firm-level data across a variety of countries, a number of recent studies examine the role of specific 'frictions' as potential sources of that misallocation. These include, for example, adjustment costs, imperfect information, financial frictions, as well as idiosyncratic firm-specific 'distortions' (e.g., due to economic policies or other institutional features), both correlated and uncorrelated with firm characteristics, e.g., size/productivity. A common methodological theme in this work has been a focus on one particular factor while abstracting from others - analyzing multiple forces in a single framework has proven challenging. However, this approach is potentially problematic – by examining a single factor in isolation, there is a risk of reaching biased conclusions of its severity and hence its contribution to misallocation.

In this paper, we outline a unified framework of firm investment to jointly analyze these factors. Our main contribution is an empirical strategy designed to precisely measure the effects of each using readily observable moments from widely available firm-level data. In particular, we augment a standard general equilibrium model of firm dynamics with a number of forces that contribute to *ex-post* misallocation, i.e., observed dispersion in marginal products.[1] In our setup, firms choose inputs facing (1) technological frictions, in the form of quadratic adjustment costs, (2) informational frictions, in the form of imperfect signals about their future fundamentals and (3) a generic class of idiosyncratic distortions *a la* Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), both correlated and uncorrelated with firm characteristics.[2] We use perturbation methods to solve the model. Apart from computational tractability, this approach also yields useful analytic expressions that allow us to make transparent the intuition underlying our empirical strategy.

To understand the difficulty in quantifying these factors, consider, as an example, the impact of convex adjustment costs. A common approach to gauge the severity of these costs is to examine the variability of firm-level investment. When adjustment costs are the only force present, this moment has an intuitive, one-to-one mapping with their magnitude - the lower is investment volatility, relative to fundamentals, the greater the adjustment cost. However,

[1]Throughout the paper we use the term misallocation to refer broadly to deviations from static marginal product equalization, whether these stem from true distortions or technological factors such as adjustment costs or imperfect information.

[2]Our baseline analysis abstracts from financial frictions, a choice motivated partly by the fact that previous studies have found only a limited role for these types of factors in leading to misallocation, for example, Midrigan and Xu (2014). We analyze the robustness of our results to the presence of liquidity constraints in Section 4.7.

suppose that there are other firm-specific factors that influence investment decisions (e.g., idiosyncratic distortions). Now, depending on the correlation of those factors with firm-level fundamentals (either demand- or supply-side) they can serve to either increase or dampen investment volatility. As a result, using this particular moment to make inferences regarding the extent of adjustment costs leads to a biased estimate of their severity. The empirically relevant case turns out to be one where these other factors are negatively correlated with fundamentals (i.e., they tend to disincentivize investment by firms with better fundamentals), implying a positive bias, that is, a model with only adjustment costs will overstate their importance. As a second example, consider the effects of firm-level uncertainty. If fundamentals are revealed only slowly, imperfect information reduces the contemporaneous correlation between investment and fundamentals. However, a low correlation could also be the result of factors orthogonal to fundamentals that enter the firm's investment problem (e.g., uncorrelated transitory distortions). Again, using this moment to measure uncertainty runs the risk of overstatement by incorrectly attributing these distortions to lower quality information on the part of firms.

We propose a strategy to overcome these challenges using only firm-level production data, namely, elements from the covariance matrix of firm-level capital and fundamentals (which we can measure using data on revenues and inputs along with the form of production function). The key insight behind our approach is that while each moment is a complicated function of multiple factors, making any single moment insufficient to identify a particular one, combining the information in a larger set of moments can be extremely helpful in disentangling them. In fact, we show that allowing these forces to act in tandem is essential in order to reconcile a broad set of moments in firm-level investment dynamics.

We use a special case of our model – when firm-level fundamentals follow a random walk – to formalize this intuition. In this case, we are able to derive analytic relationships between moments and parameters, enabling us to prove that – when examined jointly – a set of four carefully chosen moments from the production data uniquely identify the underlying structural parameters that determine the severity of each factor. Specifically, the variance of investment, its autocorrelation as well as its correlation with past fundamentals, and the covariance of the marginal product of capital (mpk) with fundamentals combine to identify the severity of adjustment costs, the extent of uncertainty and the magnitudes of the correlated and uncorrelated transitory components of distortions. We also exploit the tractability of this case to illustrate how this strategy works - when the alternative approach of examining a single factor in isolation fails – and to highlight which combinations of moments are most informative in disentangling the role of particular factors. To take the example from earlier, consider the challenge of disentangling adjustment costs from distortions that are negatively correlated with fundamentals. Both dampen the firm's incentives to respond to fundamentals and so depress the volatility of investment. However, they have opposing effects on the autocorrelation of investment - convex adjustment costs create incentives to smooth investment over time and so tend to make investment more serially correlated. A distortion that directly reduces the response to fundamentals, on the other hand, increases the relative importance of transitory factors in investment, reducing the autocorrelation. Holding all else fixed, these two moments allow us to separate the two forces. Similar arguments can be developed for the remaining factors as well. In our quantitative work, where we depart from the polar random walk case, we follow an estimation strategy guided by these findings and demonstrate numerically that the same logic carries through.

We apply our methodology to data on manufacturing firms in China from the Annual Survey of Industrial Production over the period 1998-2009. These data represent a census of all state and non-state manufacturing firms above a certain size threshold. Our results show that adjustment and informational frictions account for a relatively modest share of misallocation among Chinese firms, composing about 1% and 9% of overall dispersion in the marginal product of capital, respectively. Losses in aggregate total factor productivity (TFP) from these two sources (relative to the undistorted first-best) are 0.4% and 3%, respectively (the corresponding figures for steady state output are 1% and 4%). A substantial portion of observed misallocation in China is then due to firm-level idiosyncratic distortions, both those that are correlated with fundamentals (and therefore, vary over time with the fortunes of the firm) and ones that are essentially permanent. These lead to TFP losses of 12% and 19% and output losses of 17% and 26% respectively.[3]

We also apply our methodology to data on publicly traded US firms over the same time period. Although the two sets of firms are not directly comparable due to their differing coverage (the Chinese data are much more comprehensive), deriving results for firms in a developed economy such as the US gives us a useful benchmark to put our results for China in context. As we would expect, the overall degree of misallocation is considerably smaller for the US firms. Adjustment costs account for a larger share (about 11%) of observed *mpk* dispersion than in China, though their overall magnitude remains modest, especially relative to earlier estimates in the literature. Uncertainty and correlated distortions play a smaller role than among Chinese firms, reducing aggregate TFP by 1% and 2% respectively. However, firm-specific fixed distortions, although considerably smaller in absolute magnitude than in China, also seem to be quite significant as a share of total *mpk* dispersion, even among large firms in the US. Our estimates suggest eliminating them could increase TFP by as much as 11%. In sum, even for the US, technological and informational frictions alone cannot account for the

[3]Our analysis allows for distortions that are transitory and uncorrelated with firm characteristics. However, our estimation finds them to be negligible.

majority of observed marginal product dispersion.[4]

We then turn to a number of extensions. First, we show that our results are robust to allowing for country-specific production parameters (specifically, a higher share of capital in China). Next, we analyze a variant of our framework in which the labor input choice is subject to the same frictions/distortions that affect investment. In contrast, our baseline analysis assumes that only capital investment decisions are made subject to frictions, while labor is chosen period-by-period under full information. We find that allowing for frictional labor choice leads to broadly similar conclusions on the relative importance of various forces – adjustment costs and uncertainty account for about 13% and 11% of *mpk* dispersion, respectively, again suggesting an important role for correlated and permanent distortions. However, since both factors of production are now affected by each of the forces, their impact on aggregate TFP is substantially higher - productivity losses from adjustment costs and uncertainty are each about 30% and output losses from each about 40%. The corresponding values for correlated and permanent distortions are much higher – for example, the TFP gap between status quo and first best is as high as about 115% and 75%, respectively. We interpret this as an upper bound on the aggregate impact of these factors, with reality likely falling somewhere in between this and the baseline version with frictionless labor, which would tend to understate the aggregate implications. Despite the large differences in the aggregate effects across the two scenarios, however, it is reassuring that our main results on the composition of capital misallocation are not particularly sensitive to our assumptions about the nature of the labor choice.

Finally, we extend our baseline framework to introduce a role for financial considerations. In particular, we assume that firms must hold a certain amount of liquid assets in order to be able to undertake capital investment.[5] This allows us to capture the essential features of financial constraints/frictions, which we abstract from entirely in our baseline analysis. Our goal here is to investigate whether this omission could be a potential source of bias in our estimates of the other factors. Parameterizing the liquidity-related components of the firm's optimization problem using additional production-side moments from the Chinese data, we find that our results regarding the sources of misallocation are essentially unchanged. Moreover, the contribution of the financial friction to misallocation is also relatively modest - were that the only friction facing firms, it would account for less than 10% of total *mpk* dispersion.

The paper is organized as follows. Section 2 describes our model of production and frictional investment. Section 3 spells out our approach to identifying these frictions using the analytically

[4]We also report results using data for Colombian and Mexican firms, albeit spanning an earlier time period. The results regarding the role of the various factors in driving misallocation are quite similar to those in China.

[5]Formally, we model this as a continuous cost, i.e., firms are penalized as their liquid assets become small relative to their productive capital. By using a smooth cost function instead of a hard constraint, we are able to retain the tractability of our baseline framework and continue to use perturbation methods.

tractable random walk case, while Section 4 details our numerical analysis and presents our quantitative results. We summarize our findings and discuss directions for future research in Section 5. Details of derivations and data work are provided in the Appendix.

Related literature. Our paper relates to several existing branches of literature. We bear a direct connection to recent work focusing on measuring and quantifying the effects of resource misallocation, seminal examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), and a recent survey of which is contained in Hopenhayn (2014). Our explicit modeling of specific frictions as sources of misallocation relates our paper to Asker et al. (2014) and David et al. (2015) (henceforth, DHV) who study the role of capital adjustment costs and information frictions, respectively, as well as Buera et al. (2011), Midrigan and Xu (2014), Moll (2014) and Gopinath et al. (2015), who analyze financial frictions. Our modeling of firm-specific distortions follows the approach taken by the misallocation literature, for example, Restuccia and Rogerson (2008), Guner et al. (2008), Bartelsman et al. (2013), Buera et al. (2013), Buera and Fattal-Jaef (2016) and Hsieh and Klenow (2014). These papers emphasize the need to distinguish between correlated distortions that are correlated with firm size/productivity and ones that are orthogonal to fundamentals. We contribute to this literature by providing robust estimates of the magnitude and correlation structure of these distortions.

There is a large body of work examining firm dynamics in the presence of adjustment costs, examples of which include Cooper and Haltiwanger (2006) and Bloom (2009). Our analysis shows that accounting for additional frictions/distortions is essential in order to reconcile a broader set of micro-moments in firm investment dynamics and sheds new light on the the severity of adjustment frictions.

Our investigation of imperfect information relates to recent work on measuring firm-level uncertainty and quantifying its implications – for example, Bloom (2009), Bachmann and Elstner (2015) and Jurado et al. (2015). Our strategy for inferring the extent of uncertainty at the firm-level is related to our earlier work in DHV, where we used a combination of production and stock market data to identify information frictions. By adapting that approach to use only production-side data, we make it more widely applicable (DHV focus only on publicly traded firms). We also differ from DHV in our explicit modeling and measurement of other factors influencing firm investment decisions.

2 The Model

We consider a discrete time, infinite-horizon environment. The economy is populated by a representative household and a continuum of firms of fixed measure one that produce a single homogeneous good. The household inelastically supplies a fixed quantity of labor N and has preferences over consumption of the single good. The household side of the economy is deliberately kept simple as it plays a limited role in our study. Throughout the analysis, we focus on a stationary equilibrium in which all aggregate variables remain constant.

Production. Firms are competitive and produce output using capital and labor according to a decreasing returns to scale production function

$$Y_{it} = \tilde{A}_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2}, \quad \alpha_1 + \alpha_2 < 1 \tag{1}$$

where i indexes firms and $\tilde{A}_{it}$ is an idiosyncratic level of firm productivity. This is the only source of fundamental uncertainty in the model (i.e., we abstract from aggregate risk).

The decreasing returns to scale assumption ensures a well defined distribution of firm sizes. As is well-known in the literature, this formulation is equivalent to an alternative setup in which firms produce differentiated products and face downward sloping demand curves due to decreasing marginal utility of consumption. In that environment, $\tilde{A}_{it}$ can also be interpreted as an idiosyncratic demand shifter. For the remainder of the analysis, we simply refer to $\tilde{A}_{it}$ as a firm-specific fundamental.

Input choices. In our baseline analysis, we assume that firms hire labor period-by-period in a spot market at a competitive wage W under full information (since we focus on a stationary equilibrium, we suppress the time subscript on all aggregate variables).[6] At the end of each period, firms choose investment in new capital, which becomes available for production in the following period. Investment is subject to quadratic adjustment costs, so that the total cost of new investment is given by

$$\Phi\left(K_{it+1}, K_{it}\right) = K_{it+1} - (1-\delta) K_{it} + \frac{\xi}{2}\left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^2 K_{it} \tag{2}$$

where the term $K_{it+1} - (1-\delta) K_{it} = I_{it}$ is gross investment, δ the rate of depreciation and ξ parameterizes the severity of the adjustment cost.

Investment decisions are likely to be affected by a number of 'non-fundamental' factors (i.e., other than productivity/demand and the level of installed capital). These could originate, for example, from distortionary government policies, e.g., taxes, size restrictions or regulations, or other features of the institutional environment that influence firm decisions. To capture these

[6] In Section 4.6, we analyze an alternative setup in which labor is also subject to the same adjustment and information frictions as capital. We show that assumption leads to an optimization problem with the same structure as our baseline version with suitably re-defined fundamentals and curvature.

factors, which we loosely call distortions, we follow Hsieh and Klenow (2009) and introduce a class of idiosyncratic 'wedges' that appear in the firm's optimization problem as proportional taxes on the cost of capital, denoted T^K_{it+1}.[7]

The firm's problem in a stationary equilibrium can be represented in recursive form as

$$\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right)=\max_{N_{it},K_{it+1}}\ \mathbb{E}_{it}\left[\tilde{A}_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2}-WN_{it}-T^K_{it+1}\Phi\left(K_{it+1},K_{it}\right)+\beta\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right]$$

where $\mathbb{E}_{it}\left[\cdot\right]$ denotes the firm's expectations conditional on the information set of the firm at the time of making its period t investment choice, denoted $\mathcal{I}_{it}$. We describe this set explicitly below.

After maximizing over the choice of N_{it}, this becomes

$$\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right)=\max_{K_{it+1}}\ \mathbb{E}_{it}\left[GA_{it}K_{it}^{\alpha}-T^K_{it+1}\Phi\left(K_{it+1},K_{it}\right)+\beta\mathcal{V}\left(K_{it+1},\mathcal{I}_{it+1}\right)\right] \tag{3}$$

where $G\equiv\left(1-\alpha_2\right)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}$, $A_{it}\equiv\tilde{A}_{it}^{\frac{1}{1-\alpha_2}}$ and $\alpha\equiv\frac{\alpha_1}{1-\alpha_2}$ is the curvature of operating profits (revenues net of wages).

Equilibrium. We can now define a *stationary equilibrium* in this economy as (i) a set of value and policy functions for the firm, $\mathcal{V}\left(K_{it},\mathcal{I}_{it}\right)$, $N_{it}\left(K_{it},\mathcal{I}_{it}\right)$ and $K_{it+1}\left(K_{it},\mathcal{I}_{it}\right)$, (ii) a wage W and (iii) a joint distribution over $\left(K_{it},\mathcal{I}_{it}\right)$ such that (a) taking as given wages and the law of motion for $\mathcal{I}_{it}$, the value and policy functions solve the firm's optimization problem, (b) the labor market clears and (c) the joint distribution remains constant through time.

Characterization. We solve the model using perturbation methods. In particular, we log-linearize the firm's optimality conditions and laws of motion around the undistorted non-stochastic steady state, where $A_{it}=\bar{A}$ and $T_{it}=1$. Appendix A.1.1 derives the following log-linearized Euler equation:[8]

$$k_{it+1}\left((1+\beta)\hat{\xi}+1-\alpha\right)=\mathbb{E}_{it}\left[a_{it+1}+\tau_{it+1}\right]+\beta\hat{\xi}\mathbb{E}_{it}\left[k_{it+2}\right]+\hat{\xi}k_{it} \tag{4}$$

where $\hat{\xi}$ is a composite parameter that indexes the degree of adjustment costs and τ_{it+1} is a composite distortion that summarizes the effect of T^K on the firm's investment decision.

