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PROJECTIONS AND UNCERTAINTIES ABOUT CLIMATE CHANGE IN AN ERA OF MINIMAL CLIMATE POLICIES

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ABSTRACT

Climate change remains one of the major international environmental challenges facing nations. Yet nations have to date taken minimal policies to slow climate change. Moreover, there has been no major improvement in emissions trends as of the latest data. The current study uses the updated DICE model to present new projections and the impacts of alternative climate policies. It also presents a new set of estimates of the uncertainties about future climate change and compares the results will those of other integrated assessment models. The study confirms past estimates of likely rapid climate change over the next century if there are not major climate-change policies. It suggests that it will be extremely difficult to achieve the 2°C target of international agreements even if ambitious policies are introduced in the near term. The required carbon price needed to achieve current targets has risen over time as policies have been delayed.

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A data appendix is available at http://www.nber.org/data-appendix/w22933

I. Introduction

Background

Climate change remains the central environmental issue of today. While the Paris Agreement on climate change of 2015 (see Paris Agreement 2016) has recently been ratified, it is limited to voluntary emissions reductions for major countries. No binding agreement for emissions reductions is currently in place since the Kyoto Protocol expired in 2012. Countries have agreed on a target temperature limit of 2 °C, but this seems far removed from actual policies, and probably infeasible, as will be seen below. The fact is that most countries are on a business-as-usual (BAU) trajectory of low or no policies to reduce their emissions, taking non-cooperative policies that are in their national interest but far from ones that would be a global cooperative policy.

Given the realities of actual climate policies, it is critical to determine the trajectory the world is on, or what the BAU involves. The most recent IPCC report (IPCC Fifth Assessment Report, Science, 2013) ignores the BAU; instead, it examines stylized trajectories that are not clearly related to actual policies, output, and emissions paths. It would be difficult, reading the most recent IPCC reports, to determine what the best guess is as to future climate change in an unregulated policy space.

The present study attempts to fill this void by investigating in detail the implications of a world in the absence of climate policies. It does so with a newly revised model, the DICE-2016R model (DICE stands for Dynamic Integrated model of Climate and the Economy). In addition to standard runs, we have added a new dimension by investigating uncertainty about several important parameters to the model estimates. The results confirm and even strengthen earlier results indicating the high likelihood of rapid warming and major damages if policies continue along the unrestrained path.

The methods here employ an approach known as integrated assessment models (IAMs). These are approaches to the economics of climate change that integrate the different elements into a single interrelated model. The present study presents the results of a fully revised version of the DICE model (as of 2016). This is the first major revision since the Fifth Assessment Report of the IPCC. This study describes the changes in the model from the last round, presents updated estimates of the different variables, and compares the new estimates with other models. In addition, the analysis provides uncertainties in the projections using new estimates of the underlying uncertainties of major parameters.

Overview of results

I will not attempt to summarize the entire paper but provide some highlights. The first result is that the revised DICE model shows more rapid growth of output and a higher temperature trajectory in the baseline path compared to earlier DICE versions and most other models. This is also reflected in a major upward revision in the social cost of carbon (SCC) and the optimal carbon tax in the current period. For example, the estimate of the SCC has been revised upwards by about 50% since the last model. There are several components of this change – some of methods and some of data – but the change is not encouraging.

A second result is that the international target for climate change with a limit of 2 °C appears to be infeasible with reasonably accessible technologies – and this is the case even with very stringent and unrealistically ambitious abatement strategies. This is so because of the inertia of the climate system, of rapid projected economic growth in the near term, and of revisions in several elements of the model. A target of 2½ °C is technically feasible but would require extreme virtually universal global policy measures.

A third point is to emphasize that this study focuses on the business-as-usual trajectory, or the one that would occur without effective climate policies. The approach of studying business as usual has fallen out of favor with analysts, who concentrate on temperature- or concentration-limiting scenarios. A careful study of limited-policy or no-policy scenarios may be depressing, but it is critical in the same way a CT scan is for a cancer patient. Moreover, notwithstanding what may be called "The Rhetoric of Nations," there has been little progress in taking strong policy measures. For example, of the six largest countries or regions, only the EU has implemented national climate policies, and the policies of the EU today are very modest. Moreover, from the perspective of political economy in different countries as of December 2016, the prospects of strong policy measures appear to be dimming rather than brightening.

The paper also investigates the implications of uncertainty for climate change. When uncertainties are taken into account, the expected value of most of the major geophysical variables, such as temperature, are largely unchanged. However, the social cost of carbon (SCC) is higher (by about 15%) under uncertainty than in the certainty-equivalent case because of asymmetry in the impacts of uncertainty on the damages from climate change. We note as well that even under highly "optimistic" outcomes (by which I mean those that have the most favorable realizations of the uncertain variables) global temperature increases markedly, and there are significant damages.

An additional important finding is that the relative uncertainty is much higher for economic variables than for geophysical variables. More precisely, the dispersion of results (measured say by the standard deviation) relative to the mean is larger for emissions, output, damages, and the SCC than for concentrations or temperature. This result is primarily because of the large uncertainty about economic growth. From a statistical point of view, uncertainty about most geophysical parameters is a *level* uncertainty and is roughly constant over time; whereas the uncertainty of economic variables is a *growth-rate* uncertainty and therefore tends to grow over time. By the year 2100, this implies a greater uncertainty from economic variables.

This study makes one further important point about uncertainty. On one question there is no doubt: the scientific crystal ball is cloudy for the path of climate change and its impacts. The ranges of uncertainty for future emissions, concentrations, temperature, and damages are extremely large. This does not imply, however, that *current policy* is to wait and do nothing. To reiterate, when taking uncertainties into account, the strength of policy (as measured by the social cost of carbon or the optimal carbon tax) would increase, not decrease.

As a final point, I emphasize that many uncertainties remain. We do not know, and are unlikely soon to know, how the global economy or energy technologies will evolve; or what the exact response of geophysical systems will be to evolving economic conditions; or exactly how damaging the changes will be for the economy as well as non-market and non-human systems. We also do not know with precision how to represent the different systems in our economic and scientific models. And the best practice has evolved over time as we learn more about all these systems. But we must take stock of what we know now as well as the implications of our actions. And the bottom line here is that this most recent taking stock has more bad news than good news, and that the need for policies to slow climate change are more and not less pressing.

II. **The Structure of the DICE-2016R Model**

The analysis begins with a discussion of the DICE-2016R model, which is a revised version of the DICE-2013R model (see Nordhaus 2014, Nordhaus and Sztorc 2013 for a detailed description of the earlier version). It is the latest version of a series of models of the economics of global warming developed at Yale University by Nordhaus and colleagues. The first version of the global dynamic model was Nordhaus (1992). The discussion explains the major modules of the model, and describes the major revisions since the 2013 version. The current version of the DICE-2016R is available at *http://www.econ.yale.edu/~nordhaus/homepage/DICEmodels09302016.htm.*

The DICE model views climate change in the framework of economic growth theory. In a standard neoclassical optimal growth model known as the Ramsey model, society invests in capital goods, thereby reducing consumption today, in order to increase consumption in the future. The DICE model modifies the Ramsey model to include climate investments, which are analogous to capital investments in the standard model. The model contains all elements from economics through climate change to damages in a form that attempts to represent simplified best practice in each area.

Equations of the DICE-2016R model

Most of the analytical background is similar to that in the 2013R model, and for details readers are referred to Nordhaus and Sztorc (2013). Major revisions are discussed as the equations are described.

The model optimizes a social welfare function, *W,* which is the discounted sum of the population-weighted utility of per capita consumption. The notation here is that *V* is the instantaneous social welfare function, *U* is the utility function, *c(t)* is per capita consumption, and *L(t)* is population. The discount factor on welfare is $R(t) = (1+\rho)^{-t}$, where ρ is the pure rate of social time preference or generational discount rate on welfare.

$$(1)\quad W = \sum_{t=1}^{Tmax} V[c(t), L(t)]\, R(t) = \sum_{t=1}^{Tmax} U[c(t)]\, L(t)\, R(t)$$

The utility function has a constant elasticity with respect to per capita consumption of the form $U(c) = c^{1-\alpha}/(1-\alpha)$. The parameter α is interpreted as generational inequality aversion. In the present version, the utility discount rate is 1.5% per year and the rate of inequality aversion is 1.45. As described below, these parameters are set to calibrate real interest rates in the model.

Net output is gross output reduced by damages and mitigation costs:

$$(2)\quad Q(t) = \Omega(t)[1 - \Lambda(t)]Y(t)$$

In this specification, *Q(t)* is output net of damages and abatement, *Y(t)* is gross output, which is a Cobb-Douglas function of capital, labor, and technology. Total output is divided between total consumption and total gross investment. Labor is proportional to population, while capital accumulates according to an optimized savings rate.

The current version develops global output in greater detail than earlier versions. The global output concept is PPP (purchasing power parity) as used by the International Monetary Fund at the country level. The growth concept is the weighted growth rate of real GDP of different countries, where the weights are the country shares of world nominal GDP using current international dollars. We constructed our own version of world output, and this corresponds closely to the IMF estimate of the growth of real output in constant international (PPP) dollars. The earlier model used the World Bank growth figures, but the World Bank growth rates by region could not be replicated.

The present version substantially revised both the historical growth estimates and the projections of per capita output growth. Future growth is based largely on a survey of experts conducted by Christensen et al. (2016). Growth in per capita output over the 1980 – 2015 period was 2.2% per year. Growth in per capita output from 2015 to 2050 is projected at 2.1 % per year, while that to 2100 is projected at 1.9% per year. The revisions are updated to incorporate the latest output, population, and emissions data and projections. Population data and projections through 2100 are from the United Nations. CO_2 emissions are from the Carbon Dioxide Information Analysis Center (CDIAC) and updated using various sources. Non-CO_2 radiative forcings for 2010 and projections to 2100 are from projections prepared for the IPCC Fifth Assessment.

The additional variables in the production function are $\Omega(t)$ and $\Lambda(t)$, which represent the damage function and the abatement-cost function, respectively. The abatement cost function, $\Lambda(t)$ in equation (2) above, was recalibrated to the abatement cost functions of other IAMs as represented in the modeling uncertainty project or MUP study (Gillingham et al. 2015). The result was a slightly more costly abatement function than earlier estimates.

The model assumes the existence of a "backstop technology," which is a technology that produces energy services at a constant (but high) cost with zero GHG emissions. The backstop price in 2020 is \$550 per ton of CO_2-equivalent, and the backstop cost declines at 5% per year. Additionally, it is assumed that there are no "negative emissions" technologies initially, but that negative emissions are available after 2150. The existence of negative-emissions technologies is critical to reaching low-temperature targets, as described below.

The damage function is defined as $\Omega(t) = D(t) / [1 + D(t)]$, where

$$(3) \quad D(t) = \psi_1 T_{AT}(t) + \psi_2 [T_{AT}(t)]^2$$

Equation (3) describes the economic impacts or damages of climate change. The DICE-2016R model takes globally averaged temperature change (T_{AT}) as a sufficient statistic for damages. Equation (3) assumes that damages can be reasonably well approximated by a quadratic function of temperature change. The estimates of the coefficients of the damage function are explained below

Uncontrolled industrial CO_2 emissions are given by a level of carbon intensity or CO CO_2-output ratio, $\sigma(t)$, times gross output. Total CO_2 emissions, $E(t)$, are equal to uncontrolled emissions reduced by the emissions-reduction rate, $\mu(t)$, plus exogenous land-use emissions.

(4) $$E(t) = \sigma(t)[1 - \mu(t)]Y(t) + E_{Land}(t)$$

The model has been revised to incorporate a more rapid decline in the CO_2-output ratio (or what is called decarbonization) to reflect the last decade's observations. The decade through 2010 showed relatively slow decarbonization, with the global CO_2/GDP ratio changing at -0.8 % per year. However, the most recent data indicate a sharp downward tilt, with the global CO_2/GDP ratio changing at -2.1% per year over the 2000 - 2015 period (preliminary data). Whether this is structural or the result of policy is unclear at this point. For the DICE model, we assume that the rate of decarbonization going forward is -1.5 % per year (using the IMF output concept). Figure 1 shows the global trend in the CO_2-GDP ratio since 1960. Note the increase in the rate of decarbonization in the last few years.

The geophysical equations link greenhouse-gas emissions to the carbon cycle, radiative forcings, and climate change. Equation (5) represents the equations of the carbon cycle for three reservoirs.

(5) $$M_j(t) = \phi_{0j}E(t) + \sum_{i=1}^{3}\phi_{ij}M_i(t-1)$$

The three reservoirs are *j = AT, UP,* and *LO,* which are the atmosphere, the upper oceans and biosphere, and the lower oceans, respectively. The parameters ϕ_{ij} represent the flow parameters between reservoirs per period. All emissions flow into the atmosphere. The 2016 version incorporates new research on the carbon cycle. Earlier versions of the DICE model were calibrated to fit the short-run carbon cycle (primarily the first 100 years). Because we plan to use the model for long-run estimates, such as the impacts on the melting of large ice sheets, it was decided to change the calibration to fit the atmospheric retention of CO_2 for periods up to 4000 years. Based on studies of Archer et al. (2009), the 2016 version of the three-box model does a much better job of simulating the long-run behavior of larger models with full ocean chemistry. This change has a major impact on the long-run trends

The relationship between GHG accumulations and increased radiative forcing is shown in equation (6).

