IMPEDIMENTS TO FINANCIAL TRADE:
THEORY AND APPLICATIONS

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<u>ABSTRACT</u>

We propose a tractable model of an informationally inefficient market featuring non-revealing prices, no noise traders, and general assumptions on preferences and payoff distributions. We show the equivalence between our model and a substantially simpler model whereby investors face distortionary investment taxes depending both on their identity and the asset class. This equivalence allows us to account for such phenomena as under-diversification. We further employ the model to assess approaches to performance evaluation, and find that it provides a theoretical basis for some intuitive practices adopted by finance professionals, such as style analysis.

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1. Introduction

We present a simple, tractable framework featuring informational asymmetries in a multi-asset economy. By incorporating multiple assets and strategic behavior for a subset of investors, our framework can dispense with the common assumption of "noise" traders, and our results do not rely on a constant-absolute-risk-aversion (CARA)-normal setup. More-over, the model is particularly tractable: We show that it is isomorphic to a symmetric-information one featuring investor- and asset-class specific distortionary and re-distributive taxes, reflecting investors' abilities to distinguish between good- and bad-quality assets in the original model. Conceptualizing the impact of asymmetric information in terms of this tax equivalence makes it simpler to see how informational asymmetry can cause phenomena such as non-participation by some investors in some markets and associated risk-sharing imperfections.

To illustrate one possible application of this theoretical framework, we assess popular approaches to performance evaluation. A distinctive feature of the model compared to the literature is that it allows a natural and transparent modeling of pure selectivity skill, defined as the ability to select the better yielding assets out of a class of seemingly identical assets. We use this feature to show that Jensen's alpha may fail to identify informational advantage even though investors in our framework only have superior information about individual assets, but do not possess superior information about the returns of the market portfolio. On the positive side, we show how our model provides a theoretical basis for some simple, intuitive approaches to performance evaluation that have proved popular with practitioners, such as the "style" alpha methodology of W. Sharpe and the usage of fund-dependent benchmarks.

Specifically, we consider a model featuring different locations, or asset classes. A fraction of investors in every location are regular investors and the complement are "swindlers." Regular investors are endowed with common stocks that pay random location-dependent dividends at date one, while each swindler owns a "fraudulent" stock that pays nothing. Investors obtain signals on the type of a given stock (regular or fraudulent) in every location. Important, the quality of that signal depends on both the investor's and the firm's locations,

allowing for significant heterogeneity in information quality across investors. However, to highlight the differences from prior literature, we assume that no investor possesses any superior information with respect to the realization of regular-firm dividends in a given location.

Swindlers have a strong incentive to trade so as to equalize the price of their stock with the prices of other stocks in their location. Moreover, the swindler can manipulate the earnings of her company, which deters short selling. A pooling equilibrium emerges with all common and fraudulent stocks in a given location trading at the same price. The failure rate f of an investor's signal to identify fraudulent stocks in a given location can be equivalently viewed as a tax rate when investing in that location: A proportion f of the stocks identified by the investor's signals as regular pay nothing. Indeed, we prove an equivalence between our model and a much simpler dual (Walrasian) economy populated only with competitive investors faced with investor- and asset-specific capital taxation. The market-clearing conditions in such a dual economy need to be carefully formulated to reflect that these taxes are redistributive rather than "iceberg" costs, since trading does not destroy resources but only redistributes them from investors with inaccurate signals to swindlers.

Our setup does not require noise traders in order to avoid information revelation through prices. This is due to three assumptions: 1) the assumption of multiple asset classes introduces a straightforward non-informational motivation to trade, namely the need to diversify across locations; 2) the swindlers are not competitive, but rather take into account the effect of their trade on the price of their company; and 3) there is no short selling in equilibrium, due to an out-of-equilibrium threat of dividend manipulation by the swindlers. Taken together, these three assumptions allow us to dispense not only with noise traders, but also with the restrictive preference and distributional assumptions of a CARA-normal setup.

The investors inside the model have an incentive to bias their portfolios towards the locations where they enjoy an informational advantage, since those are the locations where they perceive lower implicit taxes. In contrast to existing literature, which we summarize below, these portfolio biases exist independently of the particular realization of the signals.

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In addition, the portfolio of any given investor is "sparse," i.e., it involves zero holdings in several individual assets (even in the locations where the investor is actively investing), and may even involve zero allocations to entire asset classes, consistent with some features of real-world portfolios.[1]

The combination of non-revealing prices (which leaves room for the better informed investors to earn superior returns) and portfolio heterogeneity makes ours a natural framework to study the validity of different performance evaluation approaches from the perspective of an uninformed econometrician. There is an established literature that has addressed this issue in noisy rational expectations models. Our framework, however, provides a novel way to capture situations where superior performance is associated purely with selectivity rather than market timing, since assets inside a location look identical to an outside econometrician, and no agent has superior information about the return distribution of the asset class itself.

We arrive at the following conclusions. When markets are informationally inefficient, Jensen's alpha may fail to identify informational advantage: passive strategies (i.e., returns obtained by simply buying the portfolio of all firms in a location ignoring any signals) generically may have alpha, and informed strategies may have negative alpha. We link these phenomena to the heterogeneity of informational inefficiency across markets.

We then address the question of how to appropriately perform performance evaluation in our setup. We show that the key feature of successful performance evaluation is to use a criterion that assigns zero alphas to linear combinations of passive investments in the asset classes in which the informed investor participates actively. Intuitively, this ensures that the return obtained by an informed investor could not have been replicated by a passive investor investing in the same asset classes.

This is the reason why the "style-alpha" approach, which was proposed by Sharpe (1992) and has proved very popular among practitioners, has several theoretical merits in our framework. Such an approach identifies skill with the alpha obtained from a regression of the investor's return on the passive returns obtained in the asset classes where the investor participates actively. We show that the alpha of such a regression provides a clear mapping to

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[1]See Koijen and Yogo (2016) for empirical evidence on portfolios held by institutional investors.

the investor's informational advantage.

We also discuss the implications of market segmentation (and more broadly portfolio specialization) for performance evaluation. We argue that when informational asymmetries result in portfolios that invest in a limited set of asset classes, the performance evaluation criterion should be investor-dependent, and focused on assigning zero alphas only to passive returns in the asset classes where the investor is actively participating. We illustrate this point with an example of a non-exploitable arbitrage, whereby it is impossible to us one pricing kernel to price all passive strategy returns, but it is still possible to evaluate performance using investor-specific evaluation criteria.

The paper relates to various strands of the literature.

The literature on noisy rational expectations models is the most popular approach to introduce informational asymmetries into finance models. This literature is too voluminous to summarize, so we provide indicative examples only. Technically, our setup borrows elements from both Grossman and Stiglitz (1980) and Akerlof (1970). The extended version of the model where all traders are strategic (analyzed in Appendix A) uses the same equilibrium concept as Kyle (1989).[2] Admati (1985) extends the noisy REE framework to a multi-asset framework. This literature typically utilizes random supply shocks ("noise") to avoid revelation[3] and a CARA-normal framework to obtain tractability, assumptions that we can avoid.

A popular application of multi-asset REE models is the explanation of portfolio biases.[4] Portfolio biases (in particular, the home bias) are especially prevalent issues in international finance, but the insights of this literature apply to understanding portfolio concentration and under-diversification more broadly. The common thread of that literature is that locals receive a signal about the aggregate performance of the local stock market. The superior signal quality makes domestic agents face lower variance when investing in local stocks,

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[2]Strategic traders are considered by many other papers in the micro-structure literature, albeit in very different contexts than the current paper. For an indicative example, see, e.g., Vayanos (2001).

[3]It is known, however, that (privately known) endowment shocks can achieve a similar outcome to random supply. See, e.g., Wang (1993).

[4]Indicative examples are Gehrig (1993) and Brennan and Cao (1997). Related, Van Nieuwerburgh and Veldkamp (2010) propose an approach relying on bounded information-processing capacity.

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leading to an *unconditional* home bias. A counterfactual implication is that conditional on a bad signal, locals should short domestic stocks. This seems at odds with the fact that the home bias is present for any given year, any given country, and for any sample period that one may consider. In our model the portfolio bias towards asset classes where one is better informed applies independently of any specific realization of the signals. The reason is that the portfolio bias is not driven by having superior information about a given location, but rather because of superior asset selection ability within the location.[5] This superior selection ability acts as a redistributive tax with obvious deterrence effects on investors who are not as well informed as locals.[6] We note in passing that the international finance literature has on occasion modeled informational advantages as taxes or transactions costs in reduced form.[7] Our paper provides the theoretical underpinning of doing so, and draws attention to the proper specification of market clearing condition to ensure the correct mapping between redistributive taxes and asymmetric information frictions.

The paper also contributes to the literature that critiques CAPM alpha, estimated from the perspective of an uninformed investor, as a measure of skill — see, e.g., Admati and Ross (1985), Dybvig and Ross (1985), and Mayers and Rice (1979), Grinblatt and Titman (1989) among many others. Our results in this vein are most closely related to those of Mayers and Rice (1979) and Dybvig and Ross (1985), and we highlight two important differences. First, these authors only evaluate portfolios that are ex-ante optimal for the informed investor in the entire universe of assets. In contrast, we are interested in ascertaining whether a market participant exhibits skill when investing in a *given* subset of asset classes and for any weights that she may choose to assign to these asset classes. We are motivated by the real-world fact that portfolio choice is routinely delegated to managers with mandates to pick good assets within a narrow set of asset classes.[8] Second, unlike in these papers, in our setting optimal

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[5]Hatchondo (2008) is closer to the setup of the current model. An important difference to his model is that we do not rely on noise trading, assuming instead the existence of strategic "swindlers." Furthermore, we can obtain the no-shorting outcome endogenously, although in the main body of the paper we impose short selling restrictions directly for simplicity.

[6]See also Kurlat (2013) for the role of information asymmetry as taxation in a different example. Li et al. (2012) discuss fraudulent assets but in a different context.

[7]See, e.g., Okawa and van Wincoop (2012) for a recent example.

[8]A voluminous literature studies how delegated portfolio choice may lead to portfolio weighting distor-

informed portfolios are not interior with respect to the effective shorting constraints, and we show that they may have negative alpha even when the information does not help predict the market return (or reference return, more generally).

Sharpe (1992) proposed style analysis as a performance evaluation criterion. In some ways our paper provides an explicit micro foundation for this criterion in an equilibrium framework. We note, though, that the specific equilibrium return properties obtaining in our model are not identical to the ones assumed by the statistical model of Sharpe (1992).