[7]Note that this class of distortions do not distort the labor choice, i.e., they do not lead to any dispersion in the marginal product of labor. In Appendix A.1.3, we show that adding distortions to the labor choice would have no effect on our analysis and results. In particular, our identification strategy and conclusions about the sources of mpk dispersion are unaffected by these additional distortions. Therefore, in light of our focus on investment choices, we abstract from them in our analysis.

[8]We use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $x_{it}=\log X_{it}$.

Stochastic processes. We assume that A_{it} follows an AR(1) process in logs with normally distributed i.i.d. innovations σ_μ^2, i.e.,

$$a_{it} = \rho a_{it-1} + \mu_{it}, \quad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \tag{5}$$

where the parameter ρ is the persistence of firm-level fundamentals.

The distortion τ_{it} is assumed to be jointly normal with a_{it}, with covariance matrix Σ. We place no restrictions on Σ, i.e., we allow distortions to covary with contemporaneous fundamentals in an arbitrary way. Then, without loss of generality, τ_{it} has the following representation:[9]

$$\tau_{it} = \gamma a_{it} + \varepsilon_{it} + \chi_i, \quad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \quad \chi_i \sim \mathcal{N}\left(0, \sigma_\chi^2\right) \tag{6}$$

where γ indexes the extent to which τ_{it} is correlated with fundamentals and ε_{it} and χ_i are uncorrelated with a_{it}. If $\gamma < 0$, the distortion discourages (encourages) investment by firms with stronger (weaker) fundamentals - arguably, the empirically relevant case. The opposite is true if $\gamma > 0$. The remaining components capture distortionary factors that are orthogonal to fundamentals. The first, ε_{it}, is i.i.d. over time while the second, χ_i, is a permanent firm-specific component.

Information. Next, we spell out $\mathcal{I}_{it}$, the information set of the firm at the time of choosing period t investment, i.e., K_{it+1}. This includes the entire history of its fundamental shock realizations through period t, i.e., $\{a_{it-s}\}_{s=0}^{\infty}$. Given the AR(1) structure of uncertainty, this history can be summarized by the most recent observation, namely a_{it}. The firm also observes a noisy signal of the following period's innovation in fundamentals

$$s_{it+1} = \mu_{it+1} + e_{it+1}, \qquad e_{it+1} \sim \mathcal{N}\left(0, \sigma_e^2\right)$$

where e_{it+1} is an i.i.d. mean-zero and normally distributed noise term. This is in essence an idiosyncratic 'news shock,' since it contains information about future fundamentals. Finally, firms also perfectly observe the uncorrelated transitory component of distortions ε_{it+1} (as well as the fixed component χ_i) at the time of choosing period t investment. They do not see the correlated component but are aware of the structure, that is, they know its covariance with the fundamental a_{it}.

Thus, the firm's information set is given by $\mathcal{I}_{it} = (a_{it}, s_{it+1}, \varepsilon_{it+1}, \chi_i)$. Direct application of

[9] See DHV for details.

Bayes' rule yields the conditional expectation of the fundamental a_{it+1}:

$$a_{it+1}|\mathcal{I}_{it} \sim N\left(\mathbb{E}_{it}\left[a_{it+1}\right], \mathbb{V}\right) \quad \text{where} \tag{7}$$

$$\mathbb{E}_{it}\left[a_{it+1}\right] = \rho a_{it} + \frac{\mathbb{V}}{\sigma_e^2} s_{it+1}, \quad \mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2}\right)^{-1} \tag{8}$$

There is a one-to-one mapping between the posterior variance $\mathbb{V}$ and the noisiness of the signal, σ_e^2 (given the volatility of fundamentals, σ_μ^2). In the absence of any learning (or 'news'), i.e., when σ_e^2 approches infinity, $\mathbb{V} = \sigma_\mu^2$, that is, all uncertainty regarding the realization of the fundamental shock μ_{it+1} remains unresolved at the time of investment. In this case, we have a standard one period time-to-build assumption with $\mathbb{E}_{it}\left[a_{it+1}\right] = \rho a_{it}$. At the other extreme, i.e., when σ_e^2 approaches zero, $\mathbb{V} = 0$ and the firm becomes perfectly informed about μ_{it+1} so that $\mathbb{E}_{it}\left[a_{it+1}\right] = a_{it+1}$. It turns out to be more convenient to work directly with the posterior variance $\mathbb{V}$ and so, for the remainder of the analysis, we will use that as our measure of uncertainty.

Optimal investment. We use a guess and verify approach to derive the firm's optimal investment policy function:[10]

$$k_{it+1} = \psi_1 k_{it} + \psi_2 \left(1+\gamma\right) \mathbb{E}_{it}\left[a_{it+1}\right] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i \tag{9}$$

where the coefficients are given by

$$\hat{\xi}\left(\beta\psi_1^2 + 1\right) = \psi_1\Big((1+\beta)\hat{\xi} + 1 - \alpha\Big) \tag{10}$$

$$\psi_2 = \frac{\psi_1}{\hat{\xi}\left(1-\beta\rho\psi_1\right)}, \qquad \psi_3 = \frac{\psi_1}{\hat{\xi}}, \qquad \psi_4 = \frac{1-\psi_1}{1-\alpha}.$$

The coefficients ψ_1-ψ_4 depend only on production parameters, including the adjustment cost, and are independent of assumptions about information and distortions. The coefficient ψ_1 is increasing and ψ_2-ψ_4 decreasing in the severity of adjustment costs, $\hat{\xi}$. If adjustment costs are 0 (i.e., $\hat{\xi} = 0$) , $\psi_1 = 0$ and $\psi_2 = \psi_3 = \psi_4 = \frac{1}{1-\alpha}$. At the other extreme, as $\hat{\xi}$ tends to infinity, ψ_1 approaches one and ψ_2-ψ_4 go to zero. Intuitively, as adjustment costs become large, the firm's choice of capital becomes more autocorrelated and less responsive to fundamentals and distortions.

Aggregation. We now turn to the aggregate economy, and in particular, measures of aggregate output and TFP. As we derive in detail in Appendix A.1.2, aggregate output can be

[10] See Appendix A.1.1 for details.

expressed as

$$\log Y \equiv y = a + \alpha_1 k + \alpha_2 n$$

where k and n represent the (logs of the) aggregate stock of capital and labor inputs, respectively. Aggregate TFP, denoted a, is given by

$$a = a^* - \frac{1}{2}\frac{\alpha_1(1-\alpha_2)}{1-\alpha_1-\alpha_2}\sigma^2_{mpk} \qquad \frac{da}{d\sigma^2_{mpk}} = -\frac{1}{2}\frac{\alpha_1(1-\alpha_2)}{1-\alpha_1-\alpha_2} \tag{11}$$

where a^* is the first-best level of TFP in the absence of all frictions and σ^2_{mpk} is the cross-sectional dispersion in (the log of) the marginal product of capital ($mpk_{it} = y_{it} - k_{it}$). Thus, aggregate productivity monotonically decreases in the extent of capital misallocation, summarized in the log-normal world by σ^2_{mpk}. In the absence of all frictions - adjustment and informational - as well as distortions, the mpk is equated is across firms and $a = a^*$. As frictions or distortions become more severe, in the sense of increasing σ^2_{mpk}, aggregate productivity falls. The effect of σ^2_{mpk} on aggregate TFP depends on the relative shares of capital and labor in production - the higher is capital's share α_1 (and so the lower labor's share α_2), the higher the cost of a given degree of misallocation in capital. Fixing the relative shares of capital and labor, the cost of misallocation is increasing in the overall returns to scale (i.e., as we move closer to constant returns). Finally, holding the aggregate factor stocks fixed, the effect on aggregate productivity a is also the effect on aggregate output y. However, the misallocation induced by these forces also reduces incentives for capital accumulation and so the aggregate stock of capital in the stationary equilibrium decreases with their severity. Incorporating this additional effect, we obtain the standard result that

$$\frac{dy}{d\sigma^2_{mpk}} = \frac{1}{1-\alpha_1}\frac{da}{d\sigma^2_{mpk}} \tag{12}$$

i.e., the output effects equal the TFP effects scaled up by a multiplier that is increasing in capital's share in production.

Equations (11) and (12) point to a natural way to quantify the aggregate consequences of particular factors - i.e., to measure the adverse effects on economic aggregates stemming from their contribution to marginal product dispersion. We will employ this strategy in our numerical analysis to provide a quantitative decomposition of aggregate TFP and output losses arising from various sources of misallocation.

3 Identification

In this section, we develop a strategy to identify the key parameters of the model using readily observable moments that are now widely available in firm-level data - specifically, revenues and investment - and so can be applied to a broad set of firms across a wide variety of datasets. The parameters of interest are the costs of capital adjustment, $\hat{\xi}$, the quality of firm-level information (summarized by $\mathbb{V}$), and the severity of distortions, parameterized by γ, σ_ε^2 and σ_χ^2. We show that, in general, these moments are each complicated functions of all the parameters, so that any attempt to identify a particular parameter – i.e., quantify a specific factor - without controlling for the others can lead to estimates that are potentially biased. This finding argues for caution in interpreting the results from previous work investigating the role of each of these forces in isolation.

Our methodology uses a set of carefully chosen elements from the covariance matrix of firm-level capital and fundamentals (the latter measured using the production function and data on revenues and inputs). Here, we outline that strategy using a tractable special case when firm-level shocks follow a random walk, i.e., $\rho = 1$. In this case, we are able to derive closed form expressions that allow us to analytically demonstrate the key intuition underlying our approach and choice of moments. When we return to our general model in the following section, we will demonstrate numerically that this intuition extends to the case with $\rho < 1$.

Because the environment with $\rho = 1$ is not stationary in levels, for the purposes of our analytical work, we work with moments computed in changes. This does mean, however, that we cannot identify the size of the fixed distortion σ_χ^2.[11] Here, we focus on the four remaining parameters, namely $\hat{\xi}$, γ, $\mathbb{V}$ and σ_ε^2, which we identify using the following four moments (1) the autocorrelation of investment, denoted $\rho_{k,k_{-1}}$, (2) the variance of investment, σ_k^2, (3) the correlation of period t investment with the innovations in fundamentals in period $t-1$, denoted $\rho_{k,a_{-1}}$ and (4) the comovement of the change in the marginal product of capital with the change in fundamentals, measured by the coefficient from a regression of Δmpk_{it} on Δa_{it}, which we denote $\lambda_{mpk,a}$. All of these moments are drawn from the covariance matrix of capital and fundamentals (which in turn can be directly computed using the production function and data on revenues/value-added). We derive closed-form expressions for each in Appendix A.4 and prove the following the result:

Proposition 1. *The parameters $\hat{\xi}$, γ, $\mathbb{V}$ and σ_ε^2 are uniquely identified by the moments $\rho_{k,k_{-1}}$, σ_k^2, $\rho_{k,a_{-1}}$ and $\lambda_{mpk,a}$.*

3.1 Intuition

The proof of Proposition 1 involves straightforward, if somewhat tedious, algebra. From the perspective of gaining intuition, however, it turns out to be more instructive to examine the

[11]In the stationary version of our model with $\rho < 1$ that we work with in our numerical analysis in the following section, we use σ_{mpk}^2, a moment computed using levels of capital and fundamentals, to pin down σ_χ^2.

factors pairwise and analyze their effects on the most relevant moments. This exercise helps to highlight the moments that are most useful in disentangling particular forces. To be clear, the goal here is simply to provide some intuition behind the identification strategy and get a sense of how the moments combine to provide information about the parameters – as mentioned earlier, the identification result in Proposition 1 makes use of the full mapping from moments to parameters, which is more complicated than this pairwise exercise.

Adjustment costs and correlated distortions. We begin by comparing adjustment costs, parameterized by $\hat{\xi}$, to the effect of correlated distortions, γ. To do so, we use the variance and autocorrelation of investment, σ_k^2 and $\rho_{k,k_{-1}}$. Both of these moments are commonly used to estimate quadratic adjustment costs in the literature - for example, Asker et al. (2014) target the variance of investment, among others, and Cooper and Haltiwanger (2006) the autocorrelation. These moments take the form

$$\sigma_k^2 = \left(\frac{\psi_2^2}{1-\psi_1^2}\right)(1+\gamma)^2\,\sigma_\mu^2 + \frac{2\psi_3^2}{1+\psi_1}\sigma_\varepsilon^2 \tag{13}$$

$$\rho_{k,k_{-1}} = \psi_1 - \psi_3^2\frac{\sigma_\varepsilon^2}{\sigma_k^2}\ , \tag{14}$$

where the ψ's are defined in equation (10); ψ_1 is increasing and ψ_2 and ψ_3 decreasing in the size of adjustment costs. To understand how these two moments are affected by $\hat{\xi}$ and γ, suppose for the moment that there are no transitory distortions, i.e., $\sigma_\varepsilon^2 = 0$. Then, from equation (13), we see that low investment volatility can be due either to more severe adjustment costs, i.e., higher $\hat{\xi}$ (which decreases $\frac{\psi_2^2}{1-\psi_1^2}$) or a stronger negative correlation of distortions with fundamentals, i.e., a more negative γ. Therefore, a strategy that uses the observed σ_k^2 to identify adjustment costs while abstracting from correlated distortions would overstate those costs. In contrast, with $\sigma_\varepsilon^2 = 0$, equation (14) implies that the autocorrelation of investment is equal to ψ_1 and so directly identifies the adjustment cost parameter $\hat{\xi}$. To see this differently, a model with only adjustment costs that targets σ_k^2 would have an upward bias in the estimate of $\hat{\xi}$, to the extent that there are correlated distortions, and therefore overstate $\rho_{k,k_{-1}}$. In conjunction, however, these two moments uniquely identify $\hat{\xi}$ and γ in the absence of transitory distortions.

More generally, with both correlated and transitory distortions, this is no longer the case, since both moments are clearly sensitive to the latter. However, they are still useful in disentangling $\hat{\xi}$ and γ. In particular, whereas the two factors have a similar dampening effect on the variance of investment, they have opposing effects on the autocorrelation - the serial correlation of investment $\rho_{k,k_{-1}}$ increases with higher adjustment costs but decreases with a more negative gamma. Intuitively, a more negative γ reduces σ_k^2, increasing the relative weight of transitory distortions (which are serially uncorrelated). Thus, while these two moments are no longer sufficient to pin down $\hat{\xi}$ and γ, they are clearly informative for that purpose.[12]

Uncertainty and correlated distortions. Next, consider the correlation of investment with past innovations in fundamentals, $\rho_{k,a_{-1}}$ and the regression coefficient $\lambda_{mpk,a}$. The expressions for these moments are:

$$\rho_{k,a_{-1}} = \left[\frac{\mathbb{V}}{\sigma_\mu^2}\left(1-\psi_1\right)+\psi_1\right]\frac{\sigma_\mu\psi_2\left(1+\gamma\right)}{\sigma_k} \tag{15}$$

$$\lambda_{mpk,a} = 1-\left(1-\alpha\right)\left(1+\gamma\right)\psi_2\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right) . \tag{16}$$

As before, the two moments are complicated functions of all the forces in the model, but they are particularly useful in disentangling uncertainty, $\mathbb{V}$, from correlated distortions, γ. To see this most clearly, assume for the moment that there are no other frictions, i.e., $\hat{\xi} = \sigma_\varepsilon^2 = 0$. This implies $\psi_1 = 0$ and $\psi_2 = \frac{1}{1-\alpha}$, so that the expressions reduce to

$$\rho_{k,a_{-1}} = \frac{\mathbb{V}}{\sigma_\mu^2} \qquad \lambda_{mpk,a} = 1-\left(1+\gamma\right)\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}\right) . \tag{17}$$

Thus, in the absence of adjustment costs and transitory distortions, the correlation of current investment with past shocks directly identifies the extent of uncertainty, independent of the degree of correlated distortions. The intuition is straightforward - high uncertainty implies that the firm will respond to fundamental shocks with a lag, leading to a higher $\rho_{k,a_{-1}}$. The invariance to correlated distortions comes from the fact that the latter simply scale the response of investment to fundamentals, leaving the correlation unchanged. This strategy - and the associated robustness properties - bears a close relationship with the approach in DHV. They show how one can draw robust conclusions about firm-level uncertainty without observing the firm's information set in its entirety. By adapting it to production-side data (DHV make use of stock prices and focus only on publicly traded firms), we make it more direct and widely applicable.[13] Given $\frac{\mathbb{V}}{\sigma_\mu^2}$, the moment $\lambda_{mpk,a}$ pins down γ. This is also intuitive - both uncertainty and correlated distortions dampen the response of investment to fundamentals. This results in under (over)-investment by firms with high (low) a, inducing a more positive covariance between mpk and a.

