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A MODEL OF FICKLE CAPITAL FLOWS AND RETRENCHMENT: GLOBAL LIQUIDITY CREATION AND REACH FOR SAFETY AND YIELD

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ABSTRACT

Gross capital flows are very large and highly cyclical. They are a central aspect of global liquidity creation and destruction. They also exhibit rich internal dynamics that shape fluctuations in domestic liquidity, such as the fickleness of foreign capital inflows and the retrenchment of domestic capital outflows during crises. In this paper we provide a model that builds on these observations to address some of the main questions and concerns in the capital flows literature. Within this model, we find that for symmetric economies, the liquidity provision aspect of capital flows vastly outweighs their fickleness cost, so that taxing capital flows, while could prove useful for a country in isolation, backfires as a global equilibrium outcome. However, if the system is heterogeneous and includes economies with abundant (DM) and with limited (EM) natural domestic liquidity, there can be scenarios when global liquidity uncertainty is high and EM's reach for safety can destabilize DMs, as well as risk-on scenarios in which DM's reach for yield can destabilize EMs.

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1. Introduction

In its "Capital for the Future," World Bank (2013) projects that annual gross capital flows will grow from approximately 15 trillion in 2016 to over 40 trillion dollars by 2030.[1] After years of a steady rise, these large flows collapsed following the subprime crisis, (Lane and Milesi-Ferretti (2012); Milesi-Ferretti and Tille (2011)), which coincided with the large and stubborn rise in most cross-country liquidity-scarcity measures, and have recovered only gradually since then (see, e.g., Borio et al. (2016) for a stark illustration of the reduced international liquidity as even the *covered* interest parity condition among the majors broke down since the subprime crisis). These gross flows are a central component of global liquidity (see e.g., Committee on the Global Financial System report, Landau (2011)). They also exhibit rich internal dynamics that influence domestic liquidity, such as the fickleness (reversal) of foreign capital inflows and the retrenchment (reversal) of domestic capital outflows during crises (see e.g., Broner et al. (2013); Forbes and Warnock (2012); Bluedorn et al. (2013)). While it is the fickleness of capital flows and its costs on domestic economies that has captured most of the attention of academics and policymakers, and is the main reason behind the perennial calls for limits on capital flows,[2] in most instances the *retrenchment* feature and its benefit to domestic economies is just the flip-side of the *fickleness* feature (see, e.g., Forbes and Warnock (2012) for an illustration of the benefit of retrenchment for countries as diverse as the US and Chile during the subprime crisis).

The main purpose of this paper is to build a global equilibrium model that puts at the core of its mechanism the liquidity creation aspect of gross capital flows and the tension between the costs of fickleness and the benefits of retrenchment. In a nutshell, gross capital flows play three roles in our model: *global liquidity creation*, *reach for safety*, and *reach for yield*. From the point of view of *financial stability*, the first role is unambiguously good, the second one is mixed, while the last one is unambiguously bad. The weight of these different roles varies across countries, such as developed and emerging, and across global economic conditions, such as risk-on and risk-off environments. However there is a systematic bias among local regulators (relative to a benevolent global planner) against capital flows, as the costs associated to reach for safety and, particularly, reach for yield, are felt directly at the local level, while the benefits of global liquidity creation are spread across the world economy.

For these reasons, we find that for symmetric economies, the liquidity provision aspect of capital flows vastly outweighs their fickleness cost, so that taxing capital flows, while could prove useful for a country in isolation, backfires as a global equilibrium outcome. However, if the system is heterogeneous and includes economies with large (DM) and with limited (EM) natural domestic liquidity, there can be scenarios when global liquidity uncertainty is high and

[1] See Figure 3.7b in World Bank (2013) for the inflows half.

[2] This was a central theme of the post World War II meetings at Bretton Woods (e.g., Forbes (2016)), and it has reemerged in earnest in the post subprime crisis era, mostly in response to the spillovers of developed markets' expansionary monetary policies onto emerging market economies (see, e.g., IMF (2012)) but also onto other developed market economies (see, e.g., Klein (2012)).

EM's reach for safety can destabilize DMs, as well as risk-on scenarios in which DM's reach for yield can destabilize EMs.

Beyond this top-down description of our core results, the model provides a coherent framework to address many positive and normative issues of relevance in the current global economy, ranging from the response of capital flows and local asset prices to shortages of safe assets and changes in global risk conditions, to the financial stability costs and benefits of coordinated and uncoordinated regulatory responses to sudden stop risks. These results are described in terms of fifteen propositions, most of which characterize equilibrium in different environments and provide a few comparative statics for the corresponding context. It is useful to summarize some of these comparative statics at this stage, to give the reader a fuller sense of the issues addressed (and not) in this paper and its connection with a wide variety of recent literatures:

- *Proposition 1:* A reduction in safe asset supply worsens fire-sale prices during crises, lowers safe interest rates, and increases gross capital flows

- *Proposition 2:* In a global equilibrium with scarcity of safe assets, a country with abundance of them will experience net capital inflows that exacerbate its own fire sales during crises

- *Proposition 3:* In the same equilibrium, a country with high returns during normal times will also experience net capital inflows that exacerbate its own fire sales during crises

- *Proposition 4:* As liquidity shocks become more correlated across countries, gross glows become less effective in providing global liquidity, a risk-premium on gross flows emerges, and safe interest rates drop.

- *Proposition 5:* As the frequency of highly correlated states rises, gross capital flows decline and safe interest rates drop.

- *Proposition 6:* As the global liquidity cycle grows in importance, safe asset producer countries not only (net) export assets, but the composition of its gross flows changes, with outflows targeting risky assets while inflows are mostly after domestic safe assets. That is, the country effectively leverages its outflows.

- *Proposition 7:* A decline in asset returns and cross-correlations (i.e., a "risk-on" environment) exacerbates capital flows to high-yielders and the size of the potential fire sale.

- *Proposition 8:* In a symmetric environment, a global planner will always choose not to tax capital flows.

- *Proposition 9:* In the same symmetric environment and with no costs of taxation (beyond its effect on capital flows), there is a unique Nash equilibrium of local regulators that has positive taxes on capital flows. This equilibrium has lower gross capital flows and safe interest rates, and worse local fires sales during crises, than under the global planner's outcome.

- *Proposition 10:* In the same environment but with convex costs of taxation, there can be multiple equilibria. The worst equilibrium has higher taxes on capital flows. In this case a reduction in the global supply of safe assets increases taxes on capital inflows, exacerbates fire sales, and lowers safe rates.

- *Proposition 11:* If local governments have some capacity to control the supply of safe assets, the global planner would maximize the utilization of this capacity.

- *Proposition 12:* In contrast, local governments would be reluctant to use this capacity.

- *Proposition 13:* In an environment where reach-for-safety dominates global liquidity creation, taxing capital inflows to safe asset producers exacerbates fire-sale prices in EMs but reduces them in DMs.

- *Proposition 14:* In an environment where EMs have lower safe liquidity but higher yields during normal times, a drop in returns in DMs may increase capital inflows into EMs and exacerbate its fire sales.

- *Proposition 15:* In a reach-for-yield dominated environment, taxing capital flows to EMs stabilizes them without worsening financial stability in DMs

Related literature. From a methodological angle, our two central ingredients are endogenous liquidity creation and fire sales. As such, our paper relates to Allen and Gale (1994) who endogenize market size and volatility in a closed economy context with entry costs. In our model liquidity is created in a manner akin to Holmström and Tirole (1998, 2001). Our context is different, as is countries and their policies rather than corporations that create the assets behind the liquidity.

The core (non reach-for-yield) reason for capital flows in our environment is the scarcity of locally safe assets to store value for domestic fire-sales stabilization. In this sense, our work is closely related to the burgeoning literature on limited availability of global assets and its macroeconomic consequences (e.g. Caballero (2006); Caballero et al. (2008, 2016); Bernanke et al. (2011); Gorton et al. (2012); Krishnamurthy and Vissing-Jorgensen (2012); Gorton (2016)).

Our model also shares elements of the limits-to-arbitrage and fire sales literature. In particular, the (limited) liquidity pricing of local assets is similar to, e.g., Allen and Gale (1994); Shleifer and Vishny (1997); Gabaix et al. (2007); Lorenzoni (2008); Krishnamurthy (2010); Gromb and Vayanos (2016); Holmström and Tirole (2001). The all or none attitude of fickle foreign investors behind the fire sales is reduced form in our model, but is intended to capture the attitude of Knightian agents facing an unfamiliar (foreign) situation, and as such it relates to Dow and da Costa Werlang (1992); Caballero and Krishnamurthy (2008); Caballero and Simsek (2013); Haldane (2013).

At some level our paper can be seen a balancing act of some of the forces highlighted in the literatures on international risk sharing (e.g., Grubel (1968); Cole and Obstfeld (1991); Obstfeld (2009); Van Wincoop (1994, 1999); Lewis (2000); Coeurdacier and Rey (2013); Lewis and Liu (2012)) and on domestic undervaluation of the costs of capital flow reversals (e.g. Caballero and Krishnamurthy (2004, 2005, 2006); Korinek (2010); Caballero and Lorenzoni (2014); Costinot et al. (2014); Brunnermeier and Sannikov (2015); Calvo (2016)). Through our fickleness assumption we take as given the core conclusions of the latter literature, and study whether the mechanisms of the former can offset the negative volatility implications once we consider the feedbacks of the global equilibrium. In this methodological sense we also relate to Scott and Uhlig (1999), who take as given the fickleness of financial investors and study the impact of this feature on economic growth.

There is an extensive debate within the international risk sharing literature on the magnitude of the welfare gains associated to it (e.g., Coeurdacier et al. (2015) argue they are small, while Colacito and Croce (2010) take the opposite view). However, the main reason for diversification in our model is different from the mostly neoclassical ones highlighted in the risk sharing literature, as in our model international liquidity is used to fund the comparative advantage of domestic arbitrageurs during fire sales, which is aligned with the evidence in Broner et al. (2013) and others (see footnote 4).

Much of the theoretical support for building policy barriers to capital flows relies on some externality, principally within the domestic financial system, which leads to an excessive credit boom, followed by destructive busts. This analysis is typically conducted from the perspective of an individual country. However, there is an increasing body of empirical literature documenting the intricate capital account linkages across countries. For example, Forbes et al. (2016) use the variation in Brazilian taxes on capital flows from 2006 to 2013 to show that increases in these taxes diverted capital flows to countries with similar exposure to a China-factor. Their evidence is confirmed and extended to a wide panel of countries from 1995 to 2009 by Giordani et al. (2014). The overt implication of these findings is the need for coordination in capital account measures, which is the focus of our normative analysis.

There is also a small but important literature that incorporates these diversions into a multilateral analysis of capital flow taxation (e.g. Ostry et al. (2012); Blanchard and Ostry (2012); Jeanne (2014); Korinek (2012)).[3] Ostry et al. (2012); Jeanne (2014); Blanchard and Ostry (2012) emphasize the importance of a multilateral analysis of capital control measures and the value of coordination in preserving the power of a domestic policy. Our section on put-policies shares some of the mechanisms and logic behind their work.

We also analyze how the presence of a global liquidity cycle affects our conclusions, which

[3] IMF (2012) refers to this multilateral approach as the Keynes-White notion of operating "at both ends of the transaction."

relates to the work of, e.g., Calvo et al. (1996); Forbes and Warnock (2012); Miranda-Agrippino and Rey (2015); Bruno and Shin (2015). This cycle is particularly relevant for EMs, which is what we capture in our asymmetric regions model. Chari et al. (2016) focus on the taper-tantrum episode of 2013 but also highlight the preceding period where capital flows to EMs tripled from 2009 to early 2013. This spillover is also the central theme of IMF (2012) (see Chapter 2), among many others.

Finally, our results on the composition of safe/risky capital flows between DMs and EMs match the important empirical work documenting the role of the U.S. as the World's venture capitalist by Gourinchas and Rey (2007); Gourinchas et al. (2010).

The rest of the paper is organized as follows. Section 2 contains a brief narrative of the workings of the model. Section 3 presents the baseline environment and equilibrium in a largely symmetric world with no global liquidity shocks. This section illustrates how symmetric capital flows help to create liquidity in our environment. It also illustrates the reach-for-safety and the reach-for-yield mechanisms by minimally departing from symmetry (in particular, considering an infinitesimal country). Section 4 revisits the previous topics after introducing aggregate liquidity shocks that create a liquidity premium for foreign financial flows. We show that increasing the correlations of liquidity shocks can naturally generate a global liquidity cycle, and investigate how the presence of this cycle interacts with the reach for safety and the reach for yield. The remaining sections focus on the policy implications. Section 5 analyzes optimal policy in our baseline environment with symmetric flows. We show that the coordinated policy outcomes sharply differ from those that would be chosen in a Nash equilibrium. Section 6 extends the baseline model to incorporate asymmetric regions, and uses this model to investigate the policy implications of the reach for safety and the reach for yield. Section 7 concludes and is followed by several appendices containing extensions and the proofs of the propositions that are not developed in the body of the paper.

2. A Narrative of the Model and its Implications

In this brief section we provide an intuitive description of the main mechanisms at work in our model. It can be skipped by those that prefer to go directly to the formal description of the model and its implications.

There are many countries that experience occasional financial crises (liquidity shocks that end up reducing prices below their fundamental value). There are investors whose ex-ante investment decisions affect the severity of these crises. The key ingredient of the model is that the investors can mitigate the crises in their home markets, but they exacerbate crises in foreign markets. Specifically, investors understand their home markets sufficiently well that they provide liquidity during a crisis (low prices induce them to buy in the local market). However, investors do not understand well the foreign markets and they flee at the first sign of trouble (low prices induce them to liquidate). We view this assumption as capturing a broad range of reasons for why foreigners tend to capitulate when local markets experience turmoil.[4] The behavior of foreigners is behind the unwanted volatility from capital flows in our model (the fickleness factor). However, foreigners are also specialists in their local markets, where they arbitrage and stabilize domestic fire sales.[5] Hence, the key tension is whether these investors with a dual role exert a stabilizing or destabilizing influence on average. This tension is isolated most cleanly in a symmetric world, which is where our analysis starts.

The first question is why investors hold foreign assets at all, if they are not specialists in those markets and will liquidate during crises. It could of course be the case that the foreign assets yield higher returns when there is no crisis—we analyze this reach-for-yield possibility later. But the model reveals that investors will want to hold foreign assets even when the countries are completely symmetric in terms of returns and fundamentals. The reason is that there is a scarcity of safe assets, and investors need some asset that is uncorrelated with the local market (as they would like to collect a large return when there is a local crisis). It turns out that a diversified portfolio in foreign markets will do the job. This portfolio will be safe(r) on average due to diversification. And it is desirable in equilibrium (despite its lower payoff due to the lack of expertise) since it will be retrenched back into the country during a local crisis to arbitrage fire sales.

That is, there will be foreign flows in the model—despite the lack of expertise and the fickleness of foreigners. The second question, then, is how do these flows affect financial stability? One could envision the answer going in either direction. On the one hand, capital inflows can exacerbate crises, as foreigners are fickle and reduce inflows precisely when there is a local crisis. On the other hand, capital outflows can mitigate crises, as they are retrenched and provide liquidity during crises. The model offers a clean answer to the second question. In a symmetric world, the retrenchment force dominates, and the flows are on net beneficial for financial stability. The intuition is that fickle foreigners sell local assets at fire-sale prices, but the local investors sell and retrench their diversified foreign assets at relatively high valuations. Since the outflows are liquidated at a higher return than inflows, symmetric capital flows end up increasing liquidity during crises. In fact, the return differential between the outflows and inflows (during a crisis) is precisely the amount of liquidity insurance the country obtains from international capital markets.

In summary, gross capital flows create global liquidity. This liquidity service is reduced when the correlation across local liquidity shocks rises, which in turn reduces the magnitude of gross

[4] The assumption is well aligned with the evidence in Cerutti et al. (2015) who document that non-resident investors are significantly more sensitive to global push factors. Also Bluedorn et al. (2013) document that capital flows are fickle for all countries, developed and emerging, although the former experience less volatility of total net inflows despite greater volatility of each component. Finally, and most directly related to our framework, Broner et al. (2013) document for a large panel of countries since the 1970s, that during expansions foreigners increase their purchase of domestic assets and domestic agents increase their investment abroad, while the opposite happens during contractions.

[5] While it simplifies the exposition to talk about domestic and foreign investors, the key distinction for us is between local (specialist) and non-local (fickle generalist) investors, which need not be perfectly aligned with domestic and foreign investors, respectively.

capital flows and increases the demand for safe assets. In terms of prices, an increase in the relative importance of global shocks results in an increase in the risk-premium associated to gross capital flows, a drop in safe interest rates, and more severe domestic fire sales.

Finally, within this symmetric world we also find that, even though flows are stabilizing from a global perspective, they are disliked by local regulators. Intuitively, every capital inflow into a country is an outflow from the perspective of some other countries. The local regulators take into account the fickleness cost of inflows, but they do not take into account the retrenchment benefit of inflows for those other countries. In an uncoordinated policy environment, this externality leads to too much protectionism—excessive taxes or restrictions on capital inflows. We also find that protectionist policies are exacerbated when there are worldwide asset shortages, as this environment leads to more severe crises and a stronger motivation for local regulators to do something about them. The protectionist policies are also strategic complements: the more some countries adopt them, the more other countries will have incentives to adopt them.

These conclusions apply in a symmetric environment, but they are qualified when there are substantial asymmetries in liquidity or investment returns across the different regions of the world. We identify two potentially destabilizing mechanisms—reach for safety and reach for yield—that apply when developed markets with substantial liquidity but relatively low returns trade flows with emerging markets with smaller liquidity but relatively high returns.

The reach-for-safety mechanism is driven by cross-country differences in liquidity. The greater liquidity in a developed country makes its assets relatively attractive for the investors in emerging markets. This induces the developed country to experience greater inflows relative to its outflows (or run current account deficits). The model further reveals that, when there is aggregate risk, the inflows into the developed country are relatively safe whereas the outflows are relatively risky. Intuitively, the investors in the developed country sell liquidity insurance (at a premium) to the emerging markets. The mismatch in the size as well as the risk composition of the flows can exacerbate the financial crises in the developed country. However, these flows also mitigate the crises in emerging markets, and thus, their net effect on worldwide financial stability is ambiguous.

The reach-for-yield mechanism is driven by cross-country differences in investment returns. If the return in developed markets is much lower than in emerging markets, then investors in developed markets hold foreign assets not only to mitigate local crises, but also to chase after the higher returns in emerging markets. These flows that are largely driven by the pursuit of higher returns are on net destabilizing, since they exacerbate crises in emerging markets without providing financial stability benefits elsewhere. Our model therefore provides a rationale for taxing certain types of flow into emerging markets even if policy can be coordinated across countries. However, the model also reveals that these types of flows happen in equilibrium only if the return differentials across the regions are sufficiently high to compensate for the developed market investors' lack of expertise (or fickleness) in emerging markets.

3. Baseline Environment and Equilibrium

In this section, we describe the baseline model in which countries are symmetric and there is no aggregate risk. We use this model to illustrate how, in a symmetric environment, global capital flows help to create liquidity and stabilize crises despite their fickleness. We also use two variants of the model to illustrate the reach-for-safety and reach-for yield mechanisms by which the global flows can be potentially destabilizing.

3.1. Liquidity Creation with Fickle Flows

Consider a model with three periods, $t \in \{0, 1, 2\}$. There is a continuum of measure one of countries denoted by superscript $j \in [0, 1]$. Each country is associated with a new investment technology—a risky asset that is supplied elastically at date 0 as we will describe below. The asset in country j always pays R dollars, but the timing of the payoff depends on the local state $\omega^j \in \{0, 1\}$ that is realized at date 1. State $\omega^j = 0$ represents the case without a liquidity shock in which the project pays off early at date 1. State $\omega^j = 1$ represents the case with a liquidity shock in which the project payoff is delayed to date 2. In this case, the asset is traded at date 1 at a price p^j that will be endogenously determined. The states are i.i.d. across countries with $\Pr\left(\omega^j = 1\right) = \pi$, where $\pi > 0$ denotes the probability of a liquidity shock within a country.

Each country is also endowed with a legacy asset with liquid payoffs (safe asset), which is supplied inelastically at date 0. Each unit of the legacy asset yields risk-free η dollars at date 1. In the baseline model, the legacy asset is traded at price η/R_f at date 0, where R_f denotes the (gross) risk-free interest rate that will be endogenously determined.

In each country j, there are two types of agents, entrepreneurs and investors. There is a mass e of entrepreneurs. They are born at date 1, with preferences given by $E\left[\tilde{c}_2\right]$. Each entrepreneur is endowed with 1 unit of the risky asset and has access to a profitable project. The output from the new project is nonpledgeable, and thus the sale of legacy assets is the only way the entrepreneurs can finance the new project. Thus, the entrepreneurs sell their endowments and invest in the project. The entrepreneurs are largely passive: their main role is to capture asset sales driven by liquidity needs and the potential welfare losses with these types of sales.

The main agents are the investors, which are denoted by the superscript j of their locality. They are endowed with all of the legacy asset supply as well as 1 dollar at date 0. They have preferences given by $E\left[u\left(c_0\right) + c_1 + c_2\right]$. Here, c_0 denotes the investors' spending in an outside option other than holding financial assets. It can be viewed as consumption or investment in an illiquid project. We assume $u\left(c_0\right)$ is an increasing and strictly concave function which also satisfies Inada-type conditions, $u'\left(0\right) = \infty$ and $u'\left(1\right) < R$, which will ensure an interior solution in our model.

The novel ingredient of the model is that investors have local habitats in financial markets. Specifically, if the foreign market is hit by a liquidity shock at date 1, then the investor is required to close her position in this market. In contrast, the local investor can take unrestricted positions in the local market. This assumption captures in reduced form the idea that investors might not feel comfortable outside of their natural markets due to unmodeled features (e.g., Knightian uncertainty) and they might flee at the first sign of trouble. The assumption also captures the concerns by policymakers that portfolio investments by outsiders tend to be fickle and might exacerbate financial instability.

The investor in country j chooses how much to spend on the outside option (or consume), c_0^j, how much to invest in the local risky asset, $x^{loc,j}$, how much to invest in risky foreign assets, $\left[x^{j',j}\right]$, and how much to invest in the legacy asset, y^j. Here, $x^{j',j}$ denotes a Lebesgue-measurable function of j' that captures the investor's foreign portfolio. We focus on symmetric equilibria in which the assets trade at the same price in all markets, $p^j \equiv p \leq R$ for each j. The investor's problem can then be written as,

$$\max_{\tilde{c}_0 . \tilde{x}^{loc}, \left[\tilde{x}^{j'}\right]_{j'}, \tilde{y} \geq 0} u\left(\tilde{c}_0\right) + \tilde{x}^{loc} R + \left(\tilde{x}\overline{R} + \tilde{y} R_f\right) M, \tag{1}$$

$$\overline{R} = (1-\pi) R + \pi p$$

$$M = 1 - \pi + \frac{R}{p}\pi$$

$$\tilde{c}_0 + \tilde{x}^{loc} + \tilde{x} + \tilde{y} = \eta / R_f + 1 \text{ and } \tilde{x} = \int x^{j'} dj'.$$

If she invests in a local asset, she holds it until maturity, which leads to return R regardless of the local shock. If instead she invests in a foreign asset or cash, she receives a financial return at date 1, which she then reinvests in the local market where she has a comparative advantage when there is a liquidity shock. The investor's expected financial return is given by $\tilde{x}\overline{R} + \tilde{y}R_f$, where $\overline{R}$ denotes the expected one-period return from the foreign portfolio. The investor's expected marginal utility is denoted by M, which combines a marginal utility of 1 in case there is no domestic liquidity shock and a marginal utility of R/p in case there is a liquidity shock. Note that the expected return from foreign investment, $\overline{R}$, is multiplied with the expected marginal utility, M, since the local and foreign liquidity shocks are uncorrelated by assumption.[6]

The market clearing condition for the legacy asset can be written as,

$$\int y^j dj = \eta / R_f. \tag{2}$$

The market clearing condition for the risky asset at date 1 in country j can be written as,

$$p^j = \min\left(R, \frac{R_f y^j + \overline{R} \int x^{j',j} dj'}{e + \int x^{j,j'} dj'}\right). \tag{3}$$

Here, the second term inside the parentheses denotes cash-per-asset in the market. The denomi-

[6] Note also that the investors cannot trade financial assets (backed by foreign investment or cash) with payoffs contingent on the realizations of local liquidity shocks, $\left\{\omega^j\right\}_j$. We allow for this possibility in Section 3.1.1.

nator of this term captures the amount of sales, which come from the liquidity sales (e) and the foreign investors' positions in this market ($\int x^{j,j'}dj'$). The numerator captures the total amount of cash in the market, which comes from the local investors' cash and foreign asset positions ($\int x^{j',j}dj'$).

An equilibrium with symmetric prices is a collection of allocations, $\left(c_0^j, x^{loc,j}, \left[x^{j',j}\right]_{j'}, y^j\right)_j$, and prices, $p^j = p \leq R$ and R_f, such that the allocations solve problem (1), and the market clearing conditions (2) $-$ (3) hold.

We analyze a symmetric equilibrium that satisfies, $c_{0,j} = c_0, x^{loc,j} = x^{loc}, x^{j',j} = x, y^j = y$. By symmetry and the market clearing condition (2), the equilibrium holdings of the legacy asset is positive and given by, $y = \eta/R_f > 0$. Plugging this observation into the budget constraint, we obtain $c_0 + x^{loc} + x = 1$. We next characterize how the investor optimally splits her dollar between outside spending, local investment, and foreign investment.

First consider the marginal utility from foreign investment, x. Using problem (1), the marginal utility is given by,

$$\overline{R}M = ((1-\pi)R + \pi p)\left(1 - \pi + \frac{R}{p}\pi\right) \equiv \mu(p), \tag{4}$$

where the second equality defines the function $\mu(p)$. The following lemma establishes the properties of this function that facilitates subsequent analysis.

Lemma 1. *The function, $\mu(p)$, is strictly decreasing in p over the range $p \in (0, R]$, and it satisfies $\mu(R) = R$. In particular, $\mu(p) > R$ for each $p \in (0, R)$.*

Note that the marginal utility from investing locally is equal to R [cf. problem (1)]. Hence, the lemma says that the local investment is weakly dominated by the foreign investment; and strictly so as long as there are fire sales, $p < R$ (which will be the case that we will focus on). Intuitively, the investor would rather invest in the local market at date 1 when she has the comparative advantage. At date 0, she would rather invest in foreign assets (or cash) as this provides her with liquidity to purchase local assets at low prices when the opportunity emerges. In line with this intuition, the lemma also illustrates that the marginal utility from holding foreign assets is greater when p is lower and local fire sales are deeper.

Note also that the marginal utility from outside spending is given by $u'(c_0)$, where $c_0 = 1 - \left(x^{loc} + x\right)$. Since the return from investment (local or foreign) is always weakly greater than R, the outside spending is always weakly smaller than $1-\underline{x}$, where $\underline{x}$ is defined as the solution to $u'(1-\underline{x}) = R$. Hence, there is a lower bound on investment, $x^{loc} + x \geq \underline{x}$. Moreover, the lower bound is strictly positive in view of the Inada condition, $u'(1) < R$. In particular, the marginal utility from investment must be equal to the marginal utility from holding the legacy asset (as both are held in positive quantities), $\mu(p) = \overline{R}M = R_f M$. Simplifying this expression, we obtain,

$$R_f = \overline{R}. \tag{5}$$

Intuitively, the legacy assets and foreign investment yield the same one-period return in equilibrium, since they are perfect substitutes for the local investor [cf. problem (1)].