We also relate to a literature that analyzes general properties of evaluation criteria and the use of stochastic discount factors for performance evaluation.[9] We differ in focus from that literature: Rather then considering any possible information structure, we make specific assumptions, which in particular allow a conceptual separation between diversifiable asset-selection risk (with agents being asymmetrically informed about it) and non-diversifiable asset-class risk (with agents being symmetrically informed about it). Our framework results in a tighter theoretical characterization of valid performance measures — indeed, in an essentially unique performance evaluation criterion. With our assumptions, an essentially sufficient condition for a valid performance evaluation criterion is to assign a zero value to any linear combination of passive strategy returns in the asset classes where the investor is actively participating.

We conclude with two caveats about our conclusions on performance evaluation: First, we abstract from timing ability, i.e., superior information about the behavior of asset classes as a whole. There is an extensive literature on timing ability, so we concentrate on the stronger results that obtain when information pertains exclusively to the relative quality of individual securities. Second, our focus is exclusively on identifying a market participant's stock-selection ability from the perspective of an econometrician. We therefore do not address how an individual investor would allocate her investment among various (potentially informed) managers, which is the central issue in the study of fund flows.[10]

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tions. We do not attempt to summarize this literature, and simply refer to Bhattacharya and Pfleiderer (1985) for an early and important contribution.

[9]See, e.g., Chen and Knez (1996), Glosten and Jagannathan (1994), for two indicative examples.

[10]For a discussion of these issues, see Berk and Green (2004), Ferson and Lin (2014), and Berk and van

The paper is organized as follows. Section 2 contains the model and the tax equivalence result. Section 3 contains the application of the model to performance evaluation. Section 4 concludes. Proofs and extensions are contained in the appendix.

2.   Model

2.1.   Locations, preferences, and firm and investor types

Time is discrete and there are two dates, $t = 0$ and $t = 1$. All trading takes place at time $t = 0$, while at $t = 1$ all payments are made and contracts are settled. There are K different locations, and each investor is located in one of the K locations. There is a continuum of investors in each location and we index a representative investor in a given location by i. Investors maximize expected utility of period-1 wealth, $E\left[U\left(W\right)\right]$, for some increasing and concave U.

Investors' time-zero endowments consist of shares in firms that are domiciled in their location. Investors in every location i are of two types, common investors and swindlers, while firms are of two types, regular and fraudulent. The number of shares in each firm is normalized to one, as are the measures of investors and firms at each location.

Common investors in location i are a fraction $\kappa \in (0, 1)$ of the population in that location. They are identically endowed with an equal-weighted portfolio of all regular firms in location i. All regular firms in location i produce the same random output D_i, and pay it out as a dividend. (Adding a firm-specific, idiosyncratic risk would be simple, but would offer no additional insights). The total measure of regular firms is κ in each location.

Swindlers are a fraction $1 - \kappa$ of the population in each location. Each swindler is endowed with the share of one fraudulent firm. Fraudulent firms produce no output or dividend $(D_i = 0)$.

For every firm in every location, there is a market for shares where any investor can submit a demand. Moreover, there exists a market for a riskless bond, available in zero net

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Binsbergen (2015) among others.
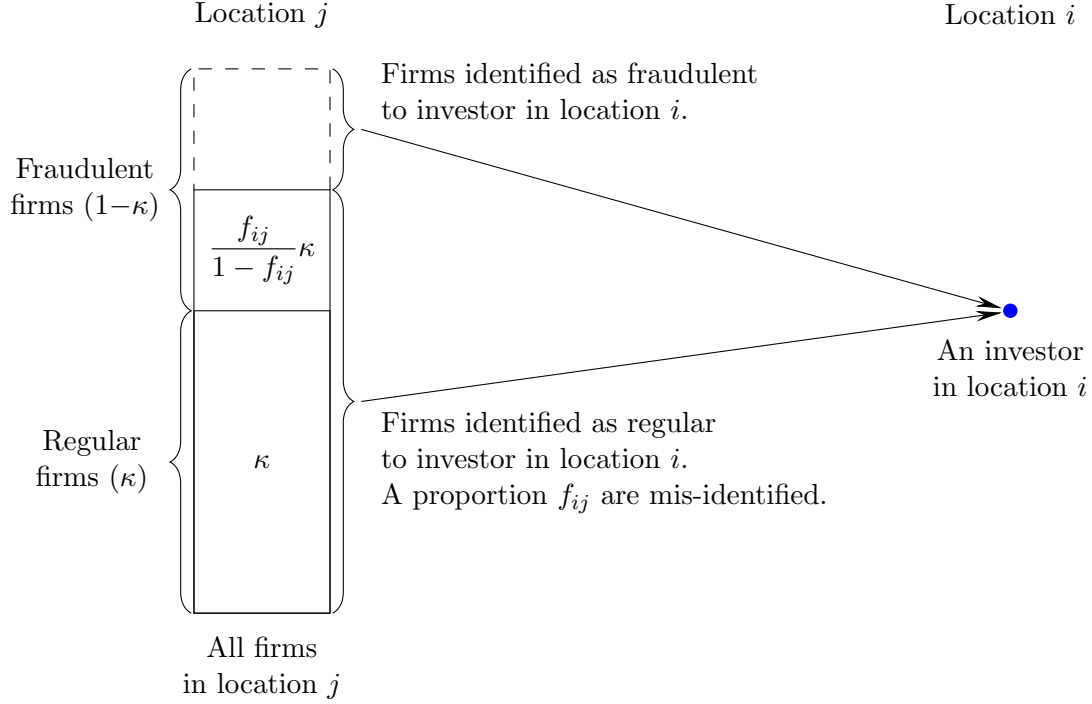
Figure 1: The figure offers a schematic representation of the information structure of the model. In location j, a fraction κ of the firms are regular and the complement $1 - \kappa$ are fraudulent. A given investor in location i receives signals about the type of each of these firms. A proportion f_{ij} of the firms identified as regular are actually fraudulent. All firms identified as fraudulent are indeed fraudulent.

supply. The interest rate is denoted by r.

2.2. Signals

Each investor obtains a binary signal of the type — regular or fraudulent — of every firm in every location. The precision of these signals depends on the locations of the investor and the firm.

Specifically, an investor in location i obtains a signal $\iota_{jk}^i \in \{0, 1\}$ about every firm k in location j. (All investors in location i obtain the same signal about any given firm, for simplicity.) This signal characterizes the firm as either regular ($\iota_{jk}^i = 1$) or fraudulent ($\iota_{jk}^i = 0$). The signal is imperfect. It correctly identifies every regular firm as such. However,

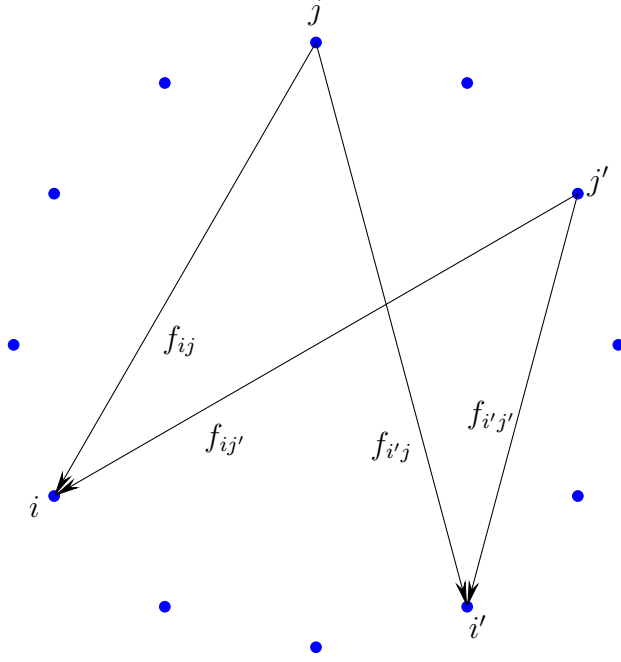Figure 2: The figure illustrates the nature of the information network. In particular, information quality, captured by the failure rate f_{ij}, depends both on the firms' and investor's location. Agents in i and i' may have signals of different qualities about location j. Similarly, agents in i may have signals of different qualities about locations j and j'.

it fails to identify all fraudulent firms: it correctly identifies a fraudulent firm with probability π_{ij} and misclassifies it as regular with probability $1 - \pi_{ij}$. For simplicity, we assume $\pi_{ii} = 1$, so that investors are fully informed about their local markets. This assumption can be easily relaxed.

Given this setup, Bayes' rule implies that the probability that a firm in location j is fraudulent given that investor i's signal identifies it as regular is given by

$$f_{ij} \equiv \frac{(1 - \pi_{ij})(1 - \kappa)}{\kappa + (1 - \pi_{ij})(1 - \kappa)}. \tag{1}$$

The law of large numbers implies then that f_{ij} can also be interpreted as the fraction of fraudulent firms among all firms in location j identified by the signal of investor i as regular.

To summarize the information structure, Figure 1 illustrates the nature of the information provided to a given agent in location i about all the firms in location j. Figure 2 emphasizes the bilateral nature of the information structure: information quality, captured by

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by f_{ij}, depends on both the firms' and investor's locations.

2.3. Budget constraints

Letting B^{ci} denote the amount that a common investor in location i invests in riskless bonds and dX_{jk}^{ci} a univariate signed measure capturing the number of shares of firm k in location j that she buys, the time-one wealth of a common investor located in i is given by

$$W_1^{ci} \equiv B^{ci}(1+r) + \sum_{j=1}^{K} \int_{k \in [0,1]} D_{jk} dX_{jk}^{ci}. \tag{2}$$

The first term on the right-hand side of (2) is the amount that the investor receives from her bond position in period 1, while the second term captures the portfolio-weighted dividends of all the firms that the investor holds. The time-zero budget constraint of a common investor in location i is given by

$$B^{ci} + \sum_{j=1}^{K} \int_{k \in [0,1]} P_{jk} dX_{jk}^{ci} = \frac{1}{\kappa} \int_{k \in [0,1]} P_{i,k} \rho_{(i,k)} dk, \tag{3}$$

where $\rho_{(i,k)}$ is an indicator function taking the value one if the firm k in location i is a regular firm and zero otherwise, and P_{jk} refers to the price of security k in location j. The left-hand side of (3) corresponds to the sum of the investor's bond and risky-security spending, while the right-hand side reflects the value of the (equal-weighted) portfolio of regular firms the investor is endowed with.

