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IS THE MONETARIST ARITHMETIC UNPLEASANT?

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ABSTRACT

The unpleasant monetarist arithmetic of Sargent and Wallace (1981) states that in a fiscally dominant regime tighter money now can cause higher inflation in the future. In spite of the qualifier 'unpleasant,' this result is positive in nature, and, therefore, void of normative content. I analyze conditions under which it is optimal in a welfare sense for the central bank to delay inflation by issuing debt to finance part of the fiscal deficit. The analysis is conducted in the context of a model in which the aforementioned monetarist arithmetic holds, in the sense that if the government finds it optimal to delay inflation, it does so knowing that it would result in higher inflation in the future. The central result of the paper is that delaying inflation is optimal when the fiscal deficit is expected to decline over time.

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1 Introduction

In "Some Unpleasant Monetarist Arithmetic," Sargent and Wallace (1981) warned that 'tighter money now can mean higher inflation eventually.' They derived this conclusion in the context of a model with a policy regime characterized by fiscal dominance. Specifically, in their formulation the fiscal authority sets an exogenous path for real primary deficits, which must be passively finance by either printing money or issuing debt. In this environment, tightening current monetary conditions requires increasing the growth of interest-bearing debt. Because the government must pay its debts eventually, at some point it has to increase the money supply to pay not only for the primary deficits but also for the increase debt and accumulated interest, entailing higher inflation than if it had not tightened monetary conditions.

The fiscally dominant regime studied by Sargent and Wallace is a realistic description of the restrictions that central banks around the world have faced at different points in history. A recent case in point is Argentina since the beginning of the Macri administration in late 2015. The government inherited a large fiscal deficit of about 5 percent of GDP and is quite limited in its ability to either cut spending or raise taxes. As a result, the fiscal authority has adopted a gradual approach to reducing the deficit, quite independently of the monetary stance. The central bank has chosen not to fully monetize the fiscal deficit. This approach has given rise to a burst of central bank debt, known, in the local jargon, as quasi-fiscal deficits. The rise in central bank debt has been the subject of much criticism by orthodox economist, who, on precisely the grounds laid out by Sargent and Wallace, warn about its consequences for future inflation.

I address the question of under what circumstances, if any, postponing inflation by failing to fully monetize the fiscal deficit can indeed be the optimal policy choice. This question is relevant not only because, as the above example testifies, we do observe policymakers in fiscally dominant regimes resorting to debt issuance to finance fiscal deficits, but also because the term 'unpleasant' in the monetarist arithmetic Sargent and Wallace refer to ought not to be necessarily understood as meaning 'welfare reducing.' In fact, Sargent's and Wallace's analysis is purely positive and therefore void of explicit normative predictions. This paper extends their contribution by placing the choices of their passive monetary authority in a welfare framework. Specifically, I ask what is the welfare maximizing monetary policy in a fiscally dominant regime.

To ensure that the present analysis is conducted in a level playing field with that of Sargent and Wallace, I build a model in which the unpleasant monetarist arithmetic holds. In particular, in the model, the fiscal authority sets an exogenous path for the primary fiscal deficit, and the central bank is limited to choosing the mix of money creation and debt issuance. In this model, failing to monetize the fiscal deficit does result in higher inflation eventually, exactly as dictated by the unpleasant monetarist arithmetic. The key departure from the analysis of Sargent and Wallace is that the central bank chooses a monetary policy that maximizes the lifetime utility of the representative household.

The central result of this paper is that whether or not in a fiscally dominant regime it is optimal to delay inflation by issuing debt depends crucially on the expected path of fiscal deficits. If fiscal deficits are expected to follow a declining path, or, more generally, are temporarily high, then it may be optimal for the central bank to fall short of full monetization of the fiscal deficit. In this case, public debt will initially rise and long-run inflation will be higher than if the central bank had refrained from initially restricting the pace of monetary expansion. If fiscal deficits are expected to grow over time, or, more generally, if fiscal deficits are temporarily low, it may indeed be optimal for the central bank to follow a monetary policy that is looser than the full monetization of the fiscal deficit would require. In this case, the long-run rate of inflation is lower than under the policy of monetizing the deficits period by period. Full monetization of the fiscal deficit emerges as the optimal policy outcome when the fiscal deficit is expected to be stable over time.

The intuition behind this result is as follows. In virtually all existing monetary models, inflation represents a distortion. Smoothing this distortion over time can be welfare increasing. In this case, the central bank will tend to set a smooth path of inflation subject to the restriction that the associated present discounted value of seignorage revenues is large enough to cover the lifetime liabilities of the government. Thus, the optimal inflation rate is dictated by the average fiscal deficit, rather than by the current one. As a result, if the current fiscal deficit is above its average value, seignorage will fall short of the fiscal deficit, and the government will need to issue debt to close the gap. This expansion in government liabilities implies higher future inflation than the alternative of printing money today to pay for the entire current fiscal deficit—the monetarist arithmetic— but is preferable because it renders a smoother path for the inflation tax.

Section 2 presents an intertemporal model in which a demand for money is motivated by assuming that real balances produce utility. Section 3 characterizes the Ramsey equilibrium. Section 4 derives conditions under which it is optimal for the central bank to increase public debt instead of fully monetizing the fiscal deficit. Section 5 analyzes an economy with long-run growth. Section 6 provides a numerical example motivated by the recent Argentine stabilization effort. Section 7 provides concluding remarks.

