NBER WORKING PAPER SERIES

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Working Paper 22793
http://www.nber.org/papers/w22793

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2016

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NBER Working Paper No. 22793
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JEL No. G10,G11,G13,N2,N21,N22

ABSTRACT

This paper analyzes a novel data set of commodity futures prices over a long sample period starting in 1877, which allows us to shed new light on several important and controversial questions. We document that commodity futures returns (1) have been positive on average; (2) vary significantly across business cycles, inflation episodes, and periods of backwardation versus contango, (3) are driven mostly by variation of spot returns and therefore closely linked to the underlying commodity spot market; (4) perform well during inflation cycles and provide more return in backwardated states; and (5) display low correlation with stocks and bonds. These long-run stylized facts imply that commodity futures can add value to a diversified portfolio from an asset allocation perspective.

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I. Introduction

From an investor's point of view, how do commodity futures fit into the asset allocation decision? To answer this question, we need to understand the statistical properties of commodity futures returns and, in particular, these properties in aggregate economic states.

A recent series of influential papers have studied the properties of returns on commodity indices. Gorton and Rouwenhorst (2006, denote GR) and Bhardwaj, Gorton and Rouwenhorst (2015, denote BGR) form an equally-weighted index of commodities over the past 50 years and document three key features of the data: (i) equity-like returns, (ii) a positive sensitivity to inflation, and (iii) a negative correlation with other major asset classes. In contrast, Erb and Harvey (2006, 2015, denote EH) provide a more cautionary note, arguing that the above statistical properties of commodity returns may be more transient. Moreover, EH find that the most important component of commodity futures returns is its carry (or income) piece, rather than the underlying spot return. We provide new perspective on this topic by analyzing a novel data set of futures prices on commodities, dating back as early as 1877.

Specifically, over the period 1877-2015, we construct returns on indices of commodity futures, decomposing these returns into the convenience yield (using the spread between adjacent futures contracts, adjusted for interest rates) and the implied excess return on the indices' underlying spot index.[1] Because this 139 year period includes (i) many more episodes of recessions (e.g., 29 versus 7 for BGR (2015)), (ii) large inflationary and deflationary cycles, and (iii) varying times of backwardation and contango, we have greater power to identify the behavior of the returns on the commodity index. Since futures require no up-front capital, one can view the returns on the commodity indices as excess returns. We therefore compare their statistical properties over the period with excess returns on stocks and bonds in the U.S.

[1]The indices include both an equal-weighted and equal risk-weighted portfolio of the available commodity futures. During the earlier part of the sample, there are fewer commodities available for inclusion in the index. For example, prior to 1945, seven commodities enter the sample. Furthermore, EH question the application of equal-weighted indices compared to other weighting schemes. These issues are discussed in Section II.

We provide long-run evidence that both components of commodity futures returns are positive: the excess-of-cash spot returns and the convenience yield. Following the practitioner literature, we denote the returns associated with the convenience yield as the interest rate adjusted carry yield. These positive returns exist both in early and later subperiods of the last 139 years. For example, breaking the sample into the period 1877-1945 and 1946-2015, average excess spot returns on an equal risk-weighted index are 2.4% and 1.6%, respectively, while the average adjusted carry yield on the same index is 2.8% and 4.4%.

The paper makes several additional contributions to the literature, all exploiting the construction of returns on a long sample of commodity futures. First, we describe the statistical behavior of commodity index returns conditional on the aforementioned aggregate states of backwardation/contango, inflation, and the economy. We find that commodity indices, such as the equal risk-weighted one, earn 7.9% versus 1.5% in periods of backwardation versus contango; 14.1% versus -4.5% in high inflation versus low inflation cycles; and 8.9% versus -7.3% in expansions versus recessions.

Second, from an asset allocation perspective, commodity futures add value to a portfolio of stocks and long-term government bonds because of their low correlation to these assets.[2] Unlike these other assets, they offer a hedge against inflation and idiosyncratic performance during backwardated periods. That said, commodities have negative exposure to market downturns and periods of low inflation. In simple asset allocation exercises of the three asset classes, we compare equal-weighted and ex post optimal portfolios over the entire 1877-2015 sample, 1877-1945 and 1946-2015 subsamples, and across the different aggregate states. For example, we find that, over the entire sample, the optimal mean-variance portfolio allocates 17% to commodities, 29% to stocks and 54% to government bonds. Moreover, a 54%/36%/10% portfolio of stocks, bonds and commodities consistently outperforms a 60%/40% allocation of just stocks and bonds.

Third, in terms of the aforementioned debate on the nature of commodity futures, this paper is generally supportive of the GR and BGR point of view. That is, commodity

[2] EH make the valid point (and provide some evidence) that the size of the commodity futures market is small relative to the size of equity and bond markets, and thus suggest caution when interpreting asset allocation exercises economy wide. In section IV, we provide some perspective regarding this argument.

futures are exposed to aggregate states with a large source of their return variation explained by commodity spot returns. The evidence does not, however, support equity-like returns for commodities. Consistent with EH's conclusion, it is interesting to note that, over the 139 year period, adjusted carry yields (roll yields plus short rates) are a larger contributor to average futures returns than are excess-of-cash spot returns, much of it coming from short rates.

This paper is organized as follows. In section 2, we provide the underlying theory for the sources of risk premia for commodity futures returns. In this context, we also describe the data on the various commodity futures from 1877-2015, including a number of interesting stylized facts. In Section 3, we construct commodity indices, based on equal-weights and equal risk-weights. Their statistical properties are explored and compared in different aggregate states of nature, including a comparison between the returns on our commodity futures indices, the U.S. aggregate stock index, and U.S. government bonds. Section 4 interprets these findings in terms of asset allocation. Section 5 concludes.

II. Data Description and Analysis

A. Theory

The starting point for models of commodity futures prices is the cost-of-carry model:[3]

$$F_{t,T} = S_t e^{(r_{t,T}+u_{t,T}-c_{t,T})(T-t)} \quad (1)$$

where S_t is the time t spot price of the commodity, $F_{t,T}$ is the futures price with maturity T, $r_{t,T}$ is the continuously-compounded riskless rate of interest from t to T, $u_{t,T}$ is the annualized storage costs as a percent of the spot price from t to T, and $c_{t,T}$ is the convenience yield over the same time period. Intuitively, the futures price reflects both the foregone interest and cost of storage of holding the commodity. These costs are potentially offset by the convenience yield c. The convenience yield arises due to potential shortages of the underlying commodity, and has been interpreted in the context of one of two theories: (i) Theory of Storage (Kaldor (1939), Working (1949) and Gorton, Hayashi and

[3] The equation assumes nonstochastic, deterministic interest rates, storage costs and convenience yields.

Rouwenhorst (2013)), and (ii) Theory of Normal Backwardation (Keynes (1930), Hicks (1939), and Gorton, Hayashi and Rouwenhorst (2013)).

Define $r_{t,t+1}^{F_T} \equiv \ln\left(F_{t+1,T} \big/ F_{t,T} \right)$, the continuously compounded return on a futures contract with maturity *T*; and $\psi_{t,T}^{*} \equiv (u_{t+1,T} - c_{t+1,T})(T-t-1) - (u_{t,T} - c_{t,T})(T-t) \approx c_{t,T} - u_{t,T}$, the convenience yield net of storage costs (assuming any change in convenience yields or storage costs are small relative to their level from *t* to *t+1*). Further define the continuously-compounded, excess return on the spot return of the commodity, $er_{t,t+1}^{S}$, as the difference between the continuously compounded spot return on the commodity, $r_{t,t+1}^{S} \equiv \ln\left(S_{t+1} \big/ S_{t} \right)$, and the continuously-compounded interest rate, $r_{t,T}^{*} \equiv r_{t+1,T}(T-t-1) - r_{t,T}(T-t) \approx r_{t,T}$ (assuming any changes in interest rates are small relative to the level of interest rates). That is, $er_{t,t+1}^{S} \equiv r_{t,t+1}^{S} - r_{t:T}^{*}$. By taking logs and rearranging, equation (1) can then be rewritten in terms of commodity futures returns:

$$r_{t,t+1}^{F_T} = er_{t,t+1}^{S} + \psi_{t,T}^{*} \qquad (2)$$

Note that, in some form or another in the prior literature, $\psi_{t,T}^{*}$ has been a focus for study. For example, practitioner researchers often report the carry of the commodity, defined by $\psi_{t,T}^{*} - r_{t:T}^{*}$, which ex post is equivalent to the roll yield of the commodity when futures contracts are rolled into the next contract as it reaches expiration (see, for example, Erb and Harvey (2006)). By adding back $r_{t:T}^{*}$ to the commodity's carry, we get back $\psi_{t,T}^{*}$. To coincide with practitioner terminology, we therefore call $\psi_{t,T}^{*}$ the interest rate adjusted carry of the commodity. We use this decomposition of returns because we find it more intuitive to focus net convenience yield and the spot returns in excess of cash instead of decomposing futures returns into spot returns and roll returns. However, we also report the latter decomposition for a few key results, and include summary statistics on the short rate of interest to show its size and impact on the decomposition.

Equation (2) provides a description of futures returns for each individual commodity in terms of their excess spot return and interest-rate adjusted carry. We also wish to consider portfolios of commodities. However, a weighted sum of the individual commodities does not aggregate in log terms, i.e., the sum of geometric returns of the individual assets does not itself equal a geometric return of the asset portfolio. It is therefore convenient to approximate equation (2) in terms of arithmetic returns which do aggregate well into portfolios. Specifically, using a first-order Taylor series approximation,[4]

$$R_{t,t+1}^{F_T} \approx eR_{t,t+1}^{S} + \psi_{t,T}^{*} + R_{t,t+1}^{S}\psi_{t,T}^{*} \tag{3}$$

where $R_{t,t+1}^{F_T}$, $R_{t,t+1}^{s}$ and $eR_{t,t+1}^{s}$ are all written in arithmetic returns.

It is well-documented that spot returns on commodities tend to be negatively correlated with the convenience yield. Hence, the returns on futures returns may be overestimated using simply the sum of excess spot returns and the convenience yield. This point aside, the goal of this paper is to better understand the properties of commodity futures returns as described by equations (2) and (3) using a very long time-series of data. Consider equation (2). The breakdown of futures returns into two components - (i) commodity spot excess returns, and (ii) an interest rate adjusted carry – is a useful way to start the analysis.

This simple futures math has clear implications: First, because the futures return is long the commodity and short the cash market, i.e., $er_{t,t+1}^{S} \equiv r_{t,t+1}^{S} - r_{t:T}^{*}$, it is immediately clear that futures represents an excess return. Because interest rates reflect expected inflation rates, and it is reasonable to assume that changes in commodity prices also reflect inflation expectations, equation (2) implies that expected inflation is netted out of futures returns. Therefore, futures' direct exposure to inflation is only through unexpected inflation shocks. To the extent these inflation shocks are likely correlated with commodity spot returns, it is transparent why commodity futures provide a hedge against inflation.