(6) $$F(t) = \eta\{log_2[M_{AT}(t) / M_{AT}(1750)]\} + F_{EX}(t)$$

F(t) is the change in total radiative forcings from anthropogenic sources such as CO_2. $F_{EX}(t)$ is exogenous forcings, and the first term is the forcings due to atmospheric concentrations of CO_2.

Forcings lead to warming according to a simplified two-level global climate model:

$$(7) \quad T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}$$

$$(8) \quad T_{LO}(t) = T_{LO}(t-1) + \xi_4[T_{AT}(t-1) - T_{LO}(t-1)]$$

In these equations, $T_{AT}(t)$ is the global mean surface temperature and $T_{LO}(t)$ is the mean temperature of the deep oceans.

The climate module has been revised to reflect recent earth system models. We have set the equilibrium climate sensitivity (ECS) using the analysis of Olsen et al. (2012). The Olsen et al. study uses a Bayesian approach, with a prior based on previous studies and a likelihood based on observational or modeled data. The reasons for using this approach are provided in Gillingham et al. (2015). The final estimate is mean warming of 3.1 °C for an equilibrium CO_2 doubling. We adjust the transient climate sensitivity or TCS (sometimes called the transient climate response) to correspond to models with an ECS of 3.1 °C, which produces a TCS of 1.7 °C.

The treatment of discounting is identical to that in DICE-2013R. We always distinguish between the welfare discount rate (ρ) and the goods discount rate (r). The welfare discount rate applies to the well-being of different generations, while the goods discount rate applies to the return on capital investments. The former is not observed, while the latter is observed in markets. When the term "discount rate" is used without a modifier, this will always refer to the discount rate on goods.

The economic assumption behind the DICE model is that the goods discount rate should reflect actual economic outcomes. This implies that the assumptions about model parameters should generate savings rates and rates of return on capital that are consistent with observations. With the current calibration, the discount rate (or equivalently the real return for on investment) averages 4¼% per year over the period to 2100. This is the global average of a lower figure for the U.S. and a higher figure for other countries and is consistent with estimates in other studies that use U.S. data.

This specification used in the DICE model is sometimes called the "descriptive approach" to discounting. The alternative approach, used in *The Stern Review* 2007 and elsewhere, is called the "prescriptive discount rate." Under this second approach, the discount rate is assumed on a normative basis and determined largely independently of actual market returns on investments.

III. Approach to estimating uncertainties

Background

The baseline DICE model uses the expected values of the parameters such as productivity growth or equilibrium temperature sensitivity. In the present study, we have examined uncertainties about major results looking at parametric uncertainties.

Developing reliable estimates that incorporate uncertainty has proven extremely challenging on both methodological and empirical grounds (see Gillingham et al. 2015). Two major sources of uncertainty are "model uncertainty" and "structural uncertainty." The difference *across models* is called model uncertainty. Using this approach, also known as ensemble uncertainty, is a convenient for estimating uncertainty because the "data," which are results of different models, are readily collected and validated. The concern with this approach is that it is conceptually incorrect and that there is a degree of arbitrariness concerning the selection of studies to include in the ensemble.[2]

Structural uncertainty, or uncertainty *within models,* arises from imprecision in knowledge of parameters or variables as well as uncertainty about model structure. For example, climate scientists are unsure about the response of climate to increasing greenhouse-gas forcings. The present study chiefly examines structural uncertainty focusing only on uncertainties about parameters.

It will be helpful to explain the structure of the current approach analytically. We can represent a model as a mapping from exogenous and policy variables and parameters to endogenous outcomes. A model can be written symbolically as follows:

(9) $$Y = H(z, \alpha, u)$$

In this schema, Y is a vector of model outputs; z is a vector of exogenous and policy variables; α is a vector of model parameters; u is a vector of uncertain parameters to be investigated; and H represents the model structure (described above for the DICE model).

The first step is to select the uncertain parameters for analysis. For the present study, we have selected five variables: The equilibrium temperature sensitivity (ETS); productivity growth; the damage function; the carbon cycle; and the rate of

[2] Ensemble uncertainty is analytically incorrect because it examines the difference in the mean values of parameters across models. To see this point, assume that models use the same data and their structures and are identical. They will have zero ensemble uncertainty.

decarbonization. For each we derive a probability density function (pdf). We label the joint distribution as $g(u_1, u_2, u_3, u_4, u_5)$. For this study, we take the distributions to be independent and denote them as $f_i(u_i)$, which implies that $g(u_1, u_2, u_3, u_4, u_5) = f_1(u_1) f_2(u_2) f_3(u_3) f_4(u_4) f_5(u_5)$.[3] We then map the distribution of the uncertain parameters into the distribution of the output variables, given schematically by $h(Y)$ as follows based on (9):

(10) $h(Y) = H[z, \alpha, f_1(u_1) f_2(u_2) f_3(u_3) f_4(u_4) f_5(u_5)]$

A critical decision is how actually to calculate the mapping in (10) for complex systems. A standard approach is a Monte Carlo sampling of the uncertain variables. This turns out to be computationally infeasible for the DICE model in the GAMS code given constraints.[4] We have therefore discretized the distributions and done a complete enumeration of the states of the world. More precisely, we took each of the uncertain variables and separated the distributions into quintiles. We next took the expected values of the uncertain variables in each quintile, obtaining discrete values for each variable of $\{u_i(1), u_i(2), u_i(3), u_i(4), u_i(5)\}$, where $u_i(k)$ is the *kth* quintile of uncertain variable *i.*

We make two adjustments of the discrete distributions for present purposes. First, we set the middle quintile equal to the expected value of the parameter. This is done so that we can easily compare the median and mean outcomes. Secondly, because the first adjustment changes the means and standard deviations for the uncertain variables, we adjust quintile values so that the means and standard deviations are preserved. This second step involved small adjustments in the non-central quintile values.

While the algorithm for estimating uncertainty is computationally efficient, an important question involves its accuracy. If the model in (10) is linear in the uncertain parameters, then the approach is exact. However, to the extent that (10) is non-linear in the parameters, then the algorithm may provide biased estimates of the variability. A

[3] The assumption of independence is clearly warranted for some of the parameters. For example, the equilibrium climate sensitivity is likely to be independent of productivity growth. However, there may be dependence of economic variables, such as the rate of decarbonization and productivity growth. We have virtually no information on such dependences, so the safest approach is to assume independence and to test for the impacts.

[4] The constraints are that the modeling should be replicable on a PC and use the GAMS code. A single run takes about 3 seconds. We estimate that it would require a sample of about 10 million runs to get a reliable Monte Carlo. GAMS software does not allow this to be done easily, and using the standard code would require about three months of computer time. Moving to other platforms would clearly speed up the computations, but this would run afoul of exportability.

simple test for bias examines the error as a function of the relationship between the uncertain parameter and the endogenous variable given by $h(Y) = H[f_i(u_i)]$ We can calculate the exact error where *H* is polynomial as in $Y = \alpha u_i^{\beta}$. For this specification, the error from the discretization of the pdfs in the Monte Carlo is in the order of *(β-1)/100,* as shown in Appendix Figure A-1. Fitting the different endogenous variables to simple polynomials yield exponents *(β)* close to one (between 0.5 and 1.5). This suggests that the approximation bias is in the order of $\pm$ 0.5%. This potential error should be weighed against the sampling error in the Monte Carlo, which can be substantial for systems with large numbers of random variables. In the end, the discretization has the advantage of computational simplicity and zero sampling error and therefore is preferred in the current situation.

For the actual computations, we did a complete enumeration of the $5^5 = 3125$ combinations of uncertain parameters for each scenario. This took about two hours on a high-end PC workstation.

Determination of the probability distributions

We have selected five probability distributions based on earlier work that suggests the most important uncertain variables (see the extensive discussion in Nordhaus 1994, 2008) as well as those in the MUP study for comparison (Gillingham et al., 2015). This section describes the derivation of the distributions.

Equilibrium temperature sensitivity (ETS). The distribution for ETS adopts the approach used in the MUP study. The following describes the reasoning: The primary estimates are from Olsen et al. (2012). This study uses a Bayesian approach, with a prior based on previous studies and a likelihood based on observational or modeled data. The best fitting distribution is a log-normal pdf. The parameters of the log-normal distribution fit to Olsen et al. are μ = 1.107 and σ = 0.264. The major summary statistics of the reference distribution in the study are the following: mean = 3.13, median = 3.03 and standard deviation = 0.843.

Productivity growth. The scholarly literature on uncertainties about long-run economic growth is thin. Here again, we followed the approach of the MUP study. This relied on a survey of experts by a team at Yale University led by Peter Christensen. The survey utilized information drawn from a panel of experts to characterize uncertainty in estimates of global output for the periods 2010-2050 and 2010-2100. Growth was defined as the average annual rate of growth of real per capita GDP, measured in purchasing power parity (PPP) terms.

The resulting estimates of growth were best fit using a normal distribution. The resulting combined normal distribution had a mean growth rate of 2.06% per year and a standard deviation of the growth rate of 1.12% per year over the period 2010-2100. The procedures and estimates will shortly be available in a working paper, and a short description is in Gillingham et al. (2015). An alternative low-frequency approach developed by Muller and Watson gives a significantly lower dispersion at long horizons (standard deviation of approximately 0.8% per year). For the uncertainty estimates we use the survey estimate.

Decarbonization. The DICE model has a highly condensed representation of the energy sector. The most important parameters are the level and trend of the global ratio of uncontrolled CO_2 emissions to output, $\sigma(t)$, as shown in equation (4). This is modeled as an initial growth rate with a slow change over time. While there are several studies that include something like this ratio (often modeled as autonomous energy efficiency improvement, AEEI), there are no consensus estimates about its uncertainty because of the differences in IAM specifications.

To estimate uncertainty, we used two alternative methods. The first was a time-series approach. For this, we looked at historical data on the global emissions/output ratio. The simplest approach is to estimate an OLS regression, using data from 1960 to 2015, and then look at the forecast error for 2100. If we include an AR1 term in the equation, the standard error of the forecast for 2100 is 13.5% of the logarithm of $\sigma(t)$. This implies an annual uncertainty of 0.149% per year.

An alternative approach is to examine the variation in models. For this, we examined the standard deviation of the growth of $\sigma(t)$ in the six MUP models for the uncontrolled run. This produced a much higher divergence, 0.32% per year from 2010 to 2100.

For the uncertainty estimates, I chose the MUP differences. This produces an uncertainty of the annual growth of $\sigma(t)$ of 0.32% per year. This is higher than the time-series numbers, but seems to capture the errors. The major advantage is that it is conceptually the correct approach and contains structural elements. The major shortcoming is that it may *underestimate* the uncertainty as many ensembles do, but it seems less prone to clustering than other variables.

Carbon cycle. The carbon cycle has several parameters, but the most important one is the size of the intermediate reservoir (biosphere and upper level of the oceans). Changes in this have major impacts on the atmospheric retention over the medium term (circa a century), while the other parameters affect primarily either the very short run or the very long run.

Since IAMs generally have primitive carbon cycles, we examined model comparisons of carbon cycles. The study by Friedlingstein et al. (2014) examined alternative predictions of 11 earth system models (ESMs) by calculating different emission-driven simulations of concentrations and temperature projections. These used the IPCC high emissions scenario (RCP 8.5). When forced by RCP8.5 CO_2 emissions, models simulate a large spread in atmospheric CO_2 concentrations, with 2100 concentrations range between 795 and 1145 ppm. The standard deviation of the 2100 concentrations (conditional on the emissions trajectory) is 97 ppm. According to the study, differences in CO_2 projections are mainly attributable to the response of the land carbon cycle, so that suggests the intermediate reservoir is the parameter to adjust. While the ensemble standard deviation is not conceptually appropriate, is a useful benchmark for the purposes at hand.

Damage function: core estimates

The damage function was revised in the 2016 DICE version to reflect new findings. The 2013 version relied on estimates of monetized damages from the Tol (2009) survey. It turns out that that survey contained several numerical errors (see the Editorial Note 2015). The current version continues to rely on existing damage studies, but these were collected by Andrew Moffat and the author and independently verified. We examined different damage estimates and used these as underlying data points and then fitted a regression to the data points. We also added an adjustment of 25 percent of the damage estimate for omitted sectors and non-market and catastrophic damages, as explained in Nordhaus and Sztorc (2014). Including all factors, the final estimate is that the damages are 2.1% of global income at 3 °C warming and 8.5% of income at 6 °C warming.

The method for estimating the damage function is the following: The new estimates start with the survey of damage estimates by Andrew Moffat and Nordhaus (in process). The survey included 26 studies. Of these, 16 contained independent damage estimates and were included, and of these 9 received full weight. Those receiving less than full weight were ones that were earlier (but different) versions of a model (for example the FUND model) or had serious shortcomings. If a study had several estimates (say, along a damage function), the sum was constrained to be 1.