2 The Model

The theoretical environment is an infinite-horizon, flexible-price, endowment economy with money in the utility function. The fiscal authority runs an exogenous stream of real primary fiscal deficits and finances them by a combination of debt issuance and money creation.

2.1 Households

Consider an economy populated by a large number of identical households with preferences for consumption and real money balances described by the following lifetime utility function

$$\int_0^\infty e^{-\rho t}[u(c_t) + v(m_t)]dt, \tag{1}$$

where c_t denotes consumption of a perishable good, m_t denotes real money balances, and $\rho > 0$ is a parameter denoting the subjective rate of discount. The subutility functions $u(\cdot)$ and $v(\cdot)$ are assumed to be strictly increasing and strictly concave.[1]

Households are endowed with an exogenous and constant stream of goods denoted $y > 0$ and receive real lump-sum transfers from the government, denoted τ_t. In addition, households can hold two types of assets, money, denoted M_t, and interest-bearing nominal bonds, denoted B_t. Bonds pay the nominal interest rate i_t, and money bears no interest. The household's flow budget constraint is then given by

$$P_t c_t + \dot{M}_t + \dot{B}_t = P_t y + P_t \tau_t + i_t B_t,$$

where a dot over a variable denotes its time derivative. Dividing through by the price level, one can write the flow budget constraint as

$$c_t + \dot{m}_t + \pi_t m_t + \dot{b}_t + \pi_t b_t = y + \tau_t + i_t b_t,$$

where $m_t \equiv M_t/P_t$ denotes real money balances, $b_t \equiv B_t/P_t$ denotes real bond holdings, and $\pi_t \equiv \dot{P}_t/P_t$ denotes the rate of inflation. Now letting

$$w_t \equiv m_t + b_t$$

denote real financial wealth and

$$r_t \equiv i_t - \pi_t$$

[1] The present study is not concerned with dynamics in which the economy falls into liquidity traps, so I need not impose weaker assumptions on $v(\cdot)$.

denote the real interest rate, one can express the flow budget constraint as

$$c_t + \dot{w}_t = y + \tau_t + r_t w_t - i_t m_t. \tag{2}$$

The right-hand side of constraint (2) represents the sources of income, given by the sum of nonfinancial income, $y + \tau_t$, and financial income $r_t w_t - i_t m_t$. The left-hand side represents the uses of income, consumption, c_t, and savings, $\dot{w}_t$. Households are also subject to the following terminal borrowing constraint that prevents them from engaging in Ponzi schemes

$$\lim_{t\to\infty} e^{-R_t} w_t \geq 0, \tag{3}$$

where $R_t \equiv \int_0^t r_t dt$ is the compounded interest rate from time 0 to time t.

The household chooses time paths $\{c_t, m_t, w_t\}$ to maximize the utility function (1), subject to the flow budget constraint (2) and the no-Ponzi-game constraint (3). Letting λ_t denote the multiplier associated with the flow budget constraint (2), the optimality conditions associated with this problem are

$$u'(c_t) = \lambda_t, \tag{4}$$

$$\frac{v'(m_t)}{u'(c_t)} = i_t, \tag{5}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t, \tag{6}$$

$$c_t + \dot{w}_t = y + \tau_t + r_t w_t - i_t m_t, \tag{7}$$

and

$$\lim_{t\to\infty} e^{-R_t} w_t = 0. \tag{8}$$

The first condition says that in the optimal plan the marginal utility of consumption must equal the shadow value of wealth. The second condition is a demand for money. It says that the desired level of real money holdings is decreasing in the nominal interest rate and increasing in consumption. Solving that optimality condition for m_t one can write

$$m_t = L(i_t, c_t), \tag{9}$$

where $L(\cdot,\cdot)$ is a liquidity preference function with partial derivatives $L_1 < 0$ and $L_2 > 0$. I assume that $iL(i, y)$ is increasing in i for the ranges of interest rates that are relevant in the present analysis. This assumption ensures that in equilibrium the government can increase seignorage revenue by raising the nominal interest rate. The third optimality condition is the Euler equation associated with bond holdings. The fourth optimality condition is the flow budget constraint. And the fifth and last optimality condition is a transversality condition given by the no-Ponzi-game constraint holding with equality.

2.2 The Government

I assume that the primary fiscal deficit, τ_t, evolves exogenously over time. An environment of this type is said to display fiscal dominance. To finance the stream of fiscal deficits, the government can either print money, $\dot{M}_t > 0$, or issue interest-bearing bonds, $\dot{B}_t > 0$. The flow budget constraint of the government is therefore given by

$$\dot{M}_t + \dot{B}_t = P_t \tau_t + i_t B_t.$$

We can write this constraint in real terms as

$$\dot{w}_t = \tau_t + r_t w_t - i_t m_t. \tag{10}$$

This expression says that the government uses increases in its total liabilities, $\dot{w}_t$, and seignorage, $i_t m_t$, to pay for the primary deficit, τ_t, and to meet interest obligations on outstanding liabilities $r_t w_t$.

2.3 Competitive Equilibrium

Combining the flow budget constraints of the household and the central bank (equations (2) and (10), respectively) yields the resource constraint

$$c_t = y, \tag{11}$$

which implies that consumption is constant over time. In turn, this result and equation (9) imply that in equilibrium the demand for money is given by

$$m_t = L(i_t, y).$$

Combining (4), (6), and (11) yields

$$r_t = \rho.$$

This expression says that in the present economy the equilibrium real interest rate is constant over time and equal to the subjective discount factor. One can break this implication in a number of ways, such as introducing nominal rigidity.