[4] The approximation can be derived as follows with the third step resulting from a first-order Taylor approximation:

$$\begin{aligned}
1 + R_{t,t+1}^{F_T} &= \frac{1 + R_{t,t+1}^{S}}{1 + R_{t:T}^{*}}\left(1 + \psi_{t,T}^{*}\right) \\
&= \frac{1 + R_{t,t+1}^{S}}{1 + R_{t:T}^{*}} + \frac{\psi_{t,T}^{*}}{1 + R_{t:T}^{*}} + \frac{R_{t,t+1}^{S}\psi_{t,T}^{*}}{1 + R_{t:T}^{*}} \\
&\approx 1 + (R_{t,t+1}^{S} + \psi_{t,T}^{*} + R_{t,t+1}^{S}\psi_{t,T}^{*}) - (R_{t:T}^{*}) \\
&\approx 1 + \left(R_{t,t+1}^{S} - R_{t:T}^{*}\right) + \psi_{t,T}^{*} + R_{t,t+1}^{S}\psi_{t,T}^{*}
\end{aligned}$$

Second, equation (2) highlights the potential risk premia associated with each source of futures returns. Specifically, the excess spot return suggests a premium corresponding to the systematic risk (if any) of the spot commodity, while the interest rate adjusted carry points to compensation for bearing inventory risk and/or providing liquidity to hedgers. This decomposition is consistent with the existing theoretical literature on futures risk premia. For example, futures risk premia are related to systematic risk by Dusak (1973) and Jagannathan (1985), among others, and to hedging pressure by Keynes (1930), Hicks (1939), Basu and Miffre (2013), Gorton, Hayashi and Rouwenhorst (2013), and, Syzmanowska, De Roon, Nijman and Goorbergh (2014), among others. Stoll (1979), Bessembinder (1992) and Hirshleifer, and de Roon, Nijman and Veld (2000) have theoretically incorporated and empirically tested models including both these sources for risk premia.[5]

In terms of the interest rate adjusted carry component, the literature has related hedging pressure and inventories to the convenience yield. The convenience yield depends negatively on storage costs and tends to be positive during periods of low inventories (Kaldor (1939) and Working (1949)), resulting in a high demand for hedging (Keynes (1930), Hicks (1939), and Cootner (1960)). The belief in this latter risk premium is controversial due to the possibility that it may be idiosyncratic in nature; nevertheless, there is empirical evidence of its existence and it can be motivated via segmented markets. Of course, since it is impractical for futures market investors to invest in one of these return sources without getting exposure to the other, this return decomposition is merely suggestive, not probative.

B. Data

In 1865, the Chicago Board of Trade (CBOT) developed the first standardized futures contracts on grain trading. Daily high and low prices for multiple contract expirations on several commodity futures are available starting January 2nd, 1877. The source for the data until 1951 is the Annual Report of the Trade and Commerce of the Chicago Board of Trade. After 1951 and before 2012, the futures prices across various

[5] Acharya, Lochstoer and Ramadorai (2013) and Hamilton and Wu (2014) discuss hedging demand using a limits to arbitrage argument.

contracts are taken from the data vendor, Commodity Systems Inc. After 2012, the futures prices are from Bloomberg. For base metals and platinum, rolled return series from S&P, Goldman Sachs, and Bloomberg are used.

Table 1 provides a description of when the data become available for each commodity, the number of contracts, and some facts with respect to the quality of the price data. The initial data period starting in 1877 includes corn, wheat, oats, lard, short ribs and pork, with cotton, soybeans, soymeal, and soyoil added at various points during the 1920s-50s. Other commodities come into the data set in various years up to the 1990s.[6] Most of the commodities data continues to December 2015. A popular measure of liquidity, namely the percentage of days of zero moves, is consistent across the commodities, irrespective of their starting date. In fact, while breaking the sample up into the 1877-1945 versus 1946-2015 time periods shows more missing data in the early period, the percentage of zero day moves is actually greater in the latter period. This is consistent with Hieronymus (1977, p.23) who shows that the nineteenth century was a period of high trading activity in commodity futures, not surpassed until the mid 1970s at least with respect to commodities such as grains, which have traded throughout the period.

We construct the annualized, arithmetic returns on each commodity futures using the following procedure.[7] At each month end, we calculate the return on each contract from the previous month end. For each month, we hold the nearest of the contracts whose delivery month is at least two months away[8]. The returns on the contracts held are spliced together on the roll dates. There are months in which the desired contract may not have a return. In those instances, we move to the next contract and follow the same procedure until there is a return or until we reach the fifth contract. If there is still no return, we then hold the contract in front of the desired contract. Using the same rolled contract series, we

[6] Opening and closing prices were not recorded in the early part of the sample, so the analysis uses high and low values, and, in particular, the average of these high and low values for the daily return series before closing price is available.
[7] We also investigate geometric returns. We do this to coincide with the existing literature such as Ibbotson and Chen (2003), Gorton and Rowenhorst (2006) and Erb and Harvey (2006).
[8] For example, we hold an April contract through the end of February. An exception is Brent oil, whose delivery month needs to be at least three months away, i.e. we hold the April contract through the end of January. This methodology is chosen to coincide with the procedure employed by the popular Goldman Sachs Commodity Index. That said, we also ran the results using commodity returns generated from the nearest to deliver contract taken from the previous month end. As the contract approaches maturity, we roll into the next nearby contract. The results are qualitatively similar.

construct a rolled price series and calculate the spot returns. Roll yield is then calculated from rolled return series and spot returns. Note that there are days with limit moves on various grains contracts, and we assume all contiguous limit moves are incorporated into the first move price.[9]

Table 2 documents the arithmetic mean and volatility for each futures return on the commodities over the full and two subsample periods (1877-1945 and 1946-2015). Table 2 also reports a breakdown of the commodity futures return into two components: (i) its excess spot return, and (ii) its interest rate adjusted carry. Because we have access to only high and low prices during the early part of the sample, for that period we use the range volatility estimator of Parkinson (1980), i.e., $\hat{\sigma}^2_{t,range} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{4\ln 2}\left(\ln\left(\frac{H_{t-i}}{L_{t-i}}\right)\right)^2$, where H and L represent the daily high and low prices, and N the number of days during the period. Having access to just high and low prices is not necessarily a drawback for volatility estimation. It is well-known that, under certain assumptions, the range volatility estimator is more efficient than using a corresponding sum of daily squared returns (e.g., Beckers (1983) and Alizadeh, Brandt and Diebold (2002)). Unfortunately, without intraday data, no such measure exists for a range correlation estimator (see Brandt and Diebold (2003)) which is discussed in the next section on index construction.

Several observations are in order. First, across 41 commodity series spread across the two subsamples, only 7 series have negative mean returns, 4 negative excess spot returns and 8 negative interest rate adjusted carry (roll yield plus short rate).[10] Second, in the full sample, of the 35 commodities represented, 21 of these commodities have higher average interest rate adjusted carry than excess spot returns. In terms of the debate between

[9] For limit day periods, we incorporate all the limit day returns into the first limit day following Roll (1984) and Boudoukh, Richardson, Shen and Whitelaw (2007). Limit days are determined by whether on that day (i) the maximum price shift across contracts of the same commodity is a round amount (before closing prices are available, the largest positive shift from high price and the largest negative shift from low price are used) (ii) two or more contracts move by this amount, (iii) if maximum price shift is from the front contract and does not meet above conditions, maximum shift of the other contracts meets above conditions (since sometimes the front contract is not subject to limits if it is considered "spot"), and (iv) this shift is equal to or higher than the official price limit set by the exchange (when available).

[10] As pointed out by GR (2006) and EH (2006), there are substantive differences between arithmetic and geometric means of commodity returns due to the high volatility of individual commodity returns. If we instead consider geometric means, then of the 41 commodity series across the two subsamples, 16 series have negative mean returns, almost all (i.e., 35) have negative excess spot returns and 8 negative adjusted convenience yields (roll yield plus short rate). Of course, the reason excess spot returns are mostly negative relative to convenience yields are the much higher volatility of the series.

GR/BGR and EH, this result is a mixed bag. On the one hand, at least for arithmetic returns, excess spot returns are a significant portion of realized returns for a subset of commodity futures. On the other hand, consistent with EH, "income," at least as measured here by convenience yields, is a key component of pricing.

Third, because the expected return component arising from spot returns is small, EH argue that spot returns are not an important component of commodity futures return and therefore commodity futures is not "a play on commodity prices". However, in describing the relation between commodity futures and spot returns, what matters is their covariation. Table 2 describes the monthly return volatility of commodity futures returns and their underlying components. Comparing the volatility of the excess spot return to the volatility of the interest rate adjusted carry (i.e., "income") component, it is clear that the driver of most of the variability is spot prices. In fact, for all commodities, either full sample or for subsamples, spot returns explain much more of the futures return variability than do interest rate adjusted carry yields. This result contrasts with EH who argue that spot returns are not a driver of futures returns at various horizon intervals. That said, EH's primary focus is on long-term holding periods, in particular, 10 years. They readily admit their analysis is subject to limited nonoverlapping observations. One of the benefits of a long sample period is that long-horizon statistics can be more efficiently estimated. Not shown in Table 2, our result that spot return variability is the majority driver of futures returns holds at all horizons measured up to 10 years over the full sample period. Using the index returns, this issue is explored in detail in the next section. Finally, there is considerable variation in the cross-section of mean returns. This suggests that commodity variation cannot be solely explained by a common component, such as short rate volatility. This result is strengthened by the especially long sample for some commodities.

Finally, Table 2 reports the average arithmetic and geometric mean across the commodities for the full and subsample periods. There are large differences between arithmetic and geometric mean returns for individual commodities due to their high level of return volatility. For example, in the full sample, the average arithmetic mean return is 5.0%, 2.9% and 2.8% for futures, excess spot returns, and interest rate adjusted carry yields, respectively. In contrast, the average geometric mean return is 1.1%, -1.4% and 2.5% for futures, excess spot returns, and interest rate adjusted carry yields, respectively.

Most of the difference between arithmetic and geometric mean returns can be explained by the fact that spot moves drive most of the futures returns variation. A significant factor, however, is also the negative correlation between spot prices and interest rate adjusted carry yields as illustrated by equations (2) and (3). That said, as shown in Section 3 below, these effects are mitigated when aggregated to the portfolio level, where the volatility is lower. (This effect, which only applies to geometric returns, has sometimes been called the "rebalancing premium".) A final comment is the similarity in results across subsamples. For example, for arithmetic mean returns, the average return across commodities is 5.0% for the full sample, 5.4% for the 1877-1945 subsample, and 4.7% for the 1946-2015 subsample.

C. Data on Macro Shocks and Aggregate Asset Classes

There is considerable interest in documenting the statistical properties of commodity indices, especially as they relate to other aggregate factors. Part of the motivation is due to the "financialization" of commodities and the increasing awareness of commodities as an asset class. As mentioned in Section II.A above, the long time series available for study in this paper makes it possible to address the long-term properties of commodities futures to coincide with an extensive literature in finance that has done similar long-horizon research on other asset classes. (See, for example, Siegel (1992), Boudoukh and Richardson (1993), Goetzmann (1993), Goetzmann, Li and Rouwenhorst (1995), Goetzmann and Jorion (1999), Hurst, Ooi and Pedersen (2012), and, more recently, Geczy and Samanov (2015).)[11]

For our analysis, we compare the commodity indices to two other asset classes - long-term U.S. government bonds and the aggregate U.S. stock market. The data is provided by Global Financial Data, and represents total returns on these asset classes. Because futures represent excess returns, these two asset classes are expressed in excess terms using a "safe" short-term rate.[12] The properties of these asset classes, relative to commodities, are discussed in the next section.