The estimates were made using four techniques. The central specification was a one-parameter quadratic equation with a zero intercept and no linear term and was therefore a one-parameter function. Unweighted least squares and median regressions generally had lower estimated damage coefficients than the weighted OLS versions. The weighted OLS had slightly higher coefficients than the weighted median regression.

Additionally, the tests were made with different lower bound thresholds from 0 to 4 °C, and upper bound estimates from 3 to 10 °C, but these made virtually no difference to the estimates. A specification with both linear and quadratic terms was extremely unstable and was rejected.

The parameter used in the model was an equation with a parameter of 0.236% loss in global income per °C squared with no linear term. This leads to a damage of 2.1% of income at 3 °C, and 8.5% of global income at a global temperature rise of 6 °C. This coefficient is slightly smaller than the parameter in the DICE-2013R model (which was 0.267% of income per °C squared). The change from the earlier estimate is due to corrections in the estimates from the Tol numbers, inclusion of several studies that had been omitted from that study, greater care in the selection of studies to be included, and the use of weighted regressions.

Damage function: uncertainties

The other key question is the uncertainty of the damage function to be used in the uncertainty analysis. One approach would be the standard error of the coefficient in the preferred equation above. The standard error from the preferred regression (including the 25% premium for omitted damages) is 0.0303% loss in global income per °C squared ($Y/°C^2$) for a central coefficient of 0.236% $Y/°C^2$. This corresponds to a t-statistic on the estimated coefficient of 7.8, so it is apparently extremely well determined.

However, this estimate does not reflect specification uncertainty, uncertainty about the studies to be included, or study dependence. We can take a broader approach to estimating the uncertainty by looking at the calculated damages for all specifications of the damage function (both linear and quadratic, weighted and unweighted, and with different temperature thresholds) at a temperature increase of 6 °C. This yields an uncertainty of 0.14% $Y/°C^2$. As a final estimate of uncertainty, we take the standard deviation of the damage coefficients of the three models used in the IAWG (DICE, FUND, and PAGE); this yields a standard deviation of the damage coefficient of 0.15 % $Y/°C^2$.

Given the different approaches, we settled on a value for the uncertainty of the damage parameter which is one-half the parameter, or 0.118% $Y/°C^2$. This reflects the great divergence today among different studies.[i]

IV. Major results for DICE-2016R

Central or certainty-equivalent (CE) estimates

It will be useful to begin with results for the central values from the revised model. For this section, we use a certainty-equivalent approach, which has been the standard for the DICE model and most IAMs. As these results relate to the next section, the results use the expected values of the parameters. The detailed results of the baseline run through 2100 are shown in Appendix Tables A-1 and A-4.

Figures 2 through 4 show the projections of emissions, concentrations, and temperature increase for four scenarios. The four scenarios are the baseline ("Base"), which is the central version of no climate policy studied here; the cost-benefit economic optimum ("Opt"), which optimizes climate policy over the indefinite future; a path that limits temperature to 2½ °C ("T<2.5"); and a policy with an extremely low discount rate as advocated by the *Stern Review* ("Stern").

Figure 2 shows the paths of emissions under the four scenarios. The baseline has rising emissions (although the path is flatter than most models, as we will see later). The two ambitious paths require zero emissions of CO_2 by mid-century, which is an extremely sharp break in trend. And the optimal trajectory has close to flat emissions for the next half-century. The two optimistic paths strain credulity for current policies.

Figure 3 shows the CO_2 concentrations paths for the four policies. The interesting feature is that the two ambitious paths require stabilization at close to today's level of 400 ppm. These are required because of the inertia in the climate system as well as because of assumed growth in non- CO_2 GHGs.

Figure 4 shows the temperature trajectories of the scenarios. The *Stern* and limit scenario asymptote to 2½ °C by the end of the 21st century. The other paths grow sharply, either because of no controls ("Base") or because of inertia even if strong policies are taken ("Opt").

A major surprise and difference from earlier versions of the DICE model is that the "optimal" trajectory is now closer to the "base" that to the ambitious scenarios. This is due to a combination of factors such as climate-system inertia and high costs of the limiting scenarios. We will return to this point when we examine the social costs of carbon (or optimal tax rates) in a later section.

Comparison with other studies

We can compare the results of the DICE CE approach with other models and studies. Figure 5 shows the projected industrial emissions of CO_2 over the coming century for baseline or no-policy scenarios. DICE-2016R is at the low end of different projections after mid-century. The reason (as explained above) is that the rate of decarbonization has increased in recent years. The lower emissions trend is reflected in the 2016 DICE version but not in most other model projections, which often reflect models constructed several years ago.

Figure 6 shows the projected temperature trajectories in five different approaches. The results for DICE-2016R are at the high end of comparable studies. The DICE results are above those of the EMF-22 modeling exercise as well as the central projections from the MUP project (Gillingham et al. 2015). The top line is the ensemble average from the Fifth Assessment Report of the IPCC (2013) for the RCP 8.5 scenario. However, the IPCC RCP 8.5 projection has a higher radiative forcing than the baseline DICE-2016R model. So the summary is that the DICE temperature projection is roughly slightly lower than the last version; is higher than most other IAMs for a baseline scenario; and is consistent (although a little lower) than the IPCC RCP8.5 ensemble average.

Social cost of carbon

A key finding from IAMs is the social cost of carbon, or SCC. This term designates the economic cost caused by an additional ton of CO_2 emissions or its equivalent. In a more precise definition, it is the change in the discounted value of economic welfare from an additional unit of CO_2–equivalent emissions. The SCC has become a central tool used in climate-change policy, particularly in the determination of regulatory policies that involve greenhouse-gas emissions. (A full discussion is contained in National Research Council 2016 and Nordhaus 2014. The discussion in this section draws upon Nordhaus 2016.) Estimates of the SCC are necessarily complex because they involve the full range of impacts from emissions, through the carbon cycle and climate change, and including economic damages from climate change. At present, there are few established integrated assessment models (IAMs) that are available for estimation of the entire path of cause and effect and can therefore calculate an internally consistent SCC. The DICE model is one of the major IAMs used by scholars and governments for estimating the SCC.

Table 1 shows alternative estimates of the SCC. The central estimate from the CE approach is $31/t CO_2 for 2015. Other estimates show the SCC for temperature limits and for different discount rates. A key set of estimates are those of the US government made by the Interagency Working Group on Social Cost of Carbon (IAWG 2015). The IAWG concept is conceptually comparable to the baseline in the first row of Table 1. The IAWG combines estimates from three models and multiple scenarios. Table 2 compares the latest round of estimates of the IAWG with estimates from the DICE-2013R and DICE-2016R models for the baseline model and different discount rates. The preferred SCC estimate of the most recent DICE model is about one-fifth lower than the IAWG's preferred SCC. At comparable discount rates, the DICE model estimate would be roughly twice that of the IAWG.

It is also useful to show the changes in the model over time. We can decompose the changes in the SCC by introducing each of the major components of the model one by one. Table 3 accounts for the changes in the SCC by major revision variable. Other than the adjustment of the damage function, other major changes had the effect on increasing the SCC between 2013 and 2016. The two major changes were the carbon cycle (discussed above) and estimated economic activity.

V. Uncertainties about climate change and policies

Results for baseline run

This section presents the results of the uncertainty analysis. To reiterate the approach, we divide the pdfs for each uncertain variable into quintiles and then take the expected value of the parameter in each quintile. For asymmetrical pdfs, we transform slightly to preserve both the mean and the standard deviation. We then take a full enumeration of all $5^5 = 3125$ equally probable states of the world.

Panel A in Table 4 shows key statistics for major variables. There are three statistics for central values and three for uncertainty. Look first at the estimates for temperature increase for 2100. The certainty equivalent (CE) for DICE is 4.10 °C, while the mean is a tiny bit higher at 4.12 and the median is lower at 4.06 °C. These suggest that the distribution is close to symmetric and that the CE gets a good approximation of the estimates with uncertainty.

Other variables display considerable asymmetry in the distributions. The SCC for 2015 has a mean of 35.6 $/t CO_2 whereas the CE is about 15% lower at 31.2 $/t CO_2. Output and CO_2 concentrations are similarly skewed. The issue of whether the CE is a reasonable approximation is important because it vastly simplifies analysis. The answer is, sometimes yes, sometimes no. Appendix Table A-3 provides a tabulation of variables and the bias from using the CE approach.

Table 4A also shows three measures of uncertainty, the standard deviation, the interquartile range (IQR), and the coefficient of variation (CV). Perhaps the most useful is the CV. The interesting feature here is that the CV is relatively low for geophysical variables such as 2100 temperature increase and carbon concentrations, but much higher for economic variables such as output, damages, and the SCC. The high uncertainty of economic variables comes largely because of output uncertainty. The long lags in geophysical variables plus lower uncertainty of geophysical parameters produces lower uncertainty in those variables.

We show box plots for several variables in the next figures. Figure 7A is temperature, 7B is CO_2 concentrations, 7C is the damage ratio, and 7D is the social cost of carbon. One important result is that even at the low fence (which is approximately ½% of outcomes) there is substantial climate change.

Brief note on uncertainty in optimal runs

The present study is primarily about the results of uncertainty for a baseline scenario. The results for the optimal run are broadly parallel, with the central optimal result shown above and the uncertainties largely similar. Panel B of Table 4 shows the basic results. The only noticeable difference is that the uncertainty and CE cases are much closer in the optimal than in the baseline scenarios. Other variables, particularly output and emissions, show much larger deviations in the uncertainty runs. Appendix Table A-2 shows the details for the optimal run with uncertainties.

One other interesting run concerns the potential for limiting temperature in the uncertain runs. We have estimated the distribution of outcomes where abatement is at its maximum (30% in the 2015 period, 70% in the 2020 period, and 100% after that). This is the outer limit of what would be feasible with maximum effort. The probability that the temperature in 2100 would be less than 2 °C is about 40% for the maximum-effort case (see Appendix Table A-5 for the results on this run).

Contributions of individual variables

Table 5 shows the contribution of the different variables to the overall uncertainty. These are calculated in two ways. Panel A starts from zero uncertainty and introduces each variable one at a time. Panel B starts from full uncertainty and reduces the uncertainty one variable at a time.

The importance of different variables differs for the endogenous variable at hand. The most important uncertainty across the board is the growth rate of productivity. This affects virtually every variable in an important way.

As a central way to view uncertainty, the most useful variable is the SCC. This is important because it indicates how stringent policy should be today, whereas many other variables are ones that relate to the distant future. For the SCC, three variables are important. The most significant is the damage coefficient. The other two, roughly equally significant, are productivity growth and the equilibrium temperature sensitivity. The carbon cycle and the emissions intensity are relatively unimportant for uncertainty about the SCC. One major surprise is that the uncertainty about carbon intensity has little effect on many variables, particularly the SCC; this arises because changes in carbon intensity affect marginal damages and marginal costs almost equally, so the two changes largely offset each other.

Comparisons with other estimates

There are several other estimates of uncertainty in IAMs. The most convenient comparison is with the estimates from the MUP project (Gillingham et al. 2015). That study presented the results of the first comprehensive study of uncertainty in climate change using multiple integrated assessment models. The study looked at model and parametric uncertainties for population, total factor productivity, and climate sensitivity. It estimated the pdfs of key output variables, including CO_2 concentrations, temperature, damages, and the social cost of carbon (SCC). The key feature was that the pdfs of the uncertain variables were standardized, while the models themselves (and the means of all driving variables) were left at the modelers' baselines.

Table 6 shows comparisons of means, standard deviations, and coefficients of variation for major variables between the current study and the MUP study. Note that the DICE model used in that study was DICE-2013R, whereas the model used here is DICE-2016R. As noted above, there have been several important changes in the specification, so for DICE there are both methodological differences and model differences between this study and the MUP study.

The most useful statistic to examine is the coefficient of variation (CV) in the bottom panel of Table 6. For the DICE model, the CVs are close but generally lower in the present study, the exception being the SCC. It is likely that the lower growth of emissions is responsible for the lower CVs for CO_2 and temperature. The increased CV for the SCC is a puzzle.

We can also examine the model differences in the bottom panel of Table 6. The striking feature here is the large differences in CVs across models. The CVs differed by a factor of 1½ to almost 3 among the six models.

The key finding of the present and earlier studies is striking: The uncertainties of geophysical variables such as CO_2 concentrations and temperature are relatively low, in the order of one-fifth of their mean values. On the other hand, the uncertainties of economic variables are much larger, with CVs ranging from around 70% to 100% for emissions, damages, output, and the SCC.

VI. Conclusion

The present study presents an updated set of results on the prospects for climate change using a revised integrated assessment model, DICE-2016R. It also develops a new and simplified method for determining the uncertainties associated with climate change and the extent to which simplified certainty-equivalent (CE) techniques provide an accurate representation of the more complex model with uncertainty.

The results pertain primarily to a world without climate policies, which is reasonably accurate for virtually the entire globe today. The results show rapidly rising accumulation of CO_2, temperatures changes, and damages. Moreover, when the major parametric uncertainties are included, there is virtually no chance that the rise in temperature will be less than the target 2 °C without climate change policies.