[12] Taken together, investment volatility could give either an over- or under-estimate of adjustment costs in the present of both correlated and uncorrelated transitory distortions, since it is decreasing in γ and increasing in σ_ε^2. The autocorrelation would tend towards an underestimate since it is decreasing in both types of distortions.

[13] The response of current actions to past shocks has been used in a similar fashion in the sticky price literature to identify information frictions - see, for example, Klenow and Willis (2007).

More generally, with all the factors operational, this logic is still present, but adjustment costs and/or transitory distortions confound these simple mappings. Intuitively, adjustment costs are an additional source of positive correlation between current investment and past shocks while uncorrelated transitory distortions are a source of orthogonal variation in investment, increasing σ_k^2 and driving down this correlation. As a result, mapping $\rho_{k,a_{-1}}$ to $\frac{\mathbb{V}}{\sigma_\mu^2}$ without controlling for adjustment costs (uncorrelated transitory distortions) runs the risk of overstating (understating) the true degree of uncertainty. However, for fixed $\hat{\xi}$ and σ_ε^2, this correlation is still monotonically increasing in $\mathbb{V}$ and decreasing in γ (which dampens the firm's incentives to respond to fundamentals, irrespective of timing).[14] In contrast, uncertainty and correlated distortions have similar effects in increasing $\lambda_{mpk,a}$. Taken together then, these moments are particularly informative in distinguishing $\mathbb{V}$ and γ.

Transitory and correlated distortions. To disentangle correlated from uncorrelated transitory distortions, consider $\lambda_{mpk,a}$ and $\rho_{k,k_{-1}}$. We have seen that the former is increasing in the severity of correlated distortions and is independent of transitory ones and that the latter is decreasing in both types of distortions - a more negative γ dampens the response to the serially correlated fundamental, while higher σ_ε^2 increases the importance of the transitory i.i.d. component. For given values of $\hat{\xi}$ and $\mathbb{V}$, the two moments can be used to pin down γ and σ_ε^2.[15]

Uncertainty and adjustment costs. Finally, what distinguishes uncertainty from adjustment costs? As shown above, an increase in the severity of either of these factors can generate sluggishness in the response of actions to fundamentals, i.e., raise the correlation of investment with past fundamental shocks $\rho_{k,a_{-1}}$, so that examining this moment is not by itself sufficient. However, the autocorrelation of investment $\rho_{k,k_{-1}}$ is independent of uncertainty and determined only by adjustment costs and distortions. Thus, for a given level of distortions, the autocorrelation of investment can be used to infer the severity of adjustment frictions. This, in combination with the correlation of actions with lagged shocks, allows us to pin down the extent of uncertainty.

[14] The influence of adjustment costs on $\rho_{k,a_{-1}}$ is not so simple. Because the first term in (15) is increasing in $\hat{\xi}$ and the second term decreasing, the correlation is not monotonic in the size of these costs. When adjustment costs are zero, $\rho_{k,a_{-1}} = \frac{\mathbb{V}}{\sigma_\mu^2}$. When adjustment costs are sufficiently, high, $\psi_1 = 1$ and $\rho_{k,a_{-1}} = 0$. The correlation as a function of the adjustment cost takes the form of an inverted-U between these two endpoints. The empirically relevant region turns out to be the upward sloping part, where using $\rho_{k,a_{-1}}$ to identify $\mathbb{V}$ while abstracting from adjustment costs overstates $\mathbb{V}$.

[15] Considering $\lambda_{mpka,a}$, Bartelsman et al. (2013) point out the usefulness of a highly related moment, the covariance of mpk and a to quantify correlated distortions (and further to disentangle them from uncorrelated ones). Buera and Fattal-Jaef (2016) parameterize the strength of correlated distortions directly to match the regression coefficient.

4 Quantitative Analysis

The analytical results in the previous section showed a tight relationship between the moments $\left(\rho_{k,a_{-1}}, \rho_{k,k_{-1}}, \sigma_k^2, \lambda_{mpk,a}\right)$ and the parameters $\left(\mathbb{V}, \hat{\xi}, \sigma_\varepsilon^2, \gamma\right)$ for the special case of $\rho = 1$. In this section, we use the insights gained from those results to develop a numerical strategy for the case where fundamentals follow a general autoregressive process, which we apply to data on Chinese manufacturing firms. This allows us to provide quantitive measures of the severity of the various forces in our model, the degree of resulting misallocation, and the impact on aggregate outcomes. We also study extensions where the firm's labor choice is subject to the same forces as investment and where the firm also faces financial frictions in the form of liquidity constraints. For purposes of comparison, we also provide results for publicly traded firms in the US and later for two additional countries, Colombia and Mexico.

4.1 Parameterization

We begin by assigning values to the more standard preference and production parameters of our model. We assume a period length of one year (our data comes at an annual frequency) and accordingly set the discount factor $\beta = 0.95$.[16] We assume a common degree of decreasing returns in production across countries, $\alpha_1 + \alpha_2 = 0.83$, quite close to the standard value of 0.85. In an environment with differentiated goods, monopolistic competition and constant returns to scale in production, our choice corresponds to an elasticity of substitution between goods of 6, roughly in the the middle of the commonly used range.[17] Assuming that capital's share of payments to factors of production is one-third and labor's share two-thirds gives values of $\alpha_1 = 0.28$ and $\alpha_2 = 0.55$. These translate into an $\alpha = 0.62$ in our baseline analysis.[18]

Next, we turn to the parameters governing the process on firm fundamentals a_{it}: the persistence ρ and the variance of the innovations σ_μ^2. We can directly compute the the fundamental for each firm (up to an additive constant) as $va_{it} - \alpha k_{it}$ where va_{it} denotes the log of value-added. We then estimate the parameters of the fundamental process by performing the autoregression implied by equation (5), additionally controlling for year by industry fixed effects to isolate the firm-specific idiosyncratic component of the innovations. The resulting coefficient represents an estimate of ρ and the variance of the residuals of σ_μ^2.

To pin down the the severity of adjustment costs $\hat{\xi}$, the quality of firm information $\mathbb{V}$, and

[16] Our explicit modeling of adjustment and other frictions allows us to perform our analysis at this relatively higher frequency, compared, for example, to DHV, who abstract from these frictions and analyze three year horizons.

[17] See DHV for details.

[18] Section 4.5 analyzes an additional case for China featuring a higher capital share, in line with the findings of a number of recent papers.

the extent of the distortions γ and σ_ε^2, we follow almost directly the strategy outlined above. Specifically, we target the correlation of investment growth with lagged shocks to fundamentals ($\rho_{i,a_{-1}}$), the autocorrelation of investment growth ($\rho_{i,i_{-1}}$), the variance of investment growth (σ_i^2) and the correlation of the marginal product of capital with fundamentals ($\rho_{mpk,a}$). We follow the literature by working with the growth rate of investment (in the analytical cases studied earlier, we used investment, i.e., the growth rate of capital) in order to at least partially cleanse the data of firm fixed effects, which are a significant component of cross-sectional differences in investment rates.[19] Finally, to infer σ_χ^2, the fixed component of distortions in equation (6), we match the overall dispersion in the marginal product of capital, σ_{mpk}^2, which is clearly increasing in σ_χ^2. Thus, by construction, our parameterized model will generate the amount of misallocation observed in the data, allowing us to decompose the role of each factor in contributing to the total. We summarize our empirical approach in Table 1.

Table 1: Parameterization - Summary

Parameter	Description	Target/Value
Common		
	Time period	1 year
α_1	Capital share	0.28
α_2	Labor share	0.55
β	Discount rate	0.95
Country-specific		
ρ	Persistence of fundamentals	Estimates of (5):
σ_μ	Shocks to fundamentals	$a_{it} = \rho a_{it-1} + \mu_{it}$
$\mathbb{V}$	Signal precision	$\rho_{i,a_{-1}}$
$\hat{\xi}$	Adjustment costs	$\rho_{i,i_{-1}}$
γ	Correlated distortions	$\rho_{mpk,a}$
σ_ε^2	Transitory distortions	σ_i^2
σ_χ^2	Permanent distortions	σ_{mpk}^2

4.2 Data

The data on Chinese manufacturing firms are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The surveys include all industrial firms that are either state-owned, or are non-state firms with sales above 5 million RMB (about \$600,000).[20]

[19] See, for example, Morck et al. (1990) and DHV.

[20] Industrial firms correspond to Chinese Industrial Classification codes 0610-1220, 1311-4392 and 4411-4620, which includes mining, manufacturing and utilities.

We use data spanning the period 1998-2009. The original data come as a repeated cross-section. A panel is constructed following almost directly the method outlined in Brandt et al. (2014), which also contains an excellent overview of the data for the interested reader. The Chinese data have been used multiple times and are by now familiar in the misallocation literature - for example, Hsieh and Klenow (2009) - although our use of the panel dimension is rather new. The data on US publicly traded firms comes from Compustat North America. We use data covering the same period as for the Chinese firms.

We measure the firm's capital stock k_{it} in each period as the value of fixed assets in China and of property, plant and equipment (PP&E) in the US, and investment as the change in the capital stock relative to the preceding period. We construct the fundamental as $a_{it} = va_{it} - \alpha k_{it}$, where we compute value-added from revenues using a share of intermediates of 0.5, and, ignoring constant terms that do not affect our calculations, measure the marginal product of capital as $mpk_{it} = va_{it} - k_{it}$. First differencing k_{it} and a_{it} gives investment and changes in fundamentals between periods. To isolate the firm-specific variation in our data series, we extract a time by industry fixed-effect from each and use the residual as the component that is idiosyncratic to the firm. In both countries, industries are classified at the 4-digit level. This is equivalent to deviating each firm from the unweighted average within its industry in each time period and serves to eliminate any aggregate components, i.e., changes in aggregate conditions, for example, or inflation, as well as render our calculations to be within-industry, which is a standard approach in the literature. After eliminating duplicates and problematic observations (for example, firms reporting in foreign currencies), outliers, observations with missing data etc., our final sample consists of 797,047 firm-year observations in China and 34,260 in the US. Appendix B provides further details on how we build our sample and construct the moments, as well as summary statistics from one year of our data, 2009.

Table 2 reports the target moments for both countries. The first two columns shows the fundamental processes, which have similar persistence but higher volatility in China. The remaining columns show that investment growth in China is more correlated with past shocks, is more volatile and less autocorrelated, that there is a higher correlation between firm fundamentals and the *mpk*, and that the overall dispersion in the *mpk* is substantially higher than among publicly traded US firms. This variation will lead us to find significant differences in the severity of investment frictions and distortions across the two sets of firms.

Table 2: Target Moments

	ρ	σ^2_μ	$\rho_{i,a_{-1}}$	$\rho_{i,i_{-1}}$	$\rho_{mpk,a}$	σ^2_i	σ^2_{mpk}
China	0.91	0.14	0.25	−0.36	0.68	0.14	0.92
US	0.93	0.08	0.13	−0.30	0.55	0.06	0.45

4.3 Identification

Before turning to the estimation results, we revisit the logic for identification. Although we no longer have analytical expressions for the mapping between moments and parameters, we can show that the intuition developed in Section 3 applies to the more general case here as well. In that section, we demonstrated how subsets of moments analyzed in pairs were particularly informative in disentangling the various forces present in the model. Here, we conduct a numerical version of that exercise. More precisely, we take two parameters at a time and plot their combinations that give rise to the empirically observed values of two moments, holding the other parameters fixed. In a slight abuse of terminology, we refer to these curves as isocorrelation curves, or isocorrs. The parameters are pinned down, at least locally, by the intersection of the isocorrs, which is the parameter configuration that is consistent with the empirical value of both moments. To illustrate this logic, we use the moments and parameter values from US publicly traded firms (reported in Tables 2 and 3) for all exercises in this section. The four plots are displayed in Figure 1. Next, we discuss each plot in turn.

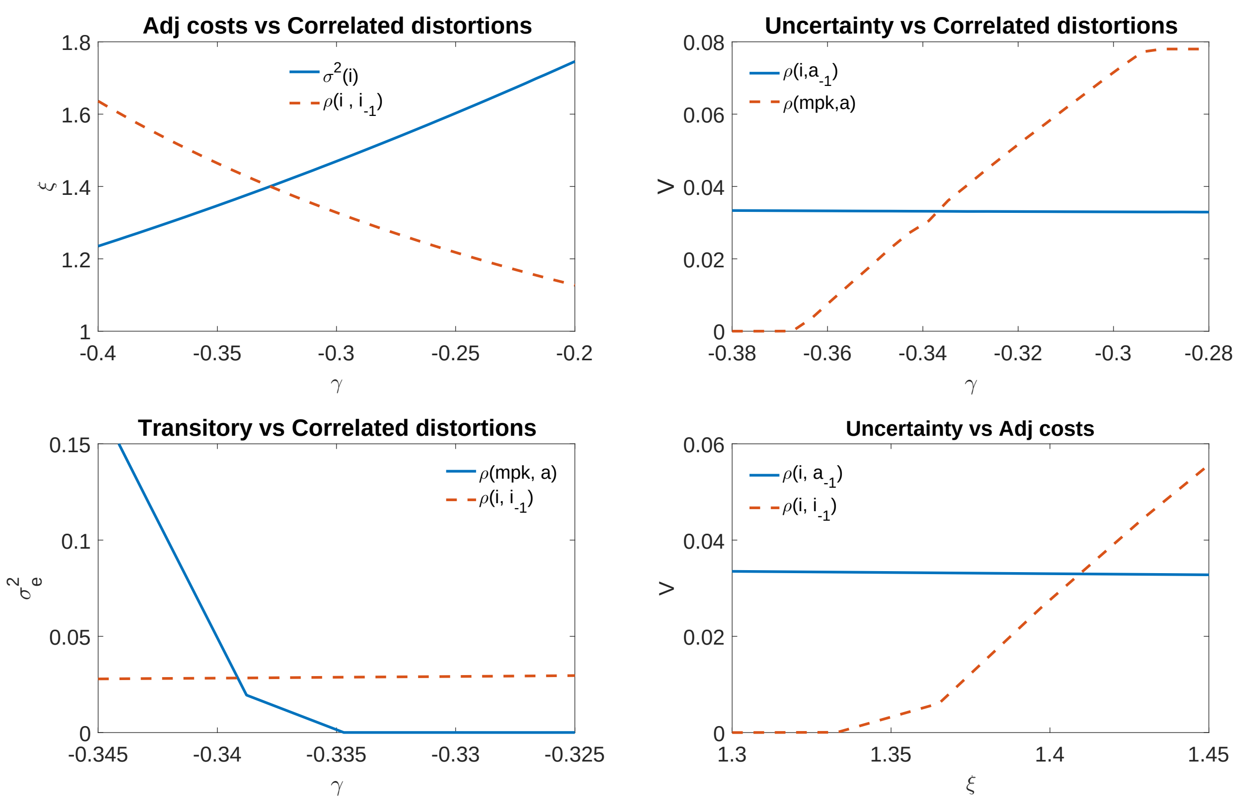

Figure 1: Identification - Quantitative Model

Adjustment costs and correlated distortions. First, to distinguish adjustment costs, $\hat{\xi}$, from the size of correlated distortions, γ, the top left panel of Figure 1 plots the isocorr curves of the variance and autocorrelation of investment growth, σ_i^2 and $\rho_{i,i_{-1}}$, respectively. The line marked σ_i^2 plots combinations of $\hat{\xi}$ and γ that generate a variance of investment equal to the observed level from the US data (holding fixed the other parameters, namely $\sigma_\varepsilon^2, \sigma_\chi^2$ and $\mathbb{V}$). Note that a more negative γ implies more severe distortions. The σ_i^2 isocorr is upward sloping since both factors depress the volatility of investment (see equation (13) for example), so that a less negative γ requires more severe adjustment costs to generate the same σ_i^2. The $\rho_{i,i_{-1}}$ isocorr, on the other hand, is downward sloping since the autocorrelation is decreasing in γ but increasing in $\hat{\xi}$ (e.g., expression (14)). Therefore, a less negative γ raises $\rho_{i,i_{-1}}$ which has to be offset by a lower $\hat{\xi}$. The two curves cross at a single point, which represents the parameter combination that matches both moments. Intuitively, the opposing effects of these factors on the moments - more severe correlated distortions dampen both the variability and autocorrelation of investment, whereas higher adjustment costs dampen the variability but increase the autocorrelation – allows us to disentangle them. To see why both forces are necessary to reconcile the two moments, note that a model featuring only adjustment costs (i.e., imposing $\gamma = 0$) parameterized to match the variability of investment would tend to overstate adjustment costs and therefore, the autocorrelation of investment (the right end of the graph). Allowing for a non-zero γ gives the model the flexibility to match both moments.[21]