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[11] Geczy and Samanov (2015) look at the 200-year plus history of statistical momentum properties of many aggregate assets, including commodities. The authors employ spot prices of 75 commodities using Global Financial Data. They do not, however, analyze investable futures prices on commodities, whose returns can differ substantially from spot returns.

[12] The U.S. government only started issuing short-term bills after 1929. The series prior to 1918 is taken from Federal Reserve Economic Data (FRED) at the St. Louis Federal Reserve Bank. The data represents New York call money rates until 1889, and the New York Times money rates until 1918. After 1918, we use Global Financial Data;

One of the goals of this paper is to better understand the means, volatilities and correlations of the various asset classes in different states of nature. We focus on three particular state variables:

- The first variable is an ex ante measure of the backwardation or contango of the commodity futures. Backwardation is defined as $F_{t,T2} < F_{t,T1}$, where $T2>T1$, where F is defined by equation (1). Contango is the opposite. As described above, there is a long literature that focuses on conditions under which a downward sloping commodity futures curve may exist. By definition, the estimated convenience yield is a function of this backwardation and theoretically reflects low inventories, high volatility of the commodity return, and increased hedging demand. We define the state as $\frac{1}{N}\sum_{i=1}^{N}(F_{it,T1} - F_{it,T2})/((T2 - \mathrm{T1}) * F_{it,T1})$ in other words, the average level of backwardation across the various commodity series.
- The second variable is the expansion and recession periods estimated by the NBER Business Cycle Dating Committee. One of the advantages of the long time series is that there have been 29 recessions (i.e., peak to trough) in the U.S. since 1877, compared to 7 since 1965 (the starting point for GR (2006) analysis) and just 3 since 1985 (another common start period). We measure recessions (peak-to-trough) and expansions (trough-to-peak) by the binary variable -1/1.
- The third variable is whether the inflation rate lies above or below its full sample mean, which is an annualized 2.3%. Post 1913, the monthly inflation rate is calculated from the U.S. consumer price index (CPI) published by the U.S. Bureau of Labor Statistics. Prior to 1913, the data comes from Shiller (2000) who uses the Warren and Pearson (1935) price index. Inflation is a particularly important state because of the well-known divergent comovements between inflation and various asset classes over short- and long-horizons (e.g., Boudoukh and Richardson (1993)).

Table 3 provides summary statistics for the three states, in particular, (i) the number of months in the state, (ii) the level of that state, (iii) the average length of a continuous episode of the state, and (iv) the continuation probability from a given period's state to the next. For example, the backwardation state occurs 47% of the time with an average value of 90 basis points. When a backwardation state occurs, its average length until turning into a contango state is 4.4 months. In contrast, recessions occur 28% of the time with an average length of 15.9 months. Note that we define expansions/recessions as 1/-1 and do not measure their severity. Finally, inflation is fairly symmetric with 48% of the time inflation lying above its long-run mean with an expected demeaned value of 0.7% per month (i.e.,

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the data represents secondary market rates on the shortest term US bonds available until 1931 and T-bills thereafter. A rolling one-year average of the short-term rate is used.

0.9% per month if not demeaned). Surprisingly, the average time inflation stays above its long-run mean is just 2.9 months, since we are using raw unsmoothed CPI, which is noisy.

One of the primary advantages of commodities over other asset classes is their hedging benefits against inflation at short horizons. Moreover, commodities are potentially sensitive to aggregate economic shocks as well as periods of low inventories and hedging demands. This paper allows us to investigate these issues due to the particularly long horizon which cover many business cycles, inflationary, and backwardated periods.

III. Commodity Index Returns and Aggregate Shocks

Since returns on various commodity futures are not highly correlated over the sample period, there are benefits to diversification. Following the literature (e.g., Bodie and Rosansky (1980), GR (2006), EH (2006), and Rallis, Miffre and Fuertes (2013), among others), we construct two indices on commodity futures using monthly rebalancing procedures:

- An equally-weighted portfolio of the available commodity futures. At the beginning of each rebalancing period, the portfolios holds $1/N$ of the N commodities.
- An equal risk-weighted portfolio of the available commodity futures. At the beginning of the monthly rebalancing period, each commodity's holding is weighted by $1/\sigma_i$, where σ_i in the earlier period is the range volatility estimator of commodity i's futures returns using daily data (defined in Section II.B), and in the later period is the realized daily returns volatility. In both cases the past year's volatility is used. Assuming equal correlation across the commodities, this approach weights each commodity by their contribution to portfolio variance.

EH suggest caution in interpreting equal-weighted commodity return indices. Their argument is based on the fact that equal-weighted commodity indices are not investable at large size because too much weight is placed on fringe commodities. Indeed, the success of the S&P Goldman Sachs Commodity Index (S&P GSCI) is likely due to the index comprising and weighting commodities by their worldwide production and liquidity in the futures markets. While this point is well taken, it does not follow that equal-weighted commodity return indices are necessarily inflating return premia relative to ones based on production-based weighting schemes. In equity markets, equal-weighted indices place extra weight on small stocks, which likely demand excess return premia. There is no similarly documented result for commodities. Indeed, a perusal of Table 2 shows that 9 of 16 commodities with less than 1% dollar weight in the S&P GSCI has lower mean returns over the sample than the equal-weighted index return.[13]

This point aside, for the majority of the paper, the use of equal-weighted indices is meant to capture the average return across commodities at any given point in time so that we can relate commodity returns to the aggregate states of nature described in section II.C. In particular, our focus is less on whether an index is investable and more about describing the risk/return properties of the average commodity. Nevertheless, we return to the investability issue when we discuss asset allocation in section IV below.

Figure 1A-B graphs the cumulative returns on each commodity index, including a breakdown of the return coming from the computed spot return and the realized roll yield (e.g., carry). The figures show the cumulative returns broken down by (i) excess spot returns and (ii) interest-rate adjusted carry (roll yield plus the short rate), as well as broken down by (i) spot returns and (ii) roll yield (with no short rate adjustment). The graph covers the sample period 1877-2015. Several observations are in order. First, the graphs for the returns on the equal-weighted and equal risk-weighted indices are qualitatively similar. Second, there are many periods of large swings in the commodity return index. Even though index returns are on average positive, long periods of flat or negative returns are not uncommon.

Third, the breakdown of the returns on the commodity index depends very much on the allocation of the short rate.[14] It is quite common to report the carry of the commodity, defined by $\psi^*_{t,T} - r^*_{t:T}$ in equation (2). This is ex post equivalent to the roll yield of the commodity when futures contracts are rolled into the next contract as it reaches expiration. This metric, on average, has earned a little less than zero over the past 139 years. If the returns are decomposed into spot returns and carry, most of the returns come from spot.

[13] Of course, some historical commodities like lard, oats, pork and short ribs, and less liquid ones like platinum and tin are less than 1% by construction as they are not represented in the GSCI.

[14] One concern might be that historical series on the short rate, especially pre U.S. T-bills in 1929, may not represent the risk-free rate either in mean or volatility terms. Thus, any breakdown of spot returns and roll yields into the mathematically equivalent excess spot return and interest rate-adjusted carry (roll yield plus short rate) may exaggerate the role of short rates. This point is addressed in Table 4A and shown not to be the source of futures return variation.

Subtracting cash rates from spot returns and adding it to carry leads to the decomposition described in equations (2) and (3). Excess-of-cash spot returns have been positive, albeit with decade long periods of negative drifts. Of course, futures returns have performed reasonably well because interest-rate adjusted carry, defined as the convenience yield, $\psi^{*}_{t,T}$, has grown with the short rate of interest.

A. Spot Versus Carry

In order to provide precision for Figure 1, Table 4A reports the mean futures return, breakdown between the excess spot return and interest rate adjusted carry yield, volatility of the returns on the various commodity indices and similar statistics for the short rate of interest. To coincide with GR (2006)/BGR (2015) and EH (2006, 2015) results are also reported for geometric means. Because no high or low value is available for the index return on any given day, the return volatility of the index series is calculated using monthly returns.

Throughout this section and the next, we will report results for the equal-weighted index only, as results are nearly identical for the equal risk weighted index. Results for both are included in the tables. In the full sample, the arithmetic mean returns are 4.8% for the equal-weighted index, with a volatility of 17.6%. In terms of the breakdown into excess spot returns versus the interest-rate adjusted carry, the excess spot return contributes 2.1% versus a higher 3.9% from adjusted carry. Note that the final row of Table 4A also reports the mean and volatility of the measured short-term interest rate over the full sample and subperiods. For example, over the entire period, the mean of the short rate is 3.6% with 0.7% volatility. The statistics on the short rate highlight the above discussion of Figure 1 with respect to interest-rate adjusted carry and roll yields. Importantly, the volatility of the short rate is seemingly unimportant for explaining variation of commodity futures returns.

As documented earlier for the volatility of individual commodities, spot return volatility is 18.2% versus adjusted carry volatility of 5.5%. The higher volatility of the spot return supports the contention that commodity futures are fundamentally about commodity prices, not just income or carry. To make sure that this is not just a short horizon phenomenon, we also looked at horizons up to 10 years, which is possible given the long data sample. The spot return volatility component of the index of futures returns also dominates at these longer horizons, though interest rate adjusted carry does play a bigger role as the horizon increases. These results are reported in Table 4B. For example, compared to the volatility of futures returns of 17.6%, 22.0%, 22.3% and 22.9% at 1-month, 1-year, 5-years and 10-years, respectively, the volatility of excess spot returns is 18.2%, 21.2%, 18.4% and 17.0% versus 5.5%, 8.7%, 13.2% and 15.3% respectively for interest rate adjusted carry. As implied by the low volatility of the short-term interest rate, these numbers barely change if we instead decompose futures returns into spot returns and roll yields.

Most interesting, however, is the relative performance of the index across the two subsample periods, 1876-1945 and 1946-2015. The performance is lower in the former subsample, that is, 3.8% versus 5.8%. The average excess spot returns are similar between the two periods, and the performance differences are driven primarily by differences in the adjusted carry yields, namely 3.2% versus 4.5%. The volatility of the index return is much higher in the first period, 20.8%, versus 13.9%. It should be noted, however, that the composition of the index changes through time, that is, the early period is mostly made up of agricultural commodities and a few meats. The results of Table 2 suggest similar volatility for the agricultural commodities in the two samples. A likely explanation for the difference in index volatility across the two subsamples is the greater diversification during the latter sample as more and more commodities join the index. To confirm this, Table 4A documents the full sample and subsample results for a portfolio made up solely of grain commodities. The commodities grain index has much more similar mean and volatility return properties across the periods. For example, the mean and volatility of the return on the equal-weighted index of grains is respectively 4.8% and 24.3% in the 1877-1945 period compared to 4.1% and 20.1% in the 1946-2015 period.

The above summary statistics for the index and its spot and carry components have been represented in terms of arithmetic means. It is also common to report the geometric mean of the series. Note that because the log portfolio return is not a weighted sum of the log individual commodity returns, equation (2) no longer strictly holds. Like the arithmetic mean, the performance of the index is lower in the earlier subsample, that is, 1.7% versus 4.9%. These differences are partially due to the higher volatility of the index in the early sample as a result of the portfolio not being diversified. Indeed, the performance is more similar when comparing the commodity grains indices, 1.9% in 1877-1945 versus 2.2% in 1946-2015.

