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Ricardo J. Caballero
Alp Simsek

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ABSTRACT

Gross capital flows are large and volatile. Their fickleness—tendency to exit distressed foreign markets—is a perennial source of concern for financial stability. Their retrenchment—tendency to exit from foreign markets when the local markets are distressed—is the less noticed stabilizing side of fickle flows. In this paper we develop a global equilibrium model of capital flows that addresses the tension between retrenchment and fickleness. We focus on certain types of flows that could be subject to fire sales (such as equity, long-term debt, or short-term debt denominated in local currency). In a symmetric world, these flows increase liquidity during local crises since the fickle flows exit the country at relatively weak local prices (or exchange rates) whereas retrenched flows are brought back at relatively high valuations. However, the fickleness of flows is not inconsequential as it ensures that the liquidity provision benefits remain largely hidden to local policymakers. We find that taxing capital flows, while could prove useful for a country in isolation, backfires as a global equilibrium outcome. We also find that capital flows are more useful and greater in magnitude when the global correlation of liquidity shocks is lower and there is a shortage of safe assets. However, the latter shortage exacerbates the severity of crises and may lead to uncoordinated policy responses that can lock the global economy into a highly inefficient autarkic equilibrium. If the system is heterogeneous and includes developed markets (DM) with abundant domestic liquidity and emerging markets (EM) with limited liquidity, there can be scenarios when global uncertainty is high and EMs' reach for safety can destabilize DMs, as well as risk-on scenarios in which DMs' reach for yield can destabilize EMs.

Ricardo J. Caballero
Department of Economics, E52-528
MIT
77 Massachusetts Avenue
Cambridge, MA 02139
and NBER
caball@mit.edu

Alp Simsek
Department of Economics, E52-552
MIT
77 Massachusetts Avenue
Cambridge, Ma. 02139
and NBER
asimsek@mit.edu

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1. Introduction

In its "Capital for the Future," World Bank (2013) projects that annual gross capital flows will grow from approximately 15 trillion in 2016 to over 40 trillion dollars by 2030. Despite their large size, the specific mechanisms behind these flows and their effect on financial stability are not entirely clear.

As described by Obstfeld (2012), the literature has been gradually migrating from a "consenting adults" view to a financial frictions perspective. The former view is a neoclassical benchmark that attempts to understand flows using standard frictionless models of international risk sharing. According to it, domestic agents hold diversified international portfolios, which generate gross flows and facilitate risk sharing. While these portfolio investments are no doubt important, they are unlikely to account for the large magnitude of the capital flows that we see in practice. The latter and newer view focuses on liquidity/capital allocation in an environment with financial frictions as the central driving force behind gross flows. This view observes that much of the gross positions are held by sophisticated financial intermediaries. Assuming that intermediaries are instrumental to mitigate financial frictions, gross flows might be key to allocate liquidity in global markets where it is most needed.

The implications of the liquidity-driven gross flows has been a cause for policy concern. First, the centrality of large levered intermediaries raises the standard concerns associated to them, especially in a context where there is no well developed permanent cross-currency lender of last resort framework. Second, and related, capital inflows are often fickle and desert countries during distress, exacerbating costly domestic fire sales. In view of these and other concerns, a growing literature emphasizes that fickle capital flows, while potentially useful for capital allocation purposes, can *increase* crisis risks and should be subject to macroprudential regulation.

In this paper we develop a framework of gross flows that embeds ingredients from both views—we offer a *qualified consenting adults view*. Ours is a qualified view in the sense that we focus on financial positions that are held by intermediaries, as in the liquidity literature, and we take as given that the flows behind these positions are fickle. Ours is also a consenting adults view in the sense that, despite their fickleness and the potential to generate financial instability, capital flows provide liquidity insurance and reduce the severity of crises. The fickleness of flows, however, is not inconsequential, as it implies that the cost of flows are felt at the local level whereas their liquidity-provision benefits obtain in general equilibrium and remain largely hidden to local policymakers. Therefore our model identifies a policy bias against fickle capital flows, and highlights the importance of policy coordination for the provision of global liquidity.

The key mechanism by which flows improve financial stability in our model is *retrenchment*: the tendency of outflows to exit from foreign markets when the local markets are distressed. The following paragraph from Obstfeld (2012) describes how retrenchment provided liquidity in the U.S. during the subprime crisis:

> Figure ... illustrates the example of the United States over the two quarters of

> intensive global deleveraging following the Lehman Brothers collapse in September 2008.... Gross capital inflows, which in previous years had been sufficient to more than cover even a 2006 net current account deficit of 6 percent of GDP, went into reverse, as foreigners liquidated \$198.5 billion in U.S. assets. In addition, the U.S. financed a current account shortfall of \$231.1 billion (down sharply from the current account deficit of \$371.4 billion over the previous two quarters). Where did the total of nearly \$430 billion in external finance come from? It came from U.S. sales of \$428.4 billion of assets held abroad—a volume so big that the dollar actually appreciated sharply through March 2009. Had these resources not been available (as a result of past gross financial outflows from the U.S.)...the U.S. current account deficit would have been compressed further, and the dollar would have slumped in currency markets.

While the fickleness of inflows by foreigners tends to exacerbate crises, the retrenchment of outflows by locals has the opposite effect. Our model is designed to capture this tension between fickleness and retrenchment. Specifically, we consider countries that experience occasional financial crises: that is, liquidity shocks that lower the asset prices below their fundamental valuations (even though we do not explicitly model exchange rates, these crises can also naturally be interpreted as fire-sale depreciations). We assume that investors (that can be thought of as intermediaries) liquidate their positions in a country that experiences a crisis regardless of the price level (the fickleness factor). We view this assumption as capturing a broad range of reasons for why foreigners tend to capitulate when local markets experience turmoil. However, investors are also specialists in their local markets, where they arbitrage and stabilize domestic fire sales (the retrenchment factor).[1] Hence, the key tension is whether these investors with a dual role exert a stabilizing or destabilizing influence on average. This tension is isolated most cleanly in a symmetric world, which is the main focus of our analysis.

The first question is why investors hold foreign assets at all, if they are not specialists in those markets and will liquidate during crises. It could of course be the case that the foreign assets yield higher returns when there is no crisis—we analyze this reach-for-yield possibility later. But the model reveals that investors will want to hold foreign assets even when the countries are completely symmetric in terms of returns and fundamentals. The reason is that there is a scarcity of safe assets, and investors need some asset that is uncorrelated with the local market (as they would like to collect a large return when there is a local crisis). It turns out that a diversified portfolio in foreign markets will do the job. This portfolio will be safe(r) on average due to diversification. And it is desirable in equilibrium despite its lower expected payoff because it will be retrenched back into the country during a local crisis.

The second question is how these flows affect the severity of crises. Our model reveals that,

[1] While it simplifies the exposition to talk about domestic and foreign investors, the key distinction for us is between local (specialist) and non-local (fickle generalist) investors, which need not be perfectly aligned with domestic and foreign investors, respectively.

in a symmetric world, retrenchment dominates fickleness and the flows are on net beneficial for financial stability. The intuition is that fickle foreigners sell local assets at fire-sale prices, but the local investors sell and retrench their diversified foreign assets at relatively high valuations. In a symmetric environment, every inflow is matched by an outflow that has equal size (that is, there are no net imbalances). Since the outflows are liquidated at a higher return than inflows, symmetric capital flows provide liquidity and increase the fire-sale prices during crises. The return differential between the outflows and inflows (during a crisis) is the amount of liquidity insurance the country obtains from international capital markets.

We envision the symmetric case of our model as roughly capturing the gross flows (held by sophisticated intermediaries) between developed countries. As the argument makes it clear, we also focus on certain types of flows that could be subject to fire sales, such as, equity, long-term debt, or short-term debt denominated in local currency. Hence, our model suggests that these types of flows *create global liquidity despite their fickleness.* We also find that, even though flows are stabilizing from a global perspective, they are disliked by local regulators. Intuitively, every capital inflow into a country is an outflow from the perspective of some other countries. The local regulators take into account the fickleness cost of inflows, but they do not take into account the retrenchment benefit of inflows for those other countries. In an uncoordinated policy environment, this externality leads to too much protectionism—excessive taxes or restrictions on capital inflows. We also find that protectionist policies are exacerbated when there are worldwide asset shortages, as this environment leads to more severe crises and a stronger motivation for local regulators to do something about them. The protectionist policies are also strategic complements: the more some countries adopt them, the more other countries will have incentives to adopt them.

Within this symmetric world, we also analyze how the global liquidity creation is affected by the presence of global liquidity shocks. The liquidity service from flows is naturally reduced when the local liquidity shocks become more correlated. Thus, an increase in the importance of global shocks—which can be thought of as capturing the aftermath of the global financial crisis—reduces the magnitude of gross flows and increases the demand for safe assets. In terms of prices, the risk premium on flows increases, the safe interest rate declines, and domestic fire sales become more severe.

These conclusions apply in a symmetric environment, but they are qualified when there are substantial asymmetries in liquidity or investment returns across the different regions of the world. We identify two potentially destabilizing mechanisms—reach for safety and reach for yield—that apply when developed markets with substantial liquidity but relatively low returns trade capital flows with emerging markets with smaller liquidity but relatively high returns.

The reach-for-safety mechanism is driven by cross-country differences in liquidity. The greater liquidity in a developed country makes its assets relatively attractive for the investors in emerging markets. This induces the developed country to experience greater inflows relative to its outflows (or run current account deficits). The model further reveals that, when there are global liquidity shocks, the inflows into the developed country are relatively safe whereas the outflows are relatively risky. Intuitively, the investors in the developed country sell liquidity insurance (at a premium) to the emerging markets. These types of reach-for-safety flows are a mixed bag for financial stability, as they exacerbate the financial crises in the developed country while mitigating the crises in emerging markets.

The reach-for-yield mechanism is driven by cross-country differences in investment returns. If the return in developed markets is much lower than in emerging markets, then investors in developed markets hold foreign assets not only to mitigate local crises, but also to chase after the higher returns in emerging markets. We show that the flows that are purely driven by the pursuit of higher returns are destabilizing, since they exacerbate crises in emerging markets without providing financial stability benefits elsewhere. Our model therefore provides a rationale for taxing certain types of flows into emerging markets even if policy can be coordinated across countries. However, the model also reveals that these types of flows happen in equilibrium only if the return differentials across the regions are sufficiently high to compensate for the developed market investors' lack of expertise (or fickleness) in emerging markets.

Our main results are described in sixteen propositions, most of which characterize equilibrium in different environments and provide a few comparative statics for the corresponding context. It is useful to summarize some of these comparative statics at this stage, in order to give the reader a fuller sense of the issues addressed (and not) in this paper and its connection with a wide variety of recent literatures, as well as of the parsimony of the framework:

- *Proposition 1:* A reduction in safe asset supply worsens fire-sale prices during crises, lowers safe interest rates, and increases gross capital flows

- *Proposition 2:* In a global equilibrium with scarcity of safe assets, a country with abundance of them will experience net capital inflows (or run current account deficits) that exacerbate its own fire sales during crises

- *Proposition 3:* In the same equilibrium, a country with high returns during normal times will also experience net capital inflows that exacerbate its own fire sales during crises

- *Proposition 4:* As liquidity shocks become more correlated across countries, gross flows become less effective in providing global liquidity, a risk-premium on gross flows emerges, and safe interest rates drop.

- *Proposition 5:* As the frequency of highly correlated states rises, gross capital flows decline and safe interest rates drop.

- *Proposition 6:* As the global liquidity cycle grows in importance, a safe asset producer country not only receives net capital inflows, but the composition of its gross flows changes, with outflows targeting risky assets while inflows are mostly after domestic safe assets. That is, the country effectively leverages its outflows.

- *Proposition 7:* A decline in asset returns and cross-correlations (i.e., a "risk-on" environment) exacerbates capital flows to high-yielders and the size of the potential fire sale.
- *Proposition 8:* In a symmetric environment, a global planner will always choose not to tax capital flows.
- *Proposition 9:* In the same symmetric environment and with no costs of taxation (beyond its effect on capital flows), there is a unique Nash equilibrium of local regulators that has positive taxes on capital flows. This equilibrium has lower gross capital flows and safe interest rates, and worse local fire sales during crises, than under the global planner's outcome.
- *Proposition 10:* In the same environment but with convex costs of taxation there can be multiple equilibria. The worst equilibrium has higher taxes on capital flows. In this case, a reduction in the global supply of safe assets increases taxes on capital inflows, exacerbates fire sales, and lowers safe rates.
- *Proposition 11:* If local governments have some capacity to inject liquidity during crises, the global planner would maximize the utilization of this capacity.
- *Proposition 12:* In contrast, local governments that can commit to their liquidity-injection policies would be reluctant to use this capacity.
- *Proposition 13:* However, local governments that decide their liquidity-injection policies after the crises are realized (without commitment) would also use this capacity. Hence, the lack of commitment is associated with a silver lining in terms of global liquidity creation.
- *Proposition 14:* In an environment where the reach for safety dominates global liquidity creation, taxing capital inflows to safe asset producers exacerbates fire-sale prices in EMs but reduces them in DMs.
- *Proposition 15:* In an environment where EMs have lower safe liquidity but higher yields during normal times, a drop in returns in DMs may increase capital inflows into EMs and exacerbate its fire sales.
- *Proposition 16:* In a reach-for-yield dominated environment, taxing capital flows to EMs stabilizes them without worsening financial stability in DMs

Related literature-Methodology. From a methodological angle, our two central ingredients are endogenous liquidity creation and fire sales. As such, our paper relates to Allen and Gale (1994) who endogenize market size and volatility in a closed economy context with entry costs. In our model liquidity is created in a manner akin to Holmström and Tirole (1998, 2001), though our context is different, as we focus on countries and their policies rather than corporations that create the assets behind the liquidity.

The core (non-reach-for-yield) reason for capital flows in our environment is the scarcity of locally safe assets to store value for domestic fire-sales stabilization. In this sense, our work is closely related to the burgeoning literature on limited availability of global assets and its macroeconomic consequences (e.g. Caballero (2006); Caballero et al. (2008, 2016); Bernanke et al. (2011); Gorton et al. (2012); Krishnamurthy and Vissing-Jorgensen (2012); Gorton (2016)).

Our model also shares elements of the limits-to-arbitrage and fire sales literature. In particular, the (limited) liquidity pricing of local assets is similar to, e.g., Allen and Gale (1994); Shleifer and Vishny (1997); Gabaix et al. (2007); Lorenzoni (2008); Krishnamurthy (2010); Gromb and Vayanos (2016); Holmström and Tirole (2001). The all or none attitude of fickle foreign investors behind the fire sales is reduced form in our model, but is intended to capture the attitude of Knightian agents facing an unfamiliar (foreign) situation, and as such it relates to Dow and da Costa Werlang (1992); Caballero and Krishnamurthy (2008); Caballero and Simsek (2013); Haldane (2013).

As we described above, our paper can be seen as a balancing act of some of the forces highlighted in the literatures on international risk sharing and financial frictions. For the former, see e.g., Grubel (1968); Cole and Obstfeld (1991); Obstfeld (2009); Van Wincoop (1994, 1999); Lewis (2000); Coeurdacier and Rey (2013); Lewis and Liu (2012). The famous equity home-bias puzzle (while declining in recent years) suggests that this kind of diversification flows are relatively small. Related, a large empirical literature that has tried to quantify the potential benefits from international risk sharing has found mixed results (e.g., Coeurdacier et al. (2015) argue they are small and Colacito and Croce (2010) take the opposite view). For the latter, the concern with the volatility of capital flows was a central theme of the post World War II meetings at Bretton Woods (e.g., Forbes (2016a)), and it has reemerged in earnest in the post subprime crisis era, mostly in response to the spillovers of developed markets' expansionary monetary policies onto emerging market economies (see, e.g., IMF (2012)) but also onto other developed market economies (see, e.g., Klein (2012)). The focus on financial frictions and liquidity allocation as the central driving force behind gross flows is in e.g., Brunnermeier et al. (2012); Bruno and Shin (2013); Miranda-Agrippino and Rey (2015). And the potential macroprudential response to these kind of flows can be found in, e.g., Caballero and Krishnamurthy (2004, 2005, 2006); Korinek (2010); Jeanne and Korinek (2010); Ostry et al. (2010); Ostry (2012); Caballero and Lorenzoni (2014); Stiglitz and Gurkaynak (2015); Brunnermeier and Sannikov (2015); Calvo (2016); Korinek and Sandri (2016).

Through our fickleness assumption we take as given the core conclusions of the latter literature, and study whether the mechanisms of the former can offset the negative volatility implications once we consider the feedbacks of the global equilibrium. In this methodological sense we also relate to Scott and Uhlig (1999), who take as given the fickleness of financial investors and study the impact of this feature on economic growth.

Much of the theoretical support for building policy barriers to capital flows relies on some externality, principally within the domestic financial system, which leads to an excessive credit boom, followed by destructive busts. This analysis is typically conducted from the perspective of an individual country. However, an increasing body of empirical literature documents that capital account restrictions divert capital flows to other countries (see, for instance, Forbes et al. (2016); Giordani et al. (2014); Ghosh et al. (2014)). There is a small but important theoretical literature that incorporates these diversions into a multilateral analysis of capital flow taxation (e.g. Ostry et al. (2012); Blanchard and Ostry (2012); Jeanne (2014); Korinek (2012)).[2] Ostry et al. (2012); Jeanne (2014); Blanchard and Ostry (2012) emphasize the importance of a multilateral analysis of capital control measures and the value of coordination in preserving the power of a domestic policy. Our analysis shares some of the mechanisms and logic behind their work but we focus on the potential global liquidity costs of these controls rather than on their benefits.

Related literature-Supporting Facts. For an exhaustive description of the large magnitude of gross flows and their fluctuations see Lane and Milesi-Ferretti (2012); Milesi-Ferretti and Tille (2011). For a report connecting these flows to the liquidity provision functioning of global intermediaries, see e.g., Committee on the Global Financial System report, Landau (2011). Also consistent with this view is the fact that the decline of gross flows in the aftermath of the global financial crisis coincided with a large and stubborn rise in cross-country liquidity-scarcity measures. See, e.g., Borio et al. (2016); Du et al. (2016) for a stark illustration of the reduced international liquidity, as even the *covered* interest parity condition among the major currency pairs broke down since the subprime crisis.

The key mechanism by which flows improve financial stability in our model is retrenchment. Forbes and Warnock (2012) document that retrenchment is a widespread phenomenon that applies for the outflows of countries as diverse as the U.S. and Chile (see also Broner et al. (2013); Bluedorn et al. (2013)).

On the other side of our balance of ingredients, the fickleness assumption is supported by the evidence in Cerutti et al. (2015) who document that non-resident investors are significantly more sensitive to global push factors. Also, Bluedorn et al. (2013) document that capital flows are fickle for all countries, developed and emerging, although the former experience less volatility of total net inflows despite greater volatility of each component. And most directly related to our framework, Broner et al. (2013) document that, for a large panel of countries since the 1970s, capital inflows and outflows increase during expansions and decrease during contractions.

We further show that changes in the (perceived) correlations of liquidity shocks naturally generate a global cycle in capital flows and asset prices (see Section 3.1), which relates to the work of, e.g., Calvo et al. (1996); Forbes and Warnock (2012); Fratzscher (2012); Rey (2015, 2016); Miranda-Agrippino and Rey (2015); Bruno and Shin (2015). We also analyze the reach-

[2] IMF (2012) refers to this multilateral approach as the Keynes-White notion of operating "at both ends of the transaction."

for-safety and the reach-for-yield mechanisms in the presence of correlated shocks (see Sections 3.2 and 3.3). This is related to the empirical work on how changes in the global cycle (as captured by the VIX) or the monetary policy in the U.S. affects the flows between DMs and EMs, e.g., Baskaya et al. (2016); Chari et al. (2016); IMF (2012).

Finally, the implications of the reach-for-safety mechanism in our model are consistent with the important empirical work by Gourinchas and Rey (2007); Gourinchas et al. (2010), who document that the U.S. serves the role of the World's venture capitalist, and estimate a valuation transfer from the U.S. to the rest of the world during the global financial crisis of close to three trillion dollar.[3]

The rest of the paper is organized as follows. Section 2 presents the baseline environment and equilibrium in a largely symmetric world with no global liquidity shocks. This section illustrates how symmetric capital flows help to create liquidity in our environment. It also illustrates the reach-for-safety and the reach-for-yield mechanisms by minimally departing from symmetry (in particular, considering an infinitesimal country). Section 3 revisits the previous topics after introducing aggregate liquidity shocks that create a liquidity premium for foreign financial flows. We show that increasing the correlations of liquidity shocks can naturally generate a global liquidity cycle, and investigate how the presence of this cycle interacts with the reach for safety and the reach for yield. The remaining sections focus on the policy implications. Section 4 analyzes optimal policy in our baseline environment with symmetric flows. We show that the coordinated policy outcomes sharply differ from those that would be chosen in a Nash equilibrium. Section 5 extends the baseline model to incorporate asymmetric regions, and uses this model to investigate the policy implications of the reach for safety and the reach for yield. Section 6 concludes and is followed by several appendices containing extensions and the proofs of the propositions that are not developed in the body of the paper.

2. Core Environment and Equilibrium

In this section we describe the baseline model in which countries are symmetric and there is no aggregate risk. Within this environment we explain the mechanism by which gross capital flows help to create liquidity and stabilize crises. We also develop two variants of the model to illustrate the reach-for-safety and reach-for yield mechanisms by which capital flows can be potentially destabilizing.

[3] In our stylized model, all valuation effects are fully exercised by fickle and retrenching investors. In practice, a significant share of the large gross position are maintained, and the insurance happens largely through capital gains and losses. In this sense, an extended interpretation of our model is perfectly aligned with the valuation literature. See, for instance, Forbes (2016b), for a careful analysis of the insurance role of valuation (and primary income) effects in the current context of the UK.

2.1. Liquidity Creation with Fickle Flows

Consider a model with three periods, $t \in \{0, 1, 2\}$, with a single consumption good that we refer to as a dollar (for simplicity, we do not explicitly model exchange rates). There is a continuum of measure one of countries denoted by superscript $j \in [0, 1]$. Each country is associated with a new investment technology—a risky asset that is supplied elastically at date 0. This asset always pays R dollars, but the timing of the payoff depends on the local state $\omega^j \in \{0, 1\}$ that is realized at date 1. State $\omega^j = 0$ represents the case without a liquidity shock in which the project pays off early at date 1. State $\omega^j = 1$ represents the case with a liquidity shock in which the project payoff is delayed to date 2. In this case, the asset is traded at date 1 at a price p^j that will be endogenously determined. When there is a shortage of liquidity, which is the case that we will focus on, the asset will be traded at a fire-sale price, $p^j < R$. Hence, we envision the liquidity shock as capturing a financial crisis in which asset prices fall below their fundamental valuations. In the baseline model, the crises are i.i.d. across countries with $\Pr\left(\omega^j = 1\right) = \pi$, where $\pi > 0$ denotes the probability of a liquidity shock within a country.

Each country is also endowed with a legacy asset with liquid payoffs (safe asset), which is supplied inelastically at date 0. Each unit of the safe asset yields η dollars at date 1. The safe asset is traded at price η/R_f at date 0, where R_f denotes the (gross) risk-free interest rate that will be endogenously determined.

In each country j, there are two types of agents, entrepreneurs and investors. There is a mass e of entrepreneurs. They are born at date 1, with preferences given by $E[\tilde{c}_2]$. Each entrepreneur is endowed with 1 unit of the risky asset at date 1, and has access to a profitable project that delivers (nonpledgeable) payoffs at date 2. Thus, each entrepreneur sells her endowment at date 1 to invest in the project. These entrepreneurs are largely passive: their main role is to capture asset sales driven by liquidity needs and the potential welfare losses with these types of sales.

The main agents are investors (with mass one), which are denoted by the superscript j of their locality. They are endowed with all of the safe legacy asset supply as well as 1 dollar at date 0. They have preferences given by $E[u(c_0) + c_1 + c_2]$. Here, c_0 denotes the investors' spending in an outside option other than holding financial assets. It can be viewed as consumption or investment in an illiquid project. We assume $u(c_0)$ is an increasing and strictly concave function that also satisfies Inada-type conditions, $u'(0) = \infty$ and $u'(1) < R$, which will ensure an interior solution.

The novel ingredient of the model is the fickleness of investors in foreign markets. Specifically, if the foreign market is hit by a liquidity shock at date 1, then the investor is required to close her position in this market. In contrast, the local investor can take unrestricted positions in the local market. This assumption captures in reduced form the idea that investors might not feel comfortable outside of their natural markets due to unmodeled features (e.g., Knightian uncertainty) and they might flee at the first sign of trouble. The assumption also captures the concerns by policymakers that portfolio investments by outsiders tend to be fickle and might exacerbate financial instability.

The investor in country j chooses how much to consume, c_0^j, how much to invest in the local risky asset, $x^{loc,j}$, how much to invest in risky foreign assets, $\left[x^{j',j}\right]$, and how much to invest in the safe asset, y^j. Here, $x^{j',j}$ denotes a Lebesgue-measurable function of j' that captures the investor's foreign portfolio. We focus on symmetric equilibria in which the assets trade at the same price in all markets, $p^j \equiv p \leq R$ for each j. The investor's problem can then be written as,

$$\max_{\tilde{c}_0.\tilde{x}^{loc},\left[\tilde{x}^{j'}\right]_{j'},\tilde{y}\geq 0} u\left(\tilde{c}_0\right)+\tilde{x}^{loc}R+\left(\tilde{x}\overline{R}+\tilde{y}R_f\right)M, \tag{1}$$
$$\overline{R}=\left(1-\pi\right)R+\pi p$$
$$M=1-\pi+\frac{R}{p}\pi$$
$$\tilde{c}_0+\tilde{x}^{loc}+\tilde{x}+\tilde{y}=\eta/R_f+1 \text{ and } \tilde{x}=\int x^{j'}dj'.$$

If she invests in a local asset, she holds it until maturity, which leads to return R regardless of the local shock. If instead she invests in a foreign asset, she obtains a liquid return at date 1, either because there is no shock in the foreign market, or there is a shock and the investor sells in view of fickleness. The variable, $\overline{R}$, denotes the expected one-period payoff from foreign investment. Likewise, if the investor holds cash, she obtains a one-period return denoted by R_f. The final return from foreign investment or cash also depends on whether there is a local shock, as the domestic shock generates a reinvestment opportunity to purchase local assets at fire-sale prices, $p < R$. The variable, M, denotes the investor's expected marginal utility from reinvestment, which combines a marginal utility of 1 in case there is no domestic shock and a marginal utility of R/p in case there is a shock. Note that the expected return from foreign investment, $\overline{R}$, is multiplied with the expected marginal utility from reinvestment, M, since the local and foreign shocks are uncorrelated.[4]

The market clearing condition for the risky asset in a country j that experiences a liquidity shock at date 1 can be written as,

$$p^j=\min\left(R,C\left(x^{in,j},x^{out,j}\right)\right), \text{ where} \tag{2}$$
$$C\left(x^{in,j},x^{out,j}\right)=\frac{R_f y^j+\overline{R}x^{out,j}}{e+x^{in,j}} \text{ and } x^{in,j}=\int x^{j,j'}dj', x^{out,j}=\int x^{j',j}dj'.$$

Here, $x^{in,j}$ denotes the inflows into country j: that is, risky asset purchases in country j made by the foreigners. Likewise, $x^{out,j}$ denotes the outflows from country j: that is, foreign asset purchases made by country j investors. The expression, $C\left(x^{in,j},x^{out,j}\right)$, denotes the cash-per-

[4] We relax this assumption in Section 3. Note also that the investors cannot trade financial assets (backed by foreign investment or cash) with payoffs contingent on the realizations of local liquidity shocks, $\left\{\omega^j\right\}_j$. We allow for this possibility in Section 2.1.1.

asset in country j at date 1 as a function of inflows and outflows. The denominator of this term captures the amount of sales, which come from liquidity-driven sales (e) and the past inflows all of which are liquidated in a crisis in view of the fickleness assumption. The numerator captures the total amount of cash in the market, which comes from the local investors' cash and foreign asset positions that are determined by the past outflows. Eq. (2) says that, if the cash-per-asset is abundant, then the asset price is determined by its fundamental value. Otherwise, there are fire sales and the asset price is determined by the cash-per-asset in the market. This equation also illustrates the key tension captured by our model: while the inflows tend to reduce the fire-sale prices during a crisis, the outflows are retrenched back into the country and help to stabilize the fire-sale prices.