The budget constraint of a swindler owning firm l in location i is similar to (3), except that the value of the agent's endowment is given by P_{il}:

$$B^{sil} + \sum_{j=1}^{K} \int_{k \in [0,1]} P_{jk} dX_{jk}^{sil} = P_{il}. \tag{4}$$

Note that the notation dX allows investors' portfolios to have atoms, which is actually important here because, in equilibrium, swindlers optimally hold a non-infinitesimal quantity of shares of their own firms. We denote the post-trade number of shares of fraudulent firm l in location i retained by the original owner by $S^{il} = dX_{il}^{sil}$.

Finally, the time-1 wealth of a swindler is

$$W_1^{sil} \equiv B^{sil} + \sum_{j=1}^{K} \int_{k \in [0,1]} D_{jk} dX_{jk}^{sil}. \tag{5}$$

2.4. Optimization problem

Common investors are price-takers. Taking a set of prices for risky assets as given for all firms in all locations and an interest rate, a common investor maximizes

$$\max_{B^{ci}, dX_{jk}^{ci}} E\left[U(W_1^{ci})|\mathcal{F}_i, P_{jk}, r\right] \tag{6}$$

subject to (3) and a short-selling constraint: $dX_{jk}^{ci} \geq 0$. Here we impose the short-selling restriction exogenously, but in the appendix we consider a simple extension in which agents endogenously refrain from selling short. Specifically, we allow the swindler to manipulate earnings — in particular, to report higher earnings than actual — which exposes anyone shorting a fraudulent firm to the risk of large losses. We relegate the details to Appendix A, and for the rest of the paper we simply exclude short sales.

The investor conditions on her own information set $\mathcal{F}_i$ (i.e., on her signals about every security), as well as on the prices of all securities in all markets.

The problem of the swindler is similar to the one of the common investor with the exception that the swindler takes into account the impact of her trading on the price of her stock.[11] Similar to a common investor, the swindler who owns firm l in location i solves

$$\max_{B^{sil}, dX_{jk}^{sil}} E\left[U(W_1^{sil})|\mathcal{F}_{il}, P_{jk}, r\right] \tag{7}$$

subject to the budget constraint (4) and $dX_{jk}^{sil} \geq 0$.

2.5. Equilibrium

An equilibrium is an interest rate r and a collection of prices $P_{i,k}$ for all risky assets, asset demands and bond holdings expressed by all investors in all locations, such that: 1) Markets

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[11]In the extension presented in Appendix A, the swindler also has the ability to manipulate the dividends of her fraudulent stock.

for all securities clear; 2) Risky-asset and bond holdings, $\left\{X_{jk}^{ci}, B^{ci}\right\}$, are optimal for regular investors in all locations given prices and the investors' expectations; 3) Bond holdings B^{sil} and asset holdings for all securities X_{jk}^{sil} (including a swindler's own holdings of her own firm S^{il}) are optimal for swindlers given their expectations; 4) All investors update their beliefs about the type of stock k in location j by using all available information to them — prices, interest rate, and private signals — and Bayes' rule, whenever possible.

Our equilibrium concept contains elements of both a rational expectations equilibrium and a Bayes-Nash equilibrium. All investors make rational inferences about the type of each security based on their signals, the equilibrium prices, and the interest rate, by using Bayes' rule and taking the optimal actions of all other investors (regular and swindlers) in all locations as given. The continuum of regular investors are price takers in all markets.

Swindlers, however, are endowed with the shares of a fraudulent company and take into account the impact of their trades on the share price. In formulating a demand for their security, swindlers have to consider how different prices might affect the perceptions of other investors about the type of their security. As is standard, Bayes' rule disciplines investors' beliefs only for demand realizations that are observed in equilibrium. As is usual in a Bayes-Nash equilibrium, there is freedom in specifying how out-of-equilibrium prices affect investor posterior distributions of security types.

We note that the distinction between regular investors who are price takers and swindlers who are strategic about the impact of their actions on the price of their firm is helpful for expediting the presentation of results, but not crucial. In Appendix A we show that our equilibrium obtains in the limit (as the number of traders approaches infinity) of a sequence of economies with finite numbers of traders — both regular and swindlers — who are strategic about their price impact, as in Kyle (1989).

By Walras' law, we need to normalize the price in one market. Since we abstract from consumption at time zero for parsimony, we normalize the price of the bond to be unity $(r = 0)$.

2.6. Tax equivalence

While our economy is seemingly complex, its equilibrium outcomes coincide with those of a much simpler Walrasian economy featuring distortionary and redistributive taxes. The intuition behind this result is quite straightforward: Conditional on investing in a location, investors optimally invest equal amounts in all assets for which they have positive signals and in no others (the only exception is the swindler investing in her firm), but the signal is imperfect. The failure rate of the signal translates into a lower payoff relative to that obtained by a local, perfectly informed investor; the proportional loss can be thought of as a tax rate, which depends on both the investor and the target location of the investment. In addition, swindlers have strict incentives to invest in their own firms so as to render them indistinguishable from regular firms, by submitting elastic demands at the prevailing price of all other assets in the location. This ensures a pooling equilibrium that justifies the behavior of the other investors. We record this result formally:

Theorem 1 *There exists an equilibrium of the original economy in which the prices of all assets in each location are equal. Furthermore, the prices P_j and aggregate positions X_j^i taken by investors located in market i when investing in market j, excluding swindlers' positions in their own firms, are given as a solution to the problem:[12]*

$$X^i \in \arg \max_{X \geq 0} EU \left(\sum_{j=1}^{K} \left((1 - f_{ij}) D_j - P_j \right) X_j + P_i \right) \tag{8}$$

$$\kappa = \sum_{i=1}^{K} (1 - f_{ij}) X_j^i. \tag{9}$$

Equation (8) formalizes the decision problem of an investor facing taxes f_{ij}, as explained above. Equation (9) is the market-clearing equation for regular firms. The left-hand side, κ, equals the supply of firms: only κ of the firms are regular. The right-hand side represents the demand for regular firms, and it depends on the tax rates: a proportion f_{ij} of the demand X_j^i is directed to fraudulent firms, leaving only the remainder to acquire regular firms. (We

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[12]In the interest of concision, we plugged in the investor's budget constraint in the objective.
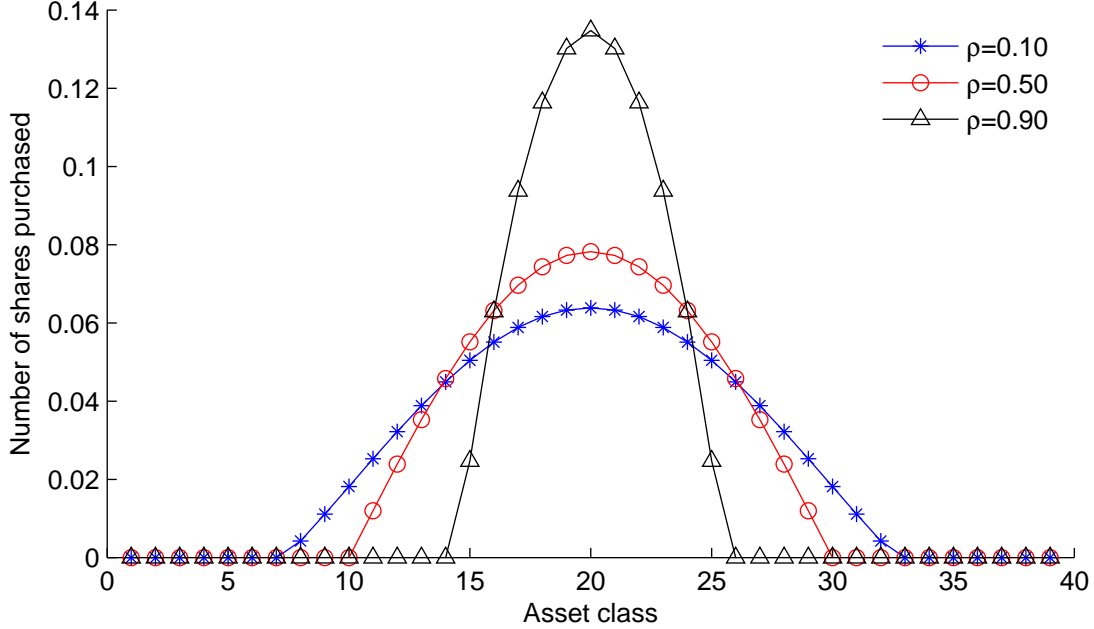
Figure 3: This figure illustrates the portfolio choice of a common investor in location $i_0 = 20$ under three alternative correlation structures. We assume $N = 39$ locations in which a proportion $\kappa = 0.99$ of assets are regular and pay normally distributed dividends with mean 1 and standard deviation 0.25; the pairwise correlation is the same for all pairs (j, j') with $j \neq j'$, and given by the parameter ρ. Agents have CARA utilities with parameter $\gamma = 2$. We set $\pi_{ij} = 1 - \frac{1}{2}(1 - \cos(2\pi d(i, j)))$ with $d(i, j) = \min\{|i - j|, N - |i - j|\}/N$, which yields f_{ij} as a decreasing function of the circular distance $d(i, j)$ between i and j. In the benchmark case $\rho = 0.5$, the expected excess passive return on an asset is 6.08%, and the lowest tax dissuading investor i from investing in location j is $f_{ij} = 0.52\%$.

note that in a pooling equilibrium the swindler submits an elastic demand for her own firm, i.e., absorbs the residual demand for her own firm at the price P_j, so that the market for fraudulent firms clears by construction.)

An obvious implication of Theorem 1 is that investors have an incentive to place a larger fraction of their wealth in locations where they are faced with lower taxes. Indeed, if the effective taxes are sufficiently severe compared to the diversification benefit, then the investors may choose to concentrate their portfolio in a subset of locations, placing zero weights in the others. A distinctive feature of the model compared to noisy REE models is that these biases apply for any realization of the signals about which firms in a location are

fraudulent and which are not. By contrast, in the typical noisy REE model a signal applies to the level of an asset class, and hence the direction of the portfolio bias depends on the realization of the signals.

Figure 3 provides an illustration of the tradeoff between diversification and information-tax avoidance. In a symmetric set-up, the higher the correlation between any two locations, the lower the threshold for f_{ij} above which agent i does not wish to invest in market j, and therefore the fewer markets the investor participates in. The precise model assumptions are listed in the caption to the figure.