For comparison purposes, Table 4C provides the arithmetic mean excess return and volatility of government bonds and aggregate stocks over the full sample and two subsample periods. Consistent with many other studies, Table 4C displays the equity premium puzzle. Across the samples, equity returns earn on average 6.7%, 6.1% and 7.4% with corresponding volatility 17.0%, 19.3% and 14.4%. Similarly, long-term government bonds earn excess returns, with means of 1.1%, 0.5% and 1.7% and volatility 5.5%, 3.1% and 7.1% across 1877-2015, 1877-1945 and 1946-2015, respectively. Of course, similar to commodities, when written in terms of geometric means, the excess return on U.S. stocks drops to 5.4%, 4.3% and 6.5% respectively for the 1877-2015, 1877-1945 and 1946-2015 periods.

B. Macro Performance of the Commodity Indices

Section III.A presents behavior of the commodity index in different periods across the entire sample. The various sample periods of course encounter different levels of business cycle effects, inflationary episodes and commodity backwardation periods. It seems worthwhile therefore to document the performance of the index conditional on these macroeconomic events.

Table 5 reports the mean and volatility of the returns on the commodity index for six states: (i) backwardation, (ii) contango, (iii) peak-to-trough (recession), (iv) trough-to-peak (expansion), (v) inflation (above its mean), and (vi) inflation (below its mean). For each of these states, we also report the β of the index against the aggregate U.S. stock market and U.S. government bonds. In brief, the table confirms that, over the 1877-2015 sample period, backwardation (as represented by the shape of the futures curve), inflation and overall state of the economy are all sources of variation in expected returns.

Consider first backwardated versus contango states. As in the previous section, because the two indices are quite similar, we just discuss the equal-weighted index findings. In periods with backwardation, the commodity index returns are higher (7.7% versus 2.1%) with marginally lower betas (0.19 and -0.29 versus 0.28 and -0.20 against stocks and bonds, respectively). While there is slightly higher volatility (18.0% versus 17.3%), the driving force behind these results are much higher interest rate adjusted carry yields (12.2% versus -3.6%). This may not be a surprise to the practitioner community. When the commodity contracts are in backwardation, the ex post carry yield does turn out to be higher. The table illustrates an additional phenomena, namely that the excess spot returns are lower in backwardation than in contango (i.e., -2.9% versus 6.5% for spot returns in backwardation versus contango). This is the well-documented relation between the mean reversion of spot prices and shape of the commodity futures curve (e.g., Bessembinder (1992)). The novel finding is that the result holds going back to the 1870s.

With respect to inflation, the differential in mean returns between high and low inflation states is dramatic, 14.8% versus -4.7%. This is especially the case given that the volatility of the index returns is essentially the same in these two states. Over the 139 year period, there are many examples of low and high inflation periods. The table confirms the standard intuition that commodity returns do well (poorly) in high inflation (low inflation) periods. Interestingly, the driver of these returns is spot commodity price returns (10.4% versus -5.8%) with interest rate adjusted carry yields only marginally different (5.5% versus 2.3%). There is a long literature, mostly in fixed income, which argues for the existence of a risk premium associated with inflation risk. The strong performance of commodities in high inflation periods provides one potential clue for why commodities might offer lower returns than stocks and bonds. Investors value the inflation hedge and therefore do not require as much compensation as other asset classes. Commodity index betas also seem to be lower in high inflation states, which provides further diversification value to commodities.

The final macro aggregate is the state of the economy as measured by the NBER dating committee. Here the long sample period is especially valuable because it allows for many more recessions than more recent data allows. The results too are fairly telling and are again contrary to the idea that commodity futures are not driven by the underlying commodity spot price. In an expansion versus a recession, mean returns on the index are 9.4% versus -7.4% with somewhat higher volatility in the recession period. While the interest rate adjusted carry yields are similar irrespective of the economic state (4.3% versus 2.9%), the mean excess spot returns differ sharply, namely 6.3% versus -9.0%. In terms of systematic risk, the equity betas are similar (0.23 in both states) while the bond betas are somewhat different (-0.27 and -0.15). The fact that the equity betas are positive and there is a distinct difference in return performance in expansions and recessions strongly suggests that the sensitivity to economic state is a source of risk premium.

The results from Table 5 support the impact of aggregate effects on the returns of commodity indices. There are two things, however, that obscure these findings. First, there may be correlation between the states of nature that lead to an omitted variables problem in interpreting the results of Table 5. For example, the average inflation rate in recessions versus expansions is -1.27% versus 3.63%**,** respectively. Second, even if the states of nature are uncorrelated, such as backwardation and the business cycle, it may be the case that there is some spurious correlation. For example, while the infamous dust bowl of the 1930s (and ensuing backwardation of grain futures) and the Great Depression were contemporaneous, few historians would argue that the drought was a major factor of the Great Depression. To address these issues, we perform a series of multivariate regressions of the return on the commodity index on inflation, backwardation/contango and the economic state.[15]

These results are reported in Table 6 for monthly returns. Consider again the equal-weighted commodity index return. The coefficients on demeaned values of the business cycle, carry and inflation are all positive and statistically significant with respective t-statistics of 3.08, 1.95 and 10.70 (the equal risk weighted index has similar t-stats, with a higher value of 3.11 on carry). The R-squared is a reasonable 8.0%. The coefficient on demeaned inflation is 1.27 which suggests an almost one-for-one relation with commodity index returns. The coefficient on the business cycle and level of backwardation (i.e., carry) are 0.0503 (i.e. 5.03%) and 0.21 respectively, consistent with the breakdown of mean returns by individual states in Table 5. Note that the positive intercept in Table 6 is partly due to the fact that expansions last much longer than recessions.

Table 6 also extends the current literature on asset returns at long versus short horizons to commodity indices. If futures returns are not autocorrelated, then the benefit of long horizons must derive from the properties of the regressors. Intuitively, in measuring the state of the business or inflationary cycle, monthly data may not be sufficient to capture

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[15] With respect to the economic state, we demean the 1/-1 measure for expansions and recessions. This way, all the state variables are effectively mean zero. This will not impact the coefficients on the economic state but does change the intercept.

the state. Therefore, we also investigate 1-year and 5-year horizons.[16] Gorton and Rouwenhorst (2005) and Erb and Harvey (2006) also perform such an analysis, but are greatly constrained by the number of non-overlapping 5-year periods. Here, because of the 139-year sample, we are able to better tie down the relation between commodity index returns and macro aggregates.

Consistent with the above intuition, at the one-year horizon, the coefficient on business cycles stays at a similar value, 0.0470 (i.e. 4.7%), while the coefficient on inflation increases to 1.76, with a jump in the R-squared to 31%. The drop in the coefficient on carry to 0.12 reflects the fact that the current state of backwardation/contango has less relation to commodity returns in the far future.[17] The large increase in R-squared is interesting. The one-year horizon likely captures the business cycle and inflationary cycle better than the one-month horizon. This point further highlights the benefit of using a long sample. The results using a 5-year horizon confirms this intuition. Only inflation remains a significant state variable for explaining commodity returns, albeit with a healthy R-squared of 37%. The reason business cycles no longer explain commodity index returns is likely because recessions are generally shorter than five years.

C. Return Predictability of the Commodity Indices

Section III.B above ran a contemporaneous regression of the commodity return index on the NBER business cycle, inflation and ex-ante carry. There is also a considerable literature documenting commodity return predictability. In particular, the focus has been on predictive variables for individual commodities based on the investment styles of momentum, value and carry.[18] One of the disadvantages of using investment styles is that there is not a sufficient time series to capture these styles. Two exceptions are Hurst, Ooi

[16] Regressions at 1-year and 5-year horizons (in both Table 6 and Table 8) are on overlapping returns at monthly frequency. A Newey–West estimator is used to calculate t-statistics.
[17] These results are based on carry averaged over the previous month. If we instead use carry averaged over the previous 12 months, for the equal risk-weighted commodity index, the coefficients on the business cycle, carry and inflation are 0.0499, 0.07, and 1.72 respectively, with t-statistics of 2.85, 0.38 and 6.55. The R-squared is 31%. Similar findings for carry hold true at the 5-year horizon.
[18] For commodities, see, for example, GR (2006), EH (2006), Miffre and Rallis (2007), Fuertes, Miffre and Rallis (2010), Moskowitz, Ooi and Pedersen (2012), Asness, Moskowitz and Pedersen (2013), Koijen, Moskowitz, Pedersen and Vrugt (2013), and Miffre and Fernandez-Perez (2015), among others.

and Pedersen (2012), and Geczy and Samanov (2015) who document momentum across many asset classes, including commodities. Nevertheless, measures of carry and value remain elusive. In this section, we combine signals on carry, momentum and value together in order to investigate the predictability of the commodity return index using investment styles over the 1877-2015 sample period.

We run a regression of monthly, annual and 5-year returns on the equal weighted commodity index on measures of carry, momentum and value. To coincide with the existing literature, we measure momentum of the index as its previous 12-month return; value as the negative 48-month return 12-months ago (i.e., long-term reversal); and carry as the backwardated/contango value of the commodity index described previously. The unique aspect of this regression is that it dates back to 1882. Of some note, the long time series allows us almost 30 independent observations on the long-term reversal, our measure of value. The results are reported in Table 7, panel A. Consider first the monthly returns. The predicted R^2 is 2% with all three coefficients on momentum (0.25), value (0.08) and carry (0.28) being of the right sign with t-statistics respectively of 3.31, 1.78 and 2.44. The annual returns produce similar results albeit with smaller (and less significant) coefficients. The R^2, however, is approximately double, at 4%. There is little predictability at the long 5-year horizon.

To test if these results are robust across aggregate states, we consider the predictive regression model of panel A in our six states of nature. Table 7, panel B, provides estimates from a regression of the realized monthly commodity index returns in the six states against the model of expected returns (using carry, momentum and value). The R^2s are similar to those of panel A. Though there is clearly variation in the coefficients, all the coefficients on carry, momentum and value are positive, irrespective of the state of nature. Approximately half of the estimates are statistically significant at conventional levels, despite the smaller sample size by construction.

IV. Using Commodities for Asset Allocation

Table 5 reports βs between the commodity index and the stock and bond markets in different states of nature. These findings suggest that the returns on stocks, commodities and bonds have responded differently to economic shocks over the last 139 years. Given these results, it seems worthwhile performing an asset allocation exercise to better understand how a portfolio of commodities, stocks and bonds might perform in different aggregate economic states.

EH argue that asset allocation exercises like this may be open to misinterpretation. They provide evidence that the size of the commodity futures market is small relative to the size of equity and bond markets. In this sense, from an economy-wide asset allocation perspective, even if the evidence pointed to a large position in commodity futures, the aggregate investor could not hold such a position. The argument suggests that the resulting asset allocation should be interpreted from an individual investor perspective, and not across the economy as a whole.[19]

For the analysis to follow, we consider an individual investor who is considering allocating across an equal-weighted index of commodity futures, the aggregate U.S. stock market and the U.S. government bond market. We make no pretense about the size of such an allocation or whether such allocations were equally feasible today versus 140 years ago. The long sample period is used so we can exploit the wide variation in aggregate economic states. We further assume that the realized mean, volatility and correlation across these asset classes were known ex ante and constant over time. Figure 2A-C graphs the mean-variance investment opportunity set for these assets in the full sample and two subsample periods. We are limiting our analysis to U.S. stocks and U.S. bonds, so the results do not reflect a global portfolio. Even though this period represents a good ex post outcome for U.S. stocks (e.g., Goetzmann and Jorion (1999)), the optimal portfolio still includes considerable holdings in commodities and government bonds.