There is also a market clearing condition for the safe legacy asset, which can be written as,

$$\int y^j dj = \eta / R_f. \tag{3}$$

An equilibrium with symmetric prices is a collection of allocations, $\left(c_0^j, x^{loc,j}, \left[x^{j',j}\right]_{j'}, y^j\right)_j$, and prices, $p^j = p \leq R$ and R_f, such that the allocations solve problem (1), and the market clearing conditions $(2) - (3)$ hold.

We analyze a symmetric equilibrium that satisfies, $c_{0,j} = c_0, x^{loc,j} = x^{loc}, x^{j',j} = x, y^j = y$. This also implies that inflows and outflows in each country are the same, $x^{in,j} = x^{out,j} = x$ for each j. The symmetry together with Eq. (3) also implies that the equilibrium holdings of the safe asset is positive and given by, $y = \eta / R_f > 0$. Plugging this observation into the investor's budget constraint, we obtain $c_0 + x^{loc} + x = 1$. It remains to characterize how the investor optimally splits her dollar between outside spending, local investment, and foreign investment.

Using problem (1), the marginal benefit from local investment is equal to R. The marginal benefit from foreign investment, x, can be calculated as,

$$\overline{R} M = \left(\left(1-\pi\right) R + \pi p\right) \left(1 - \pi + \frac{R}{p} \pi\right) \equiv \mu\left(p\right). \tag{4}$$

Here, the second equality defines the function $\mu\left(p\right)$. Note that the foreign investment delivers lower one-period return than local investment in view of the fickleness assumption, $\overline{R} < R$. On the other hand, foreign investment delivers a higher marginal reinvestment utility, $M > 1$, since it can be retrenched back into the country in a crisis to arbitrage local fire sales. The following lemma resolves this tension and shows that there will be foreign investment in equilibrium despite the fickleness element.

Lemma 1. *The marginal benefit from foreign investment, $\mu\left(p\right)$, is strictly decreasing in p over the range $p \in (0, R]$, and it satisfies $\mu\left(R\right) = R$. In particular, $\mu\left(p\right) > R$ for each $p \in (0, R)$.*

The lemma implies that, when $p < R$, local investment is strictly dominated by foreign investment, $x^{loc} = 0$. Intuitively, the scarcity of local liquidity (safe assets) induces investors to obtain liquidity insurance by holding foreign assets. Consistent with this intuition, a decline in the fire-sale price p increases the marginal benefit from foreign investment, $\mu(p)$.

The equilibrium level of foreign investment is then characterized as the solution to,

$$\begin{cases} u'(1-x) = \mu(p), & \text{if } p < R \\ x \in [0, \underline{x}] & \text{if } p = R \end{cases}, \tag{5}$$

The first line captures the case with fire sales. In this case, $x^{loc} = 0$ and the foreign investment is determined by equating its marginal benefit with the marginal utility from consumption, $c_0 = 1 - x$. The second line captures the case without fire sales. In this case, consumption is at its upper bound, $c_0 = 1-\underline{x}$, which is found by solving $u'(1-\underline{x}) = R$. Total investment is at its lower bound, $\underline{x} > 0$. The investor is indifferent between investing in the foreign and the local market, and there is a range of optimal foreign investment levels (with the residual invested in the local market, $x^{loc} = \underline{x} - x$).

Eq. (5) can also be viewed as describing the price level that is consistent with the optimality of a given amount of foreign flows, $x \in (0, 1)$. We denote this optimality relation with $p = P^{opt}(x)$. Note that this is a decreasing relation (and strictly so if $p < R$), that is, a decline in fire-sale prices increases the amount of foreign flows. Figure 1 illustrates the optimality relation for a particular parameterization (with $u(c_0) = h \log c_0$ for some $h \in (0, R)$). The strictly decreasing region corresponds to the case with fire sales, and the flat region corresponds to the case without fire sales,

Next consider the market clearing condition (2), which can be rewritten as,

$$p = \min\left(R, \frac{\eta + \overline{R}x}{e + x}\right) = \min\left(R, \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right) \equiv P^{mc}(x). \tag{6}$$

Here, the first equality substitutes the market clearing condition for the safe asset, $y = \eta/R_f$. The second equality rearranges terms to eliminate p from the right hand side. The third equality defines the market clearing relation, $p = P^{mc}(x)$, which describes the price level as a function of flows. As we noted before, greater inflows tend to decrease the fire-sale price level due to fickleness but greater outflows tend to increase it in view of retrenchment. The following lemma resolves this tension and shows that, with symmetric flows, retrenchment dominates fickleness.

Lemma 2. *The market clearing price level, $P^{mc}(x)$, is weakly increasing in symmetric gross flows, x, and strictly so if there are fire sales, $P^{mc}(x) < R$.*

The intuition for why retrenchment dominates fickleness can be understood by inspecting the first equality in (6). Note that inflows (x in the denominator) are liquidated at the fire-sale return, p. However, the outflows (x in the numerator) are retrenched back into the country at the diversified portfolio return, $\overline{R}$. If there are fire sales, $p < R$ (which is the only case in which gross flows strictly influence prices), then the fire-sale return is lower than the diversified portfolio return, $p < \overline{R} = (1-\pi)R + \pi p$. It follows that the symmetric flows on net increase

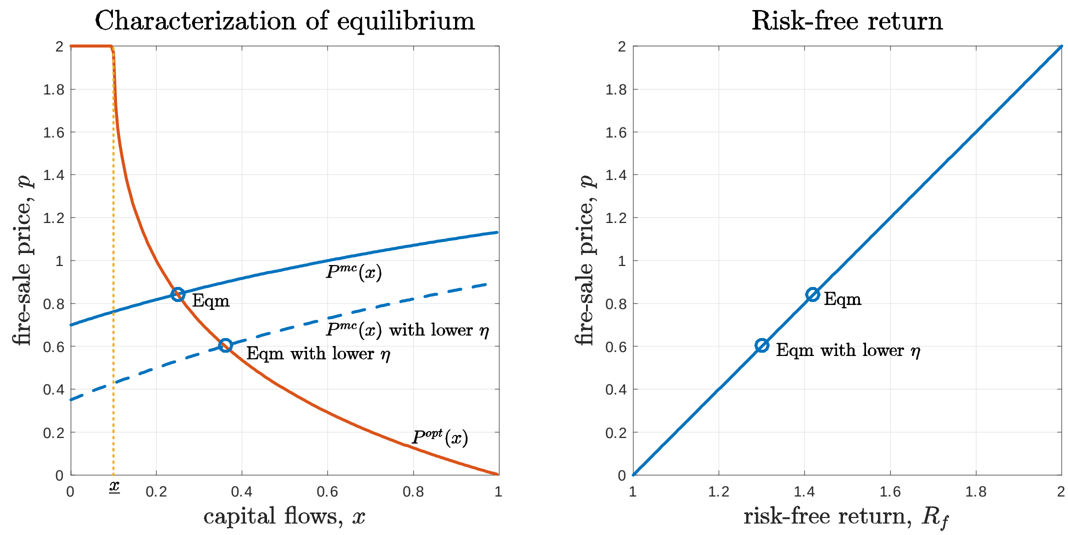

Figure 1: The left panel illustrates the characterization of equilibrium in the baseline environment. The dashed line illustrates the effect of a reduction in the liquidity from legacy assets, η. The right panel illustrates the risk-free rate in equilibrium.

liquidity and fire-sale prices. Intuitively, the fickle flows exit at weak local prices, as they are driven by local shocks, while retrenchment flows take place at favorable prices as they are driven by shocks back at home rather than globally. Hence, despite their fickleness, gross flows help to bring the excess liquidity in foreign financial markets that do not experience liquidity shocks into the local market that has a liquidity shock.

The equilibrium is characterized as the intersection of the increasing market clearing relation, $p = P^{mc}(x)$, with the decreasing optimality relation, $p = P^{opt}(x)$. Figure 1 illustrates the characterization of equilibrium for a particular parameterization. There exists an equilibrium that satisfies $x \in (\underline{x}, 1)$ and that features fire sales, $p < R$, as long as the following *domestic safe asset scarcity* condition holds—which we maintain for the rest of the analysis.

Assumption 1. $\eta < eR$.

Once the variables, x, p, are characterized, the risk-free return is characterized by the financial market equilibrium condition,

$$R_f = \overline{R}. \tag{7}$$

In the baseline model, cash and foreign investment yield the same one-period return since they are perfect substitutes [cf. problem (1)]. This feature will be modified once we introduce global liquidity shocks (see Section 3). Our next result summarizes the above discussion and establishes some properties of equilibrium.

Proposition 1. *Consider the baseline model (with Assumption 1). There exists a unique symmetric equilibrium,* $\left(\left(c, x^{loc}, x, y\right), p, R_f\right)$*, which satisfies* $yR_f = \eta, x^{loc} = 0, c = 1 - x$ *and fire-sale prices,* $p < R$*. The pair* (x, p) *is characterized by Eqs.* (5) *and* (6)*. Decreasing the local liquidity (safe assets),* η*, decreases* p *and* R_f*, and increases the capital flows,* x*. Decreasing the return,* R*, decreases* p *and* R_f*, as well as the capital flows,* x*.*

The comparative static result with respect to η follows by observing that reducing liquidity shifts the market clearing curve $p = P^{mc}(x)$ downwards, without affecting the optimality curve, $p = P^{opt}(x)$ (see Figure 1 for an illustration). Intuitively, the price declines in view of the market clearing condition (6). In turn, the lower price induces greater foreign investment; with smaller local liquidity, there is greater need for global liquidity creation. The risk-free return also declines in view of the financial optimality condition (7). Hence, the result implies that a reduction in global supply of safe assets lowers the risk-free return and increases gross capital flows. This result provides one explanation for the worldwide increase in gross capital flows in the run-up to the Global Financial Crisis (see Bluedorn et al. (2013)). From the lens of our model, the gross flows increased at least in part as a response to the global asset shortages that developed in early 2000s (see e.g. Caballero (2006)). In Section 4, we will ask the follow-up question of whether the endogenous reaction by governments exacerbates or mitigates the impact of a contraction in global liquidity.

Likewise, the comparative static result with respect to R follows by observing that decreasing R shifts market clearing curve downward as global liquidity declines. It also shifts the optimality curve downward, since investment becomes relatively unattractive. The net effect is a decline in the fire-sale price, p, as well as safe asset returns, R_f. This analysis does not help to identify the effect on x. The proof in the appendix uses a more subtle argument to show that decreasing R also decreases x. In Section 2.3, we will ask the follow-up question of how the decline in the return in developed countries, captured by R, affects flows into and fire-sale prices in emerging markets with relatively high returns.

2.1.1. Insurance markets with respect to local shocks

Note that the financial markets in our baseline model are incomplete in the sense that investors cannot trade financial contracts (backed by foreign investment or cash) whose payoffs are contingent on the realizations of the local liquidity shocks, $\left\{\omega^j\right\}_j$. This incompleteness results in an inefficient allocation of liquidity at date 1. Specifically, the investors have liquid financial wealth in states in which their country does not experience liquidity shocks which they would have ideally liked to transfer to states with local liquidity shocks. In Appendix A.1, we relax this assumption by introducing intermediaries that sell contingent contracts and invest in foreign markets as well as cash. We show that local investors purchase the contracts contingent on local liquidity shocks, and that the presence of these insurance arrangements further increases local fire-sale prices. In particular, the fire-sales are avoided for sufficiently low levels of π, as these states feature sufficient global liquidity, but they are not for higher levels of π, in which case the crises have a global scope and the local insurance markets provide little help.

Importantly, when π is sufficiently large (so that crises are sufficiently frequent), the policy tension that we identify later in our baseline model continues to apply when we allow for local insurance markets. We therefore abstract away from these insurance arrangements in our baseline analysis. This is arguably a realistic feature of the model. It could also be motivated by informational considerations: frictions such as moral hazard or adverse selection would have a particularly strong bite for insurance arrangements with respect to idiosyncratic shocks. That said, to the extent that these markets are feasible, they should be promoted. See e.g. Caballero (2003); Brunnermeier et al. (2016) for proposals in the context of emerging markets and the Eurozone, respectively.

2.2. Reach for safety

We next consider a variant of the baseline model to illustrate the reach-for-safety mechanism. Specifically, consider the same setup with the only difference that one country j (that has measure zero) has potentially different liquidity, η^j, compared to the world average, η. A developed country with deep financial markets and a large supply of safe assets—such as the U.S.—can be thought of as featuring $\eta^j > \eta$. Conversely, an emerging market country is captured by low η^j.

Suppose $\eta^j > \eta$ so that the country in consideration has a relatively developed financial market (the other case is discussed at the end of the subsection). As a benchmark, first suppose the country is in autarky. In this case, consumption and local investment in risky assets is given by respectively $c_0^j = 1 - \underline{x}$ and $x^{loc,j} = \underline{x}$, and the safe asset holding is $y^j = \eta^j / R_f$. The asset price at date 1 (conditional on a liquidity shock) is $p^j = \min\left(R, \frac{\eta^j}{e}\right)$. To obtain sharp results, we make the following safe asset abundance assumption (in addition to Assumption 1).

Assumption S. $eR < \eta^j$.

That is, country j has access to abundant domestic liquidity, which ensures that the autarky equilibrium features no fire sales, $p^j = R$.

Let us contrast this outcome with the equilibrium with free capital flows. The world equilibrium, which we continue to denote by (x, p, R_f) is the same as before. However, the equilibrium allocations in country j are potentially different. When the country experiences positive inflows—which will be the case in equilibrium—the optimality conditions for foreign investors imply,

$$(1-\pi) R + \pi p = (1-\pi) R + \pi p^j. \tag{8}$$

In particular, the fire-sale price in country j is exactly the same as in the representative country. Put differently, even though the country j has abundant liquidity and would not feature fire sales in autarky, it cannot escape fire sales in the equilibrium with free capital flows.

For intuition, consider the optimality conditions in (8). All else equal, greater liquidity in the country, η^j, would increase the fire-sale price, p^j. However, this makes the country's assets attractive to foreign investors and increase the inflows, $x^{\text{in},j}$. Foreign investors will be indifferent to invest in the country only when the inflows increase to the point at which the expected return is in line with that in the representative country. Formally, the market clearing condition in country j can be written as,

$$p^j = \min\left(R, \frac{\eta^j + x^{\text{out},j}\overline{R}}{e + x^{\text{in},j}}\right), \tag{9}$$

where $x^{\text{out},j}$ denotes the outflows from the country. Using the optimality condition for local investors, $u'\left(1 - x^{\text{out},j}\right) = \overline{R}$, the outflows are the same as in other countries, $x^{\text{out},j} = x$. Combining this with Eqs. $(9), (8)$ and (6), we obtain, $x^{\text{in},j} - x^{\text{out},j} = \left(\eta^j - \eta\right)/p > 0$. That is, the liquidity difference advantage of the country is neutralized by its greater inflows relative to outflows (i.e., capital account surplus and current account deficit). The following proposition summarizes this discussion.

Proposition 2. *Consider the baseline model in which a country has abundant local liquidity, $\eta^j > \eta$, that satisfies Assumption S (so that the country would not experience fire sales in autarky). In an equilibrium with free financial flows, the country receives more inflows than its outflows, $x^{in,j} > x^{out,j}$, and experiences fire-sales that are just as severe as those in the representative country, $p^j = p < R$.*

This result suggests that the *reach-for-safety* flows have potentially destabilizing effects. However, note that the flows are also potentially stabilizing for foreign investors that invest in the developed country. To see this, consider the mirror-image situation in which a country has relatively low liquidity compared to the world average, $\eta^j < \eta$. The equilibrium in this country is characterized by similar steps as above. With symmetric flows, this country would experience deeper fire sales in view of its low local liquidity, $p^j < p$. All else equal, these fire sales would make the country's assets relatively unattractive to foreigners, which would reduce inflows. This process continues until the fire sales are on average the same as those in the representative country, $p^j = p$ (except when $\eta^j < \eta - xp$, in which case the equilibrium features $x^{\text{in},j} = 0$ and $p^j < p$). Hence, the effect of reach-for-safety flows on worldwide financial stability is ambiguous.

2.3. Reach for yield

We next consider another variant of the model to illustrate the reach-for-yield mechanism. Specifically, consider the same setup as in Section 2.1 with the only difference that one country j (that has measure zero) has a greater fundamental return relative to the world average, $R^j > R$. Country j can be thought of as a rapidly growing or high yielding emerging market such as China, India, or Brazil. These types of countries appear to have relatively attractive fundamental returns, especially in recent years in which the asset returns in developed markets have been unusually low. In line with this interpretation, we also assume the country has (weakly) lower liquidity than the representative country, $\eta^j \leq \eta$. To obtain an interior solution, we also assume that R^j and η^j are not too far from their representative country counterparts.

Assumption Y. $R^j - R \in \left(0, \frac{\pi}{1-\pi}p\right)$ and $\eta^j - \eta \in [-px, 0]$.

The analysis parallels Section 2.2 with minor differences. With positive inflows into country j—which will be the case in equilibrium—the optimality condition for foreign investors imply,

$$(1-\pi) R + \pi p = (1-\pi) R^j + \pi p^j. \tag{10}$$

In particular, the fire-sale price in country j is lower than in the representative country, $p^j < p$.

For intuition, first imagine the country had the same investment return as the world average. As we discussed above, this country's outflows would exceed its inflows, $x^{\text{out},j} \geq x^{\text{in},j}$ (the country would run a current-account surplus), which would stabilize local financial crises, $p^j = p$. Relative to this benchmark, an increase in the country's investment return, R^j, makes its assets relatively attractive to foreigners. The inflows increase (and the current account surplus declines) up to the point at which the local fire-sales are sufficiently severe to deter the foreigners from investing further in the country.

We next analyze how a decline in asset yields in developed markets, which we capture with a decline in R, affects the equilibrium in the emerging market country j. As we noted in Proposition 1, a decline in R reduces the fire-sale price in the representative country, p (via a reduction in international liquidity). In view of this observation, Eq. (10) implies that an increase in R also decreases the fire-sale price in country j. It can further be seen that the fire-sale price declines more in country j than in the representative country, that is, $p^j - p$ declines. The following result summarizes this discussion.

Proposition 3. *Consider the baseline model in which a country has relatively high return, $R^j > R$, and satisfies Assumption Y. In an equilibrium with free financial flows, the country experiences fire sales that are more severe than in the representative country, $p^j < p$. A decrease in R reduces the fire-sale price in the country, p^j, as well as the relative fire-sale price, $p^j - p$.*

3. Environment with Aggregate Shocks

We next introduce aggregate liquidity shocks into our analysis to illustrate a number of additional mechanisms. We show that aggregate shocks result in a risk premium on capital flows (over safe assets). We then analyze the key determinants of the liquidity risk premium, and how this premium affects the reach-for-safety as well as the reach-for-yield mechanisms. We show that changes in the correlation of liquidity shocks drives a global cycle in liquidity premia, asset returns, and capital flows.

To address these issues, consider the setup in Section 2 with the only difference that there are several aggregate states denoted by $s \in S = \{1, 2, .. |S|\}$. The states differ in the probability of the liquidity shock, π_s. Throughout, we assume:

Assumption 2. π_s is increasing in s.

Hence, the states with higher s are associated with a greater probability of the liquidity shock (and thus, greater financial distress).[5] We denote the probability of the aggregate state s with γ_s, where $\gamma_s > 0$ for each s and $\sum_s \gamma_s = 1$.

We also assume that, at date 0, the agents can trade financial securities contingent on the aggregate state at date 1. Specifically, for each state $s \in S$, there is an Arrow-Debreu financial security that pays 1 dollar if state s is realized. The security is traded at date 0 competitively at price q_s. We assume the Arrow-Debreu securities are supplied by competitive intermediaries that undertake risky foreign investments at date 0. As before, $x^{in,j} \geq 0$ denotes the inflows into country j. The intermediaries' optimality conditions then imply that,

$$1 \geq \sum_s q_s \overline{R}_s^j, \text{ with equality if } x^{in,j} > 0, \tag{11}$$

where $\overline{R}_s^j = R(1-\pi_s) + p_s^j \pi_s$. Hence, the date-0 value of investment in a country is equal to its cost, normalized to 1, whenever there are positive inflows.

Similar to the earlier analysis, the investor in country j chooses how much to consume, c_0, how much to invest locally, x^{loc}, how much to invest in the legacy asset, y, and how much to invest in Arrow-Debreu securities, denoted by $(z_s)_s$. Her problem is,

$$\begin{aligned} &\max_{\tilde{c}_0, \tilde{x}^{loc}, \tilde{y}, \left(\tilde{z}_s \geq -\tilde{y} R_f\right)_s} u(c_0) + \tilde{x}^{loc} R + \sum_s \gamma_s \left(\tilde{y} R_f + \tilde{z}_s\right) M_s^j, \\ &\tilde{c}_0 + \tilde{x}^{loc} + \sum_s q_s \tilde{z}_s + \tilde{y} = \eta / R_f + 1. \end{aligned} \tag{12}$$

Here, $M_s^j = 1 - \pi_s + \frac{R}{p_s^j} \pi_s$ denotes the marginal utility from reinvestment as before. We assume the investors' holdings of the Arrow-Debreu securities satisfy, $\tilde{z}_s \geq -\tilde{y} R_f$: that is, the investor can take a short position but only if she holds the safe asset to cover the position. Unlike in problem (1), the investor does not directly choose risky investment in foreign countries. Instead, she chooses financial claims on the investments that are undertaken by competitive intermediaries as described above. To ensure continuity with the earlier analysis, we use $x^{\text{out},j} = \sum_s q_s z_s$ to denote the outflows from the country into risky investment. We also define $\overline{x}^{\text{out},j} = y^j - \eta / R^f + \sum_s q_s z_s$ as the total outflows that include the net trading of safe assets.[6]

[5] In Appendix A.4, we consider the possibility that aggregate shocks can also affect the cash flows from legacy assets and new investment, η_s and R_s. We show that these types of shocks do not change the baseline analysis in a significant way as long as η_s and R_s scale proportionally across states—which we view as a neutral assumption in our setting. In this case, the available liquidity also scales proportionally with η_s and R_s across states. In particular, the fire-sale price to return ratio, p_s/R_s, remains constant and the analysis becomes similar to the case without aggregate uncertainty.

[6] In the previous section, this distinction was not important since the safe assets and foreign investment were perfect substitutes in equilibrium, and we focused (without loss of generality) on symmetric equilibria in which the countries retained their safe asset endowments. The distinction will play some role when we revisit the reach-for-safety mechanism in Section 3.2.

The market clearing conditions can be written as,

$$\begin{aligned} p_s^j &= \min\left(R, \frac{R_f y^j + z_s^j}{e + x^{\text{in},j}}\right) \text{ for each } s \in S, \\ \int y^j dj &= \eta / R_f, \\ \int z_s^j dj &= \int x^{\text{in},j} \overline{R}_s^j dj \text{ for each } s \in S. \end{aligned} \tag{13}$$

Here, the first two equations are the analogs of the earlier market clearing conditions. The third equation is a new condition that says that the total amount of traded Arrow-Debreu securities is equal to the amount of financial payoffs from foreign investment in state s. There is also an aggregate resource constraint at date 0 which can be written as, $\int_j \left(c_0^j + x^{loc,j} + x^{\text{in},j}\right) dj = 1$. By Walras' law, this resource constraint is satisfied when all of the budget constraints hold in equilibrium.

An equilibrium with aggregate shocks is a collection of allocations, $\left(c_0^j, x^{loc,j}, y^j, \left[\left\{z_s^j\right\}_s\right]_j\right), \left[x^{\text{in},j}\right]_j$, and prices, $\left(R_f, \{q_s\}_s, \left[\left\{p_s^j\right\}_s\right]_j\right)$, such that the financial intermediaries' optimality condition (11) holds, the investors' allocations solve problem (12), and the market clearing conditions (13) hold.

The characterization of equilibrium closely parallels the baseline model without aggregate shocks. As before, we focus on symmetric equilibrium allocations and prices, which we denote by dropping the superscript j. Note that the aggregate resource constraint implies the inflows and risky outflows are equated in equilibrium, $x^{\text{in}} = \sum_s q_s z_s = x^{\text{out}}$ (which is also equal to $\overline{x}^{\text{out}}$). To ensure continuity with the earlier analysis, we use the notation $x = x^{\text{in}} = x^{\text{out}}$ to denote these symmetric flows.