Combining our observations, the optimal level of foreign investment, x, is determined by,

$$\begin{cases} u'(1-x) = \mu(p), & \text{if } p < R \\ x \in [0, \underline{x}] & \text{if } p = R \end{cases}. \tag{6}$$

Here, the first line captures the case with fire sales. In this case, the local investment is zero and the outside spending is below its upper bound, $1-\underline{x}$ (and thus, foreign investment is above $\underline{x}$). The foreign investment is determined by equating its marginal utility with the marginal utility from outside spending. The second line captures the case without fire sales. In this case, the outside spending is at it upper bound, $1-\underline{x}$. The investor is indifferent between investing in the foreign and the local market, and there is a range of optimal foreign investment levels (with the residual invested in the local market, $x^{loc} = \underline{x} - x$).

Recall that $\mu(p)$ is decreasing in p. Hence, Eq. (6) implies that the optimal amount of foreign investment, $x \in (0,1)$, is weakly decreasing in p (and strictly so if $p < R$). Intuitively, the deeper the local fire sales (lower p), the more the investor saves in liquid assets that are uncorrelated with the local market. Since local liquidity provided by the legacy assets is scarce, greater demand for liquid assets translates into greater foreign investment in equilibrium. Viewed in reverse, Eq. (6) can also be thought of as describing the price level that is consistent with the optimality of a given amount of foreign investment, $x \in (0,1)$. We denote this (decreasing) function with $p = P^{opt}(x)$. Figure 1 illustrates this curve for a particular parameterization (with $u(c_0) = h \log c_0$ for some $h \in (0, R)$). The flat region corresponds to the case without fire sales, and the strictly decreasing region corresponds to the case with fire sales.

It remains to characterize the fire-sale price level in equilibrium, p. Using symmetry and market clearing for legacy assets, $y = \eta / R_f$, the market clearing condition (3) can be rewritten as,

$$p = \min\left(R, C\left(x, \overline{R}\right)\right) \text{ where } C\left(x, \overline{R}\right) \equiv \frac{\eta + x\overline{R}}{e + x} \text{ and } \overline{R} = (1-\pi) R + \pi p. \tag{7}$$

Here, $C\left(x, \overline{R}\right)$, captures the amount of cash available per unit of asset sold in a market experiencing a liquidity shock, as a function of the foreign investment, x, and the return on foreign investment, $\overline{R}$. The denominator of this expression captures the amount of forced sales in a market. The numerator captures the total amount of liquidity in the market, which comes from local investors' holdings of legacy and foreign assets. The following lemma shows that cash per asset is increasing in the amount of foreign flows.

Lemma 2. *The function, $C\left(x, \overline{R}\right)$, is strictly increasing in x iff $C\left(x, \overline{R}\right) < \overline{R}$.*

This result might be surprising since one might have expected that fickle foreign flows should exacerbate financial distress during liquidity shocks. This effect is captured in the denominator of $C\left(x, \overline{R}\right) = \frac{\eta + x\overline{R}}{e+x}$. However, the numerator shows that the foreign investment (by locals) also

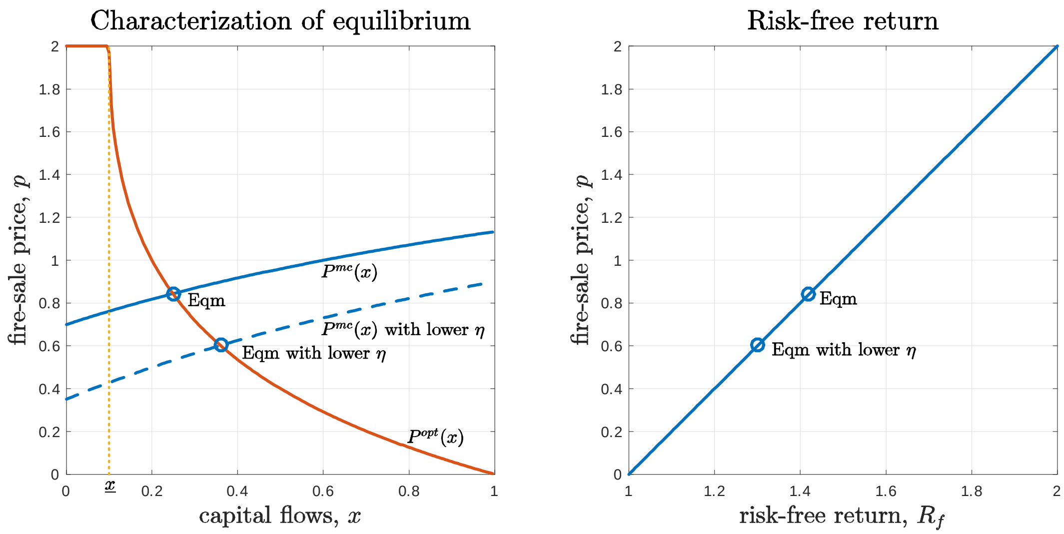

Figure 1: The left panel illustrates the characterization of equilibrium in the baseline environment. The dashed line illustrates the effect of a reduction in the liquidity from legacy assets, η. The right panel illustrates the risk-free rate in equilibrium.

helps to increase the local liquidity available during liquidity shocks. The net effect depends on the derivative, $\frac{\partial C(x,\overline{R})}{\partial x}$. The result in the lemma follows by observing that,

$$\frac{\partial C\left(x,\overline{R}\right)}{\partial x} > 0 \text{ iff } \overline{R} > C\left(x,\overline{R}\right).$$

Hence, the liquidity effect of capital flows dominates (and the cash per asset is increasing in foreign flows) whenever the condition in the lemma is satisfied.

The condition in the lemma will in fact be satisfied in *any equilibrium in which there are fire sales* ($p < R$). In such equilibria, we have $p = C\left(x,\overline{R}\right)$, which also implies, $C\left(x,\overline{R}\right) = p < \overline{R}$ (since $\overline{R} - p = (1-\pi)(R-p) > 0$). The lemma has the strong implication that, whenever there are fire sales, greater foreign flows lead to greater cash per asset—and ultimately, greater asset prices during liquidity shocks. The intuition is that the date 1 value of the asset dumped is the fire-sale price level, $p = C\left(x,\overline{R}\right) < \overline{R}$. In contrast, the date 1 value of the liquidity created is the expected return on foreign assets, $\overline{R}$. The fire-sale price is always less than the return on foreign assets (as those are diversified). Thus, the liquidity effect always dominates the fickle foreigners effect in the relevant range in which cash per asset determines the asset price. Intuitively, foreign flows help to bring the excess liquidity in foreign financial markets that do not experience shocks into the local market that has a liquidity shock.

To solve for the equilibrium, we rearrange the market clearing condition, $p =$

$\min\left(R, C\left(x, \overline{R}\right)\right)$, to obtain,

$$p = \min\left(R, \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right). \tag{8}$$

Like Eq. (7), this equation represents a weakly increasing relation between p and x (and strictly so when $p < R$). We denote this function with $p = P^{mc}(x)$. Recall that we also had a decreasing function, $p = P^{opt}(x)$, representing Eq. (6). The equilibrium is found as the intersection of these two relations. It can be checked that there exists an intersection that satisfies $x \in (0,1)$. It can also be checked that the equilibrium features fire sales, $p < R$, and zero local investment, $x^{loc} = 0$ and $x > \underline{x}$, as long as the following *domestic safe asset scarcity* condition holds—which we maintain for the rest of the analysis.

Assumption 1. $\eta < eR$.

Figure 1 illustrates the characterization of equilibrium for a particular parameterization. Once the variables, x, p, are characterized, the risk-free return is characterized from Eq. (5). Our next result summarizes the above discussion and establishes some properties of equilibrium.

Proposition 1. *Consider the baseline model (with Assumption 1). There exists a unique symmetric equilibrium,* $\left(\left(c, x^{loc}, x, y\right), p, R_f\right)$*, which satisfies* $yR_f = \eta, x^{loc} = 0, c = 1 - x$ *and fire-sale prices,* $p < R$*. The pair* (x, p) *is characterized by Eqs.* (6) *and* (8)*. Decreasing the local liquidity (safe assets),* η*, decreases* p *and* R_f*, and increases the capital flows,* x*. Decreasing the return,* R*, decreases* p *and* R_f*, as well as the capital flows,* x*.*

The comparative static result with respect to η follows by observing that reducing liquidity shifts the market clearing curve $p = P^{mc}(x)$ downwards, without affecting the optimality curve, $p = P^{opt}(x)$ (see Figure 1 for an illustration). Intuitively, the price declines in view of the market clearing condition (8). In turn, the lower price induces greater foreign investment; with smaller local liquidity, there is greater need for global liquidity creation. The risk-free return also declines in view of the financial optimality condition (5). Hence, the result implies that a reduction in global liquidity exacerbates fire sales and lowers the risk-free return. In Section 5, we will ask the follow-up question of whether the endogenous reaction by governments exacerbates or mitigates the impact of a contraction in global liquidity.

Likewise, the comparative static result with respect to R follows by observing that decreasing R shifts market clearing curve downwards, in view of a decline in global liquidity. It also shifts the optimality curve downwards, since investment becomes relatively unattractive. The net effect is a decline in the fire-sale price, p, as well as safe asset returns, R_f. This analysis does not help to identify the effect on x. The proof in the appendix uses a more subtle argument to show that decreasing R also decreases x. In Section 3.3, we will ask the follow-up question of how the decline in the return in developed countries, captured by R, affects flows into and fire-sale prices in emerging markets with relatively high returns.

3.1.1. Insurance markets with respect to local shocks

Note that the financial markets in our baseline model are incomplete in the sense that investors cannot trade financial contracts (backed by foreign investment or cash) whose payoffs are contingent on the realizations of the local liquidity shocks, $\{\omega^j\}_j$. This incompleteness results in an inefficient allocation of liquidity at date 1. Specifically, the investors have liquid financial wealth in states in which their country does not experience liquidity shocks which they would have ideally liked to transfer to states with local liquidity shocks. In Appendix A.1, we relax this assumption by introducing intermediaries that sell contingent contracts and invest in foreign markets as well as cash. We show that local investors purchase the contracts contingent on local liquidity shocks, and that the presence of these insurance arrangements further increases local fire-sale prices. In particular, the fire-sales are avoided for sufficiently low levels of π, as these states feature sufficient global liquidity, but they are not for higher levels of π, in which case the crises have a global scope and the local insurance markets provide little help.

Importantly, when π is sufficiently large (so that crises are sufficiently frequent), the policy tension that we identify later in our baseline model continues to apply when we allow for local insurance markets. We therefore abstract away from these insurance arrangements in our baseline analysis. This is arguably a realistic feature of the model. It could also be motivated by informational considerations: frictions such as moral hazard or adverse selection would have a particularly strong bite for insurance arrangements with respect to idiosyncratic shocks. That said, to the extent that these markets are feasible, they should be promoted. See e.g. Caballero (2003); Brunnermeier et al. (2016) for proposals in the context of emerging markets and the Eurozone, respectively.

3.2. Reach for safety

We next consider a variant of the baseline model to illustrate the reach for safety mechanism. Specifically, consider the same setup with the only difference that one country j (that has measure zero) has potentially different liquidity, η^j, compared to the world average, η. A developed country with deep financial markets and a large supply of safe assets—such as the US—can be thought of as featuring $\eta^j > \eta$. Conversely, an emerging market country is captured by low η^j.

Suppose $\eta^j > \eta$ so that the country in consideration has a relatively developed financial market (the other case is discussed at the end of the subsection). As a benchmark, first suppose the country is in autarky. In this case, consumption and local investment in risky assets is given by respectively $c_0^j = 1 - \underline{x}$ and $x^{loc,j} = \underline{x}$, and the safe asset holding is $y^j = \eta / R_f$. The asset price at date 1 (conditional on a liquidity shock) is $p_s^j = \min\left(R, \frac{\eta^j}{e}\right)$. To obtain sharp results, we make the following safe asset abundance assumption (in addition to Assumption 1).

Assumption S. $eR < \eta^j$.

That is, country j has access to abundant domestic liquidity, which ensures that the autarky equilibrium features no fire sales, $p_s^j = R$ for each $s \in S$.

Let us contrast this outcome with the equilibrium with free capital flows. The world equilibrium, which we continue to denote by $\left(c_0, x^{loc}, x, y\right)$, is the same as before. However, the equilibrium allocations in country j are potentially different. The optimality conditions for foreign investors (to invest in the country) imply,

$$(1-\pi) R + \pi p \geq (1-\pi) R + \pi p^j \text{ with equality if } x^{\text{in},j} > 0. \tag{9}$$

In particular, as long as the country experiences positive inflows—which will be the case in equilibrium—the fire-sale price in country j is exactly the same as in the representative country. Put differently, even though the country j has abundant liquidity and would not feature fire sales in autarky, it cannot escape fire sales in the equilibrium with free financial flows.

For intuition, consider the optimality conditions in (9). All else equal, greater liquidity in the country, η^j, would increase the fire-sale price, p^j. However, this makes the country's assets attractive to foreign investors and increase the inflows, $x^{\text{in},j}$. Foreign investors will be indifferent to invest in the country only when the inflows increase to the point at which the expected return is in line with that in the representative country. Formally, the market clearing condition in country j can be written as,

$$p^j = \min\left(R, \frac{\eta^j + x^{\text{out},j}\overline{R}}{e + x^{\text{in},j}}\right), \tag{10}$$

where $x^{\text{out},j}$ denotes the outflows from the country. Using the optimality condition for local investors, $u'\left(1 - x^{\text{out},j}\right) = \overline{R}$, the outflows are the same as in other countries, $x^{\text{out},j} = x$. Combining this with Eqs. $(10), (9)$ and (7), we obtain, $x^{\text{in},j} - x^{\text{out},j} = \left(\eta^j - \eta\right)/p > 0$. That is, the liquidity difference advantage of the country is neutralized by its greater inflows relative to outflows (i.e., capital account surplus and current account deficit). The following proposition summarizes this discussion.

Proposition 2. *Consider the baseline model in which a country has abundant local liquidity, $\eta^j > \eta$, that satisfies Assumption S (so that the country would not experience fire sales in autarky). In an equilibrium with free financial flows, the country receives more inflows than its outflows, $x^{in,j} > x^{out,j}$, and experiences fire-sales that are just as severe as those in the representative country, $p^j = p < R$ for each s.*

This result suggests that the *reach-for-safety* flows have potentially destabilizing effects. However, note that the flows are also potentially stabilizing for foreign investors that invest in the developed country. To see this, consider the mirror-image situation in which a country has relatively low liquidity compared to the world average, $\eta^j < \eta$. The equilibrium in this country is characterized by similar steps as above. With symmetric flows, this country would experience deeper fire sales in view of its low local liquidity, $p^j < p$. All else equal, these fire sales would make the country's assets relatively unattractive to foreigners, which would reduce inflows. This process continues until the fire sales are on average the same as those in the representative country, $p^j = p$ (except when $\eta^j < \eta - xp$, in which case the equilibrium features $x^{\text{in},j} = 0$ and $p^j < p$). Hence, the effect of reach-for-safety flows on worldwide financial stability is ambiguous.

3.3. Reach for yield

We next consider another variant of the model to illustrate the reach-for-yield mechanism. Specifically, consider the same setup as in Section 3.1 with the only difference that one country j (that has measure zero) has a greater fundamental return relative to the world average, $R^j > R$. Country j can be thought of as a rapidly growing emerging market such as China, India, or Mexico. These types of countries appear to have relatively attractive fundamental returns, especially in recent years in which the asset returns in developed markets are unusually low. In line with this interpretation, we also assume the country has (weakly) lower liquidity than the representative country, $\eta^j \leq \eta$. To obtain an interior solution, we also assume that R^j and η^j are not too far from their representative country counterparts.

Assumption Y. $R^j - R \in \left(0, \frac{\pi}{1-\pi}p\right)$ and $\eta^j - \eta \in [-px, 0]$.

The analysis parallels Section 3.2 with minor differences. When $x^{\text{in},j} > 0$ (which will be the case in equilibrium), the relevant equilibrium conditions in country j can be written as,

$$(1-\pi) R + \pi p = (1-\pi) R^j + \pi p^j, \tag{11}$$

$$u'\left(1 - x^{\text{out},j}\right) = \left((1-\pi) R^j + \pi p^j\right)\left(1 - \pi + \pi \frac{R^j}{p^j}\right), \tag{12}$$

$$\text{and } p^j = \min\left(R, \frac{\eta^j + x^{\text{out},j}\overline{R}}{e + x^{\text{in},j}}\right). \tag{13}$$

Eq. (11) illustrates that the fire-sale price in country j is *lower* than the fire-sale price in the representative country, $p^j < p$. Eq. (12) implies that, since $R^j/p^j > R/p$, the country's outflows exceed the level in the representative country, $x^{\text{out},j} > x$. However, Eq. (13) illustrates that the decline in fire-sale price, p^j, is brought about by a sufficiently large increase in $x^{\text{in},j}$ that dominates the increase in the outflows, $x^{\text{out},j}$.

For intuition, first imagine the country had the same investment return as the world average. As we discussed above, this country's outflows would exceed its inflows, $x^{\text{out},j} \geq x^{\text{in},j}$ (the country would or run a current-account surplus), which would stabilize local financial crises, $p^j = p$. Relative to this benchmark, an increase in the country's investment return, R^j, makes its assets relatively attractive to foreigners. The inflows increase (and the current account surplus declines) up to the point at which the local fire-sales are sufficiently severe to deter the foreigners from investing further in the country. The local investors increase their holdings of foreign financial assets (which raises outflows) in an attempt to mitigate the deeper financial crises they experience. In equilibrium, the increase in outflows are matched by a further increase in inflows that leaves the foreigners indifferent to invest in the country.

We next analyze how a decline in asset yields in developed markets, which we capture with a decline in R, affects the equilibrium in the emerging market country j. As we noted in Proposition 1, a decline in R reduces the fire-sale price in the representative country, p (via a reduction in international liquidity). In view of this observation, Eq. (11) implies that an increase in R also decreases the fire-sale price in country j. It can further be seen that the fire-sale price declines more in country j than in the representative country, that is, $p^j - p$ declines. The following result summarizes this discussion.

Proposition 3. *Consider the baseline model in which a country has relatively high return, $R^j > R$, and satisfies Assumption Y. In an equilibrium with free financial flows, the country experiences fire sales that are more severe than in the representative country, $p^j < p$. A decrease in R reduces the fire-sale price in the country, p^j, as well as the relative fire-sale price, $p^j - p$.*

4. Environment with Aggregate Shocks

We next introduce aggregate liquidity shocks into our analysis to illustrate a number of additional mechanisms. We show that aggregate shocks result in a risk premium on capital flows (over safe assets). We then analyze the key determinants of the liquidity risk premium, and how this premium affects the reach-for-safety as well as the reach-for-yield mechanisms we described in the previous section. Among other things, we show that increasing the correlation among liquidity shocks drives a global cycle in liquidity premia, asset returns, and capital flows.

To address these issues, consider the setup in Section 3 with the only difference that there are several aggregate states denoted by $s \in S = \{1, 2, .. |S|\}$. The states differ in the probability of the liquidity shock, π_s. Throughout, we assume:

Assumption 2. π_s is increasing in s.

Hence, the states with higher s are associated with a greater probability of the liquidity shock (and thus, greater financial distress).[7] We denote the probability of the aggregate state s with γ_s, where $\gamma_s > 0$ for each s and $\sum_s \gamma_s = 1$.

We also assume that, at date 0, the agents can trade financial securities contingent on the aggregate state at date 1. Specifically, for each state $s \in S$, there is an Arrow-Debreu financial security that pays 1 dollar if state s is realized. The security is traded at date 0 competitively at price q_s. We assume the Arrow-Debreu securities are supplied by competitive intermediaries that undertake risky foreign investments at date 0. We let $x^{in,j} \geq 0$ denote the inflows into country j, which also corresponds to the foreign investment in country j by the intermediaries.

[7] In Appendix A.4, we consider the possibility that aggregate shocks can also affect the cash flows from legacy assets and new investment, η_s and R_s. We show that these types of shocks do not change the baseline analysis in a significant way as long as η_s and R_s scale proportionally across states—which we view as a neutral assumption in our setting. In this case, the available liquidity also scales proportionally with η_s and R_s across states. In particular, the fire-sale price to return ratio, p_s/R_s, remains constant and the analysis becomes similar to the case without aggregate uncertainty.

The intermediaries' optimality conditions then imply that,

$$1 \geq \sum_s q_s \overline{R}_s^j, \text{ with equality if } x^{in,j} > 0, \tag{14}$$

where $\overline{R}_s^j = R(1-\pi_s) + p_s^j \pi_s$. Hence, the date-0 value of investment in a country is equal to its cost, normalized to 1, whenever there are positive inflows.

Similar to the earlier analysis, the investor in country j chooses how much to spend on the outside option, c_0, how much to invest locally, x^{loc}, how much to invest in the legacy asset, y, and how much to invest in Arrow-Debreu securities, denoted by $(z_s)_s$. Her problem can be written as,

$$\begin{aligned} &\max_{\tilde{c}_0, \tilde{x}^{loc}, \tilde{y}, \left(\tilde{z}_s \geq -\tilde{y}R_f\right)_s} u(c_0) + \tilde{x}^{loc} R + \sum_s \gamma_s \left(\tilde{y}R_f + \tilde{z}_s\right) M_s^j, \\ &\tilde{c}_0 + \tilde{x}^{loc} + \sum_s q_s \tilde{z}_s + \tilde{y} = \eta / R_f + 1. \end{aligned} \tag{15}$$

Here, $M_s^j = 1 - \pi_s + \frac{R}{p_s^j}\pi_s$ denotes the marginal utility from reinvestment as before. . We assume the investors' holdings of the Arrow-Debreu securities satisfy, $\tilde{z}_s \geq -\tilde{y}R_f$, which captures the idea that the investor can take a short position but only if she holds other liquid assets (here, the safe asset) to cover these position. Unlike in problem (1), the investor does not directly choose risky investment in foreign countries. Instead, she chooses financial claims on the investments that are undertaken by competitive intermediaries as described above. We use $x^{\text{out},j} = \sum_s q_s z_s$ to denote the outflows from the country into risky investment, and $\overline{x}^{\text{out},j} = y^j - \eta / R^f + \sum_s q_s z_s$ as the total outflows that also include the net trading of safe assets.[8]

The market clearing conditions can be written as,

$$\begin{aligned} &\int y^j dj = \eta / R_f \\ &\int z_s^j dj = \int x^{\text{in},j} \overline{R}_s^j dj \text{ for each } s \in S, \\ &\text{and } p_s^j = \min\left(R, \frac{R_f y^j + z_s^j}{e + x^{\text{in},j}}\right) \text{ for each } s \in S. \end{aligned} \tag{16}$$

Here, the first and the third conditions are the analogs of the earlier market clearing conditions. The second equation is a new condition that says that the total amount of traded Arrow-Debreu securities is equal to the amount of financial payoffs from foreign investment in state s. There is also an aggregate resource constraint at date 0 which can be written as, $\int_j \left(c_0^j + x^{loc,j} + x^{\text{in},j}\right) dj = 1$. By Walras' law, this resource constraint is satisfied when all of

[8] In the previous section, this distinction was not important since the safe assets and foreign investment were perfect substitutes in equilibrium, and we assumed without loss of generality that the countries retained their safe asset endowments. The distinction will play some role when we revisit the reach for safety mechanism in Section 4.2.

the budget constraints hold in equilibrium.

An equilibrium with aggregate shocks is a collection of allocations, $\left(c_0^j, x^{loc,j}, y^j, \left[\left\{z_s^j\right\}_s\right]_j\right), \left[x^{\text{in},j}\right]_j$, and prices, $\left(R_f, \{q_s\}_s, \left[\left\{p_s^j\right\}_s\right]_j\right)$, such that the financial intermediaries' optimality condition (14) holds, the investors' allocations solve problem (15), and the market clearing conditions (16) hold.

As before, we focus on symmetric equilibrium allocations and prices, which we denote by dropping the superscript j, e.g., x^{in} denotes the capital inflows in each country in the symmetric equilibrium. The symmetry implies the safe asset holdings are given by, $y = \eta/R_f$. The local investors' budget constraint is then given by $c_0 + x^{\text{loc}} + \sum_s q_s z_s = 1$. It remains to characterize how the local investors divide their endowments of one dollar across $c_0, x^{loc}, (z_s)_s$. Note also that the aggregate resource constraint implies the inflows and outflows are equated in equilibrium, $x^{\text{in}} = \sum_s q_s z_s = x^{\text{out}}$ (which is also equal to $\overline{x}^{\text{out}}$). When it is clear from the context, we use the notation $x = x^{\text{in}} = x^{\text{out}}$ to denote these symmetric flows.

In view of Assumption 1, we conjecture that there will be fire sales (conditional on the liquidity shock) in every state, $p_s < R$ for each s. In view of our analysis in Section 3, we also conjecture that consumption will be below its upper bound, $c_0 < 1-\underline{x}$, and local investment will be dominated, $x^{loc} = 0$, and thus, the symmetric flows will satisfy, $x > \underline{x}$. We will verify these conjectures below. Under these conjectures, the optimality condition (14) holds as equality. In addition, the date-0 resource constraint implies $c_0 = 1 - x$. Combining this with the optimality conditions for problem (15), we have,

$$u'(1-x) = \frac{\gamma_s}{q_s} M_s \text{ for each } s \in S, \tag{17}$$

which by Eq. (14) implies,

$$u'(1-x) = \sum_s \gamma_s \overline{R}_s M_s \equiv \sum_s \gamma_s \mu_s(p_s). \tag{18}$$

Here, we define the function $\mu_s(p_s) \equiv ((1-\pi_s)R + \pi_s p_s)\left(1 - \pi_s + \frac{R}{p_s}\pi_s\right)$ as in (4).

Using the market clearing conditions (16) and symmetry, we also obtain,

$$p_s = \min\left(R, \frac{\eta + x\overline{R}_s}{e + x}\right) = \min\left(R, \frac{\eta + x(1-\pi_s)R}{e + x(1-\pi_s)}\right). \tag{19}$$

This expression, together with Assumption 1, verifies our conjecture that $p_s < R$. Combining this with Eq. (18) also implies that $\mu_s(p_s) \geq R$ with strict inequality if $\pi_s \in (0,1)$. In particular, $\sum_s \gamma_s \mu_s(p_s) > R$, which verifies our conjecture that $c_0 < 1-\underline{x}$ and $x^{loc} = 0$. Note also that Eq. (19), together with Assumption 2, implies that the price, p_s, is decreasing in s: that is, states with greater financial distress (in terms of the likelihood of liquidity shocks) are associated with lower prices. Likewise, the expected payoff from foreign assets, $\overline{R}_s = (1-\pi_s)R + \pi_s p_s$, is decreasing in s.

The equilibrium is characterized by Eqs. (18) and (19), which are the analogs of Eqs. (6) and (8) in this setting. As before, it can be checked that there is a unique symmetric equilibrium with $x \in (\underline{x}, 1)$ and $p_s < R$ for each s.

Given these variables, Eq. (17) determines the Arrow-Debreu prices in financial markets. Note that the price-to-probability ratio, q_s/γ_s, corresponds to the stochastic discount factor (SDF) for state s. Using (17), we have,

$$q_s/\gamma_s = \frac{M_s}{u'(1-x)} = \frac{1}{u'(1-x)}\left(1 - \pi_s + \frac{R}{p_s}\pi_s\right). \tag{20}$$

In view of Assumption 2, and the earlier observation that p_s is decreasing in s, we also have that the SDF is increasing in s. That is, the states with greater financial distress are associated with more expensive Arrow-Debreu asset (insurance) prices. Put differently, the price of aggregate liquidity risk is positive.

Note that the risk-free return can be calculated as,

$$R_f = \frac{1}{\sum_s q_s} = \frac{u'(1-x)}{E[M_s]} = \frac{E\left[\overline{R}_s M_s\right]}{E[M_s]}. \tag{21}$$

Here, the second equality follows from Eq. (20) and the third equality follows from (17). Using this expression, the risk premium on financial assets (which can be viewed as a liquidity premium) can be calculated as,

$$E\left[\overline{R}_s\right] - R_f = \frac{-cov\left(M_s, \overline{R}_s\right)}{E[M_s]}. \tag{22}$$

Note that the covariance term is negative since expected asset payoff, $\overline{R}_s$, is decreasing in s, whereas the marginal utility, M_s (which is proportional to the SDF), is increasing in s. Thus, with aggregate liquidity risk, the risk premium on foreign financial assets is strictly positive. Intuitively, the value of the foreign assets is reduced by the fact that they pay relatively less when the liquidity is relatively scarce. The following result summarizes this discussion.