Uncertainty and correlated distortions. Next, to tease out uncertainty, $\mathbb{V}$, from correlated distortions, γ, the top right panel of Figure 1 plots the isocorr curves of the correlation of investment growth with lagged shocks and the correlation of the *mpk* with fundamentals, $\rho_{i,a_{-1}}$ and $\rho_{mpka,a}$, respectively. The former is virtually flat, implying that $\rho_{i,a_{-1}}$ is nearly independent of γ. Recall the discussion of equation (15) in Section 3 where we showed that in the absence of adjustment costs and transitory distortions, the correlation of current investment with past shocks is invariant to γ. This invariance holds here as well, albeit approximately, reflecting the fact that the adjustment costs and transitory distortions are fixed at modest levels (which is what we find in the US). On the other hand, the correlation of the *mpk* with fundamentals is increasing in both $\mathbb{V}$ and γ - the intuition is analogous to that behind (16) – leading to an upward sloping $\rho_{mpka,a}$ isocorr. A less negative γ increases the response of investment to fundamentals and so, ceteris paribus, reduces $\rho_{mpk,a}$. To offset this effect and generate the same $\rho_{mpk,a}$, uncertainty must be greater. Ignoring uncertainty and inferring γ from $\rho_{mpk,a}$ would risk

[21] Ignoring correlated distortions and parameterizing adjustment costs to match the autocorrelation of investment growth would tend to overstate the volatility of investment growth and underestimate adjustment costs.

overstating the severity of distortions.[22] On the other hand, using the standard one-period time-to-build specification, which also maximizes uncertainty by assumption, would underestimate the extent of this type of distortion.

Transitory and correlated distortions. To disentangle correlated and uncorrelated transitory distortions, γ and σ_ε^2, the bottom left panel of Figure 1 plots the isocorr curves of $\rho_{i,i_{-1}}$ and $\rho_{mpka,a}$. The autocorrelation isocorr is virtually flat - recall from (14) that in the absence of transitory distortions, the autocorrelation of investment was invariant to γ. This result approximately goes through here. The $\rho_{mpka,a}$ curve is downward sloping since the moment is increasing in γ but decreasing in σ_ε^2 (see equation (34) in Appendix A.4.1)). Uncorrelated transitory distortions reduce both the autocorrelation of investment and the correlation of the *mpk* with fundamentals. Correlated distortions also (mildly) reduce the serial correlation of investment, but increase $\rho_{mpka,a}$. As before, these opposing effects imply that there is a unique combination of γ and σ_ε^2 consistent with both moments.

Uncertainty and adjustment costs. Lastly, to separate uncertainty from adjustment costs, the bottom right panel of Figure 1 plots the $\rho_{i,a_{-1}}$ and $\rho_{i,i_{-1}}$ isocorr curves. The former is virtually flat - in this region of the parameter space, the correlation between investment and lagged fundamentals is not particularly sensitive to adjustment costs. The $\rho_{i,i_{-1}}$ isocorr curve, however, is upward sloping. Higher adjustment costs increase the autocorrelation of investment, but have little effect on its correlation with past shocks. The fact that $\rho_{i,a_{-1}}$ is high (relative to $\rho_{i,i_{-1}}$) is an indication of a departure from full information. At the other extreme, assuming that firms have no information about future shocks to fundamentals would lead to a $\rho_{i,a_{-1}}$ that is counterfactually high. Introducing additional information on the part of the firm, i.e., reducing $\mathbb{V}$, allows us to hit this target precisely.

4.4 Results

Table 3 contains our baseline results. In the top panel we display the parameter estimates. In the second panel, we report the contribution of each factor to dispersion in the *mpk*, which we denote $\Delta\sigma_{mpk}^2$.[23] These are calculated under the assumption that only the factor of interest is operational, i.e., in the absence of the others, so that the contribution of each one is measured

[22] A strategy that abstracts from correlated distortions and infers $\mathbb{V}$ using $\rho_{mpk,a}$ would, in turn, risk overstating the extent of uncertainty.

[23] For adjustment costs, we do not have an analytic mapping between the severity of these costs and misallocation, but this is a straightforward calculation to make numerically; for each of the other factors in our model, we can compute their contributions to misallocation analytically.

relative to the undistorted first-best.[24] The third panel expresses the contribution as a percent of the total *mpk* dispersion measured in the data, denoted $\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$. Because of interactions between the factors, there is no *a priori* reason to expect these relative contributions to sum to one. In practice, however, we find that the total is reasonably close to one, allowing us to interpret this exercise as a decomposition of total observed misallocation. Finally, in the bottom two panels of the table, we compute the implied losses in aggregate TFP and output stemming from each factor, again relative to the undistorted first-best level, i.e., $\Delta a = a^* - a$ and $\Delta y = y^* - y$. Once we have the contribution of each factor to *mpk* dispersion, these latter two values are simply applications of formulas (11) and (12).

Table 3: Frictions, Distortions and Misallocation

			Distortions		
	Adjustment Costs	Uncertainty	Correlated	Transitory	Permanent
Parameters	$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
China	0.16	0.09	−0.63	0.00	0.51
US	1.38	0.03	−0.33	0.03	0.29
$\Delta\sigma^2_{mpk}$					
China	0.01	0.09	0.33	0.00	0.51
US	0.05	0.03	0.06	0.03	0.29
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$					
China	1.1%	9.4%	36.1%	0.0%	55.2%
US	10.8%	7.3%	14.4%	6.3%	64.7%
Δa					
China	0.00	0.03	0.12	0.00	0.19
US	0.02	0.01	0.02	0.01	0.11
Δy					
China	0.01	0.04	0.17	0.00	0.26
US	0.03	0.02	0.03	0.01	0.15

Adjustment costs. One of our key quantitative findings is a relatively modest degree of adjustment frictions in both countries. For example, the estimate of $\hat{\xi} = 1.38$ for the US in Table 3 implies a primitive parameter ξ in the adjustment cost function (2) of 0.2, a value which is generally lower than existing estimates in the literature.[25] Asker et al. (2014), for example,

[24] An alternative would be to calculate the contribution of each factor holding the others constant at their estimated values. However, the resulting interactions between them makes it difficult to compare the severity of the individual factors across countries.

[25] The mapping is derived in equation (24) in Appendix A.1.1. We use an annual depreciation rate of $\delta = 0.10$.

report an estimate of 8.8 for their convex adjustment cost parameter estimated using data on US manufacturing firms. To interpret this difference, consider a firm that doubles its capital stock in a year. Our estimates imply that the firm would incur adjustment costs equal to about 11% of the value of the investment, whereas the corresponding number using the Asker et al. (2014) estimate would be 60%.[26] The level of costs is estimated to be even lower in China.

These results imply a limited role for adjustment costs in generating misallocation, particularly so in China - if this were the only friction, mpk dispersion in China arising from this channel would be 0.01, representing about 1% of the total σ^2_{mpk}. The contribution of adjustment costs in the US is significantly higher, where they lead to mpk dispersion of 0.05, about 11% of the total. The corresponding losses in aggregate TFP are about 0.4% and 2% in the two countries, respectively, and output losses 1% and 3%. Thus, adjustment frictions are relatively more important in the US compared to China. However, these estimates imply that adjustment costs alone cannot account for the majority of marginal product dispersion. While their effect is not trivial, it is quite modest compared to the effect of distortionary factors described below. Later, we show that this finding remains robust to a number of variants of our baseline model – where we allow for country-specific capital share parameters, frictions/distortions in labor and financial constraints.

It is important to note that one would reach a very different conclusion from examining adjustment costs in isolation. To show this, we also estimated a version of our model in which we abstract from the other forces and parameterize those costs to match a single moment in the data. First, we infer the severity of adjustment costs from the volatility of investment growth, σ^2_i. Doing so gives considerably larger estimates of their magnitude, about 60% higher in the US and almost 10 times higher in China. The resulting mpk dispersion increases by a similar amount. Tellingly, however, the implied autocorrelation of investment growth from this approach is much higher than that observed in the data, -0.17 vs a true value of -0.30 in the US and -0.20 vs -0.36 in China. These are precisely the patterns predicted by our identification arguments in Sections 3 and 4.3 – a strategy that parameterizes adjustment costs to match investment variability alone leads to estimates that substantially overstate the autocorrelation of investment. Explicitly accounting for other factors enables the model to reconcile the two moments and significantly reduces the implied value of those costs. A similar strategy targeting the autocorrelation of investment leads to the opposite conclusion – here, we would understate the magnitude of adjustment costs and predict too high a variability of investment compared to the data.

[26] For the US, our estimates are closer to those in Cooper and Haltiwanger (2006) and Bloom (2009).

Uncertainty. Table 3 shows that firms in both countries face significant uncertainty, although the data rejects the extreme assumption of no information. The severity of the informational friction is higher among Chinese firms. For example, as a share of the prior uncertainty, σ_μ^2, residual uncertainty, $\frac{\mathbb{V}}{\sigma_\mu^2}$, is 0.42 in the US and 0.63 in China. In other words, firm learning eliminates about 60% of total uncertainty in the US and about 35% in China.[27] It is straightforward to show that in an environment where imperfect information is the only friction, $\sigma_{mpk}^2 = \mathbb{V}$, so that the contribution of uncertainty alone to observed misallocation can be directly read off the second column in Table 3 - namely 0.09 in China and 0.03 in the US. These represent about 9% and 7% of total *mpk* dispersion in the two countries, respectively. The implications for aggregate TFP and output losses in China are 3% and 4%, while the corresponding values in the US are slightly lower, at 1% and 2%.

Distortions. The last three columns of Table 3 show that distortions play a significant role in generating the observed *mpk* dispersion in both countries. Turning first to the correlated component, the negative values of γ suggest that they disincentive investment by more productive firms and especially so in China. The contribution of these distortions to *mpk* dispersion is given by $\gamma^2\sigma_a^2$, which amounts to 0.33 in China, or 36% of total misallocation. The associated aggregate consequences are also quite sizable – TFP and output losses are 12% and 17%, respectively. In contrast, the estimate of γ in the US is significantly less negative than in China, suggesting that this type of size-dependent distortion is less of an issue for firms in the US, both in an absolute sense – the *mpk* dispersion from these factors in the US is 0.06, less than one-fifth that in China – and in relative terms – they account for only 14% of the observed dispersion in marginal products in the US. The corresponding TFP and output effects are also considerably smaller for the US - namely, 2% and 3%, respectively.

Next, we consider the role of distortions that are uncorrelated with firm fundamentals. Table 3 shows that purely transitory factors (measured by σ_ε^2) are negligible in both countries, but permanent firm-specific factors (measured by σ_χ^2) play a prominent role. Their contribution to *mpk* dispersion, which is also given by σ_χ^2, amounts to 0.51 in China and 0.29 in the US. Thus, their absolute magnitude in the US is considerably below that in China (just over one-half), but in relative terms, these factors do seem to account for a substantial portion of measured misallocation in both countries. The aggregate consequences of these types of distortions are also significant, with TFP losses of 19% in China and about half that in the US.

In sum, the estimation results point to the presence of substantial distortions to investment,

[27]Our values for $\frac{\mathbb{V}}{\sigma_\mu^2}$ are comparable to those in DHV, who find 0.41 in the US and 0.63 in China. Though these estimates are not directly comparable – DHV focus on longer time horizons (they analyze 3-year time intervals) and consider only publicly traded firms in China – the fact that they are quite close to each other provides a degree of validation for our methodology.

especially in China, where they disproportionately disincentivize investment by more productive firms. What patterns in the data lead us to this conclusion? Recall that the *mpk* in both countries shows significant dispersion along with a high correlation with fundamentals, indicating a dampened response of investment to fundamentals. In principle, this pattern could emerge from adjustment costs, imperfect information or correlated distortions. However, the autocorrelation of investment growth, $\rho_{i,i_{-1}}$, in the data is relatively low, which bounds the severity of adjustment frictions. Similarly, the response of investment to past shocks, $\rho_{i,a_{-1}}$, is also modest, limiting the role of the informational friction. Hence, the estimation assigns a substantial role to correlated distortions, particularly in China, as well as fixed distortions, in order to generate the observed patterns in the *mpk*.[28]

4.5 Higher Capital Share in China

In our baseline computations, we assumed a common capital share across countries of one-third, which is a standard value used in the literature for US firms. A number of recent papers have found a higher capital share in China, for example, Song et al. (2011) and Bai et al. (2006), generally equal to one-half. Table 4 presents results for this case, specifically, where we set $\alpha_1 = \alpha_2 = \frac{0.83}{2}$, which implies an $\alpha = 0.71$. The new value of α affects our estimates of firm fundamentals and through this, the target moments. We report the recomputed moments in the top panel of the table. The second panel contains the new parameter estimates, which are broadly similar to those in the baseline case (γ becomes slightly more negative and σ_χ^2 falls somewhat), as are the implications for misallocation - adjustment costs and uncertainty play limited roles, with correlated and permanent distortions accounting for a large portion of *mpk* dispersion. A higher capital share relative to labor increases the cost of misallocation in terms of TFP and output losses, i.e., the multipliers in equations (11) and (12) increase, and so the adverse effects of investment frictions/distortions on aggregate outcomes is larger than in the baseline case. Here, for example, correlated and permanent distortions each lead to productivity losses of about 30% and output losses of about 50% and the impact of uncertainty becomes larger, implying TFP and output losses of 7% and 12%. However, the increase in the aggregate effects holds across all the factors, so that our conclusions regarding the substantial role for distortions continue to hold.

[28] A high value for $\rho_{mpk,a}$ also tends to rule out uncorrelated transitory distortions as an important driver of investment decisions.

Table 4: China-Specific Capital Share

Moments	ρ	σ^2_μ	$\rho_{i,a_{-1}}$	$\rho_{i,i_{-1}}$	$\rho_{mpk,a}$	σ^2_i	σ^2_{mpk}
	0.91	0.15	0.29	−0.36	0.76	0.14	0.92
Parameters			$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
			0.13	0.10	−0.70	0.00	0.41
Aggregate Effects							
$\Delta\sigma^2_{mpk}$			0.01	0.10	0.44	0.00	0.41
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$			1.3%	10.3%	47.4%	0.0%	44.4%
Δa			0.01	0.07	0.31	0.00	0.29
Δy			0.01	0.12	0.53	0.00	0.50

4.6 Frictional Labor

Our baseline analysis makes the rather stark assumption that there are no adjustment or information frictions in labor decisions, rendering the labor input a static choice made under full information. Although this is not an uncommon assumption in the literature, it may not be an apt description of labor markets in developing economies such as China. In this section, we extend our analysis to allow for frictions in the adjustment of labor. In particular, we show in Appendix A.2.1 that when labor is subject to the same forces as capital - adjustment and informational frictions and distortions - the firm's intertemporal investment problem takes the same form as in expression (3), but where the degree of curvature is equal to $\alpha = \alpha_1 + \alpha_2$ (and with slightly modified versions of the G and A_{it} terms). Thus, the qualitative analysis of the model is unchanged, although the quantitative results will differ since we now have $\alpha = \alpha_1 + \alpha_2 = 0.83$. Table 5 reports results from this specification for the Chinese firms. The top panel of the table displays the target moments recomputed under this scenario (recall that a number of the moments depend on the value of α). A comparison to the baseline moments in Table 2 shows that under the assumption of frictional labor, the correlation of investment with lagged shocks increases, as does the correlation of the *mpk* with fundamentals. The new values imply a higher level of adjustment costs, greater uncertainty and more severe correlated distortions. As a result, a lower level of the permanent component of uncorrelated distortions is needed to mach the dispersion in the *mpk*. The estimated parameter values are reported in the second panel of Table 5.