It is worth emphasizing one further point about the impact of uncertainty on policy. The future is highly uncertain for virtually all variables, particularly economic variables such as future emissions, damages, and the social cost of carbon. It might be tempting to conclude that nations should wait until the uncertainties are resolved, or at least until the fog has lifted a little. The present study finds the opposite result. When taking uncertainties into account, the strength of policy (as measured by the social cost of carbon or the optimal carbon tax) would increase, not decrease.

References

Archer, David, et al. 2009. "Atmospheric lifetime of fossil fuel carbon dioxide," *Annual Reviews Earth Planetary Science,* 37:117–34

Christensen, Peter, Kenneth Gillingham, and William Nordhaus. 2016. "Uncertainty in forecasts of long-run productivity growth," manuscript in preparation.

Clarke, Leon, et al. 2009. "International climate policy architectures: Overview of the EMF 22 international scenarios." *Energy Economics,* 31: S64–S81.

Editorial Note. 2015. "Editorial Note: Correction to Richard S. Tol's 'the economic effects of climate change'," *Journal of Economic Perspectives,* vol. 29, no. 1, Winter, 217-20.

Friedlingstein, Pierre, Malte Meinshausen, Vivek K. Arora, Chris D. Jones, Alessandro Anav, Spencer K. Liddicoat, and Reto Knutti. 2014. "Uncertainties in CMIP5 Climate Projections due to Carbon Cycle Feedbacks," *Journal of Climate,* 27: 511-526.

Gillingham, Kenneth, William D. Nordhaus, David Anthoff, Geoffrey Blanford, Valentina Bosetti, Peter Christensen, Haewon McJeon, John Reilly, and Paul Sztorc. 2015. "Modeling uncertainty in climate change: a multi-model comparison." No. w21637. National Bureau of Economic Research.

IAWG. 2015. Interagency Working Group on Social Cost of Carbon, United States Government, *Response to Comments: Social Cost of Carbon for Regulatory Impact Analysis Under Executive Order 12866,* July, available at https://www.whitehouse.gov/sites/default/files/omb/inforeg/scc-response-to-comments-final-july-2015.pdf.

IPCC Fifth Assessment Report, *Science.* 2013. Intergovernmental Panel on Climate Change, *Climate Change 2013: The Physical Science Basis,* Contribution of Working Group I to the Fifth Assessment Report of the IPCC, available online at http://www.ipcc.ch/report/ar5/wg1/#.Ukn99hCBm71.

National Research Council. 2016. *Assessment of Approaches to Updating the Social Cost of Carbon: Phase 1 Report on a Near-Term Update.* Washington, D.C.: National Academy Press.

Nordhaus, William D. 1992. “An optimal transition path for controlling greenhouse gases,” *Science,* 258, November 20: 1315-1319.

Nordhaus, William D. 1994. *Managing the Global Commons: The Economics of Climate Change*, MIT Press, Cambridge, MA, USA.

Nordhaus, W. 2008. *A Question of Balance: Weighing the Options on Global Warming Policies*. New Haven, CT: Yale University Press.

Nordhaus, William. 2014. "Estimates of the social cost of carbon: concepts and results from the DICE-2013R model and alternative approaches." *Journal of the Association of Environmental and Resource Economists,* 1, no. 1/2: 273-312.

Nordhaus, William. 2015. “Climate clubs: Overcoming free-riding in international climate policy.” *American Economic Review*, 105.4: 1339-70

Nordhaus, William. 2016. “The social cost of carbon: Updated estimates.” *Proceedings of the U. S. National Academy of Sciences*, forthcoming.

Nordhaus, William and Paul Sztorc. 2013. *DICE 2013R: Introduction and User’s Manual,* October 2013, available at http://www.econ.yale.edu/~nordhaus/homepage/documents/DICE_Manual_100413r1.pdf.

Olsen, R., R. Sriver, M. Goes, N. Urban, D. Matthews, M. Haran, and K. Keller. 2012. "A Climate Sensitivity Estimate Using Bayesian Fusion of Instrumental Observations and an Earth System Model." *Geophysical Research Letters* 117(D04103): 1-11.

Paris Agreement. 2016. United Nations Framework Convention on Climate Change, *The Paris Agreement,* available at http://unfccc.int/paris_agreement/items/9485.php.

Stern Review. 2007. *The Economics of Climate Change: The Stern Review*, Cambridge University Press: Cambridge, UK.

Tol, R.S.J. 2009. “The Economic Impact of Climate Change,” *Journal of Economic Perspectives,* 23 (2): 29–51.

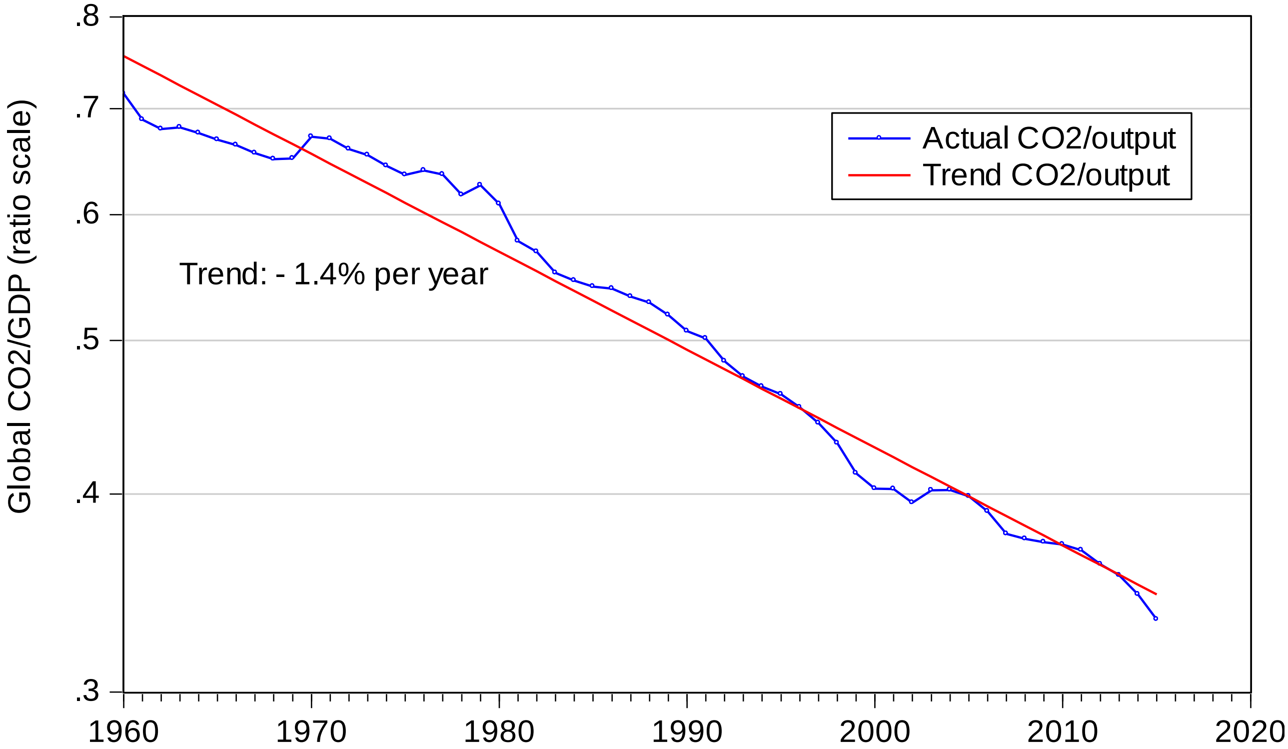


Figure 1. Global trend and actual history for CO_2/GDP ratio

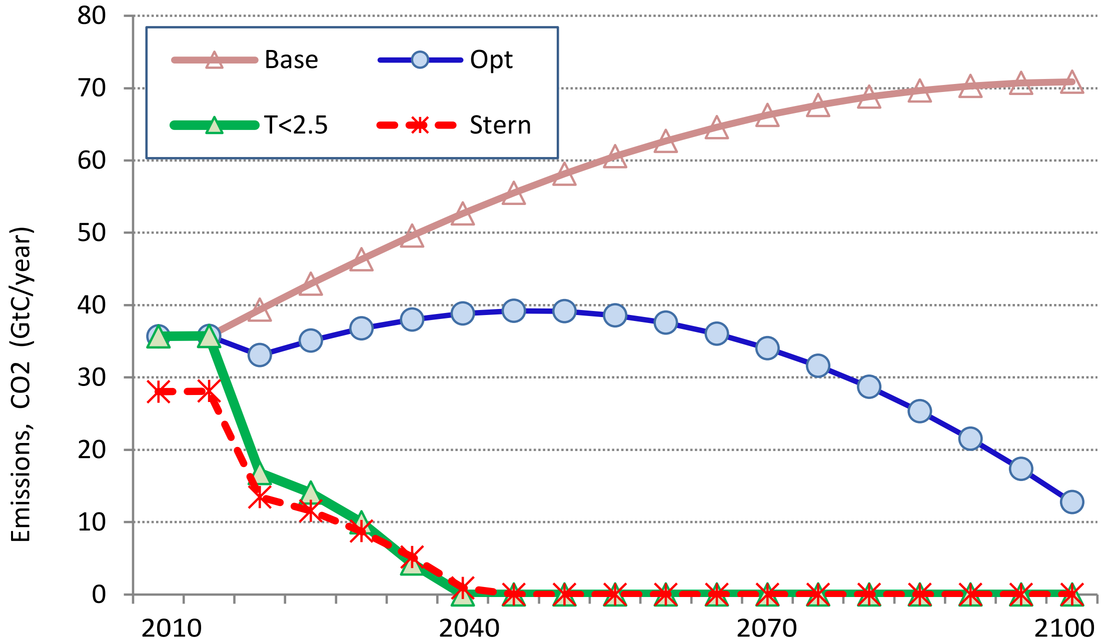


Figure 2. Emissions of CO_2 in different scenarios

The two most ambitious scenarios require zero emissions by mid-century.

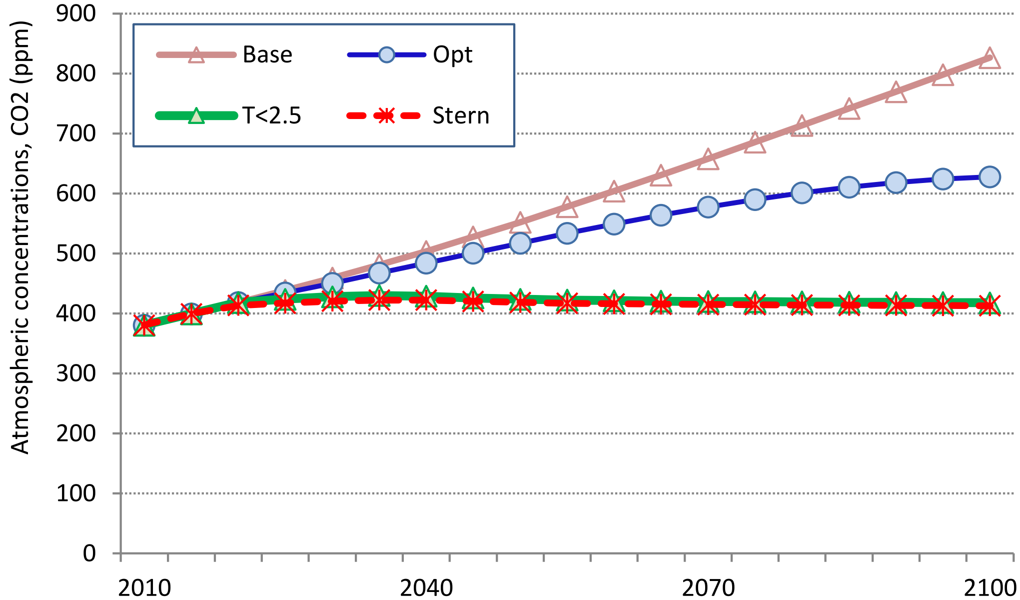


Figure 3.Concentrations of CO_2 in different scenarios

The two most ambitious scenarios require concentrations emissions close to current levels.