[19] As an aside, the size of equity markets and government bond markets does not negate the importance of commodities. Presumably, the value of government bonds depends somewhat on the ability of the government to pay down its debt which in turn depends on the resources owned by the government and its tax base. These natural resources especially are tied to commodities. Similarly, a number of equity sectors, such as energy, agribusiness, mining, etc…, represent present values of future commodity payouts. In other words, an investment in long commodity futures can be likened to an equity claim on a company tied to commodity-based assets. Commodity futures are just one, albeit small way, to get exposure to the underlying spot commodity market.

Consider the full sample period, 1877-2015, described by Figure 2A. Several observations are in order. First, investments solely in government bonds, stocks or commodities all lie far from portfolios combining these three assets. While commodity futures lie far from the optimal portfolio, this does not imply commodity futures should not be held in some amount. Second, to this point, the optimal portfolio allocates 17% to commodities, 29% to stocks and 54% to government bonds. While both commodity futures and stocks do poorly in recessions, government bonds do well, providing a hedge. Third, commodity futures are the only assets that perform well during inflationary periods. Moreover, their strong performance during backwardation provides some idiosyncratic return relative to stocks and bonds. Finally, an equal-weighted portfolio across the three asset classes performs fairly close to the optimal portfolio when combined with holdings in short-term bonds. Moreover, a 54%/36%/10% portfolio of stocks, bonds and commodity futures outperforms a 60%/40% allocation of just stocks and bonds.

The two subsamples provide contrasting results. The latter sample implies optimal holdings of 29% to commodity futures, 31% to stocks and 39% to long-term government bonds and therefore is close to an equal-weighted index. In contrast, the early period provides little weight to commodity futures (i.e., 7%), 21% to stocks, and 72% to long-term government bonds. The early sample includes a greater fraction of recessions and massive deflation during large parts of the late 19^{th} century, both of which are not particularly good states for commodities. Because long-term government bonds do well in these periods, their allocation is higher. However, in both subsample periods, a 54%/36%/10% portfolio of stocks, bonds and commodity futures outperforms a 60%/40% allocation of just stocks and bonds

In order to see why commodity futures are important to hold in a portfolio, Table 8A reports the mean, variance, and Sharpe ratio of (a) each individual asset class, (b) an equal-weighted-portfolio of the assets, (c) a 60/40 portfolio consisting of 60% stocks and 40% bonds, (d) a 54/36/10 portfolio consisting of stocks, bonds and commodity futures respectively, and (e) the optimal mean-variance portfolio. These are measured in the full sample, and conditional on the six states: (i) backwardation, (ii) contango, (iii) peak-to-trough (recession), (iv) trough-to-peak (expansion), (v) inflation (above its mean), and (vi) inflation (below its mean). By construction, the optimal mean-variance portfolio has the highest Sharpe ratio unconditionally over the entire period.

First, consider the mean return of the various assets and asset portfolios across the different states of nature. All the assets and portfolios, except long-term government bonds, have negative returns during recessions. This is precisely why long-term governments are so valuable to hold in a portfolio. Commodities tend to do poorly in low inflation periods and recessions in contrast to stocks that perform well most everywhere except recessions. Of course, commodity futures do particularly well in inflationary and backwardated environments, providing a motivation for why commodity futures help the risk/return profile of a portfolio.

Second, volatility is similar across many of the states, with arguably higher levels during recessions. The cross-sectional differences, however, are large. Return volatility for bonds is much less than stock and commodity futures return volatility. This leads to much lower volatility for the portfolio with a high weight in bonds, albeit at much lower mean returns than the other portfolios.

Finally, the pattern in Sharpe ratios of the four portfolios across the different aggregate states is quite telling. In the full sample, the optimal portfolio, equally weighted portfolio, 60/40 portfolio and 54/36/10 portfolio have Sharpe ratios of 0.48, 0.45, 0.42 and 0.45, respectively, showcasing the robustness of the equal weighting. In backwardated states, the equal weighted portfolio performs best with a Sharpe of 0.49 while in contango states the 60/40 stock-bond portfolio does marginally better than the optimal portfolio (with a Sharpe of 0.52 versus 0.51). In high inflation states, a portfolio 100% in commodity futures has the highest Sharpe ratio, i.e., 0.84, though an equal-weighted portfolio has a similar Sharpe ratio, i.e., 0.83. In low inflation states, the opposite is true, with commodity futures earning -0.27. In expansions, the equal-weight and optimal portfolios have a Sharpe ratio of 0.92 yet -0.42 and -0.33, respectively in recessions. The only portfolio to earn a positive Sharpe ratio is one with 100% weight in government bonds with a Sharpe ratio 0.23 in recessions and 0.20 in expansions.

Another way to evaluate these four portfolios is to consider the consistency of their performance through time. Table 8B documents seven independent 20-year periods from 1877 through 2015. In terms of realized Sharpe ratios, the optimal portfolio outperforms in two of the periods, equal-weight in three periods, and 60/40 in two periods. The 54/36/10 portfolio outperforms 60/40 in 4 of the 7 cases, and is only dominated by the optimal portfolio in 3 of the 7 cases.

V. Conclusion

This paper studies the return properties of commodity futures over a uniquely long sample period from 1877-2015. The long sample allows us to better identify aggregate states of nature related to business cycles, inflation episodes, and backwardation/contango periods. We are able to address the recent debate on whether commodities improve asset allocation. We provide evidence that:

- Commodity futures index returns have been positive and significant since 1877. While the performance has come both from the excess spot portion of returns as well as the interest rate adjusted carry portion of returns (i.e., roll yield plus short rate or the convenience yield), the carry component has earned the lion's share.
- Commodity futures returns vary significantly across aggregate states. Most of the variation across these states and through time comes from variation of spot returns and therefore commodity futures are "a play on commodity prices". Moreover, this sensitivity to business cycles and inflation episodes offer support to the existence of risk premiums in commodities markets.
- Commodity index returns are somewhat predictable at 1-month horizons, and less so at 1-year and 5-year horizons using investment styles of carry, value and momentum. Interestingly, these forecasted returns tend to be robust predictors across aggregate states of nature like business cycles and inflation episodes, suggesting investment styles are related to fundamentals.
- Unlike stocks and bonds, commodity futures offer a hedge against inflation and provide idiosyncratic return in backwardated states, thus providing justification for including them in a diversified portfolio of assets as illustrated in a simple asset allocation exercise.

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Table 1
Description of Commodity Futures Data

Table 1 provides a description of commodity futures data availability and summary facts regarding the quality of the price data, including the number of days without data and proportion of days with zero moves (which only includes days on which contract prices of all maturities do not move). The earliest start date is January 1877 for certain commodities, including corn, wheat, oats, lard, and pork.

	Start Date	End Date	Number of Contracts	Longest Period without Data (Days)	% of Days w/o Data	% Days of Zero Moves
ALUMINUM	1992-07-31	2015-12-31	N/A	-	-	0.0%
BRENTOIL	1988-06-23	2015-12-31	415	-	-	2.4%
CATTLE	1964-11-30	2015-12-31	331	-	-	3.4%
COCOA	1965-12-30	2015-12-31	261	-	-	4.2%
COFFEE	1972-08-16	2015-12-31	235	-	-	4.2%
COMEXCOPPER	1988-07-29	2015-12-31	345	-	-	3.8%
COPPER	1993-09-30	2015-12-31	N/A	-	-	0.0%
CORN	1877-01-02	2015-12-31	855	456	4.2%	2.1%
COTTON	1925-01-02	2015-12-31	450	2739	16.9%	3.0%
CRUDE	1983-03-30	2015-12-31	469	-	-	3.8%
FEEDERCATTLE	1971-11-30	2015-12-31	354	-	-	3.4%
GASOIL	1981-04-06	2015-12-31	487	-	-	2.6%
GOLD	1975-01-02	2015-12-31	493	-	-	3.9%
HEATOIL	1978-11-15	2015-12-31	489	-	-	3.9%
HOGS	1966-02-28	2015-12-31	379	-	-	3.5%
KANSASWHEAT	1966-05-16	2015-12-31	262	-	-	3.5%
LARD	1877-01-02	1951-12-31	569	619	10.9%	1.5%
LEAD	1995-02-28	2015-12-31	N/A	-	-	0.0%
LONDONCOCOA	1968-01-02	2015-12-31	255	-	-	3.2%
NATGAS	1990-04-03	2015-12-31	463	-	-	3.8%
NICKEL	1994-10-31	2015-12-31	N/A	-	-	0.0%
NYMEXPLATINUM	1964-01-14	2015-12-31	368	-	-	4.1%
OATS	1877-01-02	2015-12-31	803	13	4.1%	1.6%
PLATINUM	1992-02-28	2015-12-31	N/A	-	-	0.0%
PORK	1877-01-02	1922-03-27	337	116	4.3%	1.4%
SHORTRIBS	1885-01-02	1929-08-14	292	50	11.6%	2.2%
SILVER	1963-06-12	2015-12-31	627	-	-	4.3%
SOYBEANS	1937-01-04	2015-12-31	504	1599	7.7%	3.3%
SOYMEAL	1951-08-29	2015-12-31	532	-	0.0%	3.8%
SOYOIL	1950-07-17	2015-12-31	535	-	-	3.4%
SUGAR	1965-12-30	2015-12-31	266	-	-	4.2%
TIN	1995-02-28	2015-12-31	N/A	-	-	0.0%
UNLEADED	1984-12-03	2015-12-31	420	-	-	3.8%
WHEAT	1877-01-02	2015-12-31	788	1055	4.4%	1.9%
ZINC	1992-10-30	2015-12-31	N/A	-	-	0.0%

Table 2
Individual Commodity Futures Returns: Summary

Table 2 documents the arithmetic mean and volatility for each futures return on the commodities over the full and two subsample periods (1877-1945 and 1946-2015). Table 2 breaks down the commodity futures return into two components: (i) its excess spot return, and (ii) its interest rate adjusted carry (i.e., its roll yield plus the short interest rate). Because only high and low prices are available during the early part of the sample, before we have closing prices we use the range volatility estimator of Parkinson (1980), i.e., $\hat{\sigma}^2_{t,range} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{4\ln 2}\left(\ln\left(\frac{H_{t-i}}{L_{t-i}}\right)\right)^2$, where H and L represent the daily high and low prices, and N the number of days during the period. Cross-section averages for both arithmetic and geometric mean returns are also provided.