Finally, the transversality condition (8) and the flow budget constraint of the government (10) are equivalent to the following intertemporal restriction

$$\frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-\rho t}[i_t L(i_t, y) - \tau_t]dt, \tag{12}$$

where we are using the equilibrium conditions $c_t = y$ and $r_t = \rho$. Equation (12) says that the present discounted value of seignorage revenue must be equal to the sum of the present discounted value of primary deficits and the central bank's initial real liabilities. We are now ready to define the competitive equilibrium in this economy.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is an initial price level P_0 and a time path of nominal interest rates $\{i_t\}$ satisfying equation* (12)*, given the initial level of nominal government liabilities $B_0 + M_0$ and the time path of real primary fiscal deficits $\{\tau_t\}$.*

I am interested in economies in which the government is initially a net debtor and in which the fiscal authority runs a stream of primary deficits that is positive in present discounted value. Accordingly, we assume that

$$B_0 + M_0 > 0 \tag{13}$$

and

$$\int_0^\infty e^{-\rho t}\tau_t dt > 0. \tag{14}$$

These assumptions ensure that the central bank must generate a stream of seignorage income that is positive in present discounted value to meet its lifetime financial obligations. The question is what part of its obligations should it finance with seignorage and what part by issuing debt at any point in time.

3 Ramsey Optimal Central Bank Policy

I assume that the central bank is benevolent and has the ability to commit to its promises. This means that among all the interest rate paths and initial price levels that are consistent with a competitive equilibrium, the monetary authority picks the one that maximizes the representative household's lifetime welfare. I refer to such equilibrium as the Ramsey optimal equilibrium. In equilibrium, welfare is given by the following indirect lifetime utility function:

$$\int_0^\infty e^{-\rho t}[u(y) + v(L(i_t, y))]dt. \tag{15}$$

Because $v(\cdot)$ is strictly increasing and $L(\cdot,\cdot)$ is decreasing in its first argument, lifetime utility is strictly decreasing in the nominal interest rate. It then follows from equations (12)-(15) that it is optimal for the central bank to implement a policy in which $P_0 \to \infty$, that is, it is optimal to cause a hyperinflation in period 0. By doing this, the central bank inflates away all of the government's initial real liabilities, $(B_0 + M_0)/P_0 \to 0$, reducing the need to generate seignorage revenue through the (distortionary) inflation tax. To avoid this unrealistic feature of optimal policy, it is typically assumed in the related literature (see, e.g., Schmitt-Grohé and Uribe, 2004, and the references therein) that the initial price level, P_0, is given. I follow this tradition. The Ramsey optimal equilibrium is then defined as follows:

Definition 2 (Ramsey Optimal Equilibrium) *A Ramsey optimal equilibrium is a path for the nominal interest rate* $\{i_t\}$ *that maximizes the indirect utility function* (15) *subject to the intertemporal constraint* (12)*, given the initial level of real government liabilities* $(B_0 + M_0)/P_0$ *and the path of primary fiscal deficits* $\{\tau_t\}$.

The optimality conditions associated with the Ramsey problem are equation (12) and

$$v'(L(i_t, y))L_1(i_t, y) + \eta[L(i_t, y) + i_t L_1(i_t, y)] = 0, \tag{16}$$

where η denotes the Lagrange multiplier associated with the constraint (12). The Lagrange multiplier η is endogenously determined in period 0, but it is constant over time. This means that the Ramsey optimal nominal interest rate is also time invariant. Let i^* denote the Ramsey optimal nominal interest rate. Then we have that in the Ramsey equilibrium

$$i_t = i^*,$$

at all times $t \geq 0$. It follows immediately from the intertemporal constraint (12) and from the assumption that $i_t L(i_t, y)$ is increasing in i_t that i^* is increasing in both the initial level of real government liabilities, $(M_0 + B_0)/P_0$, and the present discounted value of fiscal deficits, $\int_0^\infty e^{-\rho t}\tau_t dt$. This implication is intuitive, the larger the present value of all government liabilities, the larger the amount of seignorage the central bank must generate to meet its obligations.

Because both the Ramsey optimal nominal interest rate and the equilibrium real interest rate are constant, we have that the Ramsey optimal inflation rate is also constant. Letting

π^* be the optimal rate of inflation, we have that in the Ramsey equilibrium

$$\pi_t = \pi^* \equiv i^* - \rho,$$

for all $t \geq 0$. Similarly, in the Ramsey optimal equilibrium real money balances are constant and satisfy

$$m_t = m^* \equiv L(i^*, y),$$

for all $t \geq 0$.

4 Optimal Public Debt Dynamics

We are now equipped with the necessary elements to characterize the optimal path of public debt, b_t. Recalling that $\dot{w}_t \equiv \dot{b}_t + \dot{m}_t$ and that m_t, i_t, and π_t are constant over time, we can write the government flow budget constraint given in (10) as

$$\dot{b}_t = \rho b_t + \tau_t - \pi^* m^*, \tag{17}$$

with the initial condition $b_0 = (B_0 + M_0)/P_0 - m^*$.[2] Intuitively, the Ramsey government uses a combination of debt creation, $\dot{b}_t$, and seignorage, $\pi^* m^*$, to pay the interest on the outstanding debt, ρb_t, and to finance the primary deficit, τ_t.