Aggregate TFP in this case is equal to[29]

$$a = a^* - \frac{1}{2}\frac{\alpha}{1-\alpha}\sigma^2_{mpk} \tag{18}$$

[29]Derivations are in Appendix A.2.2.

where a^* is again TFP in the frictionless, undistorted benchmark. Given the effect of misallocation on aggregate TFP, the additional effect on output as a result of a reduction in the aggregate capital stock is the same as in the baseline case, i.e., equation (12).

Expression (18) shows that the relative shares of capital and labor in production – a key determinant of the aggregate ramifications of misallocation in the baseline case with frictionless labor - no longer play a role. Only the overall returns to scale α matters and, as in the baseline case, the higher the returns to scale (i.e., the closer to constant returns), the greater the losses from misallocation. Further, it is straightforward to see that for a fixed set of parameters, the cost of misallocation is larger here than in the baseline case.

The bottom panel of Table 5 reports the contribution of each factor to total misallocation and computes the implications for aggregate TFP and output. There is a noticeable increase in the impact of adjustment costs from the baseline case - in this specification, these costs account for almost 13% of *mpk* dispersion (compared to 1% above). There is also a small increase in the impact of uncertainty (from 9% to 11%). However, both of these forces remain muted compared to correlated and permanent distortions, which continue to account for the largest portion of observed misallocation. The effects on productivity and output are much larger than in the baseline scenario - intuitively, when labor is chosen in the face of the same factors as capital, firms are unable to mitigate capital misallocation by adjusting on the labor margin, a force that was present in the baseline case. For example, adjustment costs and imperfect information now lead to TFP losses of about 25% and 30%, respectively. The corresponding figures for correlated and permanent distortions imply gaps relative to the first-best of about 115% and 75%. While the main message of our analysis remains - correlated and permanent distortions seem to play a significant role in explaining misallocation - the results here illustrate the potential for large aggregate consequences of adjustment/information frictions.

Table 5: Frictional Labor

Moments	ρ	σ^2_μ	$\rho_{i,a_{-1}}$	$\rho_{i,i_{-1}}$	$\rho_{mpk,a}$	σ^2_i	σ^2_{mpk}
	0.92	0.16	0.33	-0.36	0.81	0.14	0.94
Parameters			$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
			0.78	0.11	-0.68	0.04	0.30
Aggregate Effects							
$\Delta\sigma^2_{mpk}$			0.12	0.11	0.48	0.04	0.30
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$			12.8%	11.3%	51.2%	4.0%	32.2%
Δa			0.29	0.26	1.17	0.09	0.74
Δy			0.41	0.36	1.62	0.13	1.02

4.7 Financial Frictions

Our baseline analysis abstracts from financial frictions, a choice motivated by the fact that previous studies have found only a limited role for these types of factors in leading to misallocation, for example, Midrigan and Xu (2014). Despite this finding, a potential concern is that our baseline conclusions regarding the composition of misallocation, i.e., the relative contributions of the various forces in the model, are potentially biased by omitting financial considerations. Further, it is of interest to understand whether the large magnitude of distortions that we measure are in any significant part attributable to financial market imperfections. To address these questions, this section introduces liquidity constraints into our framework, which capture, albeit in a stylized way, some essential features of financial frictions while retaining the tractability of the baseline model. We find that including financial factors modestly increases our estimate of adjustment costs and reduces the severity of correlated distortions, but our results on the sources of misallocation are largely unchanged. Further, the role of the financial friction itself in generating misallocation, although non-negligible, is fairly modest, a result broadly in line with the findings in the existing literature.

To introduce financial frictions, we assume that firms face a liquidity cost $\Upsilon\left(K_{it+1}, B_{it+1}\right)$ that is increasing in the chosen level of capital (K_{it+1}) but decreasing in their holding of liquid assets (denoted B_{it+1}). This cost captures the idea that firms may not have frictionless access to external funds from financial markets and so must finance some portion of investment internally. Our formulation can be interpreted as a smoothed version of financing or other constraints such as that in Buera and Shin (2013) and Moll (2014). Using a continuous penalty function rather than an occasionally binding constraint allows us to continue using perturbation methods, while still capturing the influence of liquidity or financial considerations on investment decisions.

We work with a flexible parameterization of the liquidity cost function:

$$\Upsilon\left(K_{it+1}, B_{it+1}\right) = \hat{\nu}\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2}}$$

where $\hat{\nu}$, ω_1 and ω_2 are parameters. The firm's recursive problem becomes

$$\begin{aligned}\mathcal{V}\left(K_{it}, B_{it}, \mathcal{I}_{it}\right) = \max_{B_{it+1}, K_{it+1}} \quad & \mathbb{E}_{it}\left[\Pi\left(K_{it}, A_{it}\right) + RB_{it} - B_{it+1} - \Phi\left(K_{it+1}, K_{it}\right) - \Upsilon\left(K_{it+1}, B_{it+1}\right)\right] \\ + \quad & \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1}, B_{it+1}, \mathcal{I}_{it+1}\right)\right]\end{aligned}$$

where $R < \frac{1}{\beta}$ denotes the (exogenous) gross return on liquid assets, which are risk-free. This is the sense in which liquidity is costly - it requires the firm to hold a low-return asset simply for the sake of moderating its effective cost of capital. The optimal choice of B_{it+1} is given by[30]

$$B_{it+1} = \left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{1}{\omega_2+1}} K_{it+1}^{\frac{\omega_1}{\omega_2+1}}$$

and using this to replace B_{it+1}, we obtain the following version of the firm's value function,

$$\begin{aligned}\mathcal{V}\left(K_{it}, \mathcal{I}_{it}\right) = \max_{K_{it+1}} \quad & \mathbb{E}_{it}\left[\Pi\left(K_{it}, A_{it}\right) + RB_{it} - \Phi\left(K_{it+1}, K_{it}\right) - \frac{\nu}{1+\omega}K_{it+1}^{1+\omega}\right] \\ + \quad & \beta\mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1}, \mathcal{I}_{it+1}\right)\right]\end{aligned}$$

where

$$\omega = \frac{\omega_1 - (1+\omega_2)}{1+\omega_2}$$

and ν is a composite parameter that is increasing in the scale parameter, $\hat{\nu}$, and in the net cost of holding liquid assets, $1-\beta R$. The parameter ω governs the additional curvature introduced by liquidity costs into the firm's capital choice problem. If $\omega < 0$, then the marginal cost of liquidity is decreasing in K_{it+1}. The opposite is true if $\omega > 0$. This is arguably the more intuitive case, where the financial friction exerts a dampening effect on the firm's incentives to adjust K_{it+1} in response to changes in expected fundamentals (and distortions).

We can show that the linearized Euler equation takes precisely the same form as in expression (4) in our baseline setting, but with the curvature parameter α replaced by a liquidity-adjusted one, denoted $\tilde{\alpha}$, where

$$\tilde{\alpha} \equiv \alpha - \left(\frac{\nu\bar{K}^{\omega}}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}\right)\omega$$

Thus, financial frictions manifest themselves as a change in the degree of curvature in the firm's investment policy function. This can be higher or lower than the curvature from the production function, depending on the properties of the liquidity cost function, specifically, the sign of ω. When $\omega > 0$, so that the marginal cost of liquidity is increasing in K_{it+1}, we have $\tilde{\alpha} < \alpha$ and the firm's response to changes in expected fundamentals is muted. The opposite is true if $\omega < 0$.

To better understand the implications of the financial friction, consider the special case when this is the only factor distorting investment. The choice of capital (in logs) is simply $k_{it} = \frac{1}{1-\tilde{\alpha}}a_{it}$, whereas the frictionless choice is $k_{it} = \frac{1}{1-\alpha}a_{it}$. When $\tilde{\alpha} < \alpha$, the elasticity of k_{it} to a_{it} is dampened relative to the frictionless level. To see the implications for misallocation, note

[30] All derivations for this section are in Appendix A.3.

that, in this case

$$mpk_{it} = a_{it} + (\alpha - 1)\, k_{it} = \left(\frac{\alpha - \tilde{\alpha}}{1 - \tilde{\alpha}}\right) a_{it} \tag{19}$$

$$\sigma^2_{mpk} = \left(\frac{\alpha - \tilde{\alpha}}{1 - \tilde{\alpha}}\right)^2 \sigma^2_a \tag{20}$$

which is zero when $\alpha = \tilde{\alpha}$ and increasing in the difference between the two.

How can we discipline $\tilde{\alpha}$? A common strategy in the literature is to use aggregate data on the extent of external finance and/or liquidity in the economy.[31] Here, we take an alternative approach and estimate $\tilde{\alpha}$ (along with the other factors) by continuing to focus on production-side moments from the micro-data. In particular, we modify the estimation exercise from the previous sections to also target the correlation of the *mpk* with k, denoted $\rho_{mpk,k}$, as well as the variance of the change in the *mpk*, denoted $\sigma^2_{\Delta mpk}$. We pick the parameter configuration that is jointly consistent with the observed values of both of these moments (in addition to the ones used in our baseline analysis). Table 6 reports the results for China. The empirical values of the two additional moments are $\rho_{mpk,k} = -0.52$ and $\sigma^2_{\Delta mpk} = 0.16$. For purposes of comparison, the first row of the table displays the parameter estimates from our baseline analysis and the second row the estimates with financial frictions.

Table 6: Financial Frictions - Parameter Estimates

	$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ	$\tilde{\alpha}$
Baseline	0.16	0.09	-0.63	0.00	0.51	-
With Financial Frictions	0.24	0.09	-0.44	0.00	1.14	0.45

The estimate of $\hat{\xi}$ under financial frictions increases modestly, while $\mathbb{V}$ and σ^2_ε are quite close to their baseline values, implying that our finding of a relatively modest contribution to *mpk* dispersion from these sources is unchanged. The main change from adding the financial friction is a reduction in (the absolute value of) γ. This suggests that some portion of what we previously measured as correlated distortions can be attributed to financial factors. Intuitively, financial frictions and correlated distortions have a similar dampening effect on the responsiveness to expected fundamentals. Quantitatively, the *mpk* dispersion stemming from correlated distortions alone ($\Delta\sigma^2_{mpk} = \gamma^2\sigma^2_a$) falls from 0.33 to 0.16. However, both correlated and permanent distortions remain key drivers of observed misallocation.

Note that our estimate of σ^2_χ increases considerably with financial frictions. This is because, under our formulation, the liquidity cost also reduces the responsiveness of investment to χ_i, so that a higher σ^2_χ is needed to generate the empirical level of *mpk* dispersion. In the presence of

[31] See Buera et al. (2011) and Moll (2014), among others.

financial frictions, mpk dispersion from χ_i is equal to $\left(\frac{1-\alpha}{1-\hat{\alpha}}\right)^2 \sigma_\chi^2 = 0.54$, which is only slightly higher than the baseline contribution of 0.51. In other words, because of the strong interaction between financial frictions and permanent distortions, the model with financial frictions suggests a higher level of the latter.

Finally, to quantify the effects of the financial friction, we can apply equation (20) to find $\Delta\sigma^2_{mpk} = 0.08$. As the only operational friction, financial factors lead to non-negligible, but relatively modest levels of mpk dispersion, representing just under 10% of the total (recall that $\sigma^2_{mpk} = 0.92$ in China). Thus, consistent with the existing literature on financial frictions, we find that they play a somewhat limited role in generating misallocation. Moreover, these results also show that explicitly incorporating them does not significantly alter our main conclusions regarding the contribution of other factors.

4.8 Evidence from Additional Countries

Our main analysis examines the sources of misallocation in China and their aggregate implications, with results for publicly traded US firms reported as a benchmark for basis of comparison. In a further exercise, we estimated our model on two additional countries for which micro-data were available to us, although the time periods are not as recent - Colombia and Mexico. For Colombia, we have plant-level data from the Annual Manufacturers Survey over the years 1982-1998, which covers plants with more than 10 employees or sales above a certain threshold (around $35,000 in 1998). The Mexican data are also at the plant-level and are from the Annual Industrial Survey over the years 1984-1990, which covers plants of the 3200 largest manufacturing firms.[32]

Table 7 reports the estimated parameter values for these countries, the share of mpk dispersion arising from each factor and the effect on aggregate productivity.[33] In brief, the results are quite similar to those from China. The contribution of adjustment costs and uncertainty as a share of overall misallocation is rather limited, and that of uncorrelated transitory distortions negligible - a large portion of misallocation in both countries stems from correlated and permanent distortions. The TFP losses from the latter two forces are fairly large, totaling roughly 30% in both countries.

[32]For a detailed description of the Colombian data, see Eslava et al. (2004) and for Mexico, Tybout and Westbrook (1995).

[33]In Appendix C we describe the data and moments in more detail and report more detailed output from the estimation.

Table 7: Evidence from Additional Countries

Parameters	$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Colombia	0.54	0.05	-0.55	0.01	0.60
Mexico	0.13	0.04	-0.82	0.00	0.42
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$					
Colombia	2.5%	5.6%	30.9%	0.7%	61.3%
Mexico	0.5%	4.9%	44.9%	0.0%	52.8%
Δa					
Colombia	0.01	0.02	0.11	0.00	0.22
Mexico	0.00	0.01	0.13	0.00	0.16

5 Conclusion

In this paper, we have laid out a model of investment under a variety of frictions and distortions, along with an empirical strategy to disentangle them using readily observable firm-level production data. An application to Chinese manufacturing firms suggests that static measures of resource misallocation are driven only partly by technological and informational frictions. Even after accounting for these factors, 'distortions' which penalize investment and/or hiring by specific firms – particularly more efficient ones – play an important role as well. Our results are quite different from those analyzing these forces in isolation and showcase the value of using a unified framework like the one we have proposed.

Our analysis points to several directions for future work. First, it should be relatively straightforward to apply this methodology to understand labor misallocation. An analogous set of moments computed using detailed data on labor inputs can be used to quantify the relative importance of adjustment/informational frictions and policy/institutional distortions in driving the dispersion in the marginal product of labor across firms.

Our formulation of frictions/distortions is rather stylized along a number of dimensions - our modeling choices were guided in part by considerations of tractability. An important but computationally demanding next step would be to generalize these specifications and allow for a broader class of frictions. Examples could include non-convex adjustment costs, a richer informational environment and specific distortionary policies. We conjecture, however, that even in a more general environment, the main message of this paper - that using a set of carefully chosen moments is essential to disentangle the effects of the various forces influencing firm investment decisions - would be relevant and help guide future empirical work in this regard.

Our findings on the prevalence of distortionary factors across countries points to the need for further research into their underlying sources, for example, by linking them to specific policies and/or institutional features. Hopenhayn (2014) reviews a number of recent papers that have investigated specific size-dependent policies that resemble correlated distortions. Buera et al. (2013) show how irreversibility in government policy can result in fixed distortions at the firm-level. Although it is unlikely that a single policy can account for the distortions we measure, a deeper understanding of how particular policies translate into distortions to allocative efficiency would be extremely valuable.

References

Asker, J., A. Collard-Wexler, and J. De Loecker (2014): "Dynamic inputs and resource (mis) allocation," *Journal of Political Economy*, 122, 1013–1063.

Bachmann, R. and S. Elstner (2015): "Firm optimism and pessimism," *European Economic Review*, 79, 297–325.

Bai, C.-E., C.-T. Hsieh, and Y. Qian (2006): "The Return to Capital in China," *Brookings Papers on Economic Activity*, 2006, 61–101.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013): "Cross Country Differences in Productivity: The Role of Allocative Efficiency," *American Economic Review*, 103, 305–334.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

Brandt, L., J. Van Biesebroeck, and Y. Zhang (2014): "Challenges of working with the Chinese NBS firm-level data," *China Economic Review*, 30, 339–352.

Buera, F. J. and R. N. Fattal-Jaef (2016): "The Dynamics of Development: Innovation and Reallocation," *working paper*.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101, 1964–2002.

