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ABSTRACT

We examine the importance of ambiguity, or Knightian uncertainty, in the capital structure decision. We develop a static tradeoff theory model in which agents are both risk averse and ambiguity averse. The model confirms the usual idea that increased risk - the uncertainty over known possible outcomes - leads firms to use less leverage. Conversely, greater ambiguity - the uncertainty over the probabilities associated with the outcomes - leads firms to increase leverage. Our empirical analysis provides results consistent with these predictions.

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1 Introduction

We incorporate ambiguity and ambiguity aversion into the static tradeoff theory of capital structure in order to develop a more complete understanding of the leverage decisions made by corporations. Ambiguity, or Knightian uncertainty, refers to the case where not only is the event to be realized *a priori* unknown, but also the odds of the possible events are either unknown or not uniquely assigned. In contrast, risk is defined as a condition in which the event to be realized is *a priori* unknown, but the odds of the possible events are perfectly known. Throughout the paper the term "uncertainty" is used to refer to the aggregation of risk and ambiguity.

Every significant financial decision is subject to ambiguity. Ellsberg (1961) has demonstrated that individuals act as if they are averse to ambiguity.[1] Nevertheless, the canonical results in finance theory have been developed using expected utility theory, a theory that can be interpreted as either ignoring ambiguity or assuming that individual preferences are neutral to this aspect of uncertainty. Most empirical studies of leverage, including literature standards such as Titman and Wessels (1988), consider only risk and take no account of ambiguity.

Modigliani and Miller's (1958) theorem asserts that two firms generating the same distribution of future cash flow will have the same market value regardless of their debt-equity structure. We develop an extension of this theorem by introducing ambiguity. We show that when there are no taxes, no bankruptcy costs and no asymmetric information, Modigliani-Miller's irrelevance is maintained under ambiguity aversion. When taxes and bankruptcy costs are introduced into the model, the theory predicts that greater ambiguity is associated with more extensive use of leverage. On the other hand, the well known result that higher risk is associated with less extensive use of leverage is maintained.

To test the predictions of our model, we investigate whether the levels of firms' leverage are affected by the level of ambiguity and risk firms are exposed to in the previous year. In order to account for the unobservable firm specific component of leverage identified by Lemmon *et al.* (2008), we also test whether changes in the extent of ambiguity or risk are followed by a change in the use of leverage in the subsequent year. We study these relations using both the book leverage ratio and the market leverage ratio. We use firms' realized ambiguity and risk measures computed directly from stock trading data, and we also employ measures of expected ambiguity and risk as predicted by an ARMA model.

The intuition explaining our findings is that when ambiguity is relatively high the cost of debt is low relative to the cost of equity. This intuition is supported by Augustin and Izhakian (2016) who

[1] Ellsberg (1961) demonstrates that in the presence of ambiguity, individuals typically violate the independence (or "Sure-Thing Principle") axiom of expected utility theory.

find that the spreads of credit default swaps (CDS) decrease with ambiguity. Since CDS spreads reflect the interest rates on the debt, these findings imply that the cost of debt decreases with ambiguity.

Previous studies of ambiguity have focused mainly on its theoretical aspects, while not testing their models empirically.[2,3] In general, these studies employ decision models that do not provide the appropriate separations between risk and ambiguity and between tastes and beliefs that are crucial for a straightforward measurement of ambiguity.[4] The idea of the decision-making model we employ, Izhakian (2016a), is that preferences for ambiguity are applied directly to probabilities such that attitude toward ambiguity is defined as attitude toward mean-preserving spreads in probabilities—analogous to the Rothschild-Stiglitz risk attitude toward mean-preserving spreads in outcomes. As such, the degree of ambiguity can be measured by the variance of probabilities, just as the degree of risk can be measured by the variance of outcomes.[5] We employ the same methodology used by Brenner and Izhakian (2016) and Izhakian and Yermack (2016) to estimate ambiguity from stock data. The extensive tests that Brenner and Izhakian (2016) conduct rule out the concern that our ambiguity measure captures other well-known dimensions of uncertainty. We find that ambiguity and volatility measure distinctly separate aspects of financial uncertainty, as they have a simple correlation across firms that is weakly negative but very near zero.

Finally, Lee (2014) considers a similar question to that posed here and finds (theoretically and empirically) that increased ambiguity leads to a reduction in leverage. His model assumes risk neu-

[2]Special attention has been given to the implications of ambiguity for the equity premium; e.g., Cao *et al.* (2005), Nau (2006), Epstein and Schneider (2008) and Izhakian and Benninga (2011). Other studies attempt to provide explanations for puzzling asset pricing phenomena by introducing ambiguity into conventional asset pricing models. For example, Pflug and Wozabal (2007) and Boyle *et al.* (2011) suggest extensions of the mean-variance approach to incorporate ambiguity. Additional papers have developed extensions of the capital asset pricing model that incorporate ambiguity, such as Chen and Epstein (2002) and Izhakian (2012).

[3]*Parameter uncertainty* (known unknowns) assumes that the set of events is known, the nature of the probability distribution is known, but the parameters governing the distribution are unknown and the decision maker maximizes utility using posterior parameters that generate a set of priors, which can be viewed as reflecting both information (beliefs) and tastes for ambiguity; e.g., Bawa *et al.* (1979), Barry and Brown (1984), Coles and Loewenstein (1988), and Coles *et al.* (1995). Therefore, parameter uncertainty may be viewed as special case of ambiguity, in which the nature of the probability distributions is known. In this view, *model uncertainty* is also a special case of ambiguity. This class of models assumes an uncertainty about the true probability law governing the realization of states, and a decision maker, with her concerns about misclassification, looks for a robust decision-making process; e.g., Hansen *et al.* (1999), and Hansen and Sargent (2001). Other studies take an empirical view of model uncertainty (or *model risk*). In this perspective, while estimating an empirical model, there is uncertainty about the true set of predictive variables. To account for such a model misspecification, a Bayesian (predictive distribution) approach may be taken by assigning each set of variables (or model) a posterior probability; e.g., Pastor (2000), Pastor and Stambaugh (2000), Avramov (2002), and Cremers (2002).

[4]Earlier literature dealt with this limitation either by model calibration (e.g., Drechsler (2012)) or by attributing the disagreement of professional forecasters to ambiguity (e.g., Anderson *et al.* (2009)).

[5]Some studies use proxies for ambiguity like disagreement among analysts; e.g., Anderson *et al.* (2009) and Antoniou *et al.* (2015). Most empirical (behavioral) studies about ambiguity use data collected in designed experiments and focus on individuals' attitudes toward ambiguity rather than on the implications of ambiguity itself. A very few studies use field data such as stock market trading data; e.g., Williams (2015), Ulrich (2013) and Thimmea and Volkertb (2015).

trality, so it cannot consider the impact of risk. His model assumes that the firm manager is ambiguity averse but that market participants are ambiguity neutral, which seems to be the main cause of the difference between his theoretical predictions and ours. Lee's empirical results are based on an event (the 1982 Voluntary Restraint Agreement on steel import quotas) that almost certainly conflates risk and ambiguity.

The remainder of this paper is organized as follows. Section 2 presents a theoretical discussion of ambiguity and develops the model. Section 3 discusses the sample selection and empirical tests. Section 4 presents regression analysis of capital structure, and Section 5 concludes the paper. All proofs are provided in the Appendix.

2 The model

2.1 The decision theoretic framework

Ambiguity, or Knightian uncertainty, has provided the basis for a rich literature in decision theory.[6] This literature takes a variety of approaches to modeling decision-making under ambiguity (i.e., the max-min expected utility of Gilboa and Schmeidler (1989), the Choquet expected utility (CEU) of Schmeidler (1989), and many others). A fundamental issue these models have tried to address is the need for an applicable framework that can be used in empirical explorations. The key to satisfying this need is a complete separation between risk and ambiguity and between tastes and beliefs. Only when this separation is accomplished will we be able to measure the extent of ambiguity regarding a specific decision without confounding this measure with preferences over ambiguity, the amount of risk, and/or preferences regarding risk. The models identified above have not achieved this separation.

We distinguish the concepts of risk and ambiguity by using the theoretical framework of expected utility with uncertain probabilities (EUUP), proposed by Izhakian (2016a). Under EUUP individuals are thought to act as if they solve a two-stage decision-making problem. In the first stage, the decision maker (DM) forms a representation of her perceived probabilities for each relevant event. The

[6] Knight (1921) defines the concept of Knightian uncertainty as distinct from risk as conditions under which the set of events that may occur is *a priori* unknown, and the odds of these events are also either not unique or are unknown. Roughly speaking, this concept can be viewed as underpinning two branches of literature. The first is the "unawareness" literature, which assumes that decision makers may not be aware of a subset of events—formally the state space is updated consequentially to reflect learning about new states of nature (e.g., Karni and Vierø (2013)). The second is the ambiguity literature, which assumes that the set of events is perfectly known but their probabilities are either not unique or are unknown (e.g., Gilboa and Schmeidler (1989) and Schmeidler (1989)). These two literatures can be viewed as overlapping when dealing with monetary outcomes (real numbers). In these outcomes, the "uncertain"—risky and ambiguous—variable is defined by a measurable function from states into the real numbers such that there is no real monetary outcome that the decision maker is not aware of. It is possible that the decision maker is not aware of some events (the so-called black swans), which affects the uncertainty about the probabilities of some outcomes. But such uncertainty is taken into account by ambiguity—the uncertainty about the probabilities of outcomes.

perceived probabilities are determined by the ambiguity that is present and the DM's tastes regarding ambiguity. In the second stage of the process the DM considers the expected utility associated with a set of possible outcomes, taken with respect to the perceived probabilities. This structure accomplishes the desired separation between risk and ambiguity and between tastes and beliefs and facilitates theoretical and empirical analyses.

The main idea of EUUP is that in the presence of ambiguity, i.e., when probabilities are uncertain, preferences concerning ambiguity are applied solely to the probabilities such that aversion to ambiguity is defined as aversion to mean-preserving spreads in probabilities. As such, the Rothschild and Stiglitz (1970) approach, which is typically applied to outcomes when examining risk, can also be applied to probabilities when examining ambiguity, independently of risk. Thereby, as Izhakian (2016b) shows, the degree of ambiguity can be measured by the volatility of probabilities—just as the degree of risk can be measured by the volatility of outcomes.[7] Unlike other (arguable) measures of ambiguity, which are risk-dependent and consider only the variance of a single moment of the outcome distribution (i.e., the variance of the mean or the variance of variance) the measure derived by Izhakian (2016b) is risk-independent and conceptually accounts for the variance of all moments of the outcome distribution. The measure of ambiguity proposed by EUUP can be employed in empirical studies and may be implemented using equity market data. This measure has, for example, been used in studies of the risk–ambiguity–return relationship in the equity markets (Brenner and Izhakian, 2016) and of the role ambiguity plays in employees' decisions of whether to exercise their options (Izhakian and Yermack, 2016).