	Full sample						1877-1945						1946-2015					
	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility
ALUMINUM	-3.3%	-0.2%	-3.0%	20.1%	19.1%	1.4%							-3.3%	-0.2%	-3.0%	20.1%	19.1%	1.4%
BRENTOIL	9.8%	4.8%	5.1%	31.6%	30.5%	5.3%							9.8%	4.8%	5.1%	31.6%	30.5%	5.3%
CATTLE	5.0%	0.3%	5.4%	15.4%	18.2%	8.4%							5.0%	0.3%	5.4%	15.4%	18.2%	8.4%
COCOA	5.6%	3.8%	2.3%	28.6%	31.2%	7.7%							5.6%	3.8%	2.3%	28.6%	31.2%	7.7%
COFFEE	5.3%	3.9%	2.1%	33.3%	37.7%	9.8%							5.3%	3.9%	2.1%	33.3%	37.7%	9.8%
COMEXCOPPER	9.4%	3.4%	6.1%	26.5%	26.2%	3.8%							9.4%	3.4%	6.1%	26.5%	26.2%	3.8%
COPPER	8.7%	4.7%	4.0%	25.8%	25.3%	2.5%							8.7%	4.7%	4.0%	25.8%	25.3%	2.5%
CORN	2.3%	1.5%	2.1%	19.8%	27.1%	12.2%	5.4%	3.0%	4.4%	19.3%	30.1%	15.3%	-0.6%	0.1%	-0.1%	20.3%	23.9%	8.3%
COTTON	3.1%	0.7%	3.8%	20.2%	26.1%	13.9%	-0.1%	-0.9%	1.1%	17.6%	27.8%	5.5%	3.9%	1.1%	4.4%	20.7%	25.7%	15.3%
CRUDE	7.7%	2.3%	5.7%	33.5%	33.0%	6.8%							7.7%	2.3%	5.7%	33.5%	33.0%	6.8%
FEEDERCATTLE	3.7%	0.1%	3.8%	15.2%	17.3%	5.2%							3.7%	0.1%	3.8%	15.2%	17.3%	5.2%
GASOIL	6.8%	1.0%	6.2%	29.2%	30.9%	6.3%							6.8%	1.0%	6.2%	29.2%	30.9%	6.3%
GOLD	1.0%	1.5%	-0.5%	19.3%	19.4%	0.8%							1.0%	1.5%	-0.5%	19.3%	19.4%	0.8%
HEATOIL	8.0%	3.1%	5.6%	30.1%	32.6%	8.5%							8.0%	3.1%	5.6%	30.1%	32.6%	8.5%
HOGS	3.5%	1.0%	5.8%	23.7%	30.9%	18.2%							3.5%	1.0%	5.8%	23.7%	30.9%	18.2%
KANSASWHEAT	1.4%	0.3%	1.3%	22.8%	25.4%	6.9%							1.4%	0.3%	1.3%	22.8%	25.4%	6.9%
LARD	-1.6%	-0.5%	-0.2%	16.0%	25.7%	9.5%	-1.6%	-0.3%	-0.3%	15.5%	24.5%	9.7%	-2.0%	-3.1%	1.8%	20.8%	38.3%	7.9%
LEAD	6.8%	6.2%	0.5%	30.5%	28.3%	2.2%							6.8%	6.2%	0.5%	30.5%	28.3%	2.2%
LONDONCOCOA	7.0%	4.3%	3.3%	25.7%	31.6%	8.8%							7.0%	4.3%	3.3%	25.7%	31.6%	8.8%
NATGAS	-8.3%	12.4%	-16.2%	45.1%	53.5%	20.3%							-8.3%	12.4%	-16.2%	45.1%	53.5%	20.3%
NICKEL	7.2%	4.8%	2.3%	35.3%	34.8%	2.2%							7.2%	4.8%	2.3%	35.3%	34.8%	2.2%
NYMEXPLATINUM	6.3%	3.1%	3.2%	25.5%	26.9%	3.3%							6.3%	3.1%	3.2%	25.5%	26.9%	3.3%
OATS	2.6%	2.1%	2.1%	22.7%	30.6%	13.8%	4.1%	2.9%	3.6%	20.2%	32.9%	16.6%	1.1%	1.2%	0.6%	24.9%	28.3%	10.2%
PLATINUM	8.1%	3.7%	4.3%	23.2%	22.6%	1.9%							8.1%	3.7%	4.3%	23.2%	22.6%	1.9%
PORK	3.5%	4.2%	1.0%	16.7%	32.0%	14.2%	3.5%	4.2%	1.0%	16.7%	32.0%	14.2%						
SHORTRIBS	9.4%	1.9%	9.4%	14.2%	27.5%	14.7%	9.4%	1.9%	9.4%	14.2%	27.5%	14.7%						
SILVER	3.2%	4.5%	-1.3%	28.3%	31.1%	1.4%							3.2%	4.5%	-1.3%	28.3%	31.1%	1.4%
SOYBEANS	7.8%	1.7%	7.2%	20.8%	26.9%	10.7%	20.0%	9.9%	13.1%	15.8%	32.5%	16.5%	6.7%	1.0%	6.7%	21.1%	26.4%	10.1%
SOYMEAL	9.6%	2.0%	8.3%	24.4%	28.8%	9.0%							9.6%	2.0%	8.3%	24.4%	28.8%	9.0%
SOYOIL	6.6%	1.1%	5.7%	24.2%	28.4%	6.4%							6.6%	1.1%	5.7%	24.2%	28.4%	6.4%
SUGAR	4.9%	7.6%	-1.5%	38.2%	43.2%	11.5%							4.9%	7.6%	-1.5%	38.2%	43.2%	11.5%
TIN	7.0%	4.5%	2.5%	25.8%	24.3%	1.3%							7.0%	4.5%	2.5%	25.8%	24.3%	1.3%
UNLEADED	15.5%	4.9%	11.9%	32.3%	36.4%	11.1%							15.5%	4.9%	11.9%	32.3%	36.4%	11.1%
WHEAT	1.5%	0.5%	1.8%	21.3%	25.5%	10.8%	2.8%	0.9%	3.1%	20.2%	26.7%	12.2%	0.2%	0.2%	0.6%	22.3%	24.3%	9.4%
ZINC	-1.3%	1.5%	-2.7%	27.2%	25.4%	2.0%							-1.3%	1.5%	-2.7%	27.2%	25.4%	2.0%
CS Avg of Arith Mean	5.0%	2.9%	2.8%				5.4%	2.7%	4.4%				4.7%	2.7%	2.6%			
CS Avg of Geom Mean	1.1%	-1.4%	2.5%				2.2%	-1.5%	3.7%				0.7%	-1.6%	2.3%			

Table 3
Description of States of Nature

Table 3 describes six states of nature over the period 1877-2015: (i) average backwardation across commodity futures, (ii) average contango across commodity futures, (iii) high inflation (inflation above its mean), (iv) low inflation (inflation below its mean), (v) expansion (trough-to-peak based on NBER dating), and (vi) recession (i.e., peak-to-trough based on NBER dating). The weighted average backwardation and contango are provided for both an equal dollar weighted and equal risk weighted index as the weights differ across these two portfolios. The inflation values are in excess of the mean inflation over the period. The table provides summary statistics for the states, in particular, (i) the number of months in the state, (ii) the average level of that state, (iii) the average length of a continuous episode of the state, and (iv) the probability of being in a state conditional on having been in that state 1, 6, and 12 months ago.

	Equal Dollar Weighted		Equal Risk Weighted		High Inflation	Low Inflation	Expansion	Recession
	Backwardation	Contango	Backwardation	Contango				
Number of Months	791	876	778	882	808	859	1207	460
Average Level of the State	0.9%	-0.7%	0.8%	-0.6%	0.7%	-0.6%		
Average Length of Continuous Episode (Months)	4.4	4.9	4.1	4.6	2.9	3.1	40.3	15.9
Conditional Probability (1 Month)	77%	80%	75%	78%	66%	68%	98%	94%
Conditional Probability (6 Months)	57%	61%	55%	60%	59%	61%	86%	62%
Conditional Probability (12 Months)	66%	69%	66%	70%	63%	65%	75%	32%

Table 4 - Commodity Index and Aggregate Asset Returns

Table 4A - Commodity Index Returns: Summary

Table 4A reports the mean, breakdown between the excess spot return and adjusted carry yield (i.e., roll yield plus short interest rate), and volatility of the returns on equal dollar- and risk-weighted commodity futures indices, for both arithmetic and geometric means. Standard errors for the means are reported in parenthesis. The table also reports the same metrics for a commodity grain futures index.

	Full Sample						1877-1945						1946-2015					
	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility	Return Mean	Excess Spot Return Mean	Interest Rate Adjusted Carry Mean	Return Volatility	Excess Spot Return Volatility	Interest Rate Adjusted Carry Volatility
Arithmetic Mean:																		
Equal-Weight	4.8% (1.5%)	2.1% (1.5%)	3.9% (0.5%)	17.6%	18.2%	5.5%	3.8% (2.5%)	2.2% (2.6%)	3.2% (0.8%)	20.8%	21.5%	6.8%	5.8% (1.7%)	2.0% (1.7%)	4.5% (0.5%)	13.9%	14.2%	3.8%
Equal Risk-Weight	4.5% (1.4%)	2.0% (1.5%)	3.6% (0.5%)	16.8%	17.3%	5.4%	3.7% (2.4%)	2.4% (2.5%)	2.8% (0.8%)	20.1%	20.7%	6.5%	5.3% (1.5%)	1.6% (1.6%)	4.4% (0.5%)	12.8%	13.1%	3.9%
Arithmetic Mean, Grains Only:																		
Equal-Weight	4.5% (1.9%)	1.7% (2.0%)	4.1% (0.7%)	22.3%	23.1%	7.8%	4.8% (2.9%)	2.5% (3.1%)	4.2% (1.2%)	24.3%	25.6%	9.8%	4.1% (2.4%)	0.9% (2.4%)	3.9% (0.6%)	20.1%	20.4%	5.1%
Equal Risk-Weight	4.4% (1.9%)	1.9% (1.9%)	3.7% (0.6%)	21.9%	22.6%	7.6%	4.7% (2.9%)	2.8% (3.0%)	3.7% (1.2%)	23.9%	25.1%	9.5%	4.0% (2.4%)	1.0% (2.4%)	3.6% (0.6%)	19.7%	19.9%	5.1%
Geometric Mean:																		
Equal-Weight	3.3% (1.5%)	0.4% (1.5%)	3.8% (0.5%)	17.6%	18.2%	5.5%	1.7% (2.5%)	-0.1% (2.6%)	3.0% (0.8%)	20.8%	21.5%	6.8%	4.9% (1.7%)	1.0% (1.7%)	4.6% (0.4%)	13.9%	14.2%	3.8%
Equal Risk-Weight	3.2% (1.4%)	0.5% (1.5%)	3.5% (0.4%)	16.8%	17.3%	5.4%	1.7% (2.4%)	0.3% (2.5%)	2.6% (0.8%)	20.1%	20.7%	6.5%	4.6% (1.5%)	0.7% (1.6%)	4.4% (0.5%)	12.8%	13.1%	3.9%
Geometric Mean, Grains Only:																		
Equal-Weight	2.1% (1.9%)	-0.9% (1.9%)	3.8% (0.7%)	22.3%	23.1%	7.8%	1.9% (2.9%)	-0.7% (3.0%)	3.8% (1.2%)	24.3%	25.6%	9.8%	2.2% (2.4%)	-1.2% (2.4%)	3.8% (0.6%)	20.1%	20.4%	5.1%
Equal Risk-Weight	2.1% (1.8%)	-0.6% (1.9%)	3.4% (0.6%)	21.9%	22.6%	7.6%	2.0% (2.8%)	-0.3% (3.0%)	3.3% (1.1%)	23.9%	25.1%	9.5%	2.2% (2.3%)	-0.9% (2.3%)	3.6% (0.6%)	19.7%	19.9%	5.1%
Short Term Interest Rate (Arithmetic)	3.6% (0.1%)			0.7%			3.2% (0.1%)			0.5%			4.0% (0.1%)			0.8%		

Table 4B – Breakdown of Volatility of Long Horizon Returns

Table 4B provides the volatility of futures return on the commodity indices at horizons of 1-month, 1-year, 5-years and 10-years over the sample period 1877-2015. The volatility is also provided for the various components of futures returns, including the excess spot return (and raw return) of the commodity, along with the interest rate-adjusted carry (and roll yield) over the same period.