As before, the symmetry implies the safe asset holdings are given by, $y = \eta / R_f$. The local investors' budget constraint can then be written as, $c_0 + x^{\text{loc}} + x = 1$. It can also be seen that (as before) local investment is dominated, $x^{\text{loc}} = 0$ (since there are fire sales in view of Assumption 1). The optimality conditions for problem (12) then imply,

$$u'(1-x) = \frac{\gamma_s}{q_s} M_s \text{ for each } s \in S. \tag{14}$$

Combining this with by Eq. (11), which holds as equality, we obtain,

$$u'(1-x) = \sum_s \gamma_s \overline{R}_s M_s \equiv \sum_s \gamma_s \mu_s(p_s). \tag{15}$$

Here, $\mu_s(p_s) \equiv ((1-\pi_s) R + \pi_s p_s) \left(1 - \pi_s + \frac{R}{p_s} \pi_s\right)$ denotes the marginal benefit from foreign investment conditional on state s [cf. Eq. (4)]. Using the market clearing conditions (13) and symmetry, we also obtain,

$$p_s = \min\left(R, \frac{\eta + x\overline{R}_s}{e+x}\right) = \min\left(R, \frac{\eta + x(1-\pi_s)R}{e + x(1-\pi_s)}\right). \tag{16}$$

The equilibrium is characterized by Eqs. (15) and (16), which are the analogs of Eqs. (5) and (6) in this setting. It can be checked that there is a unique symmetric equilibrium with $x \in (\underline{x}, 1)$ and $p_s < R$ for each s. Using Eq. (16), note also that the price, p_s, is decreasing in s: that is, states with greater financial distress (in terms of the likelihood of crises) are associated with lower prices. Likewise, the expected payoff from foreign assets, $\overline{R}_s = (1-\pi_s)R + \pi_s p_s$, is decreasing in s.

Given the flows and fire-sale prices, (x, p), Eq. (14) determines the Arrow-Debreu prices in financial markets. Note that the price-to-probability ratio, q_s/γ_s, corresponds to the stochastic discount factor (SDF) for state s. Rewriting Eq. (14), we have,

$$q_s/\gamma_s = \frac{M_s}{u'(1-x)} = \frac{1}{u'(1-x)}\left(1 - \pi_s + \frac{R}{p_s}\pi_s\right).$$

Since π_s is increasing in s (Assumption 2) and p_s is decreasing in s, we also have that the SDF is increasing in s. That is, the states with greater financial distress are associated with more expensive Arrow-Debreu asset (insurance) prices. We also calculate the risk-free rate as,

$$R_f = \frac{1}{\sum_s q_s} = \frac{u'(1-x)}{E[M_s]} = \frac{E\left[\overline{R}_s M_s\right]}{E[M_s]}. \tag{17}$$

Here, the second equality follows from Eq. (14) and the third equality follows from Eq. (15). Using this expression, we also calculate the risk premium on financial assets (which can be viewed as a liquidity premium) as,

$$E\left[\overline{R}_s\right] - R_f = \frac{-cov\left(M_s, \overline{R}_s\right)}{E[M_s]}. \tag{18}$$

Note that the covariance term is negative since expected asset payoff, $\overline{R}_s$, is decreasing in s, whereas the marginal utility, M_s (which is proportional to the SDF), is increasing in s. Thus, with aggregate liquidity risk, the risk premium on foreign financial assets is strictly positive. Intuitively, the value of the foreign assets is reduced by the fact that they pay relatively less when the liquidity is relatively scarce. The following result summarizes this discussion.

Proposition 4. *Consider the symmetric model with aggregate risk (with Assumptions 1 and 2). There exists a unique symmetric equilibrium, $\left(\left(c_0, x^{loc}, y, \{z_s\}\right), \{q_s, p_s\}_s, x\right)$, which satisfies $c_0 = 1 - x, x^{loc} = 0, x > \underline{x}$, and fire-sale prices, $p_s < R$. The tuple $\left(x, (p_s)_s\right)$ is characterized by Eqs. $(15-16)$. The fire-sale price, p_s, is decreasing in s: that is, more distressed states with greater likelihood of liquidity shocks are associated with lower prices. The state price-to-probability ratios, $\{q_s/\gamma_s\}_s$, are characterized by Eq. (14) and are increasing in s (the degree of financial distress). The risk-free return is characterized by Eq. (17). The risk premium on* *foreign assets is positive and characterized by Eq.* (18).

3.1. Correlated Shocks and the Global Liquidity Cycle

We next use a special case of the model with correlated liquidity shocks to show that changes in correlations can naturally generate a global liquidity cycle (e.g., Calvo et al. (1996); Forbes and Warnock (2012); Rey (2015)). To this end, suppose there are three aggregate states, $s \in \{1, 2, 3\}$, that feature,

$$\pi_1 = 0 < \pi_2 = \pi < \pi_3 = 1, \tag{19}$$

for some $\pi \in (0, 1)$. In particular, state $s = 2$ corresponds to the state in the baseline analysis in which the liquidity shocks are i.i.d. across the regions. States $\{1, 3\}$ together represent a "correlated shock" state in which the liquidity shocks are perfectly correlated across the countries. Specifically, either all countries are hit (state 3) or no country is hit (state 1). We also assume the state probabilities are given by,

$$\gamma_1 = \beta(1 - \pi), \gamma_3 = \beta\pi \text{ and } \gamma_2 = 1 - \beta. \tag{20}$$

Here, the parameter β captures the extent to which the shocks are correlated—controlling for everything else in the model. The case, $\beta = 0$, corresponds to the model in the previous section with i.i.d. shocks, whereas the case $\beta = 1$ corresponds to the other limit in which the liquidity shocks are always correlated.

Note also that $\mu_1(p) = R$ and $\mu_3(p) = p \times \frac{R}{p}$ [cf. Eq. (4)]. Thus, Eq. (15) becomes,

$$u'(1 - x) = \beta R + (1 - \beta)\mu_2(p_2). \tag{21}$$

The market clearing conditions (16) imply,

$$p_2 = \frac{\eta + x(1 - \pi) R}{e + x(1 - \pi)}.$$

The last two equations determine the pair, (x, p_2). By inspecting the equations it can be seen that increasing β reduces x. Intuitively, as liquidity shocks become more correlated, the liquidity-provision benefit from capital flows declines. Note also that an increase in correlations, β, reduces the fire-sale prices even in the i.i.d. state, p_2, in view of the reduction in capital flows, x.

Using the market clearing condition (16), we also calculate the price in state 3 (with $\pi_3 = 1$) as,[7] $p_3 = \frac{\eta}{e}$. This is also the average fire-sale price conditional on a liquidity shock in the aggregate shock state, $\{1, 3\}$ (since $\pi_1 = 0$ and $\pi_3 = 1$). Note that we have, $p_3 < p_2$: that is, the aggregate shock state features deeper fire sales than the i.i.d. state. Intuitively, the aggregate

[7] We could similarly calculate the fire-sale price in state 1 as $p_1 = \frac{\eta + xR}{e + x}$. This price does not play any role in the analysis since $\pi_1 = 0$, that is, the liquidity shock happens with zero probability in state 1.

shock state has as much aggregate liquidity on average but this liquidity is not distributed appropriately across the states (state 3 has too little and state 1 has too much of it).

Next consider the effect of β on the expected return on foreign assets,

$$\begin{aligned}E\left[\overline{R}_s\right] &= \beta\left(\left(1-\pi\right)R+\pi p_3\right)+\left(1-\beta\right)\left(\left(1-\pi\right)R+\pi p_2\right)\\ &= \left(1-\pi\right)R+\pi\left(\beta p_3+\left(1-\beta\right)p_2\right).\end{aligned}$$

This expression implies that increasing the correlations reduces $E\left[\overline{R}_s\right]$, because it decreases the (unconditional) average fire-sale price, $\beta p_3+\left(1-\beta\right)p_2$.

Next consider the effect of β on the risk-free return. Using Eq. (17), we have,

$$R_f=\frac{E\left[\overline{R}_s M_s\right]}{E\left[M_s\right]}=\frac{\beta R+\left(1-\beta\right)\left(\left(1-\pi\right)R+\pi p_2\right)\left(1-\pi+\pi\frac{R}{p_2}\right)}{\beta\left(1-\pi+\pi\frac{R}{p_3}\right)+\left(1-\beta\right)\left(1-\pi+\pi\frac{R}{p_2}\right)}.$$

The last expression is decreasing in β (since the numerator is decreasing and the denominator is increasing in β). Intuitively, as the liquidity shocks become more correlated, the risk-free asset becomes more valuable as it provides liquidity when there is a global liquidity shock. Finally, consider the risk premium on foreign financial investment, $E\left[\overline{R}_s\right]-R_f$ [cf. (18)]. Since the expected return on foreign asset as well as the risk-free asset decline, the effect on the risk premium is in general ambiguous. However, recall that the risk premium is zero for $\beta=0$ (see Section 2) and becomes strictly positive for any $\beta>0$ (see Eq. (18)). Thus, the risk premium is increasing in the neighborhood of $\beta=0$. The following result summarizes this discussion.

Proposition 5. *Consider the symmetric model with the possibility of correlated liquidity shocks. Increasing β (so that the shocks become more correlated) reduces the capital flows, x, and reduces fire-sale asset price in the i.i.d. state, p_2. It reduces the expected return on foreign financial assets, $E\left[\overline{R}_s\right]$, as well as the risk-free interest rate, R_f. In the neighborhood of $\beta=0$, it also increases the risk premium on foreign assets, $E\left[\overline{R}_s\right]-R_f$.*

An increase in β in this model can be thought of as capturing a "risk-off" environment in which the investors retrench into their home markets (even at date 0, before the crises are realized). This reduces the capital flows and liquidity creation, while also reducing the risk-free rate and increasing the risk premia. This result is consistent with the large decline in gross capital flows in the aftermath of the Global Financial Crisis (see Bluedorn et al. (2013) and Lane and Milesi-Ferretti (2012); Milesi-Ferretti and Tille (2011)). From the lens of our model, the global crisis increased the (real or perceived) correlations of financial crises, which in turn reduced the usefulness and the magnitude of gross capital flows. In the next section, we will also analyze how this type of switch to a "risk-off" environment affects the reach-for-safety and the reach-for-yield mechanisms that we introduced in Section 2.

3.2. Reach for Safety with Aggregate Shocks

We next revisit the reach-for-safety mechanism we introduced in Section 2.2 in the presence of aggregate liquidity shocks. To this end, suppose a developed country j (that has measure zero) has greater liquidity than the world average, $\eta^j > \eta$. As before, suppose Assumption S holds so that the autarky equilibrium in the country would feature no fire sales, $p_s^j = R$ for each $s \in S$.

Consider the equilibrium with free capital flows. The world equilibrium is the same as in Section 3. However, the equilibrium allocations in country j are different. In particular, the optimality conditions for foreign investment implies the following analogue of Eq. (8),

$$1 = \sum_s q_s \left((1 - \pi_s) R + \pi_s p_s \right) = \sum_s q_s \left((1 - \pi_s) R + \pi_s p_s^j \right). \tag{22}$$

This equation implies $\overline{p}^j = \overline{p}$, where we define the (price-)weighted average fire-sales as respectively,

$$\overline{p} = \frac{\sum_s q_s \pi_s p_s}{\sum_s q_s \pi_s} \text{ and } \overline{p}^j = \frac{\sum_s q_s \pi_s p_s^j}{\sum_s q_s \pi_s}. \tag{23}$$

As before, the country receives positive net inflows and cannot escape fire-sales "on average" even though it would not feature fire sales in autarky.

This leaves open the possibility that the volatility of the fire-sale prices in country j could be lower than in the representative country. In fact, a naive look at the market clearing condition (16) could suggest that country j would experience relatively less severe fire sales in states with greater s, as its large endowment of the safe asset would provide some cushion from the declines in aggregate liquidity. This prediction turns out to be incorrect. To see this, note that the local investors' optimality condition is given by,

$$u'\left(c_0^j\right) = \frac{M_s^j}{q_s/\gamma_s} \text{ for each } s \in S, \text{ where } M_s^j = 1 - \pi_s + \pi_s \frac{R}{p_s^j}. \tag{24}$$

This equation, together with Eq. (22), represents a system of $|S| + 1$ equations in $|S| + 1$ unknowns, where the unknowns are the prices $\left\{p_s^j\right\}_s$ and consumption, c_0^j. The unique solution is given by, $p_s^j = p_s < R$ for each s and $c_0^j = c_0$. In particular, the fire-sale price in country j is the same as in the representative country *state-by-state*.

The naive intuition is incorrect since the local investors do not retain their initial endowments of the safe asset. Rather, as captured by Eq. (24), they trade financial assets so as to move their liquidity across aggregate states. Recall also that the states with greater s command higher risk prices, q_s/γ_s, and that the country j has relatively large endowment of liquidity in these states. Thus, the local investors sell financial claims for states with higher s (and purchase financial claims for states with lower s). These financial trades ensure that the country's liquidity—and thus, fire-sale price—is in line with that in the representative country state-by-state.

We obtain additional insights by explicitly calculating the risks of the country's outflows relative to its inflows. Recall that $\overline{x}^{\text{out},j} = y^j - \eta^j/R_f + \sum_s q_s z_s^j$ denotes the date-0 value of the country's total outflows including its net trade of the safe asset. In the appendix, we show that $\overline{x}^{\text{out},j} = x$, that is, the country has the same size of outflows as the representative country. However, the outflows have a different risk composition. To see this, let $x_s^{\text{out},j} = R_f \left(y^j - \eta^j\right) + z_s^j$ denote the payoff from the outflows conditional in state s of date 1. We show that,

$$x_s^{\text{out},j} = -\left(l^j - 1\right) x + l^j x \overline{R}_s \text{ for each } s\text{, where } l^j > 1. \tag{25}$$

That is, the local investors can be thought of as selling some of their safe asset endowments to make a leveraged investment in foreign diversified portfolio. The variable, $l^j > 1$, is a measure of the leverage ratio in outflows: the value of the the risky investments the country undertakes relative to the value of its outflows. Note also that the date-1 payoff from the inflows is given by, $x_s^{\text{in},j} = x^{\text{in},j}\overline{R}_s$ for each s, that is, the leverage ratio in inflows is equal to one. Hence, Eq. (25) implies that the country's outflows are riskier than its inflows.

It follows that, in addition to having greater inflows than outflows as in Section 2.2, $x^{\text{in},j} > \overline{x}^{\text{out},j} = x$ (which continues to hold in this setting), the country also experiences relatively safe inflows and relatively risky outflows. This difference in the composition of flows is further destabilizing, and ensures that the country experiences the same (fire-sale) asset price volatility as the representative country. These results are consistent with the empirical work by Gourinchas and Rey (2007); Gourinchas et al. (2010), who document that the outflows of the U.S. are riskier than its inflows. They also show that the U.S. earns a risk premium on capital flows in normal times, but it transferred resources and provided insurance to the rest of the world during the Global Financial Crisis. Our model suggests that these transfers are likely to have exacerbated the severity of the GFC in the U.S., while mitigating its impact in the countries that held the (relatively) safe U.S. assets.

We also analyze how the global liquidity cycle affects the level and the risk composition of the country's net inflows. To this end, consider the special case of the model with aggregate shocks described in Section 3.1. Suppose β increases so that the shocks become more correlated. As captured by Proposition 5, this decreases the symmetric flows, x, as well as the risk-free rate, R_f. In the appendix, we show that $x^{\text{in},j} - \overline{x}^{\text{out},j}$ increases: that is, the country's inflows decline less than its outflows, $\overline{x}^{\text{out},j} = x$. Furthermore, we also show that the leverage ratio of the country's outflows, l^j, increases. Intuitively, the "risk-off" induced by the increase in β makes international liquidity scarce and increases the value of safe assets that provide liquidity in global distress states. This increases the relative inflows into the developed country j, while also inducing the country to undertake foreign investment with a greater leverage ratio. The following result summarizes this discussion.

Proposition 6. *Consider the setting with aggregate risk in which a country has abundant local liquidity, $\eta^j > \eta$, that satisfies Assumption S (so that it would not experience fire sales in autarky). In an equilibrium with free financial flows, the country receives more inflows than* *its outflows, $x^{in,j} > \overline{x}^{out,j}$, and experiences fire sales with prices that are equal to those in the representative country, $p_s^j = p_s < R$ for each s. The country's outflows are riskier than its inflows, and they can be replicated as in* (25) *where $l^j > 1$ captures the leverage ratio in outflows.*

In the special case with correlated liquidity shocks, increasing β (so that the shocks become more correlated) reduces the outflows, $\overline{x}^{out,j}$, increases the inflows relative to outflows, $x^{in,j} - \overline{x}^{out,j}$, and increases the leverage ratio, l^j.

3.3. Reach for Yield with Aggregate Shocks

We next revisit the reach-for-yield mechanism we introduced in Section 2.3 in the presence of aggregate liquidity shocks. For simplicity, we focus on the special case with correlated liquidity shocks described in Section 3.1. In this setting, suppose an emerging market country j (that has measure zero) has greater fundamental return than the world average, $R^j > R$. As before, suppose also that the country has relatively low liquidity, $\eta^j \leq \eta$. We also modify Assumption Y as follows.

Assumption $\tilde{\text{Y}}$. $R^j - R \in \left(0, \overline{p}\frac{\sum_s q_s \pi_s}{\sum_s q_s(1-\pi_s)}\right)$ and $\eta^j - \eta \in \left[-x\frac{\sum_s q_s p_s}{\sum_s q_s}, 0\right]$.

When $x^{\text{in},j} > 0$ (which will be the case in equilibrium), foreign intermediaries' optimality condition implies,

$$\sum_{s\in\{1,2,3\}} q_s \left(\left(1-\pi_s\right) R + \pi_s p_s\right) = \sum_{s\in\{1,2,3\}} q_s \left(\left(1-\pi_s\right) R^j + \pi_s p_s^j\right). \tag{26}$$

Note that this equation implies $\overline{p}^j < \overline{p}$ (since $R^j > R$), where the weighted average fire-sales are defined in (23). Hence, as in Section 2.3, the country with higher return receives positive net inflows and experiences greater fire-sales "on average."

In the appendix, we complete the characterization of equilibrium and show that the country experiences deeper fire sales relative to the representative country in both distress states, $s \in \{2,3\}$. We further show that the relative depth of fire sales in country j is greater in the idiosyncratic shock state than in the aggregate shock state, that is, $\frac{R^j/p_2^j}{R/p_2} > \frac{R^j/p_3^j}{R/p_3} > 1$. Intuitively, since the crises are more frequent in state 3, the local investors in country j purchase relatively more liquidity for this state than in state 2. This helps to mitigate somewhat the fire sales caused by the reach-for-yield inflows in state 3 (the global crisis), at the expense of deepening the fire sales in state 2 (the local crises).

We next establish the comparative statics of the reach-for-yield mechanism. To this end, we combine the optimality condition (23) with Eqs. (23), (14), (19), and (20) to obtain,

$$\left(R^j - R\right)\left(1-\pi\right)\left(M_1\beta + M_2\left(1-\beta\right)\right) = \left(\overline{p} - \overline{p}^j\right)\pi\left(M_2\left(1-\beta\right) + M_3\beta\right) \tag{27}$$

This equation illustrates that investing in country j as opposed to other countries represents a trade-off between crisis and non-crisis states. The investors collect net positive returns if there is no crisis (captured by the left side), but they make net negative returns if there is a crisis (captured by the right side). The net gains are multiplied by the probability of no crisis $(1-\pi)$ and the average marginal utility conditional on no crisis. The net losses are calculated in similar fashion. The equilibrium obtains when the weighted net gains and the net losses are equated.

Eq. (27) shows that, all else equal, a decline in investment returns in other countries, R, makes investing in country j more attractive. In equilibrium, this tends to decrease the relative fire-sale price in country j—so as to counter the greater net gains with greater net losses conditional on a crisis. However, the result does not immediately follow since the marginal utilities are also endogenous and depend on R. In the appendix, we show that (for the model with three states) the endogenous effect reinforces the direct effect. In particular, a decrease in R decreases $\overline{p}^j - \overline{p}$, generalizing Proposition 3.

Eq. (27) also shows that, all else equal, a decline in the correlation parameter, β, makes investing in country j relatively more attractive: it decreases the weighting term on the right (loss) side while increasing the weighting term on the left (gain) side since $M_1 < M_2 < M_3$. In equilibrium, this reduces the relative fire-sale price in country j. As before, the result does not immediately follow since the marginal utilities are endogenous. In the appendix we show that the endogenous effect mitigates but does not overturn the direct effect. In particular, an increase in β increases $\overline{p}^j - \overline{p}$, that is, it shrinks the gap between the fire-sale prices in country j and the representative country.

It follows that a decline in R (low global returns) as well as a decline in β (risk-on) strengthens the reach-for-yield mechanism. The intuition for the latter effect is that investing in high-yielders makes losses during local crises, and these losses are less costly when the local crises are less correlated with aggregate distress states. The following proposition summarizes this discussion.

Proposition 7. *Consider the special case of the aggregate risk model with correlated liquidity shocks, in which a country has relatively high return, $R^j > R$, and Assumption $\tilde{Y}$ holds. In an equilibrium with free financial flows, the country experiences deeper fire sales than in the representative country in both distress states, but less so in the more distressed state; that is, $\frac{R^j/p_2^j}{R/p_2} > \frac{R^j/p_3^j}{R/p_3} > 1$. A decrease in R as well as a decrease in β reduces the country's relative weighted average fire-sale price, $\overline{p}^j - \overline{p}$.*

4. Optimal Policy with Symmetric Flows

In the rest of the paper we analyze the policy implications of our analysis. In this section we consider optimal capital restriction policy in the baseline model with symmetric capital flows. We show that a global planner that is concerned with financial stability encourages capital flows, in view of their liquidity creation benefits, but local planners restrict capital flows. We also analyze the liquidity-injection policies by which the planners can mitigate crises, and show that these policies are subject to a similar coordination problem. In the next section we revisit the optimal capital restriction policy in asymmetric environments that feature the reach-for-safety and the reach-for-yield mechanisms.

Recall that fire sales are costly in our setting since they reduce the financing available to entrepreneurs, each of which sells one unit of the asset at date 1 to reinvest. To analyze welfare, we need to be more specific about entrepreneurs' investment technology. We assume entrepreneurs come in two varieties that differ in the type of their projects. A fraction, ζ, of them have a project with decreasing returns to scale: Investing p dollars in this project at date 1 yields $\lambda f(p)$ dollars at date 2. Here, $f(\cdot)$ is an increasing and strictly concave function. The remaining fraction, $1-\zeta$, of entrepreneurs have a project with constant returns to scale: Investing p dollars yields λp dollars at date 2. Here, the parameter λ captures the strength of the financial stability concerns (the benefits from mitigating fire sales and increasing p). The concave function, $f(\cdot)$, captures that the the marginal benefit from financial stabilization will be greater when the prices are lower and the fire sales are deeper. The distinction between entrepreneurs with diminishing and linear scales is not important, but it provides an additional level of generality that helps to simplify some of the expressions in our optimal policy analysis.

We also suppose the planner in each country is utilitarian: she maximizes the sum of the local investors' and the local entrepreneurs' expected utilities. Since all agents are risk neutral, the social welfare function for the planner in country j can be written as,

$$W^j = u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + \lambda e \left(\begin{array}{c} \zeta\left((1-\pi) f(R) + \pi f\left(p^j\right)\right) \\ +(1-\zeta)\left((1-\pi) R + \pi p^j\right) \end{array} \right). \tag{28}$$

We focus on the special case in which the financial stability concerns are very important, $\lambda \to \infty$. In this case, the planner effectively maximizes the output per entrepreneur,

$$W^j/(\lambda e) \to \zeta\left((1-\pi) f(R) + \pi f\left(p^j\right)\right) + (1-\zeta)\left((1-\pi) R + \pi p^j\right), \tag{29}$$

which is increasing in the local fire-sale price level, p^j. In Appendix A.2, we analyze the more general case with finite λ. As our analysis there illustrates, there are in fact other welfare considerations in this model, but we envision a regulatory environment in which those considerations are dominated by concerns with financial stability.

We next investigate the desirability of various policies in settings with symmetric flows. In each setting, we consider the policies that would be chosen by a global planner that could coordinate the decisions of individual planners, and compare this outcome with the Nash equilibrium that would obtain absent coordination. We assume the global planner maximizes the sum of individual planners' objectives, $\int_j \left(W^j/(\lambda e)\right) dj$. In a symmetric equilibrium, this amounts to maximizing each individual planner's objective. We start by analyzing the desirability of capital taxes targeted towards reducing the inflows ex ante. We then analyze the desirability of liquidity injection policies targeted towards mitigating crises ex post.

4.1. Capital Restrictions with Symmetric Flows

Consider the baseline model in Section 2.1 with the only difference that the planner in each country j can impose a linear tax, $\tau^j \geq 0$, on the short-term return on foreign inflows: that is, the return on the foreign financial holdings in country j is now given by $\overline{R}\left(1-\tau^j\right)$. We assume that the tax revenues are used to purchase an equal-weighted portfolio of all financial assets. The assets that are purchased are then wasted by the planner. The latter assumption ensures that expropriating foreigners is not the rationale behind taxing capital flows. The former assumption (asset purchases) ensures that the liquidity that the government collects via taxation is injected back into the financial markets in equal proportion so that the government taxation does not directly waste liquidity. This leads to simpler expressions, but our results continue to hold if we instead assume the government wastes the tax revenues without purchasing assets.[8]

Coordinated policy. To analyze the optimal coordinated policy, consider the equilibrium in which all countries apply the same tax rate, $\tau \geq 0$. The analysis is similar to Section 2 with minor differences. One caveat is that foreign investment does not necessarily dominate local investment at date 0 since foreign investment is taxed. In the appendix, we show that the equilibrium behavior depends on a threshold tax level, $\bar{\tau}$. If the tax level is above the threshold, $\tau \geq \bar{\tau}$, then there is zero foreign investment, $x = 0$, and the fire-sale price level is given by $p = \eta/e$. If instead the tax level is below the threshold, $\tau < \bar{\tau}$, then there is positive foreign investment, $x > 0$. In this case, the equilibrium conditions can be written as,

$$
\begin{aligned}
&R_f = \overline{R}\left(1-\tau\right), \qquad\qquad (30)\\
&\begin{cases} u'\left(1-x\right) = \mu\left(p\right)\left(1-\tau\right) & \text{if } \mu\left(p\right)\left(1-\tau\right) > R \\ x \in \left[0, \underline{x}\right] & \text{if } \mu\left(p\right)\left(1-\tau\right) = R \end{cases},\\
&\text{and } p = \min\left(R, \frac{\eta + x\overline{R}\left(1-\tau\right) + x\overline{R}\tau}{e+x}\right) = \min\left(R, \frac{\eta + x\left(1-\pi\right)R}{e + x\left(1-\pi\right)}\right).
\end{aligned}
$$

These conditions are the analogues of respectively Eqs. (7), (6) and (5) in Section 2.1. The first two equations are adjusted for the presence of taxes. The market clearing condition is unchanged in view of the assumption that the taxes taken away by the planner are injected back into the market, as illustrated by the equation.