Formally, let $\mathcal{S}$ be an infinite, nonempty state space, and let $X : \mathcal{S} \to \mathbb{R}$ be a measurable function describing consumption.[8] An investor, who doesn't know the precise probabilities of the relevant outcomes, holds a set $\mathcal{P}$ of possible (additive) probability measures P on $\mathcal{S}$. In addition, she holds a second-order belief (a probability measure on $\mathcal{P}$), denoted ξ. Using the EUUP theory, the expected utility of the investor, who does not distort perceived probabilities, can be represented by

$$\begin{aligned} \mathrm{V}(X) = & \int_{z \leq 0} \left[\Upsilon^{-1} \left(\int_{\mathcal{P}} \Upsilon \left(\mathrm{P}_X \left(\mathrm{U}(x) \geq z \right) \right) d\xi \right) - 1 \right] dz + \\ & \int_{z \geq 0} \Upsilon^{-1} \left(\int_{\mathcal{P}} \Upsilon \left(\mathrm{P}_X \left(\mathrm{U}(x) \geq z \right) \right) d\xi \right) dz, \end{aligned} \tag{1}$$

[7] Other models do not permit such a derivation since either ambiguity is not distinguished from aversion to ambiguity (e.g., Schmeidler (1989), and Gilboa and Schmeidler (1989)) or aversion to ambiguity is defined as aversion to mean-preserving spreads in certainty equivalent utilities, which are subject to risk and preferences for risk (e.g., Chew and Sagi (2008)).

[8] We consider an economy with an infinite state space and an infinite set of outcomes. All our results, however, may also be applied to an economy with a finite state space. Because the model's intuition is illustrated more naturally in a finite state space setting, we introduce this presentation below and more completely in Appendix A.1.

where U is a (von Neumann-Morgenstern) utility function; and Υ is an *outlook function*, capturing attitude toward ambiguity. As usual, a concave U characterizes risk-averse behavior, and a convex U characterizes risk-loving behavior. Similarly, a concave Υ characterizes ambiguity-averse behavior, and a convex Υ characterizes ambiguity-loving behavior. The utility function is normalized such that $\mathrm{U}(k) = 0$, for some reference point k. That is, any outcome $x \leq k$ is considered unfavorable (a loss) and any outcome $x \geq k$ is considered favorable (a gain).

To illustrate the intuition of Equation (1), it might be helpful to write it for a discrete state space $\mathcal{S}$ and for finitely many probability measures in $\mathcal{P}$. In this case the outcomes of X can be written $X = (x_1, \ldots, x_n)$, where x_s stands for the outcome at state $s \in \mathcal{S}$ and $x_1, \ldots, x_n$ are listed in a non-decreasing order. The expected utility representation of the investor's preferences is then written

$$\mathrm{V}(X) = \sum_{0<s\leq k} \mathrm{Q}_X(x_s) \left[\mathrm{U}(x_s) - \mathrm{U}(x_{s+1})\right] + \sum_{k<s\leq n} \mathrm{Q}_X(x_s) \left[\mathrm{U}(x_s) - \mathrm{U}(x_{s-1})\right], \tag{2}$$

where

$$\mathrm{Q}_X(x_s) = \Upsilon^{-1}\left(\sum_{\mathrm{P}\in\mathcal{P}} \Upsilon\left(\mathrm{P}_X(x_s)\right) \xi\left(\{\mathrm{P}_X\}\right)\right) \tag{3}$$

and $\mathrm{P}_X(x_s)$ stands for $\mathrm{P}(X \geq x_s)$, i.e., the probability of the outcome being *greater* than x_s. The term $\mathrm{Q}_X(x_s)$ is the discrete equivalent of the $\Upsilon^{-1}\left(\int_{\mathcal{P}} \Upsilon\left(\mathrm{P}_X(x)\right) d\xi\right)$ in Equation (1) and represents the perceived probability that the outcome is greater than or equal to x_s. Intuitively, the *perceived probability* is the unique certain probability value that the DM is willing to accept in exchange for the uncertain probability of a given event. The term in square brackets in Equation (2) is equivalent to the dz term in Equation (1), capturing the change in utility from one outcome to the next.

If the perceived probabilities are additive, Equation (2) could be written in a more familiar form as the perceived probability of state s times the utility of the outcome in state s summed over the n states. However, because the perceived probability of state s ($\mathrm{Q}_X(x_s)$ which is used for assessing expected utility under ambiguity aversion rather than the probability of that outcome) might be non-additive, expected utility in a discrete outcome space cannot generally be written in this manner. However, this representation of the preference representation makes it clear that the model we use can be viewed as a version of CEU (Schmeidler, 1989) where Schmeidler's "capacities" are structured as perceived probabilities. The important distinction is that Izhakian's (2016a) axiomatic development of the preceived probabilities implies that EUUP accomplishes the necessary separation between risk and ambiguity and between beliefs and tastes that is not accomplished in CEU. This separation allows us to consider the comparative static response of leverage to a change in ambiguity from a theoretical perspective and also allows the development of a measure of ambiguity that may be used in empirical investigations. The apparent complexity of Equation (1) lies in the need to use Choquet integration (as in CEU) due to the non-additivity of the perceived probabilities.

Referring again to Equation (1), suppose that the outlook function is twice differentiable and satisfies

$$\left|\frac{\Upsilon''\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}{\Upsilon'\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}\right| \quad \leq \quad \frac{1}{\mathrm{Var}\left[\varphi_X\left(x\right)\right]}, \tag{4}$$

for every x, where $\varphi_X\left(x\right)$ is the marginal probability function (density or mass function) of X associated with $\mathrm{P} \in \mathcal{P}$; and $\mathrm{E}\left[\cdot\right]$ and $\mathrm{Var}\left[\cdot\right]$ are respectively the expectation and the variance of probabilities taken using the second order probability measure ξ. Namely,

$$\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right] \;=\; \int_{\mathcal{P}} \mathrm{P}_X\left(x\right) d\xi \qquad \text{and} \qquad \mathrm{Var}\left[\varphi_X\left(x\right)\right] \;=\; \int_{\mathcal{P}} \left[\varphi_X\left(x\right) - \mathrm{E}\left[\varphi_X\left(x\right)\right]\right]^2 d\xi. \tag{5}$$

Condition (4) bounds the level of aversion to ambiguity (the concavity of Υ) and the level of love for ambiguity (the convexity of Υ) to assure that the approximated marginal perceived probabilities are nonnegative and that the perceived probability of an event is not smaller than the perceived probability of any of its sub-events. By Izhakian (Theorem 2, 2016b), the dual representation of the expected utility in Equation (1), can then be approximated as[9]

$$\begin{aligned}\mathrm{W}\left(X\right) \;\approx\; & \int_{x\leq k} \mathrm{U}\left(x\right)\mathrm{E}\left[\varphi_X\left(x\right)\right]\left(1 - \frac{\Upsilon''\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}{\Upsilon'\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi_X\left(x\right)\right]\right) dx \;+ \\ & \int_{x\geq k} \mathrm{U}\left(x\right)\mathrm{E}\left[\varphi_X\left(x\right)\right]\left(1 + \frac{\Upsilon''\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}{\Upsilon'\left(\mathrm{E}\left[\mathrm{P}_X\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi_X\left(x\right)\right]\right) dx.\end{aligned} \tag{6}$$

To simplify notations, henceforth the subscript X, designating the random variable, is omitted. In addition, since X is a measurable function, we abuse the notation and write the outcome x in state s instead of state s itself. With this notation in place, the marginal perceived probability (density) of x can be written

$$\pi\left(x\right) \;=\; \begin{cases} \mathrm{E}\left[\varphi\left(x\right)\right]\left(1 + \eta\big(\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\big)\mathrm{Var}\left[\varphi\left(x\right)\right]\right), & x \leq k \\ \mathrm{E}\left[\varphi\left(x\right)\right]\left(1 - \eta\big(\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\big)\mathrm{Var}\left[\varphi\left(x\right)\right]\right), & x > k \end{cases}, \tag{7}$$

where $\eta\left(\cdot\right) = -\dfrac{\Upsilon''\left(\cdot\right)}{\Upsilon'\left(\cdot\right)}$ and, by Condition (4), $\pi(x)$ is always positive.

Based upon the functional form in Equation (6), the degree of ambiguity can be measured by the expected volatility of probabilities, across the relevant events. Formally, the measure of ambiguity is

[9]The remainder of this approximation is of order $o\big(\int \mathrm{E}\left[\left|\varphi\left(x\right) - \mathrm{E}\left[\varphi\left(x\right)\right]\right|^3\right] x dx\big)$ as $\int \left|\varphi_x\left(x\right) - \mathrm{E}\left[\varphi_x\left(x\right)\right]\right| dx \to 0$. This is equivalent to a cubic approximation, $o\left(\mathbb{E}\left[\left|x - \mathbb{E}\left[x\right]\right|^3\right]\right)$, in which the fourth and higher absolute central moments of outcomes X are of strictly smaller order than the third absolute central moment as $\left|x - \mathbb{E}\left[x\right]\right| \to 0$, and are therefore negligible.

given by[10]

$$\mho^2[X] \;=\; \int \mathrm{E}\left[\varphi(x)\right]\mathrm{Var}\left[\varphi(x)\right]dx, \tag{8}$$

where x is assumed to be *symmetrically* distributed as defined by Izhakian (Definition 7, 2016).[11] The main advantage of this measure is that it can be computed from the data and can be employed in empirical tests; e.g., Brenner and Izhakian (2016) and Izhakian and Yermack (2016). *Risk independence* is another major advantage of $\mho^2$ (mho^2); unlike risk measures, it does not depend upon the magnitudes of the associated consequences.

2.2 The asset pricing framework

The decision theoretic framework of the previous section enables the formalization of an ambiguous economy. We use a common market structure where the only variation in our analysis is the specification of probabilities. We assume markets are complete and an absence of arbitrage opportunities (the law of one price holds). There are two dates, 0 and 1. The development of the capital structure model in the next section is not dependent on the simple assumptions made regarding the market structure, and the two-period utility function employed here can be generalized to a dynamic structure following the approach taken in prior papers. The state at date 0 is known, and the states at date 1 are ordered from the lowest consumption level to the highest.

To develop the asset pricing framework, consider a representative investor, with a twice differentiable utility function U, whose initial wealth is w_0 and whose wealth at time 1 is w_1.[12] The uncertain outcomes of the single product in the economy are defined by the set of outcomes X, where $x \in X$ is the time 1 outcome in some state $s \in \mathcal{S}$. Since, the asset pricing framework is used only to extract state prices, for simplicity, we assume a single product in the economy. However, the extension to multiple assets is straightforward.

The objective function of an individual agent is standard and can then be written

$$\max_{c_0,\theta} \quad \mathrm{W}(c_0) + \mathrm{W}(c_1) \tag{9}$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x)x dx \qquad \text{and} \qquad c_1 = w_1 + \theta x,$$

[10] The measure $\mho^2$ is also applicable in finite state space. In this case, $\mho^2[X] = \sum_i \mathrm{E}\left[\varphi(x_i)\right]\mathrm{Var}\left[\varphi(x_i)\right]$, where $\varphi(\cdot)$ is a probability mass function.