	Equal-Weight:				Equal Risk-Weight			
	1-month	1-year	5-year	10-year	1-month	1-year	5-year	10-year
Futures Return	17.6%	22.0%	22.3%	22.9%	16.8%	21.4%	22.2%	23.3%
Excess Spot Return	18.2%	21.2%	18.4%	17.0%	17.3%	20.4%	18.4%	17.6%
Spot Return	18.2%	21.0%	17.5%	15.8%	17.3%	20.2%	17.5%	16.4%
Interest Rate Adjusted Carry	5.5%	8.7%	13.2%	15.3%	5.4%	8.4%	13.2%	15.4%
Roll Yield	5.5%	8.7%	13.1%	15.3%	5.3%	8.4%	12.9%	15.2%

Table 4C - Aggregate Asset Returns

Table 4C provides the arithmetic mean excess return and volatility of government bonds and aggregate stocks over the full sample and two subsample periods. Standard errors are included in parenthesis.

	Full sample		1877-1945		1946-2015	
	Return Mean	Volatility	Return Mean	Volatility	Return Mean	Volatility
U.S. Bonds	1.1%	5.5%	0.5%	3.1%	1.7%	7.1%
	(0.5%)		(0.4%)		(0.8%)	
U.S. Equities	6.7%	17.0%	6.1%	19.3%	7.4%	14.4%
	(1.4%)		(2.3%)		(1.7%)	

Table 5
Commodity Index Returns and Aggregate States

Table 5 reports the mean and volatility of the returns on the commodity futures indices for six states over the period 1877-2015: (i) backwardation, (ii) contango, (iii) high inflation (inflation above its mean), (iv) low inflation (inflation below its mean), (v) expansion (trough-to-peak based on NBER dating), and (vi) recession (peak-to-trough based on NBER dating). The returns on the commodity indices are broken down into two components: (i) its excess spot return, and (ii) its interest rate adjusted carry (i.e., its roll yield plus the short interest rate). For each of these states, the table also reports the β of the index against U.S. government bonds and the aggregate U.S. stock market.

Equal-Weight:

	Full Sample	Backwardation	Contango	High Inflation	Low Inflation	Expansion	Recession
Number of Months	1667	791	876	808	859	1207	460
Return Mean	4.8%	7.7%	2.1%	14.8%	-4.7%	9.4%	-7.4%
	(1.5%)	(2.2%)	(2.0%)	(2.2%)	(2.0%)	(1.7%)	(3.2%)
Excess Spot Return Mean	2.1%	-2.9%	6.5%	10.4%	-5.8%	6.3%	-9.0%
	(1.5%)	(2.2%)	(2.1%)	(2.2%)	(2.1%)	(1.7%)	(3.3%)
Interest Rate Adjusted Carry Mean	3.9%	12.2%	-3.6%	5.5%	2.3%	4.3%	2.9%
	(0.5%)	(0.7%)	(0.5%)	(0.7%)	(0.6%)	(0.5%)	(1.0%)
Return Volatility	17.6%	18.0%	17.3%	17.7%	17.2%	16.6%	19.8%
Beta with Bond	-0.24	-0.29	-0.20	-0.27	-0.13	-0.27	-0.15
Beta with Stock	0.25	0.19	0.28	0.18	0.29	0.23	0.23

Equal Risk-Weight:

	Full Sample	Backwardation	Contango	High Inflation	Low Inflation	Expansion	Recession
Number of Months	1660	778	882	805	855	1207	453
Return Mean	4.5%	7.9%	1.5%	14.1%	-4.5%	8.9%	-7.3%
	(1.4%)	(2.2%)	(1.9%)	(2.0%)	(2.0%)	(1.6%)	(3.1%)
Excess Spot Return Mean	2.0%	-2.1%	5.5%	9.9%	-5.5%	6.0%	-8.7%
	(1.5%)	(2.2%)	(2.0%)	(2.1%)	(2.0%)	(1.6%)	(3.1%)
Interest Rate Adjusted Carry Mean	3.6%	11.3%	-3.2%	5.2%	2.1%	4.0%	2.5%
	(0.5%)	(0.7%)	(0.5%)	(0.7%)	(0.6%)	(0.5%)	(1.0%)
Return Volatility	16.8%	17.4%	16.3%	16.8%	16.5%	15.9%	18.7%
Beta with Bond	-0.23	-0.38	-0.14	-0.26	-0.13	-0.28	-0.12
Beta with Stock	0.25	0.23	0.26	0.18	0.29	0.22	0.25

Table 6
Commodity Index Futures Returns and Aggregate States: Multivariate Analysis

We perform a series of multivariate regressions of the return of the commodity index on a 1/-1 binary variable for the business cycle, average ex ante carry, and contemporaneous inflation. Inflation and business cycle variables are demeaned. The sample period covers 1877-2015. The regressions are run at 1-month, 1-year and 5-year horizons.

Horizon: Monthly

	Business Cycle	Carry	Inflation	Intercept	R-squared
Equal-Weight, Coefficient:	0.0503	0.21	1.27	4.65%	0.08
Equal-Weight, t-Stat:	3.08	1.95	10.70	3.23	
Equal Risk-Weight, Coefficient:	0.0490	0.33	1.19	4.15%	0.08
Equal Risk-Weight, t-Stat:	3.10	3.11	10.21	3.00	

Horizon: One Year

	Business Cycle	Carry	Inflation	Intercept	R-squared
Equal-Weight, Coefficient:	0.0470	0.12	1.76	4.60%	0.31
Equal-Weight, t-Stat:	2.65	1.66	6.75	3.79	
Equal Risk-Weight, Coefficient:	0.0490	0.13	1.72	4.28%	0.31
Equal Risk-Weight, t-Stat:	2.82	1.76	6.65	3.59	

Horizon: Five Years

	Business Cycle	Carry	Inflation	Intercept	R-squared
Equal-Weight, Coefficient:	-0.0011	0.03	1.97	4.45%	0.39
Equal-Weight, t-Stat:	-0.03	0.77	4.08	3.54	
Equal Risk-Weight, Coefficient:	-0.0060	0.03	1.99	4.04%	0.40
Equal Risk-Weight, t-Stat:	-0.17	0.72	4.20	3.26	

Table 7
Commodity Index Futures Returns and Investment Styles: Multivariate Analysis

We perform a series of multivariate regressions of the return of the commodity index on the investment styles of momentum, value and carry. Momentum of the index is measured by its previous 12-month return, value measured as the negative 48-month return 12-months ago (i.e., long-term reversal), and carry measured as the backwardated/contango value of the commodity index. The sample period covers 1877-2015. Panel A reports the regressions run at 1-month, 1-year and 5-year horizons. Panel B reports regressions of realized monthly commodity index returns across six states of nature (average backwardation, average contango, high inflation, low inflation, expansions, and recessions) on the monthly prediction model estimated in panel A.

Panel A: **Predictive Regression Model of Commodity Returns on Momentum, Value and Carry**

Horizon: Monthly

	Momentum	Value	Carry	Intercept	R-squared
Equal-Weight, Coefficient:	0.25	0.08	0.28	4.47%	0.02
Equal-Weight, t-Stat:	3.31	1.78	2.44	2.89	
Equal Risk-Weight, Coefficient:	0.29	0.08	0.36	4.36%	0.02
Equal Risk-Weight, t-Stat:	3.87	2.01	3.16	2.94	

Horizon: One Year

	Momentum	Value	Carry	Intercept	R-squared
Equal-Weight, Coefficient:	0.11	0.06	0.18	4.89%	0.04
Equal-Weight, t-Stat:	1.26	1.51	2.21	3.30	
Equal Risk-Weight, Coefficient:	0.14	0.06	0.18	4.58%	0.05
Equal Risk-Weight, t-Stat:	1.64	1.34	2.18	3.14	

Horizon: Five Years

	Momentum	Value	Carry	Intercept	R-squared
Equal-Weight, Coefficient:	-0.01	0.01	0.08	5.18%	0.01
Equal-Weight, t-Stat:	-0.36	0.54	2.21	3.01	
Equal Risk-Weight, Coefficient:	-0.01	0.01	0.07	4.90%	0.01
Equal Risk-Weight, t-Stat:	-0.30	0.20	1.88	2.77	

Panel B: Predictive **Regression Model of Commodity Returns on Momentum, Value and Carry in Various Aggregate States**

Horizon: Monthly

		Full Sample	Backwardation	Contango	High Inflation	Low Inflation	Expansion	Recession
Momentum	coefficient:	0.25	0.40	0.12	0.04	0.23	0.15	0.28
	t-Stat:	3.31	3.44	1.26	0.39	2.14	1.62	1.97
Value	coefficient:	0.08	-0.02	0.16	0.06	0.14	0.04	0.12
	t-Stat:	1.78	-0.36	2.82	0.88	2.42	0.75	1.44
Carry	coefficient:	0.28	0.11	0.54	0.55	0.19	0.25	0.34
	t-Stat:	2.44	0.58	2.04	3.14	1.22	1.84	1.60
Intercept	coefficient:	4.47%	4.28%	6.35%	13.33%	-2.90%	7.62%	-2.63%
	t-Stat:	2.89	1.43	2.22	5.59	-1.37	4.37	-0.74
R-squared		0.02	0.02	0.02	0.02	0.02	0.01	0.02

Table 8
Asset Allocation Analysis

Table 8A
Asset Allocation Performance and Aggregate States

Table 8A reports the mean, volatility, and Sharpe ratio of (a) each individual asset class, (b) the optimal mean-variance portfolio, (c) an equal-weighted portfolio of the assets, (d) a 60/40 portfolio of 60% stocks and 40% bonds, and (e) a 54%/36%/10% portfolio of stocks / bonds / commodities. These are reported for the full sample, and conditional on the six states: (i) average backwardation, (ii) average contango, (iii) high inflation (inflation above its mean), (iv) low inflation (inflation below its mean), (v) expansion (trough-to-peak based on NBER dating), and (vi) recession (peak-to-trough based on NBER dating). By construction, the optimal mean-variance portfolio has the highest Sharpe ratio unconditionally over the full sample period, 1877-2015.