Proposition 4. *Consider the symmetric model with aggregate risk (with Assumptions 1 and 2). There exists a unique symmetric equilibrium,* $\left(\left(c_0, x^{loc}, y, \{z_s\}\right), \{q_s, p_s\}_s, x\right)$, *which satisfies* $c_0 = 1 - x, x^{loc} = 0, x > \underline{x}$, *and fire-sale prices,* $p_s < R$. *The tuple* $\left(x, (p_s)_s\right)$ *is characterized by Eqs.* $(18-19)$. *The fire-sale price,* p_s, *is decreasing in* s*: that is, more distressed states with greater likelihood of liquidity shocks are associated with lower prices. The state price-to-probability ratios,* $\{q_s/\gamma_s\}_s$, *are characterized by Eq.* (20) *and are increasing* s *(the degree of financial distress). The risk-free return is characterized by Eq.* (21)*. The risk premium on foreign assets is positive and characterized by Eq.* (22)*.*

4.1. Correlated Shocks and the Global Liquidity Cycle

We next use a special case of the model with aggregate shocks to show that changes in correlations can naturally generate a global liquidity cycle (e.g., Forbes and Warnock (2012); Fratzscher (2012); Rey (2016)). To this end, suppose there are three aggregate states, $s \in \{1, 2, 3\}$, that feature,

$$\pi_1 = 0 < \pi_2 = \pi < \pi_3 = 1, \tag{23}$$

for some $\pi \in (0, 1)$. In particular, state $s = 2$ corresponds to the state in the baseline analysis in which the liquidity shocks are i.i.d. across the regions. States $\{1, 3\}$ together represent an "aggregate shock" state in which the liquidity shocks are perfectly correlated across the countries. Specifically, either all countries are hit (state 3) or no country is hit (state 1). We also assume the state probabilities are given by,

$$\gamma_1 = \beta(1 - \pi), \gamma_3 = \beta\pi \text{ and } \gamma_2 = 1 - \beta. \tag{24}$$

Here, the parameter β captures the extent to which the shocks are correlated—controlling for everything else in the model. The case, $\beta = 0$, corresponds to the model in the previous section with i.i.d. shocks, whereas the case $\beta = 1$ corresponds to the other limit in which the liquidity shocks are always correlated.

Note also that $\mu_1 (p) = R$ and $\mu_3 (p) = p \times \frac{R}{p}$ [cf. Eq. (4)]. Thus, Eq. (18) becomes,

$$u' (1 - x) = \beta R + (1 - \beta) \mu_2 (p_2). \tag{25}$$

The market clearing conditions (19) imply,

$$p_2 = \frac{\eta + x (1 - \pi) R}{e + x (1 - \pi)}.$$

The last two equations determine the pair, (x, p_2). By inspecting the equations, it can be seen that increasing β reduces x. Intuitively, as liquidity shocks become more correlated, the liquidity-provision benefit from capital flows declines. Note also that an increase in correlations, β, reduces the asset price even in the i.i.d. state, p_2, in view of the reduction in capital flows, x.

Using the market clearing condition (19), we also calculate the price in state 3 (with $\pi_3 = 1$) as,[9]

$$p_3 = \frac{\eta}{e}.$$

This is also the average fire-sale price conditional on the aggregate shock state, $\{1, 3\}$, experiencing a fire sale (since $\pi_1 = 0$ and $\pi_3 = 1$). Note that we have, $p_3 < p_2$: that is, the aggregate shock state features deeper fire sales than the i.i.d. state. Intuitively, the aggregate shock state

[9] We could similarly calculate the fire-sale price in state 1 as $p_1 = \frac{\eta + xR}{e + x}$. This price does not play any role in the analysis since $\pi_1 = 0$, that is, the liquidity shock happens with zero probability in state 1.

has as much aggregate liquidity on average but this liquidity is not distributed appropriately across the states (state 3 has too little and state 1 has too much of it).

Next consider the effect of β on the expected return on foreign assets, which can be written as,

$$\begin{aligned} E\left[\overline{R}_s\right] &= \beta\left(\left(1-\pi\right)R + \pi p_3\right) + \left(1-\beta\right)\left(\left(1-\pi\right)R + \pi p_2\right) \\ &= \left(1-\pi\right)R + \pi\left(\beta p_3 + \left(1-\beta\right)p_2\right). \end{aligned}$$

This expression implies that increasing the correlations reduces the expected return on foreign assets, $E\left[\overline{R}_s\right]$, because it decreases the (unconditional) average fire-sale price, $\beta p_3 + (1-\beta)\,p_2$.

Next consider the effect of β on the risk-free return. Using Eq. (21), we have,

$$R_f = \frac{E\left[\overline{R}_s M_s\right]}{E\left[M_s\right]} = \frac{\beta R + \left(1-\beta\right)\left(\left(1-\pi\right)R + \pi p_2\right)\left(1-\pi+\pi\frac{R}{p_2}\right)}{\beta\left(1-\pi+\pi\frac{R}{p_3}\right) + \left(1-\beta\right)\left(1-\pi+\pi\frac{R}{p_2}\right)}.$$

The last expression is decreasing in β (since the numerator is decreasing and the denominator is increasing in β). Intuitively, as the liquidity shocks become more correlated, the risk-free asset becomes more valuable as it provides liquidity in case of an adverse aggregate liquidity shock. Finally, consider the risk premium on foreign financial investment, $E\left[\overline{R}_s\right] - R_f$ [cf. (22)]. Since the expected return on foreign asset as well as the risk-free asset decline, the effect on the risk premium is in general ambiguous. However, recall that the risk premium is zero for $\beta = 0$ (see Section 3) and becomes strictly positive for any $\beta > 0$ (see the previous subsection). Thus, the risk premium is increasing in the neighborhood of $\beta = 0$. The following result summarizes this discussion.

Proposition 5. *Consider the symmetric model with the possibility of correlated liquidity shocks. Increasing β (so that the shocks become more correlated) reduces the capital flows, x, and reduces fire-sale asset price in the i.i.d. state, p_2. It reduces the expected return on foreign financial assets, $E\left[\overline{R}_s\right]$, as well as the risk-free interest rate, R_f. In the neighborhood of $\beta = 0$, it also increases the risk premium on foreign assets, $E\left[\overline{R}_s\right] - R_f$.*

Hence, an increase in β in this model can be thought of as capturing a "risk-off" environment in which the investors retrench into their home markets (even at date 0, before the crises are realized). This reduces the capital flows and liquidity creation, while also reducing the risk-free rate and increasing risk premia. Next, we will analyze how the switch to a "risk-off" (or conversely, "risk-on") environment affects the reach-for-safety and the reach-for-yield mechanisms we introduced in Section 3.

4.2. Reach for Safety with Aggregate Shocks

We next revisit the reach-for-safety mechanism we introduced in Section 3.2 in the presence of aggregate liquidity risk. To this end, suppose a developed country j (that has measure zero) has greater liquidity than the world average, $\eta^j > \eta$. As before, suppose Assumption S holds so that the autarky equilibrium in the country would feature no fire sales, $p_s^j = R$ for each $s \in S$.

Consider the equilibrium with free capital flows. The world equilibrium is the same as described in Section 4. However, the equilibrium allocations in country j are different. In particular, when $x^{\text{in},j} > 0$ (which will be the case in equilibrium), the optimality conditions for the foreign investment by the intermediaries implies the following analogue of Eq. (9),

$$1 = \sum_s q_s \left(\left(1 - \pi_s\right) R + \pi_s p_s \right) = \sum_s q_s \left(\left(1 - \pi_s\right) R + \pi_s p_s^j \right). \tag{26}$$

This equation implies $\overline{p}^j = \overline{p}$, where we define the (price-)weighted average fire-sales as respectively,

$$\overline{p} = \frac{\sum_s q_s \pi_s p_s}{\sum_s q_s \pi_s} \text{ and } \overline{p}^j = \frac{\sum_s q_s \pi_s p_s^j}{\sum_s q_s \pi_s}. \tag{27}$$

Hence, the country cannot escape fire-sales "on average" even though it would not feature fire sales in autarky. Intuitively, the foreigners increase their investments in the country up to the point at which the local fire sales are sufficiently severe to deter inflows.

This leaves open the possibility that the volatility of the fire-sale prices in country j could be lower than in the representative country. In fact, a naive look at the market clearing condition (19) could suggest that country j would experience relatively less severe fire sales in states with greater s, as its large endowment of the safe asset would provide some cushion from the declines in aggregate liquidity. This prediction turns out to be incorrect. To see this, note that the local investors' optimality condition is given by,

$$u'\left(c_0^j\right) = \frac{M_s^j}{q_s/\gamma_s} \text{ for each } s \in S, \text{ where } M_s^j = 1 - \pi_s + \pi_s \frac{R}{p_s^j}. \tag{28}$$

This equation, together with Eq. (26), represents a system of $|S| + 1$ equations in $|S| + 1$ unknowns, where the unknowns are the prices $\left\{p_s^j\right\}_s$ and the outside spending, c_0^j. The unique solution is given by, $p_s^j = p_s < R$ for each s and $c_0^j = c_0$. In particular, the fire-sale price in country j is the same as in the representative country state-by-state.

The naive intuition is incorrect since the local investors do not retain their initial endowments of the safe asset. Rather, as captured by Eq. (28), they trade financial assets so as to move their liquidity across aggregate states. Recall also that the states with greater s command higher risk prices, q_s/γ_s, and that the country j has relatively large endowment of liquidity in these states. Thus, the local investors sell financial claims for states with higher s (and purchase financial claims for states with lower s). These financial trades ensure that the country's liquidity—and thus, fire-sale price—is in line with that in the representative country *state-by-state*.

These claims can be formalized by explicitly calculating the risks of the country's outflows relative to its inflows. Recall that $\overline{x}^{\text{out},j} = y^j - \eta^j/R_f + \sum_s q_s z_s^j$ denotes the date-0 value of the country's total outflows including its net trade of the safe asset. In the appendix, we show that $\overline{x}^{\text{out},j} = x$, that is, the country's outflows have the same size as in the representative country. However, the outflows have a different risk composition. To see this, let $x_s^{\text{out},j} = R_f \left(y^j - \eta^j\right) + z_s^j$ denote the payoff from the outflows conditional in state s of date 1. In the appendix, we show that,

$$x_s^{\text{out},j} = -\left(l^j - 1\right) x + l^j x \overline{R}_s \text{ for each } s, \text{ where } l^j > 1. \tag{29}$$

That is, the local investors can be thought of as selling some of their safe asset endowments to make a leveraged investment in foreign diversified portfolio. The variable, $l^j > 1$, is a measure of the leverage ratio in outflows: the value of the the risky investments the country undertakes relative to the value of its outflows. Note also that the date-1 payoff from the inflows is given by, $x_s^{\text{in},j} = x^{\text{in},j} \overline{R}_s$ for each s, that is, the leverage ratio in inflows is equal to one. Hence, Eq. (29) implies that the country's outflows are riskier than its inflows.

It follows that, in addition to having greater inflows than outflows as in Section 3.2, $x^{\text{in},j} > \overline{x}^{\text{out},j} = x$ (which continues to hold in this setting), the country also experiences relatively safe inflows and relatively risky outflows. This difference in the composition of flows is further destabilizing, and ensures that the country experiences the same (fire-sale) asset price volatility as the representative country.

We next analyze how an increase in aggregate correlations affects the level and the risk composition of the country's net inflows. To this end, consider the special case of the model with aggregate shocks described in Section 4.1. Suppose β increases so that the shocks become more correlated. As captured by Proposition 5, this decreases the symmetric flows, x, as well as the risk-free rate, R_f. In the appendix, we show that $x^{\text{in},j} - \overline{x}^{\text{out},j}$ increases: that is, the country's inflows decline less than its outflows, $\overline{x}^{\text{out},j} = x$. Furthermore, we also show that the leverage ratio of the country's outflows, l^j, increases. Intuitively, the "risk-off" induced by the increase in β makes international liquidity scarce and increases the value of safe assets that yield liquidity in high distress states. This increases the relative inflows into the developed country j, while also inducing the country to undertake foreign investment with a greater leverage ratio. The following result summarizes this discussion.

Proposition 6. *Consider the setting with aggregate risk in which a country has abundant local liquidity, $\eta^j > \eta$, that satisfies Assumption S (so that it would not experience fire sales in autarky). In an equilibrium with free financial flows, the country receives more inflows than its outflows, $x^{in,j} > \overline{x}^{out,j}$, and experiences fire sales with prices that are equal to those in the representative country, $p_s^j = p_s < R$ for each s. The country's outflows are riskier than its inflows, and they can be replicated as in* (29) *where $l^j > 1$ captures the leverage ratio in outflows.*

In the special case with correlated liquidity shocks, increasing β (so that the shocks become *more correlated) reduces the outflows $\overline{x}^{out,j}$, increases the inflows relative to outflows $x^{in,j} - \overline{x}^{out,j}$, and increases the leverage ratio, l^j.*

4.3. Reach for Yield with Aggregate Shocks

We next revisit the reach-for-yield mechanism we introduced in Section 3.3 in the presence of aggregate liquidity risk. For simplicity, we focus on the special case with correlated liquidity shocks described in Section 4.1. In this setting, suppose an emerging market country j (that has measure zero) has greater fundamental return than the world average, $R^j > R$. As before, suppose also that the country has relatively low liquidity, $\eta^j \leq \eta$. We also modify Assumption Y as follows.

Assumption $\tilde{\text{Y}}$. $R^j - R \in \left(0, \overline{p}\frac{\sum_s q_s \pi_s}{\sum_s q_s(1-\pi_s)}\right)$ and $\eta^j - \eta \in \left[-\frac{\sum_s q_s p_s}{\sum_s q_s}, 0\right]$.

When $x^{\text{in},j} > 0$ (which will be the case in equilibrium), foreign intermediaries' optimality condition implies,

$$\sum_{s\in\{1,2,3\}} q_s\left(\left(1-\pi_s\right)R + \pi_s p_s\right) = \sum_{s\in\{1,2,3\}} q_s\left(\left(1-\pi_s\right)R^j + \pi_s p_s^j\right). \tag{30}$$

Note that this equation implies $\overline{p}^j < \overline{p}$ (since $R^j > R$), where the weighted average fire-sales are defined in (27). Hence, as in Section 3.3, the county with higher return experiences greater fire-sales "on average."

The remaining question is how these fire sales are distributed across the two distress states $s \in \{2,3\}$ (recall that p_1 does not affect the equilibrium since there are no crises in state 1, $\pi_1 = 0$). The distribution of fire sales is determined by local investors' allocation of liquidity across states. Specifically, these investors' optimality conditions are given by,

$$u'\left(c_0^j\right) = \frac{M_2^j}{q_2/\gamma_2} = \frac{M_3^j}{q_3/\gamma_3}, \text{ where } M_2^j = 1-\pi+\pi\frac{R^j}{p_2^j} \text{ and } M_3^j = \frac{R^j}{p_3^j}.$$

The optimality condition for the no-distress state 1 does not appear in this expression since it is satisfied with inequality, $u'\left(c_0^j\right) > \frac{M_1^j}{q_1/\gamma_1}$. Intuitively, the investors' marginal utility in the no-crisis state 1 is relatively low, $M_1^j = 1$. In equilibrium, this induces them to hold as little liquidity as possible in this state (formally, $z_1^j + y^j R_f = 0$) so as to hold more liquidity in distress states 2 and 3.

Next note that the analogues of the optimality conditions above also hold for the representative country. In particular, we have $\frac{M_2}{q_2/\gamma_2} = \frac{M_3}{q_3/\gamma_3}$. Combining this with the above conditions and substituting the respective marginal utilities, we obtain,

$$\frac{1-\pi+\pi R^j/p_2^j}{1-\pi+\pi R/p_2} = \frac{R^j/p_3^j}{R/p_3}. \tag{31}$$

Eqs. (30) and (31) represent two equilibrium conditions in two unknowns, p_2^j, p_3^j. In the appendix, we show that the solution corresponds to an equilibrium in country j with appropriate corresponding allocations, $\left(c_0^j, \left\{z_s^j + y^j R_f\right\}_{s\in\{2,3\}}, x^{\text{in},j}\right)$, that ensure that the budget and the market clearing conditions also hold.

We next inspect the properties of the equilibrium prices in country j. Eqs. (30) and (31) imply $\frac{R^j/p_2^j}{R/p_2} > \frac{R^j/p_3^j}{R/p_3} > 1$. That is, the country experiences deeper fire sales relative to the representative country state-by-state. However, the relative depth of the fire sales is greater in the idiosyncratic shock state 2 than in the aggregate shock state 3. Intuitively, since the crises are more frequent in state 3, the local investors in country j purchase relatively more liquidity for this state than in state 2. This helps to mitigate somewhat the fire sales caused by the reach-for-yield inflows in state 3 (the global crisis), at the expense of deepening the fire sales in state 2 (the local crises).

We next analyze how a decline in R (the reach-for-yield) as well as an increase in β (the risk-off) affect the equilibrium prices in country j. To this end, note that, the intermediaries' optimality condition (30) can be combined with the definition of the average fire-sale prices in (27) to obtain,

$$\left(\overline{p} - \overline{p}^j\right)\left(\sum_s q_s \pi_s\right) = \left(R^j - R\right)\left(\sum_s q_s \left(1-\pi_s\right)\right). \tag{32}$$

The right hand side of this expression captures the relative return advantage to investing in country j. The advantage is driven by the return differences, and it is realized only if the country does not experience a crisis. Hence, the advantage is multiplied by $\sum_s q_s \left(1-\pi_s\right)$: the sum of the probabilities of not having a crisis in an aggregate state times the Arrow-Debreu price. The left hand side captures the relative disadvantage to investing in country j. The disadvantage is driven by the fire-sale differences, and it is realized when the country experiences a crisis. Consequently, the disadvantage is multiplied by $\sum_s q_s \pi_s$.

Eq. (32) suggests that a decline in R exacerbates the relative fire sales in country j. However, the result does not immediately follow since the multiplier terms are endogenous and depend on R. In the appendix, we show that (for the model with three states) the endogenous effect reinforces the direct effect. In particular, a decrease in R increases $\overline{p} - \overline{p}^j$, generalizing Proposition 3.

Next consider how an increase in β affects the reach-for-yield mechanism. To this end, we characterize the ratio of the multiplier terms in Eq. (32) further to obtain,

$$\frac{\sum_s \frac{q_s}{\gamma_s}\gamma_s \left(1-\pi_s\right)}{\sum_s \frac{q_s}{\gamma_s}\gamma_s \pi_s} = \frac{\sum_s M_s \gamma_s \left(1-\pi_s\right)}{\sum_s M_s \gamma_s \pi_s} = \frac{1-\pi}{\pi}\frac{M_1\beta + M_2\left(1-\beta\right)}{M_2\left(1-\beta\right) + M_3\beta}. \tag{33}$$

Here, the first equality substitutes for the state prices q_s/γ_s from Eq. (20), and the second equality substitutes for the probabilities in the three state model from $(23-24)$. Recall also that the marginal utilities satisfy, $M_1 < M_2 < M_3$. Hence, the equation suggests that increasing

β decreases the numerator and increases the denominator. This in turn decreases the relative fire-sale prices in country j in view of Eq. (32). The result does not immediately follow, however, since increasing β also affects the marginal utilities (in particular, it increases M_2). In the appendix, we show that the indirect effect through the marginal utilities mitigates but does not overturn the direct effect. In particular, an increase in β reduces $\overline{p} - \overline{p}^j$, that is, it shrinks the gap between the fire-sale prices in country j and the representative country.

Hence, a risk-off environment driven by high β mitigates the reach-for-yield mechanism, resulting in relatively less severe fire sales in countries with high returns. For intuition, recall from Eq. (32) that investing in country j offers an advantage if there is no local crisis but a disadvantage if there is a crisis. Greater β implies that the local crises are relatively more correlated with aggregate distress states. Since investors like payoffs relatively more in more-distressed aggregate states, this reduces the return advantage and increases the disadvantage—as captured by Eq. (33). This reduces the foreign inflows into country j, which in turn mitigates the fire sales. The following proposition summarizes this discussion.

Proposition 7. *Consider the special case of the aggregate risk model with correlated liquidity shocks, in which a country has relatively high return, $R^j > R$, and Assumption 1^Y holds. In an equilibrium with free financial flows, the country experiences deeper fire sales than in the representative country in both distress states, that is, $\frac{R^j/p_s^j}{R/p_s} > 1$ for each $s \in \{2, 3\}$. The relative depth of fire sales in country j is greater in the idiosyncratic shock state than in the aggregate shock state, that is, $\frac{R^j/p_2^j}{R/p_2} > \frac{R^j/p_3^j}{R/p_3} > 1$. A decrease in R reduces the country's relative weighted average fire-sale price, $\overline{p}^j - \overline{p}$. An increase in β increases $\overline{p}^j - \overline{p}$.*

5. Optimal Policy with Symmetric Flows

In the rest of the paper, we analyze the policy implications of our analysis. In this section, we analyze capital restrictions (and related policies) in the context of the baseline model with symmetric capital flows. We show that a global planner that is concerned with financial stability encourages capital flows, in view of their liquidity creation benefits, but local planners restrict capital flows. We also show that the planners' capital restrictions are complementary, which could lead to multiple equilibria or amplification of exogenous liquidity shocks. In the next section, we analyze the policy implications in asymmetric environments that feature the reach-for-safety and the reach-for-yield mechanisms.

Recall that fire sales are costly in our setting since they reduce the financing available to entrepreneurs, each of which sells one unit of the asset at date 1 to reinvest. To analyze welfare, we need to be more specific about entrepreneurs' investment technology. We assume entrepreneurs come in two varieties that differ in the type of their projects. A fraction, ζ, of them have a project with decreasing returns to scale: Investing p dollars in this project at date 1 yields $\lambda f(p)$ dollars at date 2. Here, $f(\cdot)$ is an increasing and strictly concave function. The remaining fraction, $1-\zeta$, of entrepreneurs have a project with constant returns to scale: Investing p dollars yields λp dollars at date 2. Here, the parameter, λ, captures the strength of the financial stability concerns (the benefits from mitigating fire sales and increasing p). The concave function, $f(\cdot)$, captures the idea that the the marginal benefit from financial stabilization will be greater when the prices are lower and the fire sales are deeper. The distinction between entrepreneurs with diminishing and linear scales is not important, but it provides an additional level of generality that helps to simplify some of the expressions in our optimal policy analysis.

We also suppose the planner in each country is utilitarian: she maximizes the sum of the local investors' and the local entrepreneurs' expected utilities. Since all agents are risk neutral, the social welfare function for the planner in country j can be written as,

$$W^j = u\left(c_0^j\right) + E\left[c_1^j + c_2^j\right] + \lambda e \left(\begin{array}{c} \zeta\left((1-\pi) f(R) + \pi f\left(p^j\right)\right) \\ +(1-\zeta)\left((1-\pi) R + \pi p^j\right) \end{array} \right). \tag{34}$$

We will focus on the special case in which the financial stability concerns are very important, $\lambda \to \infty$. In this case, the planner effectively maximizes the output per entrepreneur,

$$W^j/(\lambda e) \to \zeta\left((1-\pi) f(R) + \pi f\left(p^j\right)\right) + (1-\zeta)\left((1-\pi) R + \pi p^j\right), \tag{35}$$

which is increasing in the local fire-sale price level, p^j. In Appendix A.2, we analyze the more general case with finite λ. As our analysis there illustrates, there are in fact other welfare considerations in this model, but we envision a regulatory environment in which those considerations are dominated by concerns with financial stability.

We next investigate the desirability of various policies in settings with symmetric flows. In each setting, we will consider the policies that would be chosen by a global planner that could coordinate the decisions of individual planners, and compare this outcome with the Nash equilibrium that would obtain absent coordination. We assume the global planner maximizes the sum of individual planners' objectives, $\int_j \left(W^j/(\lambda e)\right) dj$. In a symmetric equilibrium, this amounts to maximizing each individual planner's objective. We start by analyzing the desirability of capital taxes targeted towards reducing the inflows ex ante. We then analyze the desirability of liquidity injection policies targeted towards mitigating crises ex post.

5.1. Capital Restrictions with Symmetric Flows

Consider the baseline model in Section 3.1 with the only difference that the planner in each country j can impose a linear tax, $\tau^j \geq 0$, on the short-term return on foreign inflows: that is, the return on the foreign financial holdings in country j is now given by $\overline{R}\left(1-\tau^j\right)$. We assume that the tax revenues are used to purchase an equal-weighted portfolio of all financial assets. The assets that are purchased are then wasted by the planner. The latter assumption ensures that expropriating foreigners is not the rationale behind taxing capital flows. The former assumption (asset purchases) ensures that the liquidity that the government collects via taxation is injected back into the financial markets in equal proportion so that the government taxation does not

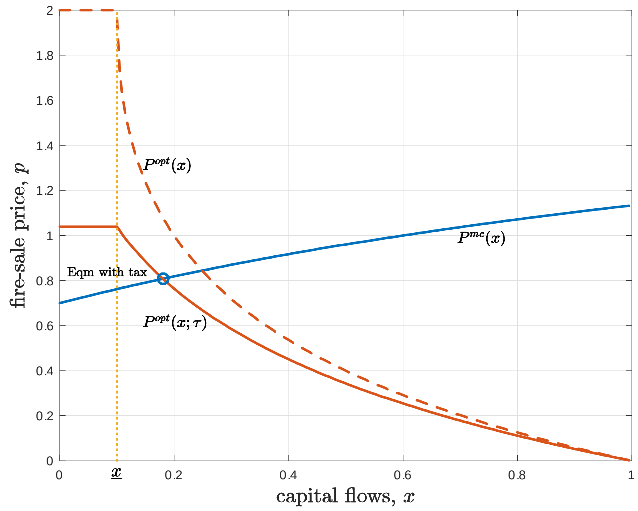

Figure 2: The characterization of equilibrium with taxes. The dashed line replicates the baseline characterization without taxes.

directly waste liquidity. This leads to simpler expressions, but our results continue to hold if we instead assume the government wastes the tax revenues without purchasing assets.[10]

Coordinated policy. To analyze the optimal coordinated policy, consider the equilibrium in which all countries apply the same tax rate, $\tau \geq 0$. The analysis is similar to Section 3 with minor differences. One caveat is that foreign investment does not necessarily dominate local investment at date 0 since foreign investment is taxed. In the appendix, we show that the equilibrium behavior depends on a threshold tax level, $\bar{\tau}$. If the tax level is above the threshold, $\tau \geq \bar{\tau}$, then there is zero foreign investment, $x = 0$, and the fire-sale price level is given by $p = \eta/e$. If instead the tax level is below the threshold, $\tau < \bar{\tau}$, then there is positive foreign investment, $x > 0$. In this case, the equilibrium conditions can be written as,

$$R_f = \overline{R}(1-\tau), \tag{36}$$
$$\begin{cases} u'(1-x) = \mu(p)(1-\tau) & \text{if } \mu(p)(1-\tau) > R \\ x \in [0, \underline{x}] & \text{if } \mu(p)(1-\tau) = R \end{cases},$$
$$\text{and } p = \min\left(R, \frac{\eta + x\overline{R}(1-\tau) + x\overline{R}\tau}{e+x}\right) = \min\left(R, \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right).$$

These conditions are the analogues of Eqs. $(5), (6), (8)$ in Section 3.1. The first two equations are adjusted for the presence of taxes. The market clearing condition is unchanged in view of the assumption that the taxes taken away by the planner are injected back into the market, as illustrated by the equation.

[10]In fact, the results become stronger since the alternative specification creates a second channel by which capital taxes reduce global liquidity and asset prices.