The optimal dynamics of public debt depend crucially on the expected future path of fiscal deficits. To see this, consider first a situation in which the primary fiscal deficit is expected to fall over time. To fix ideas, assume that the fiscal deficit evolves according to a first-order autoregressive process of the type

$$\dot{\tau}_t = -\delta \tau_t, \tag{18}$$

with $\tau_0 > 0$ and $\delta > 0$. Then, we can write the equilibrium law of motion of public debt given in equation (17) as

$$\dot{b}_t = \rho b_t + \tau_0 e^{-\delta t} - \pi^* m^*, \tag{19}$$

Because $\rho > 0$, for arbitrary values of $\pi^* m^*$ the differential equation (19) is mathematically unstable. However, it is economically stable, because the central bank chooses the level of seignorage $\pi^* m^*$ to guarantee the satisfaction of the transversality condition (8), which implies that w_t, and therefore also b_t since m_t is constant, grows at a rate less than ρ. To

[2] The implementation of the Ramsey optimal plan, if unanticipated, in general gives rise to a portfolio recomposition at time 0, because households may change their desired money holdings (and therefore decrease their desired bond holdings) in a discrete fashion.

see this, solve the difference equation (19) to obtain

$$b_t = \left[b_0 + \frac{\tau_0}{\rho+\delta} - \frac{\pi^* m^*}{\rho}\right] e^{\rho t} - \frac{\tau_0}{\rho+\delta} e^{-\delta t} + \frac{\pi^* m^*}{\rho}$$

Using equation (12) one can show that the expression within square brackets is zero,

$$b_0 + \frac{\tau_0}{\rho+\delta} - \frac{\pi^* m^*}{\rho} = 0,$$

which eliminates the unstable branch of the solution. It follows that the optimal equilibrium dynamics of public debt is given by

$$b_t = \frac{\pi^* m^*}{\rho} - \frac{\tau_0}{\rho+\delta} e^{-\delta t}. \tag{20}$$

Equation 20 delivers the main result of this paper, namely, that if the fiscal deficit is expected to fall over time, it is optimal for the government to finance it partly by issuing debt, instead of by money creation alone. This result is quite intuitive. The central bank finds it optimal to smooth seignorage revenue over time. As a result, if initially the fiscal deficit exceeds seignorage revenue, the government finances the difference by issuing new debt. Over time, the primary deficits fall, but interest obligations increase. On net, however, the sum of these two sources of outlays fall, converging to zero asymptotically. In the limit, the primary fiscal deficit is nil $\tau_t \to 0$, and interest obligations are exactly equal to seignorage revenue $\rho b_t \to \pi^* m^*$. This means that asymptotically, public debt converges to a constant, given by the present discounted value of seignorage revenue.

It is straightforward to show that if the primary fiscal deficit is temporarily low or follows an increasing path over time, as in the autoregressive form

$$\tau_t = \overline{\tau} - (\overline{\tau} - \tau_0) e^{-\delta t},$$

with $\overline{\tau} > \tau_0 > 0$ and $\delta > 0$, then the optimal path of debt is decreasing. In this case, the central bank finds it optimal to create more money than is necessary to cover the fiscal deficit, and it uses the excess seignorage to retire some debt. As time goes by, the primary deficit increases, and a larger fraction of the constant seignorage revenue is devoted to paying for it. Finally, in the intermediate case in which the primary deficit is expected to be constant over time, the government does not resort to debt issuance to finance the primary deficit.

5 A Growing Economy

Thus far, I have limited the analysis to a stationary economy. It is of interest to ascertain how the conditions under which it is optimal to delay inflation change in an environment with long-run growth. To this end, here I generalize the law of motion of the endowment to allow for secular growth as follows,

$$y_t = y\, e^{gt},$$

where $g > 0$ is a parameter defining the growth rate of output, and $y > 0$ is a parameter defining the detrended level of output. In a balanced growth path, consumption and real money holdings grow at the same rate as output in the long run. To make this possible, I assume that the subutility functions $u(\cdot)$ and $v(\cdot)$ are both homogeneous of the same degree, as in the utility function

$$u(c) + v(m) = \frac{c^{1-1/\alpha} + A^{1/\alpha} m^{1-1/\alpha}}{1 - 1/\alpha}, \tag{21}$$

where $A, \alpha > 0$ are parameters.

Let $\tilde{x}_t \equiv x_t e^{-gt}$ be the detrended version of x_t, for $x_t = c_t, m_t, \tau_t, w_t, b_t$ and let $\tilde{\lambda}_t \equiv \lambda_t e^{g\alpha t}$ be the detrended version of λ_t. We can then write the first-order conditions associated with the household's utility maximization problem, given in equations (4)-(8), in terms of detrended variables as

$$u'(\tilde{c}_t) = \tilde{\lambda}_t, \tag{22}$$

$$\frac{v'(\tilde{m}_t)}{u'(\tilde{c}_t)} = i_t,$$

$$\frac{\dot{\tilde{\lambda}}_t}{\tilde{\lambda}_t} = \rho + g\alpha - r_t, \tag{23}$$

$$\tilde{c}_t + \dot{\tilde{w}}_t = y + \tilde{\tau}_t + (r_t - g)\tilde{w}_t - i_t \tilde{m}_t,$$

and

$$\lim_{t \to \infty} e^{-(R_t - gt)} \tilde{w}_t = 0.$$

Similarly, after expressing variables in detrended form, the government flow budget constraint (10) becomes

$$\dot{\tilde{w}}_t = \tau_t + (r_t - g)\tilde{w}_t - i_t \tilde{m}_t. \tag{24}$$

In equilibrium, detrended consumption must equal detrended output

$$\tilde{c}_t = y.$$

This result together with optimality conditions (4) and (**??**) implies that the real interest rate is constant and given by

$$r_t = \tilde{\rho} \equiv \rho + g\alpha.$$

According to this expression, the real interest rate is higher in the growing economy. This is intuitive, because growth makes the marginal utility of consumption fall faster over time, causing agents to demand higher compensation for sacrificing current consumption in exchange for future consumption.