		Full Sample	Backwardation	Contango	High Inflation	Low Inflation	Expansion	Recession
Mean:	Bond	1.1%	0.3%	1.9%	0.5%	1.8%	1.1%	1.3%
	Commodity	4.8%	7.7%	2.1%	14.8%	-4.7%	9.4%	-7.4%
	Stock	6.7%	4.7%	8.5%	5.6%	7.8%	12.3%	-7.9%
	Optimal Portfolio	3.4%	2.9%	3.9%	4.4%	2.4%	5.8%	-2.9%
	Equally Weighted Portfolio	4.2%	4.2%	4.2%	7.0%	1.6%	7.6%	-4.7%
	60/40 Portfolio	4.5%	2.9%	5.9%	3.5%	5.4%	7.8%	-4.2%
	10% Commodities, 90% 60/40 Portfolio	4.5%	3.4%	5.5%	4.7%	4.4%	8.0%	-4.5%
Volatility:	Bond	5.5%	4.6%	6.1%	6.3%	4.5%	5.3%	5.7%
	Commodity	17.6%	18.0%	17.3%	17.7%	17.2%	16.6%	19.8%
	Stock	17.0%	15.3%	18.4%	14.4%	19.1%	14.8%	21.2%
	Optimal Portfolio	7.1%	6.5%	7.6%	6.6%	7.5%	6.3%	8.6%
	Equally Weighted Portfolio	9.3%	8.7%	9.8%	8.4%	9.9%	8.3%	11.1%
	60/40 Portfolio	10.6%	9.6%	11.4%	9.3%	11.6%	9.2%	13.2%
	10% Commodities, 90% 60/40 Portfolio	10.1%	9.1%	10.9%	8.8%	11.1%	8.7%	12.5%
Sharpe ratio:	Bond	0.21	0.06	0.31	0.07	0.39	0.20	0.23
	Commodity	0.27	0.43	0.12	0.84	-0.27	0.57	-0.37
	Stock	0.40	0.31	0.46	0.39	0.41	0.83	-0.37
	Optimal Portfolio	0.48	0.44	0.51	0.67	0.32	0.92	-0.33
	Equally Weighted Portfolio	0.45	0.49	0.43	0.83	0.16	0.92	-0.42
	60/40 Portfolio	0.42	0.31	0.52	0.38	0.46	0.85	-0.32
	10% Commodities, 90% 60/40 Portfolio	0.45	0.38	0.51	0.53	0.39	0.91	-0.36

Table 8B
Portfolio Sharpe Ratios in 20-year periods

Period	Optimal Portfolio	Equally Weighted Portfolio	60/40 Portfolio	10% Commodities, 90% 60/40 Portfolio
1877 - 1895	0.08	0.00	0.27	0.21
1896 - 1915	0.46	0.60	0.32	0.46
1916 - 1935	0.30	0.24	0.31	0.30
1936 - 1955	0.95	0.92	0.75	0.82
1956 - 1975	0.42	0.60	0.23	0.34
1976 - 1995	0.59	0.61	0.57	0.59
1996 - 2015	0.67	0.54	0.59	0.58

Figure 1

The Cumulative Returns on Commodity Indices

Figure 1A-B graphs the cumulative returns on each commodity index, including two versions of the breakdown of the return into spot return and the realized roll yield (i.e., carry). The figures show the cumulative returns in terms of (i) excess-of-cash spot returns and (ii) interest-rate adjusted carry (roll yield plus the short rate), as well as in terms of (i) spot returns and (ii) roll yield (with no short rate adjustment). The graph covers the sample period 1877-2015.

Figure 1A: Equal-Weighted Index

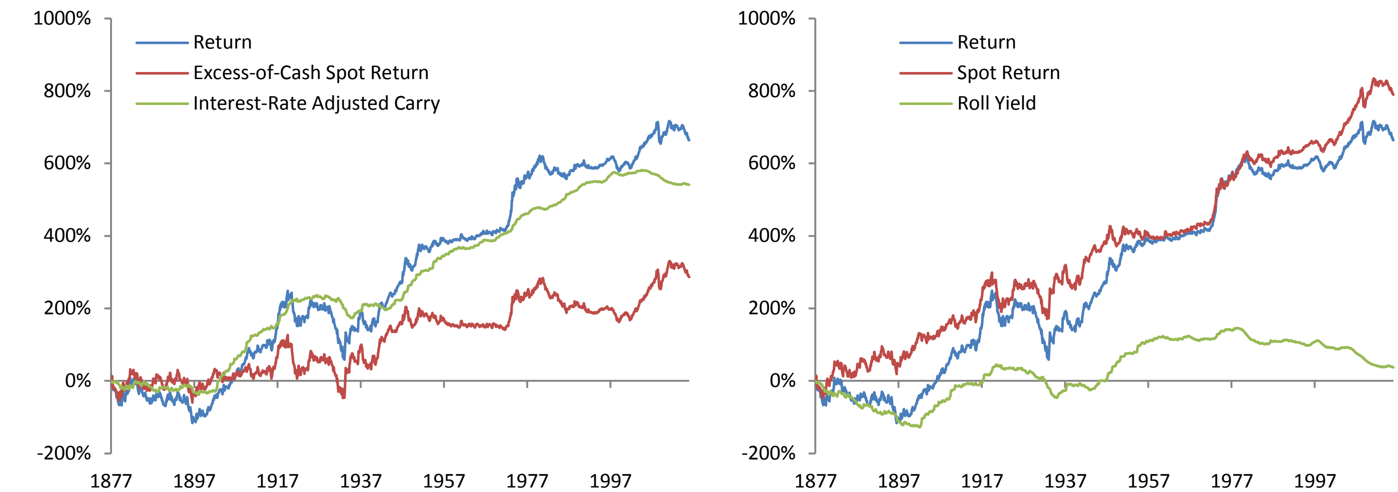


Figure 1B: Equal Risk-Weighted Index

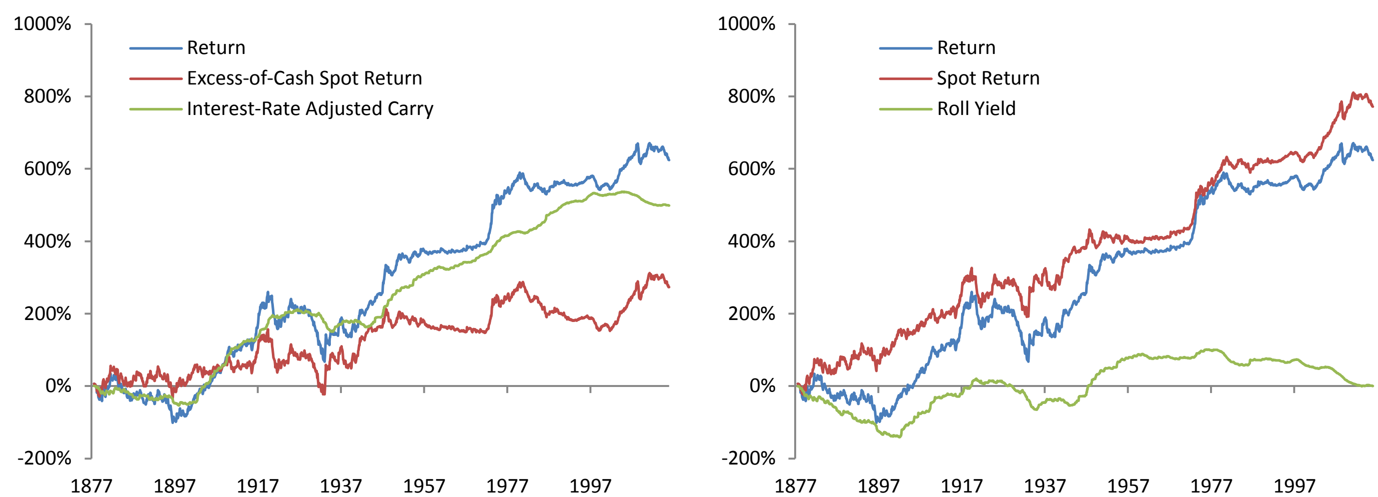
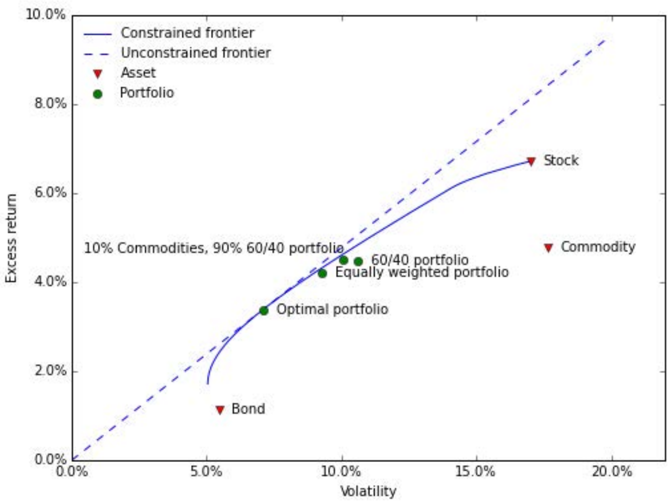


Figure 2
Asset Allocation Involving Commodity Futures

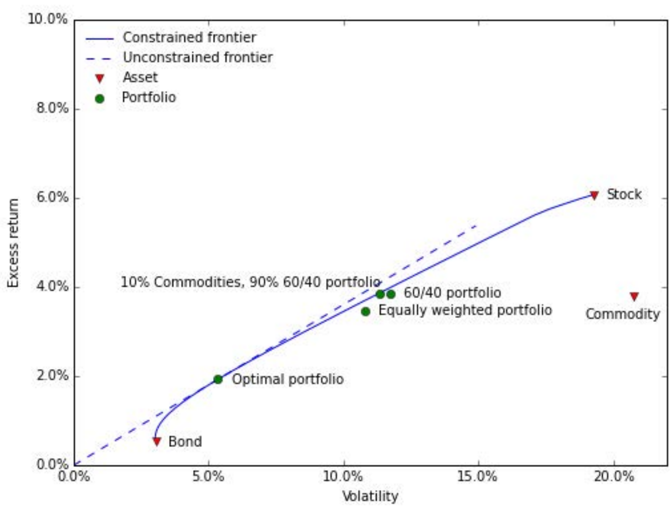
Figure 2A-C graphs the mean-variance opportunity set of long-term U.S. government bonds, commodities, and U.S. stocks over the full sample (1877-2015) and two subsample periods (1877-1945 and 1946-2015). The figures assume that the realized mean, volatility and correlation across these asset classes were known ex ante and are constant over time.

Figure 2A – Full Sample (1877-2015)



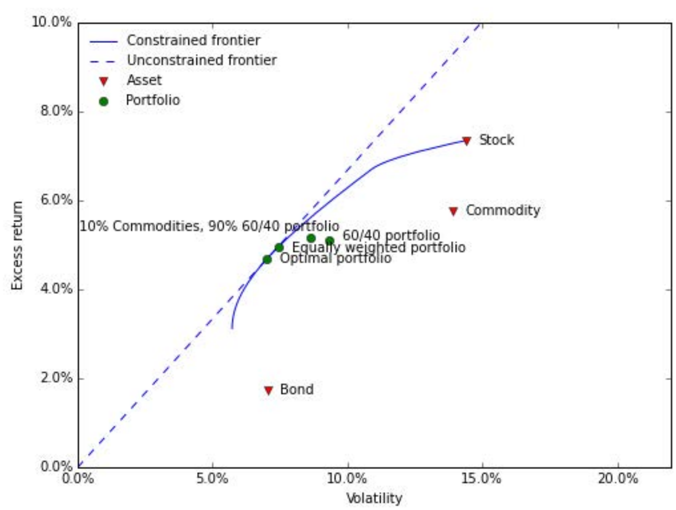
	Optimal weights
Bond	54%
Commodity	17%
Stock	29%

Figure 2B – Subsample (1877-1945)



	Optimal weights
Bond	72%
Commodity	7%
Stock	21%

Figure 2C – Subsample (1946-2015)



	Optimal weights
Bond	39%
Commodity	29%
Stock	31%