Figure 2 plots the analogs of the optimality and the market clearing curves, which we now denote by $p = P^{opt}(x; \tau)$ and $p = P^{mc}(x)$. Note that introducing (or increasing) taxes shifts the optimality curve downwards and leads to lower capital flows, x, as well as the fire-sale price, p. Intuitively, the capital taxation discourages foreign flows, which in turn decreases the cash-per-asset and the fire-sale price in local distressed markets as we discussed earlier. It follows that a global planner that coordinates countries' policies and that focuses on increasing the fire-sale price level sets zero tax on capital inflows.

For future reference, it is useful to complete the characterization of the equilibrium with taxes. Using Figure 2 and Eq. (36), the threshold tax level above which there are no foreign flows, $\overline{\tau}$, is characterized by solving $\mu(p)(1-\overline{\tau}) = R$ for $p = P^{mc}(0) = \eta/e$. Note also that there is also a lower threshold tax level, $\underline{\tau} \in (0, \overline{\tau})$, such that if the tax is above this threshold but below the higher threshold, $\tau \in (\underline{\tau}, \overline{\tau})$, then the equilibrium is in the flat part of the optimality curve: that is, outside spending is at its upper bound, $c_0 = 1 - \underline{x}$, and there is some local investment, $x^{loc} > 0$. The lower threshold is characterized as the solution to $\mu(p)(1-\underline{\tau}) = R$ for $p = \frac{\eta + \underline{x}(1-\pi)R}{e + \underline{x}(1-\pi)}$. If the tax is below the lower threshold, $\tau < \underline{\tau}$, then the outside spending is below its upper bound, $c_0 < 1 - \underline{x}$, local investment is zero, $x^{loc} = 0$, and the foreign flows exceed $\underline{x}$.

Proposition 8. *Consider the symmetric model with capital taxes in the limit as $\lambda \to \infty$ (financial stability concerns are dominant). There exists a threshold tax level $\bar{\tau} > 0$ such that, for each $\tau < \bar{\tau}$, there are positive capital flows, $x > 0$, and the equilibrium is characterized as the solution to the system in* (36). *There also exists a lower threshold tax level $\underline{\tau} \in (0, \bar{\tau})$ such that, for each $\tau < \underline{\tau}$, local investment is dominated, $x^{loc} = 0$, and the foreign flows satisfy, $x > \underline{x}$. In either case, increasing the symmetric tax level, τ, reduces the capital flows, x, and decreases p and R_f. A global planner that coordinates countries' policies sets zero tax on capital inflows, $\tau = 0$.*

Nash equilibrium. We next analyze the uncoordinated outcomes that would emerge in a Nash equilibrium in which each planner chooses its own policy taking the policies in other countries as given. To this end, consider the optimal tax rate for an individual country, $\tau^j \geq 0$, when all other countries apply the same tax rate, τ. To keep the analysis simple, suppose the taxes cannot be increased above the lower threshold characterized above, that is, $\tau^j \leq \underline{\tau}$ for each j (the case with $\tau^j > \underline{\tau}$ is slightly more complicated but does not offer much additional insight). We will establish that the only Nash equilibrium is one in which all countries set the highest allowed tax level, $\tau^j = \underline{\tau}$.

To show this, suppose the common tax level is strictly below the threshold, $\tau < \underline{\tau}$. As the above characterization illustrates, the symmetric equilibrium features $x^{loc} = 0$ and $x > \underline{x}$. Let x^j and $x^{\text{in},j}$ denote the outflows from and inflows into a particular country j when this country deviates and sets a potentially different tax level, τ^j. For sufficiently small deviations, it can be seen (by continuity) that the inflows remain positive, $x^{\text{in},j} > 0$, and the outflows exceed the lower bound, $x^j > \underline{x}$ (since $x > \underline{x}$). The foreign investors' optimality condition can then be written as,

$$R_f = \overline{R}\,(1-\tau) = \overline{R}^j\left(1-\tau^j\right), \text{ where } \overline{R}^j = (1-\pi)\,R + \pi p^j.$$

Inspecting this condition, it follows that increasing τ^j (in a neighborhood of τ) increases p^j. Intuitively, greater taxes discourage foreign investors, which reduces the inflows into the country, $x^{\text{in},j}$ and increases p^j. This process continues until p^j is sufficiently high to convince the foreigners to remain in the country. Since the local planner prefers a higher local price level p^j, it follows that there is a profitable deviation as long as the symmetric tax level is below its upper bound, $\tau < \underline{\tau}$. Hence, the unique symmetric Nash equilibrium features the highest allowed tax level, $\tau = \underline{\tau}$. Our next result summarizes this discussion.

Proposition 9. *Consider the symmetric model with capital taxes in the limit as $\lambda \to \infty$ (with the restriction that $\tau^j \in [0, \underline{\tau}]$). There exists a unique Nash equilibrium with symmetric allocations in which the individual planners set the highest allowed tax level, $\tau^j = \underline{\tau}$ for each j. The capital flows, $x = \underline{x}$, the fire-sale price, p, and the risk-free return, R_f, are lower than what would obtain in an equilibrium without taxes.*

Comparing this result with Proposition 8 illustrates that the uncoordinated Nash equilibrium generates a highly inefficient outcome at the global level. The Nash equilibrium features highest allowed level of capital taxes, whereas the globally efficient solution features zero taxes. Intuitively, a country that taxes capital inflows improves its own financial stability at the expense of reducing the global liquidity and exacerbating financial shocks in other countries. A country that sets its tax level in isolation does not take into account the negative externalities it causes on other countries by reducing global liquidity. This leads to protectionist capital policies that are inefficient at the global level.

5.1.1. Complementarities in Capital Restrictions

The result that the Nash equilibrium exhibits the highest allowed tax level helps to illustrate our point sharply. However, it is extreme and it also prevents us from analyzing how the capital market policies in one country react to other countries' policies or exogenous changes. To analyze these issues, we next consider a version of the model in which capital taxes are costly. In particular, suppose the capital taxes cannot be targeted perfectly, and some of the taxes also fall on the entrepreneurs. Since entrepreneurs sell assets to undertake productive projects, these costs reduces the planner's welfare even as $\lambda \to \infty$.

More specifically, suppose applying a tax $\tau \geq 0$ on the foreign capital reduces the returns of the entrepreneurs that have linear scale by $v(\tau) \geq 0$. Then, as $\lambda \to \infty$, the planner effectively maximizes the following analogue of the objective function in (35),

$$(1-\zeta)\left((1-\pi)\,f(R) + \pi f\left(p^j\right)\right) + \zeta \overline{R}^j\left(1 - v\left(\tau^j\right)\right). \tag{37}$$

Suppose the cost function $v(\cdot)$ is strictly increasing and convex (greater mistargeting when taxes are greater). To ensure an interior solution, suppose also that it satisfies the Inada-type conditions, $v(0) = v'(0) = 0$ and $v'(\tilde{\tau}) = \infty$ for some $\tilde{\tau} > 0$ which also satisfies $\tilde{\tau} < \underline{\tau}$ (see Proposition 8 for the definition of $\underline{\tau}$). We view the cost function as capturing in reduced form various difficulties associated with restricting capital flows in practice. The assumption that the costs hit only the entrepreneurs with linear scale, together with the assumption that the function $f(\cdot)$ is strictly concave, will capture the intuitive idea that the planner has greater incentives to intervene when there is greater distress during a liquidity shock.

To characterize the optimal tax level, note the Eq. $(B.75)$ (in the Appendix) describes a relation between the price and the tax level. Plugging this relation into Eq. (37) and taking the first order condition, we obtain,

$$V\left(\tau^{j}\right) = \frac{(1-\zeta) f'\left(p^{j}\right) + \zeta}{\zeta}, \text{ where } V\left(\tau^{j}\right) = v'\left(\tau^{j}\right)\left(1-\tau^{j}\right) + v\left(\tau^{j}\right). \tag{38}$$

Here, $V(\tau)$ is an increasing function over $[0, \tilde{\tau})$ with $V(0) = 0$ and $V(\tilde{\tau}) = \infty$. Hence, for any symmetric equilibrium price level, $p^{j} = p$, there is a unique solution to Eq. (38) that characterizes the optimal tax level for the country. Note also that the optimal tax level is decreasing in p: that is, a lower price level induces a greater tax, because it increases the (local) benefits of taxation more than its costs.

The equilibrium is characterized by solving the earlier system of equations (36) together with Eq. (38). Note that the earlier system describes the equilibrium price level as a decreasing function of τ, that is, greater taxes induce lower prices. Eq. (38) characterizes the optimal tax level as a decreasing function of p. The equilibrium corresponds to the intersection of two decreasing curves. This observation leads to the following result.

Proposition 10. *Consider the symmetric model with costly capital taxes in the limit as* $\lambda \to \infty$.

(i) There can be multiple symmetric Nash equilibria. When this is the case, the equilibrium with a lower price level leads to lower welfare for all planners.

(ii) Suppose the parameters are such that there is a unique Nash equilibrium (or consider the neighborhood of any stable equilibrium). Reducing the local liquidity, η*, increases the equilibrium tax level,* τ*, and reduces the price,* p*, as well as the risk-free return,* R_f*. Moreover, the price and the risk-free return decline more than the alternative case in which the taxes are kept at their pre-change levels.*

The intuition follows from observing that the policies that restrict capital flows represent negative externalities on other planners, and that these policies are strategic complements. A country that sets a more restrictive policy reduces global liquidity. This leads to lower fire-sale prices in other countries. The low prices reduce the welfare of other planners. They also induce those planners to set more restrictive policies. When these complementarities are sufficiently strong, there can be multiple equilibria. Even when there is a single equilibrium, the

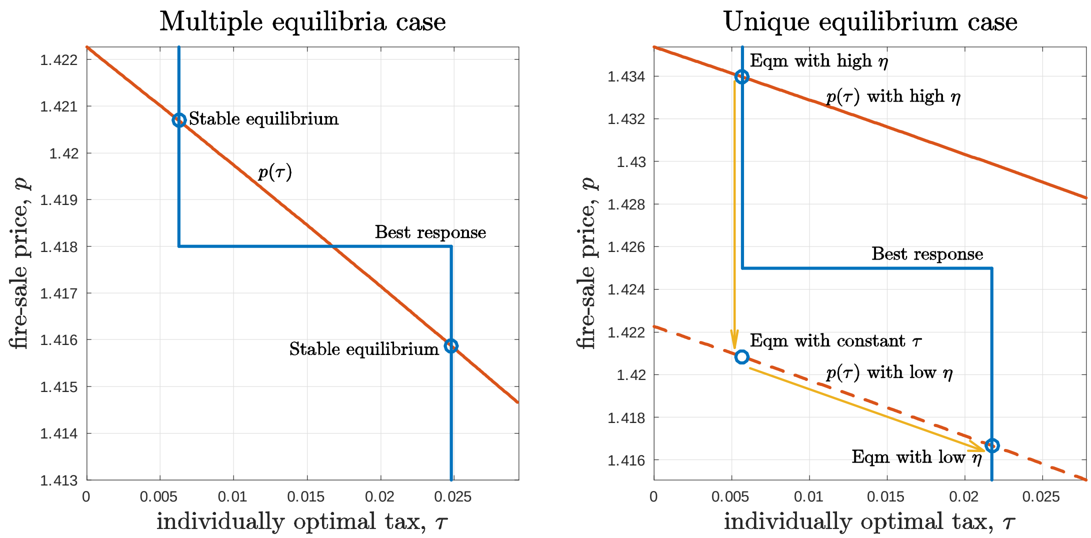

Figure 3: The left panel illustrate the equilibria with costly capital taxes for a parameterization that generates multiple equilibria. The right panel illustrates the parameterization with a unique equilibrium. The shift form the solid line to the dashed line captures the effect of decreasing local liquidity, η.

complementarities amplify the impact of exogenous shocks that reduce liquidity.

We illustrate these results using a numerical example. Suppose the utility from the outside option is given by $u(c_0) = h \log c_0$, with $h \in (0, R)$. Suppose the cost function takes the form, $v(\tau) = -k\left(\log\left(\frac{\tilde{\tau}-\tau}{\tilde{\tau}}\right) + \frac{\tau}{\tilde{\tau}}\right)$ for some $k > 0$, which satisfies the regularity conditions over $\tau \in [0, \tilde{\tau}]$. Suppose the entrepreneurs' production function takes the piecewise-linear form,[11]

$$f(p) = \begin{cases} ap, & \text{for } p \leq \overline{p} \\ bp, & \text{for } p > \overline{p} \end{cases}, \text{ for some } a > b > 0, \text{ and } \bar{p} \in (0, R).$$

The left panel of Figure 3 illustrates the possibility of multiple equilibria using a particular parameterization. The straight decreasing line plots the equilibrium price as a function of the tax. The jagged decreasing line plots the planner's optimal tax choice as a function of the price. The two intersections illustrate the stable equilibria. If the price is above the threshold, $\overline{p}$, financial stability concerns are not too significant and the planners set relatively low taxes. This leads to high global liquidity and supports fire-sale prices that are above the threshold. However, if the price level falls below the threshold, $\overline{p}$, then financial stability concerns become more important, which induces the planners to set high taxes. This leads to a reduction in

[11]This function violates the regularity conditions on $f(\cdot)$ (e.g., it is not striclty concave) but it can be made to satisfy the conditions after some smoothing and it helps to illustrate the result sharply.

global liquidity and leads to fire-sale prices below the threshold. Note that the equilibrium with the higher tax and the lower price level is dominated: it yields a lower utility for each planner than the other equilibrium.

The right panel of Figure 3 illustrates the amplification mechanism using a parameterization that leads to a unique equilibrium. The solid and the dashed lines plot the equilibrium price function with respectively higher and lower local liquidity, η. If the tax level was exogenously fixed, a reduction in local liquidity would reduce the price level as formalized in Proposition 1. When the tax level is endogenous, the price declines even more. In this example, the exogenous liquidity shock reduces the price below the threshold below which the financial stability concerns increase. This leads to higher taxes and lower prices.

In Appendix A.5, we also analyze how introducing aggregate shocks affects our analysis of capital taxation in this section. Specifically, we allow the planners to set state-contingent taxes, $\{\tau_s\}_s$, in the setting with aggregate shocks introduced in Section 4. We show that the tax rate is positive for each state, $\tau_s > 0$ for each $s \in S$, generalizing the results in this section. We also show that τ_s is increasing in $s \in S$: that is, states with greater probability of liquidity shocks are associated with higher taxes. For intuition, recall from Section 4 that the foreign investors value payoff in distressed states relatively more. Taxing them in those states provides a cheaper way of discouraging (ex-ante) inflows. Hence, the planner applies larger taxes—more protectionism—in states with greater financial distress.

5.2. Liquidity Creation with Symmetric Flows

We next analyze policies by which the government might increase the liquidity at date 1 to mitigate fire sales. In particular, suppose the planner in each country can generate additional liquidity at date 1 by taxing a third group of agents, which we refer to as nonparticipants. Nonparticipants are endowed with $\bar{\eta} > 0$ dollars at date 1 that are taxable by the government. We assume the planner can only intervene by purchasing financial assets in case of a local liquidity shock. In particular, a planner that raises $\eta^{pl,j} \in [0, \bar{\eta}]$ dollars in the low liquidity state of date 1 purchases $\eta^{pl,j}/p^j$ units of the asset, where p^j denotes the equilibrium price that obtains after the intervention. We assume that the planner can commit to implementing a particular policy, i.e., we abstract away from time-inconsistency problems.

We also assume that the assets purchased by the planner are wasted, which ensures that the rationale for intervention is not driven by the government's comparative advantage in financial markets.[12] The social welfare function in (34) is then modified by,

$$W^j = u(c_0) + E[c_1 + c_2] + \lambda e \left(\zeta\left((1-\pi) f(R) + \pi f(p)\right) + (1-\zeta)\overline{R}\right) + \bar{\eta} - \pi \eta^{pl,j}. \tag{39}$$

The last term captures the expected consumption loss due to the government liquidity creation

[12] The planner's advantage lies in its unique ability to raise tax revenues and generate liquidity as in Holmstrom and Tirole (1998).

in the low liquidity state. As $\lambda \to \infty$, the planner cares only about financial stability and effectively maximizes the same objective function (35) as before. However, for any finite λ, there are costs associated with liquidity creation, which will help to break ties when various policy choices yield the same value for the objective in (35).

Coordinated policy. As in the case of capital taxes, first consider the symmetric coordinated policy, $\eta^j = \eta^{pl}$ for each j, that would be chosen by a global planner. The characterization of equilibrium is the same as in the baseline analysis in Section 3 with the only difference that the market clearing equations (7) and (8) are replaced by,

$$p = \min\left(R, \frac{\eta + \eta^{pl} + x\overline{R}}{e + x}\right) = \min\left(R, \frac{\eta + \eta^{pl} + x(1-\pi)R}{e + x(1-\pi)}\right).$$

In particular, for any level of foreign flows, x, the asset price in each country is increased by public liquidity injection by the planner. In equilibrium, the increase in the price reduces the foreign flows x, as there is less need of private liquidity creation, but this effect does not undo the initial price increase. It follows that a global planner creates the maximum amount of liquidity.

Proposition 11. *Consider the symmetric model with public liquidity creation in the limit as $\lambda \to \infty$. Suppose the parameters satisfy, $\eta + \bar{\eta} < eR$. A global planner that coordinates countries' policies creates the maximum amount of liquidity, $\eta^{pl} = \bar{\eta}$.*

Nash equilibrium. Next consider the optimal public liquidity injection policy for the planner of a country, $\eta^{pl,j}$, when all other countries set their public liquidity injection at some level, $\eta^{pl} > 0$. For sufficiently small deviations, it can be seen (by continuity) that the inflows remain positive, $x^{\text{in},j} > 0$, and the outflows exceed the lower bound, $x^j > \underline{x}$ (since $x > \underline{x}$). The foreign investors' optimality condition can then be written as,

$$R_f = \overline{R} = \overline{R}^j, \text{ where } \overline{R}^j = (1-\pi)R + \pi p^j.$$

Inspecting this condition, it follows that the policy has no impact on the asset price, p^j. The reason is that the amount of public liquidity injection is anticipated by the financial markets and neutralized by capital inflows. If the country decides to inject more public liquidity than other countries, $\eta^{pl,j} > \eta^{pl}$, all else equal this increases the price in its financial markets, p^j. However, as in our earlier analysis with reach-for-safety, this policy also makes the country's assets more attractive compared to other countries', which in turn increases the inflows, $x^{\text{in},j}$. This process continues until the country's assets are equally attractive as other countries' assets. Formally, we show (in the appendix) that $x^{\text{in},j} - x = \left(\eta^{pl,j} - \eta^{pl}\right)/p$: the country would receive excess inflows (or run current account deficits) that would fully neutralize its excess liquidity injection.

It follows that public liquidity creation by an individual country leaves the country price unchanged and does not provide any financial stability benefits. Since the liquidity creation is costly for any finite λ (and thus, in the limit as $\lambda \to \infty$), a local planner does not have any incentive to create liquidity. The following result summarizes this discussion.

Proposition 12. *Consider the symmetric model with public liquidity creation in the limit as $\lambda \to \infty$. The uncoordinated Nash equilibrium in which planners set the tax level in their own countries features zero public liquidity injection, $\eta^j = 0$ for each j.*

Comparing this result with Proposition 11 illustrates that the coordinated and the uncoordinated equilibria sharply differ. The coordinated equilibrium calls for the maximum level of public liquidity creation, whereas the uncoordinated equilibrium features no public liquidity creation. Intuitively, when a country creates liquidity, it also attracts greater inflows. These inflows dilute the financial stability benefits to other countries, which now have access to greater liquidity to arbitrage fire sales in their own financial markets. A country that sets its policy in isolation does not take into account the positive externalities it has on other countries via global liquidity creation. This leads to too little liquidity creation during financial crises compared to a coordinated outcome.

So far, we analyzed the planners' incentives to create liquidity directly. A related question is whether the planners might also want to encourage the creation of private liquidity. We address this question in Appendix A.3 by allowing the planner to interfere with the investors' date 0 decisions. By discouraging/taxing the outside option, the planner might increase local investors' financial assets, which in turn increases liquidity and improves asset prices. We find that the policy implications of private liquidity creation is similar to public liquidity creation. Specifically, a global planner that is concerned with financial stability incentivizes local investors to hold financial assets, whereas the Nash equilibrium features no such incentives for the same reason as above. Intuitively, greater financial savings by the local investors are neutralized by greater fickle flows from abroad, leaving the local fire sales unchanged.

This discussion also suggests that a country might want to combine protectionist policies in the capital market with local liquidity-creation policies. To accomplish this, however, the country would have to use quantity restrictions in the capital market—rather than price restrictions such as taxes—as the arguments for liquidity creation continue to apply for any interior tax level (that allows some positive inflows). By setting a quantity restriction on foreign flows, the country can ensure that the additional liquidity it creates remains inside the country. Note, however, that this outcome would still not replicate the coordinated solution as it would feature too little capital flows and inefficient global liquidity creation.

Taken together, Propositions 8-12 illustrates the importance of policy coordination for managing global liquidity in an environment with fickle capital flows. These flows reduce financial stability in the receiving country, but they also help to distribute excess liquidity to countries and areas that need it relatively more. The Nash equilibrium might feature too much impediment to capital inflows and too little local global liquidity creation, because the individual countries do not take into account the external benefits of distributing the excess liquidity they have or newly create. Moreover, individual countries' decisions to restrict capital flows are complementary, which amplifies the negative liquidity shocks by restricting the endogenous global liquidity creation.

6. Optimal Policy with Asymmetric Flows

In Sections 3 and 4, we showed that the asymmetric liquidities or returns across countries naturally generate a reach-for-safety and a reach-for-yield mechanism. We developed these mechanisms in an environment in which the world was symmetric except for one country. While this approach is useful to illustrate the mechanisms, it does not allow for a meaningful welfare analysis, since a country with measure zero does not enter the global planner's welfare function. In this section, we first extend the baseline model in Section 3 (without aggregate risk) to a setting with multiple and asymmetric regions. We then use special cases of this model to analyze the policy implications of the reach-for-safety and the reach-for-yield mechanisms.

Suppose there are multiple regions of countries denoted by the superscript $k \in K = \{1, .., |K|\}$. Each region k consists of a continuum of countries that is identical to the continuum we analyzed in the baseline model in Section 3. We let m^k denote the mass of countries in region k and assume $\sum_{k \in K} m^k = 1$. The earlier analysis is the special case with a single region.

The liquidity shocks are i.i.d. within regions as well as across regions (so we abstract away from aggregate risk for simplicity). The regions are the same as one another except that the countries in each region might feature heterogeneous amounts of liquidity, $\left\{\eta^k\right\}_k$, as well as heterogeneous returns from new investment, $\left\{R^k\right\}_k$. Later, we will focus on special cases with two regions that can be thought of as corresponding to developed and emerging markets.

As before, there are two types of agents in each country, entrepreneurs and investors. The entrepreneurs are largely passive and sell e units of the asset at date 1. The investors have local habitats and are forced to liquidate foreign positions when those markets experience a liquidity shock. The difference is that the investors can take positions in multiple regions. As before, we focus on symmetric equilibria in which the investors of the same region take identical positions, and assets (of the countries) within the same region trade at the identical price denoted by p^k. As problem (40) illustrates, the latter assumption implies that the distribution of an investor's portfolio among the countries of a region is not payoff relevant—what matters is the total position in the region. Hence, without loss of generality, we also focus on symmetric allocations in which the representative investor in region k takes fully diversified positions within each region k'. We denote these positions by $\left\{x^{k',k}\right\}_{k' \in K}$. We also denote the investor's positions in legacy assets by $\left\{y^{k',k}\right\}_{k' \in K}$. As before, we use R_f to denote the (endogenous) risk-free return. The problem for the representative investor (in region k) can then be written as,

$$\max_{\tilde{c}_0.\tilde{x}^{loc},\left\{\tilde{x}^{k'},\tilde{y}^{k'}\right\}_{k'}} u\left(\tilde{c}_0\right)+\tilde{x}^{loc}R^k+\left(\sum_{k'}\tilde{x}^{k'}\overline{R}^{k'}+\tilde{y}^{k'}R_f\right)M^k, \tag{40}$$

$$\overline{R}^{k'}=\left(1-\pi\right)R^{k'}+\pi p^{k'} \text{ for each } k'\in K$$

$$M^k=1-\pi+\frac{R^k}{p^k}\pi$$

$$\tilde{c}_0+\tilde{x}^{loc}+\sum_{k'}\left(\tilde{x}^{k'}+\tilde{y}^{k'}\right)=\eta^k/R_f+1.$$

Note that the investor solves a generalized version of problem (1).

The market clearing conditions for the legacy assets in region k can be written as,

$$\sum_{k'\in K} m^{k'}y^{k,k'}=m^k\eta^k/R_f, \tag{41}$$

The market clearing condition for the risky assets in a country of region k can be written as,

$$p^k=\min\left(R^k,\frac{\sum_{k'}\left(y^{k',k}R_f+x^{k',k}\overline{R}^{k'}\right)}{e+x^{\text{in},k}}\right), \text{ where } x^{\text{in},k}=\sum_{k'}m^{k'}x^{k,k'}/m^k. \tag{42}$$

Here, we define the total inflows into the country (which is normalized by the mass of the region since $x^{k,k'}$ denotes the total flows into the region). An equilibrium with symmetric allocations and prices is a collection, $\left(c_0^k,x^{loc,k},\left\{x^{k',k},y^{k',k}\right\}_{k'}\right)_k,\left(p^k\leq R^k\right)_k,R_f$, such that the allocations solve problem (40), and the market clearing conditions (41) and (42) hold.

To characterize the equilibrium, we make a number of simplifying observations. Without loss of generality, we assume each country retains its safe asset endowment (since there is no aggregate risk, safe assets and foreign investment are perfect substitutes), that is,

$$y^{k',k}=\begin{cases}\eta^k/R_f, & \text{if } k'=k\\ 0, & \text{otherwise}\end{cases}.$$

This also ensures that the market clearing condition for legacy asset holds. It remains to characterize how the investors split their dollars between outside spending, local investment, or investment in other regions, $c_0^k+x^{loc,k}+x^{\text{out},k}=1$, where we define $x^{\text{out},k}=\sum_{k'}x^{k',k}$ as the total amount of outflows from the country.

As before, Lemma 1 implies that absent taxes local investment, $x^{\text{loc},k}$, is weakly dominated by investing in the other countries of the same region, $x^{k,k}$, and strictly so if there are local fire sales, $p^k<R^k$. Hence, whenever there are no taxes, we can focus on equilibria in which $x^{\text{loc},k}=0$ without loss of generality.[13]

[13]To see that this is without loss of generality, note that the local investment can be feasible only if $p^k=R^k$.

Next note that, by problem (40), the optimality condition for investment in region k (by any region k') implies,

$$R_f \geq \overline{R}^k, \text{ with equality if } x^{\text{in},k} > 0 \text{ (equivalently, } x^{k,k'} > 0 \text{ for some } k'). \tag{43}$$

The return in a region cannot exceed R_f since legacy assets are held in positive quantities in equilibrium. Moreover, the return is exactly equated to R_f as long as the country receives inflows from some other country. Combining these observations, the optimality condition for outflows from a country can be written as,

$$u'\left(1 - x^{\text{out},k}\right) = \begin{cases} \overline{R}^k M^k = \mu^k\left(p_k\right), & \text{if } x^{\text{in},k} > 0 \\ R_f M^k \geq \mu^k\left(p_k\right), & \text{if } x^{\text{in},k} = 0 \end{cases}. \tag{44}$$

Hence, for regions that experience inflows, the amount of total foreign investment is determined by the same equation as before [cf. Eq. (6)]. For regions that do not experience inflows, the total investment is greater than before and determined by the (higher) asset returns in other regions. This illustrates a key feature of the present setup: agents invest in other countries not only because this helps them to arbitrage the local fire sales but also because doing so might enable them to obtain greater returns than what they could obtain in their own region.