Figure 2 plots the analogs of the optimality and the market clearing curves, which we now denote by $p = P^{opt}\left(x;\tau\right)$ and $p = P^{mc}\left(x\right)$. Note that introducing (or increasing) taxes shifts the optimality curve downward and leads to lower capital flows, x. The threshold tax, $\bar{\tau}$, is the level which ensures the optimality and market clearing curves intersect at $x = 0$. As the figure illustrates, there is also a lower threshold tax, $\underline{\tau} < \bar{\tau}$, which ensures that the two curves intersect at $\underline{x} > 0$. If $\tau < \underline{\tau}$, the local investment is zero, $x^{\text{loc}} = 0$, and the foreign flows exceed $\underline{x}$. If $\tau \in \left(\underline{\tau}, \bar{\tau}\right)$, then the equilibrium is in the flat part of the optimality curve: that is, there is some

[8] In fact, the results become stronger since the alternative specification creates a second channel by which capital taxes reduce global liquidity and asset prices.

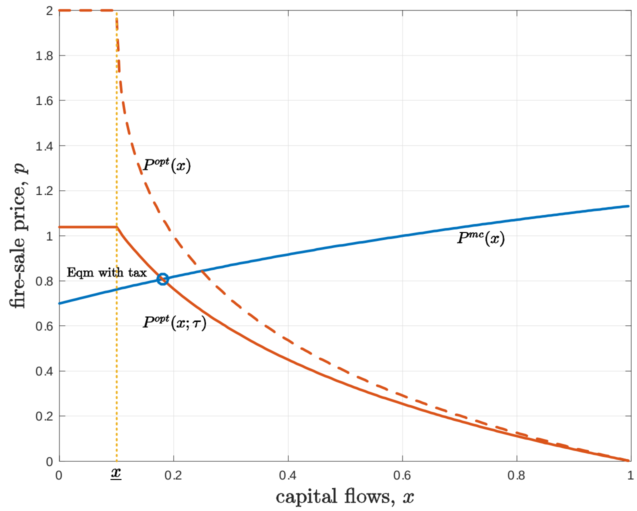

Figure 2: The characterization of equilibrium with taxes. The dashed line replicates the baseline characterization without taxes.

local investment, $x^{\text{loc}} > 0$, and the foreign flows satisfy, $x = \underline{x} - x^{\text{loc}} > 0$.

Figure 2 also illustrates that, in either case, increasing taxes leads to a lower fire-sale price level, p. Intuitively, capital taxation discourages foreign flows. This in turn decreases global liquidity and the fire-sale price in local distressed markets, because for symmetric flows retrenchment dominates fickleness as we discussed earlier. It follows that a global planner that coordinates countries' policies and that focuses on increasing the fire-sale price level sets zero tax on capital inflows. The following result summarizes this discussion.

Proposition 8. *Consider the symmetric model with capital taxes in the limit as $\lambda \to \infty$ (financial stability concerns are dominant). There exists a threshold tax level $\bar{\tau} > 0$ such that, for each $\tau < \bar{\tau}$, there are positive capital flows, $x > 0$, and the equilibrium is characterized as the solution to the system in* (30)*. There also exists a lower threshold tax level $\underline{\tau} \in (0, \bar{\tau})$ such that, for each $\tau < \underline{\tau}$, the local investment is dominated, $x^{loc} = 0$, and the foreign flows satisfy, $x > \underline{x}$. Increasing the symmetric tax level, τ, reduces the capital flows, x, and decreases p and R_f. A global planner that coordinates countries' policies sets zero tax on capital inflows, $\tau = 0$.*

Nash equilibrium. We next analyze the uncoordinated outcomes that would emerge in a Nash equilibrium in which each planner chooses its own policy taking the policies in other countries as given. To this end, consider the optimal tax rate for an individual country, $\tau^j \geq 0$, when all other countries apply the same tax rate, τ. To keep the analysis simple, suppose the taxes cannot be increased above the lower threshold characterized above, that is, $\tau^j \leq \underline{\tau}$ for each j (the case with $\tau^j > \underline{\tau}$ is slightly more complicated but does not offer much additional insight). We will establish that the only Nash equilibrium is one in which all countries set the highest allowed tax level, $\tau^j = \underline{\tau}$.

To show this, suppose the common tax level is strictly below the threshold, $\tau < \underline{\tau}$. Consider a country j that deviates and sets a potentially the tax level, τ^j. For sufficiently small deviations, the foreign investors' optimality condition can be written as,

$$R_f = \overline{R}(1-\tau) = \overline{R}^j \left(1-\tau^j\right), \text{ where } \overline{R}^j = (1-\pi) R + \pi p^j.$$

Inspecting this condition, it follows that increasing τ^j (in a neighborhood of τ) increases p^j. Intuitively, greater taxes discourage foreign inflows and increase p^j. This process continues until p^j is sufficiently high to convince the foreigners to invest in the country. Since the local planner prefers a higher local price level p^j, it follows that there is a profitable deviation as long as the symmetric tax level is below its upper bound, $\tau < \underline{\tau}$. Hence, the unique symmetric Nash equilibrium features the highest allowed tax level, $\tau = \underline{\tau}$. Our next result summarizes this discussion.

Proposition 9. *Consider the symmetric model with capital taxes in the limit as $\lambda \to \infty$ (with the restriction that $\tau^j \in [0, \underline{\tau}]$). There exists a unique Nash equilibrium with symmetric allocations in which the individual planners set the highest allowed tax level, $\tau^j = \underline{\tau}$ for each j. The capital flows, $x = \underline{x}$, the fire-sale price, p, and the risk-free return, R_f, are lower than what would obtain in an equilibrium without taxes.*

Comparing this result with Proposition 8 illustrates that the uncoordinated Nash equilibrium generates a highly inefficient outcome at the global level. The Nash equilibrium features the highest allowed level of capital taxes, whereas the globally efficient solution features zero taxes. Intuitively, a country that taxes capital inflows improves its own financial stability at the expense of reducing the global liquidity and exacerbating fire sales in other countries. The country does not take into account the negative externalities it causes on other countries by reducing global liquidity. This leads to protectionist capital policies that are inefficient at the global level.

4.1.1. Complementarities in Capital Restrictions

The result that the Nash equilibrium exhibits the highest allowed tax level helps to illustrate our point sharply. However, it is extreme and it also prevents us from analyzing how the capital market policies in one country react to other countries' policies or exogenous changes. To analyze these issues, we next consider a version of the model with convex costs of taxation, which ensure that the optimal tax level is interior.

Suppose the capital taxes cannot be targeted perfectly, and some of the taxes also fall on the entrepreneurs. Since entrepreneurs sell assets to undertake productive projects, these costs reduces the planner's welfare even as $\lambda \to \infty$. More specifically, suppose applying a tax $\tau \geq 0$ on the foreign capital reduces the returns of the entrepreneurs that have linear scale by $v(\tau) \geq 0$. Then, as $\lambda \to \infty$, the planner effectively maximizes the following analogue of the objective function in (29),

$$(1-\zeta)\left((1-\pi) f(R)+\pi f\left(p^j\right)\right)+\zeta \overline{R}^j\left(1-v\left(\tau^j\right)\right). \tag{31}$$

To ensure an interior solution, suppose the cost function $v(\cdot)$ is strictly increasing and convex and satisfies $v(0)=v'(0)=0$ and $v'(\tilde{\tau})=\infty$ for some $\tilde{\tau}\in(0,\underline{\tau})$.

We view the cost function as capturing in reduced form various difficulties associated with restricting capital flows in practice. The objective function in (31), together with the concavity of the function $f(\cdot)$, captures the idea that the planner has greater incentives to restrict flows when the fire-sale prices are lower. In particular, in the appendix, we characterize the optimal tax level as the unique solution to,

$$V\left(\tau^j\right)=\frac{(1-\zeta) f'\left(p^j\right)+\zeta}{\zeta}, \text{ where } V\left(\tau^j\right)=v'\left(\tau^j\right)\left(1-\tau^j\right)+v\left(\tau^j\right). \tag{32}$$

Here, $V(\tau)$ is an increasing function over $[0,\tilde{\tau})$ with $V(0)=0$ and $V(\tilde{\tau})=\infty$. Note that a lower price level, p^j, induces a greater tax level, τ^j, because it increases the (local) benefits of taxation more than its costs.

A symmetric Nash equilibrium is a pair, (p,τ), that satisfies the earlier competitive equilibrium conditions (30) as well as the individual planners' optimality condition (32). Recall that the competitive equilibrium describes a decreasing relation between p and τ: that is, greater taxes reduce global liquidity and fire-sale prices. The optimality condition also establishes a decreasing relation between τ and p. These observations lead to the following result.

Proposition 10. *Consider the symmetric model with costly capital taxes in the limit as* $\lambda\to\infty$.

(i) There can be multiple symmetric Nash equilibria. When this is the case, the equilibrium with a lower price level leads to lower welfare for all planners.

(ii) Suppose the parameters are such that there is a unique Nash equilibrium (or consider the neighborhood of any stable equilibrium). Reducing the local liquidity, η*, increases the equilibrium tax level,* τ*, and reduces the price,* p*, as well as the risk-free return,* R_f*. Moreover, the price and the risk-free return decline more than the alternative case in which the taxes are kept at their pre-change levels.*

The intuition follows from observing that capital restriction policies are strategic complements. A country that sets a more restrictive policy reduces global liquidity. This leads to lower fire-sale prices in other countries. The low fire-sale prices not only reduce the welfare of other planners (in view of the externality that we discussed earlier), but they also induce those planners to set more restrictive policies. When these complementarities are sufficiently strong, there can be multiple equilibria. Even when there is a single equilibrium, the complementarities amplify the impact of exogenous shocks that reduce liquidity.

We illustrate these results using a numerical example. Suppose the utility from consumption is given by $u(c_0)=h\log c_0$, with $h\in(0,R)$. Suppose the cost function takes the form, $v(\tau)=-k\left(\log\left(\frac{\tilde{\tau}-\tau}{\tilde{\tau}}\right)+\frac{\tau}{\tilde{\tau}}\right)$ for some $k>0$, which satisfies the regularity conditions over $\tau\in[0,\tilde{\tau}]$.

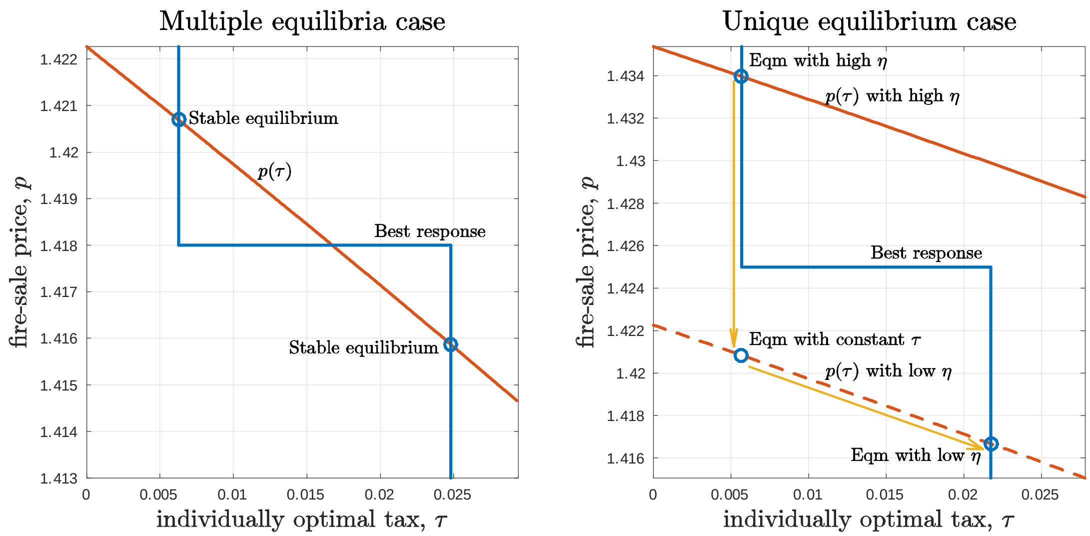

Figure 3: The left panel illustrate the equilibria with costly capital taxes for a parameterization that generates multiple equilibria. The right panel illustrates the parameterization with a unique equilibrium. The shift form the solid line to the dashed line captures the effect of decreasing local liquidity, η.

Suppose the entrepreneurs' production function takes the piecewise-linear form,[9]

$$f(p) = \begin{cases} ap, & \text{for } p \leq \overline{p} \\ bp, & \text{for } p > \overline{p} \end{cases}, \text{ for some } a > b > 0, \text{ and } \overline{p} \in (0, R).$$

The left panel of Figure 3 illustrates the possibility of multiple equilibria using a particular parameterization. The straight decreasing line plots the equilibrium price as a function of the tax. The jagged decreasing line plots the planner's optimal tax choice as a function of the price. The two intersections illustrate the stable equilibria. If the price is above the threshold, $\overline{p}$, financial stability concerns are not too significant and the planners set relatively low taxes. This leads to high global liquidity and supports fire-sale prices that are above the threshold. However, if the price falls below the threshold, $\overline{p}$, then financial stability concerns become more important, which induces the planners to set high taxes. This leads to a reduction in global liquidity and leads to fire-sale prices below the threshold. Note that the equilibrium with the higher tax and the lower price is dominated: it yields a lower utility for each planner than the other equilibrium.

The right panel of Figure 3 illustrates the amplification mechanism using a parameterization

[9] This function violates the regularity conditions on $f(\cdot)$ (e.g., it is not stricltly concave) but it can be made to satisfy the conditions after some smoothing and it helps to illustrate the result sharply.

that leads to a unique equilibrium. The solid and the dashed lines plot the equilibrium price function with respectively higher and lower local liquidity, η. If the tax level was exogenously fixed, a reduction in local liquidity would reduce the price level as formalized in Proposition 1. When the tax level is endogenous, the price declines even more. In this example, the exogenous liquidity shock reduces the price below the threshold below which the financial stability concerns increase. This leads to higher taxes and lower prices.

In Appendix A.5, we also analyze how introducing aggregate shocks affects our analysis of capital taxation in this section. Specifically, we allow the planners to set state-contingent taxes, $\{\tau_s\}_s$, in the setting with aggregate shocks introduced in Section 3. We show that the tax rate is positive for each state, $\tau_s > 0$ for each $s \in S$, generalizing the results in this section. We also show that τ_s is increasing in $s \in S$: that is, states with greater probability of liquidity shocks are associated with higher taxes. For intuition, recall from Section 3 that the foreign investors value payoff in distressed states relatively more. Taxing them in those states provides a cheaper way of discouraging (ex-ante) inflows. Hence, the planner applies larger taxes—more protectionism—in states with greater financial distress.

4.2. Crisis Mitigation Policies with Symmetric Flows

In practice, governments intervene during crises to provide liquidity and alleviate fire sales. We next analyze the potential coordination problems associated with the use of these types of policies. To this end, suppose the planner in each country can generate additional (public) liquidity at date 1 by taxing a third group of agents, which we refer to as nonparticipants. Nonparticipants are endowed with $\bar{\eta} > 0$ dollars at date 1 that are taxable by the government. We assume the planner can only intervene by purchasing financial assets in case of a local liquidity shock. In particular, a planner that raises $\eta^{pl,j} \in [0, \bar{\eta}]$ dollars in the low liquidity state of date 1 purchases $\eta^{pl,j}/p^j$ units of the asset, where p^j denotes the equilibrium price that obtains after the intervention. We start by assuming that the planner can commit to implementing a particular policy, i.e., there are no time-inconsistency problems. We will analyze the case without commitment at the end of the section.

We also assume that the assets purchased by the planner are wasted, which ensures that the rationale for intervention is not driven by the government's comparative advantage in financial markets.[10] The social welfare function in (28) is then modified by,

$$W^j = u(c_0) + E[c_1 + c_2] + \lambda e\left(\zeta\left((1-\pi) f(R) + \pi f(p)\right) + (1-\zeta)\overline{R}\right) + \bar{\eta} - \pi\eta^{pl,j}. \qquad (33)$$

The last term captures the expected consumption loss due to the government liquidity creation in the low liquidity state. As $\lambda \to \infty$, the planner cares only about financial stability and effectively maximizes the same objective function (29) as before. However, for any finite λ,

[10] The planner's advantage lies in its unique ability to raise tax revenues and generate liquidity as in Holmstrom and Tirole (1998).

there are costs associated with liquidity creation, which will help to break ties when various policy choices yield the same value for the objective in (29).

Coordinated policy. As in the case of capital taxes, first consider the symmetric coordinated policy, $\eta^j = \eta^{pl}$ for each j, that would be chosen by a global planner. The characterization of equilibrium is the same as in the baseline analysis in Section 2 with the only difference that the market clearing condition (6) is replaced by,

$$p = \min\left(R, \frac{\eta + \eta^{pl} + x\overline{R}}{e + x}\right) = \min\left(R, \frac{\eta + \eta^{pl} + x\,(1-\pi)\,R}{e + x\,(1-\pi)}\right). \tag{34}$$

In particular, for any level of foreign flows, x, the asset price in each country is increased by public liquidity injection by the planner. In equilibrium, the increase in the price reduces the foreign flows x, as there is less need of private liquidity creation, but this effect does not undo the initial price increase. It follows that a global planner creates the maximum amount of liquidity.

Proposition 11. *Consider the symmetric model with public liquidity creation in the limit as $\lambda \to \infty$. Suppose the parameters satisfy, $\eta + \bar{\eta} < eR$, and the planners can commit to liquidity-creation policies. A global planner that coordinates countries' policies creates the maximum amount of liquidity, $\eta^{pl} = \bar{\eta}$.*

Nash equilibrium. Next consider the optimal public liquidity injection policy for the planner of a country, $\eta^{pl,j}$, when all other countries set their public liquidity injection at some level, $\eta^{pl} > 0$. For sufficiently small deviations, the foreign investors' optimality condition is the same as Eq. (8) in Section 2.2. Inspecting this condition implies that the policy has no impact on the asset price, p^j. The reason is that the amount of public liquidity injection is anticipated by the financial markets and neutralized by capital inflows. If the country decides to inject more public liquidity than other countries, $\eta^{pl,j} > \eta^{pl}$, all else equal this increases the price in its financial markets, p^j. However, as in our earlier analysis with the reach for safety, this policy also makes the country's assets more attractive compared to other countries', which in turn increases the inflows, $x^{\text{in},j}$. This process continues until the country's assets are equally attractive as other countries' assets.

It follows that public liquidity creation by an individual country leaves the fire-sale price in the country unchanged and does not provide any financial stability benefits. Since the liquidity creation is costly for any finite λ (and thus, in the limit as $\lambda \to \infty$), a local planner with commitment does *not* create public liquidity during crises. This result can be viewed as a variant of the standard moral hazard argument: the planner commits not to intervene during crisis so as to deter (ex-ante) fickle flows that reduces (local) financial stability. The following result summarizes this discussion.

Proposition 12. *Consider the symmetric model with public liquidity creation in the limit as $\lambda \to \infty$. Suppose the planners can commit to liquidity-creation policies. The uncoordinated Nash equilibrium features zero public liquidity creation, $\eta^j = 0$ for each j.*

Comparing this result with Proposition 11 illustrates that the coordinated and the uncoordinated equilibria sharply differ. The coordinated equilibrium calls for the maximum public liquidity creation, whereas the uncoordinated equilibrium with commitment features zero public liquidity creation (in view of standard moral hazard concerns by individual planners). Intuitively, when a country creates liquidity, it also attracts greater inflows. While these inflows look costly from the country's perspective, they actually help to increase global liquidity and ensure that other countries now have access to greater liquidity to arbitrage fire sales in their own financial markets. Put differently, the inflows dilute the financial stability benefits of the planner's intervention to other countries. A planner that sets policy in isolation does not take into account the positive externalities on other countries, which leads to commitment to create too little liquidity.

So far, we analyzed the planners' incentives to intervene during crises so as to create public liquidity. A related question is whether the planners might also want to encourage the creation of private liquidity. We address this question in Appendix A.3 by allowing the planner to interfere with the investors' date 0 decisions. By discouraging/taxing consumption, the planner might increase local investors' financial asset holdings, which in turn increases liquidity and improves asset prices. We find that the policy implications of private liquidity creation is similar to public liquidity creation. Specifically, a global planner that is concerned with financial stability incentivizes local investors to hold financial assets, whereas the Nash equilibrium features no such incentives for the same reason as above. Greater financial savings by the local investors are neutralized by greater fickle flows from abroad, leaving the local fire sales unchanged.[11]

The results so far show that the local planners would like to commit to not create public liquidity during crises. This type of commitment might be difficult to maintain in practice, since crises tend to generate considerable pressure on policymakers to intervene. We next consider the polar opposite case in which a local planner cannot commit and chooses her liquidity-creation policy after the local liquidity shock is realized. In this case, the market clearing condition in country j can be written as,

$$p^j = \min\left(R, \frac{\eta + \eta^{pl,j} + x^{out,j}\overline{R}}{e + x^{in,j}}\right).$$

The planner chooses $\eta^{pl,j}$ taking the inflows and outflows, $x^{in,j}, x^{out,j}$, as given (as they are determined in the past). Then, the objective function in (33) implies that the planner chooses to create *the maximum* amount of liquidity, $\eta^{pl,j} = \bar{\eta}$. Intuitively, the ex-ante flows are already realized by the time the planner decides. Hence, the planner chooses to inject public liquidity

[11]This discussion also suggests that a country might want to combine protectionist policies in the capital market with local liquidity-creation policies. To accomplish this, however, the country would have to use quantity restrictions in the capital market—rather than price restrictions such as taxes—as the arguments for liquidity creation continue to apply for any interior tax level (that allows some positive inflows). By setting a quantity restriction on foreign flows, the country can ensure that the additional liquidity it creates remains inside the country. Note, however, that this outcome would still not replicate the coordinated solution as it would feature too little capital flows and inefficient global liquidity creation.

and increase the fire-sale price level.

Proposition 13. *Consider the symmetric model with public liquidity creation in the limit as $\lambda \to \infty$. Suppose the planners cannot commit and decide their liquidity-creation policies ex post. The uncoordinated Nash equilibrium features maximum public liquidity creation, $\eta^j = \bar{\eta}$ for each j.*

Comparing Propositions 11-13 illustrates that the lack of policy commitment has a silver lining: It helps to overcome the individual planner's resistance to create public liquidity, which in turn improves global liquidity and generates positive externalities on other countries. In our model, the lack of commitment completely solves the coordination problem for public-liquidity creation and replicates the outcome that would be chosen by a global planner with commitment. However, this feature is driven by rather special features of the model (e.g., as $\lambda \to \infty$, the benefit of liquidity creation always dominates its cost), and it is unlikely to hold generally. The robust message here is that the lack of commitment for crisis-intervention policies improves global liquidity and tends to mitigate the coordination problem among local planners.

More broadly, Propositions 8-12 illustrate the importance of policy coordination for managing global liquidity in an environment with fickle capital flows. These flows reduce financial stability in the receiving country, but they also help to distribute excess liquidity to countries and areas that need it relatively more. The Nash equilibrium might feature too much impediment to capital inflows and too little local global liquidity creation, because the individual countries do not take into account the external benefits of distributing the excess liquidity they have or newly create. Moreover, individual countries' decisions to restrict capital flows are complementary, which amplifies the negative liquidity shocks by restricting the endogenous global liquidity creation.

5. Optimal Policy with Asymmetric Flows

In Sections 2 and 3, we showed that the asymmetric liquidities or returns across countries naturally generate a reach-for-safety and a reach-for-yield mechanism. We developed these mechanisms in an environment in which the world was symmetric except for one country. While this approach is useful to illustrate the mechanisms, it does not allow for a meaningful welfare analysis, since a country with measure zero does not enter the global planner's welfare function. In this section, we first extend the baseline model in Section 2 (without aggregate risk) to a setting with multiple and asymmetric regions. We then use special cases of this model to analyze the policy implications of the reach-for-safety and the reach-for-yield mechanisms.

Suppose there are multiple regions of countries denoted by the superscript $k \in K = \{1, .., |K|\}$. Each region k consists of a continuum of countries that is identical to the continuum we analyzed in the baseline model in Section 2. We let m^k denote the mass of countries in region k and assume $\sum_{k \in K} m^k = 1$. The earlier analysis is the special case with a single region.

The liquidity shocks are i.i.d. within regions as well as across regions (so we abstract away from aggregate risk for simplicity). The regions are the same as one another except that the countries in each region might feature heterogeneous amounts of liquidity, $\left\{\eta^k\right\}_k$, as well as heterogeneous returns from new investment, $\left\{R^k\right\}_k$. Later, we will focus on special cases with two regions that can be thought of as corresponding to developed and emerging markets.

As before, investors are fickle and are forced to liquidate positions during a foreign crisis—even in the countries that might be in the same region as their own country. The investors can now take positions in multiple regions. We focus on symmetric equilibria in which the investors of the same region take identical positions, and assets (of the countries) within the same region trade at the identical price denoted by p^k. As problem (35) illustrates, the latter assumption implies that the distribution of an investor's portfolio among the countries of a region is not payoff relevant—what matters is the total position in the region. Hence, without loss of generality, we also focus on symmetric allocations in which the representative investor in region k takes fully diversified positions within each region k'. We denote these positions by $\left\{x^{k',k}\right\}_{k'\in K}$. We also denote the investor's positions in safe assets supplied by region k' as $\left\{y^{k',k}\right\}_{k'\in K}$. The problem for the representative investor (in region k) can then be written as,

$$\max_{\tilde{c}_0.\tilde{x}^{loc},\left\{\tilde{x}^{k'},\tilde{y}^{k'}\right\}_{k'}} u\left(\tilde{c}_0\right)+\tilde{x}^{loc}R^k+\left(\sum_{k'}\tilde{x}^{k'}\overline{R}^{k'}+\tilde{y}^{k'}R_f\right)M^k, \tag{35}$$
$$\overline{R}^{k'}=\left(1-\pi\right)R^{k'}+\pi p^{k'} \text{ for each } k'\in K$$
$$M^k=1-\pi+\frac{R^k}{p^k}\pi$$
$$\tilde{c}_0+\tilde{x}^{loc}+\sum_{k'}\left(\tilde{x}^{k'}+\tilde{y}^{k'}\right)=\eta^k/R_f+1.$$

Note that the investor solves a generalized version of problem (1).

The market clearing condition for the risky assets in a country of region k can be written as,

$$p^k=\min\left(R^k,\frac{\sum_{k'}y^{k',k}R_f+\sum_{k'}x^{k',k}\overline{R}^{k'}}{e+x^{\text{in},k}}\right) \tag{36}$$
$$\text{where } x^{\text{in},k}=\sum_{k'}m^{k'}x^{k,k'}/m^k \text{ and } x^{\text{out},k}=\sum_{k'}x^{k',k}.$$

Here, $x^{\text{in},k}$ and $x^{\text{out},k}$ respectively denote the inflows into and the outflows from the country. The inflows are normalized by the mass of the region, m_k, because $x^{k,k'}$ denotes the total flows into the region (as opposed to the country). There are also market clearing conditions for the safe assets supplied by each region k,

$$\sum_{k'\in K}m^{k'}y^{k,k'}=m^k\eta^k/R_f. \tag{37}$$

An equilibrium with symmetric allocations and prices is a collection, $\left(c_0^k, x^{loc,k}, \left\{x^{k',k}, y^{k',k}\right\}_{k'}\right)_k, \left(p^k \leq R^k\right)_k, R_f$, such that the allocations solve problem (35), and the market clearing conditions (36) and (37) hold.