The market clearing condition (9) highlights that the implicit taxes in our setup are redistributive, rather than "iceberg costs." Indeed, if we multiply both sides of equation (9) by D_j, we obtain

$$\kappa D_j = \sum_{i=1}^{K} (1 - f_{ij}) D_j X_j^i.$$

In words, the aggregate dividends κD_j in location j are all paid to investors in proportion to their holdings of regular firms in this location, and no dividend gets lost.

Proposition 1 provides a micro-foundation to the common practice (especially in international economics, but also more broadly) of using taxes (or "wedges") as a reduced-form way of modeling informational frictions, as long as these taxes are redistributive, rather than iceberg costs.

For the purposes of the analysis that follows, Proposition 1 makes the description of an equilibrium relatively easy, as we illustrate in the following section. In addition, it provides one with an intuitive language to talk about the degree to which any investor is at a disadvantage when investing in any given market.

3. Informationally Inefficient Markets: Implications

In this section we exploit the equivalence formalized in Proposition 1 between informational frictions and taxes to study the ability of popular performance-evaluation approaches to

appropriately identify investors with "skill," i.e., investors who select stocks based on infor-
mative signals. Throughout we envisage an econometrician, by definition uninformed, who
observes the return obtained by an investor on her portfolio and is trying to infer if that
investor had valuable signals in choosing her portfolio.

The first question we address (Section 3.1.–3.2.) is whether CAPM alphas provide an
appropriate measure of an investor's informational advantage. Specifically, in Section 3.1.
we make assumptions to ensure that the CAPM would hold in the absence of informational
asymmetries, and we solve for equilibrium prices. Using these equilibrium prices, in Section
3.2. we obtain an explicit expression for equilibrium alphas inside the model and conclude
that they are problematic: investors with no skill may have positive alpha, while investors
with skill may have negative alpha. This shows that even though in our model the only
skill is a stock-selection skill, the CAPM alphas do not provide an appropriate measure of
this skill. Sections 3.3. and 3.4., on the other hand, analyze what is essentially the unique
meaningful such measure in our model, and discuss it in the context of the literature.

3.1.   Equilibrium prices

To ensure that the CAPM would hold in the absence of informational frictions, we assume in
sections 3.1. and 3.2. that the dividends D_j are jointly normal. For simplicity we also assume
that they have the same mean, which we normalize to unity. To obtain explicit expressions
for equilibrium prices, we endow investors with CARA utilities, $U(W) = -e^{-\gamma W}$.

We let $\lambda_{ij} \geq 0$ denote the Lagrange multiplier associated with $X_j^i \geq 0$, and $p_{ij} := 1 - f_{ij}$
be the effective payoff to investing in assets of location j. Note that p_{ij} is the probability
that security j is regular given that the signal of investor i identifies it as such. Clearly,
$p_{ij} \geq \kappa$, with strict inequality if the investor's signal is valuable. Given the CARA-normal
setup, the first-order condition of an investor in location i faced with problem (8) is

$$\gamma \text{cov}\left(p_{ij} D_j, \sum_{k=1}^{K} p_{ik} D_k X_k^i \right) = p_{ij} - P_j + \lambda_{ij}. \tag{10}$$

Dividing this equation by p_{ij} and summing over all agents i yields

$$\gamma \text{cov}\left(D_j, \kappa D^a\right) = 1 - \frac{P_j}{K}\sum_{i=1}^{K} p_{ij}^{-1} + \frac{1}{K}\sum_{i=1}^{K} p_{ij}^{-1}\lambda_{ij}, \tag{11}$$

where we introduced the notation D^a for the average dividend $D^a = \frac{1}{K}\sum_{j=1}^{K} D_j$.

We note that, by (9) and exchanging the order of the summation,

$$\sum_{i=1}^{K}\sum_{k=1}^{K} p_{ik} D_k X_k^i = \sum_{k=1}^{K} D_k \sum_{i=1}^{K} p_{ik} X_k^i = \kappa K D^a. \tag{12}$$

The price P_j is consequently expressed as

$$
\begin{aligned}
P_j &= \left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}^{-1}\right)^{-1} \times \left(1 - \gamma \text{cov}\left(D_j, \kappa D^a\right) + \frac{1}{K}\sum_{i=1}^{K}\lambda_{ij} p_{ij}^{-1}\right) \\
&= \left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}\right) \times \left(1 - \gamma \text{cov}\left(D_j, \kappa D^a\right) + \frac{1}{K}\sum_{i=1}^{K}\lambda_{ij} p_{ij}^{-1}\right) \times \frac{\left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}^{-1}\right)^{-1}}{\left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}\right)},
\end{aligned}
\tag{13}
$$

which provides a natural formula. The first term captures the average post-tax payoff to investors, the second the risk adjustment and the effect of the shorting constraint, while the third measures dispersion in p_{ij} across agents. Equation (13) shows that two asset classes may be priced differently even when containing the same amount of aggregate risk (i.e., $\text{cov}(D_j, D^a)$ is the same for all j) and being held in positive amounts by all agents ($\lambda_{ij} = 0$). As long as $p_{ij} \neq p_{ij'}$ for some i for two asset classes j and j', it is possible that $P_j \neq P_{j'}$. This observation will prove useful in the next section.

3.2. Alpha does not measure skill

We next obtain some implications of the model for CAPM alphas. By CAPM alphas we mean the estimates of the constant in a regression of the excess return obtained by an investment strategy on the excess return of the market portfolio. Throughout the paper, we do not concern ourselves with estimation issues. We focus exclusively on the implications of

our theory for the moments of such regressions.

To start, we define R_j^p as the gross return of a passive (or index) return in location j. This is the gross return obtained by simply buying all the firms in location j. (This would be the return of an uninformed investor, who doesn't have access to any private signals). Given the assumption of the model this return is given by $R_j^p = \frac{\kappa D_j}{P_j}$, with expectation $\frac{\kappa}{P_j}$. Similarly, define average price $P^a = \frac{1}{K} \sum_{k=1}^{K} P_k$ and the return on an index replicating the market portfolio is $R^p = \frac{\kappa D^a}{P^a}$. Using these observations, and recalling that the interest rate is normalized to zero, the regression of the observed (passive) return of the index in location j on the market portfolio return yields a constant ("alpha") of

$$
\begin{aligned}
\alpha_j &= \frac{\kappa}{P_j} - 1 - \frac{\operatorname{cov}\left(\frac{\kappa D_j}{P_j}, (P^a)^{-1} \kappa D^a\right)}{(P^a)^{-2} \kappa^2 \sigma_a^2} \left(\frac{\kappa}{P^a} - 1\right) \\
&= \left(\beta_j^D \frac{P^a}{P_j} - 1\right) + \frac{\kappa}{P_j}\left(1 - \beta_j^D\right),
\end{aligned} \tag{14}
$$

where β_j^D is the "cash-flow beta"

$$
\beta_j^D = \frac{\operatorname{cov}(D_j, D^a)}{\operatorname{var}(D^a)}. \tag{15}
$$

In the special case in which there is no asymmetric information ($p_{ij} = \kappa$) equations (13) and (14) imply the usual CAPM relation ($\alpha_j = 0$).[13]

However, in the presence of informational asymmetries, $\alpha_j \neq 0$ in general, even for passive strategies. To see this in the simplest possible case, consider a world with $\beta_j^D = 1$ for all j. Accordingly, $\alpha_j = \frac{P^a}{P_j} - 1$. In addition, equation (13) implies that some asset classes may still exhibit lower (or higher) than average prices, despite all assets having the same exposure to aggregate risk and the same expected dividend. For instance, a lower overall quality of information in asset class j (low values of p_{ij} compared to other asset classes) translates into a lower than average price for that class; since $\alpha_j = \frac{P^a}{P_j} - 1$, even an index investment in

---

[13]To see this, notice that equation (13) implies that $P_j = \kappa - \gamma \kappa^2 \beta_j^D \sigma_a^2$. Then it follows from (14) that $\alpha = \left(\beta_j^D \frac{\kappa - \gamma \kappa^2 \sigma_a^2}{P_j} - 1\right) + \frac{\kappa}{P_j}\left(1 - \beta_j^D\right) = \frac{\kappa - \gamma \kappa^2 \beta_j^D \sigma_a^2}{P_j} - 1 = 0$.

such a class has positive alpha.[14]

If uninformed (passive) strategies command alphas, then alphas cannot be an accurate measure of an investor's skill. Indeed, continuing with the assumption that $\beta_j^D = 1$ for all j, the alpha resulting from a regression of the return that an investor i obtains when investing in location j on the return of the market portfolio is given by

$$\alpha_{ij} = \frac{p_{ij}}{\kappa} \frac{P^a}{P_j} - 1. \tag{16}$$

Hence, even an investor who has an informational advantage $p_{ij} > \kappa$ might exhibit a negative alpha when that informational advantage happens to be in an asset class that is comparatively more expensive than the average asset class, i.e., $P^a < P_j$.

The reason why the CAPM fails to assign zero alpha even to passive strategies is qualitatively different from the arguments that have been proposed so far. Unlike elsewhere in the literature, in our setup investors don't possess any signals on the realization of D_j, so they are on equal footing about predicting the return of an asset class. It is tempting to attribute the failure of the CAPM in our model to the fact that different investors hold different mean-variance efficient portfolios, so that the market portfolio is not mean-variance efficient for any investor. This fact, however, is not sufficient to render CAPM alpha an inaccurate measure of skill: Suppose, for instance that all prices across all asset classes are equal ($P_j = P$), which would occur for instance if the informational advantages are symmetric ($p_{ij} = p$ for all $i \neq j$ and some positive $p < 1$), and all betas are unity. In that case investors still choose different mean-variance efficient portfolios, depending on their locations. Yet, equation (14) shows that alphas are zero for passive strategies, while equation (16) shows that informed investors have positive alphas.

What makes alpha a valid measure of performance in this special case? As we show in more generality in Section 3.4., the key feature is that the market portfolio is mean-variance

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[14]An additional, less interesting observation is that mispricing is not related to the amount of risk. Consider for instance the case $p_{ij} = p$. All indexes, including the market, earn negative excess returns $\kappa - p$ (compared to an informed strategy) before the risk adjustments, but a high β_j^D index is benchmarked against a leveraged market index, thus one with even higher negative returns, and therefore has positive alpha. Conversely, low β_j^D is associated with negative alpha. We shut down this channel by assuming $\beta_k^D = 1$ for all k.

efficient from the perspective of an uninformed investor. However, this property is special to this example. In general the market portfolio is not mean-variance efficient even from the perspective of an uninformed agent.