Finally, using these observations, the market clearing condition (42) can be rewritten as,

$$p^k = \min\left(R^k, \frac{\eta^k + x^{\text{out},k} R_f}{e + x^{\text{in},k}}\right). \tag{45}$$

In view of the aggregate resource constraint, the total outflows and the inflows satisfy the conservation equation,

$$\sum_k m^k x^{\text{in},k} = \sum_k m^k x^{\text{out},k}. \tag{46}$$

The equilibrium is then characterized by a collection of inflows into and outflows from the representative countries within regions, $\left(x^{\text{in},k}, x^{\text{out},k}\right)_k$, and prices $\left(p_k\right)_k, R_f$, that solve Eqs. $(43-46)$. Note that there are $3\,|K|+1$ equations in $3\,|K|+1$ unknowns (although some of the equations take the form of complementary slackness).

6.1. Optimal Policy with Reach for Safety

We next consider a special case of the model to analyze the policy implications for the reach for safety. We assume $\pi = 1$ so that the liquidity shocks happen with certainty. This ensures that risky capital flows do not provide any liquidity, shutting down the benefits we identified in Section 5 and enabling us to focus on the policy implications that are purely driven by

In this case, it can be checked that if there is an equilibrium with $x^{loc,k} > 0$, then there is also an equilbrium with $\tilde{x}^{loc,k} = 0$ and $\tilde{x}^{k,k} = x^{k,k} + x^{loc,k}$: that is, the local investment can be substituted for investment in the other countries of the same region without changing any of the equilibrium conditions.

reach-for-safety.

For concreteness, we also assume there are two regions, $k \in \{D, E\}$, where $k = D$ corresponds to developed financial markets and $k = E$ corresponds to emerging markets. We assume region D has sufficient liquidity that it would avoid fire sales in autarky (similar to Assumption S in Section 3.2). We also assume that there is a worldwide scarcity of liquidity, that is, we modify Assumption 1 as follows.

Assumption 1S. $\eta^D > eR^D > \eta^E$, and $\eta^D m^D + \eta^E m^E < e \min\left(R^D, R^E\right)$. In addition, $\underline{x}^{\text{out},E} \geq e\left(1 - \frac{\eta^E}{\eta^D m^D + \eta^E m^E}\right)$.

Note that the autarky prices in regions D and E are then, respectively, given by R^D and $\eta^E/e < R^E$. In the last part of the assumption, $\underline{x}^{\text{out},E}$ denotes the minimum level of outflows form region E, characterized as the solution to, $u'\left(1 - \underline{x}^{\text{out},E}\right) = R^E$. The assumption does not play an important role beyond simplifying the analysis (by ensuring that there is an equilibrium with positive flows into both regions).[14]

With these assumptions, the characterization of equilibrium is relatively simple. In the appendix, we obtain a closed form solution (to Eqs. $(43-46)$) that satisfies,

$$
\begin{aligned}
p^D = p^E = \frac{\eta^D m^D + \eta^E m^E}{e} &< \min\left(R^D, R^E\right), \qquad (47)\\
x^{\text{in},k} - x^{\text{out},k} = e\left(\frac{\eta^k}{\eta^D m^D + \eta^E m^E} - 1\right) &\text{ for each } k \in \{D, E\}.
\end{aligned}
$$

These equations generalize the reach for safety result in Section 3.2. The first equation shows that, with free financial flows, region D also experiences fire sales, even though it would not feature fire sales in autarky. The second equation illustrates that this outcome obtains because region D receives more inflows relative to its outflows, $x^{\text{in},D} - x^{\text{out},D} > 0$ (that is, it is running a current account deficit).

Conversely, since $\frac{\eta^D m^D + \eta^E m^E}{e} > \frac{\eta^E}{e}$, the first equation in (47) shows that financial flows improve the fire-sale prices in region E. The second equation illustrates that this outcome obtains because region E has more outflows than its inflows, $x^{\text{out},E} - x^{\text{in},E} > 0$ (that is, it is running a current account surplus).

These observations suggest that capital restrictions in this setting will have costs as well as benefits. To investigate further, suppose the planner in each country j can impose a linear tax, τ^j, on inflows. We assume that the planner injects the tax receipts back into different regions (via equal-weighted asset purchases in that region) according to the fraction of investment in the country that comes from each region. For instance, if the fraction, $\alpha \in (0, 1)$, of the inflows into the country, $x^{\text{in},j}$, come from region E, then the planner injects $\alpha x^{\text{in},j} \overline{R}^j \tau^j$ into region E and $(1-\alpha)\, x^{\text{in},j} \overline{R}^j \tau^j$ into region D. This ensures that taxation does not affect the liquidity in

[14] Note that we allow the regions to have potentially different returns. This assumption does not affect the analysis in this section since $\pi = 0$ and thus foreigners in one region cannot realize the higher return in the other region.

either region, ensuring continuity with our earlier analysis.

Consider the equilibrium with symmetric taxes within each region, $\left\{\tau^k \geq 0\right\}_{k\in\{D,E\}}$. In the appendix, we show that taxes in region E do not affect the equilibrium prices. In particular, we can take $\tau^E = 0$ without loss of generality. The taxes in region D, however, affect the equilibrium. When τ^D is sufficiently small, the equilibrium prices now satisfy the indifference condition, $p^E = p^D\left(1-\tau^D\right)$. The appendix completes the analysis and shows that the prices have a closed form solution,

$$p^D = \frac{\eta^D m^D + \eta^E m^E / \left(1-\tau^D\right)}{e} \text{ and } p^E = \frac{\eta^D m^D \left(1-\tau^D\right) + \eta^E m^E}{e}. \tag{48}$$

In particular, increasing τ^D increases the fire-sale price in region D at the expense of reducing the fire-sale price in region E. Hence, the optimal tax for the global planner is ambiguous as it depends on the relative cost of fire sales in respective regions. Our model does not help to resolve this ambiguity since we capture the relative cost of fire sales in reduced form using the functions $f^D(p), f^E(p)$ (which might in principle differ across the regions). The following result summarizes this discussion.

Proposition 13. *Consider the asymmetric model with developed and emerging market regions that satisfy Assumption 1S. When $\tau^E = \tau^D = 0$, the equilibrium prices and the flows satisfy* (47). *Increasing τ^E does not affect the equilibrium prices. Starting with zero taxes, increasing the tax in the developed region τ^D increases the fire-sale price in this region, p^D, and decreases the fire-sale price in the emerging market region, p^E.*

6.2. Optimal Policy with Reach for Yield

We next consider another special case of the model with asymmetric regions to analyze the policy implications for the reach for yield. As in the previous case, we consider two regions, $k \in \{D, E\}$. We depart from the previous case by assuming $\pi < 1$. This enables for flows to be driven at least in part by the return differentials, but it also makes the analysis less tractable. We make a number of assumptions that bring back analytical tractability. First, we relax the earlier assumptions on $u(\cdot)$ and assume instead that investors receive no utility from outside investment.

Assumption 0. $u(c_0) = 0$ for each c_0.

This assumption ensures that $c_0^k = 0$ and $x^{\text{out},k} = 1$; that is, the outflow from each country is exogenously fixed and equal to one (we could also make this into an exogenous parameter $x \in (0,1)$ by slightly changing the assumption). This helps to drop Eqs. (44) from the equilibrium conditions and replace $x^{\text{out},k}$ in the remaining conditions by 1. The assumption does not play an important role beyond simplifying the analysis, since our goal in this section is to analyze the direction of the global capital flows as opposed to their magnitudes.

Second, we assume that η^D is sufficiently large so that $p^D = R^D$ regardless of the flows in equilibrium: that is, the developed markets have abundant liquidity to prevent fire sales. We also assume η^E is relatively small so that $p^E < R^E$ in any equilibrium with positive inflows into E: that is, the emerging markets have relatively low liquidity and are subject to fire sales. The following assumption specifies the exact parametric conditions.

Assumption 1Y. $\eta^D > \left(e + 1/m^D - 1\right) R^D$, and $\eta^E < eR^E - \max\left(R^E, R^D\right)$.[15]

With these assumptions, we have $x^{\text{out},D} = x^{\text{out},E} = 1$ and $p^D = R^D$. To characterize the rest of the equilibrium, first consider the case in which there are positive flows into both markets, $x^{\text{in},D}, x^{\text{in},E} > 0$. The conditions for this type of equilibrium can be written as,

$$\begin{aligned} R_f &= (1-\pi) R^E + \pi p^E = R^D, \qquad (49) \\ p^E &= \frac{\eta^E + \overline{R}^E}{e + x^{\text{in},E}} = \frac{\eta^E + (1-\pi) R^E}{e + x^{\text{in},E} - \pi}. \\ m^D x^{\text{in},D} &+ m^E x^{\text{in},E} = 1. \end{aligned}$$

Inspecting the second condition, this type of equilibrium features a price level in E that lies in an interval, $p^E \in \left(p^E_{low}, p^E_{high}\right)$, where

$$p^E_{low} \equiv \frac{\eta^E + (1-\pi) R^E}{e + 1/m^E - \pi} \text{ and } p^E_{high} \equiv \frac{\eta^E + (1-\pi) R^E}{e - \pi}.$$

Note also that any price in between can be obtained by adjusting the amount of flows that go into region E and letting the residual flows go into region D. Using this observation in the first equation of (49), we obtain that an equilibrium with positive flows into both markets exists if and only if the return in developed markets lies in an interval, $R^D \in \left(R^D_{low}, R^D_{high}\right)$, where

$$R^D_{low} \equiv (1-\pi) R^E + \pi p^E_{low} \text{ and } R^D_{high} = (1-\pi) R^E + \pi p^E_{high}. \qquad (50)$$

It is then easy to check that the equilibrium takes one of three forms depending on the return in region D. If $R^D \geq R^D_{high}$, then there are zero flows into region E, $x^{\text{in},E} = 0$, and all flows go into region D. If $R^D \leq R^D_{low}$, then there are zero flows into region D, $x^{\text{in},D} = 0$, and all flows go into region E. If $R^D \in \left(R^D_{low}, R^D_{high}\right)$, there are flows in both directions as described above. Note also that $R^D_{high} < R^E$: it takes a strictly lower return in region D than in region E to ensure some flows will go into region E. This is because region E is subject to fire sales, in view of its low liquidity, whereas region D is not.

We next analyze the comparative statics of equilibrium with respect to the return in developed markets, R^D. For simplicity, consider the case with interior flows in both directions, $R^D \in \left(R^D_{low}, R^D_{high}\right)$. As the first equation in (49) illustrates, this leads to a decline in p^E. As

[15] Note that the latter condition also implies $e > 1$, which in turn implies $e > \pi$ (since $1 > \pi$). This observation might help to follow some of the susbequent analysis.

the second equation illustrates, this decline is brought about by an increase in fickle inflows into region E, $x^{\text{in},E}$. Intuitively, a reduction in R^D makes the assets in emerging markets relatively more attractive, which induces more of the global financial flows to flow into this region, generalizing Proposition 3 to this setting. The following result summarizes this discussion.

Proposition 14. *Consider the asymmetric model with developed and emerging market regions. The equilibrium depends on the comparison of the return in the developed region, R^D, with the thresholds R^D_{low}, R^D_{high} characterized by* (50) *that satisfy $R^D_{low} < R^D_{high} < R^E$. If $R^D \leq R^D_{low}$, then $x^{in,D} = 0$. If instead $R^D \geq R^D_{high}$, then $x^{in,E} = 0$. If $R^D \in \left(R^D_{low}, R^D_{high}\right)$, then there are positive flows in each region and the equilibrium is characterized by the system in* (49)*. When $R^D \in \left(R^D_{low}, R^D_{high}\right)$, a decrease in the return in region D, R^D, increases the inflows into region E, $x^{in,E}$, and decreases the fire-sale price in this region, p^E.*

We next analyze the desirability of policies directed toward restricting capital flows. As before, suppose the planner in each country j can impose a linear tax, τ^j, on inflows. The planner injects the taxed liquidity back into the regions in which the flows come from as described in Section 6.1. We also assume that taxation is costly as in Section 5.1.1. Specifically, adopting the tax level τ reduces the return of the entrepreneurs that have linear scale by $v(\tau) \geq 0$, where $v(\cdot)$ is a convex function that satisfies the Inada-type conditions, $v(0) = v'(0) = 0$ and $v'(1) = \infty$. As $\lambda \to \infty$, the planner's objective function is given by (37).

Consider a global planner that can coordinate tax policies across countries and regions. The planner chooses two tax rates, τ^D, τ^E, to be applied in the countries in, respectively, region D and region E. The global planner's problem can be written as,

$$\max_{\tau^D, \tau^E \geq 0} \sum_{k \in \{D,E\}} m^k \left((1-\zeta) \left((1-\pi) f\left(R^k\right) + \pi f\left(p^k\right) \right) + \zeta \overline{R}^k \left(1 - v\left(\tau_k\right)\right) \right). \tag{51}$$

It can be seen that the planner always sets $\tau^D = 0$. This is because there are no fire sales in region D, and taxing the flows into region D does not help to increase the price in region E. However, the planner might want to set a positive tax rate in region E. We next characterize the equilibrium with tax levels, $\tau^D = 0, \tau^E \geq 0$, and analyze the optimal tax rate.

One caveat is that local investment in region E might not be dominated by foreign investment in view of the taxes on foreign flows in this region. In the appendix, we show that this does not happen if we restrict attention to the cases with, $R^D > R^D_{low}$ (so that there is some investment in region D absent taxes) and if the parametric condition in the following assumption holds:

Assumption 3. $R^D > R^D_{low}$ and $\left(1 - \pi + \pi \frac{R^E}{p^{E,\max}}\right) R^D \geq R^E$, where $p^{E,\max} = \frac{\eta^E + R^D}{e}$.

Here, $p^{E,\max}$ is the maximum price level that can obtain in country E when $R^D > R^D_{low}$ (see below). The parametric condition ensures that, even when the price is maximized, investors in

E will invest in foreign assets (as opposed to investing locally) to arbitrage local fire sales. The condition holds as long as R^D exceeds a threshold which is strictly below R^E. Moreover, the threshold can be made arbitrarily small by increasing the forced sales, e (and reducing η^E). We maintain this condition for the rest of the analysis.

With Assumption 3, the characterization of the equilibrium with $\tau^E \geq 0$ parallels the analysis in the previous section. First consider the case with relatively high return in region D, $R^D \geq R^D_{high}$. In this case, the equilibrium without taxes features zero flows into region E, $x^{\text{in},E} = 0$. Increasing the tax level on these flows has no effect on equilibrium (they continue to remain at zero). The planner optimally sets a zero tax level, $\tau^E = 0$.

Next consider the case with lower return in region D, $R^D \in \left(R^D_{low}, R^D_{high}\right)$. In this case, the equilibrium without taxes features positive inflows in both directions. When the planner applies the tax level, $\tau^E \geq 0$, the equilibrium conditions in (49) are modified as (assuming positive inflows in both directions),

$$
\begin{aligned}
R_f &= \left(1-\tau^E\right)\left(\left(1-\pi\right)R^E + \pi p^E\right) = R^D, \\
p^E &= \frac{\eta^E + R^D}{e + x^{\text{in},E}}, \\
m^D x^{\text{in},D} &+ m^E x^{\text{in},E} = 1.
\end{aligned} \tag{52}
$$

As the first equation illustrates, the planner might benefit from setting $\tau^E > 0$. These taxes make the assets in region E relatively unattractive. This in turn induces foreign investors to exit this region, lowering $x^{\text{in},E}$ and increasing p^E. The inflows stop declining when p^E increases sufficiently to leave the foreign investors indifferent.

As the second equation in (52) illustrates, the planner can increase the price up to the level, $p^{E,\max} = \frac{\eta^E + R^D}{e}$, which obtains when all flows exit region E, $x^{\text{in},E} = 0$. We let $\tau^{E,\max} > 0$ denote the tax level that brings about this price level: specifically, $\tau^{E,\max}$ is the solution to

$$
\left(1-\tau^{E,\max}\right)\left(\left(1-\pi\right)R^E + \pi p^{E,\max}\right) = R^D, \text{ where } p^{E,\max} = \frac{\eta^E + R^D}{e}. \tag{53}
$$

Increasing the taxes beyond $\tau^{E,\max}$ does not affect the equilibrium, since it leaves the flows into region E unchanged at zero. Thus, the planner chooses $\tau^E \in \left[0, \tau^{E,\max}\right]$ to maximize the objective function in (51), subject to the condition that the price level solves the first equation in (52). Taking the first order condition, the optimal tax level is given by, $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$, where $\tau^{E,*} \in (0,1)$ is the unique solution to,

$$
V\left(\tau^{E,*}\right) = \frac{\left(1-\zeta\right) f'\left(p^E\right) + \zeta}{\zeta}. \tag{54}
$$

Here, $V\left(\tau^j\right) = v'\left(\tau^j\right)\left(1-\tau^j\right) + v\left(\tau^j\right)$ is a convex function that satisfies the appropriate boundary conditions as in Section 5.1.1. Note that $\tau^E = \tau^{E,\max}$ corresponds to a corner solution in which the planner reduces the inflows to zero, $x^{\text{in},E} = 0$, whereas $\tau^E = \tau^{E,*} < \tau^{E,\max}$ correspond to an interior solution in which the planner leaves some inflows, $x^{\text{in},E} > 0$, due to costly taxation. Note also that $\tau^{E,\max}$ and $\tau^{E,*}$ are both strictly positive, and thus, the optimal tax rate is positive in either case.

It is also instructive to characterize the comparative statics of the optimal tax rate when there is an interior solution, $\tau^E = \tau^{E,*} < \tau^{E,\max}$. As Eq. (54) illustrates, comparative statics that decrease the fire-sale price level, p^E, increase the optimal tax rate, $\tau^{E,*}$. Recall from Proposition 14 that a reduction in R^D decreases p^E. It can then be seen that this also induces the global planner to set a higher tax rate, $\tau^{E,*}$. The following result summarizes this discussion.

Proposition 15. *Consider the asymmetric model with developed and emerging market regions and costly taxation in the limit as $\lambda \to \infty$ (with Assumptions 0, 1Y and 3). Consider a global planner that coordinates countries' policies. The optimal tax rate in the D region is zero, $\tau^D = 0$. The optimal tax rate in the E region is also zero, $\tau^E = 0$, when the return in the D region exceeds a threshold, R^D_{high} (which is strictly below the return in the E region, R^E), but it is strictly positive for lower levels of return, $R^D \in \left(R^D_{low}, R^D_{high}\right)$. If the optimal tax rate is positive and corresponds to an interior solution (more specifically, if $x^{in,E} > 0$ at the optimum tax level), then a decrease in R^D increases the optimal tax rate, τ^E, and reduces the equilibrium price, p^E.*

The result qualifies some of our earlier conclusions about the desirability of capital taxes (e.g., Proposition 8). Specifically, in an environment with asymmetric returns and liquidity needs, taxing capital flows might be justified even for a global planner. Intuitively, the flows from region D into region E are driven by the pursuit of higher returns in this region—as opposed to the liquidity needs in region D. These flows exacerbate the fire sales in region E without providing financial stability benefits elsewhere. The global planner optimally applies capital taxes in region E so as to discourage these types of destabilizing flows.

Note, however, that the presence of asymmetries per se is not sufficient to justify capital taxes. The result shows that taxes are positive only if the return in region D is sufficiently below the return in region E (since $R^D_{high} < R^E$). Capital taxes are justified but only if the reach-for-yield phenomenon is sufficiently strong to generate substantial flows into regions that experience fire sales—despite the fact that foreigners make losses during fire sales.

Finally, the comparative statics in the result suggest that—when a positive tax level is justified—the strength of the optimal intervention also depends on the strength of the reach-for-yield phenomenon. A further reduction in returns in region D strengthens the reach for yield and induce greater destabilizing flows. When this happens, the global planner optimally increases the tax level in region E so as to lean against these destabilizing flows. Likewise, although we do not analyze optimal policy with aggregate shocks, our earlier analysis (specifically, Proposition 7) suggests a "risk-on" environment driven by a decrease in correlations would also increase the destabilizing flows and induce a higher optimal tax.

7. Final Remarks

In the core of the paper we selected a configuration of parameters where local and global regulators worry exclusively about financial stability. From this perspective, gross capital flows play three roles in our model: global liquidity creation, reach for safety, and reach for yield. The first role is unambiguously good, the second one is a mixed bag, while the last one is unambiguously bad. The weight of these different roles varies across countries and across global risk and return conditions. However there is a systematic bias among local regulators against capital flows (relative to a benevolent global planner), as the costs associated to reach for safety and, particularly, reach for yield, are felt directly at the local level, while the benefits of global liquidity creation are spread across the world economy.

While actual policymakers do focus on financial stability, it is important to note that there could be additional welfare considerations. To explore some of these, in Appendix A.2 we focus on the polar opposite case in which there are no financial stability concerns (by assuming the entrepreneurs' projects merely break even). In this context, the fire-sale prices do not reduce social welfare—as they are merely transfers among the agents. The global planner is not concerned with fire sales, and she discourages liquidity creation via foreign investment. However, investors continue to undertake foreign investment, so as to exploit and profit from the local fire sales. Hence, absent financial stability concerns, the model features too much liquidity creation and too much foreign investment (similar to Hart and Zingales (2011)). In addition, this version of the model features a different type of coordination problem among planners. While the global planner dislikes foreign flows, local regulators encourage foreign inflows into their countries, as they realize that some of these investments will be appropriated by the local investors (who will purchase them at fire-sale prices). This captures the broader notion that, absent concerns with financial stability and fire sales, the countries would actually welcome foreign flows as some of the returns from foreign investment would accrue to the locals.

There are many other important topics in the capital flow taxation debate that we omitted from our analysis. Perhaps the most significant one is the differentiation of the kinds of capital flows (e.g., equity vs fixed income, short term vs long term). While our model is not designed to address these issues directly, there are insights that carry over to that discussion. The key mechanism by which fickle capital flows generate liquidity in our model is the gap between the return received by local investors on their diversified international portfolios and the fire sale returns received by fickle foreign investors withdrawing their funds from local turmoil. However, if capital inflows take the form of short term debt, then the fire sale and return-gap is limited and so is the liquidity service of these flows. Hence, a global planner that coordinates policies might discourage the short-term flows more than longer-term flows. We leave an exploration of these issues for future work.

Similarly, while in our model all foreign investors are fickle, in practice some foreign investors are not (conversely, some local investors are fickle). Our model can accommodate this extension naturally, at least in the positive economics sections, as our concept of local is just that of an investor that has enough expertise in a market to attempt to arbitrage domestic fire sales rather than running away from them. Of course, the nationality of such investor has practical implications for the mechanism used to tax and identify fickle capital flows.

Another strong assumption we made is that investors have rational and thus common beliefs at the ex-ante investment stage (although we motivated the ex-post fickleness of foreigners with an unmodeled belief friction). Introducing belief disagreements would generate a tension between speculation and risk sharing, similar to Simsek (2013), that would qualify some of our conclusions. In particular, an investor who is relatively optimistic about a foreign market can invest there even though she does not need liquidity and is not an expert in the foreign market. These speculative flows would be destabilizing for the foreign market without providing insurance benefits elsewhere—just like the reach-for-yield flows in our environment. In fact, the speculative flows can also be categorized as reaching for yield as they are driven by high perceived returns in the minds of the investors. We thus conjecture that heterogeneous beliefs would strengthen the reach for yield channel and create a stronger rationale for taxing capital flows (even if the planner respects the investors' heterogeneous beliefs, since the rationale for taxation would be driven by fire-sale externalities).

There are two other extensions that we leave for future work. The first one is to add an investment margin at date 0 to entrepreneurs. In this case capital inflows at date 0 may increase the size of the illiquid assets and the potential fire sales, but also allow for a larger domestic investment. In fact, the single-country literature typically focuses on this particular trade-off, which serves to highlight that our mechanisms and externalities are distinct from those in the standard capital flow taxation literature. Second, in our model we assume that the intermediaries are competitive and face no capital constraints. In practice some of the most significant global crises stem from shocks to intermediary capital that are correlated with the global cycle, as considered by the stringent macro-stress tests applied to most large banks around the world in the aftermath of the subprime crisis.

References

Allen, F., Gale, D., 1994. Limited market participation and volatility of asset prices. The American Economic Review, 933–955.

Bernanke, B. S., Bertaut, C. C., DeMarco, L., Kamin, S. B., 2011. International capital flows and the return to safe assets in the united states, 2003-2007. FRB International Finance Discussion Paper (1014).

Blanchard, O., Ostry, J., 2012. The multilateral approach to capital controls. VoxEU. org 11.

Bluedorn, M. J. C., Duttagupta, R., Guajardo, J., Topalova, P., 2013. Capital flows are fickle: Anytime, anywhere. No. 13-183. International Monetary Fund.

Borio, C., McCauley, R., McGuire, P., Sushko, V., et al., 2016. Covered interest parity lost: understanding the cross-currency basis. BIS Quarterly Review, 45.

Broner, F., Didier, T., Erce, A., Schmukler, S. L., 2013. Gross capital flows: Dynamics and crises. Journal of Monetary Economics 60 (1), 113–133.

Brunnermeier, M. K., Langfield, S., Pagano, M., Reis, R., Van Nieuwerburgh, S., Vayanos, D., 2016. Esbies: Safety in the tranches.

Brunnermeier, M. K., Sannikov, Y., 2015. International credit flows and pecuniary externalities. American Economic Journal: Macroeconomics 7 (1), 297–338.

Bruno, V., Shin, H. S., 2015. Capital flows and the risk-taking channel of monetary policy. Journal of Monetary Economics 71, 119–132.

Caballero, R. J., 2003. The future of the imf. The American Economic Review 93 (2), 31–38.

Caballero, R. J., Nov. 2006. On the macroeconomics of asset shortages. In: Beyer, A., Reichlin, L. (Eds.), The Role of Money - Money and Monetary Policy in the Twenty-First Century. European Central Bank, pp. 272–283.

Caballero, R. J., Farhi, E., Gourinchas, P.-O., 2016. Safe asset scarcity and aggregate demand. The American Economic Review 106 (5), 513–518.

Caballero, R. J., Farhi, E., Gourinchas, P.-O., et al., 2008. An equilibrium model of "global imbalances" and low interest rates. American Economic Review 98 (1), 358–93.

Caballero, R. J., Krishnamurthy, A., 2004. Smoothing sudden stops. Journal of Economic Theory 119 (1), 104–127.

Caballero, R. J., Krishnamurthy, A., May 2005. Exchange rate volatility and the credit channel in emerging markets: A vertical perspective. International Journal of Central Banking 1 (1), 207–245.

Caballero, R. J., Krishnamurthy, A., 2006. Bubbles and capital flow volatility: Causes and risk management. Journal of monetary Economics 53 (1), 35–53.

Caballero, R. J., Krishnamurthy, A., 2008. Collective risk management in a flight to quality episode. The Journal of Finance 63 (5), 2195–2230.

Caballero, R. J., Lorenzoni, G., 2014. Persistent appreciations and overshooting: A normative analysis. IMF Economic Review 62 (1), 1–47.

Caballero, R. J., Simsek, A., 2013. Fire sales in a model of complexity. The Journal of Finance 68 (6), 2549–2587.

Calvo, G. A., 2016. Macroeconomics in Times of Liquidity Crises: Searching for Economic Essentials. The MIT Press.

Calvo, G. A., Leiderman, L., Reinhart, C. M., 1996. Inflows of capital to developing countries in the 1990s. The Journal of Economic Perspectives 10 (2), 123–139.

Cerutti, E., Claessens, S., Puy, M. D., 2015. Push factors and capital flows to emerging markets: Why knowing your lender matters more than fundamentals. No. 15-127. International Monetary Fund.

Chari, A., Stedman, K. D., Lundblad, C., 2016. Taper tantrums: Qe, its aftermath and emerging market capital flows.

Coeurdacier, N., Rey, H., 2013. Home bias in open economy financial macroeconomics. Journal of Economic Literature 51 (1), 63–115.