By an analysis similar to that applied in the economy without growth, we can deduce that a competitive equilibrium in the economy with long-run growth is an initial price level P_0 and a time path of nominal interest rates $\{i_t\}$ satisfying the intertemporal constraint

$$\frac{B_0 + M_0}{P_0} = \int_0^\infty e^{-(\tilde{\rho}-g)t}[i_t L(i_t, y) - \tilde{\tau}_t]dt, \tag{25}$$

given the initial level of nominal government liabilities $B_0 + M_0$ and the time path of real detrended primary fiscal deficits $\{\tilde{\tau}_t\}$.

With long-run growth, the indirect utility function (15) takes the form

$$\int_0^\infty e^{-(\tilde{\rho}-g)t}[u(y) + v(L(i_t, y))]dt. \tag{26}$$

A Ramsey optimal equilibrium in the growing economy is then a path for the nominal interest rate $\{i_t\}$ that maximizes the indirect utility function (26) subject to the intertemporal constraint (25), given the initial level of real government liabilities $(B_0 + M_0)/P_0$ and the path of real detrended primary fiscal deficits $\{\tilde{\tau}_t\}$. The first-order condition with respect to i_t associated with this optimization problem is identical to its counterpart in the stationary economy, namely, equation (16). This implies, in particular, that the Ramsey optimal nominal interest rate is constant over time in the growing economy.

Finally, assume, as we did in the stationary economy, that the primary fiscal deficit obeys law of motion

$$\dot{\tilde{\tau}}_t = -\delta \tilde{\tau}_t,$$

with $\tilde{\tau}_0 = \tau_0 > 0$ and $\delta > 0$. That is, the primary fiscal deficit as a fraction of output falls gradually over time. Following the same steps as in the economy without growth, we can deduce that the Ramsey optimal path of public debt is given by

$$\tilde{b}_t = \frac{(\pi^* + g)m^*}{\tilde{\rho} - g} - \frac{\tau_0}{\tilde{\rho} - g + \delta} e^{-\delta t},$$

which says that if the detrended primary fiscal deficit is expected to fall over time, then it is optimal for the government not to fully monetize the fiscal deficit and instead allow the public debt to grow over time as a fraction of output. We therefore conclude that the central result of this paper is robust to allowing for secular growth.

6 A Numerical Illustration: Fiscal Gradualism

To illustrate the implications of optimal policy for the dynamics of public debt, consider the following numerical example. It is motivated by developments in Argentina since the beginning of the Macri administration in December of 2015. Table 1 summarizes the calibration of the model. The time unit is a year. The fiscal authority inherited a large fiscal deficit of about 5 percentage points of GDP Thus, I set the initial condition $\tau_0 = 0.05\, y$. The government is committed to reduce the fiscal deficit, albeit at a gradual pace. As a baseline case, I assume that τ_t has a half life of 3 years. This means that at the beginning of the last year of the Macri administration (December of 2018) the primary fiscal deficit will be 2.5 percentage points of GDP. The assumed half life of 3 years implies that the parameter δ in the law of motionn (18) takes the value 0.23. With these parameter values, the primary fiscal deficit evolves according to the expression

$$\tau_t = 0.05\, e^{-0.23t}.$$

I set the initial total government liabilities equal to 38.9 percent of GDP, or

$$w_0 \equiv \frac{M_0 + B_0}{/P_0} = 0.389\, y.$$

I arrive at this number as follows. I set the liabilities of the treasury to 22.9 percent of GDP, and the liabilities of the central bank at 16 percent of GDP at the beginning of 2016. I calculate the liabilities of the treasury as the difference between the total debt of the treasury of 53.6 percent of GDP and the debt of the treasury with other government agencies of 30.7 percent of GDP (Informe Ministerio de Hacienda y Finanzas, first quarter 2016). I measure the liabilities of the central bank by the sum of the monetary base and the stock of Lebac bonds outstanding. These two aggregates stood at 960 billion pesos at the beginning of 2016 (Informe Diario del BCRA, 2016). Annual GDP in Argentina is estimated to be about 400 billion dollars, and the nominal exchange rate at the beginning of 2016 fluctuated around 15 pesos per dollar. Taken together, these figures imply liabilities of the central bank of 16 percent of GDP.

Table 1: Calibration to Argentina

Parameter	Value	Description
ρ	0.0392	Subjective discount factor
g	0.0198	Growth rate
α	0.13	Parameter of money demand function $L(i,y) = A\,y\,i^{-\alpha}$
A	0.0882	Parameter of money demand function $L(i,y) = A\,y\,i^{-\alpha}$
τ_0	0.05	Initial deficit-to-GDP ratio
δ	0.23	Half life of deficit (3 years)
w_0/y	0.389	Initial government liabilities as share of GDP

Note. The time unit is one year.

I assume a money demand function of the form

$$L(i, c) = A\,c\,i^{-\alpha},$$

where $A, \alpha > 0$ are parameters. This liquidity preference specification is implied by the period utility function given in equation (21). I set $\alpha = 0.13$ using the estimate of the interest-rate semielasticity of the demand for money in Argentina produced by Kiguel and Neumeyer (1995).[3] This value is in line with estimates for other countries including low-inflation ones (see, for example, Inagaki, 2009, for the United States and Japan). I calibrate the scale parameter A to match a monetary-base-to-GDP ratio of about 10 percent of GDP and an interest rate of 38 percent as observed in Argentina at the beginning of 2016. Thus, I set A to satisfy $0.1 = A \cdot 0.38^{-0.13}$, or $A = 0.0882$.