[11] Formally, outcomes are said to be symmetrically distributed around a point of symmetry k if for any $x, y \in X$ that satisfy $|x-k| = |y-k|$, both $\mathrm{E}\left[\varphi(x)\right] = \mathrm{E}\left[\varphi(y)\right]$ and $\mathrm{Var}\left[\varphi(x)\right] = \mathrm{Var}\left[\varphi(y)\right]$ hold.

[12] À la Constantinides (1982), a representative investor can be defined as one whose tastes and beliefs are such that, if all investors in the economy had identical tastes and beliefs, the equilibrium in the economy would remain unchanged.

where $q(x)$ is the date 0 price of the contingent claim on state s with consumption x and θ is the holding of the single asset X.[13] Using the functional form of expected utility in Equation (6), the state prices can be extracted as follows.

Theorem 1 *Suppose a time-separable utility function and an outlook function* Υ *satisfying the conditions of Izhakian (Theorem 2, 2016b).*[14] *The state price of state* x *is then*

$$q(x) \;=\; \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{10}$$

where $q(x)$ *is always positive.*

The state price $q(x)$ is the price of a pure state contingent claim or Arrow security of state s with outcome x.[15] That is, it is the price of a claim to one unit of consumption contingent on the occurrence of state s. Since markets are complete and the law of one price holds, the payoff pricing functional assigns a unique price to each state contingent claim. When there is no ambiguity, i.e., probabilities are perfectly known, the state price $q(x)$ reduces to the conventional state price $q(x) = \varphi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.[16] The following corollary is an immediate consequence of Theorem 1.

Corollary 1 *The risk neutral probability of state* x *is*

$$\pi^*(x) = \frac{q(x)}{\displaystyle\int q(x)dx} = \frac{\pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}}{\displaystyle\int \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx}, \tag{11}$$

and the risk-free rate of return is

$$r_f \;=\; \frac{1}{\displaystyle\int q(x)dx} - 1 \;=\; \frac{1}{\displaystyle\int \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx} - 1. \tag{12}$$

The definition of the risk-neutral probabilities and the risk-free rate of return in an ambiguous economy allows us to define the price of an asset as its discounted expected payoff. That is,

$$p \;=\; \int q(x)xdx \;=\; \frac{1}{1+r_f}\int \pi^*(x)xdx \;=\; \frac{1}{1+r_f}\mathbb{E}^*\left[x\right], \tag{13}$$

where $\mathbb{E}^*$ is the expectation taken with respect to the risk neutral probabilities π^*, defined in Corollary 1. Notice that the double-struck capital font is used to designate expectation or variance of

[13] Formally, the consumptions, c_0 and c_1, need not be restricted to positive values, since in the EUUP's settings, as in cumulative prospect theory (Tversky and Kahneman, 1992), the utility function can obtain negative values.

[14] As in Equation (4), Theorem 2 in Izhakian (2016b) bounds the concavity and the convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and that they satisfy set monotonicity.

[15] In a related study, Chapman and Polkovnichenko (2009) extracts the state prices in a rank-dependent expect utility framework.

[16] Similarly, in the presence of ambiguity but with ambiguity neutral agents the state price $q(x)$ reduces to $q(x) = \mathrm{E}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.

outcomes, taken with respect to the expected probabilities or with respect to risk-neutral probabilities, while the regular straight font is used to designate expectation or variance of *probabilities*, taken with respect to the second-order probabilities.

2.3 Capital structure decision

The capital structure model is based on the canonical tradeoff theory model of Kraus and Litzenberger (1973). Consider a one-period project that requires a capital investment I at time 0. The payoff of this project, obtained at time 1, is given by the risky and ambiguous variable X, having a bounded support. It is assumed that the information is symmetric in the sense that all agents agree upon the set $\mathcal{P}$ of possible probability distributions of X and therefore they also agree on the expected probability of any specific payoff x.

For simplicity, it is assumed that the owner of the project is a representative investor, whose set of priors $\mathcal{P}$ is identical to the set of priors of each investor in the economy, and her belief (second-order probability distribution) over $\mathcal{P}$ is also identical to that of each investor in the economy over. At time $t = 0$, a decision to invest in a project is made if and only if

$$\frac{1}{1+r_f}\mathbb{E}^*\left[X\right] > I.$$

Given the decision to invest in the project, its owner must decide at time 0 what mix of debt and equity financing the firm will maintain to run the project. The objective of this decision is to maximize the value of the firm:

$$\begin{aligned}
\max_{F} \quad & S_0\left(F\right) + D_0\left(F\right) \\
s.t. \quad S_0 &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left(X - F\ ,\ 0\right)\right] \\
D_0 &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ X\right)\right],
\end{aligned}$$

where S_0 is the market value of the common shares; D_0 is the market value of debt and F is the debt's face value. Without loss of generality the debt is assumed to be a one-period zero-coupon bond. The optimization problem can be written more explicitly as

$$\begin{aligned}
\max_{F} \quad & S_0\left(F\right) + D_0\left(F\right) \\
s.t. \quad S_0 &= \frac{1}{1+r_f}\int_F^{\infty} \pi^*(x)\left(x - F\right)dx \\
D_0 &= \frac{1}{1+r_f}\left(\int_0^k \pi^*(x)xdx + \int_k^F \pi^*(x)xdx + \int_F^{\infty} \pi^*(x)Fdx\right).
\end{aligned}$$

Any $x < F$ is considered a default state and the reference point is assumed to satisfy $k < F$. In this maximization problem, the default states $\{x \mid k \leq x \leq F\}$ are still considered favorable in the view of the representative investor. This assumption is consistent with the notion that corporate debt is not riskless. This assumption is required in our analysis, since otherwise, no rational decision maker will consider investing in the firm's debt.

Theorem 2 *Suppose that there are no taxes, no bankruptcy costs and no asymmetric information. The Modigliani-Miller's irrelevance is then maintained under ambiguity.*

Proposition 1 in Modigliani and Miller (1958) shows that, in an economy without taxes bankruptcy costs or asymmetric information, the capital-structure choice of a firm is irrelevant to its market value. Their proof is based on the idea the an investor can generate the return to holding the levered firm's equity by holding the equity of an equivalent unlevered firm and a loan in the same proportion as the leverage ratio of the levered firm. The same argument holds true in an ambiguous economy.

2.4 Taxes and bankruptcy costs

Consider now an economy with taxes and bankruptcy costs. Suppose that any state $x < F$ is considered a default state. The bankruptcy costs are assumed to be proportional to the firm's payoff in the event of default

$$B(F) = \begin{cases} \alpha x, & x < F \\ 0, & x \geq F \end{cases},$$

where $0 < \alpha < 1$. The tax benefit associated with the use of debt is, for simplicity, based on the entire debt service in non-default states,

$$T(F) = \begin{cases} 0, & x < F \\ \tau F, & x \geq F \end{cases},$$

where τ is the corporate income tax rate. In such an economy, firm value is a function of the capital structure, in particular the amount of debt F issued, and can be written

$$\begin{aligned} V(F) &= S_0(F) + D_0(F) \\ &= \frac{1}{1+r_f}\left(\begin{array}{l} \int_F^\infty \pi^*(x)\,(1-\tau)\,(x-F)\,dx + \\ \int_0^k \pi^*(x)\,(1-\alpha)\,x dx + \int_k^F \pi^*(x)\,(1-\alpha)\,x dx + \int_F^\infty \pi^*(x) F dx \end{array}\right). \end{aligned} \tag{14}$$

The capital structure choice problem becomes

$$\begin{aligned} \max_{F} \quad & S_0(F) + D_0(F) \\ s.t. \quad S_0 &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)(x-F)\ ,\ 0\right)\right] \\ D_0 &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ (1-\alpha)x\right)\right]. \end{aligned}$$

With the capital structure choice problem defined as above, the next theorem identifies the optimal level of leverage for the firm.

Theorem 3 *The optimal leverage satisfies*

$$F = \frac{\tau}{\pi^*(F)\alpha}\int_F^\infty \pi^*(x)dx. \tag{15}$$

We turn now to analyze the determinants of the optimal leverage. It can be immediately observed from Equation (15) that a higher tax rate (τ) or a lower bankruptcy cost (α) positively affect the use of leverage. Our focus is, however, on the implications of changes in the components of uncertainty (ambiguity and risk) for capital structure choice. Proposition 1 describes the consequence of a comparative static change in ambiguity.

Proposition 1 *The higher is the degree of ambiguity, the higher is the optimal leverage.*

Similarly to the comparative static concerning the effect of ambiguity on the optimal leverage, we may also study the effect of a change in risk. To this end, the notation $\partial_{xxx}\mathrm{U}$ stands for the third derivative of U with respect to x. It is important to note that risk, denoted $\mathcal{R}$, is not assumed to be measured by the variance of returns or payoffs, since it is not assumed that returns are normally distributed or that utility is quadratic. As in Rothschild and Stiglitz (1970), an asset is said to become risker if its payoffs at time t can be written as mean-preserving spread of its outcomes at time $t-1$.

Proposition 2 *Suppose $\partial_{xxx}\mathrm{U} \geq 0$.*[17] *Then the higher is the degree of risk, the lower is the optimal leverage.*

Propositions 1 and 2 demonstrate that increased ambiguity has a *positive* effect on the chosen level of leverage, while increased risk has a *negative* effect. The intuition for the result in Proposition 2 is standard: Greater volatility in the project payoff increases the expected cost of bankruptcy at the margin, leaving the expected marginal tax benefit of debt fixed. It can be tempting to suggest that

[17] This property is satisfied by many well known utility functions including constant relative risk aversion (CRRA) and constant absolute risk aversion (CARA).

this intuition applied to an increase in ambiguity as well as more *uncertain* cash flows can be thought to have the same effect. An increase in ambiguity, however, represents an increased uncertainty regarding probabilities however this increased uncertainty is stake independent (unlike an increase in risk) which implies the standard intuition cannot be applied. A more appropriate intuition comes from the effect of ambiguity and ambiguity aversion that is common across most models of ambiguity averse preferences; agents act as if they put more weight on “worse” outcomes and less on “better” outcomes. This implies that equity becomes unambiguously (no pun intended) less valuable when ambiguity is increased. The relative cost of debt versus equity financing for the firm therefore decreases, resulting in an increased use of debt finance.[18]

3 Empirical design

We turn now to test the statements of Propositions 1 and 2 empirically. For consistency with the majority of the empirical capital structure literature, we examine how annual measures of levels and changes in firms' leverage ratios are affected by annual measures of risk and ambiguity.

3.1 Sample selection

We estimate the relation between ambiguity and the leverage ratio using a sample of all nonfinancial firm-year observations in the annual Compustat database between 1993-2015. The time window 1993-2015 is dictated by the intraday stock data available on TAQ. This data is used for estimating the degree of ambiguity associated with each firm. After a process of data cleaning and filtering described below, we analyze $53,369$ annual observations for $7,577$ unique firms over the 23 years between 1993 and 2015.