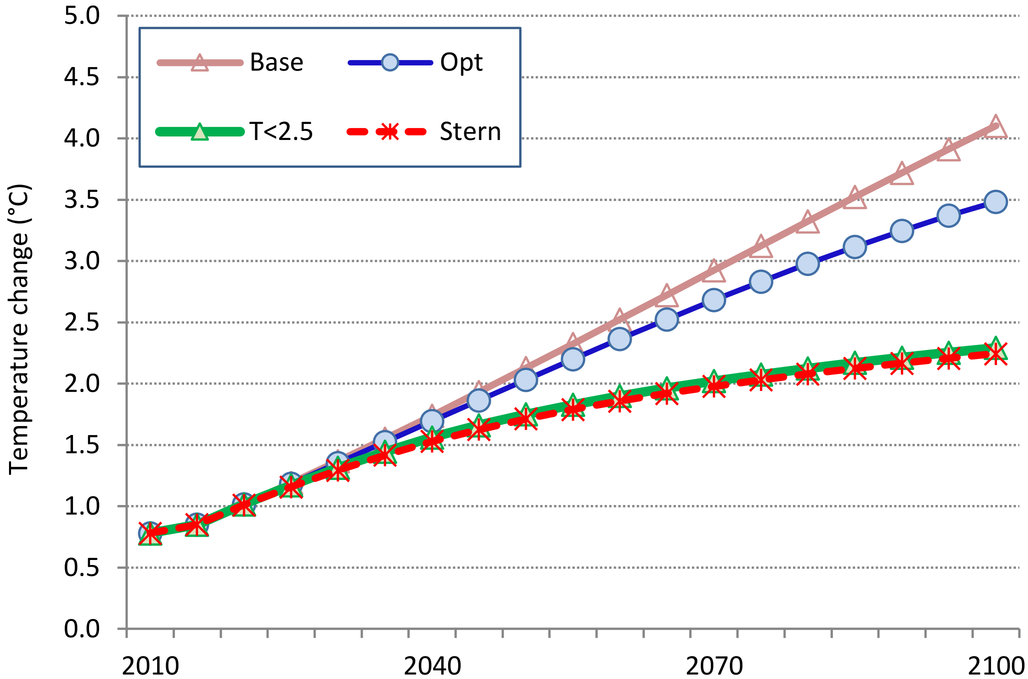


Figure 4. Temperature change in different scenarios

The most ambitious scenarios cannot limit temperature to 2 ½ °C, and the cost-benefit optimum with standard parameters has sharply rising temperatures.[ii]

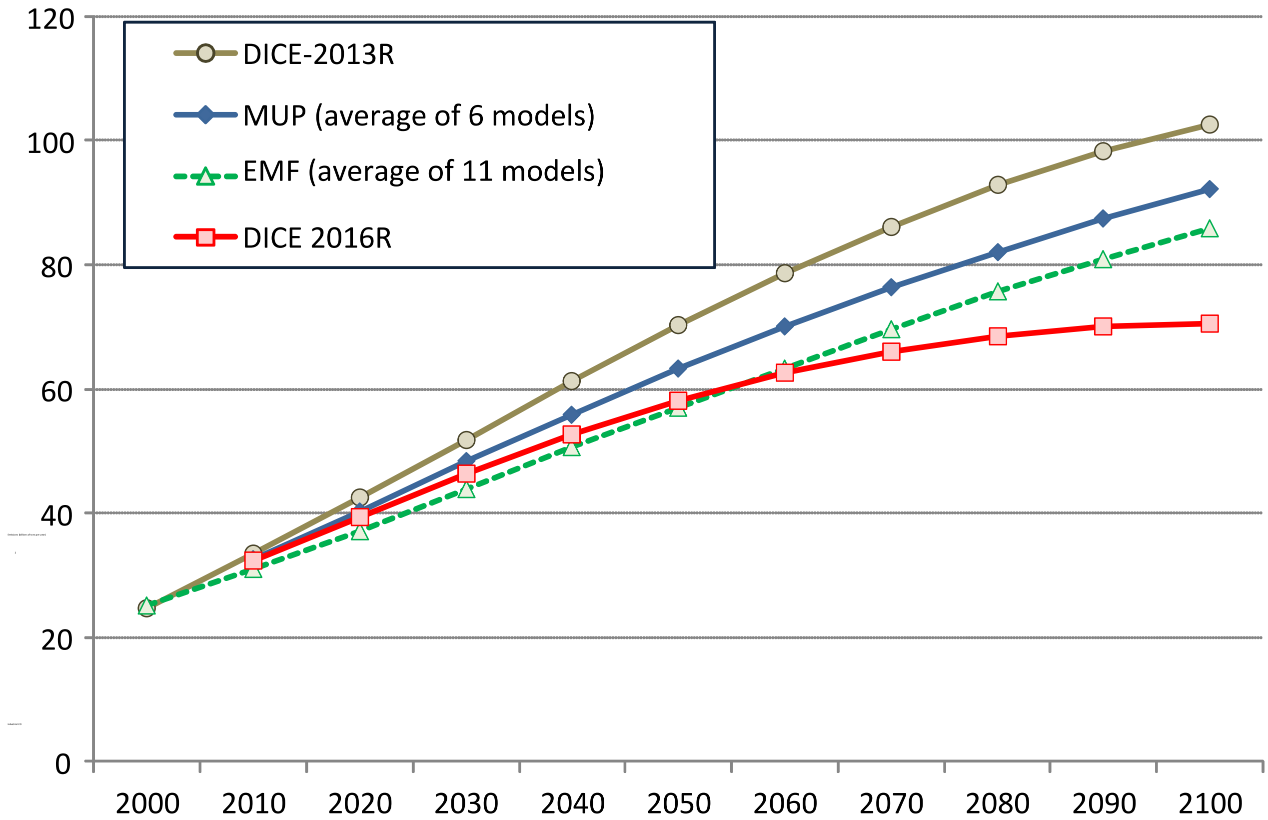


Figure 5. Projected industrial CO_2 emissions in baseline scenario

The figure compares the projections of the most recent DICE models and two model comparison exercises. The estimates from the MUP project are from Gillingham et al. (2015), while the EMF-22 estimates are from Clarke et al. (2009).

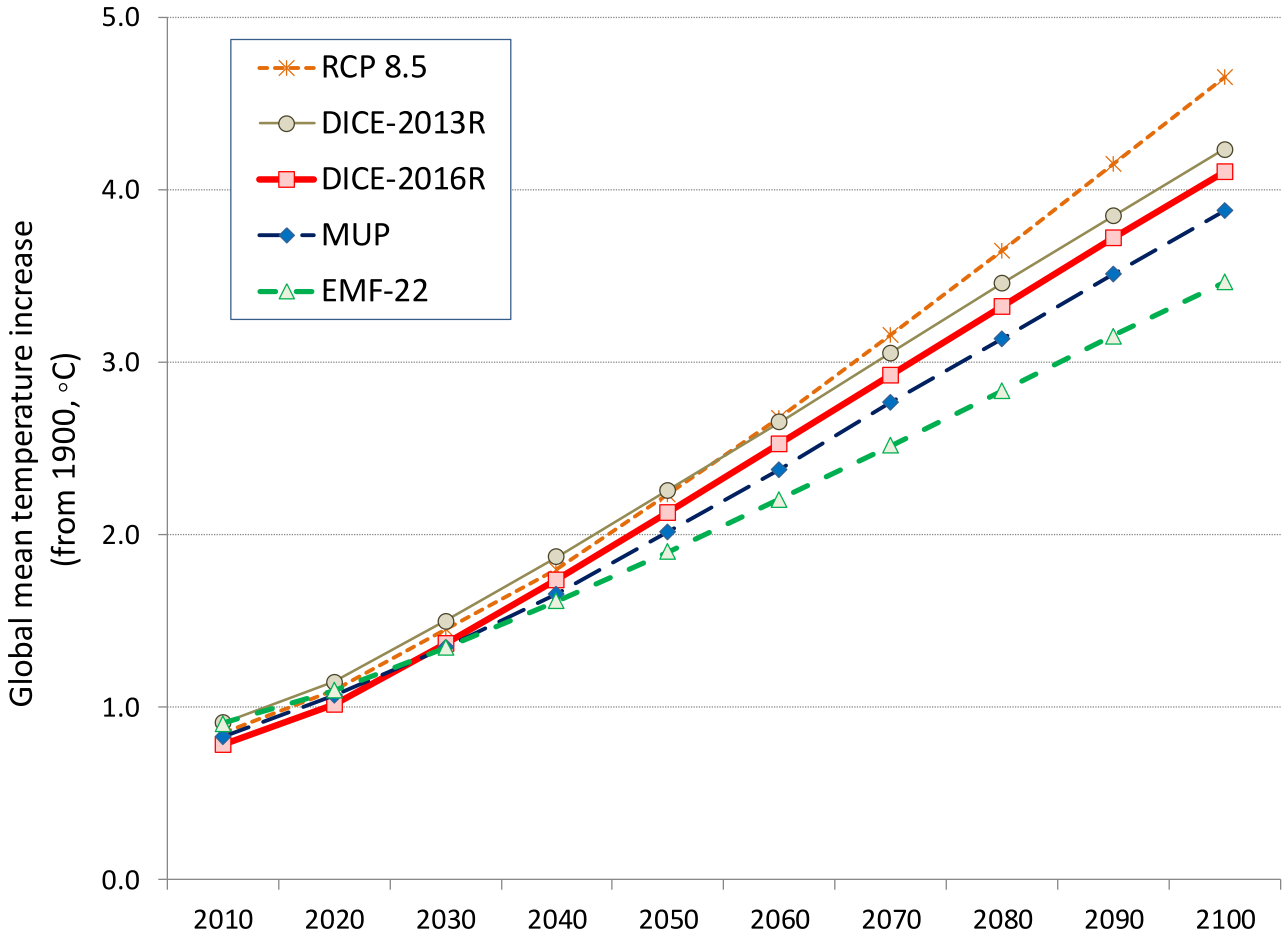
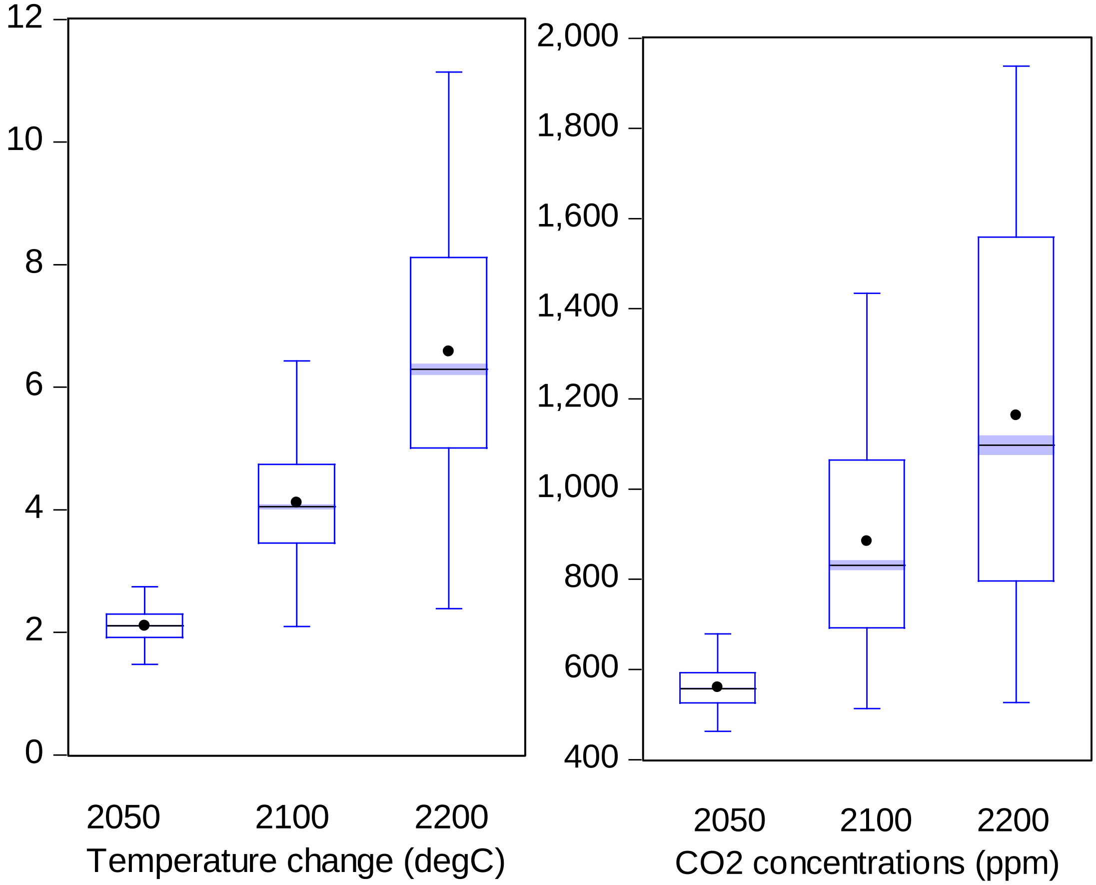


Figure 6. Global mean temperature increase as projected by IPCC scenarios and integrated assessment economic models

The figure compares the projections of the most recent DICE models, the IPCC RCP high scenario (8.5), and two model comparison exercises.