Coeurdacier, N., Rey, H., Winant, P., 2015. Financial integration and growth in a risky world. Tech. rep., National Bureau of Economic Research.

Colacito, R., Croce, M. M., 2010. The short-and long-run benefits of financial integration. In: American Economic Review, Papers and Proceedings.

Cole, H. L., Obstfeld, M., 1991. Commodity trade and international risk sharing: How much do financial markets matter? Journal of monetary economics 28 (1), 3–24.

Costinot, A., Lorenzoni, G., Werning, I., 2014. A theory of capital controls as dynamic terms-of-trade manipulation. Journal of Political Economy 122 (1), 77–128.

Dow, J., da Costa Werlang, S. R., 1992. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. Econometrica: Journal of the Econometric Society, 197–204.

Forbes, K., Fratzscher, M., Kostka, T., Straub, R., 2016. Bubble thy neighbour: portfolio effects and externalities from capital controls. Journal of International Economics 99, 85–104.

Forbes, K. J., 2016. Capital controls. In: Banking Crises. Springer, pp. 39–43.

Forbes, K. J., Warnock, F. E., 2012. Capital flow waves: Surges, stops, flight, and retrenchment. Journal of International Economics 88 (2), 235–251.

Fratzscher, M., 2012. Capital flows, push versus pull factors and the global financial crisis. Journal of International Economics 88 (2), 341–356.

Gabaix, X., Krishnamurthy, A., Vigneron, O., 2007. Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. The Journal of Finance 62 (2), 557–595.

Giordani, P. E., Ruta, M., Weisfeld, H., Zhu, L., 2014. Capital flow deflection.

Gorton, G., 2016. Safe assets. Annual Review of Economics forthcoming.

Gorton, G., Lewellen, S., Metrick, A., 2012. The safe-asset share. The American Economic Review 102 (3), 101–106.

Gourinchas, P.-O., Rey, H., 2007. From world banker to world venture capitalist: Us external adjustment and the exorbitant privilege. In: G7 Current Account Imbalances: Sustainability and Adjustment. University of Chicago Press, pp. 11–66.

Gourinchas, P.-O., Rey, H., Govillot, N., et al., 2010. Exorbitant privilege and exorbitant duty. Tech. rep., Institute for Monetary and Economic Studies, Bank of Japan.

Gromb, D., Vayanos, D., 2016. Arbitrage and Its Limits. Palgrave Macmillan UK, London, pp. 1–3. URL http://dx.doi.org/10.1007/978-1-349-94848-2$_6$64 − 1

Grubel, H. G., 1968. Internationally diversified portfolios: welfare gains and capital flows. The American Economic Review 58 (5), 1299–1314.

Haldane, A. G., 2013. Rethinking the financial network. In: Fragile stabilität–stabile fragilität. Springer, pp. 243–278.

Holmström, B., Tirole, J., 1998. Private and public supply of liquidity. The Journal of Political Economy 106 (1), 1–40.

Holmström, B., Tirole, J., 2001. Lapm: A liquidity-based asset pricing model. the Journal of Finance 56 (5), 1837–1867.

IMF, 2012. The liberalization and management of capital flows: An institutional view. Tech. rep., IMF Staff Review.

Jeanne, O., 2014. Macroprudential policies in a global perspective. Tech. rep., National Bureau of Economic Research.

Klein, M. W., 2012. Switzerland may find itself in a currency war. Brookings Op-Ed.

Korinek, A., 2010. Regulating capital flows to emerging markets: An externality view. Available at SSRN 1330897.

Korinek, A., 2012. Capital controls and currency wars.

Krishnamurthy, A., 2010. Amplification mechanisms in liquidity crises. American Economic Journal: Macroeconomics 2 (3), 1–30.

Krishnamurthy, A., Vissing-Jorgensen, A., 2012. The aggregate demand for treasury debt. Journal of Political Economy 120 (2), 233–267.

Landau, J., 2011. Global liquidity-concept, measurement and policy implications. CGFS Papers 45.

Lane, P. R., Milesi-Ferretti, G. M., 2012. External adjustment and the global crisis. Journal of International Economics 88 (2), 252–265.

Lewis, K. K., 2000. Why do stocks and consumption imply such different gains from international risk sharing? Journal of international Economics 52 (1), 1–35.

Lewis, K. K., Liu, E. X., 2012. International consumption risk is shared after all: an asset return view. Tech. rep., National Bureau of Economic Research.

Lorenzoni, G., 2008. Inefficient credit booms. The Review of Economic Studies 75 (3), 809–833.

Milesi-Ferretti, G.-M., Tille, C., 2011. The great retrenchment: international capital flows during the global financial crisis. Economic Policy 26 (66), 289–346.

Miranda-Agrippino, S., Rey, H., 2015. World asset markets and the global financial cycle. Tech. rep., National Bureau of Economic Research.

Obstfeld, M., 2009. International finance and growth in developing countries: What have we learned? IMF staff papers 56 (1), 63–111.

Ostry, M. J. D., Ghosh, M. A. R., Korinek, M. A., 2012. Multilateral aspects of managing the capital account. International Monetary Fund.

Rey, H., 2016. International channels of transmission of monetary policy and the mundellian trilemma. IMF Economic Review 64 (1), 6–35.

Scott, A., Uhlig, H., 1999. Fickle investors: An impediment to growth? European Economic Review 43 (7), 1345–1370.

Shleifer, A., Vishny, R. W., 1997. The limits of arbitrage. The Journal of Finance 52 (1), 35–55.

Simsek, A., 2013. Speculation and risk sharing with new financial assets. The Quarterly Journal of Economics, qjt007.

Van Wincoop, E., 1994. Welfare gains from international risksharing. Journal of Monetary Economics 34 (2), 175–200.

Van Wincoop, E., 1999. How big are potential welfare gains from international risksharing? Journal of International Economics 47 (1), 109–135.

World Bank, 2013. Global development horizons - Capital for the future: Saving and investment in an interdependent world.

Appendix A: Extensions

A.1. Insurance for Local Liquidity Shocks

In the baseline model, we assumed the investors cannot trade financial contracts contingent on the realization of idiosyncratic risks. In this section, we relax this assumption by introducing intermediaries that sell contingent contracts.

Consider the model in Section 3 with the difference that investors can also purchase insurance with respect to idiosyncratic liquidity shocks. Specifically, there is an insurance contract that pays 1 dollar if the country has a crisis at date 1. The contract is traded at date 0, and it costs f dollars (the fee/or the premium) to be paid at date 1. Hence, the net payoff from the contract is $1-f$ dollars if there is a crisis and $-f$ dollars if there is no crisis. We assume the insurance market is competitive, which implies that the insurance is actuarially fair. In a symmetric equilibrium, the fee is equal to the probability of the liquidity shock, $f=\pi$.

We also require the investor to hold sufficient liquid assets at date 1 to back up her insurance premiums. Specifically, letting z^j denote the amount of insurance country j purchases, we require $x^j R + y^j R_f \geq z^j f$. The investor's problem (1) is then modified as,

$$\max_{\tilde{c}_0 . \tilde{x}^{loc}, \left[\tilde{x}^{j'}\right]_{j'}, \tilde{y} \geq 0} u\left(\tilde{c}_0\right) + \tilde{x}^{loc} R + \left[\begin{array}{c} (1-\pi)\left(\tilde{x} R + \tilde{y} R_f - \tilde{z} f\right) \\ +\pi\left(\tilde{x} p + \tilde{y} R_f + \tilde{z}(1-f)\right)(R/p) \end{array}\right],$$

$$\tilde{c}_0 + \tilde{x}^{loc} + \tilde{x} + \tilde{y} = \eta/R_f + 1 \text{ where } \tilde{x} = \int x^{j'} dj' \text{ and } \tilde{z} f \leq \tilde{x} R + \tilde{y} R_f.$$

A symmetric-price equilibrium is defined as in Section 3, with the additional condition that the insurance contracts break even, $f=\pi$.

As before, we focus on equilibria that feature symmetric allocations. We conjecture that under an appropriate parametric assumption (that we specify below), the equilibrium features fire sales, $p<R$. In this equilibrium, the investor's net return from the insurance purchase is given by,

$$-(1-\pi) + \pi(1-f)(R/p) > 0.$$

Here, the inequality follows from $f=\pi$ and $p<R$. Hence, the investors purchase the maximum amount of insurance, $z = \left(xR + yR_f\right)/f$. As before, we also have $y = \eta/R_f, x^{\mathrm{loc}} = 0$, and $c_0 = 1-x$. It remains to characterize the amount of foreign investment, x.

To this end, first consider the return to foreign investment. Note that one dollar of foreign investment enables the investor to purchase R/π units of insurance. This induces the investor to pay $R/\pi \times f = R$ dollars when there is no crisis, and receive $R/\pi - R$ dollars when there is a crisis. Recall also that the foreign asset has a direct payoff during a crisis given by p. Combining these observations, the return from foreign investment in this setting is given by,

$$\pi\left(p + R\left(1/\pi - 1\right)\right) R/p = \overline{R} R/p, \tag{A.55}$$

where $\overline{R} = (\pi p + (1-\pi) R)$ as before. Note that this expression is greater than the return in the baseline setting, $\mu(p) = \overline{R} M$ (since $R/p > M = 1 - \pi + \pi R/p$). Intuitively, the insurance market enables the investors to transfer their payoffs in the no crisis states to the crisis states, which makes foreign investment more valuable. The amount of foreign investment is determined by the condition,

$$u'(1-x) = \overline{R}R/p. \tag{A.56}$$

As before, this describes a decreasing relation between x and p.

Next note that he asset market clearing condition can also be written as,

$$p = \min\left(R, \frac{\eta + (p + R(1/\pi - 1))x}{e + x}\right) = \min\left(R, \frac{\eta + xR(1/\pi - 1)}{e}\right). \tag{A.57}$$

As before, this describes a decreasing relation between x and p.

The equilibrium is the intersection of Eqs. $(A.56)$ and $(A.57)$. The following strengthening of Assumption 1 ensures that there is an equilibrium with fire sales, $p < R$.

Assumption 1^{I}. $\pi > \underline{\pi} \equiv \frac{\underline{x}R}{\underline{x}R + eR - \eta}$.

Finally, we characterize the risk-free return, R_f. Like the foreign investment, the investor uses the payoff from the risk-free asset in the no crisis state to purchase insurance. A similar analysis as above then implies that the return from risk-free investment is given by $R_f R/p$. Equating this with the return in foreign investment in $(A.55)$ gives $R_f = \overline{R}$. In particular, condition (5) in the baseline model continues to apply in this setting.

Next consider how the presence of the insurance market affects the baseline analysis. Comparing Eqs. $(A.56 - A.57)$ with $(6 - 8)$ in the baseline setting, note that both curves (in the $x - p$ space) are shifted upwards in this setting. It follows that the presence of the insurance market increases the fire-sale price, p. Intuitively, as captured by Eq. $(A.57)$, the insurance market transfers the excess liquidity in the countries that do not experience crises to countries with crises. As captured by Eq. $(A.56)$, the insurance market (by utilizing the foreign liquidity more effectively) also induces the local investors to undertake greater foreign investment conditional on a given level of fire-sales. Both effects increase the fire-sale price in equilibrium.

The above analysis also illustrates that there will be fire sales—despite the presence of insurance markets—as long as the liquidity shocks are sufficiently frequent. Moreover, when $p < R$, the qualitative features of the equilibrium are very similar to the baseline setting. For instance, an increase in local liquidity, η, increases p and decreases x as in Proposition 1. Likewise, when regulators tax capital flows without costs, the global regulator sets zero tax as in Proposition 8, but the local regulators in an uncoordinated equilibrium set prohibitively high taxes and implement $x = 0$ as in Proposition 9.

A.2. Welfare Analysis with Weaker Financial Stability Concerns

Our welfare analysis in the main text focused on the special case in which $\lambda \to \infty$ so that financial stability concerns dominated all other concerns. In this appendix, we investigate the case with finite λ and illustrate the other forces at play. To keep the analysis simple, we focus on the baseline model analyzed in Section 3. Recall that the social welfare in a country is given by,

$$\begin{aligned} W^j &= u(c_0) + E[c_1 + c_2] + \lambda E[\tilde{c}_2] \qquad \text{(A.58)} \\ &= u(c_0) + Rx^{\text{loc},j} + \left(\int_{j'} x^{j',j}\overline{R}^{j'} dj' + y^j\right) M^j + \lambda e \left(\begin{array}{c} \zeta\left((1-\pi) f(R) + \pi f\left(p^j\right)\right) \\ +(1-\zeta)\overline{R}^j \end{array}\right), \end{aligned}$$

where $\overline{R}^j = (1-\pi) R + \pi p^j, M^j = 1 - \pi + \pi \frac{R}{p^j}$.

The global welfare is the aggregation of this expression over all countries, $W = \int W^j dj$. We first describe the forces that influence the global welfare, and then turn to the forces that influence the welfare in an individual country.

Determinants of Global Welfare We first start by analyzing the determinants of global welfare. In a symmetric allocation without taxes or other interventions, the global welfare can be simplified further,

$$W = u\left(1 - x^{\text{loc}} - x\right) + \eta + R\left(x^{\text{loc}} + x + e\right) - e\overline{R} + e\lambda \left(\begin{array}{c} \zeta\left((1-\pi) f(R) + \pi f(p)\right) \\ +(1-\zeta)\overline{R} \end{array}\right) \qquad \text{(A.59)}$$

Here, the first term follows from the resource constraints at date 0. The remaining two terms follow from the sum of the resource constraints at dates 1 and 2. At these dates, the investors consume the available resources in the economy plus the expected net profits that are generated by the entrepreneurs' investment, as captured by the last two terms.

To understand the forces that influence welfare, it is useful to analyze a special case with $\zeta = 0$ and $\lambda = 1$. In this case, the entrepreneurs break even from their investments, and the last two terms in $(A.59)$ disappear. As this happens, the fire-sale price, p, also disappears from $(A.59)$. It can further be checked that the world welfare is maximized when the outside spending is at its upper bound, $c_0 = 1 - \underline{x}$, and total investment (local or foreign) is at its lower bound, $x + x^{loc} = \underline{x}$ (recall that $\underline{x}$ solves $u'(1-\underline{x}) = R$). In particular, in the special case with $\zeta = 0$ and $\lambda = 1$, the baseline competitive equilibrium characterized in Section 3 which features $x^{loc} = 0$ and $x > \underline{x}$ is inefficient. Moreover, the equilibrium features *too much foreign investment*, and the global planner would like to reduce the foreign flows—the opposite of what we emphasized in the main text.

Intuitively, the special case $\zeta = 0$ and $\lambda = 1$ captures the opposite situation in which the global planner has no financial stability concerns (increasing the price does not increase the planner's utility since the entrepreneurs break even). Hence, the force that we emphasized in the main text is completely shut down. Instead, another force comes into play and generates too much liquidity creation, which translates into too much foreign investment in this model. Intuitively, investors in a competitive equilibrium have greater incentives to invest in liquid assets (compared to the planner), because they perceive they will make high returns in states with fire sales. The planner without financial stability concerns views these fire sales as harmless transfers among the agents in the economy, and thus, she does not perceive a particularly high return from arbitraging them. Hence, the planning allocation features less liquidity creation and deeper fire sales compared to the competitive equilibrium.[16]

Our goal is to understand the regulation of capital flows in an environment in which the planners are

[16] Technically, in this case, the investors exert fire sale externalities on one another as opposed to the entrepreneurs. By investing one more unit in liquid assets, the investor increases the price and hurts other investors but she does not internalize these effects.

concerned with asset price volatility and fire sales. Therefore, in the main text we abstract away from this counterforce by focusing on cases in which λ is sufficiently large (specifically, the limit as $\lambda \to \infty$) so that the planning allocation features more stable prices relative to the competitive equilibrium.

Determinants of Local Welfare Now consider the determinants of welfare in an individual country. Unlike the global welfare, we cannot simplify this expression much further than in (A.58) without specifying particular policies and characterizing the equilibrium. For concreteness, consider the extension with capital taxes we analyzed in Section 5.1. In particular, suppose all other countries set the tax level, $\tau > 0$, and country j deviates to a tax level, τ^j, in a sufficiently small neighborhood of τ so that the characterization in (36) applies. The resulting welfare in country j can be written as,

$$W^j\left(\tau^j\right) = u\left(1 - x^j\right) + \left(\eta + x^j \overline{R}\right) M^j + e\lambda \begin{pmatrix} \zeta\left(\left(1-\pi\right) f\left(R\right) + \pi f\left(p^j\right)\right) \\ + \left(1-\zeta\right)\overline{R}^j \end{pmatrix},$$

$$\text{where } M^j = 1 - \pi + \pi \frac{R}{p^j} \text{ and } \overline{R}^j = 1 - \pi + \pi p^j.$$

Here, x^j, p^j, as well as $x^{\text{in},j}$ (which does not directly appear in the welfare function) are implicit functions of τ^j as describes by the equation system (36). Taking the first order condition and using the Envelope Theorem, we obtain,

$$\begin{aligned} \frac{dW^j}{d\tau^j} &= \frac{\partial W^j}{\partial p^j}\frac{dp^j}{d\tau^j}, \\ &= \pi\left[\left(\lambda\left(\zeta f'\left(p^j\right) + \left(1-\zeta\right)\right) - 1\right) e - \frac{R}{p^j}\left(e + x^{\text{in},j}\right)\right]\frac{dp^j}{d\tau^j}. \end{aligned}$$

Recall that the term, $\frac{dp^j}{d\tau^j}$, is weakly positive, that is, taxing capital flows increases the fire-sale price. The bracketed term captures the effect of the increase in the price on the welfare in the country. A greater price level yields financial stability benefits, as captured by the first term. However, it also reduces the expected return of the investors, as captured by the second term. In fact, in the special case with $\zeta = 0$ and $\lambda = 1$ (no net financial stability benefits), the the bracketed term is negative. That is, increasing the local price level via taxes reduces the local welfare via a reduction of the investors' welfare. In this special case, the planner that acts in isolation choose lower taxes and encourage greater capital flows—the opposite of what we emphasized in the main text.

Intuitively, a local planner without financial stability concerns would like to increase the inflows into the country because some of the payoffs from these investments are ultimately appropriated by locals (as the foreigners liquidate in case of a liquidity shock). Note, however, that the mechanism by which local investors (with limited liquidity) appropriate greater inflows by foreigners is a reduction in asset prices and a deepening of fire sales. Thus, these beneficial effects would be arguably second order for a planner that has financial stability concerns and dislikes fire sales. Therefore, in the main text we abstract away from this counterforce by focusing on cases in which λ is sufficiently large.

A.3. Private Liquidity Creation

In the main text, we analyzed the planners' incentives to create public liquidity. Instead of creating liquidity directly, the planner might also encourage the private sector to hold more liquid assets. In this section, we analyze this set of policies and show that they have similar implications as public liquidity creation.

Suppose the planner can tax the outside spending/consumption of local investors so as to incentivize them to hold more financial assets. Specifically, suppose spending c_0 dollars on the outside option yields $(1-\tau_c)\,c_0$ dollars of consumption. The utility from outside spending is now given by $u\left((1-\tau_c)\,c_0\right)$. As before, the government wastes the tax revenues it collects, $\tau_c c_0$.

First consider the symmetric coordinated policy, $\tau_c^j = \tau_c$ for each j, that would be chosen by a worldwide planner. The characterization of equilibrium parallels the analysis in Section 3. The main difference is that Eq. (6), is replaced by,

$$\begin{cases} (1-\tau_c)\,u'\left((1-x)\,(1-\tau_c)\right) = \mu\,(p)\,, & \text{if } p < R \\ x \in [0, \underline{x}\,(\tau_c)] & \text{if } p = R \end{cases},$$

where the lower bound on investment, $\underline{x}\,(\tau_c)$, is now an increasing function of the taxes on outside spending. The same steps as earlier imply that there exists a unique equilibrium with $x \in (\underline{x}\,(\tau_c)\,, 1)$ and $p < R$. Moreover, the tax on outside spending increases foreign flows, x, as well as the asset price, p.

By taxing the illiquid outside spending, the planner encourages liquidity creation and mitigates fire sales. Since the safe asset is in scarce supply, global liquidity is created via greater foreign flows in equilibrium. Intuitively, greater flows help to utilize the countries' excess liquidity more effectively. The implication is that a global planner with financial stability concerns ($\lambda \to \infty$) sets prohibitively high level of taxes, $\tau_c = 1$, and creates the maximum amount of private liquidity, $x = 1$.

Next consider optimal private liquidity policy for the planner, τ_c^j, when all other countries set their private liquidity policies at some level, τ_c. When the deviation is in a sufficiently small neighborhood of τ_c, the equilibrium conditions can now be written as,

$$\begin{aligned} R_f = \overline{R} = \overline{R}^j, &\text{ where } \overline{R}^j = (1-\pi)\,R + \pi p^j \\ \left(1-\tau_c^j\right) u'\left(\left(1-x^j\right)\left(1-\tau_c^j\right)\right) = \overline{R} M^j, &\text{ where } M^j = 1 - \pi + \pi \frac{R}{p^j} \\ \text{and } p^j = \min &\left(R, \frac{\eta + x^j \overline{R}}{e + x^{\text{in},j}}\right). \end{aligned}$$

As in the case of public liquidity creation, private liquidity policy does not affect the local fire-sale price, p^j. Intuitively, the private liquidity creation in country j is anticipated and neutralized by financial markets. The implication is the Nash equilibrium features too little private liquidity creation relative to the coordinated solution.

A.4. Aggregate Shocks to Cash Flows

In the main text, we analyzed the effect of aggregate liquidity shocks, π_s. In this section, we analyze other sources of aggregate uncertainty that affect cash flows. Specifically, suppose the payoff from the legacy asset, η_s, as well as the return from new investment, R_s, can now depend on the realization of the state $s \in S = \{1, .., |S|\}$. Throughout, we assume the liquidity shocks are constant across states, $\pi_s = \pi$, so as to focus on shocks to cash flows. We also maintain the following assumptions about cash flows.

Assumption 1$^{\text{C}}$. $\eta_s < eR_s$ for each $s \in S$.

Assumption 2$^{\text{C}}$. There exist variables, $\{\kappa_s > 0\}_s\,, R, \eta > 0$, such that $\eta_s = \eta\kappa_s$ and $R_s = R\kappa_s$ for each $s \in S$. The weights, κ_s, are decreasing in s and satisfy $\sum_s \gamma_s \kappa_s = 1$.

The first assumption a strengthening of Assumption 1 for aggregate uncertainty. The second assumption says that the cash flows from legacy assets and new investment scale proportionally as the aggregate state changes. We view this as a natural starting point. We will discuss the implications of relaxing this assumption at the end of the section. As before, η denotes the expected payoff from the legacy asset (since $E[\eta_s] = \sum_s \gamma_s \kappa_s \eta = \eta$), and R denotes the expected return from new investment (since $E[R_s] = \sum_s \gamma_s \kappa_s R = R$). The assumption that κ_s is decreasing in s captures that greater s corresponds to greater "distress" as in the analysis in the main text.

Note that the legacy asset is no longer risk-free. Hence, we use R_l (as opposed to R_f) denote the expected return on the legacy asset. In particular, the legacy asset is traded at a price η/R_l that will be endogenously determined. Note that $R_l \kappa_s$ denotes the return of the legacy asset conditional on state s, and $R\kappa_s$ denotes the return on new investment conditional on state s.

As in Section 4, the investors can trade financial securities contingent on the aggregate state at date 1 that are provided by competitive intermediaries. The intermediaries' optimality condition is still given by (14), with the modification that the expected payoff is adjusted for the uncertainty about cash flows, $\overline{R}_s^j = R\kappa_s (1-\pi) + p_s^j \pi$. The investors' problem is given by the following analogue of problem (15),

$$\max_{\tilde{c}_0, \tilde{x}^{loc}, \tilde{y}, (\tilde{z}_s \geq 0)_s} u(c_0) + \tilde{x}^{loc} R + \sum_s \gamma_s (\tilde{y} R_l \kappa_s + \tilde{z}_s) M_s^j,$$
$$\tilde{c}_0 + \tilde{x}^{loc} + \tilde{y} + \sum_s q_s \tilde{z}_s = \eta/R_l + 1,$$

where $M_s^j = 1 - \pi + \frac{R\kappa_s}{p_s^j}\pi$. The market clearing conditions are given by the following analogues of (16),

$$\int y^j dj = \eta/R_l$$
$$\int z_s^j dj = \int x^{\text{in},j} \overline{R}_s^j dj \text{ for each } s \in S,$$
$$\text{and } p_s^j = \min\left(R\kappa_s, \frac{R_l \kappa_s y^j + z_s^j}{e + x^{in,j}}\right) \text{ for each } s \in S.$$

Note that the last market clearing condition takes into account the state dependence in the cash flows.

The characterization of the symmetric equilibrium parallels the analysis in the main text. Eqs. $(17-18)$ continue to apply. The main difference concerns the market clearing condition (16). Following similar steps, we now obtain,

$$p_s = \min\left(R\kappa_s, \frac{\eta\kappa_s + x\overline{R}_s}{e+x}\right) = \min\left(R\kappa_s, \frac{\eta\kappa_s + x(1-\pi)R\kappa_s}{e + x(1-\pi)}\right) \text{ for each } s.$$

Hence, we have $p_s = p\kappa_s$, where we define

$$p = \min\left(R, \frac{\eta + x(1-\pi)R}{e + x(1-\pi)}\right).$$

That is, the price scales proportionally with cash flows as the aggregate state changes. More specifically, the price to return ratio, $p_s/(R\kappa_s)$, is constant across states. This also implies that the marginal utility is constant across states, $M_s = M \equiv 1 - \pi + \frac{R}{p}\pi$ for each s. Plugging this into Eq. (18), and using the notation $\overline{R} = (1-\pi) R + \pi p$, we obtain,

$$u'(1-x) = E\left[\overline{R}_s\right] M = \overline{R} M = \mu(p).$$

The last two equations determine the pair, (p, x), from which the rest of the equilibrium can be obtained. Note that these equations are identical to Eqs. (6) and (8) in the baseline setting.

Hence, under Assumptions 1 and 2, introducing aggregate shocks to cash flows leaves the baseline analysis largely unchanged. Intuitively, when the payoffs to legacy and new assets scale proportionally, the liquidity—and thus, the fire-sale price level—scale by the same proportion. Consequently, the investors' marginal utility remains constant across states and the analysis reduces to the setting without aggregate uncertainty.

A.5. Capital Flow Restrictions with Aggregate Shocks

In Section 4, we generalized our baseline model to incorporate aggregate liquidity shocks, which we then used to investigate a number of issues. In this section, we investigate how the presence of aggregate shocks affect the planners' incentives to restrict capital flows. To this end, consider the setup with arbitrary aggregate states, $s \in S$. Suppose the planner in each country j can impose a state-contingent linear tax, $\left\{\tau_s^j \geq 0\right\}_s$, on date 1 payoff from foreign inflows: that is, the return on the inflows (by the intermediaries) in country j is now given by $\overline{R}_s\left(1-\tau_s^j\right)$. As before, the tax revenues are used to purchase an equal-weighted portfolio of all financial assets, which are then wasted by the planner.

Note that we allow the planner to make the tax rate (or more broadly, capital restrictions) contingent on the aggregate state.[17] Our goal is to understand how the optimal tax rate differs across aggregate states, $s \in S$. To this end, we assume taxation is costly as in Section 5.1.1. Specifically, applying the tax rate $\tau_s \geq 0$ on foreign financial flows reduces the return of the entrepreneurs that have linear scale by $v(\tau_s) \geq 0$, where $v(\cdot)$ is a convex function that satisfies the Inada type conditions as before. As $\lambda \to \infty$, the planner effectively maximizes the objective function,

$$\sum_s \gamma_s \left(\begin{array}{c} (1-\zeta)\left((1-\pi_s) f(R) + \pi_s f(p_s)\right) \\ +\zeta \overline{R}_s \left(1 - v(\tau_s)\right) \end{array} \right). \tag{A.60}$$

The equilibrium is defined as before, with the difference that the optimality condition for the intermediaries is now adjusted for the presence of taxes [cf. Eq. (14)],

$$1 \leq \sum_s q_s \overline{R}_s^j \left(1-\tau_s^j\right) \text{ for each } j\text{, with equality if } x^{in,j} > 0.$$

The portfolio problem (15) remains unchanged since the investors are not directly affected by the presence of taxes (they hold financial assets indirectly through intermediaries). The market clearing conditions (16) are adjusted by the presence of taxes and the asset purchases by the government. As before, in a symmetric allocation, the market clearing condition for risky assets will remain unchanged and given by Eq. (19).