To characterize the equilibrium, we make a number of simplifying observations. First, we restrict attention to equilibria in which each country retains its safe asset endowment, that is,

$$y^{k',k} = \begin{cases} \eta^k/R_f, & \text{if } k' = k \\ 0, & \text{otherwise} \end{cases}.$$

This is without loss of generality since there is no aggregate risk, which implies that safe assets and foreign investment are perfect substitutes. This restriction also ensures that the market clearing condition for legacy asset holds. It remains to characterize how the investors split their dollars between outside spending, local investment, or investment in other regions, $c_0^k + x^{loc,k} + x^{\text{out},k} = 1$.

As before, Lemma 1 implies that absent taxes local investment, $x^{\text{loc},k}$, is weakly dominated by investing in the other countries of the same region, $x^{k,k}$, and strictly so if there are local fire sales, $p^k < R^k$. Hence, whenever there are no taxes, we also restrict attention to equilibria in which $x^{\text{loc},k} = 0$ without loss of generality.[12]

Next note that, by problem (35), the optimality condition for investment in region k (by any region k') implies,

$$R_f \geq \overline{R}^k, \text{ with equality if } x^{\text{in},k} > 0 \text{ (equivalently, } x^{k,k'} > 0 \text{ for some } k'). \tag{38}$$

The return in a region cannot exceed R_f since safe assets are held in positive quantities in equilibrium. Moreover, the return is exactly equated to R_f as long as the country receives inflows from some other country. Combining these observations, the optimality condition for outflows from a country can be written as,

$$u'\left(1 - x^{\text{out},k}\right) = \begin{cases} \overline{R}^k M^k = \mu^k\left(p_k\right), & \text{if } x^{\text{in},k} > 0 \\ R_f M^k \geq \mu^k\left(p_k\right), & \text{if } x^{\text{in},k} = 0 \end{cases}. \tag{39}$$

Hence, for regions that experience inflows, the size of the foreign flows are determined by the same equation as before [cf. Eq. (5)]. For regions that do not experience inflows, the foreign flows are greater than before and determined by the (higher) asset returns in other regions. This illustrates a key feature of the present setup: investors hold foreign positions not only because it helps them to arbitrage local fire sales but also because doing so might enable them to obtain greater returns than what they could obtain in their own region.

[12] To see that this is without loss of generality, note that the local investment can be feasible only if $p^k = R^k$. In this case, it can be checked that if there is an equilibrium with $x^{loc,k} > 0$, then there is also an equlibrium with $\tilde{x}^{loc,k} = 0$ and $\tilde{x}^{k,k} = x^{k,k} + x^{loc,k}$: that is, the local investment can be substituted for investment in the other countries of the same region without changing any of the equilibrium conditions.

Finally, using these observations, the market clearing condition (36) can be rewritten as,

$$p^k = \min\left(R^k, \frac{\eta^k + x^{\text{out},k} R_f}{e + x^{\text{in},k}}\right). \tag{40}$$

In addition, the total outflows and the inflows satisfy the conservation equation,

$$\sum_k m^k x^{\text{in},k} = \sum_k m^k x^{\text{out},k}. \tag{41}$$

The equilibrium is then characterized by a collection of inflows into and outflows from the representative countries within regions, $\left(x^{\text{in},k}, x^{\text{out},k}\right)_k$, and prices $\left(p_k\right)_k, R_f$, that solve Eqs. $(38-41)$. Note that there are $3\left|K\right|+1$ equations in $3\left|K\right|+1$ unknowns (although some of the equations take the form of complementary slackness).

5.1. Optimal Policy with Reach for Safety

We next consider a special case of the model to analyze the policy implications for the reach for safety. We assume $\pi = 1$ so that the liquidity shocks happen with certainty. This ensures that risky capital flows are not driven by liquidity-insurance considerations (since the shocks are correlated) or return differentials (since $\pi = 1$ implies that foreigners cannot realize the higher returns in other regions). That is, we abstract away from the liquidity-insurance benefits we emphasized in Section 4, while also shutting down the reach-for-yield motive for flows, which enables us to focus on the welfare effects that are purely driven by the reach for safety.

For concreteness, we also assume there are two regions, $k \in \{D, E\}$, where $k = D$ corresponds to developed financial markets and $k = E$ corresponds to emerging markets. We assume region D has sufficient liquidity that it would avoid fire sales in autarky (similar to Assumption S in Section 2.2). We also assume that there is a worldwide scarcity of liquidity, that is, we modify Assumption 1 as follows.

Assumption 1S. $\eta^D > eR^D > \eta^E$, and $\eta^D m^D + \eta^E m^E < e\min\left(R^D, R^E\right)$. In addition, $\underline{x}^{\text{out},E} \geq e\left(1 - \frac{\eta^E}{\eta^D m^D + \eta^E m^E}\right)$.

Note that the autarky prices in regions D and E are then, respectively, given by R^D and $\eta^E/e < R^E$. In the last part of the assumption, $\underline{x}^{\text{out},E}$ denotes the minimum level of outflows form region E, characterized as the solution to, $u'\left(1 - \underline{x}^{\text{out},E}\right) = R^E$. The assumption does not play an important role—it ensures that there is an equilibrium with positive flows into both regions.

With these assumptions, the characterization of equilibrium is relatively simple. In the appendix, we obtain a closed form solution (to Eqs. $(38-41)$) that satisfies,

$$p^D = p^E = \frac{\eta^D m^D + \eta^E m^E}{e} < \min\left(R^D, R^E\right), \tag{42}$$
$$x^{\text{in},k} - x^{\text{out},k} = e\left(\frac{\eta^k}{\eta^D m^D + \eta^E m^E} - 1\right) \text{ for each } k \in \{D, E\}.$$

These equations generalize the reach-for-safety result in Section 2.2. The first equation shows that, with free financial flows, region D also experiences fire sales, even though it would not feature fire sales in autarky. The second equation illustrates that this outcome obtains because the countries in region D receive more inflows relative to their outflows, $x^{\text{in},D} - x^{\text{out},D} > 0$ (that is, they run current account deficits).

Conversely, since $\frac{\eta^D m^D + \eta^E m^E}{e} > \frac{\eta^E}{e}$, the first equation in (42) shows that financial flows improve the fire-sale prices in region E. The second equation illustrates that this outcome obtains because the countries in region E have more outflows than their inflows, $x^{\text{out},E} - x^{\text{in},E} > 0$ (that is, they runs current account surpluses).

These observations suggest that capital restrictions in this setting have costs as well as benefits. To investigate further, suppose the planner in each country j can impose a linear tax, τ^j, on inflows. We assume that the planner injects the tax receipts back into different regions (via equal-weighted asset purchases in that region) according to the fraction of investment in the country that comes from each region. For instance, if the fraction, $\alpha \in (0,1)$, of the inflows into the country, $x^{\text{in},j}$, come from region E, then the planner injects $\alpha x^{\text{in},j} \overline{R}^j \tau^j$ into region E and $(1-\alpha) x^{\text{in},j} \overline{R}^j \tau^j$ into region D. This ensures that taxation does not affect the liquidity in either region, ensuring continuity with our earlier analysis.

Consider the equilibrium with symmetric taxes within each region, $\left\{\tau^k \geq 0\right\}_{k \in \{D,E\}}$. In the appendix, we show that taxes in region E do not affect the equilibrium prices. In particular, we can take $\tau^E = 0$ without loss of generality. The taxes in region D, however, affect the equilibrium. When τ^D is sufficiently small, the equilibrium returns now satisfy the indifference condition, $p^E = p^D\left(1-\tau^D\right)$. The appendix completes the analysis and shows that the prices have a closed form solution,

$$p^D = \frac{\eta^D m^D + \eta^E m^E / \left(1-\tau^D\right)}{e} \text{ and } p^E = \frac{\eta^D m^D \left(1-\tau^D\right) + \eta^E m^E}{e}. \tag{43}$$

In particular, increasing τ^D increases the fire-sale price in region D at the expense of reducing the fire-sale price in region E. Hence, the optimal tax for the global planner is ambiguous as it depends on the relative cost of fire sales in respective regions. Our model does not help to resolve this ambiguity since we capture the relative cost of fire sales in reduced form using the functions $f^D(p), f^E(p)$ (which might in principle differ across the regions). The following result summarizes this discussion.

Proposition 14. *Consider the asymmetric model with developed and emerging market regions* *that satisfy Assumption 1S. When $\tau^E = \tau^D = 0$, the equilibrium prices and the flows satisfy (42). Increasing τ^E does not affect the equilibrium prices. Starting with zero taxes, increasing the tax in the developed region τ^D increases the fire-sale price in this region, p^D, and decreases the fire-sale price in the emerging market region, p^E.*

5.2. Optimal Policy with Reach for Yield

We next consider another special case of the model with asymmetric regions to analyze the policy implications for the reach for yield. As in the previous case, we consider two regions, $k \in \{D, E\}$. We depart from the previous case by assuming $\pi < 1$. This enables for flows to be driven at least in part by the return differentials, but it also makes the analysis less tractable. We make a number of assumptions that bring back analytical tractability. First, we relax the earlier assumptions on $u(\cdot)$ and assume instead that investors receive no utility from consumption.

Assumption 0. $u(c_0) = 0$ for each c_0.

This assumption ensures that $c_0^k = 0$ and $x^{\text{out},k} = 1$; that is, the outflow from each country is exogenously fixed. This helps to drop Eqs. (39) from the equilibrium conditions and replace $x^{\text{out},k}$ in the remaining conditions by 1. The assumption does not play an important role beyond simplifying the analysis, since our goal in this section is to analyze the direction of the global capital flows as opposed to their magnitudes.

Second, we assume that η^D is sufficiently large so that $p^D = R^D$ regardless of the flows in equilibrium: that is, the developed markets have abundant liquidity to prevent fire sales. We also assume η^E is relatively small so that $p^E < R^E$ in any equilibrium with positive inflows into E: that is, the emerging markets have relatively low liquidity and are subject to fire sales. The following assumption specifies the exact parametric conditions.

Assumption 1Y. $\eta^D > \left(e + 1/m^D - 1\right) R^D$, and $\eta^E < eR^E - \max\left(R^E, R^D\right)$.[13]

With these assumptions, we have $x^{\text{out},D} = x^{\text{out},E} = 1$ and $p^D = R^D$. To characterize the rest of the equilibrium, first consider the case in which there are positive flows into both markets, $x^{\text{in},D}, x^{\text{in},E} > 0$. The conditions for this type of equilibrium can be written as,

$$\begin{aligned} R_f &= (1-\pi) R^E + \pi p^E = R^D, \\ p^E &= \frac{\eta^E + \overline{R}^E}{e + x^{\text{in},E}} = \frac{\eta^E + (1-\pi) R^E}{e + x^{\text{in},E} - \pi}. \\ m^D x^{\text{in},D} &+ m^E x^{\text{in},E} = 1. \end{aligned} \tag{44}$$

In the appendix, we show that these conditions are satisfied as long as the return in developed markets lies in an interval, $R^D \in \left(R^D_{low}, R^D_{high}\right)$ (see Eq. $(B.74)$). We also check that the equilibrium takes one of three forms depending on the return in region D. If $R^D \in \left(R^D_{low}, R^D_{high}\right)$,

[13] Note that the latter condition also implies $e > 1$, which in turn implies $e > \pi$ (since $1 > \pi$). This observation might help to follow some of the susbequent analysis.

there are flows in both directions as described above. If $R^D \geq R^D_{high}$, then there are zero flows into region E, $x^{\text{in},E} = 0$, and all flows go into region D. If $R^D \leq R^D_{low}$, then there are zero flows into region D, $x^{\text{in},D} = 0$, and all flows go into region E. We also show that $R^D_{high} < R^E$: it takes a strictly lower return in region D than in region E to ensure some flows will go into region E. This is because region E is subject to fire sales, in view of its low liquidity, whereas region D is not.

We next analyze the comparative statics of equilibrium with respect to the return in developed markets, R^D. For simplicity, consider the case with flows in both directions. As the first equation in (44) illustrates, a decline in R^D leads to a decline in p^E. As the second equation illustrates, this decline is brought about by an increase in fickle inflows into region E, $x^{\text{in},E}$. Intuitively, a reduction in R^D makes the assets in emerging markets relatively more attractive, which induces more of the global financial flows to flow into this region, generalizing Proposition 3 to this setting. The following result summarizes this discussion.

Proposition 15. *Consider the asymmetric model with developed and emerging market regions. The equilibrium depends on the comparison of the return in the developed region, R^D, with two thresholds R^D_{low}, R^D_{high} that satisfy $R^D_{low} < R^D_{high} < R^E$. If $R^D \leq R^D_{low}$, then $x^{in,D} = 0$. If instead $R^D \geq R^D_{high}$, then $x^{in,E} = 0$. If $R^D \in \left(R^D_{low}, R^D_{high}\right)$, then there are positive flows in each region and the equilibrium is characterized by the system in (44). When $R^D \in \left(R^D_{low}, R^D_{high}\right)$, a decrease in the return in region D, R^D, increases the inflows into region E, $x^{in,E}$, and decreases the fire-sale price in this region, p^E.*

We next analyze the desirability of policies directed toward restricting capital flows. As before, suppose the planner in each country j can impose a linear tax, τ^j, on inflows. The planner injects the taxed liquidity back into the regions in which the flows come from as described in Section 5.1. We also assume that taxation is costly as in Section 4.1.1. Specifically, adopting the tax level τ reduces the return of the entrepreneurs that have linear scale by $v(\tau) \geq 0$, where $v(\cdot)$ is a convex function with the same properties as before.

We consider a global planner that can coordinate tax policies across countries and regions. The planner chooses two tax rates, τ^D, τ^E, to be applied in the countries in, respectively, region D and region E. As $\lambda \to \infty$, the global planner's problem can be written as [cf. (31)],

$$\max_{\tau^D, \tau^E \geq 0} \sum_{k \in \{D,E\}} m^k \left((1-\zeta) \left((1-\pi) f\left(R^k\right) + \pi f\left(p^k\right) \right) + \zeta \overline{R}^k \left(1 - v\left(\tau^k\right)\right) \right). \tag{45}$$

It can be seen that the planner always sets $\tau^D = 0$. This is because there are no fire sales in region D, and taxing the flows into region D does not help to increase the price in region E. However, the planner might want to set a positive tax rate in region E. We next characterize the equilibrium with tax levels, $\tau^D = 0, \tau^E \geq 0$, and analyze the optimal tax rate.

One caveat is that investors in the countries of region E might choose to invest locally in view of the taxes on foreign flows in region E. In the appendix, we show that this does not happen if we assume $R^D > R^D_{low}$ (so that there is some investment in region D absent taxes) as well as the following parametric condition.[14]

Assumption 3. $R^D > R^D_{low}$ and $\left(1 - \pi + \pi \frac{R^E}{p^{E,\max}}\right) R^D \geq R^E$, where $p^{E,\max} = \frac{\eta^E + R^D}{e}$.

The characterization of the equilibrium with $\tau^E \geq 0$ then parallels the analysis in the previous section. First consider the case with relatively high return in region D, $R^D \geq R^D_{high}$. In this case, the equilibrium without taxes features zero flows into region E, $x^{\text{in},E} = 0$. Increasing the tax level on these flows has no effect on equilibrium (they continue to remain at zero). The planner optimally sets a zero tax level, $\tau^E = 0$.

Next suppose $R^D \in \left(R^D_{low}, R^D_{high}\right)$ so that the equilibrium without taxes features flows into both regions. For sufficiently small levels of taxes, $\tau^E \geq 0$, the foreigners' optimality condition in (44) is modified as,

$$R_f = \left(1 - \tau^E\right)\left((1 - \pi) R^E + \pi p^E\right) = R^D, \tag{46}$$

This expression illustrates that setting a greater tax level in region E increases the fire-sale prices in this region, p^E. As before, greater taxes discourage foreign inflows into region E, which in turn improves the local fire-sale prices.

We next characterize the optimal tax rate. In the appendix, we show that the planner can increase the price up to the level, $p^{E,\max} = \frac{\eta^E + R^D}{e}$ (which obtains when $x^{\text{in},E} = 0$). We let $\tau^{E,\max} > 0$ denote the tax level that brings about this price level (see Eq. $(B.76)$). Increasing the taxes beyond $\tau^{E,\max}$ does not affect the equilibrium, since it leaves the flows into region E unchanged at zero. Thus, the planner's problem is to choose $\tau^E \in \left[0, \tau^{E,\max}\right]$ to maximize the objective function in (45), subject to Eq. (46). The optimal tax level is given by, $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$, where $\tau^{E,*} \in (0,1)$ is the unique solution to,

$$V\left(\tau^{E,*}\right) = \frac{(1 - \zeta) f'\left(p^E\right) + \zeta}{\zeta}. \tag{47}$$

Here, $V\left(\tau^j\right)$ is an increasing and convex function that satisfies the appropriate boundary conditions as in Section 4.1.1. Note that $\tau^E = \tau^{E,\max}$ corresponds to a corner solution in which the planner reduces the inflows to zero, $x^{\text{in},E} = 0$, whereas $\tau^E = \tau^{E,*} < \tau^{E,\max}$ correspond to an interior solution in which the planner leaves some inflows, $x^{\text{in},E} > 0$, due to costly taxation. Note also that comparative statics that decrease the fire-sale price level, p^E—for instance, a decline in R^D as in Proposition 15, increases the optimal interior tax rate. The following result

[14] Here, $p^{E,\max}$ is the maximum price level that can obtain in country E when $R^D > R^D_{low}$ (see below). The parametric condition ensures that, even when the price is maximized, investors in E will invest in foreign assets (as opposed to investing locally) to arbitrage local fire sales. The condition holds as long as R^D exceeds a threshold which is strictly below R^E. Moreover, the threshold can be made arbitrarily small by increasing the forced sales, e (and reducing η^E).

summarizes this discussion.

Proposition 16. *Consider the asymmetric model with developed and emerging market regions and costly taxation in the limit as $\lambda \to \infty$ (with Assumptions 0, 1Y and 3). Consider a global planner that coordinates countries' policies. The optimal tax rate in the D region is zero, $\tau^D = 0$. The optimal tax rate in the E region is also zero, $\tau^E = 0$, when the return in the D region exceeds a threshold, R^D_{high} (which is strictly below the return in the E region, R^E), but it is strictly positive for lower levels of return, $R^D \in \left(R^D_{low}, R^D_{high}\right)$. If the optimal tax rate is positive and corresponds to an interior solution (with $x^{in,E} > 0$), then a decrease in R^D increases the optimal tax rate, τ^E, and reduces the equilibrium price, p^E.*

The result qualifies some of our earlier conclusions about the undesirability of capital taxes (e.g., Proposition 8). Specifically, in an environment in which capital flows are *purely* driven by reach-for-yield considerations, taxing capital flows might be justified even for a global planner. Intuitively, the pure reach-for-yield flows exacerbate the fire sales in region E without providing financial stability benefits elsewhere. The global planner optimally applies capital taxes in region E so as to lean against these types of destabilizing flows, and more so, if the return in developed regions is low and the reach-for-yield phenomenon is strong.[15]

We note a couple caveats about applying Proposition 16 in practical settings. First, the analysis in this section abstracts away from flows that are driven by liquidity-insurance considerations. In practice, the aggregate flows are likely to be driven by a combination of liquidity-insurance and reach-for-yield motives. The composition of flows should be carefully inspected, and (only) those that are predominantly driven by the reach for yield should be subject to closer scrutiny. Second, and related, the analysis shows that the presence of asymmetric returns is not sufficient to generate pure reach-for-yield flows. Foreign capital flows into region E only if the return in region D is sufficiently below the return in region E (since $R^D_{high} < R^E$) so as to compensate the fickleness of foreigners. This suggests that the destabilizing pure reach-for-yield flows are likely to be a relatively rare phenomenon.

6. Final Remarks

In the core of the paper we selected a configuration of parameters where local and global regulators worry exclusively about financial stability. From this perspective, gross capital flows play three roles in our model: global liquidity creation, reach for safety, and reach for yield. The first role is unambiguously good, the second one is a mixed bag, while the last one is unambiguously bad. The weight of these different roles varies across countries and across global risk and return conditions. However there is a systematic bias among local regulators against capital flows (relative to a benevolent global planner), as the costs associated to reach for safety and, particularly,

[15] Although we do not analyze optimal policy with aggregate shocks, our earlier analysis (specifically, Proposition 7) suggests a "risk-on" environment driven by a decrease in correlations would also increase the destabilizing flows and induce a higher optimal tax.

reach for yield, are felt directly at the local level, while the benefits of global liquidity creation are spread across the world economy.

While actual policymakers do focus on financial stability, it is important to note that there could be additional welfare considerations. To explore some of these, in Appendix A.2 we focus on the polar opposite case in which there are no financial stability concerns (by assuming the entrepreneurs' projects merely break even). In this context, the fire-sale prices do not reduce social welfare—as they are merely transfers among the agents. The global planner is not concerned with fire sales, and she discourages liquidity creation via foreign investment. However, investors continue to undertake foreign investment, so as to exploit and profit from the local fire sales. Hence, absent financial stability concerns, the model features too much liquidity creation and too much foreign investment (similar to Hart and Zingales (2011)). In addition, this version of the model features a different type of coordination problem among planners. While the global planner dislikes foreign flows, local regulators encourage foreign inflows into their countries, as they realize that some of these investments will be appropriated by the local investors (who will purchase them at fire-sale prices). This captures the broader notion that, absent concerns with financial stability and fire sales, the countries would actually welcome foreign flows as some of the returns from foreign investment would accrue to the locals.

There are many other important topics in the capital flow taxation debate that we omitted from our analysis. Perhaps the most significant one is the differentiation of the kinds of capital flows (e.g., equity vs fixed income, short term vs long term). While our model is not designed to address these issues directly, there are insights that carry over to that discussion. The key mechanism by which fickle capital flows generate liquidity in our model is the gap between the return received by local investors on their diversified international portfolios and the fire sale returns received by fickle foreign investors withdrawing their funds from local turmoil. However, if capital inflows take the form of short term debt denominated in foreign currency, then the fire sale and return-gap is limited and so is the liquidity service of these flows. Hence, a global planner that coordinates policies might discourage the short-term flows more than longer-term flows. We leave an exploration of these issues for future work.

Similarly, while in our model all foreign investors are fickle, in practice some foreign investors are not (conversely, some local investors are fickle). Our model can accommodate this extension naturally, at least in the positive economics sections, as our concept of local is just that of an investor that has enough expertise in a market to attempt to arbitrage domestic fire sales rather than running away from them. Of course, the nationality of such investor has practical implications for the mechanism used to tax and identify fickle capital flows.

Another strong assumption we made is that investors have rational and thus common beliefs at the ex-ante investment stage (although we motivated the ex-post fickleness of foreigners with an unmodeled belief friction). Introducing belief disagreements would generate a tension between speculation and risk sharing, similar to Simsek (2013), that would qualify some of our conclusions. In particular, an investor who is relatively optimistic about a foreign market can invest there even though she does not need liquidity and is not an expert in the foreign market. These speculative flows would be destabilizing for the foreign market without providing insurance benefits elsewhere—just like the reach-for-yield flows in our environment. In fact, the speculative flows can also be categorized as reaching for yield as they are driven by high perceived returns in the minds of the investors. We thus conjecture that heterogeneous beliefs would strengthen the reach-for-yield channel and create a stronger rationale for taxing capital flows (even if the planner respects the investors' heterogeneous beliefs, since the rationale for taxation would be driven by fire-sale externalities).

There are two other extensions that we leave for future work. The first one is to add an investment margin at date 0 to entrepreneurs. In this case capital inflows at date 0 may increase the size of the illiquid assets and the potential fire sales, but also allow for a larger domestic investment. In fact, the single-country literature typically focuses on this particular trade-off, which serves to highlight that our mechanisms and externalities are distinct from those in the standard capital flow taxation literature. Second, in our model foreign investment is (ultimately) undertaken by unleveraged investors. In practice, leveraged intermediaries play a central role in facilitating foreign investment, and some of the most significant global crises stem from shocks to intermediary capital, as considered by the stringent macro-stress tests applied to most large banks around the world in the aftermath of the subprime crisis (see, e.g., Bruno and Shin (2015)).

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Appendix A: Extensions

A.1. Insurance for Local Liquidity Shocks

In the baseline model we assumed the investors cannot trade financial contracts contingent on the realization of idiosyncratic risks. In this section we relax this assumption by introducing intermediaries that sell contingent contracts.

Consider the model in Section 2 with the difference that investors can also purchase insurance with respect to idiosyncratic liquidity shocks. Specifically, there is an insurance contract that pays 1 dollar if the country has a crisis at date 1. The contract is traded at date 0, and it costs f dollars (the fee/or the premium) to be paid at date 1. Hence, the net payoff from the contract is $1-f$ dollars if there is a crisis and $-f$ dollars if there is no crisis. We assume the insurance market is competitive, which implies that the insurance is actuarially fair. In a symmetric equilibrium, the fee is equal to the probability of the liquidity shock, $f=\pi$.

We also require the investor to hold sufficient liquid assets at date 1 to back up her insurance premiums. Specifically, letting z^j denote the amount of insurance country j purchases, we require $x^j R + y^j R_f \geq z^j f$. The investor's problem (1) is then modified as,

$$\max_{\tilde{c}_0 . \tilde{x}^{loc}, \left[\tilde{x}^{j'}\right]_{j'}, \tilde{y} \geq 0, \tilde{z}} u\left(\tilde{c}_0\right)+\tilde{x}^{loc} R+\left[\begin{array}{c}(1-\pi)\left(\tilde{x} R+\tilde{y} R_f-\tilde{z} f\right) \\ +\pi\left(\tilde{x} p+\tilde{y} R_f+\tilde{z}(1-f)\right)(R / p)\end{array}\right],$$

$$\tilde{c}_0+\tilde{x}^{loc}+\tilde{x}+\tilde{y}=\eta / R_f+1 \text{ where } \tilde{x}=\int x^{j'} dj' \text{ and } \tilde{z} f \leq \tilde{x} R+\tilde{y} R_f .$$

A symmetric-price equilibrium is defined as in Section 2, with the additional condition that the insurance contracts break even, $f=\pi$.