12
10
8
6
4
2
0
2050
2100
2200
Temperature change (degC)
2,000
1,800
1,600
1,400
1,200
1,000
800
600
400
2050
2100
2200
CO2 concentrations (ppm)

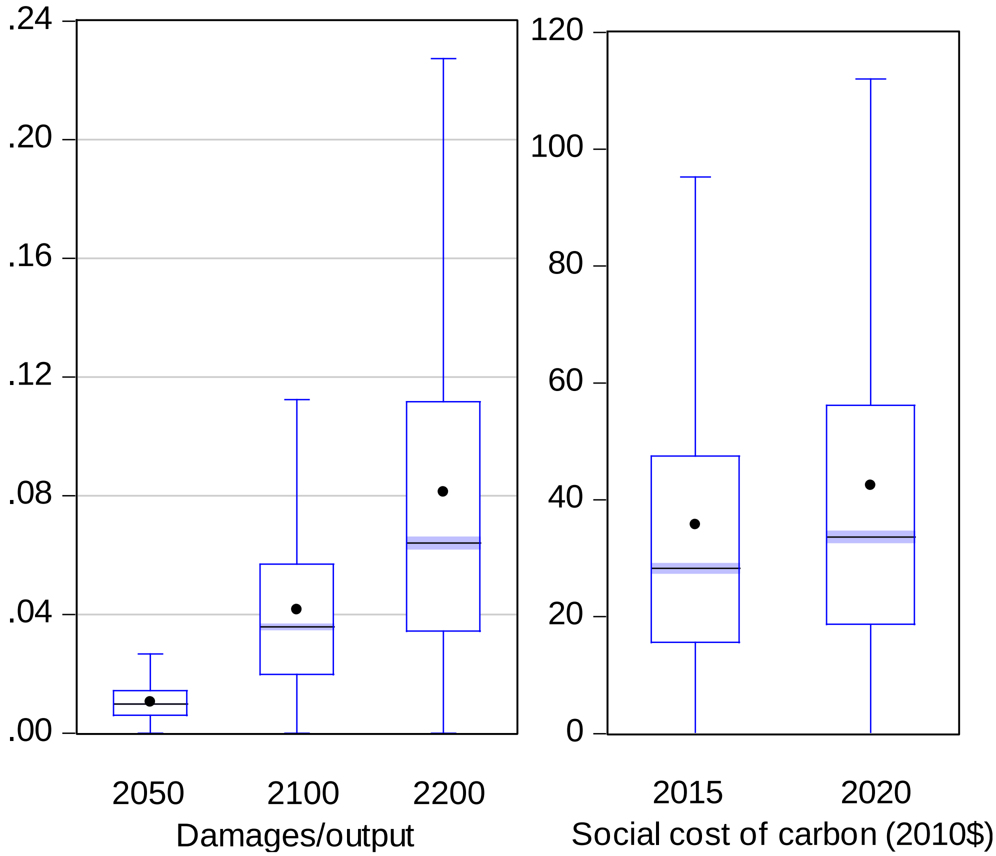


Figure 7. Boxplots for major uncertain variables.

The interpretation is that the line in the middle of the box is the median, while the shaded region around the line is the standard error of the median. The dot is the mean. The box shows the interquartile range, IQR (= Q3-Q1), while the fences are at Q1 – 1.5×IQR and Q3 + 1.5×IQR. For a normal distribution, the fences contain 99.2% of the distribution.[iii]

Scenario	2015	2020	2025	2030	2050
Base parameters					
Baseline (a)	31.2	37.3	44.0	51.6	102.5
Optimal controls (b)	30.7	36.7	43.5	51.2	103.6
2.5 degree maximum					
Maximum (b)	184.4	229.1	284.1	351.0	1,006.2
Max for 100 years (b)	106.7	133.1	165.1	203.7	543.3
Stern Review discounting					
Uncalibrated (b)	197.4	266.5	324.6	376.2	629.2
Alternative discouny rates (a)					
2.5%	128.5	140.0	324.6	376.2	629.2
3%	79.1	87.3	152.0	164.6	235.7
4%	40.9	45.8	95.9	104.9	156.6
5%	19.7	22.6	51.1	56.6	88.7

Table 1. Global social cost of carbon by different assumptions

The social cost of carbon is measured in 2010 international US dollars. The years at the top refer to the date at which emissions take place. Therefore, $31.2 is the cost of emissions in 2015 in terms of consumption in 2015. (a) Calculation along the reference path with current policy. In the baseline calculation, welfare is maximized as in (1) but when damages are set to zero for the optimization but included in the ex post calculation. (b) Calculation along the optimized emissions path. Note that for the temperature ceilings, the damages are included. By putting a temperature cap, this implicitly assumes that the damages are infinite beyond that limit.

Model and scenario					
	5%	DICE-base	4%	3%	2.5%
A. Estimates of 2020 SCC from US Working Group, 2013 (2010$)					
DICE-2010	12			40	59
PAGE	23			74	105
FUND	3			22	37
Average	13			***45***	67
B. Estimates for different DICE model versions (2010$)					
DICE-2013R	15	24	26	50	74
DICE-2016R	23	***37***	41	87	140

Table 2. Estimates of the social cost of carbon for 2020 from US Interagency Working Group and Comparison with DICE model in 2010 US$

Panel A shows estimates of the 2020 SCC from the IAWG. The three models have harmonized outputs, emissions, populations, and equilibrium temperature sensitivity (ETS) distribution and use constant discount rates. Panel B shows the results of the estimates from the two latest versions of the DICE model for the baseline (see Table 1) and using constant discount rates. The estimates in shaded boxes show the preferred estimate for US regulatory purposes and the DICE-2016R estimate.

	Model	*SCC (2015)*	*Change*
1	Dice-2016	31.23	
2	1 + Old economics	24.68	-27%
3	2 +Old population	23.21	-6%
4	3 + Old temp sensitivity	21.30	-9%
5	4 + Old damage	24.26	12%
6	5 + Old carbon cycle	18.21	-33%
7	DICE-2013R	17.03	-7%

Table 3. Accounting for changes in SCC from DICE-2013R

The table shows the impact of introducing model changes starting with the 2016 model and ending with the 2013 model in a step fashion. The last column shows the change moving from a later specification to an earlier one. A negative number in the last column is a decrease from 2016 to 2013. For example, introducing "old economics" in version 2 lowers the SCC by 26% relative to DICE-2016R. The two major changes are economic assumptions and the carbon cycle (see the text for a discussion).

A. Results for Baseline Scenario[iv]

Variable	Mean	DICE cert	50%ile	St Dev	IQ range	Coef of Var
SCC, 2015	35.8	31.2	28.4	28.0	41.5	0.78
Temperature, 2100 (°C)	4.12	4.10	4.06	0.89	2.01	0.22
Carbon concentrations, 2100 (ppm)	885	826	833	234	488	0.12
World output, 2100 (trillions 2010 $)	867	757	761	581	1,056	0.67
Emissions 2100	82.9	70.9	71.0	52.6	114.5	0.63
Damages, 2100 (% output)	4.2%	4.0%	3.6%	2.8%	4.9%	0.67
Real interest rate, 2100 (%/yr)	3.0%	3.6%	2.9%	1.0%	2.2%	0.33
Objective (trillions, 2010$)	3,884	4,486	4,497	2,419	6,069	0.62

B. Results for Optimal Scenario[v]

Variable	Mean	DICE cert	50%ile	St Dev	IQ range	Coef of Var
SCC, 2015	32.1	30.7	26.6	23.8	37.8	0.74
Temperature, 2100 (°C)	4.12	3.48	4.06	0.89	2.01	0.22
Carbon concentrations, 2100 (ppm)	885	628	833	234	488	0.12
World output, 2100 (trillions 2010 $)	867	764	761	581	1,056	0.67
Emissions 2100	82.9	12.7	71.0	52.6	114.5	0.63
Damages, 2100 (% output)	4.2%	2.9%	3.6%	2.8%	4.9%	0.67
Real interest rate, 2100 (%/yr)	3.0%	3.6%	2.9%	1.0%	2.2%	0.33
Objective (trillions, 2010$)	3,884	4,517	4,497	2,419	6,069	0.62

Table 4. Statistics for major variables

The table shows statistics for major variables from the discretized uncertainty analysis for baseline (panel A) and optimal scenario (panel B). For a more complete tabulation, see appendix tables. "DICE Cert" is the certainty equivalent of DICE, which sets the uncertain parameters at their expected value.

A. From zero uncertainty

Fraction (if only uncertainty)	SCC, 2015	Temp, 2100	CO2 conc, 2100	Output, 2100	Emissions, 2100	Damage fraction, 2100	Interest rate, 2100	Global income (PV)
Productivity	44%	75%	108%	107%	113%	49%	104%	112%
Damage	63%	1%	1%	4%	2%	75%	9%	3%
Equil. Temp. Sens.	40%	77%	1%	3%	1%	46%	7%	2%
Carbon cycle	7%	29%	25%	1%	0%	18%	2%	1%
Emissions intensity	0%	21%	30%	1%	38%	13%	3%	1%
All	100%	100%	100%	100%	100%	100%	100%	100%

B. From full uncertainty

Fraction (if only uncertainty reduced)	SCC, 2015	Temp, 2100	CO2 conc, 2100	Output, 2100	Emissions, 2100	Damage fraction, 2100	Interest rate, 2100	Global income (PV)
Productivity	27%	24%	65%	95%	66%	15%	88%	96%
Damage	43%	0%	-1%	-2%	-1%	52%	-1%	0%
Equil. Temp. Sens.	24%	27%	-1%	-1%	-1%	11%	0%	0%
Carbon cycle	9%	3%	2%	-1%	-1%	2%	0%	0%
Emissions intensity	2%	1%	1%	5%	0%	0%	6%	-1%

Table 5. Impact on uncertainty of individual uncertain variables

Table shows the contribution (as fraction of total uncertainty) for each uncertainty. Panel A starts from zero uncertainty, where each variable is introduced with zero uncertainty as a base. Panel B shows the reduction in uncertainty if only the variable is set a zero uncertainty while other variables have full uncertainty.[vi]

	Mean of variable							
Variable	This study DICE-2016	DICE-2013	FUND	GCAM	IGSM	MERGE	WITCH	Average
SCC, 2015	35.81	21.87	2.75	na	na	na	15.47	13.36
Temperature, 2100 (°C)	4.12	3.88	3.72	3.94	3.60	4.31	3.75	3.87
Carbon concentrations, 2100 (ppm)	884.8	939.3	906.9	860.7	810.8	998.6	854.1	895.1
Emissions 2100	82.9	127.7	142.7	90.2	71.3	168.7	90.5	115.2

	Standard deviation of variable							
Variable	This study DICE-2016	DICE-2013	FUND	GCAM	IGSM	MERGE	WITCH	Average
SCC, 2015	27.98	15.25	2.17	na	na	na	4.46	7.30
Temperature, 2100 (°C)	0.89	1.10	0.77	1.02	0.81	1.01	0.73	0.91
Carbon concentrations, 2100 (ppm)	233.8	318.3	353.8	222.1	130.9	325.1	134.2	247.4
Emissions 2100	52.6	92.5	145.8	52.7	29.8	130.0	34.6	80.9

	Coefficient of variation of variable							
Variable	This study DICE-2016	DICE-2013	FUND	GCAM	IGSM	MERGE	WITCH	Average
SCC, 2015	0.78	0.70	0.79	na	na	na	0.29	0.55
Temperature, 2100	0.22	0.28	0.21	0.26	0.23	0.23	0.19	0.23
Carbon concentrations, 2100 (ppm)	0.26	0.34	0.39	0.26	0.16	0.33	0.16	0.28
Emissions 2100	0.63	0.72	1.02	0.58	0.42	0.77	0.38	0.70

Table 6. Comparative statistics from current study and other models[vii]

Appendix.

We can calculate exactly the errors of approximation from discretizing the probability distributions if the relationship is polynomial. Figure A-1 shows the approximation error of discretizing a pdf where the model function is $Y = \alpha u_i^{\beta}$. The horizontal axis is the exponent *(β)* in the model function. The error is calculated by integrating the polynomial over the unit interval.

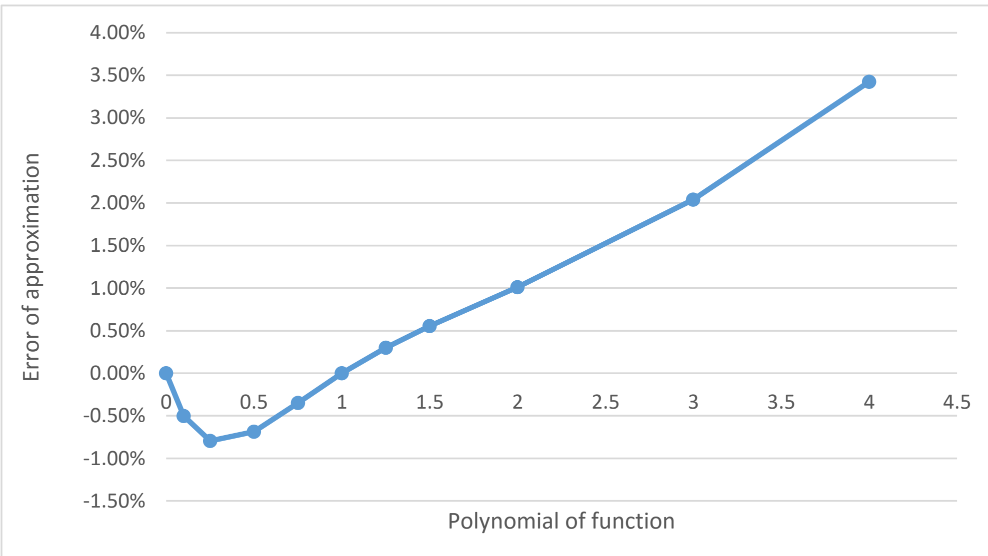


Figure A-1. Approximation error of discretization of pdfs.

The approximation error is a function of the exponent in the polynomial function. The approximation is exact for linear functions (β = 1). The exponent can be estimated using the model calculations over the range in the model runs and is usually in the range of 0.5 to 1.5.[viii]

The detailed statistics of the major variables are as follows in Table A-1 for the baseline run and for the optimal case in Table A-2.