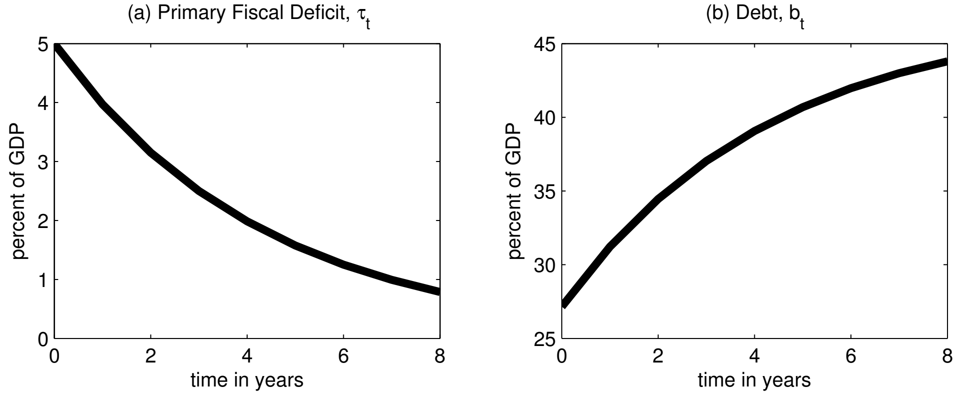
Finally, I set the subjective discount factor to 4 percent, or $\rho = \ln(1.04)$, and the long-run growth rate of output per capita at 2 perccent, or $g = \ln(1.02)$.

Under this calibration, the model predicts an optimal inflation rate of 6.8 percent per year ($\pi^* = 0.068$). The associated optimal nominal interest rate is 11 percent ($i^* = 0.11$). The optimal rate of inflation generates seignorage revenue equal to 0.8 percent of GDP ($\pi^* m^* = 0.008$).

Evaluating equation (20) at the parameter values given in table 1, one can trace out the

[3] The interest-rate semielasticity of the demand for money is defined as $\partial \ln L(i,y)/\partial i$. Kiguel and Neumeyer denote this object a_1. They report an average estimate of a_1 of -0.041 (arithmetic mean of all the estimates reported in their table 2). To derive the value of α implied by this estimate of a_1, one must apply two transformations. First, in their specification, the opportunity cost of money is measured in percent per month, so one must rescale a_1 by the factor 100/12. Second, to convert the semielasticity a_1 into an elasticity, one must multiply by the opportunity cost of money, i. For this purpose, I apply the interest rate on Lebacs prevailing in Argentina at the beginning of 2016 of 38 percent per year. This yields $\alpha = 0.1298 = 0.041 \times (100/12) \times 0.38$.

Figure 1: Ramsey Optimal Debt Dynamics



Note. τ_t and b_t are measured in percent of annual GDP. Replication file qf.m in uribe_monetarist_arithmetic.zip

path of central bank debt under the Ramsey optimal policy. Panel (b) of figure 1 displays the equilibrium dynamics of central bank debt, b_t, as a fraction of GDP. The initial financing needs of the central bank, $\tilde{\rho}w_0 + \tau_0$ amount to 6.6 percent of GDP. As mentioned above, under the optimal monetary policy seignorage revenue is only 0.8 percent of GDP. The central bank closes this gap by increasing public debt. Thus, $\dot{b}_t$ is positive. As time goes by, the dynamics of public debt are dictated by two opposing forces. On the one hand, the stock of debt, b_t, grows over time, so interest obligations, given by $\tilde{\rho}b_t$, also increase. On the other hand, by assumption, the primary fiscal deficit gradually falls. It turns out that the latter force dominates. This is reflected in a concave shape of the path of central bank debt (figure 1). Asymptotically, the financing needs of the central bank converge to a value exactly equal to seignorage revenue. At this limiting point, the central bank does not need to increase public debt to finance its outlays. In the present example, the interest-bearing debt of the consolidated government starts at around 27 percent of GDP and reaches 43 percent of GDP after 8 years (by the end of a potential second term of the Macri administration). In the long run, debt stabilizes at 47 percent of GDP.

Table 2 displays the key prediction of the model for various levels of the half life of the primary fiscal deficit and its initial level. Intuitively, the more gradual is the elimination of fiscal deficits and the larger is the initial deficit, the larger is the level of public debt in the long run, the higher is the inflation rate, and the larger is the amount of resources collected via seignorage. For example, as the half life of deficits increases from 3 to 4 years and the initial primary deficit increases from 5 to 6 percent of GDP, the interest-bearing debt of the consolidated government in year 8 increases from 43.8 to 50.5 percent of GDP, inflation increases from 6.8 to 9.2 percent per year, and seignorage revenue ($\pi^* m^*$) increases from 0.8 to 1 percent of GDP.