[17] We could also allow the planner to condition the tax level on the idiosyncratic state. With the assumptions we made, it can be seen that the planner would not use this conditionality. Intuitively, the taxes only affect the outcomes through the foreigners who only care about the average tax level across idiosyncratic realizations. Hence, conditioning the tax level on the idiosyncratic state would not increase the benefits, but it would increase the costs of taxation since $v(\tau)$ is convex.

To characterize the equilibrium, first consider the symmetric case in which all planners choose the same tax policies, $\tau_s^j = \tau_s$ for each j. In the appendix, we show that there exists $\underline{\tau} > 0$ such that, if $\tau_s \in [0, \underline{\tau})$ for each $s \in S$, then outside spending is below its lower bound, $c_0 < 1 - \underline{x}$, and local investment is dominated in equilibrium, $x^{loc} = 0$ (and thus, foreign investment satisfies $x > \underline{x}$). We assume that $v'(\tilde{\tau}) = \infty$ for some $\tilde{\tau} < \underline{\tau}$ so that the equilibrium always falls in this region. The analogue of Eq. (18) is then given by,

$$u'(1-x) = \sum_s \gamma_s \overline{R}_s M_s (1-\tau_s) \equiv \sum_s \gamma_s \mu_s (p_s) (1-\tau_s). \tag{A.61}$$

The equilibrium is the intersection of Eq. $(A.61)$ and Eqs. (19). Once we solve for $(x, (p_s)_s)$, the asset prices are determined by Eq. (20) as before. It can also be seen that increasing the tax level in any state, τ_s, reduces the capital flows, x, and the fire-sale price level in all states, $(p_s)_{s \in S}$, as well as the risk-free interest rate, R_f. Hence, similar to the earlier analysis, the global planner optimally chooses zero taxes in all states, $\tau_s = 0$ for each $s \in S$.

Next suppose an individual country sets the tax policy, $\left\{\tau_s^j\right\}_{s \in S}$, when all other countries apply the same tax policy, $\{\tau_s\}_{s \in S}$. When $\left\{\tau_s^j\right\}_{s \in S}$ is in a neighborhood of $\{\tau_s\}_{s \in S}$, the equilibrium in country j is characterized by the system of equations $(B.81)$ listed in the proof of Proposition 16 (in the proofs appendix). To characterize the optimal tax policy, it suffices to analyze the following subset of those equations,

$$1 = \sum_s q_s \overline{R}_s^j \left(1 - \tau_s^j\right), \text{ where } \overline{R}_s^j = (1-\pi_s) R + \pi_s p_s^j, \tag{A.62}$$

$$\text{and } q_s/\gamma_s = \frac{M_s^j}{M_0^j} \text{ for each } s \in S, \text{ where } M_s^j = 1 - \pi_s + \pi_s \frac{R}{p_s^j}.$$

Here, $M_0^j = u'\left(1 - x^j\right)$ is a constant independent of state s. Note that the country takes the Arrow-Debreu prices, $(q_s)_s$, as given. Hence, Eq. $(A.62)$ represents $|S| + 1$ equations in $|S| + 1$ unknowns, $\left(p_s^j\right)_s, M_0^j$. After factoring out M_0^j, it can be thought of as $|S|$ equations in the $|S|$ unknown prices. Intuitively, the first equation determines the "weighted average" level for the prices. This equation follows from the foreign investors' optimality condition to invest in the country. The second set of equations determines the relative fire-sales across different states, p_s^j. This equation follows from the local investors' optimality condition to trade financial securities across states. Note also that, when the country sets the same taxes as other countries (which will be the case in Nash equilibrium), the unique solution is the same as the symmetric equilibrium described above, that is, $p_s^j = p_s$ for each s.

Next consider the optimal tax policy for country j. The planner chooses the tax policy, $\left\{\tau_s^j\right\}_{s \in S}$, to maximize the objective function in $(A.60)$ subject to the equilibrium conditions in $(A.62)$. Given the prices $\{q_s, p_s\}_s$, the optimal tax rate is characterized as the solution to the equation system $(A.63)$ in the proof of Proposition 16. In turn, in a symmetric Nash equilibrium, the prices are functions of the symmetric tax policies, $\{\tau_s\}_s$, as described above. The Nash equilibrium is found as the intersection of these two systems. As before, there can also be multiple stable Nash equilibria. The following result summarizes this discussion and establishes the properties of taxes in any Nash equilibrium.

Proposition 16. *Consider the symmetric model with aggregate risk and costly (and state-contingent) capital taxes in the limit as $\lambda \to \infty$. A global planner that coordinates countries' policies sets zero tax in each state, $\tau_s = 0$ for each s. In any Nash equilibrium, the tax rate is positive for each state, $\tau_s > 0$ for* *each $s \in S$. Moreover, the tax rates satisfy,*

$$\frac{v'(\tau_s)}{v'(\tau_{s'})} = \frac{q_s/\gamma_s}{q_{s'}/\gamma_{s'}} \text{ for each } s, s'. \tag{A.63}$$

In particular, the tax rate is increasing in $s \in S$: that is, states with greater probability of liquidity shocks are associated with higher taxes.

The last claim in the proposition follows from Eq. $(A.63)$ after observing that q_s/γ_s, is increasing in s. In turn, Eq. $(A.63)$ follows from an individual planner's optimality condition. The intuition is that foreign investors value payoff in distressed states relatively more. Taxing them in these states provides a cheaper way of discouraging foreign investment at date 0. Hence, the planner applies larger taxes—more protectionism—in states with greater financial distress.

Appendix B: Proofs

Proof of Lemma 1. We have,

$$\begin{aligned}\mu'(p) &= \pi\left(1-\pi+\frac{R}{p}\pi\right) - \pi\frac{R}{p^2}\left((1-\pi)R+\pi p\right)\\ &= \pi(1-\pi)\left(1-\frac{R^2}{p^2}\right).\end{aligned}$$

Hence, $\mu(p)$ is strictly decreasing over the range $p \in (0, R)$. The result follows after observing that $\mu(R) = R$. □

Proof of Lemma 2. We have,

$$\frac{\partial C\left(x, \overline{R}\right)}{\partial x} = \frac{\overline{R} - \frac{\eta + x\overline{R}}{e+x}}{e+x} = \frac{\overline{R} - C\left(x, \overline{R}\right)}{e+x},$$

which implies that $\frac{\partial C\left(x,\overline{R}\right)}{\partial x} > 0$ iff $\overline{R} > C\left(x, \overline{R}\right)$. □

Proof of Proposition 1. Let $P^{opt} : [0, 1) \to [0, R]$ and $P^{mc} : [0, 1] \to [0, R]$ denote the functions that are defined in the main text: that is, $P^{opt}(x)$ corresponds to the optimality condition for foreign investment (6), and $P^{mc}(x)$ corresponds to the market clearing condition (8). Note that $P^{opt}(x)$ is strictly increasing, in view of Lemma 2, and $P^{mc}(x)$ is weakly decreasing, in view of Lemma 1. We also have that $P^{mc}(x) \in (0, R)$ for each x in view of Assumption 1. In addition, we have $\lim_{x\to 1} P^{opt}(x) = 0$ and $P^{opt}(\underline{x}) = R$, where recall that $\underline{x} > 0$ denotes the threshold below which $P^{opt}(x) = R$ and there is some local investment. In view of the boundary conditions, there exists $x \in (\underline{x}, 1)$ and $p \in (0, R)$ such that $p = P^{mc}(x) = P^{opt}(x)$. The pair (x, p) corresponds to the equilibrium.

Next consider the comparative statics. Increasing η strictly increases the curve $P^{mc}(x)$, for each $x \in [0, 1]$, while leaving the curve, $P^{opt}(x)$, unchanged. This increases p and reduces x in equilibrium. Using condition (5), it also increases the risk-free return, R_f. Likewise, increasing R strictly decreases both curves $P^{mc}(x)$ and $P^{opt}(x)$ for each $x \in [0, 1]$. This reduces p as well as R_f.

It remains to show that decreasing R also decreases x. To this end, define the variable $\tilde{p} = p/R$ as the price-to-return ratio. Eqs. (6) and (8) can then be written in terms of $(\tilde{p}, x)$ as,

$$u'(1-x) = R(1-\pi+\pi\tilde{p})\left(1-\pi+\pi\frac{1}{\tilde{p}}\right),$$
$$\text{and } \tilde{p} = \min\left(1, \frac{\eta/R + x(1-\pi)}{e + x(1-\pi)}\right).$$

As before, the first equation describes $\tilde{p}$ as a decreasing function of x, the second function describes $\tilde{p}$ as an increasing function of x, and the equilibrium corresponds to the intersection. Note also that decreasing R strictly decreases the first curve for each x, and strictly increases the second curve for each x. This implies that decreasing R also reduces x, completing the proof. $\square$

Proof of Proposition 2. Most of the proof is provided in the main text. It remains to check that the conjectured allocations, $x^{\text{out},j} = x$ and $x^{\text{in},j} = x + \left(\eta^j - \eta\right)/p$, satisfy the market clearing condition (10). Plugging the expressions for $x^{\text{out},j}$ and $x^{\text{in},j}$ into the market clearing condition, we obtain,

$$\begin{aligned} p^j &= \min\left(R, \frac{p\left(\eta^j + x\overline{R}\right)}{(e+x)p + \eta^j - \eta}\right) \\ &= \max\left(R, \frac{\frac{\eta + x\overline{R}}{e+x}\left(\eta^j + x\overline{R}\right)}{\eta + x\overline{R} + \eta^j - \eta}\right) = \min\left(R, \frac{\eta + x\overline{R}}{e+x}\right) = p. \end{aligned}$$

Here, the second line uses the market clearing condition for the representative country (7). This verifies the conjecture that $p^j = p$. Note also that $x^{\text{in},j} > x^{\text{out},j} = x > 0$, which completes the proof. $\square$

Proof of Proposition 3. Let $p^j = \frac{(1-\pi)\left(R-R^j\right)+\pi p}{\pi}$, which lies in the interval $(0,p)$ in view of Assumption Y. With this price level, the optimality condition (11) holds as equality. Next let, $x^{\text{out},j}$ denote the solution to Eq. (12). Note that $x^{\text{out},j} > x$ since $(1-\pi)R^j + \pi p^j = (1-\pi)R + \pi p$ and $R^j/p^j > R/p$. Let $x^{\text{in},j} = \frac{\eta^j + x^{\text{out},j}\overline{R}}{p^j} - e$, and note that $x^{\text{in},j} > 0$ since $p^j < p, x^{\text{out},j} > x$, and $\eta^j \geq \eta - px$ by Assumption Y. With this level of inflows, the market clearing condition (13) holds. Thus, the constructed tuple, $\left(p^j, x^{\text{in},j}, x^{\text{out},j}\right)$, corresponds to an equilibrium for the country. Next consider a decrease in R. By Proposition 1, this decreases x and p. Since $p^j - p = \frac{(1-\pi)\left(R-R^j\right)}{\pi}$, it also decreases $p^j - p$, which in turn implies that it decreases p^j.This completes the proof. $\square$

Proof of Proposition 4. Most of the proof is provided in the main text. It remains to check that there exists a solution to Eqs. (18) and (19), which satisfies $x \in (\underline{x}, 1)$. To this end, define the function,

$$F(x) = u'(1-x) - \sum_s \gamma_s \mu_s(p_s), \text{ where } p_s = \frac{\eta + x(1-\pi_s)R}{e + x(1-\pi_s)} \text{ for each } s.$$

Note that $F(\underline{x}) = R - \sum_s \gamma_s \mu_s p_s < 0$, and $F(1) = \infty$. Note also that $F(x)$ is strictly increasing in x. By continuity, there exists a unique solution to the equation, $F(x) = 0$, over the range, $x \in (\underline{x}, 1)$. This completes the proof. $\square$

Proof of Proposition 5. The proof is provided in the main text.

Proof of Proposition 6. The equilibrium in the country is determined by the optimality conditions (26) and (28), together with the conditions

$$c_0^j + \sum_s q_s z_s^j + y^j = 1 + \eta^j / R^f. \tag{B.64}$$

$$\text{and } p_s^j = \min\left(R, \frac{R_f y^j + z_s^j}{e + x^{\text{in},j}}\right) \text{ for each } s.$$

Here, the first equation is the the budget constraint at date 0 and the equations in the second line capture the market-clearing conditions in state s of date 1. We conjecture (and verify) that the prices and outside spending is given by

$$p_s^j = p_s \text{ for each } j, \text{ and } c_0^j = c_0, \tag{B.65}$$

and the inflows and the local investor's financial portfolio satisfy respectively,

$$x^{\text{in},j} = l^j (e + x) - e, \tag{B.66}$$

$$\text{and } R_f y^j + z_s^j = l^j \left(\eta + x\overline{R}_s\right) \text{ for each } s. \tag{B.67}$$

Here, we define the leverage ratio as

$$l^j = \frac{\eta^j / R_f + x}{\eta / R_f + x}. \tag{B.68}$$

To verify that these allocations satisfy the equilibrium conditions, note that Eqs. (26) and (28) hold as described in the main text. Next note that Eq. $(B.67)$ determines the investor's portfolio (up to multiplicity that does not affect the total payoffs). In particular, in view of no arbitrage, the date-0 value of the investor's portfolio is given by,

$$y^j + \sum_s q_s z_s^j = l^j (\eta / R_f + x) = \eta^j / R_f + x. \tag{B.69}$$

Combining this expression with $c_0^j = c_0$ implies the budget constraint in $(B.64)$ (since $c_0 = 1 - x$). The market clearing conditions in $(B.64)$ also hold since,

$$p_s^j = \min\left(R, \frac{l^j \left(\eta + x\overline{R}_s\right)}{e + x^{\text{in},j}}\right) = \min\left(R, \frac{\eta + x\overline{R}_s}{e + x}\right) = p_s \text{ for each } s.$$

Here, the first equality uses $(B.67)$, the second equality uses the definition of $x^{\text{in},j}$ in $(B.66)$, and the last equality uses the market clearing condition for the representative country. Hence, the allocations described by Eq. $(B.65)$ and $(B.66 - B.67)$ correspond to the equilibrium in country j.

We next establish the properties of the inflows and outflows in this equilibrium. Note that the date-0 value of the outflows in the country is the same as in the representative country since Eq. $(B.69)$ implies, $x^{\text{out},j} = y^j + \sum_s q_s z_s^j - \eta^j / R_f = x$.Combining this with Eq. $(B.66)$, the difference between the inflows and the outflows is given by,

$$x^{\text{in},j} - x^{\text{out},j} = (e + x) \frac{\left(\eta^j - \eta\right) / R_f}{\eta / R_f + x} > 0. \tag{B.70}$$

In particular, the inflows exceed outflows. Next note that, using $(B.67)$, the date-1 payoff from the outflows is given by,

$$x_s^{\text{out},j} = y^j R_f + z_s^j - \eta^j = \left(l^j \eta - \eta^j\right) + x l^j \overline{R}_s = -x\left(l^j - 1\right) R_f + x l^j \overline{R}_s,$$

which proves (29). Here, the second equality uses Eq. $(B.67)$, and the last equality uses the valuation equation $(B.69)$. Note also that $l^j > 1$ since $\eta^j > \eta$ [cf. Eq. $(B.68)$].

Next consider the special case with correlated shocks described in Section 4.1. Consider an increase in β. As described by Proposition 5, this reduces x and R_f. Since $x^{\text{out},j} = x$, the outflows from country j also decline. Since xR_f declines, Eq. $(B.70)$ implies that $x^{\text{in},j} - x^{\text{out},j}$ increases: that is, the inflows decline less than the outflows. Finally, note that Eq. $(B.68)$ implies

$$l^j = \frac{\eta^j + xR_f}{\eta + xR_f} = 1 + \frac{\eta^j - \eta}{\eta + xR_f}.$$

Since xR_f declines, l^j increases, completing the proof of the proposition. $\square$

Proof of Proposition 7. Under Assumption $\tilde{\text{Y}}$, there is a unique positive solution to Eqs. (30) and (31) that satisfies, $p_2^j \in (0, p_2)$ and $p_3^j \in (0, p_3)$. Next note the optimality condition for outside spending is given by $u'\left(c_0^j\right) = \frac{M_3^j}{q_3/\gamma_3} = \frac{R_3/p_3^j}{q_3/\gamma_3}$. Given p_3^j, there is a unique $c_0^j > 0$ that solves this expression. Since $p_3^j < p_3$, we also obtain $c_0^j < x$ by comparing the equation with its counterpart for the representative country.

We next prove our conjecture that $z_1^j + y^j R_f = 0$. Suppose, to reach a contradiction, that $z_1^j + y^j R_f > 0$. Since $\pi_1 = 0$, we have $M_1 = M_1^j = 1$. Then, deriving the analogue of Eq. (31) for states 1 and 3, we obtain, $1 = \frac{R^j/p_3^j}{R/p_3}$. This yields a contradiction since $R^j > R$ and $p_3^j < p_3$.

Next note that the market clearing constraints and the budget constraint in the country can be respectively written as,

$$p_s^j = \frac{z_s^j + y^j R_f}{e + x^{\text{in},j}} \text{ for each } s \in \{2,3\}, \tag{B.71}$$

$$\sum_{s \in \{2,3\}} q_s \left(z_s^j + y^j R_f\right) = \eta^j / R_f + 1 - c_0^j. \tag{B.72}$$

Note that, for each $s \in \{2,3\}$, the first equation defines $z_s^j + y^j R_f$ as a function of $x^{\text{in},j}$. Plugging this into the second equation, we obtain,

$$\left(e + x^{\text{in},j}\right) \sum_{s \in \{2,3\}} q_s p_s^j = \eta^j / R_f + 1 - c_0^j,$$

Using the same steps for the representative country, we also obtain

$$(e + x) \sum_s q_s p_s = \eta / R_f + 1 - x.$$

Subtracting these equations and using $c_0^j < x$, we obtain,

$$x^{\text{in},j} \sum_{s \in \{2,3\}} q_s p_s^j > \left(\eta^j - \eta\right) \sum_s q_s + x \sum_s q_s p_s.$$

In view of Assumption $\tilde{Y}$, this equation implies $x^{\text{in},j} > 0$. It follows that there exist unique allocations, $\left(c_0^j, \left\{z_s^j + y^j R_f\right\}_{s\in\{2,3\}}, x^{\text{in},j}\right)$, that ensure that the optimality, budget, and the market clearing conditions hold for the prices that solve Eqs. (30) and (31).

We next analyze the comparative statics of the equilibrium in country j with respect to R and β. First consider a decrease in R. Let $\tilde{p}_2 = p_2/R$ denote the price-to-return ratio in state 2. Then, the optimality condition (25) and the market clearing condition (19) can be written in terms of $(\tilde{p}_2, x)$ as,

$$u'(1-x) = R\left(\beta + (1-\beta)(1-\pi+\pi\tilde{p}_2)\left(1-\pi+\pi\frac{1}{\tilde{p}_2}\right)\right),$$
$$\text{and } \tilde{p}_2 = \min\left(1, \frac{\eta/R + x(1-\pi)}{e + x(1-\pi)}\right).$$

As before, the first equation describes $\tilde{p}_2$ as a decreasing function of x, the second equation describes $\tilde{p}_2$ as an increasing function of x, and the equilibrium corresponds to the intersection. Moreover, decreasing R strictly decreases the first curve for each x, and (under Assumption 1) strictly increases the second curve for each x. It follows that decreasing R decreases the equilibrium level of foreign investment, x. Thus, decreasing R also decreases the price level, $p_2 = \min\left(R, \frac{\eta + Rx(1-\pi)}{e + x(1-\pi)}\right)$, while leaving $p_3 = \min\left(R, \frac{\eta}{e}\right)$ unchanged.

Next note that combining Eqs. (32) and (33), we obtain,

$$\frac{\overline{p} - \overline{p}^j}{R^j - R} = \frac{1-\pi}{\pi}\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta) + M_3\beta}. \tag{B.73}$$

This implies that $\frac{\overline{p}-\overline{p}^j}{R^j - R} < \frac{1-\pi}{\pi}$ since $M_1 < M_2 < M_3$. After substituting for M_1, M_2, M_3, the equation can also be written as,

$$\frac{\overline{p} - \overline{p}^j}{R^j - R}\frac{\pi}{1-\pi} = \frac{\beta\frac{1}{R} + (1-\beta)\zeta(R)}{(1-\beta)\zeta(R) + \beta\frac{1}{p_3}}, \text{ where } \xi(R) = (1-\pi)\frac{1}{R} + \pi\frac{1}{p_2}.$$

It can be checked that increasing $\xi(R)$ increases the right hand side (since it is less than one). It follows that decreasing R increases $\frac{\overline{p}-\overline{p}^j}{R^j-R}$, both directly via the $1/R$ term in the numerator, and indirectly by decreasing $\xi(R) = (1-\pi)\frac{1}{R} + \pi\frac{1}{p_2}$. It follows that decreasing in R decreases $\overline{p}^j - \overline{p}$.

Next consider an increase in β. By Proposition 5, this decreases x, which in turn decreases p_2 and leaves p_3 unchanged. Thus, it also increases M_2 and leaves M_1 and M_3 unchanged. Inspecting Eq. $(B.73)$ illustrates that increasing β tends to decrease $\frac{\overline{p}-\overline{p}^j}{R^j-R}$ by increasing the weight on the smaller marginal utility (M_1) in the numerator and by increasing the weight on the larger marginal utility (M_3) in the denominator. However, increasing β also generates an indirect effect since it also increases M_2 (as in the above analysis). As it turns out, the indirect effect tends to increase $\frac{\overline{p}-\overline{p}^j}{R^j-R}$, counteracting the direct effect. We conjecture that the indirect effect does not overturn the direct effect, that is, $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^j}{R^j-R}\right) < 0$, which in turn implies that increasing β increases $\overline{p}^j - \overline{p}$.

To prove this conjecture, we differentiate Eq. $(B.73)$ with respect to β, which implies that $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^j}{R^j-R}\right) < 0$ if and only if,

$$\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta) + M_3\beta} > \frac{M_1 + \frac{d}{d\beta}(M_2(1-\beta))}{M_3 + \frac{d}{d\beta}(M_2(1-\beta))}$$

We make a second conjecture that $\frac{d}{d\beta}(M_2(1-\beta)) < 0$. Under this conjecture, the above inequality holds because,

$$\frac{M_1\beta + M_2(1-\beta)}{M_2(1-\beta)+M_3\beta} > \frac{M_1}{M_3} > \frac{M_1 + \frac{d}{d\beta}(M_2(1-\beta))}{M_3 + \frac{d}{d\beta}(M_2(1-\beta))}.$$

Here, the first equality follows from $M_1 < M_2 < M_3$, and the second inequality uses $M_1 < M_3$ together with $\frac{d}{d\beta}(M_2(1-\beta)) < 0$.

Hence, it remains to prove the second conjecture, $\frac{d}{d\beta}(M_2(1-\beta)) < 0$. To this end, note that Eq. (25) in Section 4.1 implies,

$$u'(1-x) = R\beta + ((1-\pi)R + \pi p_2)(1-\beta)M_2.$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)}{d\beta} < 0$ (since increasing β decreases x), we obtain,

$$R + \pi\frac{dp_2}{d\beta}(1-\beta)M_2 + ((1-\pi)R + \pi p_2)\frac{d}{d\beta}(M_2(1-\beta)) < 0.$$

From here, note that $R + \pi\frac{dp_2}{d\beta}(1-\beta)M_2 > 0$ implies that $\frac{d}{d\beta}(M_2(1-\beta)) < 0$. That is, our second conjecture follows from a third conjecture,

$$(1-\beta)\pi\left(\frac{-dp_2}{d\beta}\right)M_2 < R. \tag{B.74}$$

To prove the third conjecture, note that Eq. (25) can also be written as,

$$\frac{u'(1-x)}{R} = \beta + (1-\beta)\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right).$$

Taking the derivative with respect to β, and using $\frac{du'(1-x)/R}{d\beta} < 0$, we obtain,

$$(1-\beta)\pi\left(\frac{-dp_2}{d\beta}\right)\frac{M_2}{R} < \frac{\left(1-\pi+\pi\frac{p_2}{R}\right)\left(1-\pi+\pi\frac{R}{p_2}\right)-1}{\frac{1-\pi+\pi\frac{p_2}{R}}{(1-\pi)\frac{1}{R}+\pi\frac{1}{p_2}}\frac{R}{p_2^2}-1} < 1$$

Hence, the last inequality follows since it is equivalent to, $\left(1-\pi+\pi\frac{R}{p_2}\right)\left((1-\pi)\frac{1}{R}+\pi\frac{1}{p_2}\right) < \frac{R}{p_2^2}$, which in turn holds since $1-\pi+\pi\frac{R}{p_2} < \frac{R}{p_2}$ and $(1-\pi)\frac{1}{R}+\pi\frac{1}{p_2} < \frac{1}{p_2}$. This establishes the third conjecture in $(B.74)$, which in turn implies $\frac{d}{d\beta}\left(\frac{\overline{p}-\overline{p}^j}{R^j-R}\right) < 0$. This completes the proof of the proposition. $\square$

Proof of Proposition 8. First consider the case in which the equilibrium features $x > 0$ (despite the presence of taxes). Let $P^{opt}(x;\tau)$ correspond to the solution to Eq. (6), which describes the optimality condition for foreign investment. As before $P^{opt}(x;\tau)$ is weakly decreasing with a flat part for $x \leq \underline{x}$ and a strictly decreasing part for $x > \underline{x}$. However, the value of the flat part is slightly different and given by the unique solution to $\mu(p)(1-\tau) = R$ over the range $p \in [0, R]$. Note that the value of the flat part is strictly decreasing in τ. Let $\overline{\tau} \in (0,1)$ denote the tax level such that the equality, $\mu(p)(1-\overline{\tau}) = R$, holds with $p = \eta/e$. For $\tau < \overline{\tau}$, we have $P^{opt}(0;\tau) > \eta/e = P^{mc}(0)$. A similar argument to that in the Proposition 1 then implies that there exists $x \in (0,1)$ and $p \in (0, \overline{p}(\tau)]$ such that $p = P^{mc}(x) = P^{opt}(x;\tau)$. The pair (x,p) corresponds to the equilibrium with taxes $\tau < \overline{\tau}$.

Next let $\underline{\tau}$ denote the tax level such that the equality, $\mu(p)(1-\underline{\tau}) = R$, holds with $p = P^{mc}(\underline{x}) = \frac{\eta+\underline{x}(1-\pi)R}{e+\underline{x}(1-\pi)}$. Note that this threshold is lower than the previous threshold, $\underline{\tau} \in (0, \overline{\tau})$. If $\tau < \underline{\tau}$, then the equilibrium features $x > \underline{x}$, and thus, $x^{loc} = 0$. If instead $\tau \in (\underline{\tau}, \bar{\tau})$, then the equilibrium features $x \in (0, \underline{x})$ and $x^{loc} = \underline{x} - x \in (0, \underline{x})$.

This completes the characterization of the equilibrium for $\tau < \bar{\tau}$. For completeness, consider also the remaining case with $\tau \geq \bar{\tau}$. In this case, we have a corner solution $x = 0$ and $x^{loc} = \underline{x}$. In addition, the first equation in (36) is replaced by $R_f M = R$ (as opposed to $R_f = \overline{R}(1-\tau)$) since the foreign investment is strictly dominated and the legacy asset is priced by equating its marginal utility with that of local investment.