It is worth comparing our results to those of Mayers and Rice (1979) and Dybvig and Ross (1985), who show that a mean-variance efficient portfolio utilizing private information has a positive alpha as long as the moments of the market portfolio (more generally, of the reference portfolio) do not depend on the information (they refer to this as "pure selectivity"). There are two notable differences with our work. First, we place emphasis on the return obtained by an investor when investing in a given asset class, rather than the return of an informed investor's mean-variance efficient portfolio. We are interested in the former return because in real life investors routinely delegate portfolio choice to managers with the mandate to find good investment opportunities within a pre-specfied set of asset classes, which may well be a subset of the investment universe.

Second, even the returns obtained by an investor on her entire mean-variance efficient portfolio may have negative alpha with respect to an uninformed reference portfolio in our model;[15] the reason is that the Dybvig and Ross (1985) result is predicated on an interior, unconstrained choice of all assets, whereas in our model investors are effectively bound by shorting constraints for the stocks that their signal identifies as fraudulent.

3.3. General properties of evaluation measures and style alphas

The previous section shows that the CAPM fails to assign zero alpha to passive strategies. We show here that assigning zero alpha to passive strategies is actually a sufficient condition for a performance measure to be valid. As a practical illustration of the results, we show that the style alpha measure proposed by W. Sharpe is a valid performance measure.

To start, we let g be a functional mapping random variables into the space of real numbers. Let $R_j^{e,p}$ the excess passive return in location j. We assume that g is a linear functional and that $g(R_j^{e,p}) = 0$ for all j. We also require $g(1) > 0$, that is, a riskless, positive excess return

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[15]Such examples are available from the authors upon request.

is assigned a positive value.

Assuming the existence of such a functional g, we next show that it is a valid performance measure, in the sense that it correctly identifies an investor's informational advantage. To see this, note that the excess return of an informed investor i in our model can be written as

$$R^{e,i} = \sum_{j=1}^{K} (q_{ij} R_j - 1) \, w_j^i,$$

where $q_{ij} \equiv \frac{p_{ij}}{\kappa} \geq 1$. Hence

$$\begin{aligned}
g\left(R^{e,i}\right) &= g\left(\sum_{j=1}^{K} q_{ij}\left(R_j - 1\right) w_j^i + \sum_{j=1}^{K} \left(q_{ij} - 1\right) w_j^i\right) \\
&= 0 + g\left(1\right) \sum_{j=1}^{K} \left(q_{ij} - 1\right) w_j^i \tag{17} \\
&\geq 0.
\end{aligned}$$

We note that this derivation also shows that the measure g is essentially unique — any two measures g and g' differ at most by a multiplicative constant.

One way to construct the measure g is the so-called style analysis, proposed by Sharpe (1992). According to this approach, the return of each manager is regressed on the passive returns of all possible asset classes. Moreover, to interpret the betas as portfolio weights, one additionally requires that the betas on the passive strategies add up to one. (They are also restricted to be positive, to satisfy the no-shorting constraints faced by mutual-fund managers.) The constant (alpha) of such a regression is interpreted as a manager's skill.

Viewing style analysis as mapping the (excess) return of a manager to a value of alpha, it is straightforward to show that it satisfies all the aforementioned properties of the functional g. We record the result formally:

Proposition 2 *Let $w_j^i = \frac{P_j X_j^i}{W_0^{ci}}$ be the portfolio weight of the investment in location j by an investor in location i. Consider the style regression of the gross return obtained by such an investor on the passive returns, including the risk-free one. The constant α_i^s in this regression*

is the portfolio-weighted informational advantage of investor i across all markets in which she invests:

$$\alpha_i^s = \sum_{j=1}^{K} \left(\frac{p_{ij}}{\kappa} - 1 \right) w_j^i. \tag{18}$$

An alternative way of formulating the functional g is as follows. Let Σ denote the covariance matrix of passive excess returns $R_j^{e,p}$, $E\left(R^{e,p}\right)$ the vector of expected excess passive returns, $w = \Sigma^{-1} E\left(R^{e,p}\right)$ a mean-variance efficient portfolio from the perspective of an uninformed econometrician, and $R^{MVE} = w^\top R^{e,p}$ the excess return of the portfolio. Then the functional $g(R^e) \equiv E(R^e) - \text{cov}(R^e, R^{MVE})$ satisfies all the requirements of the functional g, since it is linear, satisfies $g(1) = 1$, and most importantly assigns the value zero to all passive excess returns. This observation formalizes the claim we made in Section 3.2.: The reason for the inadequacy of CAPM alphas is that the market portfolio is not mean-variance efficient even from the perspective of an uninformed econometrician.[16]

Our analysis in this section is related to Chen and Knez (1996), who characterize performance measures satisfying a reasonable minimal set of requirements in a general payoff-and-information environment. Our special model structure implies a tighter characterization — essentially, our performance measure is unique, assigns positive alpha to informed strategies, and it can be thought of as a style alpha.

3.4. Investor-specific performance evaluation

A key requirement for a valid functional g is that it assign zero alpha to passive strategies. An issue that we did not address in the previous section is that the requirement need only apply with respect to the locations in which a given investor i participates. Indeed, equation (17) continues to hold even if the values $g(R_j^{e,p})$ are set arbitrarily whenever the investor chooses $w_j^i = 0$.

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[16]See also Ferson and Siegel (2001) for the properties of unconditionally efficient portfolios for the purposes of performance evaluation.

This observation is of practical importance because in the real world many portfolios are concentrated in only a few asset classes, and virtually all shun some asset classes. It also helps explain the widespread use of heterogeneous benchmarks. Thus, if the goal is to evaluate the stock-picking skills of an asset manager who only invests in Finnish stocks, then our analysis provides a justification for regressing her return only on the Finnish stock market index rather than some global index, or a set of indices from several countries. We also note that adding more classes not only does not help, but in fact hurts by deteriorating the quality of inference with finite data.

The above discussion helps us illustrate an additional point of some theoretical interest: One can find valid, investor-specific g_i even when a functional g pricing all passive strategies does not. The easiest way to illustrate this point is by using a minimal example whereby an equilibrium features an unexploitable arbitrage. For instance, consider an economy in which *(i)* the passive portfolios in two locations (say, locations j and j') have the same dividends from the perspective of a passive investor ($\kappa D_j = \kappa D_{j'}$) but different prices ($P_j \neq P_{j'}$);[17] *(ii)* investor j invests only in market j, because $\frac{p_{jj}}{P_j} > \frac{p_{j'}}{P_{j'}}$ and similarly investor j' only invests in market j'. The prohibitive risks of shorting make this arbitrage opportunity compatible with equilibrium. A global measure g applying simultaneously to $R_j^{e,p}$ and $R_{j'}^{e,p}$ does not exist, yet individual measures g_j and $g_{j'}$ are easy to construct, e.g., by regressing each investor's return on the passive returns in the asset classes in which she invests.

3.5. Summing up alphas

Here we take a closer look at the cross-section of portfolio performance in our model. The starting point is the general observation that, relative to the market, the average alpha must be zero by construction. However, in our model all the investors have some information, which should allow them to improve on the market portfolio and consequently display into positive alphas.

To discuss this issue, we revisit Section 3.2. and concentrate on an economy that is

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[17]This could occur in equilibrium, for instance, because the investors in a third location j'' are better informed about one of these two locations, resulting in a higher price for its securities.

symmetric with respect to the various locations. In this economy, $P_j = P = P^a$, and equation (16) gives $\alpha_{ij} = \frac{p_{ij}}{\kappa} - 1 > 0$. Since all individual alphas are positive, it would appear that the portfolio-weighted average of alphas (across investors) is strictly positive. This conclusion is not correct, though, because the analysis so far has ignored the swindlers' investment in their own firms. Indeed, these agents invest a non-zero fraction of their portfolio in an asset costing $P > 0$ and paying back zero, i.e., offering a net return of -100%.

One can see explicitly the negative return to the swindlers' retained holdings in their own firms in the market-clearing equation from Theorem 1. Focusing on a single market, recalling that $1 - f_{ij} = p_{ij}$, and summing across investors i expresses equation (9) as

$$0 = \sum_{i=1}^{K} \left(\frac{p_{ij}}{\kappa} - 1 \right) X_j^i + (-1) \times \left(1 - \sum_{i=1}^{K} X_j^i \right). \tag{19}$$

The right-hand side of equation (19) contains two terms. The first term is positive and captures the intuition of aggregate positive alphas. The second term, though, is negative, because $\sum_{i=1}^{K} X_j^i < 1$: the difference $1 - \sum_{i=1}^{K} X_j^i$ represents the swindlers' position in their own firms in location j, and -1 is the associated net return.

The alphas realized by the swindlers combine the -100% on their own firms with the positive values on the rest of their portfolios, but are negative in the aggregate. This is despite the fact that the swindlers possess superior information. Given their endowment of worthless stock, swindlers are actually better off retaining some of their shares in their effort to pool with the regular stock. The reason for their negative alpha is not suboptimal behavior, but rather the nature of their initial endowment.

This observation illustrates an additional caveat to using alpha, namely that it depends on endowments, or initial positions. An investor's return may appear inferior simply because her initial position is overvalued, and disposing of it would deteriorate (mark-to-market) wealth. In such situations, one would obtain a better measure of the investor's information from the return on the *change* in the investor's portfolio, under the implicit assumption that this change would have been zero absent any information. Implementing such a procedure,

though, requires knowledge of portfolios (trades), which is far harder to come by.

4. Conclusion

We develop a multiple-market, multiple-investor model, whereby informational asymmetries act as distortionary and redistributive capital taxes. By explicitly modeling the incentive to diversify across asset classes, and introducing strategic-trading considerations for some traders, we can dispense with noise trading, yet keep prices non-revealing. Moreover, the duality between the model and a tax economy makes the model quite tractable to analyze, without requiring CARA utilities and normal dividends.

By drawing a distinction between asset classes (sets of stocks that seem identical from the perspective of an uninformed agent) and individual assets within asset classes, the model can account for portfolio biases towards specific asset classes for any realization of the signals about the quality of individual assets. Hence the model provides a simple and analytically convenient framework to model persistent portfolio biases toward a set of asset classes, under-diversification, and portfolios with non-interior (zero) holdings of individual assets.

To illustrate the analytical tractability of the model we revisit an established literature that analyzes the properties of popular performance evaluation measures. Our framework allows a particularly clean distinction between pure selection and timing abilities. Without trivializing the possible importance of timing information, we show that the specific informational assumptions we adopt provide a simple and intuitive theoretical basis for portfolio evaluation criteria such as style analysis and fund-dependent choice of benchmarks, which have proved popular amongst practitioners.