Variable	Mean	St Dev	1%ile	5%ile	10%ile	25%ile	50%ile	75%ile	90%ile	95%ile	99%ile	IQ range	Coef of Var	DICE cert
Damage parameter	0.00236	0.00112	0.00071	0.00071	0.00071	0.00173	0.00236	0.00299	0.00401	0.00401	0.00401	0.00228	0.47	0.00236
ETS	3.092	0.762	2.028	2.028	2.028	2.603	3.100	3.454	4.275	4.275	4.275	1.426	0.25	3.100
Productiivity parameter	0.076	0.047	0.006	0.006	0.006	0.049	0.076	0.103	0.146	0.146	0.146	0.096	0.62	0.076
Carbon cycle parameter	361.5	84.8	246.5	246.5	246.5	305.7	360.0	398.7	496.6	496.6	496.6	152.2	0.23	360.0
Carbon intensity parameter	(0.0150)	0.0030	(0.0193)	(0.0193)	(0.0193)	(0.0168)	(0.0152)	(0.0134)	(0.0105)	(0.0105)	(0.0105)	0.0059	(0.20)	(0.0152)
SCC, 2015	35.8	28.0	3.5	6.0	8.3	15.7	28.4	47.5	73.6	93.4	131.3	41.5	0.78	31.2
SCC, 2020	42.5	33.2	4.3	7.2	10.0	18.7	33.8	56.2	86.1	110.1	163.3	49.0	0.78	37.3
Temperature, 2050 (°C)	2.11	0.26	1.54	1.66	1.74	1.92	2.12	2.30	2.45	2.53	2.66	0.64	0.13	2.13
Temperature, 2100 (°C)	4.12	0.89	2.36	2.73	2.96	3.46	4.06	4.74	5.34	5.67	6.16	2.01	0.22	4.10
Temperature, 2200 (°C)	6.59	2.07	2.78	3.46	3.96	5.01	6.31	8.12	9.46	10.44	11.03	4.65	0.31	6.71
Carbon concentrations, 2050 (GtC)	1,194	98	998	1,040	1,068	1,122	1,192	1,263	1,326	1,363	1,422	223	0.08	1,177
Carbon concentrations, 2100 (GtC)	1,885	498	1,133	1,228	1,293	1,475	1,775	2,268	2,665	2,769	2,951	1,040	0.26	1,760
Carbon concentrations, 2200 (GtC)	2,479	873	1,178	1,283	1,379	1,697	2,342	3,321	3,672	3,851	4,095	2,038	0.35	2,306
Carbon concentrations, 2050 (ppm)	561	46	468	488	501	527	560	593	623	640	668	105	0.04	552
Carbon concentrations, 2100 (ppm)	885	234	532	577	607	692	833	1,065	1,251	1,300	1,385	488	0.12	826
Carbon concentrations, 2200 (ppm)	1,164	410	553	602	647	797	1,100	1,559	1,724	1,808	1,923	957	0.17	1,082
World output, 2050 (trillions 2010 $)	313.3	117.8	163.1	165.4	166.6	233.0	292.8	368.5	511.8	520.7	530.8	203.1	0.38	292
World output, 2100 (trillions 2010 $)	867.0	581.1	189.0	195.7	199.3	442.3	760.8	1,252.1	1,721.0	2,043.7	2,467.4	1,056.4	0.67	757
Emissions 2050	62.9	24.3	28.6	31.2	33.1	44.0	58.2	77.0	102.0	107.4	115.5	45.8	0.39	58
Emissions 2100	82.9	52.6	13.1	16.0	18.4	36.6	71.0	130.6	162.9	165.3	168.3	114.5	0.63	70.9
Emissions 2200	61.7	40.0	5.0	6.9	8.6	23.0	60.8	95.7	113.6	124.1	127.2	88.8	0.65	60.6
Damages, 2050 (% output)	1.1%	0.6%	0.2%	0.3%	0.3%	0.6%	1.0%	1.4%	1.9%	2.1%	2.6%	1.2%	0.55	0.0
Damages, 2100 (% output)	4.2%	2.8%	0.5%	0.8%	1.1%	2.0%	3.6%	5.7%	8.0%	9.6%	12.7%	4.9%	0.67	0.0
Damages, 2200 (% output)	8.1%	6.2%	0.7%	1.3%	1.9%	3.4%	6.4%	11.2%	16.8%	20.7%	28.5%	9.8%	0.76	0.1
Emissions intensity, 2050 (CO2/$)	0.350	0.000	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	-	0.00	0.350
Emissions intensity, 2100 (CO2/$)	0.106	0.026	0.072	0.072	0.072	0.089	0.101	0.117	0.149	0.149	0.149	0.045	0.25	0.101
Emissions intensity, 2200 (CO2/$)	0.056	0.022	0.030	0.030	0.030	0.042	0.051	0.064	0.093	0.093	0.093	0.034	0.38	0.051
Real interest rate, 2015 (%/yr)	5.1%	0.9%	3.6%	3.7%	3.7%	4.5%	5.1%	5.7%	6.5%	6.5%	6.6%	2.0%	0.18	5.1%
Real interest rate, 2100 (%/yr)	3.0%	1.0%	1.4%	1.5%	1.6%	2.1%	2.9%	3.7%	4.3%	4.4%	4.5%	2.2%	0.33	3.6%
Objective (trillions, 2010$)	3,884	2,419	(516)	(359)	(275)	2,949	4,497	5,711	6,677	6,850	7,024	6,069	0.62	4,486

Table A-1. Statistics of major variables from uncertainty analysis for the baseline run. “DICE cert” is the certainty equivalent or standard DICE model.[ix]

Variable	Mean	St Dev	1%ile	5%ile	10%ile	25%ile	50%ile	75%ile	90%ile	95%ile	99%ile	IQ range	Coef of Var	DICE cert	% error (CE/mean)
Damage parameter	0.00236	0.00112	0.00071	0.00071	0.00071	0.00173	0.00236	0.00299	0.00401	0.00401	0.00401	0.00228	0.47	0.00236	0.0%
ETS	3.092	0.762	2.028	2.028	2.028	2.603	3.100	3.454	4.275	4.275	4.275	1.426	0.25	3.100	0.3%
Productiivity parameter	0.076	0.047	0.006	0.006	0.006	0.049	0.076	0.103	0.146	0.146	0.146	0.096	0.62	0.076	0.0%
Carbon cycle parameter	361.5	84.8	246.5	246.5	246.5	305.7	360.0	398.7	496.6	496.6	496.6	152.2	0.23	360.0	-0.4%
Carbon intensity parameter	(0.0150)	0.0030	(0.0193)	(0.0193)	(0.0193)	(0.0168)	(0.0152)	(0.0134)	(0.0105)	(0.0105)	(0.0105)	0.0059	(0.20)	(0.0152)	1.1%
SCC, 2015	32.1	23.8	-	5.3	7.8	14.7	26.6	43.2	63.4	78.3	112.7	37.8	0.74	30.7	-4.5%
SCC, 2020	38.4	27.1	4.2	7.2	9.8	18.3	32.4	51.5	74.8	91.0	128.4	44.3	0.71	36.7	-4.5%
Temperature, 2050 (°C)	2.02	0.25	1.49	1.61	1.68	1.84	2.03	2.20	2.34	2.42	2.56	0.59	0.12	2.03	0.6%
Temperature, 2100 (°C)	3.47	0.65	2.20	2.49	2.67	3.02	3.43	3.87	4.31	4.62	5.31	1.38	0.19	3.48	0.3%
Temperature, 2200 (°C)	3.51	1.52	-	0.04	0.89	2.94	3.66	4.43	5.23	5.76	6.59	4.40	0.43	3.94	11.0%
Carbon concentrations, 2050 (GtC)	1,123	97	931	973	1,000	1,053	1,119	1,189	1,252	1,291	1,359	216	0.09	1,102	-1.9%
Carbon concentrations, 2100 (GtC)	1,398	322	980	1,048	1,095	1,186	1,319	1,512	1,788	2,046	2,625	464	0.23	1,338	-4.5%
Carbon concentrations, 2200 (GtC)	1,430	377	976	1,045	1,091	1,188	1,329	1,550	1,913	2,184	2,909	505	0.26	1,303	-9.7%
Carbon concentrations, 2050 (ppm)	527	45	437	457	470	494	525	558	588	606	638	101	0.09	517	-1.9%
Carbon concentrations, 2100 (ppm)	656	151	460	492	514	557	619	710	840	960	1,232	218	0.23	628	-4.5%
Carbon concentrations, 2200 (ppm)	672	177	458	491	512	558	624	728	898	1,025	1,366	237	0.26	612	-9.7%
World output, 2050 (trillions 2010 $)	320.7	129.2	163.2	165.4	166.6	233.1	292.9	370.6	542.8	547.1	551.1	205.3	0.40	293	-9.6%
World output, 2100 (trillions 2010 $)	1,195.8	1,091.3	192.8	197.4	200.2	448.1	767.5	1,336.3	3,235.7	3,292.2	3,384.2	1,138.9	0.91	764	-56.4%
Emissions 2050	43.8	19.9	15.4	19.4	22.4	28.6	39.4	54.5	72.7	84.4	102.1	35.1	0.45	39	-11.9%
Emissions 2100	18.90	28.15	0.00	0.00	0.00	0.00	10.80	23.77	49.36	70.91	144.62	23.77	1.49	12.74	-48.3%
Emissions 2200	3.51	7.55	0.00	0.00	0.00	0.00	0.00	4.28	11.07	18.12	39.03	4.28	2.15	na	na
Damages, 2050 (% output)	1.0%	0.5%	0.2%	0.3%	0.3%	0.6%	0.9%	1.3%	1.6%	1.9%	2.2%	1.0%	0.52	0.0	1.5%
Damages, 2100 (% output)	2.7%	1.3%	0.5%	0.8%	1.0%	1.7%	2.7%	3.6%	4.4%	4.9%	6.0%	2.8%	0.47	0.0	5.2%
Damages, 2200 (% output)	3.7%	1.6%	0.7%	1.2%	1.6%	2.5%	3.5%	4.7%	5.8%	6.5%	7.9%	3.4%	0.44	0.0	4.8%
Emissions intensity, 2050 (CO2/$)	0.350	0.000	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	-	0.00	0.350	0.0%
Emissions intensity, 2100 (CO2/$)	0.106	0.026	0.072	0.072	0.072	0.089	0.101	0.117	0.149	0.149	0.149	0.045	0.25	0.101	-4.4%
Emissions intensity, 2200 (CO2/$)	0.06	0.02	0.03	0.03	0.03	0.04	0.05	0.06	0.09	0.09	0.09	0.03	0.38	0.05	-9.7%
Real interest rate, 2015 (%/yr)	5.1%	1.0%	3.6%	3.7%	3.7%	4.5%	5.1%	5.7%	6.6%	6.6%	6.7%	2.0%	0.19	5.1%	-0.6%
Real interest rate, 2100 (%/yr)	3.7%	1.4%	1.6%	1.6%	1.7%	2.9%	3.7%	4.5%	5.9%	5.9%	5.9%	2.9%	0.39	3.6%	-2.0%
Objective (trillions, 2010$)	4,043	2,561	(437)	(326)	(260)	2,994	4,525	5,749	7,222	7,245	7,266	6,075	0.63	4,517	10.5%

Table A-2. Statistics of major variables from uncertainty analysis for the optimal run.[x]

Variable	Mean	50%ile	DICE cert	Error of certainty equivalent
Damage parameter	0.00236	0.00236	0.00236	0.0%
ETS	3.092	3.100	3.100	0.3%
Productiivity parameter	0.076	0.076	0.076	0.0%
Carbon cycle parameter	361.5	360.0	360.0	-0.4%
Carbon intensity parameter	(0.0150)	(0.0152)	(0.0152)	1.1%
SCC, 2015	35.8	28.4	31.2	-12.8%
SCC, 2020	42.5	33.8	37.3	-12.4%
Temperature, 2050 (°C)	2.11	2.12	2.13	0.9%
Temperature, 2100 (°C)	4.12	4.06	4.10	-0.3%
Temperature, 2200 (°C)	6.59	6.31	6.71	1.9%
Carbon concentrations, 2050 (GtC)	1,194	1,192	1,177	-1.5%
Carbon concentrations, 2100 (GtC)	1,885	1,775	1,760	-6.6%
Carbon concentrations, 2200 (GtC)	2,479	2,342	2,306	-7.0%
Carbon concentrations, 2050 (ppm)	561	560	552	-1.5%
Carbon concentrations, 2100 (ppm)	885	833	826	-6.6%
Carbon concentrations, 2200 (ppm)	1,164	1,100	1,082	-7.0%
World output, 2050 (trillions 2010 $)	313.3	292.8	292	-6.6%
World output, 2100 (trillions 2010 $)	867.0	760.8	757	-12.7%
Emissions 2050	62.9	58.2	58	-7.6%
Emissions 2100	82.9	71.0	70.9	-14.6%
Emissions 2200	61.7	60.8	60.6	-1.8%
Damages, 2050 (% output)	1.1%	1.0%	0.0	0.6%
Damages, 2100 (% output)	4.2%	3.6%	0.0	-4.6%
Damages, 2200 (% output)	8.1%	6.4%	0.1	-6.1%
Emissions intensity, 2050 (CO2/$)	0.350	0.350	0.350	0.0%
Emissions intensity, 2100 (CO2/$)	0.106	0.101	0.101	-4.2%
Emissions intensity, 2200 (CO2/$)	0.056	0.051	0.051	-8.8%
Real interest rate, 2015 (%/yr)	5.1%	5.1%	5.1%	0.0%
Real interest rate, 2100 (%/yr)	3.0%	2.9%	3.6%	20.0%
Objective (trillions, 2010$)	3,884	4,497	4,486	15.5%