Table 2: Ramsey Optimal Long-Run Debt, Inflation, and Seignorage as Revenue Functions of the Initial Level and Half Life of the Primary Fiscal Deficit

Half Life of Deficit	Initial Deficit τ_0	Long-run Debt b_8	Inflation Rate π^*	Seignorage Revenue $\pi^* m^*$
2.00	4.00	37.03	4.91	0.59
2.00	5.00	39.67	5.48	0.65
2.00	6.00	42.30	6.05	0.72
3.00	4.00	40.35	5.95	0.71
3.00	5.00	43.80	6.80	0.80
3.00	6.00	47.24	7.65	0.89
4.00	4.00	42.54	6.96	0.82
4.00	5.00	46.52	8.07	0.93
4.00	6.00	50.49	9.19	1.05

Note. The half life of primary deficits is measured in years, π^* is measured in percent per year, and τ_0, b_8, and $\pi^* m^*$ are measured in percentage points of GDP. b_8 denotes debt 8 years after the implementation of the Ramsey policy. All parameters of the model, except for δ and τ_0 take the values displayed in table 1.

6.1 Optimal Policy Versus Full Monetization

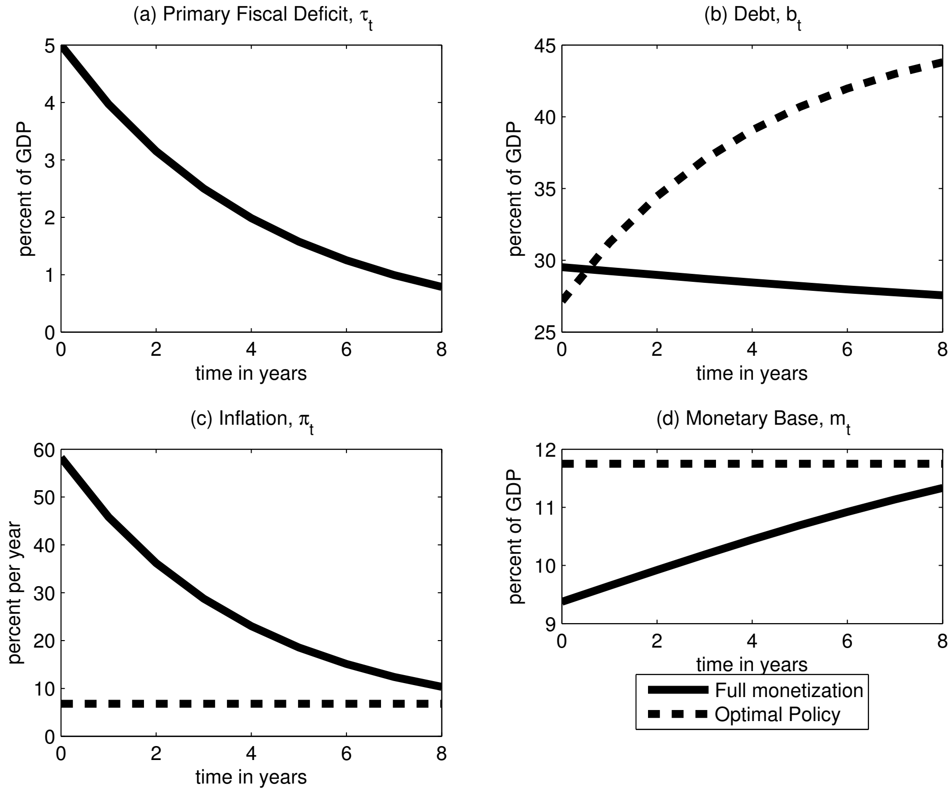
It is of interest to compare the Ramsey optimal dynamics with those implied by a monetary policy that, through money creation, keeps total government liabilities from growing over time, $\dot{w}_t = 0$. I refer to this policy as full monetization of the fiscal deficit. This is the policy that results from interpreting the adjective 'unpleasant' attached to the monetarist arithmetic as meaning that in a fiscally dominant regime it is counterproductive to delay inflation by financing the fiscal deficit with debt. Formally, setting $\dot{w}_t = 0$, we can rewrite the central bank's flow budget constraint (24) as

$$i_t L(i_t, y) = \tilde{\tau}_0 e^{-\delta t} + (\tilde{\rho} - g)\tilde{w}_0. \tag{27}$$

According to equation (27), under full monetization the government pays for the primary fiscal deficit and interest on its outstanding liabilities with seignorage revenue as they accrue. Using the assumed functional form for $L(\cdot, \cdot)$, equation (27) can be solved for the nominal interest rate to get

$$i_t = \left(\frac{\tau_0 e^{-\delta t} + (\tilde{\rho} - g)\tilde{w}_0}{A} \right)^{\frac{1}{1-\alpha}}.$$

Figure 2: Equilibrium Dynamics Under Full Monetization



Note. τ_t, b_t, and m_t are measured in percent of annual GDP, and π_t is measured in percent per year.

Since $\alpha < 1$, we have that under full monetization the nominal interest rate is increasing in the current level of the primary deficit. With the path of i_t at hand, one can readily derive the equilibrium paths of all other endogenous variables of the model.