To construct our sample, we begin with a download of all records from the Compustat fundamentals annual database. We drop all financial firms as well as all government entities. We also drop all observations missing one of the following descriptors: fiscal year end (FYR), total assets (AT), debt in current liabilities (DLC), long-term debt (DLTT) or stock price (PRCC). Finally, we drop all observations with negative values for sales, total assets or debt, and all observations with a negative leverage ratio or leverage ratio greater than 100% (book or market). This leaves us with $88,209$ observations. In addition, we drop all duplicate records or records that we cannot match with identifiers to the CRSP or TAQ stock price databases. All observations for which the annual degree of ambiguity or risk cannot be estimated are also dropped, which leaves us with $53,369$ observations for regression

[18] The impact on the cost of debt as ambiguity increases is more subtle. In particular, it is dependent on the riskiness of the debt and so the relation between the debt's face value and the reference point k. However, Proposition 1 indicates that the relative cost of debt versus equity reduces as ambiguity is increased.

analysis.

We estimate the annual degree of ambiguity and risk for each firm and each month as detailed below. We then use their average over the fiscal year as the annual risk and ambiguity associated with each firm-year observation. We apply the same method for estimating the annual expected ambiguity and risk. That is, for each firm and each month we estimate the expected values based on only the preceding 36 months, as detailed below, and then take the average of these values over the fiscal year. To check for robustness against potential biases that might be caused by outlier observations, we test all our hypotheses while winsorizing all variables at the lowest and the highest 5% level.

3.2 Estimating ambiguity

To assess the monthly ambiguity of a firm we estimate the ambiguity of its equity. To estimate the ambiguity of an asset using the model in Equation (8), one first needs to elicit the possible probability distributions of returns. To do so, we assume a representative agent whose set of priors is an aggregation of the sets of priors of all investors in the economy, and who acts as if all priors within that set are equally likely. Intraday observed returns on an asset in the market are assumed to be a realization of one prior out of the set of priors. That is, every day is characterized by a different distribution of returns, and the set of these distributions over a month represents the agent's set of priors.

To estimate the probabilities of returns from trading data, one must make assumptions about the nature of the distribution of returns. We assume that returns on assets are log-normally distributed; therefore, the degree of ambiguity of the return r_j on the equity j can be measured by

$$\mho^2 [r_j] = \int \mathrm{E} [\phi (r_j; \mu_j, \sigma_j)] \, \mathrm{Var} [\phi (r_j; \mu_j, \sigma_j)] \, dr_j, \tag{16}$$

where $\phi (r_j; \mu_j, \sigma_j)$ stands for the normal probability density function of r_j conditional upon the mean μ_j and the variance σ_j^2. It is important to note that our empirical tests use a measure of the degree of ambiguity, defined by Equation (8), which is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter, which is a matter of tastes, is endogenously determined by the empirical estimations.

We employ the empirical method developed by Brenner and Izhakian (2016) to estimate the degree of ambiguity using intraday stock trading data from the TAQ database. We compute the degree of ambiguity, given in Equation (16), for each stock and for each month, by applying the following procedure. We sample the price of the stock every five minutes starting from 9:30 until 16:00. The decision to use five-minute time intervals is motivated in part by Andersen *et al.* (2001), who show this time interval is sufficient to eliminate microstructure effects. In cases for which there is no trade during a specific time interval, we take the volume-weighted average of the closest trading price. Using these prices we compute five-minute returns, 78 returns for each day at most. Note that this procedure implies that we ignore returns between each closing price and opening price on the following day, thereby eliminating the impact of overnight price changes and dividend distributions. For each stock, we drop all trading days that do not have at least 15 quotes representing different five minute time intervals, and we drop all trading months that do not have at least 15 days that satisfy this condition. Observations with extreme price changes (plus or minus 10 percent log returns) within five minutes are also dropped, because many of them are due to mistaken orders that were cancelled by the stock exchange.[19]

For a given stock each day, we compute the normalized (by the number of intraday observations) mean and variance of the return, denoted μ_j and σ_j^2 respectively. As in French *et al.* (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading proposed by Scholes and Williams (1977).[20] Based upon the assumption that the intraday returns are normally distributed, for each stock j we construct the set of priors $\mathcal{P}_j$, where each prior P_j within the set $\mathcal{P}_j$ is defined by a pair of μ_j and σ_j. It is important to mention that in our approach, the sets of priors that underlie ambiguity are endogenously derived.

The set $\mathcal{P}_j$ of (normal) probability distributions of each stock j for a given month consists of 15 to 22 different probability distributions. To compute the monthly degree of ambiguity of a given asset, specified in Equation (16), we represent each daily return distribution by a histogram. To this end, we divide the range of daily returns, from -40% to 40%, into 160 intervals (bins), each of width 0.5%. For each day, we compute the probability of the return being in each bin. In addition, we compute the probability of the return being lower than -40% and the probability of the return being higher than $+40\%$. Using these probabilities, we compute the mean and the variance of probabilities for each of the 162 bins separately, where the histograms are assigned with equal weights, i.e., each probability distribution in the set $\mathcal{P}_j$ is equally likely. This is equivalent to assuming that the daily ratios $\frac{\mu_j}{\sigma_j}$ are student's-t distributed.[21] Then, we estimate the degree of ambiguity of each stock j for each month

[19] Testing our hypotheses while including observations with extreme price changes shows that the effect of ambiguity is even more significant than while excluding these observations.

[20] Scholes and Williams' (1977) adjustment for non-synchronous trading suggests that the volatility of returns takes the form $\sigma_t^2 = \frac{1}{N_t}\sum_{i=1}^{N_t}\left(r_{t,i} - \mathrm{E}\left[r_{t,i}\right]\right)^2 + 2\frac{1}{N_t - 1}\sum_{i=2}^{N_t}\left(r_{t,i} - \mathrm{E}\left[r_{t,i}\right]\right)\left(r_{t,i-1} - \mathrm{E}\left[r_{t,i-1}\right]\right)$. We also test our model without the Scholes-Williams correction for non-synchronous trading. The results are essentially the same.

[21] When $\frac{\mu}{\sigma}$ is Student's t-distributed, cumulative probabilities are uniformly distributed. See, for example, Kendall and Alan (2010, page 21, Proposition 1.27). This is consistent with the idea that the representative investor does not have

by the discrete form

$$\mho^2[r_j] = \frac{1}{w(1-w)} \left(\begin{array}{l} \mathrm{E}\left[\Phi(r_{j,0};\mu_j,\sigma_j)\right] \mathrm{Var}\left[\Phi(r_{j,0};\mu_j,\sigma_j)\right] + \\ \sum_{i=1}^{160} \begin{array}{l} \mathrm{E}\left[\Phi(r_{j,i};\mu_j,\sigma_j) - \Phi(r_{j,i-1};\mu_j,\sigma_j)\right] \times \\ \mathrm{Var}\left[\Phi(r_{j,i};\mu_j,\sigma_j) - \Phi(r_{j,i-1};\mu_j,\sigma_j)\right] + \end{array} \\ \mathrm{E}\left[1-\Phi(r_{j,40};\mu_j,\sigma_j)\right] \mathrm{Var}\left[1-\Phi(r_{j,40};\mu_j,\sigma_j)\right] \end{array} \right), \tag{17}$$

where $\Phi(\cdot)$ stands for the cumulative normal probability distribution, $r_0 = -0.40$, $w = r_i - r_{i-1} = 0.005$, and $\frac{1}{w(1-w)}$ scales the weighted-average volatilities of probabilities to the bins' size.[22] This scaling, which is analogous to Sheppard's correction, has been tested to verify that it minimizes the effect of the selected bin size on the values of $\mho^2$; see Brenner and Izhakian (2016).

We don't investigate whether $\mho^2$ captures other well-known uncertainty factors because this verification has been done in Brenner and Izhakian (2016). That study conducts an exhaustive set of tests to rule out the concern that the degree of ambiguity, measured by $\mho^2$, is related to other "uncertainty" factors including skwness, kurtosis, variance of variance, variance of mean, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), investors' sentiment and others. It also conducts tests that rule out the concern that observed returns are generated by a single (additive) probability distribution. In our empirical exploration, data in Table 2 indicate that the correlation of ambiguity with risk is relatively low.

3.3 Estimating expected ambiguity

Next, we estimate the expected ambiguity of every stock in every sample month. We assume that individuals form an expectation about future ambiguity by taking into account the evolution of a given stock's past ambiguity over time. The average monthly ambiguity $\mho$ (over firms) is positively skewed. To adjust for skewness and to avoid negative values of conditional ambiguity, we examine the natural logarithm of $\mho_{j,t}$.

The estimated autocorrelations of $\mho_{j,t}$ suggest using the autoregressive moving average (ARMA) model for estimating expected ambiguity. Using the formal link between realized and conditional volatilities, as provided by Andersen *et al.* (2003), we estimate the expected ambiguity (conditional volatility of probabilities) by substituting the realized ambiguity (realized volatility of probabilities), $\mho$, for the latent monthly ambiguity.[23] To do so, for each company in each month, $\widehat{\ln \mho_{j,t+1}}$ is computed

any information indicating which of the possible probability distributions is more likely, and thus he acts as if he assigns an equal weight to each possibility.

[22]We find that this scale improves the pervious scale $\frac{1}{w \ln \frac{1}{w}}$ for the bin's size, used in Izhakian and Yermack (2016), in the sense that it reduces the sensitivity of the estimated $\mho^2$ to the selection of the bin's size.

[23]Andersen *et al.* (2003) provide the theoretical framework for integrating high-frequency intraday data into the measurement of daily volatility, and they show that long-memory Gaussian vector autoregression for the logarithmic

using the coefficients estimated by the time-series ARMA(p, q) model

$$\ln \mho_{j,t} \quad = \quad \psi_0 + \epsilon_{j,t} + \sum_{i=1}^{p} \psi_i \cdot \ln \mho_{j,t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{j,t-1} \tag{18}$$

with the minimal Akaike information criterion (AIC). The expected volatility is then calculated as

$$\left(\mho^2_{j,t+1}\right)^E \quad = \quad \exp\left(2 \ln \widehat{\mho_{j,t+1}} + 2 \mathrm{Var}\left[u_{j,t+1}\right]\right). \tag{19}$$

where $\mathrm{Var}\left[u_{j,t+1}\right]$ is the minimal predicted variance of the error term. For every month t, using the data of only the 36 preceding months, i.e., from month $t-36$ to month $t-1$, the time-series regression given by Equation (18) is estimated for each $p = 1, \ldots, 10$ and $q = 1, \ldots, 10$; in total $p \times q = 100$ combinations. The coefficients of the model that attains the minimal AIC (the highest-quality model) are then used for the estimation of the expected ambiguity, $\left(\mho^2_{j,t+1}\right)^E$, by Equation (19). Note that these estimates of expected ambiguity are out-of-sample estimates.