As before, we focus on equilibria that feature symmetric allocations. We conjecture that under an appropriate parametric assumption (that we specify below), the equilibrium features fire sales, $p<R$. In this equilibrium the investor's net return from the insurance purchase is given by,

$$-(1-\pi) f+\pi(1-f)(R / p)>0 .$$

The inequality follows from $f=\pi$ and $p<R$. Hence, the investors purchase the maximum amount of insurance, $z=\left(x R+y R_f\right) / f$. As before, we also have $y=\eta / R_f, x^{\mathrm{loc}}=0$, and $c_0=1-x$. It remains to characterize the amount of foreign investment, x.

To this end, first consider the return to foreign investment. Note that one dollar of foreign investment enables the investor to purchase R / π units of insurance. This induces the investor to pay $R / \pi \times f=R$ dollars when there is no crisis, and receive $R / \pi-R$ dollars when there is a crisis. Recall also that the foreign asset has a direct payoff during a crisis given by p. Combining these observations, the return from foreign investment in this setting is given by,

$$\pi(p+R(1 / \pi-1)) R / p=\overline{R} R / p, \tag{A.48}$$

where $\overline{R}=(\pi p+(1-\pi) R)$ as before. Note that this expression is greater than the return in the baseline setting, $\mu(p)=\overline{R} M$ (since $R / p>M=1-\pi+\pi R / p$). Intuitively, the insurance market enables the investors to transfer their payoffs in the no crisis states to the crisis states, which makes foreign investment more valuable. The amount of foreign investment is determined by the condition,

$$u'(1-x) = \overline{R}R/p. \tag{A.49}$$

As before, this describes a decreasing relation between x and p.

Next note that the asset market clearing condition can also be written as,

$$p = \min\left(R, \frac{\eta + (p + R(1/\pi - 1))x}{e + x}\right) = \min\left(R, \frac{\eta + xR(1/\pi - 1)}{e}\right). \tag{A.50}$$

As before, this describes an increasing relation between x and p.

The equilibrium is the intersection of Eqs. $(A.49)$ and $(A.50)$. The following strengthening of Assumption 1 ensures that there is an equilibrium with fire sales, $p < R$.

Assumption $\mathbf{1}^{\mathrm{I}}$. $\pi > \underline{\pi} \equiv \frac{\underline{x}R}{\underline{x}R + eR - \eta}$.

Finally, we characterize the risk-free return, R_f. Like the foreign investment, the investor uses the payoff from the risk-free asset in the no crisis state to purchase insurance. A similar analysis as above then implies that the return from risk-free investment is given by $R_f R/p$. Equating this with the return in foreign investment in $(A.48)$ gives $R_f = \overline{R}$. In particular, condition (7) in the baseline model continues to apply in this setting.

Next consider how the presence of the insurance market affects the baseline analysis. Comparing Eqs. $(A.49 - A.50)$ with $(5 - 6)$ in the baseline setting, note that both curves (in the $x - p$ space) are shifted upwards in this setting. It follows that the presence of the insurance market increases the fire-sale price, p. Intuitively, as captured by Eq. $(A.50)$, the insurance market transfers the excess liquidity in the countries that do not experience crises to countries with crises. As captured by Eq. $(A.49)$, the insurance market (by utilizing the foreign liquidity more effectively) also induces the local investors to undertake greater foreign investment conditional on a given level of fire-sales. Both effects increase the fire-sale price in equilibrium.

The above analysis also illustrates that there will be fire sales—despite the presence of insurance markets—as long as the liquidity shocks are sufficiently frequent. Moreover, when $p < R$, the qualitative features of the equilibrium are very similar to the baseline setting. For instance, an increase in local liquidity, η, increases p and decreases x as in Proposition 1. Likewise, when regulators tax capital flows without costs, the global regulator sets zero tax as in Proposition 8, but the local regulators in an uncoordinated equilibrium set prohibitively high taxes and implement $x = 0$ as in Proposition 9.

A.2. Welfare Analysis with Weaker Financial Stability Concerns

Our welfare analysis in the main text focused on the special case in which $\lambda \to \infty$ so that financial stability concerns dominated all other concerns. In this appendix we investigate the case with finite λ and illustrate the other forces at play. To keep the analysis simple, we focus on the baseline model analyzed in Section 2. Recall that the social welfare in a country is given by,

$$\begin{aligned} W^j &= u(c_0) + E[c_1 + c_2] + \lambda e E[\tilde{c}_2] \\ &= u(c_0) + R x^{\text{loc},j} + \left(\int_{j'} x^{j',j} \overline{R}^{j'} dj' + y^j \right) M^j + \lambda e \left(\begin{array}{c} \zeta \left((1-\pi) f(R) + \pi f(p^j) \right) \\ + (1-\zeta) \overline{R}^j \end{array} \right), \end{aligned} \tag{A.51}$$

where $\overline{R}^j = (1-\pi) R + \pi p^j, M^j = 1 - \pi + \pi \frac{R}{p^j}$.

The global welfare is the aggregation of this expression over all countries, $W = \int W^j dj$. We first describe the forces that influence the global welfare, and then turn to the forces that influence the welfare in an individual country.

Determinants of Global Welfare We first start by analyzing the determinants of global welfare. In a symmetric allocation without taxes or other interventions, the global welfare can be simplified further,

$$W = u\left(1 - x^{\text{loc}} - x\right) + \eta + R\left(x^{\text{loc}} + x + e\right) - e\overline{R} + e\lambda \left(\begin{array}{c} \zeta \left((1-\pi) f(R) + \pi f(p) \right) \\ + (1-\zeta) \overline{R} \end{array} \right) \tag{A.52}$$

Here, the first term follows from the resource constraints at date 0. The remaining two terms follow from the sum of the resource constraints at dates 1 and 2. At these dates, the investors consume the available resources in the economy plus the expected net profits that are generated by the entrepreneurs' investment, as captured by the last two terms.

To understand the forces that influence welfare, it is useful to analyze a special case with $\zeta = 0$ and $\lambda = 1$. In this case, the entrepreneurs break even from their investments, and the last two terms in $(A.52)$ disappear. As this happens, the fire-sale price, p, also disappears from $(A.52)$. It can be checked that the world's welfare is maximized when the outside spending is at its upper bound, $c_0 = 1 - \underline{x}$, and total investment (local or foreign) is at its lower bound, $x + x^{loc} = \underline{x}$ (recall that $\underline{x}$ solves $u'(1-\underline{x}) = R$). In particular, in the special case with $\zeta = 0$ and $\lambda = 1$, the baseline competitive equilibrium characterized in Section 2 which features $x^{loc} = 0$ and $x > \underline{x}$ is inefficient. Moreover, the equilibrium features *too much foreign investment*, and the global planner would like to reduce the foreign flows—the opposite of what we emphasized in the main text.

Intuitively, the special case $\zeta = 0$ and $\lambda = 1$ captures the opposite situation in which the global planner has no financial stability concerns (increasing the price does not increase the planner's utility since the entrepreneurs break even). Hence, the force that we emphasized in the main text is completely shut down. Instead, another force comes into play and generates too much liquidity creation, which translates into too much foreign investment in this model. Intuitively, investors in a competitive equilibrium have greater incentives to invest in liquid assets (compared to the planner), because they perceive they will make high returns in states with fire sales. The planner without financial stability concerns views these fire sales as harmless transfers among the agents in the economy, and thus, she does not perceive a particularly high return from arbitraging them. Hence, the planning allocation features less liquidity creation and deeper fire sales compared to the competitive equilibrium.[16]

Our goal is to understand the regulation of capital flows in an environment in which the planners are

[16] Technically, in this case, the investors exert fire sale externalities on one another as opposed to the entrepreneurs. By investing one more unit in liquid assets, the investor increases the price and hurts other investors but she does not internalize these effects.

concerned with asset price volatility and fire sales. Therefore, in the main text we abstract away from this counterforce by focusing on cases in which λ is sufficiently large (specifically, the limit as $\lambda \to \infty$) so that the planning allocation features more stable prices relative to the competitive equilibrium.

Determinants of Local Welfare Now consider the determinants of welfare in an individual country. Unlike the global welfare, we cannot simplify this expression much further than in (A.51) without specifying particular policies and characterizing the equilibrium. For concreteness, consider the extension with capital taxes we analyzed in Section 4.1. In particular, suppose all other countries set the tax level, $\tau > 0$, and country j deviates to a tax level, τ^j, in a sufficiently small neighborhood of τ so that the characterization in Eq. (B.69) in Appendix A.5 applies. The resulting welfare in country j can be written as,

$$W^j\left(\tau^j\right) = u\left(1-x^j\right) + \left(\eta + x^j\overline{R}\left(1-\tau\right) + x\overline{R}\tau\right)M^j + e\lambda\left(\begin{array}{c}\zeta\left(\left(1-\pi\right)f\left(R\right)+\pi f\left(p^j\right)\right)\\ +\left(1-\zeta\right)\overline{R}^j\end{array}\right),$$

where $M^j = 1-\pi+\pi\dfrac{R}{p^j}$ and $\overline{R}^j = \left(1-\pi\right)R + \pi p^j$.

Here, x^j, p^j, as well as $x^{\text{in},j}$ (which does not directly appear in the welfare function) are implicit functions of τ^j as describes by the equation system (30). Taking the first order condition and using the Envelope Theorem, we obtain,

$$\begin{aligned}\frac{dW^j}{d\tau^j} = \frac{\partial W^j}{\partial p^j}\frac{dp^j}{d\tau^j} &= \pi\left[e\lambda\left(\zeta f'\left(p^j\right)+\left(1-\zeta\right)\right) - \frac{R}{\left(p^j\right)^2}\left(\eta + x^j\overline{R}\left(1-\tau\right)+x\overline{R}\tau\right)\right]\frac{dp^j}{d\tau^j}\\ &= \pi\left[e\lambda\left(\zeta f'\left(p^j\right)+\left(1-\zeta\right)\right) - \frac{R}{p^j}\left(e + x^{\text{in},j}\right)\right]\frac{dp^j}{d\tau^j}.\end{aligned}$$

Here, the second line uses the pricing equation, $p^j = \frac{\eta + x^j\overline{R}(1-\tau)+x\overline{R}\tau}{e+x^{\text{in},j}}$, to substitute the local liquidity with the total asset sales. Recall that the term, $\frac{dp^j}{d\tau^j}$, is weakly positive, that is, taxing capital flows increases the fire-sale price. The bracketed term captures the effect of the increase in the price on the welfare in the country. A greater price yields financial stability benefits, as captured by the first term inside the brackets. However, it also reduces the expected return of the investors, as captured by the second term. In fact, in the special case with $\zeta = 0$ and $\lambda = 1$ (no net financial stability benefits), the bracketed term is positive since $p^j < R$ and $x^{\text{in},j} > 0$. That is, increasing the local price level via taxes reduces the local welfare via a reduction of the investors' welfare. In this special case, the planner that acts in isolation chooses lower taxes and encourages greater capital flows—the opposite of what we emphasized in the main text.

Intuitively, a local planner without financial stability concerns would like to increase the inflows into the country because some of the payoffs from these investments are ultimately appropriated by locals (as the foreigners liquidate in case of a liquidity shock). Note, however, that the mechanism by which local investors (with limited liquidity) appropriate greater inflows by foreigners is a reduction in asset prices and a deepening of fire sales. Thus, these beneficial effects would be arguably second order for a planner that has financial stability concerns and dislikes fire sales. Therefore, in the main text we abstract away from this counterforce by focusing on cases in which λ is sufficiently large.

A.3. Private Liquidity Creation

In the main text, we analyzed the planners' incentives to create public liquidity. Instead of creating liquidity directly, the planner might also encourage the private sector to hold more liquid assets. In this section, we analyze this set of policies and show that they have similar implications as public liquidity creation.

Suppose the planner can tax the outside spending/consumption of local investors so as to incentivize them to hold more financial assets. Specifically, suppose spending c_0 dollars on the outside option yields $(1-\tau_c)\,c_0$ dollars of consumption. The utility from outside spending is now given by $u\left((1-\tau_c)\,c_0\right)$. As before, the government wastes the tax revenues it collects, $\tau_c c_0$.

First consider the symmetric coordinated policy, $\tau_c^j = \tau_c$ for each j, that would be chosen by a worldwide planner. The characterization of equilibrium parallels the analysis in Section 2. The main difference is that Eq. (5), is replaced by,

$$\begin{cases} (1-\tau_c)\,u'\left((1-x)\,(1-\tau_c)\right) = \mu\,(p)\,, & \text{if } p < R \\ x \in [0, \underline{x}\,(\tau_c)] & \text{if } p = R \end{cases},$$

where the lower bound on investment, $\underline{x}\,(\tau_c)$, is now an increasing function of the taxes on outside spending. The same steps as earlier imply that there exists a unique equilibrium with $x \in (\underline{x}\,(\tau_c)\,, 1)$ and $p < R$. Moreover, the tax on outside spending increases foreign flows, x, as well as the asset price, p.

By taxing the illiquid outside spending, the planner encourages liquidity creation and mitigates fire sales. Since the safe asset is in scarce supply, global liquidity is created via greater foreign flows in equilibrium. Intuitively, greater flows help to utilize the countries' excess liquidity more effectively. The implication is that a global planner with financial stability concerns ($\lambda \to \infty$) sets prohibitively high level of taxes, $\tau_c = 1$, and creates the maximum amount of private liquidity, $x = 1$.

Next consider optimal private liquidity policy for the planner, τ_c^j, when all other countries set their private liquidity policies at some level, τ_c. When the deviation is in a sufficiently small neighborhood of τ_c, the equilibrium conditions can now be written as,

$$R_f = \overline{R} = \overline{R}^j, \text{ where } \overline{R}^j = (1-\pi)\,R + \pi p^j$$

$$\left(1-\tau_c^j\right) u'\left(\left(1-x^j\right)\left(1-\tau_c^j\right)\right) = \overline{R}M^j, \text{ where } M^j = 1-\pi+\pi\frac{R}{p^j}$$

$$\text{and } p^j = \min\left(R, \frac{\eta + x^j\overline{R}}{e + x^{\text{in},j}}\right).$$

As in the case of public liquidity creation, private liquidity policy does not affect the local fire-sale price, p^j. Intuitively, the private liquidity creation in country j is anticipated and neutralized by financial markets. The implication is the Nash equilibrium features too little private liquidity creation relative to the coordinated solution.

A.4. Aggregate Shocks to Cash Flows

In the main text, we analyzed the effect of aggregate liquidity shocks, π_s. In this section, we analyze other sources of aggregate uncertainty that affect cash flows. Specifically, suppose the payoff from the legacy asset, η_s, as well as the return from new investment, R_s, can now depend on the realization of the state $s \in S = \{1, .., |S|\}$. Throughout, we assume the liquidity shocks are constant across states, $\pi_s = \pi$, so as to focus on shocks to cash flows. We also maintain the following assumptions about cash flows.

Assumption 1^{C}. $\eta_s < eR_s$ for each $s \in S$.

Assumption 2^{C}. There exist variables, $\{\kappa_s > 0\}_s, R, \eta > 0$, such that $\eta_s = \eta\kappa_s$ and $R_s = R\kappa_s$ for each $s \in S$. The weights, κ_s, are decreasing in s and satisfy $\sum_s \gamma_s \kappa_s = 1$.

The first assumption a strengthening of Assumption 1 for aggregate uncertainty. The second assumption says that the cash flows from legacy assets and new investment scale proportionally as the aggregate state changes. We view this as a natural starting point. We will discuss the implications of relaxing this assumption at the end of the section. As before, η denotes the expected payoff from the legacy asset (since $E[\eta_s] = \sum_s \gamma_s \kappa_s \eta = \eta$), and R denotes the expected return from new investment (since $E[R_s] = \sum_s \gamma_s \kappa_s R = R$). The assumption that κ_s is decreasing in s captures that greater s corresponds to greater "distress" as in the analysis in the main text.

Note that the legacy asset is no longer risk-free. Hence, we use R_l (as opposed to R_f) denote the expected return on the legacy asset. In particular, the legacy asset is traded at a price η/R_l that will be endogenously determined. Note that $R_l\kappa_s$ denotes the return of the legacy asset conditional on state s, and $R\kappa_s$ denotes the return on new investment conditional on state s.

As in Section 3, the investors can trade financial securities contingent on the aggregate state at date 1 that are provided by competitive intermediaries. The intermediaries' optimality condition is still given by (11), with the modification that the expected payoff is adjusted for the uncertainty about cash flows, $\overline{R}_s^j = R\kappa_s (1-\pi) + p_s^j \pi$. The investors' problem is given by the following analogue of problem (12),

$$\begin{aligned}
&\max_{\tilde{c}_0, \tilde{x}^{loc}, \tilde{y}, (\tilde{z}_s \geq -\tilde{y}R_l)_s} u(c_0) + \tilde{x}^{loc} R + \sum_s \gamma_s (\tilde{y} R_l \kappa_s + \tilde{z}_s) M_s^j, \\
&\tilde{c}_0 + \tilde{x}^{loc} + \tilde{y} + \sum_s q_s \tilde{z}_s = \eta/R_l + 1,
\end{aligned}$$

where $M_s^j = 1 - \pi + \frac{R\kappa_s}{p_s^j}\pi$. The market clearing conditions are given by the following analogues of (13),

$$\begin{aligned}
&\int y^j dj = \eta/R_l \\
&\int z_s^j dj = \int x^{\text{in},j} \overline{R}_s^j dj \text{ for each } s \in S, \\
&\text{and } p_s^j = \min\left(R\kappa_s, \frac{R_l \kappa_s y^j + z_s^j}{e + x^{in,j}}\right) \text{ for each } s \in S.
\end{aligned}$$

Note that the last market clearing condition takes into account the state dependence in the cash flows.

The characterization of the symmetric equilibrium parallels the analysis in the main text. Eqs. $(14-15)$ continue to apply. The main difference concerns the market clearing condition (13). Following similar steps, we now obtain,

$$p_s = \min\left(R\kappa_s, \frac{\eta\kappa_s + x\overline{R}_s}{e+x}\right) = \min\left(R\kappa_s, \frac{\eta\kappa_s + x(1-\pi)R\kappa_s}{e + x(1-\pi)}\right) \text{ for each } s.$$

Hence, we have $p_s = p\kappa_s$, where we define

$$p = \min\left(R, \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right).$$

That is, the price scales proportionally with cash flows as the aggregate state changes. More specifically, the price to return ratio, $p_s/(R\kappa_s)$, is constant across states. This also implies that the marginal utility is constant across states, $M_s = M \equiv 1-\pi+\frac{R}{p}\pi$ for each s. Plugging this into Eq. (15), and using the notation $\overline{R} = (1-\pi)R + \pi p$, we obtain,

$$u'(1-x) = E\left[\overline{R}_s\right] M = \overline{R}M = \mu(p).$$

The last two equations determine the pair, (p, x), from which the rest of the equilibrium can be obtained. Note that these equations are identical to Eqs. (5) and (6) in the baseline setting.

Hence, under Assumptions 1 and 2, introducing aggregate shocks to cash flows leaves the baseline analysis largely unchanged. Intuitively, when the payoffs to legacy and new assets scale proportionally, the liquidity—and thus, the fire-sale price level—scale by the same proportion. Consequently, the investors' marginal utility remains constant across states and the analysis reduces to the setting without aggregate uncertainty.

A.5. Capital Flow Restrictions with Aggregate Shocks

In Section 3, we generalized our baseline model to incorporate aggregate liquidity shocks, which we then used to investigate a number of issues. In this section, we investigate how the presence of aggregate shocks affect the planners' incentives to restrict capital flows. To this end, consider the setup with arbitrary aggregate states, $s \in S$. Suppose the planner in each country j can impose a state-contingent linear tax, $\left\{\tau_s^j \geq 0\right\}_s$, on date 1 payoff from foreign inflows: that is, the return on the inflows (by the intermediaries) in country j is now given by $\overline{R}_s\left(1-\tau_s^j\right)$. As before, the tax revenues are used to purchase an equal-weighted portfolio of all financial assets, which are then wasted by the planner.

Note that we allow the planner to make the tax rate (or more broadly, capital restrictions) contingent on the aggregate state.[17] Our goal is to understand how the optimal tax rate differs across aggregate states, $s \in S$. To this end, we assume taxation is costly as in Section 4.1.1. Specifically, applying the tax rate $\tau_s \geq 0$ on foreign financial flows reduces the return of the entrepreneurs that have linear scale by $v(\tau_s) \geq 0$, where $v(\cdot)$ is a convex function that satisfies the Inada type conditions as before. As $\lambda \to \infty$, the planner effectively maximizes the objective function,

$$\sum_s \gamma_s \left(\begin{array}{c} (1-\zeta)\left((1-\pi_s) f(R) + \pi_s f(p_s)\right) \\ +\zeta \overline{R}_s \left(1-v(\tau_s)\right) \end{array} \right). \tag{A.53}$$

The equilibrium is defined as before, with the difference that the optimality condition for the intermediaries is now adjusted for the presence of taxes [cf. Eq. (11)],

$$1 \geq \sum_s q_s \overline{R}_s^j \left(1-\tau_s^j\right) \text{ for each } j \text{, with equality if } x^{in,j} > 0.$$

The portfolio problem (12) remains unchanged since the investors are not directly affected by the presence of taxes (they hold financial assets indirectly through intermediaries). The market clearing conditions

[17] We could also allow the planner to condition the tax level on the idiosyncratic state. With the assumptions we made, it can be seen that the planner would not use this conditionality. Intuitively, the taxes only affect the outcomes through the foreigners who only care about the average tax level across idiosyncratic realizations. Hence, conditioning the tax level on the idiosyncratic state would not increase the benefits, but it would increase the costs of taxation since $v(\tau)$ is convex.

(13) are adjusted by the presence of taxes and the asset purchases by the government. As before, in a symmetric allocation, the market clearing condition for risky assets will remain unchanged and given by Eq. (16).

To characterize the equilibrium, first consider the symmetric case in which all planners choose the same tax policies, $\tau_s^j = \tau_s$ for each j. In Appendix A.5, we show that there exists $\underline{\tau} > 0$ such that, if $\tau_s \in [0, \underline{\tau})$ for each $s \in S$, then outside spending is below its lower bound, $c_0 < 1 - \underline{x}$, and local investment is dominated in equilibrium, $x^{loc} = 0$ (and thus, foreign investment satisfies $x > \underline{x}$). We assume that $v'(\tilde{\tau}) = \infty$ for some $\tilde{\tau} < \underline{\tau}$ so that the equilibrium always falls in this region. The analogue of Eq. (15) is then given by,

$$u'(1-x) = \sum_s \gamma_s \overline{R}_s M_s (1-\tau_s) \equiv \sum_s \gamma_s \mu_s (p_s)(1-\tau_s). \tag{A.54}$$

The equilibrium is the intersection of Eq. $(A.54)$ and Eqs. (16). Once we solve for $(x, (p_s)_s)$, the asset prices are determined by Eq. (14) as before. It can also be seen that increasing the tax level in any state, τ_s, reduces the capital flows, x, and the fire-sale price level in all states, $(p_s)_{s\in S}$, as well as the risk-free interest rate, R_f. Hence, similar to the earlier analysis, the global planner optimally chooses zero taxes in all states, $\tau_s = 0$ for each $s \in S$.

Next suppose an individual country sets the tax policy, $\left\{\tau_s^j\right\}_{s\in S}$, when all other countries apply the same tax policy, $\{\tau_s\}_{s\in S}$. When $\left\{\tau_s^j\right\}_{s\in S}$ is in a neighborhood of $\{\tau_s\}_{s\in S}$, the equilibrium in country j is characterized by the system of equations $(B.78)$ listed in the proof of Proposition 17 (in the proofs appendix). To characterize the optimal tax policy, it suffices to analyze the following subset of those equations,

$$1 = \sum_s q_s \overline{R}_s^j \left(1-\tau_s^j\right), \text{ where } \overline{R}_s^j = (1-\pi_s) R + \pi_s p_s^j, \tag{A.55}$$

$$\text{and } q_s/\gamma_s = \frac{M_s^j}{M_0^j} \text{ for each } s \in S, \text{ where } M_s^j = 1 - \pi_s + \pi_s \frac{R}{p_s^j}.$$

Here, $M_0^j = u'\left(1-x^j\right)$ is a constant independent of state s. Note that the country takes the Arrow-Debreu prices, $(q_s)_s$, as given. Hence, Eq. $(A.55)$ represents $|S|+1$ equations in $|S|+1$ unknowns, $\left(p_s^j\right)_s, M_0^j$. After factoring out M_0^j, it can be thought of as $|S|$ equations in the $|S|$ unknown prices. Intuitively, the first equation determines the "weighted average" level for the prices. This equation follows from the foreign investors' optimality condition to invest in the country. The second set of equations determines the relative fire-sales across different states, p_s^j. This equation follows from the local investors' optimality condition to trade financial securities across states. Note also that, when the country sets the same taxes as other countries (which will be the case in Nash equilibrium), the unique solution is the same as the symmetric equilibrium described above, that is, $p_s^j = p_s$ for each s.

Next consider the optimal tax policy for country j. The planner chooses the tax policy, $\left\{\tau_s^j\right\}_{s\in S}$, to maximize the objective function in $(A.53)$ subject to the equilibrium conditions in $(A.55)$. Given the prices $\{q_s, p_s\}_s$, the optimal tax rate is characterized as the solution to the equation system $(A.56)$ in the proof of Proposition 17. In turn, in a symmetric Nash equilibrium, the prices are functions of the symmetric tax policies, $\{\tau_s\}_s$, as described above. The Nash equilibrium is found as the intersection of these two systems. As before, there can also be multiple stable Nash equilibria. The following result summarizes this discussion and establishes the properties of taxes in any Nash equilibrium.

Proposition 17. *Consider the symmetric model with aggregate risk and costly (and state-contingent)* *capital taxes in the limit as $\lambda \to \infty$. A global planner that coordinates countries' policies sets zero tax in each state, $\tau_s = 0$ for each s. In any Nash equilibrium, the tax rate is positive for each state, $\tau_s > 0$ for each $s \in S$. Moreover, the tax rates satisfy,*

$$\frac{v'(\tau_s)}{v'(\tau_{s'})} = \frac{q_s/\gamma_s}{q_{s'}/\gamma_{s'}} \text{ for each } s, s'. \tag{A.56}$$

In particular, the tax rate is increasing in $s \in S$: that is, states with greater probability of liquidity shocks are associated with higher taxes.

The last claim in the proposition follows from Eq. $(A.56)$ after observing that q_s/γ_s, is increasing in s. In turn, Eq. $(A.56)$ follows from an individual planner's optimality condition. The intuition is that foreign investors value payoff in distressed states relatively more. Taxing them in these states provides a cheaper way of discouraging foreign investment at date 0. Hence, the planner applies larger taxes—more protectionism—in states with greater financial distress.