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Appendix

A   Strategic Agents and Dividend Manipulation

Here we build a model extension designed to capture two desired phenomena. First, we model a finite economy populated by agents who behave strategically and show that the equilibrium approaches the one in the benchmark (continuum) economy as the number of agents grows without bound. Second, we also obtain no shorting as an endogenous consequence of allowing swindlers to manipulate the cash flows from their firms, in a sense made precise below.

In order to illustrate the point as quickly and easily to convey as possible, we make a number of (dispensable) simplifying assumptions.

Consider two locations, $\mathcal{L} = \{1, 2\}$, each populated by N agents and hosting N firms. A number κN of the agents[18] in each location are common, the others being swindlers. Similarly, there are κN regular and $N - \kappa N$ fraudulent firms per location. Agents have CARA preferences with risk-aversion parameter γN, and each regular firm in location j has output and dividend D_j/N, where $\mathrm{E}[D_j] = 1$ and $\mathrm{Var}(D_j) = \sigma^2$. Let Ω denote the variance of (D_1, D_2). Dividends in different locations are assumed to be independent. Fraudulent firms have output equal to zero. All agents are endowed with an equal number of shares of regular firms in their locations. Each swindler also owns entirely a fraudulent firm.

The information structure is as in the main text, with some simplifications. As in the text, we maintain that $p_{ii} = 1$, and also impose symmetry, i.e., $p_{12} = p_{21} \equiv p$. We also impose that some quantities, such as the proportion of fraudulent firms mis-identified as regular equal their ex-ante averages. More precisely, agent 1 receives $p^{-1}\kappa N$ good signals for the firms in location 2, of which exactly κN correspond to the regular firms and the remainder to fraudulent firms. The set of $(p^{-1} - 1)\kappa N$ of mis-identified firms is chosen from a uniform distribution on the set of cardinality-$(p^{-1} - 1)\kappa N$ subsets of the set of fraudulent

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[18]The number of agents must be an integer, of course. We therefore adopt the convention that all necessary quantities are rounded in some reasonable fashion. Alternatively, we restrict κ to be rational and N to an appropriate (unbounded) set. The same for $p^{-1}\kappa N$.

firms.

The action space for common investors consists of demand functions $X(\bar{P})$ that give the numbers of shares in the $2N$ securities that a given investor is willing to purchase given the $2N$-dimensional price vector $\bar{P}$. Swindlers must take an additional action, which is the amount L that they borrow and divert into the firm to increase its liquidation value.

Specifically, we assume that each swindler has the ability to borrow any amount L of her choosing at time 0, divert these funds into the firm, and report earnings equal to $L(1+r) = L$ in period 1. (Equivalently, we could assume that the swindler can take an action to produce earnings L by incurring a personal non-pecuniary cost of effort, which would have a value L in monetary terms.) Given the possibility of such a diversion, equation (5), giving the swindler's time-1 wealth, becomes

$$W_1^{sil} \equiv B^{sil} + \sum_{j=1}^{K} \int_{k\in[0,1]} D_{jk} dX_{jk}^{sil} + L^{il}\left(S^{il} - 1\right). \tag{A.1}$$

We note that the difference to (5) is the term $L^{il}\left(S^{il} - 1\right)$, which is intuitive. If $S^{il} - 1 < 0$, i.e., if the swindler reduces her ownership of shares by being a net seller, then she has no incentive to perform earnings diversion, since she will recover only a fraction of the funds she diverted into the company. If, however, the swindler is a net buyer of her own security $\left(S^{il} - 1 > 0\right)$, then the ability to manipulate earnings becomes infinitely valuable, since L^{il} can be chosen to be an arbitrarily large number. Intuitively, the swindler can report arbitrarily large profits at the expense of outside investors who hold negative positions (short sellers) in the fraudulent firm. This feature discourages any other agent from shorting: with non-zero probability all other agents know that the firm is fraudulent and don't buy any shares, so that any shorting results in $S^{il} > 1$.

Given that we are considering a sequential game — the swindler's decision to manipulate is taken after the asset market clears — of incomplete information, we are looking for a *perfect Bayesian Nash* equilibrium. Loosely speaking, this concept requires that all actions — demands and manipulation decisions — be optimal given beliefs, while beliefs be updated according to Bayes' rule wherever possible. Note that, unlike in the main body of the paper,

all agents take into account their potential impact on the price, and on the other agents' beliefs, when submitting their demand functions.

We concentrate on the sub-class of symmetric equilibria, in which all agents in a given market behave identically conditional on their type and signals, while the differences in behavior between market-1 and market-2 agents come down to index permutations in the natural way. Important, we are interested in the existence of pooling equilibria, in which all securities in a given location, and therefore in the entire economy by symmetry, have the same price.

Proposition 3 *A perfect Bayesian Nash equilibrium exists with the following properties.*

(i) *The equilibrium is symmetric.*

(ii) *All security prices are equal.*

(iii) *Common investors and swindlers have the same portfolios in equilibrium, with the exception of the swindler's holding of her own firm.*

(iv) *There is no shorting.*

(v) *There is no dividend manipulation.*

Furthermore, as N increases, equilibrium prices and aggregate holdings of agents in any location i of all assets in location j converge to the competitive-strategic equilibrium in Theorem 1.

B Proofs

Proof of Theorem 1.   The proof proceeds in a number of steps. We start with an equilibrium in the simplified competitive tax economy, and use it to construct demands in the original economy. Second, we specify out-of-equilibrium beliefs in the original economy that support the equilibrium. In a third step, we verify that all agents, regular as well as swindlers, find it optimal to submit the demands specified given prices and their beliefs. Finally, we verify that markets clear.

Consider a solution to the simplified problem (8)–(9). The demands in the original economy are defined naturally based on this solution:

$$dX_{jk}^{ci} = (1 - f_{ij}) \kappa^{-1} X_j^i \mathbf{1}_{\iota_{jk}^i} \mathbf{1}_{(P_{jk}=P_j)} dk \tag{B.1}$$

$$dX_{jk}^{sil} = (1 - f_{ij}) \kappa^{-1} X_j^i \mathbf{1}_{\iota_{jk}^i} \mathbf{1}_{(P_{jk}=P_j)} dk, \quad (i,l) \neq (j,k) \tag{B.2}$$

$$dX_{il}^{sil} = \begin{cases} [0, \infty) & \text{if} \quad P_{il} = P_i \\ 0 & \text{if} \quad P_{il} \neq P_i \end{cases}. \tag{B.3}$$

The conjectured prices are $P_{jk} = P_j$ for all j and k. Note that dX_{jk}^{ci} and dX_{jk}^{sil} are themselves demand curves, i.e., functions of the prices $\{P_j\}_j$.

In words, all investors buy the same number of shares in each market as in the tax economy, but they split this position (equally) only among the firms about which they receive a good signal — note that the multiplicative factor $(1 - f_{ij}) \kappa^{-1}$ equals the reciprocal of the probability that a given signal is good. Another proviso is that the price equal the pooling price P_j; for any other price, the agents shun the asset. The only exception to this behavior is provided by the insiders of fraudulent firms, who submit an elastic demand at $P_{jl} = P_j$.

In equilibrium, only prices P_j are realized, and therefore prices are not informative. We postulate that all agents believe that any firm k in market j that has price $P_{jk} \neq P_j$ is fraudulent with probability one.

To see that X_{jk}^{ci} is optimal, start by writing the expected utility for the agent as

$$\mathrm{E}\left[U \left(\sum_{j=1}^{K} \int_k (D_{jk} - P_j) \, dX_{jk}^{ci} + P_i \right) | \iota^i \right] = \mathrm{E}\left[U \left(\sum_{j=1}^{K} \int_k (\rho_{(jk)} D_j - P_j) \, dX_{jk}^{ci} + P_i \right) | \iota^i \right] \tag{B.4}$$

and note that, by Jensen's inequality, this utility is maximized by choosing dX_{jk}^{ci}, for fixed j, to be measurable with respect to ι_{jk}^i — in words the agent invests identically in all assets in market j in which she received the same signal. Furthermore, the portfolio of assets with

higher signals ($\iota^i_{jk} = 1$) strictly dominates the portfolio with low signals ($\iota^i_{jk} = 0$). Let $\hat{X}^{ci}_j = \frac{dX^{ci}_{jk}}{dk}$ denote the mass of shares in each asset in market j in which the investor has a positive signal. Consequently, (B.4) is equal to

$$\text{E}\left[U\left(\sum_{j=1}^{K}\int_k \left(\rho_{(jk)}D_j - P_j\right)\mathbf{1}_{\left(\iota^i_{jk}=1\right)}dk\hat{X}^{ci}_j + P_i\right)|\iota^i\right] \tag{B.5}$$

$$= \text{E}\left[U\left(\sum_{j=1}^{K}\left((1 - f_{ij})D_j - P_j\right)Pr\left(\iota^i_{jk} = 1\right)\hat{X}^{ci}_j + P_i\right)\right].$$

It follows that the optimal position is

$$\hat{X}^{ci}_j = Pr\left(\iota^i_{jk} = 1\right)^{-1}X^i_j = (1 - f_{ij})\kappa^{-1}X^i_j. \tag{B.6}$$

Equation (B.1) is immediate.

The same argument holds for the choice that a swindler makes with respect to all assets but her own. When choosing the position in her own asset, the only consideration is the time-zero revenue $(1 - dX^{sil}_{il})P_{il}$, since the asset pays zero. Given the other investors' demands, the insider must ensure that $P_{il} = P_i$. To that end she submits a demand that fails to clear the market at $P_{il} \neq P_i$, and is willing to take any position at $P_{il} = P_i$.

To see that markets clear at prices P_j, start from (9) and a consider a regular firm k in market j. Since by assumption we have $\iota^i_{jk} = 1$, the total demand follows from adding (B.1) and (B.2) over all i, which gives

$$\kappa\sum_{i=1}^{K}dX^{ci}_{jk} + (1 - \kappa)\sum_{i=1}^{K}dX^{sil}_{jk} = \sum_{i=1}^{K}(1 - f_{ij})\kappa^{-1}X^i_j dk = dk \tag{B.7}$$

by (9). The markets for fraudulent assets clear due to the elastic demands submitted by insiders.