3.4 Estimating expected volatility

Along with ambiguity, volatility serves as the most important explanatory variable in our analysis. We compute *monthly* volatility with standard methods, using daily log returns adjusted for dividends obtained from the CRSP database. Since probabilities are uncertain, volatilities can be viewed as computed using expected probabilities, as theoretically justified in Izhakian (2016b). For each individual stock in a given month, we calculate the standard deviation, $\mathbb{S}\mathrm{td}_{j,t}$, of the stock's daily returns over that month, again applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.[24] As with ambiguity, the average volatility measures $\mathbb{S}\mathrm{td}_{j,t}$ (over firms) are also positively skewed. To adjust for skewness and to avoid negative values of conditional volatility, we examine the natural logarithm of $\mathbb{S}\mathrm{td}_{j,t}$. Again we assume that investors form an expectation about future volatility based upon its evolution over time for a given stock. The average autocorrelations (over firms) of $\ln\left(\mathbb{S}\mathrm{td}_{j,t}\right)$ decay beyond lag 6, suggesting that $\ln\left(\mathbb{S}\mathrm{td}_{j,t}\right)$ follows an autoregressive process. Similarly to the procedure applied to ambiguity, for every month t, the expected volatility is also estimated with ARMA(p, q) for each equity j. Namely, using realized volatility, the parameter $\ln \widehat{\mathbb{S}\mathrm{td}_{j,t+1}}$ is estimated using the coefficients of the time-series model

$$\ln \mathbb{S}\mathrm{td}_{j,t} \quad = \quad \psi_0 + \epsilon_{j,t} + \sum_{i=1}^{p} \psi_i \cdot \ln \mathbb{S}\mathrm{td}_{j,t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{j,t-1} \tag{20}$$

daily realized volatilities performs admirably. We apply the same approach to ambiguity, since it uses probabilities that are estimated from intraday data.

[24] See, for example, French *et al.* (1987).

that attains the minimal AIC out of the $p \times q = 100$ combinations of coefficients obtained from this regression.[25] The expected volatility is then calculated as

$$\mathbb{V}\mathrm{ar}^{E}_{j,t+1} \quad = \quad \exp\left(2\ln\widehat{\mathbb{S}\mathrm{td}_{j,t+1}} + 2\mathrm{Var}\left[u_{j,t+1}\right]\right), \tag{21}$$

where $\mathrm{Var}\left[u_{j,t+1}\right]$ is the minimal predicted variance of the error term. As with expected ambiguity, the obtained estimates of expected volatility are out-of-sample.

3.5 Firm leverage and specific characteristics

The annual leverage ratio of each firm is the main dependent variable in our empirical tests, and we estimate both the book value and the market value versions of this ratio. Book leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total book value of book assets. Market leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt".

In addition to the leverage ratios, we obtain a set of firm annual characteristics using variables that are standard in capital structure research. Firm size is measured by the log of the sales normalized by gross domestic product (GDP).[26] Firm profitability is measured by operating income before depreciation divided by book assets. Asset tangibility is measured by property, plant and equipment divided by book assets. The market to book ratio is measured by market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is measured by R&D expenses relative to sales normalized by $10,000$. In addition, to explore the firms' leverage ratios relative to their industry, we also include industry median annual leverage, where firms are classified by their four-digit SIC codes. Expected marginal tax rates are obtained from John Graham's website. When tax rate is missing, we use the industry median annual marginal tax rate if available or the market-wide median annual marginal tax rate if necessary.

Table 1 presents summary statistics for the key variables used in the paper.

[Table 1]

Table 2 presents a correlation matrix for these variables. The two measures of uncertainty, expected volatility and expected ambiguity, are only weakly negatively related to one another, with an average

[25] Hansen and Lunde (2005) also estimate the expected volatilities by substituting the realized volatilities for the latent volatilities.

[26] Alternatively, firm size can be measured by the log of book assets normalized to the annual GDP level. We test our hypotheses using this measure of firm size. The results are essentially the same.

estimated correlation of -0.0857. This suggests that the two variables capture economically distinct aspects of financial uncertainty, a conclusion that coincides with the comprehensive tests of Brenner and Izhakian (2016).

[Table 2]

3.6 Regression tests

To test the statements of Propositions 1 and 2, we use two main empirical models. The first model explores firm-level variation in the leverage ratio L. This model uses the time series/cross sectional regression

$$L_{j,t} = \alpha + \beta_1 \cdot \mho^2_{j,t-1} + \beta_2 \cdot \mathbb{V}\mathrm{ar}_{j,t-1} + \gamma \cdot Z_{j,t-1} + \varepsilon_{j,t}, \tag{22}$$

where j designates firm and t designates year. The vector Z consists of the standard controls in the literature for firm characteristics as described in Section 3.5. The main goal of this regression test is to examine the explanatory power (beyond the standard control variables) provided by ambiguity for the leverage ratio.

The second explores the effect of firm level change in ambiguity on the change in firm leverage. To this end, we estimate

$$\Delta L_{i,t} = \alpha + \beta_1 \cdot \Delta\mho^2_{j,t-1} + \beta_2 \cdot \Delta\mathbb{V}\mathrm{ar}_{j,t-1} + \gamma \cdot \Delta Z_{j,t-1} + \varepsilon_{j,t}, \tag{23}$$

where $\Delta L_{i,t}$ is the change in variable L between time $t-1$ and t and $\Delta Z_{i,t-1}$ is the change in the vector of variables Z between time $t-2$ and time $t-1$.

Both models are tested using both book leverage and market leverage. They are also tested using both realized ambiguity and risk and for expected ambiguity and risk, estimated as detailed above. All reported standard errors are clustered by year and by firm.

4 Empirical findings

4.1 Main findings

First, we test the basic model in Equation (22). That is, we test whether the level of leverage ratio is affected by the level of ambiguity and risk in the previous year. Table 3 presents the findings of these regression tests. Panel A shows estimates for the book leverage ratio, while Panel B shows estimates for the market leverage ratio. In both panels, the actual values of ambiguity and risk are used in the first four columns, while the expected values, as defined earlier, are used in the right four columns. In each case we present estimates for leverage as a function of ambiguity alone, then as a function of risk alone, then in a model with both explanatory variables included, and finally in a model with a full range of additional controls.

Table 3 shows a consistent pattern of results, with just one exception. In every model, ambiguity (or expected ambiguity) exhibits a positive, statistically significant association with leverage. In every model but one, risk (or expected risk) exhibits a negative and significant association with leverage.

[Table 3]

Next, as a control for the appearant endogeneity problems inherent in the standard levels regression (Lemmon *et al.*, 2008) we test whether leverage is adjusted to changes in the extent of ambiguity and risk. That is, we test the model in Equation (23), which suggests that a change in the extent of ambiguity or risk are followed by a change in the use of leverage in the subsequent year. As with the level regressions, we test both book leverage ratio (Panel A) and market leverage ratio (Panel B). Estimates are arranged identically to those in Table 3. Again we find that in every model, the ambiguity (or expected ambiguity) variable has a positive and significant estimate, while the estimates for risk (or expected risk) are uniformly negative and almost always significant.

Control variables in Tables 3 and 4 generally have estimates in line with prior research. Leverage appears to be higher in larger firms and also in firms with higher marginal tax rates, at least in most of the estimations. Firms with higher asset tangibility also borrow more. Leverage is lower when profitability is high and also when the market-to-book ratio is high; both of these relations are partly mechanical, since these firms tend to accumulate large amounts of equity in their capital structures. One surprising result in our model is the positive association estimated between research & development spending and leverage. Most studies, such as Mackie-Mason (1990) and Berger, Ofek and Yermack (1997), have tended to find the opposite, in line with the Myers (1977) prediction that growth opportunities will be financed mostly by equity.

[Table 4]

We note that our estimates for the ambiguity variable—the central focus of our study—remain positive and significant regardless of the controls used, the inclusion of the risk variable, or whether we estimate our regressions in levels or changes. The estimates for the effect of ambiguity upon leverage appear to be more consistent and more reliably significant than the estimates for the risk variable.[27] The somewhat erractic significance of the risk variable can been seen as consistent with

[27]Using firm fixed effects, rather than first differences to control for the firm specific unobserved heterogeneity in leverage, provides very similar results. Specifically, the estimated coefficients on ambiguity or expected ambiguity are positive and significant while the estimated coefficients on risk or expected risk are negative but not always statistically significant.

Frank and Goyal (2009), who find that risk, measured as the variance of stock returns, is not one of the core factors determining leverage, although their analysis is based on the incremental contribution to R-squared rather than the magnitude or significance of coefficient estimates.

To investigate the economic significance of ambiguity in explaining capital-structure decision, Table 5 presents the economic significance of all the independent variables. The table is constructed by multiplying the coefficient estimate of each explanatory variable from Table 3 (Panels A and B) and from Table 4 (Panels A and B), shown in the first column, by its standard deviation reported in table 1, shown in the second column, with the product of these two quantities displayed in the third column under the heading significance. The table shows several patterns. First, ambiguity and risk have approximately equal and opposite effects upon the leverage choice. Second, the measures of the expected future values of risk and of ambiguity appear to have much larger impacts on leverage than do lagged values of the variables. Finally, other variables seem more important than uncertainty, with the prevailing industry leverage rate and the size of each firm standing out as the two variables with greatest economic significance.

[Table 5]

5 Conclusion

Uncertainty plays a role in capital structure choice and many other important financial decisions. Until recently, most studies controlled for uncertainty simply by considering risk, typically measured as the standard deviation or variance of returns to common stock. However, ambiguity, or Knightian uncertainty, represents a separate and distinct type of uncertainty. We contribute to a growing empirical literature by showing that ambiguity and risk both play roles in capital structure choice.

We present a model generating hypotheses of a positive relation between ambiguity and leverage, and we test these hypotheses using cross-sectional data and first differences in a dataset covering more than 53,000 firm-year observations from more than 7,500 individual Compustat firms. As predicted, we find a positive association between ambiguity and leverage along with a negative association between equity risk (as a proxy for the risk of the cash flows) and leverage.

Our results are consistent with other recent papers analyzing variables affected by financial uncertainly, including the pricing of credit default swaps and the timing of stock option exercises. Together, these studies suggest that the role of uncertainty in financial decisions is richer and more nuanced than previously believed, and that further investigations of amibiguity have the promise of yielding additional insights that may improve our understanding of basic financial decision-making.

References

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens (2001) "The distribution of realized stock return volatility," *Journal of Financial Economics*, Vol. 61, No. 1, pp. 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2003) "Modeling and Forecasting Realized Volatility," *Econometrica*, Vol. 71, No. 2, pp. 579–625.

Anderson, E. W., E. Ghysels, and J. L. Juergens (2009) "The Impact of Risk and Uncertainty on Expected Returns," *Journal of Financial Economics*, Vol. 94, No. 2, pp. 233–263.

Antoniou, C., R. D. Harris, and R. Zhang (2015) "Ambiguity aversion and stock market participation: An empirical analysis," *Journal of Banking & Finance*, Vol. 58, pp. 57–70.