Appendix B: Proofs

Proof of Lemma 1. We have,

$$\begin{aligned}
\mu'(p) &= \pi\left(1 - \pi + \frac{R}{p}\pi\right) - \pi\frac{R}{p^2}\left((1-\pi)R + \pi p\right) \\
&= \pi(1-\pi)\left(1 - \frac{R^2}{p^2}\right).
\end{aligned}$$

Hence, $\mu(p)$ is strictly decreasing over the range $p \in (0, R)$. The result follows after observing that $\mu(R) = R$. $\square$

Proof of Lemma 2. If $\eta \geq eR$, then $P^{mc}(x) = R$, which does not depend on x. Suppose $\eta < eR$, in which case $P^{mc}(x) = \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}$. Taking the derivative, we obtain,

$$\frac{d}{dx}\left(\frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right) = \frac{(1-\pi)}{e + x(1-\pi)}\left(R - P^{mc}(x)\right) > 0.$$

Here, the inequality follows since $\eta < eR$ implies $P^{mc}(x) = \frac{\eta + x(1-\pi)R}{e + x(1-\pi)} < R$, which completes the proof. $\square$

Proof of Proposition 1. Let $P^{opt} : [0, 1) \to [0, R]$ and $P^{mc} : [0, 1] \to [0, R]$ denote the functions that are defined in the main text: that is, $P^{opt}(x)$ corresponds to the optimality condition for foreign investment (5), and $P^{mc}(x)$ corresponds to the market clearing condition (6). Note that $P^{opt}(x)$ is strictly increasing, in view of Lemma 2, and $P^{mc}(x)$ is weakly decreasing, in view of Lemma 1. We also have that $P^{mc}(x) \in (0, R)$ for each x in view of Assumption 1. In addition, we have $\lim_{x \to 1} P^{opt}(x) = 0$ and $P^{opt}(\underline{x}) = R$, where recall that $\underline{x} > 0$ denotes the threshold below which $P^{opt}(x) = R$ and there is some local investment. In view of the boundary conditions, there exists $x \in (\underline{x}, 1)$ and $p \in (0, R)$ such that $p = P^{mc}(x) = P^{opt}(x)$. The pair (x, p) corresponds to the equilibrium.

Next consider the comparative statics. Increasing η strictly increases the curve $P^{mc}(x)$, for each $x \in [0, 1]$, while leaving the curve, $P^{opt}(x)$, unchanged. This increases p and reduces x in equilibrium.

Using condition (7), it also increases the risk-free return, R_f. Likewise, decreasing R strictly decreases both curves $P^{mc}(x)$ and $P^{opt}(x)$ for each $x \in [0,1]$. This reduces p as well as R_f.

It remains to show that decreasing R also decreases x. To this end, define the variable $\tilde{p} = p/R$ as the price-to-return ratio. Eqs. (5) and (6) can then be written in terms of $(\tilde{p}, x)$ as,

$$u'(1-x) = R(1-\pi+\pi\tilde{p})\left(1-\pi+\pi\frac{1}{\tilde{p}}\right),$$
$$\text{and } \tilde{p} = \min\left(1, \frac{\eta/R + x(1-\pi)}{e + x(1-\pi)}\right).$$

As before, the first equation describes $\tilde{p}$ as a decreasing function of x, the second function describes $\tilde{p}$ as an increasing function of x, and the equilibrium corresponds to the intersection. Note also that decreasing R strictly decreases the first curve for each x, and strictly increases the second curve for each x. This implies that decreasing R also reduces x, completing the proof. □

Proof of Proposition 2. Most of the proof is provided in the main text. It remains to check that the conjectured allocations, $x^{\text{out},j} = x$ and $x^{\text{in},j} = x + (\eta^j - \eta)/p$, satisfy the market clearing condition (9). Plugging the expressions for $x^{\text{out},j}$ and $x^{\text{in},j}$ into the market clearing condition, we obtain,

$$\begin{aligned} p^j &= \min\left(R, \frac{p\left(\eta^j + x\overline{R}\right)}{(e+x)p + \eta^j - \eta}\right) \\ &= \max\left(R, \frac{\frac{\eta + x\overline{R}}{e+x}\left(\eta^j + x\overline{R}\right)}{\eta + x\overline{R} + \eta^j - \eta}\right) = \min\left(R, \frac{\eta + x\overline{R}}{e+x}\right) = p. \end{aligned}$$

Here, the second line uses the market clearing condition for the representative country (6). This verifies the conjecture that $p^j = p$. Note also that $x^{\text{in},j} > x^{\text{out},j} = x > 0$, which completes the proof. □

Proof of Proposition 3. We conjecture an equilibrium with $x^{\text{in},j} > 0$. Under this conjecture, the equilibrium in country j is characterized by condition (10) in addition to the following equations,

$$u'\left(1 - x^{\text{out},j}\right) = ((1-\pi)R + \pi p)\left(1 - \pi + \pi\frac{R^j}{p^j}\right), \tag{B.57}$$
$$\text{and } p^j = \min\left(R, \frac{\eta^j + x^{\text{out},j}\overline{R}}{e + x^{\text{in},j}}\right). \tag{B.58}$$

Here, the first line captures the optimality condition for the local investors, and the second line is the market clearing condition.

We next show that there exists a tuple, $\left(p^j, x^{\text{out},j}, x^{\text{in},j}\right)$, that satisfies the equilibrium conditions and that features positive inflows. Let $p^j = \frac{(1-\pi)\left(R-R^j\right)+\pi p}{\pi}$, which lies in the interval $(0,p)$ in view of Assumption Y. With this price level, the optimality condition (10) holds as equality. Next let $x^{\text{out},j}$ denote the solution to Eq. $(B.57)$ and note that $x^{\text{out},j} > x$ (since $R^j/p^j > R/p$). We finally let $x^{\text{in},j} = \frac{\eta^j + x^{\text{out},j}\overline{R}}{p^j} - e$ so that the market clearing condition $(B.58)$ also holds. We also note that $x^{\text{in},j} > 0$ since $p^j < p, x^{\text{out},j} > x$, and $\eta^j \geq \eta - px$ by Assumption Y. Thus, the constructed tuple, $\left(p^j, x^{\text{in},j}, x^{\text{out},j}\right)$, corresponds to an equilibrium in country j.

Next consider a decrease in R. By Proposition 1, this decreases x and p. Since $p^j - p = \frac{(1-\pi)\left(R-R^j\right)}{\pi}$, it also decreases $p^j - p$, which in turn implies that it decreases p^j.This completes the proof. □

Proof of Proposition 4. Most of the proof is provided in the main text. It remains to check that there exists a solution to Eqs. (15) and (16), which satisfies $x \in (\underline{x}, 1)$. To this end, define the function,

$$F(x) = u'(1-x) - \sum_s \gamma_s \mu_s(p_s), \text{ where } p_s = \frac{\eta + x(1-\pi_s) R}{e + x(1-\pi_s)} \text{ for each } s.$$

Note that $F(\underline{x}) = R - \sum_s \gamma_s \mu_s(p_s) < 0$, and $F(1) = \infty$. Note also that $F(x)$ is strictly increasing in x. By continuity, there exists a unique solution to the equation, $F(x) = 0$, over the range, $x \in (\underline{x}, 1)$. This completes the proof. $\square$

Proof of Proposition 5. The proof is provided in the main text.

Proof of Proposition 6. The equilibrium in the country is determined by the optimality conditions (22) and (24), together with the conditions

$$c_0^j + \sum_s q_s z_s^j + y^j = 1 + \eta^j / R^f. \tag{B.59}$$

$$\text{and } p_s^j = \min\left(R, \frac{R_f y^j + z_s^j}{e + x^{\text{in},j}}\right) \text{ for each } s.$$

Here, the first equation is the the budget constraint at date 0 and the equations in the second line capture the market-clearing conditions in state s of date 1. We conjecture (and verify) that the prices and outside spending is given by

$$p_s^j = p_s \text{ for each } j, \text{ and } c_0^j = c_0, \tag{B.60}$$

and the inflows and the local investor's financial portfolio satisfy respectively,

$$x^{\text{in},j} = l^j(e+x) - e, \tag{B.61}$$

$$\text{and } R_f y^j + z_s^j = l^j\left(\eta + x\overline{R}_s\right) \text{ for each } s. \tag{B.62}$$

Here, we define the leverage ratio as

$$l^j = \frac{\eta^j / R_f + x}{\eta / R_f + x}. \tag{B.63}$$

To verify that these allocations satisfy the equilibrium conditions, note that Eqs. (22) and (24) hold as described in the main text. Next note that Eq. $(B.62)$ determines the investor's portfolio (up to multiplicity that does not affect the total payoffs). In particular, in view of no arbitrage, the date-0 value of the investor's portfolio is given by,

$$y^j + \sum_s q_s z_s^j = l^j(\eta/R_f + x) = \eta^j / R_f + x. \tag{B.64}$$

Combining this expression with $c_0^j = c_0$ implies the budget constraint in $(B.59)$ (since $c_0 = 1 - x$). The market clearing conditions in $(B.59)$ also hold since,

$$p_s^j = \min\left(R, \frac{l^j\left(\eta + x\overline{R}_s\right)}{e + x^{\text{in},j}}\right) = \min\left(R, \frac{\eta + x\overline{R}_s}{e + x}\right) = p_s \text{ for each } s.$$

Here, the first equality uses $(B.62)$, the second equality uses the definition of $x^{\text{in},j}$ in $(B.61)$, and the last equality uses the market clearing condition for the representative country. Hence, the allocations described by Eq. $(B.60)$ and $(B.61-B.62)$ correspond to the equilibrium in country j.

We next establish the properties of the inflows and outflows in this equilibrium. Note that the date-0 value of the outflows in the country is the same as in the representative country since Eq. $(B.64)$ implies, $x^{\text{out},j}=y^j+\sum_s q_s z_s^j-\eta^j/R_f=x$.Combining this with Eq. $(B.61)$, the difference between the inflows and the outflows is given by,

$$x^{\text{in},j}-x^{\text{out},j}=(e+x)\frac{\left(\eta^j-\eta\right)/R_f}{\eta/R_f+x}>0. \tag{B.65}$$

In particular, the inflows exceed outflows. Next note that, using $(B.62)$, the date-1 payoff from the outflows is given by,

$$x_s^{\text{out},j}=y^jR_f+z_s^j-\eta^j=\left(l^j\eta-\eta^j\right)+xl^j\overline{R}_s=-x\left(l^j-1\right)R_f+xl^j\overline{R}_s,$$

which proves (25). Here, the second equality uses Eq. $(B.62)$, and the last equality uses the valuation equation $(B.64)$. Note also that $l^j>1$ since $\eta^j>\eta$ [cf. Eq. $(B.63)$].

Next consider the special case with correlated shocks described in Section 3.1. Consider an increase in β. As described by Proposition 5, this reduces x and R_f. Since $x^{\text{out},j}=x$, the outflows from country j also decline. Since xR_f declines, Eq. $(B.65)$ implies that $x^{\text{in},j}-x^{\text{out},j}$ increases: that is, the inflows decline less than the outflows. Finally, note that Eq. $(B.63)$ implies

$$l^j=\frac{\eta^j+xR_f}{\eta+xR_f}=1+\frac{\eta^j-\eta}{\eta+xR_f}.$$

Since xR_f declines, l^j increases, completing the proof of the proposition. $\square$

Proof of Proposition 7. We conjecture an equilibrium with $x^{\text{in},j}>0$. The optimality condition for foreign investors is given by Eq. (26) in the main text. As we discuss there, this condition implies that the country experiences greater fire sales "on average."

The remaining question is how these fire sales are distributed across the two distress states $s\in\{2,3\}$ (recall that p_1 does not affect the equilibrium since there are no crises in state 1, $\pi_1=0$). The distribution of fire sales is determined by local investors' allocation of liquidity across states. Specifically, these investors' optimality conditions for states $\{2,3\}$ are given by,

$$u'\left(c_0^j\right)=\frac{M_2^j}{q_2/\gamma_2}=\frac{M_3^j}{q_3/\gamma_3}\text{, where } M_2^j=1-\pi+\pi\frac{R^j}{p_2^j}\text{ and } M_3^j=\frac{R^j}{p_3^j}.$$

Next note that the analogues of the optimality conditions above also hold for the representative country. In particular, we have $\frac{M_2}{q_2/\gamma_2}=\frac{M_3}{q_3/\gamma_3}$. Combining this with the above conditions and substituting the respective marginal utilities, we obtain,

$$\frac{1-\pi+\pi R^j/p_2^j}{1-\pi+\pi R/p_2}=\frac{R^j/p_3^j}{R/p_3}. \tag{B.66}$$

Eqs. (26) and $(B.66)$ represent two equilibrium conditions in two unknowns, p_2^j,p_3^j. Under Assumption $\tilde{\text{Y}}$, there is a unique positive solution to these equations that satisfy, $p_2^j\in(0,p_2)$ and $p_3^j\in(0,p_3)$. Inspecting the equations also implies that the solution satisfies, $\frac{R^j/p_2^j}{R/p_2} > \frac{R^j/p_3^j}{R/p_3} > 1$.

We also show that the optimality condition for the no-distress state 1 is satisfied with inequality, $u'\left(c_0^j\right) = \frac{M_3^j}{q_3/\gamma_3} > \frac{M_1^j}{q_1/\gamma_1}$. To see this, note that applying the same condition for the representative country implies $\frac{M_3}{q_3/\gamma_3} = \frac{M_1}{q_1/\gamma_1}$. Combining this with $M_3^j > M_3$ and $M_1^j = M_1 = 1$ proves the claim. This in turn implies the investors in country j hold zero liquidity in the no-distress state, $z_1^j + y^j R_f = 0$. Intuitively, he investors' marginal utility in this state is relatively low, $M_1^j = 1$. In equilibrium, this induces them to hold as little liquidity as possible so as to hold more liquidity in distress states 2 and 3.

We next show that the prices characterized above correspond to an equilibrium in country j with appropriate corresponding allocations, $\left(c_0^j, \left\{z_s^j + y^j R_f\right\}_{s\in\{2,3\}}, x^{\text{in},j}\right)$ that satisfies $x^{\text{in},j} > 0$ as well as the following market clearing and budget constraints,

$$p_s^j = \frac{z_s^j + y^j R_f}{e + x^{\text{in},j}} \text{ for each } s \in \{2,3\},$$
$$\sum_{s\in\{2,3\}} q_s\left(z_s^j + y^j R_f\right) = \eta^j/R_f + 1 - c_0^j.$$

Note that, for each $s \in \{2,3\}$, the market clearing condition defines $z_s^j + y^j R_f$ as a function of $x^{\text{in},j}$. Plugging this into the budget constraint, we obtain,

$$\left(e + x^{\text{in},j}\right)\sum_{s\in\{2,3\}} q_s p_s^j = \eta^j/R_f + 1 - c_0^j.$$

Using the same steps for the representative country, we also obtain

$$(e+x)\sum_s q_s p_s = \eta/R_f + 1 - c_0.$$

Subtracting these equations and using $c_0^j < c_0$ (since $u'\left(c_0^j\right) > u'(c_0)$), we obtain,

$$x^{\text{in},j}\sum_{s\in\{2,3\}} q_s p_s^j > \left(\eta^j - \eta\right)\sum_s q_s + x\sum_s q_s p_s.$$

In view of Assumption $\tilde{\text{Y}}$, this equation implies $x^{\text{in},j} > 0$. It follows that the allocations, $\left(c_0^j, \left\{z_s^j + y^j R_f\right\}_{s\in\{2,3\}}, x^{\text{in},j}\right)$, together with the prices characterized earlier, (p_2, p_3) (and $z_1^j + y^j R_f = 0$) correspond to the equilibrium in country j.

We next establish the comparative statics of the equilibrium. First consider a decrease in R. Let $\tilde{p}_2 = p_2/R$ denote the price-to-return ratio in state 2. Then, the optimality condition (21) and the market clearing condition (16) can be written in terms of $(\tilde{p}_2, x)$ as,

$$u'(1-x) = R\left(\beta + (1-\beta)(1-\pi+\pi\tilde{p}_2)\left(1-\pi+\pi\frac{1}{\tilde{p}_2}\right)\right),$$
$$\text{and } \tilde{p}_2 = \min\left(1, \frac{\eta/R + x(1-\pi)}{e + x(1-\pi)}\right).$$

As before, the first equation describes $\tilde{p}_2$ as a decreasing function of x, the second equation describes $\tilde{p}_2$ as an increasing function of x, and the equilibrium corresponds to the intersection. Moreover, decreasing R strictly decreases the first curve for each x, and (under Assumption 1) strictly increases the second curve for each x. It follows that decreasing R decreases the equilibrium level of foreign investment, x. Thus, decreasing R also decreases the price level, $p_2 = \min\left(R, \frac{\eta + Rx(1-\pi)}{e + x(1-\pi)}\right)$, while leaving $p_3 = \min\left(R, \frac{\eta}{e}\right)$ unchanged.

Next note that Eq. (27) implies,

$$\frac{\overline{p} - \overline{p}^j}{R^j - R} = \frac{1-\pi}{\pi} \frac{M_1 \beta + M_2 (1-\beta)}{M_2 (1-\beta) + M_3 \beta}. \tag{B.67}$$

This implies that $\frac{\overline{p} - \overline{p}^j}{R^j - R} < \frac{1-\pi}{\pi}$ since $M_1 < M_2 < M_3$.After substituting for $M_s = 1 - \pi_s + \pi_s \frac{R}{p_s}$, the equation can also be written as,

$$\frac{\overline{p} - \overline{p}^j}{R^j - R} \frac{\pi}{1-\pi} = \frac{\beta \frac{1}{R} + (1-\beta) \xi(R)}{(1-\beta) \zeta(R) + \beta \frac{1}{p_3}}, \text{ where } \xi(R) = (1-\pi) \frac{1}{R} + \pi \frac{1}{p_2}.$$

It can be checked that increasing $\xi(R)$ increases the right hand side (since it is less than one). Thus, decreasing R increases $\frac{\overline{p} - \overline{p}^j}{R^j - R}$, both directly via the $1/R$ term in the numerator, and indirectly by increasing $\xi(R) = (1-\pi) \frac{1}{R} + \pi \frac{1}{p_2}$. It follows that decreasing R decreases $\overline{p}^j - \overline{p}$.

Next consider a decrease in β. By Proposition 5, this increases x, which in turn increases p_2 and leaves p_3 unchanged. Thus, it also decreases M_2 and leaves M_1 and M_3 unchanged.

Inspecting Eq. $(B.67)$ illustrates that decreasing β tends to increase $\frac{\overline{p} - \overline{p}^j}{R^j - R}$ by decreasing the weight on the smaller marginal utility (M_1) in the numerator as well as the weight on the larger marginal utility (M_3) in the denominator. However, decreasing β also generates an indirect effect since it also decreases M_2. As it turns out, the indirect effect tends to decrease $\frac{\overline{p} - \overline{p}^j}{R^j - R}$, counteracting the direct effect. We conjecture that the indirect effect does not overturn the direct effect, that is, $\frac{d}{d\beta}\left(\frac{\overline{p} - \overline{p}^j}{R^j - R}\right) < 0$, which in turn implies that decreasing β decreases $\overline{p} - \overline{p}^j$ (equivalently, increases $\overline{p}^j - \overline{p}$).

To prove this conjecture, we differentiate Eq. $(B.67)$ with respect to β, which implies that $\frac{d}{d\beta}\left(\frac{\overline{p} - \overline{p}^j}{R^j - R}\right) < 0$ if and only if,

$$\frac{M_1 \beta + M_2 (1-\beta)}{M_2 (1-\beta) + M_3 \beta} > \frac{M_1 + \frac{d}{d\beta}(M_2 (1-\beta))}{M_3 + \frac{d}{d\beta}(M_2 (1-\beta))}$$

We make a second conjecture that $\frac{d}{d\beta}(M_2 (1-\beta)) < 0$. Under this conjecture, the above inequality holds because,

$$\frac{M_1 \beta + M_2 (1-\beta)}{M_2 (1-\beta) + M_3 \beta} > \frac{M_1}{M_3} > \frac{M_1 + \frac{d}{d\beta}(M_2 (1-\beta))}{M_3 + \frac{d}{d\beta}(M_2 (1-\beta))}.$$

Here, the first equality follows from $M_1 < M_2 < M_3$, and the second inequality uses $M_1 < M_3$ together with $\frac{d}{d\beta}(M_2 (1-\beta)) < 0$.

Hence, it remains to prove the second conjecture, $\frac{d}{d\beta}(M_2 (1-\beta)) < 0$. To this end, note that Eq. (21) in Section 3.1 implies,

$$u'(1-x) = R\beta + ((1-\pi) R + \pi p_2)(1-\beta) M_2.$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)}{d\beta} < 0$ (since increasing β decreases x), we obtain,

$$R + \pi \frac{dp_2}{d\beta} (1-\beta) M_2 + ((1-\pi) R + \pi p_2) \frac{d}{d\beta}(M_2 (1-\beta)) < 0.$$

From here, note that $R+\pi\frac{dp_2}{d\beta}(1-\beta)M_2>0$ implies that $\frac{d}{d\beta}(M_2(1-\beta))<0$. That is, our second conjecture follows from a third conjecture,

$$(1-\beta)\pi\left(\frac{-dp_2}{d\beta}\right)M_2<R. \tag{B.68}$$

To prove the third conjecture, note that Eq. (21) can also be written as,

$$\frac{u'(1-x)}{R}=\beta+(1-\beta)\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right).$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)/R}{d\beta}<0$, we obtain,

$$(1-\beta)\pi\left(\frac{-dp_2}{d\beta}\right)\frac{M_2}{R}<\frac{\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right)-1}{\frac{1-\pi+\pi\frac{p_2}{R}}{(1-\pi)\frac{1}{R}+\pi\frac{1}{p_2}}\frac{R}{p_2^2}-1}<1$$

Hence, the last inequality follows since it is equivalent to, $\left(1-\pi+\pi\frac{R}{p_2}\right)\left((1-\pi)\frac{1}{R}+\pi\frac{1}{p_2}\right)<\frac{R}{p_2^2}$, which in turn holds since $1-\pi+\pi\frac{R}{p_2}<\frac{R}{p_2}$ and $(1-\pi)\frac{1}{R}+\pi\frac{1}{p_2}<\frac{1}{p_2}$. This establishes the third conjecture in $(B.68)$, which in turn implies $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^j}{R^j-R}\right)<0$. This completes the proof of the proposition. $\square$

Proof of Proposition 8. First consider the case in which the equilibrium features $x>0$ (despite the presence of taxes). Let $P^{opt}(x;\tau)$ correspond to the solution to Eq. (5), which describes the optimality condition for foreign investment. As before $P^{opt}(x;\tau)$ is weakly decreasing with a flat part for $x\leq\underline{x}$ and a strictly decreasing part for $x>\underline{x}$. However, the value of the flat part is slightly different and given by the unique solution to $\mu(p)(1-\tau)=R$ over the range $p\in[0,R]$. Note that the value of the flat part is strictly decreasing in τ. Let $\overline{\tau}\in(0,1)$ denote the tax level such that the equality, $\mu(p)(1-\overline{\tau})=R$, holds with $p=\eta/e$. For $\tau<\overline{\tau}$, we have $P^{opt}(0;\tau)>\eta/e=P^{mc}(0)$. A similar argument to that in the Proposition 1 then implies that there exists $x\in(0,1)$ and $p\in(0,\overline{p}(\tau)]$ such that $p=P^{mc}(x)=P^{opt}(x;\tau)$. The pair (x,p) corresponds to the equilibrium with taxes $\tau<\overline{\tau}$.

Next let $\underline{\tau}$ denote the tax level such that the equality, $\mu(p)(1-\underline{\tau})=R$, holds with $p=P^{mc}(\underline{x})=\frac{\eta+\underline{x}(1-\pi)R}{e+\underline{x}(1-\pi)}$. Note that this threshold is lower than the previous threshold, $\underline{\tau}\in(0,\overline{\tau})$. If $\tau<\underline{\tau}$, then the equilibrium features $x>\underline{x}$, and thus, $x^{loc}=0$. If instead $\tau\in(\underline{\tau},\overline{\tau})$, then the equilibrium features $x\in(0,\underline{x})$ and $x^{loc}=\underline{x}-x\in(0,\underline{x})$.

This completes the characterization of the equilibrium for $\tau<\overline{\tau}$. For completeness, consider also the remaining case with $\tau\geq\overline{\tau}$. In this case, we have a corner solution $x=0$ and $x^{loc}=\underline{x}$. In addition, the first equation in (30) is replaced by $R_fM=R$ (as opposed to $R_f=\overline{R}(1-\tau)$) since the foreign investment is strictly dominated and the legacy asset is priced by equating its marginal utility with that of local investment.

Next consider the comparative statics with respect to taxes. If $\tau\geq\overline{\tau}$, increasing the tax level further has no effect on the equilibrium. Consider the case with $\tau<\overline{\tau}$. Using Eq. (5) and Lemma 1, increasing the tax level shifts the curve $p=P^{opt}(x;\tau)$ downwards. Since the curve $p=P^{mc}(x)$ is strictly increasing and unaffected by the taxes, it follows that increasing the tax level strictly reduces both p and x. It also reduces R_f through the first equation in (30).

Finally, consider the optimal coordinated tax level set by a global planner. The planner's welfare is inversely proportional to the symmetric fire-sale price level in all countries, p. Since increasing the tax level reduces p, the planner optimally sets $\tau = 0$. $\square$

Proof of Proposition 9. First consider the case in which all countries set the tax level, $\tau \in [0, \bar{\tau})$. We prove that there exists a sufficiently small neighborhood of τ such that, when τ^j is in this neighborhood, the equilibrium in country j is characterized as the unique solution to the following system of equations,

$$R_f = \overline{R}(1-\tau) = \overline{R}^j \left(1-\tau^j\right), \text{ where } \overline{R}^j = (1-\pi) R + \pi p^j. \tag{B.69}$$
$$u'\left(1-x^j\right) = M^j \overline{R}(1-\tau), \text{ where } M^j = 1 - \pi + \pi \frac{R}{p^j}$$
$$\text{and } p^j = \min\left(R, \frac{\eta + x^j \overline{R}(1-\tau) + x\overline{R}\tau}{e + x^{\text{in},j}}\right).$$

To see this, first note that the first equation describes p^j as an implicit function of τ^j. Then note that the second equation describes x^j as an implicit function of the pair, $\left(\tau^j, p^j\right)$. Finally, note that the last equation describes $x^{\text{in},j}$ as an implicit function of τ^j, p^j, x^j. It follows that there exists a sufficiently small neighborhood, $(\tau - \varepsilon, \tau + \varepsilon)$, such that there is a unique solution to the system in $(B.69)$ when $\tau^j \in (\tau - \varepsilon, \tau + \varepsilon)$. Moreover, ε can be taken to be sufficiently small so that $x^j > \underline{x}$ and $x^{\text{in},j} > 0$ (since the equilibrium with symmetric taxes, $\tau^j = \tau$, satisfies $x = x^{\text{in}} > \underline{x}$). When this is the case, the solution corresponds to an equilibrium in country j (since $x^j > \underline{x}$ implies $u'\left(1-x^j\right) > R$ and local investment is dominated as implicitly assumed by $(B.69)$).