Table A-3. Error of certainty equivalent approach to DICE model
The last column shows the error from using a certainty-equivalent rather than the uncertain version of the DICE model. The error is small where the distribution of the variable is close to symmetrical, but errors arise from skewed distributions.[xi]

Year	2015	2020	2025	2030	2035	2040	2045	2050	2055	2060	2065	2070	2075	2080	2085	2090	2095	2100
Industrial Emissions GTCO2 per year	35.7	39.4	42.9	46.3	49.6	52.7	55.5	58.2	60.6	62.7	64.6	66.3	67.7	68.8	69.7	70.3	70.7	70.9
Atmospheric concentration C (ppm)	399.5	418.5	438.3	459.1	481.0	503.9	527.7	552.4	577.8	604.0	630.8	658.1	685.7	713.7	741.8	770.1	798.3	826.4
Atmospheric Temperature	0.85	1.02	1.19	1.37	1.55	1.74	1.93	2.13	2.32	2.52	2.72	2.92	3.13	3.32	3.52	3.72	3.91	4.10
Net output	105	125	147	172	198	227	259	292	328	367	407	451	496	544	593	646	700	757
Climate Damages fraction output	0.0017	0.0024	0.0033	0.0044	0.0057	0.0071	0.0088	0.0107	0.0128	0.0150	0.0175	0.0202	0.0231	0.0261	0.0293	0.0326	0.0361	0.0398
Consumption Per Capita	10.50	11.84	13.30	14.89	16.61	18.46	20.45	22.58	24.86	27.28	29.85	32.56	35.43	38.46	41.63	44.95	48.43	52.05
Carbon Price (per t CO2)	2.00	2.21	2.44	2.69	2.97	3.28	3.62	4.00	4.42	4.88	5.38	5.94	6.56	7.25	8.00	8.83	9.75	10.77
Emissions Control Rate	0.03	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.06	0.07	0.08	0.08	0.09	0.10	0.10	0.11
Social cost of carbon	31.23	37.25	44.04	51.62	60.03	69.29	79.44	90.49	102.48	115.42	129.32	144.21	160.09	176.98	194.88	213.79	233.73	254.70
Interest Rate	0.051	0.050	0.049	0.048	0.047	0.046	0.045	0.044	0.043	0.042	0.041	0.040	0.039	0.039	0.038	0.037	0.036	0.036
Population	7,403	7,853	8,265	8,639	8,977	9,280	9,550	9,791	10,004	10,193	10,359	10,505	10,633	10,745	10,844	10,929	11,004	11,069
TFP	5.12	5.54	5.98	6.44	6.93	7.45	7.98	8.54	9.12	9.73	10.36	11.01	11.68	12.38	13.10	13.84	14.60	15.38
Gross output	105	125	148	173	200	229	261	296	333	372	415	460	508	558	611	668	727	788
Change tfp	0.076	0.074	0.072	0.071	0.069	0.067	0.065	0.064	0.062	0.061	0.059	0.058	0.056	0.055	0.054	0.052	0.051	0.050
Capital	223	268	318	375	437	505	579	660	746	840	940	1,047	1,160	1,280	1,407	1,541	1,682	1,830
Savings rate	0.26	0.26	0.25	0.25	0.25	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
Investment	27.25	32.04	37.32	43.09	49.38	56.19	63.52	71.40	79.82	88.79	98.32	108.41	119.07	130.29	142.07	154.43	167.34	180.81
Gross output, net of damages	105	125	147	172	198	228	259	292	328	367	408	451	496	544	593	646	700	757
Damages	0.18	0.31	0.49	0.76	1.13	1.64	2.30	3.16	4.24	5.60	7.26	9.28	11.70	14.56	17.91	21.79	26.25	31.34
Damage fraction	0.0017	0.0024	0.0033	0.0044	0.0057	0.0071	0.0088	0.0107	0.0128	0.0150	0.0175	0.0202	0.0231	0.0261	0.0293	0.0326	0.0361	0.0398
Abatement costs	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.04
Emissions intensity	0.35	0.32	0.30	0.28	0.26	0.24	0.22	0.21	0.19	0.18	0.17	0.15	0.14	0.13	0.13	0.12	0.11	0.10
Total forcings	2.46	2.74	3.01	3.29	3.57	3.84	4.12	4.39	4.66	4.92	5.18	5.44	5.69	5.93	6.16	6.39	6.61	6.82
Other Forcings	0.50	0.53	0.56	0.59	0.62	0.65	0.68	0.71	0.74	0.76	0.79	0.82	0.85	0.88	0.91	0.94	0.97	1.00
Period utilty	0.45	0.49	0.53	0.56	0.59	0.62	0.65	0.68	0.70	0.72	0.74	0.76	0.78	0.79	0.81	0.82	0.83	0.85
Consumption	77.74	93.00	109.96	128.65	149.10	171.31	195.31	221.09	248.66	278.03	309.17	342.09	376.78	413.22	451.40	491.30	532.89	576.16
Objective	4,485.74																	
Land emissions	2.60	2.30	2.04	1.80	1.59	1.41	1.25	1.11	0.98	0.87	0.77	0.68	0.60	0.53	0.47	0.42	0.37	0.33
Cumulative industrial emissions	400	449	502	561	624	692	764	839	919	1,001	1,087	1,175	1,265	1,358	1,452	1,547	1,642	1,739
Cumulative total emissions	500	552	609	670	736	806	880	957	1,038	1,122	1,209	1,298	1,389	1,482	1,577	1,672	1,769	1,866
Atmospheric concentrations GtC	851	891	934	978	1,025	1,073	1,124	1,177	1,231	1,287	1,344	1,402	1,461	1,520	1,580	1,640	1,700	1,760
Atmospheric concentrations ppm	400	418	438	459	481	504	528	552	578	604	631	658	686	714	742	770	798	826
Total Emissions GTCO2 per year	38	42	45	48	51	54	57	59	62	64	65	67	68	69	70	71	71	71
Atmospheric concentrations upper	460	471	485	501	519	539	561	585	610	636	664	693	723	754	786	819	852	886
Atmospheric concentrations lower	1,740	1,741	1,741	1,742	1,743	1,744	1,746	1,747	1,748	1,750	1,752	1,754	1,756	1,759	1,762	1,765	1,768	1,771
Atmospheric fraction since 1850	0.53	0.55	0.57	0.58	0.59	0.60	0.61	0.61	0.62	0.62	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63
Atmospheric fraction since 2010	-	0.77	0.76	0.74	0.73	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69	0.68	0.68	0.67	0.67	0.67

Table A-4. Detailed results for baseline run

Variable	Mean	St Dev	1%ile	5%ile	10%ile	25%ile	50%ile	75%ile	90%ile	95%ile	99%ile	IQ range	Coef of Var
Damage parameter	0.00236	0.00112	0.00071	0.00071	0.00071	0.00173	0.00236	0.00299	0.00401	0.00401	0.00401	0.00228	0.47
ETS	3.092	0.762	2.028	2.028	2.028	2.603	3.100	3.454	4.275	4.275	4.275	1.426	0.25
Productiivity parameter	0.076	0.047	0.006	0.006	0.006	0.049	0.076	0.103	0.146	0.146	0.146	0.096	0.62
Carbon cycle parameter	361.5	84.8	246.5	246.5	246.5	305.7	360.0	398.7	496.6	496.6	496.6	152.2	0.23
Carbon intensity parameter	(0.0150)	0.0030	(0.0193)	(0.0193)	(0.0193)	(0.0168)	(0.0152)	(0.0134)	(0.0105)	(0.0105)	(0.0105)	0.0059	(0.20)
SCC, 2015	32.1	23.8	-	5.3	7.8	14.7	26.6	43.2	63.4	78.3	112.7	37.8	0.74
SCC, 2020	38.4	27.1	4.2	7.2	9.8	18.3	32.4	51.5	74.8	91.0	128.4	44.3	0.71
Temperature, 2050 (°C)	2.02	0.25	1.49	1.61	1.68	1.84	2.03	2.20	2.34	2.42	2.56	0.59	0.12
Temperature, 2100 (°C)	3.47	0.65	2.20	2.49	2.67	3.02	3.43	3.87	4.31	4.62	5.31	1.38	0.19
Temperature, 2200 (°C)	3.51	1.52	-	0.04	0.89	2.94	3.66	4.43	5.23	5.76	6.59	4.40	0.43
Carbon concentrations, 2050 (GtC)	1,123	97	931	973	1,000	1,053	1,119	1,189	1,252	1,291	1,359	216	0.09
Carbon concentrations, 2100 (GtC)	1,398	322	980	1,048	1,095	1,186	1,319	1,512	1,788	2,046	2,625	464	0.23
Carbon concentrations, 2200 (GtC)	1,430	377	976	1,045	1,091	1,188	1,329	1,550	1,913	2,184	2,909	505	0.26
Carbon concentrations, 2050 (ppm)	527	45	437	457	470	494	525	558	588	606	638	101	0.09
Carbon concentrations, 2100 (ppm)	656	151	460	492	514	557	619	710	840	960	1,232	218	0.23
Carbon concentrations, 2200 (ppm)	672	177	458	491	512	558	624	728	898	1,025	1,366	237	0.26
World output, 2050 (trillions 2010 $)	320.7	129.2	163.2	165.4	166.6	233.1	292.9	370.6	542.8	547.1	551.1	205.3	0.40
World output, 2100 (trillions 2010 $)	1,195.8	1,091.3	192.8	197.4	200.2	448.1	767.5	1,336.3	3,235.7	3,292.2	3,384.2	1,138.9	0.91
Emissions 2050	43.8	19.9	15.4	19.4	22.4	28.6	39.4	54.5	72.7	84.4	102.1	35.1	0.45
Emissions 2100	18.90	28.15	0.00	0.00	0.00	0.00	10.80	23.77	49.36	70.91	144.62	23.77	1.49
Emissions 2200	3.51	7.55	0.00	0.00	0.00	0.00	0.00	4.28	11.07	18.12	39.03	4.28	2.15
Damages, 2050 (% output)	1.0%	0.5%	0.2%	0.3%	0.3%	0.6%	0.9%	1.3%	1.6%	1.9%	2.2%	1.0%	0.52
Damages, 2100 (% output)	2.7%	1.3%	0.5%	0.8%	1.0%	1.7%	2.7%	3.6%	4.4%	4.9%	6.0%	2.8%	0.47
Damages, 2200 (% output)	3.7%	1.6%	0.7%	1.2%	1.6%	2.5%	3.5%	4.7%	5.8%	6.5%	7.9%	3.4%	0.44
Emissions intensity, 2050 (CO2/$)	0.350	0.000	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	-	0.00
Emissions intensity, 2100 (CO2/$)	0.106	0.026	0.072	0.072	0.072	0.089	0.101	0.117	0.149	0.149	0.149	0.045	0.25
Emissions intensity, 2200 (CO2/$)	0.06	0.02	0.03	0.03	0.03	0.04	0.05	0.06	0.09	0.09	0.09	0.03	0.38
Real interest rate, 2015 (%/yr)	5.1%	1.0%	3.6%	3.7%	3.7%	4.5%	5.1%	5.7%	6.6%	6.6%	6.7%	2.0%	0.19
Real interest rate, 2100 (%/yr)	3.7%	1.4%	1.6%	1.6%	1.7%	2.9%	3.7%	4.5%	5.9%	5.9%	5.9%	2.9%	0.39
Objective (trillions, 2010$)	4,043	2,561	(437)	(326)	(260)	2,994	4,525	5,749	7,222	7,245	7,266	6,075	0.63

Table A-5. Detailed results for maximum abatement run[xii]

References for figures and Tables:

[i] (dam est for eviews-090916-up112816.xlsx, page unc_coefs)
[ii] unc-SCC-tabfig-101516-120216.xlsx
[iii] unc-113016.wf1
[iv] DiceResults-loopv23I_II_113016.xlsx, DiceResults-loopv23I
[v] DiceResults-loopv23I-OPT-120216.xlsx
[vi] marginal sigma 113016.xlsx; page singles and table.
[vii] DiceResults-loopv23I_II_113016.xlsx, DiceResults-loopv23I
[viii] approx error 112916.xlsx
[ix] DiceResults-loopv23I_II_113016.xlsx, DiceResults-loopv23I
[x] DiceResults-loopv23I-OPT-120216.xlsx
[xi] DiceResults-loopv23I_II_113016.xlsx, page CE
[xii] DiceResults-loopv23I-max.xlsx