Augustin, P. and Y. Izhakian (2016) "Ambiguity, Volatility and Credit Risk," *SSRN eLibrary*, 2776377.

Avramov, D. (2002) "Stock return predictability and model uncertainty," *Journal of Financial Economics*, Vol. 64, No. 3, pp. 423–458.

Barry, C. B. and S. Brown (1984) "Differential information and the small firm effect," *Journal of Financial Economics*, Vol. 13, No. 2, pp. 283–294.

Bawa, V. S., S. Brown, and R. Klein (1979) *Estimation Risk and Optimal Portfolio Choice* in , Studies in Bayesian Econometrics, No. 3, Amsterdam: North-Holland.

Berger, P. G., E. Ofek, and D. L. Yermack (1997) "Managerial entrenchment and capital structure decisions," *The Journal of Finance*, Vol. 52, No. 4, pp. 1411–1438.

Boyle, P. P., L. Garlappi, R. Uppal, and T. Wang (2011) "Keynes Meets Markowitz: The Tradeoff Between Familiarity and Diversification," *Management Science*, Vol. 58, pp. 1–20.

Brenner, M. and Y. Izhakian (2016) "Asset Prices and Ambiguity: Empirical Evidence," *SSRN eLibrary*, 1996802.

Cao, H. H., T. Wang, and H. H. Zhang (2005) "Model Uncertainty, Limited Market Participation, and Asset Prices," *The Review of Financial Studies*, Vol. 18, No. 4, pp. 1219–1251.

Chapman, D. A. and V. Polkovnichenko (2009) "First-Order Risk Aversion, Heterogeneity, and Asset Market Outcomes," *The Journal of Finance*, Vol. 64, No. 4, pp. 1863–1887.

Chen, Z. and L. Epstein (2002) "Ambiguity, Risk, and Asset Returns in Continuous Time," *Econometrica*, Vol. 70, No. 4, pp. 1403–1443.

Chew, S. H. and J. S. Sagi (2008) "Small Worlds: Modeling Attitudes Toward Sources of Uncertainty," *Journal of Economic Theory*, Vol. 139, No. 1, pp. 1–24.

Coles, J. and U. Loewenstein (1988) "Equilibrium pricing and portfolio composition in the presence of uncertain parameters," *Journal of Financial Economics*, Vol. 22, No. 2, pp. 279–303.

Coles, J., U. Loewenstein, and J. Suay (1995) "On Equilibrium Pricing under Parameter Uncertainty," *Journal of Financial and Quantitative Analysis*, Vol. 30, No. 03, pp. 347–364.

Constantinides, G. M. (1982) "Intertemporal Asset Pricing with Heterogeneous Consumers and without Demand Aggregation," *The Journal of Business*, Vol. 55, No. 2, pp. 253–267.

Cremers, K. M. (2002) "Stock return predictability: A Bayesian model selection perspective," *Review of Financial Studies*, Vol. 15, No. 4, pp. 1223–1249.

Drechsler, I. (2012) "Uncertainty, Time-Varying Fear, and Asset Prices," *The Journal of Finance*, Forthcoming.

Ellsberg, D. (1961) "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics*, Vol. 75, No. 4, pp. 643–669.

Epstein, L. G. and M. Schneider (2008) "Ambiguity, Information Quality, and Asset Pricing," *The Journal of Finance*, Vol. 63, No. 1, pp. 197–228.

Frank, M. Z. and V. K. Goyal (2009) "Capital structure decisions: which factors are reliably important?" *Financial management*, Vol. 38, No. 1, pp. 1–37.

French, K. R., G. W. Schwert, and R. F. Stambaugh (1987) "Expected stock returns and volatility," *Journal of Financial Economics*, Vol. 19, No. 1, pp. 3–29.

Gilboa, I. and D. Schmeidler (1989) "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, Vol. 18, No. 2, pp. 141–153.

Hansen, L. P. and T. J. Sargent (2001) "Robust Control and Model Uncertainty," *American Economic Review*, Vol. 91, No. 2, pp. 60–66.

Hansen, L. P., T. J. Sargent, and T. D. Tallarini (1999) "Robust Permanent Income and Pricing," *The Review of Economic Studies*, Vol. 66, No. 4, pp. 873–907.

Hansen, P. R. and A. Lunde (2005) "A forecast comparison of volatility models: does anything beat a GARCH(1,1)?" *Journal of Applied Econometrics*, Vol. 20, No. 7, pp. 873–889.

Izhakian, Y. (2012) "Capital Asset Pricing under Ambiguity," *SSRN eLibrary*, 2007815.

——— (2016a) "Expected Utility with Uncertain Probabilities Theory," *SSRN eLibrary*, 2017944.

——— (2016b) "A Theoretical Foundation of Ambiguity Measurement," *SSRN eLibrary*, 1332973.

Izhakian, Y. and S. Benninga (2011) "The Uncertainty Premium in an Ambiguous Economy," *The Quarterly Journal of Finance*, Vol. 1, pp. 323–354.

Izhakian, Y. and D. Yermack (2016) "Risk, Ambiguity, and the Exercise of Employee Stock Options," *The Journal of Financial Economics*, Forthcoming.

Karni, E. and M.-L. Vierø (2013) ""Reverse Bayesianism": A Choice-Based Theory of Growing Awareness," *American Economic Review*, Vol. 103, No. 7, pp. 2790–2810.

Kendall, M. and A. Stuart (2010) "The Advanced Theory of Statistics. Vol. 1: Distribution Theory," *London: Griffin, 2010, 6th ed.*, Vol. 1.

Knight, F. M. (1921) *Risk, Uncertainty and Profit*, Boston: Houghton Mifflin.

Kraus, A. and R. H. Litzenberger (1973) "A state-preference model of optimal financial leverage," *The journal of finance*, Vol. 28, No. 4, pp. 911–922.

Lee, S. (2014) "Knightian uncertainty and capital structure: Theory and evidence," *SSRN eLibrary*, 2374679.

Lemmon, M. L., M. R. Roberts, and J. F. Zender (2008) "Back to the beginning: persistence and the cross-section of corporate capital structure," *The Journal of Finance*, Vol. 63, No. 4, pp. 1575–1608.

Mackie-Mason, J. K. (1990) "Do taxes affect corporate financing decisions?" *The journal of finance*, Vol. 45, No. 5, pp. 1471–1493.

Modigliani, F. and M. H. Miller (1958) "The cost of capital, corporation finance and the theory of investment," *The American economic review*, Vol. 48, No. 3, pp. 261–297.

Myers, S. C. (1977) "Determinants of corporate borrowing," *Journal of financial economics*, Vol. 5, No. 2, pp. 147–175.

Nau, R. F. (2006) "Uncertainty Aversion with Second-Order Utilities and Probabilities," *Management Science*, Vol. 52, pp. 136–145.

Pástor, L. (2000) "Portfolio selection and asset pricing models," *The Journal of Finance*, Vol. 55, No. 1, pp. 179–223.

Pástor, L. and R. F. Stambaugh (2000) "Comparing asset pricing models: an investment perspective," *Journal of Financial Economics*, Vol. 56, No. 3, pp. 335–381.

Pflug, G. and D. Wozabal (2007) "Ambiguity in Portfolio Selection," *Quantitative Finance*, Vol. 7, No. 4, pp. 435–442.

Rothschild, M. and J. E. Stiglitz (1970) "Increasing Risk: I. A Definition," *Journal of Economic Theory*, Vol. 2, No. 3, pp. 225–243.

Schmeidler, D. (1989) "Subjective Probability and Expected Utility without Additivity," *Econometrica*, Vol. 57, No. 3, pp. 571–587.

Scholes, M. and J. Williams (1977) "Estimating betas from nonsynchronous data," *Journal of Financial Economics*, Vol. 5, No. 3, pp. 309–327.

Thimmea, J. and C. Völkertb (2015) "Ambiguity in the Cross-Section of Expected Returns: An Empirical Assessment," *Journal of Business & Economic Statistics*, Vol. 33, pp. 418–429.

Titman, S. and R. Wessels (1988) "The determinants of capital structure choice," *The Journal of finance*, Vol. 43, No. 1, pp. 1–19.

Tversky, A. and D. Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, Vol. 5, No. 4, pp. 297–323.

Ulrich, M. (2013) "Inflation ambiguity and the term structure of U.S. Government bonds," *Journal of Monetary Economics*, Vol. 60, No. 2, pp. 295–309.

Williams, C. D. (2015) "Asymmetric Responses to Earnings News: A Case for Ambiguity," *Accounting Review*, Vol. 90, pp. 785–817.

A Appendix

A.1 Discrete state space

Let $\mathcal{S}$ be a finite state space and $X : \mathcal{S} \to \mathbb{R}$ be a consumption. The outcomes of X can then be written $X = (x_1, \ldots, x_n)$, where x_s stands for the outcome at state s and $x_1, \ldots, x_n$ are listed in a non-decreasing order. The investor, who doesn't know the precise probabilities of these outcomes, holds a set $\mathcal{P}$ of possible probability measures P on a σ-algebra of subsets of $\mathcal{S}$. In addition, she holds a second-order belief ξ (a probability measure on a σ-algebra of subsets of $\mathcal{P}$).

The expected utility of the investor can then by represented by

$$\begin{aligned} \mathrm{V}(X) \;=\; & \sum_{0<s\leq k} \Upsilon^{-1}\left(\sum_{\mathrm{P}\in\mathcal{P}} \Upsilon\left(\mathrm{P}_X\left(x_s\right)\right)\xi\left(\{\mathrm{P}_X\}\right)\right)\left[\mathrm{U}\left(x_s\right)-\mathrm{U}\left(x_{s+1}\right)\right]+ \\ & \sum_{k<s\leq n} \Upsilon^{-1}\left(\sum_{\mathrm{P}\in\mathcal{P}} \Upsilon\left(\mathrm{P}_X\left(x_s\right)\right)\xi\left(\{\mathrm{P}_X\}\right)\right)\left[\mathrm{U}\left(x_s\right)-\mathrm{U}\left(x_{s-1}\right)\right], \end{aligned} \tag{24}$$

where U is a (von Neumann-Morgenstern) utility function; and Υ is an outlook function, capturing attitude toward ambiguity. As usual, a concave U characterizes risk-averse behavior, and a convex U characterizes risk-loving behavior. Similarly, a concave Υ characterizes ambiguity-averse behavior, and a convex Υ characterizes ambiguity-loving behavior. The utility function is normalized such that $\mathrm{U}(x_k) = 0$, for some reference point $0 \leq k$. That is, any outcome indexed by $s \leq k$ is considered unfavorable (a loss) and any outcome indexed $k < s$ is considered favorable (a gain).