Buera, F. J., B. Moll, and Y. Shin (2013): "Well-intended policies," *Review of Economic Dynamics*, 16, 216–230.

Buera, F. J. and Y. Shin (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121, 221–272.

Cooper, R. W. and J. C. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *The Review of Economic Studies*, 73, 611–633.

David, J. M., H. A. Hopenhayn, and V. Venkateswaran (2015): "Information, Misallocation and Aggregate Productivity," *Quarterly Journal of Economics, forthcoming.*

Eslava, M., J. Haltiwanger, A. Kugler, and M. Kugler (2004): "The effects of structural reforms on productivity and profitability enhancing reallocation: evidence from Colombia," *Journal of Development Economics*, 75, 333–371.

Gopinath, G., S. Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez (2015): "Capital allocation and productivity in south europe," Tech. rep., National Bureau of Economic Research.

Guner, N., G. Ventura, and Y. Xu (2008): "Macroeconomic implications of size-dependent policies," *Review of Economic Dynamics*, 11, 721–744.

Hopenhayn, H. A. (2014): "Firms, misallocation, and aggregate productivity: A review," *Annu. Rev. Econ.*, 6, 735–770.

Hsieh, C. and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

Hsieh, C.-T. and P. J. Klenow (2014): "The Life Cycle of Plants in India and Mexico," *The Quarterly Journal of Economics*, 129, 1035–1084.

Jurado, K., S. C. Ludvigson, and S. Ng (2015): "Measuring Uncertainty," *The American Economic Review*, 105, 1177–1216.

Klenow, P. J. and J. L. Willis (2007): "Sticky information and sticky prices," *Journal of Monetary Economics*, 54, 79–99.

Midrigan, V. and D. Y. Xu (2014): "Finance and misallocation: Evidence from plant-level data," *The American Economic Review*, 104, 422–458.

Moll, B. (2014): "Productivity losses from financial frictions: can self-financing undo capital misallocation?" *The American Economic Review*, 104, 3186–3221.

Morck, R., A. Shleifer, R. W. Vishny, M. Shapiro, and J. M. Poterba (1990): "The Stock Market and Investment: Is the Market a Sideshow?" *Brookings Papers on Economic Activity*, 1990, 157–215.

Restuccia, D. and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11, 707–720.

Song, Z., K. Storesletten, and F. Zilibotti (2011): "Growing like china," *The American Economic Review*, 101, 196–233.

Tybout, J. R. and M. D. Westbrook (1995): "Trade liberalization and the dimensions of efficiency change in Mexican manufacturing industries," *Journal of International Economics*, 39, 53–78.

Appendix

A Derivations

A.1 Baseline

In this section, we provide detailed derivations for the model solution and aggregation results in our baseline analysis.

A.1.1 Model Solution

The first order condition and envelope conditions associated with (3) are, respectively,

$$\begin{aligned} T^K_{it+1}\Phi_1\left(K_{it+1}, K_{it}\right) &= \beta\mathbb{E}_{it}\left[\mathcal{V}_1\left(K_{it+1}, \mathcal{I}_{it+1}\right)\right] \\ \mathcal{V}_1\left(K_{it}, \mathcal{I}_{it}\right) &= \Pi_1\left(K_{it}, A_{it}\right) - T^K_{it+1}\Phi_2\left(K_{it+1}, K_{it}\right) \end{aligned}$$

and combining yields the Euler equation

$$\mathbb{E}_{it}\left[\beta\Pi_1\left(K_{it+1}, A_{it+1}\right) - \beta T^K_{it+2}\Phi_2\left(K_{it+2}, K_{it+1}\right) - T^K_{it+1}\Phi_1\left(K_{it+1}, K_{it}\right)\right] = 0 \tag{21}$$

where

$$\begin{aligned}
\Pi_1\left(K_{it+1}, A_{it+1}\right) &= \alpha G A_{it+1} K_{it+1}^{\alpha-1} \\
\Phi_1\left(K_{it+1}, K_{it}\right) &= 1+\xi\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right) \\
\Phi_2\left(K_{it+1}, K_{it}\right) &= -(1-\delta)-\xi\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right)\frac{K_{it+1}}{K_{it}}+\frac{\xi}{2}\left(\frac{K_{it+1}}{K_{it}}-(1-\delta)\right)^2 \\
&= -(1-\delta)+\frac{\xi}{2}(1-\delta)^2-\frac{\xi}{2}\left(\frac{K_{it+1}}{K_{it}}\right)^2
\end{aligned}$$

In the undistorted $\left(\bar{T}^K=1\right)$ non-stochastic steady state, these are equal to

$$\begin{aligned}
\bar{\Phi}_1 &= 1+\xi\delta \\
\bar{\Phi}_2 &= -(1-\delta)+\frac{\xi}{2}(1-\delta)^2-\frac{\xi}{2} \\
\bar{\Pi}_1 &= \alpha\bar{G}\bar{A}\bar{K}^{\alpha-1}
\end{aligned}$$

Log-linearizing the Euler equation around this point yields

$$\mathbb{E}_{it}\left[\beta\bar{\Pi}_1\pi_{1,it+1}-\beta\bar{\Phi}_2\left(\phi_{2,it+1}+\tau_{it+2}^K\right)-\bar{\Phi}_1\left(\phi_{1,it}+\tau_{it+1}^K\right)\right]=0 \tag{22}$$

where

$$\begin{aligned}
\bar{\Pi}_1\pi_{1,it+1} &\approx \alpha\bar{G}\bar{A}\bar{K}^{\alpha-1}\left(a_{it+1}+(\alpha-1)k_{it+1}\right) \\
\bar{\Phi}_1\phi_{1,it} &\approx \xi\left(k_{it+1}-k_{it}\right) \\
\bar{\Phi}_2\phi_{2,it+1} &\approx -\xi\left(k_{it+2}-k_{it+1}\right)
\end{aligned}$$

Let

$$\tau_{it+1} \equiv -\left(\frac{\beta\bar{\Phi}_2\tau_{it+2}^K+\bar{\Phi}_1\tau_{it+1}^K}{\bar{\Pi}_1}\right) \tag{23}$$

Substituting into (22) and rearranging leads to expression (4).

The steady state Euler equation yields an expression for $\hat{\xi} \equiv \frac{\xi}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}$ as a function of parameters:

$$\beta\bar{\Pi}_1-\beta\bar{\Phi}_2=\bar{\Pi}_1 \quad\Rightarrow\quad \beta\alpha\bar{G}\bar{A}\bar{K}^{\alpha-1}-\beta\left(-(1-\delta)+\frac{\xi}{2}(1-\delta)^2-\frac{\xi}{2}\right)=1+\xi\delta$$

and rearranging,

$$\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}=1-\beta(1-\delta)+\xi\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)$$

so that

$$\hat{\xi}=\frac{\xi}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}=\frac{\xi}{1-\beta\left(1-\delta\right)+\xi\delta\left(1-\beta\left(1-\frac{\delta}{2}\right)\right)} \tag{24}$$

It is straightforward to show $\frac{\partial\hat{\xi}}{\partial\xi}>0$.

To derive the investment policy function, we conjecture that it takes the form in expression (9). Then,

$$\begin{aligned}
k_{it+2} &= \psi_1 k_{it+1}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it+1}a_{it+2}+\psi_3\varepsilon_{it+2}+\psi_4\chi_i\\
\mathbb{E}_{it}\left[k_{it+2}\right] &= \psi_1 k_{it+1}+\psi_2\left(1+\gamma\right)\rho\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_4\chi_i\\
&= \psi_1\left(\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i\right)+\psi_2\left(1+\gamma\right)\rho\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_4\chi_i\\
&= \psi_1^2 k_{it}+\left(\psi_1+\rho\right)\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_1\psi_3\varepsilon_{it+1}+\psi_4\left(1+\psi_1\right)\chi_i
\end{aligned}$$

where we have used $\mathbb{E}_{it}\left[\varepsilon_{it+2}\right]=0$ and $\mathbb{E}_{it}\left[\mathbb{E}_{it+1}\left[a_{it+2}\right]\right]=\rho\mathbb{E}_{it}\left[a_{it+1}\right]$. Substituting and rearranging,

$$\begin{aligned}
&\left(1+\beta\hat{\xi}\psi_4\left(1+\psi_1\right)\right)\chi_i+\left(1+\beta\hat{\xi}\psi_1\psi_3\right)\varepsilon_{it+1}\\
+&\left(1+\beta\hat{\xi}\left(\psi_1+\rho\right)\psi_2\right)\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\hat{\xi}\left(1+\beta\psi_1^2\right)k_{it}\\
=&\left(\left(1+\beta\right)\hat{\xi}+1-\alpha\right)\left(\psi_1 k_{it}+\psi_2\left(1+\gamma\right)\mathbb{E}_{it}\left[a_{it+1}\right]+\psi_3\varepsilon_{it+1}+\psi_4\chi_i\right)
\end{aligned}$$

Finally, matching coefficients gives

$$\begin{aligned}
\hat{\xi}\left(\beta\psi_1^2+1\right) &= \psi_1\left((1+\beta)\hat{\xi}+1-\alpha\right)\\
1+\beta\hat{\xi}\left(\psi_1+\rho\right)\psi_2 &= \psi_2\left((1+\beta)\hat{\xi}+1-\alpha\right)\Rightarrow\psi_2=\frac{1}{1-\alpha+\beta\hat{\xi}\left(1-\psi_1-\rho\right)+\hat{\xi}}\\
1+\beta\hat{\xi}\psi_1\psi_3 &= \psi_3\left((1+\beta)\hat{\xi}+1-\alpha\right)\Rightarrow\psi_3=\frac{1}{1-\alpha+\left(1-\psi_1\right)\beta\hat{\xi}+\hat{\xi}}\\
1+\beta\hat{\xi}\psi_4\left(1+\psi_1\right) &= \psi_4\left((1+\beta)\hat{\xi}+1-\alpha\right)\Rightarrow\psi_4=\frac{1}{1-\alpha+\hat{\xi}\left(1-\beta\psi_1\right)}
\end{aligned}$$

A few lines of algebra yields the expressions in (10).

A.1.2 Aggregation

To derive aggregate TFP and output, substitute the firm's optimality condition for labor

$$N_{it}=\left(\frac{\alpha_2}{W}\tilde{A}_{it}K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}$$

into the production function (1) to get

$$Y_{it} = \left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} \tilde{A}_{it}^{\frac{1}{1-\alpha_2}} K_{it}^{\frac{\alpha_1}{1-\alpha_2}} = \left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} A_{it} K_{it}^{\alpha}$$

Labor market clearing implies

$$\int N_{it} di = \int \left(\frac{\alpha_2}{W}\right)^{\frac{1}{1-\alpha_2}} A_{it} K_{it}^{\alpha} di = N$$

so that

$$\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} = \left(\frac{N}{\int A_{it} K_{it}^{\alpha} di}\right)^{\alpha_2} \Rightarrow Y_{it} = \frac{A_{it} K_{it}^{\alpha}}{\left(\int A_{it} K_{it}^{\alpha} di\right)^{\alpha_2}} N^{\alpha_2}$$

By definition,

$$MPK_{it} = \alpha \frac{A_{it} K_{it}^{\alpha-1}}{\left(\int A_{it} K_{it}^{\alpha} di\right)^{\alpha_2}} N^{\alpha_2}$$

so that

$$K_{it} = \left(\frac{\alpha A_{it}}{MPK_{it}}\right)^{\frac{1}{1-\alpha}} \left(\frac{N}{\int A_{it} K_{it}^{\alpha} di}\right)^{\frac{\alpha_2}{1-\alpha}}$$

and capital market clearing implies

$$K = \int K_{it} di = \alpha^{\frac{1}{1-\alpha}} \left(\frac{N}{\int A_{it} K_{it}^{\alpha} di}\right)^{\frac{\alpha_2}{1-\alpha}} \int A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di$$

The latter two equations give

$$K_{it}^{\alpha} = \left(\frac{A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}}}{\int A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di} K\right)^{\alpha}$$

Substituting into the expression for Y_{it} and rearranging, we can derive

$$Y_{it} = \frac{\dfrac{A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{\alpha}{1-\alpha}}}{\left(\int A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\alpha}}}{\left(\dfrac{\int A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{\alpha}{1-\alpha}} di}{\left(\int A_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di\right)^{\alpha}}\right)^{\alpha_2}} K^{\alpha_1} N^{\alpha_2}$$

Aggregating gives expressions for aggregate output and TFP:

$$Y = \int Y_{it} di = A K^{\alpha_1} N^{\alpha_2}$$

where

$$A=\left(\frac{\int A_{it}^{\frac{1}{1-\alpha}}MPK_{it}^{-\frac{\alpha}{1-\alpha}}di}{\left(\int A_{it}^{\frac{1}{1-\alpha}}MPK_{it}^{-\frac{1}{1-\alpha}}di\right)^{\alpha}}\right)^{1-\alpha_2}$$

or in logs,

$$a=(1-\alpha_2)\left[\ln\left(\int A_{it}^{\frac{1}{1-\alpha}}MPK_{it}^{-\frac{\alpha}{1-\alpha}}\right)-\alpha\ln\left(\int A_{it}^{\frac{1}{1-\alpha}}MPK_{it}^{-\frac{1}{1-\alpha}}\right)\right]$$

The first term in brackets is equal to

$$\frac{1}{1-\alpha}\overline{a}-\frac{\alpha}{1-\alpha}\overline{mpk}+\frac{1}{2}\left(\frac{1}{1-\alpha}\right)^2\sigma_a^2+\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)^2\sigma_{mpk}^2-\frac{\alpha}{(1-\alpha)^2}\sigma_{mpk,a}$$

and the second,

$$\frac{\alpha}{1-\alpha}\overline{a}-\frac{\alpha}{1-\alpha}\overline{mpk}+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_a^2+\frac{1}{2}\alpha\left(\frac{1}{1-\alpha}\right)^2\sigma_{mpk}^2-\frac{\alpha}{(1-\alpha)^2}\sigma_{mpk,a}$$

Combining,

$$\begin{aligned}
a &= (1-\alpha_2)\left[\overline{a}+\frac{1}{2}\frac{1}{1-\alpha}\sigma_a^2-\frac{1}{2}\frac{\alpha}{1-\alpha}\sigma_{mpk}^2\right]\\
&= a^*-\frac{1}{2}(1-\alpha_2)\frac{\alpha}{1-\alpha}\sigma_{mpk}^2\\
&= a^*-\frac{1}{2}\frac{\alpha_1(1-\alpha_2)}{1-\alpha_1-\alpha_2}\sigma_{mpk}^2
\end{aligned}$$

which is equation (11) in the text.

To compute the effect on output, notice that the aggregate production function is

$$y=\alpha_1k+\alpha_2n+a$$

so that

$$\begin{aligned}
\frac{dy}{d\sigma_{mpk}^2} &= \alpha_1\frac{dk}{da}\frac{da}{d\sigma_{mpk}^2}+\frac{da}{d\sigma_{mpk}^2}\\
&= \frac{da}{d\sigma_{mpk}^2}\left(1+\alpha_1\frac{dk}{da}\right)
\end{aligned}$$

In the stationary equilibrium, the aggregate marginal product of capital must be a constant, denote it by $\bar{R}$, i.e., $\ln \alpha_1 + y - k = \bar{r}$ so that

$$k = \frac{1}{1-\alpha_1}\left(\ln \alpha_1 + \alpha_2 n + a - \bar{r}\right)$$

and

$$\frac{dk}{da} = \frac{1}{1-\alpha_1}$$

Combining,

$$\frac{dy}{d\sigma^2_{mpk}} = \frac{da}{d\sigma^2_{mpk}}\left(1 + \frac{\alpha_1}{1-\alpha_1}\right) = \frac{da}{d\sigma^2_{mpk}}\frac{1}{1-\alpha_1}$$

A.1.3 Labor Market Distortions

In this section, we add labor distortions to our setup and show that they change our interpretation of the fundamental but otherwise have no effect on our analysis. We show that these distortions do not lead to measured *mpk* dispersion and so our strategy for disentangling the various sources of capital misallocation and our estimates for their magnitudes go through unchanged.