The final observation to make is that X^i_j is indeed equal to the aggregate positions taken by investors located in market i when investing in market j, excluding swindlers' positions

in their own firms, i.e.,

$$X_j^i = \left(\kappa \int dX_{jk}^{ci} + (1 - \kappa) \int 1_{((i,l) \neq (j,k))} \, dX_{jk}^{sil} \right) Pr \left(\iota_{jk}^i = 1 \right). \tag{B.8}$$

∎

Proof of Proposition 2. The result follows immediately from the fact that any informed excess return is given by a linear combination of passive excess returns plus an additive constant which is given by (18). Alternatively, one can also check directly that the style alpha satisfies the properties required of a measure g, with $g(1) = 1$. ∎

Proof of Proposition 3.

To construct an equilibrium, we proceed in a number of steps. We first construct demand functions under the postulate, later verified, that all assets in the same location have the same price, shorting is prohibited, and there is no dividend manipulation. In a second step we extend the demand functions to cover all other price configurations, while in subsequent steps we address out-of-equilibrium beliefs, dividend manipulation, and, in a final step, shorting. We focus on a particular investor 1 and use symmetry throughout.

Step 1: Two-asset equilibrium. Suppose that P_j exists such that $P_{jk} = P_j$ for all firms jk. Consider the investment problem of any agent 1, common investor or swindler. We note that, given her signals on any location j, the agent (i) excludes from consideration all firms with bad signals — these have zero payoff — and (ii) invests equally in all the others, thus minimizing idiosyncratic risk.[19] At this stage, we also assume that the beliefs about the asset qualities are given by the signals, and are not updated based on the price. We address out-of-equilibrium beliefs in a later step.

This problem is a relatively standard, perfect-information one, featuring mean-variance investors facing differential taxes who invest strategically in multiple assets in the presence

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[19]The agent is perfectly informed about location 1, and thus does not face any idiosyncratic risk, but equal weighting is, of course, still optimal, albeit only weakly.

of shorting constraints. For the sake of completeness, we sketch proofs of both existence and convergence towards the competitive outcome as the number of agents grows to infinity.

Given that the asset payoffs in the two locations are independent and preferences are CARA, investments in the two assets do not interact. Market clearing, however, involves the demands of both agents, so we choose to write the problem in matrix form, even if all endogenous matrices are diagonal.

Let Π be diagonal with $\Pi_{jj} = p_{1j}$. Agent 1 faces a two-asset universe with expected payoffs diag(Π) and variance-covariance matrix of payoffs $\Pi\Omega\Pi$. With X the portfolio choice of the agent, it is convenient to focus on the quantity $Y = \Pi X$. We are looking for an equilibrium in which all demands, as functions of the two prices, are piece-wise linear — in fact, linear truncated at zero. Relying on symmetry, we need to parameterize only the demand of agent 1, as

$$Y = (A - BP)^+ = Z(A - BP), \tag{B.9}$$

where $P \in \mathbb{R}^2$ denotes the vector of prices per share in the two markets, and Z is a diagonal matrix with $Z_{jj} = 1$ if entry j of $A - BP$ is positive and $Z_{jj} = 0$ otherwise. Z is a function of P. A requirement of the equilibrium is that, taking all other demand schedules as given, an agent 1's optimal portfolio choice, denoted $\hat{X}$ (respectively $\hat{Y}$), and therefore P, is optimal subject to the restriction $\hat{X} \geq 0$. The type of equilibrium we are looking for requires that $\hat{Y} = Z(A - BP)$.

Consider price setting for a regular firm. We note that if an agent 1 demands X ($\in \mathbb{R}^2$) total shares in each location, the total demand for regular-firm shares is only $\Pi X = Y$. The reason is that, in market j, a proportion $1 - p_{1j}$ of the demand flows to fraudulent firms.

Given (B.9), it follows that the residual demand faced by an agent 1 for the aggregate regular asset in each location is also linear, given as

$$Y^{(r)} = A^{(r)} - B^{(r)}P, \tag{B.10}$$

with $A^{(r)} = (N-1)ZA + N\mathcal{R}ZA$ and $B^{(r)} = (N-1)ZB + N\mathcal{R}ZB\mathcal{R}$, where the matrix $\mathcal{R}$ implements the permutation $1 \leftrightarrow 2$ applied to all indices to capture the demand of agents in location 2; that is,

$$
\mathcal{R} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.
$$

An agent 1 maximizes

$$
\hat{Y}^\top \left(\mathbf{1} - \Pi^{-1}P \right) + \frac{\kappa}{N} e_1^\top P - \frac{\gamma N}{2} \hat{Y}^\top \Omega \hat{Y}, \tag{B.11}
$$

where $e_1 = (1,0)^\top$ is a vector that selects market 1, to capture the agent's endowment, and $\mathbf{1} = (1,1)^\top$. Note that the agent's endowment represents a fraction $1/N$ of the total endowment in market 1, and his risk tolerance a fraction $1/N$ of the aggregate risk tolerance of agents in location 1. The agent takes into account that the price depends on her demand through the market-clearing condition

$$
\hat{Y} + Y^{(r)}(P) = \kappa \mathbf{1}. \tag{B.12}
$$

The logic of the argument is familiar. The agent can be thought of as choosing the quantity $\hat{Y}$ and via (B.12) the price vector P, taking the residual demand as given. Before we compute the optimal demand, we remark, based on (B.11), that (i) if $P_1 < 1$, then $\hat{Y}_1 > 0$, and (ii) $\hat{Y}_2 > 0$ if and only if $P_2 < p$. Since the market for asset 1 must clear, $Y_1 > 0$ in equilibrium, while Y_2 may or not be positive.

The first-order condition for the Lagrangean associated with (B.11) is

$$
0 = 1 - \Pi^{-1}P - (D_{\hat{Y}}P)\Pi^{-1}\hat{Y} + (D_{\hat{Y}}P)e_1 \frac{\kappa}{N} - \gamma N \Omega \hat{Y} + \lambda, \tag{B.13}
$$

where λ is the Lagrange-multiplier vector attached to the no-shorting condition, while dif-

ferentiating (B.12) with respect to $\hat{Y}$ gives

$$
\begin{aligned}
0 &= 1 + D_P Y^{(r)} D_{\hat{Y}} P \\
&= 1 - B^{(r)} D_{\hat{Y}} P.
\end{aligned}
\tag{B.14}
$$

Note that, via Z, $A^{(r)}$ and $B^{(r)}$ are actually functions of P. Since they are step functions, though, we may treat them as constants when evaluating first-order conditions. We verify later that the first-order approach generates an equilibrium in our context.

Putting (B.13) and (B.14) together provides a candidate demand schedule for agent 1:

$$
\hat{Y} = \hat{Z} \left(\left(B^{(r)} \right)^{-1} \Pi^{-1} + \gamma N \Omega \right)^{-1} \left(\mathbf{1} - \Pi^{-1} P + \left(B^{(r)} \right)^{-1} e_1 \frac{\kappa}{N} \right).
\tag{B.15}
$$

We'll define the coefficients of the linear demand of the agent 1 under consideration based on (B.15), but we first determine the value that the matrix Z, and therefore $\hat{Z}$, takes in equilibrium. Let P^* denote the equilibrium price and suppose that $P_1^* > p$, so that agent 1 is the only one investing in market 1. Then, from (B.15) market clearing implies

$$
\kappa = N\hat{Y} = \left(N^{-1} \left(B_{11}^{(r)} \right)^{-1} + \gamma \sigma^2 \right)^{-1} \left(1 - P_1^* + N^{-1} \left(B_{11}^{(r)} \right)^{-1} \kappa \right),
\tag{B.16}
$$

which gives upon rearrangement

$$
P_1^* = 1 - \gamma \kappa \sigma^2.
\tag{B.17}
$$

Thus, if $p < 1 - \gamma \kappa \sigma^2$ and the equilibrium satisfies (B.15), then agent 1 doesn't invest in market 2.

Suppose now that the agent invests in both markets. We use again market clearing from

(B.15),

$$\kappa = \left(N^{-1}\left(B_{11}^{(r)}\right)^{-1} + \gamma\sigma^2\right)^{-1}\left(1 - P_1^* + N^{-1}\left(B_{11}^{(r)}\right)^{-1}\kappa\right) + \tag{B.18}$$
$$\left(N^{-1}\left(B_{22}^{(r)}\right)^{-1}p^{-1} + \gamma\sigma^2\right)^{-1}\left(1 - p^{-1}P_2^*\right),$$

which, using $P_1^* = P_2^*$, leads to

$$P_1^* = 1 - \gamma\kappa\sigma^2 + \frac{\left(N^{-1}\left(B_{11}^{(r)}\right)^{-1} + \gamma\sigma^2\right)}{\left(N^{-1}\left(B_{22}^{(r)}\right)^{-1}p^{-1} + \gamma\sigma^2\right)}\left(1 - p^{-1}P_1^*\right), \tag{B.19}$$

further rewritten as

$$(p - P_1^*)(1 + a_0) = p - \left(1 - \gamma\kappa\sigma^2\right) \tag{B.20}$$

with

$$a_0 = \frac{\left(N^{-1}\left(B_{11}^{(r)}\right)^{-1} + \gamma\sigma^2\right)}{\left(N^{-1}\left(B_{22}^{(r)}\right)^{-1} + p\gamma\sigma^2\right)} > 0.$$

Thus $p > P_1^*$, implying active participation of agent 2 in market 1 if and only if $p > 1 - \gamma\kappa\sigma^2$. We have therefore shown that $Z_{22}(P^*) = 1_{(p>1-\gamma\kappa\sigma^2)}$, should an equilibrium exist.

Let $Z^* \equiv Z(P^*)$. We define the demand of agent 1 under consideration based on the coefficients

$$\phi(B) = Z^*\left(\left(B^{(r)}\right)^{-1} + \gamma N\Omega\right)^{-1}\Pi^{-1} \tag{B.21}$$
$$\psi(B) = \phi(B)\Pi\left(\mathbf{1} + \left(B^{(r)}\right)^{-1}e_1\frac{\kappa}{N}\right). \tag{B.22}$$

Note that we used the known value of Z^* in this definition, as defined above.

Suppose that the mapping $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ admitted a non-zero, positive fixed point. Then

let $A = \psi(B)$ and compute the price from the market-clearing condition. It follows from equation (B.15) that $\hat{Y} = \psi(B) - \phi(B)P^*$ as long as $\hat{Z}(P^*) = Z^*$, i.e., as long as (i) $(\psi(B) - \phi(B)P^*)_{11} = (A - BP^*)_{11}$ is positive, and (ii) $(\psi(B) - \phi(B)P^*)_{22} = (A - BP^*)_{22}$ is positive if and only if $Z_{22}^* = 0$, thus $p > 1 - \gamma\kappa\sigma^2$.