By Izhakian (Theorem 2, 2016b), the dual representation of the expected utility in Equation (24), can be approximated to obtain

$$\begin{aligned} \mathrm{W}(X) \;\approx\; & \sum_{s\leq k} \mathrm{U}\left(x_s\right)\mathrm{E}\left[\varphi_X\left(x_s\right)\right]\left(1-\frac{\Upsilon''\left(\mathrm{E}\left[\mathrm{P}_X\left(x_s\right)\right]\right)}{\Upsilon'\left(\mathrm{E}\left[\mathrm{P}_X\left(x_s\right)\right]\right)}\mathrm{Var}\varphi_X\left(x_s\right)\right)+ \\ & \sum_{s>k} \mathrm{U}\left(x_s\right)\mathrm{E}\left[\varphi_X\left(x_s\right)\right]\left(1+\frac{\Upsilon''\left(\mathrm{E}\left[\mathrm{P}_X\left(x_s\right)\right]\right)}{\Upsilon'\left(\mathrm{E}\left[\mathrm{P}_X\left(x_s\right)\right]\right)}\mathrm{Var}\varphi_X\left(x_s\right)\right), \end{aligned} \tag{25}$$

where $\varphi_X(x)$ is the probability mass function of X being *greater* than x; $\mathrm{P}_X(x)$ is a cumulative probability $\mathrm{P} \in \mathcal{P}$ of X; and $\mathrm{E}[\cdot]$ and $\mathrm{Var}[\cdot]$ are respectively the expectation and the variance of probabilities taken using the second order probability ξ. To simplify notations, the subscript X, designating the random variable, is omitted. Using this functional form, the state prices of state $s \in \mathcal{S}$ can be defined by

$$q_s \;=\; \mathrm{E}\left[\varphi(s)\right]\Lambda(s)\frac{\partial_s \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{26}$$

where

$$\Lambda(s) = \begin{cases} 1+\eta\left(\mathrm{E}\left[\mathrm{P}\left(s\right)\right]\right)\mathrm{Var}\left[\varphi\left(s\right)\right], & s \le k \\ 1-\eta\left(\mathrm{E}\left[\mathrm{P}\left(s\right)\right]\right)\mathrm{Var}\left[\varphi\left(s\right)\right], & s > k \end{cases}, \tag{27}$$

Accordingly, the risk neutral probability of state s is

$$\pi_s^* = \frac{q_s}{\sum_s q_s} = \frac{\mathrm{E}\left[\varphi\left(s\right)\right]\Lambda\left(s\right)\frac{\partial_s \mathrm{U}}{\partial_0 \mathrm{U}}}{\sum_s \mathrm{E}\left[\varphi\left(s\right)\right]\Lambda\left(s\right)\frac{\partial_s \mathrm{U}}{\partial_0 \mathrm{U}}}, \tag{28}$$

and the risk-free rate of return is

$$r_f = \frac{1}{\sum_s q_s} - 1 = \frac{1}{\sum_s \mathrm{E}\left[\varphi\left(s\right)\right]\Lambda\left(s\right)\frac{\partial_s \mathrm{U}}{\partial_0 \mathrm{U}}} - 1. \tag{29}$$

Let F be the face value of a zero-coupon debt and d is index. The objective function can then be written

$$\begin{aligned} \max_F \quad & S_0\left(F\right) + D_0\left(F\right) \\ s.t. \quad S_0 &= \frac{1}{1+r_f}\sum_{d<s}\pi_s^*\left(1-\tau\right)\left(x_s - F\right) \\ D_0 &= \frac{1}{1+r_f}\left[\sum_{s\le k}\pi_s^*\left(1-\alpha\right)x + \sum_{k<s\le d}\pi_s^*\left(1-\alpha\right)x + \sum_{d<s}\pi_s^* F\right]. \end{aligned}$$

A.2 Proofs

Proof of Theorem 1.

Assume that $x \le k$, i.e., an unfavorable outcome. Substituting the budget constraints into the objective function in Equation (9) and solving the maximization problem, differentiation with respect to θ conditional on a given x (state of nature), provides

$$q(x)\partial_0 \mathrm{U} = \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\eta\left(\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)\mathrm{Var}\left[\varphi\left(x\right)\right]\right)\partial_x \mathrm{U}.$$

Organizing terms completes the proof. The proof for $k < x$, i.e., a favorable outcome, is similar. ■

Proof of Theorem 2.

Obtained from the same arguments of the Modigliani-Miller's (1958) Proposition 1. ■

Proof of Theorem 3.

The first order condition, obtained by differentiating the firm value in Equation (14) with respect to its leverage, is

$$\frac{\partial V(\cdot)}{\partial F} = \frac{1}{1+r_f}\left(\begin{array}{l} -\int_F^\infty \pi^*(x)\left(1-\tau\right)dx \\ +\pi^*(F)F\left(1-\alpha\right) + \int_F^\infty \pi^*(x)dx - \pi^*(F)F \end{array}\right) = 0.$$

Rearranging terms provides

$$F = \frac{\tau}{\pi^*(F)\alpha} \int_F^\infty \pi^*(x) dx.$$

■

Proof of Proposition 1.

Substitute the explicit expression of $\pi^*(\cdot)$ into the optimal leverage in Equation (15) to obtain

$$F = \frac{\tau}{\alpha q(F)} \int_F^\infty q(x) dx. \tag{30}$$

The first order condition (FOC) can be written by the implicit function[28]

$$G\left(F, \eta, \mho^2, \gamma, \mathcal{R}\right) = F - \frac{\tau}{\alpha q(F)} \int_F^\infty q(x) dx. \tag{31}$$

Suppose that the ambiguity increases such that the variance $\text{Var}[\varphi(x)]$ of the probability of some $x \in X$ increases. Differentiating the implicit function in Equation (31) with respect to $\text{Var}[\varphi(x)]$, provides

$$\frac{\partial G}{\partial \text{Var}[\varphi(x)]} = -\frac{\tau}{\alpha q(F)} \frac{\partial q(x)}{\partial \text{Var}[\varphi(x)]}.$$

By the second-order condition,

$$\frac{\partial G}{\partial F} < 0.$$

By implicit function theorem,

$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial \text{Var}[\varphi(x)]} d\text{Var}[\varphi(x)] = 0.$$

Therefore,

$$\frac{dF}{d\text{Var}[\varphi(x)]} = -\frac{\frac{\partial G}{\partial \text{Var}[\varphi(x)]}}{\frac{\partial G}{\partial F}}.$$

For $F \leq x$, $\frac{\partial q(x)}{\partial \text{Var}\varphi(x)} = -\eta(\cdot) \frac{\partial_x \text{U}}{\partial_0 \text{U}} \leq 0$ (aversion to ambiguity). Therefore, $\frac{\partial G}{\partial \text{Var}[\varphi(x)]} \geq 0$, which implies that $\frac{dF}{d\text{Var}[\varphi(x)]} \geq 0$. ■

Proof of Proposition 2.

As in Rothschild and Stiglitz (1970), add an additional element of risk by assuming a mean-preserving spread in a given outcome x. Namely, there is an $0 \leq \epsilon \leq x$, which is mean-independent of x, such that $\mathbb{E}[\epsilon] = 0$ and $\text{P}(-\epsilon) = \text{P}(\epsilon) = \frac{1}{2}$. Differentiating the implicit function in Equation (31) with respect to ϵ, provides

$$\frac{\partial G}{\partial \epsilon} = \frac{\tau}{\alpha q(F)} \frac{1}{2} \text{E}[\varphi(x)] \left(1 - \eta(\text{E}[\text{P}(x)]) \text{Var}[\varphi(x)]\right) \left(\frac{\partial_{xx} \text{U}(x-\epsilon)}{\partial_0 \text{U}} - \frac{\partial_{xx} \text{U}(x+\epsilon)}{\partial_0 \text{U}}\right)$$

[28] In the notation $G\left(F, \eta, \mho^2, \gamma, \mathcal{R}\right)$, γ stands for the risk aversion coefficient and η for the ambiguity aversion coefficient.

Since, by risk aversion, $\partial_{xx}\mathrm{U} < 0$ and $\partial_{xxx}\mathrm{U} \geq 0$, $\frac{\partial G}{\partial \epsilon} \leq 0$. Because, by the second-order condition, $\frac{\partial G}{\partial F} < 0$,

$$\frac{dF}{d\epsilon} = -\frac{\frac{\partial G}{\partial \epsilon}}{\frac{\partial G}{\partial F}} \leq 0.$$

■

A.3 Tables and Figures

Table 1: **Descriptive statistics**

The full sample includes 53,369 annual observations associated with 7,577 individual firms between 1993 and 2015, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ration is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Firm profitability is the operating income before depreciation divided by book assets. Firm tangibility is the plant property divided by book assets. Market to book ration is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry.

Variable	Mean	Std Dev	Minimum	Median	Maximum	N
Book Leverage	0.1923	0.1999	0.0000	0.1428	0.9993	53369
Market Leverage	0.1690	0.2051	0.0000	0.0898	0.9906	53369
Ambiguity	0.5638	0.2613	0.0807	0.5119	2.5080	53369
Risk	0.1679	0.1205	0.0018	0.1401	6.8647	53369
Expected Ambiguity	0.5492	0.2301	0.0771	0.5065	2.3655	37291
Expected Risk	0.1330	0.0747	0.0082	0.1146	1.5013	37291
Median Book Lev.	0.1587	0.1531	0.0000	0.1117	0.9906	53369
Median Market Lev.	0.1368	0.1597	0.0000	0.0720	0.9793	53369
Profitability	0.0284	0.3722	-18.6098	0.1088	3.2533	53369
Tangibility	0.2465	0.2250	0.0000	0.1705	0.9931	53369
Market to Book	2.1155	2.6664	0.0022	1.3909	132.6424	53369
R&D	0.0003	0.0134	0.0000	0.0000	2.5159	53369
Sales	5.6702	2.3013	0.0000	5.8076	12.9966	53369
Tax Rate	1.6196	2.5133	0.0000	0.3500	9.9995	53369

Table 2: **Correlation matrix**

The full sample includes 53,369 annual observations associated with 7,577 individual firms between 1993 and 2015, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ration is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Firm profitability is the operating income before depreciation divided by book assets. Firm tangibility is the plant property divided by book assets. Market to book ration is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. p-values appear in parentheses.