We introduce labor distortions as proportional labor 'taxes', denoted T^N_t. The firm's problem becomes

$$\mathcal{V}\left(K_{it}, \mathcal{I}_{it}\right) = \max_{N_{it}, K_{it+1}} \ \mathbb{E}_{it}\left[\tilde{A}_{it} K^{\alpha_1}_{it} N^{\alpha_2}_{it} - W T^N_{it} N_{it} - T^K_{it+1}\Phi\left(K_{it+1}, K_{it}\right) + \beta \mathcal{V}\left(K_{it+1}, \mathcal{I}_{it+1}\right)\right]$$

The labor choice satisfies the first order condition

$$N_{it} = \left(\alpha_2 \frac{\tilde{A}_{it} K^{\alpha_1}_{it}}{W T^N_{it}}\right)^{\frac{1}{1-\alpha_2}}$$

Substituting, we can derive output/revenues as

$$Y_{it} = \tilde{A}_{it} K^{\alpha_1}_{it}\left(\alpha_2 \frac{\tilde{A}_{it} K^{\alpha_1}_{it}}{W T^N_{it}}\right)^{\frac{\alpha_2}{1-\alpha_2}} = \frac{\tilde{A}^{\frac{1}{1-\alpha_2}}_{it}}{\left(T^N_{it}\right)^{\frac{\alpha_2}{1-\alpha_2}} W^{\frac{\alpha_2}{1-\alpha_2}}} K^{\alpha}_{it}$$

and operating profits (revenues net of total wages) as

$$
\begin{aligned}
\tilde{A}_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2} - WT_{it}^N N_{it} &= \tilde{A}_{it}K_{it}^{\alpha_1}\left(\alpha_2 \frac{\tilde{A}_{it}K_{it}^{\alpha_1}}{WT_{it}^N}\right)^{\frac{\alpha_2}{1-\alpha_2}} - WT_{it}^N\left(\alpha_2 \frac{\tilde{A}_{it}K_{it}^{\alpha_1}}{WT_{it}^N}\right)^{\frac{1}{1-\alpha_2}} \\
&= \frac{\tilde{A}_{it}^{\frac{1}{1-\alpha_2}}\left(K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}}{\left(WT_{it}^N\right)^{\frac{\alpha_2}{1-\alpha_2}}}\left[\alpha_2^{\frac{\alpha_2}{1-\alpha_2}} - \alpha_2^{\frac{1}{1-\alpha_2}}\right] \\
&= \frac{\tilde{A}_{it}^{\frac{1}{1-\alpha_2}}\left(K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}}{\left(T_{it}^N\right)^{\frac{\alpha_2}{1-\alpha_2}} W^{\frac{\alpha_2}{1-\alpha_2}}}(1-\alpha_2) = G\frac{\tilde{A}_{it}^{\frac{1}{1-\alpha_2}}}{\left(T_{it}^N\right)^{\frac{\alpha_2}{1-\alpha_2}}}K_{it}^{\alpha}
\end{aligned}
$$

which is the same form as in the baseline version, except now the transformed fundamental A_{it} also incorporates the effect of the labor tax on net revenues and is defined by[34]

$$
A_{it} \equiv \left(\frac{\tilde{A}_{it}}{T_{it}^{N\alpha_2}}\right)^{\frac{1}{1-\alpha_2}}
$$

With this re-interpretation, the firm's dynamic investment decision is still given by (3). To see that this also implies that labor distortions do not contibute to *mpk* dispersion, suppose that they are the only friction, i.e., the capital choice is made under full information with no adjustment costs or uncertainty. The capital choice is then static and given by

$$
K_{it} = \left(\frac{\alpha G \tilde{A}_{it}^{\frac{1}{1-\alpha_2}}}{\left(T_{it}^N\right)^{\frac{\alpha_2}{1-\alpha_2}}}\right)^{\frac{1}{1-\alpha}}
$$

Combining this with the expression for revenues, the measured *mpk* is equal to

$$
\begin{aligned}
mpk_{it} &= \text{Const} + y_{it} - k_{it} \\
&= \text{Const} + \frac{-\alpha_2}{1-\alpha_2}\tau_{it}^N + \frac{1}{1-\alpha_2}\tilde{a}_{it} + (\alpha-1)\frac{-\alpha_2}{1-\alpha_2}\frac{1}{1-\alpha}\tau_{it}^N + (\alpha-1)\frac{1}{1-\alpha_2}\frac{1}{1-\alpha}\tilde{a}_{it} \\
&= \text{Const}
\end{aligned}
$$

So, T_{it}^N does lead to any measured dispersion in the *mpk*.

A.2 Frictional Labor

In this section, we provide detailed derivations for the model solution and aggregation results in the case of frictional labor.

[34] This is also the a_{it} we would measure from the data using the definition $a_{it} = va_{it} - \alpha k_{it}$.

A.2.1 Model Solution

When labor is chosen under the same frictions as capital, the firm's value function takes the form

$$\begin{aligned}\mathcal{V}\left(K_{it}, N_{it}, \mathcal{I}_{it}\right) = \max_{K_{it+1}, N_{it+1}} \quad & \mathbb{E}_{it}\left[\tilde{A}_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - T_{it+1}\Phi\left(K_{it+1}, K_{it}\right) - T_{it+1} W \Phi\left(N_{it+1}, N_{it}\right)\right] \\ + \quad & \mathbb{E}_{it}\left[\beta \mathcal{V}\left(K_{it+1}, N_{it+1}, \mathcal{I}_{it+1}\right)\right] \end{aligned} \tag{25}$$

where the adjustment cost function $\Phi\left(\cdot\right)$ is as defined in expression (2). Because the firm makes a one-time payment to hire incremental labor, the cost of labor W is now to be interpreted as the present discounted value of wages. Capital and labor are both subject to the same adjustment friction, the same distortions, denoted T_{it+1}, and are chosen under the same information set, though the cost of labor adjustment is denominated in labor units.

The first order and envelope conditions yield two Euler equations:

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\Phi_1\left(K_{it+1}, K_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_1\tilde{A}_{it+1}K_{it+1}^{\alpha_1-1}N_{it+1}^{\alpha_2} - \beta T_{it+2}\Phi_2\left(K_{it+2}, K_{it+1}\right)\right] \\ \mathbb{E}_{it}\left[W T_{it+1}\Phi_1\left(N_{it+1}, N_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_2\tilde{A}_{it+1}K_{it+1}^{\alpha_1}N_{it+1}^{\alpha_2-1} - \beta W T_{it+2}\Phi_2\left(N_{it+2}, N_{it+1}\right)\right]\end{aligned}$$

To show that this setup leads to an intertemporal investment problem that takes the same form as (3), we prove that there exists a constant η such that $N_{it+1} = \eta K_{it+1}$ which leads to the same solution as if the firm were choosing only capital facing a degree of curvature $\alpha = \alpha_1 + \alpha_2$.

Under this conjecture, we can rewrite the firm's problem in (25) as

$$\tilde{\mathcal{V}}\left(K_{it}, \mathcal{I}_{it}\right) = \max_{K_{it+1}} \mathbb{E}_{it}\left[\frac{\eta^{\alpha_2}}{1+W\eta}\tilde{A}_{it}K_{it}^{\alpha_1+\alpha_2} - T_{it+1}\Phi\left(K_{it+1}, K_{it}\right) + \beta\tilde{\mathcal{V}}\left(K_{it+1}, \mathcal{I}_{it+1}\right)\right]$$

Let $\{K_{it}^*\}$ be the solution to this problem. By definition, it must satisfy the following optimality condition

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\Phi_1\left(K_{it+1}^*, K_{it}^*\right)\right] &= \mathbb{E}_{it}\left[\beta\frac{\left(\alpha_1+\alpha_2\right)\tilde{A}_{it+1}K_{it+1}^{*\alpha_1+\alpha_2-1}\eta^{\alpha_2}}{1+W\eta}\right] \qquad (26) \\ &- \mathbb{E}_{it}\left[\beta T_{it+2}\Phi_2\left(K_{it+2}^*, K_{it+1}^*\right)\right] \\ & \qquad (27)\end{aligned}$$

Now substitute the conjecture that $N_{it}^* = \eta K_{it}^*$ into the optimality condition for labor from the original problem and rearrange to get:

$$\mathbb{E}_{it}\left[T_{it+1}\Phi_1\left(K^*_{it+1},K^*_{it}\right)\right]=\mathbb{E}_{it}\left[\beta\frac{\alpha_2\tilde{A}_{it+1}K^{*\alpha_1+\alpha_2-1}_{it+1}\eta^{\alpha_2}}{W\eta}-\beta T_{it+2}\Phi_2\left(K^*_{it+2},K^*_{it+1}\right)\right] \tag{28}$$

If η satisfies

$$\frac{\alpha_1+\alpha_2}{1+W\eta}=\frac{\alpha_2}{W\eta} \qquad \Rightarrow \qquad W\eta=\frac{\alpha_2}{\alpha_1} \tag{29}$$

then (28) is identical to (26). In other words, under (29), the sequence $\{K^*_{it},N^*_{it}\}$ satisfies the optimality condition for labor from the original problem. It is straightforward to verify that this also implies that $\{K^*_{it},N^*_{it}\}$ satisfy the optimality condition for capital from the original problem:

$$\begin{aligned}\mathbb{E}_{it}\left[T_{it+1}\Phi_1\left(K^*_{it+1},K^*_{it}\right)\right] &= \mathbb{E}_{it}\left[\beta\alpha_1A_{it+1}K^{*\alpha_1+\alpha_2-1}_{it+1}\eta^{\alpha_2}-\beta T_{it+2}\Phi_2\left(K^*_{it+2},K^*_{it+1}\right)\right]\\ &= \mathbb{E}_{it}\left[\beta\frac{\alpha_2A_{it+1}K^{*\alpha_1+\alpha_2-1}_{it+1}\eta^{\alpha_2}}{W\eta}-\beta T_{it+2}\Phi_2\left(K^*_{it+2},K^*_{it+1}\right)\right]\end{aligned}$$

Thus, we can analyze this environment in an analogous fashion to the baseline specification in the text of the paper, where the firm's intertemporal optimization problem takes the same form as expression (3), with $\alpha=\alpha_1+\alpha_2$, $G=\frac{\eta^{\alpha_2}}{1+W\eta}$ and $A_{it}=\tilde{A}_{it}$.

A.2.2 Aggregation

To derive aggregate output and TFP for this case, we use the fact that, as shown above, $N_{it}=\eta K_{it}$ where $\eta=\frac{\alpha_2}{\alpha_1 W}$. Substitituting into the production function (1) gives

$$Y_{it}=\tilde{A}_{it}\eta^{\alpha_2}K^{\alpha_1+\alpha_2}_{it}=\tilde{A}_{it}\eta^{\alpha_2}K^{\alpha}_{it} \tag{30}$$

By definition,

$$MPK_{it}=\alpha\tilde{A}_{it}\eta^{\alpha_2}K^{\alpha-1}_{it}$$

so that

$$K_{it}=\left(\frac{\alpha\tilde{A}_{it}\eta^{\alpha_2}}{MPK_{it}}\right)^{\frac{1}{1-\alpha}}$$

and

$$K=\int K_{it}di=\alpha^{\frac{1}{1-\alpha}}\eta^{\frac{\alpha_2}{1-\alpha}}\int\tilde{A}^{\frac{1}{1-\alpha}}_{it}MPK^{-\frac{1}{1-\alpha}}_{it}di \tag{31}$$

and substituting into (30),

$$Y_{it}=\alpha^{\frac{\alpha}{1-\alpha}}\eta^{\frac{\alpha_2}{1-\alpha}}\tilde{A}^{\frac{1}{1-\alpha}}_{it}MPK^{-\frac{\alpha}{1-\alpha}}_{it}$$

so that aggregate output is

$$Y = \alpha^{\frac{\alpha}{1-\alpha}} \eta^{\frac{\alpha_2}{1-\alpha}} \int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{\alpha}{1-\alpha}} di \tag{32}$$

Using (31), we have

$$K^{\alpha_1} N^{\alpha_2} = \eta^{\alpha_2} K^{\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} \eta^{\frac{\alpha_2}{1-\alpha}} \left(\int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di \right)^{\alpha} \tag{33}$$

and from (32) and (33) we can derive aggregate TFP to be

$$A = \frac{Y}{K^{\alpha_1} N^{\alpha_2}} = \frac{\int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{\alpha}{1-\alpha}} di}{\left(\int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di \right)^{\alpha}}$$

or in logs,

$$a = \ln \left(\int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{\alpha}{1-\alpha}} di \right) - \alpha \ln \left(\int \tilde{A}_{it}^{\frac{1}{1-\alpha}} MPK_{it}^{-\frac{1}{1-\alpha}} di \right)$$

Following similar steps as in the baseline case, we can derive

$$\begin{aligned} a &= \overline{a} + \frac{1}{2} \frac{1}{1-\alpha} \sigma_{\tilde{a}}^2 - \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma_{mpk}^2 \\ &= a^* - \frac{1}{2} \frac{\alpha}{1-\alpha} \sigma_{mpk}^2 \end{aligned}$$

The output effects are the same as in the baseline case.

A.3 Financial Frictions

Including the liquidity constraint (and abstracting from distortions for ease of notation, but which are straightforward to include), the firm's problem can be written

$$\begin{aligned} \mathcal{V}\left(K_{it}, B_{it}, \mathcal{I}_{it}\right) = \max_{B_{it+1}, K_{it+1}} & \mathbb{E}_{it}\left[\Pi\left(K_{it}, A_{it}\right) + RB_{it} - B_{it+1} - \Phi\left(K_{it+1}, K_{it}\right) - \Upsilon\left(K_{it+1}, B_{it+1}\right)\right] \\ + & \beta \mathbb{E}_{it}\left[\mathcal{V}\left(K_{it+1}, B_{it+1}, \mathcal{I}_{it+1}\right)\right] \end{aligned}$$

The first order conditions are given by

$$\begin{aligned} \mathbb{E}_{it}\left[\beta \Pi_1\left(K_{it+1}, A_{it+1}\right) - \beta \Phi_2\left(K_{it+2}, K_{it+1}\right)\right] &= \Phi_1\left(K_{it+1}, K_{it}\right) + \Upsilon_1\left(K_{it+1}, B_{it+1}\right) \\ -1 - \Upsilon_2\left(K_{it+1}, B_{it+1}\right) + \beta R &= 0 \end{aligned}$$

Let

$$
\Upsilon\left(K_{it+1}, B_{it+1}\right) = \hat{\nu}\left(\frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2}}\right)
$$

$$
\Rightarrow \quad \Upsilon_2\left(K_{it+1}, B_{it+1}\right) = -\hat{\nu}\omega_2 \frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2+1}}, \qquad \Upsilon_1\left(K_{it+1}, B_{it+1}\right) = \hat{\nu}\omega_1 \frac{K_{it+1}^{\omega_1-1}}{B_{it+1}^{\omega_2}}
$$

Using the FOC for B_{it+1}

$$
\begin{aligned}
1 &= \hat{\nu}\omega_2 \frac{K_{it+1}^{\omega_1}}{B_{it+1}^{\omega_2+1}} + \beta R \quad \Rightarrow \quad B_{it+1} = \left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{1}{\omega_2+1}} K_{it+1}^{\frac{\omega_1}{\omega_2+1}} \\
\Upsilon_1\left(K_{it+1}, B_{it+1}\right) &= \hat{\nu}\omega_1 \frac{K_{it+1}^{\omega_1-1}}{B_{it+1}^{\omega_2}} = \hat{\nu}\omega_1 \frac{K_{it+1}^{\omega_1-1}}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}} K_{it+1}^{\frac{\omega_2\omega_1}{\omega_2+1}}} = \frac{\hat{\nu}\omega_1}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}}} \left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}} K_{it+1}^{\frac{\omega_1-(\omega_2+1)}{\omega_2+1}} \\
&= \nu K_{it+1}^{\omega}
\end{aligned}
$$

where

$$
\begin{aligned}
\nu &\equiv \frac{\hat{\nu}\omega_1}{\left(\frac{\hat{\upsilon}\omega_2}{1-\beta R}\right)^{\frac{\omega_2}{\omega_2+1}}} \left(1-\beta R\right)^{\frac{\omega_2}{\omega_2+1}} \\
\omega &\equiv \frac{\omega_1 - (\omega_2+1)}{\omega_2+1}
\end{aligned}
$$

Approximating,

$$
\begin{aligned}
\bar{\Upsilon}_1 + \bar{\Upsilon}_1 \upsilon_{1t+1} &\approx \nu \bar{K}^{\omega} + \nu \bar{K}^{\omega} \omega k_{it+1} \\
\bar{\Upsilon}_1 \upsilon_{1t+1} &\approx \nu \bar{K}^{\omega} \omega k_{it+1}
\end{aligned}
$$

and subsituting into the FOC,

$$
\mathbb{E}_{it}\left[\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}\left(a_{it+1} + (\alpha-1)\, k_{it+1}\right) + \beta\xi\left(k_{it+2} - k_{it+1}\right)\right] = \xi\left(k_{it+1} - k_{it}\right) + \nu\bar{K}^{\omega}\omega k_{it+1}
$$

or

$$
k_{it+1}\left(\left(1+\beta\right)\hat{\xi} + 1 - \tilde{\alpha}\right) = \mathbb{E}_{it}\left[a_{it+1}\right] + \beta\hat{\xi}\mathbb{E}_{it}\left[k_{it+2}\right] + \hat{\xi}k_{it}
$$

where

$$
\tilde{\alpha} \equiv \alpha - \frac{\nu\bar{K}^{\omega}\omega}{\alpha\beta\bar{G}\bar{A}\bar{K}^{\alpha-1}}
$$

Adding distortions in the same manner as in the baseline case gives the same expression as (4), but with α replaced by $\tilde{\alpha}$.