Suppose $\tau^j \in (\tau - \varepsilon, \tau + \varepsilon)$ and consider the comparative statics for the equilibrium in country j. Increasing the tax level increases the price, p^j, in view of the first equation in $(B.69)$. It follows that the planner in country j strictly prefers to increase the tax level, τ^j. Thus, the symmetric allocation with $\tau \in [0, \bar{\tau})$ does not correspond to a Nash equilibrium.

Next consider the case in which all countries set the tax level, $\tau = \underline{\tau}$. In this case, the symmetric equilibrium features flows, $\underline{x}$, and the corresponding price level, $P^{mc}(\underline{x})$. We claim that there is no profitable deviation for an individual planner. Note that the tax level cannot be increased further (by assumption). Suppose the planner lowers the tax level to an arbitrary, $\tau^j \in [0, \underline{\tau})$. Suppose τ^j is not too low so that there is a solution to the first equation with $p^j > 0$ (otherwise, the equilibrium features $p^j = 0$, which does not correspond to a profitable deviation). Then, the same argument as above applies and shows that there is a unique solution to the system in $(B.69)$. Moreover, reducing the tax level decreases p^j, increases x^j, and increases $x^{\text{in},j}$. In particular, we have $x^j > \underline{x}$, which ensures that the solution corresponds to an equilibrium in country j. Note also that $p^j < P^{mc}(\underline{x})$, which shows that the deviation is not profitable for the planner in country j. Thus, the symmetric allocation with the tax level, $\tau = \underline{\tau}$, corresponds to a Nash equilibrium.

Finally, let (x, p) denote the equilibrium without taxes and note that $x > \underline{x}$ and $p > P^{mc}(\underline{x})$. This proves that the capital flows and the fire-sale price in the Nash equilibrium are lower than what would obtain in an equilibrium without taxes. By the first equation in (30), the risk-free return is also lower, completing the proof. $\square$

Proof of Proposition 10.

Part (i). The possibility of multiple equilibria is illustrated in the left panel of Figure 3. The example features a discontinuous function $f(\cdot)$, but the multiple equilibria in the figure would remain if we were to approximate $f(\cdot)$ with a smooth function. Next suppose there are multiple symmetric equilibria and consider their welfare ranking. Recall that the system in $(A.55)$ describes the equilibrium price as a decreasing function of the tax level. Thus, an equilibrium with lower price level is also associated with a higher tax level. Given the welfare function in (31), this equilibrium is dominated for each planner by an equilibrium with a higher price level and a lower tax level.

Part (ii). Suppose there is a unique equilibrium. The equilibrium is characterized as the intersection of two decreasing curves. Moreover, the intersection is such that the best response curve crosses the equilibrium price curve, $p(\tau)$, from above (as illustrated in the right panel of Figure 3). Inspecting the equilibrium system in (30) shows that decreasing η shifts the equilibrium price curve, $p(\tau)$, downwards. Combining these observations, it follows that reducing η reduces p and increases τ in the unique Nash equilibrium. The risk-free return also declines from the first equation in (30). The last part of the proposition follows by combining the observation that τ increases with the comparative statics of the increase in τ established in Proposition 8. $\square$

Proof of Proposition 11. Let $P^{mc}\left(x;\eta^{pl}\right) = \min\left(R, \frac{\eta+\eta^{pl}+x(1-\pi)R}{e+x(1-\pi)}\right)$ denote the market clearing curve when the planner injects liquidity, η^{pl}. As in the proof of Proposition 1, the equilibrium is characterized as the intersection of the strictly increasing curve, $p = P^{mc}\left(x;\eta^{pl}\right)$, and the decreasing curve, $p = P^{opt}(x)$. Moreover, the intersection is in the strictly decreasing range of $p = P^{opt}(x)$ (with $x > \underline{x}$). Note that increasing η^{pl} shifts the market clearing curve upwards without affecting the optimality curve. Hence, it leads to a higher price, p, and lower capital flows, x. Since the global planner prefers higher prices, she creates the maximum amount of liquidity, $\eta^{pl} = \bar{\eta}$. $\square$

Proof of Proposition 12. Consider the case in which all countries create positive liquidity, $\eta^{pl} > 0$. We prove that, when $\eta^{pl,j}$ is in a sufficiently small neighborhood of η^{pl}, then the equilibrium in country j is characterized as the unique solution to the following system of equations,

$$\begin{aligned} R_f = \overline{R} = \overline{R}^j, &\text{ where } \overline{R}^j = (1-\pi)R + \pi p^j \\ \text{and } u'\left(1-x^j\right) = \overline{R}M^j, &\text{ where } M^j = 1-\pi+\pi\frac{R}{p^j} \\ \text{and } p^j = \min&\left(R, \frac{\eta+\eta^{pl,j}+x^j\overline{R}}{e+x^{\text{in},j}}\right). \end{aligned} \tag{B.70}$$

To see this, note that the first equation determines $p^j = p$ as independent of η^j. The second equation determines $x^j = x$ as independent of η^j. The third equation then implies that

$$x^{\text{in},j} - x = \left(\eta^{pl,j} - \eta^{pl}\right)/p.$$

Hence, there exists a sufficiently small neighborhood, $\left(\eta^{pl}-\varepsilon, \eta^{pl}+\varepsilon\right)$, such that there is a unique solution to the system in $(B.70)$ with $x^{\text{in},j} > 0$. The solution corresponds to an equilibrium since $x^j = x > \underline{x}$ (and thus, $u'\left(1-x^j\right) > R$ and the local investment is dominated as implicitly assumed by the system in $(B.70)$).

Next suppose $\eta^{pl,j} \in \left(\eta^{pl}-\varepsilon, \eta^{pl}+\varepsilon\right)$ and consider the comparative statics for the equilibrium in country j. Decreasing $\eta^{pl,j}$ has no effect on the price. However, for any finite λ, it helps to economize on the liquidity-creation costs (see the social welfare function in (33)). Hence, the symmetric allocation with $\eta^{pl} > 0$ does not correspond to a Nash equilibrium for any finite λ, and thus, also as $\lambda \to \infty$.

Next consider the case in which all countries create zero liquidity, $\eta^{pl} = 0$. We claim that there is no profitable deviation for an individual planner. Suppose the planner deviates to $\eta^{pl,j} > 0$. The same argument as above implies that, for any $\eta^{pl,j} > 0$, the local equilibrium is characterized by $p^j = p, x^j = x$, and $x^{\text{in},j} = x + \eta^{pl,j} > 0$. The deviation is not profitable since it does not change the price but it increases liquidity-creation costs for any finite λ. This proves that the symmetric allocation with $\eta^{pl} = 0$ corresponds to a Nash equilibrium for any finite λ, and thus, also as $\lambda \to \infty$. $\square$

Proof of Proposition 13. The proof is provided in the main text.

Proof of Proposition 14. First consider the case without taxes. Under Assumption 1S, we conjecture an equilibrium in which $x^{\text{in},D} > 0, x^{\text{in},E} \geq 0$, and conditions (38) and (39) are satisfied as equalities (even at the corner case, $x^{\text{in},E} = 0$). Under this conjecture, combining condition (38) with $\pi = 1$ implies $R_f = p^D = p^E$. Likewise, combining condition (39) with $\pi = 1$ implies,

$$u'\left(1 - x^{\text{out},k}\right) = \overline{R}^k M^k = p^k \frac{R^k}{p^k} = R^k \text{ for each } k.$$

In particular, the foreign outflows from each region are fixed at their minimum levels, $x^{\text{out},k} = \underline{x}^{\text{out},k}$ (defined as the solution to $u'\left(1 - \underline{x}^{\text{out},k}\right) = R^k$).

We next plug $\pi = 1$ into the market clearing conditions (40), and use $p^D = p^E$, to obtain,

$$p^k = \frac{\eta^k + x^{\text{out},k} p^k}{e + x^{\text{in},k}} = \frac{\eta^k}{e + x^{\text{in},k} - x^{\text{out},k}} \text{ for each } k \in \{D, E\}. \tag{B.71}$$

After multiplying these inequalities with m^k/p^k and aggregating, we obtain,

$$m^D\left(e + x^{\text{in},D} - x^{\text{out},D}\right) + m^E\left(e + x^{\text{in},E} - x^{\text{out},E}\right) = \eta^D \frac{m^D}{p^D} + \eta^E \frac{m^E}{p^E}. \tag{B.72}$$

After using $m^D + m^E = 1$ and the conservation equation (41), the left hand side becomes e. Using $p^D = p^E$ on the right hand side, this implies,

$$p^D = p^E = \frac{\eta^D m^D + \eta^E m^E}{e}.$$

Combining this with the market clearing condition in $(B.71)$, we obtain,

$$x^{\text{in},k} - x^{\text{out},k} = e\left(\frac{\eta^k}{\eta^D m^D + \eta^E m^E} - 1\right) \text{ for each } k \in \{D, E\}. \tag{B.73}$$

Since $x^{\text{out},k} = \underline{x}^{\text{out},k}$, this equation determines $x^{\text{in},D}, x^{\text{in},E}$ in terms of the parameters of the problem. Note also that $x^{\text{in},D} > \underline{x}^{\text{out},D} > 0$ since $\eta^D > \eta^E$, and $x^{\text{in},E} < \underline{x}^{\text{out},E}$ since $\eta^E < \eta^D$. In addition $x^{\text{in},E} \geq 0$ since $\underline{x}^{\text{out},E} \geq e\left(1 - \frac{\eta^E}{\eta^D m^D + \eta^E m^E}\right)$ by Assumption 1S. This verifies our conjecture and completes the characterization of equilibrium without taxes. In particular, the equilibrium prices and flows satisfy Eqs. (42) in the main text.

It is also useful to note that in equilibrium investors are indifferent between local and foreign investment. The equilibrium characterized above corresponds to zero local investment and $x^{\text{out},k} = \underline{x}^{\text{out},k}$ for each k. However, there are also equilibria with $x^{\text{out},k} \in \left[0, \underline{x}^{\text{out},k}\right]$ for each k. The only requirement is that the outflows from the region E exceed a minimum level, $x^{\text{out},E} \geq e\left(1 - \eta^k / \left(\eta^D m^D + \eta^E m^E\right)\right)$. For each pair of outflows, $x^{\text{out},D}, x^{\text{out},E}$, that satisfies these conditions, there exists an equilibrium in which the inflows are determined by Eq. $(B.73)$ and the prices are determined by Eq. (9). In particular, the indeterminacy does not affect the equilibrium prices.

Next consider the equilibrium with taxes. The equilibrium conditions $(38-41)$ are slightly modified since investing locally is no longer weakly dominated. First suppose $\tau^E > 0$ and $\tau^D = 0$. In this case, it is easy to check that the equilibrium prices are unchanged. The taxes in region E imply the inflows into region E are zero, $x^{\text{in},E} = 0$, and the outflows from region E (which go into region D) are at their minimum level, $x^{\text{out},E} = e\left(1 - \eta^k / \left(\eta^D m^D + \eta^E m^E\right)\right)$. Hence, the taxes in region E help to partially resolve the indeterminacy described above, but they do not affect the equilibrium prices or net inflows.

Next suppose $\tau^E = 0$ and $\tau^D > 0$, where τ^D is in a sufficiently small neighborhood of 0. We conjecture an equilibrium in which the after-tax returns are equated, $R_f = p^D\left(1 - \tau^D\right) = p^E$. Note that the market clearing conditions $(B.71)$ remain unchanged (since the taxed liquidity is injected back into the investing regions by assumption). The aggregated condition $(B.72)$ also remains unchanged. As before, the left hand side of this equation is equal to e, which implies,

$$\eta^D \frac{m^D}{p^D} + \eta^E \frac{m^E}{p^E} = e.$$

The equilibrium is determined by solving this equation together with $p^E = p^D\left(1 - \tau^D\right)$. The prices have a closed form solution given by Eq. (43) in the main text. To obtain the corresponding flows, first note that combining Eq. (43) with the market clearing conditions $(B.71)$implies,

$$\begin{aligned} x^{\text{in},D} - x^{\text{out},D} &= e\left(\frac{\eta^D}{\eta^D m^D + \eta^E m^E / \left(1 - \tau^D\right)} - 1\right), \\ \text{and } x^{\text{in},E} - x^{\text{out},E} &= e\left(\frac{\eta^E}{\eta^D m^D \left(1 - \tau^D\right) + \eta^E m^E} - 1\right). \end{aligned}$$

Next note that $x^{\text{out},D} = 0$, since investing in other countries of region D is dominated by local investment in region D. Finally, note that $x^{\text{out},E} \geq e\left(1 - \frac{\eta^E}{\eta^D m^D(1-\tau^D)+\eta^E m^E}\right)$ and $x^{\text{out},E} \leq \underline{x}^{\text{out},E}$. Given any choice of $x^{\text{out},E}$ in this interval and $x^{\text{out},D} = 0$, the flows are uniquely pinned down by the above displayed equations.

Finally, consider the case with $\tau^E > 0$ and $\tau^D > 0$, where τ^D is in a sufficiently small neighborhood of 0. The equilibrium prices in this case are exactly as in the previous case. The only difference (as before) is that the taxes in region E imply the inflows into region E are zero, $x^{\text{in},E} = 0$, and the outflows from region E (which go into region D) are at their minimum level, $x^{\text{out},E} = e\left(1 - \frac{\eta^E}{\eta^D m^D(1-\tau^D)+\eta^E m^E}\right)$. This completes the proof. $\square$

Proof of Proposition 15. We first complete the characterization of equilibrium. The return thresholds are given by,

$$\begin{aligned} R^D_{low} &\equiv (1-\pi) R^E + \pi p^E_{low} \text{ and } R^D_{high} = (1-\pi) R^E + \pi p^E_{high}, \\ \text{with } p^E_{low} &\equiv \frac{\eta^E + (1-\pi) R^E}{e + 1/m^E - \pi} \text{ and } p^E_{high} \equiv \frac{\eta^E + (1-\pi) R^E}{e - \pi}. \end{aligned} \tag{B.74}$$

These thresholds are obtained from the system in (44) by noting that any price between p^E_{low} and p^E_{high} can be obtained by adjusting the flows that go into region E and letting the residual flows go into region D. It follows that an equilibrium with flows into both regions (that satisfies the system in (44)) exists if and only if $R^D \in \left(R^D_{low}, R^D_{high}\right)$.

Next suppose $R^D \leq R^D_{low}$. Then, the equilibrium system (44) is replaced by,

$$R_f = (1-\pi) R^E + \pi p^E \geq R^D,$$
$$x^{\text{in},D} = 0, x^{\text{in},E} = 1/m^E \text{ and } p^E = \frac{\eta^E + (1-\pi) R^E}{e + 1/m^E - \pi}.$$

Suppose instead $R^D \geq R^D_{high}$. Then, the equilibrium is system is replaced by,

$$R_f = (1-\pi) R^E + \pi p^E \leq R^D,$$
$$x^{\text{in},D} = 1/m^D, x^{\text{in},E} = 0 \text{ and } p^E = \frac{\eta^E + (1-\pi) R^E}{e - \pi}.$$

In either case, there is a solution to the system in view of the definition of the thresholds R^D_{low}, R^D_{high} in $(B.74)$.

Next suppose $R^D \in \left(R^D_{low}, R^D_{high}\right)$ and consider the comparative statics for the system (44). A decrease in R^D decreases p^E via the first equation. This increases $x^{\text{in},E}$ via the second equation. This in turn decreases $x^{\text{in},D}$ via the third equation. □

Proof of Proposition 16. The case $R^D \geq R^D_{high}$ is straightforward since the taxes in region E have no effect on the equilibrium. Consider the case $R^D \in \left(R^D_{low}, R^D_{high}\right)$. Note that this case always features flows into region D (but flows into region E might be driven to zero by taxes in this region). Hence, the local investor's return from investing in other countries is equal to $M^E R^D = \left(1 - \pi + \pi \frac{R^E}{p^E}\right) R^D$. Note also that the maximum price level in this case is given by $p^{E,\max} = \frac{\eta^E + R^D}{e}$ (which obtains when all of the flows exit region E). Hence, as long as the condition in Assumption 3 holds, we have $M^E R^D = \left(1 - \pi + \pi \frac{R^E}{p^E}\right) R^D \geq R^E$.

Thus, local investment in region E is dominated, $x^{loc,E} = 0$. We also continue to assume the local investment in region D is zero, $x^{\text{loc},D} = 0$, which is without loss of generality as before. Using these observations, an equilibrium with flows in both directions is characterized by the system,

$$\begin{aligned} R_f &= \left(1 - \tau^E\right) \left((1-\pi) R^E + \pi p^E\right) = R^D, \\ p^E &= \frac{\eta^E + R^D}{e + x^{\text{in},E}}, \\ m^D x^{\text{in},D} &+ m^E x^{\text{in},E} = 1. \end{aligned} \tag{B.75}$$

The second equation suggests that the planner can increase the fire-sale prices in region E up to the level, $p^{E,\max} = \frac{\eta^E + R^D}{e}$, which obtains when the inflows are zero, $x^{\text{in},E} = 0$. We let $\tau^{E,\max}$ denote the tax level that brings about this outcome, that is, $\tau^{E,\max}$ is the solution to,

$$\left(1 - \tau^{E,\max}\right) \left((1-\pi) R^E + \pi p^{E,\max}\right) = R^D, \text{ where } p^{E,\max} = \frac{\eta^E + R^D}{e}. \tag{B.76}$$

Below, we verify that this equation has a positive solution, $\tau^{E,\max} > 0$. It follows that, as long as $\tau^E \leq \tau^{E,\max}$, there is a solution to the system in $(B.75)$ with $x^{\text{in},E} \geq 0$ and $p^E \leq p^{E,\max}$, which corresponds to the equilibrium.

To verify that Eq. $(B.76)$ has a positive solution, note that the solution is a strictly decreasing function of R^D (since $e/\pi < 1$ in view of Assumption 3), that is, lower returns in region D require higher taxes to eliminate all flows into region E. Note also that $R^D = R^D_{high}$ implies $p^E_{high} = p^{E,\max}$, which in turn implies $\tau^{E,\max} = 0$ (since $R^D_{high} = (1-\pi) R^E + \pi p^E_{high}$ by definition). It follows that $\tau^{E,\max} > 0$ for each $R^D \in \left(R^D_{low}, R^D_{high}\right)$.

Next note that the planner's optimality condition implies Eq. (47) where $V\left(\tau^j\right) = v'\left(\tau^j\right)\left(1-\tau^j\right) + v\left(\tau^j\right)$. In view of the Inada conditions on the cost function, $v(\cdot)$, there is a unique interior optimal tax level, $\tau^{E,*} \in (0,1)$. The unconstrained optimal tax level is given by $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$, which is strictly positive since $\tau^{E,\max}, \tau^{E,*} > 0$. The equilibrium pair, $\left(\tau^{E,}, p^E\right)$, is found by solving Eq. (46) together with the optimality condition $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$.

Next suppose the optimal tax level takes an interior value, $\tau^E = \tau^{E,*} < \tau^{E,\max}$ (so that $x^{in,E} > 0$). In this case, Eq. (46) describes p^E as a strictly increasing function τ^E (since higher taxes increase the price level). Condition (47) represents p^E as a strictly decreasing function of τ^E (since lower prices induce greater taxes). The optimal tax level is the intersection of these two curves. A decline in R^D shifts the first (increasing) curve downwards without affecting the second (decreasing) curve. This leads to a greater tax level, τ^E, as well as a lower price level, p^E, completing the proof. $\square$

Proof of Proposition 17. Let $\underline{\tau} \in (0,1)$ denote the unique solution to,

$$u'(1-\underline{x}) = R = \sum_s \gamma_s \mu_s \left(\frac{\eta + \underline{x}(1-\pi_s) R}{e + \underline{x}(1-\pi_s)}\right)(1-\underline{\tau}). \tag{B.77}$$

Suppose $\tau_s < \underline{\tau}$ for each $s \in S$. We prove that the equilibrium is characterized by the unique solution to Eq. (15) and Eqs. (16). To this end, define the function,

$$F\left(x; (\tau_s)_s\right) = u'(1-x) - \sum_s \gamma_s \mu_s (p_s)(1-\tau_s), \text{ where } p_s = \frac{\eta + x(1-\pi_s) R}{e + x(1-\pi_s)} \text{ for each } s.$$

Note that $F\left(\underline{x}; (\tau_s)_s\right) < 0$ in view of Eq. $(B.77)$ and $\tau_s < \underline{\tau}$ for each $s \in S$. Note also that $F\left(1; (\tau_s)_s\right) = \infty$. Since $F\left(x; (\tau_s)_s\right)$ is continuous and strictly increasing in x, there exists a unique solution, $x \in (\underline{x}, 1)$. Since $x > \underline{x}$ (and thus, $u'(1-x) > R$), the solution corresponds to the equilibrium.

Next suppose $\tau_s < \underline{\tau}$ for each $s \in S$ and consider the comparative statics for taxes. Note that increasing τ_s for any s shifts the function, $F\left(x; (\tau_s)_s\right)$, upwards. This reduces the equilibrium foreign investment, x (characterized as the solution to $F\left(x; (\tau_s)_s\right) = 0$). By Eq. (16), this also reduces the fire-sale price level, p_s, in every state. It follows that the global planner sets, $\tau_s = 0$, for each s.

Next consider the Nash equilibrium. Consider an allocation with symmetric taxes, $\tau_s < \tilde{\tau} < \underline{\tau}$, for each s (where recall that $\tilde{\tau}$ is an upper bound on the taxes in view of the assumption that $v'(\tilde{\tau}) = \infty$). Suppose a planner deviates and sets a different tax policy, $\left(\tau^j_s\right)_s$. We let $x^{\text{in},j}$ denote the inflows into the country j, and $x^j = \sum q_s z^j_s$ denote the total outflows from the country. When $\left(\tau^j_s\right)_s$ is in a sufficiently small neighborhood of $(\tau_s)_s$, we conjecture that the equilibrium will feature $x^{\text{in},j} > 0, z^j_s > 0$ for each s, and $x^j > \underline{x}$. The conditions for such an equilibrium can be written as,

$$1 = \sum_s q_s \overline{R}^j_s \left(1-\tau^j_s\right), \text{ where } \overline{R}^j_s = (1-\pi_s) R + \pi_s p^j_s, \tag{B.78}$$

$$\text{and } q_s/\gamma_s = \frac{M^j_s}{u'(1-x^j)} \text{ for each } s, \text{ where } M^j_s = 1 - \pi_s + \pi_s \frac{R}{p^j_s},$$

$$\text{and } p^j_s = \min\left(R, \frac{\eta + z^j_s + x\overline{R}_s \tau_s}{e + x^{\text{in},j}}\right) \text{ for each } s.$$

It can be checked that there exists a sufficiently small neighborhood of $(\tau_s)_s$, denoted by $B((\tau_s)_s) \subset \mathbb{R}^{|S|}$, such that if $\left(\tau_s^j\right)_s \in B((\tau_s)_s)$, then there exists a unique solution to this system, $\left(p_s^j, z_s^j, x^{\text{in},j}\right)$, that satisfies the conjecture (since $x^{\text{in}} = x > 0$ and $z_s = x^{\text{in}}\overline{R}_s > 0$ in the symmetric allocation). The solution corresponds to the equilibrium given taxes since $x^j > \underline{x}$ (so that $u'\left(1 - x^j\right) > R$ and local investment is dominated as implicitly assumed by the second set of equations). This establishes that, when $\left(\tau_s^j\right)_s \in B((\tau_s)_s)$, the prices, $\left(p_s^j\right)_s$, are determined as the unique solution to the reduced system $(A.55)$ in the main text.

Next suppose the planner solves the constrained optimization problem,

$$\max_{\left\{\tau_s^j\right\}_s \in B\left((\tau_s)_s\right)} \sum_s \gamma_s \left((1-\zeta)\left((1-\pi_s) f(R) + \pi_s f\left(p_s^j\right)\right) + \zeta \overline{R}_s^j \left(1 - v\left(\tau_s^j\right)\right)\right).$$

The taxes can be optimal for the planner only if the first order conditions for this problem are satisfied so that there is no profitable deviation within the neighborhood. Taking the first order condition with respect to τ_s^j, we obtain,

$$\zeta \gamma_s \overline{R}_s^j v'\left(\tau_s^j\right) = \sum_{\tilde{s}} \gamma_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta\left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} \text{ for each } s.$$

Differentiating the first equation in $(A.55)$ with respect to τ_s^j, we also obtain,

$$q_s \overline{R}_s^j = \sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} \text{ for each } s.$$

Differentiating the second equation in $(A.55)$ for state $\tilde{s} \in S$ with respect to τ_s^j, we obtain,

$$\pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} = -\frac{dM_0^j}{d\tau_s^j} \frac{q_{\tilde{s}}}{\gamma_{\tilde{s}}} \frac{\left(p_{\tilde{s}}^j\right)^2}{R} \text{ for each } \tilde{s}.$$

Plugging the last equation into the previous two equations, we obtain,

$$\zeta \gamma_s \overline{R}_s^j v'\left(\tau_s^j\right) = -\frac{dM_0^j}{d\tau_s^j} \sum_{\tilde{s}} q_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta\left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R},$$

$$\text{and } q_s \overline{R}_s^j = -\frac{dM_0^j}{d\tau_s^j} \sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \left(q_{\tilde{s}}/\gamma_{\tilde{s}}\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R},$$

for each $s \in S$. Taking the ratio of these expressions, we have,

$$\zeta \frac{\gamma_s}{q_s} v'\left(\tau_s^j\right) = \frac{\sum_{\tilde{s}} q_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta\left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R}}{\sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \left(q_{\tilde{s}}/\gamma_{\tilde{s}}\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R}} \text{ for each } s. \tag{B.79}$$

This expression describes the optimal tax rate in state s in terms of the other tax rates and the endogenous variables. For the special case in which S is a singleton, the equation reduces to Eq. (32) that describes the optimal tax rate. In the more general case, the equations for all tax rates, $\{\tau_s\}_s$, are jointly solved taking the prices, $\{q_s, p_s\}_s$, as given. Note also that the prices are determined as a function of the tax rates, $\{\tau_s\}_s$, as described in the main text. The Nash equilibrium must satisfy both of these equation systems. As before, there can be multiple stable equilibria.

Note that, since $v'(0) = v(0) = 0$, Eq. $(B.79)$ cannot be satisfied for $\tau_s = 0$. This proves that the taxes are strictly positive in any Nash equilibrium.

Now consider a particular equilibrium. Note that the right hand side of Eq. $(B.79)$ does not depend on state s. Then, taking the ratio of these equations for two arbitrary states we obtain Eq. $(A.56)$ in the main text, completing the proof. $\square$