Figure 2 displays with solid lines the equilibrium dynamics of debt, inflation, and money holdings under full monetization. For comparison, the figure also displays with dashed lines the Ramsey optimal dynamics. Under full monetization, initially the central bank must generate large seignorage revenues to finance the elevated primary deficit. This results in a high initial inflation rate of 58 percent per year, more than 8 times as high as the Ramsey optimal rate of inflation of 6.8 percent (panel (c) of figure 2). Because under full monetization the central bank prints money instead of issuing debt to pay for both the primary deficit and the interest on the government's outstanding liabilities, the path of interest-bearing debt stays relatively flat at about 28.5 percent of GDP (panel (b) of figure 2).[4] By contrast,

[4]Indeed b_t displays a slightly downward sloping path under full monetization. The reason is that the sum of debt and money, $b_t + m_t$, is constant for all t and equal to w_0, while m_t increases steadily over time, as inflation falls. Thus, along the transition to the steady state, households continuously substitute money for bonds in their portfolios.

under the Ramsey policy debt increases continuously, reaching 47 percent of GDP in the long run.[5] As a result, in the long run, the inflation tax necessary to pay the interest on the debt is lower under full monetization than under the optimal policy. The long-run inflation rate is 2.7 percent under full monetization versus 6.8 percent under the optimal policy. This shows that the monetarist arithmetic is at work: the Ramsey policy delivers a lower rate of inflation than full monetization in the short run, but a higher one in the long run. However, in this case the monetarist arithmetic is not unpleasant. Panel (d) of figure 2 shows why. Under the Ramsey policy real money balances display a much smoother pattern than under full monetization. Because the period utility function is concave in real money balances, households prefer the smoother path of money.

6.2 Discussion

In the present model, it is optimal for the central bank to jump to the long-run interest rate from the outset. In reality, however, central banks change interest rates in small steps. This has also been the case during the episode that motivates the present numerical example. The Argentine central bank has been reducing the interest rate gradually, from around 38 percent in December of 2015 to about 25 percent by the end of 2016. This path lies in between the one associated with full monetization and the Ramsey optimal one.

Although it is outside the scope of the present model to explain why central banks tend to move their policy instrument sluggishly, it is worth mention a number of factors that may contribute to this practice. One is uncertainty about the actual path of fiscal deficits. In an uncertain environment, it may be optimal for the central bank to move slowly, waiting until new information about the evolution of fiscal variables emerges. A second factor that might explain why disinflation often takes place gradually is nominal rigidity and backward-looking indexation. In environments withnominal frictions large declines in the rate of inflation can create disequilibria in product markets and involuntary unemployment in factor markets.

Finally, central banks have been shown to move even more gradually than is call for by uncertainty or nominal rigidity. This characteristic of monetary policy is known as interest-rate smoothing and has been extensively documented in developed countries (see, for example, Goodfriend, 1991; and Sack, 2000), but is also present in emerging countries, including those experiencing high inflation (Argentina since late 2015 being an example). Motivated by the empirical relevance of interest-rate smoothing, a number of studies have introduced this fea-

[5] At time 0, debt is higher under full monetization than under the Ramsey policy (panel (b) of figure 2). This is because at time 0 total government liabilities of the government, w_0, are fixed and equal in both regimes. But their compositions (money and bonds) are endogenously determined. Because at time 0 the nominal interest rate is higher under full monetization, in this regime households hold less money and more bonds than under the Ramsey optimal policy.

ture in dynamic models. An early contribution is Barro 1988). Woodford (2003) shows that when the central bank lacks commitment, adding lagged interest rates to its loss function can be desirable, as it brings the resulting optimal policy under discretion closer to its counterpart under commitment. To introduce interest-rate smoothing, consider a modified version of the indirect utility function (15) of the form $\int_0^\infty e^{-\rho t}[u(y) + v((L(i_t, y)) - \gamma(i_t - z_t)^2]dt$, where $\gamma > 0$ is a parameter, and z_t denotes a weighted average of past interest rates that obeys the law of motion $\dot{z}_t = i_t - \theta z_t$, for $\theta > 0$. The parameter γ measures the degree of interest-rate smoothing. The model collapses to the baseline specification when γ equals 0. The parameter θ measures the degree of backward-looking behavior in the central bank's monetary conduct. The variable z_t is, of course, a state variable. The problem of the central bank is to choose a path for the nominal interest rate $\{i_t\}$ to maximize its modified objective function subject to the intertemporal constraint (12) and to the law of motion of z_t. This specification of the model is likely to deliver more realistic paths for the nominal interest rate and inflation—i.e., more gradual disinflation dynamics—while preserving the desirability of debt creation along the stabilization path.

7 Concluding Remarks

In economies in which the monetary-fiscal regime is fiscally dominant, the central bank does not control inflation. This is because under this policy regime money creation must passively finance the present discounted value of fiscal deficits plus the government's initial liabilities. The central bank can, however, choose when to create money. The unpleasant monetarist arithmetic states that tight current monetary conditions imply higher inflation in the future. This paper does not quarrel with this dictum, but with the conclusion—implicit in the qualifier 'unpleasant'—that in a fiscally dominant regime tight monetary policy, understood as financing part of the fiscal deficit by issuing debt, is undesirable. Arriving at such a conclusion requires a normative analysis. In this paper I attempt to fill this gap, by characterizing the welfare maximizing path of public debt in a monetary economy characterized by fiscal dominance.

The main result derived from this analysis is that in a fiscally dominant regime tight money may not have unpleasant consequences, but, on the contrary, be optimal. This result obtains when the fiscal deficit is expected to fall over time or is temporarily high. The intuition is simple. Among all the inflation paths that generate enough seignorage revenue to pay for the present discounted value of the government's liabilities, the monetary authority prefers a flat one to smooth out the distortions created by the inflation tax. In turn, a flat path of inflation induces a flat path of seignorage revenue. This means that in periods in which fiscal deficits are above average, the central bank will find it optimal to issue debt to finance part of the fiscal deficit. The central bank prefers to issue debt even though it knows that it will cause higher inflation in the future than the alternative of financing the entire current deficit by printing money. In this case, the monetarist arithmetic obtains, but is not unpleasant.

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