Note, however, that $Z_{22}^* = 0$ implies $B_{22} = A_2 = 0$, and therefore $\hat{Z}_{22}(P^*) = 0$. Given symmetry and market clearing, it follows that $A_1 - B_{11}P_1^* > 0$. If $Z_{22}^* = 1$ and both investors invest in both markets, then we saw above that market clearing leads to (B.20), which implies $P_1^* < p$. From the definitions,

$$A - BP^* = \left(\left(B^{(r)}\right)^{-1} + \gamma N\Omega\right)^{-1} \left(\mathbf{1} + \left(B^{(r)}\right)^{-1} e_1 \frac{\kappa}{N} - \Pi^{-1}P^*\right). \tag{B.23}$$

Since $P_1^* < p$, $\mathbf{1} - \Pi^{-1}P^* > 0$, and therefore $(A - BP^*)_{jj} > 0$, thus $\hat{Z}_{jj}^* > 0$, for $j \in \{1,2\}$.

One can use Brouwer's theorem to show that ϕ has a strictly positive fixed point, as follows. It is convenient to concentrate on the mapping $\phi_N(NB) \equiv N\phi(B)$. To invoke this theorem, we restrict attention to $B_1 > 0$ (and $B_2 \geq 0$) and note that, as a consequence, the image of ϕ_N is bounded above (in the operator sense) — uniformly in N, in fact. We also see that $\delta > 0$ exists such that, if $NB_1 \geq \delta$, then $(\phi_N(NB))_1 > \delta$. Specifically, from (B.21) it follows that δ must obey

$$N \left((2N - 1)^{-1} \delta^{-1} + \gamma N\sigma^2\right)^{-1} \geq \delta, \tag{B.24}$$

which holds, for instance, for

$$\delta = \frac{1}{2} \left(\gamma\sigma^2\right)^{-1}. \tag{B.25}$$

Thus, we have verified that the continuous mapping ϕ_N maps a compact set into itself, and therefore has a fixed point B characterized by $B_1 > \delta > 0$, $B_2 \geq 0$.

The last fact that must be established before concluding that we have an equilibrium is that the agents' portfolios are, indeed, optimal. We constructed them to satisfy first-order conditions, but the agents' objectives (B.11) are not concave in general. Given equilibrium

residual demands, though, they are concave.

To make the matters clear, we rewrite (B.11) as

$$\hat{Y}^\top \mathbf{1} - \left(\hat{Y}^\top \Pi^{-1} - \frac{\kappa}{N} e_1^\top \right) P - \frac{\gamma N}{2} \hat{Y}^\top \Omega \hat{Y}, \tag{B.26}$$

Since P is not a constant, this function is not quadratic in $\hat{Y}$. The implicit function $P(\hat{Y})$, though, defined via (B.12), is (piece-wise linear and) convex. To obtain concavity, it is sufficient that $P(\hat{Y})$ be linear whenever $\hat{Y}^\top \Pi^{-1} - \frac{\kappa}{N} e_1^\top < 0$. Differently put, that for $\hat{Y} < \frac{\kappa}{N} \Pi e_1 = \frac{\kappa}{N} e_1$, both agents are long in both markets given equilibrium demands and $P(\hat{Y})$.

Obviously, there is no problem concerning asset 2 (since $e_{12} = 0$: agent 1 has no endowment of asset 2). For asset 1, consider first the case in which agents only participate in their home markets in equilibrium. Since we defined $A_2 = B_{22} = 0$, i.e., constant, in this case, there is no issue. In the other case, suppose that the agent increases $\hat{Y}_1$ until agents 2 drop out of the market, i.e., the price becomes $P_1 = p$. Each other agent 1, at this price, holds

$$A_1 - B_{11}p = \left((B_{11}^{(r)})^{-1} + \gamma N \sigma^2 \right)^{-1} \left(1 + (B_{11}^{(r)})^{-1} \frac{\kappa}{N} - p \right) \tag{B.27}$$

$$< \left((B_{11}^{(r)})^{-1} + \gamma N \sigma^2 \right)^{-1} \left((B_{11}^{(r)})^{-1} \frac{\kappa}{N} + \gamma \kappa \sigma^2 \right) \tag{B.28}$$

$$= \frac{\kappa}{N}, \tag{B.29}$$

where we used $p > 1 - \gamma \kappa \sigma^2$. The residual demand is therefore linear until a point where $\hat{Y}_1 = \kappa - (N-1)(A_1 - B_{11}p) > \frac{\kappa}{N}$.

Let's turn now to the behavior of this equilibrium as N grows large. Since NB is bounded below by $\delta > 0$ (independently of N), (B.21) implies

$$\bar{B} = Z^* (\gamma \Omega)^{-1} \Pi^{-1}, \tag{B.30}$$

with $\bar{B}$ denoting the limit of NB.

We note that $\lim_{N\to\infty} B^{(r)} = \bar{B} + \mathcal{R}\bar{B}\mathcal{R}$. It follows, using (B.22), that the limit of NA is

$$\bar{A} = \bar{B}\Pi\mathbf{1} = Z^* (\gamma\Omega)^{-1}\mathbf{1} \tag{B.31}$$

and that the limit price per share is

$$\bar{P} = \left(\bar{B} + \mathcal{R}\bar{B}\mathcal{R}\right)^{-1} (\bar{A} + \mathcal{R}\bar{A} - \kappa\mathbf{1}). \tag{B.32}$$

The quantities $\bar{A}$ and $\bar{B}$ also correspond to the solution to (B.11) taking the price as given. The price $\bar{P}$ is therefore the competitive price in the two-asset, short-sale constrained equilibrium. The term $\left(\bar{B} + \mathcal{R}\bar{B}\mathcal{R}\right)^{-1} (\bar{A} + \mathcal{R}\bar{A})$ captures the aggregate expected payoff from an asset one invests in. The remaining term is the risk adjustment, with $\kappa\mathbf{1}$ the supply of the asset, and $\left(\bar{B} + \mathcal{R}\bar{B}\mathcal{R}\right)^{-1}$ accounting for the covariance between one asset the aggregate investor purchases and one unit of the total supply.

Step 2: The other demands. Returning to the finite-N case, we consider now the demand of a swindler in her own firm. Given that the only way to generate a positive demand in her firm — given the other agents' equilibrium strategies, described below — is to ensure that its price is P_1, the swindler submits a perfectly elastic demand at the price P_1 at which all other firms clear, as long as the demand is a quantity that does not exceed one. At all other prices, the swindler submits a demand $(1, \infty)$, i.e., stands ready to clear the market as long as she takes a gross position higher than one. This case can only obtain when another agent is willing to short at the respective price, and the optimal reaction of the swindler is to accommodate the shorter and manipulate dividends, as we describe below.

Formally, the demand of the swindler for her own firm l is

$$X^{sl}(P_{1l}) = \begin{cases} (-\infty, \infty) & \text{if} \quad \exists P_1 \text{ \& } P_{1l} = P_1 \\ (1, \infty) & \text{if} \quad \nexists P_1 \text{ or } P_{1l} \neq P_1 \end{cases}. \tag{B.33}$$

We also note that, in equilibrium, the swindler never shorts her own firm, since the demand for it is lower than the demand for a regular firm, about which the signals are better.

All agents demand a zero amount of shares in firms in which they have bad signals. Local agents know precisely all regular firms. If the set of prices of regular firms in market 1 is not a singleton, then we specify the demand of agent 1 as $\hat{Y}_{1j} = \infty$ for all j such that $P_{1j} < \max_k P_{1k}$, and $\hat{Y}_{1j} = -\infty$ for all j such that $P_{1j} = \max_k P_{1k}$. If all positive-signal firms in market 2 do not have the same price, then agent 1 maximizes utility conditional on his out-of-equilibrium beliefs, stated below. The important feature to note is that no deviation by a swindler can prevent the regular firms having the same price — this is ensured by the demands of the local common investors — and therefore the only relevant belief concerns the case in which the price P_{2j} of a fraudulent firm is not equal to that of at least κN other firms in location 2.[20]

Step 3: Out-of-equilibrium beliefs. Investor 1 knows all types in market 1. Suppose that she observes prices P_{2k}. Given $\kappa > \frac{1}{2}$, there are only two possibilities. First, at least κN firms of the $p^{-1}\kappa N$ ones about which the investor has positive signals have the same price (there is only one price level for which this statement is true). Then the agent assigns probability one that all firms with different price are fraudulent. Second, there is no such subset of firms. Then the agent believes that the prices are entirely uninformative.

As remarked above, given the prescribed strategies, no swindler can bring about the second case, and it is optimal for each swindler to induce the pooling outcome, as it carries zero cost and unilateral deviation exposes the firm as certainly fraudulent.

Finally, common agents do not have an incentive to adjust demand for the regular stocks in market 1 in the hope of signaling, given the swindlers' equilibrium demand (which ensures the same price for the fraudulent firm as for the regular ones).

Step 4: Dividend manipulation. The swindler's action also includes the amount of dividend manipulation she engages in, subsequent to asset-market clearing. If the swindler borrows the amount $F \geq 0$ that she diverts in the firm, then she makes profit

$$X^{sl}\left(F - P_{1l}\right) - F + P_{il} = P_{il}\left(1 - X^{sl}\right) + F\left(X^{sl} - 1\right). \tag{B.34}$$

---

[20]To simplify exposition, we make the parametric assumption $\kappa > \frac{1}{2}$, which excludes the possibility that there are two or more disjoint sets of firms of size κN, which may have the same price.

Clearly, if $X^{sl} > 1$, the swindler benefits from borrowing an arbitrarily large amount L to divert in the firm, pushing its liquidating price arbitrarily high and making arbitrarily high profits. As long as there is no shorting, however, there is no manipulation.

Step 5: Shorting. Agent 1 is perfectly informed about assets in market 1, and does not short regular assets in market 1, as discussed in Step 1. She also does not short fraudulent assets in market 1 because agent 2 may also know that they are fraudulent, and therefore the swindler would be the only buyer, would end up being a net buyer of the asset and manipulate the dividends to an arbitrary extent. Agent 1 does not short an asset in market 2 that is fraudulent, for everyone knows that it is fraudulent. Finally, she may consider shorting an asset in market 2 about which she has a positive signal. If the equilibrium demand for market 2 by agent 1 in the shorting-constrained economy without manipulation is zero, then there is a positive probability that the asset is fraudulent and no one else invests in it. If this demand is actually strictly positive, then taking a negative position would be suboptimal even in the absence of the manipulation threat. ∎