A.4 Identification

In this Appendix we derive analytic expressions for the four moments in the random walk case, i.e., when $\rho = 1$, and prove proposition 1.

A.4.1 Moments

From expression (9), we have the firm's investment policy function

$$k_{it+1} = \psi_1 k_{it} + \psi_2 (1+\gamma) \mathbb{E}_{it} [a_{it+1}] + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

and substituting for the expectation and defining $\hat{\psi}_2 \equiv \psi_2 (1+\gamma)$ to ease notation,

$$k_{it+1} = \psi_1 k_{it} + \hat{\psi}_2 (a_{it} + \phi (\mu_{it+1} + e_{it+1})) + \psi_3 \varepsilon_{it+1} + \psi_4 \chi_i$$

where $\phi = \frac{\mathbb{V}}{\sigma_e^2}$ so that $1 - \phi = \frac{\mathbb{V}}{\sigma_\mu^2}$. Then,

$$\Delta k_{it+1} = \psi_1 \Delta k_{it} + \hat{\psi}_2 ((1-\phi) \mu_{it} + \phi \mu_{it+1} + \phi (e_{it+1} - e_{it})) + \psi_3 (\varepsilon_{it+1} - \varepsilon_{it})$$

We will use the fact that

$$\begin{aligned} \mathrm{cov}(\Delta k_{it+1}, \mu_{it+1}) &= \hat{\psi}_2 \phi \sigma_\mu^2 \\ \mathrm{cov}(\Delta k_{it+1}, e_{it+1}) &= \hat{\psi}_2 \phi \sigma_e^2 \\ \mathrm{cov}(\Delta k_{it+1}, \varepsilon_{it+1}) &= \psi_3 \sigma_\varepsilon^2 \end{aligned}$$

Now,

$$\begin{aligned} \mathrm{var}(\Delta k_{it+1}) &= \psi_1^2 \mathrm{var}(\Delta k_{it}) + \hat{\psi}_2^2 (1-\phi)^2 \sigma_\mu^2 + \hat{\psi}_2^2 \phi^2 \sigma_\mu^2 + 2\hat{\psi}_2^2 \phi^2 \sigma_e^2 + 2\psi_3^2 \sigma_\varepsilon^2 \\ &+ 2\psi_1 \hat{\psi}_2 (1-\phi) \mathrm{cov}(\Delta k_{it}, \mu_{it}) - 2\psi_1 \hat{\psi}_2 \phi \mathrm{cov}(\Delta k_{it}, e_{it}) - 2\psi_1 \psi_3 \mathrm{cov}(\Delta k_{it}, \varepsilon_{it}) \end{aligned}$$

where substituting, rearranging and using the fact that the moments are stationary gives

$$\sigma_k^2 \equiv \mathrm{var}(\Delta k_{it}) = \frac{(1+\gamma)^2 \psi_2^2 \sigma_\mu^2 + 2(1-\psi_1) \psi_3^2 \sigma_\varepsilon^2}{1-\psi_1^2}$$

which can be rearranged to yield expression (13).

Next,

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \Delta k_{it}\right) &= \psi_1 \text{var}\left(\Delta k_{it}\right) + \hat{\psi}_2 \left(1-\phi\right) \text{cov}\left(\Delta k_{it}, \mu_{it}\right) \\
&- \hat{\psi}_2 \phi \text{cov}\left(\Delta k_{it}, e_{it}\right) - \psi_3 \text{cov}\left(\Delta k_{it}, \varepsilon_{it}\right) \\
&= \psi_1 \text{var}\left(\Delta k_{it}\right) - \psi_3 \text{cov}\left(\Delta k_{it}, \varepsilon_{it}\right) \\
&= \psi_1 \sigma_k^2 - \psi_3^2 \sigma_\varepsilon^2
\end{aligned}$$

so that

$$\rho_{k,k_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta k_{it-1}\right) = \psi_1 - \psi_3^2 \frac{\sigma_\varepsilon^2}{\sigma_k^2}$$

which is expression (14).

Similarly,

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \Delta a_{it}\right) &= \text{cov}\left(\Delta k_{it+1}, \mu_{it}\right) \\
&= \psi_1 \text{cov}\left(\Delta k_{it}, \mu_{it}\right) + \hat{\psi}_2 \left(1-\phi\right) \sigma_\mu^2 \\
&= \psi_1 \hat{\psi}_2 \phi \sigma_\mu^2 + \hat{\psi}_2 \left(1-\phi\right) \sigma_\mu^2 \\
&= \left(1 - \phi\left(1-\psi_1\right)\right) \psi_2 \left(1+\gamma\right) \sigma_\mu^2
\end{aligned}$$

and from here it is straightforward to derive

$$\rho_{k,a_{-1}} \equiv \text{corr}\left(\Delta k_{it}, \Delta a_{it-1}\right) = \left[\frac{\mathbb{V}}{\sigma_\mu^2}\left(1-\psi_1\right) + \psi_1\right] \frac{\sigma_\mu \psi_2 \left(1+\gamma\right)}{\sigma_k}$$

as in expression (15).

Finally,

$$mpk_{it} = \text{Const} + y_{it} - k_{it} = \text{Const} + a_{it} + \alpha k_{it} - k_{it} = \text{Const} + a_{it} - \left(1-\alpha\right) k_{it}$$

so that

$$\Delta mpk_{it} = \Delta a_{it} - \left(1-\alpha\right) \Delta k_{it} = \mu_{it} - \left(1-\alpha\right) \Delta k_{it}$$

which implies

$$\text{cov}\left(\Delta mpk_{it}, \mu_{it}\right) = \left(1 - \left(1-\alpha\right)\left(1+\gamma\right) \psi_2 \phi\right) \sigma_\mu^2$$

and

$$\lambda_{mpka,a} \equiv \frac{\mathrm{cov}\left(\Delta mpk_{it}, \mu_{it}\right)}{\sigma_\mu^2} = 1 - (1-\alpha)(1+\gamma)\psi_2\phi$$

$$= 1 - (1-\alpha)(1+\gamma)\psi_2\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right)$$

which is expression (16).

To see that the correlation $\rho_{mpka,a}$ is decreasing in σ_ε^2, we derive

$$\begin{aligned}
\mathrm{var}\left(\Delta mpk_{it}\right) &= \sigma_\mu^2 + (1-\alpha)^2\sigma_k^2 - 2(1-\alpha)\,\mathrm{cov}\left(\Delta k_{it}, \mu_{it}\right) \\
&= \sigma_\mu^2 + (1-\alpha)^2\left(\frac{\hat{\psi}_2^2\sigma_\mu^2 + 2(1-\psi_1)\psi_3^2\sigma_\varepsilon^2}{1-\psi_1^2}\right) - 2(1-\alpha)\hat{\psi}_2\phi\sigma_\mu^2 \\
&= \frac{1}{1-\psi_1^2}\left(\left(\left(1-\psi_1^2\right)(1-2(1-\alpha)(1+\gamma)\psi_2\phi) + (1-\alpha)^2(1+\gamma)^2\psi_2^2\right)\sigma_\mu^2\right) \\
&+ \frac{1}{1-\psi_1^2}\left(2(1-\alpha)^2(1-\psi_1)\psi_3^2\sigma_\varepsilon^2\right)
\end{aligned}$$

so

$$\rho_{mpk,a} = \frac{(1-(1-\alpha)(1+\gamma)\psi_2\phi)\,\sigma_\mu\sqrt{1-\psi_1^2}}{\sqrt{\left(\left(1-\psi_1^2\right)(1-2(1-\alpha)(1+\gamma)\psi_2\phi) + (1-\alpha)^2(1+\gamma)^2\psi_2^2\right)\sigma_\mu^2 + 2(1-\alpha)^2(1-\psi_1)\psi_3^2\sigma_\varepsilon^2}} \tag{34}$$

A.4.2 Proof of Proposition 1

Write the variance of investment as

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + (1+\gamma)^2\psi_2^2\sigma_\mu^2 + 2(1-\psi_1)\psi_3^2\sigma_\varepsilon^2$$

To rewrite the last term as a function of an observable moment, use the autocovariance of investment,

$$\sigma_{k,k_{-1}} = \psi_1\sigma_k^2 - \psi_3^2\sigma_\varepsilon^2 \tag{35}$$

and substitution yields

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + (1+\gamma)^2\psi_2^2\sigma_\mu^2 + 2(1-\psi_1)\left(\psi_1\sigma_k^2 - \sigma_{k,k_{-1}}\right) \tag{36}$$

To eliminate the second term, use the equation for $\lambda_{mpk,a}$ to solve for

$$(1+\gamma)\,\psi_2\phi = \frac{1-\lambda_{mpk,a}}{1-\alpha} = \tilde{\lambda} \tag{37}$$

where $\tilde{\lambda}$ is a decreasing function of $\lambda_{mpk,a}$ that depends only on the known parameter α. Substituting into the expression for the covariance of investment with the lagged shock, $\sigma_{k,a_{-1}}$, and rearranging yields

$$(1+\gamma)\,\psi_2 = \frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}\,(1-\psi_1) \tag{38}$$

which is an equation in ψ_1 and observable moments. Substituting into (36) gives

$$\sigma_k^2 = \psi_1^2\sigma_k^2 + \left(\frac{\sigma_{k,a_{-1}}}{\sigma_\mu^2} + \tilde{\lambda}\,(1-\psi_1)\right)^2 \sigma_\mu^2 + 2\,(1-\psi_1)\left(\psi_1\sigma_k^2 - \sigma_{k,k_{-1}}\right)$$

and rearranging, we can derive

$$0 = \left(\hat{\lambda}^2 - 1\right)(1-\psi_1)^2 + 2\left(\hat{\lambda}\rho_{k,a_{-1}} - \rho_{k,k_{-1}}\right)(1-\psi_1) + \rho_{k,a_{-1}}^2 \tag{39}$$

where

$$\hat{\lambda} = \frac{\sigma_\mu}{\sigma_k}\tilde{\lambda} = \frac{\sigma_\mu}{\sigma_k}\left(\frac{1-\lambda_{mpk,a}}{1-\alpha}\right)$$

Equation (39) represents a quadratic equation in a single unknown, $1-\psi_1$, or equivalently, in ψ_1. The solution features two positive roots, one greater than one and one less. The smaller root corresponds to the true ψ_1 that represents the solution to the firm's investment policy. The value of ψ_1 pins down the adjustment costs parameter $\hat{\xi}$ as well as ψ_2 and ψ_3. We can then back out γ from (38), ϕ (and so $\mathbb{V}$) from (37) and finally, σ_ε^2 from (35).

B Data

As described in the text, our Chinese data are from the Annual Surveys of Industrial Production conducted by the National Bureau of Statistics. The data span the period 1998-2009 and are built into a panel following quite closely the method outlined in Brandt et al. (2014). We measure the capital stock as the value of fixed assets and calculate investment as the change in the capital stock relative to the preceding period. We construct firm fundamentals a_{it} as the log of value-added less α multiplied by the log of the capital stock and (the log of) the marginal product of capital (up to an additive constant) as the log of value-added less the log of the capital stock. We compute value-added from revenues using a share of intermediates of 0.5 (our data does not include a direct measure of value-added in all years). We first difference the investment and fundamental series to compute investment growth and changes in fundamentals. To extract the firm-specific variation in our variables, we regress each on a year by time fixed-effect and work with the residual. Industries are defined at the 4-digit level. This eliminates the industry-wide component of each series common to all firms in an industry and time period (as well the aggregate component common across all firms) and leaves only the idiosyncratic variation. To estimate the parameters governing firm fundamentals, i.e., the persistence ρ and variance of the innovations σ_μ^2, we perform the autoregression implied by (5), again including industry by year controls. We eliminate duplicate observations (firms with multiple observations within a single year) and trim the 3% tails of each series. We additionally exclude observations with excessively high variability in investment (investment rates over 100%). Our final sample in China consists of 797,047 firm-year observations.

Our US data are from Compustat North America and again span the period 1998-2009. We measure the capital stock using gross property, plant and equipment. We treat the data in exactly the same manner as just described for the set of Chinese firms. We additionally eliminate firms that are not incorporated in the US and/or do not report in US dollars. Our final sample in the US consists of 34,260 firm-year observations.

Table 8 reports a number of summary statistics from one year of our data, 2009: the number of firms (with available data on sales), the share of GDP they account for, and average sales and capital.

Table 8: Sample Statistics 2009

	No. of Firms	Share of GDP	Avg. Sales ($M)	Avg. Capital ($M)
China	303623	0.65	21.51	8.08
US	6177	0.45	2099.33	1811.35

C Additional Countries

In this Appendix, we apply our empirical methodology to two additional countries for which we have firm-level data - Colombia and Mexico. As described in the text, the Colombian data come from the Annual Manufacturers Survey (AMS) and span the years 1982-1998. The AMS contains plant-level data and covers plants with more than 10 employees, or sales above a certain threshold (around $35,000 in 1998, the last year of the data). We use data on output and capital, which includes buildings, structures, machinery and equipment. The construction of these variables is described in detail in Eslava et al. (2004). Plants are classified into industries defined at a 4-digit level. The Mexican data are from the Annual Industrial Survey over the years 1984-1990, which covers plants of the 3200 largest manufacturing firms. They are also at the plant-level. We use data on output and capital, which includes machinery and equipment, the value of current construction, land, transportation equipment and other fixed capital assets. A detailed description is in Tybout and Westbrook (1995). Plants are again classified into industries defined at a 4-digit level. For both countries, we compute the target moments following the same methodology as outlined in the text of the paper for China and the US. Our final sample for Colombia consists of 44,909 plant-year observations; for Mexico, 3,208.

Table 9: Additional Countries

Moments	ρ	σ^2_μ	$\rho_{i,a_{-1}}$	$\rho_{i,i_{-1}}$	$\rho_{mpk,a}$	σ^2_i	σ^2_{mpk}
Colombia	0.95	0.09	0.28	−0.35	0.61	0.07	0.98
Mexico	0.93	0.07	0.17	−0.39	0.69	0.02	0.79
Parameters			$\hat{\xi}$	$\mathbb{V}$	γ	σ^2_ε	σ^2_χ
Colombia			0.54	0.05	−0.55	0.01	0.60
Mexico			0.13	0.04	−0.82	0.00	0.42
$\Delta\sigma^2_{mpk}$							
Colombia			0.02	0.05	0.30	0.01	0.60
Mexico			0.00	0.04	0.36	0.00	0.42
$\frac{\Delta\sigma^2_{mpk}}{\sigma^2_{mpk}}$							
Colombia			2.5%	5.6%	30.9%	0.7%	61.3%
Mexico			0.5%	4.9%	44.9%	0.0%	52.8%
Δa							
Colombia			0.01	0.02	0.11	0.00	0.22
Mexico			0.00	0.01	0.13	0.00	0.16
Δy							
Colombia			0.01	0.03	0.16	0.00	0.31
Mexico			0.00	0.02	0.18	0.00	0.22