	Book Leverage	Leverage Leverage	Ambiguity Ambiguity	Risk Risk	Expected Ambiguity	Expected Risk	Profitability	Tangibility	Market to Book	R&D	Sales	Tax Rate
Book Leverage	1											
Market Leverage	0.8229	1										
	(<.0001)											
Ambiguity	0.1296	0.0715	1									
	(<.0001)	(<.0001)										
Risk	-0.0144	-0.0089	-0.0857	1								
	(.0009)	(.0391)	(<.0001)									
Expected Ambiguity	0.1298	0.0418	0.9560	-0.2875	1							
	(<.0001)	(<.0001)	(<.0001)	(<.0001)								
Expected Risk	-0.0622	-0.0224	-0.3417	0.7122	-0.3426	1						
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)							
Profitability	0.0804	0.1019	0.2155	-0.1019	0.2580	-0.3701	1					
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)						
Tangibility	0.3380	0.3454	0.1412	-0.0353	0.1053	-0.0879	0.1503	1				
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)					
Market to Book	-0.1749	-0.3119	-0.1056	0.0334	-0.0744	0.0949	-0.3001	-0.1588	1			
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)				
R&D	-0.0043	-0.0108	-0.0136	0.0016	-0.0137	0.0079	-0.0463	-0.0072	0.0101	1		
	(.3237)	(.0127)	(.0017)	(.7200)	(.0080)	(.1251)	(<.0001)	(.0980)	(.0202)			
Sales	0.2994	0.3423	0.4363	-0.1200	0.4420	-0.3426	0.4864	0.2444	-0.2816	-0.0531	1	
	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)	(<.0001)		
Tax Rate	0.0331	0.0348	-0.0451	0.0075	-0.0569	0.0526	-0.0757	-0.0137	0.0050	0.0030	-0.0382	1
	(<.0001)	(<.0001)	(<.0001)	(.0821)	(<.0001)	(<.0001)	(<.0001)	(.0015)	(.2525)	(.4959)	(<.0001)	

Table 3: **Level regression estimates of capital structure**

Linear model regression estimates of the level of annual leverage ratio explained by one-year lagged variables. The full sample includes 53,369 annual observations associated with 7,577 individual firms between 1993 and 2015, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ration is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Firm profitability is the operating income before depreciation divided by book assets. Firm tangibility is the plant property divided by book assets. Market to book ration is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. *t*-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

Panel A: Book leverage

Intercept	0.164	0.197	0.166	0.020	0.165	0.210	0.171	0.001
	(114.96)	(172.80)	(109.44)	(6.00)	(90.55)	(109.99)	(66.44)	(0.17)
Ambiguity	0.077		0.075	0.014				
	(30.88)		(29.68)	(6.81)				
Risk		-0.070	-0.037	0.031				
		(-3.98)	(-4.08)	(0.79)				
Expected Ambiguity					0.097		0.090	0.026
					(25.58)		(21.00)	(2.81)
Expected Risk						-0.506	-0.192	-0.168
						(-6.94)	(-3.74)	(-4.29)
Median Leverage				0.645				0.646
				(90.87)				(79.14)
Profitability				-0.038				-0.059
				(-7.03)				(-10.11)
Tangibility				0.084				0.076
				(19.84)				(15.46)
Market to Book				-0.003				-0.002
				(-6.62)				(-4.69)
R&D				0.164				0.161
				(7.28)				(7.74)
Sales				0.010				0.012
				(19.07)				(20.49)
Tax Rate				0.002				0.003
				(7.88)				(7.44)
N	42890	42890	42890	42890	31316	31316	31316	31316
R-Square	0.0226	0.0022	0.0233	0.3463	0.0236	0.0070	0.0245	0.3439

Panel B: Market leverage

Intercept	0.152	0.176	0.154	0.017	0.159	0.188	0.167	0.000
	(105.79)	(155.87)	(100.45)	(4.86)	(87.97)	(111.02)	(67.64)	(0.08)
Ambiguity	0.0547		0.0529	0.033				
	(22.76)		(21.65)	(14.65)				
Risk		-0.057	-0.034	0.025				
		(-3.77)	(-3.5)1	(0.50)				
Expected Ambiguity					0.056		0.047	0.030
					(15.96)		(12.08)	(9.33)
Expected Risk						-0.396	-0.232	-0.214
						(-6.84)	(-4.74)	(-5.46)
Median Leverage				0.640				0.636
				(81.89)				(70.66)
Profitability				-0.047				-0.062
				(-7.58)				(-11.90)
Tangibility				0.088				0.094
				(18.81)				(17.53)
Market to Book				-0.009				-0.011
				(-12.04)				(-8.33)
R&D				0.088				0.085
				(6.78)				(6.83)
Sales				0.014				0.015
				(23.73)				(22.58)
Tax Rate				0.002				0.003
				(7.55)				(7.13)
N	42890	42890	42890	42890	31316	31316	31316	31316
R-Square	0.01037	0.001348	0.01084	0.3987	0.007114	0.003921	0.00828	0.40200

Table 4: **Changes regression estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 53,369 annual observations associated with 7,577 individual firms between 1993 and 2015, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ration is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Firm profitability is the operating income before depreciation divided by book assets. Firm tangibility is the plant property divided by book assets. Market to book ration is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. t-statistics clustered by firm and year appear in parentheses below each coefficient estimate. All errors are double clustered by firm and year.

Panel A: Book leverage

Intercept	0.008	0.008	0.008	0.006	0.007	0.007	0.007	0.006
	(14.77)	(14.75)	(14.74)	(11.90)	(11.59)	(11.55)	(11.59)	(10.23)
Ambiguity	0.005		0.005	0.005				
	(2.51)		(2.47)	(2.68)				
Risk		-0.004	-0.004	-0.003				
		(-1.50)	(-2.42)	(-2.32)				
Expected Ambiguity					0.018		0.018	0.019
					(6.36)		(6.31)	(6.53)
Expected Risk						-0.003	-0.007	-0.002
						(-0.09)	(-2.23)	(2.06)
Median Leverage				-0.016				-0.013
				(-1.94)				(-1.32)
Profitability				0.000				-0.002
				(-0.05)				(-0.34)
Tangibility				0.070				0.071
				(5.09)				(4.59)
Market to Book				-0.001				-0.001
				(-2.47)				(-2.91)
R&D				0.097				0.101
				(4.46)				(4.91)
Sales				0.011				0.011
				(6.26)				(4.62)
Tax Rate				0.000				0.000
				(-0.56)				(-0.77)
N	35323	35323	35323	35323	25961	25961	25961	25961
R-Square	0.0002	0.0000	0.0002	0.0052	0.0013	0.0000	0.0013	0.0061

Panel B: Market leverage

Intercept	0.010	0.010	0.010	0.008	0.008	0.008	0.008	0.007
	(17.99)	(17.92)	(17.89)	(13.71)	(13.01)	(12.52)	(12.58)	(10.77)
Ambiguity	0.014		0.014	0.013				
	(5.83)		(5.71)	(5.41)				
Risk		-0.017	-0.016	-0.015				
		(-3.29)	(-3.28)	(-3.21)				
Expected Ambiguity					0.034		0.032	0.031
					(9.23)		(8.76)	(8.39)
Expected Risk						-0.131	-0.114	-0.114
						(-3.38)	(-3.10)	(-3.23)
Median Leverage				-0.029				-0.031
				(-3.22)				(-3.09)
Profitability				0.000				-0.004
				(-0.02)				(-1.22)
Tangibility				0.057				0.051
				(4.18)				(2.95)
Market to Book				0.000				0.000
				(1.39)				(1.34)
R&D				0.068				0.068
				(6.99)				(7.26)
Sales				0.022				0.019
				(11.19)				(8.68)
Tax Rate				0.000				0.000
				(1.74)				(0.92)
N	35323	35323	35323	35323	25961	25961	25961	25961
R-Square	0.0012	0.0006	0.0017	0.0098	0.0036	0.0015	0.0048	0.0101

Table 5: **Economic significance of the estimates of capital structure**

The full sample includes 53,369 annual observations associated with 7,577 individual firms between 1993 and 2015, using records from the Compustat database. Expected volatility, based on CRSP daily stock price records, and expected ambiguity, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ration is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Firm profitability is the operating income before depreciation divided by book assets. Firm tangibility is the plant property divided by book assets. Market to book ration is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales.

Panel A: Book leverage levels

	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.2613	0.0142	0.0037	1.0000	1.5966
Risk	0.1205	0.0314	0.0038	0.9828	1.5692
Expected Ambiguity	0.2301	0.0258	0.0059	0.6263	1.0000
Expected Risk	0.0747	-0.1684	-0.0126	0.2950	0.4710
Median Leverage	0.1531	0.6463	0.0989	0.0375	0.0599
Profitability	0.3722	-0.0377	-0.0140	0.2649	0.4229
Tangibility	0.2250	0.0843	0.0190	0.1957	0.3124
Market to Book	2.6664	-0.0031	-0.0083	0.4449	0.7103
R&D	0.0134	0.1637	0.0022	1.6934	2.7036
Sales	2.3013	0.0104	0.0240	0.1546	0.2468
Tax Rate	2.5133	0.0025	0.0063	0.5919	0.9451

Panel B: Market leverage levels

	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.2613	0.0328	0.0086	1.0000	0.8120
Risk	0.1205	0.0254	0.0031	2.8030	2.2760
Expected Ambiguity	0.2301	0.0302	0.0070	1.2315	1.0000
Expected Risk	0.0747	-0.2135	-0.0160	0.5365	0.4357
Median Leverage	0.1597	0.6360	0.0974	0.0880	0.0714
Profitability	0.3722	0.6395	0.2380	0.0360	0.0292
Tangibility	0.2250	-0.0470	-0.0106	0.8098	0.6575
Market to Book	2.6664	0.0880	0.2347	0.0365	0.0296
R&D	0.0134	-0.0093	-0.0001	68.9041	55.9507
Sales	2.3013	0.0877	0.2019	0.0424	0.0344
Tax Rate	2.5133	0.0141	0.0355	0.2413	0.1960

Panel C: Book leverage changes

	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.2613	0.0053	0.0004	1.0000	2.7049
Risk	0.1205	-0.0034	-0.0001	7.2502	19.6108
Expected Ambiguity	0.2301	0.0185	0.0010	0.3697	1.0000
Expected Risk	0.0747	-0.0019	0.0000	33.9156	91.7373
Median Leverage	0.1531	-0.0127	-0.0003	1.2189	3.2970
Profitability	0.3722	-0.0001	0.0000	17.9990	48.6851
Tangibility	0.2250	0.0700	0.0035	0.1022	0.2765
Market to Book	2.6664	-0.0009	-0.0064	0.0566	0.1530
R&D	0.0134	0.0975	0.0000	20.7183	56.0404
Sales	2.3013	0.0114	0.0607	0.0060	0.0161
Tax Rate	2.5133	-0.0001	-0.0007	0.5477	1.4814

Panel D: Market leverage changes

	Std Dev	Coefficient	Significance	Ambiguity Relative to	Ex. Ambiguity Relative to
Ambiguity	0.2613	0.0131	0.0009	1.0000	1.8204
Risk	0.1205	-0.0149	-0.0002	4.1397	7.5357
Expected Ambiguity	0.2301	0.0307	0.0016	0.5493	1.0000
Expected Risk	0.0747	-0.1140	-0.0006	1.4029	2.5537
Median Leverage	0.1531	-0.0312	-0.0007	1.2234	2.2270
Profitability	0.3722	-0.0291	-0.0040	0.2221	0.4043
Tangibility	0.2250	-0.0001	0.0000	266.2574	484.6844
Market to Book	2.6664	0.0567	0.4035	0.0022	0.0040
R&D	0.0134	0.0002	0.0000	20731.2296	37738.3093
Sales	2.3013	0.0680	0.3603	0.0025	0.0045
Tax Rate	2.5133	0.0224	0.1418	0.0063	0.0115