Next consider the comparative statics with respect to taxes. If $\tau \geq \bar{\tau}$, increasing the tax level further has no effect on the equilibrium. Consider the case with $\tau < \bar{\tau}$. Using Eq. (6) and Lemma 1, increasing the tax level shifts the curve $p = P^{opt}(x;\tau)$ downwards. Since the curve $p = P^{mc}(x)$ is strictly increasing and unaffected by the taxes, it follows that increasing the tax level strictly reduces both p and x. It also reduces R_f through the first equation in (36).

Finally, consider the optimal coordinated tax level set by a global planner. The planner's welfare is inversely proportional to the symmetric fire-sale price level in all countries, p. Since increasing the tax level reduces p, the planner optimally sets $\tau = 0$. □

Proof of Proposition 9. First consider the case in which all countries set the tax level, $\tau \in [0, \bar{\tau})$. We prove that there exists a sufficiently small neighborhood of τ such that, when τ^j is in this neighborhood, the equilibrium in country j is characterized as the unique solution to the following system of equations,

$$\begin{aligned} R_f = \overline{R}(1-\tau) = \overline{R}^j\left(1-\tau^j\right), \text{ where } \overline{R}^j = (1-\pi)R + \pi p^j. \\ u'\left(1-x^j\right) = M^j\overline{R}(1-\tau), \text{ where } M^j = 1-\pi+\pi\frac{R}{p^j} \\ \text{and } p^j = \min\left(R, \frac{\eta + x^j\overline{R}(1-\tau) + x\overline{R}\tau}{e + x^{\text{in},j}}\right). \end{aligned} \tag{B.75}$$

To see this, first note that the first equation describes p^j as an implicit function of τ^j. Then note that the second equation describes x^j as an implicit function of the pair, $\left(\tau^j, p^j\right)$. Finally, note that the last equation describes $x^{\text{in},j}$ as an implicit function of τ^j, p^j, x^j. It follows that there exists a sufficiently small neighborhood, $(\tau-\varepsilon, \tau+\varepsilon)$, such that there is a unique solution to the system in $(B.75)$ when $\tau^j \in (\tau-\varepsilon, \tau+\varepsilon)$. Moreover, ε can be taken to be sufficiently small so that $x^j > \underline{x}$ and $x^{\text{in},j} > 0$ (since the equilibrium with symmetric taxes, $\tau^j = \tau$, satisfies $x = x^{\text{in}} > \underline{x}$). When this is the case, the solution corresponds to an equilibrium in country j (since $x^j > \underline{x}$ implies $u'\left(1-x^j\right) > R$ and local investment is dominated as implicitly assumed by $(B.75)$).

Suppose $\tau^j \in (\tau-\varepsilon, \tau+\varepsilon)$ and consider the comparative statics for the equilibrium in country j. Increasing the tax level increases the price, p^j, in view of the first equation in $(B.75)$. It follows that the planner in country j strictly prefers to increase the tax level, τ^j. Thus, the symmetric allocation with $\tau \in [0, \bar{\tau})$ does not correspond to a Nash equilibrium.

Next consider the case in which all countries set the tax level, $\tau = \underline{\tau}$. In this case, the symmetric equilibrium features flows, $\underline{x}$, and the corresponding price level, $P^{mc}(\underline{x})$. We claim that there is no profitable deviation for an individual planner. Note that the tax level cannot be increased further (by assumption). Suppose the planner lowers the tax level to an arbitrary, $\tau^j \in [0, \underline{\tau})$. Suppose τ^j is not too low so that there is a solution to the first equation with $p^j > 0$ (otherwise, the equilibrium features $p^j = 0$, which does not correspond to a profitable deviation). Then, the same argument as above applies and shows that there is a unique solution to the system in $(B.75)$. Moreover, reducing the tax level decreases p^j, increases x^j, and increases $x^{\text{in},j}$. In particular, we have $x^j > \underline{x}$, which ensures that the solution corresponds to an equilibrium in country j. Note also that $p^j < P^{mc}(\underline{x})$, which shows that the deviation is not profitable for the planner in country j. Thus, the symmetric allocation with the tax level, $\tau = \underline{\tau}$, corresponds to a Nash equilibrium.

Finally, let (x, p) denote the equilibrium without taxes and note that $x > \underline{x}$ and $p > P^{mc}(\underline{x})$. This proves that the capital flows and the fire-sale price in the Nash equilibrium are lower than what would obtain in an equilibrium without taxes. By the first equation in (36), the risk-free return is also lower, completing the proof. □

Proof of Proposition 10.

Part (i). The possibility of multiple equilibria is illustrated in the left panel of Figure 3. The example features a discontinuous function $f(\cdot)$, but the multiple equilibria in the figure would remain if we were to approximate $f(\cdot)$ with a smooth function. Next suppose there are multiple symmetric equilibria and consider their welfare ranking. Recall that the system in ($A.62$) describes the equilibrium price as a decreasing function of the tax level. Thus, an equilibrium with lower price level is also associated with a higher tax level. Given the welfare function in (37), this equilibrium is dominated for each planner by an equilibrium with a higher price level and a lower tax level.

Part (ii). Suppose there is a unique equilibrium. The equilibrium is characterized as the intersection of two decreasing curves. Moreover, the intersection is such that the best response curve crosses the equilibrium price curve, $p(\tau)$, from above (as illustrated in the right panel of Figure 3). Inspecting the equilibrium system in (36) shows that decreasing η shifts the equilibrium price curve, $p(\tau)$, downwards. Combining these observations, it follows that reducing η reduces p and increases τ in the unique Nash equilibrium. The risk-free return also declines from the first equation in (36). The last part of the proposition follows by combining the observation that τ increases with the comparative statics of the increase in τ established in Proposition 8. □

Proof of Proposition 11. Let $P^{mc}\left(x; \eta^{pl}\right) = \min\left(R, \frac{\eta + \eta^{pl} + x(1-\pi)R}{e + x(1-\pi)}\right)$ denote the market clearing curve when the planner injects liquidity, η^{pl}. As in the proof of Proposition 1, the equilibrium is characterized as the intersection of the strictly increasing curve, $p = P^{mc}\left(x; \eta^{pl}\right)$, and the decreasing curve, $p = P^{opt}(x)$. Moreover, the intersection is in the strictly decreasing range of $p = P^{opt}(x)$ (with $x > \underline{x}$). Note that increasing η^{pl} shifts the market clearing curve upwards without affecting the optimality curve. Hence, it leads to a higher price, p, and lower capital flows, x. Since the global planner prefers higher prices, she creates the maximum amount of liquidity, $\eta^{pl} = \bar{\eta}$. □

Proof of Proposition 12. Consider the case in which all countries create positive liquidity, $\eta^{pl} > 0$. We prove that, when $\eta^{pl,j}$ is in a sufficiently small neighborhood of η^{pl}, then the equilibrium in country j is characterized as the unique solution to the following system of equations,

$$\begin{gathered} R_f = \overline{R} = \overline{R}^j, \text{ where } \overline{R}^j = (1-\pi) R + \pi p^j \\ \text{and } u'\left(1 - x^j\right) = \overline{R} M^j, \text{ where } M^j = 1 - \pi + \pi \frac{R}{p^j} \\ \text{and } p^j = \min\left(R, \frac{\eta + \eta^{pl,j} + x^j \overline{R}}{e + x^{\text{in},j}}\right). \end{gathered} \tag{B.76}$$

To see this, note that the first equation determines $p^j = p$ as independent of η^j. The second equation determines $x^j = x$ as independent of η^j. The third equation then implies that

$$x^{\text{in},j} - x = \left(\eta^{pl,j} - \eta^{pl}\right)/p.$$

Hence, there exists a sufficiently small neighborhood, $\left(\eta^{pl} - \varepsilon, \eta^{pl} + \varepsilon\right)$, such that there is a unique solution to the system in $(B.76)$ with $x^{\text{in},j} > 0$. The solution corresponds to an equilibrium since $x^j = x > \underline{x}$ (and thus, $u'\left(1 - x^j\right) > R$ and the local investment is dominated as implicitly assumed by the system in $(B.76)$).

Next suppose $\eta^{pl,j} \in \left(\eta^{pl} - \varepsilon, \eta^{pl} + \varepsilon\right)$ and consider the comparative statics for the equilibrium in country j. Decreasing $\eta^{pl,j}$ has no effect on the price. However, for any finite λ, it helps to economize on the liquidity-creation costs (see the social welfare function in (39)). Hence, the symmetric allocation with $\eta^{pl} > 0$ does not correspond to a Nash equilibrium for any finite λ, and thus, also as $\lambda \to \infty$.

Next consider the case in which all countries create zero liquidity, $\eta^{pl} = 0$. We claim that there is no profitable deviation for an individual planner. Suppose the planner deviates to $\eta^{pl,j} > 0$. The same argument as above implies that, for any $\eta^{pl,j} > 0$, the local equilibrium is characterized by $p^j = p, x^j = x$, and $x^{\text{in},j} = x + \eta^{pl,j} > 0$. The deviation is not profitable since it does not change the price but it increases liquidity-creation costs for any finite λ. This proves that the symmetric allocation with $\eta^{pl} = 0$ corresponds to a Nash equilibrium for any finite λ, and thus, also as $\lambda \to \infty$. $\square$

Proof of Proposition 13. First consider the case without taxes. Under Assumption 1S, we conjecture an equilibrium in which $x^{\text{in},D} > 0, x^{\text{in},E} \geq 0$, and conditions (43) and (44) are satisfied as equalities (even at the corner case, $x^{\text{in},E} = 0$). Under this conjecture, combining condition (43) with $\pi = 1$ implies $R_f = p^D = p^E$. Likewise, combining condition (44) with $\pi = 1$ implies,

$$u'\left(1 - x^{\text{out},k}\right) = \overline{R}^k M^k = p^k \frac{R^k}{p^k} = R^k \text{ for each } k.$$

In particular, the foreign outflows from each region are fixed at their minimum levels, $x^{\text{out},k} = \underline{x}^{\text{out},k}$ (defined as the solution to $u'\left(1 - \underline{x}^{\text{out},k}\right) = R^k$).

We next plug $\pi = 1$ into the market clearing conditions (45), and use $p^D = p^E$, to obtain,

$$p^k = \frac{\eta^k + x^{\text{out},k} p^k}{e + x^{\text{in},k}} = \frac{\eta^k}{e + x^{\text{in},k} - x^{\text{out},k}} \text{ for each } k \in \{D, E\}. \tag{B.77}$$

After multiplying these inequalities with m^k/p^k and aggregating, we obtain,

$$m^D\left(e + x^{\text{in},D} - x^{\text{out},D}\right) + m^E\left(e + x^{\text{in},E} - x^{\text{out},E}\right) = \eta^D \frac{m^D}{p^D} + \eta^E \frac{m^E}{p^E}. \tag{B.78}$$

After using $m^D + m^E = 1$ and the conservation equation (46), the left hand side becomes e. Using $p^D = p^E$ on the right hand side, this implies,

$$p^D = p^E = \frac{\eta^D m^D + \eta^E m^E}{e}.$$

Combining this with the market clearing condition in $(B.77)$, we obtain,

$$x^{\text{in},k} - x^{\text{out},k} = e\left(\frac{\eta^k}{\eta^D m^D + \eta^E m^E} - 1\right) \text{ for each } k \in \{D, E\}. \tag{B.79}$$

Since $x^{\text{out},k} = \underline{x}^{\text{out},k}$, this equation determines $x^{\text{in},D}, x^{\text{in},E}$ in terms of the parameters of the problem. Note also that $x^{\text{in},D} > \underline{x}^{\text{out},D} > 0$ since $\eta^D > \eta^E$, and $x^{\text{in},E} < \underline{x}^{\text{out},E}$ since $\eta^E < \eta^D$. In addition $x^{\text{in},E} \geq 0$ since $\underline{x}^E \geq e\left(1 - \frac{\eta^E}{\eta^D m^D + \eta^E m^E}\right)$ by Assumption 1S. This verifies our conjecture and completes the characterization of equilibrium without taxes. In particular, the equilibrium prices and flows satisfy Eqs. (47) in the main text.

It is also useful to note that in equilibrium investors are indifferent between local and foreign investment. The equilibrium characterized above corresponds to zero local investment and $x^{\text{out},k} = \underline{x}^{\text{out},k}$ for each k. However, there are also equilibria with $x^{\text{out},k} \in \left[0, \underline{x}^{\text{out},k}\right]$ for each k. The only requirement is that the outflows from the region E exceed a minimum level, $x^{\text{out},E} \geq e\left(1 - \eta^k / \left(\eta^D m^D + \eta^E m^E\right)\right)$. For each pair, $\left\{x^{\text{out},k}\right\}_{k \in \{D,E\}}$, that satisfies these conditions, there exists an equilibrium in which the inflows are determined by Eq. $(B.79)$ and the prices are determined by Eq. $(B.77)$. In particular, the indeterminacy does not affect the equilibrium prices.

Next consider the equilibrium with taxes. The equilibrium conditions $(43 - 46)$ are slightly modified since investing locally is no longer weakly dominated. First suppose $\tau^E > 0$ and $\tau^D = 0$. In this case, it is easy to check that the equilibrium prices are unchanged. The taxes in region E imply the inflows into region E are zero, $x^{\text{in},E} = 0$, and the outflows from region E (which go into region D) are at their minimum level, $x^{\text{out},E} = e\left(1 - \frac{\eta^k}{\eta^D m^D + \eta^E m^E}\right)$. Hence, the taxes in region E help to partially resolve the indeterminacy described above, but they do not affect the equilibrium prices or net inflows.

Next suppose $\tau^E = 0$ and $\tau^D > 0$, where τ^D is in a sufficiently small neighborhood of 0. We conjecture an equilibrium in which the after-tax returns are equated, $R_f = p^D\left(1 - \tau^D\right) = p^E$. Note that the market clearing conditions $(B.77)$ remain unchanged (since the taxed liquidity is injected back into the investing regions by assumption). The aggregated condition $(B.78)$ also remains unchanged. As before, the left hand side of this equation is equal to e, which implies,

$$\eta^D \frac{m^D}{p^D} + \eta^E \frac{m^E}{p^E} = e.$$

The equilibrium is determined by solving this equation together with $p^E = p^D\left(1 - \tau^D\right)$. The prices have a closed form solution given by Eq. (48) in the main text. To obtain the corresponding flows, first note that combining Eq. (48) with the market clearing conditions $(B.77)$implies,

$$\begin{aligned} x^{\text{in},D} - x^{\text{out},D} &= e\left(\frac{\eta^D}{\eta^D m^D + \eta^E m^E / \left(1 - \tau^D\right)} - 1\right), \\ \text{and } x^{\text{in},E} - x^{\text{out},E} &= e\left(\frac{\eta^E}{\eta^D m^D \left(1 - \tau^D\right) + \eta^E m^E} - 1\right). \end{aligned}$$

Next note that $x^{\text{out},D} = 0$, since investing in other countries of region D is dominated by local investment in region D. Finally, note that $x^{\text{out},E} \geq e\left(1 - \frac{\eta^E}{\eta^D m^D (1-\tau^D) + \eta^E m^E}\right)$ and $x^{\text{out},E} \leq \underline{x}^{\text{out},E}$. Given any choice of $x^{\text{out},E}$ in this interval and $x^{\text{out},D} = 0$, the flows are uniquely pinned down by the above displayed equations.

Finally, consider the case with $\tau^E > 0$ and $\tau^D > 0$, where τ^D is in a sufficiently small neighborhood of 0. The equilibrium prices in this case are exactly as in the previous case. The only difference (as before) is that the taxes in region E imply the inflows into region E are zero, $x^{\text{in},E} = 0$, and the outflows from region E (which go into region D) are at their minimum level, $x^{\text{out},E} = e\left(1 - \frac{\eta^E}{\eta^D m^D (1-\tau^D) + \eta^E m^E}\right)$. This completes the proof. $\square$

Proof of Proposition 14.

Part (i). The characterization for the case $R^D \in \left(R^D_{low}, R^D_{high}\right)$ is provided in the main text. Suppose $R^D \leq R^D_{low}$. Then, the equilibrium system (49) is then replaced by,

$$R_f = (1-\pi) R^E + \pi p^E \geq R^D,$$
$$x^{\text{in},D} = 0, x^{\text{in},E} = 1/m^E \text{ and } p^E = \frac{\eta^E + (1-\pi) R^E}{e + 1/m^E - \pi}.$$

Suppose instead $R^D \geq R^D_{high}$. Then, the equilibrium is system is replaced by,

$$R_f = (1-\pi) R^E + \pi p^E \leq R^D,$$
$$x^{\text{in},D} = 1/m^D, x^{\text{in},E} = 0 \text{ and } p^E = \frac{\eta^E + (1-\pi) R^E}{e - \pi}.$$

In either case, there is a solution to the system in view of the definition of the thresholds R^D_{low}, R^D_{high} in (50).

Part (ii). Suppose $R^D \in \left(R^D_{low}, R^D_{high}\right)$ and consider the system (49). An decrease in R^D decreases p^E via the first equation. This increases $x^{\text{in},E}$ via the second equation. This in turn decreases $x^{\text{in},D}$ via the third equation. □

Proof of Proposition 15. The case $R^D \geq R^D_{high}$ is straightforward since the taxes in region E have no effect on the equilibrium. Consider the case $R^D \in \left(R^D_{low}, R^D_{high}\right)$. Note that this case always features flows into region D (but flows into region E might be driven to zero by taxes in this region). Hence, the local investor's return from investing in other countries is equal to $M^E R^D = \left(1 - \pi + \pi \frac{R^E}{p^E}\right) R^D$. Note also that the maximum price level in this case is given by $p^{E,\max} = \frac{\eta^E + R^D}{e}$ (which obtains when all of the flows exit region E). Hence, as long as the condition in Assumption 3 holds, we have $M^E R^D = \left(1 - \pi + \pi \frac{R^E}{p^E}\right) R^D \geq R^E$.

Thus, local investment in region E is dominated, $x^{loc,E} = 0$. We also continue to assume the local investment in region D is zero, $x^{\text{loc},D} = 0$, which is without loss of generality as before. Using these observations, an equilibrium with flows in both directions is characterized by the system in (52). Note that there is a solution to this system with $x^{\text{in},E} > 0$ as long as $\tau^E \leq \tau^{E,\max}$. Note that Eq. (53) defines $\tau^{E,\max}$ as an implicit function of R^D, which we denote by $T\left(R^D\right)$. Moreover, $T\left(R^D\right)$ is a strictly decreasing function of R^D (since $e/\pi < 1$ in view of Assumption 3), that is, lower returns in region D require higher taxes to eliminate all flows into region E. Finally, note that $R^D = R^D_{high}$ implies $p^E_{high} = p^{E,\max}$, which in turn implies $\tau^{E,\max} = 0$ (since $R^D_{high} = (1-\pi) R^E + \pi p^E_{high}$ by definition). These observations verify that $\tau^{E,\max} > 0$ for each $R^D \in \left(R^D_{low}, R^D_{high}\right)$.

Next note that there is a unique solution to Eq. (54) with $\tau^{E,*} \in (0,1)$ in view of the Inada conditions on the cost function, $v(\cdot)$. The optimal tax satisfies $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$ in view of the concavity of $f(\cdot)$ and the convexity of $v(\cdot)$. This is strictly positive since $\tau^{E,\max}, \tau^{E,*} > 0$. The equilibrium pair, $\left(\tau^{E,}, p^E\right)$, is found by solving the first equation in (52) together with the optimality condition $\tau^E = \min\left(\tau^{E,\max}, \tau^{E,*}\right)$.

Next suppose the optimal tax level takes an interior value, $\tau^E = \tau^{E,*} < \tau^{E,\max}$ (so that $x^{in,E} > 0$). In this case, the first equation in (52) describes p^E as a strictly increasing function τ^E (since higher taxes increase the price level). Condition (54) represents p^E as a strictly decreasing function of τ^E (since lower prices induce greater taxes). The optimal tax level is the intersection of these two curves. A decline in R^D shifts the first (increasing) curve downwards without affecting the second (decreasing) curve. This leads to a greater tax level, τ^E, as well as a lower price level, p^E, completing the proof. $\square$

Proof of Proposition 16. Let $\underline{\tau} \in (0,1)$ denote the unique solution to,

$$u'(1-\underline{x}) = R = \sum_s \gamma_s \mu_s \left(\frac{\eta + \underline{x}(1-\pi_s) R}{e + \underline{x}(1-\pi_s)} \right) (1-\underline{\tau}). \tag{B.80}$$

Suppose $\tau_s < \underline{\tau}$ for each $s \in S$. We prove that the equilibrium is characterized by the unique solution to Eq. (18) and Eqs. (19). To this end, define the function,

$$F\left(x; (\tau_s)_s\right) = u'(1-x) - \sum_s \gamma_s \mu_s (p_s)(1-\tau_s), \text{ where } p_s = \frac{\eta + x(1-\pi_s) R}{e + x(1-\pi_s)} \text{ for each } s.$$

Note that $F\left(\underline{x}; (\tau_s)_s\right) < 0$ in view of Eq. $(B.80)$ and $\tau_s < \underline{\tau}$ for each $s \in S$. Note also that $F\left(1; (\tau_s)_s\right) = \infty$. Since $F\left(x; (\tau_s)_s\right)$ is continuous and strictly increasing in x, there exists a unique solution, $x \in (\underline{x}, 1)$. Since $x > \underline{x}$ (and thus, $u'(1-x) > R$), the solution corresponds to the equilibrium.

Next suppose $\tau_s < \underline{\tau}$ for each $s \in S$ and consider the comparative statics for taxes. Note that increasing τ_s for any s shifts the function, $F\left(x; (\tau_s)_s\right)$, upwards. This reduces the equilibrium foreign investment, x (characterized as the solution to $F\left(x; (\tau_s)_s\right) = 0$). By Eq. (19), this also reduces the fire-sale price level, p_s, in every state. It follows that the global planner sets, $\tau_s = 0$, for each s.

Next consider the Nash equilibrium. Consider an allocation with symmetric taxes, $\tau_s < \tilde{\tau} < \underline{\tau}$, for each s (where recall that $\tilde{\tau}$ is an upper bound on the taxes in view of the assumption that $v'(\tilde{\tau}) = \infty$). Suppose a planner deviates and sets a different tax policy, $\left(\tau_s^j\right)_s$. We let $x^{\text{in},j}$ denote the inflows into the country j, and $x^j = \sum q_s z_s^j$ denote the total outflows from the country. When $\left(\tau_s^j\right)_s$ is in a sufficiently small neighborhood of $(\tau_s)_s$, we conjecture that the equilibrium will feature $x^{\text{in},j} > 0, z_s^j > 0$ for each s, and $x^j > \underline{x}$. The conditions for such an equilibrium can be written as,

$$1 = \sum_s q_s \overline{R}_s^j \left(1-\tau_s^j\right), \text{ where } \overline{R}_s^j = (1-\pi_s) R + \pi_s p_s^j, \tag{B.81}$$

$$\text{and } q_s/\gamma_s = \frac{M_s^j}{u'(1-x^j)} \text{ for each } s, \text{ where } M_s^j = 1 - \pi_s + \pi_s \frac{R}{p_s^j},$$

$$\text{and } p_s^j = \min\left(R, \frac{\eta + z_s^j + x\overline{R}_s \tau_s}{e + x^{\text{in},j}}\right) \text{ for each } s.$$

It can be checked that there exists a sufficiently small neighborhood of $(\tau_s)_s$, denoted by $B\left((\tau_s)_s\right) \subset \mathbb{R}^{|S|}$, such that if $\left(\tau_s^j\right)_s \in B\left((\tau_s)_s\right)$, then there exists a unique solution to this system, $\left(p_s^j, z_s^j, x^{\text{in},j}\right)$, that satisfies the conjecture (since $x^{\text{in}} = x > 0$ and $z_s = x^{\text{in}}\overline{R}_s > 0$ in the symmetric allocation). The solution corresponds to the equilibrium given taxes since $x^j > \underline{x}$ (so that $u'\left(1-x^j\right) > R$ and local investment is dominated as implicitly assumed by the second set of equations). This establishes that, when $\left(\tau_s^j\right)_s \in B\left((\tau_s)_s\right)$, the prices, $\left(p_s^j\right)_s$, are determined as the unique solution to the reduced system $(A.62)$ in the main text.

Next suppose the planner solves the constrained optimization problem,

$$\max_{\left\{\tau_s^j\right\}_s \in B\left((\tau_s)_s\right)} \sum_s \gamma_s \left((1-\zeta)\left((1-\pi_s) f(R) + \pi_s f\left(p_s^j\right)\right) + \zeta \overline{R}_s^j \left(1 - v\left(\tau_s^j\right)\right) \right).$$

The taxes can be optimal for the planner only if the first order conditions for this problem are satisfied so that there is no profitable deviation within the neighborhood. Taking the first order condition with respect to τ_s^j, we obtain,

$$\zeta \gamma_s \overline{R}_s^j v'\left(\tau_s^j\right) = \sum_{\tilde{s}} \gamma_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta \left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} \text{ for each } s.$$

Differentiating the first equation in $(A.62)$ with respect to τ_s^j, we also obtain,

$$q_s \overline{R}_s^j = \sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} \text{ for each } s.$$

Differentiating the second equation in $(A.62)$ for state $\tilde{s} \in S$ with respect to τ_s^j, we obtain,

$$\pi_{\tilde{s}} \frac{dp_{\tilde{s}}^j}{d\tau_s^j} = -\frac{dM_0^j}{d\tau_s^j} \frac{q_{\tilde{s}}}{\gamma_{\tilde{s}}} \frac{\left(p_{\tilde{s}}^j\right)^2}{R} \text{ for each } \tilde{s}.$$

Plugging the last equation into the previous two equations, we obtain,

$$\begin{aligned}
\zeta \gamma_s \overline{R}_s^j v'\left(\tau_s^j\right) &= -\frac{dM_0^j}{d\tau_s^j} \sum_{\tilde{s}} q_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta \left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R}, \\
\text{and } q_s \overline{R}_s^j &= -\frac{dM_0^j}{d\tau_s^j} \sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \left(q_{\tilde{s}} / \gamma_{\tilde{s}}\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R},
\end{aligned}$$

for each $s \in S$. Taking the ratio of these expressions, we have,

$$\zeta \frac{\gamma_s}{q_s} v'\left(\tau_s^j\right) = \frac{\sum_{\tilde{s}} q_{\tilde{s}} \left((1-\zeta) f'\left(p_{\tilde{s}}^j\right) + \zeta \left(1 - v\left(\tau_{\tilde{s}}^j\right)\right)\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R}}{\sum_{\tilde{s}} q_{\tilde{s}} \left(1 - \tau_{\tilde{s}}^j\right) \left(q_{\tilde{s}} / \gamma_{\tilde{s}}\right) \frac{\left(p_{\tilde{s}}^j\right)^2}{R}} \text{ for each } s. \tag{B.82}$$

This expression describes the optimal tax rate in state s in terms of the other tax rates and the endogenous variables. For the special case in which S is a singleton, the equation reduces to Eq. (38) that describes the optimal tax rate. In the more general case, the equations for all tax rates, $\{\tau_s\}_s$, are jointly solved taking the prices, $\{q_s, p_s\}_s$, as given. Note also that the prices are determined as a function of the tax rates, $\{\tau_s\}_s$, as described in the main text. The Nash equilibrium must satisfy both of these equation systems. As before, there can be multiple stable equilibria.

Note that, since $v'(0) = v(0) = 0$, Eq. $(B.82)$ cannot be satisfied for $\tau_s = 0$. This proves that the taxes are strictly positive in any Nash equilibrium.

Now consider a particular equilibrium. Note that the right hand side of Eq. $(B.82)$ does not depend on state s. Then, taking the ratio of these equations for two arbitrary states we obtain Eq. $(A.63)$ in the main text, completing the proof. □